Part I:
Collective flow in high-energy heavy-ion collisions and nuclear mean-field

Part II:
Hadron-string cascade vs hydrodynamics in heavy-ion collisions at RHIC

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Part I:
Basics in high-energy heavy-ion collisions

Part II:
Collective flows from AGS to RHIC energies
--- Cascade vs Hydrodynamics: When and where is QGP formed ? ---

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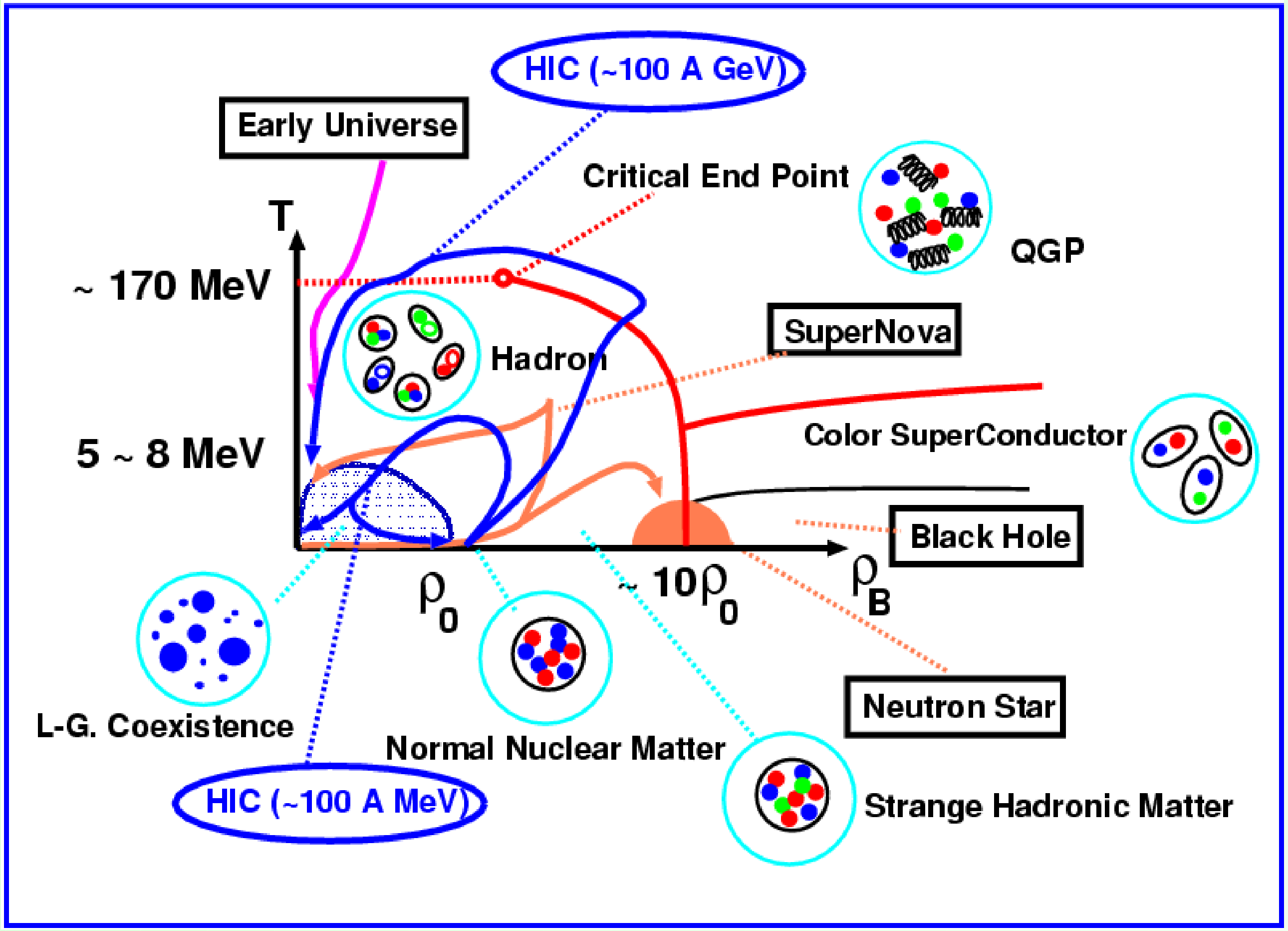
Hadronic Matter Phase Diagram



HIC (~ A few 100 A MeV) = Little Supernova
HIC (100+100 A GeV) = Little Big Bang

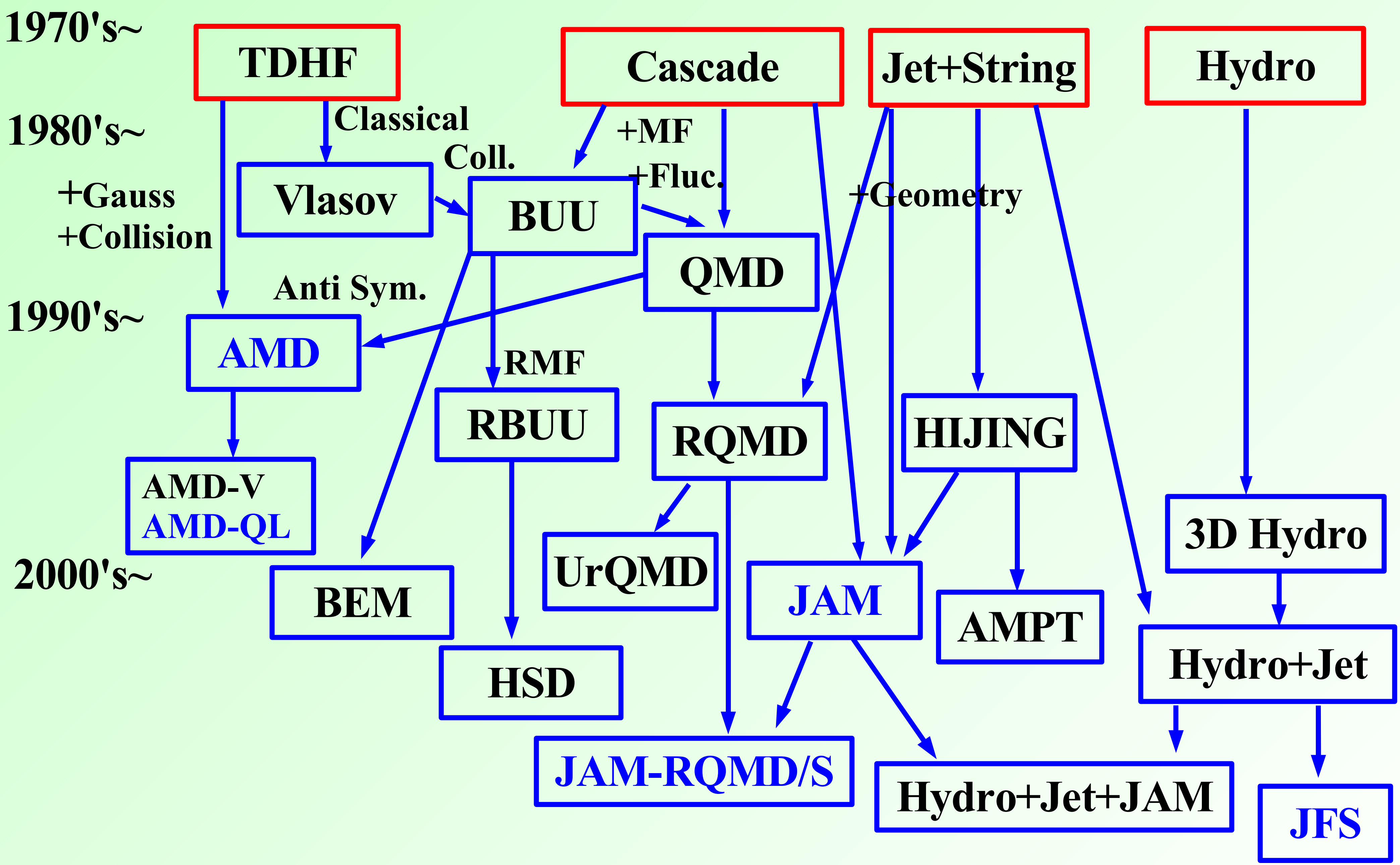
Part I: Basics in high-energy heavy-ion collisions

- Nuclear Mean Field Dynamics for HIC
- Hadronic Cascade and Hadron-Hadron Cross Section
- Regge, Strings, and Partonic Interactions
- Relativistic Hydrodynamics
- Mean Field adopted in HIC

HIC Models: Major Four Origins

- *Nuclear Mean Field Dynamics*
  - Basic Element of Low Energy Nuclear Physics,
    and Critically Determines High Density EOS / Collective Flows
  - TDHF → Vlasov → BUU
- *NN two-body (residual) interaction*
  - Main Source of Particle Production
  - Intranuclear Cascade Models
- *Partonic Interaction and String Decay*
  - Main Source of high pT Particles at Collider Energies
  - JETSET + (previous) PYTHIA (Lund model) → (new) PYTHIA
- *Relativistic Hydrodynamics*
  - Most Successful Picture at RHIC

HIC Models: History



Nuclear Mean Field Models for Heavy-Ion Collisions

TDHF and Vlasov Equation

- **Time-Dependent Mean Field Theory (e.g., TDHF)** $i\hbar \dfrac{\partial \phi_i}{\partial t} = h\phi_i$
- **Density Matrix**

$$\rho(r, r') = \sum_i^{Occ} \phi_i(r)\phi_i^*(r') \quad \rightarrow \quad \boldsymbol{\rho_W = f} \text{ (phase space density)}$$

- **TDHF for Density Matrix**

$$i\hbar \frac{\partial \rho}{\partial t} = [h, \rho] \quad \rightarrow \quad \frac{\partial f}{\partial t} = \{h_W, f\}_{P.B.} + O(\hbar^2)$$

- **Wigner Transformation and Wigner-Kirkwood Expansion (Ref.: Ring-Schuck)**

$$O_W(r, p) \equiv \int d^3 s \exp(-i p \cdot s/\hbar) < r + s/2 | O | r - s/2 >$$

$$(AB)_W = A_W \exp(i\hbar\Lambda) B_W \quad \Lambda \equiv \nabla'_r \cdot \nabla_p - \nabla'_p \cdot \nabla_r \quad (\nabla' \text{ acts on the left})$$

$$[A, B]_W = 2\mathrm{i} A_W \sin(\hbar\Lambda/2) B_W = i\hbar\{A_W, B_W\}_{P.B.} + O(\hbar^3)$$

Test Particle Method

- **Vlasov Equation**

$$\frac{\partial f}{\partial t} - \{h_W, f\}_{P.B.} = \frac{\partial f}{\partial t} + v \cdot \nabla_r f - \nabla U \cdot \nabla_p f = 0$$

- **Classical Hamiltonian**

$$h_W(r,p) = \frac{p^2}{2\mathrm{m}} + U(r,p)$$

- **Test Particle Method (C. Y. Wong, 1982)**

$$f(r,p) = \frac{1}{N_0} \sum_i^{A N_0} \delta(r - r_i)\,\delta(p - p_i) \quad \rightarrow \quad \frac{dr_i}{dt} = \nabla_p h_w, \quad \frac{dp_i}{dt} = -\nabla_r h_w,$$

Mean Field Evolution can be simulated by Classical Test Particles
→ Opened a possibility to Simulate High Energy HIC including Two-Body Collisions in Cascade

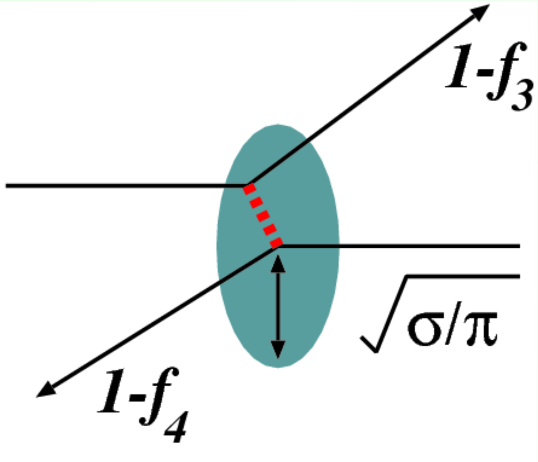
BUU (Boltzmann-Uehling-Uhlenbeck) Equation

- **BUU Equation (Bertsch and Das Gupta, Phys. Rept. 160(88), 190)**

$$\frac{\partial f}{\partial t} + v \cdot \nabla_r f - \nabla U \cdot \nabla_p f = I_{coll}[f]$$

$$I_{coll}[f] = -\frac{1}{2} \int \frac{d^3 p_2 d\Omega}{(2\pi\hbar)^3} v_{12} \frac{d\sigma}{d\Omega} \times [f f_2 (1-f_3)(1-f_4) - f_3 f_4 (1-f)(1-f_2)]$$

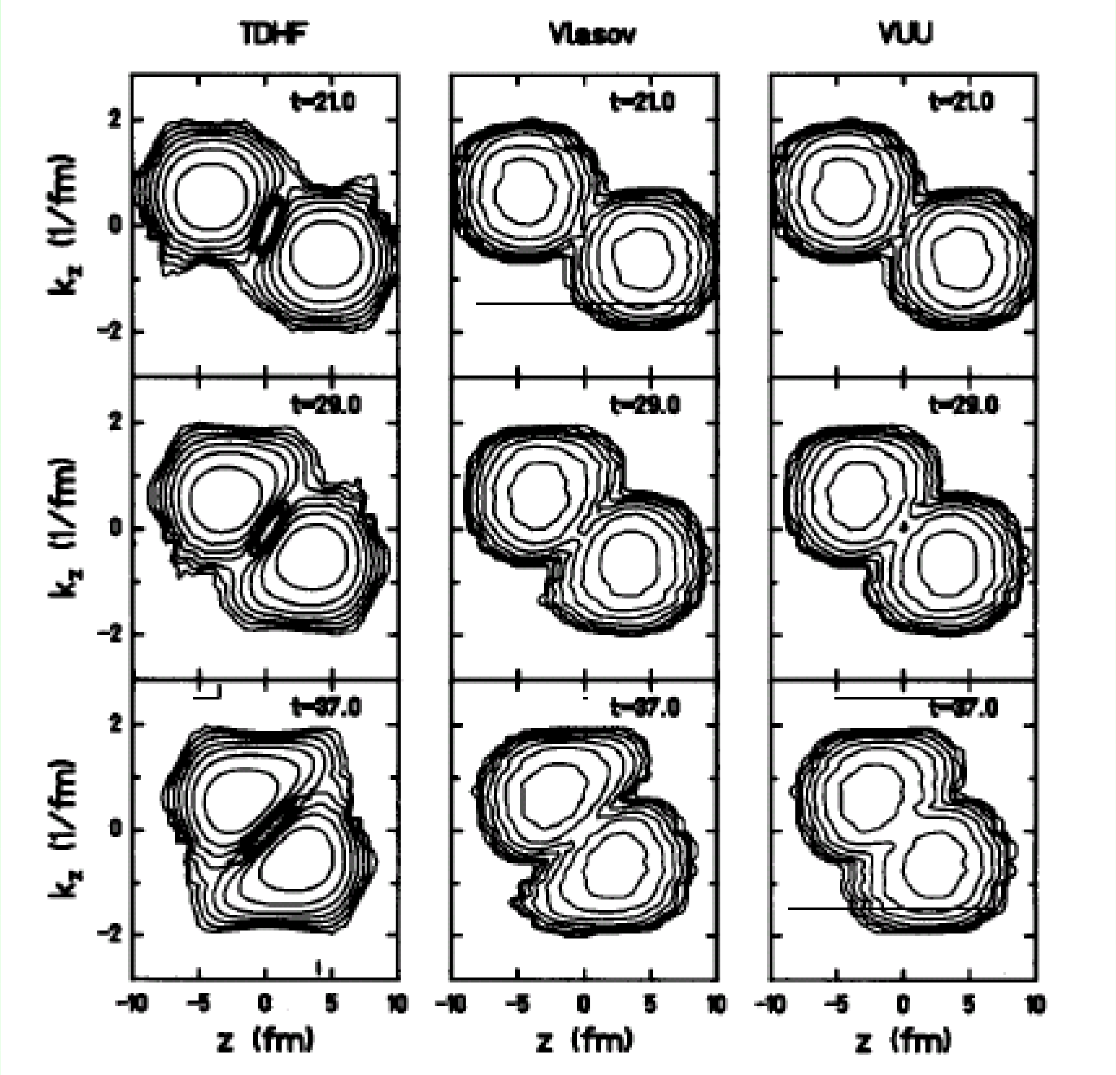
- **Incorporated Physics in BUU**
  - **Mean Field Evolution**
  - **(Incoherent) Two-Body Collisions**
  - **Pauli Blocking in Two-Body Collisions**



○ *One-Body Observables (Particle Spectra, Collective Flow, ..)*
X *Event-by-Event Fluctuation (Fragment, Intermittency, ...)*

Comarison of TDHF, Vlasov and BUU(VUU)

- Ca+Ca, 40 A MeV
  (Cassing-Metag-Mosel-Niita, Phys. Rep. 188 (1990) 363).



Exercise (1)

- Prove that the spatial integral of the Wigner function *f(x,p)* gives a momentum distribution of nucleons.
- Prove that the Wigner function with test particles satisfy the Vlasov equation when the test particle follows the classical EOM.
- Prove that the collision term becomes zero (i.e. gain and loss terms cancel) in equilibrium.
- Derive the collision term for bosons, which disappears in equilibrium.
- *(ADVANCED)* Prove the relation of the commutator and Poisson bracket. (It takes a long time ....)
- *(ADVANCED)* Prove that the Wigner function can be negative. (Therefore, the probability interpretation is not always possible.)

AMD (Antisymmetrized Molecular Dynamics)

(Ono, Horiuchi, Maruyama, AO, PTP(1992).

- **Gaussian Approximation for single particle wave function**

$$|\Psi\rangle = A\prod|\psi_i\rangle \quad , \quad \psi_i = \phi(r;Z_i)\chi(\sigma,\tau) \quad , \qquad Z = \sqrt{\nu}\,D + \frac{i}{2\hbar\sqrt{\nu}}K$$

$$\phi(r;Z) = \left(\frac{2\nu}{\pi}\right)^{3/4} \exp\left(-\nu(r - Z/\sqrt{\nu})^2 + Z^2/2\right) \propto \exp\left(-\nu(r-D)^2 + i\,K\cdot(r-D)/\hbar\right)$$

- **Time-dependent Variational Principle → Equations of Motion**

$$L = \frac{\langle\Psi|i\hbar\partial/\partial t - H|\Psi\rangle}{\langle\Psi|\Psi\rangle} \quad , \quad \frac{d}{dt}\frac{\partial L}{\partial(d\bar{Z}_i/dt)} - \frac{\partial L}{\partial\bar{Z}_i} = 0 \rightarrow \quad i\hbar C_{i\alpha,j\beta}\frac{dZ_i}{dt} = \frac{\partial H}{\partial\bar{Z}_i}$$

- **Ignoring Antisymmetrization**
  → Quantum Molecular Dynamics EOM (= Classical EOM)

$$C = \delta \quad \rightarrow \quad \frac{dD_i}{dt} = \frac{\partial H}{\partial K_i} \quad , \quad \frac{dK_i}{dt} = -\frac{\partial H}{\partial D_i}$$

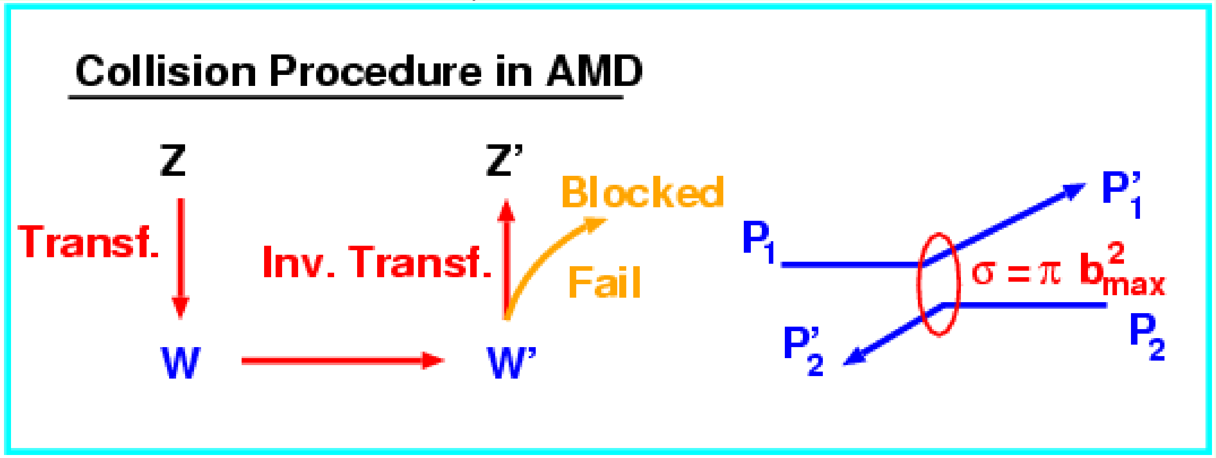
Classical-type EOM is obtained through Gaussian + TDVP

Collision Term in AMD

- **Approximate Canonical Variables**

$$W_i = \sqrt{Q}_{ij} Z_j = \sqrt{\nu}\, R_i + \frac{i}{\sqrt{\nu}\hbar} P_i \quad , \quad Q_{ij} \equiv B_{ij} \;\; B_{ij}^{-1} \quad , \quad B_{ij} = \langle \psi_i | \psi_j \rangle$$

$$\mathit{Example} \quad \langle \mathbf{L} \rangle = \sum_{ij} B_{ji}^{-1} B_{ij} \frac{1}{i} \overline{Z}_i \times Z_j = \sum_i \overline{W}_i \times W_i$$



Physics included in AMD

Time Evolution of Anti-Symmetrized Wave Function

Collision Term = “Canonical" Variable + Classical Analogy

Event-by-Event Fluctuation

Problems: Non-Rela., Classical Analogy of Collision term,CPU cost

Relativistic QMD/Simplified (RQMD/S)

- RQMD = Constraint Hamiltonian Dynamics
  (Sorge, Stocker, Greiner, Ann. of Phys. 192 (1989), 266.)
- Constraints: φ ≈ 0 (Satisfied on the realized trajectory, by Dirac)
  - Variables in Covariant Dynamics = 8N phase space: q_μ, p_μ
  - Variables in EOM = 6N phase space
    → We need 2N constraints to get EOM
- On Mass-Shell Constraints

$$H_i \equiv p_i^2 - m_i^2 - 2m_i V_i \approx 0$$

- Time-Fixation in RQMD/S

$$\chi_i \equiv \hat{a}\cdot(q_i - q_N) \approx 0 \left(i = 1, \sim N-1\right) \quad , \quad \chi_N \equiv \hat{a}\cdot q_N - \tau \approx 0$$

$$\hat{a} = \text{Time-like unit vector in the Calculation Frame}$$

(Tomoyuki Maruyama et al., Prog. Theor. Phys. 96(1996), 263.)

RQMD/S (cont.)

- **Hamiltonian is made of constraints**

$$H=\sum_i u_i \phi_i \quad (\phi_i = H_i (i=1 \sim N), \chi_{i-N} (i=N+1 \sim 2N))$$

- **Time Development** $\dfrac{df}{d\tau} = \dfrac{\partial f}{\partial \tau} + \{f, H\} \quad , \quad \{q_\mu, p_\nu\} = g_{\mu\nu}$
- **Lagrange multipliers are determined to keep constraints**
  → *We can solve obtain the multipliers analytically in RQMD/S*

$$\frac{d\phi_i}{d\tau} \approx 0 \;\rightarrow\; \delta_{i,2N} + \sum_j u_j \{\phi_i, \phi_j\} \approx 0$$

- **Equations of Motion**

$$H=\sum_i (p_i^2 - m_i^2 - 2m_i V_i)/2p_i^0 \quad , \quad p_i^0 = E_i = \sqrt{\vec{p}_i^{\,2} + m_i^2 + 2m_i V_i}$$

$$\frac{d\vec{r}_i}{d\tau} \approx -\frac{\partial H}{\partial \vec{p}_i} = \frac{\vec{p}}{p_i^0} + \sum_j \frac{m_j}{p_j^0} \frac{\partial V_j}{\partial \vec{p}_i} \quad , \quad \frac{d\vec{p}_i}{d\tau} \approx \frac{\partial H}{\partial \vec{r}_i} = -\sum_j \frac{m_j}{p_j^0} \frac{\partial V_j}{\partial \vec{r}_i}$$

We can include MF in an almost covariant way in molecular dynamics

Particle "DISTANCE"

$$r_{Tij}{}^2 \equiv r_\mu r^\mu - \left(r_\mu P_{ij}^\mu\right)^2 / P_{ij}^2 = \vec{r}^{\,2} \qquad (\texttt{in CM})$$

$$P_{ij} \equiv p_i + p_j \quad , \qquad r \equiv r_i - r_j$$

Particle "Momentum Difference"

$$p_{Tij}{}^2 \equiv p_\mu p^\mu - \left(p_\mu P_{ij}^\mu\right)^2 / P_{ij}^2 = \vec{p}^{\,2} \qquad (\texttt{in CM})$$

$$p \equiv p_i - p_j$$

Lorentz Invariant, and Becomes Normal Distance in CM !

Exercise (2)

- **Prove that the TDVP (time-dependent variational principle) gives the Schrodinger equation when the wave function is not restricted, for example to a Slater determinant.**
- *(ADVANCED)* **Prove that the AMD wave function is equivalent to harmonic oscillator shell model wave function when all Z's goes to zero. (This tells you why the Slater determinant of (s-wave) Gaussians can describe nuclei above s-shell.)**
- *(ADVANCED)* **Obtain the Lagrange multiplier in RQMD/S.**

Cascade Model
Hadron-Hadron Collisions

AA collisions at High E.

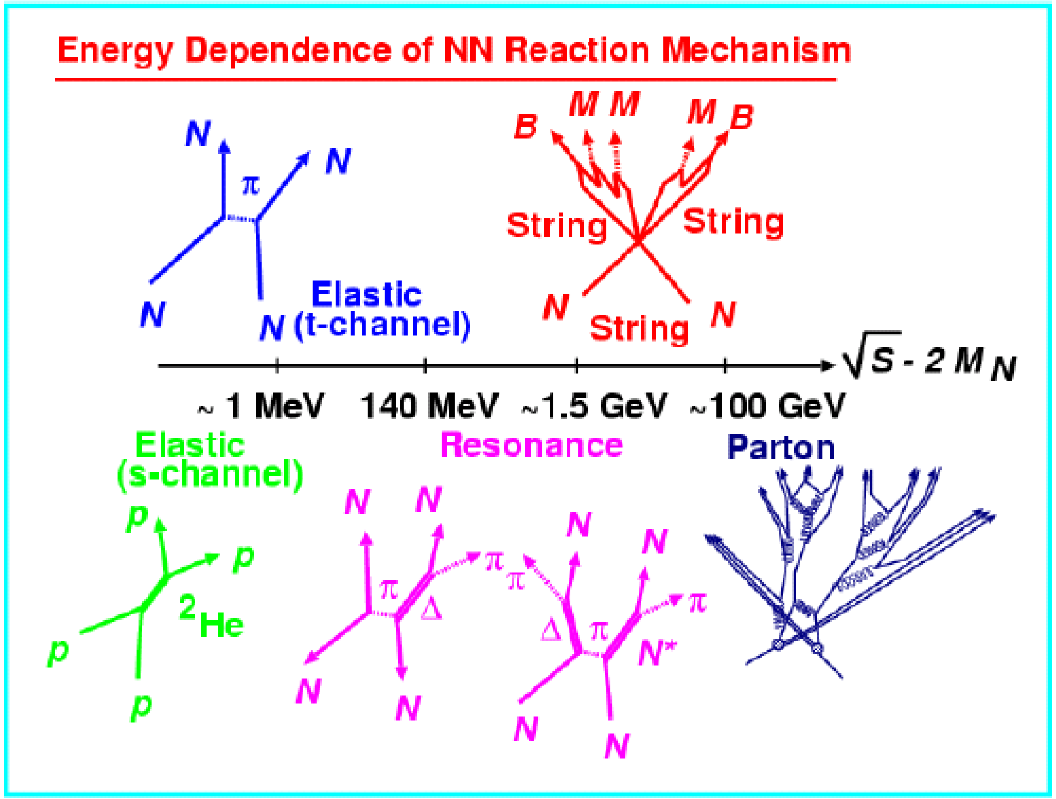
~ Sum of (Multistep) NN collisions (Cascade)

+ *Interesting Physics*

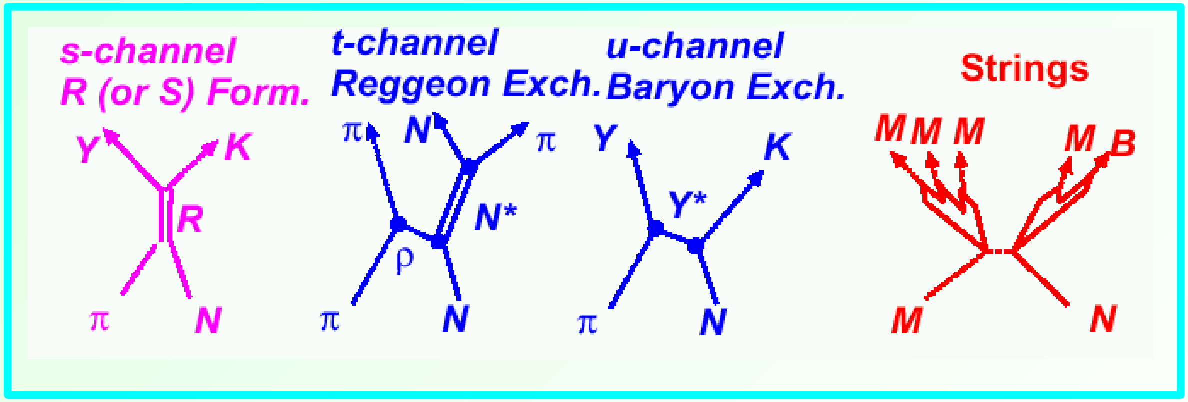
→ Cascade gives the "baseline" of evaluation !

Baryon-Baryon and Meson-Baryon Collisions

- NN collision mechanism
  - Elastic
  - → Resonance
  - → String
  - → Jet

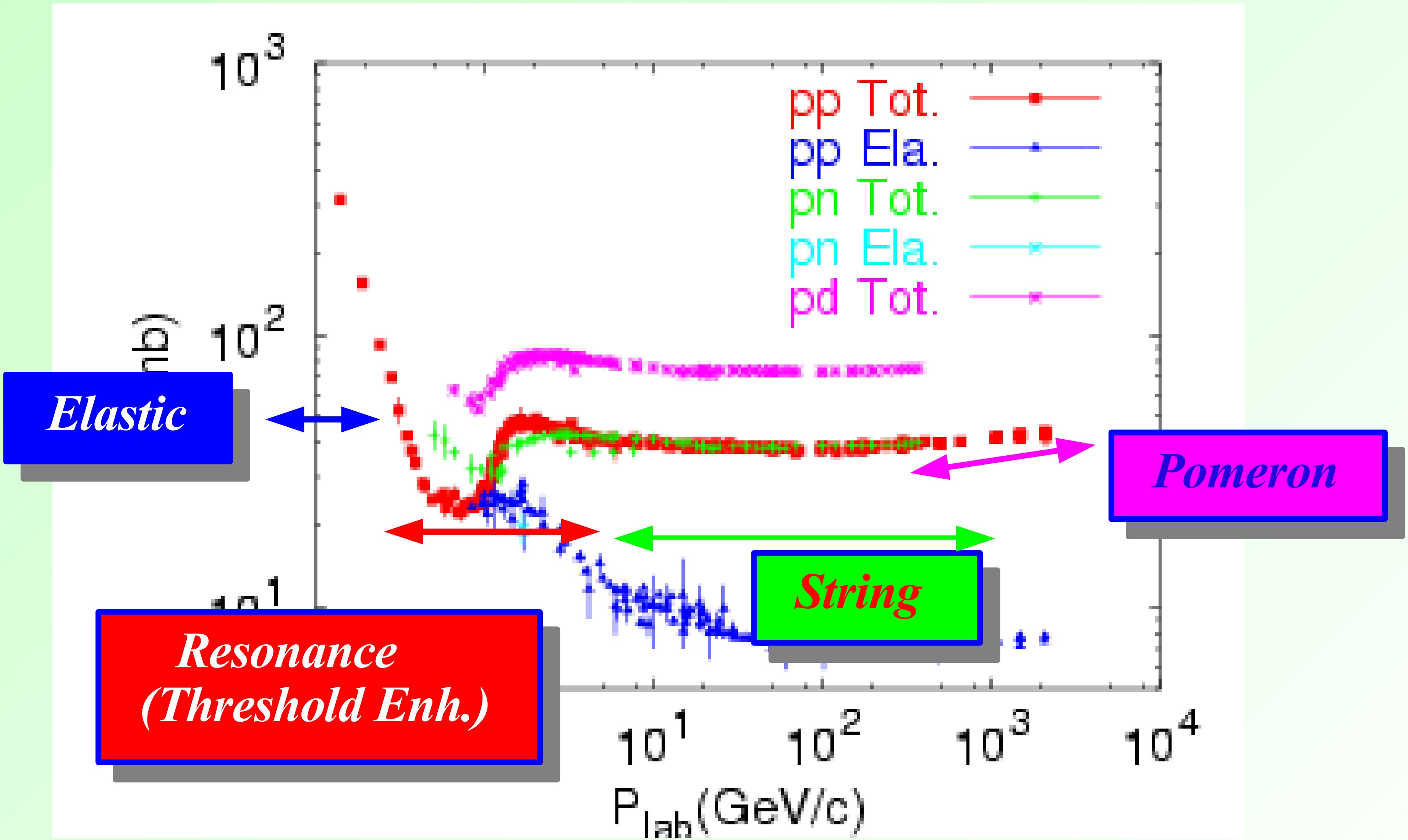


- Meson-Nucleon Collision
  - → s-channel Resonance
  - → t-(u-) channel Res.
  - → String formation

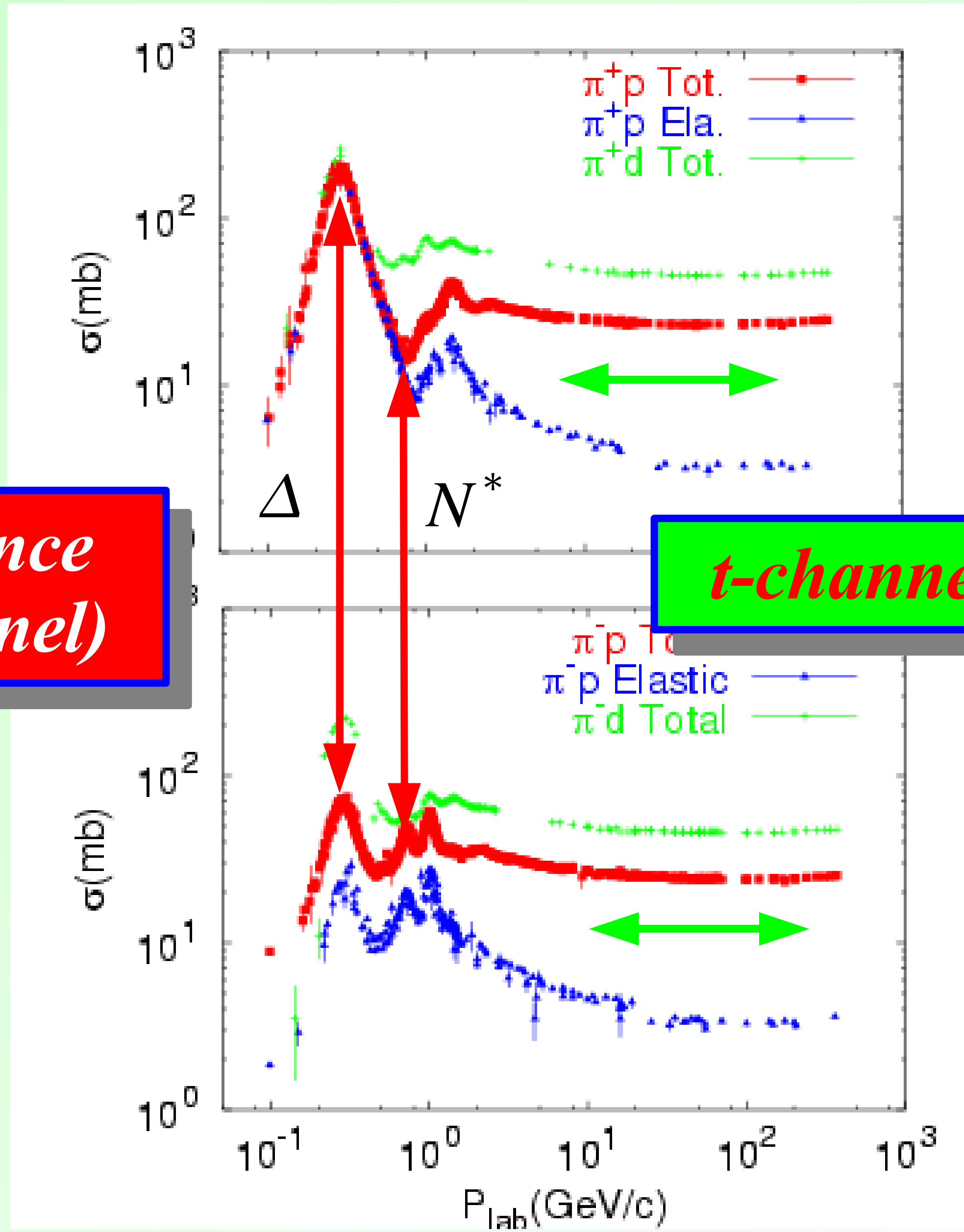


NN Cross Sections

From Particle Data Group



Meson-Baryon Cross Section



Resonance (s-channel)

t-channel and String

Regge, String, and Jet
--- High Energy hh Collisions ---

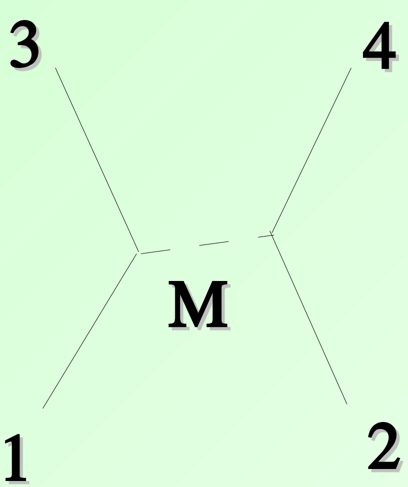
Reggeon Exchange

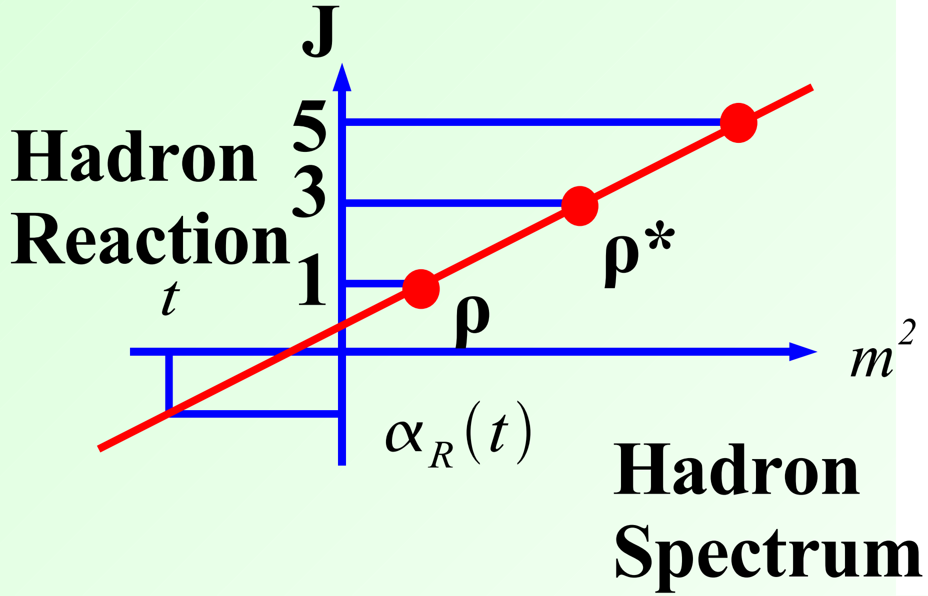
(Barger and Cline (Benjamin, 1969), H. Sorge, PRC (1995), RQMD2.1)

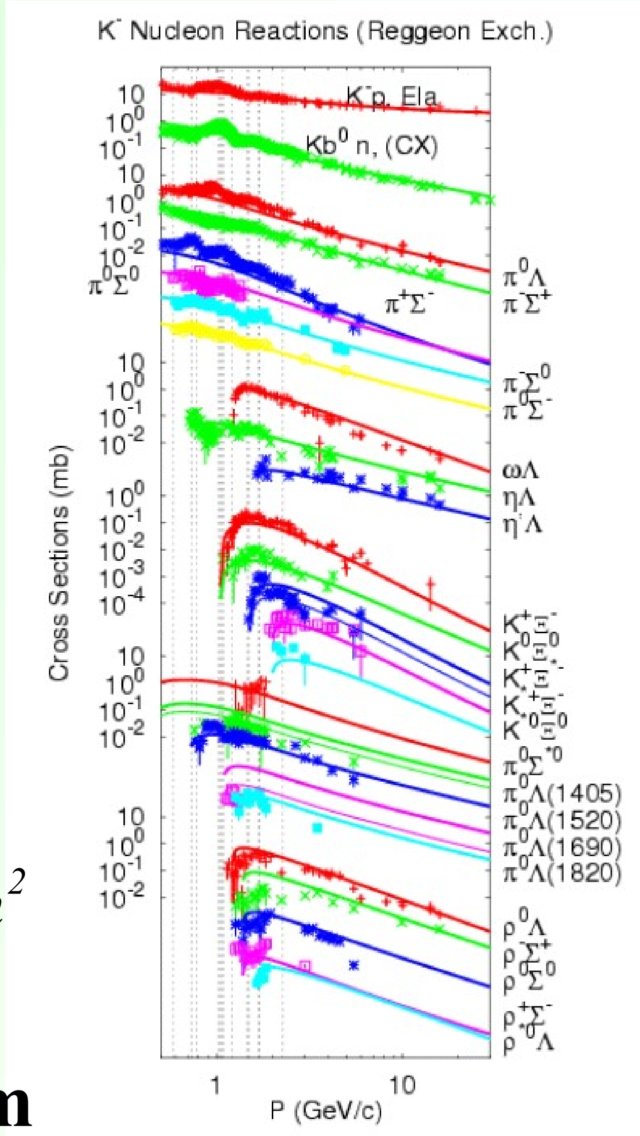
- Regge Trajectory $J = \alpha_R(t) \sim \alpha_R(0) + \alpha'_R(0)\, t$
- 2 to 2 Cross Section

$$\frac{d\sigma}{d\Omega} = \frac{p_f}{64\pi s\, p_i} |M(s,t)|^2$$

$$M(s,t) \sim \sum_R \frac{(p_i p_f)^J}{t - M_R} \sim F(t) \exp\left[\alpha_R(t) \log(s/s_0)\right]$$



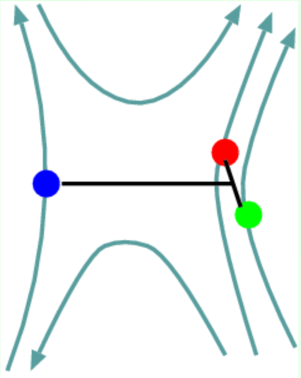




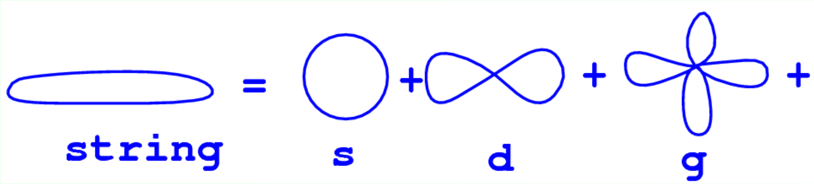
String formation and decay

- **What does the regge trajectory suggest ?**
  → Existence of (color- or hadron-)String !

$$M = 2\int_0^R \frac{\kappa\, dr}{\sqrt{1-(r/R)^2}} = \pi\kappa R \quad , \quad J = 2\int_0^R r \times \frac{\kappa\, dr}{\sqrt{1-(r/R)^2}}\frac{r}{R} = \frac{\pi\kappa R^2}{2}\pi$$



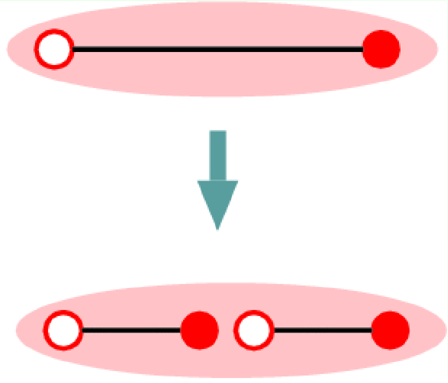
$$\rightarrow \quad J = \frac{M^2}{2\pi\kappa}$$



- **String Tension**

$$\frac{1}{2\pi\kappa} = \alpha'_R(0) \approx 0.9\, GeV^{-2} \quad \rightarrow \quad \kappa \approx 1\, GeV/fm$$

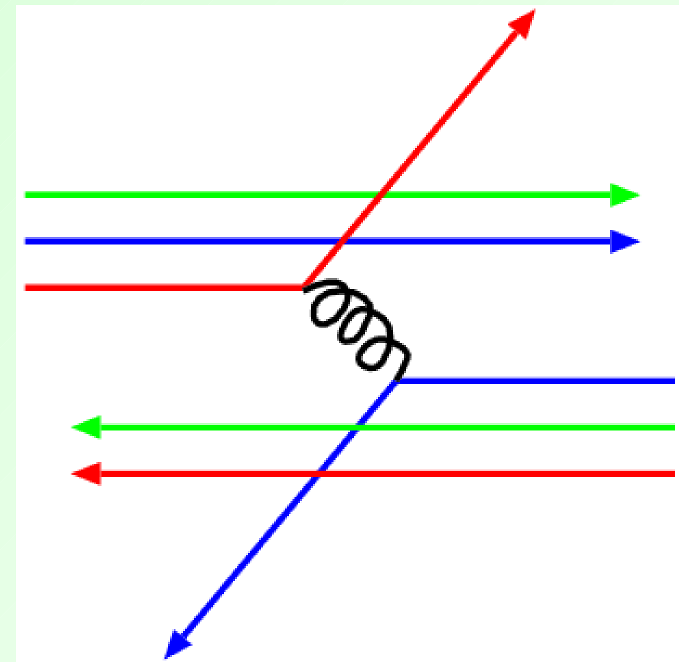
- **String decay**
  Extended String
  → Large E stored
  → q qbar pair creation (Schwinger mech.)



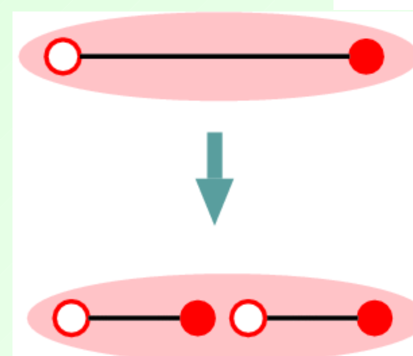
String = Coherent superposition of hadron resonances with various J

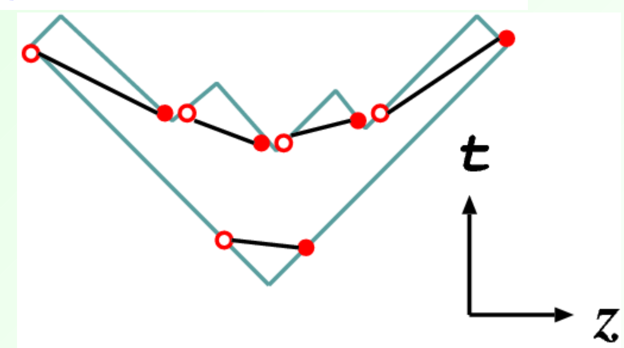
Jet Production

- **Elastic Scattering of Partons (mainly) with One Gluon Exch.**
- **Color Exch. between Hadrons**
  → Complex color flux starting from leading partons
  → many hadron production
  → Jet production



- ***PYTHIA***
  - **Event Generator of High Energy Reactions**
    → Jet production +String decay
    for QCD processes



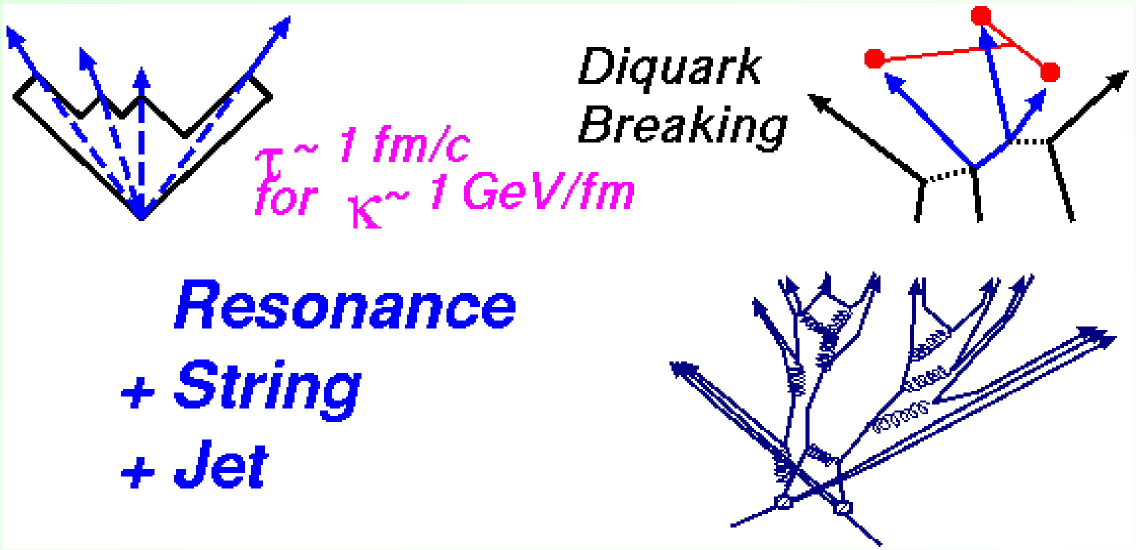


(T. Sjostrand et al., Comput. Phys. Commun. 135 (2001), 238.)

JAM (Jet AA Microscopic transport model)

Nara, Otuka, AO, Niita, Chiba, Phys. Rev. C61 (2000), 024901.

- **Hadron-String Cascade with Jet production**
  - **hh collision with Res. up to m < 2 GeV (3.5 GeV) for M (B)**
  - **String excitation and decay**
  - **String-Hadron collisions are simulated by hh collisions in the formation time.**
  - **jet production is incl. using PYTHIA**
  - **Secondary partonic int.:**
    NOT incl.
  - **Color transparency:**
    NOT taken care of



Exercise (3)

- **Prove that the sum of Mandelstam variables becomes a constant.**
  $s = (p_1+p_2)^2,\ \ t = (p_1-p_3)^2,\ \ u = (p_1-p_4)^2,$
  in 1+2 → 3+4 reaction.
- **Draw the Feynman diagram of $K^-+p \to \pi^+ + \Sigma^-$. You will be able to guess that the angular distribution becomes backward peaked due to the u-channel dominance.**
- **Explain why we have peak structures in MB collisions and we do not see peaks in BB collisions.**
- *(If you already learned QCD,)* **Obtain the squared Feynman amplitude of qq → qq in the tree level averaged over the color and spin. (You can ignore quark mass.) You will see the cross section is divergent at forward angle. Explain why we do not see this divergent behavior in NN collisions.**

Relativistic Hydrodynamics

Relativistic Hydrodynamics

- EOM: Conservation Laws

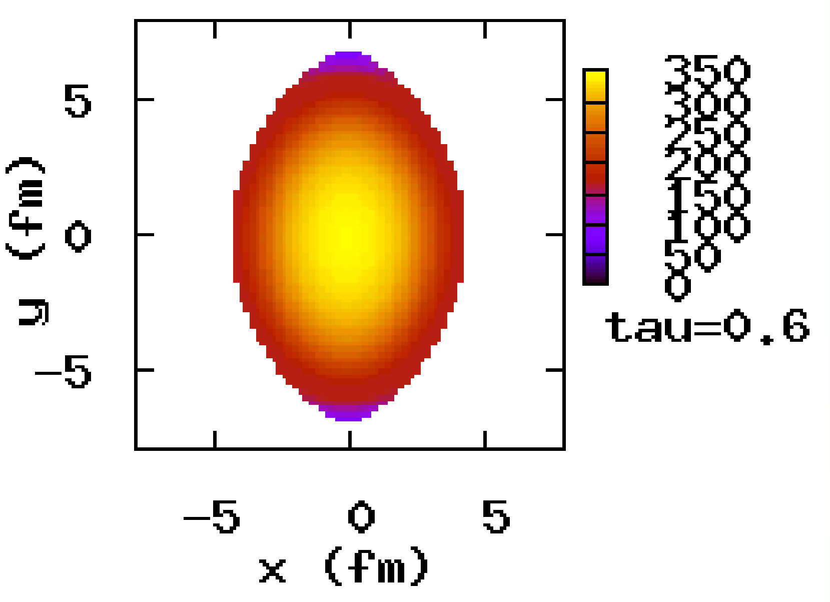
$\partial_\mu T^{\mu\nu} = 0$ Energy Momentum Conservation

$\partial_\mu n_i u^\mu = 0$ Conservation of Charge (Baryon, Strangeness, ...)

$$T^{\mu\nu} = (e+P)u^\mu u^\nu - P g^{\mu\nu}$$

e : energy density, *P: pressure,*

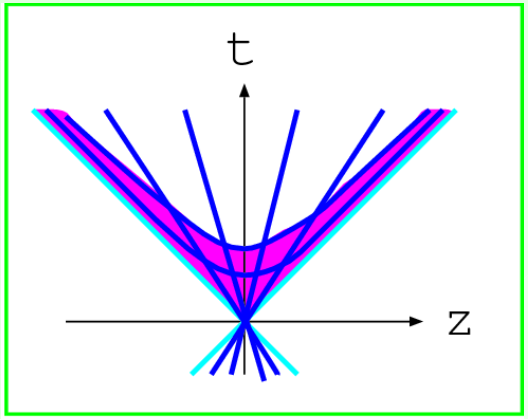
u^μ: :four velocity $\gamma(1,v)$, n_i: :number density



T. Hirano, Y. Nara,
Nucl. Phys. A743, 305 (2004)
T. Hirano, K. Tsuda,
Phys. Rev. C 66, 054905(2002)

Relativistic Hydrodynamics (II)

- One more condition is necessary
  → *Equation of State $P = P(e, n_i)$ is needed*
  - Independent Variables: $e, P, v, n_i \to 6$
  - Independent Equations: 4+1 =5



- Solve Hydro. in Bjorken Variables (τ, η_s, x, y) → Save CPU a lot !
  - Most of the Dynamics is govered by τ during τ < 10 fm/c
  - η_s approximately corresponds to η, and fixed by inc. E.
- Parameters
  - τ_0 (Thermalization time), T^{ch} (chemical F.O.) → Au+Au $dN/d\eta$ fit
  - T^{th}: Free Parameter
- Initial Condition: Glauber type or Color Glass Condensate

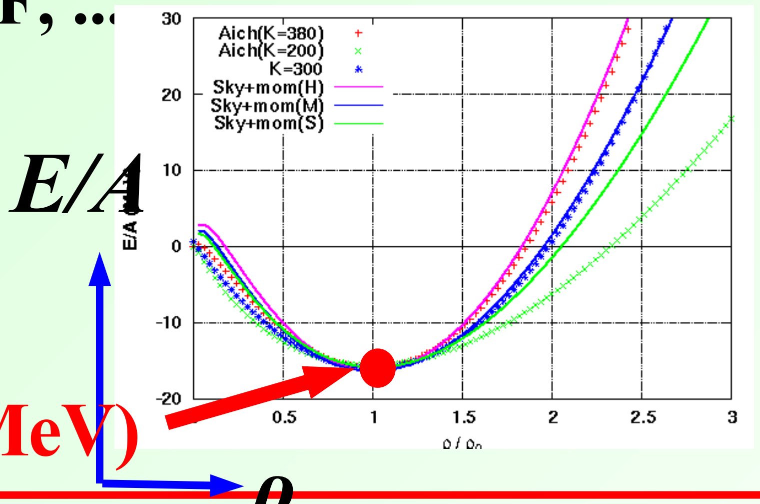
Nuclear Mean Field for HIC
--- Density and Momentum Deps. ---

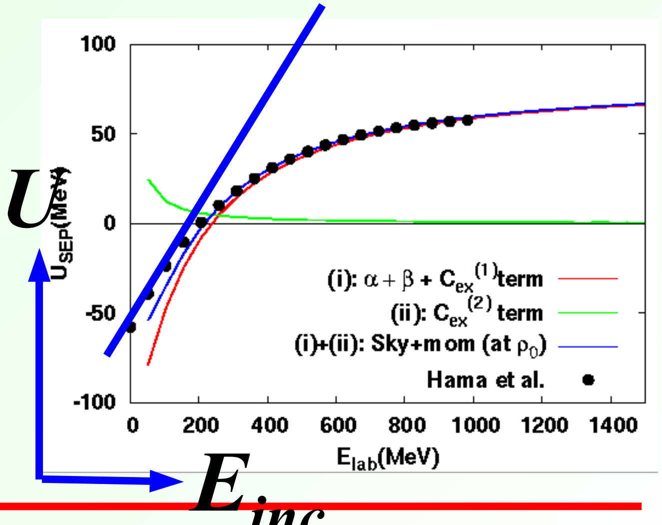
Nuclear Mean Field

- **MF has on both of ρ and p-deps.**
  - **ρ dep.: (ρ_0, E/A) = (0.15 fm^{-3}, -16.3 MeV) is known**
    Stiffness is not known well
  - **p dep.: Global potential up to E=1 GeV is known from pA scattering**
    $U(\rho_0, E) = U(\rho_0, E{=}0)+0.3\,E$
- **Ab initio Approach; LQCD, GFMC, DBHF, G-matrix, ...**
  → Not easy to handle, Not satisfactory for phen. purposes
- **Effective Interactions (or Energy Functionals)**
  : Skyrme HF, RMF, ...

U(E)=U(0)+0.3E

(ρ_0 , E/A)
=(0.15 fm^{-3} , -16.3 MeV)





Skyrme Hartree-Fock

(c.f. Talk by Van Giai and Lynch, See Ring-Schuck for details)

- **Zero-Range Two- and Three-Body Interaction**

$$v_{ij}=t_0\delta(r_i-r_j)+\frac{1}{2}\left[\delta(r_i-r_j)k^2+k^2\delta(r_i-r_j)\right]$$
$$+t_2 k\,\delta(r_i-r_j)k+iW_0\left[\sigma_i+\sigma_j\right]\times\delta(r_i-r_j)k$$
$$k=\frac{1}{2i}\left(\nabla_i-\nabla_j\right)$$
$$v_{ijk}=t_3\delta(r_i-r_j)\delta(r_j-r_k)$$

- **Energy Density (Even-Even, N=Z)**

$$H(r)=\frac{\hbar^2}{2m^*(\rho)}\tau+\frac{3}{8}t_0\rho^2+\frac{1}{16}t_3\rho^3+Deriv.\ \ terms\to\rho\left[\frac{3}{5}\ \frac{\hbar^2k_F^2}{2m^*(\rho)}+\frac{3}{8}t_0\rho+\frac{1}{16}t_3\rho^2\right]$$

$$\tau=\sum_i|\nabla\phi_i|^2\ ,\quad \frac{\hbar^2}{2m^*(\rho)}=\frac{\hbar^2}{2m}+\frac{1}{16}(3t_1+5t_2)\rho$$

Problems in Skyrme HF (in Dense Nuclear Matter/High Energy)
Repulsive Zero-Range 3-body Int.: → Ferromagnetism
Energy Dep. = Linear (m* term) → Too Repulsive at High E

Relativistic Mean Field (I)

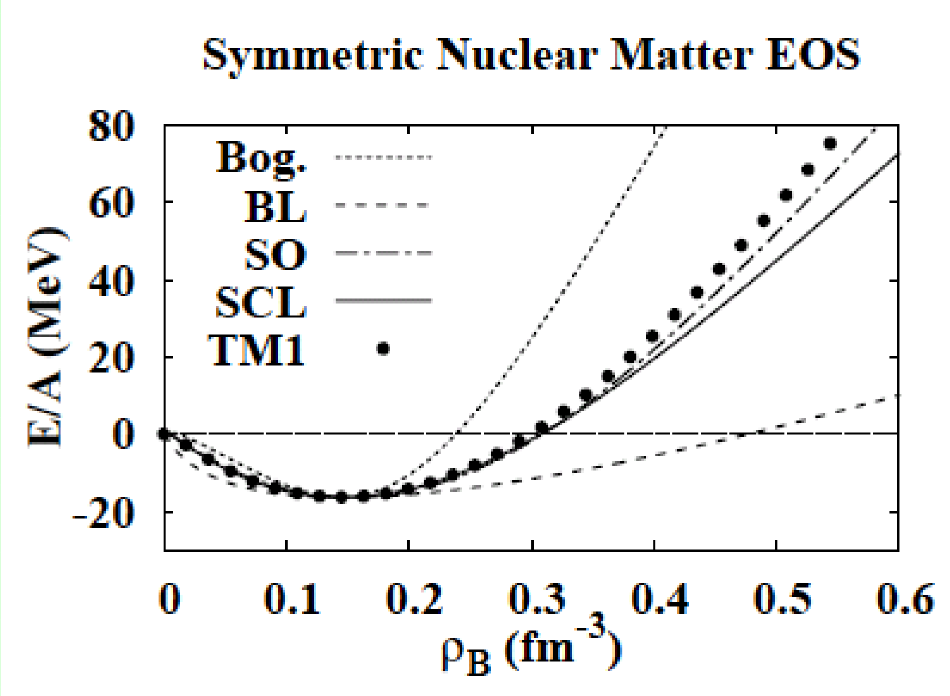
(c.f. Talk by Peter Ring, See e.g Walecka text book)

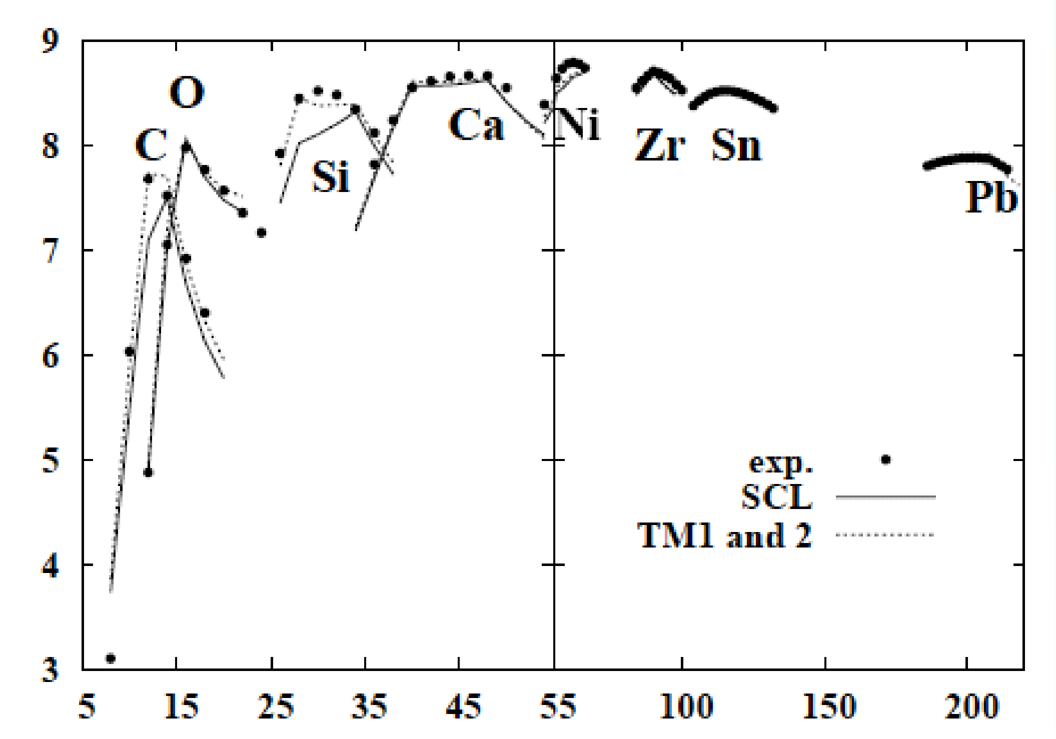
- **Describe nuclear energy functional in meson and baryon fields**
  - **Fit B.E. of Stable as well as Unstable (n-rich) Nuclei**
  - **Has been successfully applied to Supernova Explosion**
  - **Three Mesons (σ,ω,ρ) are included**
  - **Meson Self-Energy Term (σ,ω)**

$$
\begin{aligned}
\mathcal{L} =& \bar{\psi}_N \left(i\partial\!\!\!/ - M - g_\sigma \sigma - g_\omega \omega\!\!\!/ - g_\rho \tau^a \rho\!\!\!/^a\right) \psi_N \\
&+ \frac{1}{2}\partial^\mu \sigma \partial_\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4 \\
&- \frac{1}{4} W^{\mu\nu} W_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega^\mu \omega_\mu - \frac{1}{4} R^{a\mu\nu} R^a_{\mu\nu} + \frac{1}{2} m_\rho^2 \rho^{a\mu} \rho^a_\mu + \frac{1}{4} c_3 \left(\omega_\mu \omega^\mu\right)^2 \\
&+ \bar{\psi}_e \left(i\partial\!\!\!/ - m_e\right) \psi_e + \bar{\psi}_\nu i\partial\!\!\!/ \psi_\nu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \ , \\
W_{\mu\nu} =& \partial_\mu \omega_\nu - \partial_\nu \omega_\mu \ , \\
R^a_{\mu\nu} =& \partial_\mu \rho^a_\nu - \partial_\nu \rho^a_\mu + g_\rho \epsilon^{abc} \rho^{b\mu} \rho^{c\nu} \ , \\
F_{\mu\nu} =& \partial_\mu A_\nu - \partial_\nu A_\mu \ . \qquad (2)
\end{aligned}
$$

Nuclear Matter EOS and Nuclear Binding E in TM

- Example: TM1 parameter set
  (Sugahara and Toki, Nucl. Phys. A579 (1994), 557.)
  - Nuclear Matter: σ4 and ω4 terms soften EOS (K ~ 280 MeV)
  - Finite nuclei: Explains B.E. from C to Pb isotopes





c.f. SCL=Chiral RMF with log σ term.

(K. Tsubakihara and AO, in preparation)

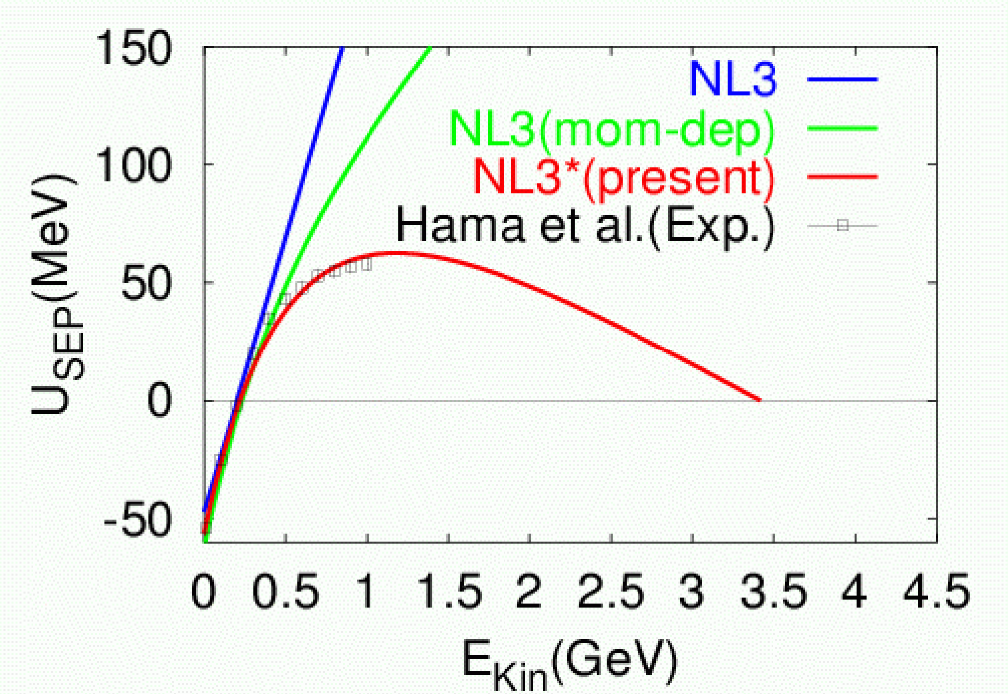
Relativistic Mean Field (II)

- **Dirac Equation** $\left(i\gamma\partial-\gamma^0 U_v-M-U_s\right)\psi=0$, $U_v=g_\omega\omega$, $U_s=-g_\sigma\sigma$
- **Schroedinger Equivalent Potential**

$$\begin{pmatrix} E-U_v-M-U_s & -i\sigma\cdot\nabla \\ i\sigma\cdot\nabla & -E+U_v-M-U_s \end{pmatrix}\begin{pmatrix} f \\ g \end{pmatrix}=0$$

$$U_{sep}\sim U_s+\frac{E}{m}U_V=-g_\sigma\sigma+\frac{E}{m}g_\omega\omega$$

$$=-\frac{g_\sigma^2}{m_\sigma^2}\rho_s+\frac{E}{m}\frac{g_\omega^2}{m_\omega^2}\rho_B$$



Saturation: -Scalar+Baryon Density
Linear Energy Dependence: Good at Low Energies,
Bad at High Energies (We need cut off !)

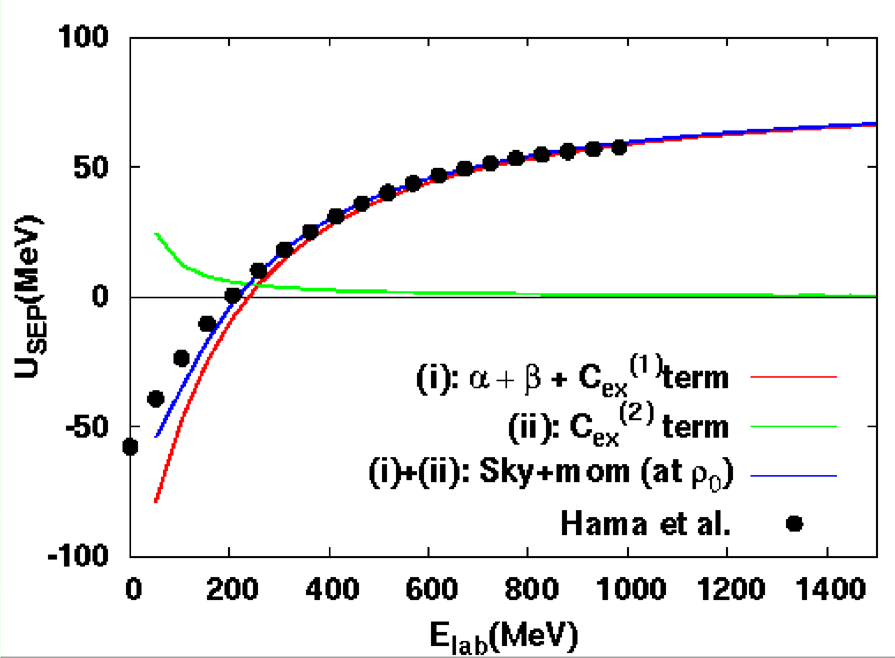
(Sahu, Cassing, Mosel, AO, Nucl. Phys. A672 (2000), 376.)

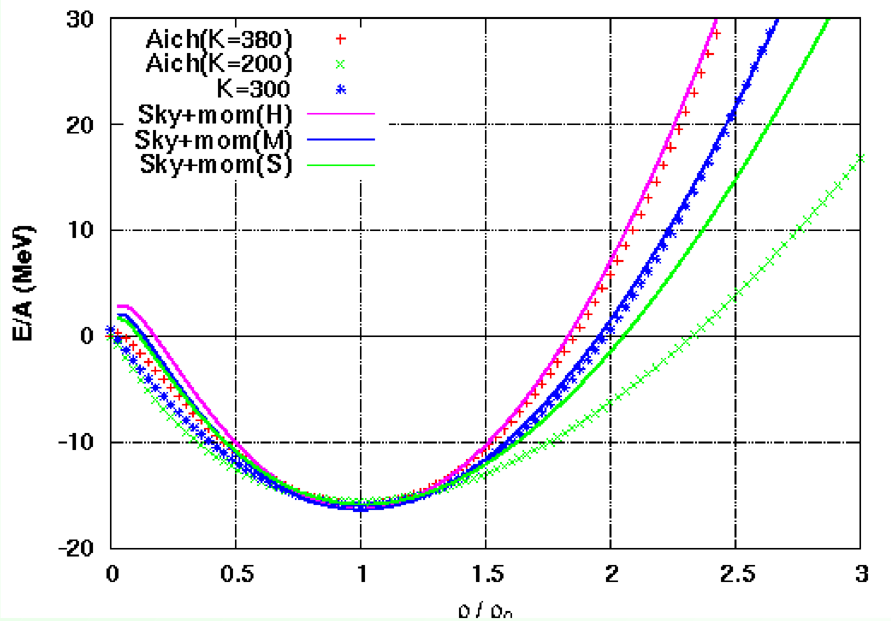
Phenomenological Mean Field

- **Skyrme type ρ-Dep. + Lorentzian p-Dep. Potential**

$$V=\sum_i V_i=\int d^3r\left[\frac{\alpha}{2}\left(\frac{\rho}{\rho_0}\right)^2+\frac{\beta}{\gamma+1}\left(\frac{\rho}{\rho_0}\right)^{\gamma+1}\right]$$

$$+\sum_k\int d^3r\,d^3p\,d^3p'\,\frac{C_{ex}^{(k)}}{2\rho_0}\,\frac{f(r,p)\,f(r,p')}{1+(p-p')^2/\mu_k^2}$$





Isse, AO, Otuka, Sahu, Nara, Phys.Rev. C 72 (2005), 064908

Exercise (4)

- Prove that the single particle potential with Skyrme interaction has a linear dependence on energy. From NA elastic scattering, the energy dependence is found to be

  $$U(\rho_0, E) \sim U(\rho_0, E=0) + 0.3\,E$$

  at low energies. Obtain the value of m*/m which explains the above energy dependence.

- Obtain the form of the Schrodinger equivalent potential in RMF. You will find that the spin-orbit potential appears as a sum of scalar and vector potential.

Summary

- **Basic ingredients in HIC models are explained.**
  - **Mean field dynamics**
  - **Two-body hadron-hadron collisions**
  - **String formation and Jet production**
  - **Hydrodynamics**
- **While nuclear MF at low energies are well investigated, it is not trivial how to apply these MFs to higher energy reactions.
  At present, phenomenologically parametrized potentials are frequently used.**
- **Students interested in HIC up to 1 A GeV should understand mean-field dynamics and NN cross sections (and π productions).
  Students interested in RHIC physics should understand parton dynamics and strings, and hydrodynamics.**

Part II: Collective flows from AGS to RHIC energies --- Cascade vs Hydrodynamics: When and where is QGP formed ? ---

Akira Ohnishi @ Hokkaido Univ.

in Collaboration with
K. Yoshino (Hokkaido U.), M.Isse(Hokkaido U.→Osaka U.),
T.Hirano (U-Tokyo), Y.Nara (Frankfurt), P.K.Sahu (IOP, India)

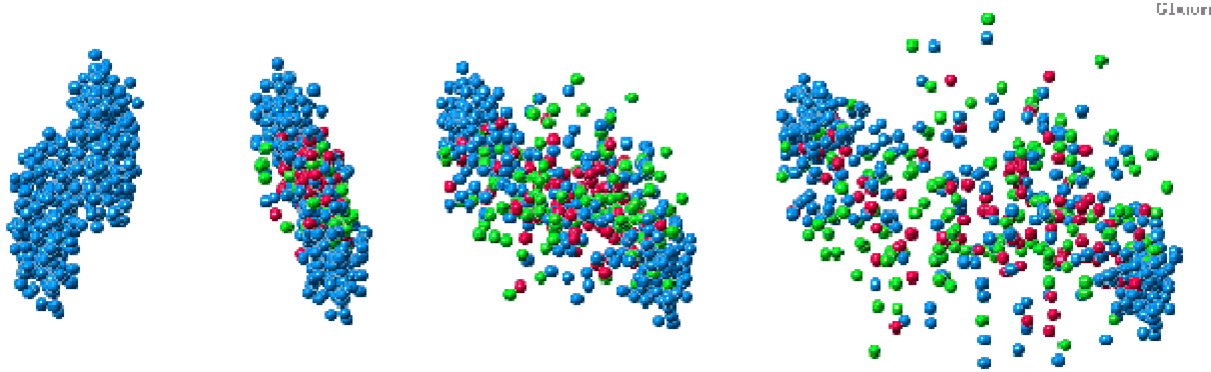
- **Collective Flows from AGS to SPS Energies**
  Isse, AO, Otuka, Sahu, Nara, Phys.Rev. C 72 (2005), 064908
- **Hydro. vs Cascade Comparison at RHIC**
  Hirano, Isse, Nara, AO, Yoshino, Phys. Rev. C 72(2005), 041901
  Sahu, Isse, Otuka, AO, Pramana, 2006, in press.
  Isse, Ph.D Thesis
- **Jet-Fluid String formation and decay at RHIC**
  Hirano, Isse, Nara, AO, Yoshino, in preparation

Collective Flows at AGS and SPS Energies

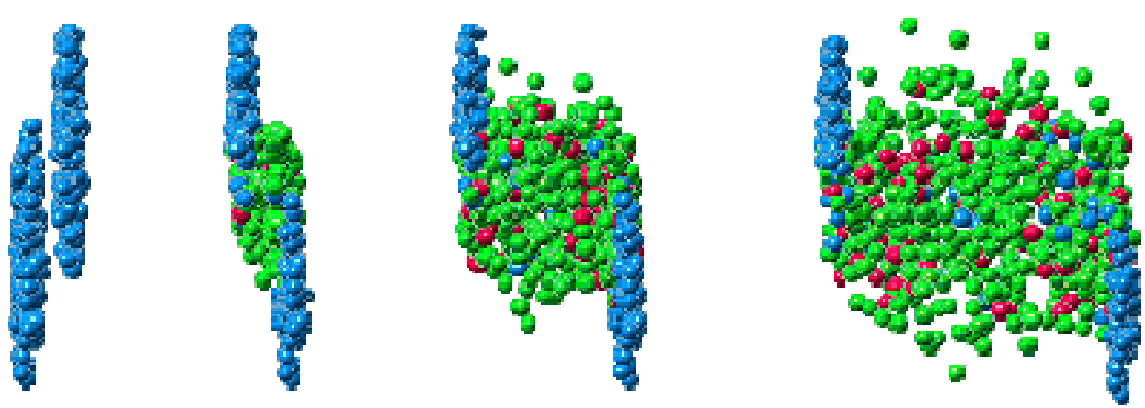
HIC at AGS and SPS Energies

JAMming on the Web, linked from http://www.jcprg.org/

AGS



SPS



What is Collective Flow ?

(Directed) Flow (dP_X/dY)

Stiffness (Low E)
+ Time Scale (High E)

Elliptic Flow (V_2)

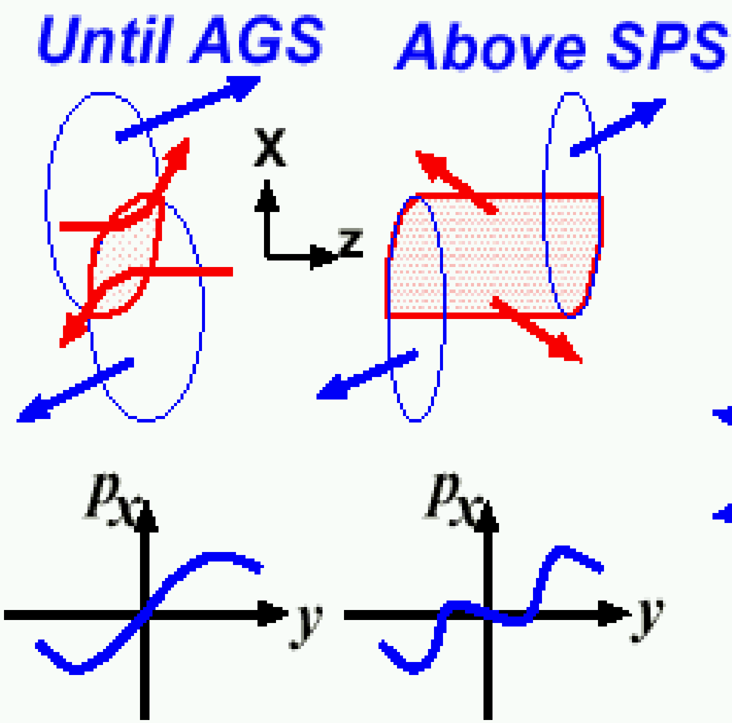
Thermalization
& Pressure Gradient

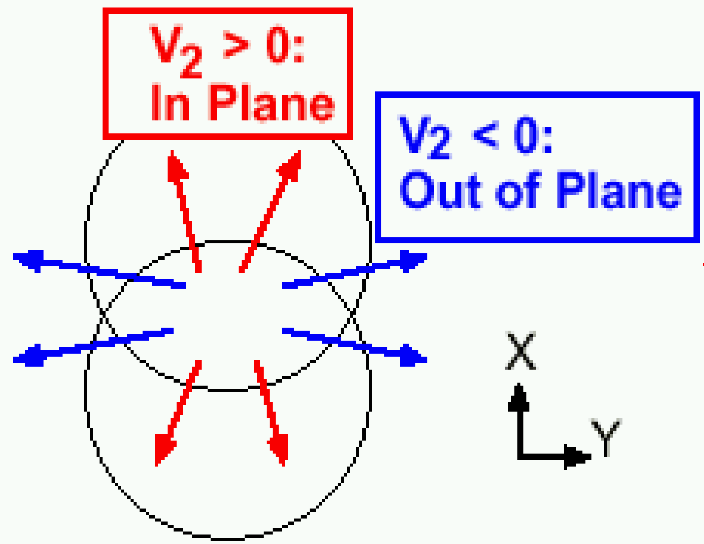
Radial Flow (β_T)

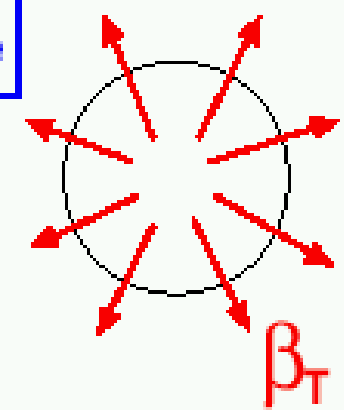
Pressure History

$$\epsilon \frac{D\boldsymbol{V}}{Dt} = -\boldsymbol{\nabla} P$$

$$\rightarrow V = \int_{path} \frac{-\boldsymbol{\nabla} P dt}{\epsilon}$$

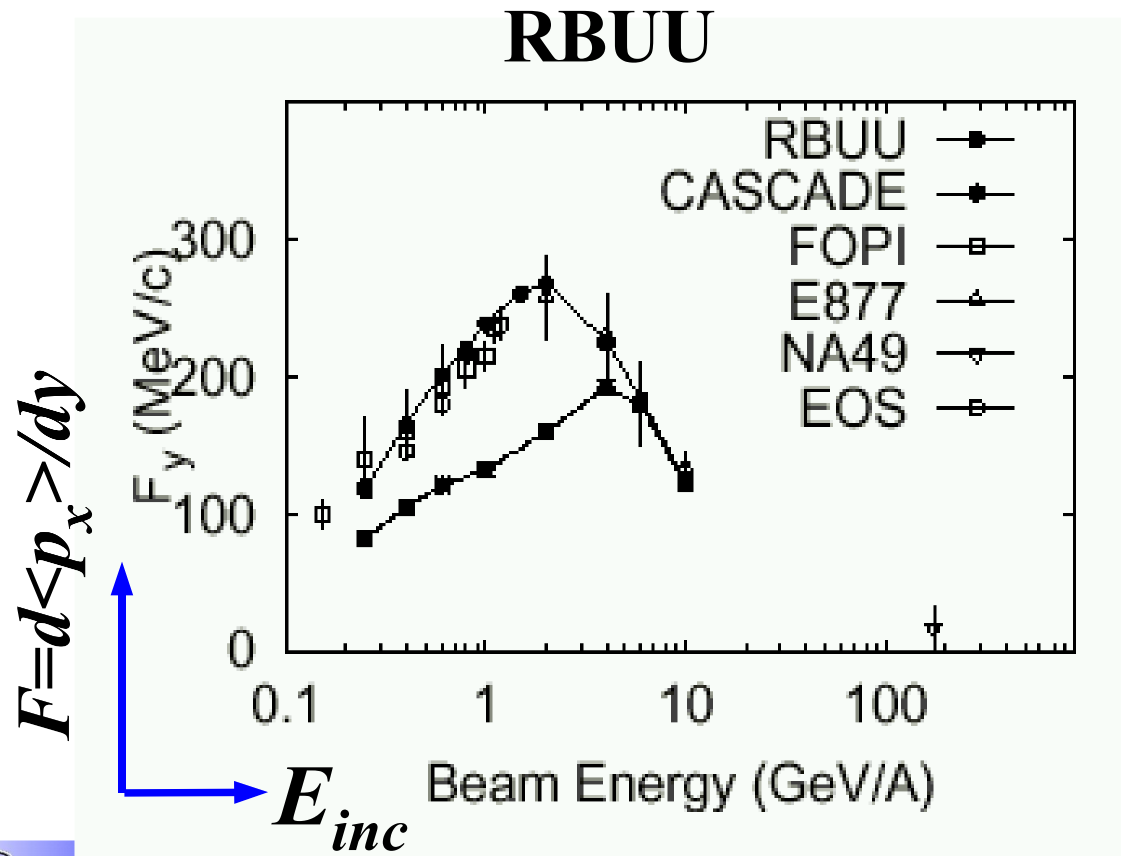


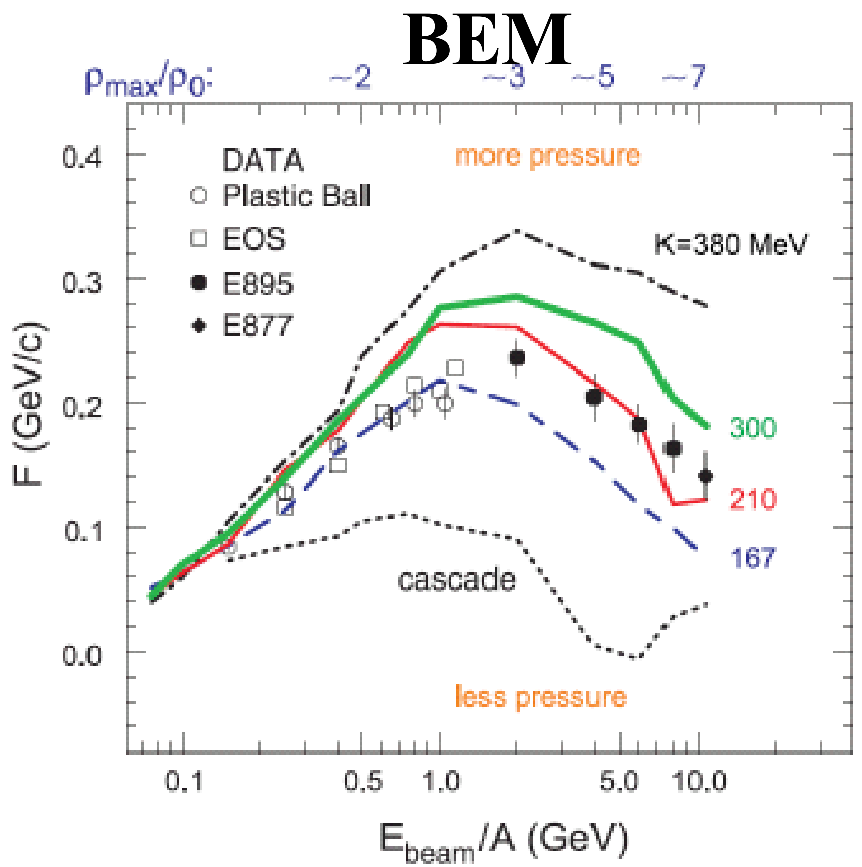




Side Flow at AGS Energies

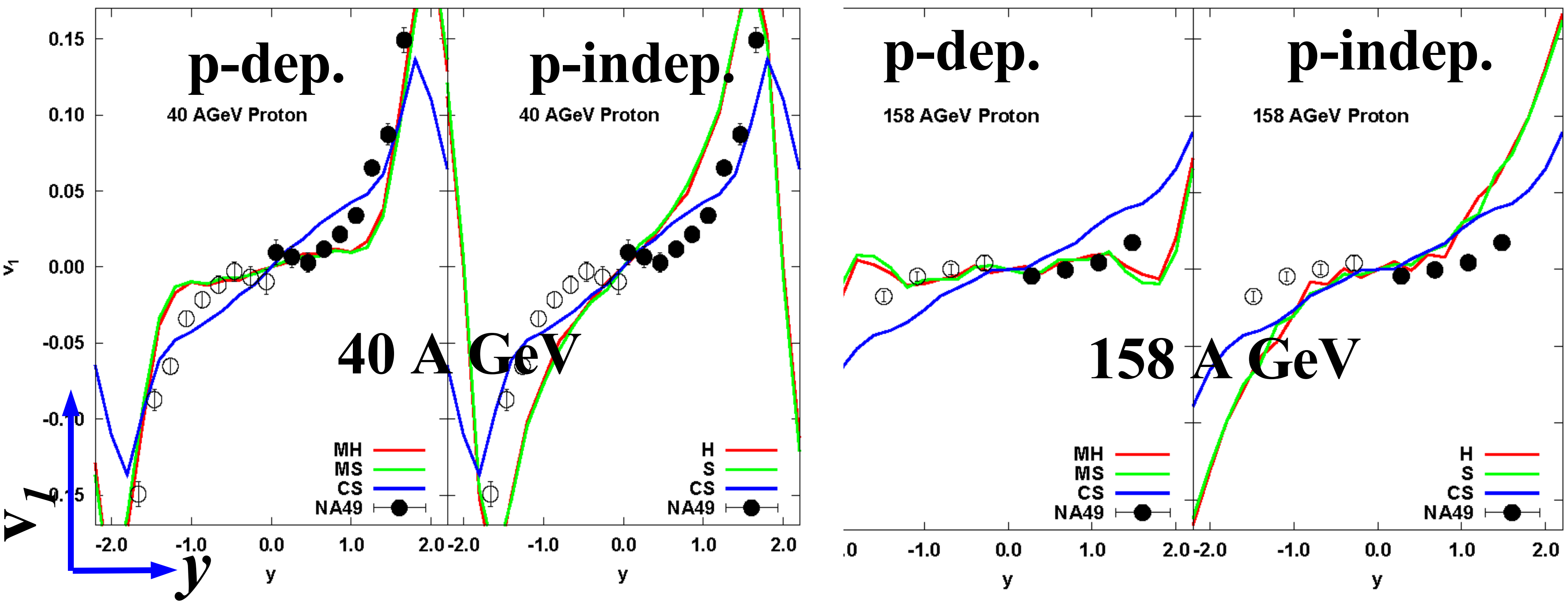
- **Relativistic BUU (RBUU) model: *K ~ 300 MeV***
  (Sahu, Cassing, Mosel, AO, Nucl. Phys. A672 (2000), 376.)
- **Boltzmann Equation Model (BEM): *K=167~210 MeV***
  (P. Danielewicz, R. Lacey, W.G. Lynch, Science 298(2002), 1592.)





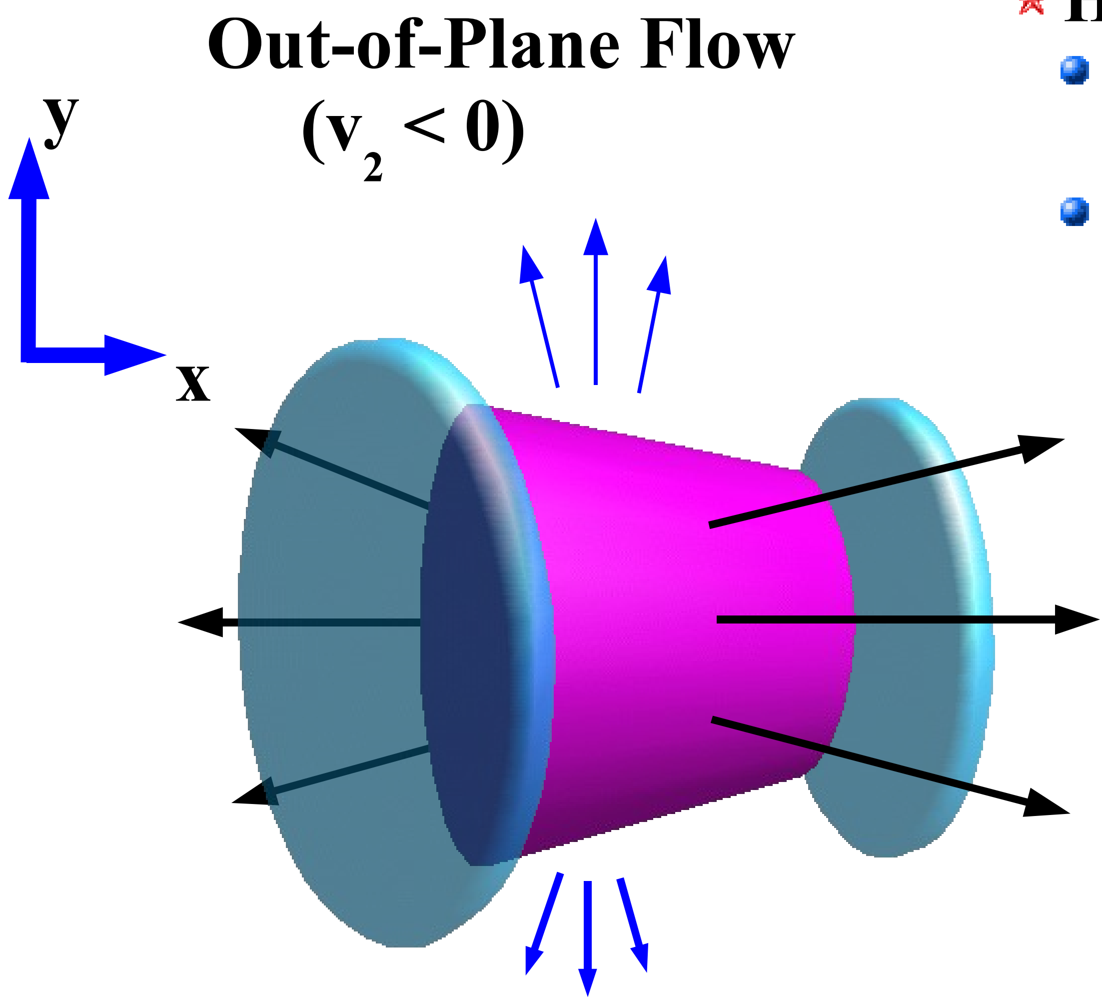
Directed flow v_1 at SPS

- **JAM-RQMD/S** (*Isse, AO, Otuka, Sahu, Nara, Phys.Rev. C 72 (2005), 064908*)
  - **p-dep. (indep.) MF suppresses (enhances) v_1. $v_1 = \langle \cos\phi \rangle = \langle p_x / p_T \rangle$**
  - **"Wiggle" behavior appears with p-dep. MF at 158 A GeV.**



Elliptic Flow

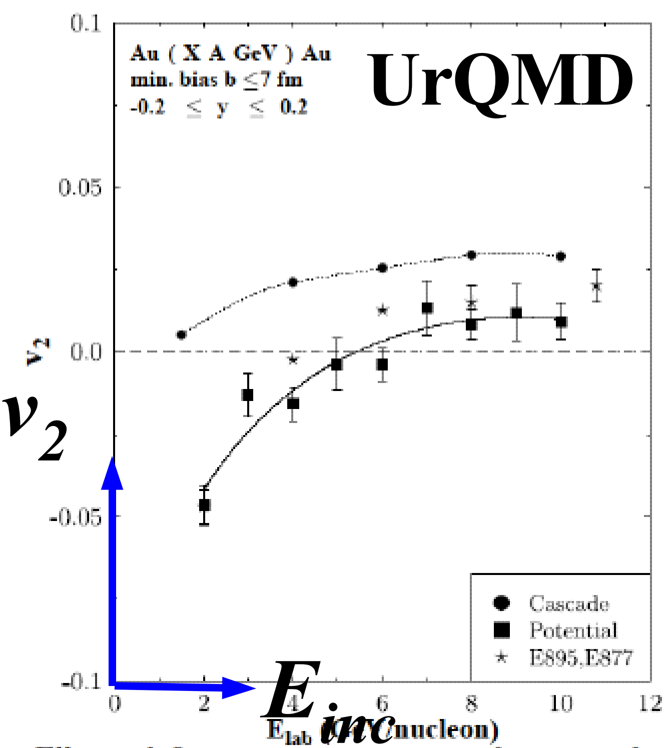
- What is Elliptic Flow ?
  - Anisotropy in P space
- Hydrodynamical Picture
  - Sensitive to the Pressure Anisotropy in the Early Stage
  - Early Thermalization is Required for Large V_2



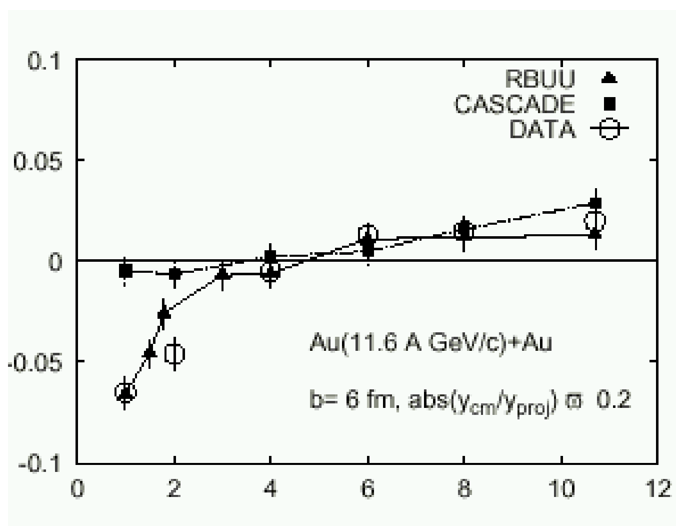
$$v_2 \equiv \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle = \langle \cos 2\phi \rangle$$

Elliptic Flow at AGS

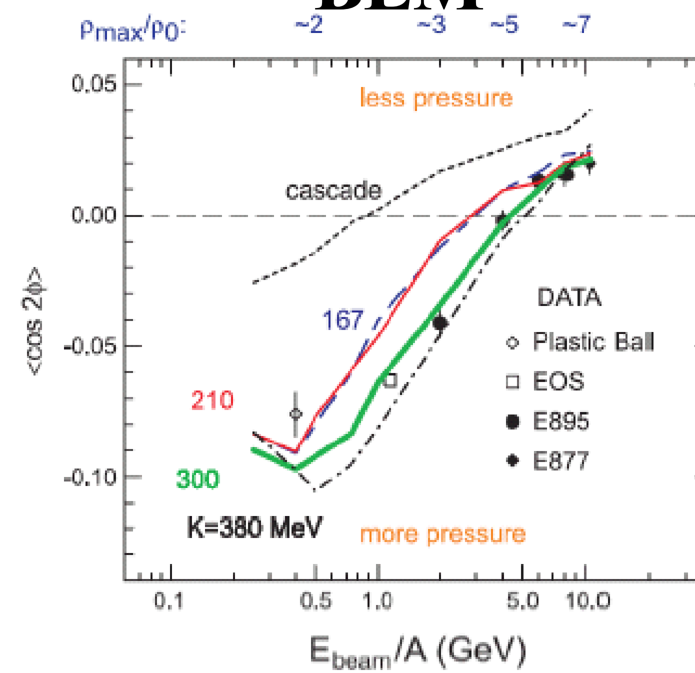
- Strong Squeezing Effects at low E (2-4 A GeV)
  - UrQMD: Hard EOS (S.Soff et al., nucl-th/9903061)
  - RBUU (Sahu2000): K ~ 300 MeV
  - BEM(Danielewicz2002): K = 167 → 300 MeV



RBUU

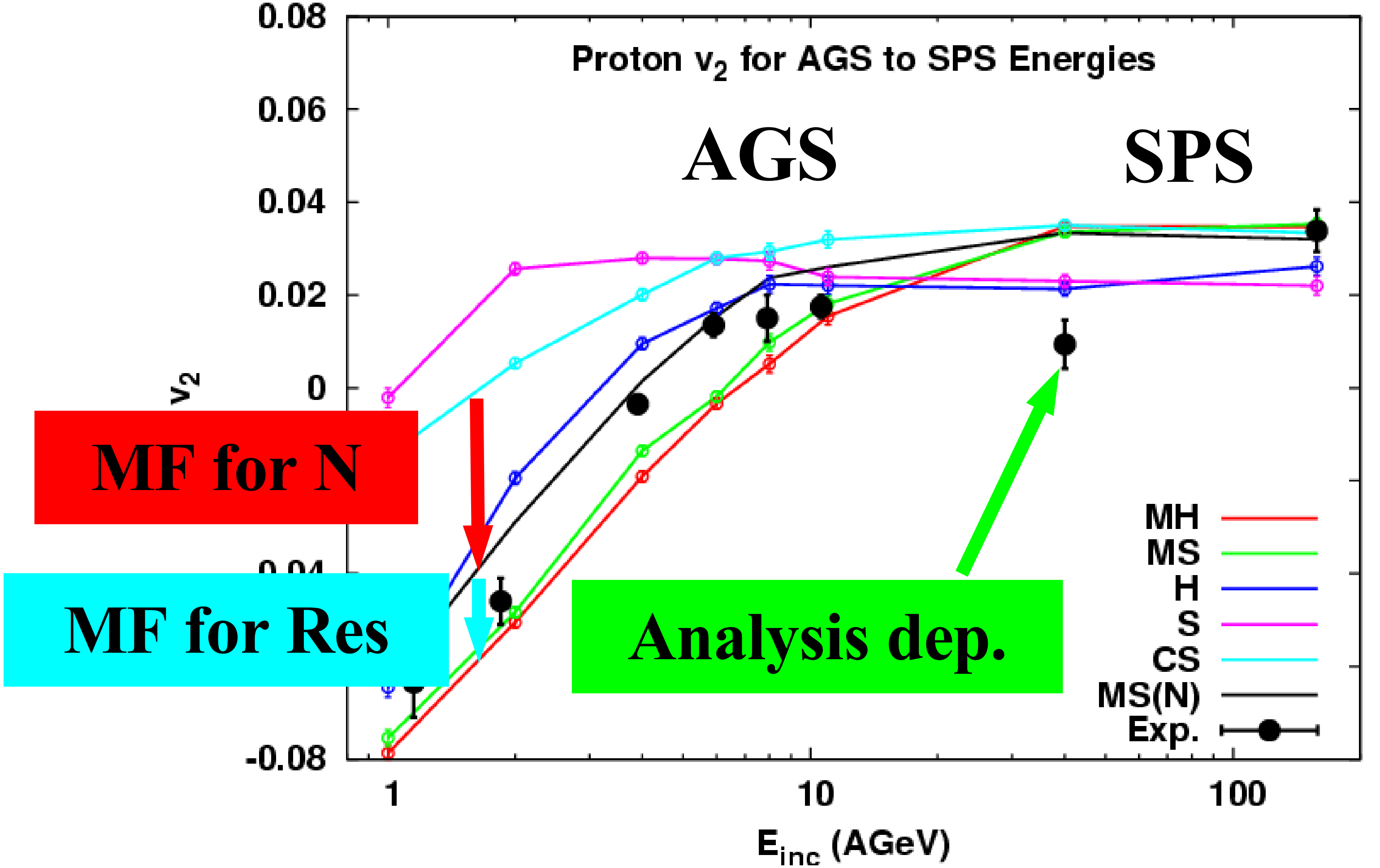


BEM



Elliptic Flow from AGS to SPS

- JAM-MF with p dep. MF explains proton v2 at 1-158 A GeV
  - v2 is not very sensitive to K (incompressibility)
  - Data lies between MS(B) and MS(N)



Flow and EOS; to be continued

- **In addition to the ambiguities in in-medium cross sections, Res.-Res. cross sections, we have model dependence.**
  - **RBUU (e.g. Sahu, Cassing, Mosel, AO, Nucl. Phys. A672 (2000), 376.)**
    - **In RMF, Strong cut-off for meson-N coupling in RMF → Smaller EOS dep.**
  - **Scalar potential interpretation in BUU**

    Larionov,Cassing,Greiner,Mosel, PRC62,064611('00), Danielewicz, NPA673,375('00)

    $$\varepsilon(\boldsymbol{p},\rho)=\sqrt{[m+U_s(\boldsymbol{p},\rho)]^2+\boldsymbol{p}^2}\quad=\sqrt{m^2+\boldsymbol{p}^2}+U(\boldsymbol{p},\rho)$$

    - **Due to the Scalar potential nature, EOS dependence is smaller.**
  - **Scalar / Vector Combination**

    Danielewicz, Lacey, Lynch, Science 298(2002), 1592

    $$\varepsilon(p,\rho)=m+\int_0^p dp'v^*(p',\rho)+\widetilde{U}(\rho),\quad v^*(p,\rho)=\frac{p}{\sqrt{p^2+[m^*(p,\rho)]^2}}.$$

    - **Relatively strong EOS dependence even at high energy**
  - **JAM-RQMD/S (*Isse, AO, Otuka, Sahu, Nara, PRC 72 (2005), 064908)***
    - **Similar to the Scalar model BUU**

Cascade vs Hydro @ RHIC

Why do we have QCD Phase Transition (I)

- **Pressure of mass less particles: Stefan-Boltzmann Law** **(cf S.H.Lee)**

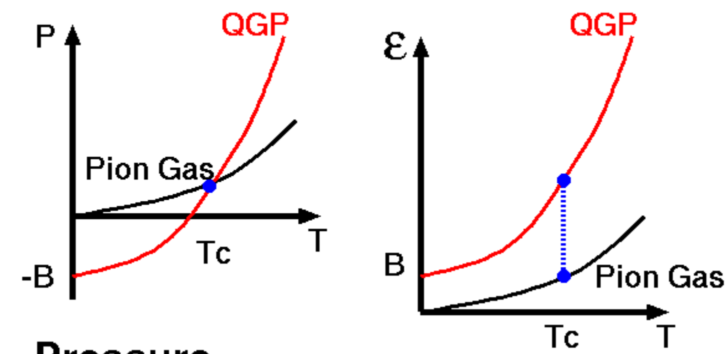
$$\Omega = -\frac{\pi^2 V}{90}\left(\sum_B g_B + \frac{7}{8}\sum_F g_F\right) T^4,$$

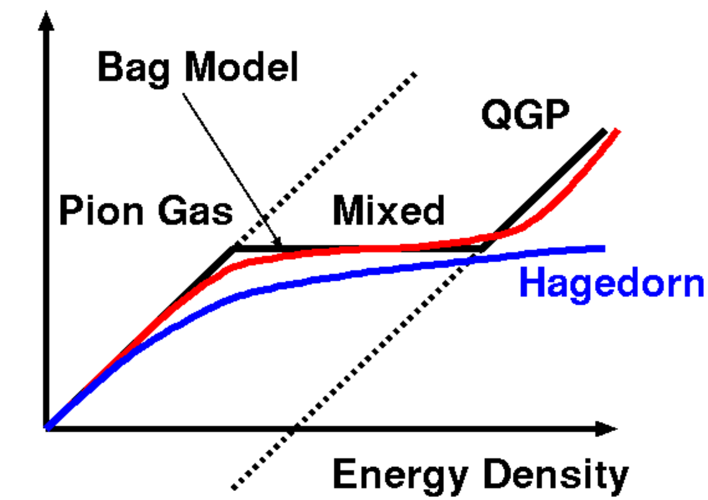
- **Hadron Phase ~ Three massless pions**

$$P_\pi = \frac{\pi^2}{30} T^4 \quad , \quad \epsilon_\pi = \frac{\pi^2}{10} T^4$$

- **QGP ~ Massless partons**
  and vacuum modification

$$P_{QGP} = \frac{37\pi^2}{90} T^4 - B \qquad \epsilon_{QGP} = \frac{37\pi^2}{30} T^4 + B$$





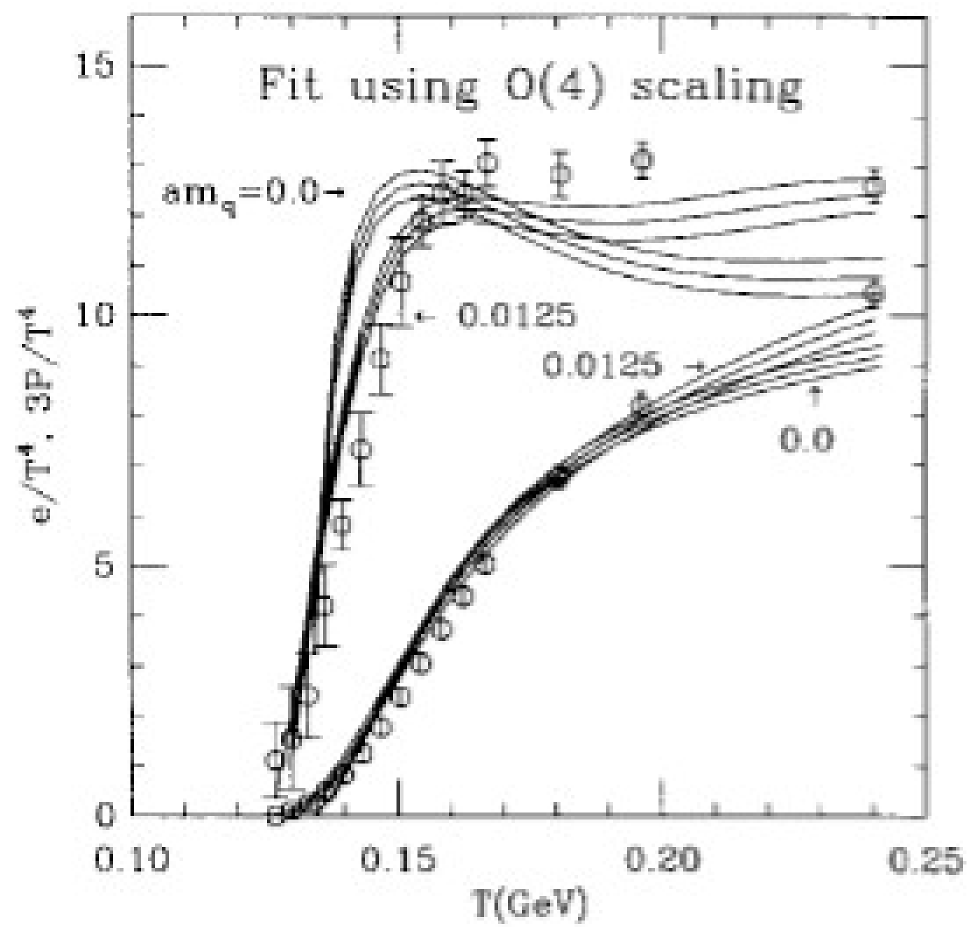
QCD phase transition=DOF change + Vacuum change

$$DOF = 2(\mathrm{spin}) \times 2(q,\bar{q}) \times 3(\mathrm{color}) \times 2(\mathrm{flavor}) \times 7/8(\mathrm{Fermion}) + 2(\mathrm{spin}) \times 8(\mathrm{color})$$

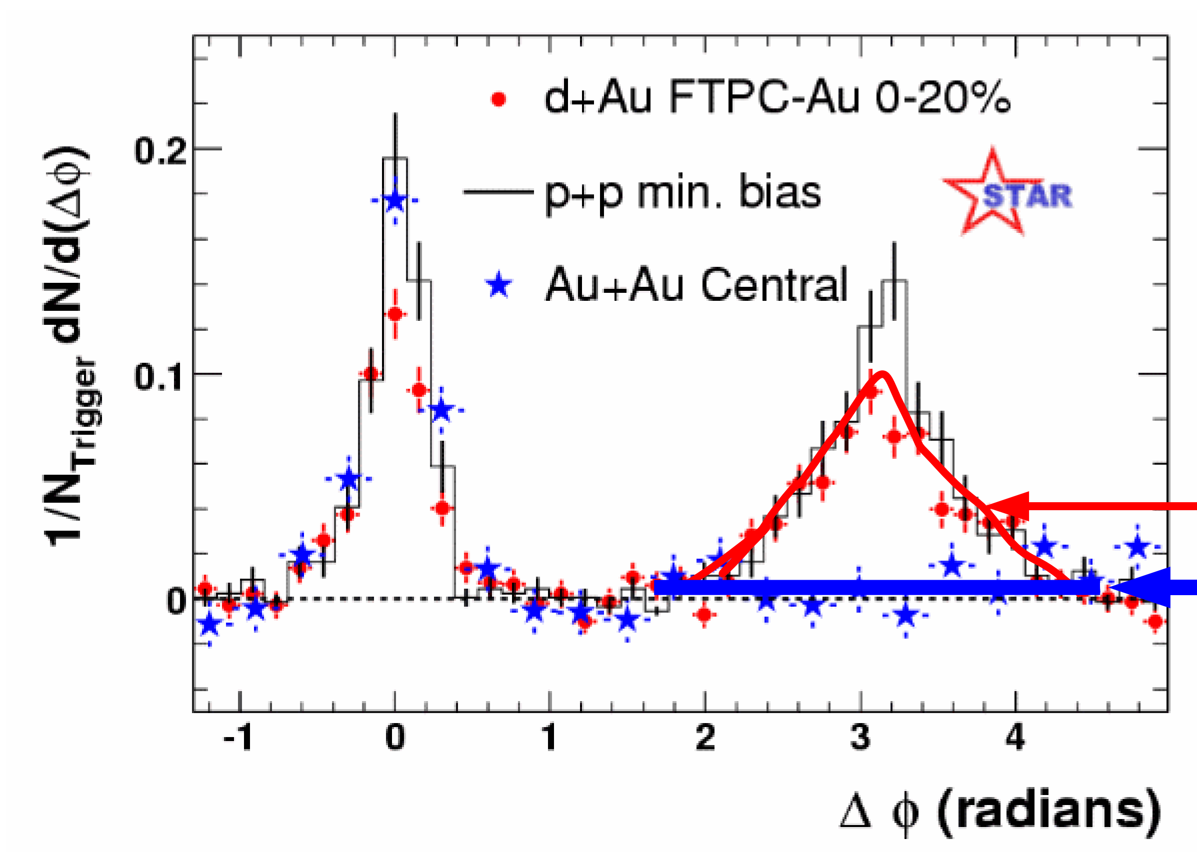
Why do we have QCD Phase Transition (II)

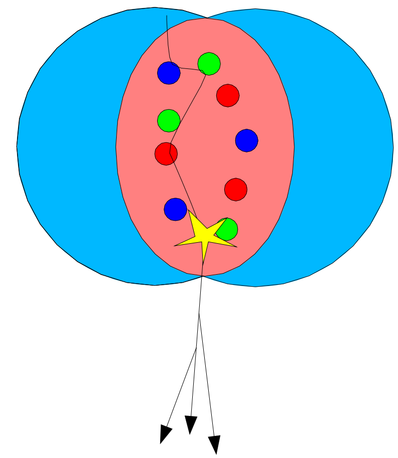
(cf S.H.Lee)

- Lattice QCD simulation
  - Figure: $(E/V)/T^4$, P/T^4
  - Sudden change of (E/V) around T = 150-200 MeV
  - Smoother change of P around T = 150-200 MeV → Phase transition to QGP



QGP Signals (I): Jet Quenching





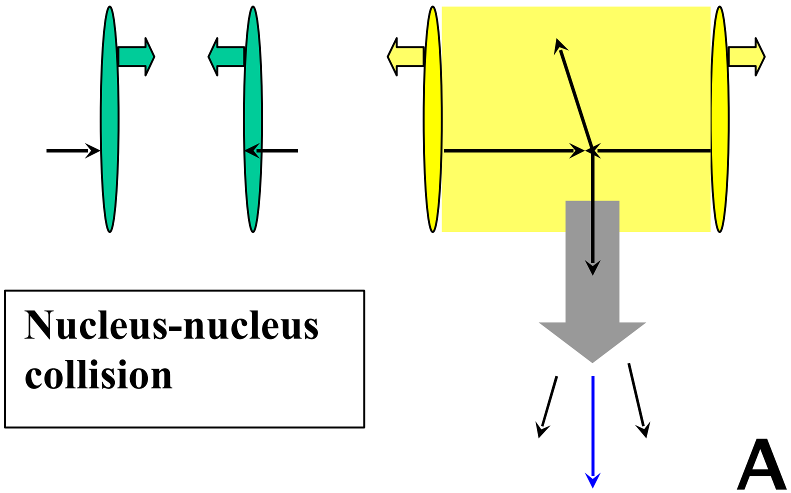
d + Au: Backward Peak

Au + Au: No Backward Peak

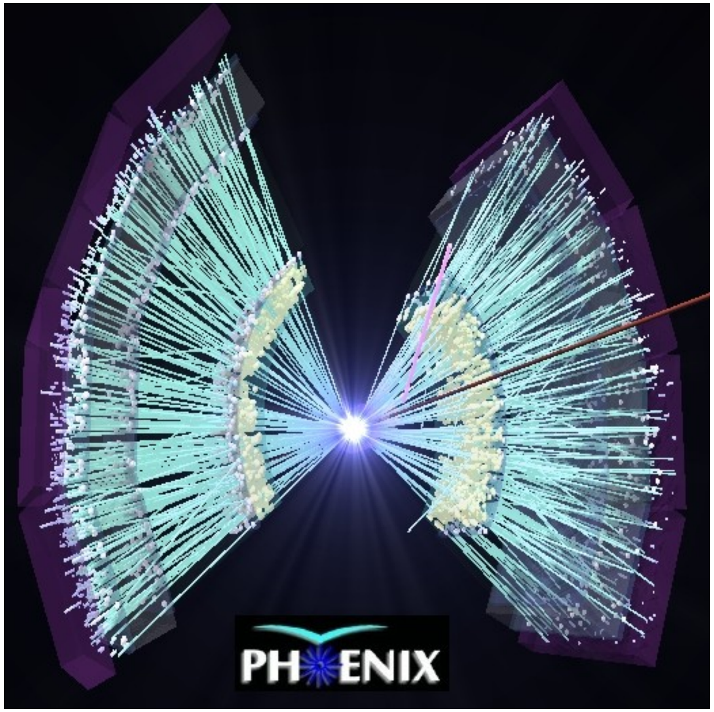
STAR (nucl-ex/0306024)

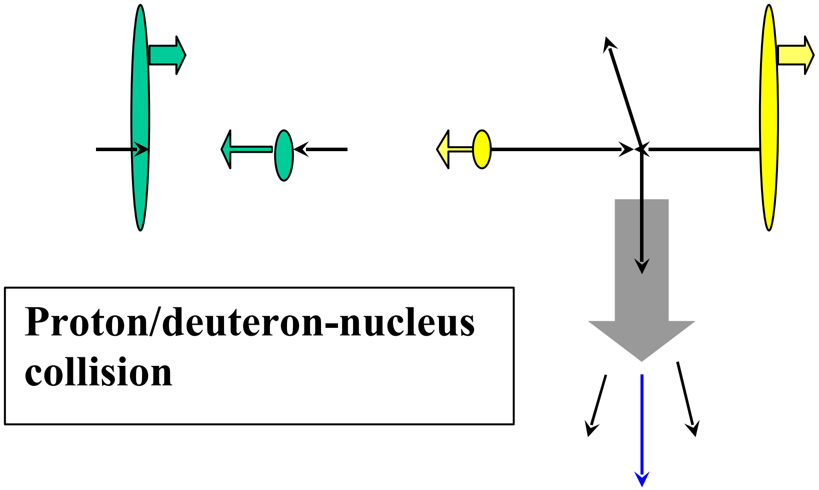
Jet Energy Loss also lead to reduction of back-to-back correlation

by Esumi

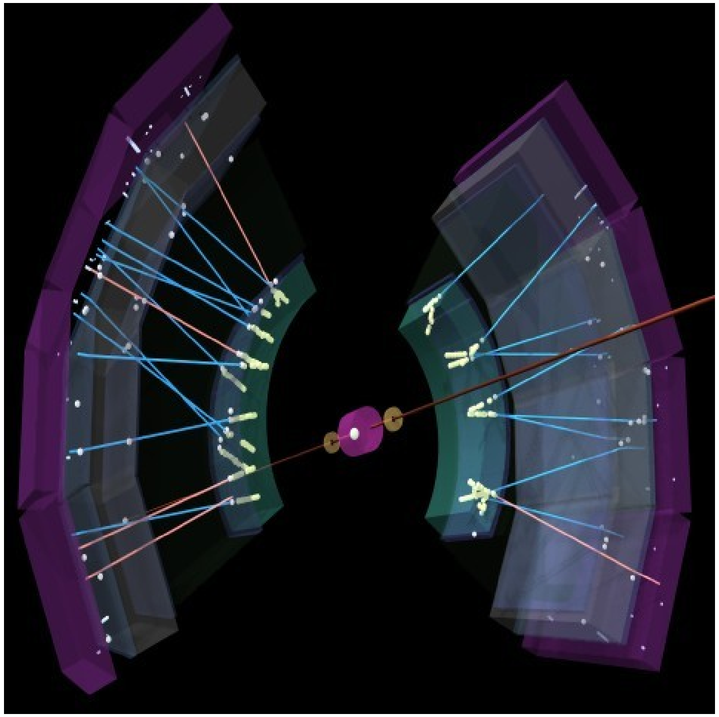


Au+Au





d+Au

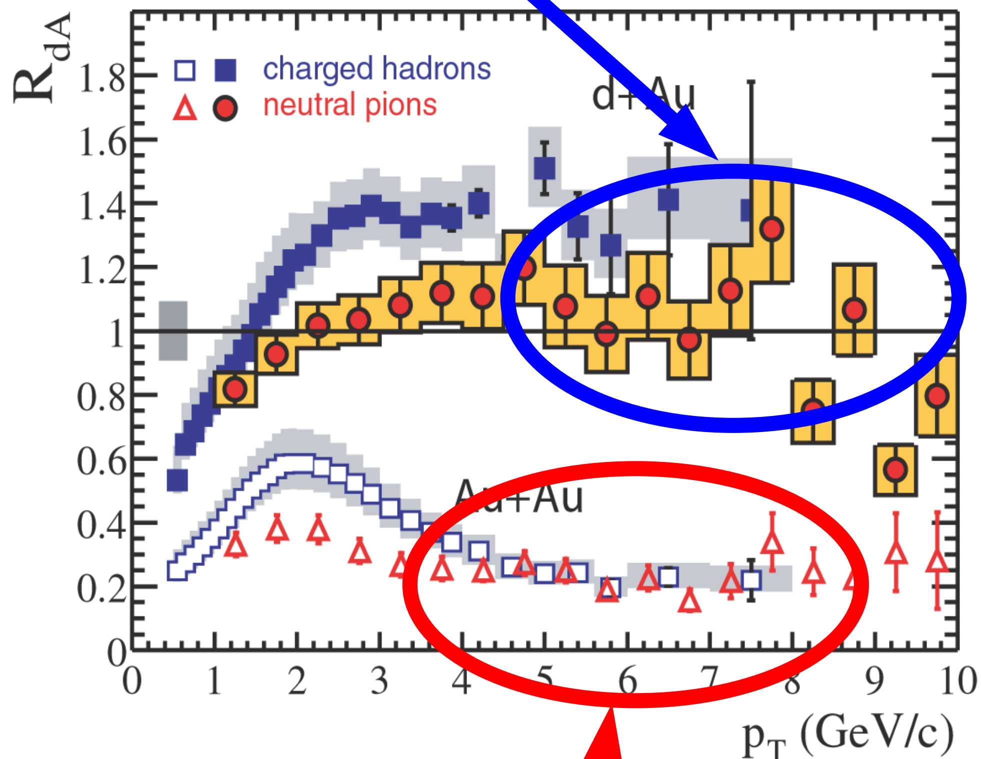


QGP Signals (II): Nuclear Modification Factor

- **Do we really see suppression of high energy particles at RHIC ?**
  → YES for Au+Au Collisions, and NO for d+Au Collisions !
- **Nuclear Modification Factor**

$$R_{AB}(p_T) = \frac{d^2N/dp_T d\eta}{T_{AB}\, d^2\sigma^{pp}/dp_T d\eta}$$

d + Au: Initial State Effects



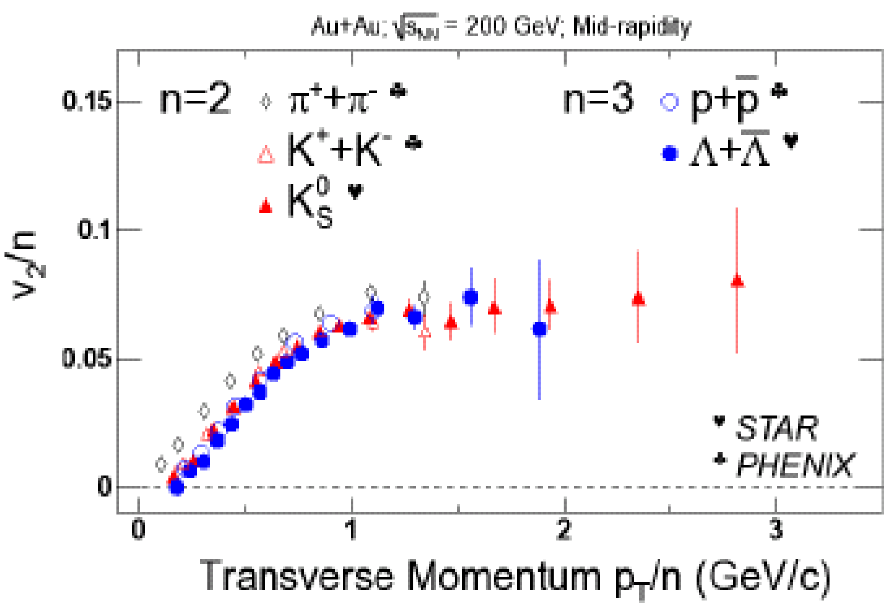
High Energy Particles are suppressed in Au + Au Collisions but NOT suppressed in d + Au Collisions at RHIC compared to p+p collisions !

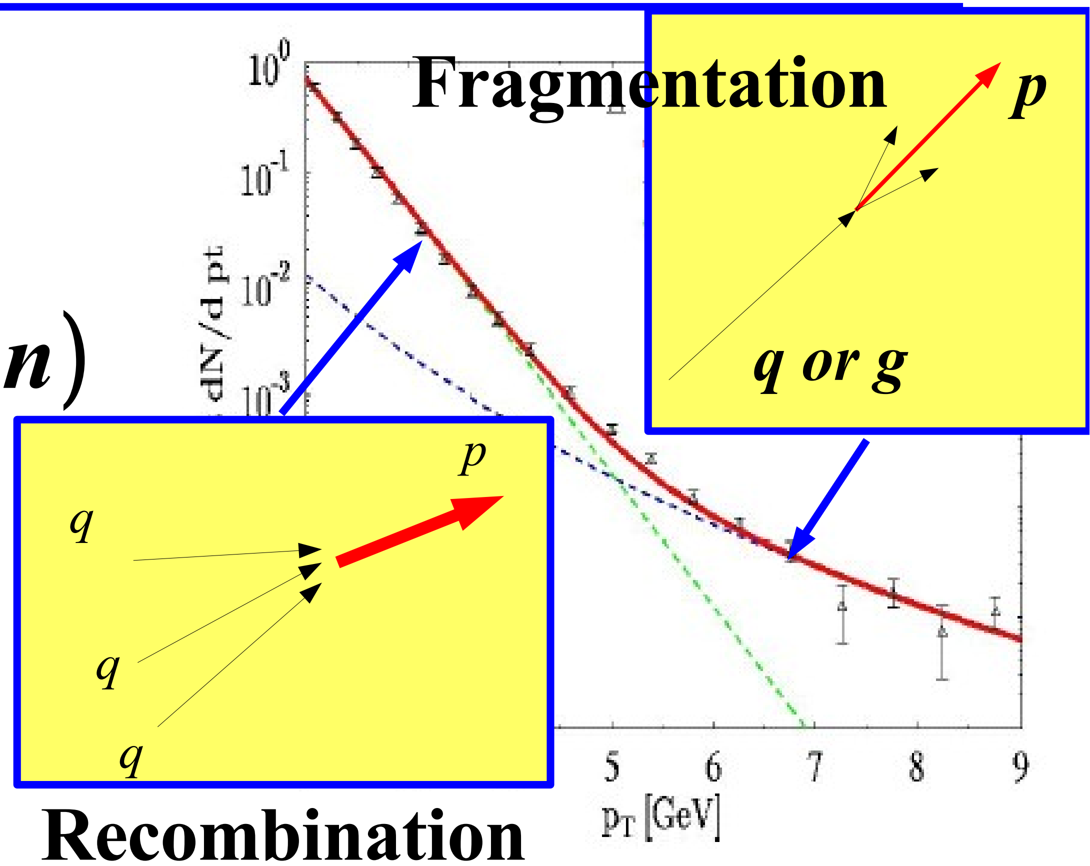
Au + Au: Initial State + Final State Effects

QGP Signals (III): Quark Number Scaling

- When *n* quarks recombines to a hadron, v2 is enhanced by *n* times.

$$v_2^{Hadron}(P_T) = n\, v_2^{Parton}(P_T/n)$$





Fries et al. PRL 90 (2003), 202303
Nonaka et al., nucl-th/0308051

Recombination Picture seems to work well
... Parton Elliptic Flow

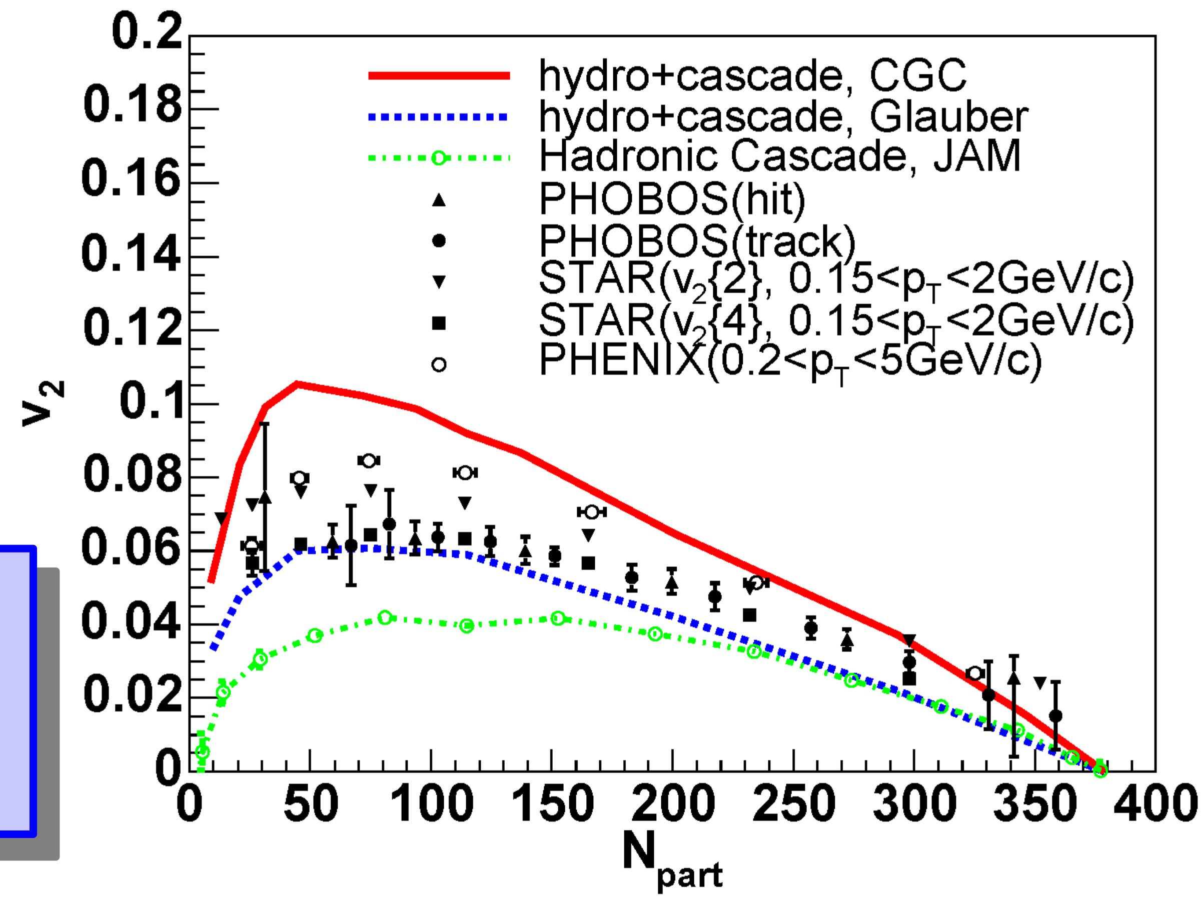
When and where is QGP formed ?

- Incident Energy
  - AGS: Strangeness Enh. (High baryon ρ effect ?)
  - SPS:
    J/ψ suppression (QGP?), Low mass dilepton enh. (chiral sym.)
    Hydro overestimate v_2 data
  - RHIC:
    Jet quenching, Strong v_2, Quark number scaling of v_2, ...
    Hadronic Cascade underestimate v_2 data

  → Bulk QGP formation seems to start between SPS and RHIC
- Proj./Targ. Mass dependence
  - Au+Au: v_2(Casc.) < v_2(hydro) ~ v_2(data)
  - Cu+Cu: Recently Measured

Cascade vs Hydro @ RHIC: Au+Au

- Comparison of v2 as a function of N_{part}
  - Cascade predict smaller v2 in peripheral collisions
  - Data lies between hydro results with two different initial condition CGC (Color Glass Condensate) and Glauber type initial condition.



Hydro is better,
CGC may be realized
in central collisions.

ZI-WEI LIN AND C. M. KO PHYSICAL REVIEW C 65 034904

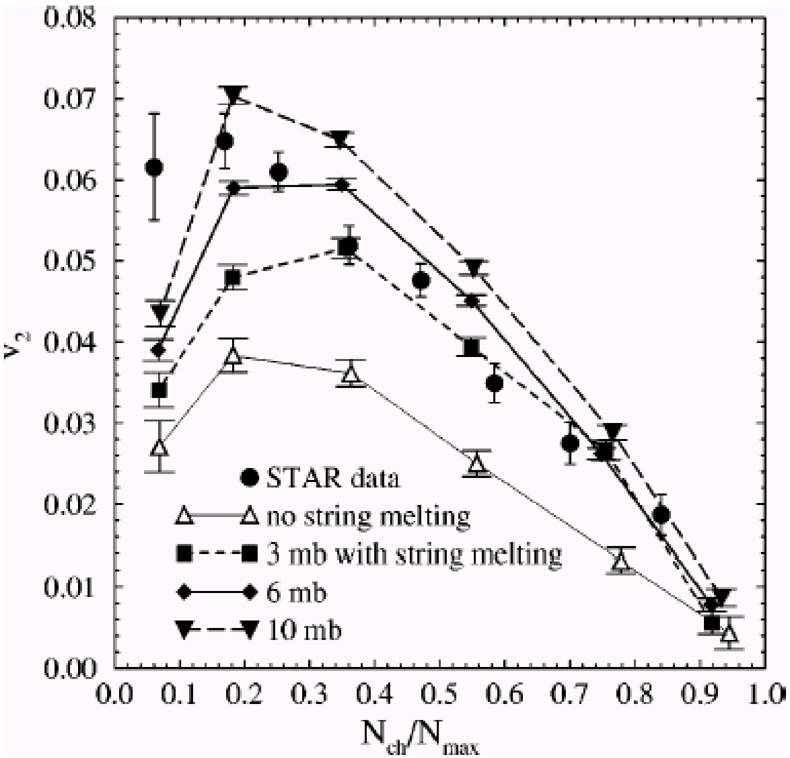


FIG. 3. Impact parameter dependence of elliptic flow at 130 *A* GeV. The data from the STAR collaboration [7] are shown by filled circles, while the theoretical results for different partonic dynamics are given by curves.

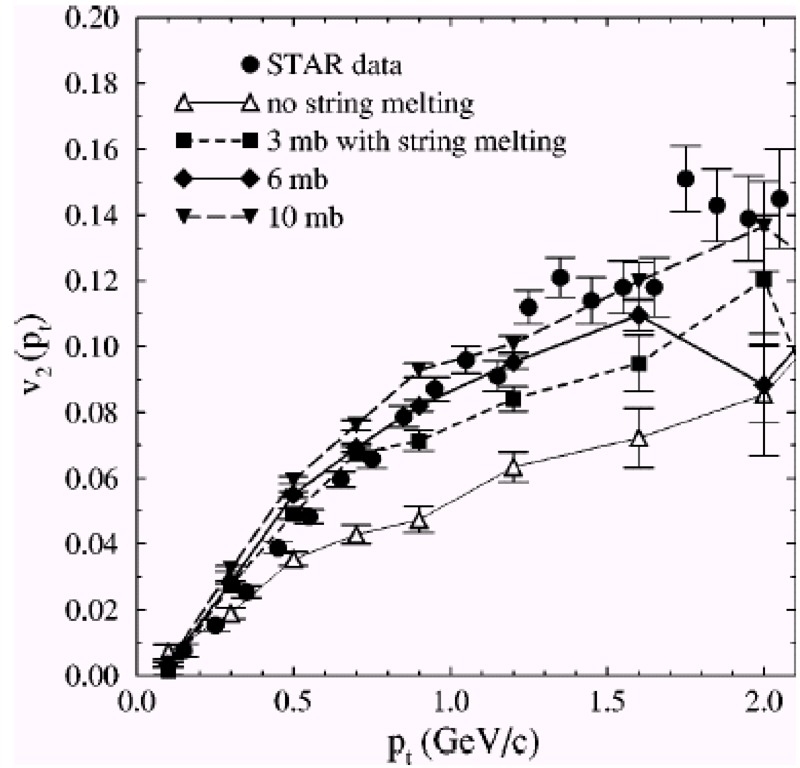
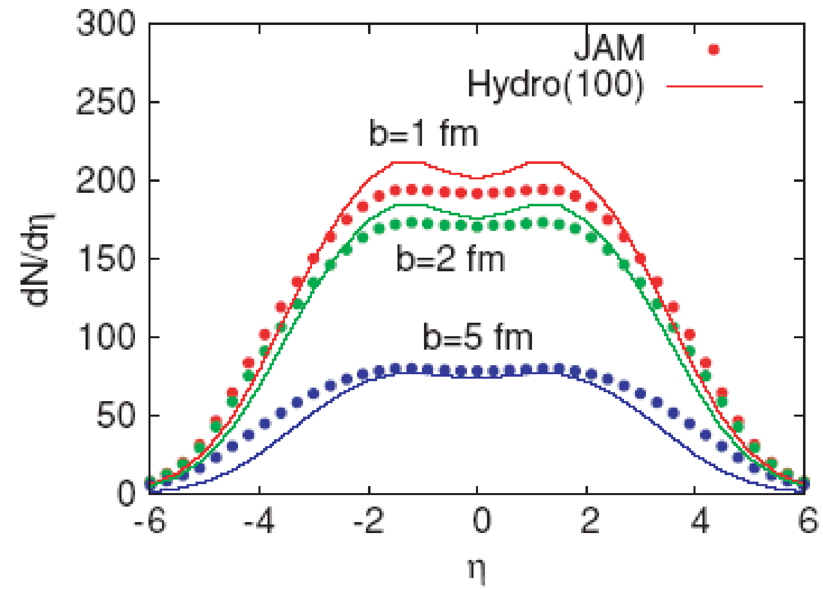


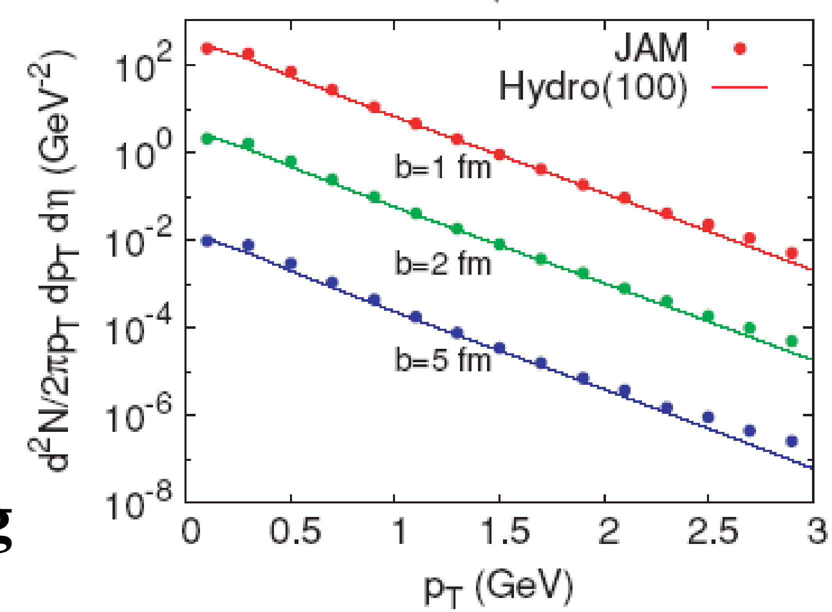
FIG. 4. Transverse momentum dependence of elliptic flow at 130 *A* GeV. Circles are the STAR data for minimum-bias Au+Au collisions [7], and curves represent the minimum-bias results for charged particles within $\eta \in (-1.3, 1.3)$ from the AMPT model.

Unexpectedly high parton cross sections of σ=5-6 mb have to be assumed in parton cascades in order to reproduce the elliptic flow.

Predictions of Cu+Cu Collisions @ RHIC (I)

- **Single particle spectra**
  - **Cascade (JAM) and Hydro predict almost the same single particle spectra**
    $dN/d\eta$, $d^2N/p_T dp_T d\eta$
- **Surprising ?**
  - **Initial Cond. of Hydro is tuned to fit *$dN/d\eta$* (~ Energy per rapidity)**
  - **Cascade use fitted σ_{NN}**
  - **Themailzation is expected at Low p_T (long time before particle production)**
    → Coincidence may not be surprising

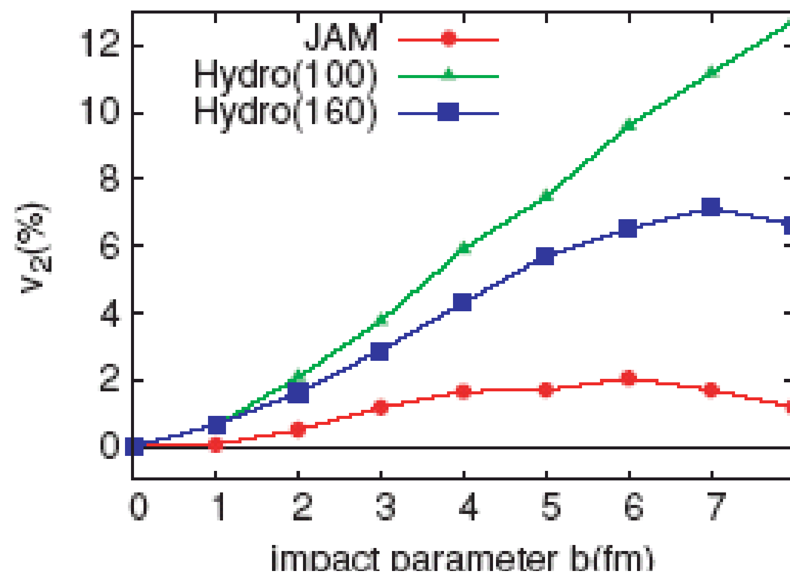


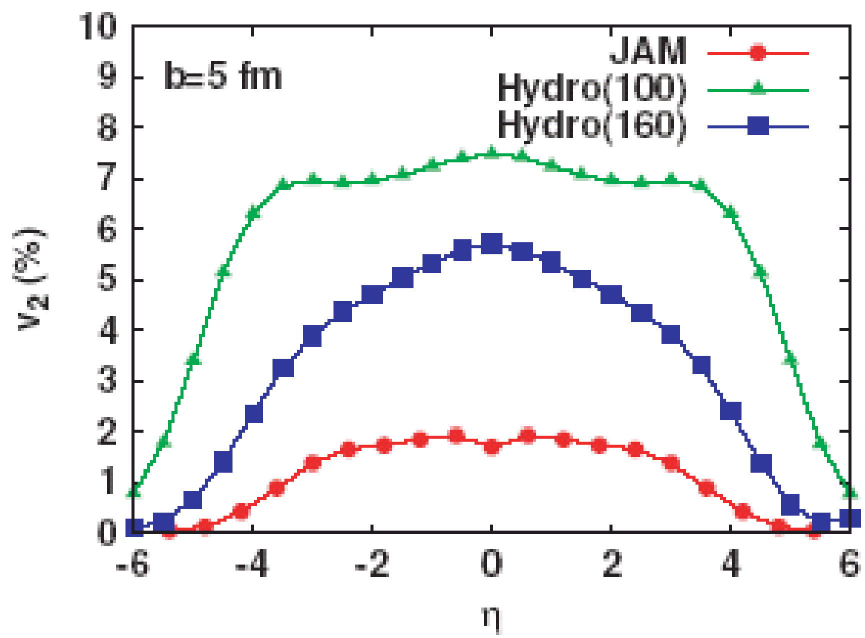


Hirano, Isse, Nara, AO, Yoshino, Phys. Rev. C 72(2005), 041901

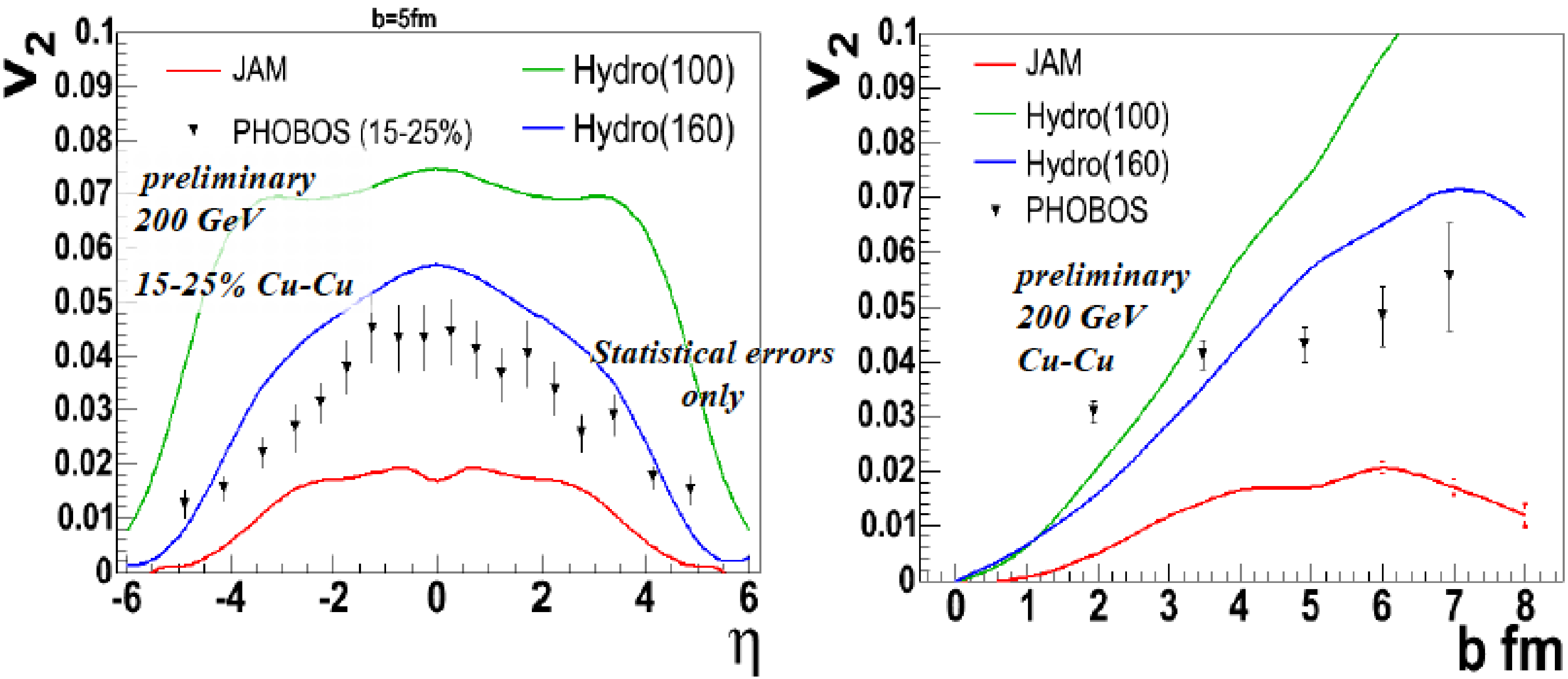
Predictions of Cu+Cu Collisions @ RHIC (II)

- **Calculations were done BEFORE the data are opened to public.**
- **Cascade and Hydro predict very different Elliptic Flow !**
  - **Cascade: small v2**
    → Small int. in the early stage
  - **Hydro: large v2**
    → Strong int. after $\tau=\tau_0 \sim 0.6$ fm/c
- **T^{th} dependence**
  - **T^{th} = 160 MeV ~ Tc = 170 MeV**
    → short time of expansion in the hadron phase
  - **T^{th} = 100 MeV < Tc = 170 MeV**
    → long time of expansion





Compared to JAM Model

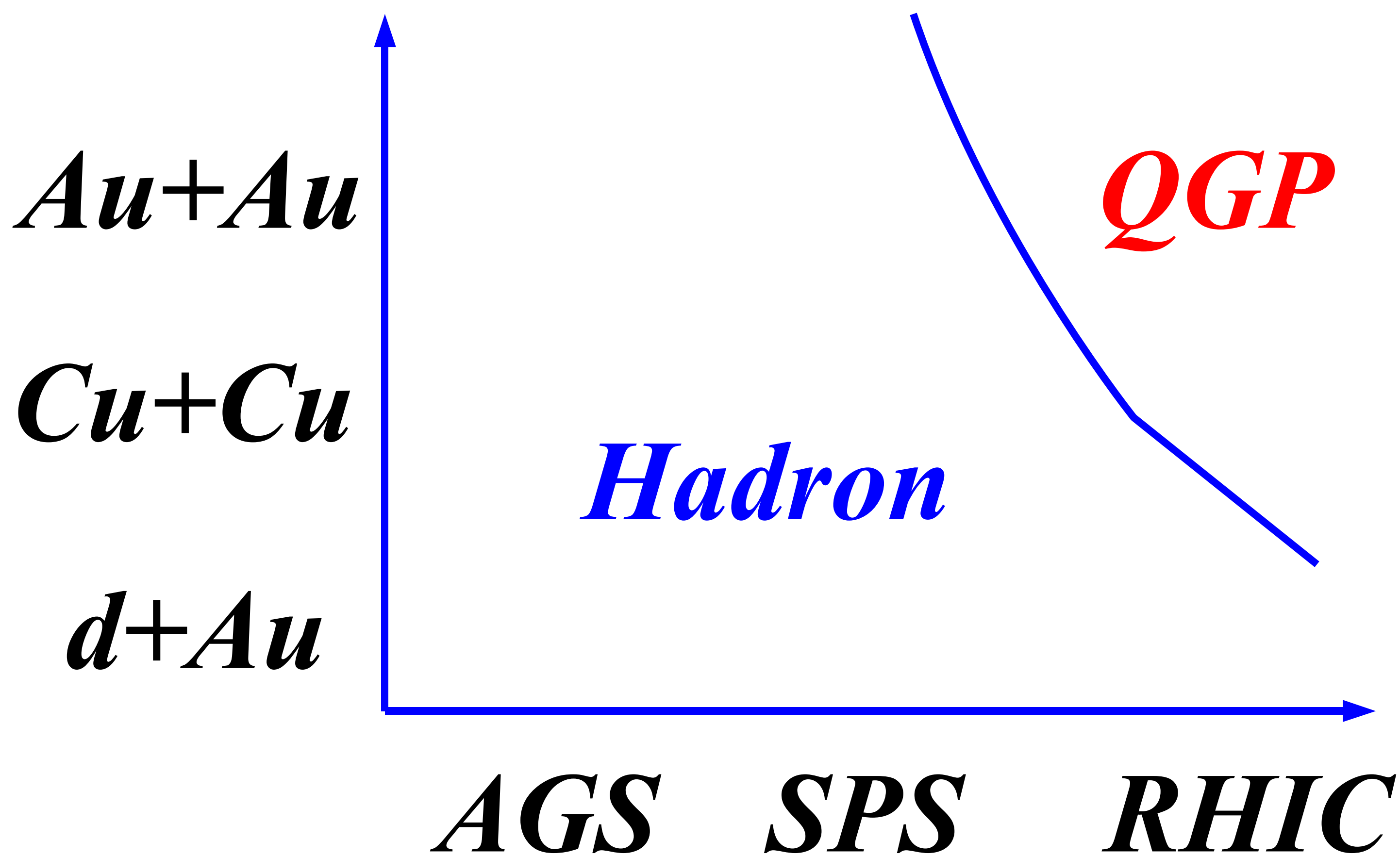


Cu-Cu more like Hydro than JAM hadron string cascade model

Here JAM uses a 1 fm/c formation time. Hydro (160) has kinetic freezeout temperature at 160 MeV

After Data are opened, ...

- Hydro wins Cascade at RHIC even for Cu+Cu collisions in the initial stage evolution.....
- "Reaction Phase Diagram" seems to be .....



Jet-Fluid String Formation and Decay at RHIC

Hirano, Isse, Nara, AO, Yoshino, in preparation

Hadronization Mechanism at RHIC

- *High p_T: Indep. Frag. of Jet Partons* *(E.g. Hirano-Nara)*
  - **O** Explains pT spectrum when E-loss is included.
  - **X** Elliptic Flow v_2 is small at high p_T ←*This Talk*
- *Medium p_T: Recombination* *(E.g. Duke-Osaka-Nagoya)*
  - **O** Explains Baryon Puzzle and Quark Number Scaling of v_2
  - **X** Entropy decreases in "$n \to 1$" process
- *Low p_T: Equil. Fluid Hadronization* *(E.g. Hirano-Gyulassy)*
  - **O** Explains p_T spec. and v_2 at low p_T
  - **X** Results depends on the Freeze-Out Conditions

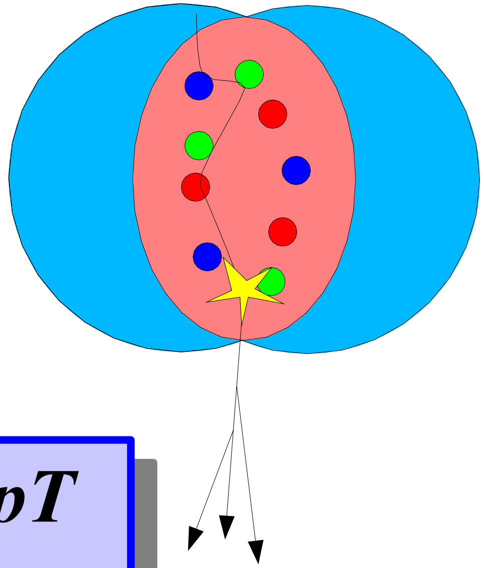
QGP Signals are understood separately,
and they are not necessarily consistent.
→ Further Ideas are required !

How can we get large v_2 at high p_T ?

- Quark Recombination → Combined Objects have larger v2

$$f(p, \varphi)=(1 + 2\, v_2(p/2) \cos \varphi) \times (1 + 2\, v_2(p/2) \cos \varphi)$$
$$\approx 1 + 2 \times 2\, v_2(p/2) \;\cos \varphi$$

- Energy Loss in QGP generates v2
  - Large/Small suppression in y/x directions



Plausible Hadronization giving large v2 at high pT
- *Combination of several partons*
- *Large Energy Loss*
  → Jet parton picks up Fluid parton and forms a string (Jet-Fluid String)

Jet-Fluid String Formation and Decay

Jet production: **pQCD(LO) × K-factor (PYTHIA6.3, K=1.8, *pp* fit)**

$$\sigma_{jet} = K \sigma_{jet}^{pQCD(LO)}$$

Jet propagation in QGP

3D Hydro + Simplified GLV 1st order formula × *C*

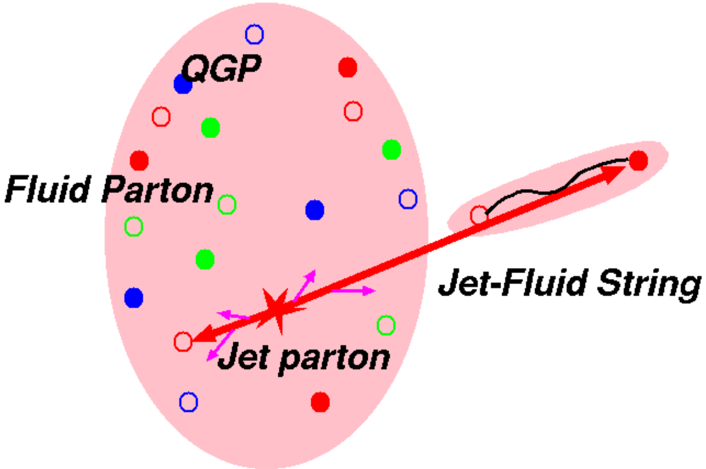
(Hirano-Nara, NPA743('04)305, Hirano-Tsuda, PRC 66('02)054905. Web version!
Gylassy-Levai-Vitev, PRL85('00)5535)

$$\Delta E = C \times 9\pi \frac{\alpha_s^3}{4} C_R \int d\tau (\tau - \tau_0) \rho_{\mathrm{eff}} \log\left(\frac{2E_0}{\mu^2 L}\right)$$

Jet-Fluid String formation

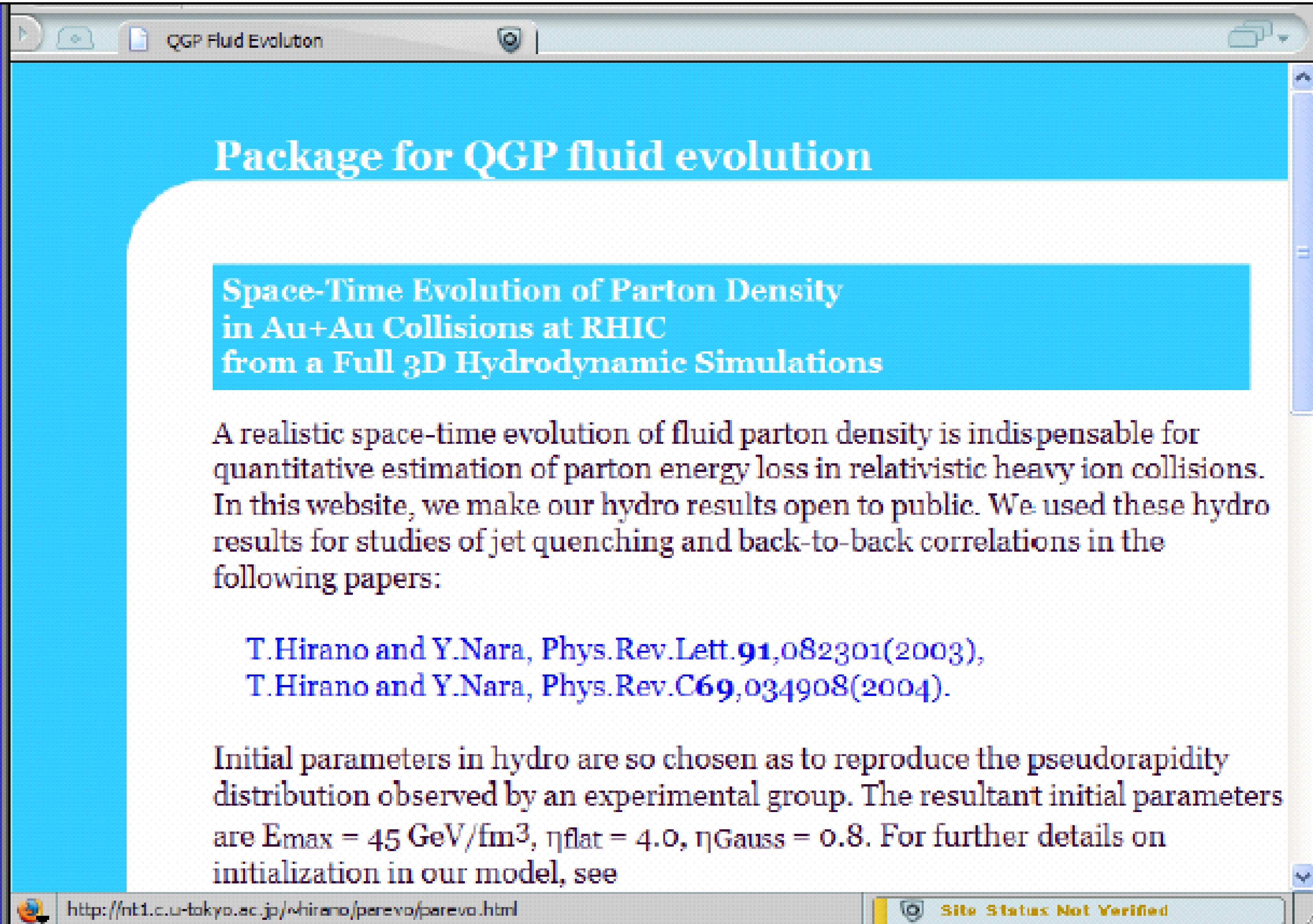
Fluid parton breaks color flux, according to string spectral func.

$$P(\sqrt{s}) \propto \Theta(\sqrt{s} - \sqrt{s_0}) \quad (\sqrt{s_0} = 2\,\mathrm{GeV})$$



Only g and light q (qbar) are considered.

http://nt1.c.u-tokyo.ac.jp/~hirano/parevo/parevo.html



QGP Fluid Evolution

Package for QGP fluid evolution

Space-Time Evolution of Parton Density in Au+Au Collisions at RHIC from a Full 3D Hydrodynamic Simulations

A realistic space-time evolution of fluid parton density is indispensable for quantitative estimation of parton energy loss in relativistic heavy ion collisions. In this website, we make our hydro results open to public. We used these hydro results for studies of jet quenching and back-to-back correlations in the following papers:

T.Hirano and Y.Nara, Phys.Rev.Lett.**91**,082301(2003),
T.Hirano and Y.Nara, Phys.Rev.C**69**,034908(2004).

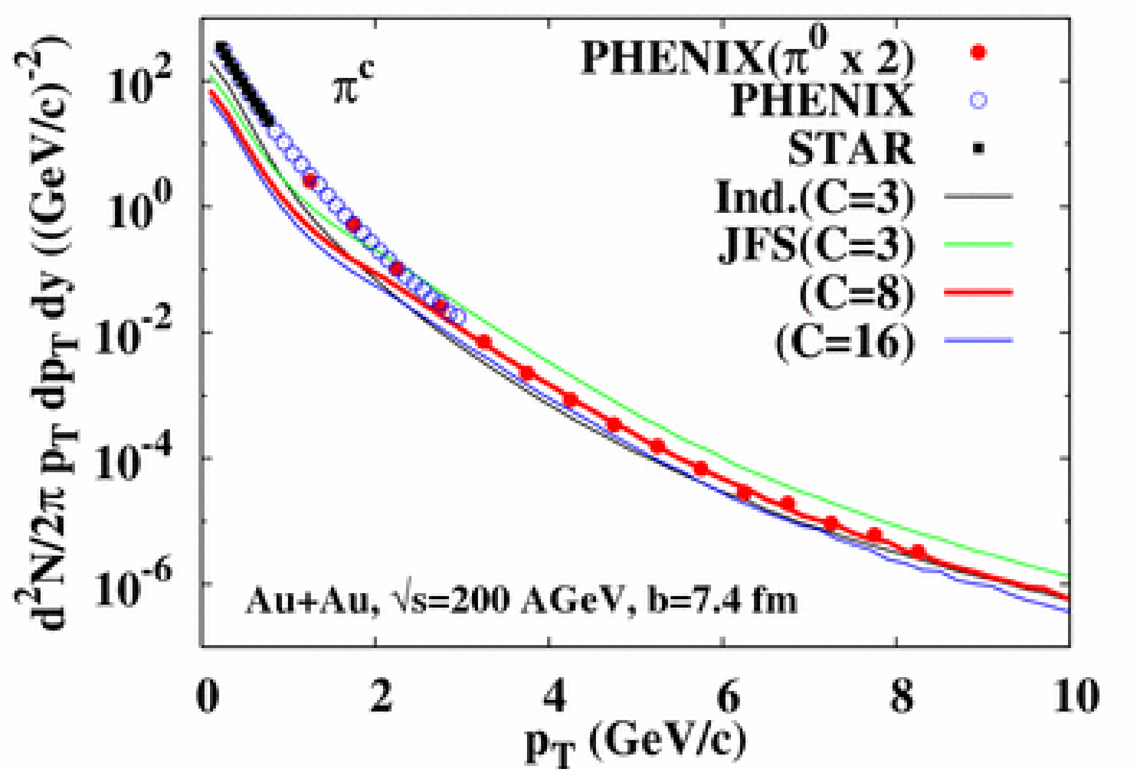
Initial parameters in hydro are so chosen as to reproduce the pseudorapidity distribution observed by an experimental group. The resultant initial parameters are E_{max} = 45 GeV/fm^3, η_{flat} = 4.0, η_{Gauss} = 0.8. For further details on initialization in our model, see

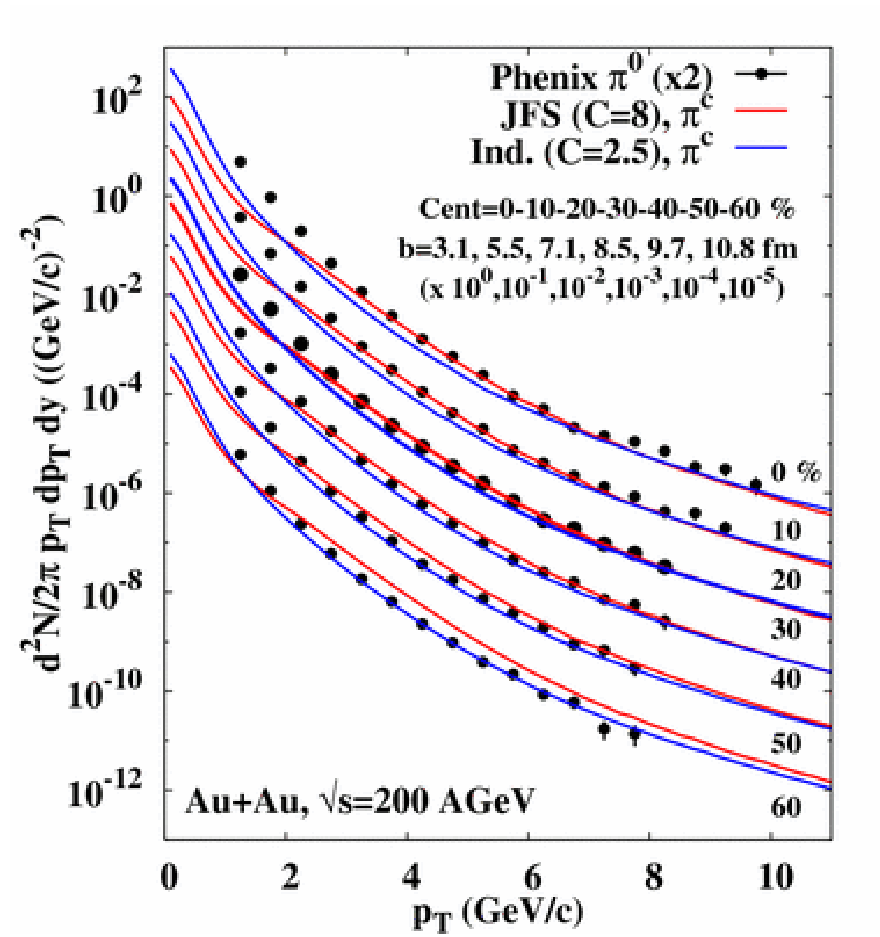
http://nt1.c.u-tokyo.ac.jp/~hirano/parevo/parevo.html

Site Status Not Verified

Energy Loss Factor C : p_T Spectrum Fit

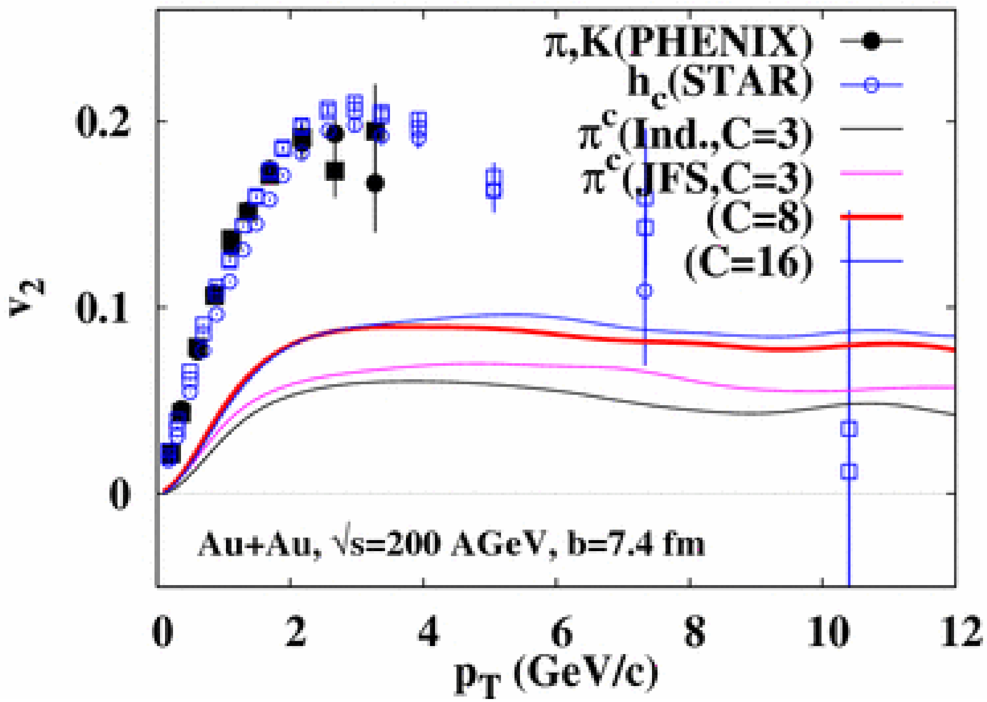
- For the same C → $dN_{JFS}(\text{high } p_T) > dN_{Ind}(\text{high } p_T)$
- p_T spec. fit → Ind. Frag.: $C \approx (2.5\text{-}3)$, JFS: $C \approx 8$
  → *Large Energy Loss is necessary / allowed in JFS*

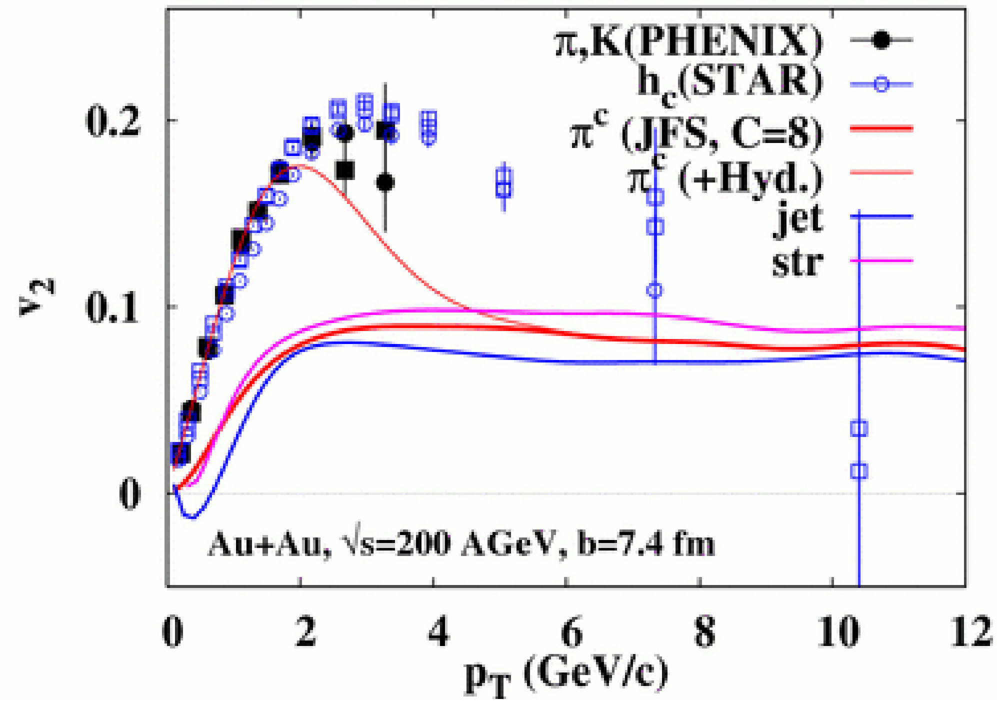




Elliptic Flow: p_T Deps.

- High pT v_2 : ~ 5 % in Ind. ($C = 3$) ↔ ~ 8 % in JFS ($C = 8$)

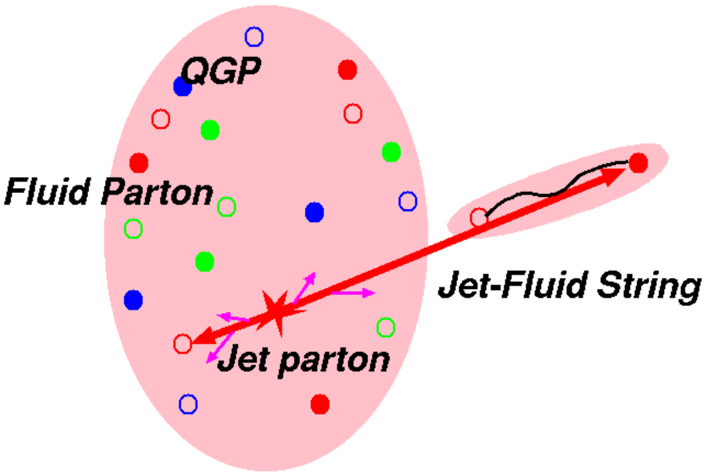




Origin of Large v_2 = Large E-loss factor C + Fluid parton v_2

JFS Summary

- *Jet-Fluid String (JFS) formation and decay* **is proposed as a mechanism to produce high p_T hadrons.**
  - **Effecitve to produce high p_T hadrons**
  - **Event-by-Event Energy-Mom. conservation ↔ Ind. Frag.**
  - **Entropy does not decreases, but increases. ↔ Reco.**
- **When we FIT p_T spectrum,** *large v_2 emerges at high p_T*
  - **Large E-loss+fluid parton v_2**
- **Problems and Homeworks**
  - **Mechanism of large E-loss**
  - **d+Au fit → Cronin Effects**
  - **s-quarks, string spectral func.**



Summary

- Heavy-ion collisions up to SPS energies seems to be reasonably described by using hadron-string cascade such as JAM model, while HIC at RHIC requires earlier thermalization (larger anisotropic pressure) even in lighter nuclear collisions such as Cu+Cu collisions.
- There are many things to do in high-energy heavy-ion collision physics.
  - AGS-FAIR-SPS energies
    Nuclear matter EOS, Baryon rich QGP, Strangeness enh., ...
  - RHIC-LHC energies
    Detailed studies of QGP properties have just started
    → Consistent understandings are not yet achieved,
    and we still have many puzzles

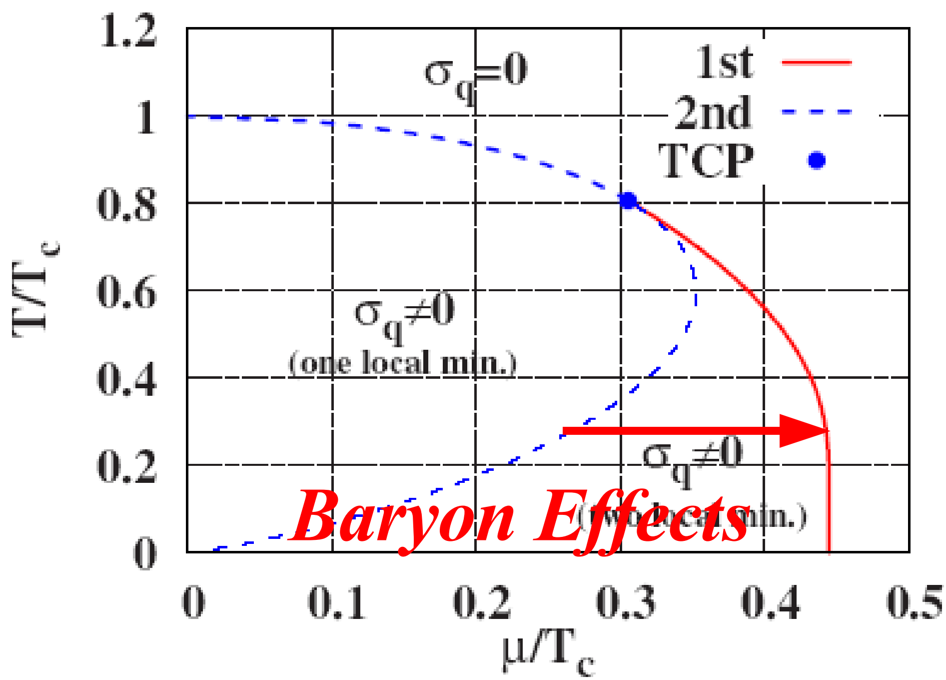
Sağ ol for listening !

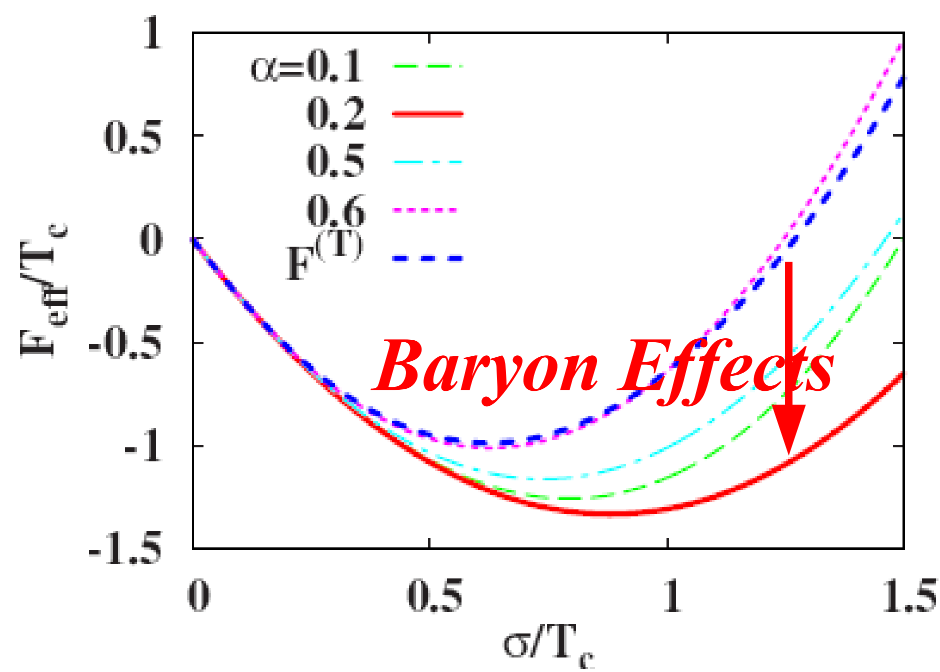
Backups

Effective Free Energy with Baryonic Effects

- **Effective Free Energy**

$$\boxed{\mathcal{F}_{\rm eff}(\sigma_q) \;=\; \frac{\sigma_q^2}{2\alpha^2} + F_{\rm eff}^{(b)}(g_\sigma \sigma_q) + F_{\rm eff}^{(q)}(\sigma_q; T, \mu)}$$





Baryons Gain Free Energy
→ Extention of Hadron Phase to Larger μ !

RMF with σ Self Energy from SCL-LQCD

- **σ Self Energy from simple Strong Coupling Limit LQCD**

$$
\begin{aligned}
S \quad &\to \quad -\tfrac{1}{2}(M, V_M M) && (1/d \text{ expansion}) \\
&\to \quad b\sigma^2 + (\bar{\chi}\ \sigma\chi) && (\text{auxiliary field}) \\
&\to \quad b\sigma^2 \boxed{- a \log \sigma^2} && (\text{Fermion Integral})
\end{aligned}
$$

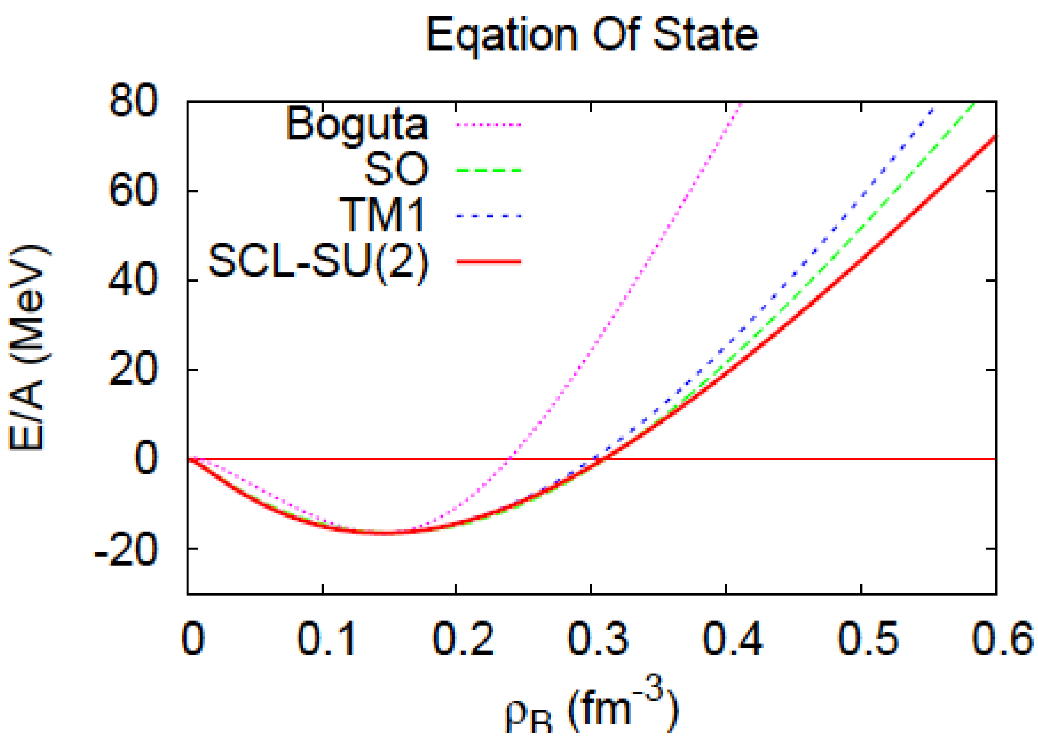
- **RMF Lagrangian** **Non-Analytic Type σ Self Energy**
  - **σ is shifted by f_π , and small explicit χ breaking term is added.**

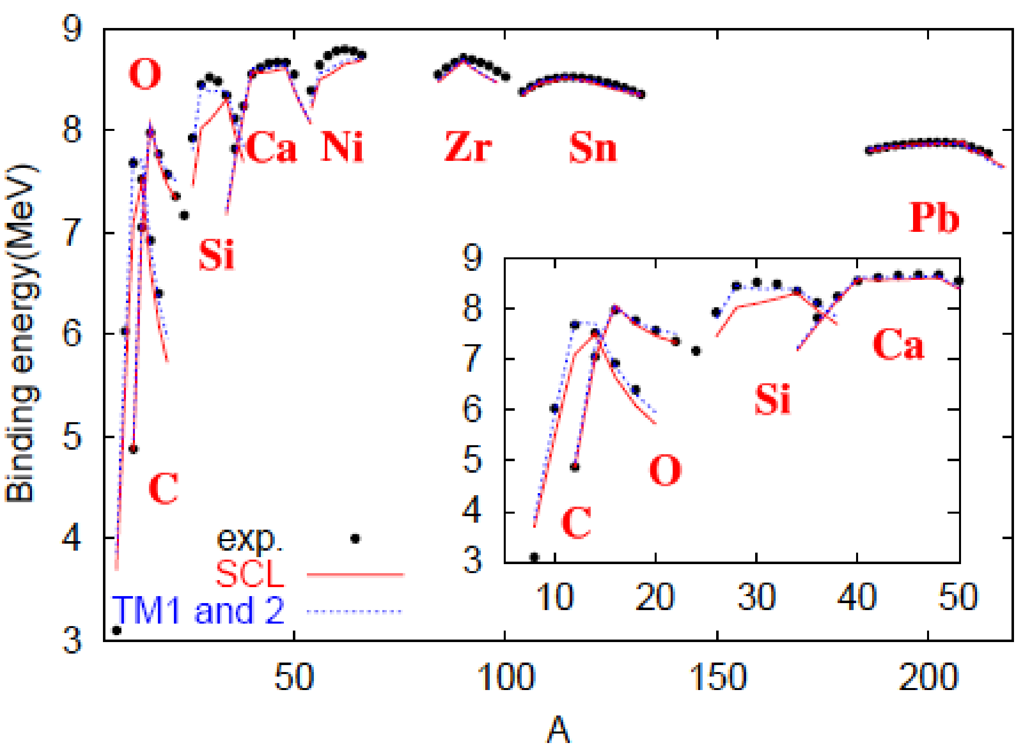
$$
\mathcal{L} = \bar{\psi}\left(i\gamma^\mu\partial_\mu - \gamma^\mu V_\mu - M + g_\sigma\sigma\right)\psi + \mathcal{L}_\sigma^{(0)} + \mathcal{L}_\omega^{(0)} + \mathcal{L}_\rho^{(0)}
$$
$$
-U_\sigma + \frac{\lambda}{4}(\omega_\mu\omega^\mu)^2
$$

$$
U_\sigma(\sigma) = 2a\, f(\sigma/f_\pi),\quad f(x) = \frac{1}{2}\left[-\log(1+x) + x - \frac{x^2}{2}\right],\quad a = \frac{f_\pi^{\ 2}}{2}\left(m_\sigma^2 - m_\pi^2\right)
$$

Nuclear Matter and Finite Nuclei

- Nuclear Matter: By tuning λ, $g_{\omega N}$, m_σ, *EOS can be Soft !*
- Finite Nuclei: By tuning $g_{\rho N}$, Global behavior of B.E. is reproduced, except for j-j closed nuclei (C, Si, Ni).





High pT v2 puzzle ?

- **V2 data at high pT seems to exceed the strong quenching limit (Hard Sphere or Hard Shell)**
  → We need to have a mechanism to go beyond this limit.

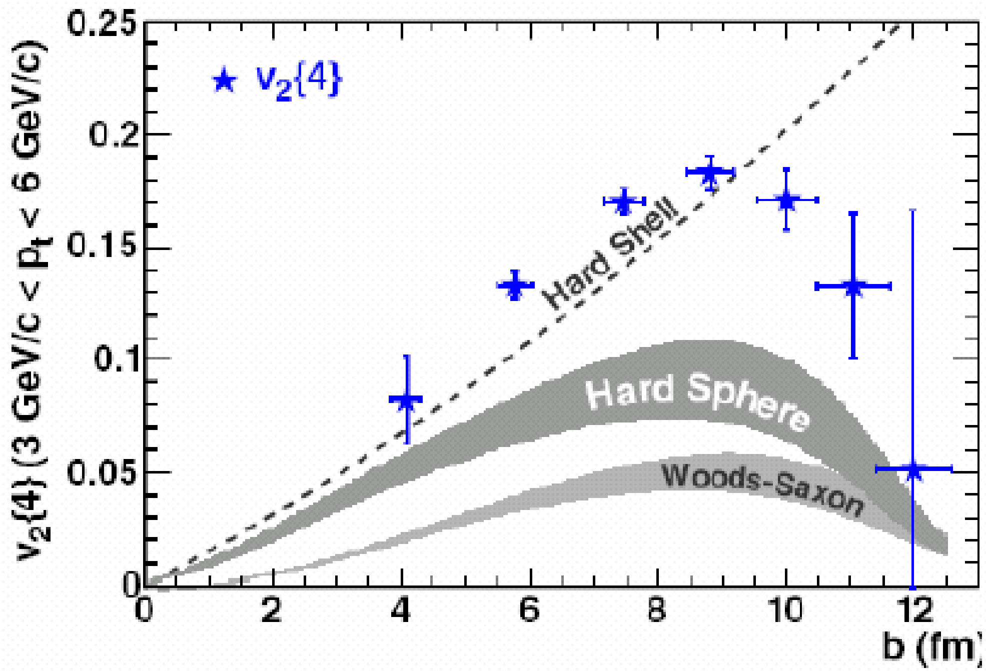
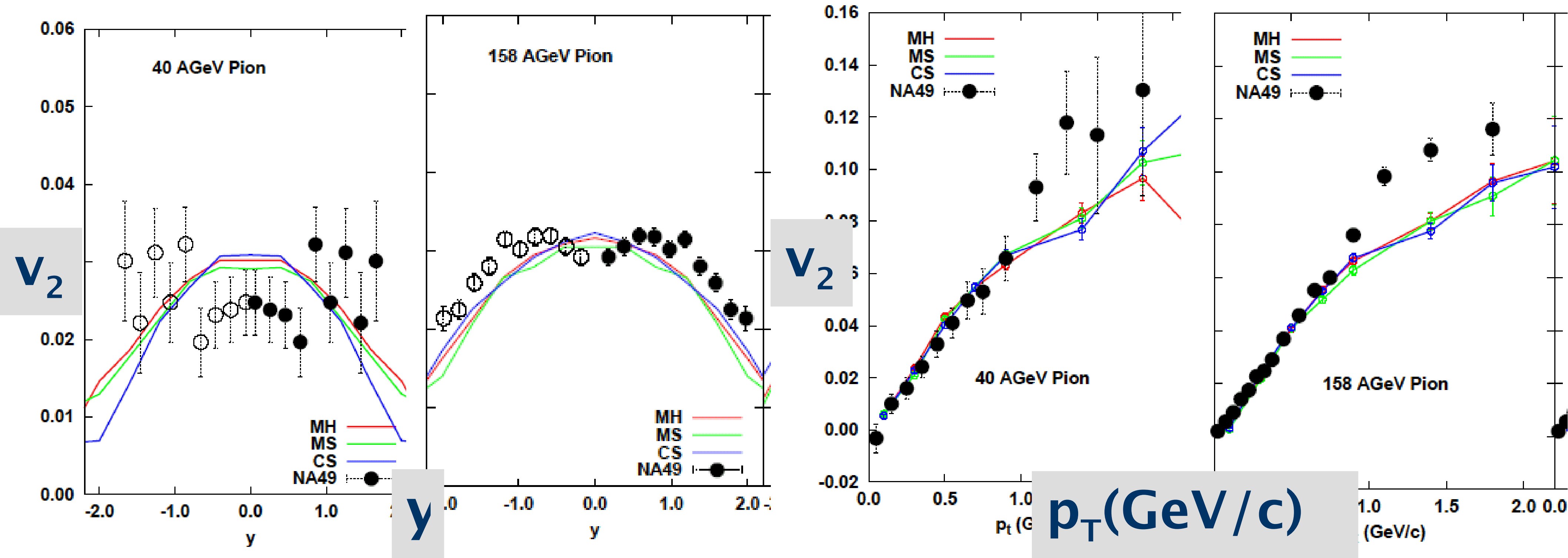


FIG. 4 (color online). v_2 at $3 \le p_t \le 6$ GeV/c versus impact parameter, b, compared to models of particle emission by a static source (see text).

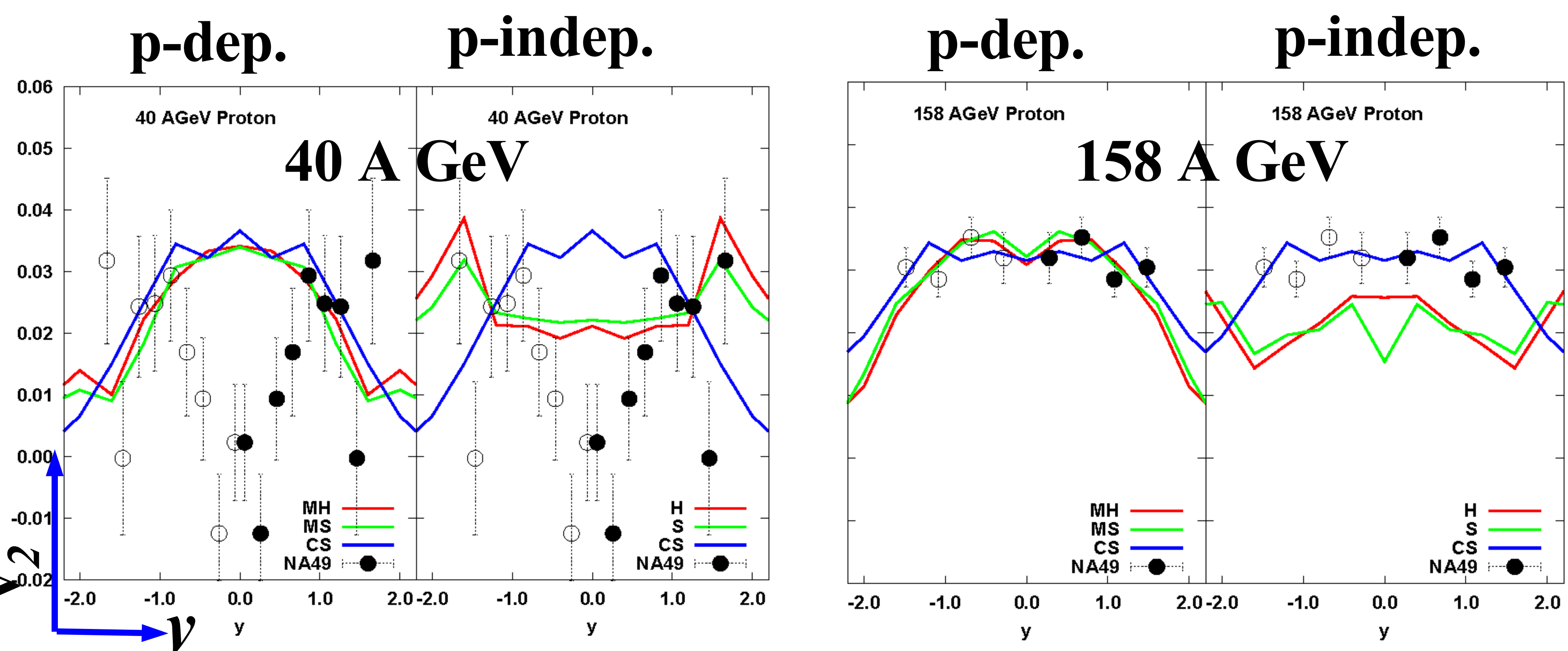
STAR, PRL93, 252301('04)

Pion v_2@SPS 40, 158 AGeV



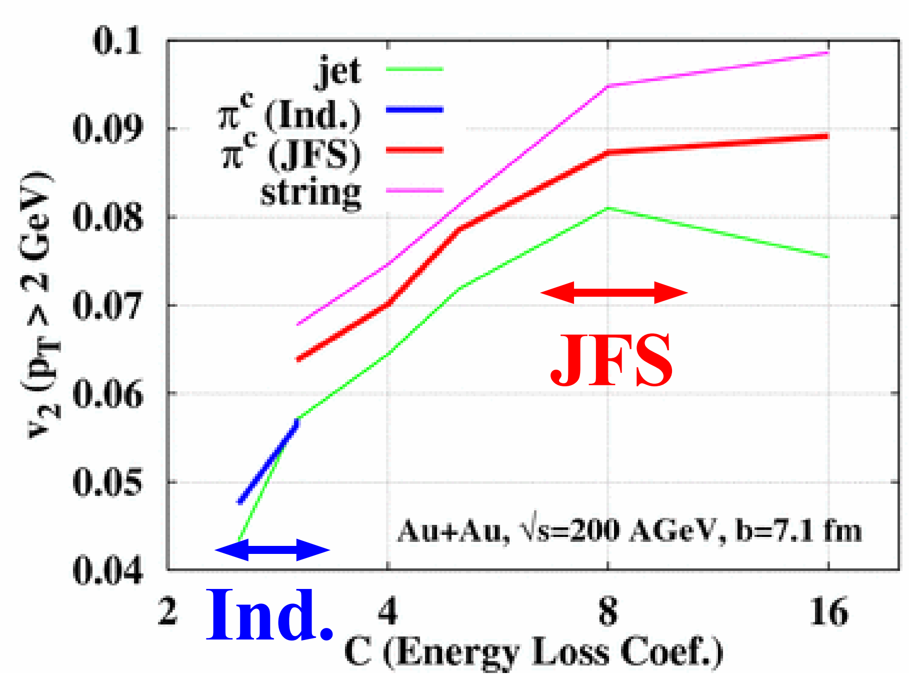
Elliptic flow v_2 at SPS

- **Rapidity dependence of proton v_2**
  → 158 A GeV data are well explained,
  but the collapse at 40 A GeV cannot be explained.



Elliptic Flow: Parameter Deps.

- v_2(jet): saturating behavior
  (large E-loss limit) ~ 8 %
- v_2 (string): grows up to ~ 10 %
  larger than v_2(jet, limit)
- v_2 (h): string decay reduces v_2
  $\rightarrow v_2(\text{jet}) < v_2(\text{h}) < v_2(\text{string})$

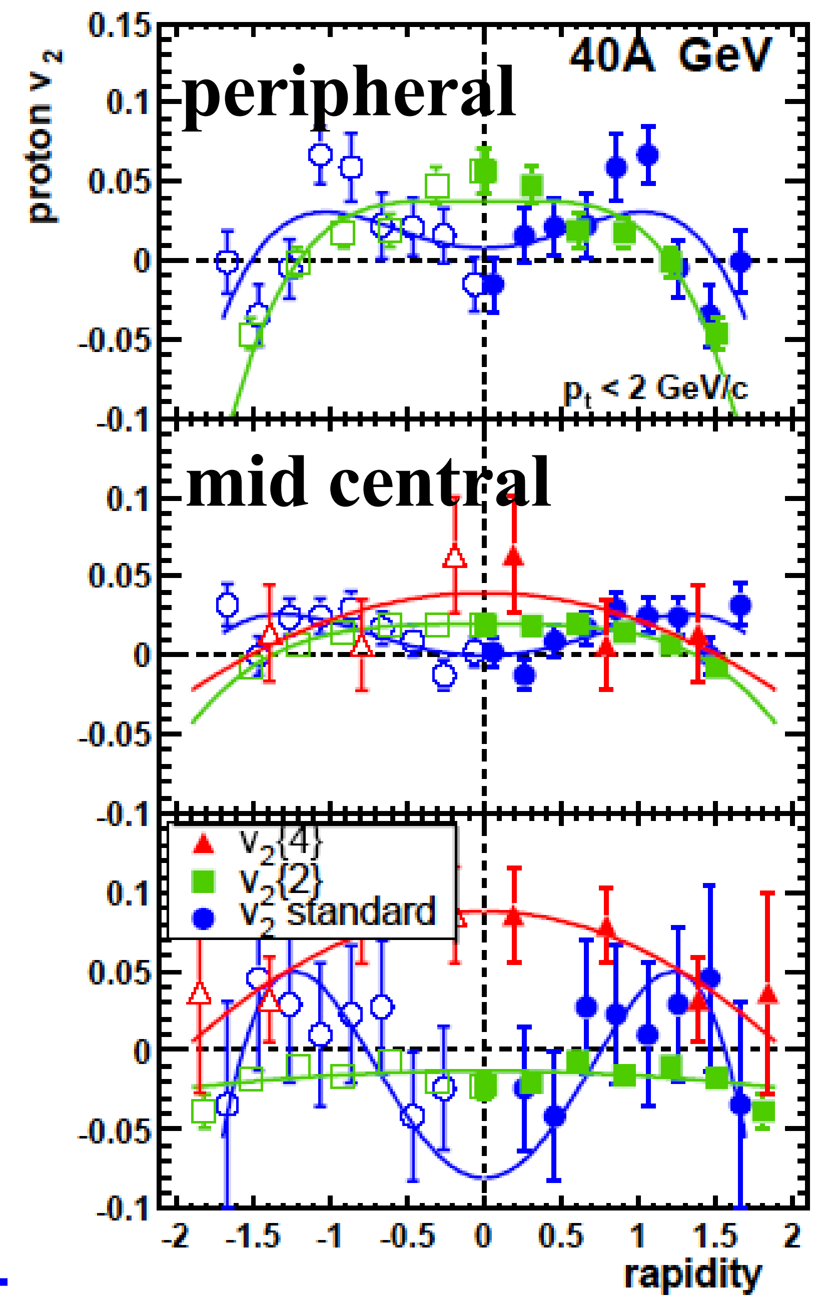


For p_T>2GeV ($p_T \approx$ 10 GeV)

Ind. Frag. with C = 2.5	$\rightarrow$	$v_2 \approx 5$ % (4 %)
Large E-loss factor C	$\rightarrow$	+ 3 %
Fluid parton v_2	$\rightarrow$	+ 1 %
JFS with C = 8	$\rightarrow$	$v_2 \approx 9$ % (8 %)

Dip of V2 at 40 A GeV: Phase Transition ?

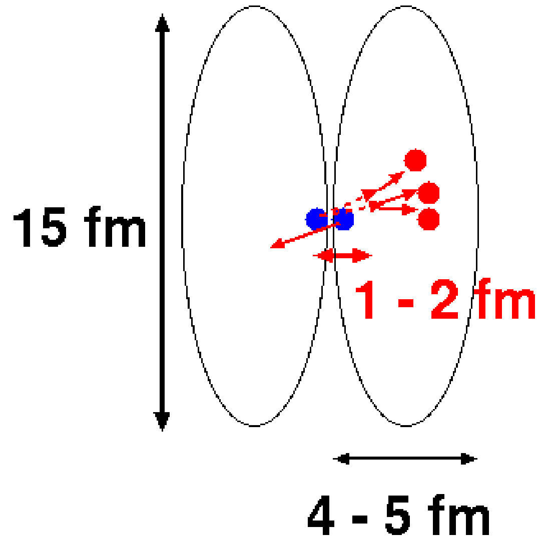
- Dip of V2 at 40 A GeV may be a signal of QCD phase transition at high baryon density.
- However, the data is too sensitive to the way of the analysis (reaction plane/two particle correlation).
  - We have to wait for better data.



Hadron Formation Time

JHF Energies

$\gamma_{cm} \simeq 3.5$, $\tau \simeq$ 0.5 - 1 fm/c

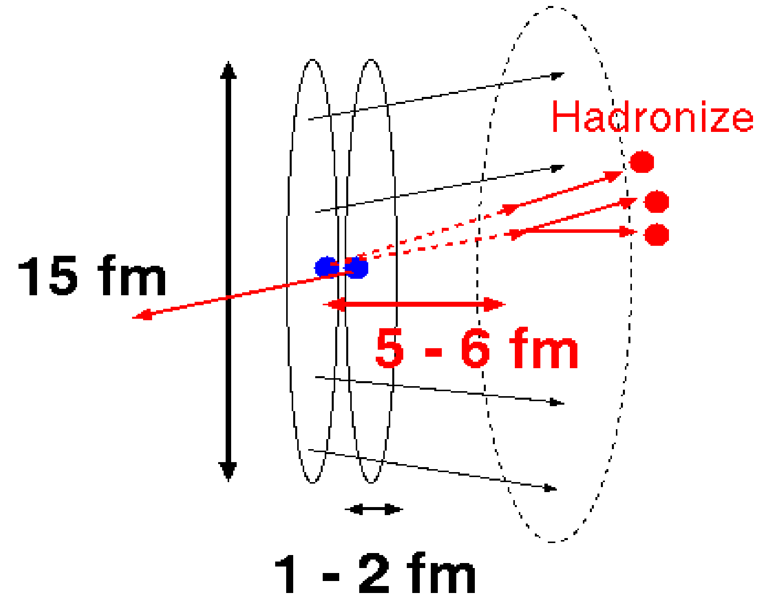


Multiple Hadron-Hadron Collisions

☞ (Approx.) Thermalized Hadron Gas

SPS Energies

$\gamma_{cm} \simeq 10$, $\tau \simeq$ 0.5 - 1 fm/c



String-String, String-Hadron Int.
+ Int. within Co-Movers

It takes τ 1 fm for hadrons to be formed (and thus to interact)
→ Pre-Hadronic Interactions are necessary at SPS & RHIC
→ Hot & Dense Hadronic Matter would be formed at AGS & JHF

Comparison with Previous Works

- J. Casalderrey-Solana, E.V. Shuryak, hep-ph/0305160
  - Quarks, diquarks and gluons in QGP cut color flux (~ JFS).
  - Large E-loss is generated by "phaleron"
  - *Large E-loss leads "surface emission" → large v_2*
- Recombination (Duke-Osaka-(Minesota)-Nagoya)
  - Predicts large v_2 (~ 10 %) at high-pT
    - Sharply edged density dist. → E-loss $\propto L \rightarrow v_2 \approx 10$ %
    - Woods-Saxon density dist. → $v_2 \approx 5$ %
  - Entropy problem: S(QGP) ≈ S(H) requires Res. and Strings
  - *Spectral Func.: δ func. ↔ θ func. in JFS*

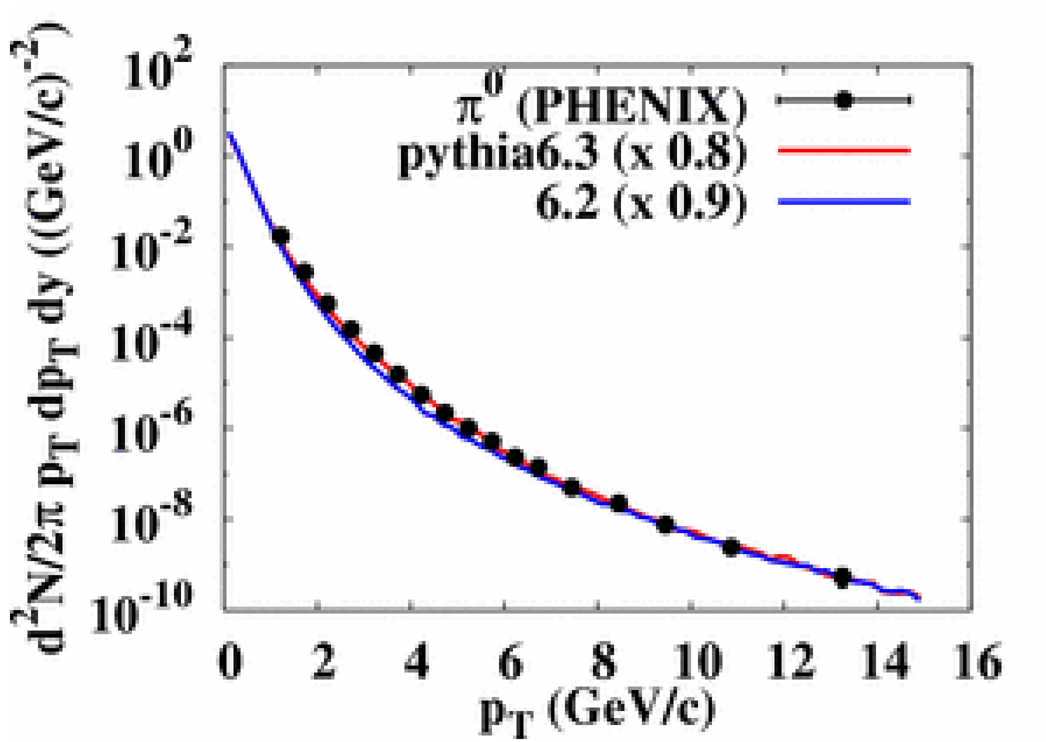
K-factor

- K-factor → absolute value of σ_{jet}
- Experimental Data: pp → π^0 @ $\sqrt{s_{NN}}$ = 200 GeV (PHENIX)

$$\frac{1}{\sigma^{\exp}}\frac{d^2\sigma^{\exp}}{2\pi p_T d p_T dy} = K \frac{\sigma^{pQCD(1st)}}{\sigma^{\exp}}\frac{d^2 N^{pQCD(1st)}}{2\pi p_T d p_T dy} \qquad A = K\frac{\sigma^{pQCD(1st)}}{\sigma^{\exp}}$$

$\sigma^{Exp.}$ = 21.8 mb (trigger)
$\sigma^{pQCD(1st)}$ = 9.9 mb

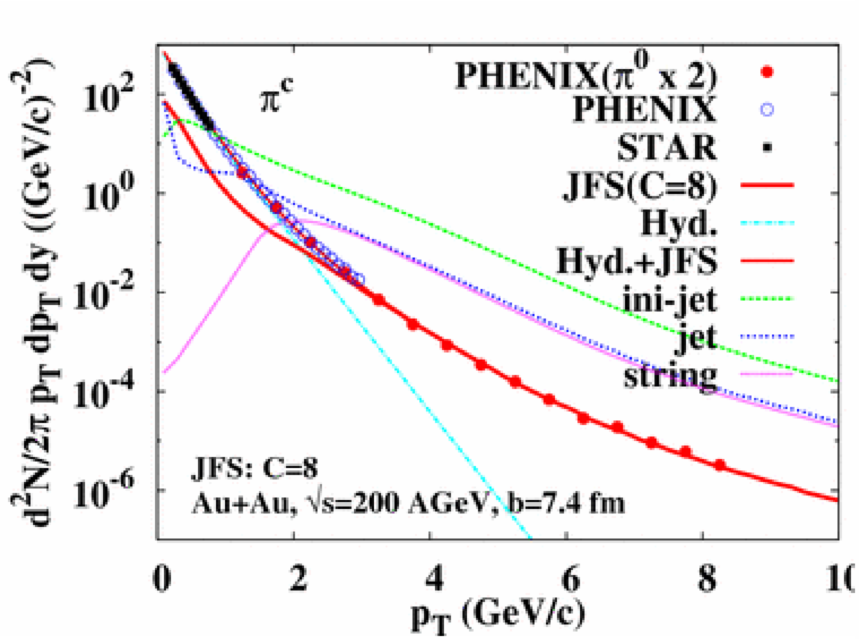
- pythia6.3 fit:
  A ≈ 0.8 → K = 1.8
  (σ_{jet} (p_T^{hard}>2Gev/c)≈ 17.5 mb)
- pythia6.2 fit:
  A ≈ 0.9 → K= 2.0
  (σ_{jet} ≈ 19.6 mb)

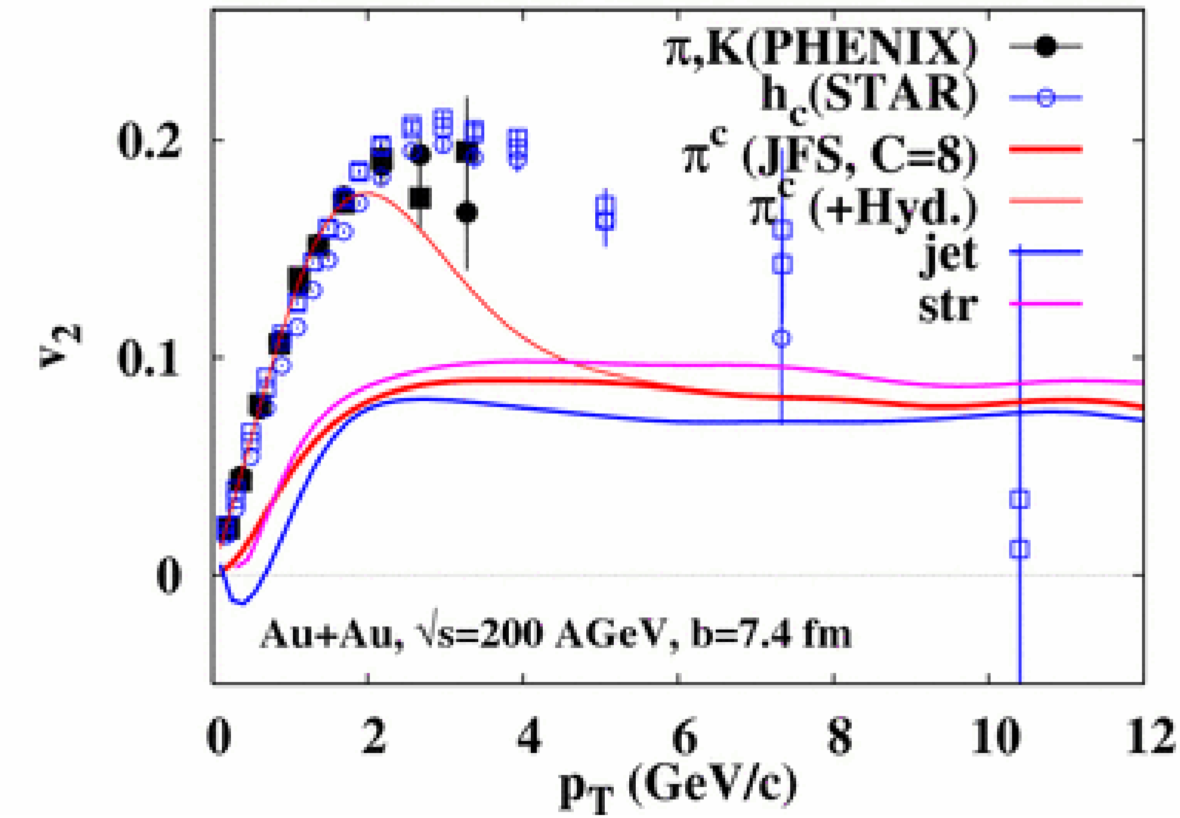


Combined with Low p_T spectrum

- Low pT spectrum is assumed and combined.

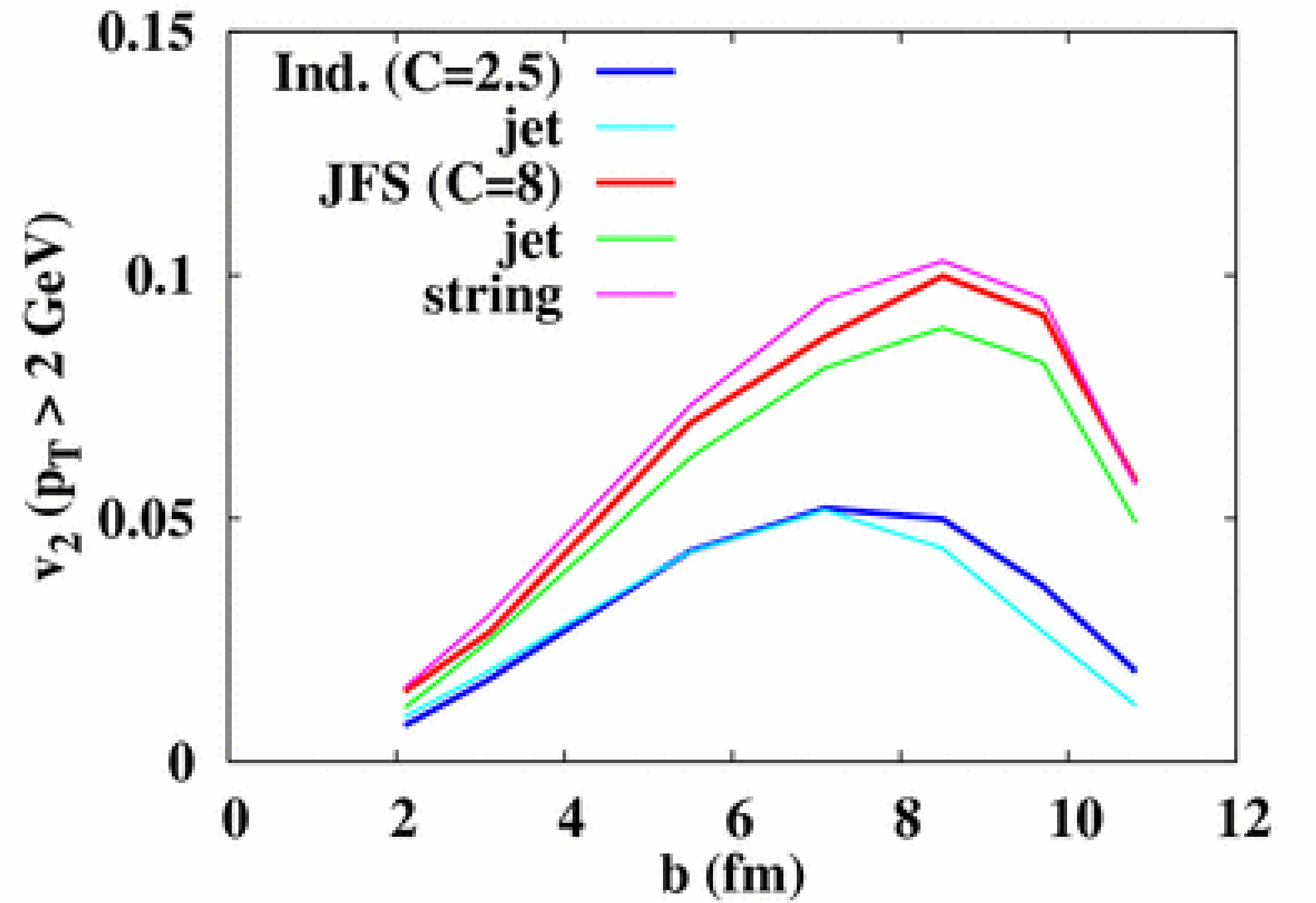
$$E\frac{d^3N_{Hyd}}{dp^3}(p_T)=A\exp(-p_T/T)(1+B/(1+(p_T/p_0)^8)) \qquad v_2^{Hyd}(p_T)=0.14\,p_T$$





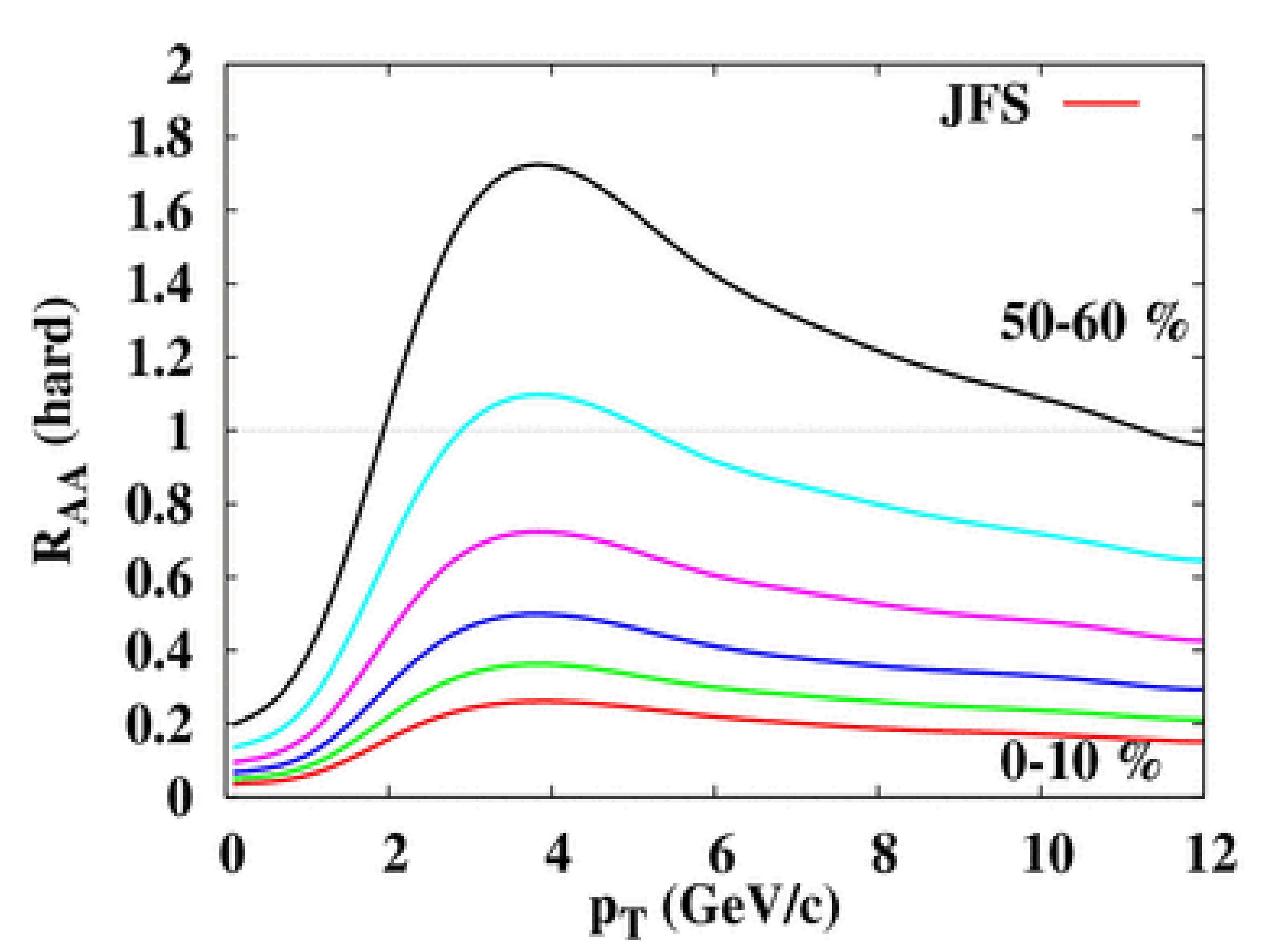
Elliptic Flow: Centrality Deps.

- Ind. (C=3): $v_2 \sim 5$ % at b ≈ 7 fm
- JFS (C=8): $v_2 \sim 10$ % at b ≈ 8.5 fm



Nuclear Modification Factor

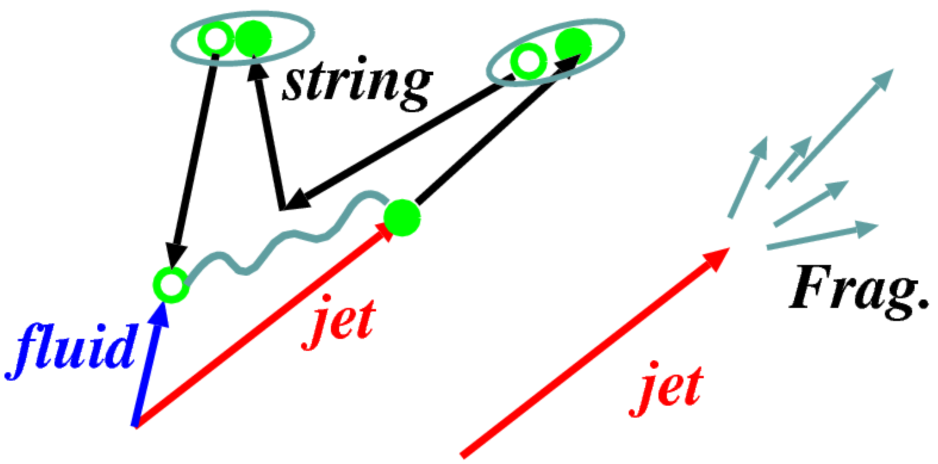
- Centrality Deps.



Discussion

- Mechanism to produce high p_T hadrons in JFS
  - String Decay from Lorenz boosted fluid
  - Relative momentum is relatively small
    → Smaller number of hadrons with high p_T are formed

↔ Independent Frag. (Large no. of Low p_T hadrons)



Energy Loss Factor

- Additional Factor for Energy Loss → High p_T hadron yield
- Exp. Data: p_T spectra of π in Au+Au (PHENIX,STAR)

$$\frac{d^2 N^{Exp.}}{2\pi p_T d p_T dy} = N_{jet} \frac{1}{N_{jet}} \frac{d^2 N^{JFS}(C)}{2\pi p_T d p_T dy}$$

→ Determining N_{jet} is important !

Ncoll = 373 @ b=7.4 fm (PHENIX estimate)

σ^{NN}_{jet} = 17.5 mb (pp fit pythia 6.3), σ^{NN}_{tot} = 47. 4 mb (JAM)

$$N_{jet} = \sigma^{NN}_{jet} \int d^{2r}_T T_A(r_T + b/2) T_B(r_T - b/2) = \frac{\sigma^{NN}_{jet}}{\sigma^{NN}_{tot}} N_{coll}$$

$$T_A(r_T) = \int dz \, \rho(r_T, z)$$

Further Problems

- Very large energy loss is required to explain p_T spectrum.
  - $C \approx 8$ in JFS $\leftrightarrow$ $C \approx 2.7$ in Hydro+Jet model (Hirano-Nara)

  Is it possible to justify this large energy loss ?
- Elliptic flow at medium pT is underestimated.
  → Fluid-Fluid String would be necessary to consider.
- Large baryon yield at medium pT may not be explained.
  → Three parton string ? (Jet-Fluid-Fluid, Fluid-Fluid-Fluid)
- String formation probability should be evaluated
  in pQCD matrix element + string level density.
- Strange hadrons