3rd RIKEN-INFN, Oct. 13, 1997

Quantum Fluctuation Effects on Nuclear Multi-Fragmentation

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1. **Introduction**
2. **Basic idea to include Quantum Fluctuation**
   - ⋆ **Quantum Langevin Model**
3. **Quantum Fluctuation Effects on Nuclear Statistical Properties**
   - ⋆ **Caloric Curve and Fragment Distribution**
4. **Quantum Fluctuation Effects on Nuclear Multi-Fragmentation**
   - ⋆ **IMF formation from Au+Au Collision**
   - ⋆ **Twin Hyperfragment Formation from Ξ^- Absorption at Rest**
5. **Summary and Outlook**

A.O. and J. Randrup, PRL 75('95), 596

A.O. and J. Randrup, AP 253('97), 279

A.O. and J. Randrup, PL B394('97), 260

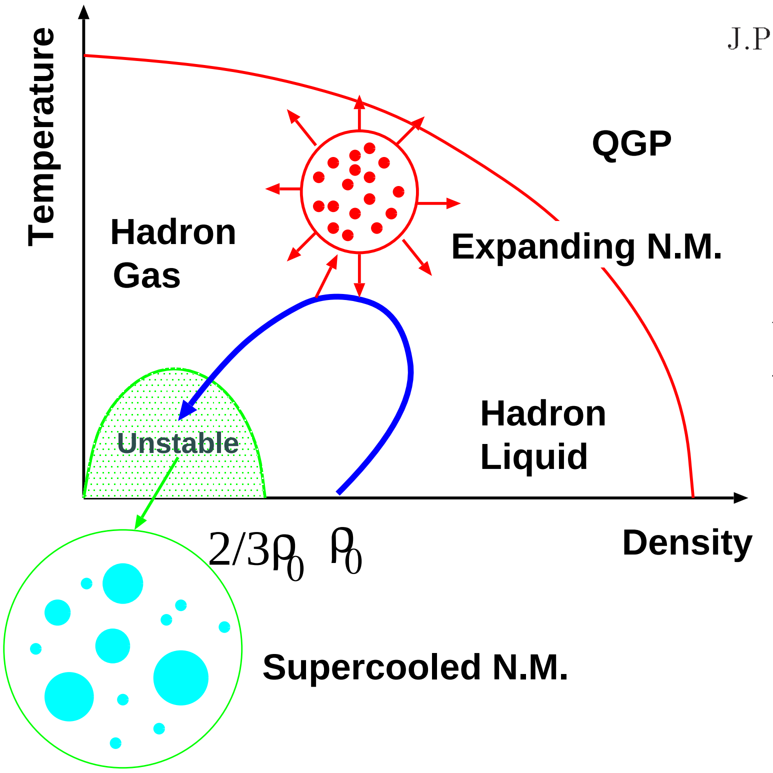
A.O. and J. Randrup, PRA 55('97), 3315R

A.O. et al., NN97 Proc, NPA ('97), in press

Y.Hirata, Y.Nara, A.O., T.Harada, and J.Randrup, in preparation

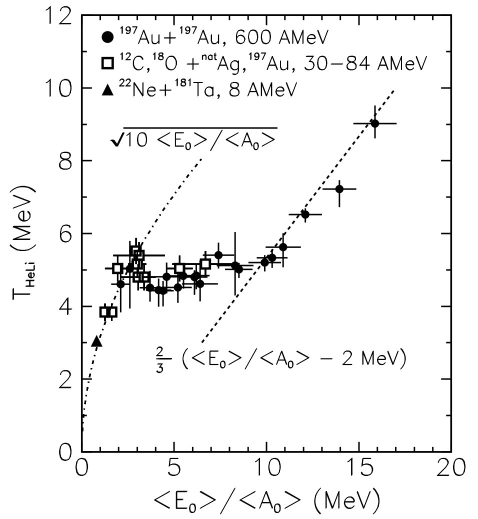
Nuclear Liquid-Gas Phase Transition

- **Phase Diagram**



- **Caloric Curve**

J.Pochadzalla et al.,PRL75('95),1040.



(Low-T: $E^*/A = aT^2$

$\rightarrow$ High-T: $E^* = 1.5T + c$)

Microscopic Approaches to Nuclear Multi-Fragmentation

$\cdots$ Statistical Property is Essential.

- **(Semi-)Classical M.D.-type Models**

  $\star$ $\mathcal{Z} = \int d\Gamma \exp(-\beta\mathcal{H})$ $\rightarrow$ $E^* \propto T$ even at low T

  $\star$ Too Small T ($\simeq$ Strength of Flucts.) at a given E

- **Transport Models with Fluctuations**

1. Boltzmann-Langevin (c.f. Maria Colonna)
2. AMD-V (Ono-Horiuchi)
3. Quantum Langevin Model

Quantum Stat. Mech. of Wave Packets

Energy Fluctuation of Wave Packets

$\sigma_E^2 = \langle \hat{H}^2 \rangle - \langle \hat{H} \rangle^2 \neq 0$

modifies Statisitcal Weight !

- **Partition Function**

$$\mathcal{Z}_\beta \equiv \mathrm{Tr}\left(\exp(-\beta \hat{H})\right) = \int d\Gamma \, \mathcal{W}_\beta(\mathbf{Z})$$

$$\mathcal{W}_\beta(\mathbf{Z}) \equiv \langle \mathbf{Z} | \exp(-\beta \hat{H}) | \mathbf{Z} \rangle \neq \exp(-\beta \langle \hat{H} \rangle)$$

- **Thermal Average**

$$\prec \hat{O} \succ_\beta \equiv \frac{1}{\mathcal{Z}_\beta} \mathrm{Tr}\left(\hat{O} \exp(-\beta \hat{H})\right) = \frac{1}{\mathcal{Z}_\beta} \int d\Gamma \, \mathcal{W}_\beta(\mathbf{Z}) \, \mathcal{O}_\beta(\mathbf{Z})$$

$$\mathcal{O}_\beta(\mathbf{Z}) \equiv \frac{\langle \mathbf{Z}_{\beta/2} | \hat{O} | \mathbf{Z}_{\beta/2} \rangle}{\langle \mathbf{Z}_{\beta/2} | \mathbf{Z}_{\beta/2} \rangle} \neq \langle \hat{O} \rangle$$

$$|\mathbf{Z}_{\beta/2}\rangle \equiv \exp(-\beta \hat{H}/2) |\mathbf{Z}\rangle \neq |\mathbf{Z}\rangle$$

- **Harmonic Approximation**

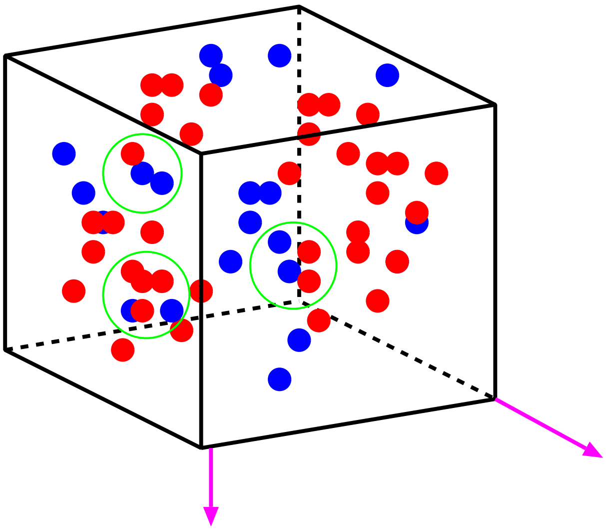
$$\mathcal{W}_\beta(\mathbf{Z}) \approx \exp\left[-\frac{\mathcal{H}}{D}\left(1 - e^{-\beta D}\right)\right] = \exp(-\beta \mathcal{H} + \beta^2 \sigma_E^2/2 + \cdots)$$

$$D(\mathbf{Z}) \equiv \sigma_E^2 / \mathcal{H}$$

$$\mathcal{H}_\beta(\mathbf{Z}) \equiv -\frac{\partial \log \mathcal{W}_\beta(\mathbf{Z})}{\partial \beta} \approx \mathcal{H}(\mathbf{Z}) \, e^{-\beta D}$$

$\rightarrow$ **Improved β Expansion**

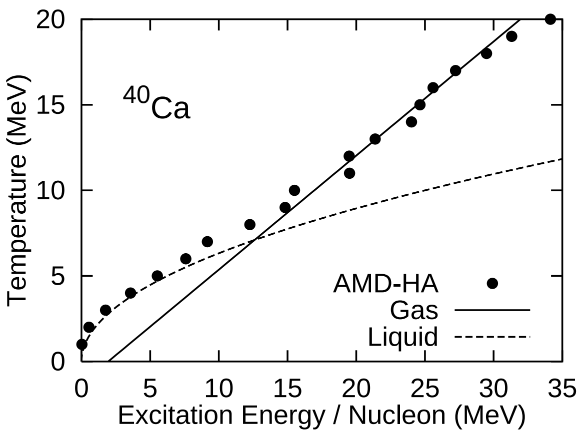
Statistical Properties of Nuclei



- **Caloric Curve**

(A.O. and J.Randrup, PRL 75('95), 596;
A.O. et al., Proc. NN97, NPA, in press.)

AMD-H.A., Volkov



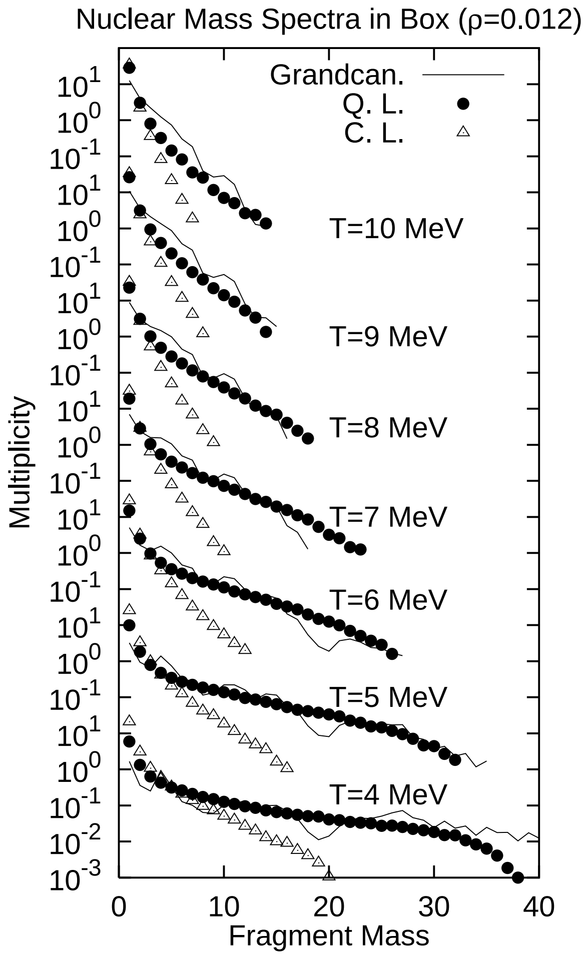
$$\mathbf{G:}\quad T = \frac{2}{3}(E/A - 2)\ \mathbf{MeV}$$

$$\mathbf{L:}\quad T = 2\sqrt{E/A}\ \mathbf{MeV}$$

- **Thermal Fragmentation**

(A.O. and J. Randrup, PL B394('97), 260)

QMD-QL, Gogny+Pauli pot.



$$\mathbf{Q.L.:}\quad \exp\left(-\frac{\mathcal{H}}{D}(1 - e^{-\beta D})\right)$$

$$\mathbf{C.L.:}\quad \exp(-\beta\mathcal{H})$$

From Quantum Statistics to Dynamics with Fluctuation

- **Equilibrium Distribution ··· Q. Microcan.**

$$\phi_{\rm eq}(\mathbf{Z}) \equiv \exp(-\mathcal{F}(\mathbf{Z})) \propto \langle \mathbf{Z}|\delta(E-\hat{H})|\mathbf{Z}\rangle$$

- **Fokker-Planck Equation: $\phi_{\rm eq}$ = Static Solution**

$$\frac{D\phi(\mathbf{Z};t)}{Dt} = \frac{\partial}{\partial \mathbf{q}} \cdot \left(\mathbf{M} \cdot \frac{\partial \mathcal{F}}{\partial \mathbf{q}} + \mathbf{M} \cdot \frac{\partial}{\partial \mathbf{q}}\right) \phi \ , \quad \{\mathbf{q}\} = \{\mathbf{r}, \mathbf{p}\}$$

- **Equivalent Langevin Equation at Fixed E**

$$\begin{aligned} \dot{\mathbf{p}} &= \mathbf{f} - \beta_{\mathcal{H}} \mathbf{M}^p \cdot (\mathbf{v} - \mathbf{u}) &+ \mathbf{g}^p \cdot \zeta^p \ , \\ \dot{\mathbf{r}} &= \mathbf{v} + \beta_{\mathcal{H}} \mathbf{M}^r \cdot \mathbf{f} &+ \mathbf{g}^r \cdot \zeta^r \ , \end{aligned}$$

Drift **Diffusion**

$\mathbf{v} = \partial\mathcal{H}/\partial\mathbf{p}$, $\mathbf{f} = -\partial\mathcal{H}/\partial\mathbf{r}$
$\mathbf{u}$: Local Collective Velocity = Classical
$\mathbf{M} = \mathbf{g}\cdot\mathbf{g}$: Mobility Tensor

⋆ **Effective Inverse Temperature:**

$$\beta_{\mathcal{H}} \equiv \frac{\partial \mathcal{F}}{\partial \mathcal{H}} = \frac{\mathcal{H} - E}{\sigma_E^2}$$

··· **Drift Term Acts as a Energy Recovering Force**

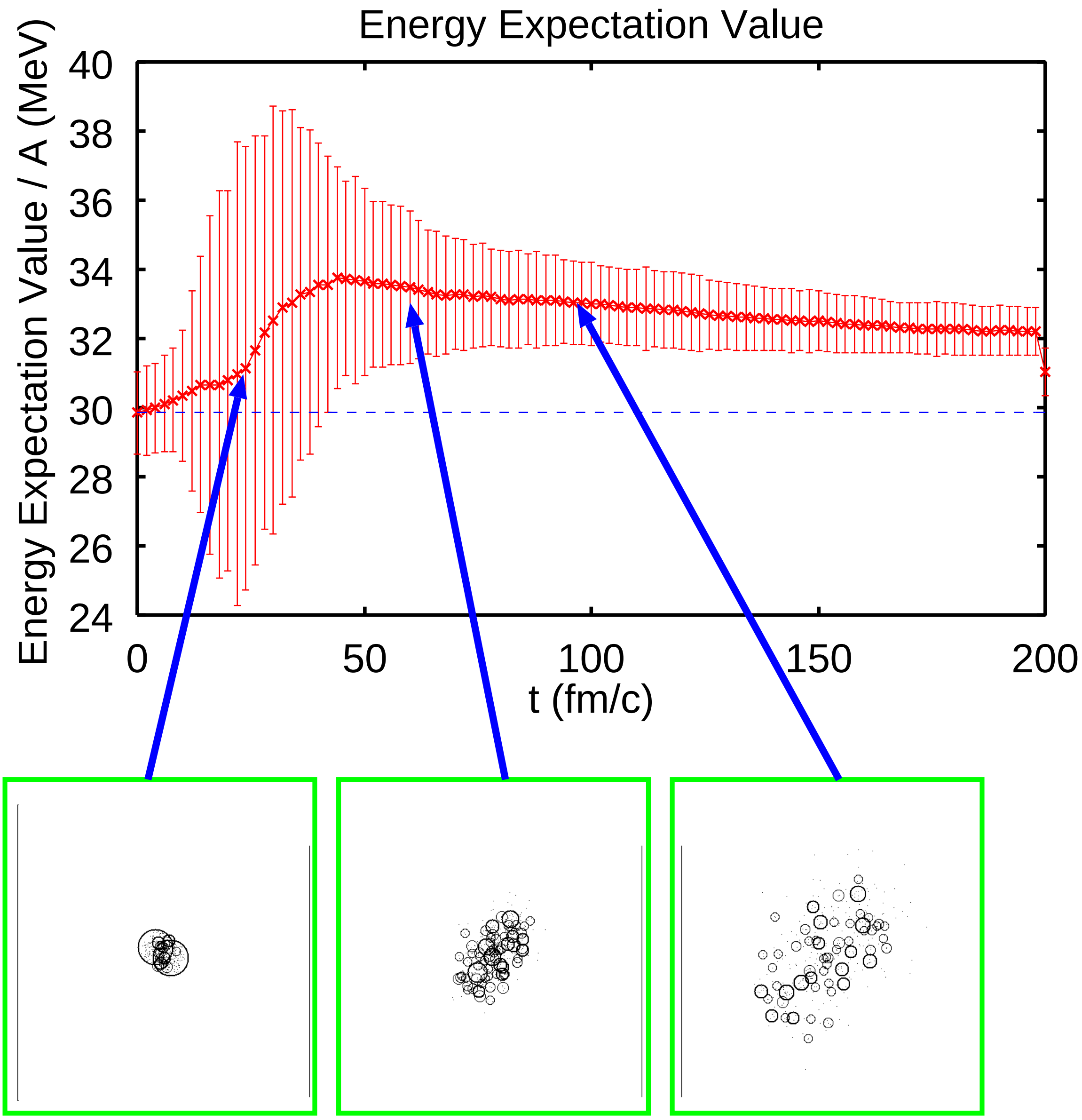
⋆ **Classical Limit = Classical Canonical Eq.**
··· $\phi_{\rm eq} = \delta(\mathcal{H} - E) \quad \leftrightarrow \dot{\mathbf{p}} = \mathbf{f},\ \dot{\mathbf{r}} = \mathbf{v}$

- **Intrinsic Distortion of Wave Packets**

$$\frac{d\mathbf{p}}{d\tau} = -\frac{2\Delta p^2}{\hbar}(\mathbf{v} - \mathbf{u}) \ , \quad \frac{d\mathbf{r}}{d\tau} = \frac{2\Delta r^2}{\hbar}\mathbf{f}$$

until $\mathcal{H} = E$ before making an observation

- **Example of Energy Fluctuation**

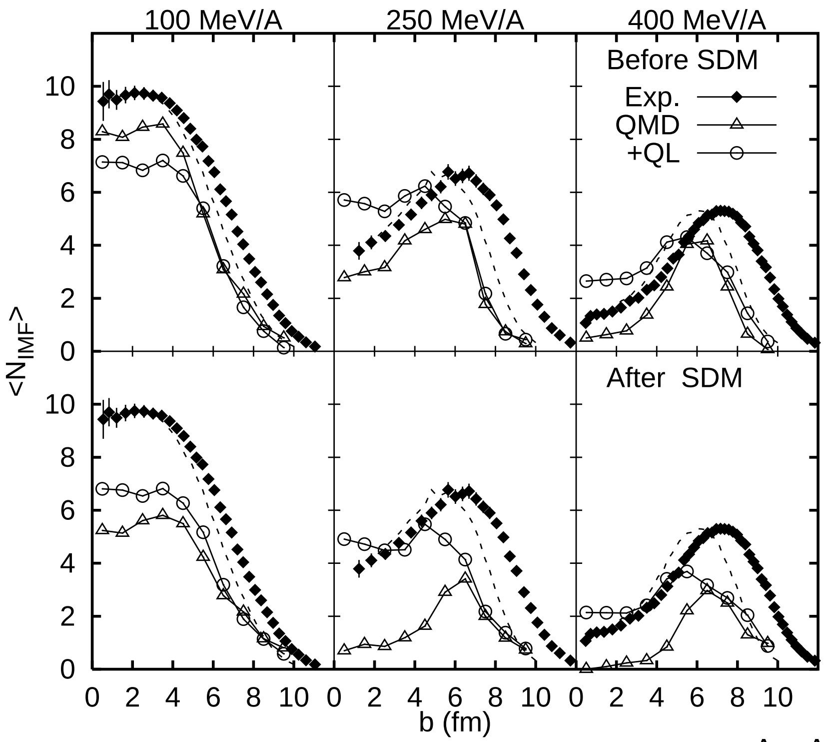


Multifragmentation from Au+Au

- **MSU/ALADIN Data**

M.B.Tsang et al., PRL 71 ('93), 1502.

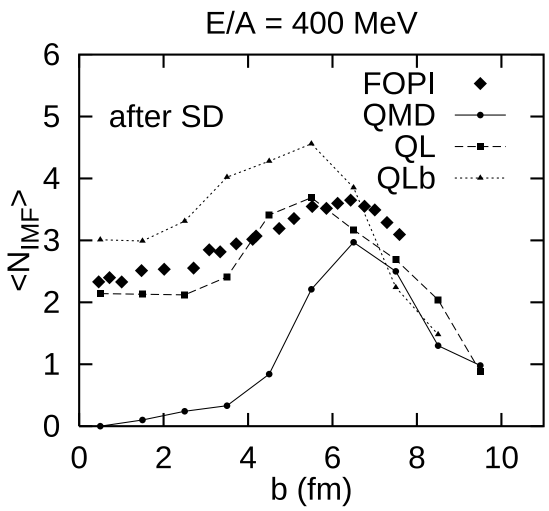
A.O. and J. Randrup, PL B394('97), 260.

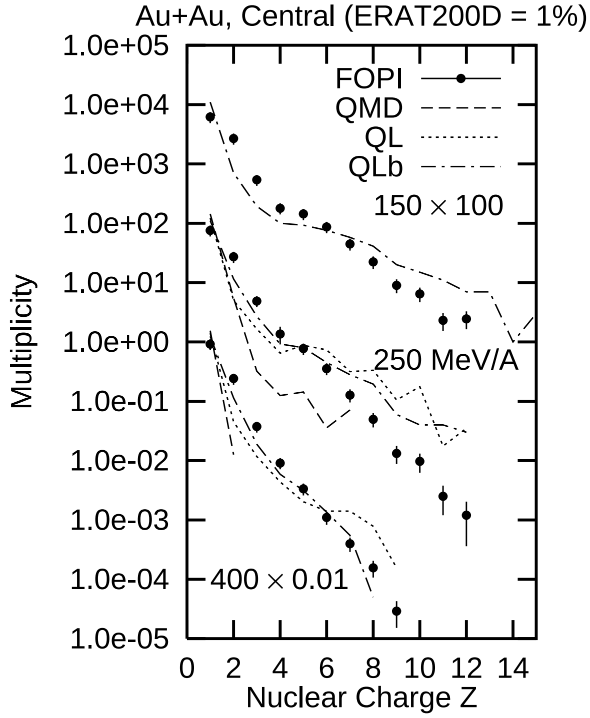


- ⋆ b_{imp} **sort** $=$ **PM**
- ⋆ $3 \le Z_{imf} \le 30$
- ⋆ **QMD: Gogny+Pauli**
- ⋆ **No Det. Eff. is incl. in calc.**

- **FOPI Data**

W. Reisdorf et al., NP A612 ('97), 493





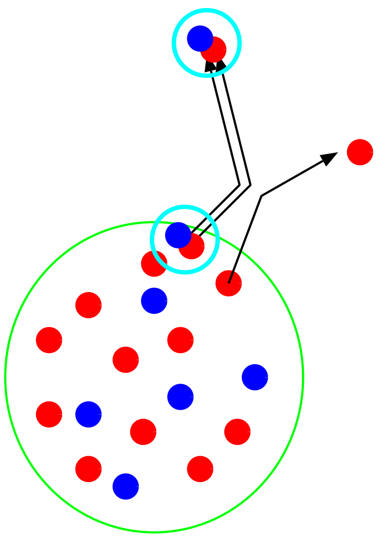
⋆ b_{imp} **sort** $=$ **PM,** $3 \le Z_{imf} \le 15$

• Cluster-Cluster Scattering

Danielewicz and Bertsch, NP A533 ('91), 712: (d, t, h)

Ono et al., PRC 47 ('91), 2652: (Nα)

Y. Nara et al. PL B346 ('95), 217: ($K^{-}\alpha \to \pi {}^{4}_{\Lambda}$H)

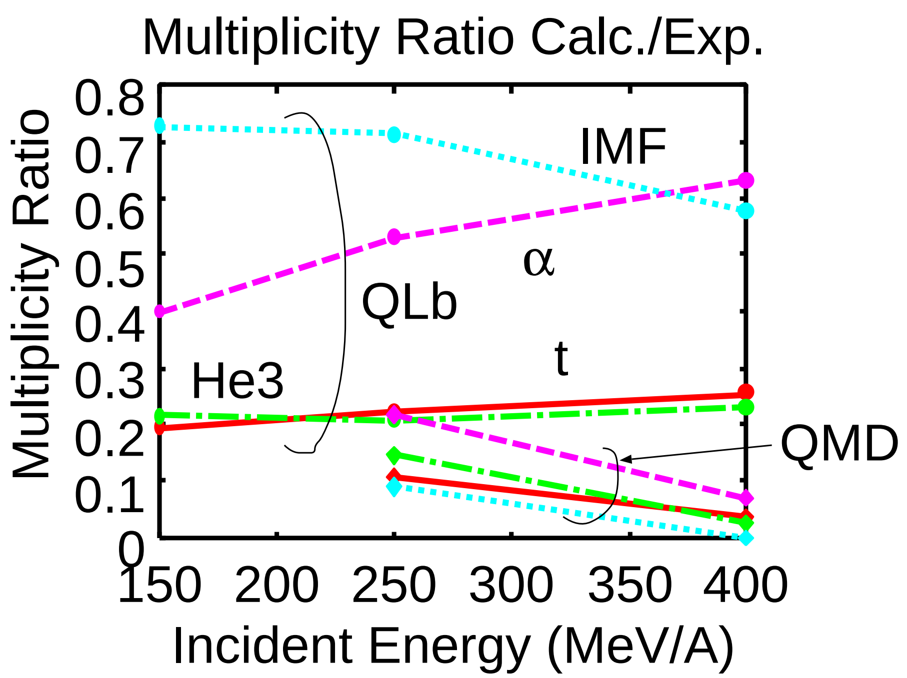


Cluster-Cluster (or N) Scattering

* Black Disc Ang. Dist. & σ are assumed
* Seed of IMFs
* Only 0s clusters are considered

• Light Charged Particle Multiplicity

$\cdots$ **Large underestimate for A=3**



SUMMARY & OUTLOOK

- **Quantal Langevin Model**

  ⋆ Based on the energy fluctuations of wave packets, which are not energy eigen states.

  ⋆ Dynamical Relaxation to Quantum Stat. Equil.

  ⋆ Larger Fluctuations (Quantum & Statistical)
  +Intrinsic Distortion (Smaller Excitation Energy)
  → Enhancement of Stable Dynamical Fragments

- **Achievements**

  a. Caloric Curve (Liquid → Gas)

  b. Thermal Fragmentation (Critical behavior)

  c. Dynamical Fragmentation (Au+Au, Ξ^- Absorption)

- **Remaining Problems**

  ⋆ Mobility Tensor M cannot be determined only from stat. requirements.

  ⋆ Light Charged Particle (LCP) formation ($d, t, {}^3\mathrm{He}, \alpha$)
  Underesitmate by a factor of 4 ∼ 10 for $A = 3$
  → Coalescence ?