Nuclear EOS in Chiral sigma Model

A. Ohnishi and K. Naito

- Introduction
- Chiral sigma models
  - Chiral symmetry
  - Φ^4 Theory
  - NJL model
  - Boguta Scenario
- Soft Nuclear EOS in Chiral sigma Model
  - ωN Form Factor
  - ω^4 Term
  - Short range qq interaction effects
- Summary

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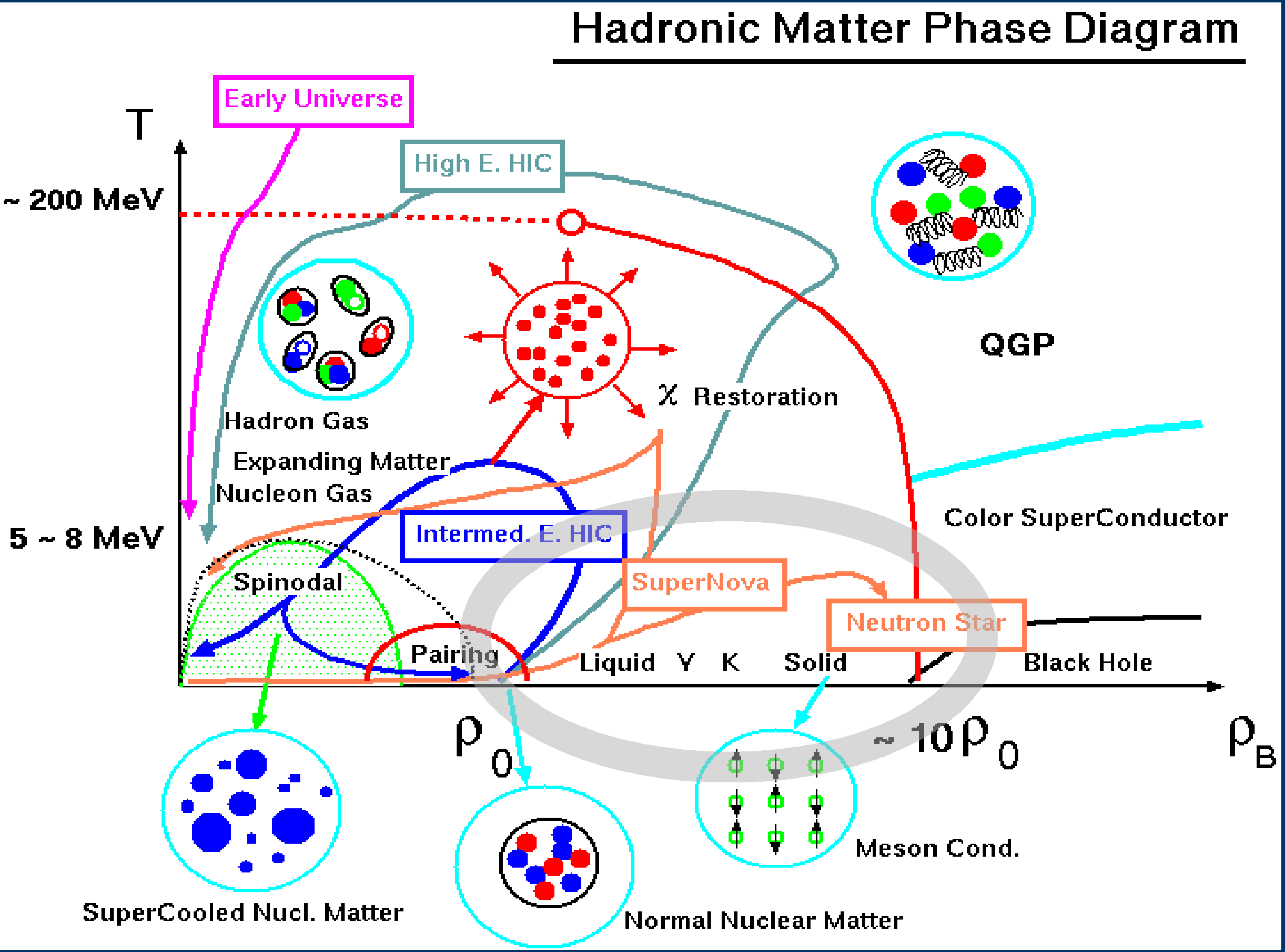
Introduction

- Nuclear EOS
  - Important in Various Nuclear/Astrophysics Contexts
  - Experiments:
    - ➔ Heavy-Ion Collisions
    - ➔ Precise Measurement of Nuclear Radii
    - ➔ Giant Monopole Resonance
  - Theory
    - ➔ Ab Initio Calculation
    - ➔ G-matrix/Effective Interaction Approach
    - ➔ Mean Field approximation
    - ➔ Transport Theory
  - How to Determine EOS far from Normal Nuclear Matter
    - ➔ High ρ, Yp (=Z/A) far from 0.5
  - We Need Models
    - ➔ Based on Well-Defined Physics Motivation
    - ➔ With Small Number of Parameters to be Extapolated
    - ➔ Simple Enough to be Applied in Various Contexts

Hadronic Matter Phase Diagram



A. Ohnishi, Colloquium, 2004/10/26

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Chiral Symmetry

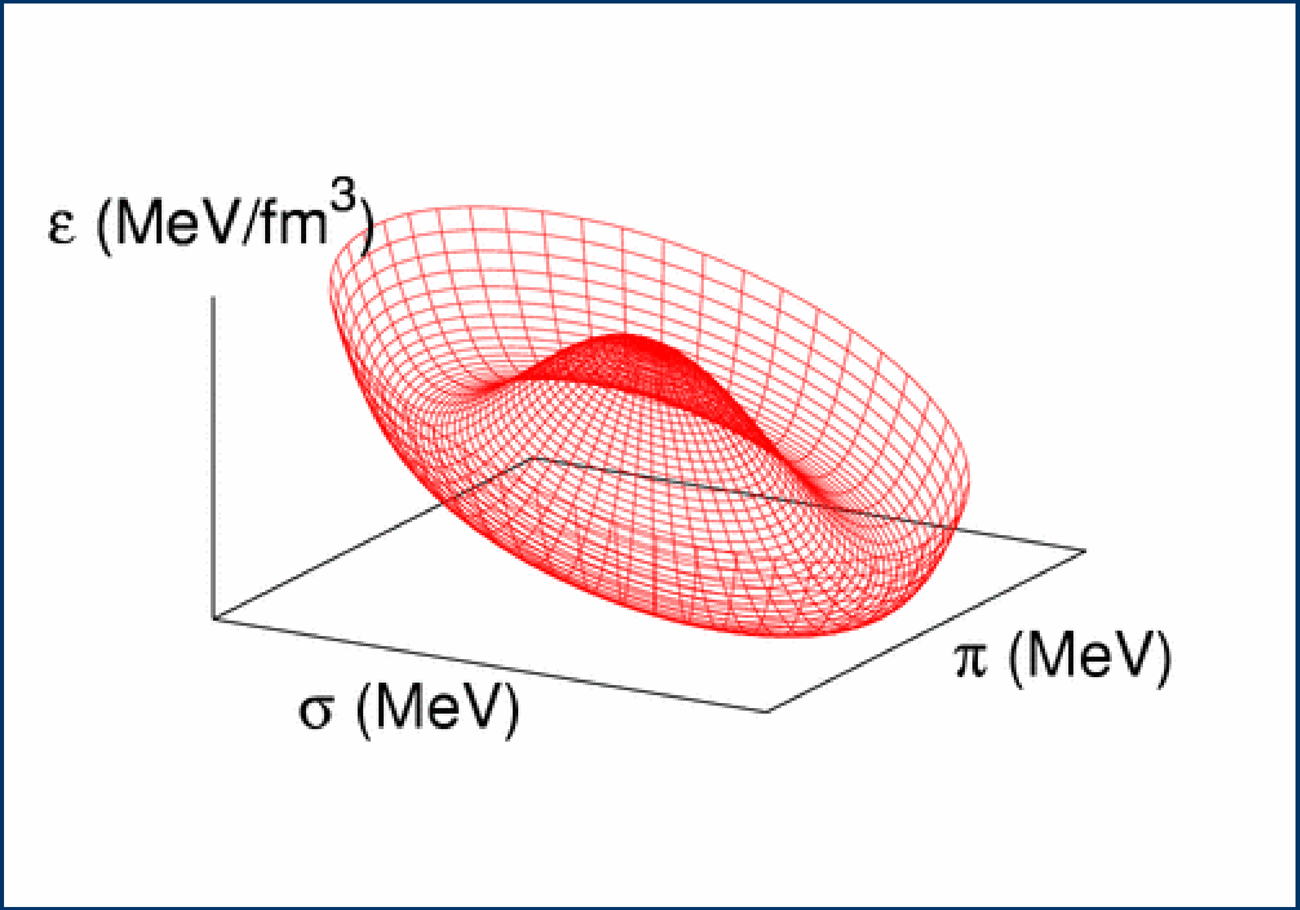
- Good (approximate) Symmetry in QCD
  - In Flavor SU(2), only the small current quark mass term breaks chiral sym.
  - Should persist also in the hadronic world
  - Explains the small mass of pions, as Nambu-Goldstone particle of the chiral symmetry, and many other low energy hadronic properties.
- Schematic model: Linear σ model
  - Wine bottle shape of the effective potential
    - ➔ Spontaneous breaking of χ symmetry
  - Expectation Value of σ
    - ➔ Nucleon Mass

$$L = \frac{1}{2}\left(\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \pi \partial^\mu \pi\right) - \frac{\lambda}{4}\left(\sigma^2 + \pi^2\right)^2 + \frac{\mu^2}{2}\left(\sigma^2 + \pi^2\right) + c\sigma$$

$$+ \overline{N} i \partial_\mu \gamma^\mu N - g_\sigma \overline{N}\left(\sigma + i\pi\tau\gamma_5\right) N$$

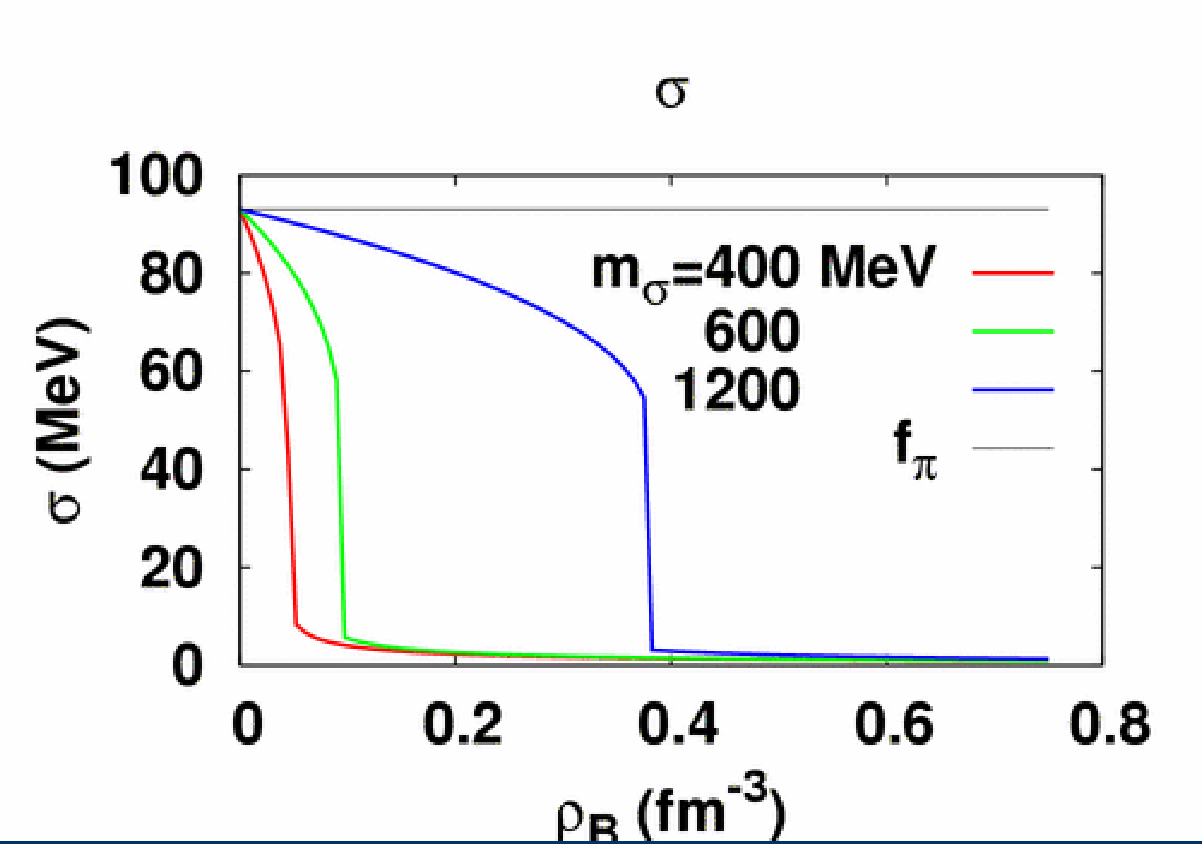
Chiral Linear σ Model: Energy Surface

$$m_\sigma{}^2 = \frac{\partial^2 \varepsilon}{\partial \sigma^2}\Big|_{vac} \ (\text{Large}), \quad m_\pi{}^2 = \frac{\partial^2 \varepsilon}{\partial \pi^2}\Big|_{vac} \ (\text{Small})$$



Chiral Linear σ Model at Fintite ρ_B (I)

- Serious problem:
  - Sudden chiral phase transition at relatively low baryon density. (Below ρ_0 if σ mass = 600 MeV)
    - ➔ Why ?



Chiral Linear σ Model at Fintite ρ_B (II)

- "Vacuum" condition = Energy Minimum State

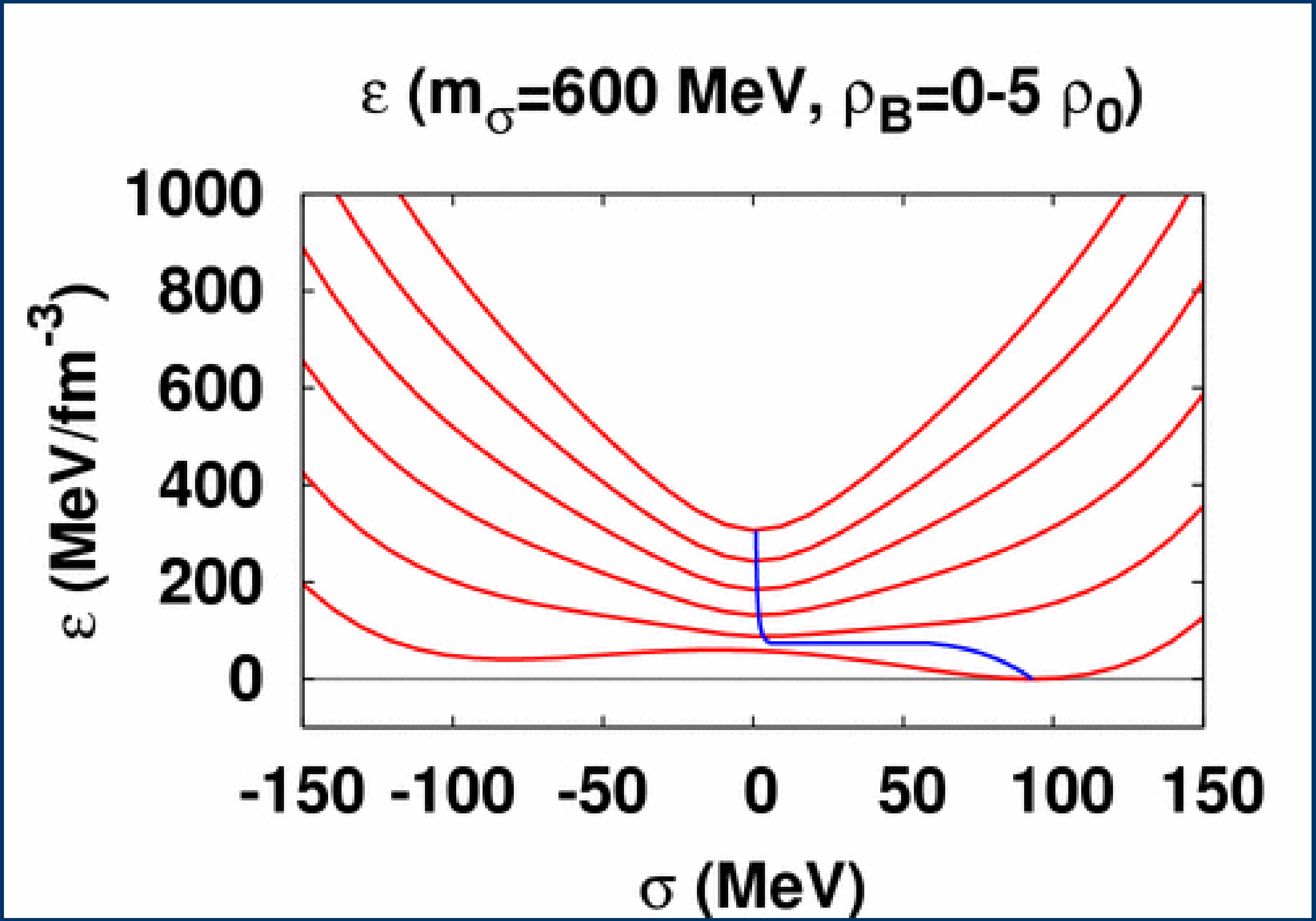
$$V = V_\sigma + E_N = \frac{\lambda}{4}\left(\sigma^2+\pi^2\right)^2 - \frac{\mu^2}{2}\left(\sigma^2+\pi^2\right) - c\sigma$$

$$+\int \frac{\gamma d^3 p}{(2\pi)^3}\sqrt{p^2+(g_\sigma \sigma)^2}$$

$$\rightarrow \quad \frac{\partial V}{\partial \sigma} = \frac{\partial V_\sigma}{\partial \sigma} + g_\sigma \rho_s = 0$$

  - Large Nucleon Energy Gain for small $\langle \sigma \rangle$ due to mass decrease.

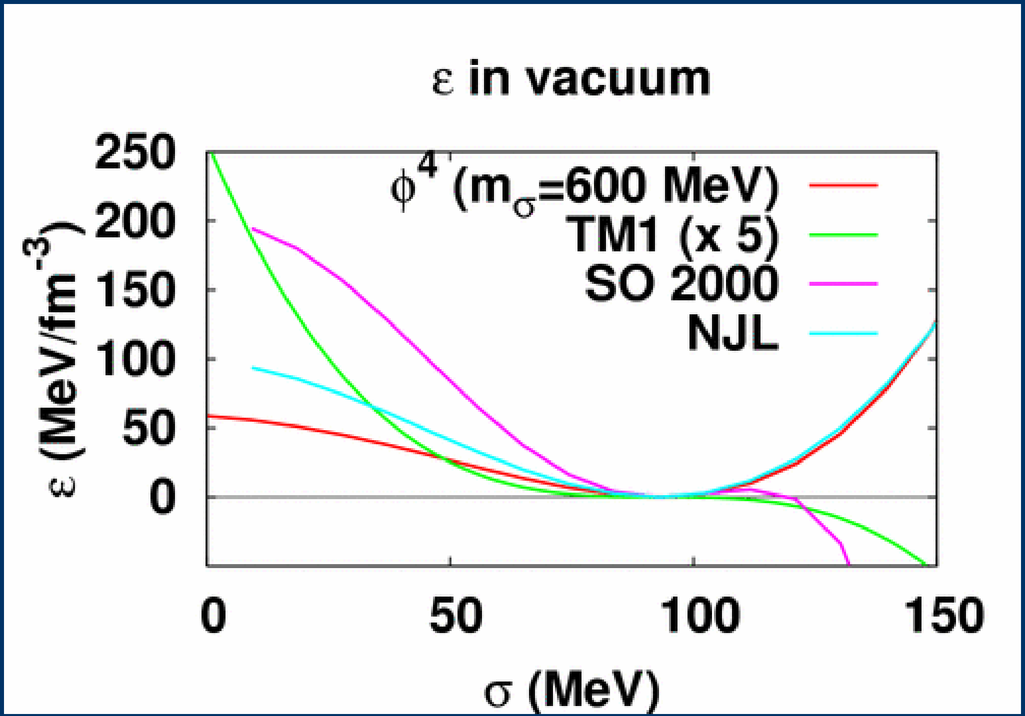
Chiral Linear σ Model at Fintite ρ_B (III)

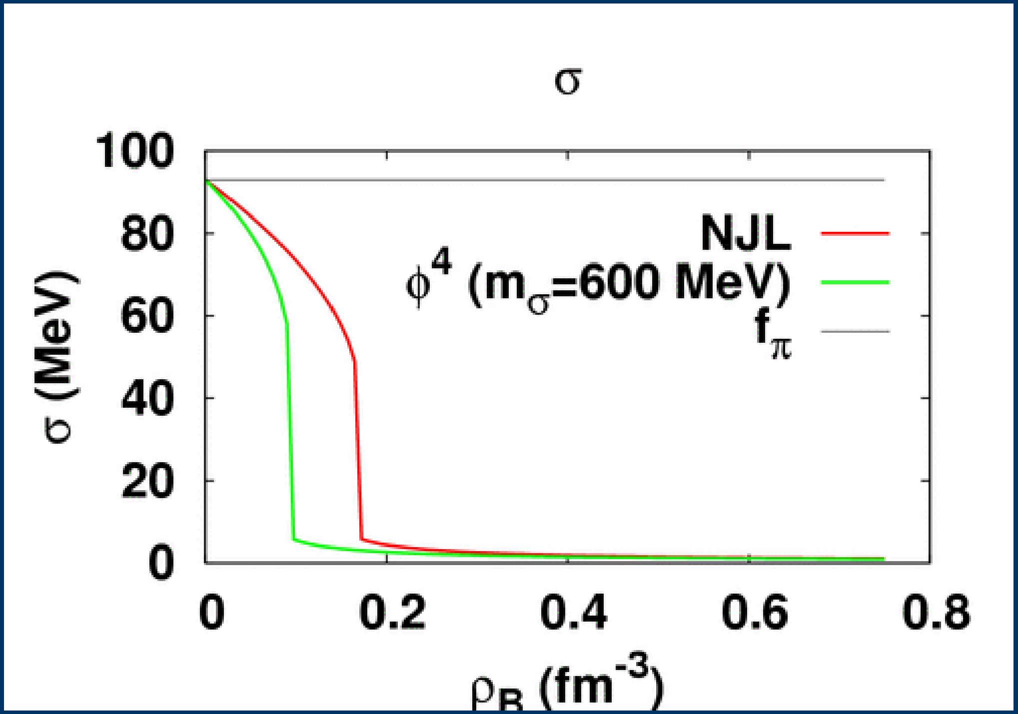


- We cannot avoid this sudden change even if we introduce ω meson-Nucleon coupling (indep. on $\langle\sigma\rangle$)
  - ➔ Why do RMF models succeed ?
  - ➔ How about NJL model ?

Other Effective Models of σ Meson

- Other Effective Models
  - Relativistic Mean Field models
    - ➔ σω model, TM1, ... DO NOT have χ symmetry
  - Nambu-Jona-Lasino model
    - ➔ has steeper increase at small ⟨σ⟩, then it is a little more stable than Φ^4 model. However, it is still unstable below ρ_0.





Boguta's Scenario (I)

J. Boguta, PLB120,34/PLB128,19

- To avoid the sudden transition to χ restored phase, it is necessary to include "stabilization potential" at finite ρ_B which grows as $\langle\sigma\rangle$ increases.
- Boguta proposed to include $\sigma\omega$ coupling

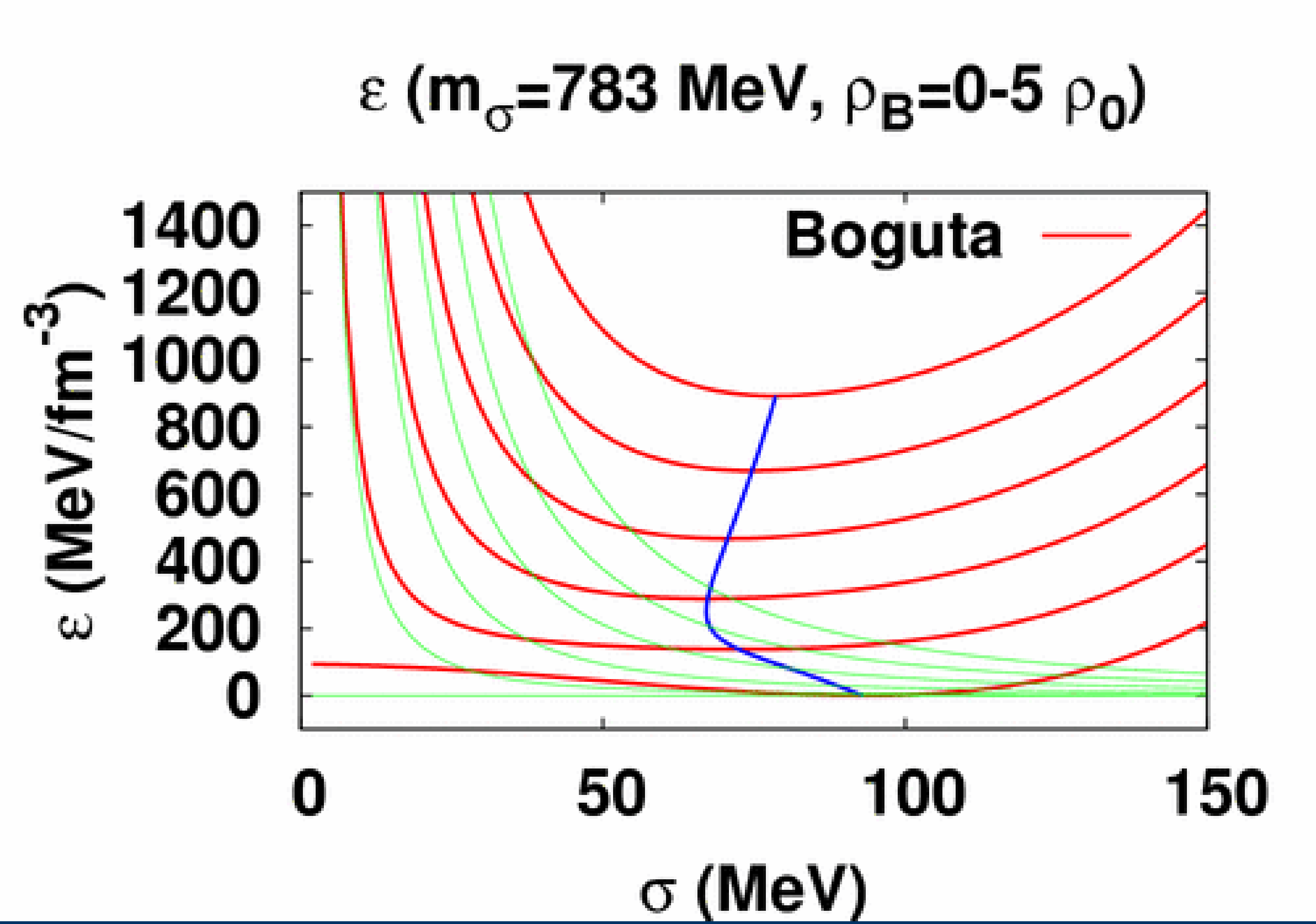
$$L_{\omega\sigma} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}C_{\sigma\omega}\sigma^2\omega^2 - g_\omega \bar{N}\gamma_\mu\omega^\mu N$$

$$\omega = g_\omega\rho_B / C_{\sigma\omega}\sigma^2 \quad \to \quad V_{\sigma\omega} = \frac{g_\omega^2\rho_B^2}{2C_{\sigma\omega}\sigma^2}$$

➔ Leads to large repulsion at around $\langle\sigma\rangle \approx 0$

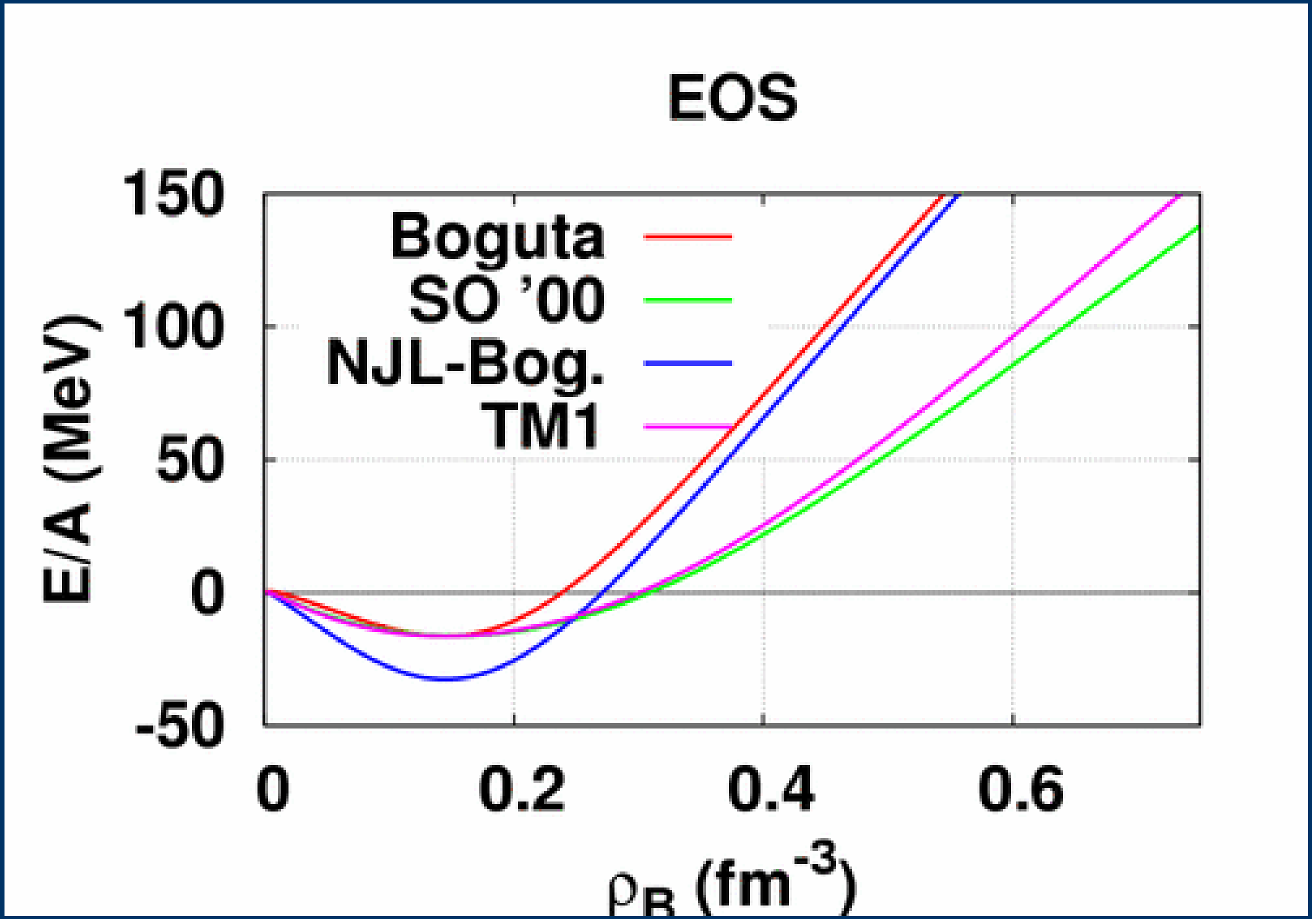
Boguta's Scenario (II)
--- χ Cond. Stabilization ---

- χ cond. does not dissapear at any high ρ_B.
- χ cond. grows again at higher density than $2\rho_0$.



Boguta's Scenario (III)
--- Stiff EOS ---

- $\sigma\omega$ coupling acts as the repulsive potential.
- σ down at medium ρ_B enhances repulsion.



Solution of "Soft" Chiral Model

- There are many proposals, but as far as we understand, there is no simple satisfactory model yet.
  - RMF-TM1
    - ➔ No χ sym., Vacuum is not the usual vac.
    - ➔ ω^4 term ... does not couple to σ (No stabilization)
  - Sahu-Ohnishi 2000
    - ➔ σ^6, σ^8 terms, Coef. are negative.
  - NJL
    - ➔ phase transition at $\rho_B < \rho_0$, or it gives too stiff EOS with $\sigma\omega$ coupling.
  - SU(3) chiral linear σ model at finite ρ_B.
    - ➔ Naito-AO: EOS is still stiff also in SU(3)
  - Dilatation Field
    - ➔ Requires to include unobserved particle
  - Vacuum polarization due to Nucleon (Anti-N) Loop
    - ➔ $V(\sigma)$ is made from quark loops. We should evaluate quark loop modification first.
  - Vacuum polarization due to π loops.
    - ➔ maybe ....

Phenomenological Approach: Form Factor

- Origin of Stiff EOS in Boguta's Scenario
  - Linear rize of Nucleon Vector Potential
    - ➔ $U_v(N) = g_\omega \omega$, $\omega = g_\omega \rho_B / m_\omega^2$.
  - Vector potential should be suppressed at high E (Sahu, Cassing, Mosel, Ohnishi, 2000)
    - ➔ It should be suppressed also at high ρ_B.
- Introducing Form Factor

$$L_{\omega N} = -g_\omega \bar{N} \gamma_\mu \omega^\mu N$$

$$\rightarrow \quad L_{\omega N} = -g_\omega \bar{N} \gamma_\mu \omega^\mu N F(\omega) \quad , \quad F(\omega) = \frac{1}{1+\omega/\omega_{cut}}$$

  - Energy density will have linear (not quadratic) dependence on ρ_B.
  - "Backward" shift of χ cond. may be avoided.

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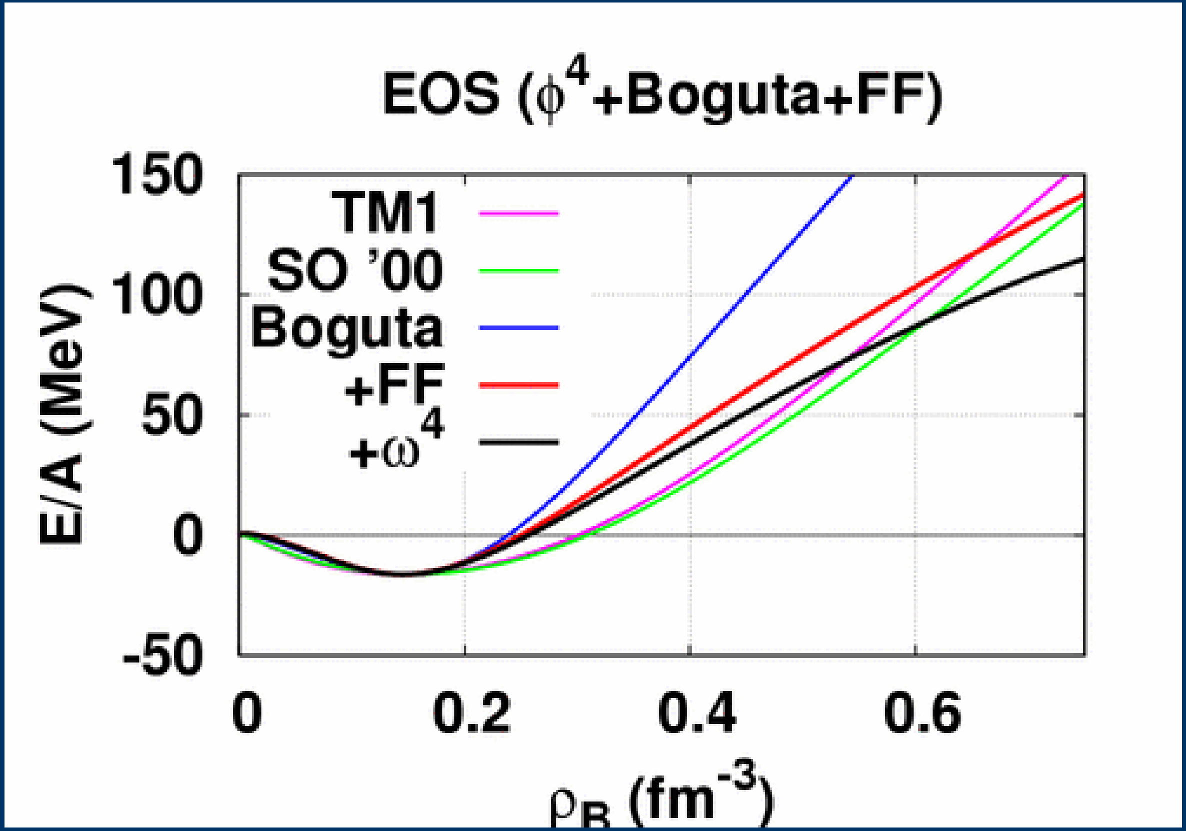
EOS with ωN Form Factor (Φ^4+Boguta+FF+ω^4)

- Intoducing ωN form factor clearly soften EOS
- Furthermore with ω4 term, it becomes softer than TM1 or SO-2000 EOS.
  - How about the behavior at around ρ_0 ?



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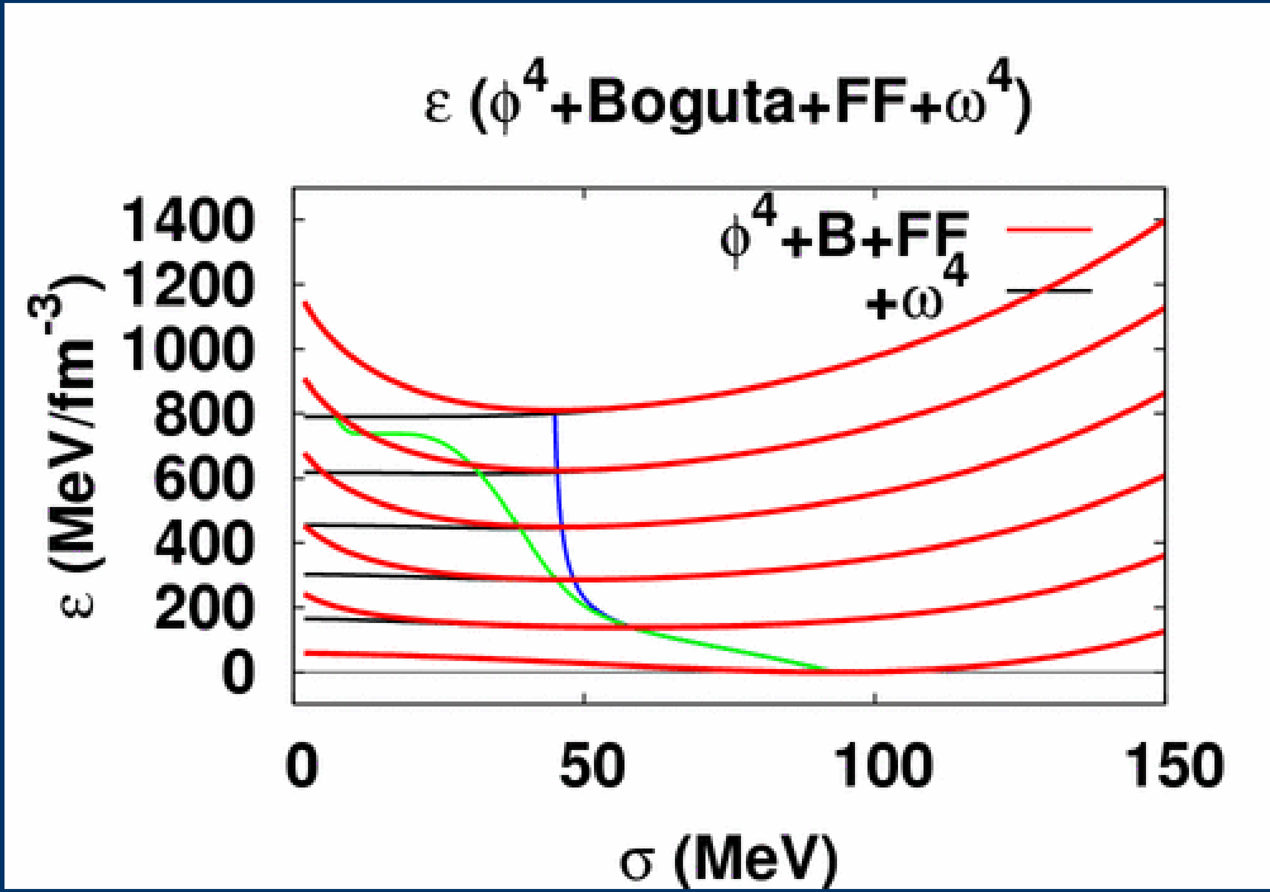
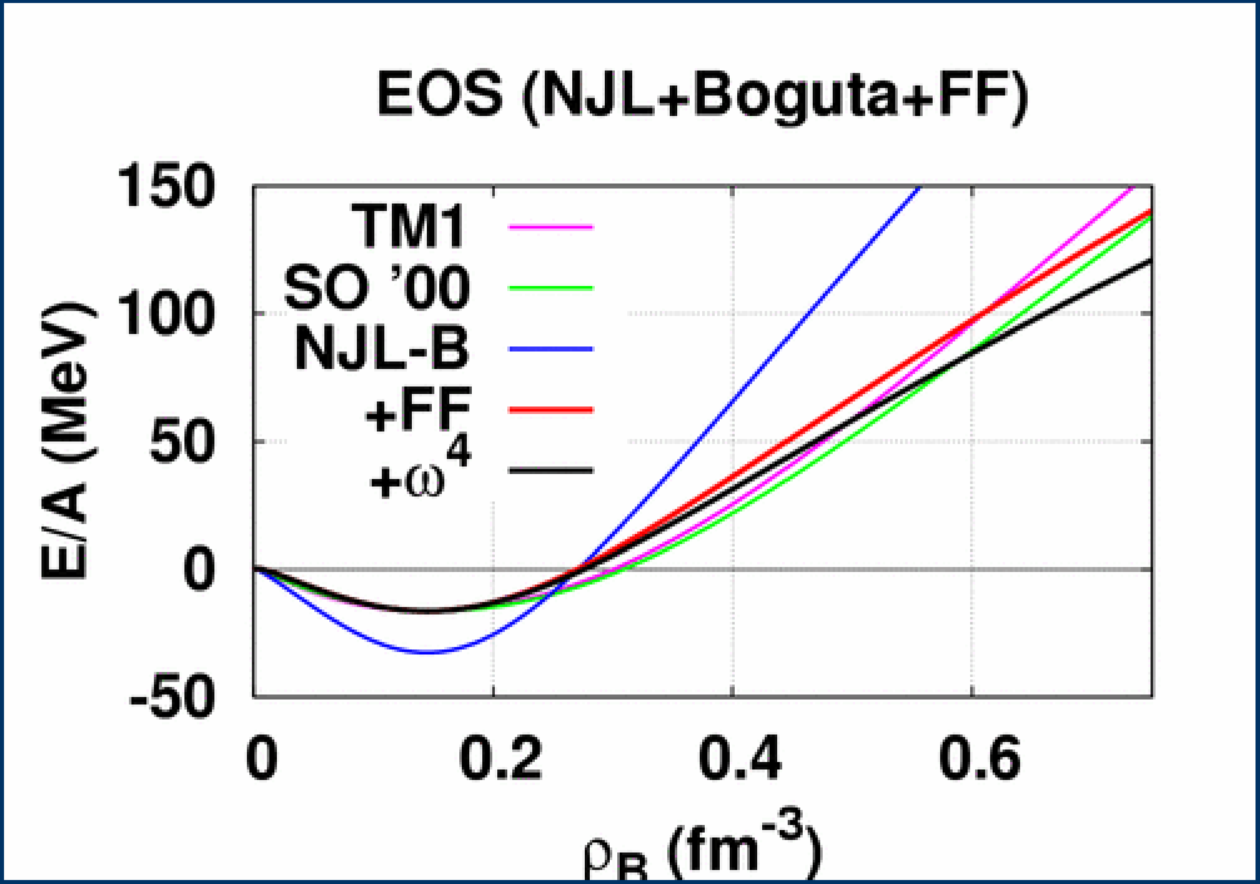
Effective Potential (Φ^4+Boguta+F.F.+ω^4)

- ωN form factor → Repulsive pot. linear in ρ_B.
- ω^4 term → Suppresses divergence at $\sigma \approx 0$
  - Vacuum can be unstable when it is too strong.



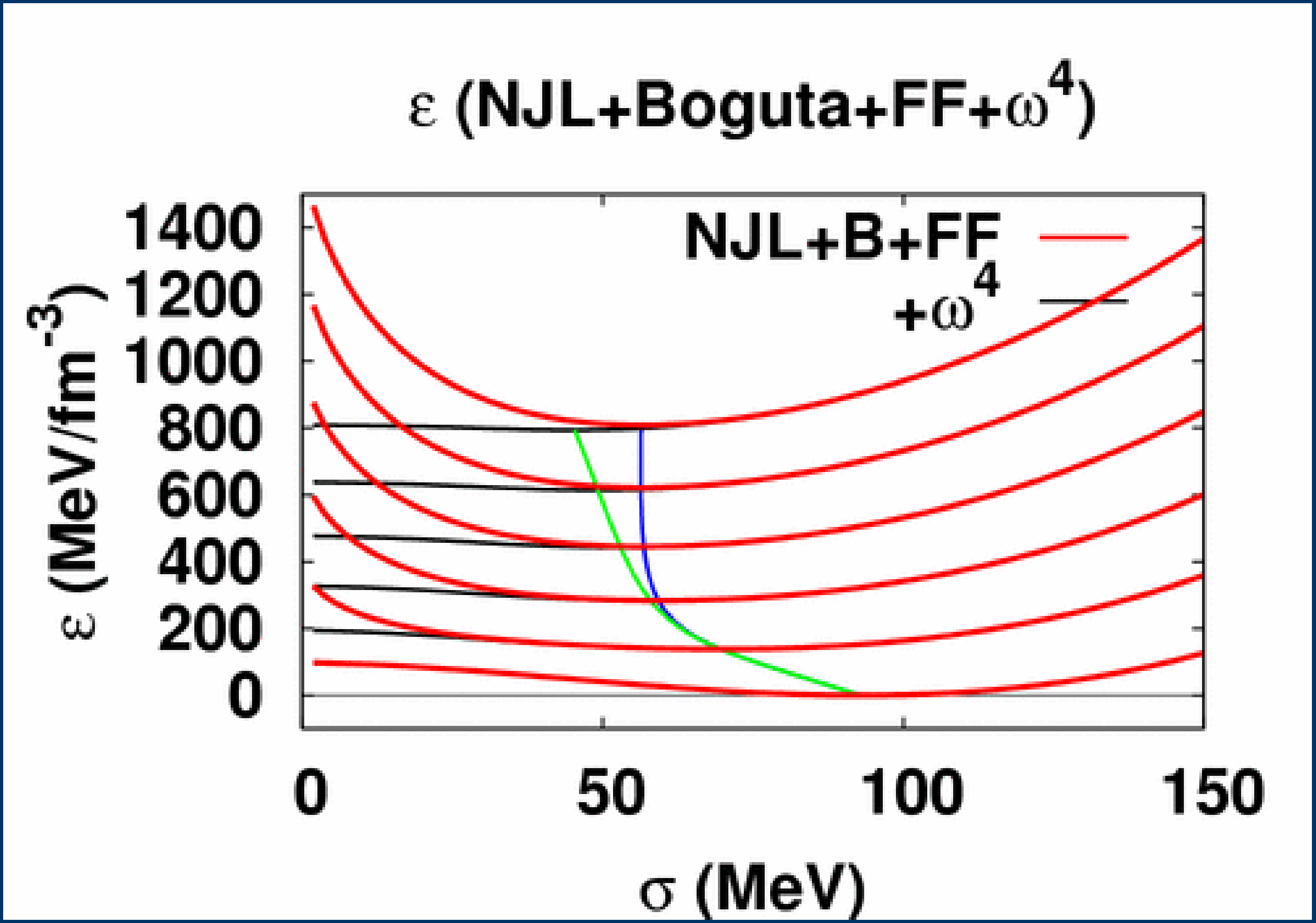
EOS with ωN Form Factor (NJL+Boguta+FF+ω^4)

- Steeper rize of V_σ(NJL) at $\sigma \approx 0$
  - a little more stable than Φ^4 model.
  - We can make softer EOS based on V_σ(NJL).



Effective Potential (NJL+Boguta+F.F.+ω^4)

- Smoother change to χ restored state



Further Consideration: Short Range Int.

- Nucleon mass is fully made of σ ? → NO !
  - Current quark mass: $m_q \approx 5.5$ MeV → very small
  - Short range qq interaction: One Gluon Exchange (OGE)
    ➔ Responsible for NΔ mass splitting (≈ 300 MeV)

$$M_B = M_0 + \sum_i \left(M_i + \frac{k}{M_i} \right) + \sum_{ij} \frac{\alpha \sigma_i \cdot \sigma_j}{M_i M_j}$$

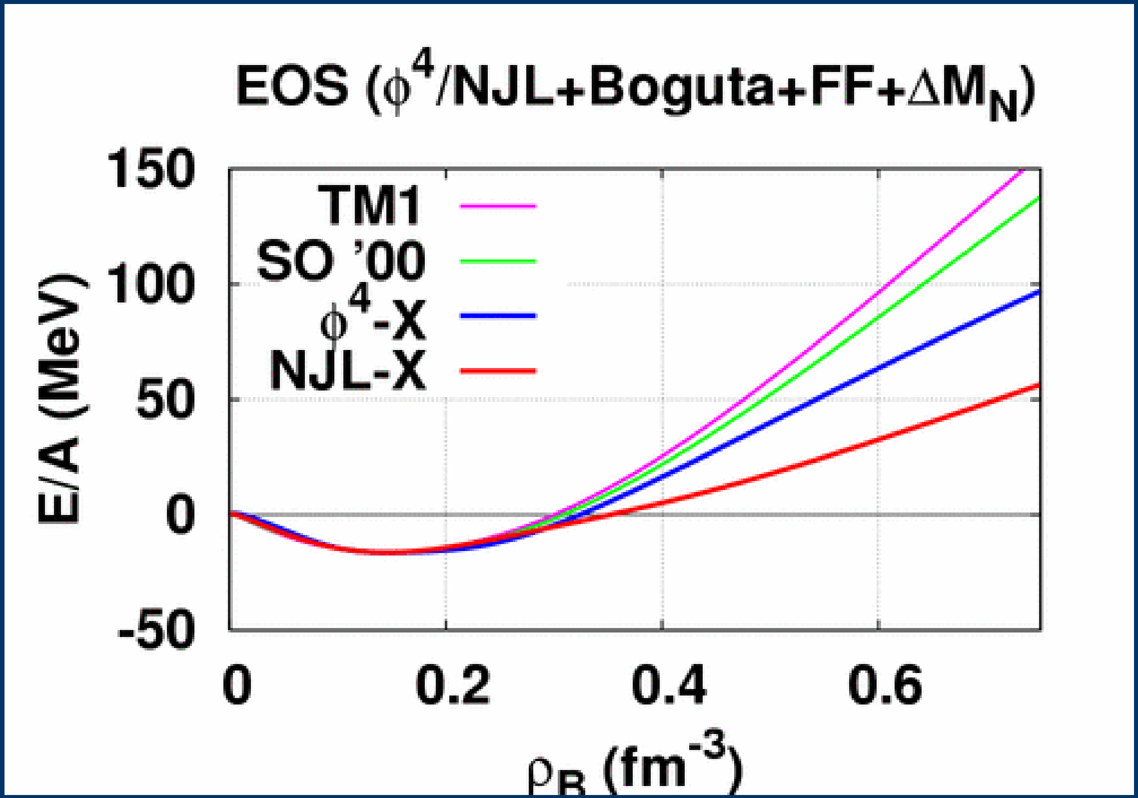
➔ String, Const. Quark Mass (σ), K.E. of Const. Q uarks, OGE

  - Nucleaon Mass would be less than the sum of Const. Q uark Mass

$$M_N = \Delta M + g_\sigma \sigma \quad (\Delta M < 0)$$

Nuclear EOS in the extended model

- Negative M_0 allows us to increase g_σ.
  - ➔ Attractive potential can be large.
- We can make very "Soft" EOS by considering
  - Boguta's $\sigma\omega$ coupling, ωN form factor, ω^4 term, and Short range qq interaction effects,



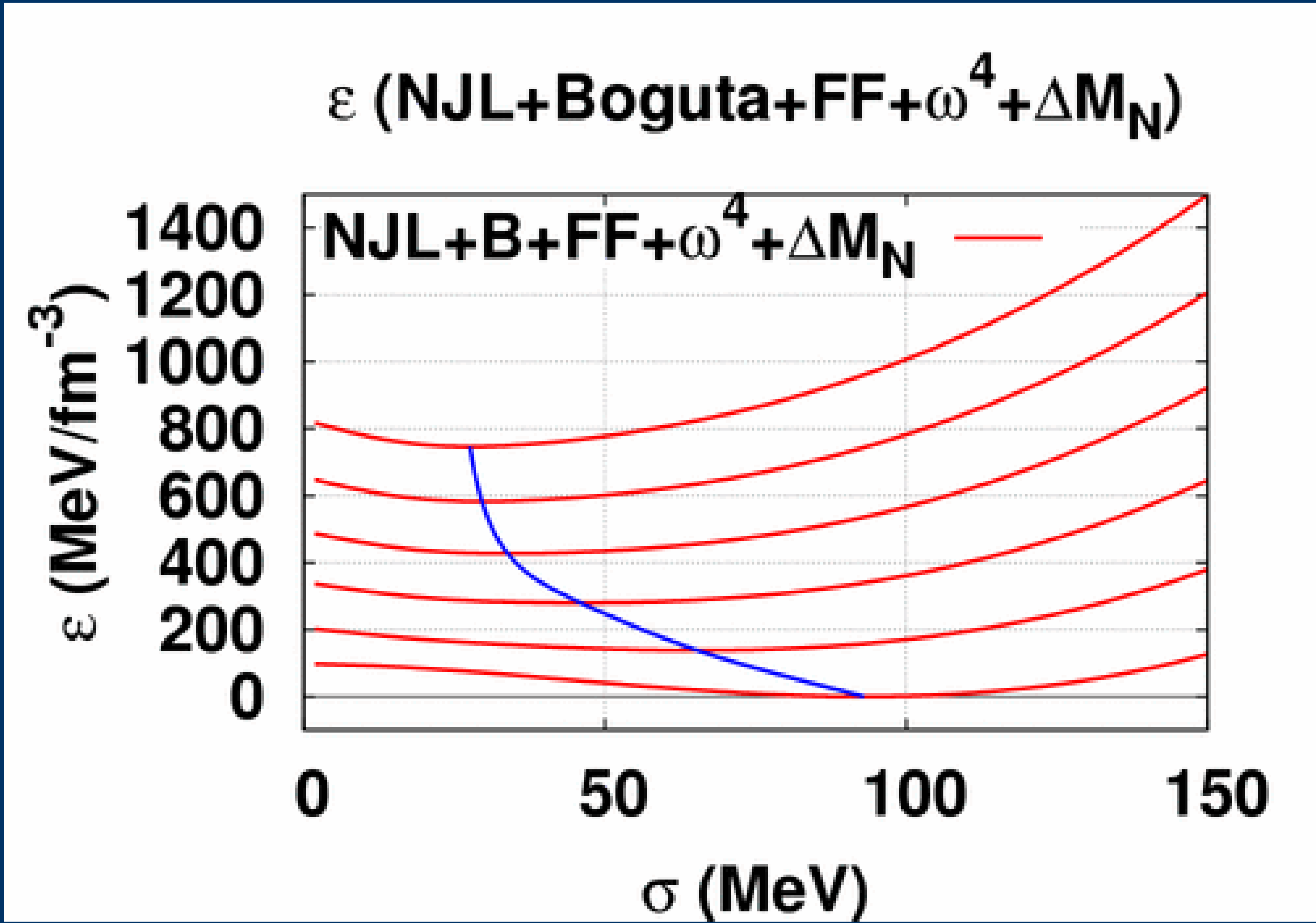
Lagrangian and Parameters

- Lagrangian

$$L = L_\sigma + \bar{N}\left(\gamma^\mu (i\partial_\mu - \omega_\mu F(\omega)) - M_0 - g_\sigma(\sigma + i\tau\pi\gamma_5)\right)N$$

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{m_\omega^2}{2f_\pi^2}\sigma^2\omega^2 + \frac{d_\omega}{4}\left(\omega_\mu\omega^\mu\right)^2$$

- Parameters
  - With NJL σ Lagrangian
    - ➔ ω_{cut} = 142 MeV, M_0 = -200 MeV, d_ω = 20, g_ω = 11.38
    - ➔ ρ_0 = 0.145 fm^{-3}(Fit), $E/A(\rho_0)$=-16.3 MeV (Fit)
    - ➔ K = 303 MeV
  - With Φ^4 Lagrangian
    - ➔ ω_{cut} = 118 MeV, M_0 = -200 MeV, d_ω = 25, g_ω = 14.2
    - ➔ m_σ= 600 MeV
    - ➔ ρ_0 = 0.145 fm^{-3}(Fit), $E/A(\rho_0)$=-16.4 MeV (Fit)
    - ➔ K = 210 MeV



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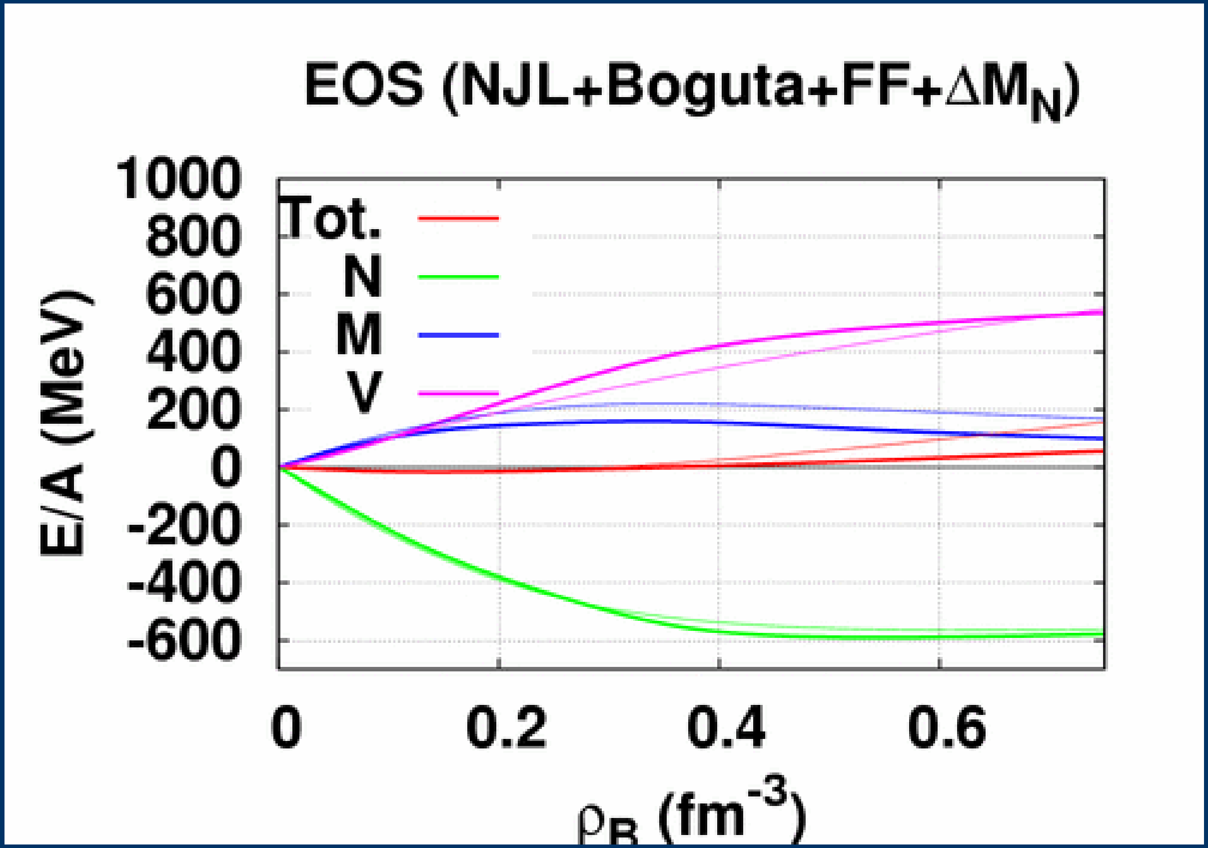
Comparison of Scalar and Vector Pot. with RMF-TM1

- Obtained Lagrangian gives similar behavior of Nucleon Scalar and Vector Potentials to RMF-TM1
  - To be verified in Finite Nuclei / Neutron Stars / Supernovae !



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Summary

- Problems of Nuclear EOS in Chiral Linear σ Models are overviewed.
- Difficulty lies in the dilemma between
  - Vacuum stability: Smooth reduction of χ cond. upto at least ρ_0.
  - Reduction of Vector potential: Boguta's σω coupling gives too much repulsion.
- We have considered following ingredients.
  - σ Lagrangian dependence (Φ^4, NJL, SO-2000, ...)
  - Boguta's σω coupling
  - ωN coupling with form factor
  - ω^4 term, used in RMF-TM1 Lagrangian
  - Short range qq interaction effects (nucleon bare mass)
- By choosing parameters appropriately, we can construct chiral linear σ model giving soft EOS, and the consequent nucleon scalar and vector potential seems to match those in RMF-TM1, which is phenomenologically very successful model.

Time-up

- References: To be shown later
- Future works: You can guess
- Acknowledgements: First to Naito-san, and other members of this lab., and Hatsuda-san.
- NJL explanation; Sorry. Wait for the next time.
- .....