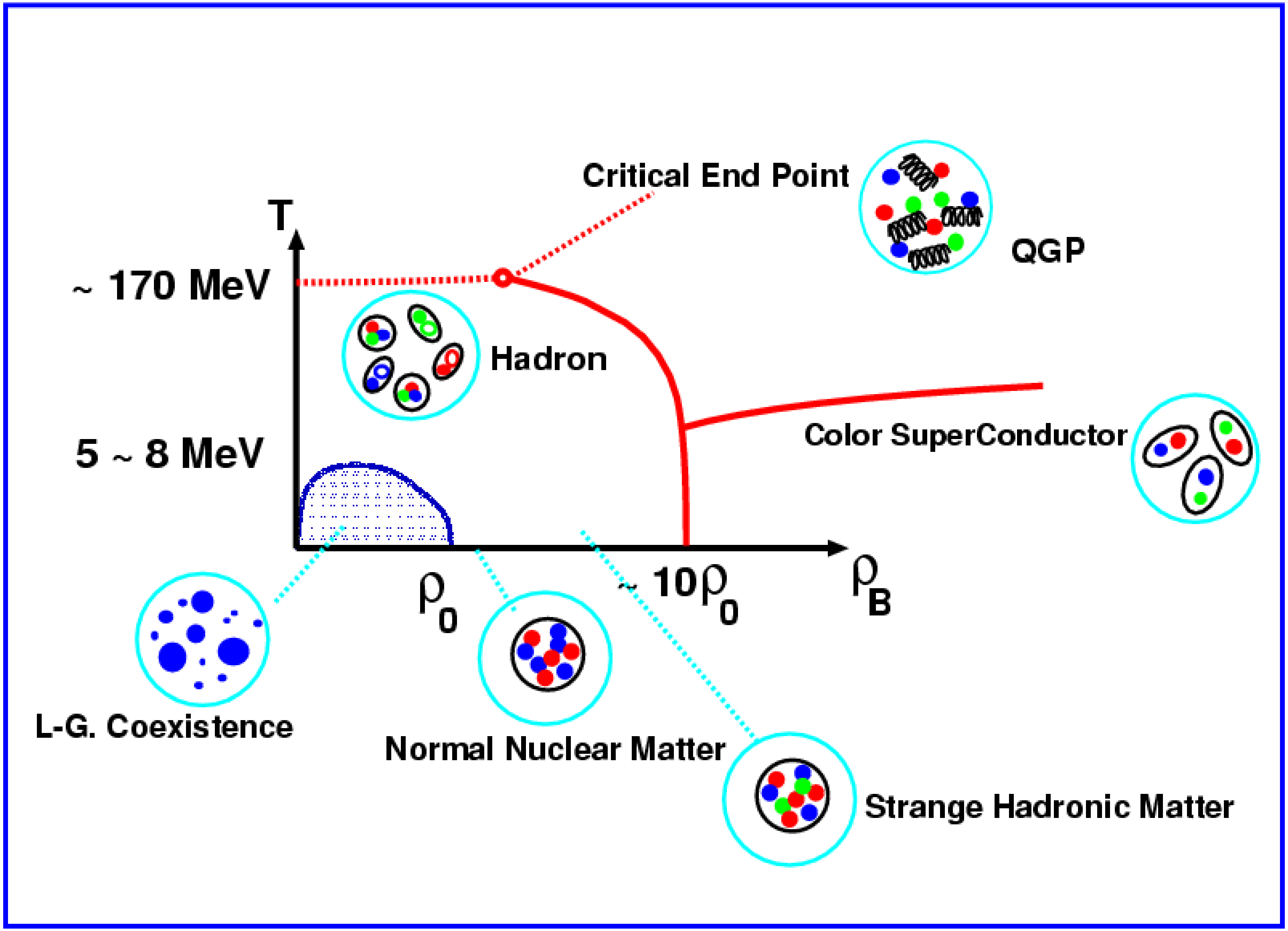
Fundamental and Phenomenological Approaches to High Density Hadronic Matter

Akira Ohnishi *in Collaboration with*
N. Kawamoto, K.Miura, T.Ohnuma, K. Tsubakihara,
M.Isse, Y.Nara, N.Otuka, P.K.Sahu, T. Hirano, K. Yoshino

- **Introduction --- Approaches to High Density Matter**
- **Strong Couling Limit Lattice QCD**
  (N. Kawamoto, K. Miura, AO, T. Ohnuma, hep-lat/051223)
- **Chiral Symmetric RMF (K.Tsubakihara,AO, in preparation)**
- **Collective Flow in High-Energy Heavy-Ion Collisions**
  (M.Isse, AO, N.Otuka, P.K.Sahu, Y.Nara, PRC72(2005),064908;
  T.Hirano, M.Isse, Y.Nara, AO, K. Yoshino, PRC72(2005),041901(R);
  P.K.Sahu,AO,M.Isse, N.Otuka, S.C.Phatak, submitted)
- **Summary**

Hadronic Matter Phase Diagram



- **Physics @ J-PARC: High Density QCD → How to attack it ?**

Physics of Dense Matter

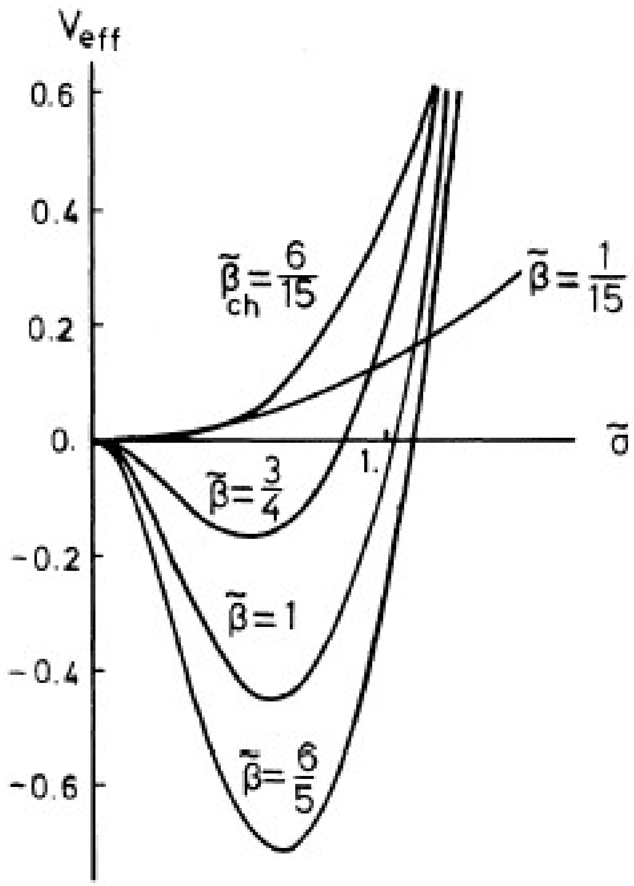
- High T → well studied theoretically and experimentally
  - Lattice QCD Monte-Carlo simulation / RHIC, SPS
- High Density Matter→ Interesting but Difficult in QCD
  - Exp't: FAIR(GSI), SPS(20-80 AGeV), AGS (10 A GeV)
  - Theor.: Weight becomes complex at finite μ in Lattice QCD
    → *Model/Approximate approaches are necessary !*
  - Monte-Carlo calc. of Lattice QCD (c.f. Ejiri's talk)
    Improved ReWeighting (Fodor-Katz)
    Taylor Expansion (Bielefeld U.)
    Analytic Continuation (de Forcrand-Philipssen), ...
  - Approximate / Model / Phenomenological Approaches:
    Strong Coupling Limit of Lattice QCD
    NJL (Hatsuda-Kunihiro, ...)
    Kaon/pion condensation (Lee, Muto, ...)
    Relativistic Mean Field, HIC simulation

Strong Coupling Limit of Lattice QCD for Color SU(3) with Baryon Effects

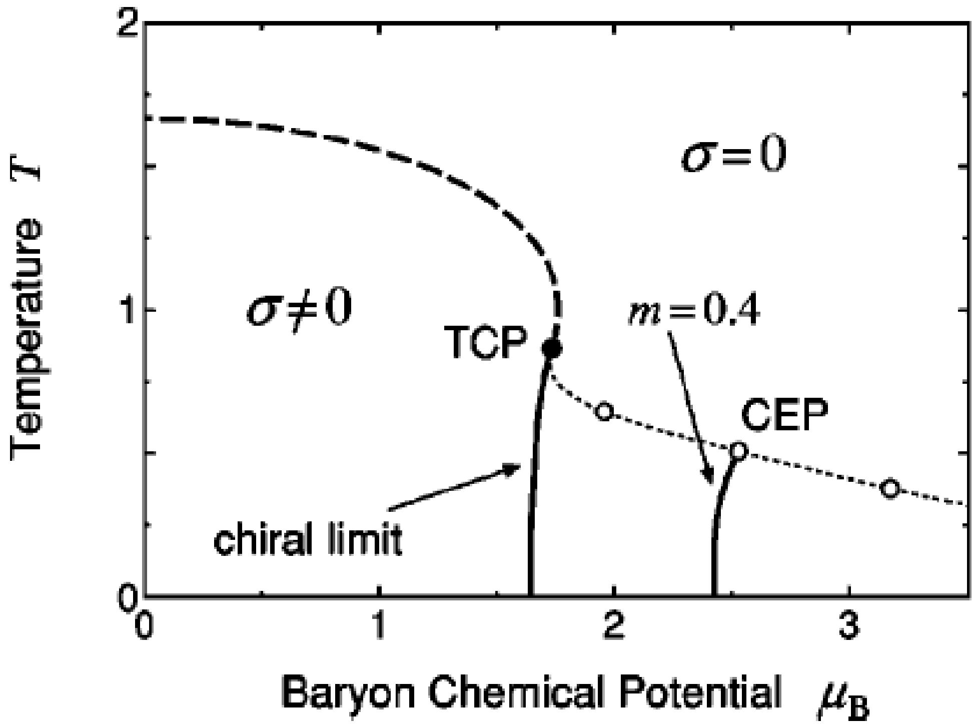
N. Kawamoto, K. Miura, AO, T. Ohnuma, hep-lat/051223

Strong Coupling Limit of Lattice QCD

- **Chiral Restoration at μ=0.**
  - **Damgaard, Kawamoto, Shigemoto, PRL53(1984),2211**



- **Phase Diagram with Nc=3**
  - **Nishida, PRD69, 094501 (2004)**



Previous Works in Strong Coupling Limit LQCD

- **Strong Coupling Limit Lattice QCD re-attracts interests**
  c.f. Nakamura @ JHF Symp. for high density matter

Ref	T	μ	Nc	Baryon	CSC	Nf
Damgaard-Kawamoto-Shigemoto('84)	*Finite*	*0*	*U(Nc)*	*X*	*X*	*1*
Damgaard-Hochberg-Kawamoto('85)	*0*	*Finite*	*3*	*Yes*	*X*	*1*
Bilic-Karsch-Redlich('92)	*Finite*	*Finite*	*3*	*X*	*X*	*1 ~ 3*
Azcoiti-Di Carlo-Galante-Laliena('03)	*0*	*Finite*	*3*	*Yes*	*Yes*	*1*
Nishida-Fukushima-Hatsuda('04)	*Finite*	*Finite*	*2*	*Yes (*)*	*Yes (*)*	*1*
Nishida('04)	*Finite*	*Finite*	*3*	*X*	*X*	*1~2*
Kawamoto-Miura-AO-Ohnuma('05)	***Finite***	***Finite***	***3***	***Yes***	***Yes (+)***	***1***

**: bosonic baryon=diquark in SU(2)*
+: analytically included, but ignored in numerical calc.

- ***Baryonic Composite will be important at High Densities, but they have been ignored in finite T treatments !***
  → This work: Nc = 3, Baryonic Composite, Finite T and μ

Strong Coupling Limit without Baryonic Effects

- **Lattice Action (staggered fermion)**
- **Spatial Link Integral**
- **Bosonization (HS transf.)**
- **Quark and U_0 Integral**
- **Mesonic and Baryonic Composites**

Strong Coupling (the S_G term is crossed out)

$$Z = \int \mathcal{D}[\chi,\bar\chi,U_0,U_1,U_2,U_3] \exp\left[-S_F^{(U_0)} - \sum_{j=1}^{3} S_F^{(U_j)} - S_F^{(m)} - S_G\right]$$

1/d Expansion (1/√d) (the $(\bar B, V_B B)$ term is crossed out)

$$\simeq \int \mathcal{D}[\chi,\bar\chi,U_0] \exp\left[\frac{1}{2}(M,V_M M) + (\bar B, V_B B) - S_F^{(U_0)} - S_F^{(m)}\right]$$

$$\simeq \int \mathcal{D}[\chi,\bar\chi,U_0,\sigma] \exp\left[-\frac{1}{2}(\sigma,V_M^{-1}\sigma) \underbrace{-(\sigma,M) - S_F^{(U_0)} - S_F^{(m)}}_{(\bar\chi, G^{-1}(\sigma)\chi)}\right]$$

$$\simeq \int \mathcal{D}[\sigma] \exp\left[-\frac{1}{2}a_\sigma \sigma^2\right] \underbrace{\prod_{\mathrm{x}} \int dU_0 \ \mathrm{Det}\ \left[G^{-1}(\sigma)\right]}_{\exp\left[-L^3 \beta F^q(\sigma)\right]}$$

$$\simeq \exp\left[-L^3 \beta F_{\mathrm{eff}}(\sigma)\right]$$

$$M(x) = \delta_{ab}\bar\chi^a(x)\chi^b(x) \ ,$$

$$B(x) = \frac{1}{6}\varepsilon_{abc}\chi^a(x)\chi^b(x)\chi^c(x) \ , \quad \bar B(x) = \frac{1}{N_c!}\varepsilon_{abc}\bar\chi^c(x)\bar\chi^b(x)\bar\chi^a(x)$$

Decomposition of Baryonic Composite Action

- **Introducing Auxiliary Baryon Field**

$$\exp(\bar{B}, V_B B) = \det V_B \int \mathcal{D}[\bar{b}, b] \exp\left[-(\bar{b}, V_B^{-1} b) + (\bar{b}, B) + (\bar{B}, b)\right]$$

- **Decomposition of coupling of baryon and 3 quarks with Diquark Composite** (Azcoit et al., JHEP 0309, 014 (2003))

$$\bar{b}B = \underbrace{\bar{b}\chi^a}_{\text{antibaryon-quark}} \times \underbrace{\chi^b\chi^c}_{\text{diquark}} \times \varepsilon_{abc}/6$$

$D^\dagger D$ makes $\bar{b}B$

$$D_a = \frac{\gamma}{2}\varepsilon_{abc}\chi^b\chi^c + \frac{1}{3\gamma}\bar{\chi}^a b\ , \quad D_a^\dagger = \frac{\gamma}{2}\varepsilon_{abc}\bar{\chi}^c\bar{\chi}^b + \frac{1}{3\gamma}\bar{b}\chi^a$$

$$\exp(\bar{b}B + \bar{B}b) = \int d[\phi_a, \phi_a^\dagger] \exp\left[-\phi_a^\dagger\phi_a + (\phi_a^\dagger D_a + D_a^\dagger\phi_a) \underbrace{-\frac{\gamma^2}{2}M^2 + M\bar{b}b/9\gamma^2}_{\bar{B}b + \bar{b}B - D_a^\dagger D_a}\right]$$

Effective Action is not yet bilinear in fermions

- ***four fermi interaction terms, M^2 and $Mb\bar{b}$***
- ***diquark-quark-antibaryon coupling***

Bosonization of Four Fermi Interactions

- $M\bar{b}b$ term → Baryon potential auxiliary field ω

$$\exp(M\bar{b}b/9\gamma^2) = \int d[\omega] \exp\left[-\omega^2/2 - \omega(\alpha M + g_\omega \bar{b}b) - \alpha^2 M^2/2\right]$$

  - $(\bar{b}b)^2 = 0$ in One species of Staggered Fermion

- M^2 and $(M, V_M M)$ terms → Chiral Condensate σ

$$\frac{1}{2}(M, V_M M) - \frac{1}{2}(\gamma^2 + \alpha^2) M^2 = \frac{1}{2}(M, \tilde{V}_M M)$$

$$\exp\left[\frac{1}{2}(M, \tilde{V}_M M)\right] = \int \mathcal{D}[\sigma] \exp\left[-\frac{1}{2}(\sigma, \tilde{V}_M^{-1}\sigma) - (\sigma, M)\right]$$

  - By absorbing "Mass" in the Hopping Term,
    We can replace both of the terms simultaneously !

Effective Action in bilinear form of Fermions !

Effective Free Energy at Zero Diquark Condensate

- **Effective Action**

Zero Diquark Condensate

$$S_F = (\bar{b}, \widetilde{V}_B^{-1} b) + \frac{1}{2}(\omega,\omega) + \frac{1}{2}(\sigma, \widetilde{V}_M^{-1}\sigma) + (\sigma_q, M) + S_F^{(U_0)} + S_F^{(m)}$$

$$\cancel{+ (\phi^\dagger,\phi) + \frac{1}{3\gamma}\left[(\bar{\chi}^a, \phi_a^\dagger b) + (\bar{b}\phi_a, \chi^a)\right] + \frac{\gamma}{2}\varepsilon_{cab}\left[(\phi_c^\dagger, \chi^a\chi^b) + (\bar{\chi}^b\bar{\chi}^a, \phi_c)\right]}$$

- **After Quark, U_0, Baryon Integral at zero diquark cond.**

$$\mathcal{F}_{\text{eff}} = \frac{1}{2}a_\sigma\sigma^2 + \frac{1}{2}\omega^2 + F_{\text{eff}}^{(b)}(g_\omega\omega) + F_{\text{eff}}^{(q)}(\sigma_q) \quad a_\sigma = \left[\frac{d}{2N_c} - (\gamma^2+\alpha^2)\right]^{-1}$$

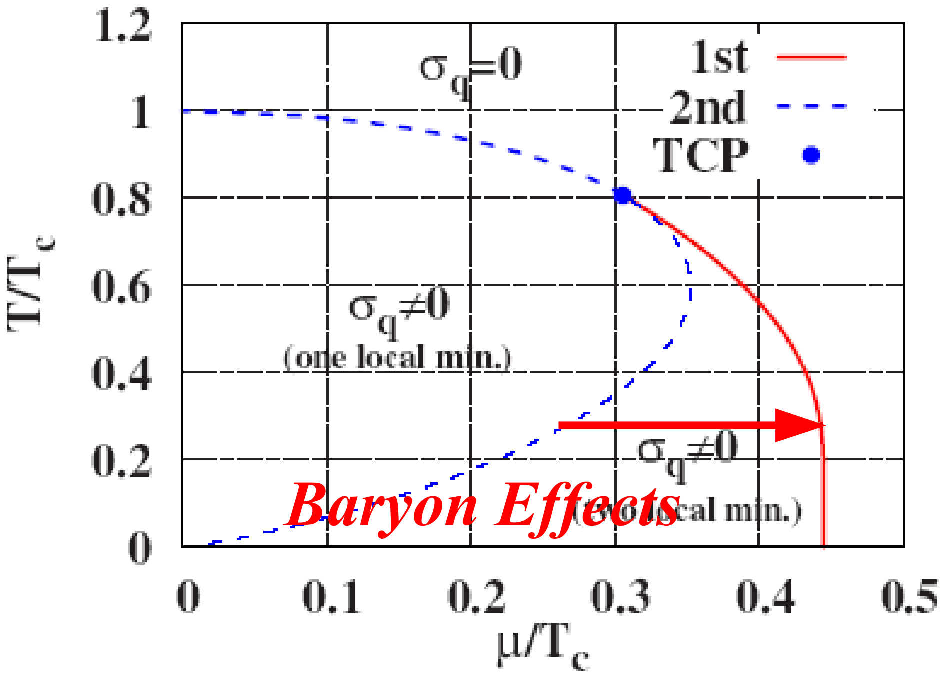
and adopting convenient parameters
(γ and ω are removed),
we get an analytical expression of Effective Free Energy

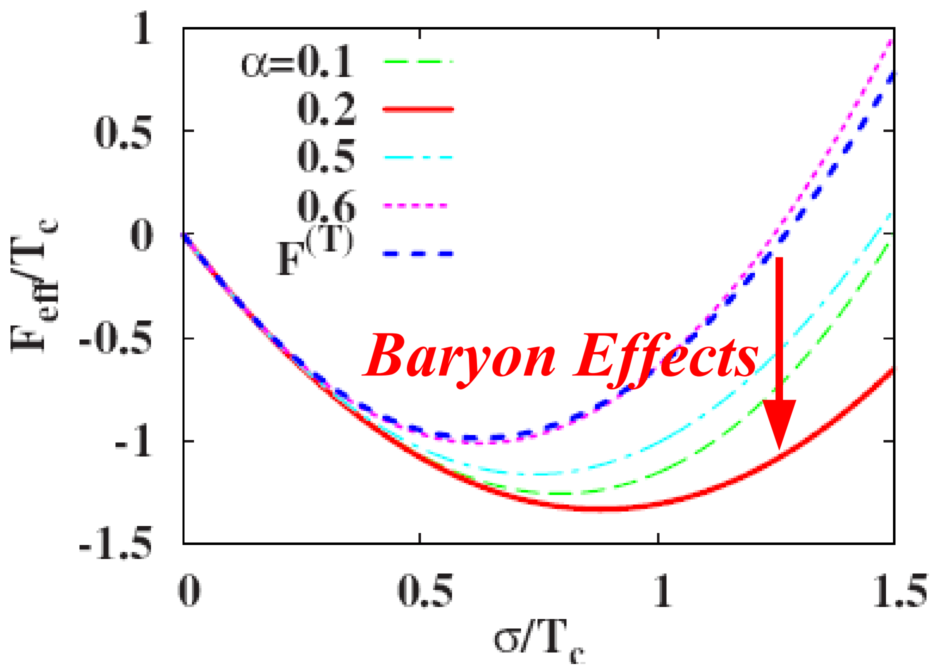
$$\mathcal{F}_{\text{eff}}(\sigma_q) = \frac{\sigma_q^2}{2\alpha^2} + F_{\text{eff}}^{(b)}(g_\sigma\sigma_q) + F_{\text{eff}}^{(q)}(\sigma_q; T, \mu)$$

Effective Free Energy with Baryonic Effects

- **Effective Free Energy**

$$\mathcal{F}_{\rm eff}(\sigma_q) = \frac{\sigma_q^2}{2\alpha^2} + F_{\rm eff}^{(b)}(g_\sigma \sigma_q) + F_{\rm eff}^{(q)}(\sigma_q; T, \mu)$$

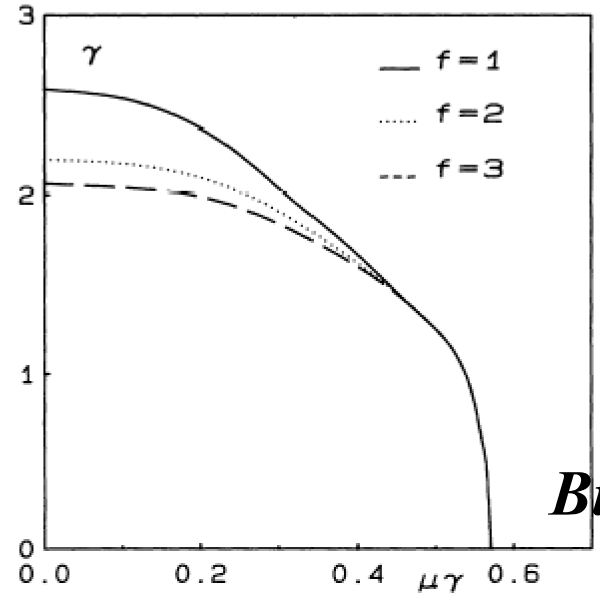




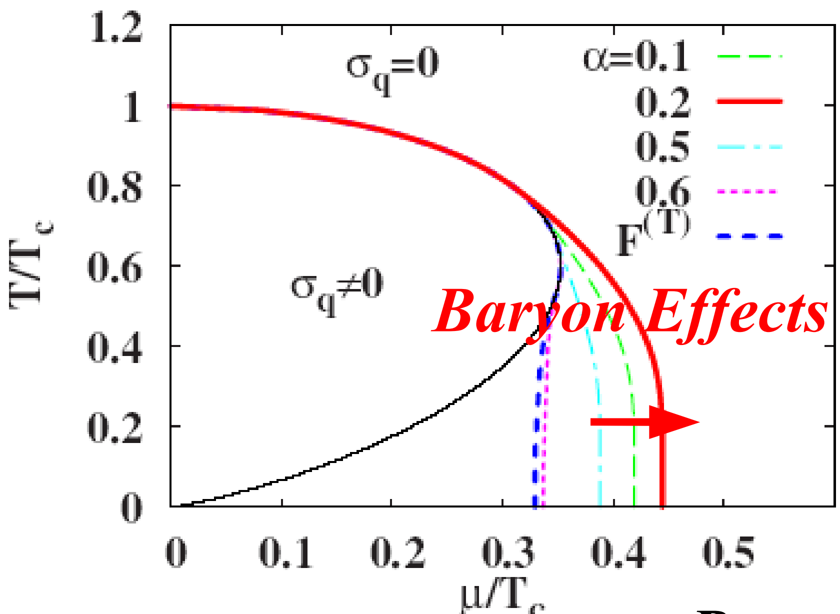
Baryons Gain Free Energy
→ Extention of Hadron Phase to Larger μ !

Small Critical μ : Common in SCL-LQCD ?

- **Strong Coupling Limit**
  - **Damgaard,Hochberg,Kawamoto ('85):**
    $\mu_B^c(0)/T_c(0) \sim 1.6$ (T=0, T≠0)
  - **T≠0, No B:** $\mu_B^c(0)/T_c(0) \sim 1.0$
    (Nishida2004, Bilic et al 1992(Bielefeld), ...)
  - **Present:** $\mu_B^c(0)/T_c(0) < 1.5$
    (Parameter dep.)
- **Monte-Carlo:** $\mu_B^c(0)/T_c(0) >> 1$
  - **Fodor-Katz, Bielefeld, de Forcrand-Philipsen, ...**
- **Real World:** $\mu_B^c(0)/T_c(0) > 7$



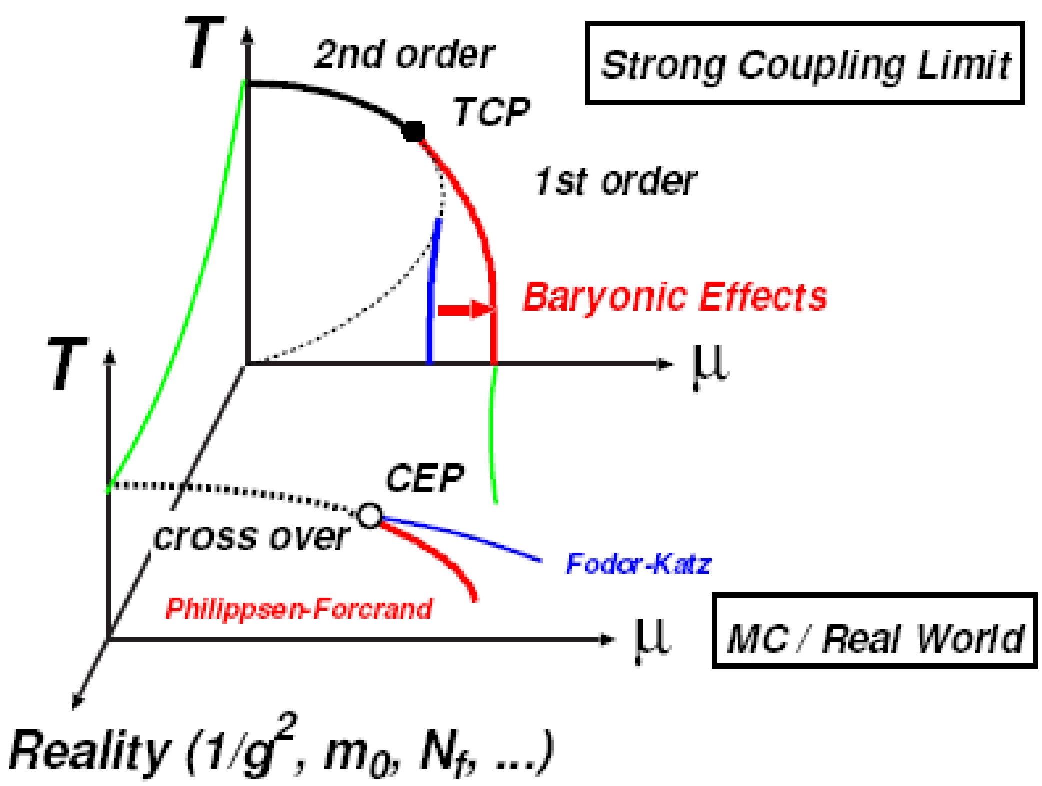
Bilic et al.



Present

Towards Realistic Understanding

- "Reality" Axis
  - Strong Coupling Limit → $1/g^2$ corrections → Smaller T_c
  - Number of Flavors → 2(ud)+1(s) → Smaller T_c
  - Chiral Limit → Finite m_q → Larger μ_c



Strangeness may play an important role to Extend Hadron Phase to Larger μ

Chirally Symmetric Relativistic Mean Field and Its Application

K. Tsubakihara and AO, in preparation.

RMF with Chiral Symmetry

- **Good Sym. in QCD, and Spontaneous breaking generates hadron masses.**
- **Schematic model: Linear σ model**

$$L=\frac{1}{2}\left(\partial_\mu\sigma\,\partial^\mu\sigma+\partial_\mu\pi\,\partial^\mu\pi\right)-\frac{\lambda}{4}\left(\sigma^2+\pi^2\right)^2+\frac{\mu^2}{2}\left(\sigma^2+\pi^2\right)+c\sigma$$
$$+\bar{N}i\partial_\mu\gamma^\mu N-g_\sigma\bar{N}\left(\sigma+i\pi\tau\gamma_5\right)N$$

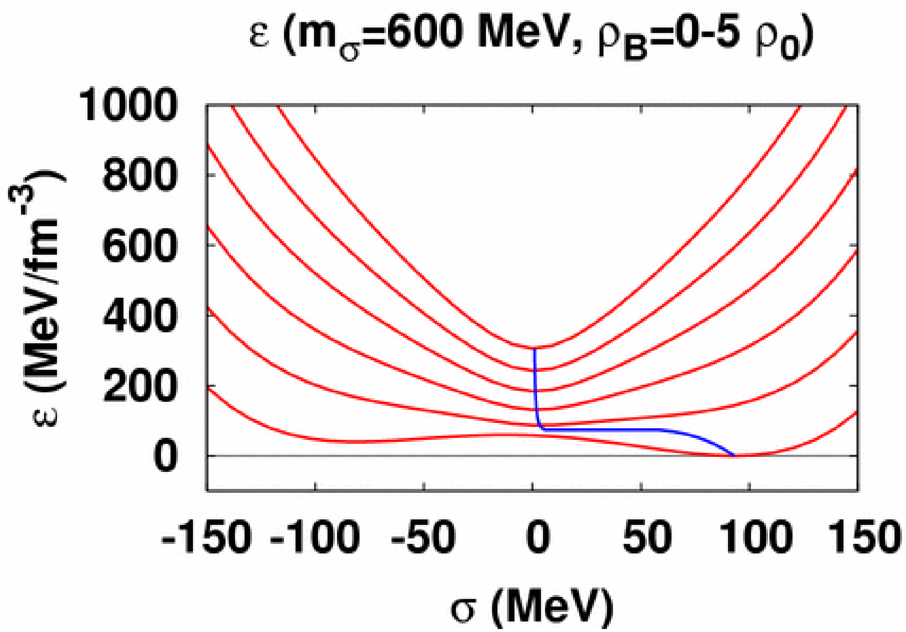
- **Many Problems**
  - **χ Sym. is restored at a very small density. σω Coupling stabilizes normal vacuum, but gives too stiff EOS.** *(Boguta PLB120,34, Ogawa et al. PTP111(2004)75)*
  - **Loop effects** *(N.K. Gledenning, NPA480,597; M. Prakash and T. L. Ainsworth, PRC36, 346; Tamenaga et al.)*
  - **Higher order terms** *(Hatsuda-Prakashi 1989, Sahu-Ohnishi, 2000)*
  - **Dielectric Field** *(Papazoglou et al. (Frankfurt), 1998)*

Problems in RMF with Chiral Symmetry

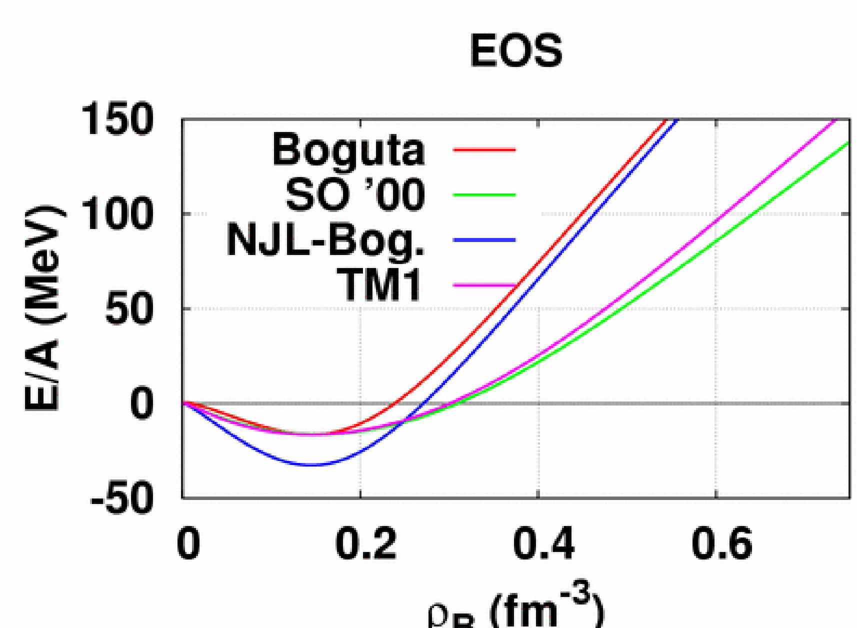
- **Sudden Change of <σ>**
- **σ ω Coupling**

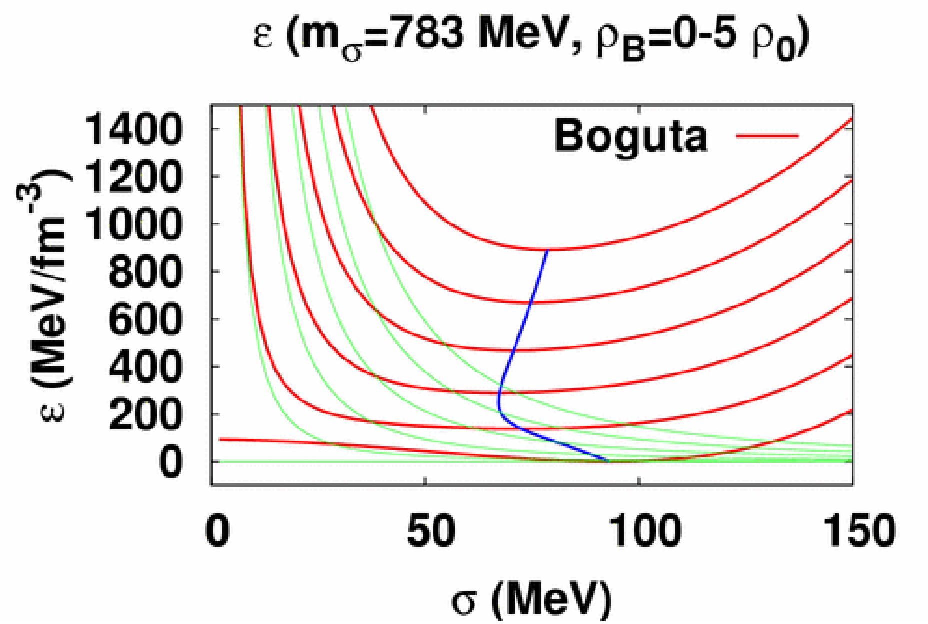
$$L_{\omega\sigma} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} C_{\sigma\omega} \sigma^2 \omega^2 - g_\omega \overline{N} \gamma_\mu \omega^\mu N$$

$$\omega = g_\omega \rho_B / C_{\sigma\omega} \sigma^2 \quad \rightarrow \quad V_{\sigma\omega} = \frac{g_\omega^2 \rho_B^2}{2 C_{\sigma\omega} \sigma^2}$$



- **Stiff EOS**





RMF with σ Self Energy from SCL-LQCD

- **σ Self Energy from simple Strong Coupling Limit LQCD**

$$
\begin{aligned}
S &\to -\tfrac{1}{2}(M, V_M M) && (1/d \text{ expansion}) \\
&\to b\sigma^2 + (\bar{\chi}\,\sigma\chi) && (\text{auxiliary field}) \\
&\to b\sigma^2 \boxed{- a \log \sigma^2} && (\text{Fermion Integral})
\end{aligned}
$$

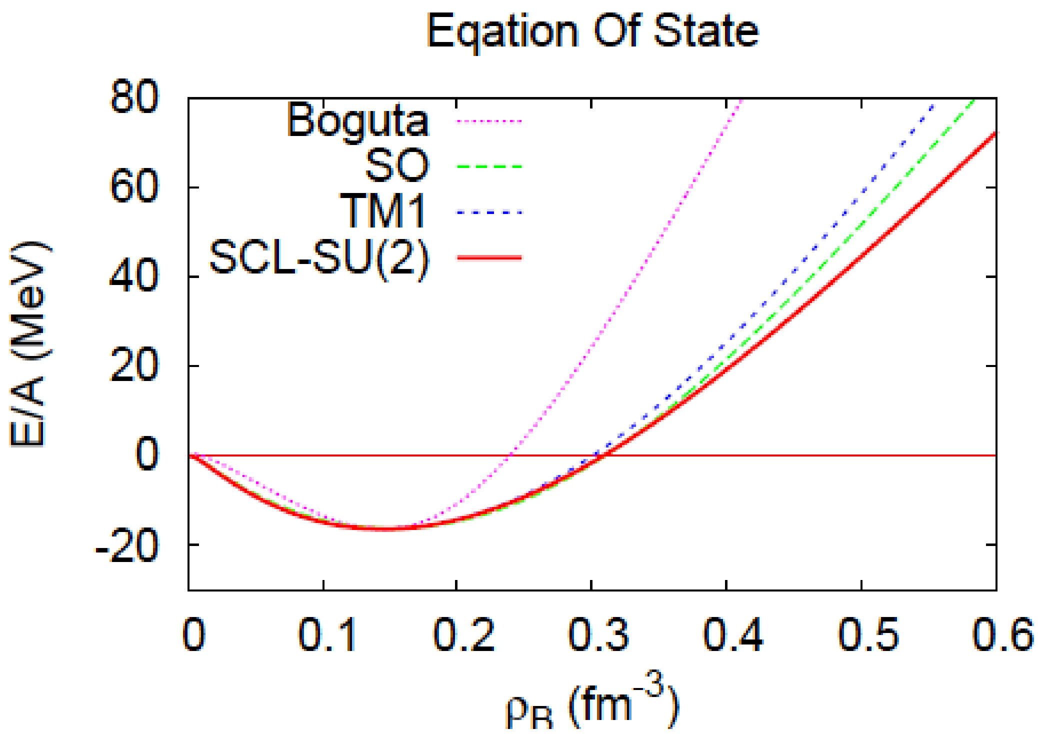
- **RMF Lagrangian** *Non-Analytic Type σ Self Energy*
  - **σ is shifted by f_π, and small explicit χ breaking term is added.**

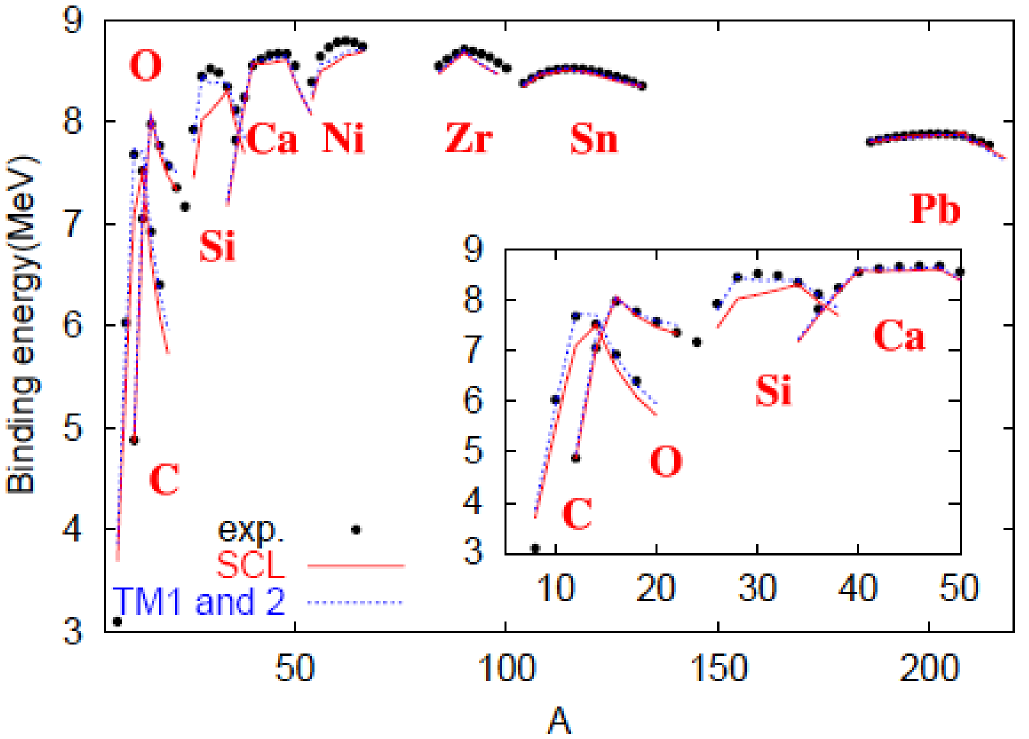
$$
\mathcal{L} = \bar{\psi}\left(i\gamma^\mu \partial_\mu - \gamma^\mu V_\mu - M + g_\sigma \sigma\right)\psi + \mathcal{L}_\sigma^{(0)} + \mathcal{L}_\omega^{(0)} + \mathcal{L}_\rho^{(0)}
$$
$$
- U_\sigma + \frac{\lambda}{4}(\omega_\mu \omega^\mu)^2
$$

$$
U_\sigma(\sigma) = 2a\, f(\sigma/f_\pi),\quad f(x) = \frac{1}{2}\left[-\log(1+x) + x - \frac{x^2}{2}\right],\quad a = \frac{f_\pi^2}{2}\left(m_\sigma^2 - m_\pi^2\right)
$$

Nuclear Matter and Finite Nuclei

- **Nuclear Matter: By tuning λ, $g_{\omega N}$, m_σ, *EOS can be Soft !***
- **Finite Nuclei: By tuning $g_{\rho N}$, Global behavior of B.E. is reproduced, except for j-j closed nuclei (C, Si, Ni).**





Extention to Chiral SU(3)

- **Strong Coupling Limit LQCD guess**

$$F_{eff} = b\,Tr(M^+ M) - a \log det(M^+ M) - c_\sigma \sigma - c_\zeta \zeta \; + d(det\, M^+ + det\, M)$$

Bosonization + Quark integral + Explicit breaking + $U_A(1)$ anomaly

$$M = \Sigma + i\Pi = diag(\sigma/\sqrt{2}, \sigma/\sqrt{2}, \zeta)\,(in\ MFA)$$

$$= a\left[2f(\sigma/f_\pi) + \frac{1}{2} f(\zeta/f'_\zeta)\right] + \frac{m_\sigma^2}{2}\sigma^2 + \frac{m_\zeta^2}{2}\zeta^2 + \xi\sigma\zeta + const.$$

(after shifting $\sigma \to f_\pi + \sigma$, $\zeta \to f_\zeta + \zeta$)

$$f(x) = \frac{1}{2}\left[-\log(1+x) + x + \frac{x^2}{2}\right], \quad a = \frac{f_\pi^2}{2}(m_\sigma^2 - m_\pi^2)$$

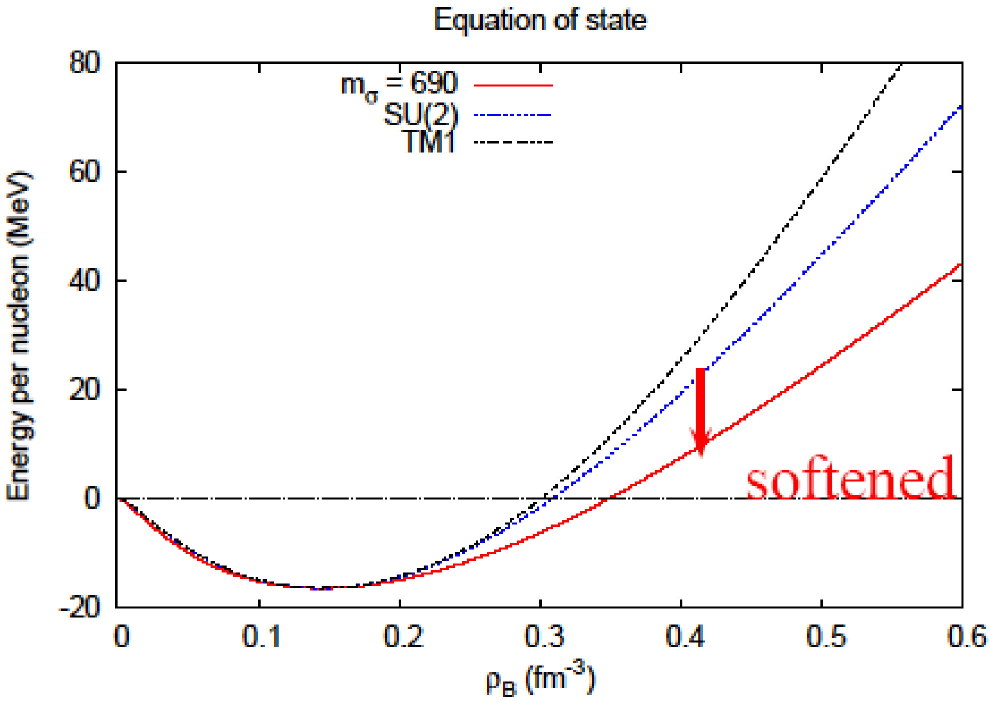
most of the parameters are determined to fit meson masses !

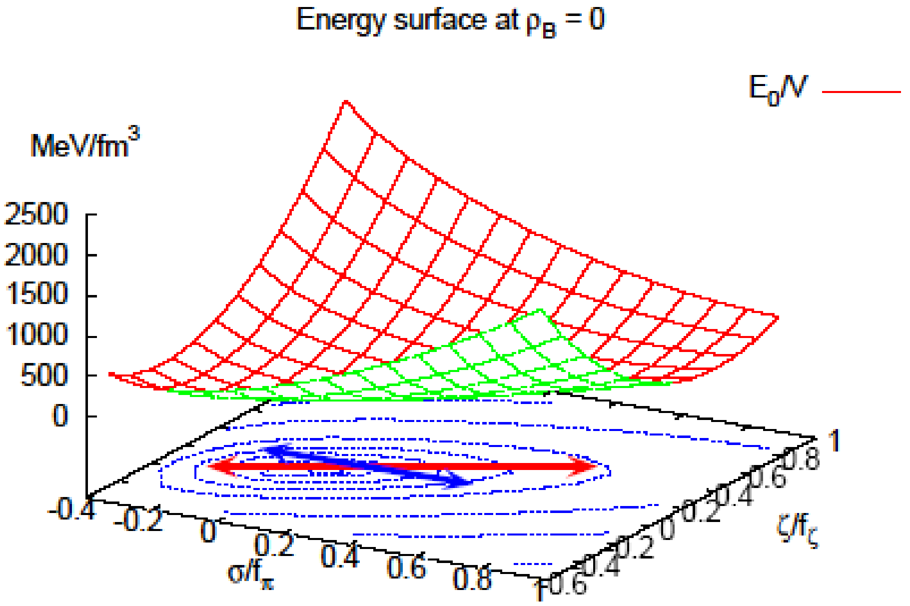
→ One parameter m_σ

Is it consistent with Nuclear Matter and Finite Nuclei ?

Symmetric Nuclear Matter in Chiral SU(3) RMF

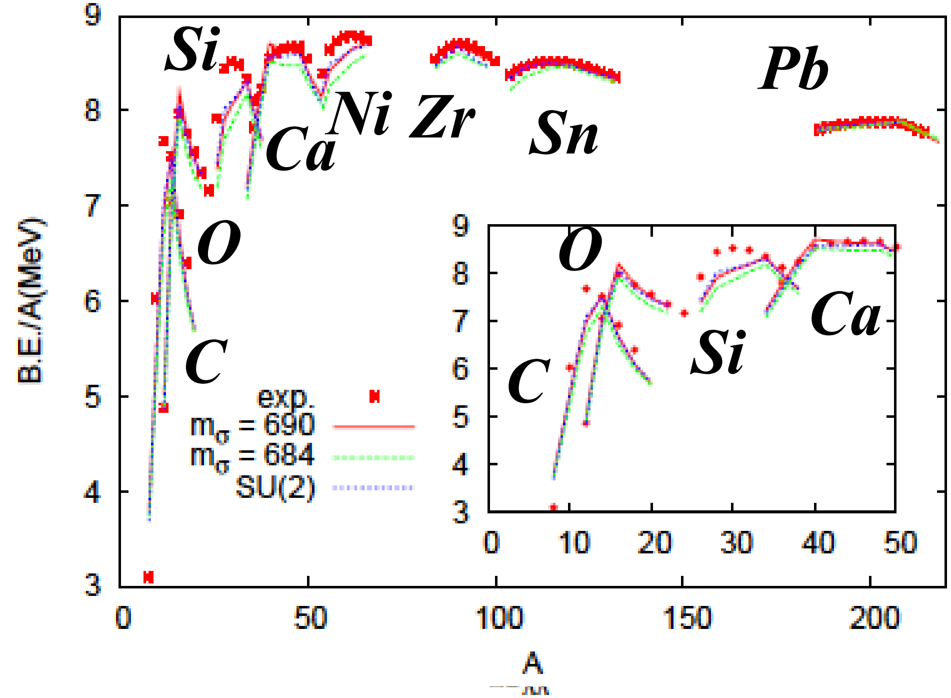
- **Soft EOS in Chiral SU(3) RMF**
  - **σ-ζ mixing → Evolution along σ-ζ valley**
  - **K= 216 MeV @ m_σ = 690 MeV → Consistent with K=210 ± 30 MeV**

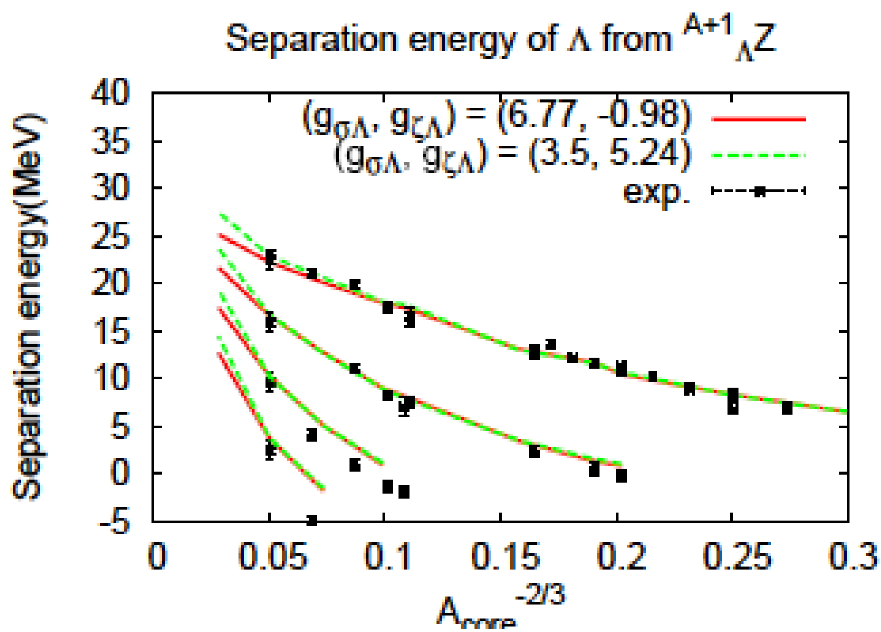


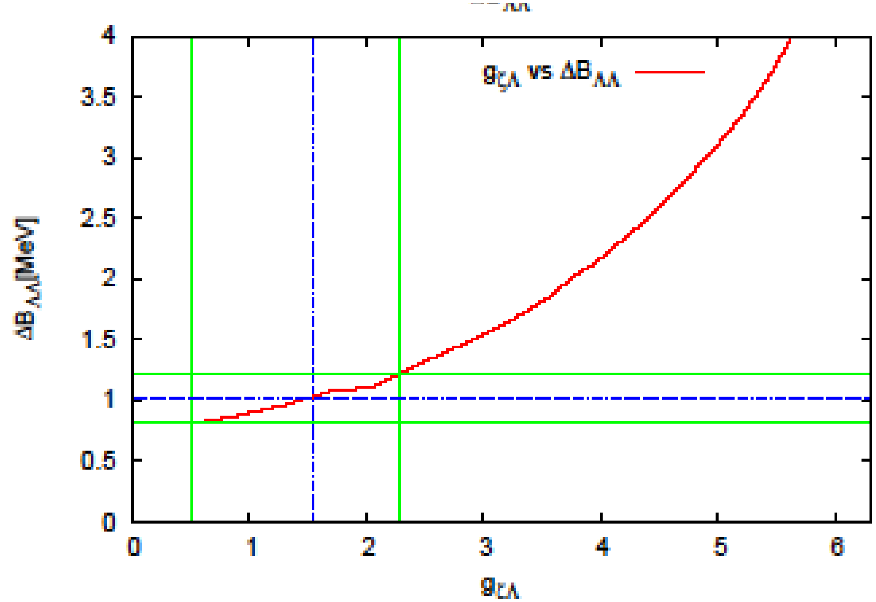


Other Model Parameters

- $g_{\rho N}$ → Normal Nuclei
- $(g_{\sigma\Lambda}, g_{\zeta\Lambda})$ → Single Λ Nuclei
- $g_{\zeta\Lambda}$ → ${}^{6}_{\Lambda\Lambda}He$
  ($SU_V(3)$ is assumed for $g_{V\Lambda}$)







Collective Flow in High-Energy Heavy-Ion Collision

"Mean-field effects on collective flow in high-energy heavy-ion collisions at 2-158A GeV energies"

M. Isse, A. Ohnishi, N. Otuka, P. K. Sahu, Y. Nara, Phys. Rev. C, Phys. Rev. C 72, 064908 (2005).

ハドロン輸送模型による高エネルギー重イオン衝突での核物質状態方程式とクォーク･グルーオン･プラズマの探求

第19回北海道原子核理論研究会「江別‘05」(平成18年2月10日)

北海道大学大学院理学研究科　物理学専攻　*D3*

一瀬 昌嗣

本研究は、以下三論文に基づいています。

(1) *"Mean-field effects on collective flow in high-energy heavy-ion collisions at 2-158A GeV energies"*
M. Isse, A. Ohnishi, N. Otuka, P. K. Sahu, Y. Nara,
Phys. Rev. C, Phys. Rev. C 72, 064908 (2005).

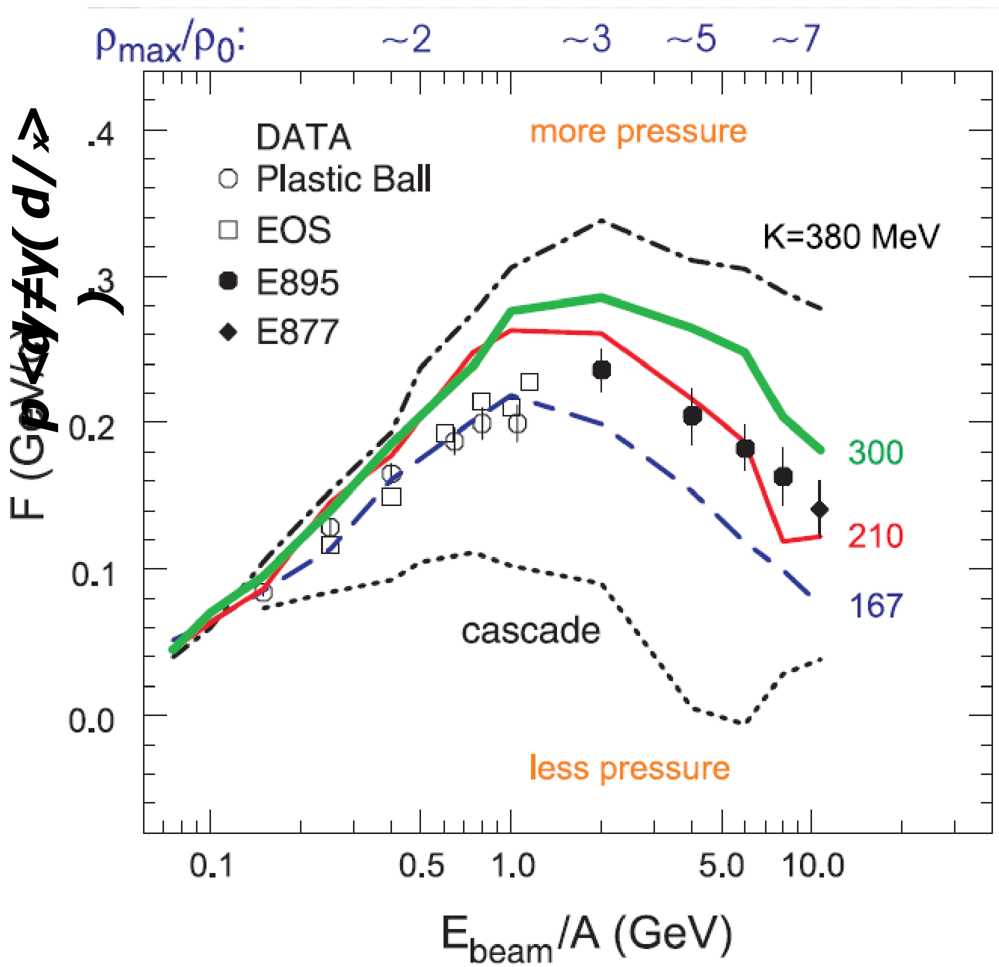
(2) *"Hadron-string cascade versus hydrodynamics in Cu+Cu collisions at $\sqrt{s_{NN}}$=200 GeV"*
T. Hirano, M. Isse, Y. Nara, A. Ohnishi, K. Yoshino,
Phys. Rev. C 72, 041901(R) (2005).

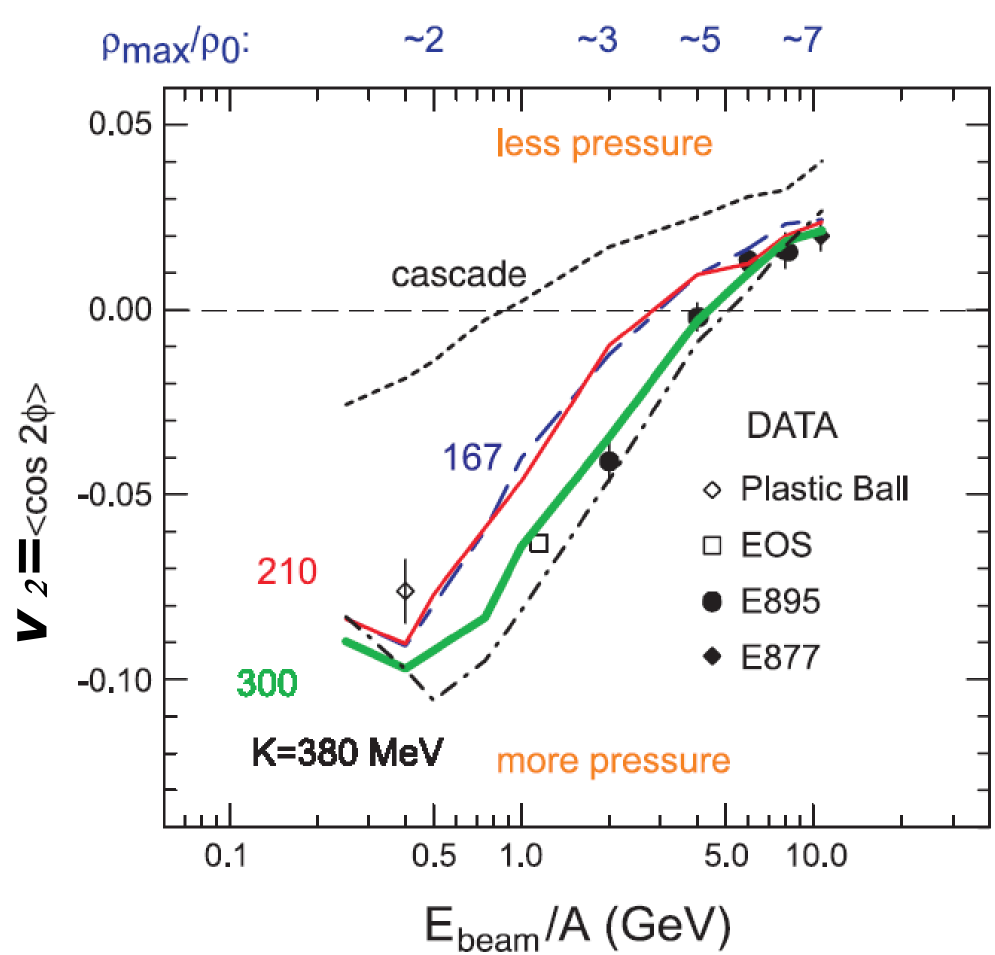
(3) *"Elliptic Flow in a Hadron String Cascade Model at 130 GeV Energy"*
P. K. Sahu, A. Ohnishi, M. Isse, N. Otuka, S. C. Phatak,
Submitted to Pramana — Journal of Physics (Indian Academy of Science).

Can we determine Nuclear EOS from Collective Flow ?

Example: P. Danielewicz et al., Science 298,1592(2002)

- SIS ~ AGS Energies (0.1~11 A GeV) → We need Mean Field
- With one value of K, we cannot explain v_1 and v_2 simultaneously.
  → EOS is not yet determined !





Flow study from AGS to SPS in JAM-RQMD/S

- EOS (or K) cannot be uniqely determined at $E_{inc} < 11\,A$ GeV
  → Higher density may be achieved at higher E_{inc}.
- Hadronic Transport Model JAM-RQMD/S
  - JAM: Particle DOF and Cross sections (Nara et al., 2000)
  - RQMD/S:
    Constraint Hamiltonian Dynamics + Simplified Time-Fixation
    (Sorge et al., Maruyama, 1998)
  - Nuclear Mean Field:
    Momentum Dep. Hard/Soft (MH, MS)
    Momentum Indep. Hard/Soft (H, S)
    Common MF for All Baryons (B) / MF only for Nucleons (N)

First Explore in Collective Flows at SPS with MF Effects

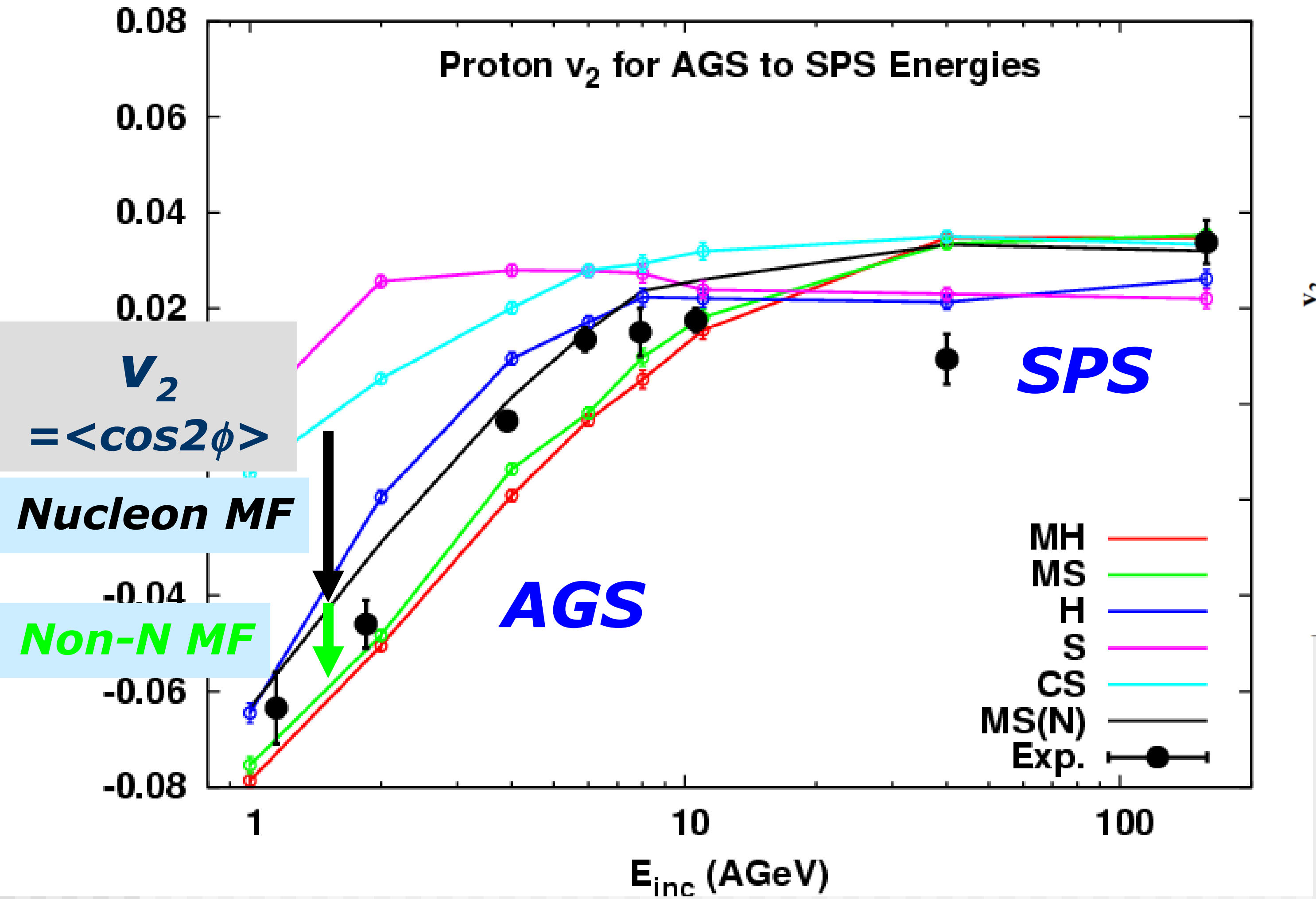
Comparison of Hadronic Transport Models

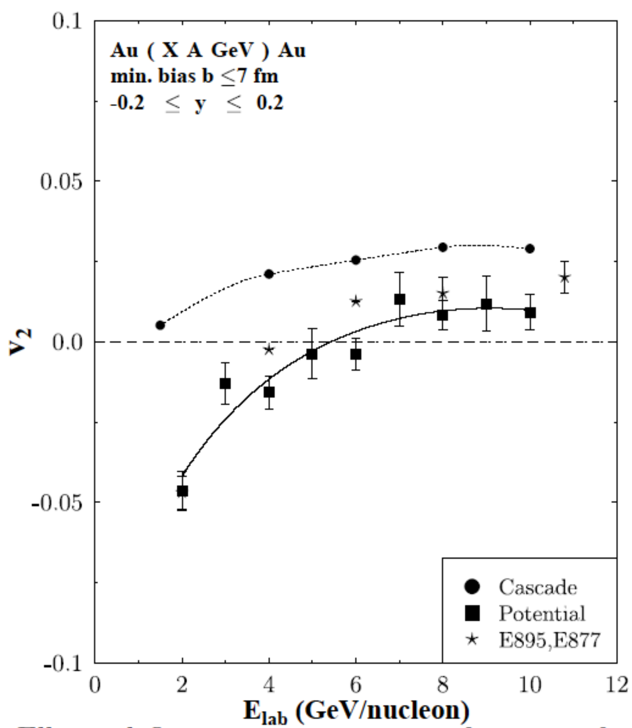
Model	Res. DOF	Rel. desc. of MF	Mom. Dep. MF	Flow @ AGS	Flow @ SPS	Flow @ RHIC
RQMD(1989)	○	○	×	○	△	△
RBUU(1994)	○	○	○	○	**?**	?
UrQMD(1996)	○	○	×	○	△	△
HSD(1996)	△	△	○	○	△	?
BEM(2000)	△	△	◎	◎	**×**	×
JAM+RQMD/S	○	○	○	○	○	△

Incident Energy Deps. of V_2

by Isse

- JAM-MF with momentum dep. MF explains proton v_2 at 1-158 A GeV energies
- *v_2 is not very sensitive to K (incompressibility)*
- Data lies between MS(B) and MS(N)
- Results with H ~ UrQMD (S.Soff et al., nucl-th/9903061)



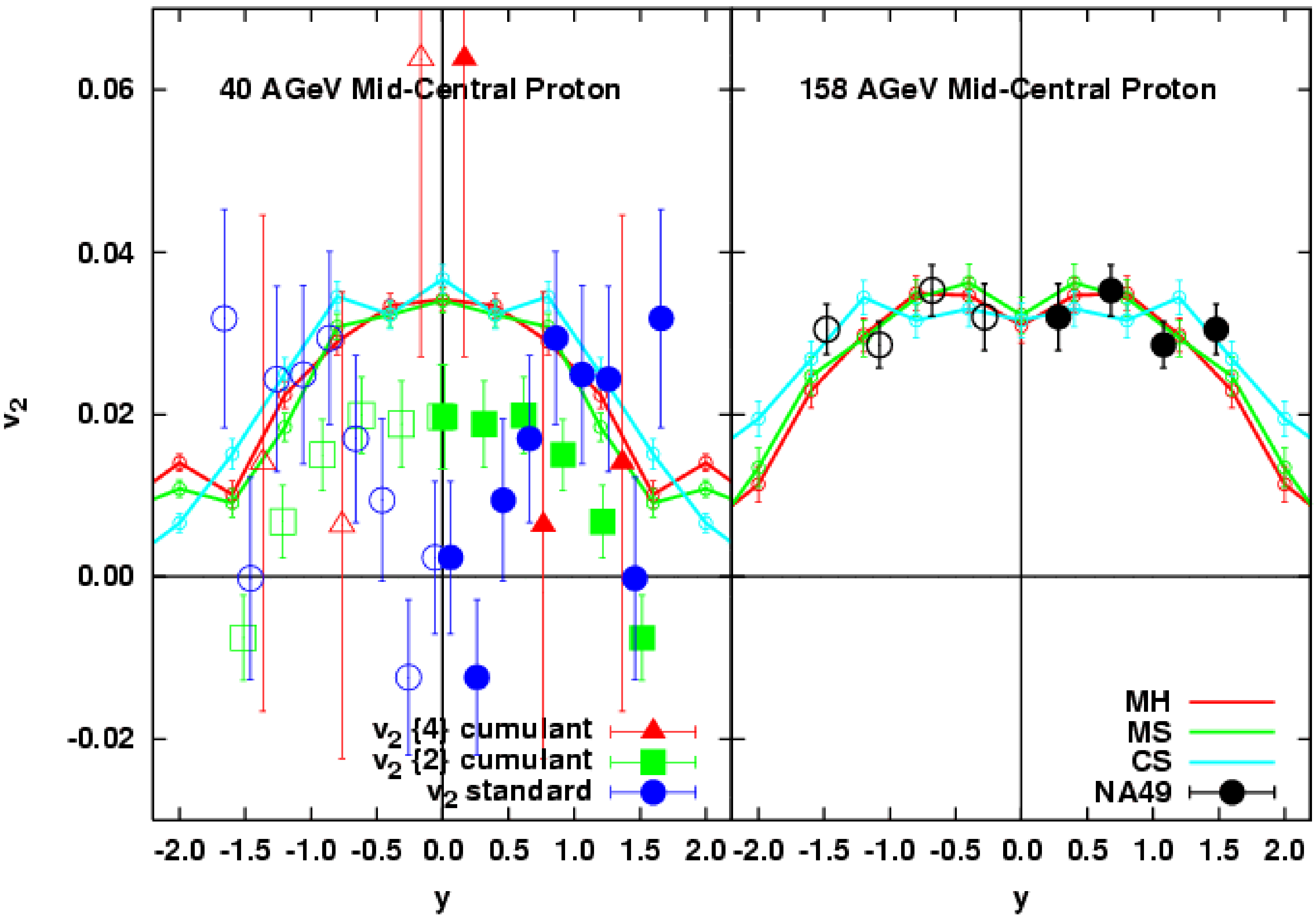


Summary

- **High Density Matter is interesting and important, but it requires various approaches at present.**
- **In the *strong coupling limit of lQCD*, baryons would modify the phase boundary at high density and low T.**
  - **Dependence on parameters introduced through bosonization identities.**
- **By using a QCD motivated σ self-energies, we can construct *chiral symmetric RMF* giving soft EOS.**
  - **Neutron Star / Pion & Kaon mass in dense matter.**
- ***Collective flow* data upto SPS are qualitatively explained in hadronic transport model with mom. dep. MF.**
  - **K (incompressibility) cannot be uniquely determined yet.**
  - **Model deps., especially MF during formation time / for Res.**

by Isse

Proton v_2 vs y @ SPS



Momentum dependent MF well suppress the proton v2.
Large uncertainty for analysis method at 40 AGeV.
We see standard v2 are wrong but good agreement to 2nd cumulant v2 at mid-rapidity.

Color Angle Average

- **Problem: Diquark Condensates induce quark-baryon coupling, and Baryon integral becomes difficult.**
  → Solution: Color Angle Average
  - **Integral of "Color Angle Variables"**

$$\int \mathcal{D}[\phi_a, \phi_a^\dagger] \exp\left\{\phi_a^\dagger D_a + D_a^\dagger \phi_a\right\} = \int \mathcal{D}[v] \exp\left\{\frac{v^2}{3} D_a^\dagger D_a + \frac{v^4}{162} M^3 \bar{b}b\right\}$$

  - **Three-Quark and Baryon Coupling is ReBorn !**

$$D_a^\dagger D_a = Y + \bar{b}B + \bar{B}b\,, \quad Y = \frac{\gamma^2}{2} M^2 - \frac{1}{9\gamma^2} M\bar{b}b$$

  - **Solve "Self-Consistent" Equaton**

$$\exp(\bar{b}B + \bar{B}b) \simeq \exp\left[-v^2 - Y + \frac{v^2}{3}(\bar{b}B + \bar{B}b + Y) + \frac{v^4}{162} M^3 \bar{b}b\right]$$

$$\simeq \exp\left[-\frac{v^2}{R_v} + \frac{v^4 M^3 \bar{b}b}{162 R_v} - Y\right] \quad (R_v = 1 - v^2/3)$$

Effective Free Energy with Diquark Condensate

- **Bosonization of** $M^k \bar{b} b$ → **Introduce k bosons**

$$\exp M^k \bar{b} b = \int d\omega_k \exp[-\frac{1}{2}(\omega_k + \alpha_k M + 1/\alpha_k M^{k-1} \bar{b} b)^2 + M^k \bar{b} b]$$
$$= \int d\omega_k \exp[-\omega_k^2/2 - \omega(\alpha_k M + 1/\alpha_k M^{k-1} \bar{b} b) - \alpha_k^2 M^2/2]$$

- **Effective Free Energy**

$$\mathcal{F}_{\text{eff}}^{(Tbv)} = F_X(\sigma, v, \omega_i) + F_{\text{eff}}^{(b)}(g_\omega \omega) + F_{\text{eff}}^{(q)}(\sigma_q)$$

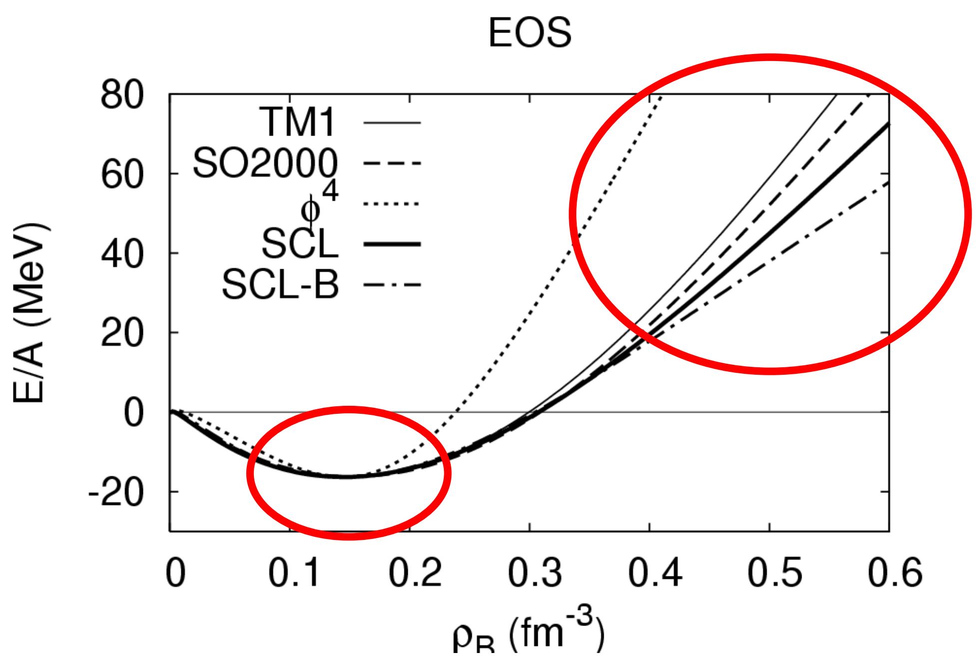
$$F_X = \frac{1}{2}(a_\sigma \sigma^2 + \omega^2 + \omega_1^2 + \omega_2^2) + \frac{v^2}{R_v} \qquad g_\omega = \frac{1}{9\alpha\gamma^2}\left[1 + \frac{\gamma^2 v^4 \omega_1 \omega_2}{18\alpha_1 \alpha_2 R_v}\right]$$

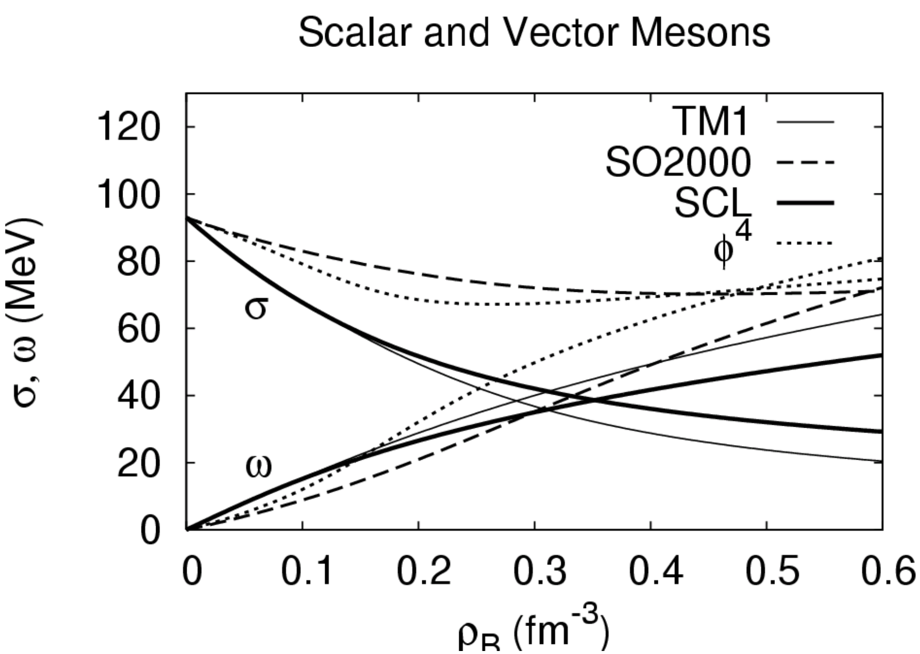
$$a_\sigma = \left(\frac{1}{2} - \gamma^2 - \alpha^2 - \alpha_1^2 - \alpha_2^2\right)^{-1} \qquad \sigma_q = \sigma + \alpha\omega + \alpha_1 \omega_1 + \alpha_2 \omega_2$$

Similar form to the previous one at v=0.
Diquark Effects in interaction start from v^4.

Nuclear Matter

- Satulation property 等の計算結果





•*Satulation property* を満たし、かつ軟らかい状態方程式を得る事が出来た

•σ、ω の振る舞い
⇒*TM1* との類似
⇒ 有限核の計算にも期待

Comparison with Other Treatments

- **T=0, without or with baryons (e.g., NK-Smit1981, Damgaard-Hochberg-NK 1985)**

$$\mathcal{F}_{\rm eff}^{(0)} = \frac{N_c\sigma^2}{d+1} - N_c \log\widetilde{\sigma} \qquad \mathcal{F}_{\rm eff}^{(0\rm b)} = \frac{N_c\sigma^2}{d+1} + F_{\rm eff}^{(b\mu)}(4\widetilde{\sigma}^3; T,\mu)$$

- **T=0, with b and diquark (ACGL2002)**

$$\Theta = \frac{1}{3}\left(R_v^2 - \frac{R_v\widetilde{\sigma}^2}{\gamma^2} + \frac{2}{9}v^2\right),$$

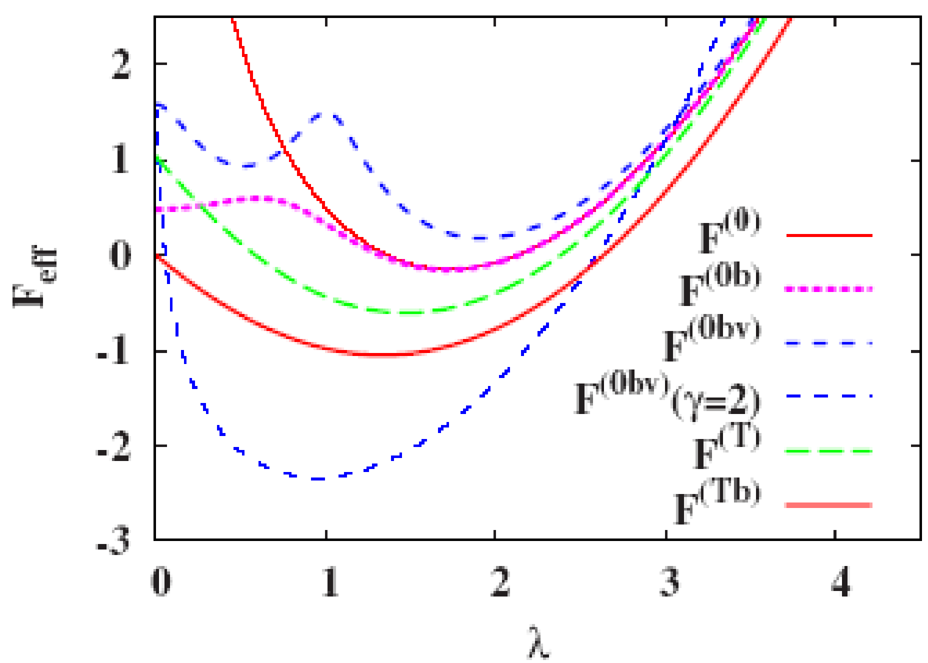
$$\mathcal{F}_{\rm eff}^{(0\rm bv)} = \frac{N_c\sigma^2}{d+1} + v^2 - \log\Theta + F_{\rm eff}^{(b\mu)}(m; T,\mu) \qquad m = \frac{4\widetilde{\sigma}\left(3\gamma^2 R_v - \widetilde{\sigma}^2\right)}{\Theta}$$

- **T ≠0 , no baryons (e.g., Nishida2004**

$$\mathcal{F}_{\rm eff}^{(T)} = \frac{N_c\sigma^2}{d} + F_{\rm eff}^{(q)}(\widetilde{\sigma})$$

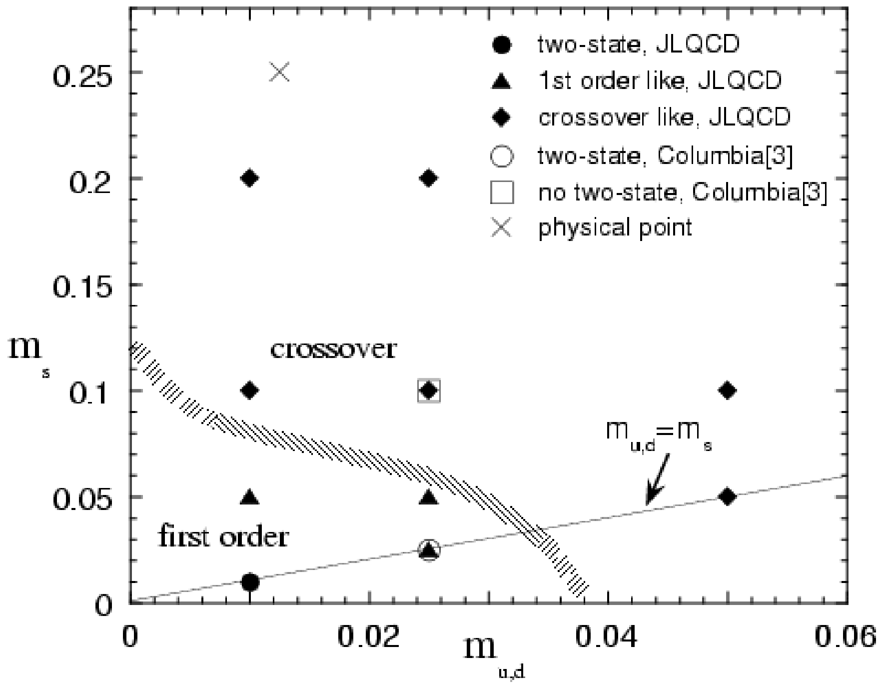
Fixing asymptotic behavior
→ $F^{(Tb)}$ is smaller

$$\mathcal{F}_{\rm eff}^{(Tb)}(\lambda = \sigma_q/\alpha) \to \frac{\lambda^2}{2} - N_c\log\lambda + F_{\rm eff}^{(b)}(g_\omega\alpha\lambda)$$



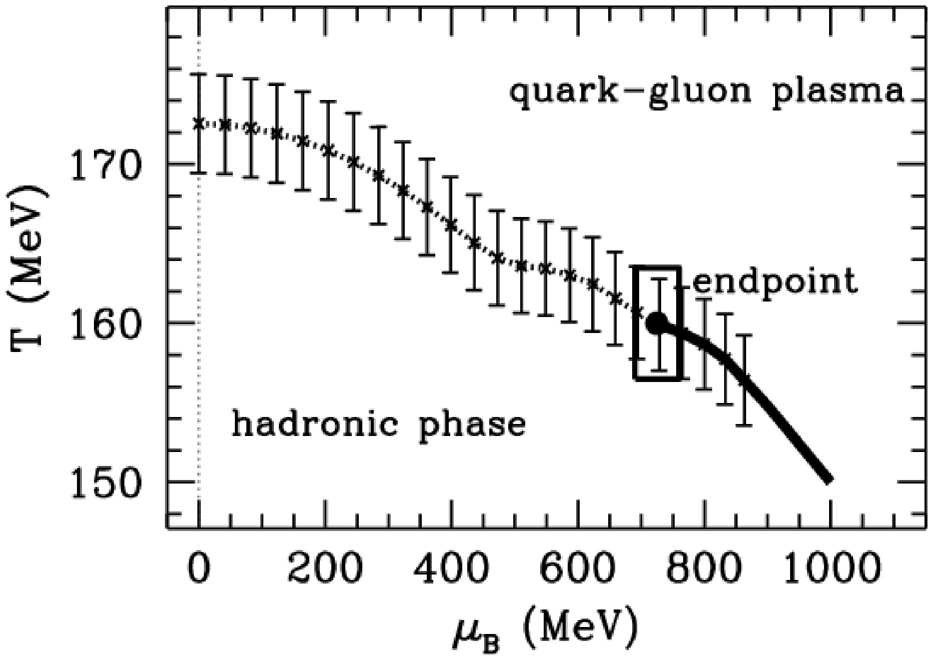
QCD Phase Diagram from Lattice QCD

- Zero Chem. Pot.



- JLQCD Collab. (S. Aoki et al.), Nucl. Phys. Proc. Suppl. 73 (1999), 459.

- Finite Chem. Pot.

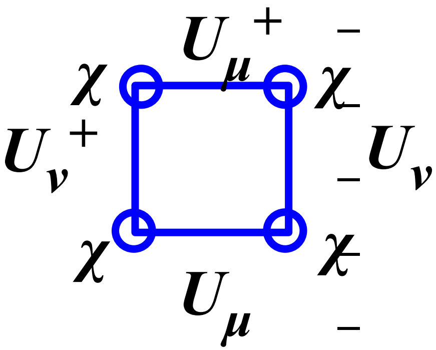


- Fodor & Katz, JHEP 0203 (2002), 014.

Zero Chem. Pot. : Cross Over
Finite Chem. Pot.: Critical End Point

Lattice Action in SCL-LQCD (1)

- **Lattice Action with staggered Fermions**



$$S[U,\chi,\bar{\chi}] = S_G[U] + S_F[U,\chi,\bar{\chi}] \ ,$$

$$S_G[U] = \frac{2N_c}{g^2}\sum_{x,\mu,\nu}\left\{1 - \frac{1}{N_c}\mathrm{ReTr}U_{\mu\nu}(x)\right\}$$

$g \to \infty$ → **0**

$$U_{\mu\nu}(x) = U_\nu^\dagger(x)U_\mu^\dagger(x+\hat{\nu})U_\nu(x+\hat{\mu})U_\mu(x) \ ,$$

Chem. Pot.

$$\begin{aligned}
S_F[U,\chi,\bar{\chi}] &= S_F^{(m)}[\chi,\bar{\chi}] + S_F^{(j)}[U_j,\chi,\bar{\chi}] + S_F^{(U0)}[U_0,\chi,\bar{\chi}] \ , \\
S_F^{(m)}[\chi,\bar{\chi}] &= m\sum_x \bar{\chi}^a(x)\chi^a(x) \ , \\
S_F^{(j)}[U_j,\chi,\bar{\chi}] &= \frac{1}{2}\sum_x\sum_{j=1}^d \eta_j(x)\left\{\bar{\chi}(x)U_j(x)\chi(x+\hat{j}) - \bar{\chi}(x+\hat{j})U_j^\dagger(x)\chi(x)\right\} \ , \\
S_F^{(U0)}[U_0,\chi,\bar{\chi}] &= \frac{1}{2}\sum_x \eta_0(x)\left\{\bar{\chi}(x)e^{\mu}U_0(x)\chi(x+\hat{0}) - \bar{\chi}(x+\hat{0})U_0^\dagger(x)e^{-\mu}\chi(x)\right\} \ .
\end{aligned}$$

- **In the Strong Coupling Limit (g → ∞), we can ignore SG, and semi-analytic calculation becomes possible.**

Details of Functions

$$\sigma_q = \sigma + \alpha\omega$$

$$\widetilde{V}_B^{-1}(x,y) = V_B^{-1}(x,y) + g_\omega \omega \delta_{x,y} \ , \quad g_\omega = \frac{1}{9\alpha\gamma^2}$$

$$F_{\rm eff}^{(b)}(g_\omega\omega) = \frac{1}{\beta L^3}\log \mathrm{Det}\left[1 + g_\omega \omega V_B\right] = -\frac{1}{2L^3}\sum_{\mathbf{k}} \log\left[1 + \frac{g_\omega^2\omega^2 s^2}{16}\right] \simeq -a_0^{(b)} f^{(b)}\left(\frac{g_\omega \omega \Lambda}{4}\right)$$

$$f^{(b)}(x) = \frac{3}{2x^3}\int_0^x k^2 dk \log(1+k^2)$$

$$F_{\rm eff}^{(q)}(\sigma_q) = -T\log\left[\frac{4}{3}\left(C_\sigma^3 - \frac{1}{2}C_\sigma + \frac{1}{4}C_{3\mu}\right)\right]$$

$$F_{\rm eff}^{(q)} = -T\log\left\{\frac{\sinh[(N_c+1)E/T]}{\sinh[E/T]} + 2\cosh N_c\mu\right\}$$

$$\mathcal{F}_{\rm eff} = \frac{1}{2}a_\sigma\sigma^2 + \frac{1}{2}\omega^2 + F_{\rm eff}^{(b)}(g_\omega\omega) + F_{\rm eff}^{(q)}(\sigma_q)$$

$$\gamma^2 + \alpha^2 = \frac{1}{2} - \epsilon \quad (\epsilon \to +0\ , \quad a_\sigma \to +\infty)$$

$$\mathcal{F}_{\rm eff}(\sigma_q) = \frac{\sigma_q^2}{2\alpha^2} + F_{\rm eff}^{(b)}(g_\sigma\sigma_q) + F_{\rm eff}^{(q)}(\sigma_q; T, \mu)$$

Several Analytic Results / Comparison

- **Critical Temperature**

$$T_c(0) = T_c^{(2nd)}(\mu=0) = \frac{5}{3b_\sigma}$$

  - **b_σ = curvature of** $\frac{\sigma_q^2}{2\alpha^2} + F_{\rm eff}^{(b)}(g_\sigma\sigma_q)$

- **2nd order critical μ**

$$\mu_c^{(2nd)}(T) = \frac{T}{3}\cosh^{-1}\left(\frac{3T_c(0)}{T} - 2\right)$$

  - **same as Nishida, 2004**

- **TriCritical Point**

$$\frac{T_{\rm TCP}}{T_c(0)} = \frac{41}{25}\left[1+\sqrt{1+\frac{164}{625}T_c^2(0)\left(5+9T_c(0)c_4^{(b)}\right)}\right]^{-1}$$

  - **$C_4^{(b)}$= coef. of σ^4 in** $\frac{\sigma_q^2}{2\alpha^2} + F_{\rm eff}^{(b)}(g_\sigma\sigma_q)$

DKS1984
Tc=5/2 (U(3))
Nishida2004
Tc=5/3 (SU(3))
Bilic et al.
Tc~ 2.5 (f=1), 2.0 (f=3)

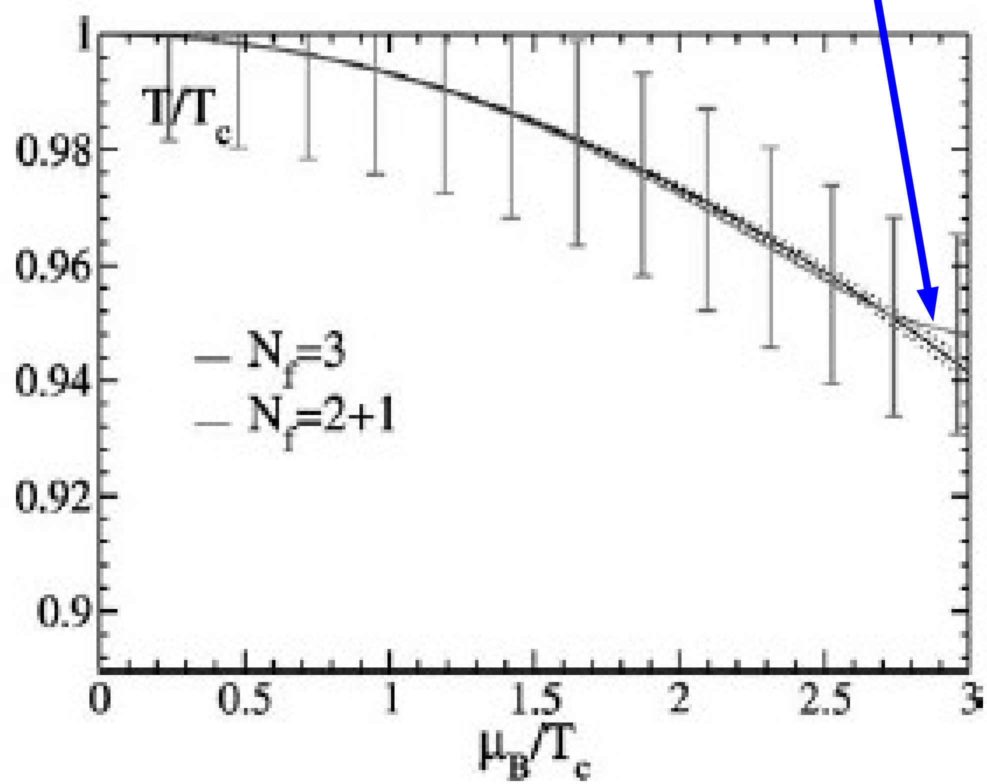
- ***Why we have $d\mu_c^{(1st)}/dT_c = d\mu_c^{(2nd)}/dT_c$***

$$F_{eff} = c_2\sigma^2 + c_4\sigma^4 + c_6\sigma^6$$
$$\to 4c_2c_6 = c_4^2$$
$$\to c_2 \simeq 0 \quad \text{around} \quad T_{tcp}$$

Debate ?

- **Tc(μ) smoothly goes down or not.**
  - **de Forcrand-Philipsen → Analytic Continuation predicts smooth decrease.**
  - **Fodor-Katz → at TCP, the phase boundary seems to have a kink**
- ***Why we have $d\mu_c^{(1st)}/dT_c = d\mu_c^{(2nd)}/dT_c$ → kink at TCP may suggest the exsitence of other order parameter(s)***

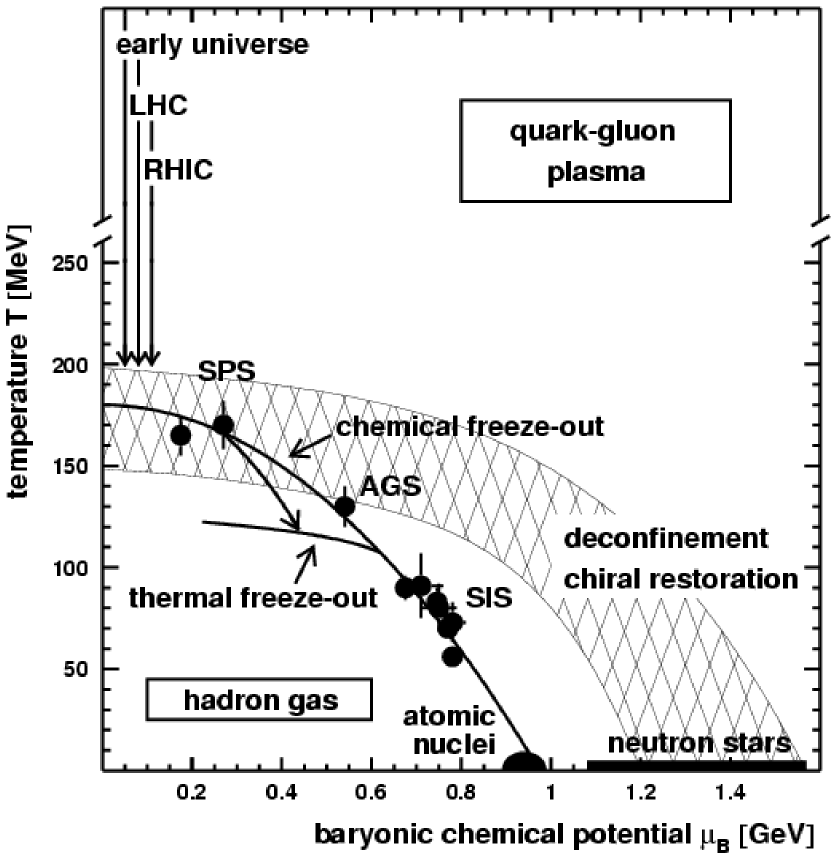
Fodor-Katz



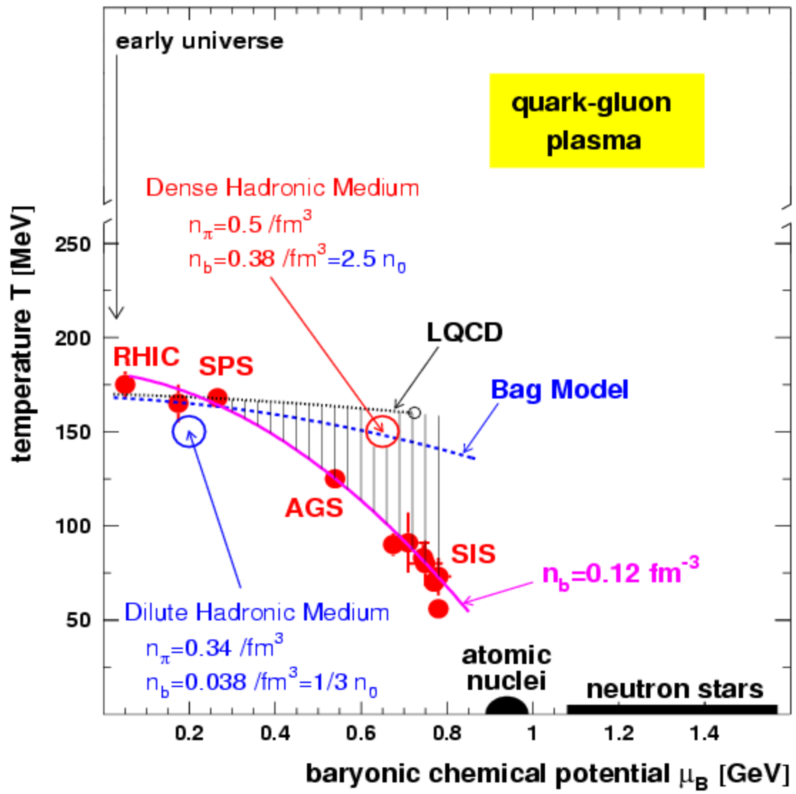
de Forcrand et al., NPB673,170(2003)

$$F_{eff} = c_2\sigma^2 + c_4\sigma^4 + c_6\sigma^6$$
$$\to 4c_2c_6 = c_4^2$$
$$\to c_2 \simeq 0 \quad around \quad T_{tcp}$$

Experimentally Estimated Phase Diagram



1998 (J. Stachel et al.)



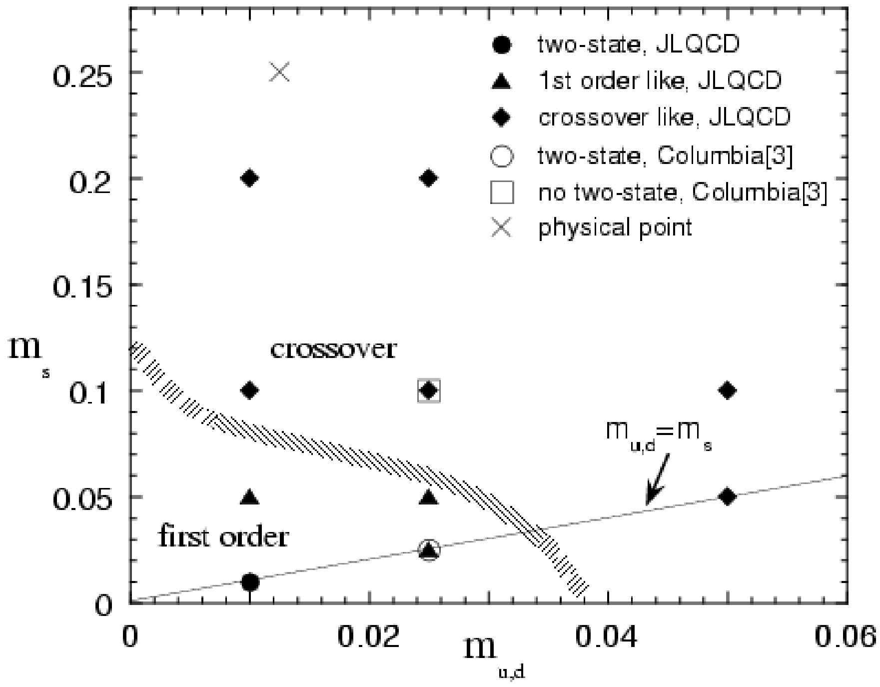
2002 (Braun-Munzinger et al.
J. Phys. G28 (2002) 1971.)

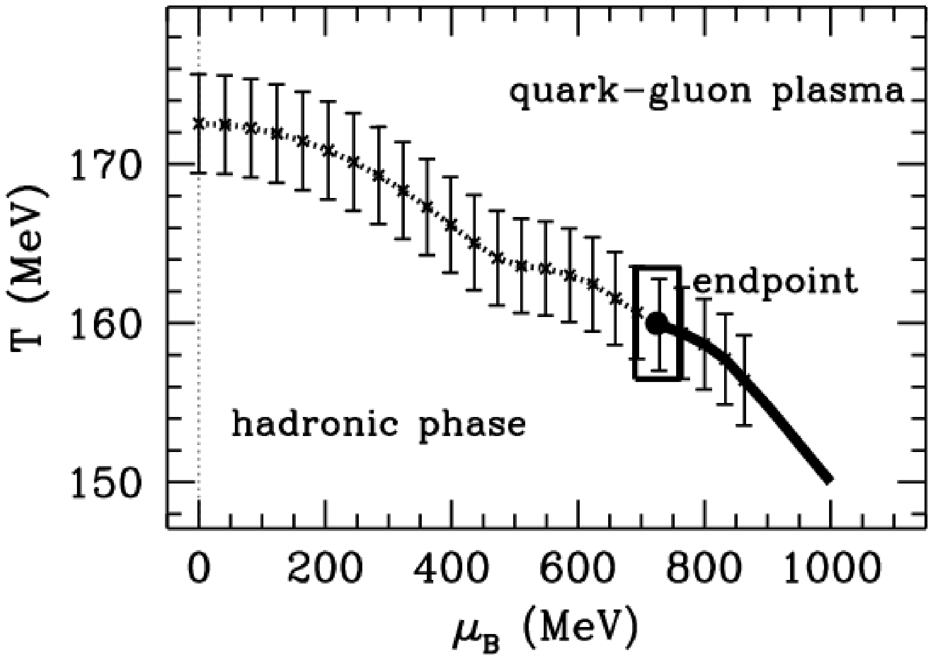
Chem. Freeze-Out Points are very Close to Expected QCD Phase Transition Boundary

Theoretically Expected QCD Phase Diagram

Zero Chem. Pot.

Finite Chem. Pot.



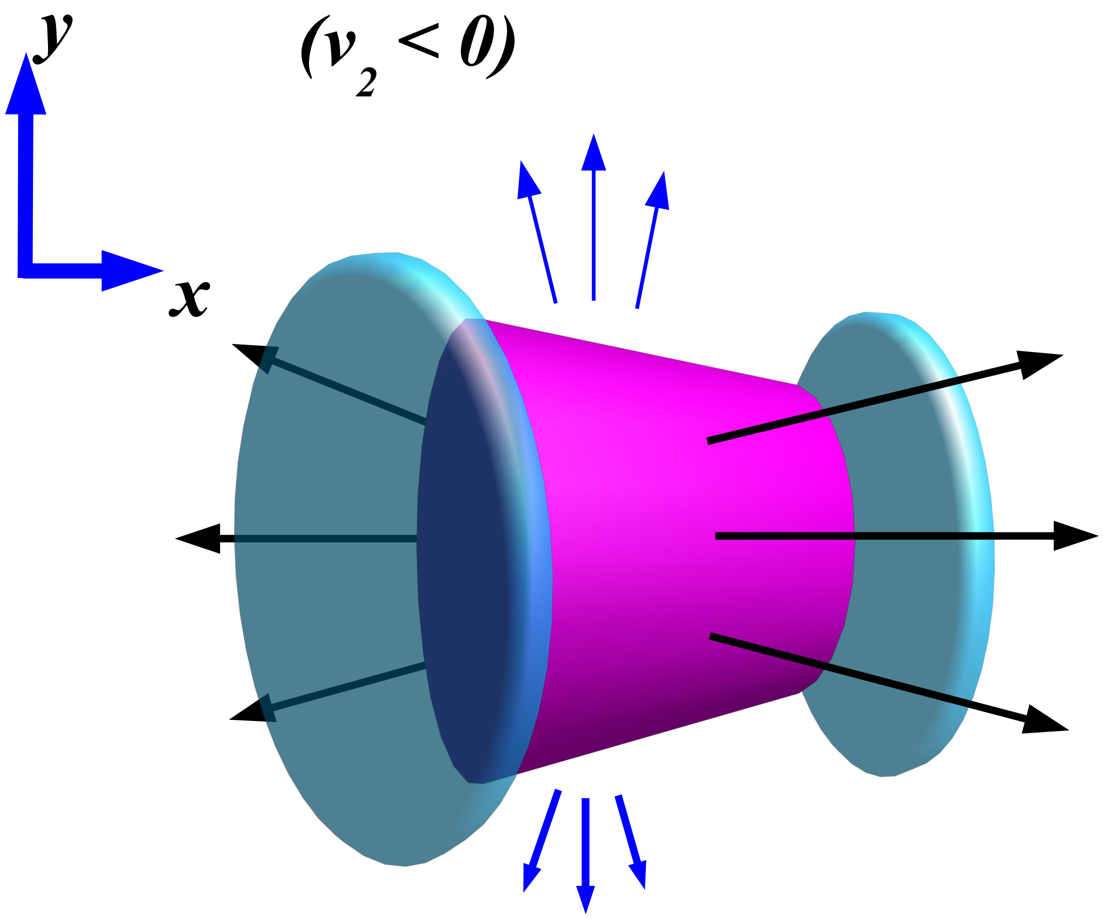


JLQCD Collab. (S. Aoki et al.), Nucl. Phys. Proc. Suppl. 73 (1999) 459.

Finite μ : Fodor & Katz, JHEP 0203 (2002), 014.

Zero Chem. Pot. : Cross Over
Finite Chem. Pot.: Critical End Point

Elliptic Flow (I)



- ⭑ *What is Elliptic Flow ?*
  - *Anisotropy in P space*
- ⭑ *Hydrodynamical Picture*
  - *Sensitive to the Pressure Anisotropy in the Early Stage*
  - *Early Thermalization is Required for Large* V_2

$$v_2 \equiv \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle = \langle \cos 2\phi \rangle$$

- **Lattice QCD action**

$$S_F^{(U_j)} = \frac{1}{2}\sum_x \eta_j(x)\left[\bar{\chi}(x)U_\mu(x)\chi(x+\hat{\mu}) - \bar{\chi}(x+\hat{\mu})U_\mu^\dagger(x)\chi(x)\right]$$

$$S_F^{(U_0)} = \frac{1}{2}\sum_x \left[\bar{\chi}(x)e^{\mu}U_\mu(x)\chi(x+\hat{\mu}) - \bar{\chi}(x+\hat{\mu})e^{-\mu}U_\mu^\dagger(x)\chi(x)\right]$$

$$S_F^{(m)} = m_0\sum_x \bar{\chi}^a(x)\chi^a(x)\ ,$$

- **Mesonic and Baryonic Composites**

$$M(x) = \delta_{ab}\bar{\chi}^a(x)\chi^b(x)\ ,$$

$$B(x) = \frac{1}{6}\varepsilon_{abc}\chi^a(x)\chi^b(x)\chi^c(x)\ ,\quad \bar{B}(x) = \frac{1}{N_c!}\varepsilon_{abc}\bar{\chi}^c(x)\bar{\chi}^b(x)\bar{\chi}^a(x)$$

- **Fermion Integral**

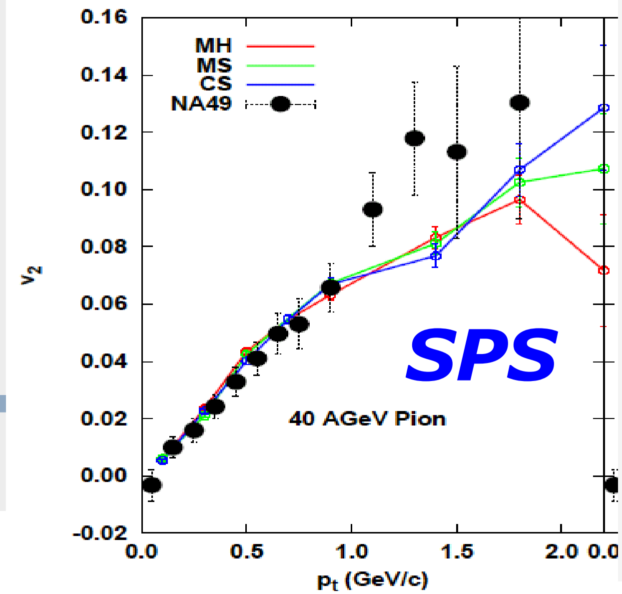
$$\int \mathcal{D}[U_0,\chi,\bar{\chi}]\exp\left[-\sum_t \sigma M - S_F^{(U_0)}\right] = \int \mathcal{D}[U_0,\chi,\bar{\chi}]\prod_k \exp\left[-\bar{\chi}_k G(k)\chi_k/2\right]$$

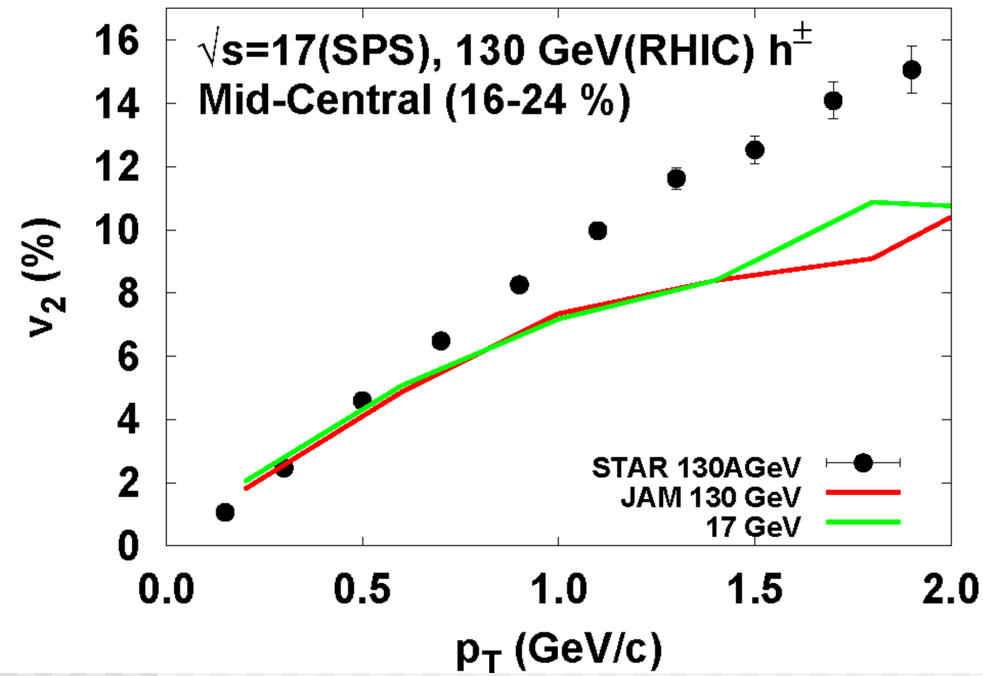
$$= \cdots = C_\sigma^3 - \frac{1}{2}C_\sigma + \frac{1}{4}\cosh(3\beta\mu)$$

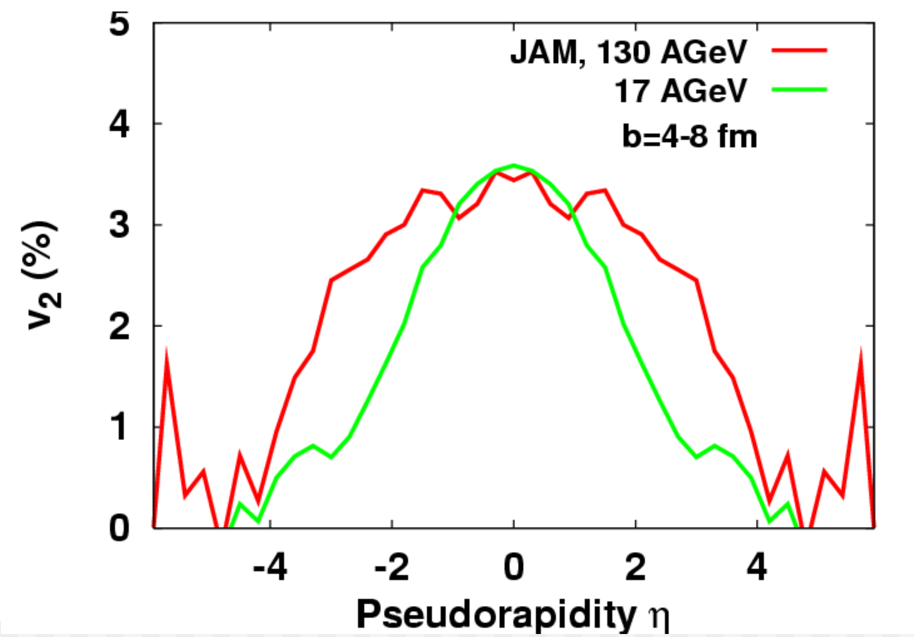
$$F_{\rm eff}^{(q)}(\sigma_q) = -T\log\left[\frac{4}{3}\left(C_\sigma^3 - \frac{1}{2}C_\sigma + \frac{1}{4}C_{3\mu}\right)\right] \qquad C_\sigma = \cosh\left[\beta \operatorname{arcsinh}\tilde{\sigma}\right]$$

by Isse

$dN/d\eta$, $v_2(\eta)$, $v_2(p_T)$ @RHIC 130GeV, Au+Au @SPS 17 GeV, Pb+Pb







- ***JAM*** での ***mid-central*** での ***SPS*** と ***RHIC*** の比較では、$v_2(\eta)$ の中心付近、$v_2(p_T)$ はほとんど同じ値を示す。
- ***JAM*** では、衝突初期の配置により v_2 のほとんどは決定されると考えられる。

by Isse

相対論的流体模型

$\partial_\mu T^{\mu\nu} = 0$　エネルギー運動量保存

$\partial_\mu n_i u^\mu = 0$　カレントの保存　*(baryon, strangeness,…)*

e : エネルギー密度
P : 圧力
u^μ: :4元速度　□ *(1,v)*
n_i: : 密度

$$T^{\mu\nu} = (e+P)u^\mu u^\nu - Pg^{\mu\nu}$$

τ_0, T^{ch} :Au+Au の dN/dη を fit, T^{th}: 可変

5 本の独立な方程式
6 個の独立変数
e, P, n_i, v

↓

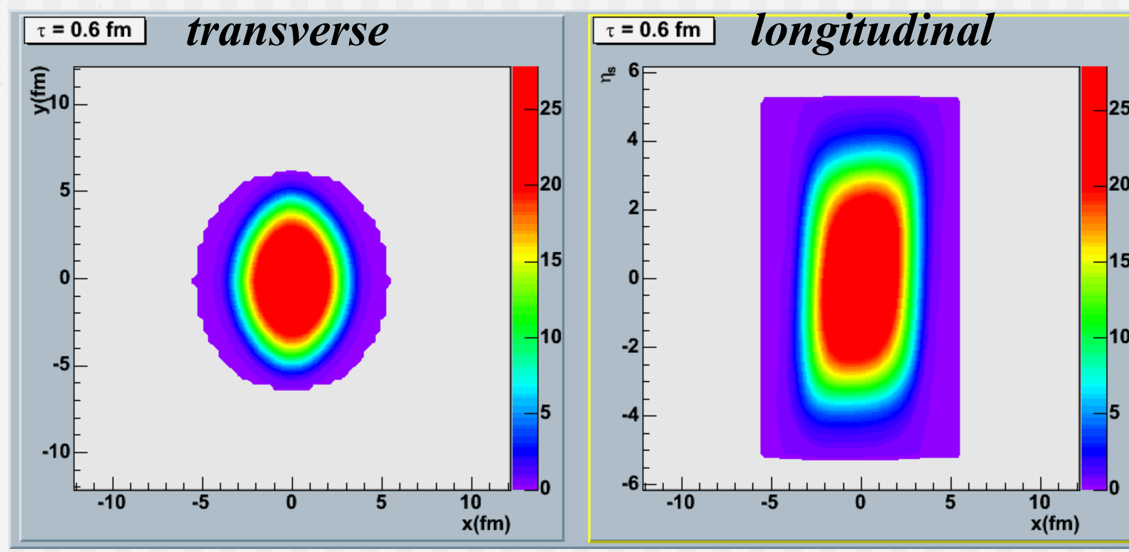
状態方程式 $P(e,n_i)$ を仮定

初期条件を与え、**Bjorken** 座標 (τ, η_s, x, y) で解く。

↓

$$\tau = \sqrt{t^2 - z^2}$$

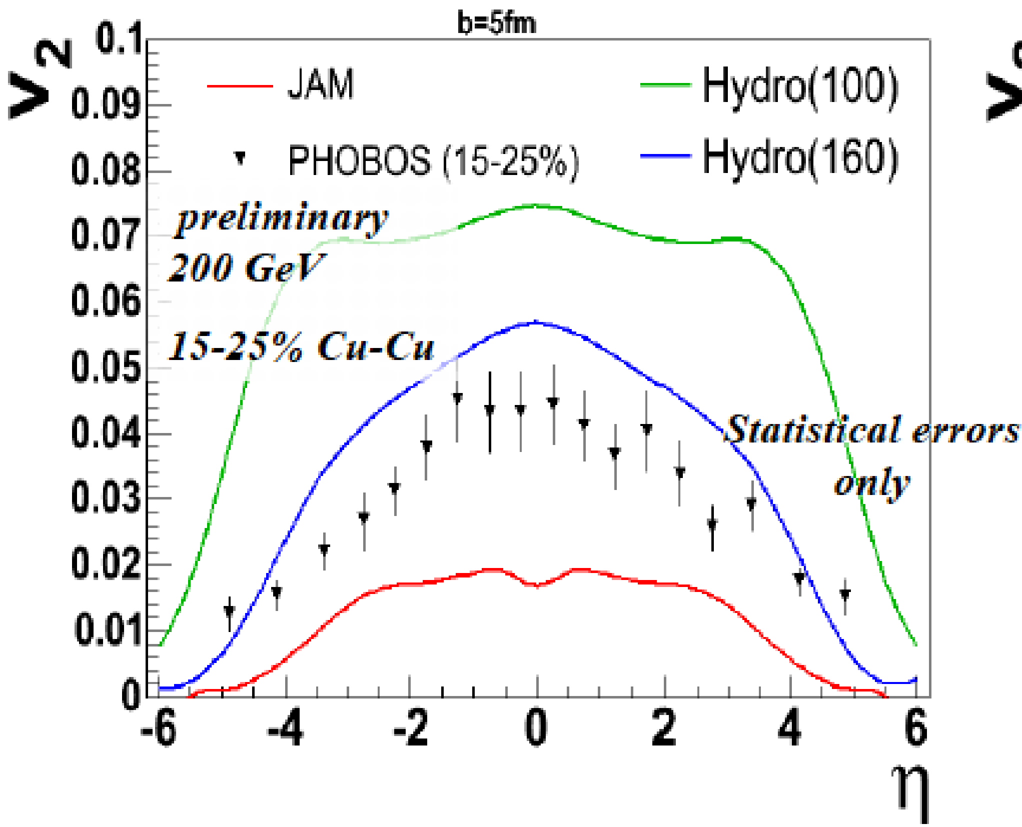
$$\eta_s = \frac{1}{2}\log\frac{t+z}{t-z}$$

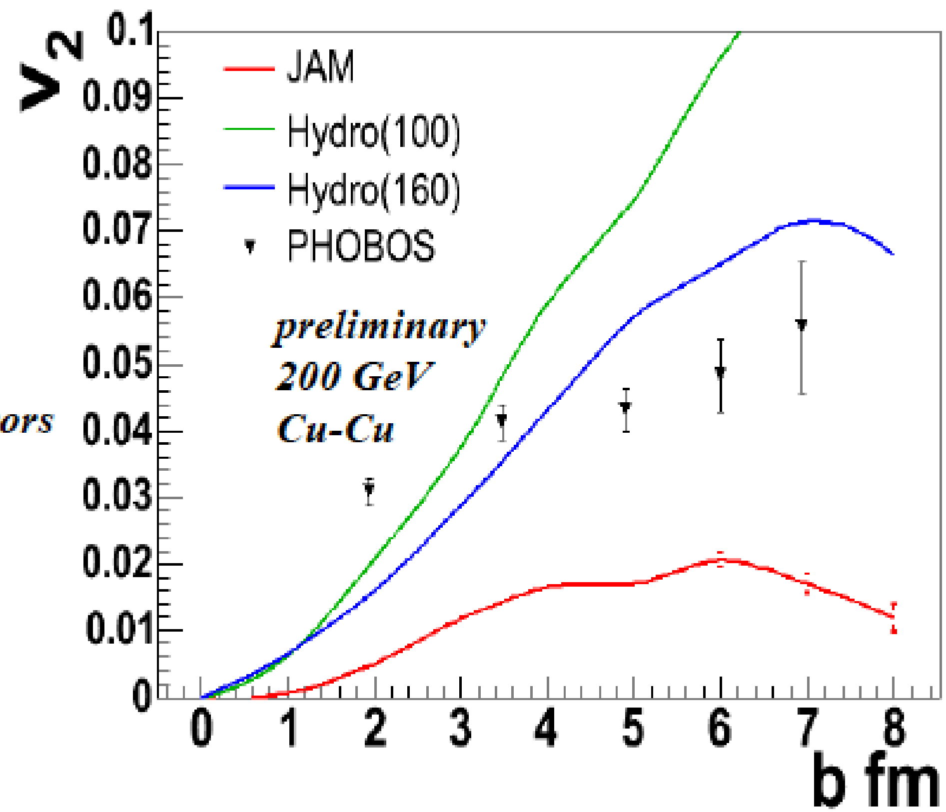


T. Hirano, Y. Nara, Nucl. Phys. A743, 305 (2004)
T. Hirano, K. Tsuda, Phys. Rev. C 66, 054905(2002)

PHOBOS の講演より

Compared to JAM Model





Cu-Cu more like Hydro than JAM hadron string cascade model

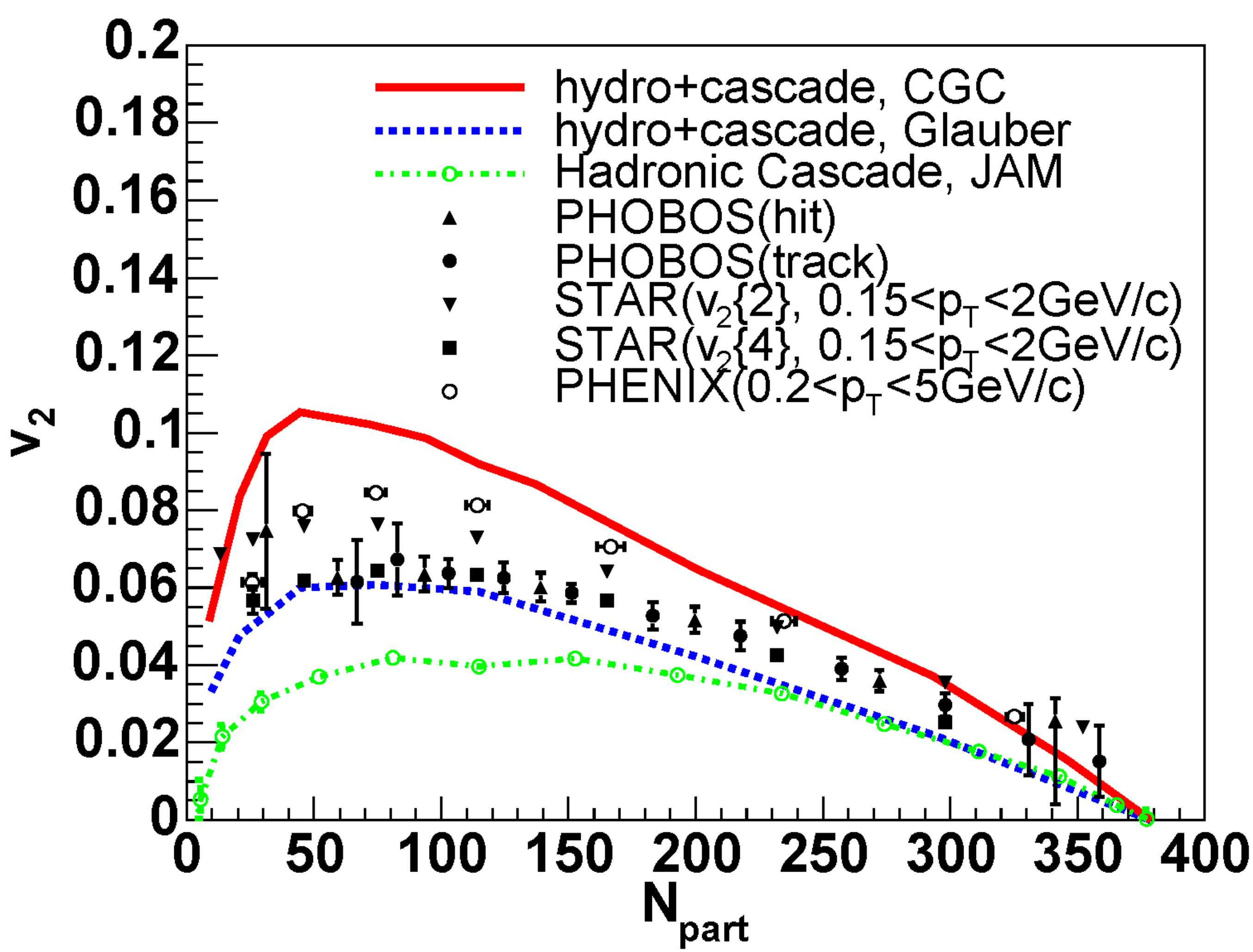
Here JAM uses a 1 fm/c formation time. Hydro (160) has kinetic freezeout temperature at 160 MeV

PHOBOS

Division of Nuclear Phys

- **v_2は、両模型の違いが顕著。**
- **流体模型でも T^{th} (⇔*Freeze out* の早さ) で違いがある。**

v_2(Centrality) @RHIC 200GeV, Au+Au



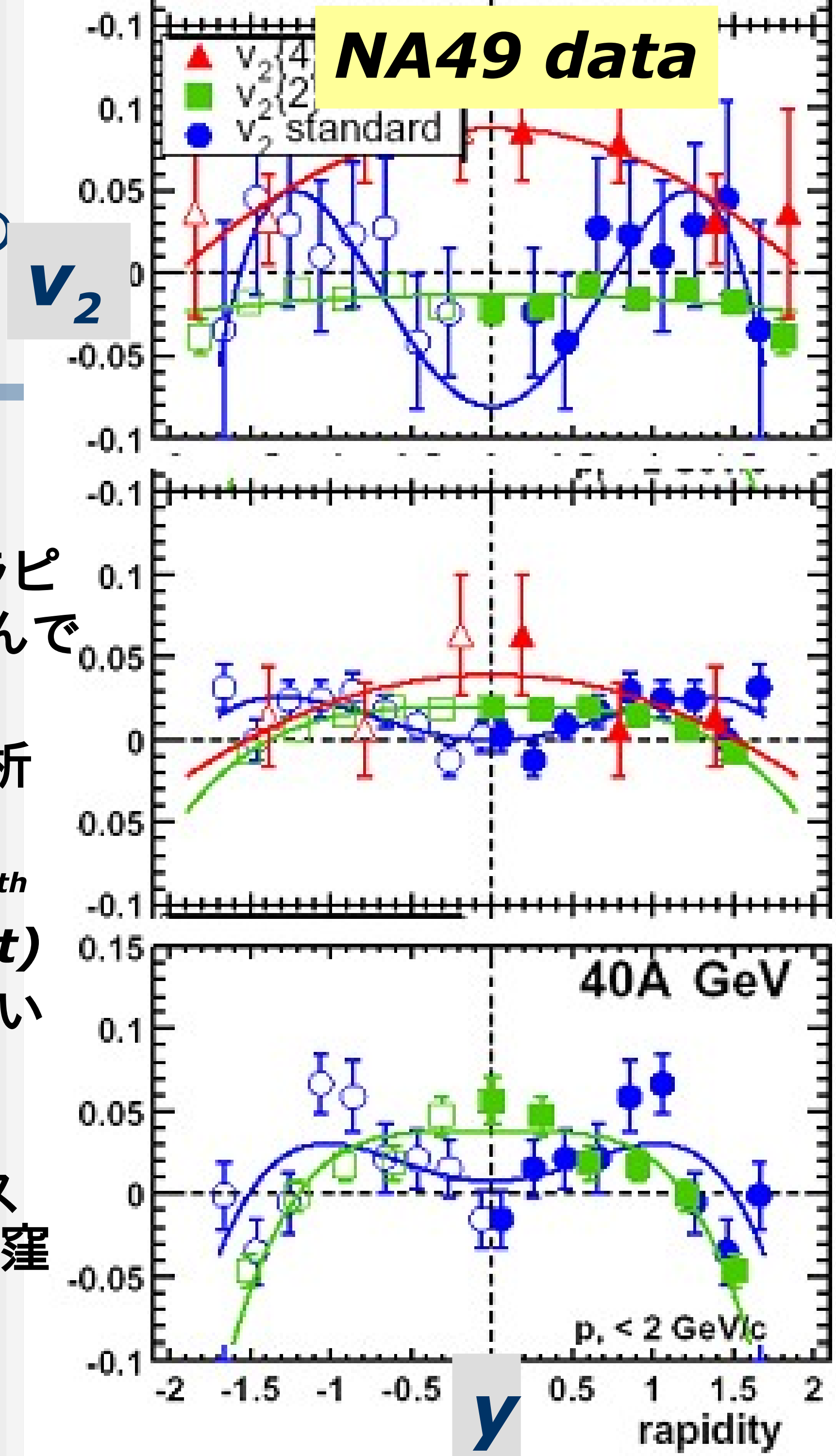
- カスケード模型 JAM は、周辺衝突になるにつれて (図の左側) 、小さくなってしまう。
- 実験値は、流体模型+カスケードに CGC または Glauber 近似を仮定したものの間に位置する。

⇒ 流体描像が妥当

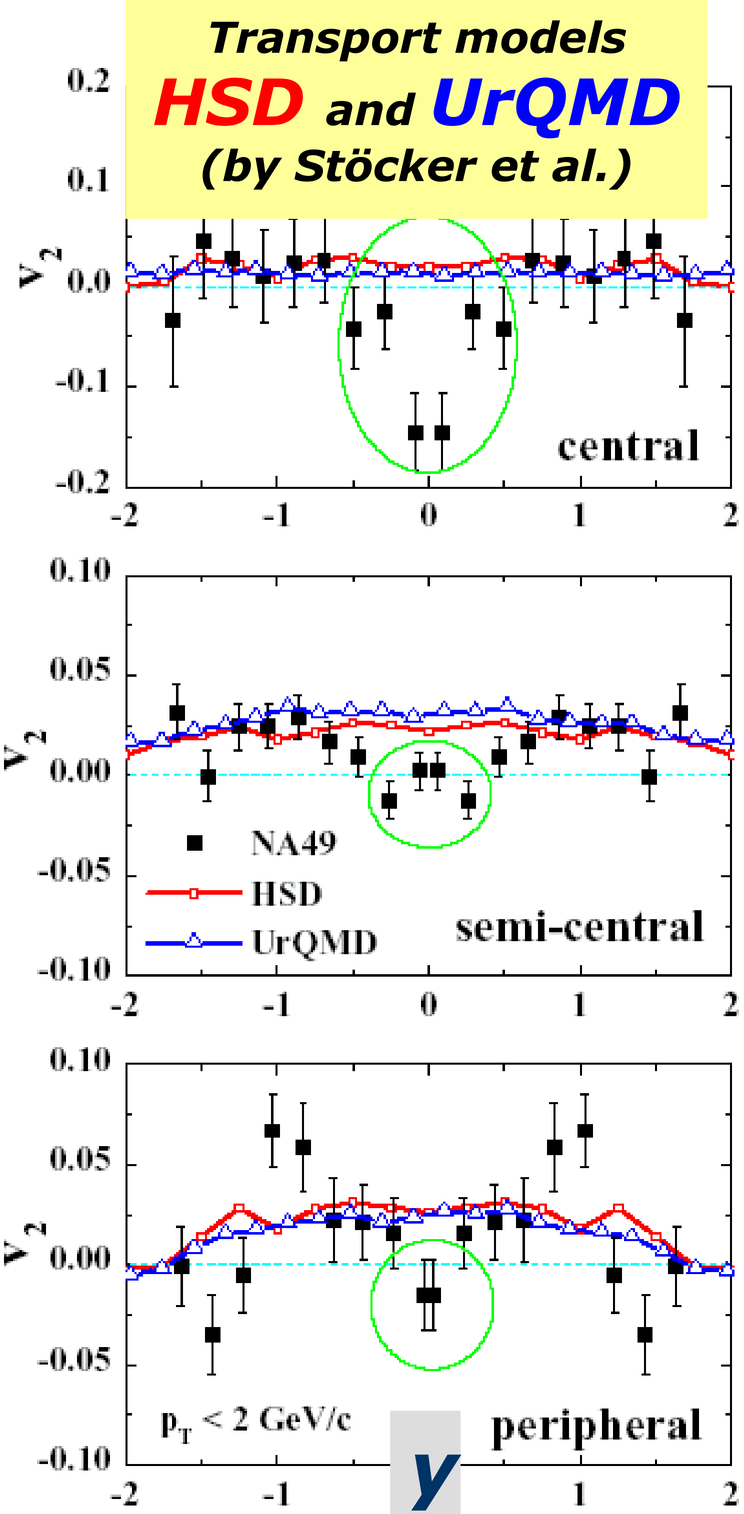
⇒ 部分的に CGC

40AGeV での Proton $v_2(y)$ のふるまい

♣ ***NA49*** の ***"Standard"*** のデータでは、中心ラピディティで v_2 が窪んでいる。
- 実験データは解析方法に大きく依存 ***(Standard, 2nd/4th order ofcumulant)***
- 相転移の証拠という主張もある。 ***(Stöcker et al.)***
- 他のハドロンカスケード模型も、この窪みを再現できない。



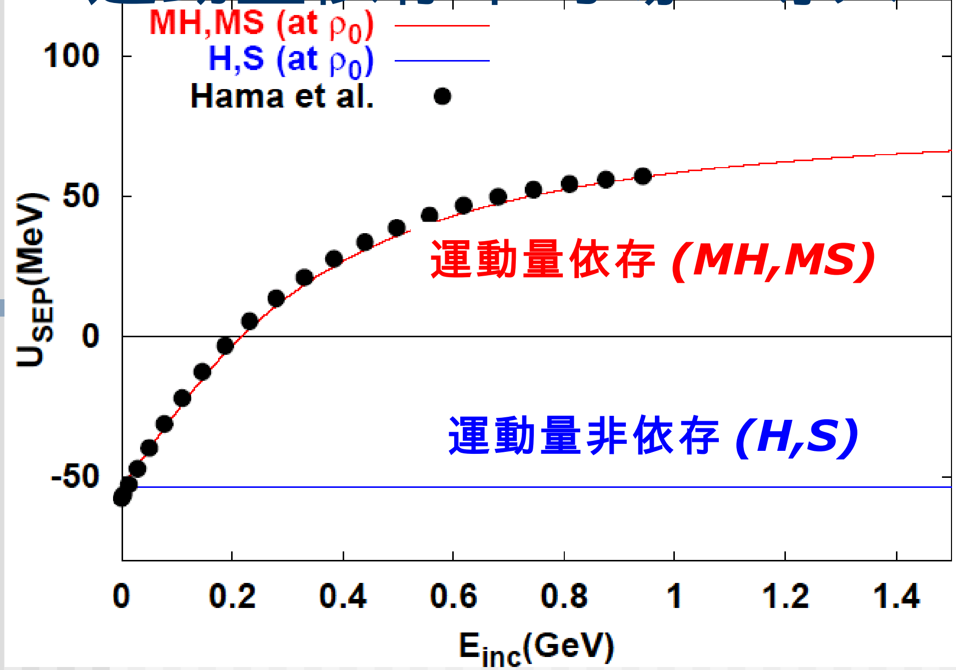
Phys. Rev. C68,034903(2003)

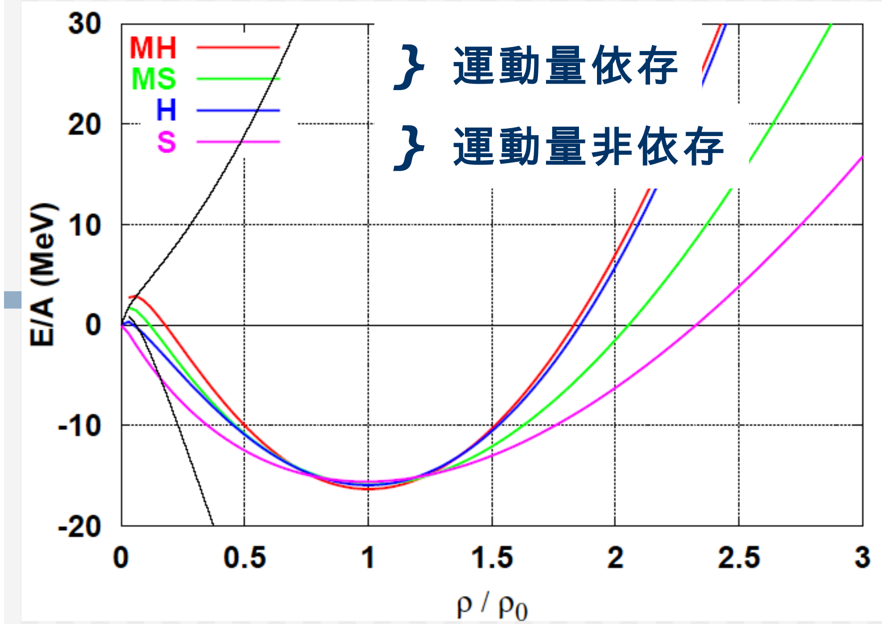


J. Phys. G31,S929(2005)

運動量依存平均場の導入

by Isse





パラメーター□□□ $C_{ex}^{(k)}$, μ_k を四種選ぶ。

♠ ***MH,MS:*** 運動量依存性は、***proton+*** 原子核 ***(***^{12}***C,*** ^{40}***Ca,*** ^{208}***Pb)*** 散乱から得られた光学ポテンシャル U_{SEP} を合わせるように決める。

[S. Hama et al, Phys. Rev. C 41, 2737 (1990)] (左図)

▯ 密度依存性は、ゼロ温度で原子核が飽和性質する性質 (右図) を再現するように決める。

K は決まっていないので 2 種づつ選んだ

MH:K=448MeV
MS:K=314MeV
H: K=380MeV
S: K=200MeV

$$E/A = \frac{3}{5}\frac{p_F(\rho)^2}{2m} + V(\rho)$$

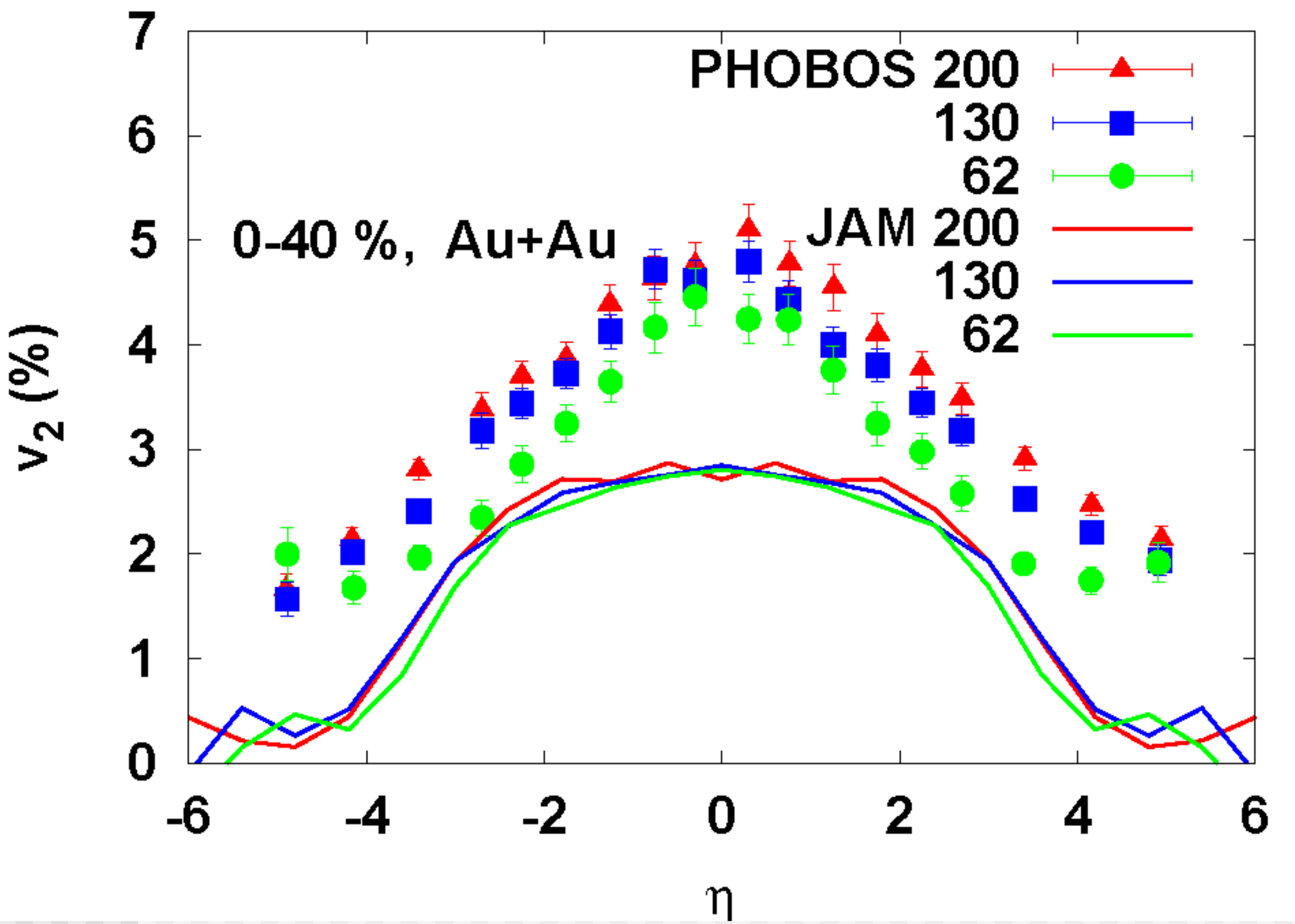
$U_{SEP}=\delta V/\delta f$ で、1 粒子あたりのポテンシャル

$$V = \int dr\left[\frac{\alpha\rho^2(r)}{2\rho_0} + \frac{\beta\rho^{\gamma+1}(r)}{(1+\gamma)\rho_0^{\gamma}}\right] + \sum_{k=1}^{2}\frac{C_{\rm ex}^{(k)}}{2\rho_0}\int dr\,dp\,dp'\,\frac{f(r,p)f(r,p')}{1+[(p-p')/\mu_k]^2}$$

但し、$\int f(r,p)dp=\rho(r)$

by Isse

入射エネルギーごとのハドロンカスケードでの比較 (0-40%:central~mid-central) @RHIC 62,130,200GeV,Au+Au



ハドロンカスケードでの *v_2* は、入射エネルギーによらず、ほぼ同じ値をとる。実験値には満たない。

実験値にある、*v_2* の微増、□’でのスケーリングは、得られない。

⇒ ハドロン相互作用起源でない *v_2* の増加 *(~QGP)*