Strong Coupling Limit Lattice QCD Approach to Nuclear Matter and Finite Nuclei

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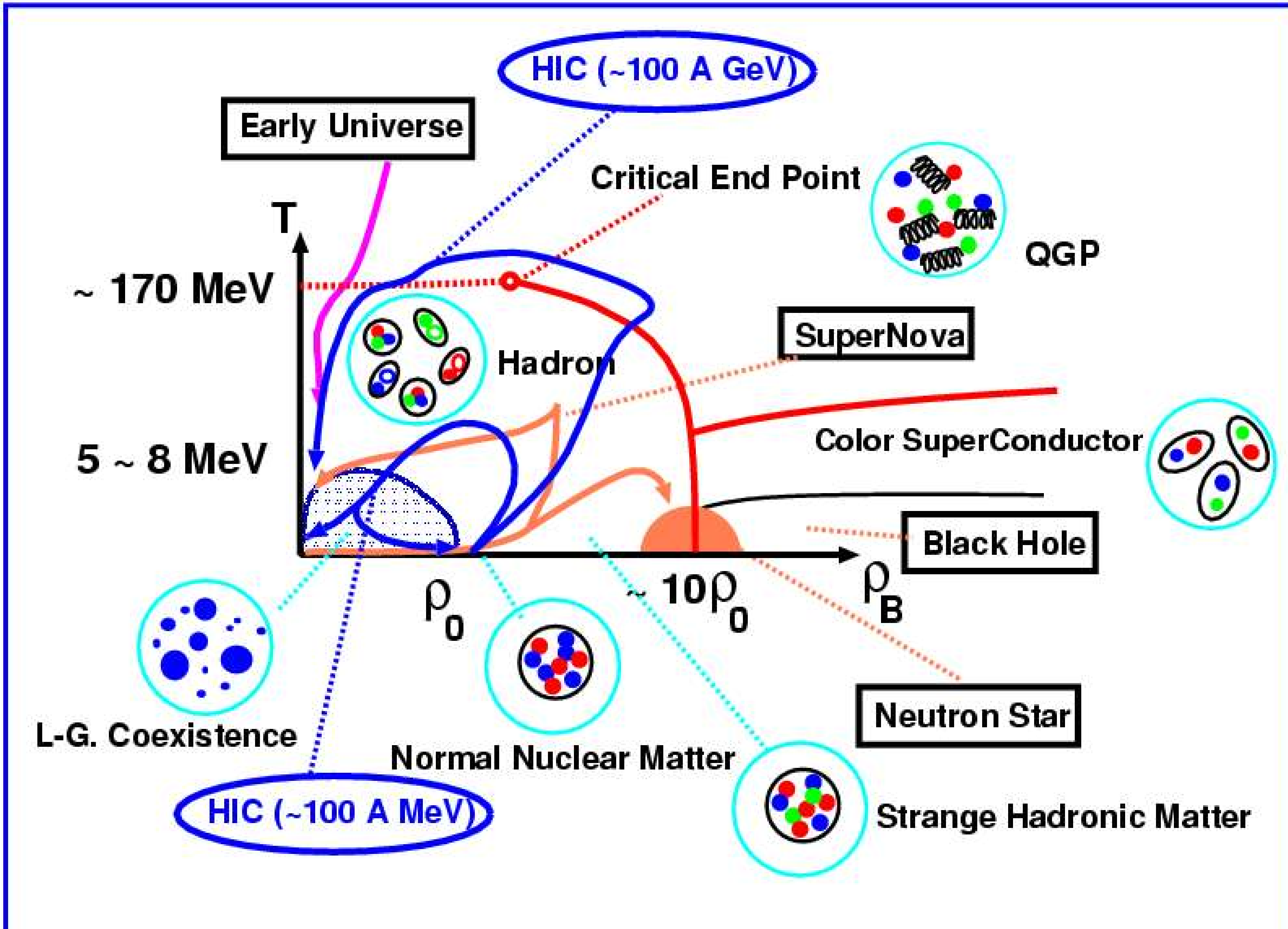
Hadronic Matter Phase Diagram



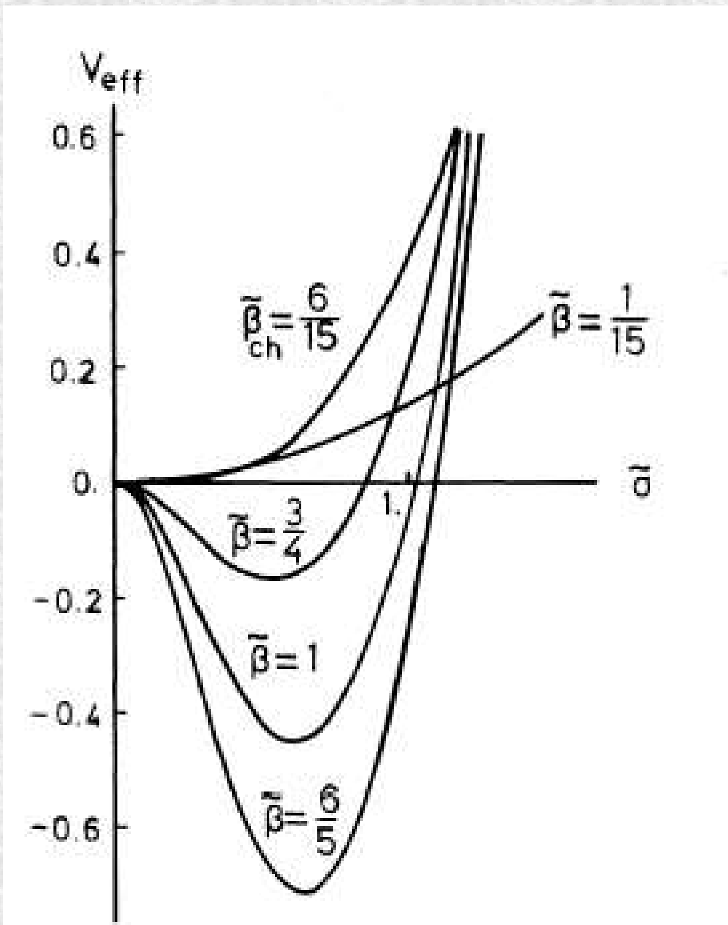
Phase Diagram in Strong Coupling Limit Lattice QCD with N_c=3

Strong Coupling Limit Lattice QCD (1)

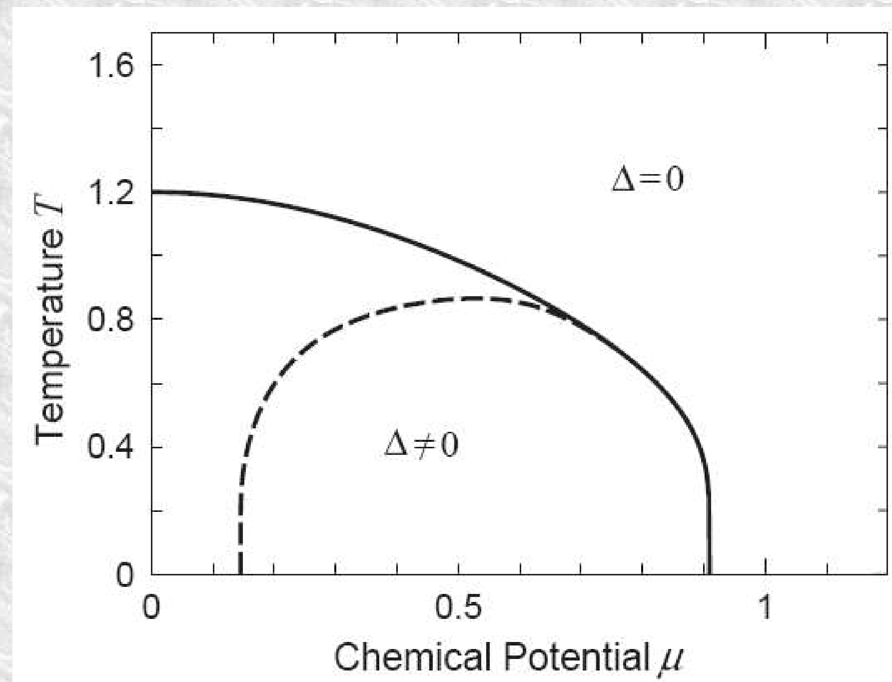
- ***Full Lattice QCD at large μ and low T is not possible***
  - Fermion Det. becomes complex → Monte-Carlo breaks down
  - Small μ → Re-Weighting / Expansion in μ
- ***Strong Coupling Limit: g → ∞***
  - Semi-analytic analyses become possible.
  - At μ=0, Chiral Restoration at high *T* is explained.
    - **Damgaard, Kawamoto, Shigemoto, PRL53(1984),2211**
  - At $\mu \neq 0$ and $N_c = 2$, Phase diagram is drawn.
    → ***Baryon = Boson with $N_c = 2$***
    - **Nishida, Fukushima, Hatsuda, PRept 394(2004),281.**
  - At $\mu \neq 0$ and $N_c = 3$, U_0 integral is done only approximately.
    - **Azcoiti, Di. Carlo, Galante, Laliena, hep-lat/0307019.**

Strong Coupling Limit Lattice QCD (2)

- ***Chiral Restoration at μ=0.***
  - Damgaard, Kawamoto, Shigemoto, PRL53(1984),2211



- ***Phase Diagram with Nc=2***
  - Nishida, Fukushima, Hatsuda, PRept 394(2004),281.



$$F_{\rm eff}[\sigma,\Delta]=\frac{d}{2}\sigma^2+\frac{d}{2}|\Delta|^2-T\log\{1+4\cosh(E_+/T)\cosh(E_-/T)\}$$

$$E_\pm=\mathrm{arccosh}\left(\sqrt{(1+M^2)\cosh^2\mu+(d/2)^2|\Delta|^2}\pm M\sinh\mu\right)$$

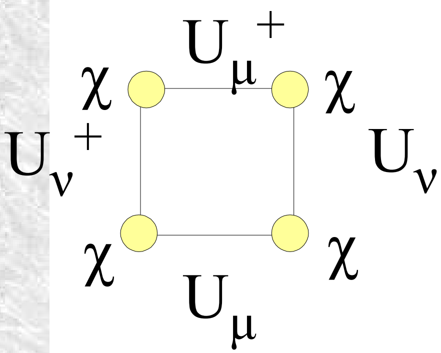
Strong Coupling Limit Lattice QCD (3)

- ***Proper Understanding of QCD phase diagram with $N_c = 3$ is not achieved yet.***
  - Nc=2: Diquark = Color Singlet Boson = Baryon
    → No Fermi Energy for Baryons ?
  - Nc=3: 1/d Expansion also for U_0 term.
    → Conversion would be bad.
    - **Azcoiti, Di. Carlo, Galante, Laliena, hep-lat/0307019.**

- ***This work:***
  - N_c=3: Baryon Integral is required
  - EXACT integral of U_0 term
  - Diquark condensate is tentatively ignored.

Lattice Action in SCL-LQCD (1)

- *Lattice Action with staggered Fermions*



$$S[U,\chi,\bar{\chi}] = S_G[U] + S_F[U,\chi,\bar{\chi}] \ ,$$

$$S_G[U] = \frac{2N_c}{g^2} \sum_{x,\mu,\nu} \left\{ 1 - \frac{1}{N_c} \mathrm{Re}\mathrm{Tr} U_{\mu\nu}(x) \right\}$$

$g \to \infty$ → **0**

$$U_{\mu\nu}(x) = U_\nu^\dagger(x) U_\mu^\dagger(x+\hat{\nu}) U_\nu(x+\hat{\mu}) U_\mu(x) \ ,$$

Chem. Pot.

$$\begin{aligned}
S_F[U,\chi,\bar{\chi}] &= S_F^{(m)}[\chi,\bar{\chi}] + S_F^{(j)}[U_j,\chi,\bar{\chi}] + S_F^{(U0)}[U_0,\chi,\bar{\chi}] \ , \\
S_F^{(m)}[\chi,\bar{\chi}] &= m \sum_x \bar{\chi}^a(x) \chi^a(x) \ , \\
S_F^{(j)}[U_j,\chi,\bar{\chi}] &= \frac{1}{2} \sum_x \sum_{j=1}^{d} \eta_j(x) \left\{ \bar{\chi}(x) U_j(x) \chi(x+\hat{j}) - \bar{\chi}(x+\hat{j}) U_j^\dagger(x) \chi(x) \right\} \ , \\
S_F^{(U0)}[U_0,\chi,\bar{\chi}] &= \frac{1}{2} \sum_x \eta_0(x) \left\{ \bar{\chi}(x) e^{\mu} U_0(x) \chi(x+\hat{0}) - \bar{\chi}(x+\hat{0}) U_0^\dagger(x) e^{-\mu} \chi(x) \right\} \ .
\end{aligned}$$

- *In the Strong Coupling Limit ($g \to \infty$), we can ignore SG, and semi-analytic calculation becomes possible.*

Lattice Action in SCL-LQCD (2)

Integral over U_j (1/d expansion)

- Expand exp(-S_F), and perform U_j integral.

$$\int \mathcal{D}[U] U_{ij} U^\dagger_{kl} = \frac{1}{N_c}\delta_{il}\delta_{jk} \,, \quad \int \mathcal{D}[U] U_{ij} U_{kl} U_{mn} = \frac{1}{N_c!}\varepsilon_{ikm}\varepsilon_{jln}$$

$$S_F^{(j)}[\chi^a, \bar{\chi}^a] = -\frac{1}{2}(M, V_M M) - (\bar{B}, V_B B)$$

$$M(x) = \delta_{ab}\bar{\chi}^a(x)\chi^b(x) \,,$$

$$B(x) = \frac{1}{6}\varepsilon_{abc}\chi^a(x)\chi^b(x)\chi^c(x) \,, \quad \bar{B}(x) = \frac{1}{N_c!}\varepsilon_{abc}\bar{\chi}^c(x)\bar{\chi}^b(x)\bar{\chi}^a(x)$$

Bosonization (Auxiliary Field)

$$\begin{aligned} S_F^{(q)} &= (\bar{b}, \widetilde{V}_B^{-1} b) + L^3\beta\left(\sigma^2/2\alpha^2 + |\Phi|^2/4\gamma^2\right) \\ &+ (\bar{\chi}\sigma\chi) + \frac{1}{12\gamma^2}\left[(\bar{\chi}^a, \phi_a^\dagger b) + (\bar{b}\phi_a, \chi^a)\right] + \frac{1}{2}\varepsilon_{cab}\left[(\phi_c^\dagger, \chi^a\chi^b) - (\bar{\chi}^a\bar{\chi}^b, \phi_c)\right] + S_F^{(U0)} \end{aligned}$$

- Action of quark (χ), gluon (U_0), sigma (σ), Diquark (φ), ***baryon (b).***
- Bi-Linear in Grassmann variables (χ and b)
  → Pfaffian Integral, Two -log (det) Terms in Effective Action

Lattice Action in SCL-LQCD (3)

- ***Fermion Integral, Matsubara Freq. Sum, and U_0 Integral***

  → Effective Action at Zero Diquark Condensate

$$F_{\rm eff} = \frac{\sigma^2}{2\alpha^2} + F_{\rm eff}^{(q)} + F_{\rm eff}^{(b)}$$

$$F_{\rm eff}^{(q)} = -T \log\left(C_\sigma^3 - \frac{1}{6} C_\sigma C_\mu^2 - \frac{1}{3} C_\sigma + \frac{3}{4} C_\mu^3 - \frac{1}{2} C_\mu \right)$$

$$C_\sigma = \cosh\left[\beta \sinh^{-1}\sigma\right] \ , \quad C_\mu = \cosh\beta\mu$$

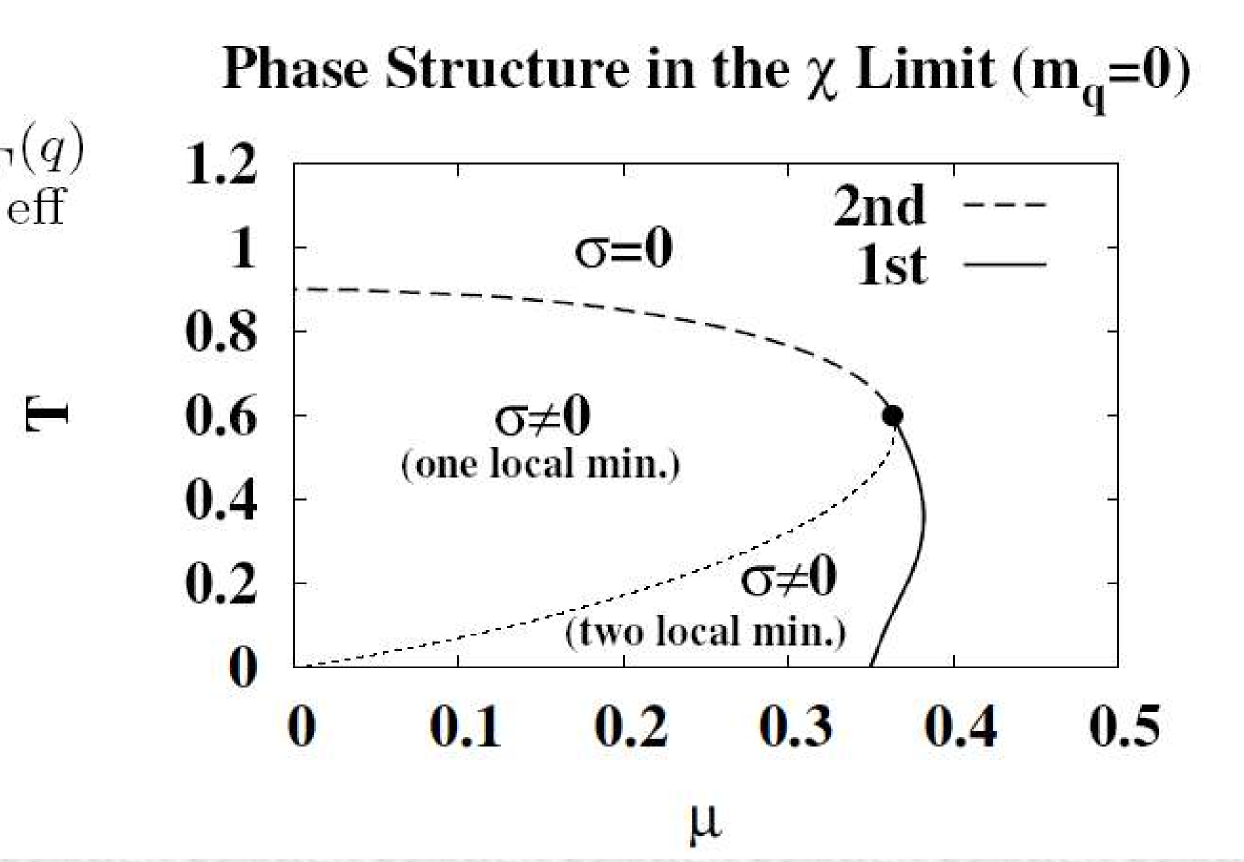
$$F_{\rm eff}^{(b)} \simeq -a_0^{(b)} f^{(b)}(c\Lambda) \ , \quad f^{(b)}(x) = \frac{3}{2x^3}\int_0^x k^2 dk \log(1+k^2)$$

- ***Two Types of Fermion Log(Det) Terms !***

Phase Diagram

- *Minimum of Effective Action → Phase Diagram*
  - Kawamoto, Miura, AO, Ohnuma, in preparation.

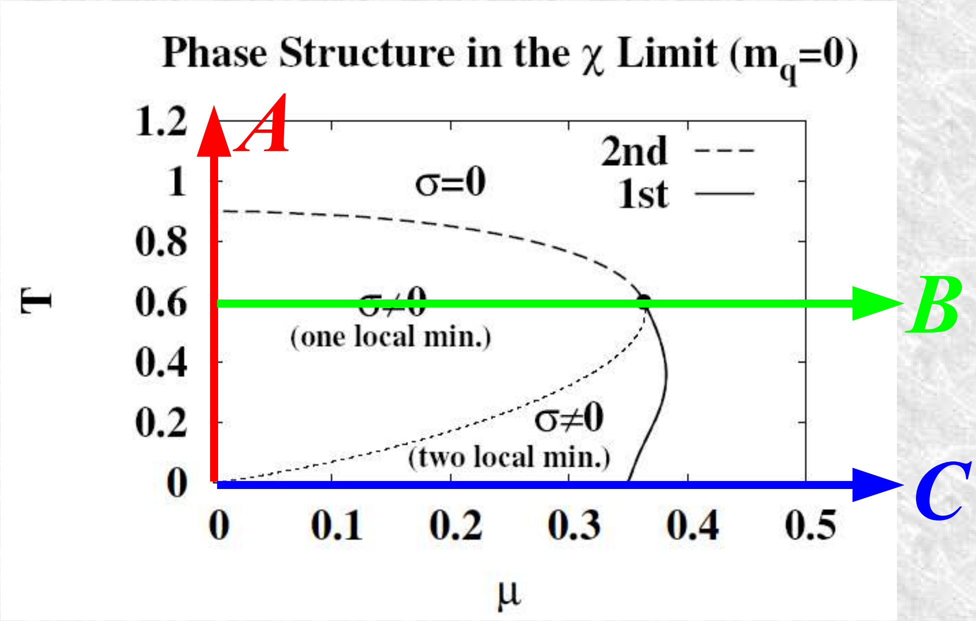
$$F_{\rm eff} = \frac{\sigma^2}{2\alpha^2} + F_{\rm eff}^{(b)} + F_{\rm eff}^{(q)}$$

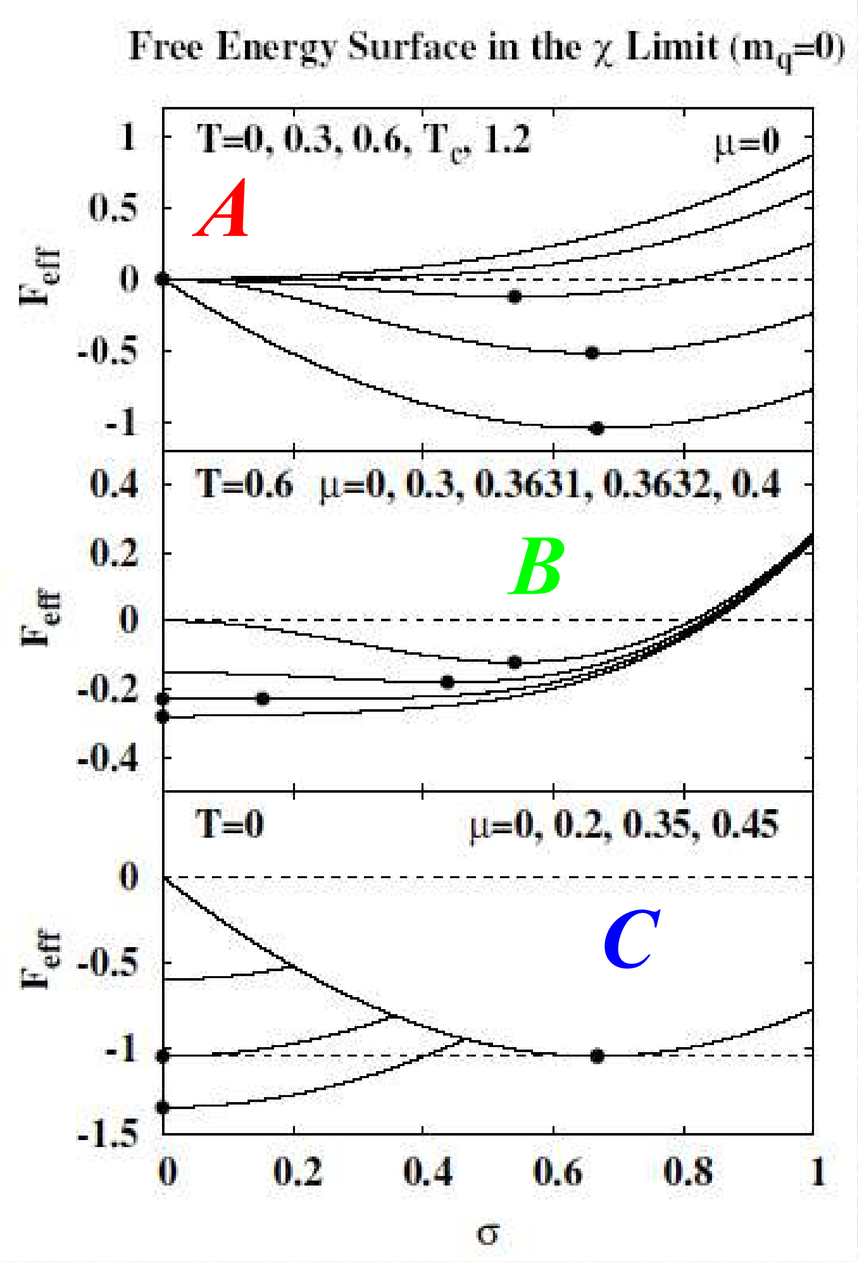


- *Change from 2nd order to 1st order at Finite μ*

Free Energy Surface

- *At $\mu \neq 0$, quark can gain Free Energy even at $\sigma = 0$*
  → Two Min. Structure
  → First Order





RMF with σ Self Energy from Strong Coupling Limit Lattice QCD

RMF with Chiral Symmetry (1)

- ***Good (approximate) Symmetry in QCD***
  - Only the current quark mass terms break chiral sym.
  - Spontaneously Broken, and $\langle \bar{q} q \rangle$ determines hadron masses
- ***Schematic model: Linear σ model***

$$L = \frac{1}{2}\left(\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \pi \partial^\mu \pi\right) - \frac{\lambda}{4}\left(\sigma^2 + \pi^2\right)^2 + \frac{\mu^2}{2}\left(\sigma^2 + \pi^2\right) + c\,\sigma$$
$$+ \bar{N} i \partial_\mu \gamma^\mu N - g_\sigma \bar{N}\left(\sigma + i \pi \tau \gamma_5\right) N$$

- ***Problem: χ Sym. is restored at a very small density.***
  - Smaller Nucleon Mass Energies are preferred
  - σω Coupling stabilizes normal vacuum, but gives *Too Stiff EOS*
    - **J. Boguta, PLB120,34/PLB128,19.**
    - **Ogawa et al. PTP 111 (2004)75.**

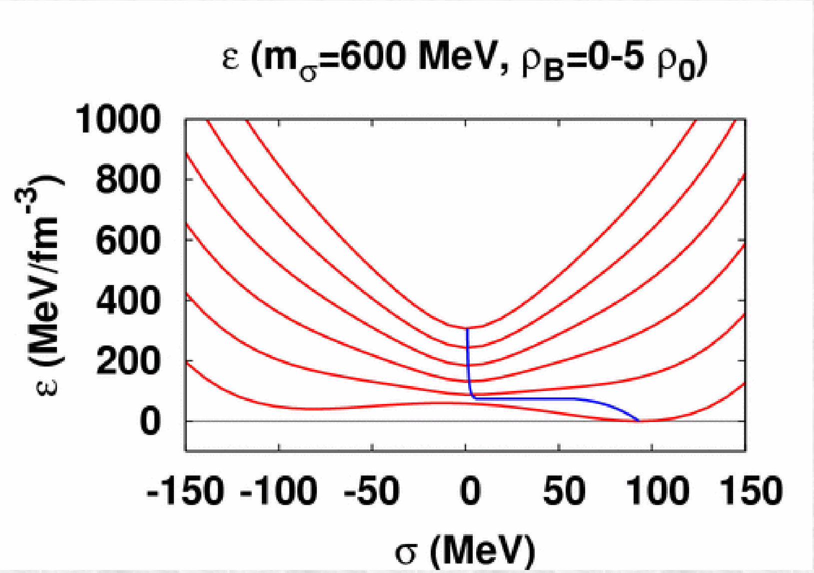
RMF with Chiral Symmetry (2)

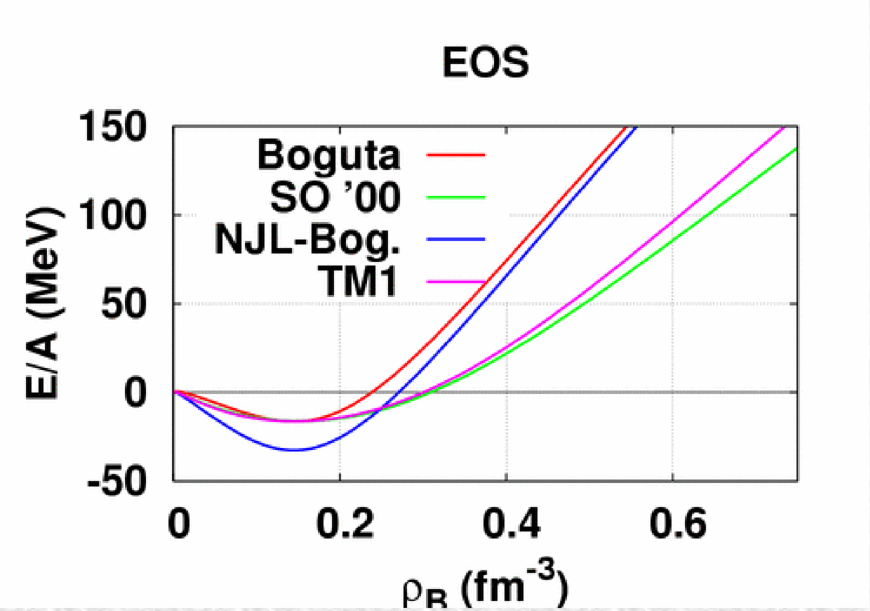
- **Sudden Change of <σ>**
- **σ ω Coupling**

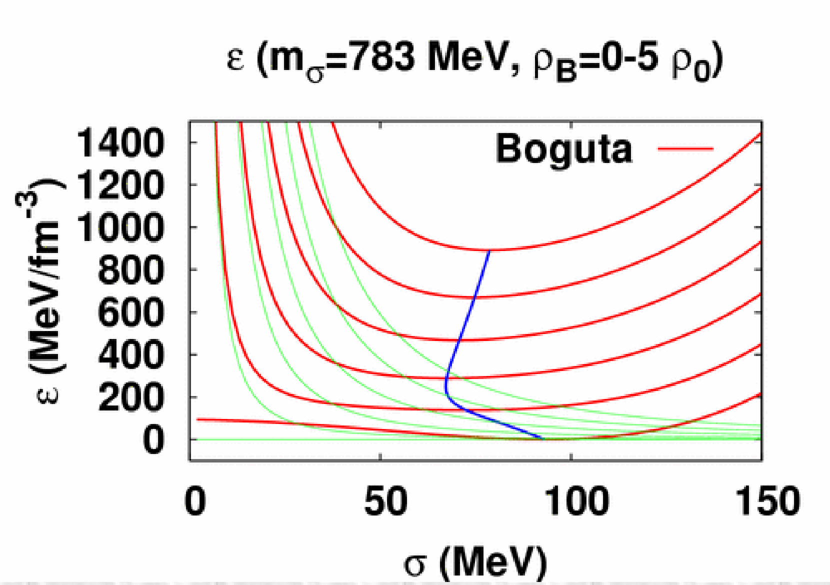
$$L_{\omega\sigma} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} C_{\sigma\omega} \sigma^2 \omega^2 - g_\omega \overline{N} \gamma_\mu \omega^\mu N$$

$$\omega = g_\omega \rho_B / C_{\sigma\omega} \sigma^2 \quad \rightarrow \quad V_{\sigma\omega} = \frac{g_\omega^2 \rho_B^2}{2 C_{\sigma\omega} \sigma^2}$$

- **Stiff EOS**







RMF with σ Self Energy from SCL-LQCD

- **σ Self Energy from simple Strong Coupling Limit LQCD**

$$
\begin{aligned}
S \quad &\to \quad -\tfrac{1}{2}(M, V_M M) && (1/d \text{ expansion})\\
&\to \quad b\sigma^2 + (\bar{\chi}\ \sigma\chi) && (\text{auxiliary field})\\
&\to \quad b\sigma^2 - a\log\sigma^2 && (\text{Fermion Integral})
\end{aligned}
$$

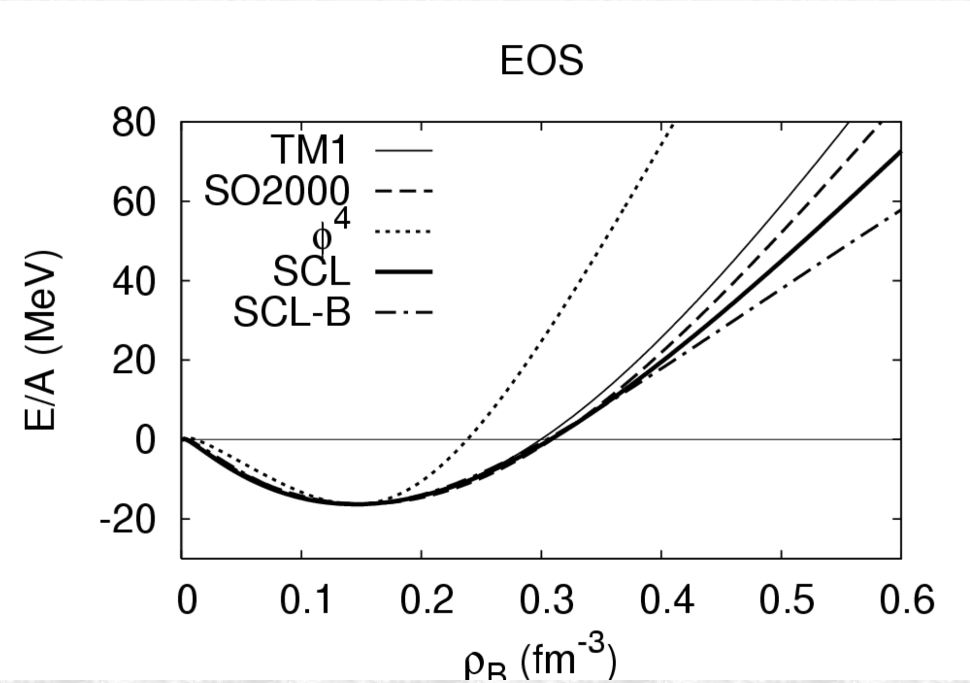
- **RMF Lagrangian**
  - σ is shifted by f_π , and small explicit χ breaking term is added.

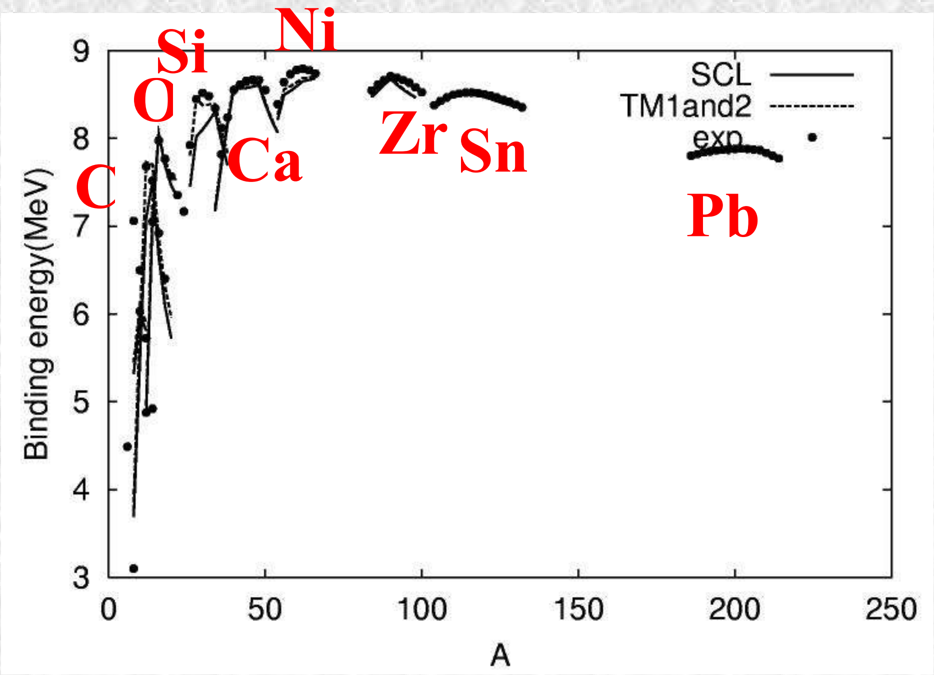
$$
\mathcal{L} = \bar{\psi}\left(i\gamma^\mu\partial_\mu - \gamma^\mu V_\mu - M + g_\sigma\sigma\right)\psi + \mathcal{L}_\sigma^{(0)} + \mathcal{L}_\omega^{(0)} + \mathcal{L}_\rho^{(0)} - U_\sigma + \frac{\lambda}{4}(\omega_\mu\omega^\mu)^2
$$

$$
U_\sigma = -af\left(\frac{\sigma}{f_\pi}\right) , \quad f(x) = 2\log(1+x) - 2x + x^2 , \quad a = \frac{f_\pi^2}{4}(m_\sigma^2 - m_\pi^2)
$$

Nuclear Matter and Finite Nuclei

- ***Nuclear Matter***
  - By tuning λ, $g_{\omega N}$, m_σ, ***Soft EOS*** can be obtained in Chirally Symmetric RMF
- ***Finite Nuclei***
  - By tuning gρN, Global behavior of Nuclear B.E. is reproduced, except for j-j closed nuclei. (C, Si, Ni)





Summary

- ***While full Lattice QCD is not (yet?) applicable to study low T and high ρ matter, we can obtain qualitative feature of the Phase Diagram with the Strong Coupling Limit of LQCD with Nc=3.***
  - With Nc = 3, Two Fermion Integrals would give different results from Nc=2 case.
  - 2nd order → 1st order as μ increases. (Chiral Limit)
- ***While many chiral symmetric RMFs give too stiff EOS, log σ type self energy seems to give softer EOS.***
  - log σ term prevents the normal vacuum to collapse.
  - It is also applicable to Hypernuclei (cf. Tsubakihara @ JPS)
- ***Interplay between quark nuclear physics and nucleon nuclear physics will be very beneficial !***

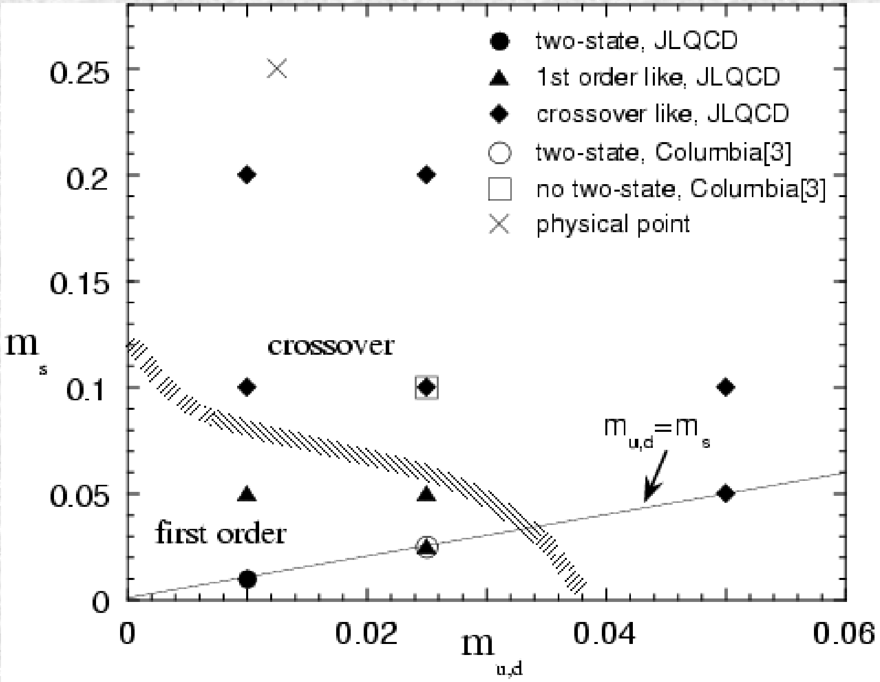
Collaborators

- ***T. Ohnuma (M1)***
- ***N. Kawamoto (Hokkaido U.)***
- ***K. Miura (M2)***
- ***K. Tsubakihara (M1)***
- ***K. Naito***

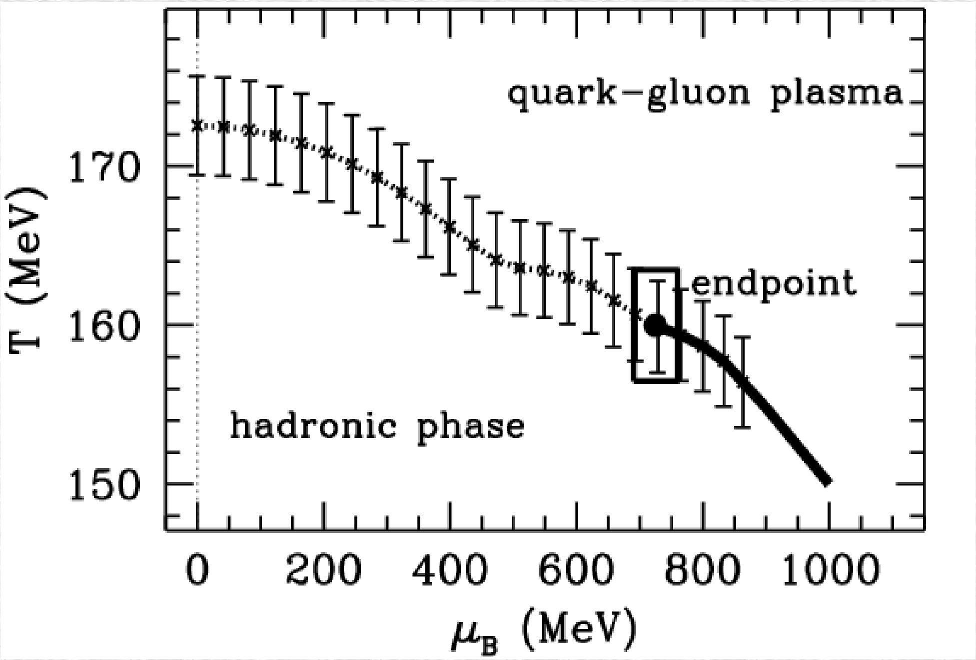
Thank You !

QCD Phase Diagram from Lattice QCD

- *Zero Chem. Pot.*



- *Finite Chem. Pot.*



☆ JLQCD Collab. (S. Aoki et al.), Nucl. Phys. Proc. Suppl. 73 (1999), 459.

☆ Fodor & Katz, JHEP 0203 (2002), 014.

Zero Chem. Pot. : Cross Over
Finite Chem. Pot.: Critical End Point

Lattice Action in SCL-LQCD (2')

- *Auxiliary Fields*

$$\exp\left[(\bar{B}, V_B B)\right] = \det V_B \int \mathcal{D}[\bar{b}, b] \exp\left[-(\bar{b}, V_B^{-1} b) + (\bar{b}, B) + (\bar{B}, b)\right]$$

$$\exp\left(\bar{b}B + \bar{B}b\right)$$
$$= \int \mathcal{D}[\phi_a, \phi_a^\dagger] \exp\left\{-\frac{1}{4\gamma^2}\phi_a^\dagger\phi_a - \frac{1}{2\gamma}(\phi_a^\dagger D_a + D_a^\dagger\phi_a) + \frac{1}{36\gamma^2}M\bar{b}b - 2\gamma^2 M^2\right\}$$

$$D_a = \gamma\varepsilon_{abc}\chi^b\chi^c + \frac{1}{6\gamma}\bar{\chi}^a b\ , \quad D_a^\dagger = -\gamma\varepsilon_{abc}\bar{\chi}^b\bar{\chi}^c + \frac{1}{6\gamma}\bar{b}\chi^a$$

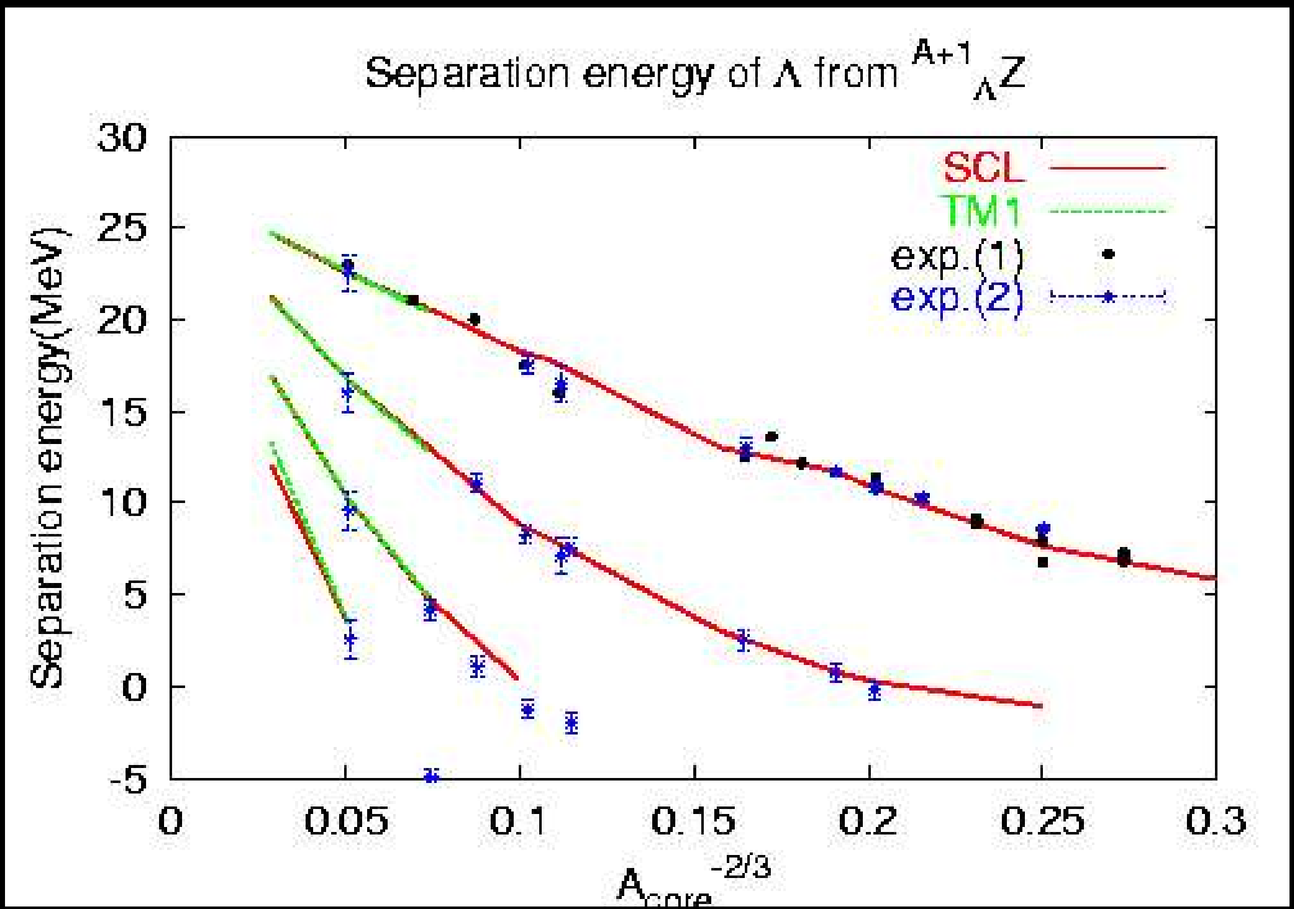
$$\exp\left[\frac{1}{36\gamma^2}M\bar{b}b\right] = \int \mathcal{D}[\omega] \exp\left[-\frac{1}{2g_\omega^2}\omega^2 - \frac{\omega}{g_\omega}(\alpha M + g_\omega\bar{b}b) - \frac{1}{2}\alpha^2 M^2\right]$$

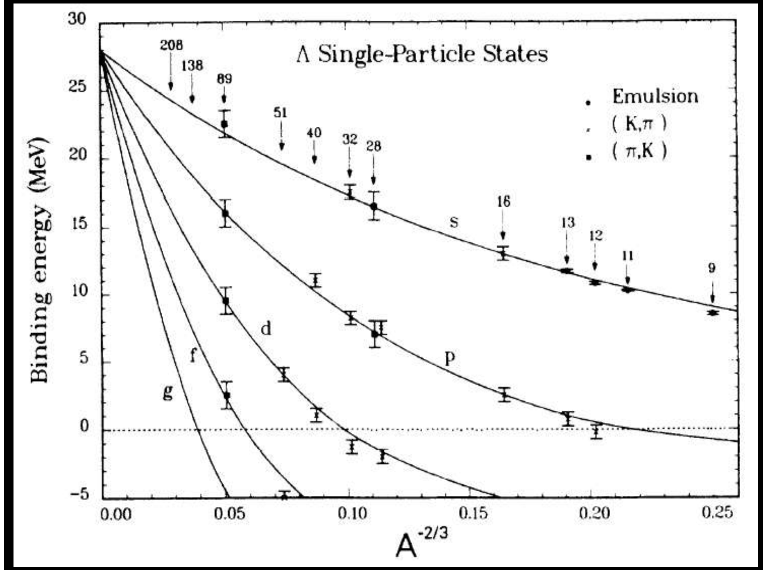
$$\exp\left[\frac{1}{2}(M, \widetilde{V}_M M) - \frac{\alpha}{g_\omega}(\omega, M)\right] = \int \mathcal{D}[\sigma] \exp\left[-\frac{1}{2}(\sigma', \widetilde{V}_M^{-1}\sigma') - (\sigma, M)\right]$$

$$\sigma'(x) = \sigma(x) - \alpha\omega(x)/g_\omega\ .$$

Single Λ Hypernucleus

- Λ の分離エネルギー





•各一粒子状態の分離エネルギーを良く再現する事が出来ている