Fundamental and Phenomenological Approaches to High Density Hadronic Matter

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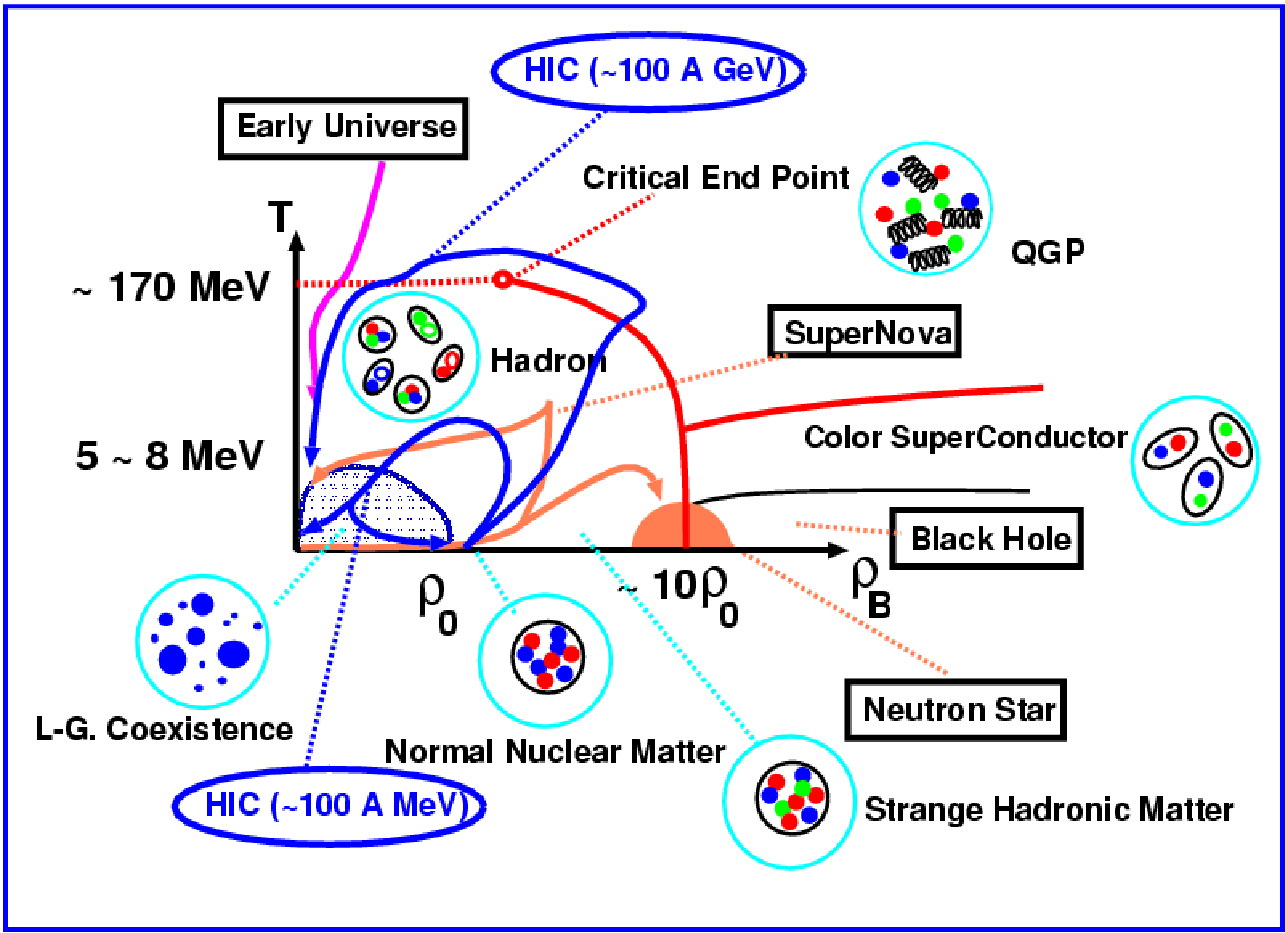
N. Kawamoto, K.Miura, T.Ohnuma, K. Tsubakihara

- **Introduction --- QGP Signals**
- **Jet-Fluid String Formation and Decay in High-Energy Heavy-Ion Collisions (T. Hirano, M. Isse, Y. Nara, AO, K. Yoshino, in preparation)**
- **Strong Couling Limit Lattice QCD (N. Kawamoto, K. Miura, AO, T. Ohnuma, hep-lat/051223)**

Introduction : QGP Signals at RHIC

Hadronic Matter Phase Diagram

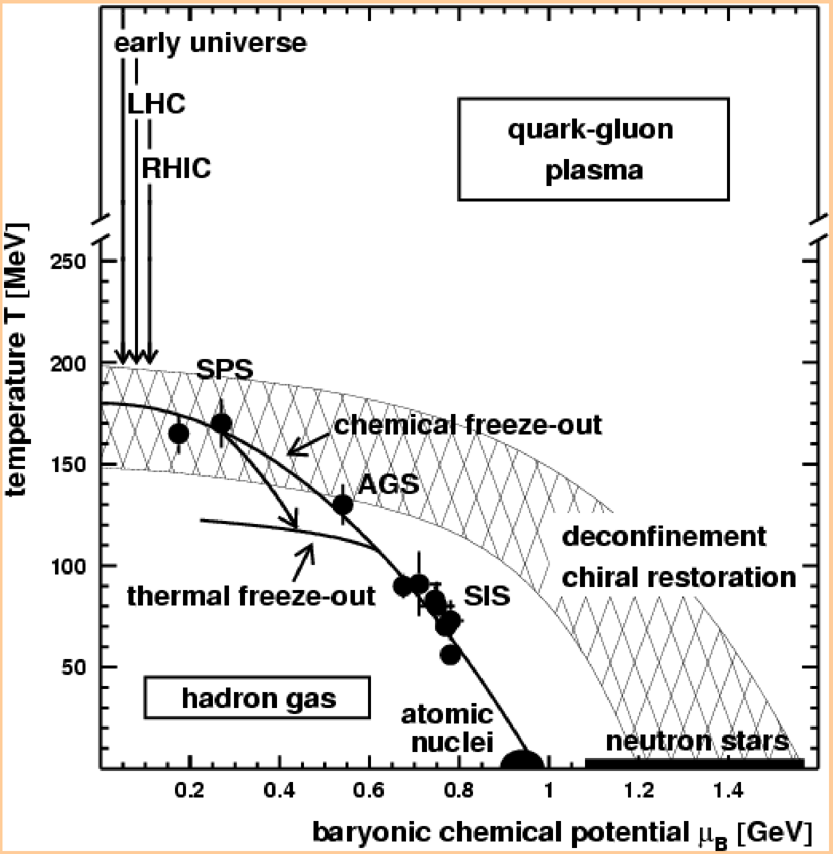


High-Energy Heavy-Ion Collision Experiments

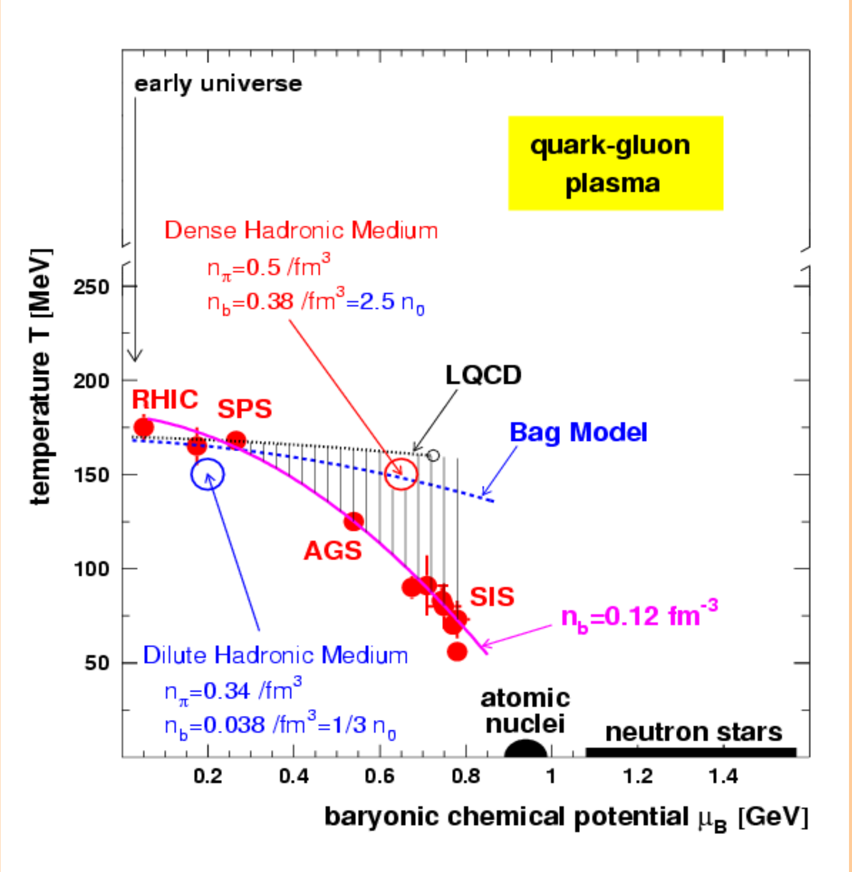
Heavy-ion physisists wanted to create QGP for a long time ...

LBL-Bevalac:
800 A MeV
GSI-SIS:
1-2 A GeV
BNL-AGS (1987-):
10 A GeV
CERN-SPS (1987-):
160 A GeV
BNL-RHIC (2000-):
100+100 A GeV
CERN-LHC (2007(?)-):
3 + 3 A TeV

Experimentally Estimated Phase Diagram



1998 (J. Stachel et al.)



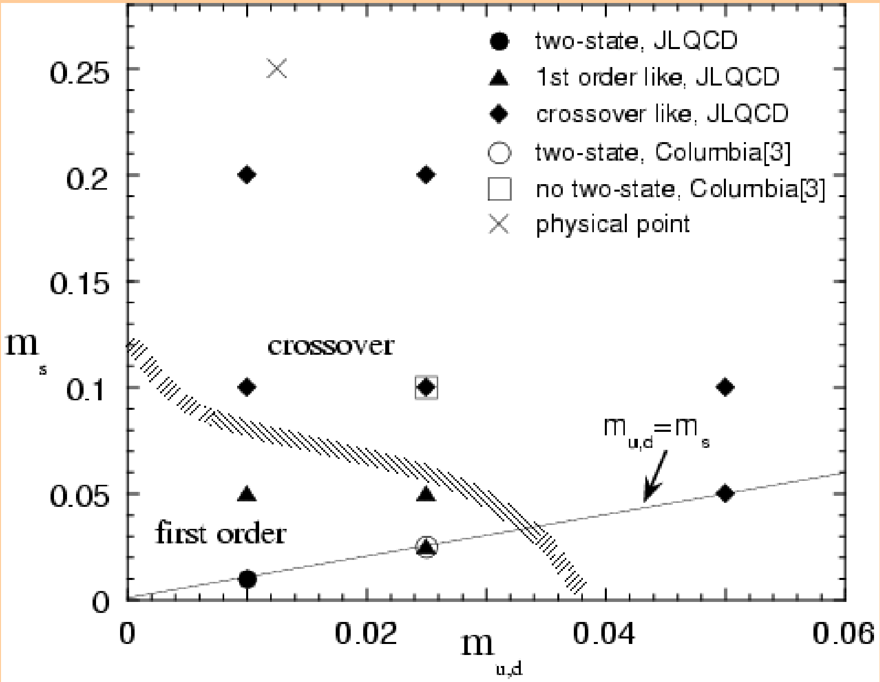
2002 (Braun-Munzinger et al.
J. Phys. G28 (2002) 1971.)

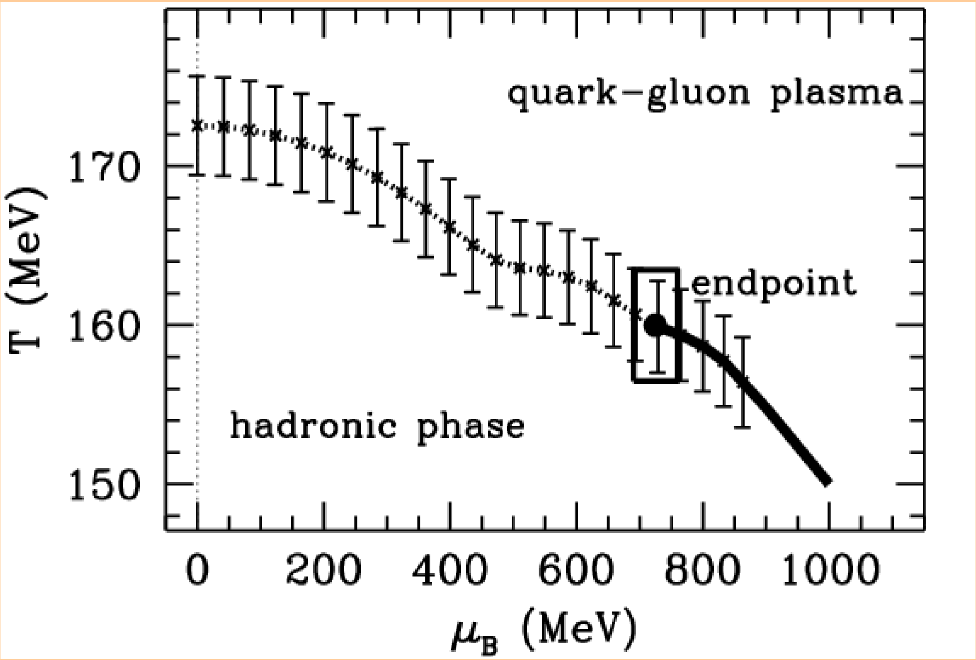
Chem. Freeze-Out Points are very Close to Expected QCD Phase Transition Boundary

Theoretically Expected QCD Phase Diagram

Zero Chem. Pot.

Finite Chem. Pot.



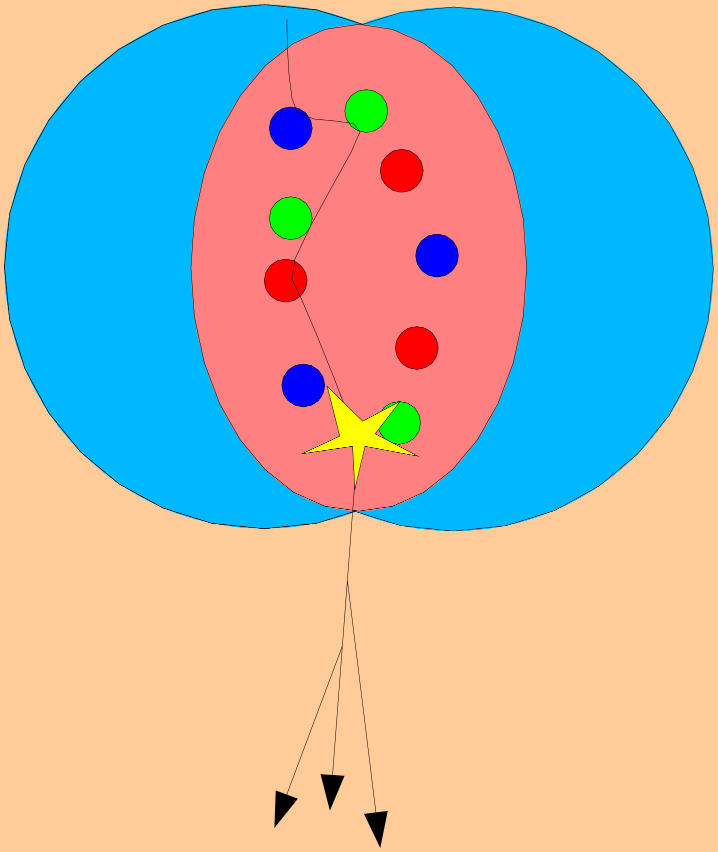


JLQCD Collab. (S. Aoki et al.), Nucl. Phys. Proc. Suppl. 73 (1999) 459.

Finite μ : Fodor & Katz, JHEP 0203 (2002), 014.

Zero Chem. Pot. : Cross Over
Finite Chem. Pot.: Critical End Point

Jet Energy Loss at RHIC (I)



2003/06/18 Press Release

Colored partons will lose energy in colored gas environment (=QGP)

Since High Energy Particles are expected to come from Jet Fragmentation, they are suppressed if QGP is formed.

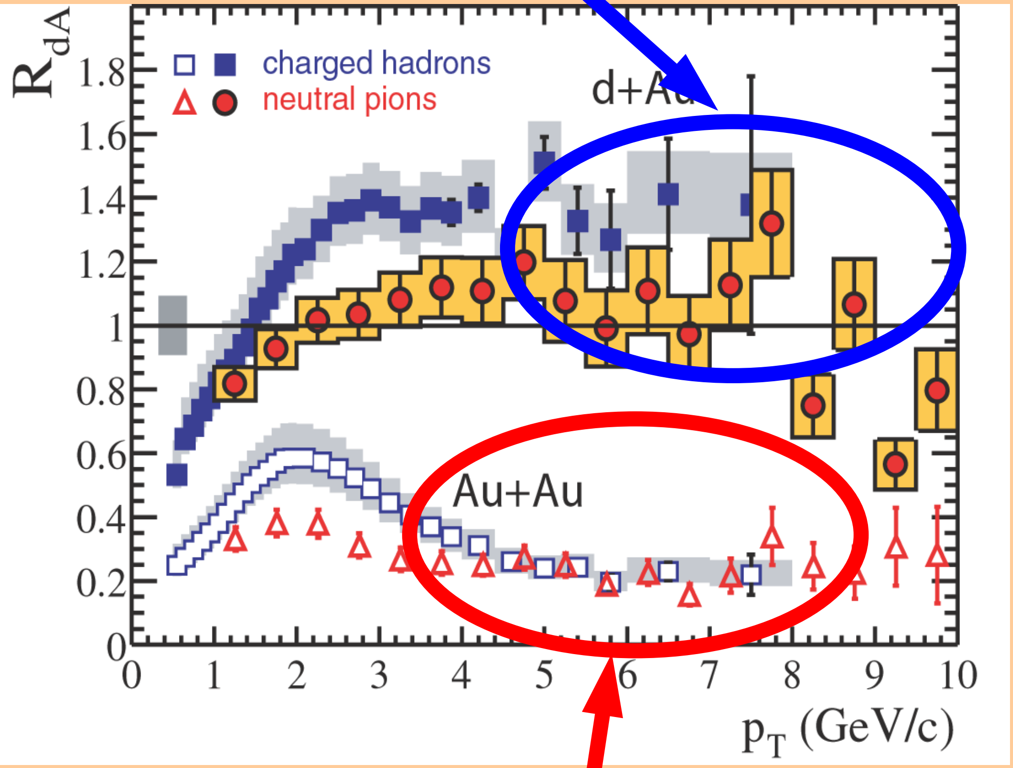
Jet Quenching at RHIC (II)

by Esumi, Matter03

Do we really see suppression of high energy particles at RHIC ?
→ YES for Au+Au Collisions,
and NO for d+Au Collisions !

$$R_{AB}(p_T) = \frac{d^2N/dp_T d\eta}{T_{AB}\, d^2\sigma^{pp}/dp_T d\eta}$$

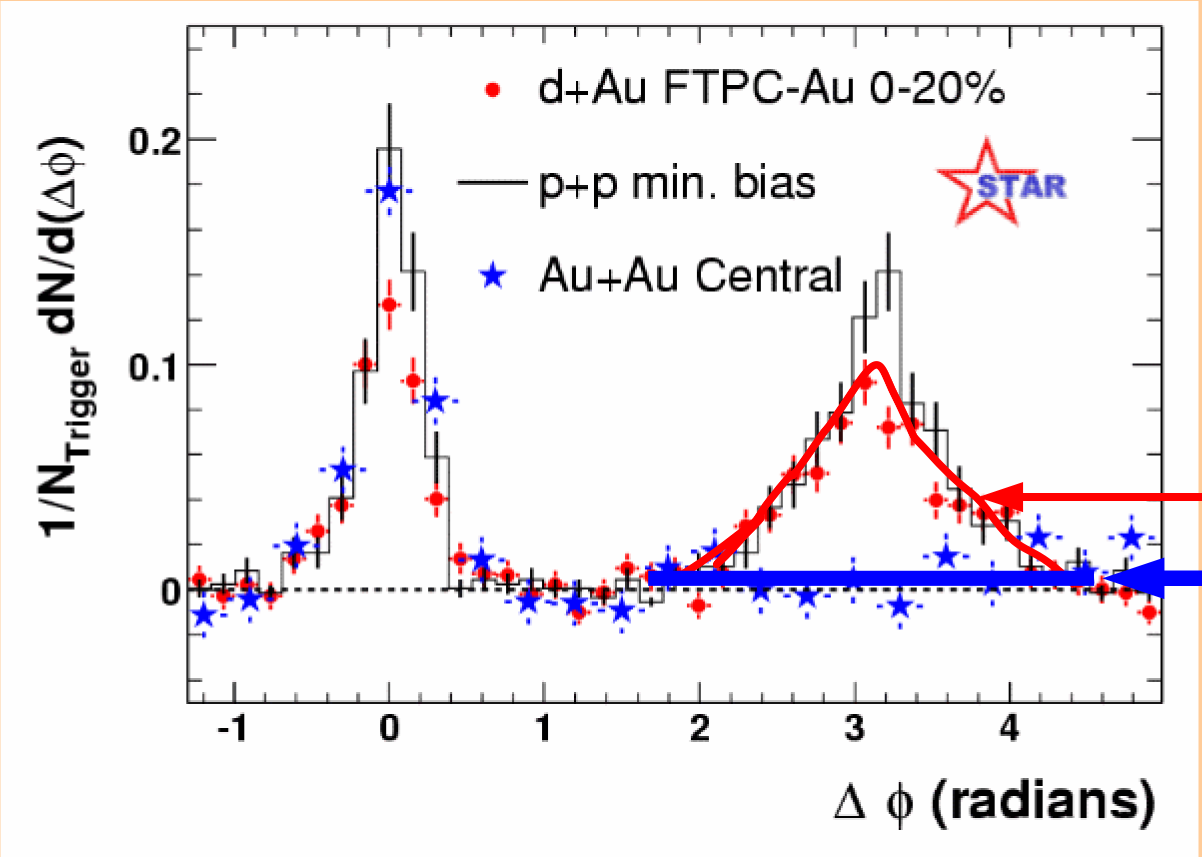
d + Au: Initial State Effects

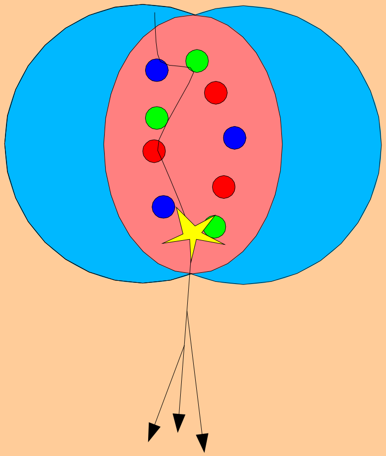


High Energy Particles are suppressed in Au + Au Collisions but NOT suppressed in d + Au Collisions at RHIC compared to p+p collisions !

Au + Au:
Initial State
+ Final State Effects

Jet Quenching at RHIC (III)





d + Au: Backward Peak

Au + Au:
No Backward Peak

STAR (nucl-ex/0306024)

Jet Energy Loss also lead
to reduction of back-to-back correlation

What is Collective Flow ?

(Directed) Flow (dP_X/dY)

Stiffness (Low E)
+ Time Scale (High E)

Elliptic Flow (V_2)

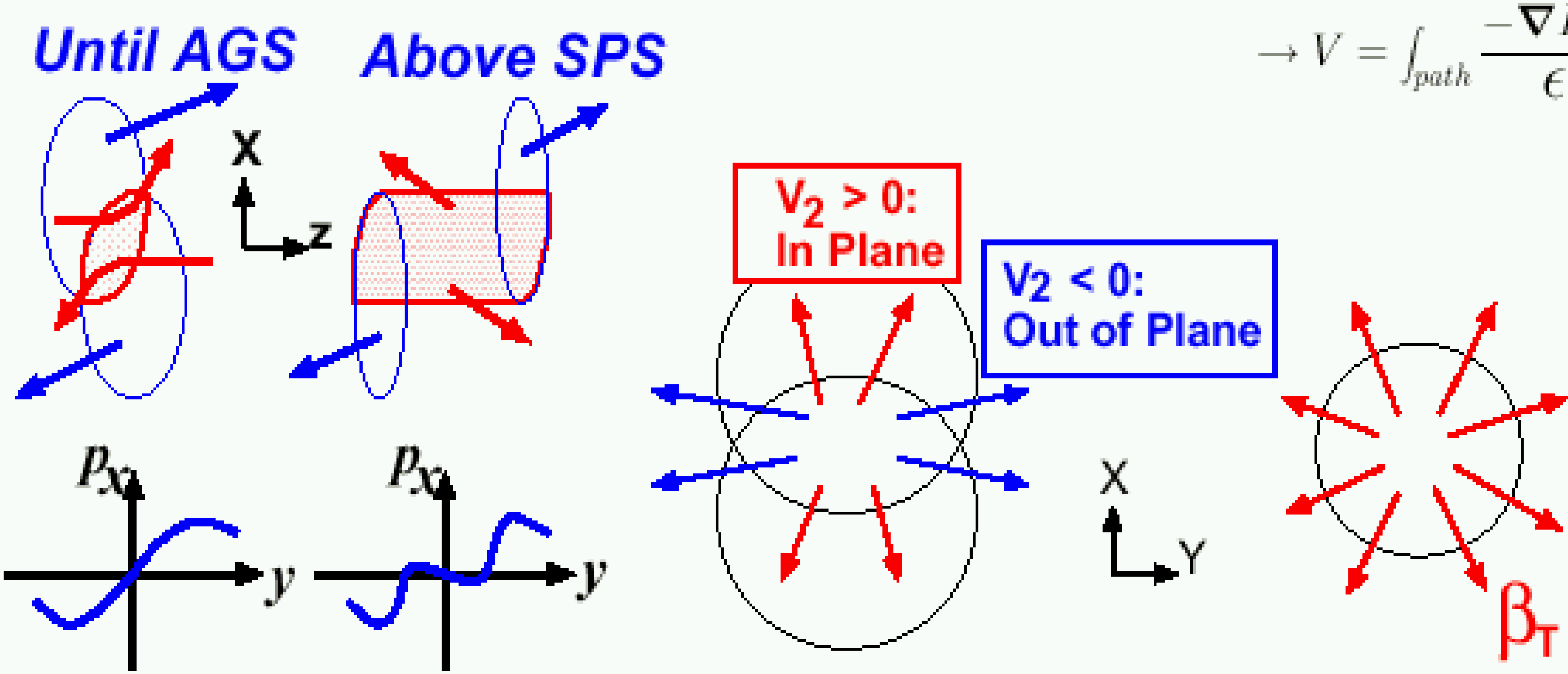
Thermalization
& Pressure Gradient

Radial Flow (β_T)

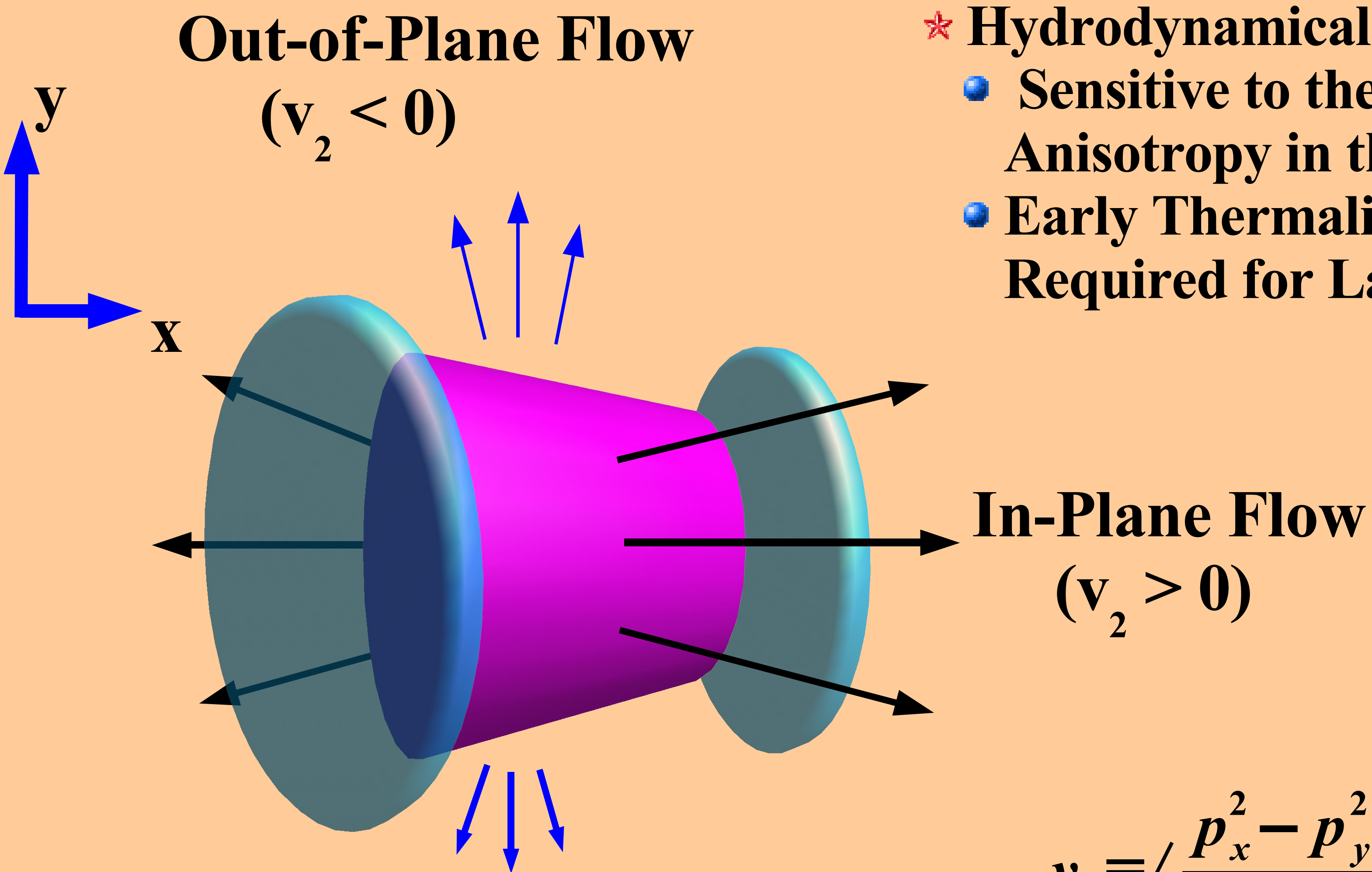
Pressure History

$$\epsilon \frac{D\mathbf{V}}{Dt} = -\nabla P$$

$$\rightarrow V = \int_{path} \frac{-\nabla P dt}{\epsilon}$$



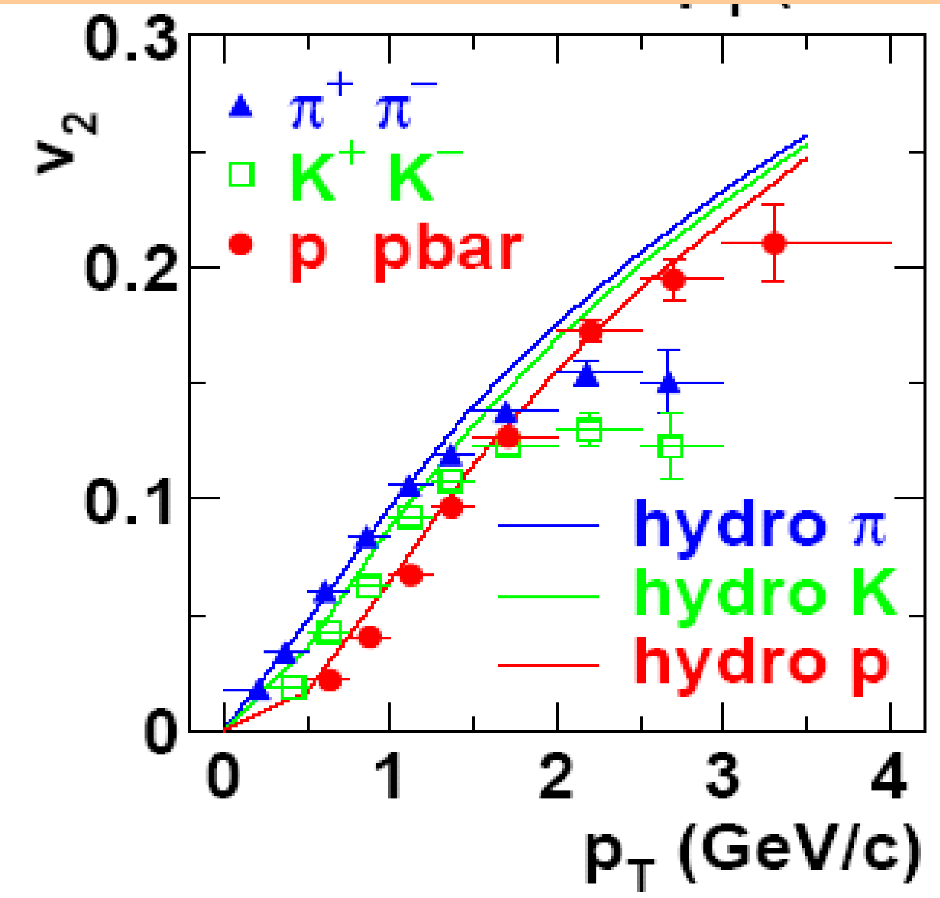
Elliptic Flow (I)

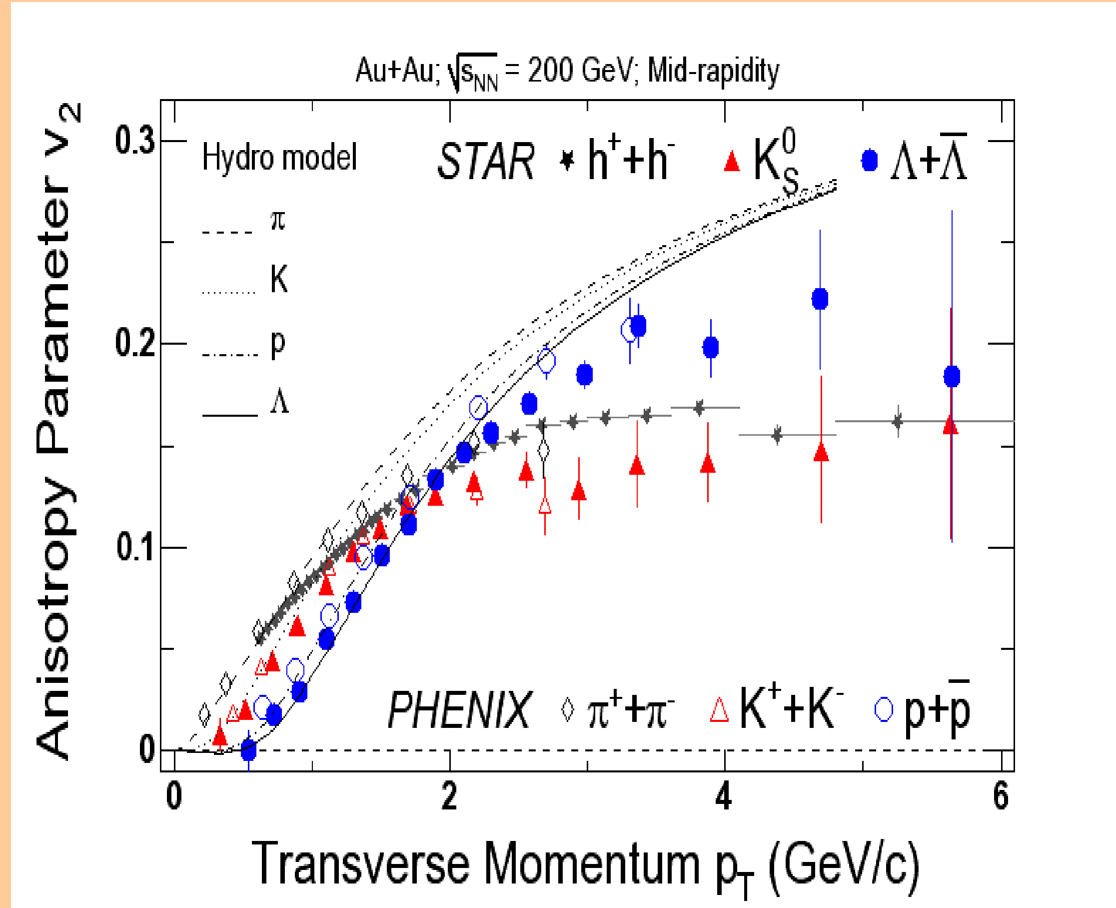


- What is Elliptic Flow ?
  - Anisotropy in P space
- Hydrodynamical Picture
  - Sensitive to the Pressure Anisotropy in the Early Stage
  - Early Thermalization is Required for Large V_2

$$v_2 \equiv \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle = \langle \cos 2\phi \rangle$$

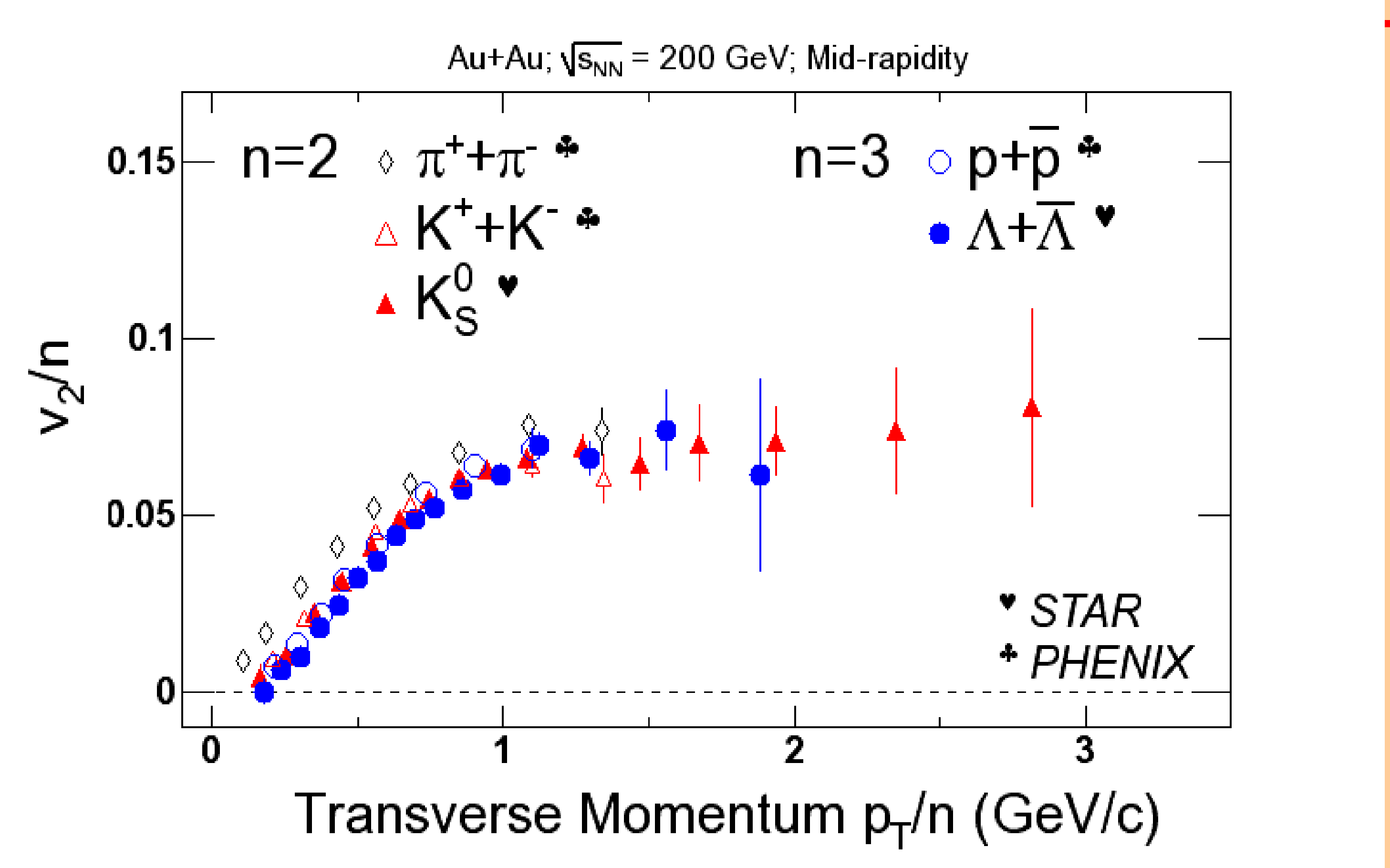
by Esumi, Matter03





Low Momentum : Hydrodynamical calc.
with Early Thermalization
High Momentum : Reduction from Hydro. calc.

by Esumi, Matter03



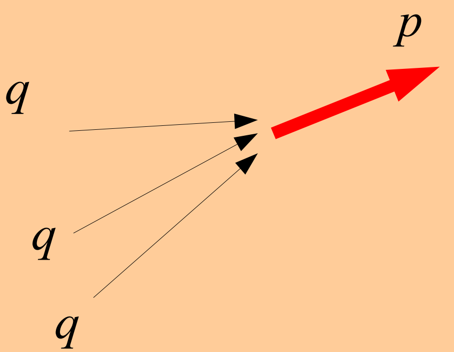
Coalescence (Recombination) Picture

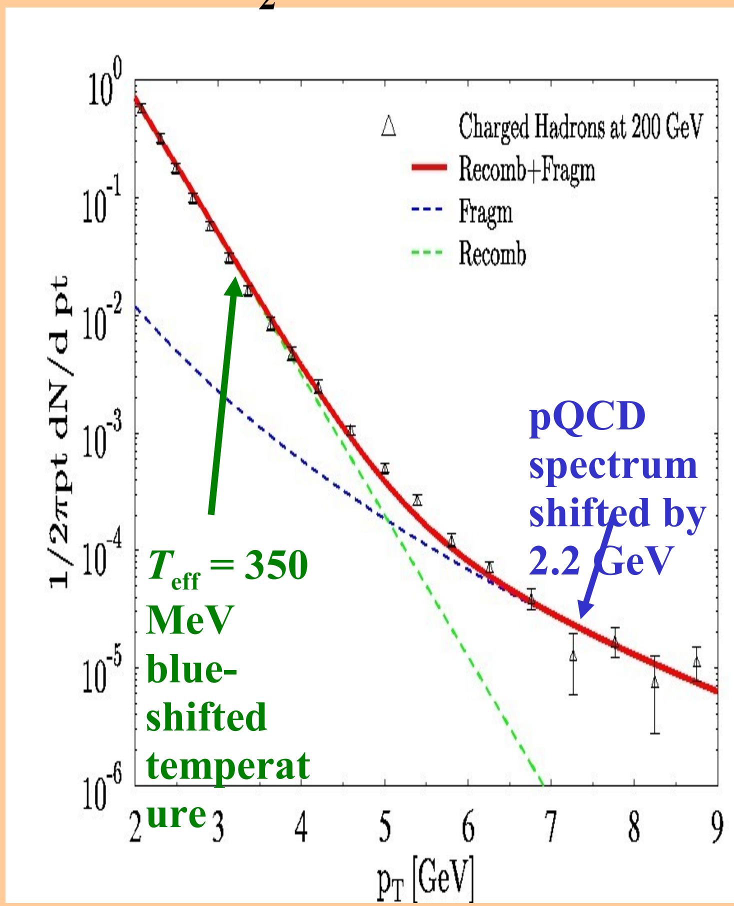
$$v_2^{Hadron}(P_T) = n\, v_2^{Parton}(P_T/n)$$

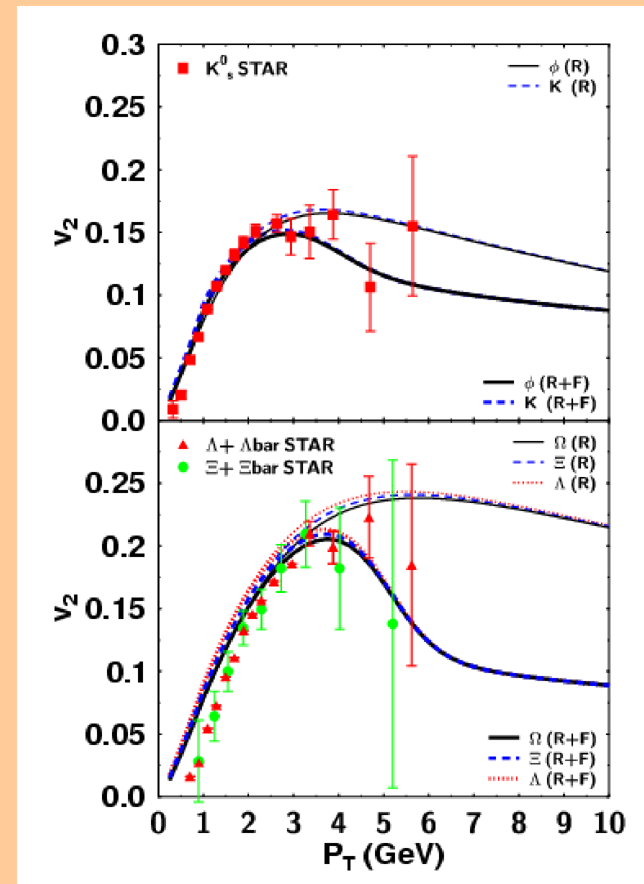
Recombination Picture seems to work well
... Parton Elliptic Flow

Frag. and Recombination (Duke U. Group)

Recombination Enhances Intermed. P_T Hadrons and Baryon V_2.







Fries et al. PRL 90 (2003), 202303, Nonaka et al., nucl-th/0308051

Jet-Fluid String Formation and Decay in High-Energy Heavy-Ion Collisions

Akira Ohnishi
in Collaboration with
T.Hirano,M.Isse,Y.Nara,K.Yoshino

- Introduction
- Jet-Fluid String (JFS) model
- Results
- Summary

Hadronization Mechanism at RHIC

- ***High p_T: Indep. Frag. of Jet Partons*** ***(E.g. Hirano-Nara)***

  O Explains pT spectrum when E-loss is included.

  X Elliptic Flow v_2 is small at high p_T *←This Talk*

- ***Medium p_T: Recombination*** ***(E.g. Duke-Osaka-Nagoya)***

  O Explains Baryon Puzzle and Quark Number Scaling of v_2

  X Entropy decreases in "$n \to 1$" process

- ***Low p_T: Equil. Fluid Hadronization*** ***(E.g. Hirano-Gyulassy)***

  O Explains p_T spec. and v_2 at low p_T

  X Results depends on the Freeze-Out Conditions

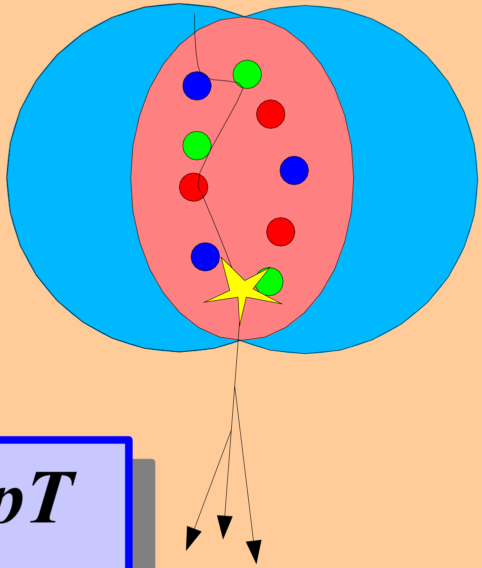
QGP Signals are understood separately,
and they are not necessarily consistent.
→ Further Ideas are required !

How can we get large v_2 at high p_T ?

- **Quark Recombination → Combined Objects have larger v2**

$$f(p, \varphi)=(1 + 2\, v_2(p/2) \cos\varphi) \times (1 + 2\, v_2(p/2) \cos\varphi)$$
$$\approx 1 + 2 \times 2\, v_2(p/2)\ \cos\varphi$$

- **Energy Loss in QGP generates v2**
  - **Large/Small suppression in y/x directions**



Plausible Hadronization giving large v2 at high pT
- ***Combination of several partons***
- ***Large Energy Loss***
  → Jet parton picks up Fluid parton and forms a string (Jet-Fluid String)

Jet-Fluid String Formation and Decay

Jet production: **pQCD(LO) × K-factor (PYTHIA6.3, K=1.8, *pp* fit)**

$$\sigma_{jet} = K\,\sigma_{jet}^{pQCD(LO)}$$

Jet propagation in QGP

3D Hydro + Simplified GLV 1st order formula × *C*

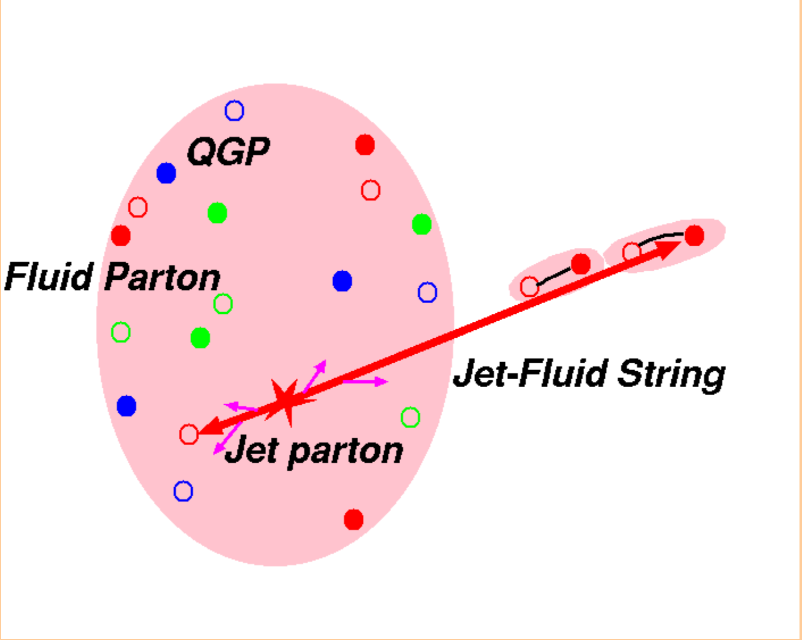
(Hirano-Nara, NPA743('04)305, Hirano-Tsuda, PRC 66('02)054905. Web version!
Gylassy-Levai-Vitev, PRL85('00)5535)

$$\frac{dE}{d\tau} = 3\pi\alpha_s^3 F_{color}\, C\,(\tau-\tau_0)\log\left(\frac{2E_0}{\mu^2 L}\right)$$

Jet-Fluid String formation

Fluid parton breaks color flux, according to string spectral func.

$$P(\sqrt{s}) \propto \Theta(\sqrt{s}-\sqrt{s_0}) \quad (\sqrt{s_0}=2\,GeV)$$



Only g and light q (qbar) are considered.

相対論的流体模型

$\partial_\mu T^{\mu\nu} = 0$　エネルギー運動量保存
$\partial_\mu n_i u^\mu = 0$　カレントの保存 (baryon, strangeness,…)

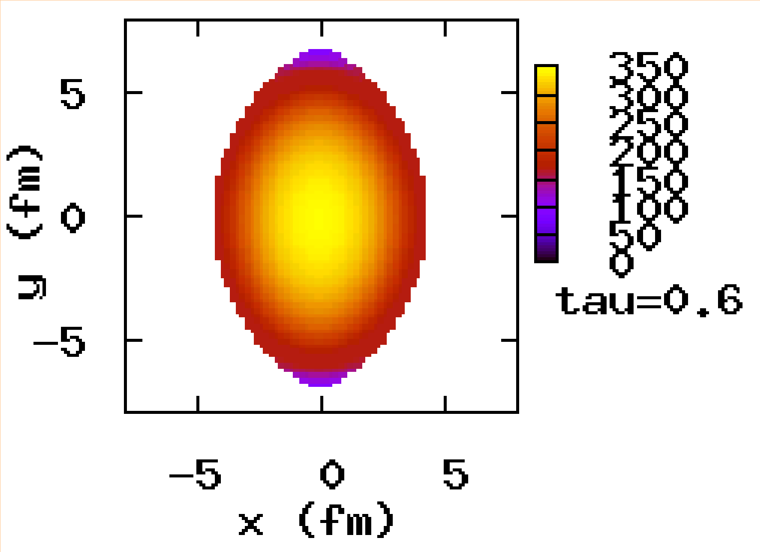
e : エネルギー密度
P : 圧力
u^{μ} :4 元速度　▪(1,v)
n_i : 密度

$$T^{\mu\nu} = (e+P) u^\mu u^\nu - P g^{\mu\nu}$$

τ_0, T^{ch} :Au+Au の dN/dη を fit, T^{th}: 可変

5 本の独立な方程式
6 個の独立変数 e,P,n_i,v
↓
状態方程式 P(e,n_i) を仮定
↓
初期条件を与え、Bjorken 座標 (τ,η_s,x,y) で解く。

$$\tau = \sqrt{t^2 - z^2} \quad , \quad \eta_s = \frac{1}{2} \log \frac{t+z}{t-z}$$

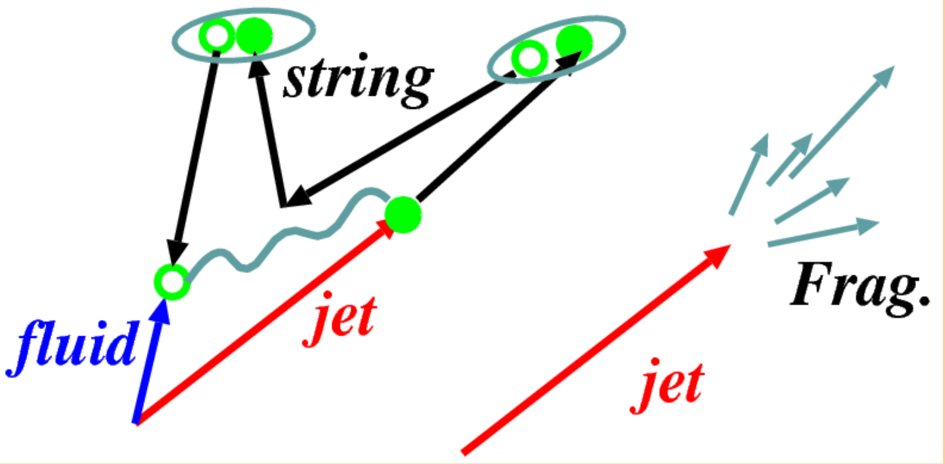


T. Hirano, Y. Nara, Nucl. Phys. A743, 305 (2004)
T. Hirano, K. Tsuda, Phys. Rev. C 66, 054905(2002)

Discussion

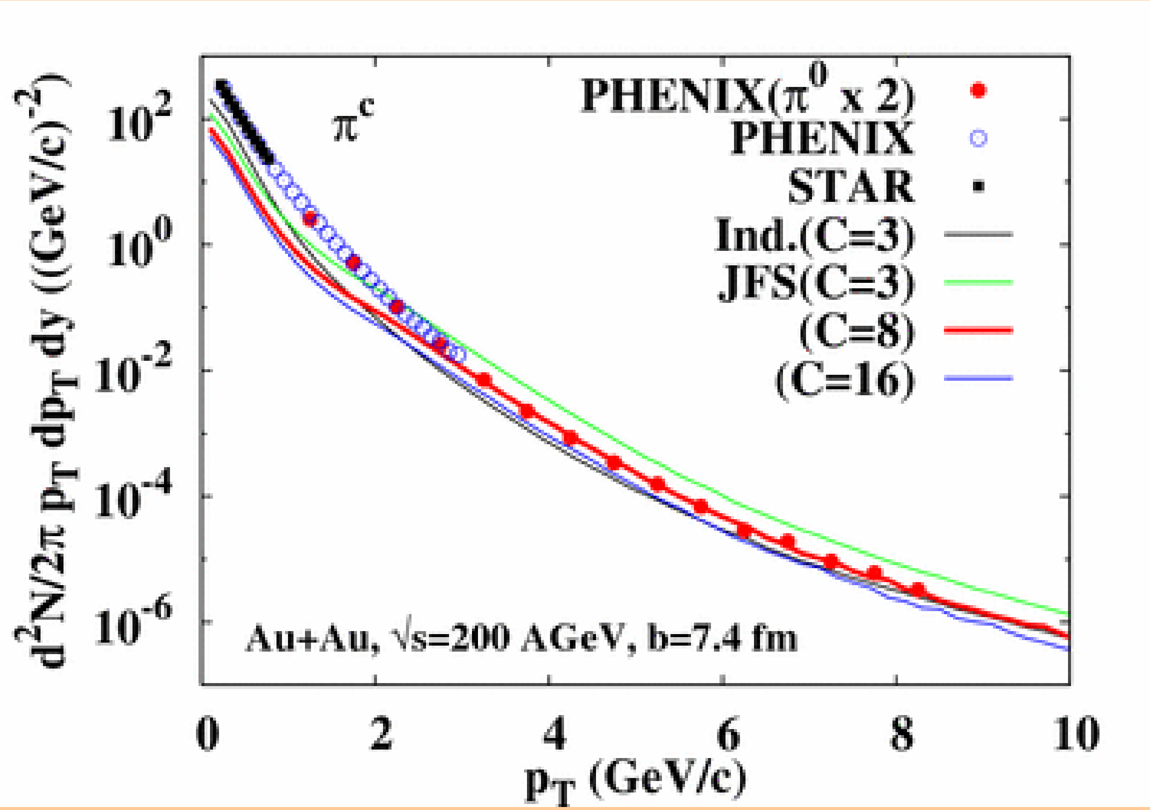
- Mechanism to produce high p_T hadrons in JFS
  - String Decay from Lorenz boosted fluid
  - Relative momentum is relatively small
    → Smaller number of hadrons with high p_T are formed

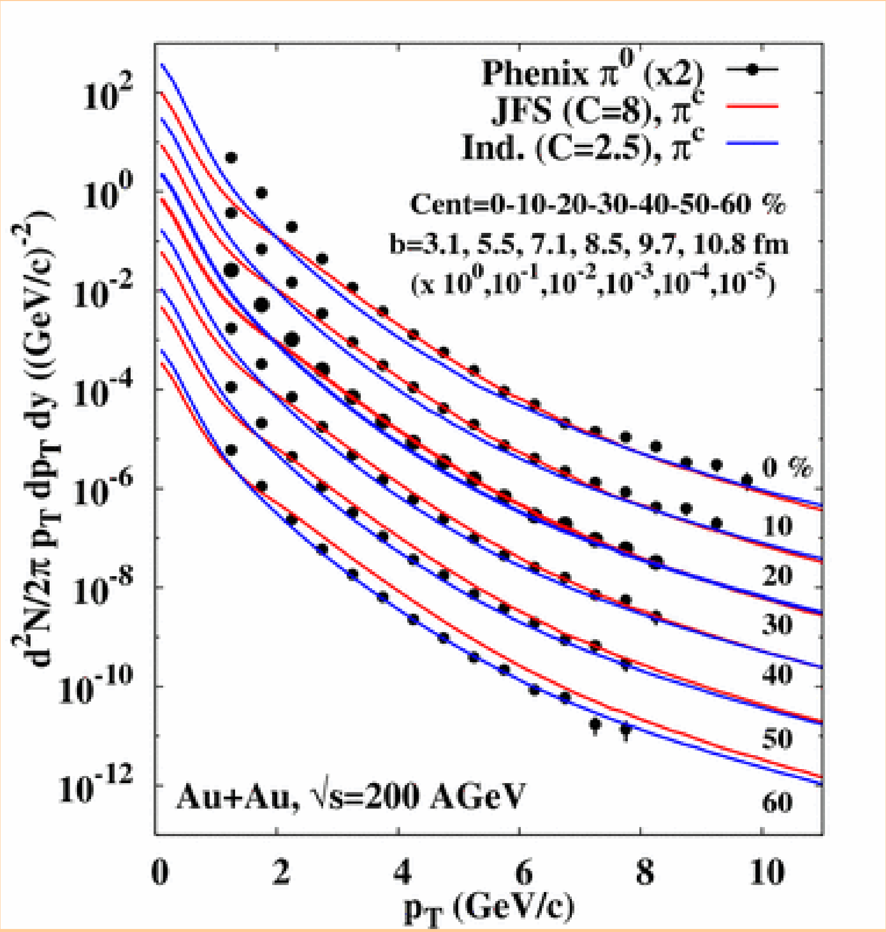
↔ Independent Frag. (Large no. of Low p_T hadrons)



Energy Loss Factor C : p_T Spectrum Fit

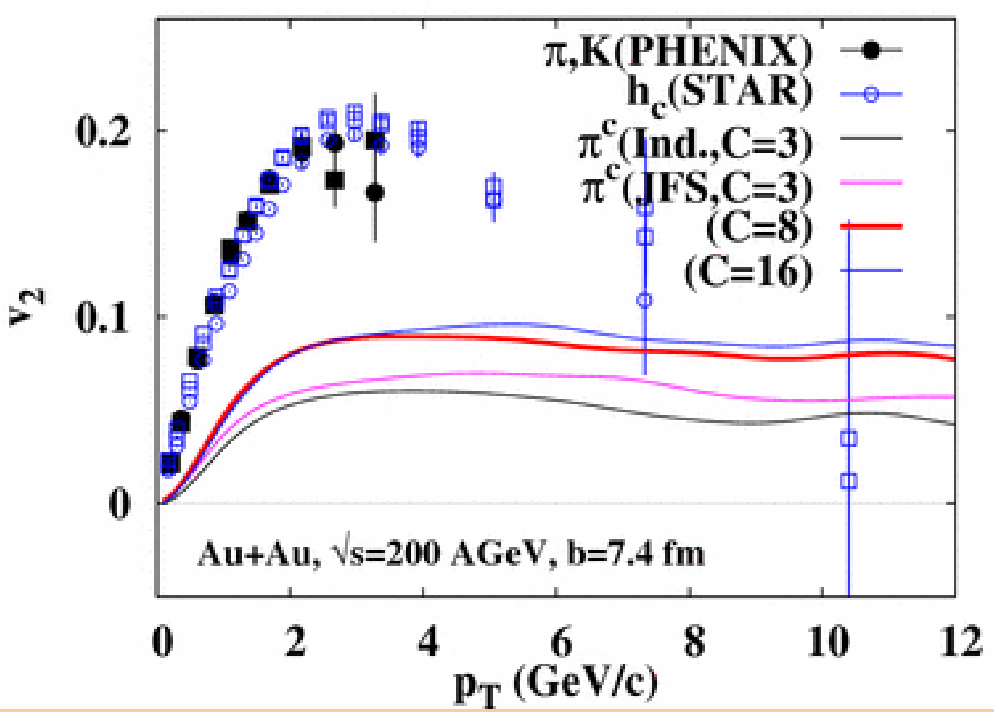
- For the same C → $dN_{JFS}(high\ p_T) > dN_{Ind}(high\ p_T)$
- p_T spec. fit → Ind. Frag.: $C \approx (2.5\text{-}3)$, JFS: $C \approx 8$
  → *Large Energy Loss is necessary / allowed in JFS*

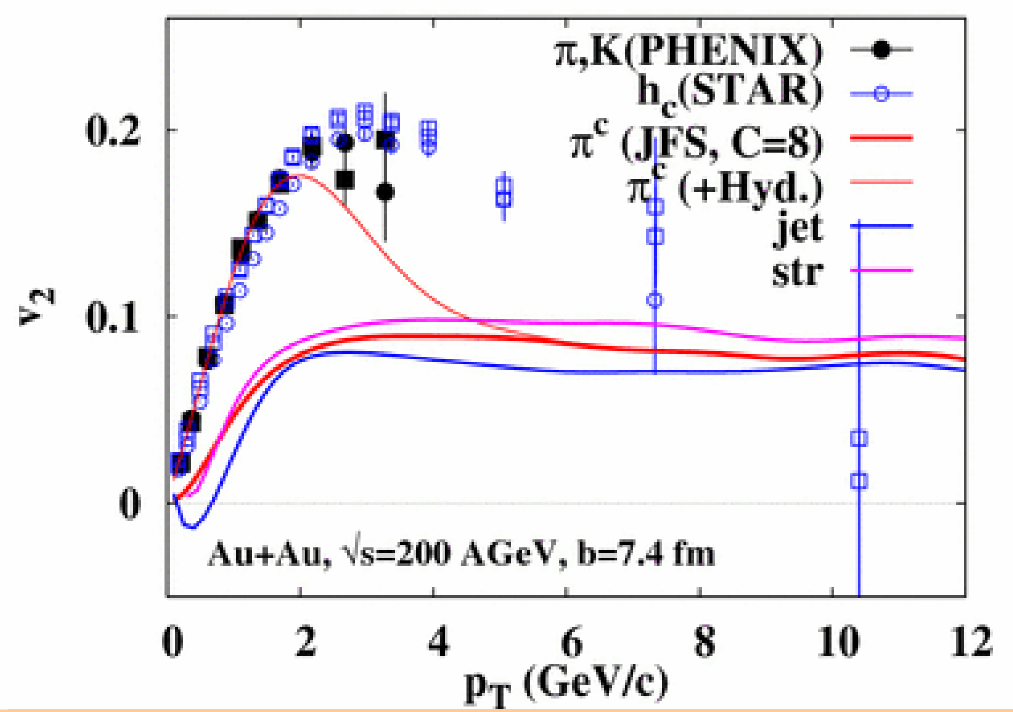




Elliptic Flow: p_T Deps.

- High pT v_2 : ~ 5 % in Ind. (*C = 3*) ↔ ~ 8 % in JFS (*C = 8*)

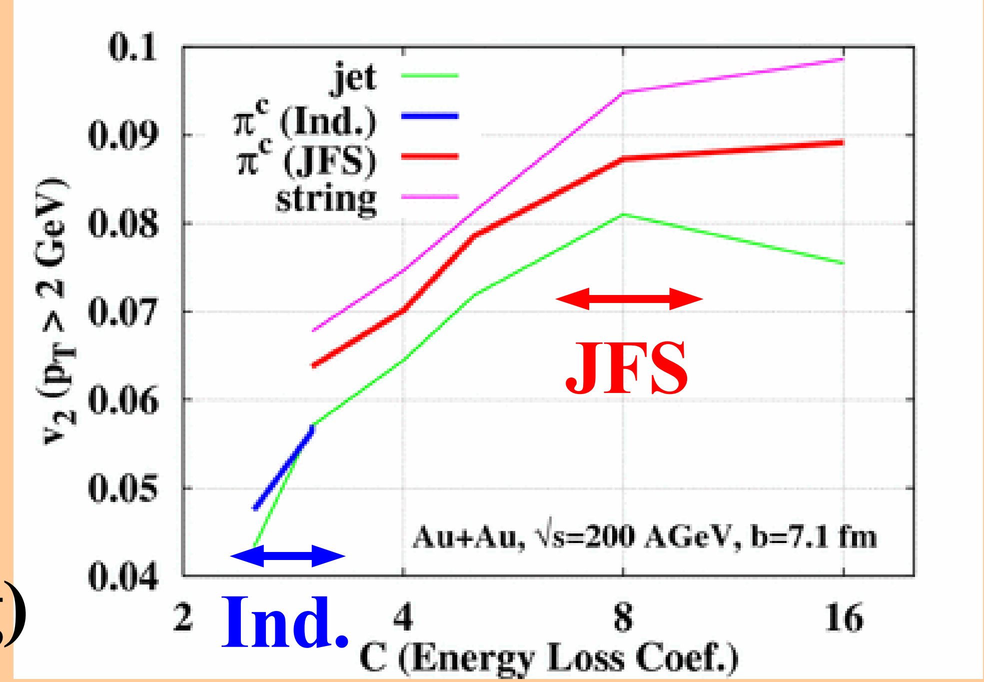




Origin of Large v_2 = Large E-loss factor C + Fluid parton v_2

Elliptic Flow: Parameter Deps.

- v_2(jet): saturating behavior
  (large E-loss limit) ~ 8 %
- v_2 (string): grows up to ~ 10 %
  larger than v_2(jet, limit)
- v_2 (h): string decay reduces v_2
  → v_2(jet) < v_2 (h) < v_2 (string)

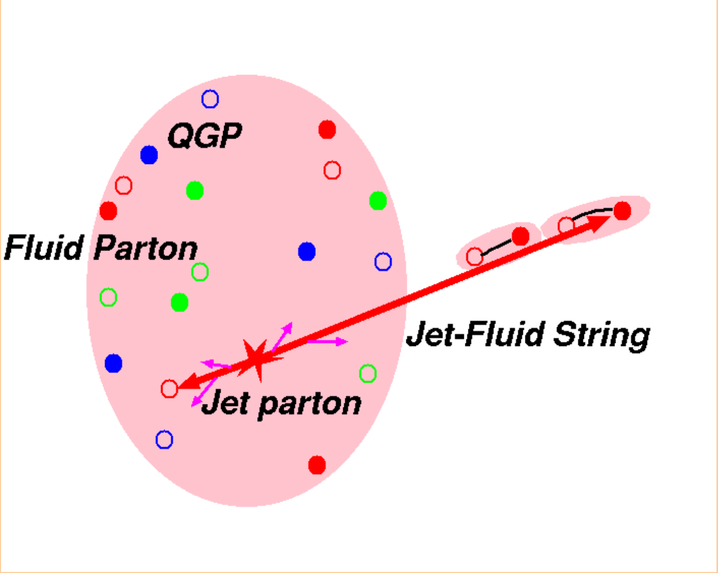


For p_T>2GeV ($p_T \approx$ 10 GeV)

Ind. Frag. with C = 2.5	→ v_2	≈ 5 % (4 %)
Large E-loss factor C	→	+ 3 %
Fluid parton v_2	→	+ 1 %
JFS with C = 8	→ v_2	≈ 9 % (8 %)

Summary

- ***Jet-Fluid String (JFS) formation and decay*** **is proposed as a mechanism to produce high p_T hadrons.**
  - **Effecitve to produce high p_T hadrons**
  - **Event-by-Event Energy-Mom. conservation ↔ Ind. Frag.**
  - **Entropy does not decreases, but increases. ↔ Reco.**
- **When we FIT p_T spectrum,** ***large v_2 emerges at high p_T.***
  - **Large E-loss+fluid parton v_2**
- **Problems and Homeworks**
  - **Mechanism of large E-loss**
  - **d+Au fit → Cronin Effects**
  - **s-quarks, string spectral func.**



Comparison with Previous Works

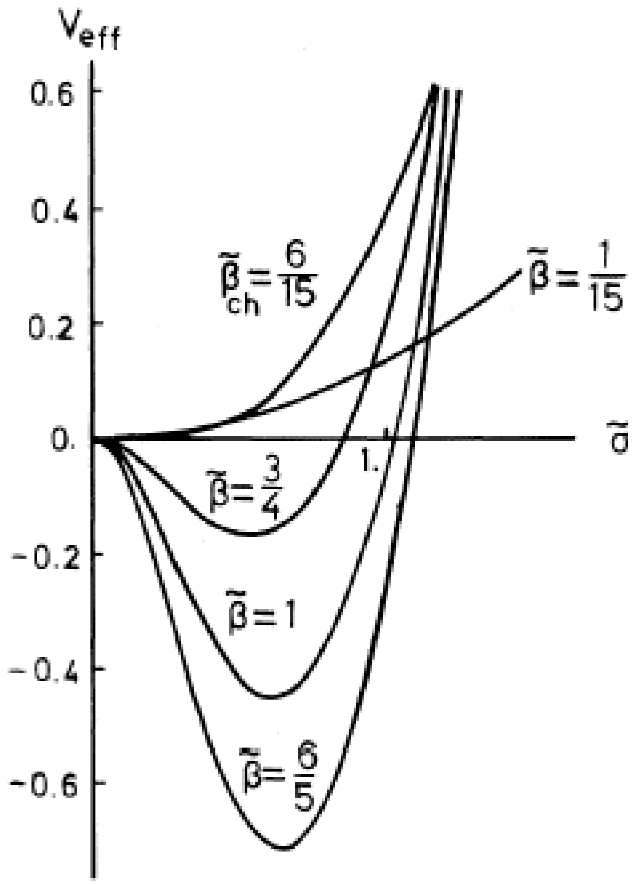
- **J. Casalderrey-Solana, E.V. Shuryak, hep-ph/0305160**
  - **Quarks, diquarks and gluons in QGP cut color flux (~ JFS).**
  - **Large E-loss is generated by "phaleron"**
  - ***Large E-loss leads "surface emission" → large v_2***
- **Recombination (Duke-Osaka-(Minesota)-Nagoya)**
  - **Predicts large v_2 (~ 10 %) at high-pT**
    - **Sharply edged density dist. → E-loss ∝ L → $v_2 \approx 10$ %**
    - **Woods-Saxon density dist. → $v_2 \approx 5$ %**
  - **Entropy problem: S(QGP) ≈ S(H) requires Res. and Strings**
  - ***Spectral Func.: δ func. ↔ θ func. in JFS***

Strong Coupling Limit of Lattice QCD for Color SU(3) with Baryon Effects

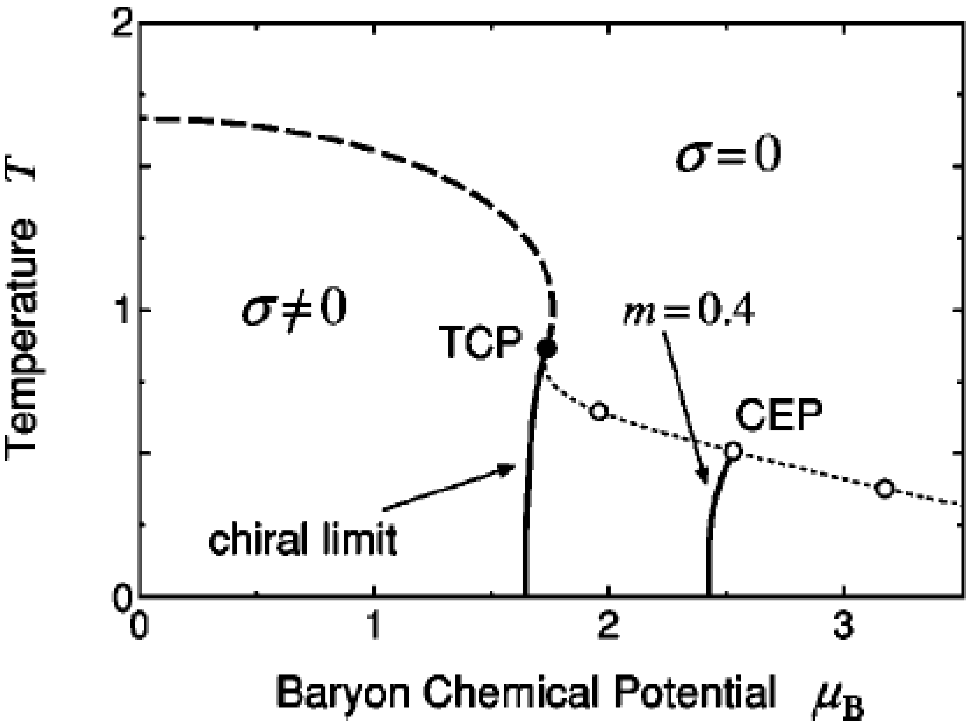
N. Kawamoto, K. Miura, AO, T. Ohnuma, hep-lat/051223

Strong Coupling Limit of Lattice QCD

- **Chiral Restoration at μ=0.**
  - **Damgaard, Kawamoto, Shigemoto, PRL53(1984),2211**



- **Phase Diagram with Nc=3**
  - **Nishida, PRD69, 094501 (2004)**



Previous Works in Strong Coupling Limit LQCD

- **Strong Coupling Limit Lattice QCD re-attracts interests**
  c.f. Nakamura @ JHF Symp. for high density matter

Ref	T	μ	Nc	Baryon	CSC	Nf
Damgaard-Kawamoto-Shigemoto('84)	*Finite*	*0*	*U(Nc)*	*X*	*X*	*1*
Damgaard-Hochberg-Kawamoto('85)	*0*	*Finite*	*3*	*Yes*	*X*	*1*
Bilic-Karsch-Redlich('92)	*Finite*	*Finite*	*3*	*X*	*X*	*1 ~ 3*
Azcoiti-Di Carlo-Galante-Laliena('03)	*0*	*Finite*	*3*	*Yes*	*Yes*	*1*
Nishida-Fukushima-Hatsuda('04)	*Finite*	*Finite*	*2*	*Yes (*)*	*Yes (*)*	*1*
Nishida('04)	*Finite*	*Finite*	*3*	*X*	*X*	*1~2*
Kawamoto-Miura-AO-Ohnuma('05)	***Finite***	***Finite***	***3***	***Yes***	***Yes (+)***	***1***

*: bosonic baryon=diquark in SU(2)
+: analytically included, but ignored in numerical calc.

- ***Baryonic Composite will be important at High Densities, but they have been ignored in finite T treatments !***
  → This work: Nc = 3, Baryonic Composite, Finite T and μ

Strong Coupling Limit without Baryonic Effects

Strong Coupling

- **Lattice Action (staggered fermion)**

$$Z = \int \mathcal{D}[\chi, \bar{\chi}, U_0, U_1, U_2, U_3] \exp\left[-S_F^{(U_0)} - \sum_{j=1}^{3} S_F^{(U_j)} - S_F^{(m)} - S_G\right]$$

- **Spatial Link Integral**

$$\simeq \int \mathcal{D}[\chi, \bar{\chi}, U_0] \exp\left[\frac{1}{2}(M, V_M M) + (\bar{B}, V_B B) - S_F^{(U_0)} - S_F^{(m)}\right]$$

1/d Expansion (1/√d)

- **Bosonization (HS transf.)**

$$\simeq \int \mathcal{D}[\chi, \bar{\chi}, U_0, \sigma] \exp\left[-\frac{1}{2}(\sigma, V_M^{-1}\sigma) \underbrace{-(\sigma, M) - S_F^{(U_0)} - S_F^{(m)}}_{(\bar{\chi}, G^{-1}(\sigma)\chi)}\right]$$

- **Quark and U_0 Integral**

$$\simeq \int \mathcal{D}[\sigma] \exp\left[-\frac{1}{2}a_\sigma \sigma^2\right] \underbrace{\prod_{\mathbf{x}} \int dU_0 \ \mathrm{Det}\ \left[G^{-1}(\sigma)\right]}_{\exp\left[-L^3 \beta F^q(\sigma)\right]}$$

$$\simeq \exp\left[-L^3 \beta F_{\mathrm{eff}}(\sigma)\right]$$

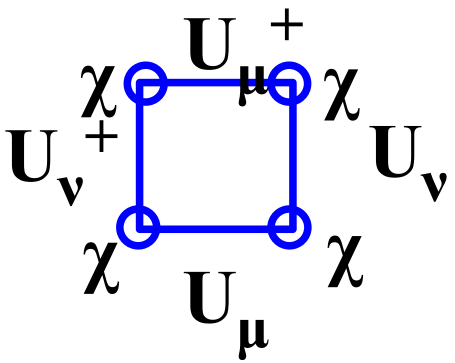
- **Mesonic and Baryonic Composites**

$$M(x) = \delta_{ab} \bar{\chi}^a(x) \chi^b(x) \ ,$$

$$B(x) = \frac{1}{6}\varepsilon_{abc}\chi^a(x)\chi^b(x)\chi^c(x) \ , \quad \bar{B}(x) = \frac{1}{N_c!}\varepsilon_{abc}\bar{\chi}^c(x)\bar{\chi}^b(x)\bar{\chi}^a(x)$$

Lattice Action in SCL-LQCD

- Lattice Action with staggered Fermions



$$S[U,\chi,\bar{\chi}] = S_G[U] + S_F[U,\chi,\bar{\chi}] \ ,$$

$$S_G[U] = \frac{2N_c}{g^2}\sum_{x,\mu,\nu}\left\{1 - \frac{1}{N_c}\mathrm{ReTr}U_{\mu\nu}(x)\right\}$$

$g \to \infty$ → 0

$$U_{\mu\nu}(x) = U_\nu^\dagger(x)U_\mu^\dagger(x+\hat{\nu})U_\nu(x+\hat{\mu})U_\mu(x) \ ,$$

Chem. Pot.

$$\begin{aligned}
S_F[U,\chi,\bar{\chi}] &= S_F^{(m)}[\chi,\bar{\chi}] + S_F^{(j)}[U_j,\chi,\bar{\chi}] + S_F^{(U0)}[U_0,\chi,\bar{\chi}] \ , \\
S_F^{(m)}[\chi,\bar{\chi}] &= m\sum_x \bar{\chi}^a(x)\chi^a(x) \ , \\
S_F^{(j)}[U_j,\chi,\bar{\chi}] &= \frac{1}{2}\sum_x\sum_{j=1}^{d}\eta_j(x)\left\{\bar{\chi}(x)U_j(x)\chi(x+\hat{j}) - \bar{\chi}(x+\hat{j})U_j^\dagger(x)\chi(x)\right\} \ , \\
S_F^{(U0)}[U_0,\chi,\bar{\chi}] &= \frac{1}{2}\sum_x \eta_0(x)\left\{\bar{\chi}(x)e^{\mu}U_0(x)\chi(x+\hat{0}) - \bar{\chi}(x+\hat{0})U_0^\dagger(x)e^{-\mu}\chi(x)\right\} \ .
\end{aligned}$$

- In the Strong Coupling Limit ($g \to \infty$), we can ignore SG, and semi-analytic calculation becomes possible.

details

- **Lattice QCD action**

$$S_F^{(U_j)} = \frac{1}{2}\sum_x \eta_j(x)\left[\bar\chi(x)U_\mu(x)\chi(x+\hat\mu) - \bar\chi(x+\hat\mu)U_\mu^\dagger(x)\chi(x)\right]$$

$$S_F^{(U_0)} = \frac{1}{2}\sum_x \left[\bar\chi(x)e^{\mu}U_\mu(x)\chi(x+\hat\mu) - \bar\chi(x+\hat\mu)e^{-\mu}U_\mu^\dagger(x)\chi(x)\right]$$

$$S_F^{(m)} = m_0\sum_x \bar\chi^a(x)\chi^a(x) \ ,$$

- **Mesonic and Baryonic Composites**

$$M(x) = \delta_{ab}\bar\chi^a(x)\chi^b(x) \ ,$$

$$B(x) = \frac{1}{6}\varepsilon_{abc}\chi^a(x)\chi^b(x)\chi^c(x) \ , \quad \bar B(x) = \frac{1}{N_c!}\varepsilon_{abc}\bar\chi^c(x)\bar\chi^b(x)\bar\chi^a(x)$$

- **Fermion Integral**

$$\int \mathcal{D}[U_0,\chi,\bar\chi]\exp\left[-\sum_t \sigma M - S_F^{(U_0)}\right] = \int \mathcal{D}[U_0,\chi,\bar\chi]\prod_k \exp\left[-\bar\chi_k G(k)\chi_k/2\right]$$

$$= \cdots = C_\sigma^3 - \frac{1}{2}C_\sigma + \frac{1}{4}\cosh(3\beta\mu)$$

$$F_{\rm eff}^{(q)}(\sigma_q) = -T\log\left[\frac{4}{3}\left(C_\sigma^3 - \frac{1}{2}C_\sigma + \frac{1}{4}C_{3\mu}\right)\right] \qquad C_\sigma = \cosh\left[\beta \operatorname{arcsinh}\tilde\sigma\right]$$

Decomposition of Baryonic Composite Action

- **Introducing Auxiliary Baryon Field**

$$\exp(\bar{B}, V_B B) = \det V_B \int \mathcal{D}[\bar{b}, b] \exp\left[-(\bar{b}, V_B^{-1} b) + (\bar{b}, B) + (\bar{B}, b)\right]$$

- **Decomposition of coupling of baryon and 3 quarks with Diquark Composite (Azcoiti et al., JHEP 0309, 014 (2003))**

$$\bar{b}B = \underbrace{\bar{b}\chi^a}_{\text{antibaryon-quark}} \times \underbrace{\chi^b\chi^c}_{\text{diquark}} \times \varepsilon_{abc}/6$$

$D^\dagger D$ makes $\bar{b}B$

$$D_a = \frac{\gamma}{2}\varepsilon_{abc}\chi^b\chi^c + \frac{1}{3\gamma}\bar{\chi}^a b\ , \quad D_a^\dagger = \frac{\gamma}{2}\varepsilon_{abc}\bar{\chi}^c\bar{\chi}^b + \frac{1}{3\gamma}\bar{b}\chi^a$$

$$\exp(\bar{b}B + \bar{B}b) = \int d[\phi_a, \phi_a^\dagger] \exp\left[-\phi_a^\dagger\phi_a + (\phi_a^\dagger D_a + D_a^\dagger \phi_a) \underbrace{-\frac{\gamma^2}{2}M^2 + M\bar{b}b/9\gamma^2}_{\bar{B}b + \bar{b}B - D_a^\dagger D_a}\right]$$

Effective Action is not yet bilinear in fermions

- ***four fermi interaction terms, M^2 and $M\bar{b}b$***
- ***diquark-quark-antibaryon coupling***

Bosonization of Four Fermi Interactions

- $M\bar{b}b$ **term → Baryon potential auxiliary field ω**

$$\exp(M\bar{b}b/9\gamma^2) = \int d[\omega] \exp\left[-\omega^2/2 - \omega(\alpha M + g_\omega \bar{b}b) - \alpha^2 M^2/2\right]$$

  - $(\bar{b}b)^2 = 0$ **in One species of Staggered Fermion**

- M^2 **and** $(M, V_M M)$ **terms → Chiral Condensate σ**

$$\frac{1}{2}(M, V_M M) - \frac{1}{2}(\gamma^2 + \alpha^2)M^2 = \frac{1}{2}(M, \widetilde{V}_M M)$$

$$\exp\left[\frac{1}{2}(M, \widetilde{V}_M M)\right] = \int \mathcal{D}[\sigma] \exp\left[-\frac{1}{2}(\sigma, \widetilde{V}_M^{-1}\sigma) - (\sigma, M)\right]$$

  - **By absorbing "Mass" in the Hopping Term,**
    We can replace both of the terms simultaneously !

Effective Action in bilinear form of Fermions !

Effective Free Energy at Zero Diquark Condensate

- **Effective Action**

Zero Diquark Condensate

$$S_F = (\bar{b}, \widetilde{V}_B^{-1} b) + \frac{1}{2}(\omega, \omega) + \frac{1}{2}(\sigma, \widetilde{V}_M^{-1}\sigma) + (\sigma_q, M) + S_F^{(U_0)} + S_F^{(m)}$$

$$+ (\phi^\dagger, \phi) + \frac{1}{3\gamma}\left[(\bar{\chi}^a, \phi_a^\dagger b) + (\bar{b}\phi_a, \chi^a)\right] + \frac{\gamma}{2}\varepsilon_{cab}\left[(\phi_c^\dagger, \chi^a\chi^b) + (\bar{\chi}^b\bar{\chi}^a, \phi_c)\right]$$

- **After Quark, U_0, Baryon Integral at zero diquark cond.**

$$\mathcal{F}_{\rm eff} = \frac{1}{2}a_\sigma \sigma^2 + \frac{1}{2}\omega^2 + F_{\rm eff}^{(b)}(g_\omega \omega) + F_{\rm eff}^{(q)}(\sigma_q) \quad a_\sigma = \left[\frac{d}{2N_c} - (\gamma^2 + \alpha^2)\right]^{-1}$$

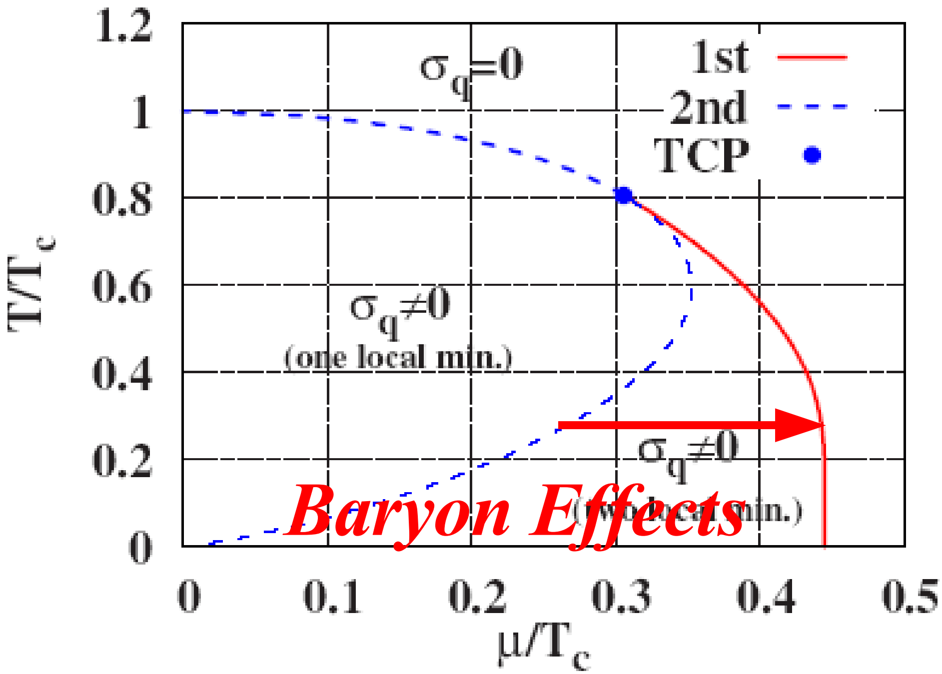
and adopting convenient parameters
(γ and ω are removed),
we get an analytical expression of Effective Free Energy

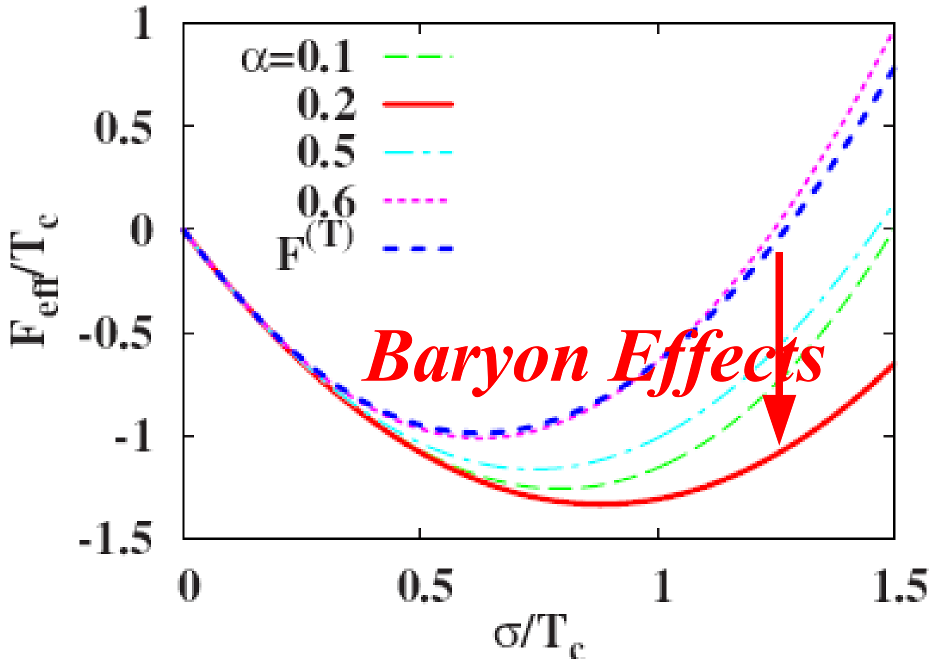
$$\mathcal{F}_{\rm eff}(\sigma_q) = \frac{\sigma_q^2}{2\alpha^2} + F_{\rm eff}^{(b)}(g_\sigma \sigma_q) + F_{\rm eff}^{(q)}(\sigma_q; T, \mu)$$

Effective Free Energy with Baryonic Effects

- **Effective Free Energy**

$$\mathcal{F}_{\rm eff}(\sigma_q) = \frac{\sigma_q^2}{2\alpha^2} + F_{\rm eff}^{(b)}(g_\sigma \sigma_q) + F_{\rm eff}^{(q)}(\sigma_q; T, \mu)$$

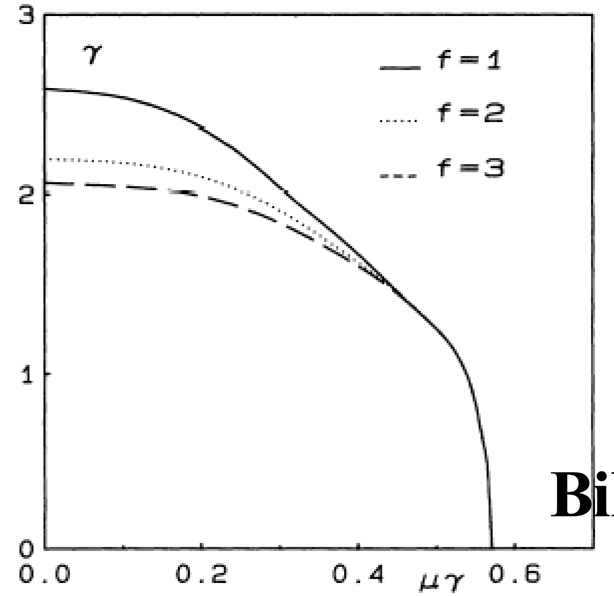




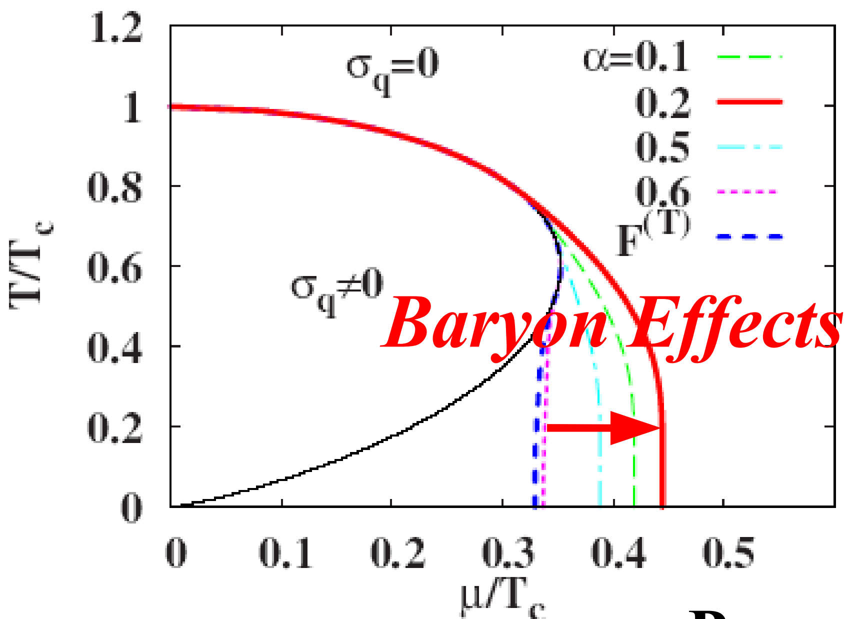
Baryons Gain Free Energy
→ Extention of Hadron Phase to Larger μ !

Small Critical μ : Common in SCL-LQCD ?

- **Strong Coupling Limit**
  - **Damgaard,Hochberg,Kawamoto ('85):**
    $\mu_B^c(0)/T_c(0) \sim 1.6$ **(T=0, T≠0)**
  - **T≠0, No B:** $\mu_B^c(0)/T_c(0) \sim 1.0$ **(Nishida2004, Bilic et al 1992(Bielefeld), ...)**
  - **Present:** $\mu_B^c(0)/T_c(0) < 1.5$ **(Parameter dep.)**
- **Monte-Carlo:** $\mu_B^c(0)/T_c(0) >> 1$
  - **Fodor-Katz, Bielefeld, de Forcrand-Philipsen, ...**
- **Real World:** $\mu_B^c(0)/T_c(0) > 7$



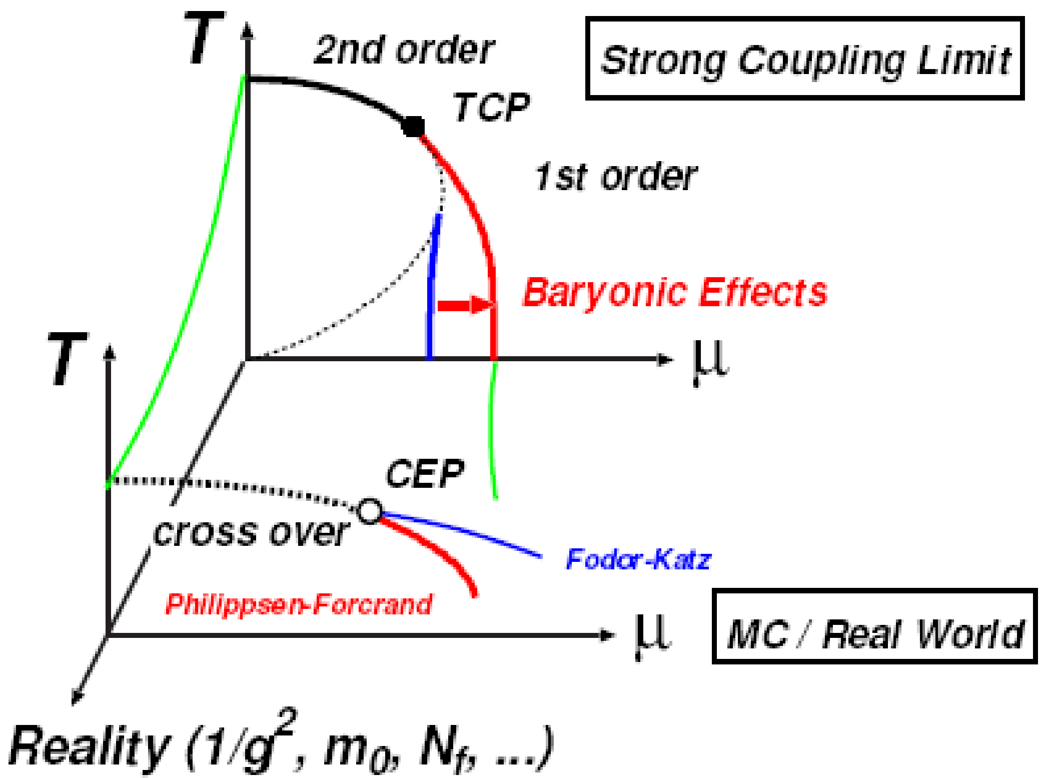
Bilic et al.



Present

Towards Realistic Understanding

- "Reality" Axis
  - Strong Coupling Limit → $1/g^2$ corrections → Smaller T_c
  - Number of Flavors → 2(ud)+1(s) → Smaller T_c
  - Chiral Limit → Finite m_q → Larger μ_c



Multi-Flavors or Strangeness may play an important role to Extend Hadron Phase to Larger μ

Color Angle Average

- **Problem: Diquark Condensates induce quark-baryon coupling, and Baryon integral becomes difficult.**
  → Solution: Color Angle Average
  - **Integral of "Color Angle Variables"**

$$\int \mathcal{D}[\phi_a, \phi_a^\dagger] \exp\left\{\phi_a^\dagger D_a + D_a^\dagger \phi_a\right\} = \int \mathcal{D}[v] \exp\left\{\frac{v^2}{3} D_a^\dagger D_a + \frac{v^4}{162} M^3 \bar{b} b\right\}$$

  - **Three-Quark and Baryon Coupling is ReBorn !**

$$D_a^\dagger D_a = Y + \bar{b}B + \bar{B}b\ , \quad Y = \frac{\gamma^2}{2} M^2 - \frac{1}{9\gamma^2} M\bar{b}b$$

  - **Solve "Self-Consistent" Equaton**

$$\exp(\bar{b}B + \bar{B}b) \simeq \exp\left[-v^2 - Y + \frac{v^2}{3}(\bar{b}B + \bar{B}b + Y) + \frac{v^4}{162} M^3 \bar{b}b\right]$$

$$\simeq \exp\left[-\frac{v^2}{R_v} + \frac{v^4 M^3 \bar{b}b}{162 R_v} - Y\right] \quad (R_v = 1 - v^2/3)$$

Effective Free Energy with Diquark Condensate

- Bosonization of $M^k \bar{b} b$ → Introduce k bosons

$$\exp M^k \bar{b} b = \int d\omega_k \exp[-\frac{1}{2}(\omega_k + \alpha_k M + 1/\alpha_k M^{k-1} \bar{b} b)^2 + M^k \bar{b} b]$$

$$= \int d\omega_k \exp[-\omega_k^2/2 - \omega(\alpha_k M + 1/\alpha_k M^{k-1} \bar{b} b) - \alpha_k^2 M^2/2]$$

- Effective Free Energy

$$\mathcal{F}_{\rm eff}^{(Tbv)} = F_X(\sigma, v, \omega_i) + F_{\rm eff}^{(b)}(g_\omega \omega) + F_{\rm eff}^{(q)}(\sigma_q)$$

$$F_X = \frac{1}{2}(a_\sigma \sigma^2 + \omega^2 + \omega_1^2 + \omega_2^2) + \frac{v^2}{R_v} \qquad g_\omega = \frac{1}{9\alpha\gamma^2}\left[1 + \frac{\gamma^2 v^4 \omega_1 \omega_2}{18\alpha_1\alpha_2 R_v}\right]$$

$$a_\sigma = \left(\frac{1}{2} - \gamma^2 - \alpha^2 - \alpha_1^2 - \alpha_2^2\right)^{-1} \qquad \sigma_q = \sigma + \alpha\omega + \alpha_1\omega_1 + \alpha_2\omega_2$$

Similar form to the previous one at v=0.
Diquark Effects in interaction start from v^4.

Summary

- We have obtained an analytical expression of effective free energy *at finite T and finite μ* with *baryonic composite action* effects in the strong coupling limit of lattice QCD.
- In order to achive above, several techniques are developed.
  - Auxiliary *baryon potential ω* is introduced, using $(\bar{b}\,b)^2 = 0$
  - Mesonic propagator is modified *to absorb M^2* terms.
- Baryonic composite action is found to result in *Free Energy Gain* and *Extension of Hadron Phase to Larger μ.*
- Problem: Too small μ_c / T_c in the Strong Coupling Limit.
  → Strangeness may play decisive role.
- Application to Finite Nuclei and Nuclear Matter
  → Talk by K. Tsubakihara (SCL action in RMF)