SU(3) chiral linear σ model for positive and negative parity baryons in dense matter

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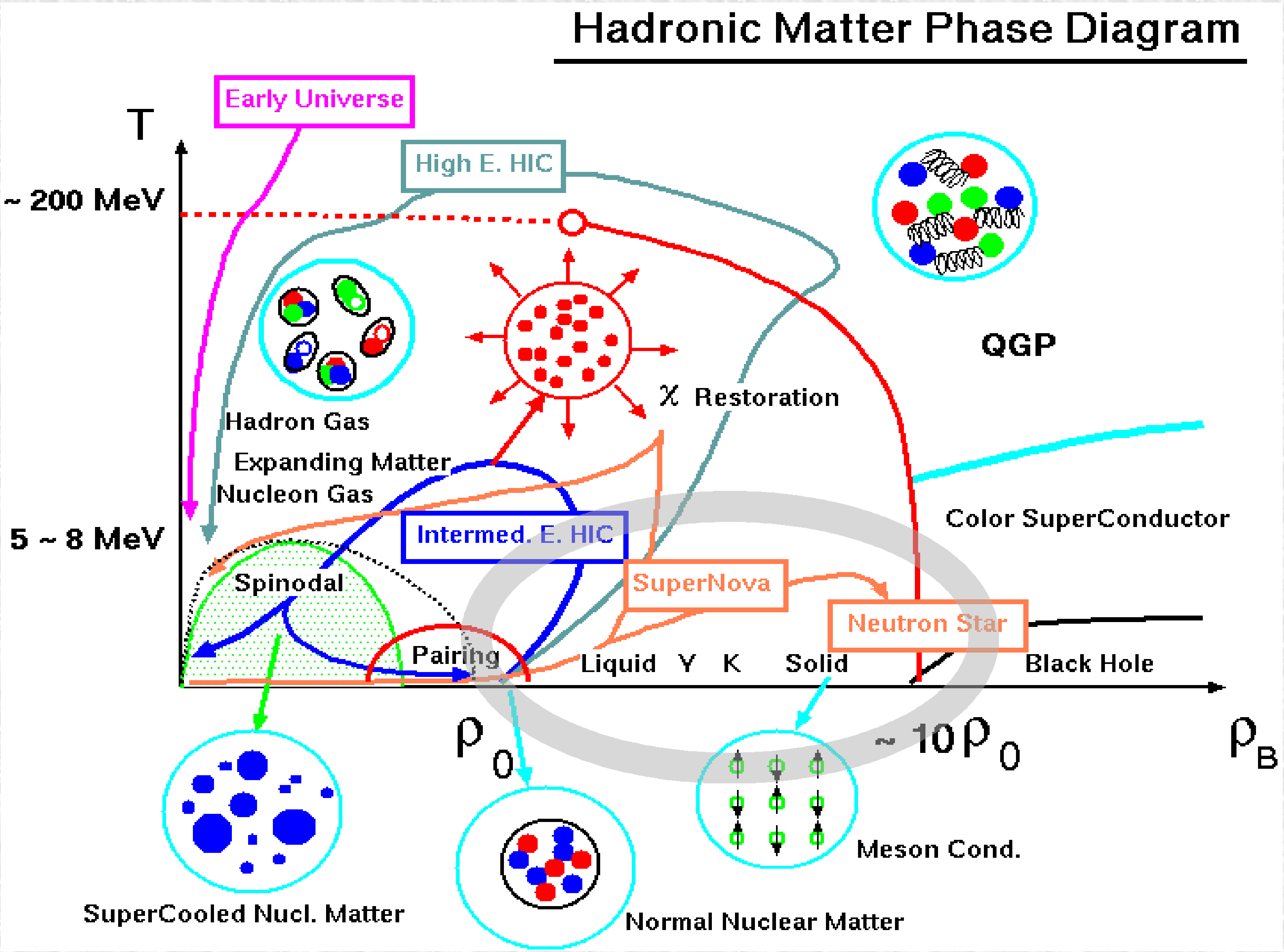
- *Introduction: Hyperons in Dense Matter*
- *SU(3) Chiral sigma model with baryons*
- *Application to Symmetric Nuclear Matter*
- *Summary*

Hadronic Matter Phase Diagram



Hyperons in Dense Matter

- ***Hyperons in Neutron Star (cf Talk by Bombaci, Vidana)***
  - *Tsuruta-Cameron (66), Langer-Rosen (70), Pand-haripande (71), Itoh(75), Glendenning, Weber-Weigel, Sugahara-Toki, Schaffner-Mishustin, Balberg-Gal, Baldo et al., Vidana et al., Nishizaki-Yamamoto-Takatsuka, Kohno-Fujiwara et al., ...*
- ***Hyperons during Supernova Explosion***
  - *Supernova explode in pure 1D hydro, but with ν transport shock stalls.*
  - *3 %increase of ν flux revive shock wave (Janka et al.)*
  - *Hyperons increase explosion energy by around 4 % (Ishizuka, AO, Sumiyoshi,Yamada, in preparation)*

Hyperons play crutial roles in dense matter, such as in neutron stars and supernova explosion.

Hyperon Potentials at High Densities

- Hyperon Potentials at around ρ_0

  $U(\Lambda) \sim -30$ MeV

  $U(\Xi) \sim -(14-16)$ MeV (KEK-E224, BNL-E885, BNL-E906)

  $U(\Sigma) \sim (-30 \sim +150)$ **MeV** (Pararell Session 1, 3)

- Hyperon Potentials at high densities (V. Koch's talk)

  Exp't Info. : Hyperon flow, K^+/π^+ enhancement, ...

  Theor. Prediction: *Strongly depends on the model* (Shinmura's Talk)

We need reliable models with smaller number of free parameters and/or derived from the first principle.

→ Chiral Symmetry

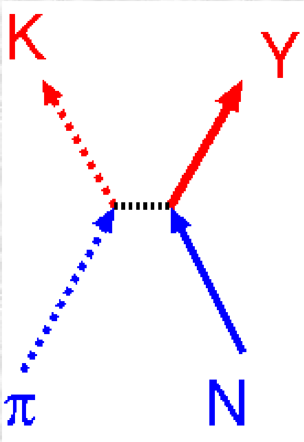
Nuclear Matter in SU(2) Chiral Linear σ Model

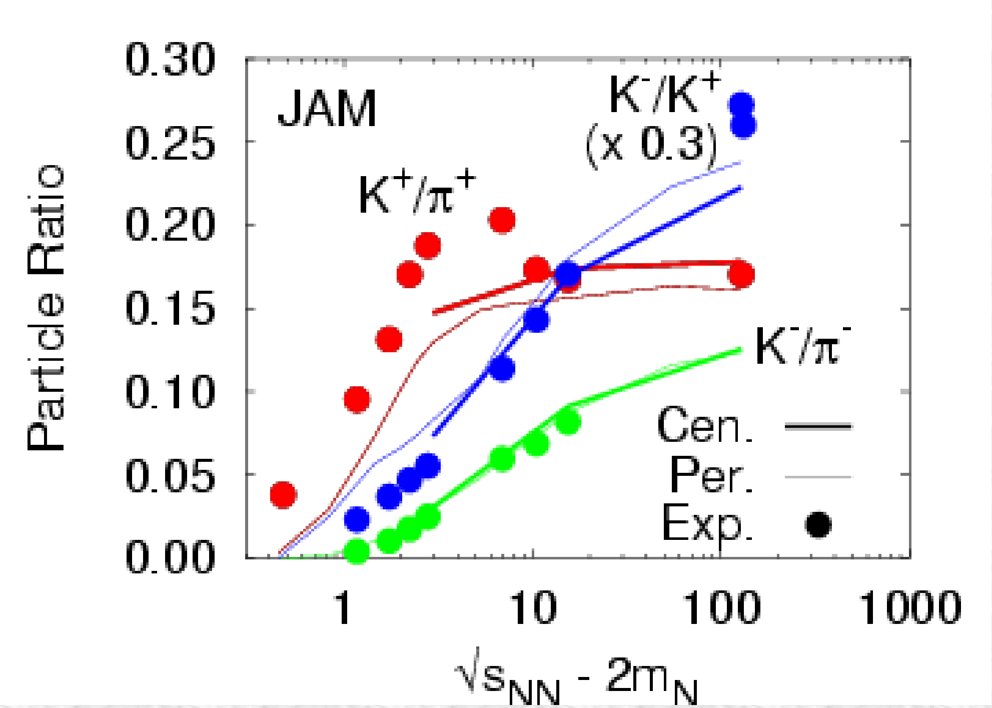
- ## *Chiral Linear σ Model*
  - *Good model in describing hadron properties.*
  - *Dynamical change of σ condensate*
    → suitable for nuclear matter study
- ## *Problems in Nuclear Matter*
  - *Naive model leads to sudden change of condensate,* $\sigma \sim f_\pi \to 0$
    → Dynamical generation of ω meson mass (σω coupling)
    (J. Boguta, PLB120,34/PLB128,19)
  - *Equation of State is too stiff.*
    → Loop Effects (vacuum renormalization)
    (N.K. Gledenning, NPA480,597,
    M. Prakash and T. L. Ainsworth, PRC36, 346)
  - *Higher order terms (σ6, σ8) (P.K. Sahu and AO, PTP104,1163)*

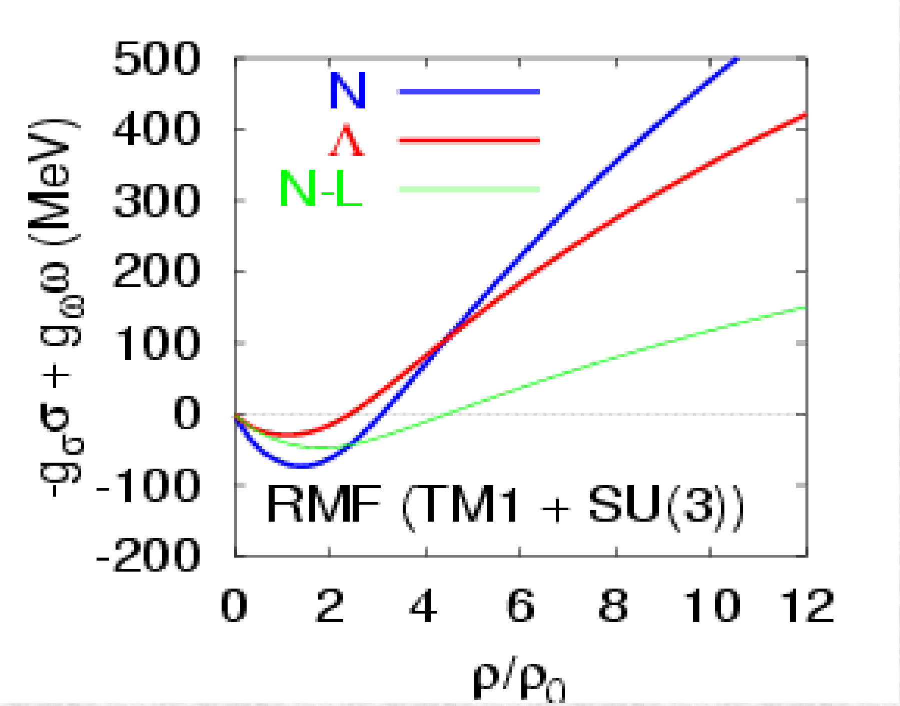
Can we soften the EOS with Hyperons ?

Does Hyperon Potential Help It ?

- *Rescattering of Resonances/Strings (RQMD)*
- *Baryon Rich QGP Formation*
- *High Baryon Density Effect (Associated Prod. of Y)*







At $\rho > 4\rho_0$, Hyperon Feels More Attractive Potential than N

SU(3) Chiral Linear σ Model with Baryons

BBM coupling in SU(2) chiral linear σ model

- **Hadron transformation**
  - **Baryons: fundamental repr.**

$$N = \begin{pmatrix} p \\ n \end{pmatrix} \quad , \quad N_L \to L\, N_L \quad , \quad N_R \to R\, N_R$$

  - **Mesons: Adjoint repr.**

$$M = \Sigma + i\,\Pi \to L\, M\, R^+$$

- **Chiral Invariant Coupling**

$$L_{BBM} = g\left(N_L^+ M\, N_R + N_R^+ M^+ N_L\right)$$
$$\to g\left(N_L^+ L^+ L\, M\, R^+ R\, N_R + c.c.\right)$$

→ *How about in SU(3) ?*

Mesons and Baryons in SU(3)

- **Meson Matrix**

$$M = \begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{s} & s\bar{d} & s\bar{s} \end{pmatrix}$$

$$M_{PS} = \Pi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta'}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta'}{\sqrt{3}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} + \frac{\eta'}{\sqrt{3}} \end{pmatrix}$$

- **Baryon Matrix**

$$\Psi = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}$$

BBM Coupling in SU(3) Chiral Linear σ Model

- *Mesons: Transforms as in SU(2)*

$$M = \Sigma + i\,\Pi \rightarrow L\,M\,R^{+}$$

- *Baryons: Two kind of transformations are proposed*

  - *Type 1: Stokes-Rijken '97, Gasiolowiz-Geffen 60's*

$$\Psi_L^{(1)} \rightarrow L\,\Psi_L^{(1)}\,L^{+} \quad , \quad \Psi_R^{(1)} \rightarrow R\,\Psi_R^{(1)}\,R^{+}$$

Lowest order chiral invariant coupling = BMBM

$$\mathrm{Tr}\left(\overline{\Psi}_L^{(1)}\,M\,\Psi_R^{(1)}\,M^{+}\right) \rightarrow \mathrm{Tr}\left(L\,\overline{\Psi}_L^{(1)}\,L^{+}\,L\,M\,R^{+}\,R\,\Psi_R^{(1)}\,R^{+}\,R\,M^{+}\,L^{+}\right)$$

  - *Type 2: Papazoglou et., 98*

$$\Psi_L^{(2)} \rightarrow L\,\Psi_L^{(2)}\,R^{+} \quad , \quad \Psi_R^{(2)} \rightarrow R\,\Psi_R^{(2)}\,L^{+}$$

Lowest order chiral invariant coupling = D-type coupling

$$\mathrm{Det}'\left(\overline{\Psi}_R^{(2)}, M, \Psi_L^{(2)}\right) \equiv \epsilon_{ijk}\,\epsilon_{lmn}\,\overline{\Psi}_{R\,il}^{(2)}\,M_{jm}\,\Psi_{kn}^{(2)} \rightarrow |R||L|\,\mathrm{Det}'\left(\overline{\Psi}_R^{(2)}, M, \Psi_L^{(2)}\right)$$

Relation to Quark Field

- **Spin half 3 quark field**
  - **Positive Parity, Having NR Limit (8 states)**

$$B^{(1)}_{dc} = N^{-3}\left(q^T_{i,a} C \gamma_5 q_{j,b}\right) q_{k,d} \epsilon_{ijk} \epsilon_{abc}$$

  - **Negative Parity, No NR Limit (9 states)**

$$B^{(2)}_{dc} = N^{-3}\left(q^T_{i,a} C q_{j,b}\right) q_{k,d} \epsilon_{ijk} \epsilon_{abc}$$

(ijk: color, abcd: flavor)

- **Transformation properties**

$$\Psi^{(1)} \equiv \left(B^{(1)} + \gamma_5 B^{(2)}\right)/\sqrt{2} \quad , \quad \Psi^{(2)} \equiv \left(B^{(1)} - \gamma_5 B^{(2)}\right)/\sqrt{2}$$

$$\Psi^{(1)}_L \to L \Psi^{(1)}_L L^+ \quad , \quad \Psi^{(1)}_R \to R \Psi^{(1)}_R R^+$$

$$\Psi^{(2)}_L \to L \Psi^{(2)}_L R^+ \quad , \quad \Psi^{(2)}_R \to R \Psi^{(2)}_R L^+$$

SU(3) Chiral Invariant BBM Coupling

- *Trace type (G.A. Christos, PRD35 (1987), 330).*

$$-L_{BM}^{Tr} = \frac{g_{tr}}{\sqrt{2}} \mathrm{Tr}\left(\overline{\Psi}_L^{(1)} M \Psi_R^{(2)} + \overline{\Psi}_R^{(1)} M^{+} \Psi_L^{(2)}\right) + h.c.$$

$$= g_{tr}\left(d_{abc} + i f_{abc}\right)\left(\overline{\psi}^{1a} m^{b} \psi^{2c} + h.c.\right)$$

- *Determinant type (Papazoglou et. al. PRC57 ('98) 2576)*

$$-L_{BM}^{Det} = \sqrt{2}\, g^{det}\left(\mathrm{Det}'\left(\overline{\Psi}_R^{(2)}, M, \Psi_L^{(2)}\right) + h.c.\right)$$

$$= 2\, g_{det}\, d'_{abc}\, \overline{\psi}^{2a} m^{+b} \psi^{2c}$$

$$\left(m^{a} \equiv \sigma^{a} + i \gamma_5 \pi^{a}\right)$$

In order to have both of D and F type BBM Coupling,
We need two types of baryons !

Baryon Masses

Explicit Breaking term

- ***Mean Field Approx. + Diagonalization***

$$-L_{BM}^{MF}=\sum_i \left(\bar{\psi}_i^1 \quad \bar{\psi}_i^2\right)\begin{pmatrix} 0 & g_{tr}\sigma_i + n_i^s m_s \\ g_{tr}\sigma_i + n_i^s m_s & -g_{det}\sigma'_i \end{pmatrix}\begin{pmatrix}\psi_i^1 \\ \psi_i^2\end{pmatrix}$$

$$=\sum_i \left(\bar{\psi}_i^{[+]} \quad -\bar{\psi}^{[-]}{}_i\right)\begin{pmatrix} M_i^{[+]} & 0 \\ 0 & -M_i^{[-]} \end{pmatrix}\begin{pmatrix}\psi_i^{[+]} \\ \psi_i^{[-]}\end{pmatrix}$$

- ***Positive and Negative Parity Octet Baryon Masses***

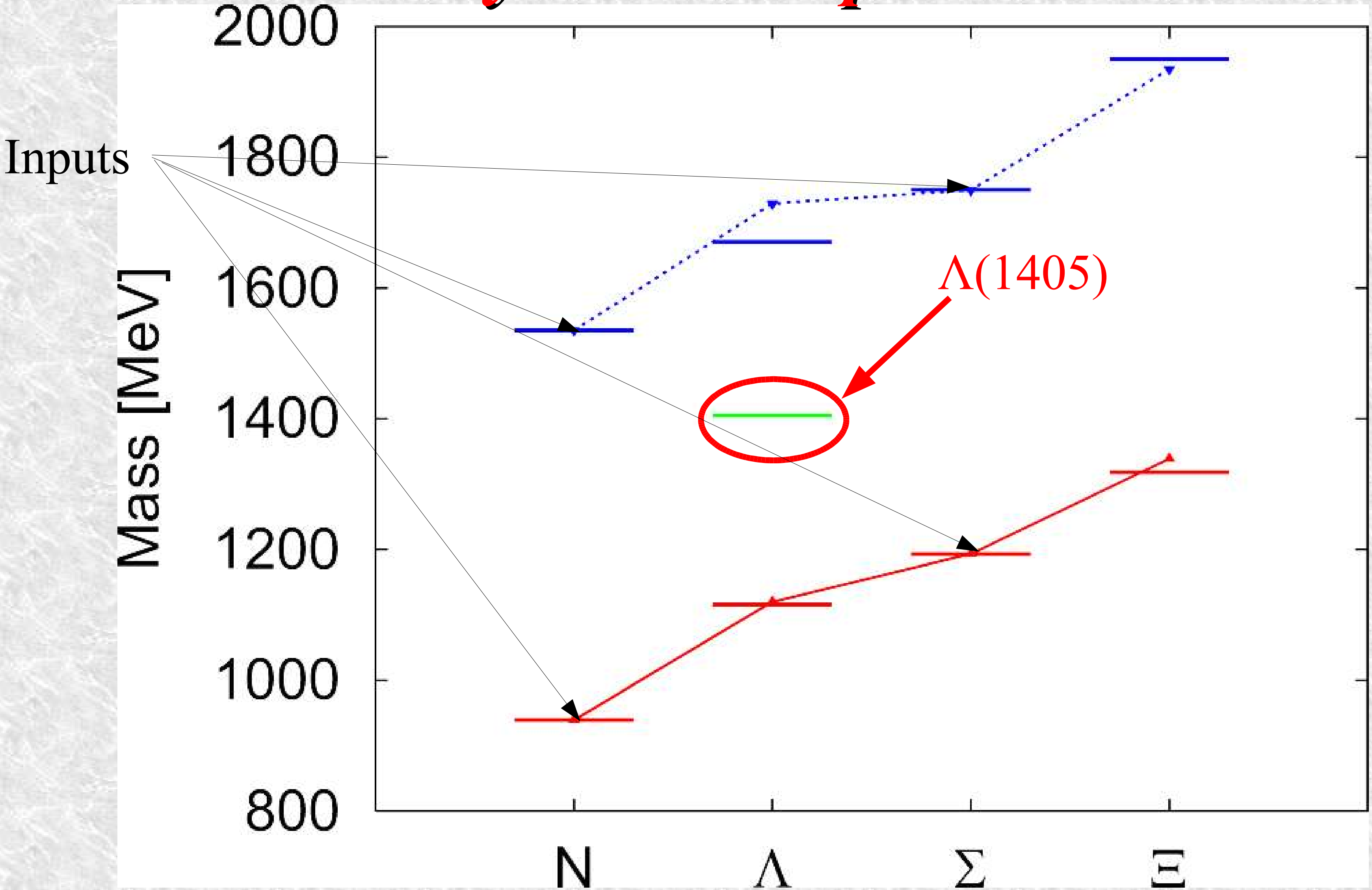
$$\sigma_i \equiv \sqrt{\frac{2}{3}}\left(\sigma_0 + a_i \sigma_8\right) \quad , \quad \sigma'_i \equiv \sqrt{\frac{2}{3}}\left(\sigma_0 + b_i \sigma_8\right)$$

$$\begin{pmatrix}\psi_i^1 \\ \psi_i^2\end{pmatrix}=\begin{pmatrix}\cos\theta_i & -\sin\theta_i \\ \sin\theta_i & \cos\theta_i\end{pmatrix}\begin{pmatrix}\psi_i^{[+]} \\ \gamma_5\psi_i^{[-]}\end{pmatrix}$$

$$M_i^{[\pm]}=\sqrt{\left|g_{tr}\sigma_i + n_i^s m_s\right|^2 + \left(g_{det}\sigma'_i/2\right)^2} \pm g_{det}\sigma'_i/2$$

Four Free Parameters < Numbers of Baryon Masses to be fitted

Baryon Mass Spectrum



Positive and Negative Spin-half Baryon Masses are well reproduced except for Λ (1405)

Meson Lagrangian: Standard

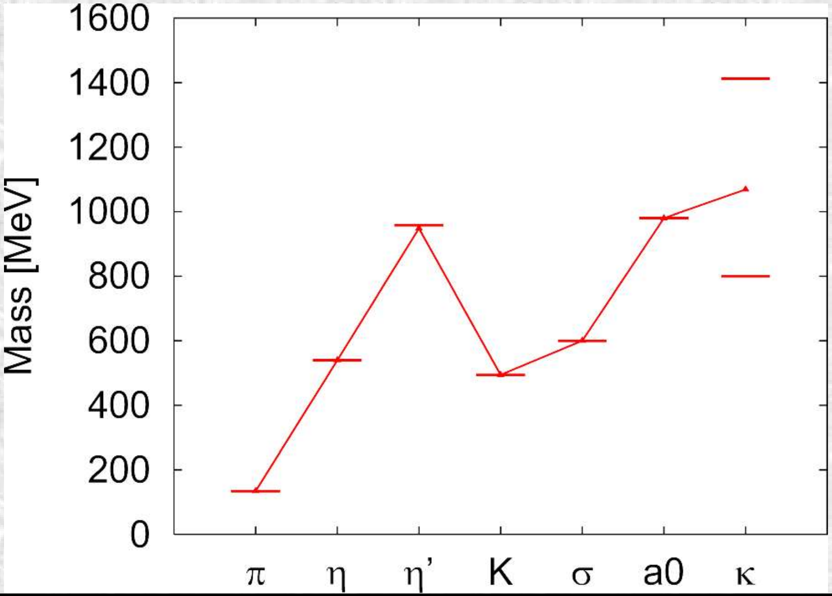
$$\mathcal{L} = \frac{1}{2}\mathrm{tr}[\partial_\mu M \partial^\mu M^\dagger] - \frac{1}{2}\mu^2 \mathrm{tr}[M M^\dagger]$$
$$- \lambda \mathrm{tr}[M M^\dagger M M^\dagger] - \lambda' \mathrm{tr}[M M^\dagger]^2 + c(\det M + \det M^\dagger)$$
$$+ \frac{\sqrt{3}D}{4}\left\{\mathrm{tr}[M M^\dagger \lambda^8] + \mathrm{tr}[M \lambda^8 M^\dagger]\right\} + c_\sigma \sigma + c_\zeta \zeta$$

Kinetic+ Second order + Fourth order + Det. Int. + Explicit breaking

Seven Free Parameters

Fitting 2 Decay constants (π and K) and 4 Meson masses (π, K, η, a_0)

→ One parameter = σ Mass



Application to Symmetric Nuclear Matter

Mean Field Lagrangian

$$L^{MF} = \frac{\mu^2}{2}\left(\sigma^2+\zeta^2\right) - \frac{\lambda}{4}\left(\sigma^2+\zeta^2\right)^2 - \frac{\lambda'}{4}\left(\sigma^4+2\zeta^4\right) + c\sigma^2\zeta$$
$$-\nu\zeta^2 + H_\sigma\sigma + H_\zeta\zeta$$
$$+\sum_i \left(\sum_{k=1,2} \overline{\psi}_i^k i\partial_\mu\gamma^\mu\psi_i^k\right)$$
$$+\sum_i \left[g_{tr}\left(\overline{\psi}_i^1\psi_i^2+\overline{\psi}_i^2+\psi_i^1\right)\left(\sigma_i+n_i^s m_s\right) - g_{det}\overline{\psi}_i^2\psi_i^2\sigma'_i\right]$$

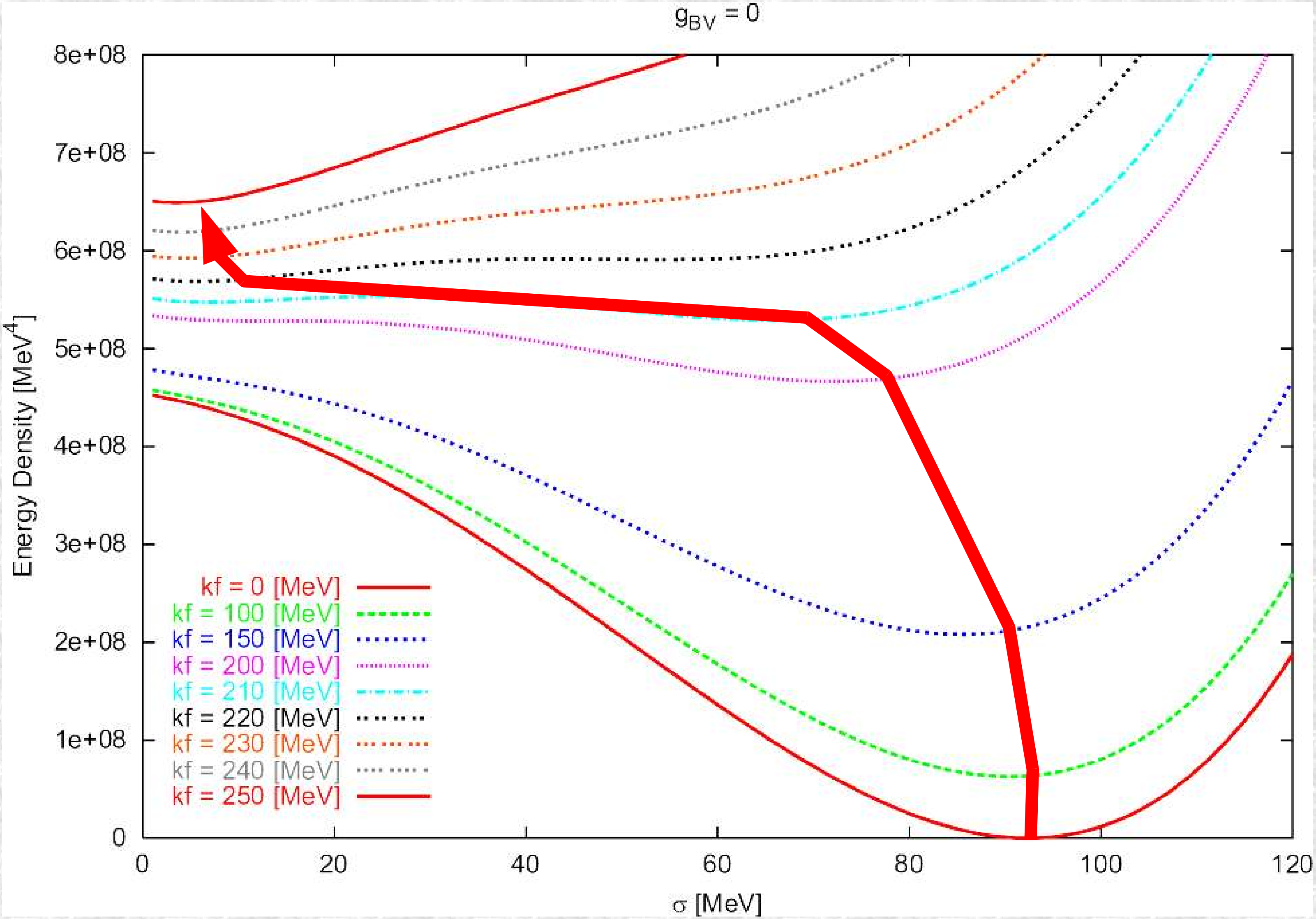
- Equation of Motion

$$\frac{\partial L}{\partial\sigma} = \mu^2\sigma - \lambda\left(\sigma^2+\zeta^2\right)\sigma - \lambda'\sigma^3 + 2c\sigma\zeta + H_\sigma - g_\sigma^{[+]}\rho_s = 0$$

In Symmetric Nuclear Matter,

$$g_\sigma^{[+]} = 2g_{tr}\sin\theta_N\cos\theta_N - g_{det}\cos\theta_N^2$$

Free Energy (1): without σ ω Coupling



Sudden Chage of σ Value → Chiral Phase Transition below ρ_0

Mean Field Lagrangian

$$L^{MF} = \frac{\mu^2}{2}\left(\sigma^2+\zeta^2\right) - \frac{\lambda}{4}\left(\sigma^2+\zeta^2\right)^2 - \frac{\lambda'}{4}\left(\sigma^4+2\zeta^4\right) + c\sigma^2\zeta$$
$$-\nu\zeta^2 + H_\sigma\sigma + H_\zeta\zeta$$
$$+\sum_i\left(\sum_{k=1,2}\overline{\psi}_i^k i\partial_\mu\gamma^\mu\psi_i^k\right)$$
$$+\sum_i\left[g_{tr}\left(\overline{\psi}_i^1\psi_i^2+\overline{\psi}_i^2+\psi_i^1\right)\left(\sigma_i+n_i^s m_s\right) - g_{det}\overline{\psi}_i^2\psi_i^2\sigma'_i\right]$$

$$-g_{VB}\,\omega\,\rho_B + \lambda_{VS}\,\sigma^2\omega^2/2$$

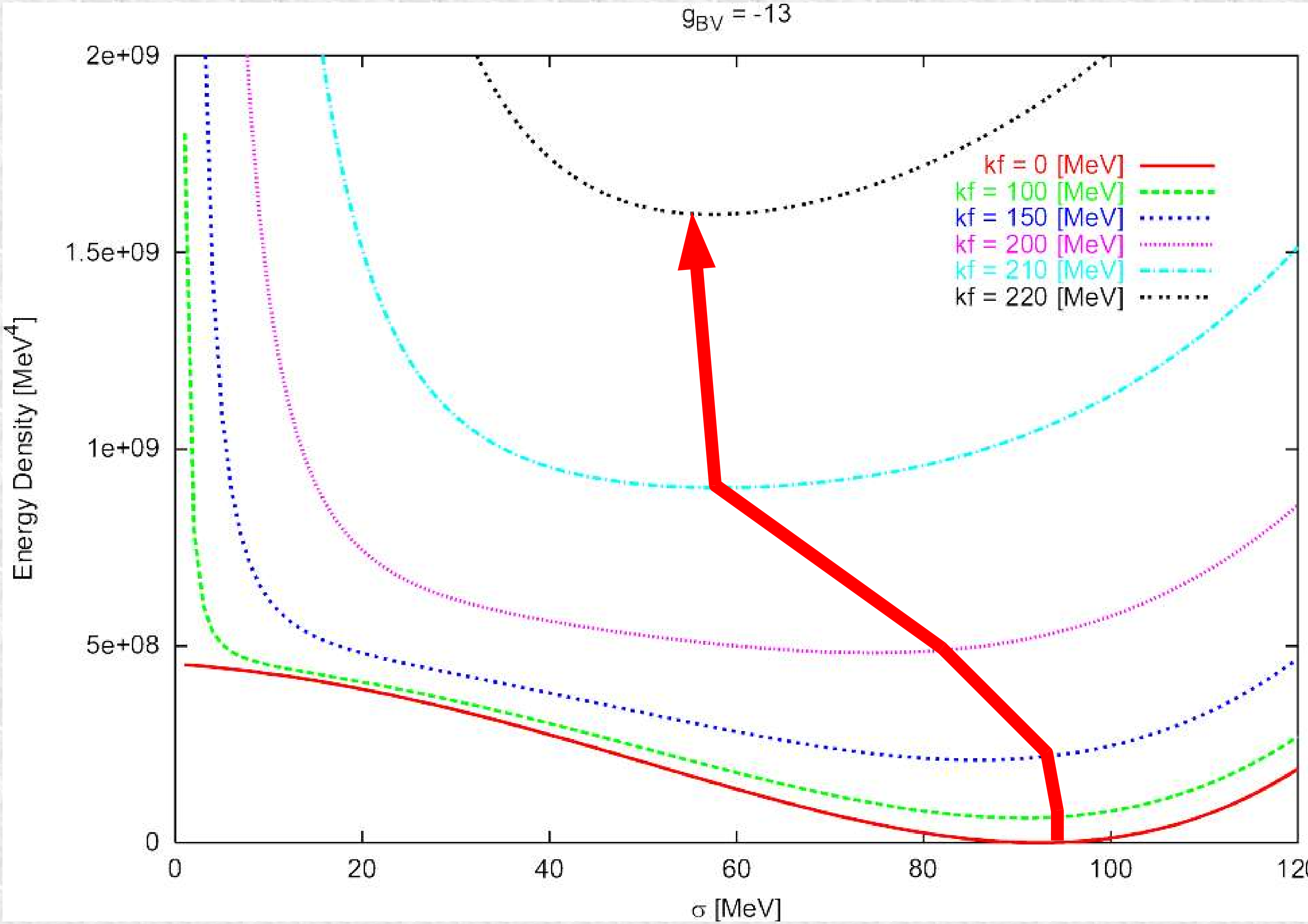
- Equation of Motion

$$\frac{\partial L}{\partial\sigma} = \mu^2\sigma - \lambda\left(\sigma^2+\zeta^2\right)\sigma - \lambda'\sigma^3 + 2c\sigma\zeta + H_\sigma - g_\sigma^{[+]}\rho_s + \lambda_{VS}\sigma\omega^2 = 0$$

$$\frac{\partial L}{\partial\omega} = -g_{VB}\rho_B + \lambda_{VS}\sigma^2\omega = 0 \;\rightarrow\; \omega = g_{VB}\rho_B/\lambda_{VS}\sigma^2$$

$$m_\omega^2 = \lambda_{VS}\sigma^2 = 782\,\text{MeV}$$ (Boguta)

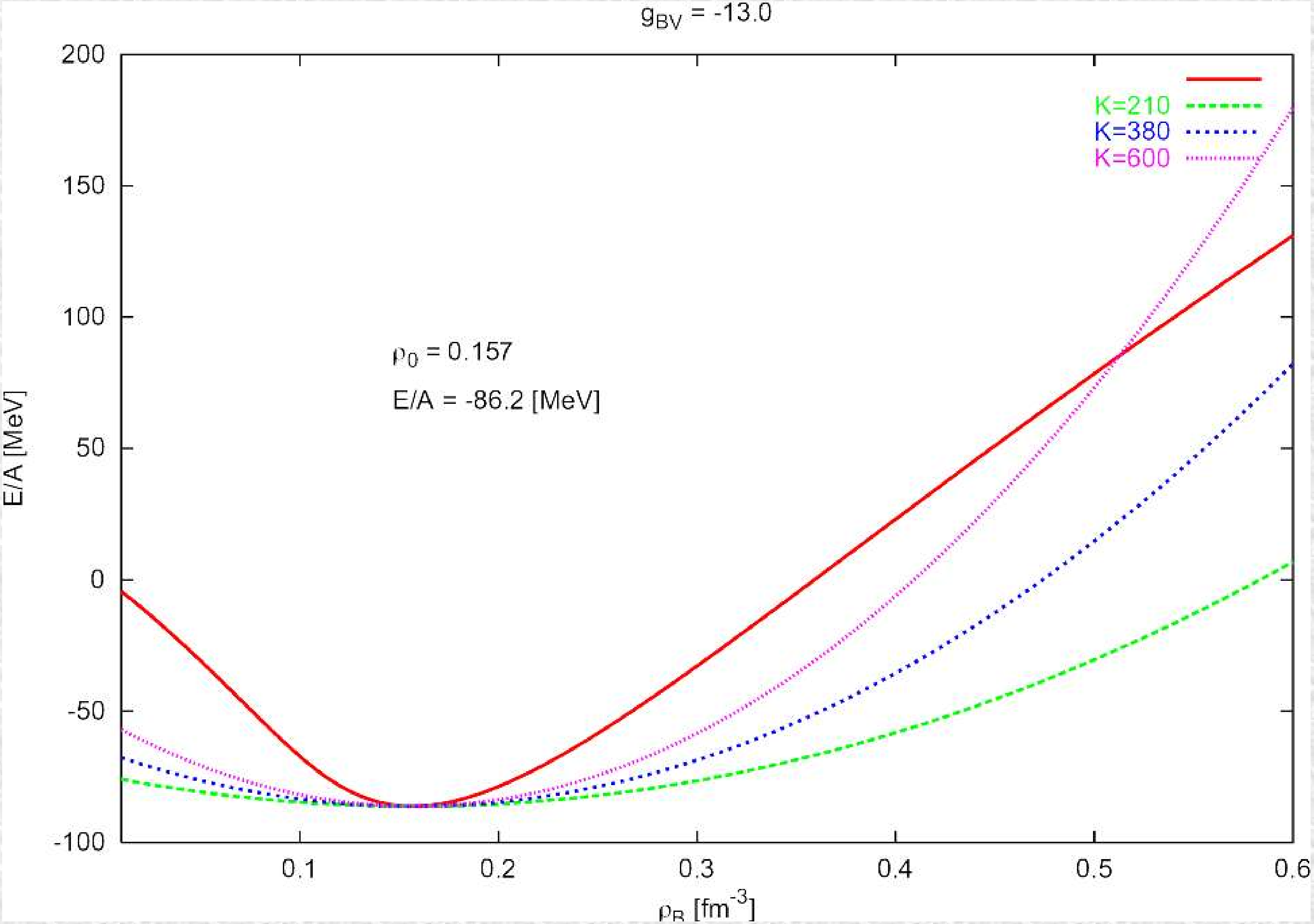
Free Energy (2): with σ ω Coupling



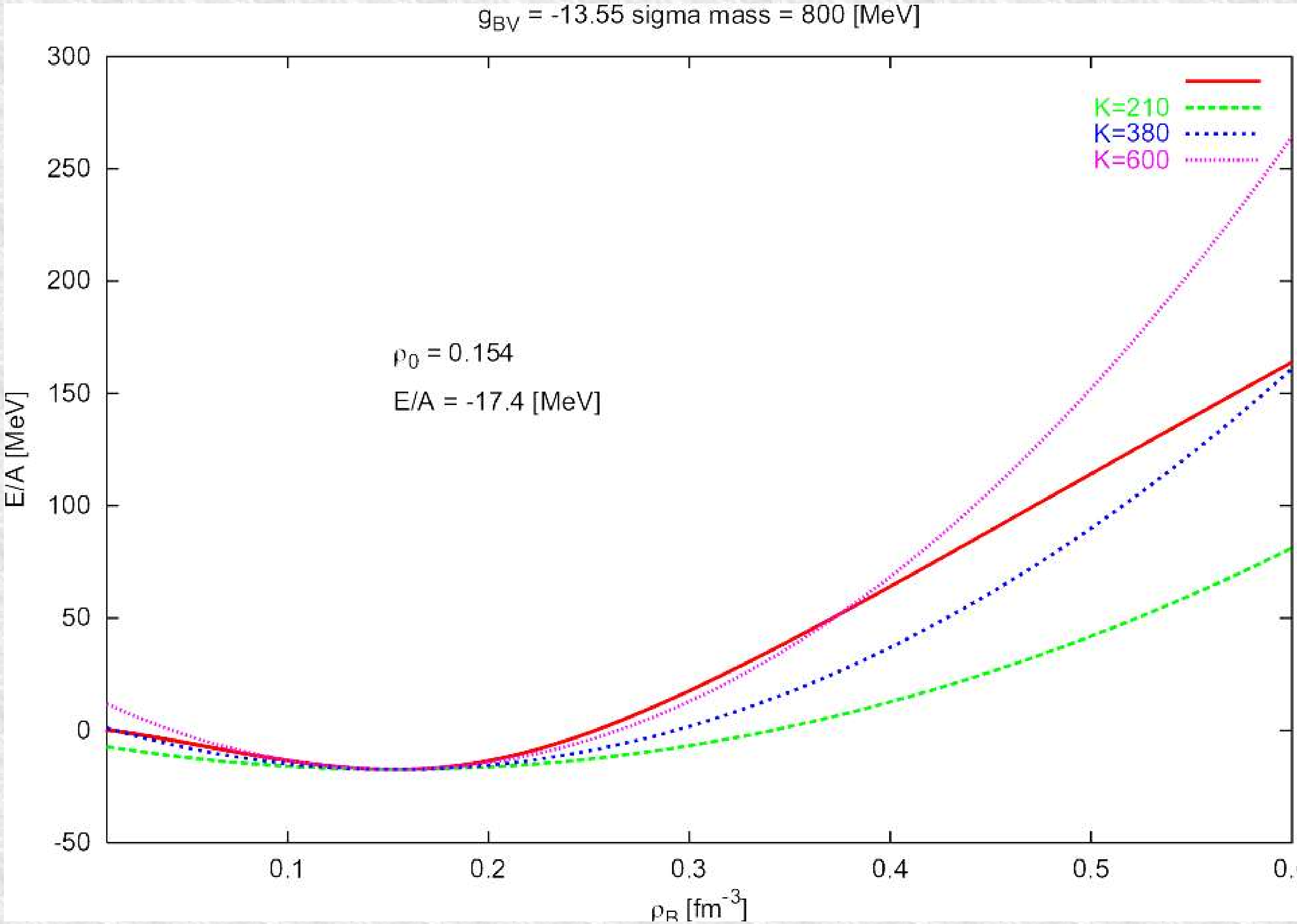
***Smooth Chage of* σ Value / Only One Local Minima**

→ Stability of Normal Vacuum

Equation of State (1): M_σ = 600 MeV



Equation of State (2): M_σ *= 800 MeV*



We can fit ρ_0 and E/A by adjusting g_{BV} and M_σ but EOS becomes too stiff.

Summary

- ***An SU(3) chiral sigma model with baryons is presented.***
  - ***Two types of transformation, B(1) and B(2)   (Christos)***
  - ***Two types of Lowest order BBM coupling*** (Christos / Papazoglou et. al.)
  - ***Explicit breaking term (Strange quark mass) → Octet Baryon Mass***
  - ***Meson Lagrangian : Standard***

Positive and Negative parity baryons are necessarily couple in constructing the lowest order chirally invariant Lagrangian having D as well as F coupling.

- ***This model is applied to symmetric nuclear matter.***
  - ***Coupling of $\sigma\omega$: Dyn. generation of vector meson mass (Boguta)***
  - ***BV coupling: Repulsive NN interaction***
  - ***EOS = Too Stiff ! → One problem in SU(2) model is not solved yet !***

Problems and Future Directions

- How can we make EOS softter ?

  "Classical" Interaction BMBM
  Loop (Gledenning / Prakash-Ainsworth)
  Higher order terms (Sahu-AO)
  Dilatation Field (Papazoglou et. al.)
  Vector Realization (Sasaki-Harada)
  Non-Linear Realization
  ....

- How does the model predict Hyperon Potentials in Dense Matter ?

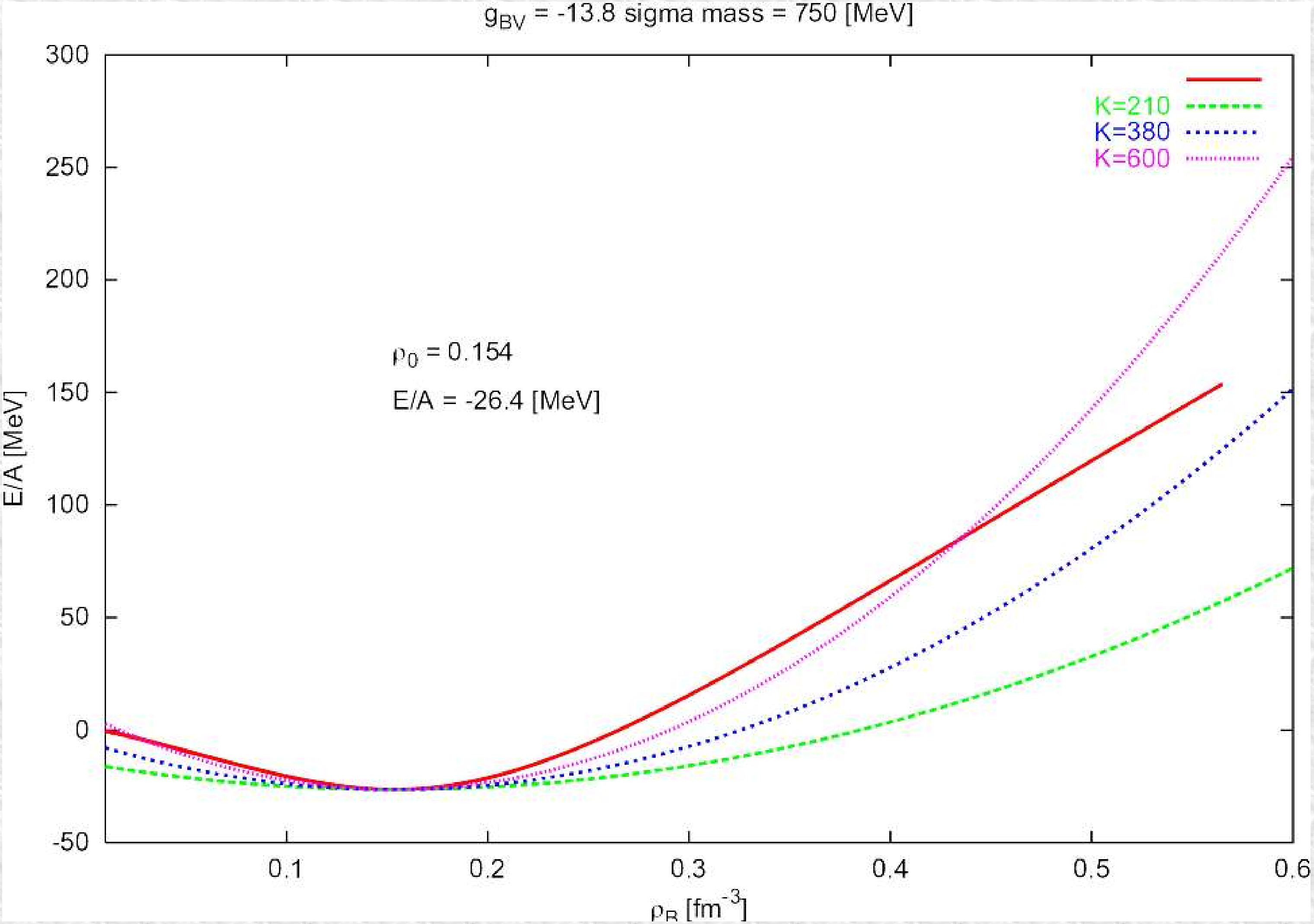
- Behavior of Negative Parity Baryons in Nuclear Matter

- F/D Ratio in Pseudoscalar BB coupling = 1.6 !

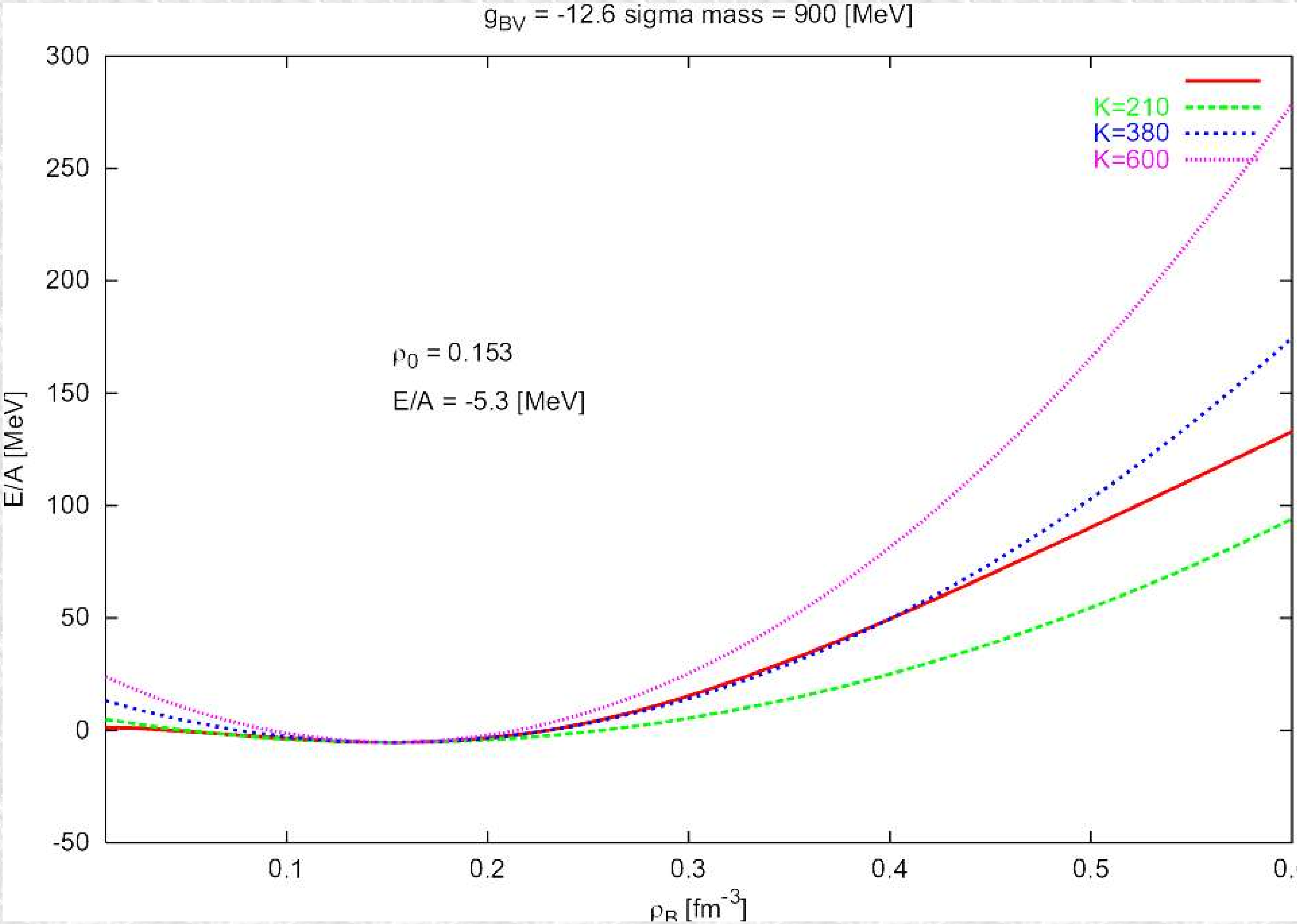
  (← emprically 0.5-0.7)

- Perturbative contribution to condensate in baryon and meson sectors.

Equation of State (2): M_σ = 750 MeV



Equation of State (4): M_σ = 900 MeV



		Transf.	Repr., (L,R)
Quarks	q_L, q_R	Lq_L, Rq_R	$(3,1), (1,3)$
Mesons	$M = \lambda_a(\bar{q}_R \lambda_a q_L)$	$LMR^\dagger$	$(3, 3^*)$
Baryons	Ψ_L^1, Ψ_R^1	$L\Psi_L^1 L^\dagger, R\Psi_R^1 R^\dagger$	$(8,1), (1,8)$
	Ψ_L^2, Ψ_R^2	$L\Psi_L^2 R^\dagger, R\Psi_R^2 L^\dagger$	$(3, 3^*), (3^*, 3)$

	$\Psi_L^1(8,1)$	$\Psi_R^1(1,8)$	$\Psi_L^2(3,3^*)$	$\Psi_R^2(3^*,3)$
$\bar{\Psi}_L^1(8,1)$	—	—	0	$\mathrm{Tr}\left[\bar{\Psi}_L^1 M \Psi_R^2\right]$
$\bar{\Psi}_R^1(1,8)$	—	—	$\mathrm{Tr}\left[\bar{\Psi}_R^1 M^\dagger \Psi_L^2\right]$	0
$\bar{\Psi}_L^2(3^*,3)$	0	$\mathrm{Tr}\left[\bar{\Psi}_L^2 M \Psi_R^1\right]$	—	$\mathrm{Det}'\left[\bar{\Psi}_L^2, M^\dagger, \Psi_R^2\right]$
$\bar{\Psi}_R^2(3,3^*)$	$\mathrm{Tr}\left[\bar{\Psi}_R^2 M^\dagger \Psi_L^1\right]$	0	$\mathrm{Det}'\left[\bar{\Psi}_R^2, M, \Psi_L^2\right]$	—

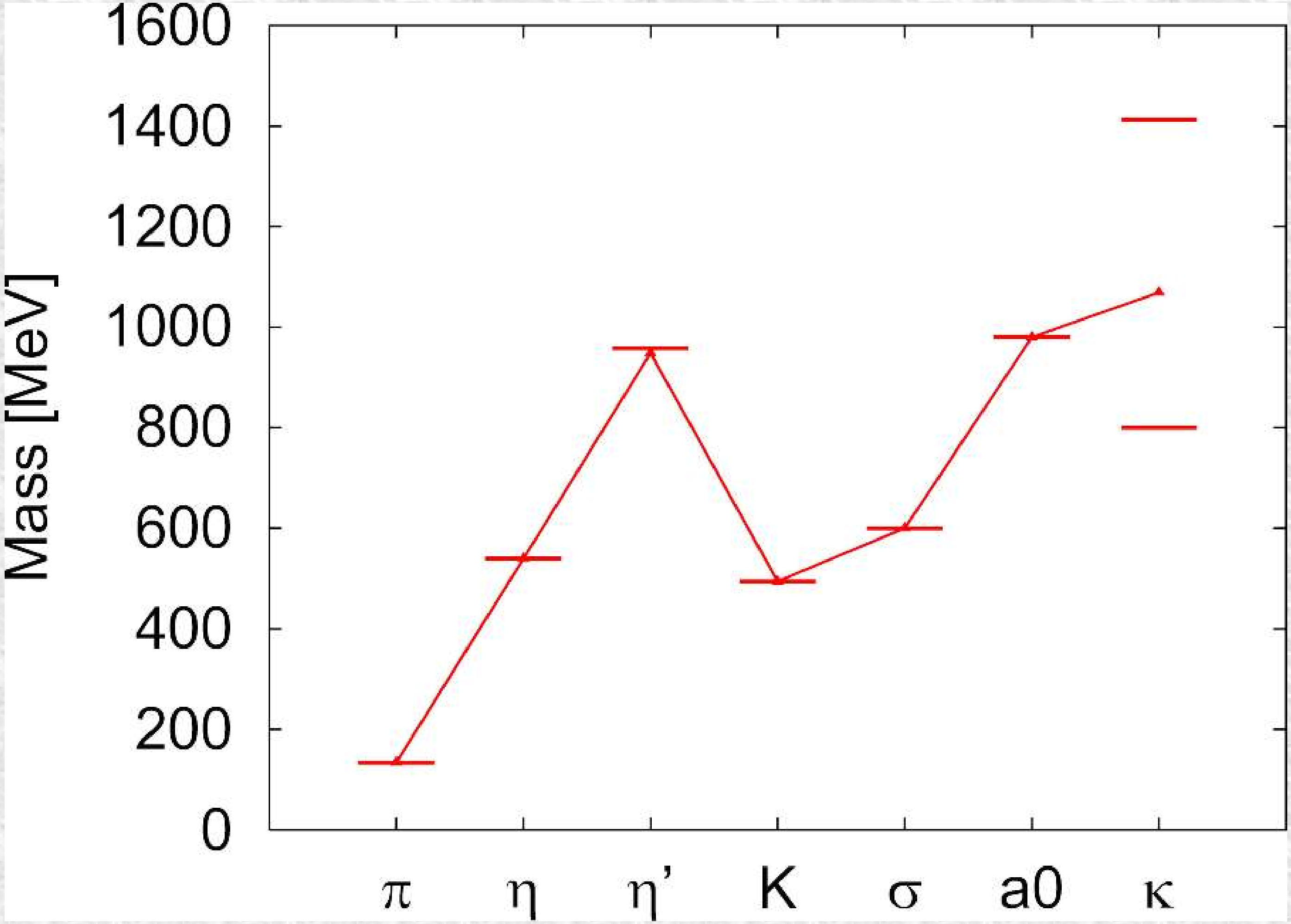
$$B^1_{lk} = \mathcal{N}^{-3}(q^T_{a,i} C\gamma_5 q_{b,j})\, q_{c,l}\epsilon_{abc}\epsilon_{ijk}\ , \qquad B^2_{lk} = \mathcal{N}^{-3}(q^T_{a,i} C q_{b,j})\, q_{c,l}\epsilon_{abc}\epsilon_{ijk}\ ,$$

$$\Psi^1 = \left(B^1 + \gamma_5 B^2\right)/\sqrt{2} \quad : \quad \Psi^1_L \to L\Psi^1_L L^\dagger\ , \quad \Psi^1_R \to R\Psi^1_R R^\dagger\ ,$$

$$\Psi^2 = \left(B^1 - \gamma_5 B^2\right)/\sqrt{2} \quad : \quad \Psi^2_L \to L\Psi^2_L R^\dagger\ , \quad \Psi^2_R \to R\Psi^2_R L^\dagger\ ,$$

$$d'_{abc} \equiv \frac{1}{4}\epsilon_{ijk}\,\epsilon_{lmn}\lambda^a_{il}\lambda^b_{jm}\lambda^c_{kn} = d_{abc} - \frac{\sqrt{6}}{2}\left[\delta_{a0}\delta_{bc} + \delta_{b0}\delta_{ca} + \delta_{c0}\delta_{ab} - 3\delta_{a0}\delta_{b0}\delta_{c0}\right]$$

Meson Mass Spectrum



Good Agreement with Data except for κ

Isoscalar Vector Meson ω

Dynamical generation of ω mass: **σ**ω Coupling

$$m_\omega^2 = \lambda_{VS}\sigma^2$$

782 MeV

Coupling to Baryon: Repulsive BB interaction

$$g_{BV_1}\left\{\mathrm{tr}\left[\overline{\Psi}_L^1 l_\mu \gamma^\mu \Psi_L^1\right] + \mathrm{tr}\left[\overline{\Psi}_R^1 r_\mu \gamma^\mu \Psi_R^1\right]\right\}$$
$$+ g_{BV_2}\left\{\mathrm{tr}\left[\overline{\Psi}_L^2 l_\mu \gamma^\mu \Psi_L^2\right] + \mathrm{tr}\left[\overline{\Psi}_R^2 r_\mu \gamma^\mu \Psi_R^2\right]\right\}$$

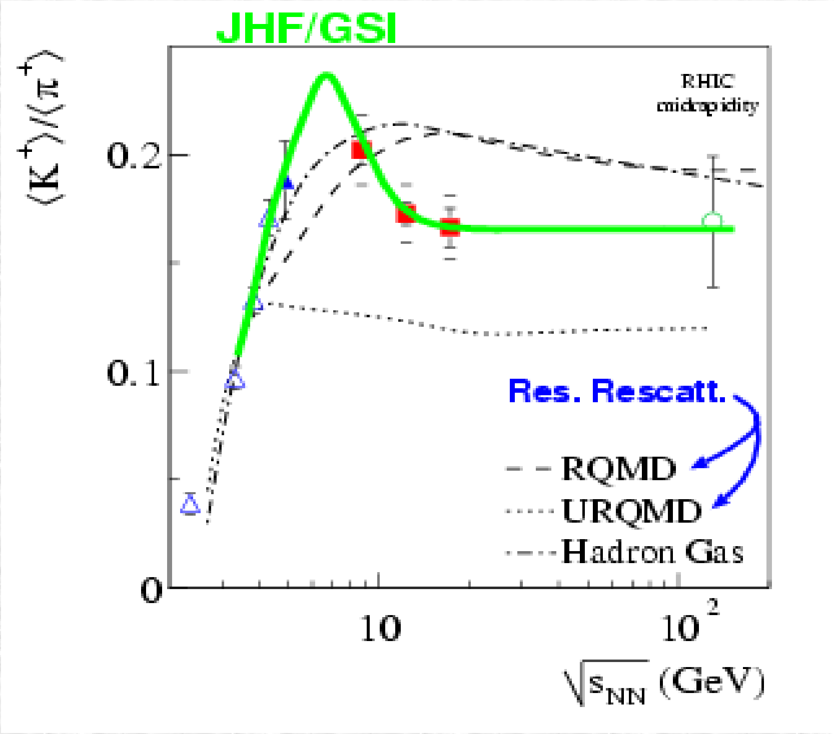
$$\lambda\omega_0\sigma^2 + \frac{g_{BV_1}\cos^2\theta_N + g_{BV_2}\sin^2\theta_N}{2}\left(\langle p^\dagger p\rangle_F + \langle n^\dagger n\rangle_F\right)$$
$$= \quad 0$$

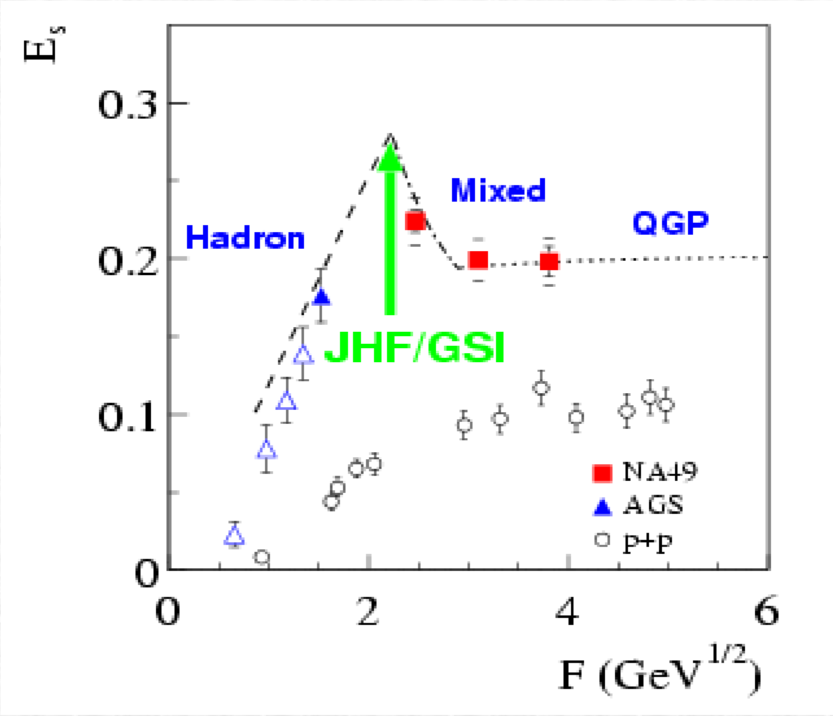
$\boldsymbol{g_{BV} \equiv g_{BV1}\cos^2\theta_N + g_{BV2}\sin^2\theta_N}$: Free Parameter !

Strangeness Enhancement: Rescattering, Potential, or Phase Transition ?

Strangeness is Enhanced Sharply at Einc = 10 ~ 40 GeV/A !

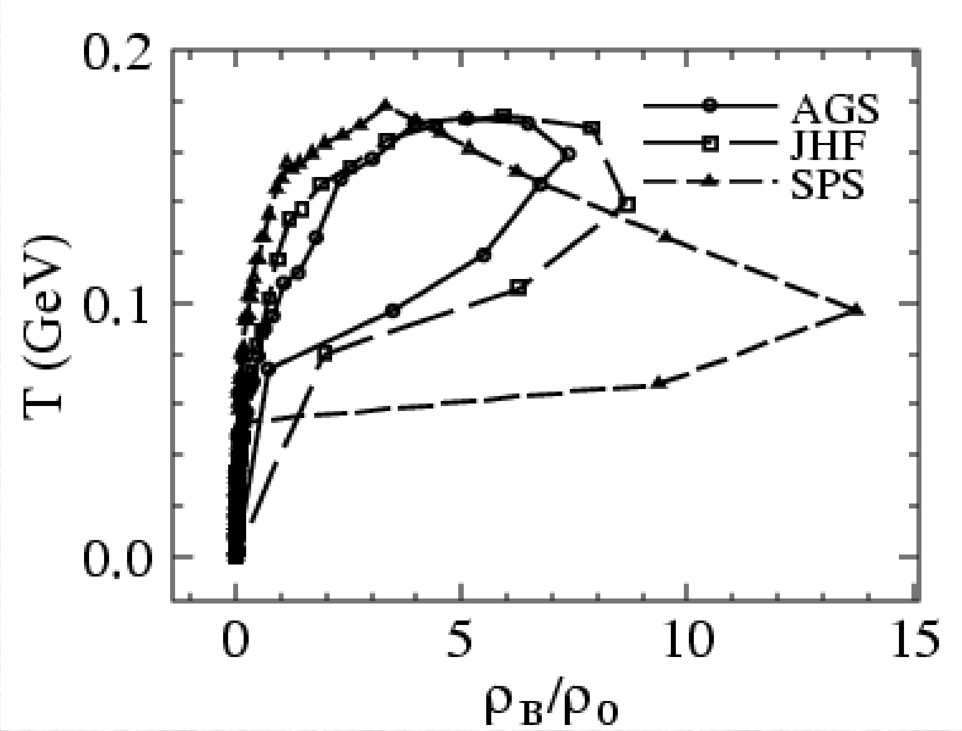
NA49 (nucl-ex/0205002)





JHF Energy: ~ Maximum K/π ratio

Thermal Evolution from AGS to SPS Energies



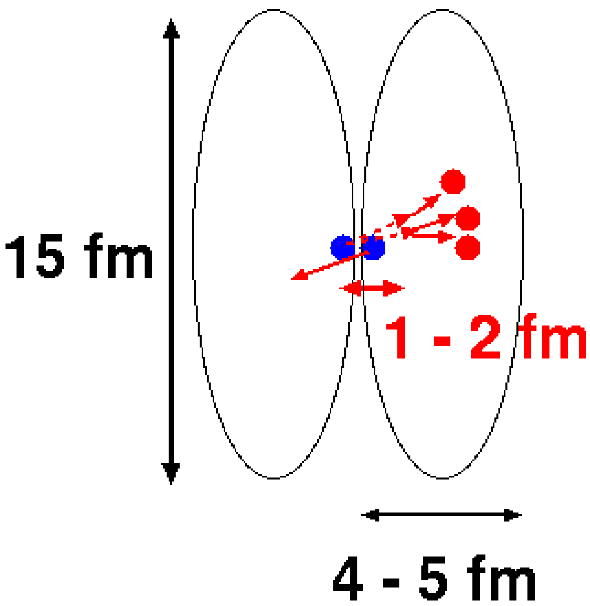
- AGS (11 A GeV), JHF (25 A GeV)
  - Smooth Evolution in (ρ, T)
  - $\rho_{max} > 2\gamma\rho_0$
- SPS (200 A GeV), RHIC
  - Sudden Jump in (ρ, T)
  - $\rho_{max} < 2\gamma\rho_0$

(JAM Calc., Y. Nara,FRONP99, 8/2-4, 1999 at JAERI)

Hadron Formation Time

JHF Energies

$\gamma_{cm} \simeq 3.5, \quad \tau \simeq 0.5 - 1$ fm/c

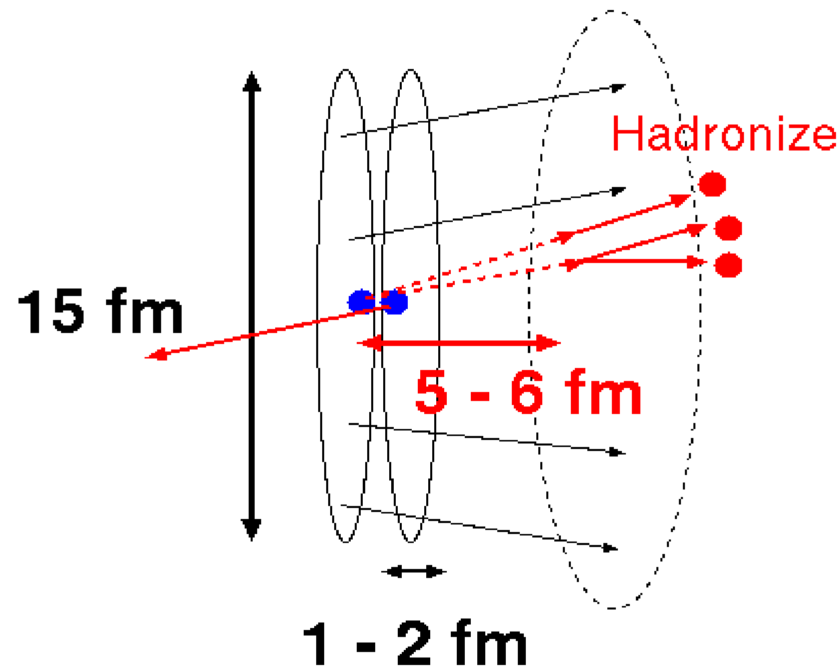


Multiple Hadron-Hadron Collisions

☞ (Approx.) Thermalized Hadron Gas

SPS Energies

$\gamma_{cm} \simeq 10, \quad \tau \simeq 0.5 - 1$ fm/c



String-String, String-Hadron Int.

+ Int. within Co-Movers

It takes τ 1 fm for hadrons to be formed (and thus to interact)

→ Pre-Hadronic Interactions are necessary at SPS & RHIC

→ Hot & Dense Hadronic Matter would be formed at AGS & JHF