Phases and Equation of State in Nuclear and Hadronic Matter

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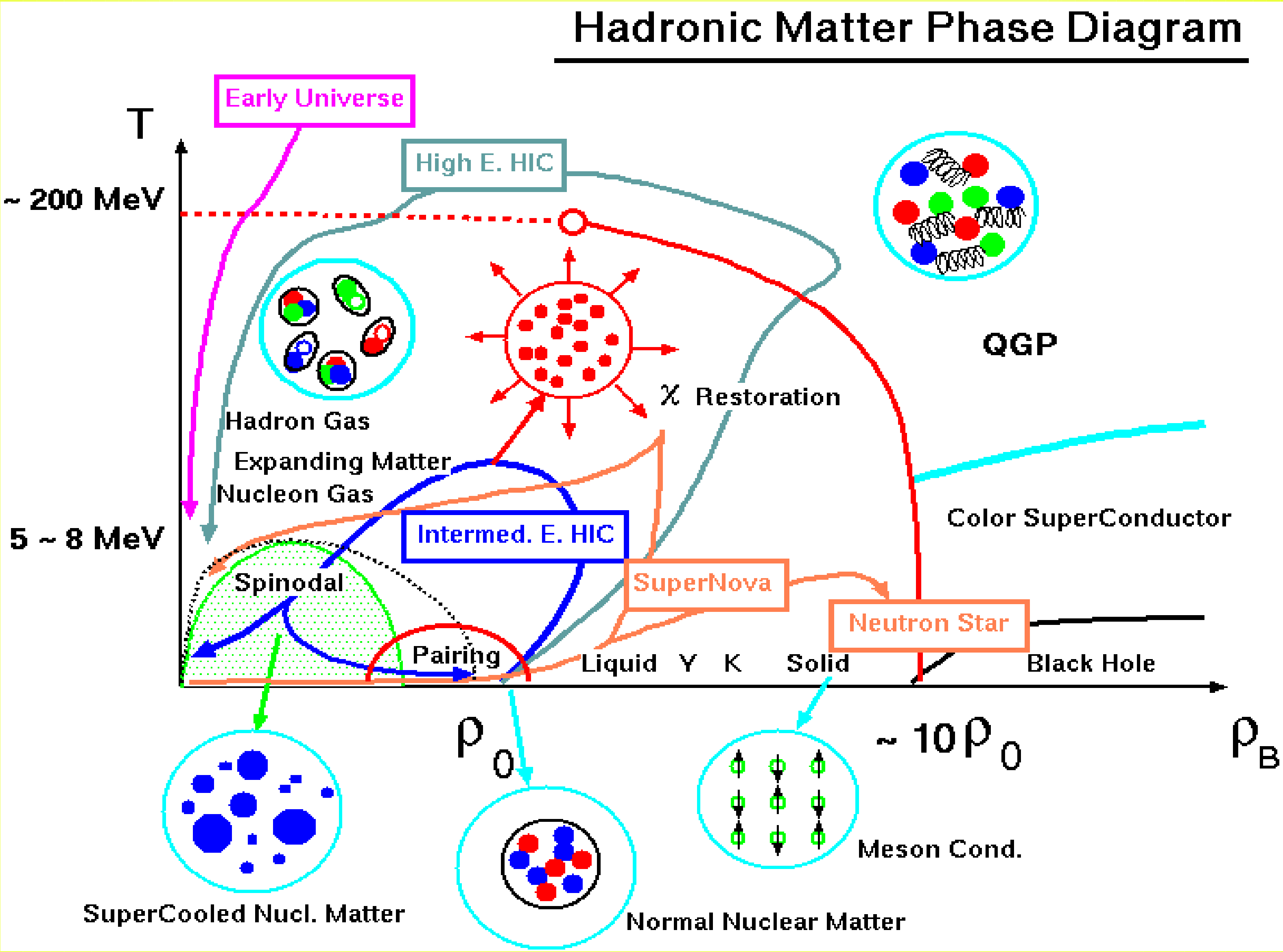
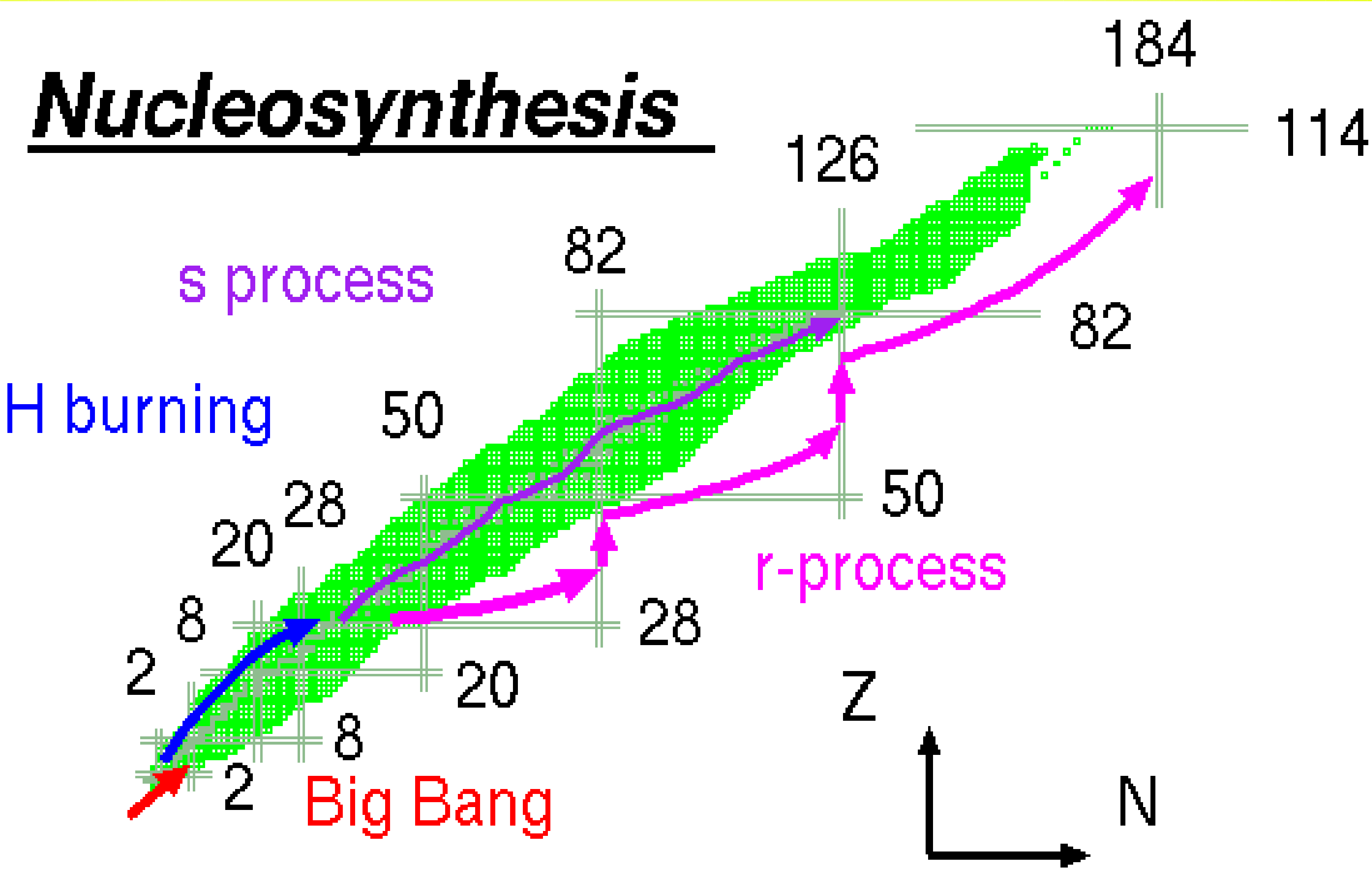
1. Hadronic Matter Equation of State,
   and Its Relation to Supernova Explosion
2. Low-Density & Low-Temperature Region: Fragmentation
   Expanding Supernova Matter and Nucleosynthesis
3. High-Density and/or High-Density Region: Heavy-Ion Collisions
   Compressed Supernova Matter
   Supernova Remnant (Neutron or Quark Star)
4. Cold-Dense Matter: Strangeness Nuclear Physics
5. Summary

Hadronic Matter Phase Diagram



Nucleosynthesis



Nuclear Physics in Supernova

- **Nuclear Reaction Rate**
- **Mass, Life-time, Excited Levels of Unstable (esp. n-rich) Nuclei**

r-process path and element abundance

Physics of Nuclear/Hadronic Matter

- **Nuclear Matter Equation of State → Hydrodynamical Evolution**

$\rho_B = (10^{-9} - 5)\rho_0 \qquad (10^5 - 10^{15}\,\mathrm{g/cc})$

$T = (0.1 - 30)\,\mathrm{MeV} \qquad (10^9 - 3\times 10^{11}\,\mathrm{K})$

- **Particle/Fragment Composition → Various Reaction Rates**

$Y_p, \; Y_L, \; Y_\alpha, \; Y_S, \; Y(^{56}\mathrm{Fe}), ...$

- **Neutrino Interaction on Nucleon and Nuclei**

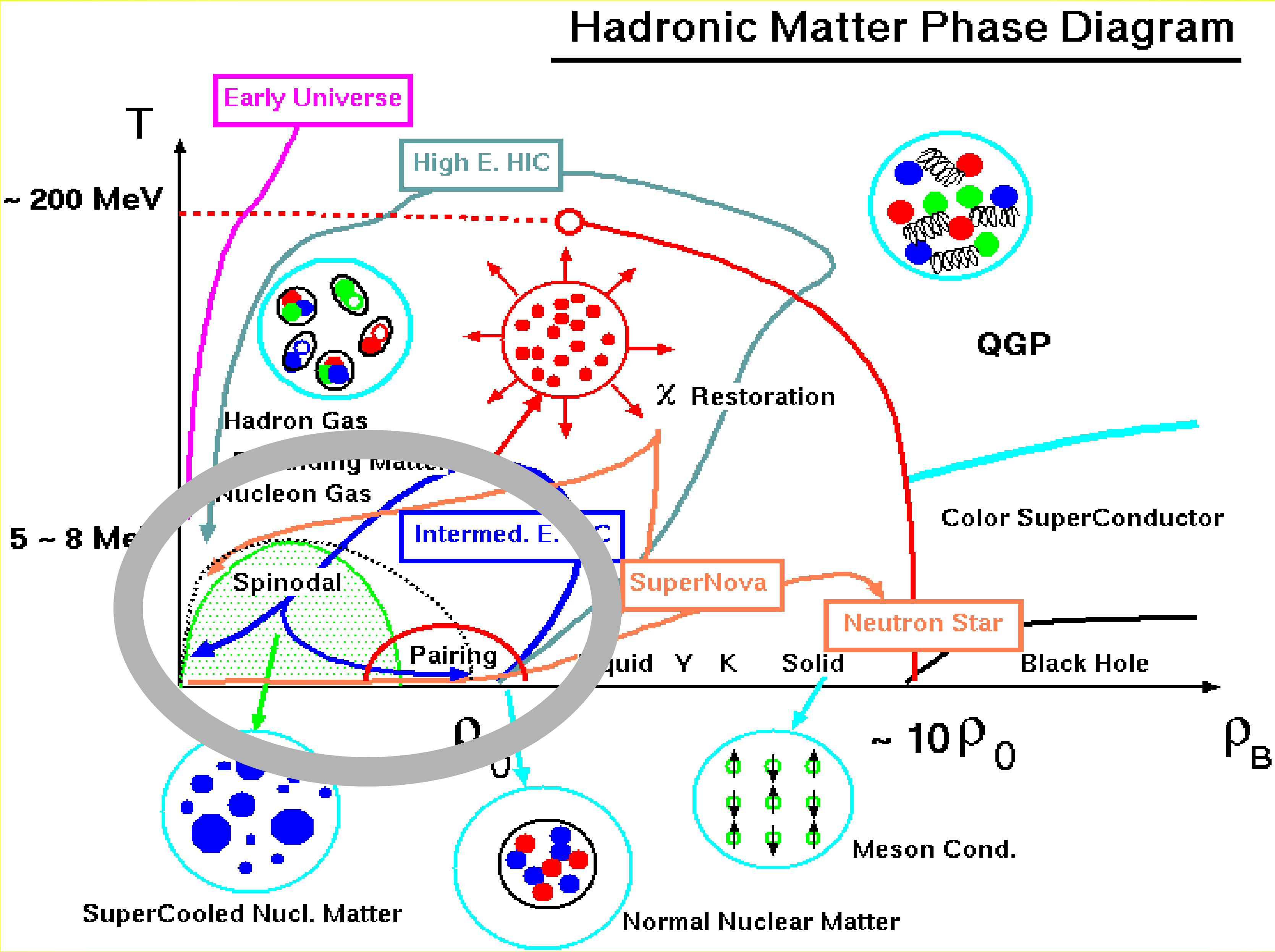
→ Initial Electron Density and Later Opacity

$e + A \to \nu + B, \;\; \nu + A \to e + B, \;\; \nu + A \to \nu' + N + B, \;\; ...$

(Physics at K2K Near Detector !)

Low Density Supernova Matter and Fragment Formation

EOS and Composition at Low Density

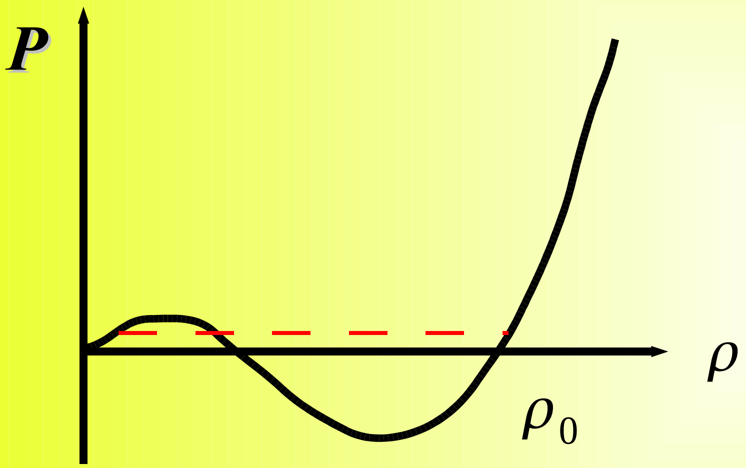


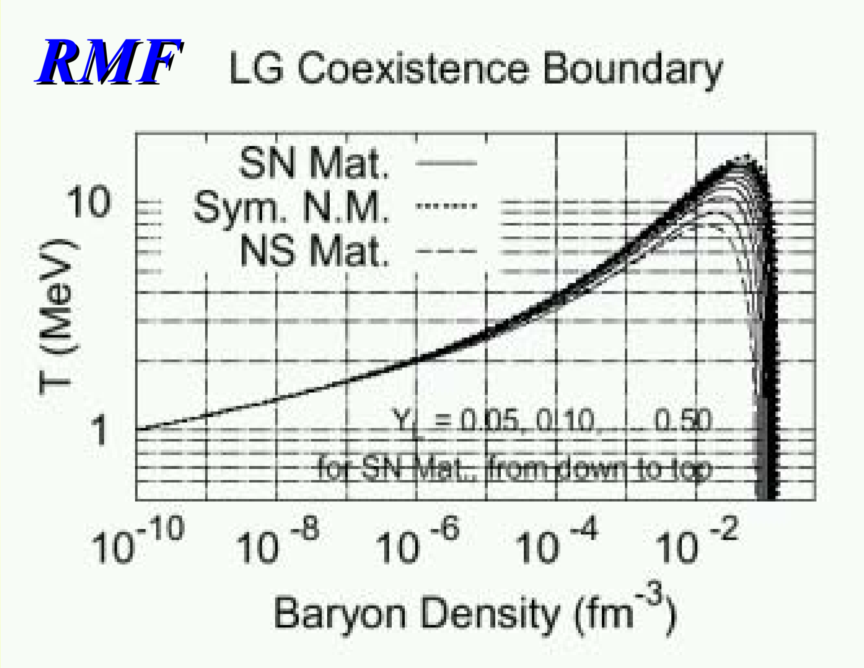
Contents

Nuclear Liquid-Gas Phase Transition

Nuclear Int. Van der Waals Int.

→ *LG Phase Transition is expected.*



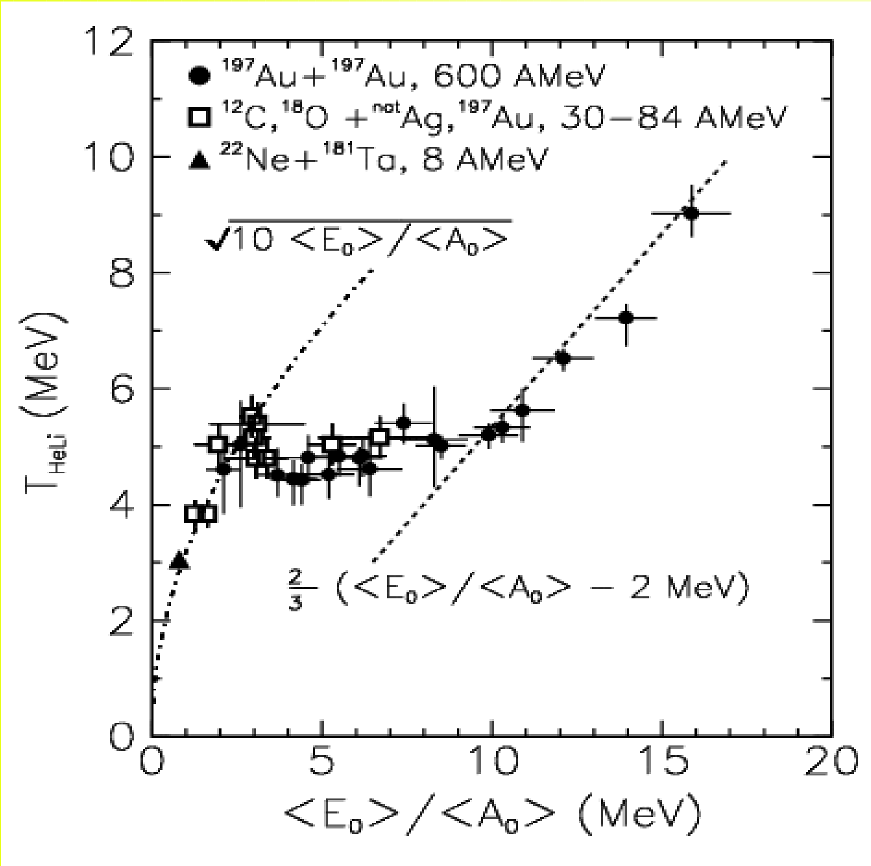


Recent Experimental Progress

Two indep. exp. on two indep. Observables show the Existence of First Order L.-G. Phase Transition.

Nuclear Caloric Curve

J. Pochadzalla et al., Phys. Rev. Lett. 75 (1995) 1040.
(GSI-ALLADIN collab.)



Boiling Temperature is Clearly Seen

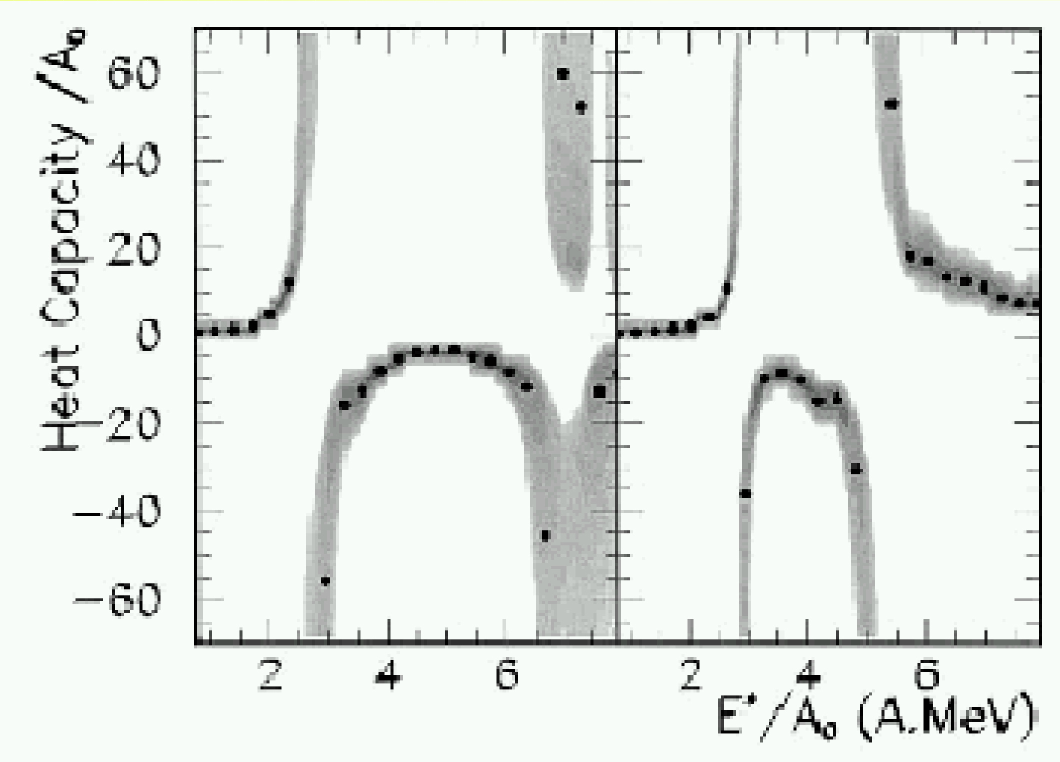
Fragment Yields are assumed to follow Equilibrium Statistics

$$Y_f \propto g_f \exp((B_f + Z\mu_p + N\mu_n)/T)$$

$$\rightarrow \frac{Y(^4He)/Y(^3He)}{Y(^7Li)/Y(^6Li)} \propto \exp(\Delta B/T)$$

Negative Heat Capacity

M. D Agostino et al., PLB 473 (2000) 219.
(MSU Exp./INFN-IN2P3 Collab.)



Negative Heat Capacity
→ First Order

T **and** E^* **are determined from *Fragment Multiplicity* and *Kinetic Energy* based on Theoretical Model**

What has been Understood ?

- **LG Phase Transition is of First Order (Exp.).**
- **It can be understood in Microscopic MD qualitatively, e.g. Fragment Yield.**
- **At around T_{Boil},**

  Statistical Ensemble of Various Fragment Configurations

  (e.g. Power-law like behavior in Mass Distribution)

  is important rather than

  One Dominant Fragment Configuration

  (Standard Treatment in Supernova Matter (e.g. Lattimer et al.)).

What Happens in Supernova Matter ?

Fragment Distribution in Supernova Matter

C. Ishizuka, AO, K. Sumiyoshi,
Prog. Theor. Phys. Suppl. (2002) (Proc. of YKIS 2001)
Proc. of PostYK01
nucl-th/0208020 (submitted).

Investigation of the properties of
Liquid-Gas phase transition of supernova matter
and its influences
on supernova physics by using
Relativistic Mean Field
an ***Fragment-based Statistical Model (NSE)***

Relativistic Mean Field

TM1 parameter set (Sugahara and Toki, Nucl. Phys. A579 (1994), 557.)

- **Fit B.E. of Stable as well as Unstable (n-rich) Nuclei**
- **Has been successfully applied to Supernova Explosion**
- **Three Mesons (σ,ω,ρ) are included**
- **Meson Self-Energy Term (σ,ω)**

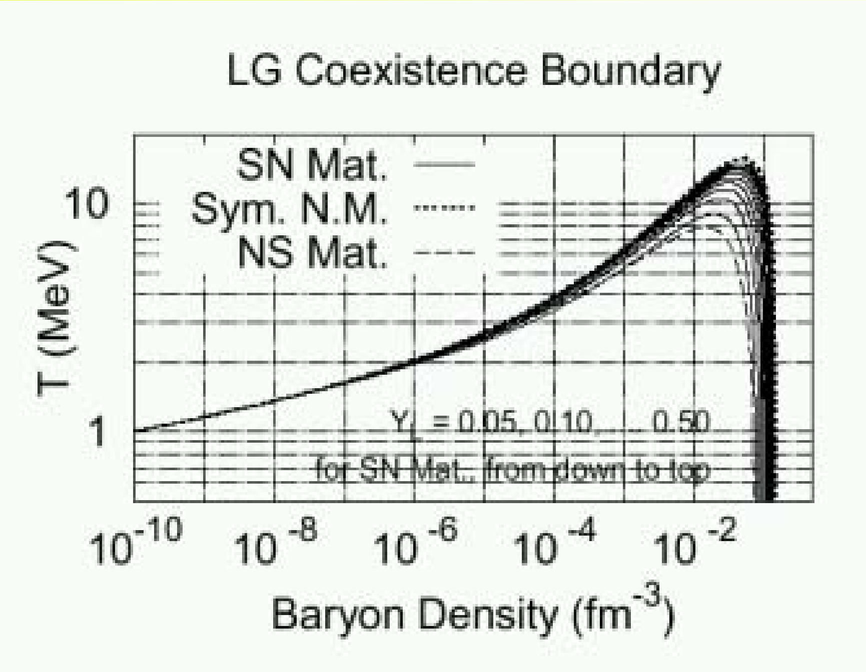
Lagrangian

$$
\begin{aligned}
\mathcal{L} =& \bar{\psi}_N \left(i\partial\!\!\!/ - M - g_\sigma \sigma - g_\omega \omega\!\!\!/ - g_\rho \tau^a \rho\!\!\!/^a \right) \psi_N \\
&+ \frac{1}{2}\partial^\mu \sigma \partial_\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4 \\
&- \frac{1}{4} W^{\mu\nu} W_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega^\mu \omega_\mu - \frac{1}{4} R^{a\mu\nu} R^a_{\mu\nu} + \frac{1}{2} m_\rho^2 \rho^{a\mu} \rho^a_\mu + \frac{1}{4} c_3 \left(\omega_\mu \omega^\mu \right)^2 \\
&+ \bar{\psi}_e \left(i\partial\!\!\!/ - m_e \right) \psi_e + \bar{\psi}_\nu i\partial\!\!\!/ \psi_\nu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \ , \\
W_{\mu\nu} =& \partial_\mu \omega_\nu - \partial_\nu \omega_\mu \ , \\
R^a_{\mu\nu} =& \partial_\mu \rho^a_\nu - \partial_\nu \rho^a_\mu + g_\rho \epsilon^{abc} \rho^{b\mu} \rho^{c\nu} \ , \\
F_{\mu\nu} =& \partial_\mu A_\nu - \partial_\nu A_\mu \ . \qquad (2)
\end{aligned}
$$

Two-Phase Coexistence in RMF

Liquid-Gas Coexistence Condition = Minimizing Free Energy

$$(1-\alpha)\,\rho_k^{Liq.} + \alpha\rho_k^{Gas} = \rho_k \;, \quad \mu_k^{Liq.} = \mu_k^{Gas} \;, \quad P^{Liq.} = P^{Gas} \;,$$



Critical Temperature

$T_c \approx 16\,\mathrm{MeV}$

(around B.E. / nucleon in Matter)

Large Coexisting Region

$T_{Boil} \geq 1\,\mathrm{MeV}$ for $\rho_B \geq 10^{-10}\,\mathrm{fm}^{-3}$

Fragment-based Statistical Model (NSE)

Statistical Equilibrium of Constituents

$$\Omega = -PV = -VT\sum_i \rho_f - P_\ell V - P_\gamma V \ ,$$

$$\rho_f = \zeta_f(T)\left(\frac{M_f T}{2\pi\hbar^2}\right)^{3/2} \exp\left(\frac{B_f+\mu_f}{T}\right) \ ,$$

$$\mu_f = Z_f(\mu_p - m_N) + N_f(\mu_n - m_N) \ ,$$

Constituents = Lepton, gamma, Nucleon, and *NUCLEI*

Mass Table: MS 1995 (9000 Nuclei)

Nuclear Level Density

$$\zeta_f(T) = \sum_i g_f^{(i)} \exp\left(-E_f^{*(i)}/T\right)$$

$$\simeq g_f^{(g.s.)} + \frac{c_1}{A_f^{5/3}} \int_0^\infty dE^* e^{-E^*/T} \exp(2\sqrt{a_f E^*}) \ , \qquad (9)$$

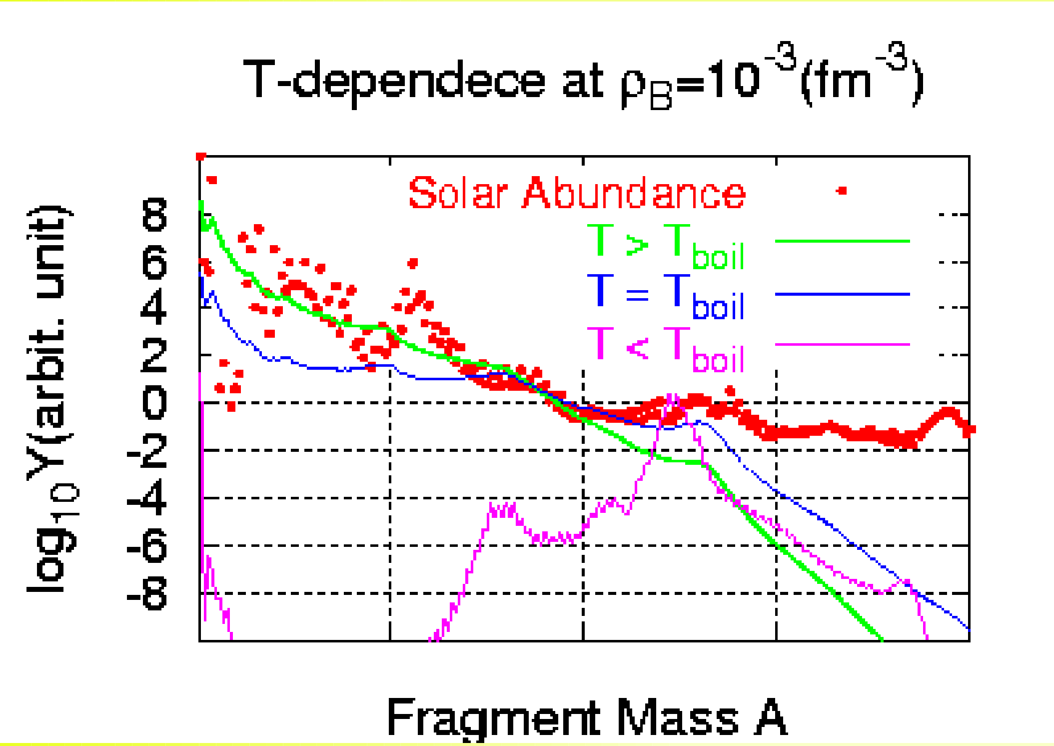
$$a_f = \frac{A_f}{8}\left(1 - c_2 A_f^{-1/3}\right)\ (\mathrm{MeV}^{-1})\ , \quad c_1 = 0.2\ (\mathrm{MeV}^{-1})\ , \quad c_2 = 0.8\ ,$$

Mass Modification from Electron Screening

$$B_f(\rho_e) = B_f(0) - \Delta V_f^{Coul}(\rho_e)\ ,$$

$$\Delta V_f^{Coul} = -\frac{3}{5}\frac{Z_f^2 e^2}{R_0}\left(\frac{3}{2}\eta_f - \frac{1}{2}\eta_f^3\right)\ , \quad \eta_f \equiv \frac{R_{0f}}{R_{ef}} = \left(\frac{\rho_e}{Z_f\rho_0/A_f}\right)^{1/3}\ ,$$

T-dependence of Mass Distribution in NSE



$T > T_{boil}$

Boltzmann Dist. $(e^{-\mu A/T})$

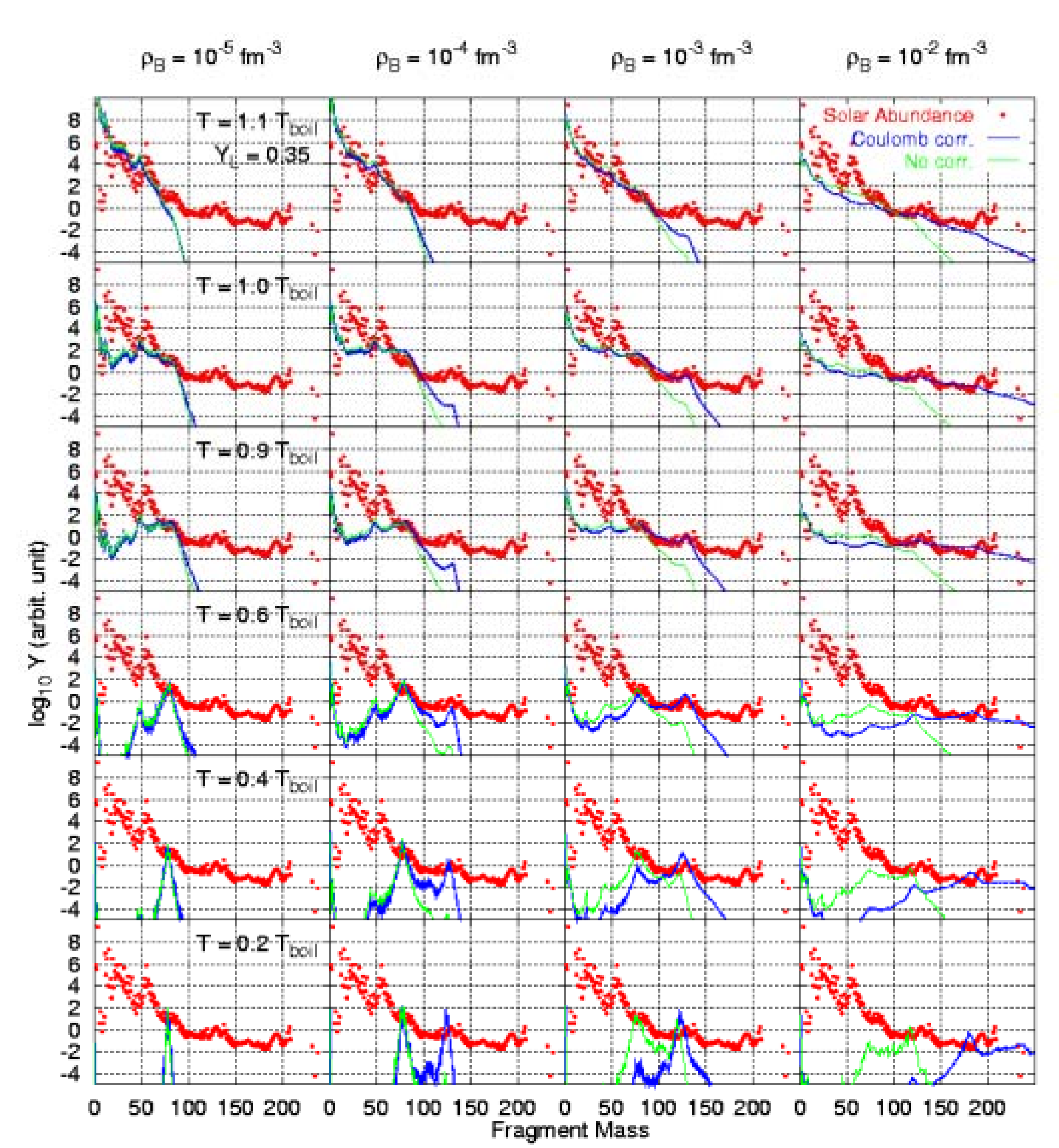
$T \approx T_{boil}$

Power-law-like $(A^{-\tau})$

$T < T_{boil}$

One Dominant Nuclei

- **Heavy Elements are most effectively formed at around** $T \approx T_{boil}$
- **Even at** $T < T_{boil}$ **, Various configurations would appear with some probability.**

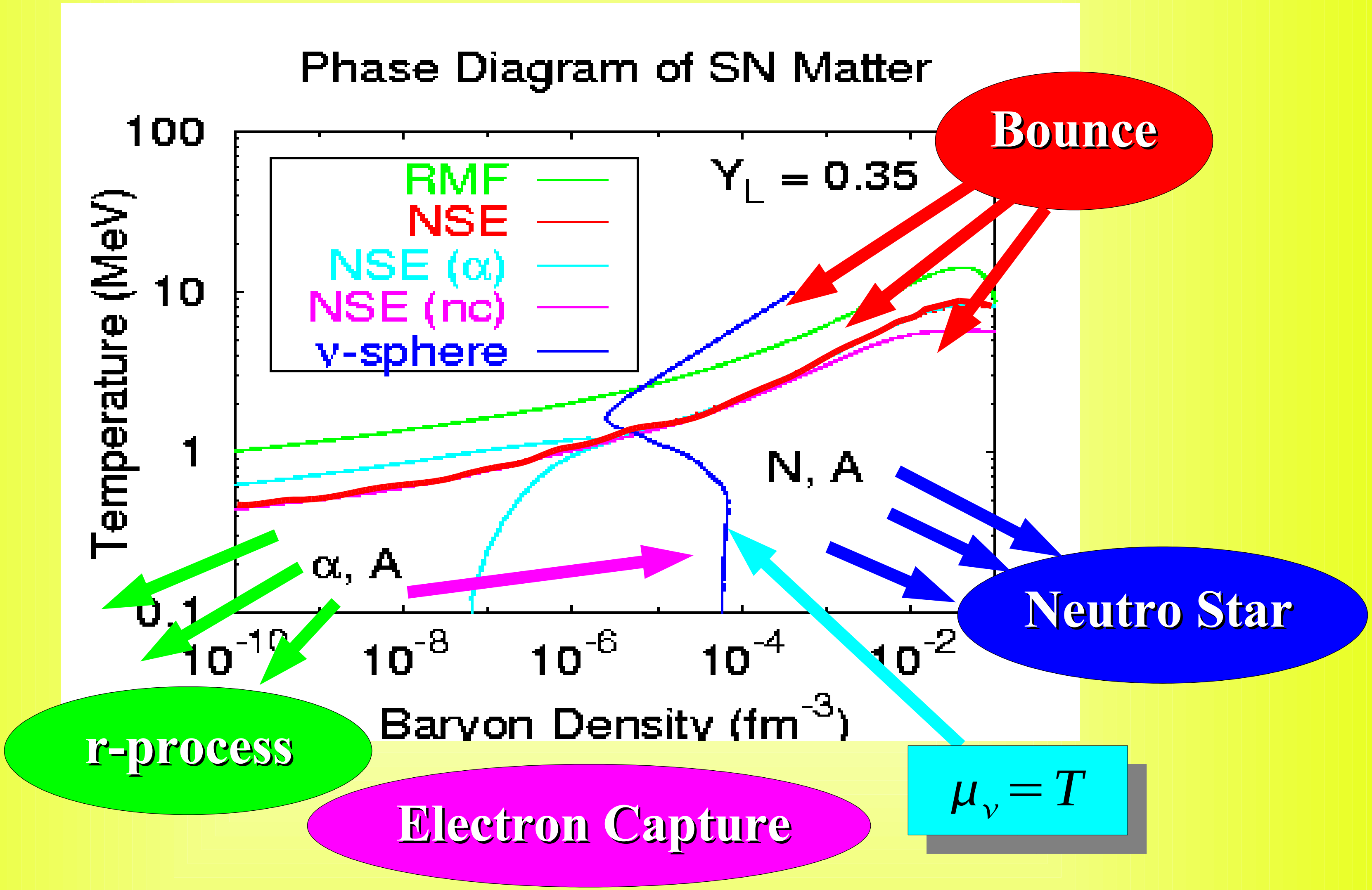


Inside of the ν-Sphere

$Y_L = 0.35$

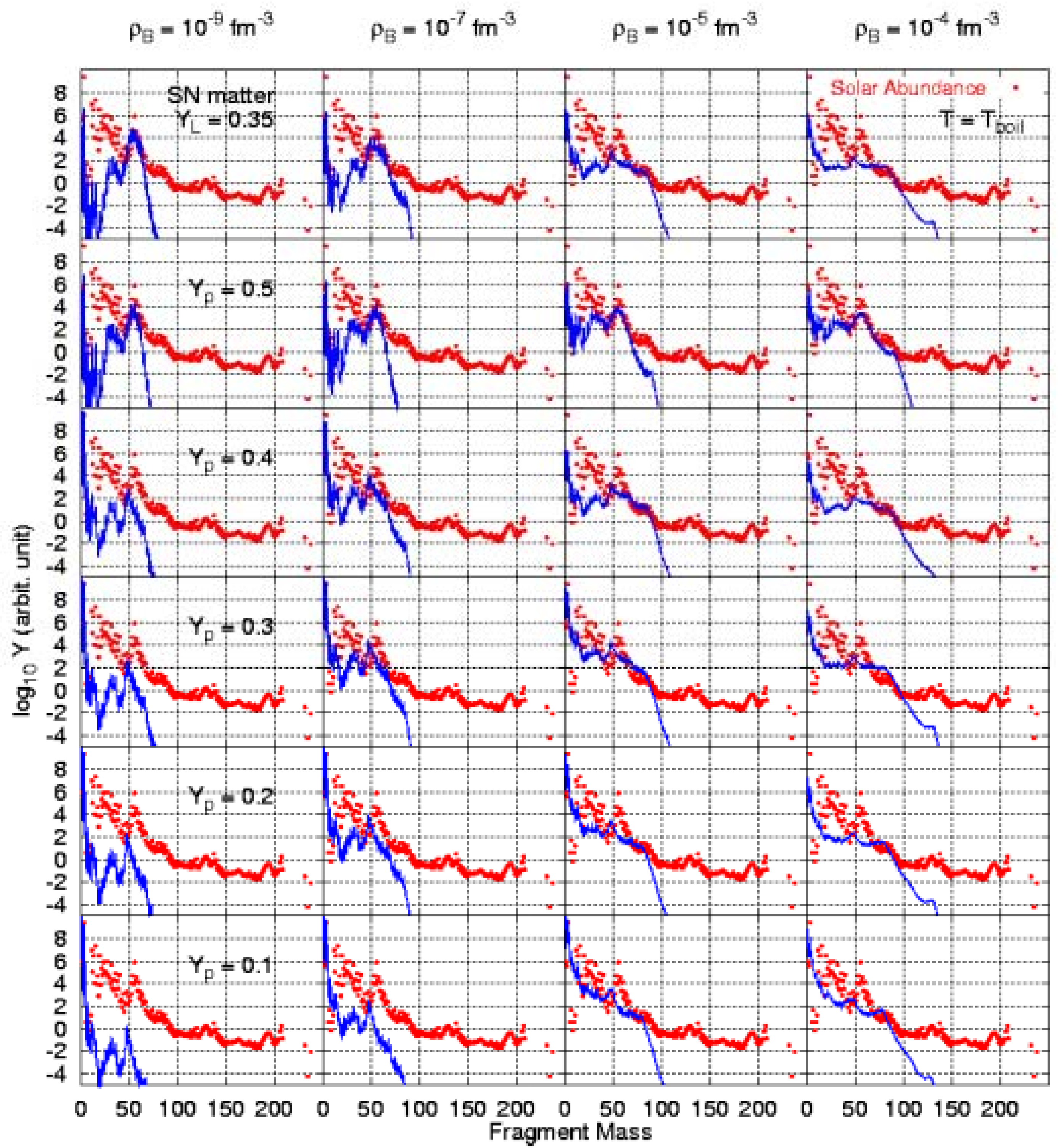
$\rho_B(\mathrm{fm}^{-3})$	$T_{boil}(\mathrm{MeV})$
10^{-5}	**1.44**
10^{-4}	**2.20**
10^{-3}	**3.89**
10^{-2}	**6.89**

Phase Diagram in RMF and NSE



Effect of Finite Nuclei: Reduction of T_c, Existence of αA region

Outside of the ν-Sphere

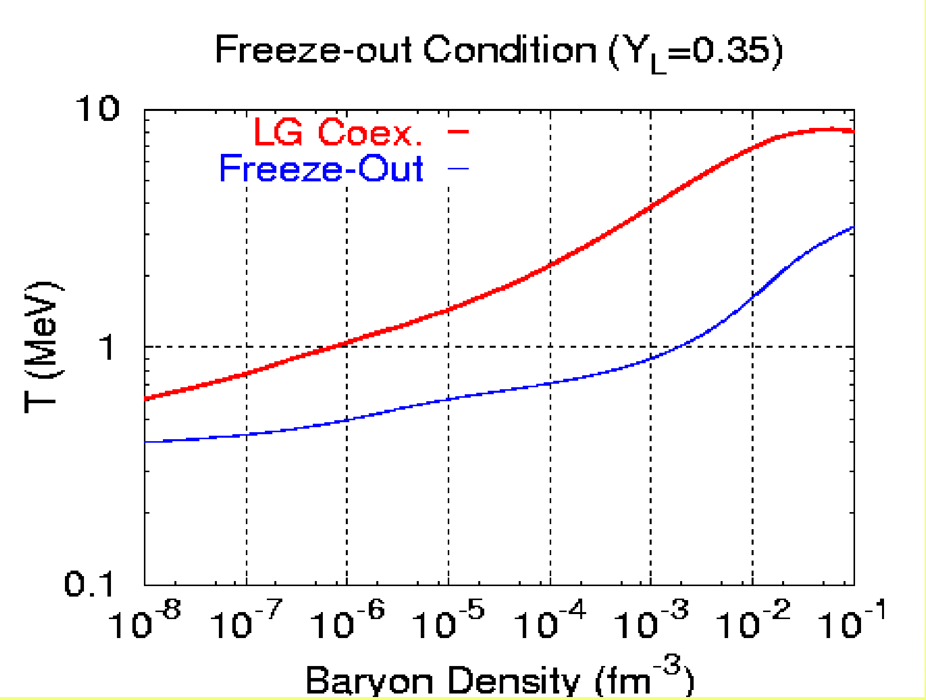


Y_p	ρ_B (fm^{-3})	T_{boil} (MeV)
0.1	10^{-9}	0.60
	10^{-7}	0.86
	10^{-5}	1.47
	10^{-4}	2.20
0.2	10^{-9}	0.60
	10^{-7}	0.86
	10^{-5}	1.49
	10^{-4}	2.25
0.3	10^{-9}	0.60
	10^{-7}	0.86
	10^{-5}	1.49
	10^{-4}	2.23
0.4	10^{-9}	0.60
	10^{-7}	0.94
	10^{-5}	1.40
	10^{-4}	2.10
0.5	10^{-9}	0.52
	10^{-7}	0.73
	10^{-5}	1.16
	10^{-4}	1.68

Where can we assume Equilibrium ?

Charged Particle Nuclear Reactions

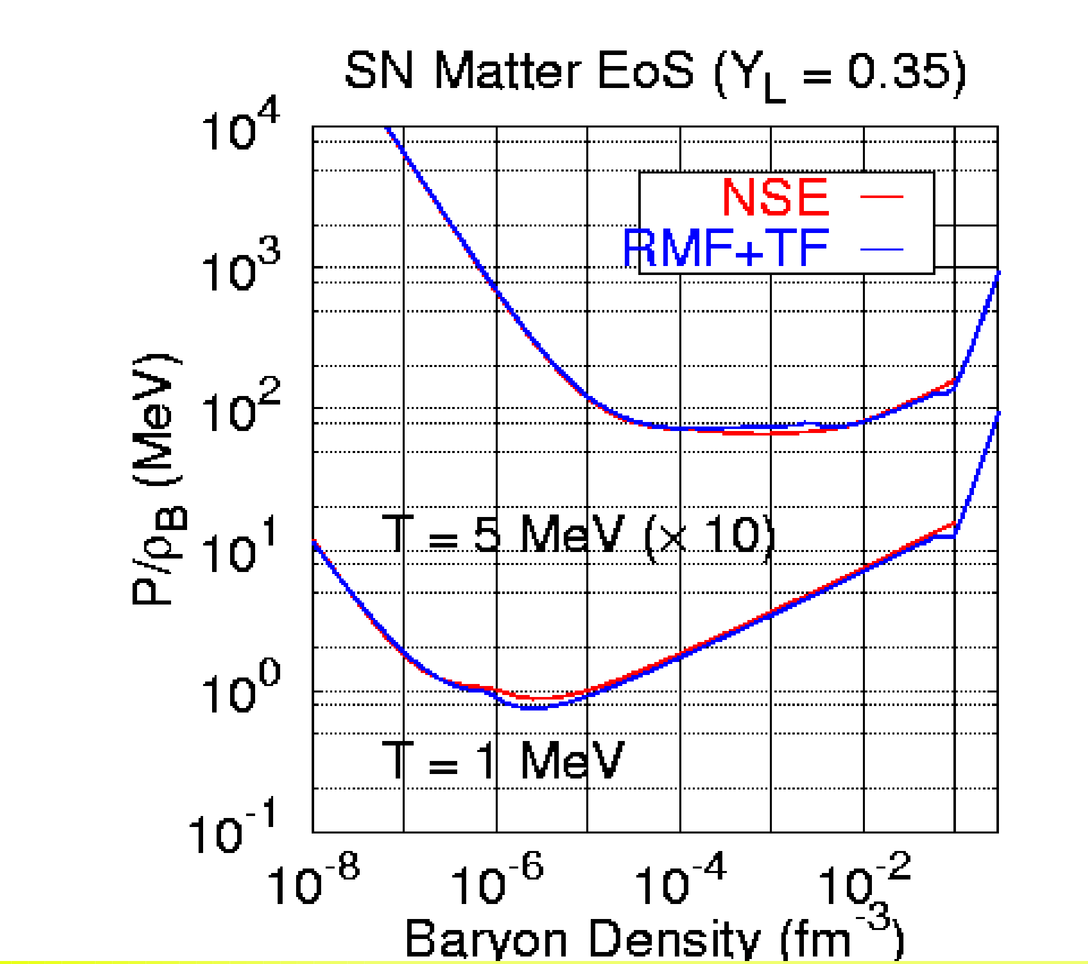
- **Reaction Rate: MOST**
  (Audi & Wapstra, Statistical Model based on HFB-2)
- **Time Scale: Free Fall Time**
- **Nuclear Population: NSE**



Charge Equilibrium is kept for $T \geq 0.4\,\mathrm{MeV}$

Comparison of EOS in RMF and NSE

RMF + TF: Shen, Sumiyoshi, Oyamatsu, Toki



Two Models give similar EOS except for

phase boundary region

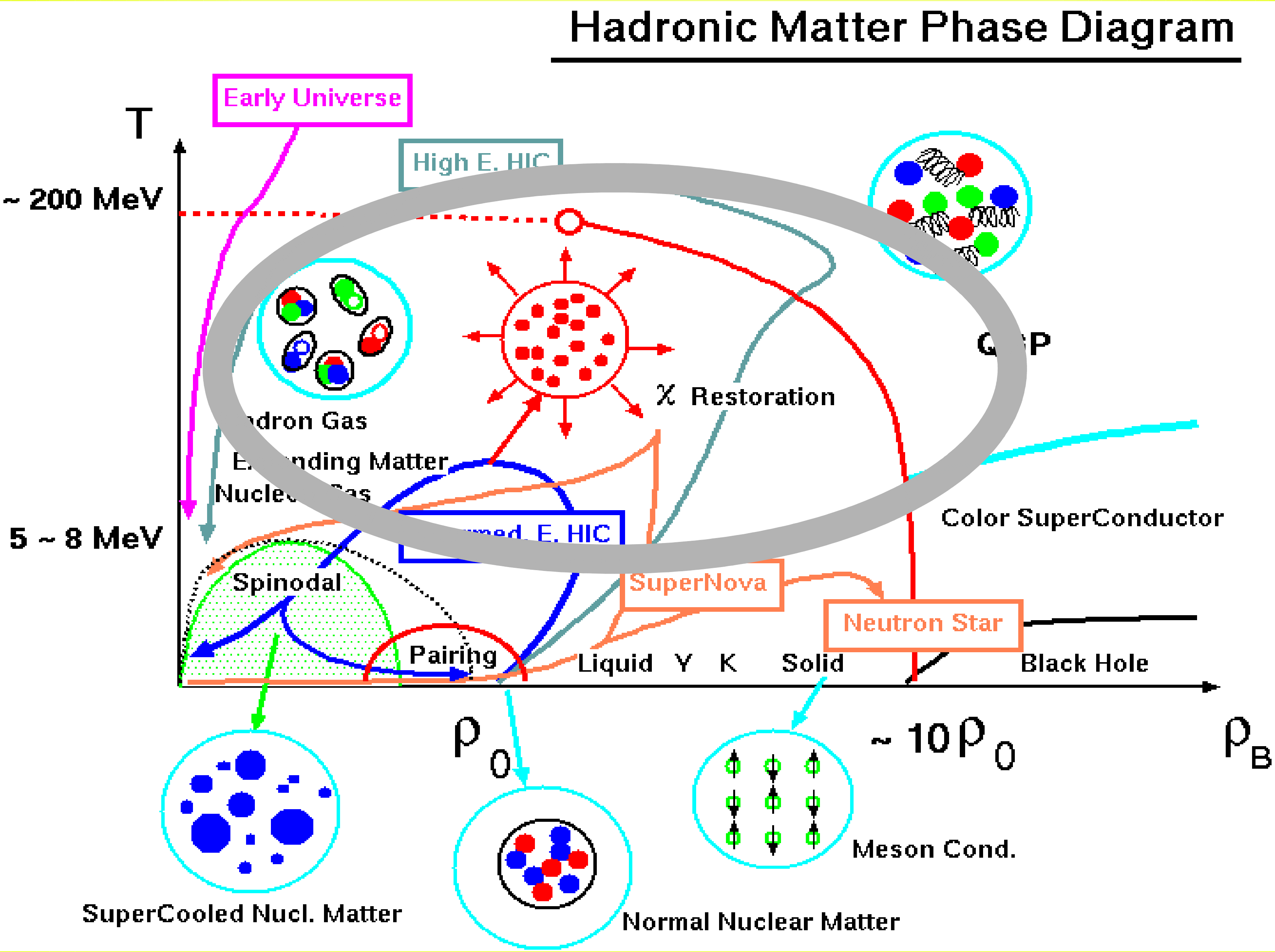
Summary

- Liquid-Gas Phase Transition of Supernova Matter may be important in Supernova Exlosion
  - *Statistical Distribution of Various Fragment Configurations*
    → Reduction of Boiling T, Modification of EOS, ...
  - *Larger Mass Elements can be formed*
    → First Peak of r-process, "Seed" Nuclei of r-process ?
- Support the "Standard" Picture
  - *Freeze out of charged particle reaction T ~ 0.5 MeV*
  - *Quick Nuclear Formation between T = 1 – 0.1 MeV*
  - *Justification of "One Configuration" EOS*
- Fragment Distribution would be very important in estimating "Electron Capture Rate"

  → Strong (Z,N) Dependence
  Small amount of "Effective" Nuclei can modify the bulk capture rate

Dense Baryonic Matter and High Energy Heavy-Ion Collisions

Nuclear Matter EOS at High Density

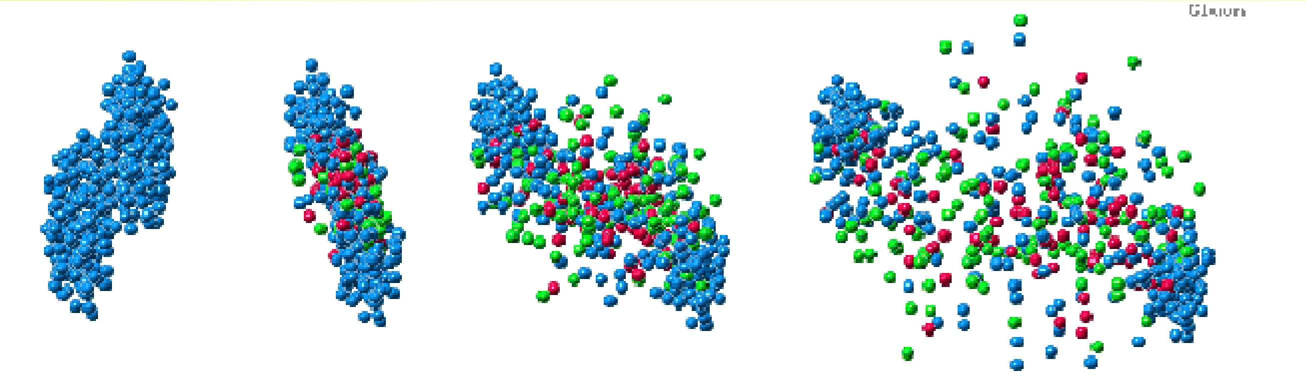


Contents

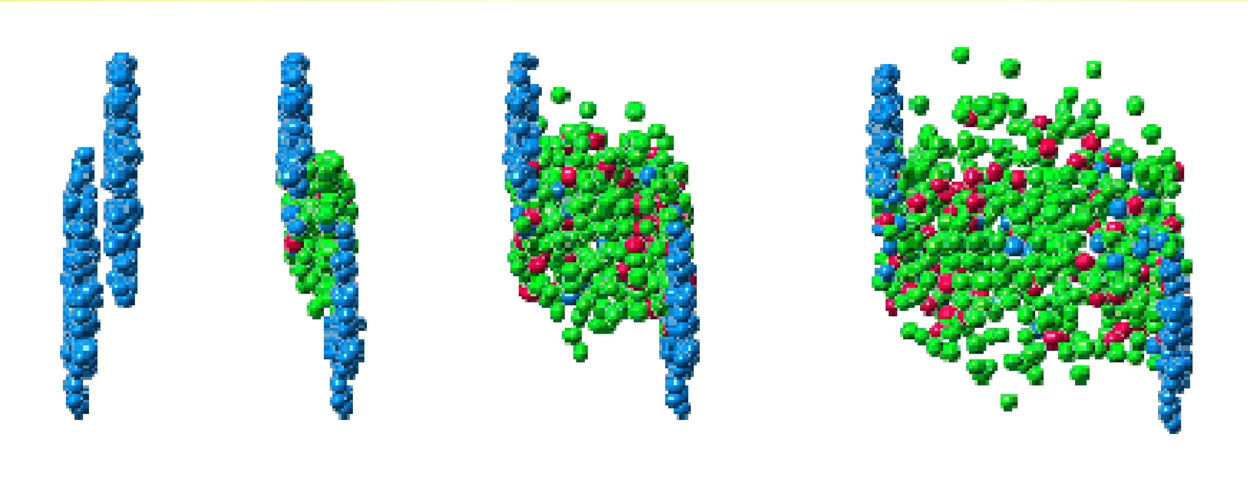
JAMming on the Web

http://nova.sci.hokudai.ac.jp/~ohtsuka/

AGS

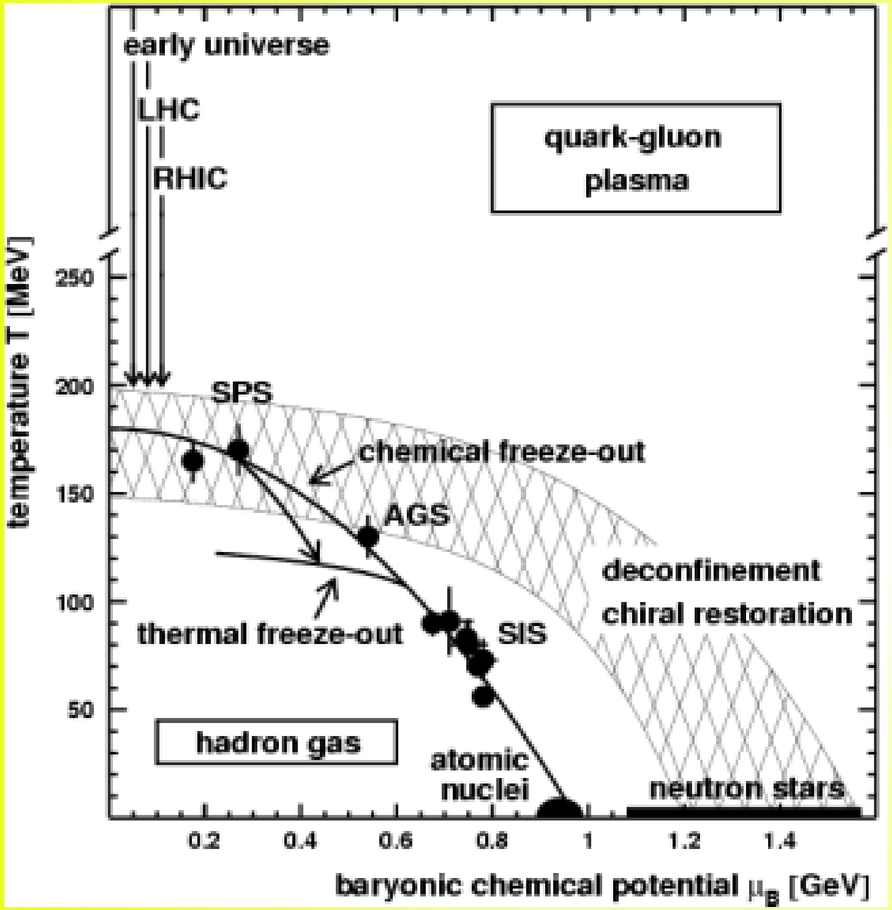


SPS

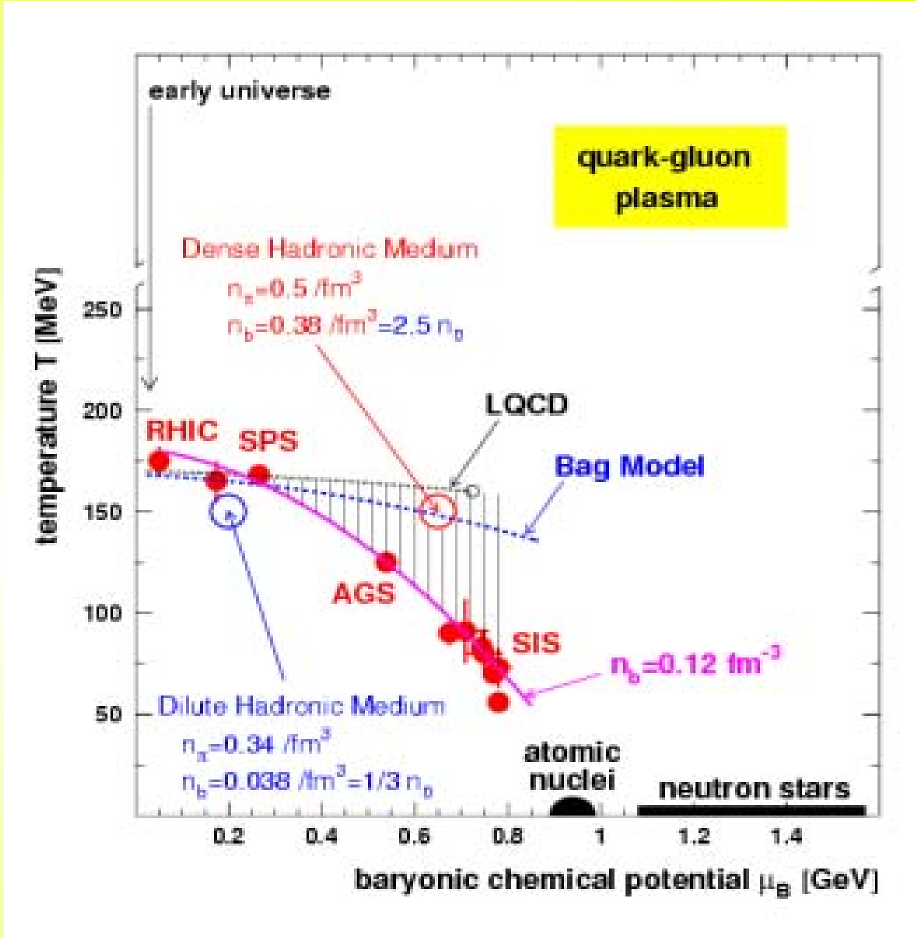


Experimentally Estimated Phase Diagram

Chemical Freeze-Out Points in High-Energy Heavy-Ion Collisions



1998 (J. Stachel et al.)

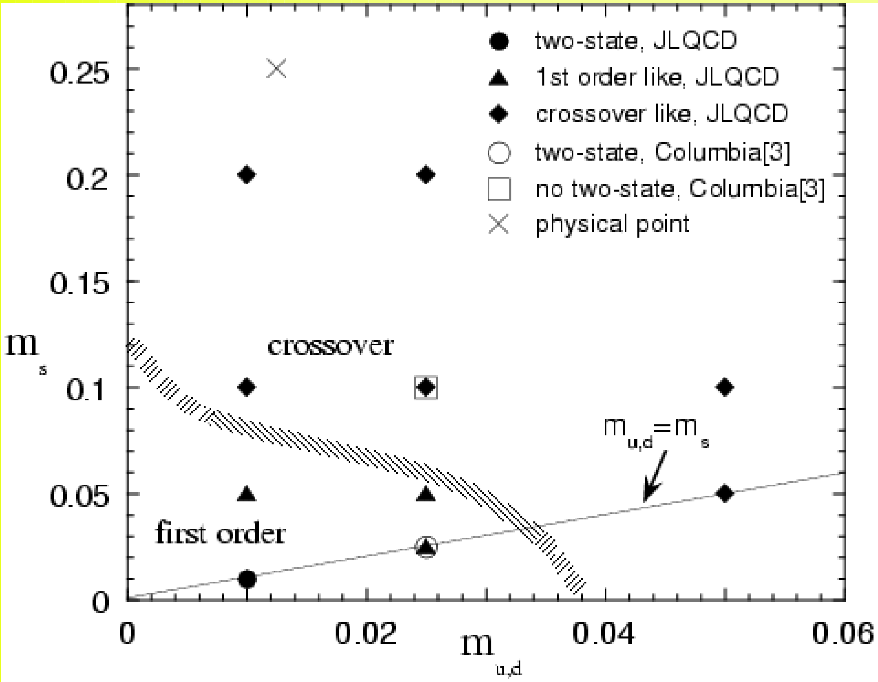


2002 (Braun-Munzinger et al. J. Phys. G28 (2002) 1971.)

Chem. Freeze-Out Points are very Close to Expected QCD Phase Transition Boundary

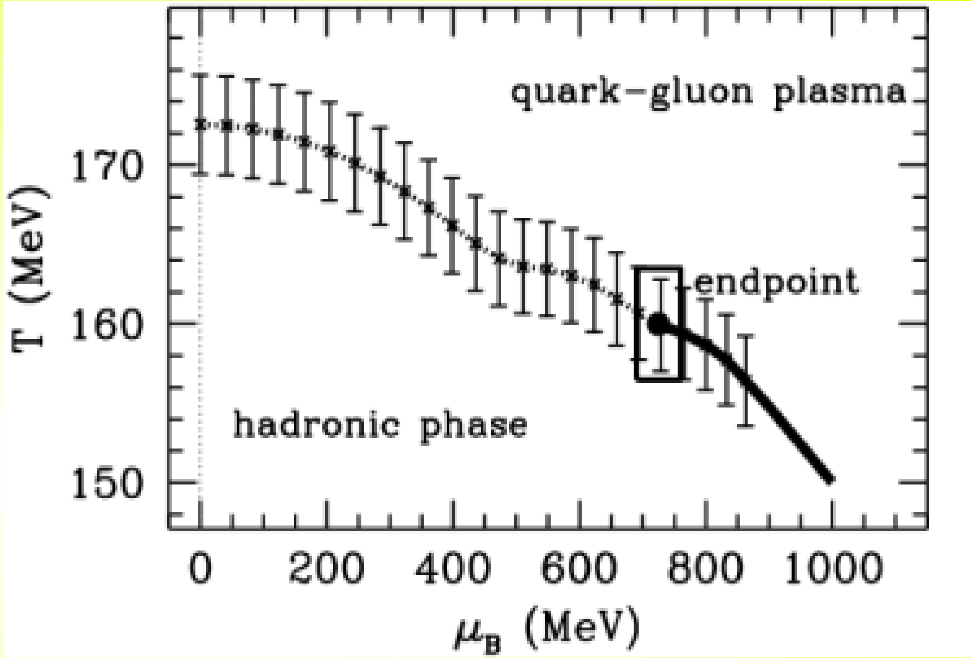
Theoretically Expected QCD Phase Diagram

Zero Chem. Pot.



JLQCD Collab. (S. Aoki et al.),
Nucl. Phys. Proc. Suppl. 73 (1999) 459.

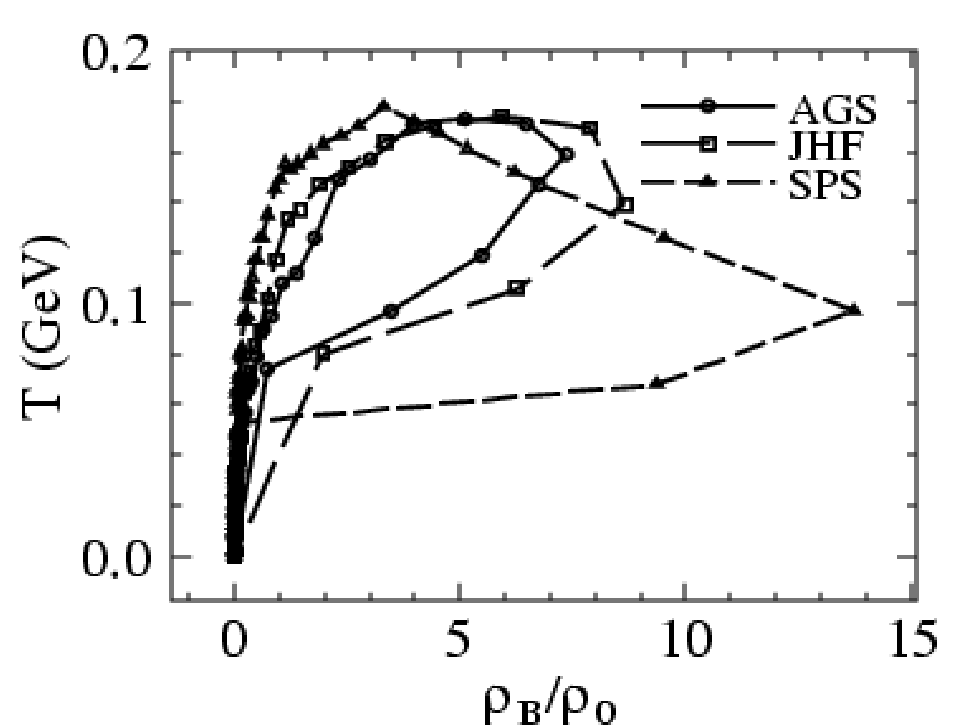
Finite Chem. Pot.



Finite μ: Fodor & Katz,
JHEP 0203 (2002), 014.

Zero Chem. Pot. : *Cross Over*
Finite Chem. Pot.: *Critical End Point*

Thermal Evolution from AGS to SPS Energies



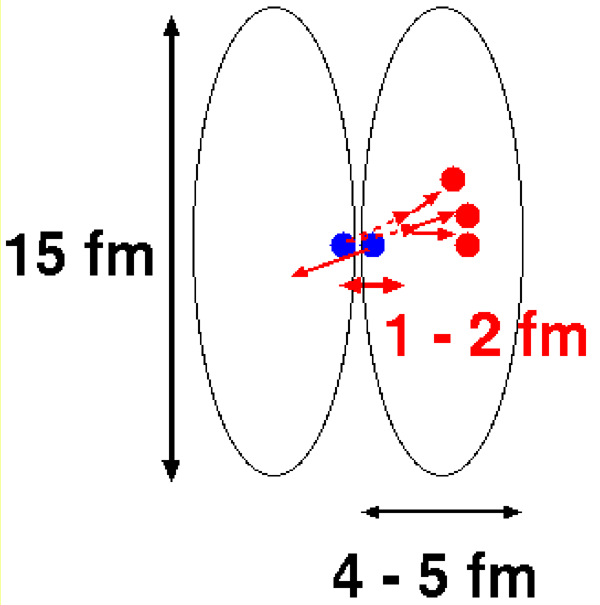
- **AGS (11 A GeV), JHF (25 A GeV)**
  - **Smooth Evolution in (ρ, T)**
  - $\rho_{max} > 2\gamma\rho_0$
- **SPS (200 A GeV), RHIC**
  - **Sudden Jump in (ρ, T)**
  - $\rho_{max} < 2\gamma\rho_0$

(JAM Calc., Y. Nara, FRONP99, 8/2-4, 1999 at JAERI)

Hadron Formation Time

JHF Energies

$\gamma_{cm} \simeq 3.5, \quad \tau \simeq$ 0.5 - 1 fm/c

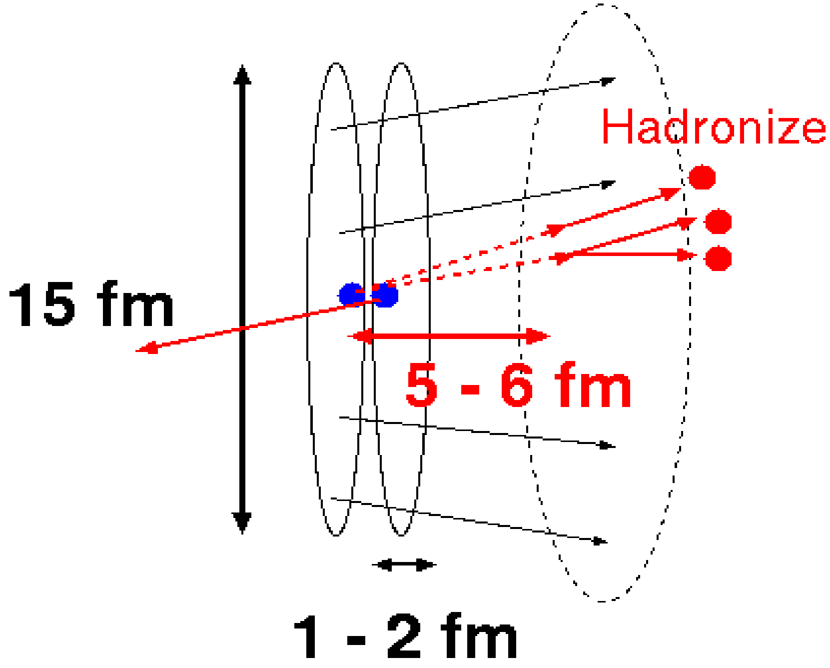


Multiple Hadron-Hadron Collisions

☞ (Approx.) Thermalized Hadron Gas

SPS Energies

$\gamma_{cm} \simeq 10, \quad \tau \simeq$ 0.5 - 1 fm/c



String-String, String-Hadron Int.
+ Int. within Co-Movers

It takes $\tau \sim$ 1 fm for hadrons to be formed (and thus to interact)

→ *Pre-Hadronic* Interactions are necessary at SPS & RHIC

→ *Hot & Dense Hadronic* Matter would be formed at AGS & JHF

Is QGP Formed at AGS, SPS and/or RHIC ?

Proposed and/or Measured Signals

- **High-Mass Lepton Pair (Yes @ SPS, Preliminary @ RHIC)**
  J/Ψ Suppresion at High Temperature
- **Jet Energy Loss (@ RHIC)**
  Parton Dynamics at High (Freed) Gluon Density

High Baryon Density !

- **Collective Flow (AGS, SPS, RHIC)**
  EOS modification / Thermalization Degree
- **Low-Mass Lepton Pair (Yes @ SPS, Not Yet @ RHIC)**
  Partial Restoration at High Temperature/De nsity
- **Strangeness Enhancement (Yes @ AGS, Lower E. SPS, No @ RHIC)**
  Rescattering or Potential at High Density or QGP

Later on, I mainly Discuss *Collective Flows*

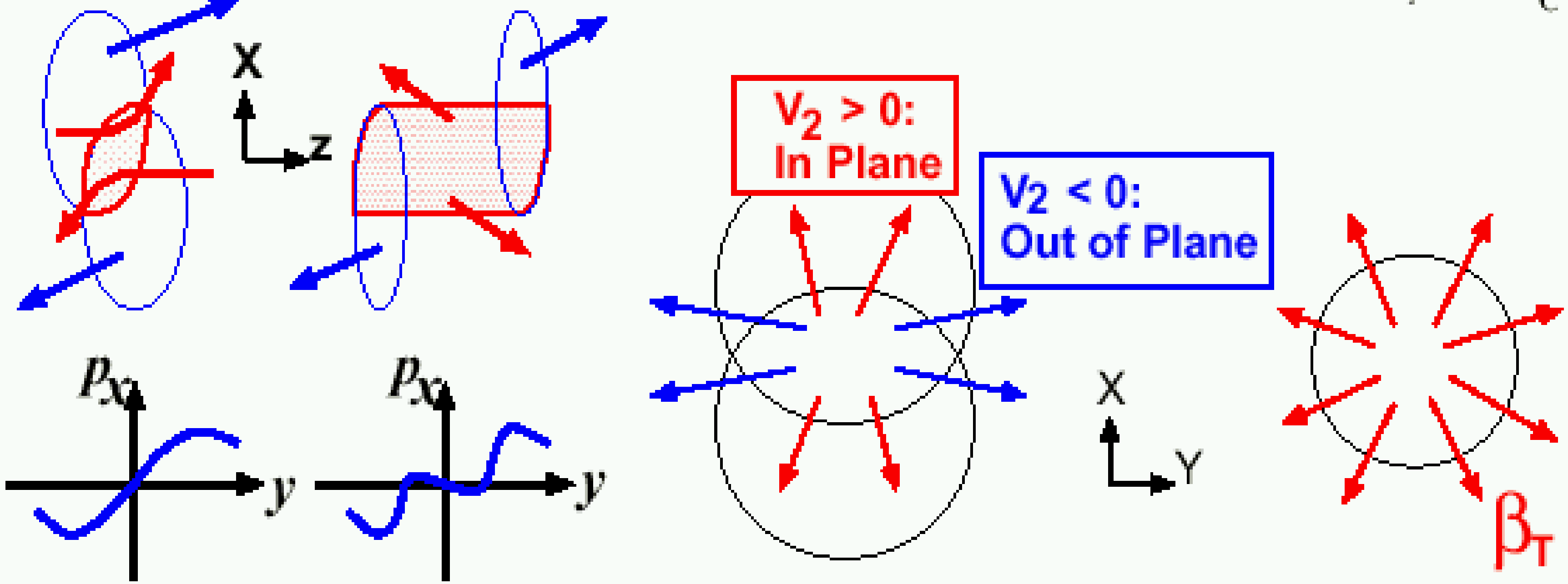
What is Collective Flow ?

(Directed) Flow (dP_X/dY)	Elliptic Flow (V_2)	Radial Flow (β_T)
Stiffness (Low E) + Time Scale (High E)	Thermalization & Pressure Gradient	Pressure History

$$\epsilon \frac{D\mathbf{V}}{Dt} = -\nabla P$$

$$\rightarrow V = \int_{path} \frac{-\nabla P dt}{\epsilon}$$

Until AGS Above SPS



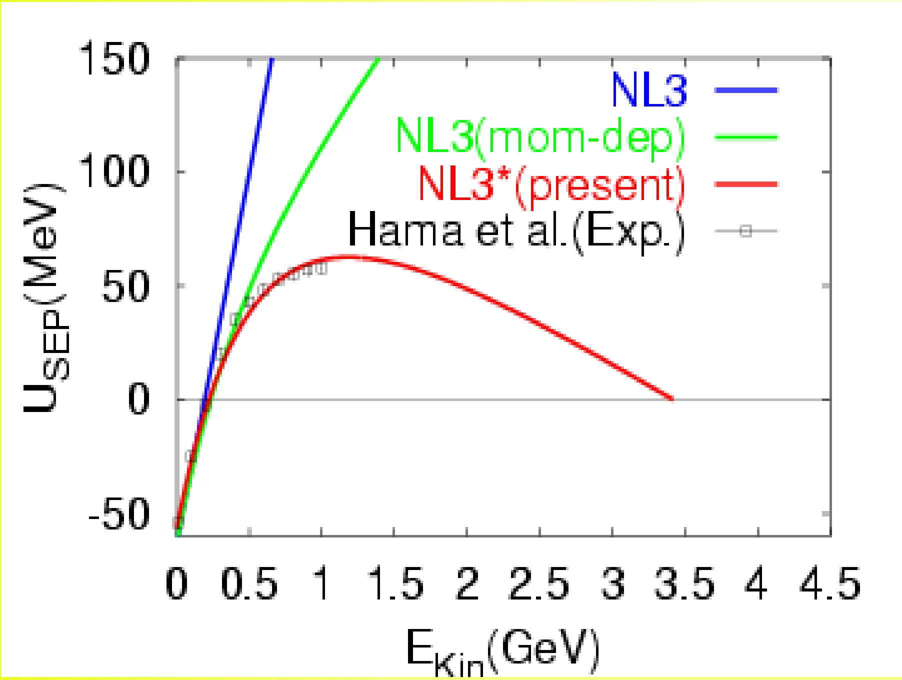
Complex Observables, but Closely Related to EOS

EOS (I): Energy Deps. of the Potential

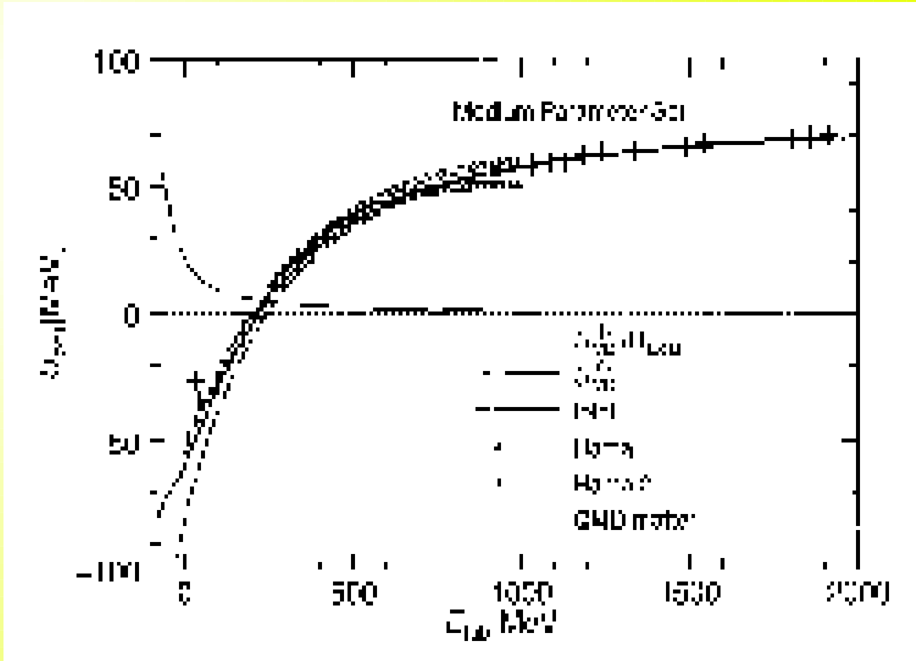
Nuclear Potential is Energy Dependent !

RMF: Vector Pot. gives rise to E-Linear dep. Pot.
Non-Rel. Models: Exchange (Fock) term generate P-dep. pot.

Sahu, Cassing, Mosel, AO (2000)



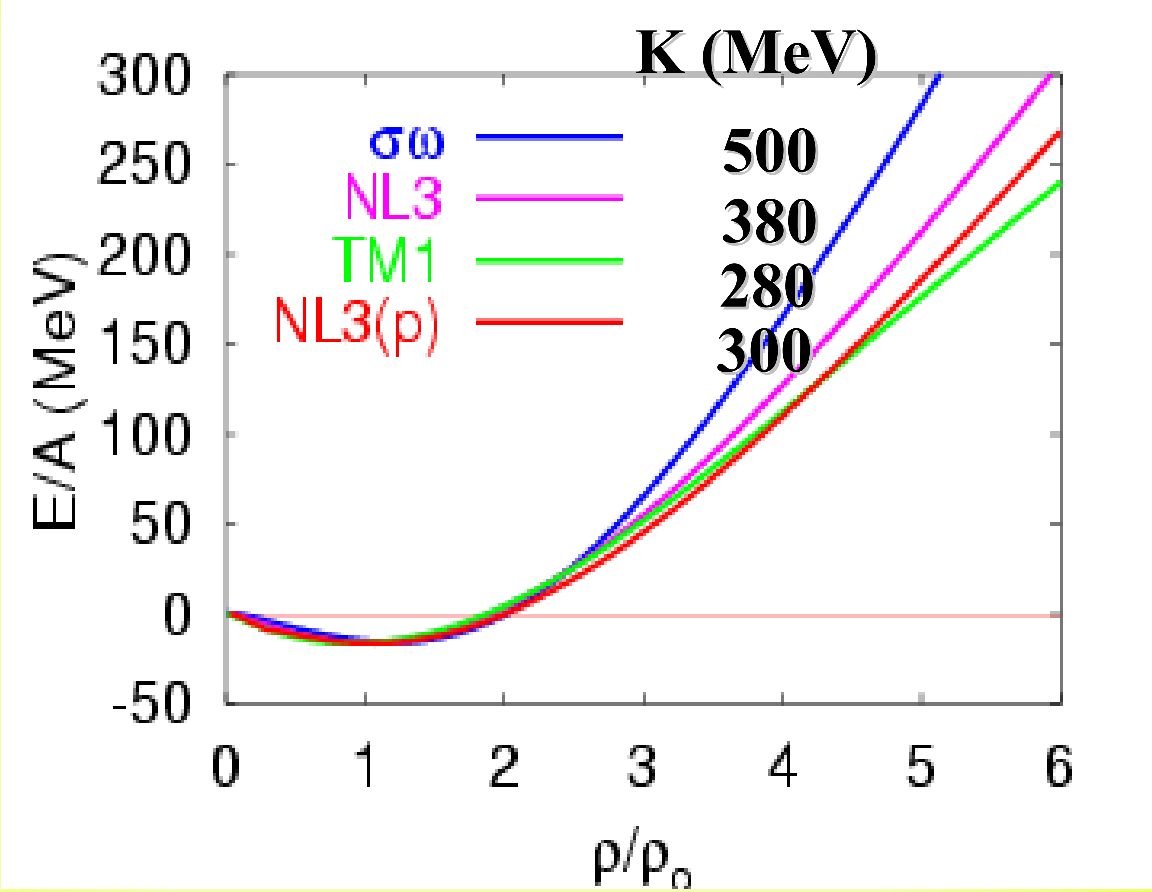
Maruyama et al.



(Form Factor in Meson-Baryon Coupling)

How does it affect EOS ?

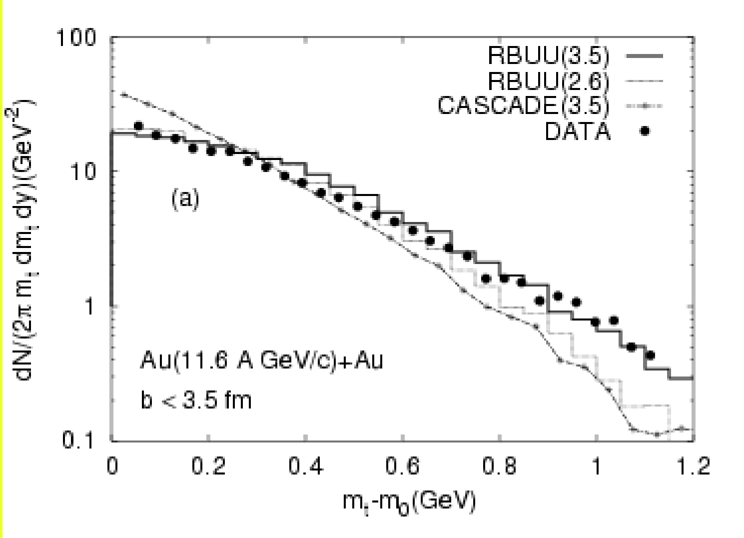
EOS(II): Density Deps.



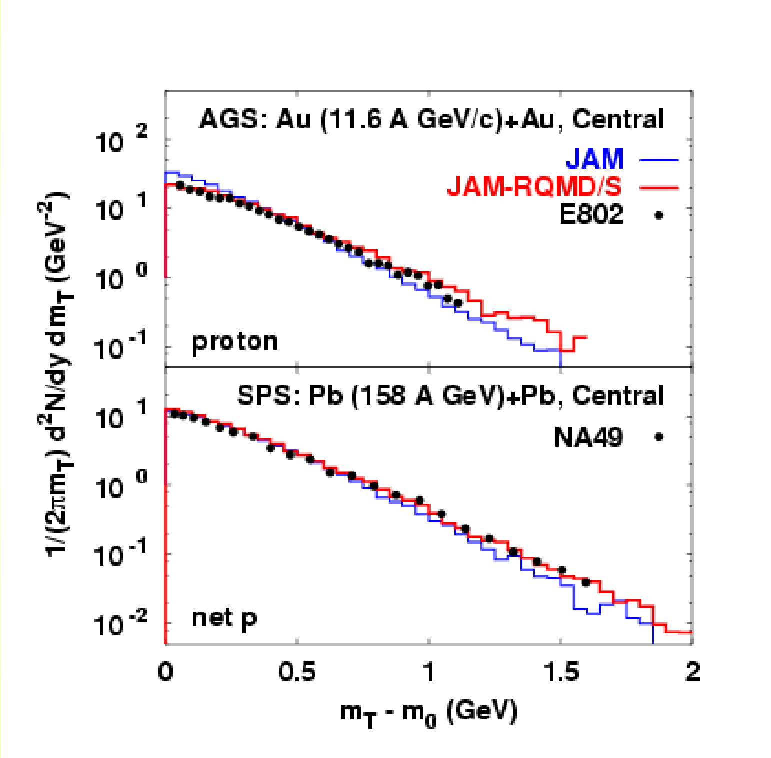
Form Factor significantly modifies EOS. !
→ How does it affct Flows ?

Mean Field Effects on Hadron Mt Spectra

Sahu et al., NPA 2000



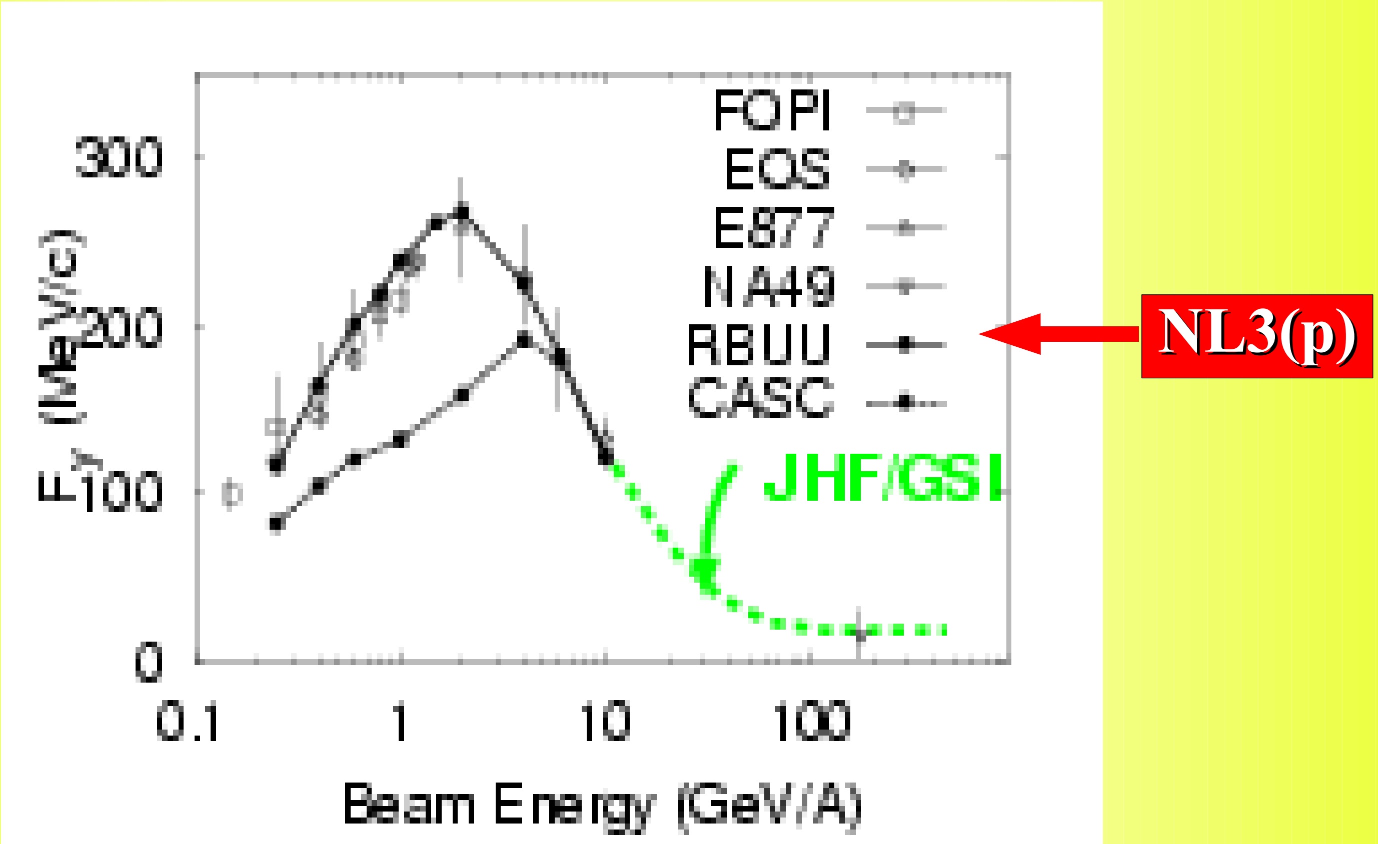
Isse et al., in preparation



Mean Field Stiffens Hadron Mt Spectrum
→ How about Anisotropic Flows ?

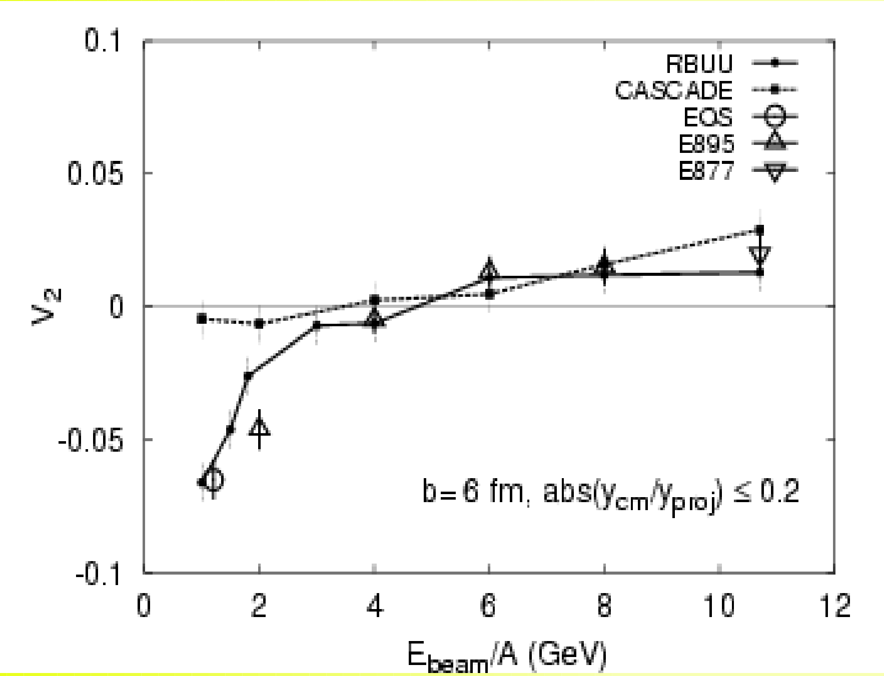
Sideflow from GSI-SIS to BNL-AGS

(Sahu, Cassing, Mosel, AO, NPA(2000))

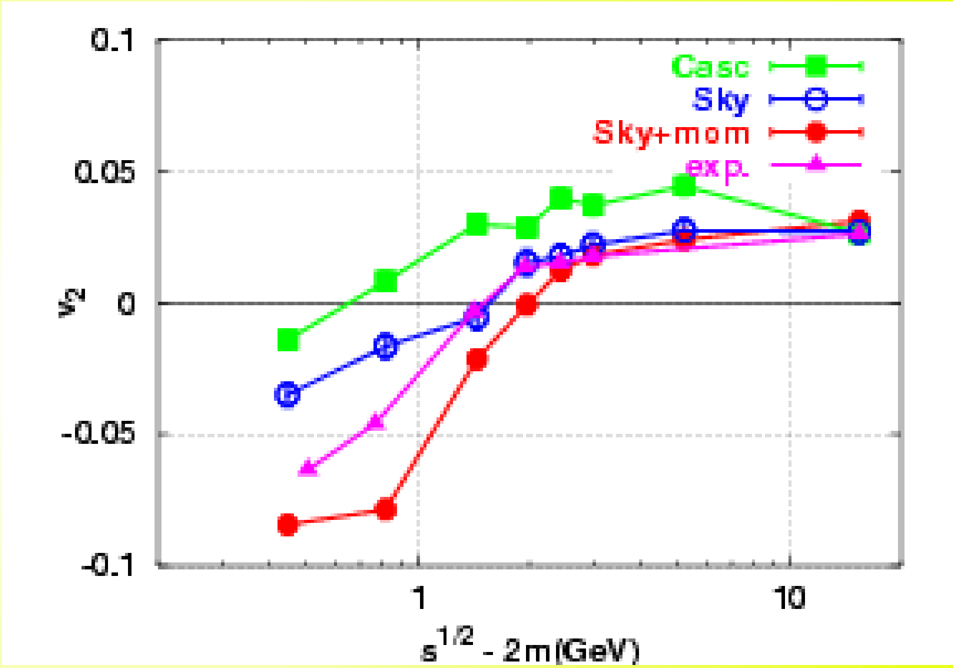


Elliptic Flow from SIS to SPS

Sahu et al., 2000



M. Isse, Master Thesis

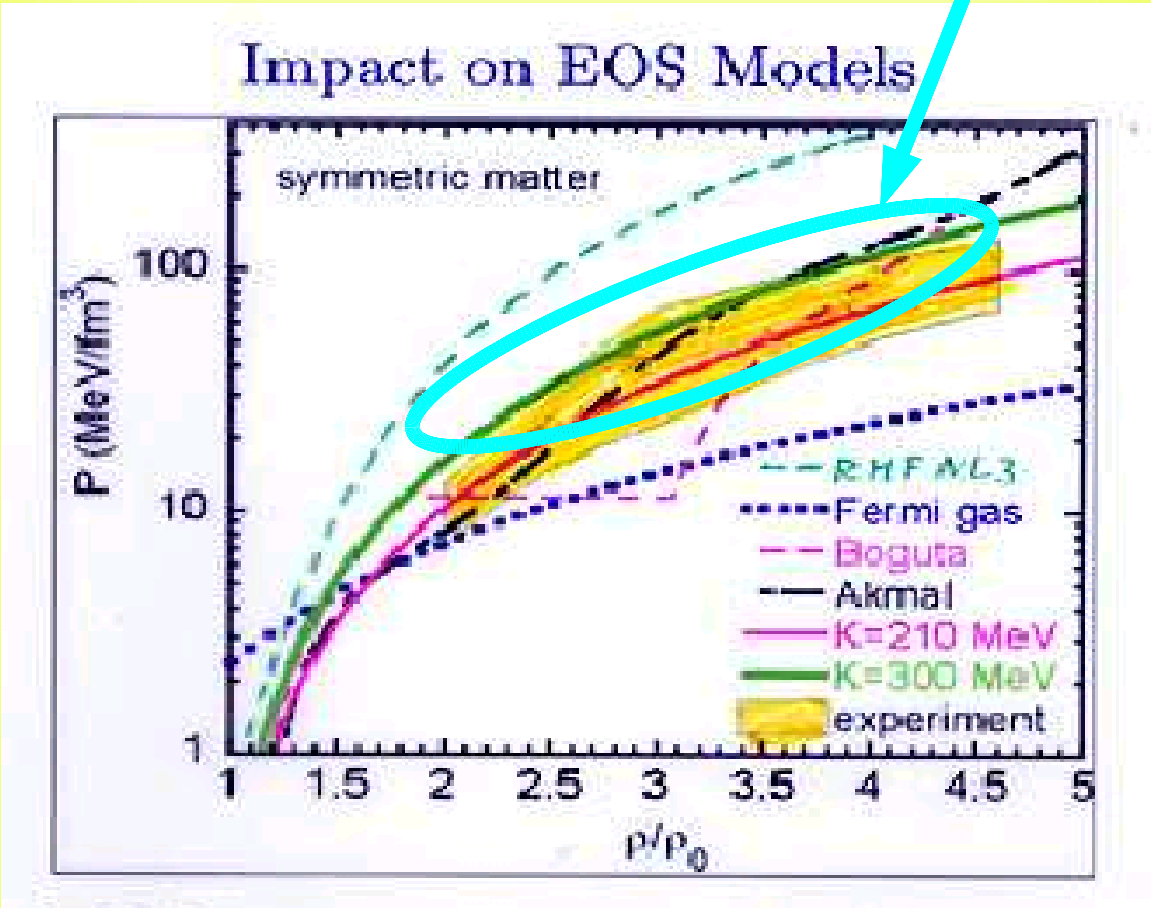


Strong "Squeezing" Effects from Mean Field from GSI-SIS (1 A GeV) to AGS (11 A GeV) energies

Probed Range in ρ-P

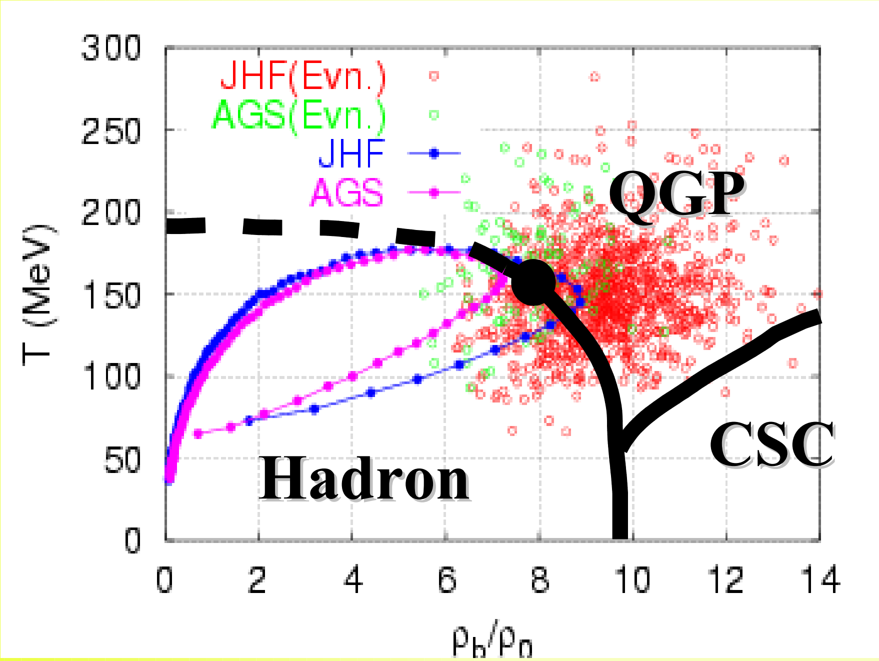
P. Danielewicz, GSI workshop (2002)

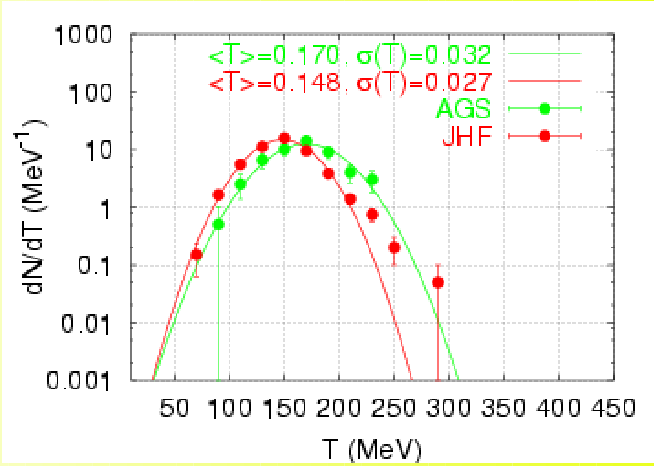
Sahu-Cassing, 2003



Possibility of Creating CSC in Heavy-Ion Collisions

Heavy-Ion Collisions at E_{inc} = 10-40 A GeV may Create "Cold-Baryonic" Matter ......





Color Superconductor: New Form of Matter !
Stony Brook Group, Iwasaki, Hatsuda et al., Iida et al., Kitazawa-Kunihiro et al, ...

Summary

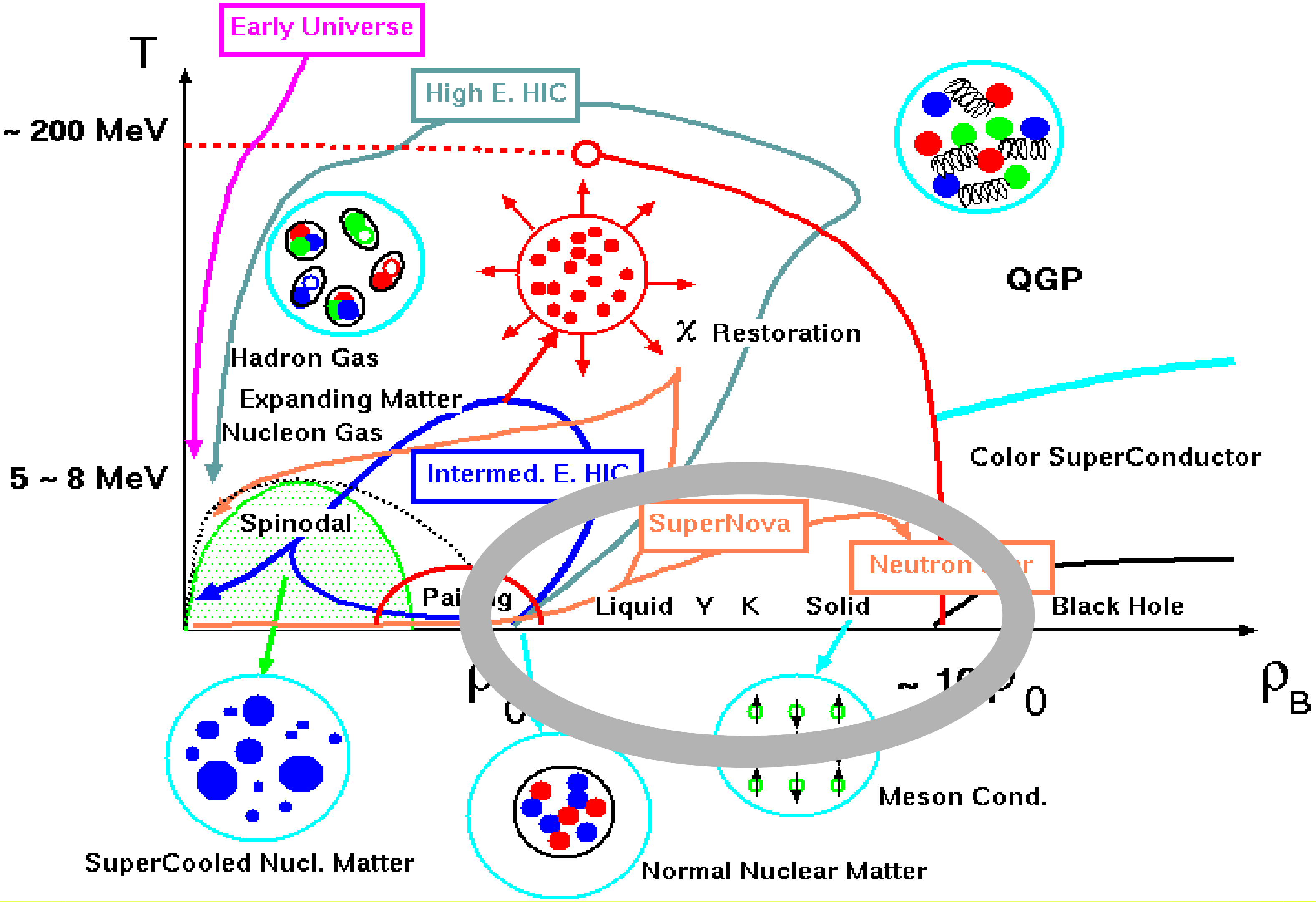
- **Quantitative Understanding of Collective Flows in Heavy-Ion Collisions Requires**
  Repulsive Nuclear Interactions (Mean Field) **!**
- **Momentum (or Incident Energy) Dependence is *Essential* !**

  RMF description: Reduction of Meson-Baryon Coupling
  Non-Rel. Pot. Description: Saturation (Fock) or Explicit Reduction

- **It suggest the EOS to be**
  Soft in *Hot Symmetric Matter* (Supernova)
  and
  Stiff in *Cold Asymmetric Matter* (Neutron Star)
- **Strangeness DOF have to be examined separately.**

Roles of Strangeness in Cold Dense Matter

Hadronic Matter Phase Diagram



What is Expected in the Neutron Star Core ?

Nucleon Superfluid $({}^1S_0, {}^3P_2)$

Pion Condensation

Strangeness

Hyperon Matter

Tsuruta-Cameron (66), Langer-Rosen (70), Pand-haripande (71), Itoh(75), Glendenning, Weber-Weigel, Sugahara-Toki, Schaffner-Mishustin, Balberg-Gal, Baldo et al., Vidana et al., Nishizaki-Yamamoto-Takatsuka, Kohno-Fujiwara et al., ...

Kaon Condensation

Kaplan-Nelson(88), Forkel-Rho et al.(SUNY), Davidson-Miller, Claymans et al., Politzer-Wise, Miller et al., Muto-Tatsumi, Brown-Thorsson-Lee-Rho-Min, Fujii et al., Yabu et al, Maruyama et al., Ellis-Knorren-Prakashi (with Y), Li-Ning, Li-Brown, Tiwari-Prasad-Singh, Glendenning-Schaffner, ...

Quark-Gluon Plasma

We cannot understand Highly Dense Hadronic Matter without the Knowledges of Strangeness Nuclear Physics

Why is Strangeness important in Dense Matter ?

Constituents:

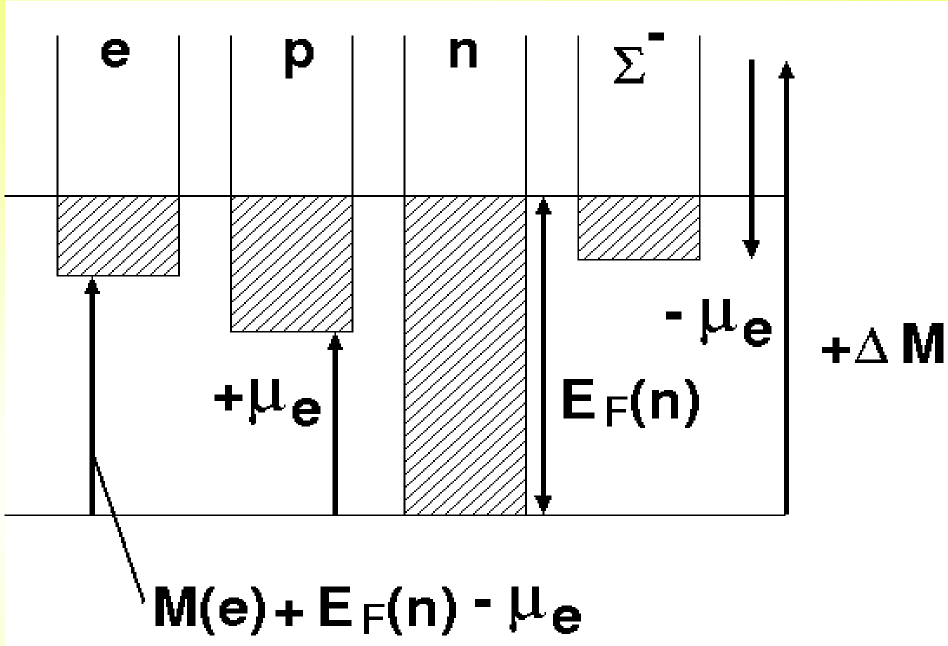
$$p, n, e^{\pm}, \mu^{\pm}, \Lambda, \Sigma^{\pm,0}, \ldots$$

Chemical Equilibrium:

× **Strangeness (Weak)**

× **Lepton (ν Emission)**

$$\mu_i = B_i \mu_B + Q_i \mu_Q$$

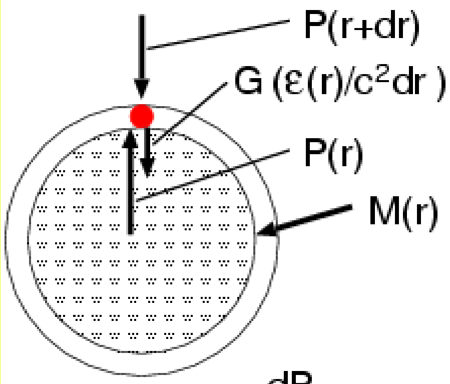


Negatively Chaged or Neutral Baryons are Favored

$$E_F^*(n) + U(n) + \mu_e = M^*(\Sigma^-) + U(\Sigma^-) \qquad \Sigma \text{ appears}$$

$$E_F^*(n) + U(n) = M^*(\Lambda) + U(\Lambda) \qquad \Lambda \text{ appears}$$

TOV Equation: Balance of Pressure and Gravitation



$$\frac{dP}{dr} = -G\frac{\varepsilon(r)/c^2 \quad M(r)}{r^2}$$

$$\frac{dP}{dr} = -G\frac{(\varepsilon/c^2 + P/c^2)(M + 4\pi r^3 P/c^2)}{r^2(1 - 2GM/rc^2)}$$

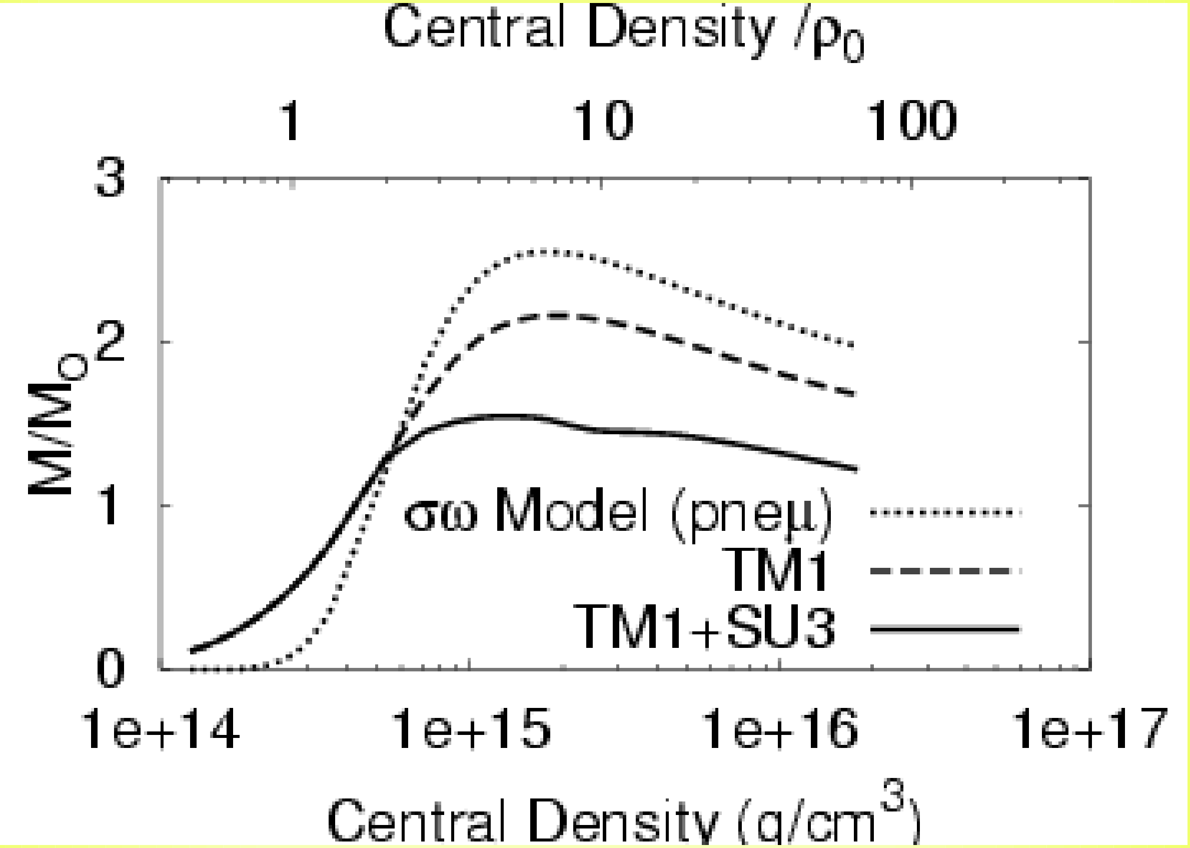
$$\frac{dM}{dr} = 4\pi r^2 \varepsilon/c^2 \;, \quad \frac{dP}{dr} = \frac{dP}{d\varepsilon}\frac{d\varepsilon}{dr}$$

$$P = P(\varepsilon) \;, \quad \frac{dP}{d\varepsilon} = \frac{dP}{d\varepsilon}(\varepsilon) \quad \text{(EOS)}$$

Neutron Star Mass = M(R), where P(R) = 0

When You Make a New EOS, Please Check Neutron Star Mass !

Neutron Star Max. Mass



A. Isshiki, AO, JPS @ Akita; Serot-Walecka ($\sigma\omega$); Sugahara-Toki (TM1); Schaffner-Mishustin (TM1+SU3); Glendenning, ...

Maximum Mass Reduction ~ 0.5-1.0 M_{sun}

Σ Potential Effects on Neutron Star Matter

- **Potential for Λ; Relatively Well Known**

  $U(\Lambda) \sim -30\ MeV$ **(Many Single Hypernuclei)**

- **Potential for Ξ; Recently Suggested from (K^-, K^+) Experiments**

  $U(\Xi) \sim -(14-16)\ MeV$

  (KEK-E224, BNL-E885, BNL-E906)

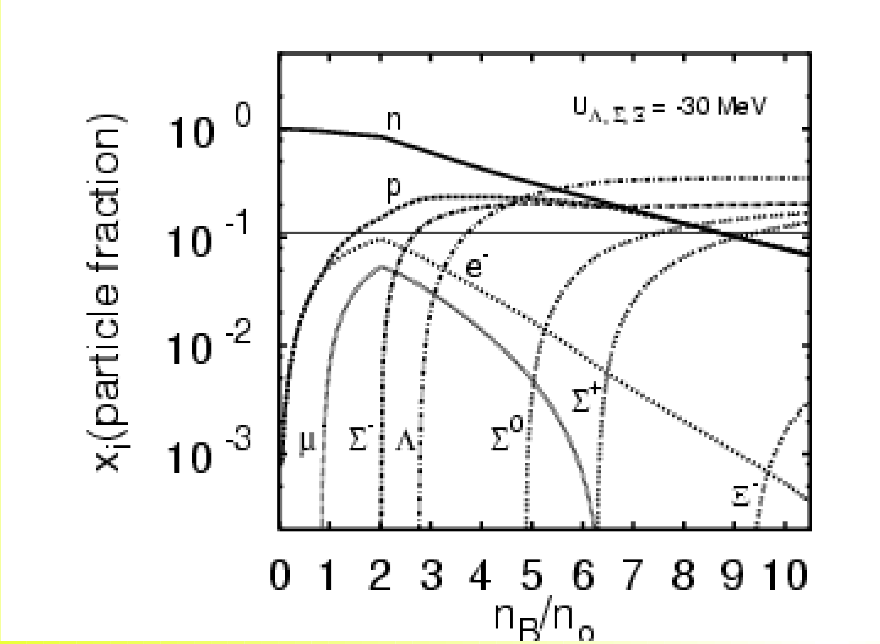
→ ***Potential Depth $\propto$ Number of ud Quarks ?***

- **Potential for Σ: Contradicting Conjectures**

  $U(\Sigma) \sim -(24-30)\ MeV$ **(Old Conjectures)**

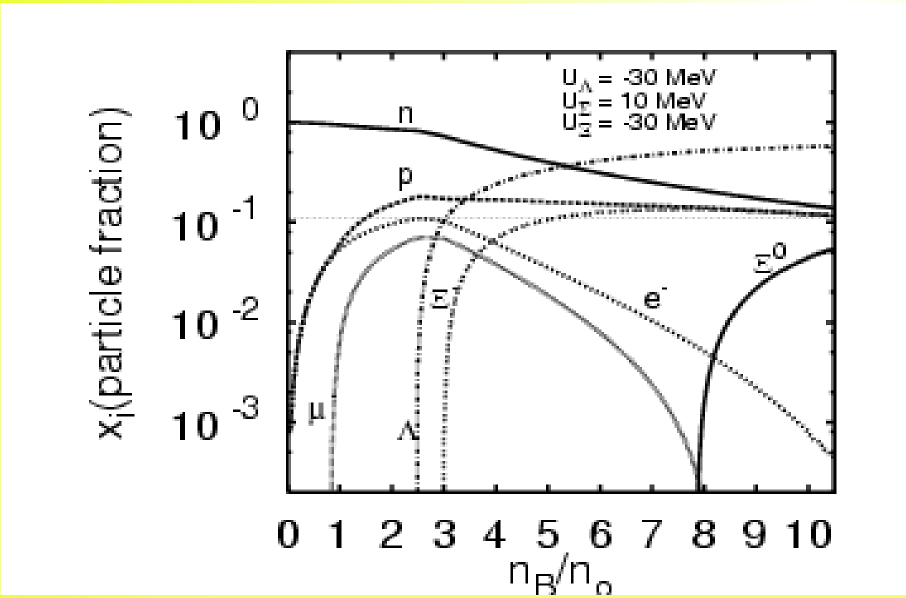
  $U(\Sigma) > 0$

  (Dabrowski, Yamamoto et al., Kohno-Fujiwara et al.)



Attractive Potential for ∑
→ ∑ appears at around

$$\rho \approx 2\rho_0$$



Repulsive Potential for ∑
→ ∑ does not appear

(RMF: Sahu, Ohnishi Nucl. Phys. A691 (2001), 439.)

What is Already Known ?

- **Light Single Λ Hypernuclear Shell/Cluster Structure**
- **Bare Λ N Interaction**
  Germanium γ -ray Detector(Tamura et al.)
  +Precise Few-Body Calculation (Hiyama et al., Nemura et al.)
- **Structure of $^{4}_{\Sigma}He$:**
  Coherent ΛΣ Coupling
  (Harada-Akaishi-Shinmura-Myint, Hiyama et al.)
- **ΛΛ Interaction in Nuclei = Weakly Attractive**
  Recent Experiment KEK-E373 (Nagara Event)

What is Still Unknown ?

- **Properties of Hyperons (All) at Higher Densities.**
- **Σ Potential at ρ_0 and Higher Densities**
- **$\Lambda\Lambda$ Interaction in "Free" Space**

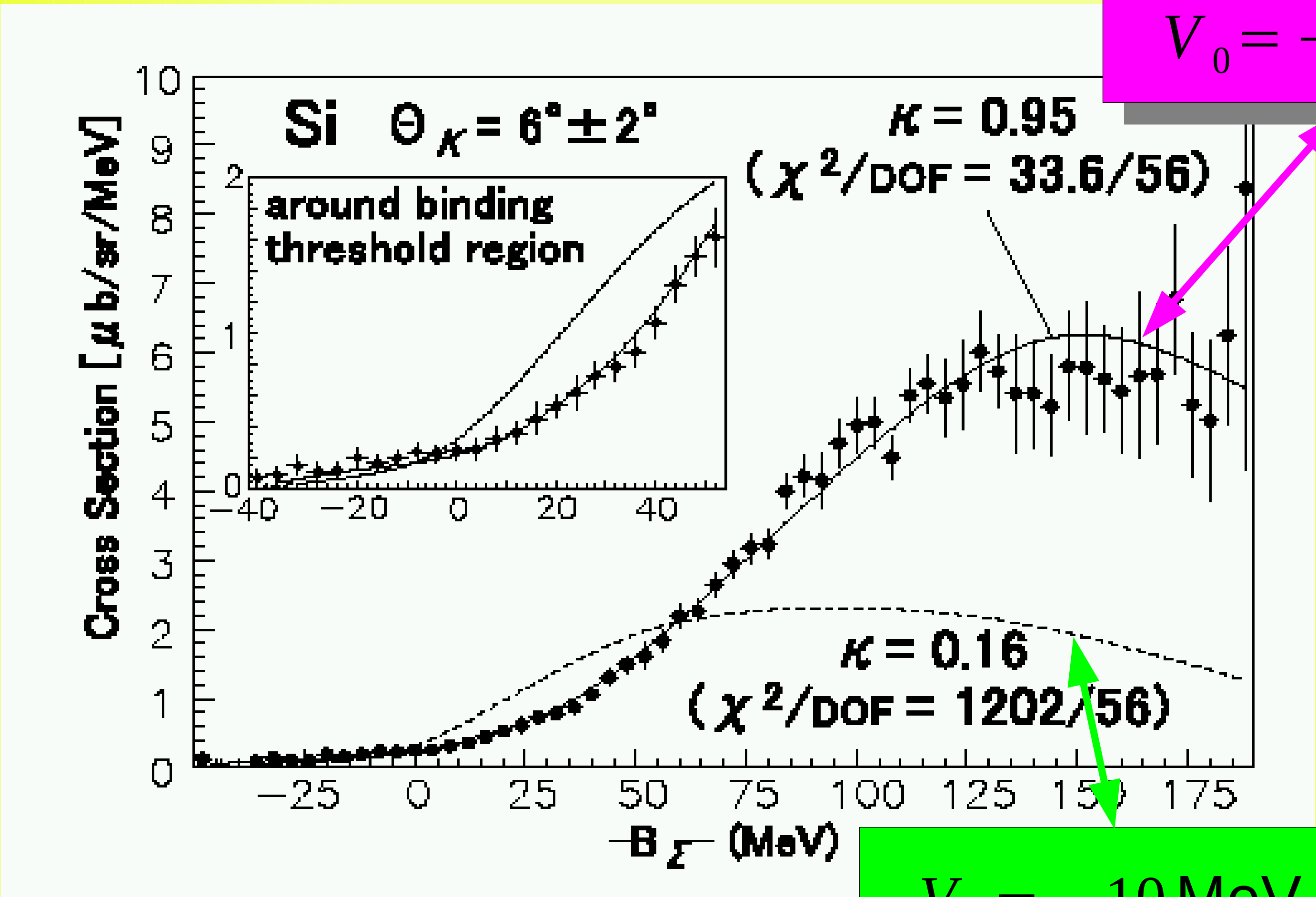
➡ **Very Recent Experiments !**

- **Direct Quasi-Free Production of Σ (Noumi et al.)**
- **Strangeness Enhancement in HIC at SPS (NA49)**

Phenomenological Determination would be possible !

Does ∑ Feel +150 MeV (Repulsive) in Nuclei ?

Noumi et al., Phys. Rev. Lett. 89 (2002) , 072301.



$V_0 = +150$ MeV

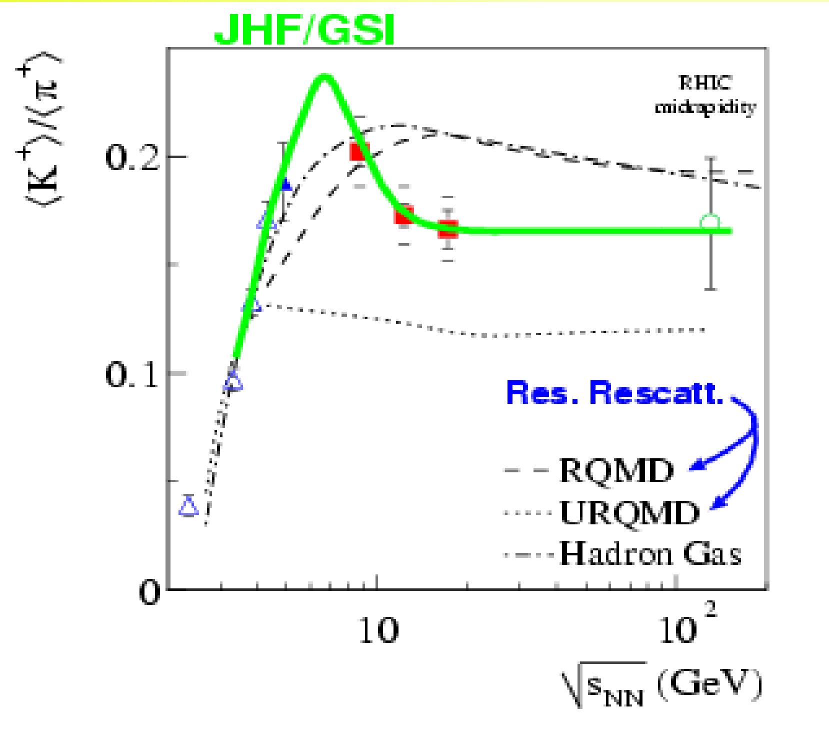
$V_0 = -10$ MeV

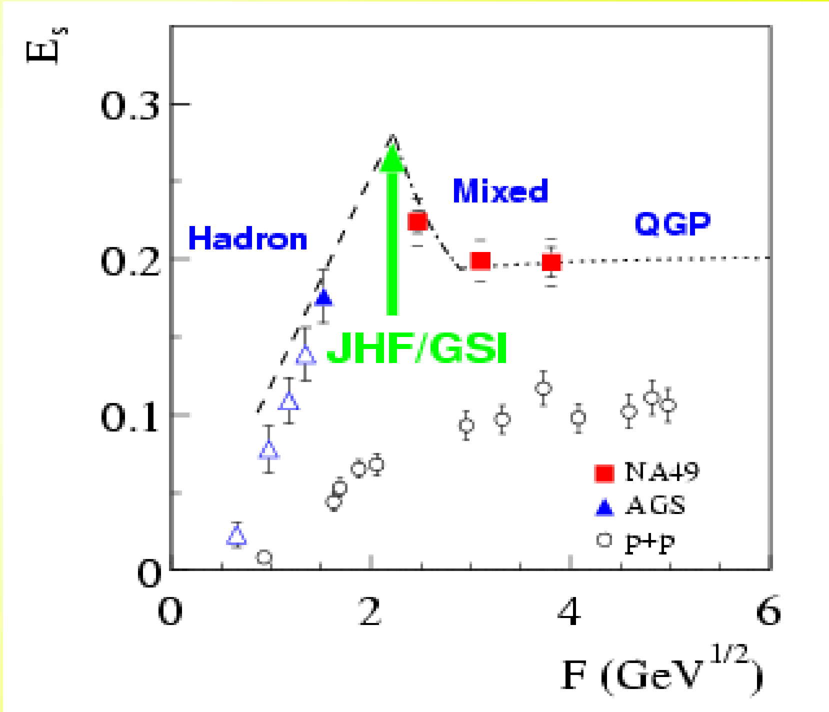
No Theoretical Model Support $V_0 = +150$ MeV **! ➡ Big Puzzle !!**

Strangeness Enhancement: Rescattering, Potential, or Phase Transition ?

Strangeness is Enhanced Sharply at Einc = 10 ~ 40 GeV/A !

NA49 (nucl-ex/0205002)

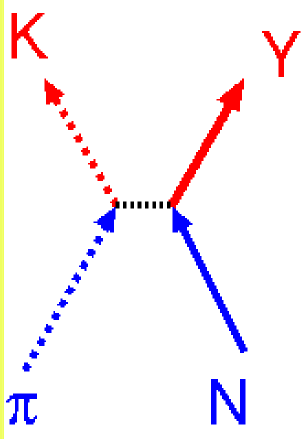


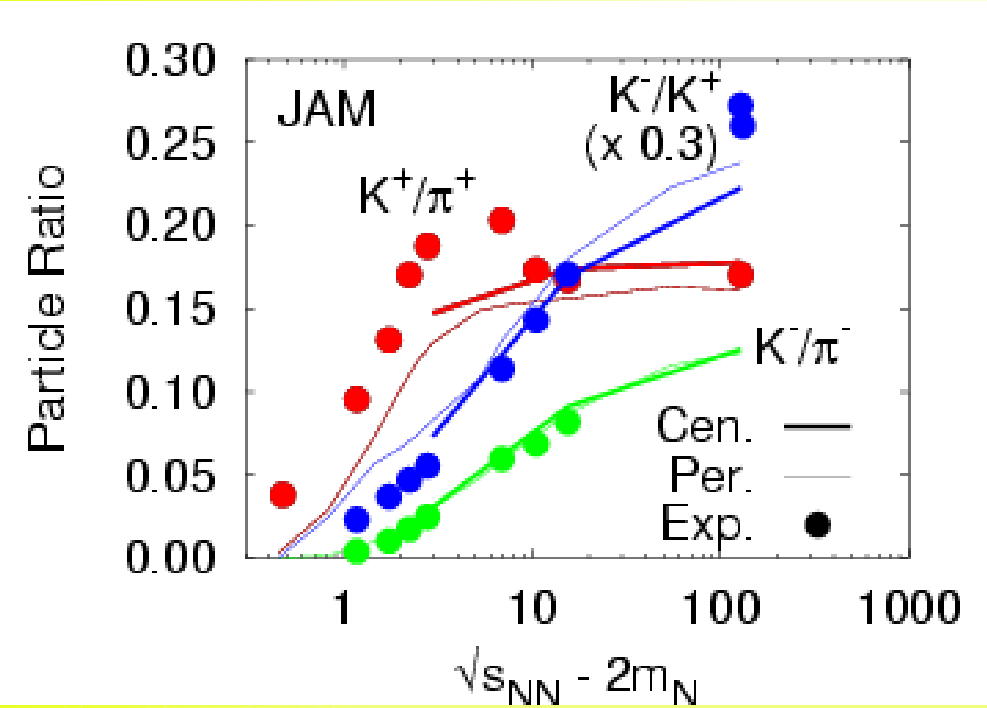


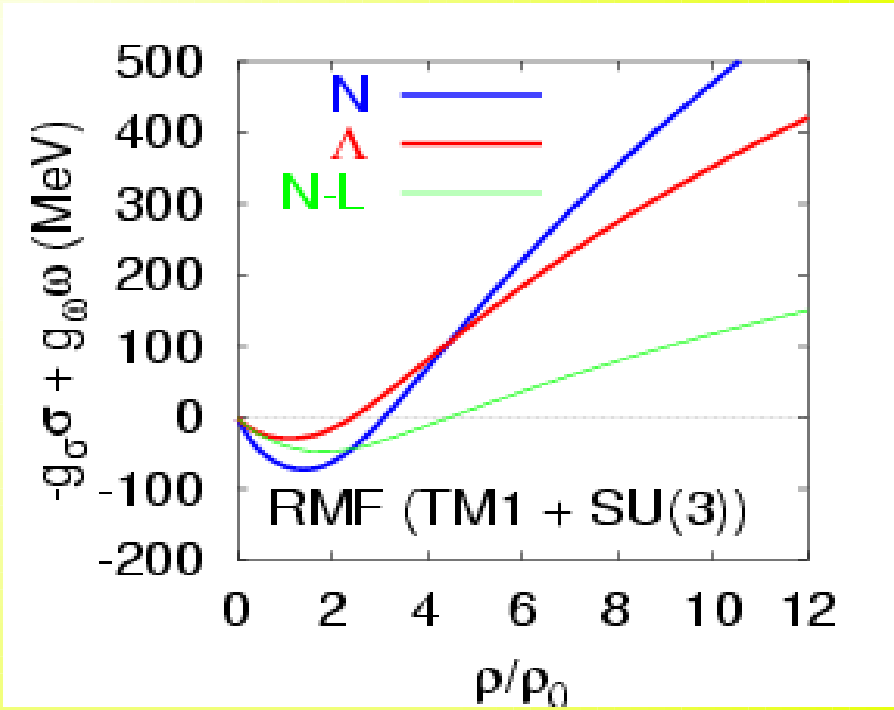
JHF Energy: ~ Maximum K/π ratio

Does Hyperon Potential Help It ?

- **Rescattering of Resonances/Strings (RQMD)**
- **Baryon Rich QGP Formation**
- **High Baryon Density Effect (Associated Prod. of Y)**







At $\rho > 5\rho_0$ Hyperon Feels More Attractive Potential than N

Is Lambda-Lambda Interaction Really Weak ?

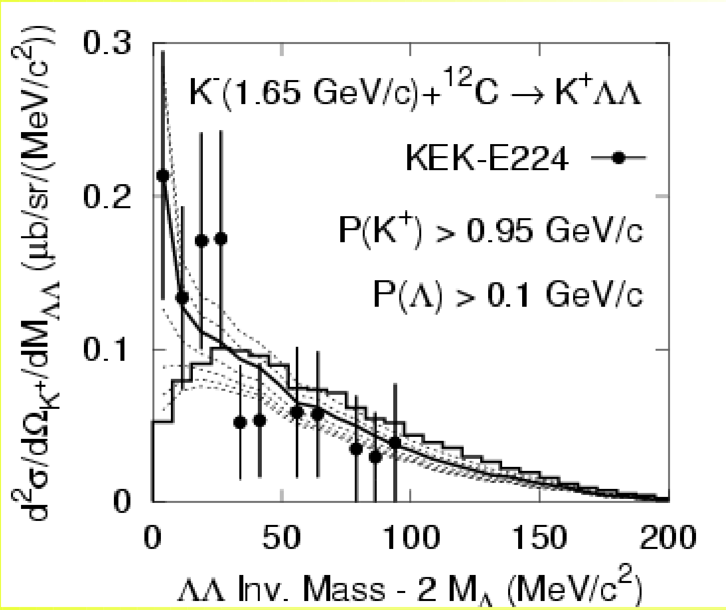
Khin Swe Myint, Shinmura, Akaishi, Euro. Phys. J. A16 (2003) 21.

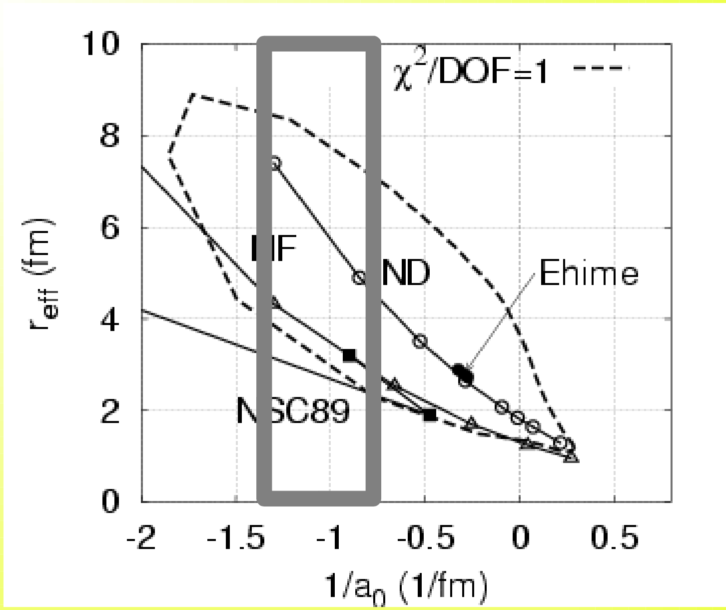
From Nagara Event,

$a_{\Lambda\Lambda} = 0.7$ **fm (Weak $\Lambda\Lambda - \Xi N$ Coupling)**

~ 1.3 **fm (Strong Coupling, Pauli Suppressed in Nuclei)**

Momentum Correlation of $\Lambda\Lambda$





We have the range of overlap !

Summary

1. **Strangeness is important in dense matter such as in neutron star core.**

   Strangeness changes the max. mass of neutron star,
   modifies the order of QCD phase transition,
   probes deeply inside the nucleus,
   mixes elementary particles in nuclei.

2. **Hypernuclear spectroscopy have developed a lot in these years, but we need more data for the understanding of dense matter.**

   Σ Potential, $\Lambda\Lambda$ Interaction, ΛN-ΣN and $\Lambda\Lambda$-ΞN Coupling,
   Hyperon Potential in Dense Matter, ...

3. **Recent Data would be Helpful to Understand Hyperons in Dense Matter based on *Real Data***

   Quasi Free Σ Production, Kaon Enhancement, $\Lambda\Lambda$ Nuclei,
   $\Lambda\Lambda$ Correlation, ...

There may be many things to do from nuclear physics side.

1. **Fragment "Distribution" Effects in Supernova Explosion**
2. **Equation of State in High Density Hadronic Matter**
3. **Strangeness DOF in High Density Matter**
4. **....**

Too Many Things to do. Depressive ?

No, at least for Young Physicists !

Collaborators

Phase transition of Supernova Matter:

C. Ishizuka, K. Sumiyoshi

High Energy Heavy-Ion Collisions:

M. Isse, N. Otuka, Y. Nara, P.K. Sahu, W. Cassing, U. Mosel

Strangeness Nuclear Physics:

K. Maekawa, A. Isshiki, P.K. Sahu,
Y. Hirata, Y. Nara, S. Shinmura, Y. Akaishi