Phase diagram at finite temperature and quark density in the strong coupling limit of lattice QCD for color SU(3)

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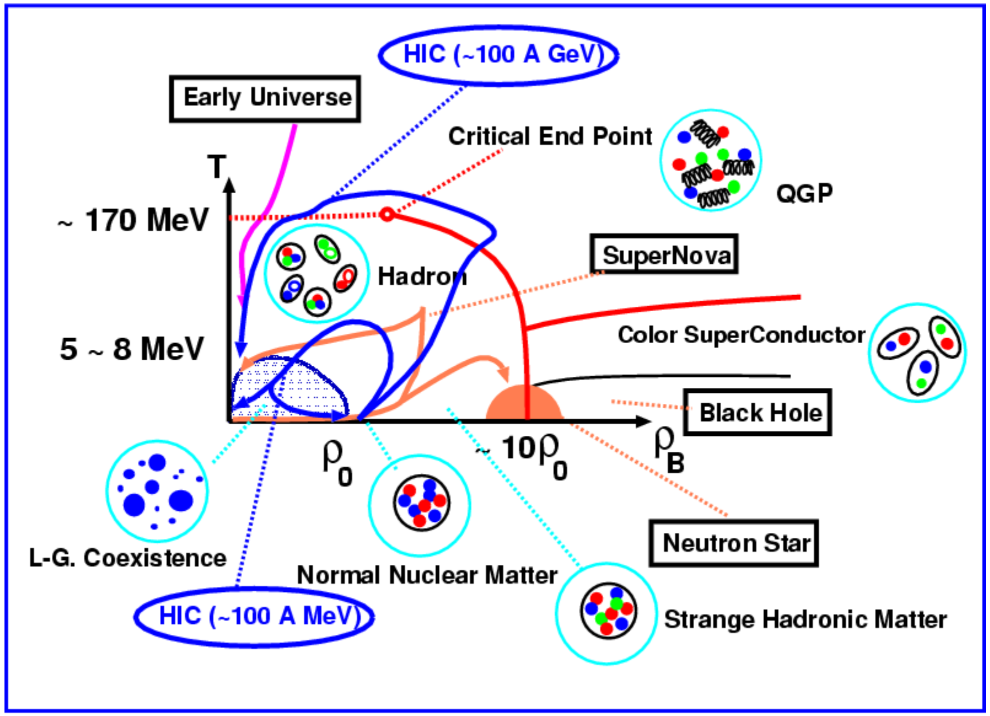
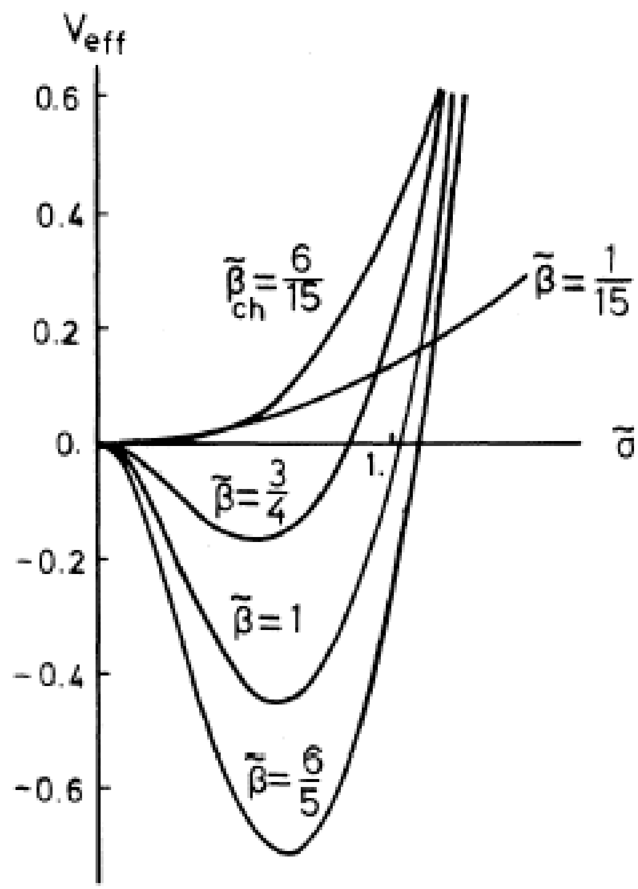
Quark and Hadronic Matter Phase Diagram

- **Dense quark & hadronic matter contains rich physics, but Lattice QCD simulation is not yet reliable.**
  → *Model/Approximate approaches are necessary !*
  - **Monte-Carlo calc. of Lattice QCD:**
    Improved ReWeighting Method (Fodor-Katz)
    Taylor Expansion in μ (Bielefeld U.)
    Analytic Continuation (de Forcrand-Philipssen)
  - **Model / Phen. Approaches:**
    NJL, QMC, RMF, ...
  - ***Strong Coupling Limit of Lattice QCD***

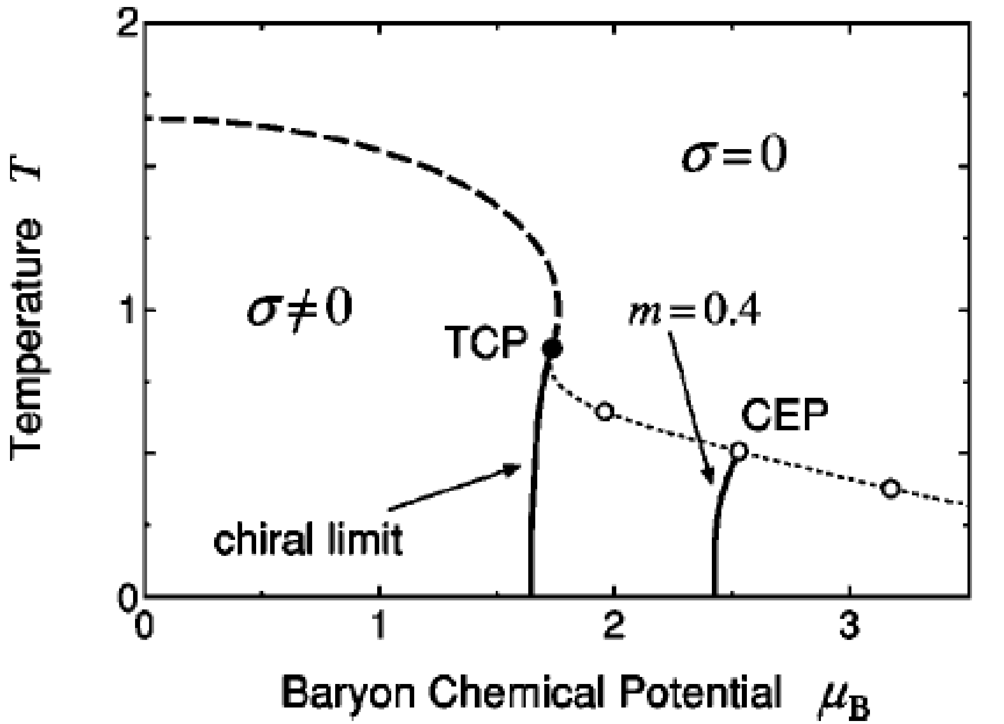


Strong Coupling Limit of Lattice QCD

- **Chiral Restoration at μ=0.**
  - **Damgaard, Kawamoto, Shigemoto, PRL53(1984),2211**



- **Phase Diagram with Nc=3**
  - **Nishida, PRD69, 094501 (2004)**



Previous Works in Strong Coupling Limit LQCD

- **Strong Coupling Limit Lattice QCD re-attracts interests**

Ref	T	μ	Nc	Baryon	CSC	Nf
Damgaard-Kawamoto-Shigemoto('84)	*Finite*	***0***	*U(Nc)*	***X***	*X*	*1*
Damgaard-Hochberg-Kawamoto('85)	***0***	*Finite*	*3*	*Yes*	*X*	*1*
Bilic-Karsch-Redlich('92)	*Finite*	*Finite*	*3*	***X***	*X*	*1 ~ 3*
Azcoiti-Di Carlo-Galante-Laliena('03)	***0***	*Finite*	*3*	*Yes*	*Yes*	*1*
Nishida-Fukushima-Hatsuda('04)	*Finite*	*Finite*	***2***	*Yes (*)*	*Yes (*)*	*1*
Nishida('04)	*Finite*	*Finite*	*3*	***X***	*X*	*1~2*
Kawamoto-Miura-AO-Ohnuma('05)	***Finite***	***Finite***	***3***	***Yes***	***Yes (+)***	***1***

*: bosonic baryon=diquark in SU(2)
+: analytically included, but ignored in numerical calc.

- ***Baryons should be important at High Baryon Densities, but they have been ignored in finite T treatments !***
→ This work: Baryonic effects at Finite T (and μ) for $SU_c(3)$

Strong Coupling Limit Lattice QCD

- **QCD Lattice Action**

$$Z \simeq \int D[\chi,\bar{\chi},U] \exp\left[-\left(S_G + S_F^{(s)} + S_F^{(t)} + m_0 M\right)\right]$$

$$S_G = \frac{1}{g^2} \sum_{x\mu\nu} \left[\mathrm{Tr}\, U_{\mu\nu} + \mathrm{Tr}\, U_{\mu\nu}^{+}\right]$$

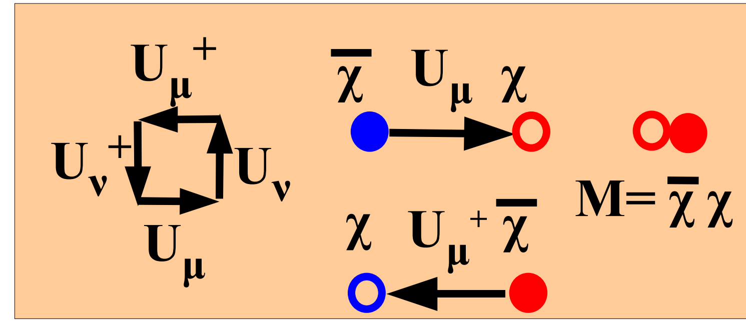
$$S_F^{(s)} = \frac{1}{2} \sum_{x,j} \eta_j(x) \left(\bar{\chi}_x U_j(x) \chi_{x+\hat{j}} - \bar{\chi}_{x+\hat{j}} U_j^{+}(x) \chi_x\right)$$

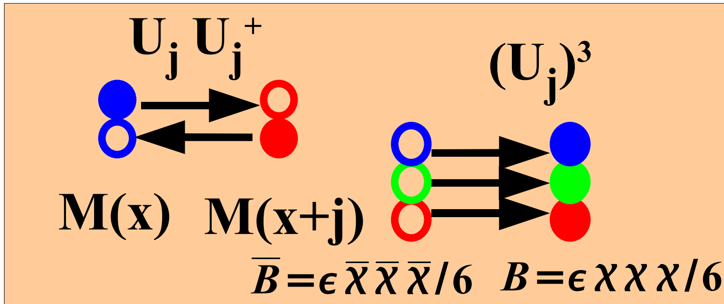
$$S_F^{(t)} = \frac{1}{2} \sum_{x} \left(e^{\mu} \bar{\chi}_x U_0(x) \chi_{x+\hat{0}} - e^{-\mu} \bar{\chi}_{x+\hat{0}} U_0^{+}(x) \chi_x\right)$$

- **Strong Coupling Limit: g→∞**
  - **We can ignore S_G and perform one-link integral after 1/d expansion.**

$$S_F^{(s)} \to -\frac{1}{2}(M V_M M) - (\bar{B} V_B B)$$

$$= -\frac{1}{4N_c} \sum_{x,j>0} M_x M_{x+\hat{j}} + \sum_{x,j>0} \frac{\eta_j}{8} \left[\bar{B}_x B_{x+\hat{j}} - \bar{B}_{x+\hat{j}} B_x\right]$$





$$\int dU\, U_{ab} U_{cd}^{+} = \frac{1}{N_c} \delta_{ad} \delta_{bc}$$

$$\int dU\, U_{ab} U_{cd} U_{ef} = \frac{1}{6} \epsilon_{ace} \epsilon_{bdf}$$

SCL-LQCD w/o Baryons

Damgaad-Kawamoto-Shigemoto 1984, Faldt-Petersson 1986, Bilic-Karsch-Redlich 1992, Nishida 2004, ...

- **Lattice Action (staggered fermion) in SCL**

$$Z \simeq \int D[\chi, \bar{\chi}, U] \exp\left[-S_F^{(s)} - S_F^{(t)} - m_0 \bar{\chi}\chi - S_G\right]$$

Strong Coupling

- **Spatial Link Integral**

$$\simeq \int D[\chi, \bar{\chi}, U_0] \exp\left[\frac{1}{2}(M, V_M M) + (\bar{B}, V_B B) - (\bar{\chi} G_0 \chi)\right]$$

1/d Expansion (1/√d)

- **Bosonization (Hubburd-Stratonovich transformation)**

$$\simeq \int D[\chi, \bar{\chi}, U_0, \sigma] \exp\left[-\frac{1}{2}(\sigma, V_M \sigma) - (\sigma, V_M M) - (\bar{\chi} G_0 \chi)\right]$$

$$(\bar{\chi} G(\sigma) \chi)$$

- **Quark and U_0 Integral**

$$\simeq \exp\left(-N_S^3 N_\tau \left[\frac{1}{2} a_\sigma \sigma^2 - T \log G_U(\sigma)\right]\right) = \exp(-N_S^3 F_{\text{eff}}/T)$$

Local Bi-linear action in quarks → Effective Free Energy

SCL-LQCD with Baryons

$$M = \bar{\chi}_a \chi^a$$
$$B = \epsilon_{abc} \chi^a \chi^b \chi^c / 6$$

- **Effective Action up to O(1/√d)**

$$Z \simeq \int D[\chi, \bar{\chi}, U_0] \exp\left[\frac{1}{2}(M, V_M M) + (\bar{B}, V_B B) - (\bar{\chi} G_0 \chi)\right]$$
$$= \int D[\chi, \bar{\chi}, U_{0,} b, \bar{b}] \exp\left[\frac{1}{2}(M V_M M) - (\bar{b} V_B^{-1} b) + (\bar{b}, B) + (\bar{B}, b) - (\bar{\chi} G_0 \chi)\right]$$

- *Decomposition of bB by using diquark condensate (Azcoiti et al., 2004)*

$$\exp[(\bar{b}, B) + (\bar{B}, b)] = \exp\left[\frac{1}{6}(\bar{b}, \epsilon \chi\chi\chi) + \frac{1}{6}(\epsilon \bar{\chi}\bar{\chi}\bar{\chi}, b)\right]$$
$$= \int D[\phi_a, \phi_a^*] \exp\left[-\phi^* \phi + \phi^* \left(\frac{\gamma}{2}\epsilon \chi\chi + \frac{\bar{\chi} b}{3\gamma}\right) + \phi\left(\frac{\gamma}{2}\epsilon \bar{\chi}\bar{\chi} + \frac{\bar{b}\chi}{3\gamma}\right)\right]$$
$$\times \exp(-\gamma M^2/2 + M \bar{b} b / 9\gamma^2)$$

- *Decomposition of Mbb using baryon potential field ω*

$$\exp(M \bar{b} b / 9\gamma^2) = \int D[\omega] \exp\left[\frac{1}{2}\omega^2 - \omega\left(\alpha M + \frac{\bar{b} b}{9\alpha\gamma^2}\right) - \frac{\alpha^2}{2} M^2\right]$$

  - *note:* $(\bar{b} b)^2 = 0$ *with one species of staggered fermion !*

Effective Free Energy with Baryon Effects

- **Effective Action in local bilinear form of quarks**

$$S_F = -\frac{1}{2}(M \tilde{V}_M M) + \frac{1}{2}(\omega,\omega) + (\bar{b}, \tilde{V}_B^{-1}(g_\omega \omega) b) + \alpha(\omega, M) + (\bar{\chi} G_0 \chi)$$

$$+(\phi^* \phi) + (\phi^* D) + (D^+ \phi)$$

Bosonization + MFA
+No diquark cond.

$$= \frac{N_s^3 N_\tau}{2}\left(a_\sigma \sigma^2 + \omega^2\right) + (a_\sigma \sigma + \alpha\omega, M) + (\bar{\chi} G_0 \chi) + (\bar{b}, \tilde{V}_B^{-1}(g_\omega \omega) b)$$

quark & gluon int. *b int.*

$$F_{\rm eff}(\sigma,\omega) = \frac{1}{2} a_\sigma \sigma^2 + \frac{1}{2}\omega^2 + F_{\rm eff}^{(q)}(a_\sigma \sigma + \alpha\omega) + F_{\rm eff}^{(b)}(g_\omega \omega)$$

$O(\omega^2)$ $O(\omega^4)$

$$= \frac{1}{2} a_\sigma \sigma^2 + \frac{1}{2} a_\omega \omega^2 + F_{\rm eff}^{(q)}(a_\sigma \sigma + \alpha\omega) + \Delta F_{\rm eff}^{(b)}(g_\omega \omega)$$

Linear Approx. $(\omega \sim \alpha\sigma/a_\omega)$

$$F_{\rm eff}(\sigma) = \frac{1}{2} b_\sigma \sigma^2 + F_{\rm eff}^{(q)}(b_\sigma \sigma) + \Delta F_{\rm eff}^{(b)}(g_\sigma \sigma)$$

Effective Free Energy with Baryon Effects

(Kawamoto-Miura-AO-Ohnuma, hep-lat/0512023)

$$F_{\mathrm{eff}}(\sigma)=\frac{1}{2}b_\sigma\sigma^2+F_{\mathrm{eff}}^{(q)}(b_\sigma\sigma;T,\mu)+\Delta F_{\mathrm{eff}}^{(b)}(g_\sigma\sigma)$$

is analytically derived based on many previous works, including

- **Strong Coupling Limit** *(Kawamoto-Smit, 1981)*
- **1/d expansion** *(Kluberg-Stern-Morel-Petersson, 1983)*
- **Lattice chemical potential** *(Hasenfratz-Karsch, 1983)*
- **Quark and time-like gluon analytic integral**
  (Damgaad-Kawamoto-Shigemoto, 1984, Faldt-Petersson, 1986)

$$F_{\mathrm{eff}}^{(q)}(\sigma;T,\mu)=-T\log\left(C_\sigma^3-\frac{1}{2}C_\sigma+\frac{1}{4}C_{3\mu}\right)\quad C_\sigma=\cosh(\sinh^{-1}\sigma/T)\quad C_{3\mu}=\cosh(3\mu/T)$$

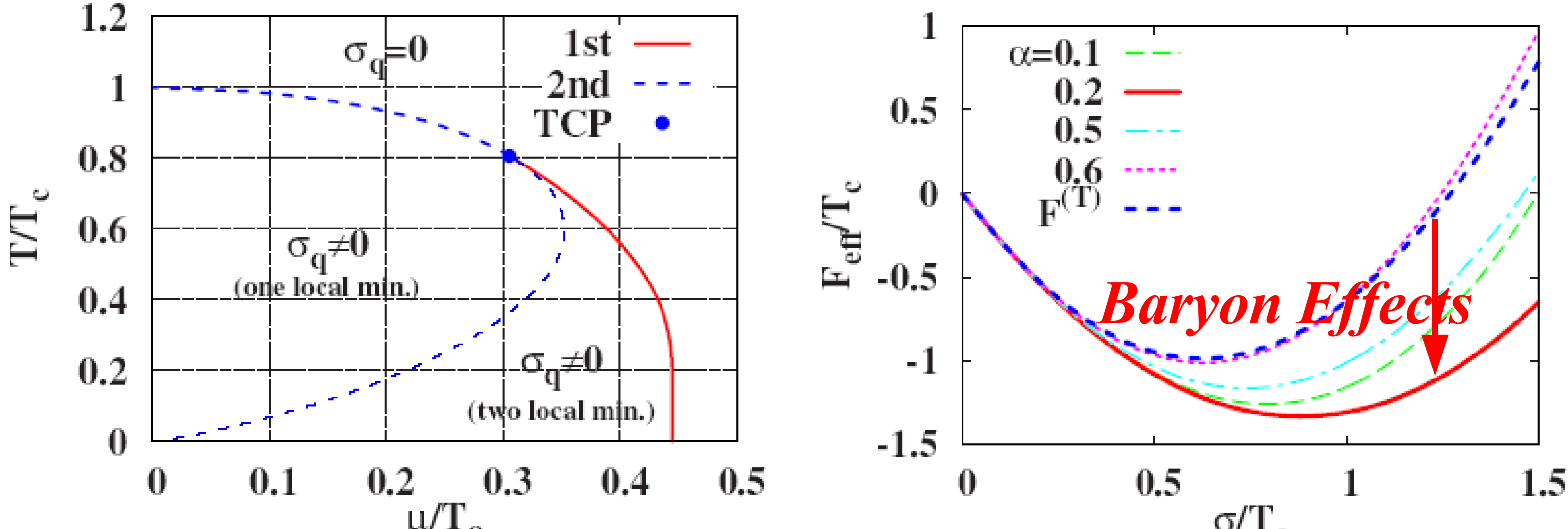
- **Decomposition of baryon-3 quark coupling**
  (Azcoiti-Di Carlo-Galante-Laliena, 2003)

and auxiliary baryon potential and baryon integral

Phase diagram in SCL-LQCD with Baryons

(Kawamoto-Miura-AO-Ohnuma, hep-lat/0512023)

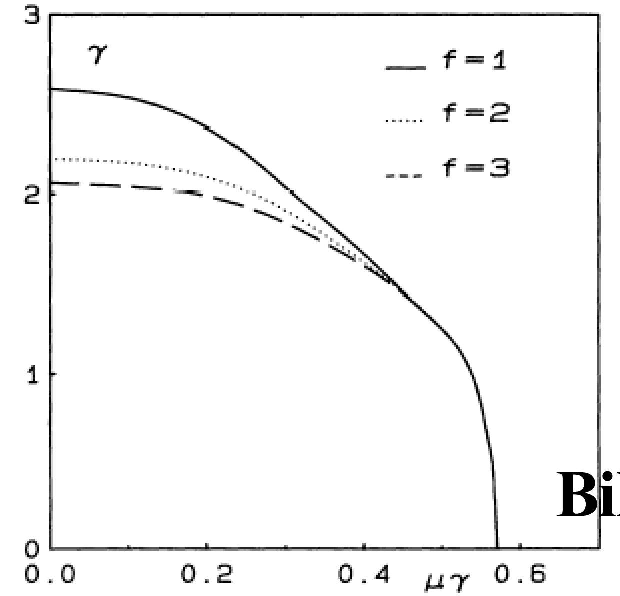
- **What is the baryonic composite effect on phase diagram ?**
  - **Auxiliary baryon integral & diquarks generate terms ∝ M^2**
    → modifies the *energy scale* !
    → Compare the phase diagram scaled with T_c.



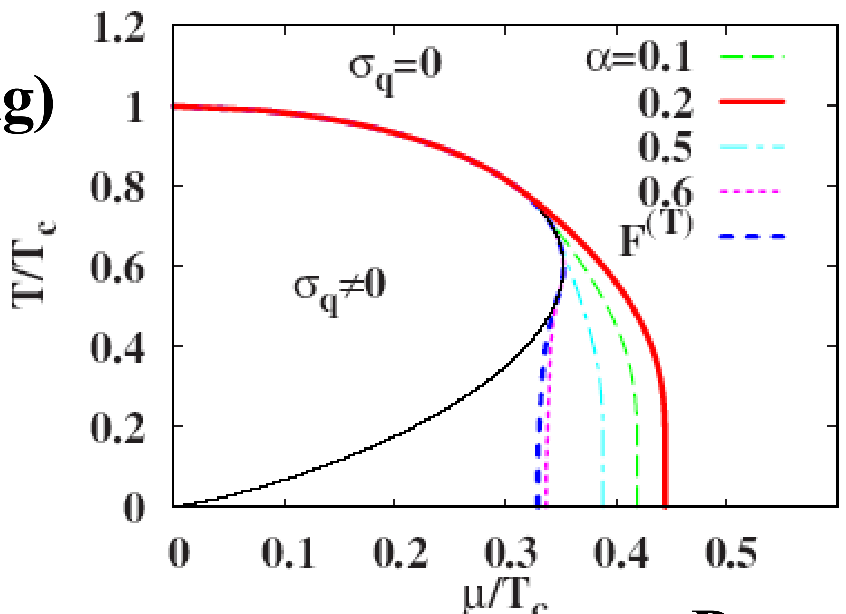
Baryons Gain Free Energy
→ Extention of Hadron Phase to Larger μ !

Small Critical μ : Common in SCL-LQCD ?

- Finite *T* SCL-LQCD
  - No B: $\mu_c(0)/T_c(0) \sim (0.2\text{-}0.35)$
    (Nishida2004,
    Bilic et al. 1992(Bielefeld), ...)
  - Present: $\mu_c(0)/T_c(0) < 0.44$
    (Parameter dep.)
- Monte-Carlo: $\mu_c(0)/T_c(0) > 1$
  - Fodor-Katz (Improved Reweighting)
    Bielefeld (Taylor expansion),
    de Forcrand-Philipsen (AC), ...
- Real World: $\mu_c(0)/T_c(0) > 3$
  - $T_c(0) \sim 170$ MeV, $\mu_c(0) > 330$ MeV



Bilic et al.



Present

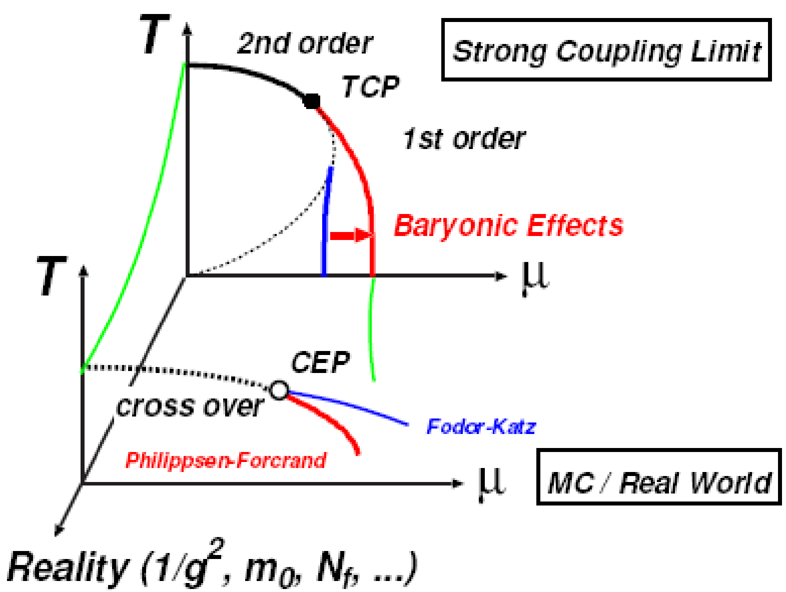
Towards Realistic Understanding

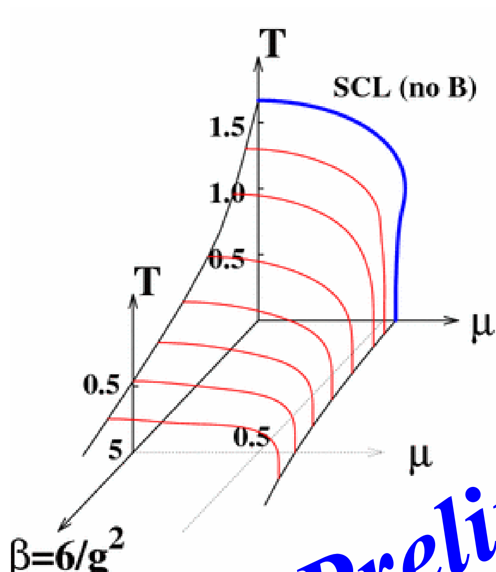
- "Reality" Axis: $1/g^2$, n_f, m_0, ... would enhance μ_c/T_c ratio
- Example: $1/g^2$ correction enhances μ_c/T_c by a factor ~(2-3).

$$\exp\left[\frac{1}{g^2}\mathrm{Tr}\,U_{0j}(x)\right]\sim\exp\left[-V_xV^+_{x+\hat{j}}/4N_c^2g^2\right]\to\exp\left[-\left(\varphi^2+2\varphi\left(V_x-V^+_{x+\hat{j}}\right)\right)/16N_c^2g^2\right]$$

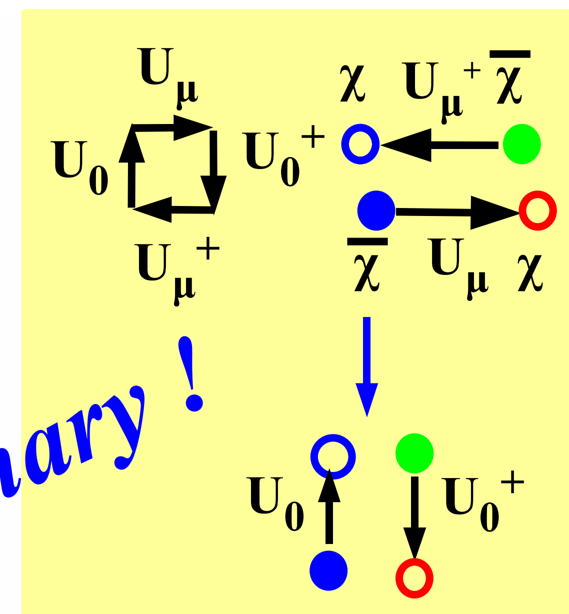
$$S_F^{(t)}=\frac{1}{2}\left(e^{\mu}V_x-e^{-\mu}V_x^+\right)\to\frac{\alpha}{2}\left(\exp\tilde{\mu}V_x-\exp(-\tilde{\mu})V_x^+\right)\qquad\left(V_x=\bar{\chi}_xU_0(x)\chi_{x+\hat{0}}\right)$$

Time-like plaquetts can modify effective chemical potential





Preliminary !



Summary

- **We obtain an analytical expression of effective free energy *at finite T and finite μ* with *baryonic composite* effects in the strong coupling limit of lattice QCD for color SU(3).**
  - *MFA, QG integral, 1/d expansion (NLO, O(1/√d)), bosonization with diquarks and baryon potential field using* $(\bar{b}b)^2=0$ *, Linear approx.,* ***zero diquark cond.(Color Angle Average), variational parameter choice***
- **Baryonic action is found to result in *Free Energy Gain* and *Extension of Hadron Phase to Larger μ by around 30 %.***
  - **Problem: Too small μ_c/T_c in the Strong Coupling Limit.**
    → $1/g^2$ correction and other may help.
- **Strong Coupling Limit is useful to understand Dense Matter**
  - **RG evolution→ $\exp(-c/g^2)$ deps. , $1/g^2$ correction seems to work well**
  - **Application to chiral RMF (K. Tsubakihara, AO, nucl-th/0607046)**

Color Angle Average

- **Problem: Diquark Condensates induce quark-baryon coupling, and Baryon integral becomes difficult.**
  → Solution: *Color Angle Average*

$$D=\frac{\gamma}{2}\epsilon\chi\chi+\frac{\bar{\chi}b}{3\gamma}$$

  - **Integral of "Color Angle Variables"**

$$\int \mathcal{D}[\phi_a,\phi_a^\dagger]\exp\left\{\phi_a^\dagger D_a + D_a^\dagger\phi_a\right\} = \int \mathcal{D}[v]\exp\left\{\frac{v^2}{3}D_a^\dagger D_a + \frac{v^4}{162}M^3\bar{b}b\right\}$$

  - **Three-Quark and Baryon Coupling is ReBorn !**

$$D_a^\dagger D_a = Y + \bar{b}B + \bar{B}b\ , \quad Y = \frac{\gamma^2}{2}M^2 - \frac{1}{9\gamma^2}M\bar{b}b$$

  - **Solve "Self-Consistent" Equaton**

$$\exp(\bar{b}B+\bar{B}b) \simeq \exp\left[-v^2 - Y + \frac{v^2}{3}(\bar{b}B+\bar{B}b+Y) + \frac{v^4}{162}M^3\bar{b}b\right]$$

$$\simeq \exp\left[-\frac{v^2}{R_v} + \frac{v^4M^3\bar{b}b}{162R_v} - Y\right] \quad (R_v = 1 - v^2/3)$$

Effective Free Energy with Diquark Condensate

- **Bosonization of** $M^k \bar{b} b$ **→ Introduce k bosons**

$$\exp M^k \bar{b} b = \int d\omega_k \exp[-\frac{1}{2}(\omega_k + \alpha_k M + 1/\alpha_k M^{k-1} \bar{b} b)^2 + M^k \bar{b} b]$$

$$= \int d\omega_k \exp[-\omega_k^2/2 - \omega(\alpha_k M + 1/\alpha_k M^{k-1} \bar{b} b) - \alpha_k^2 M^2/2]$$

- **Effective Free Energy**

$$\mathcal{F}_{\mathrm{eff}}^{(Tbv)} = F_X(\sigma, v, \omega_i) + F_{\mathrm{eff}}^{(b)}(g_\omega \omega) + F_{\mathrm{eff}}^{(q)}(m_q)$$

$$F_X = \frac{1}{2}(a_\sigma \sigma^2 + \omega^2 + \omega_1^2 + \omega_2^2) + \frac{v^2}{R_v} \qquad m_q = a_\sigma \sigma + \alpha\omega + \alpha_1 \omega_1 + \alpha_2 \omega_2 + m_0$$

$$a_\sigma = \frac{1}{2} - \gamma^2 - \alpha^2 - \alpha_1^2 - \alpha_2^2 \qquad g_\omega = \frac{1}{9\alpha\gamma^2}\left[1 + \frac{\gamma^2 v^4 \omega_1 \omega_2}{18\alpha_1 \alpha_2 R_v}\right]$$

The same F_{eff} *is obtained at v=0.*
Diquark Effects in interaction start from v^4*.*

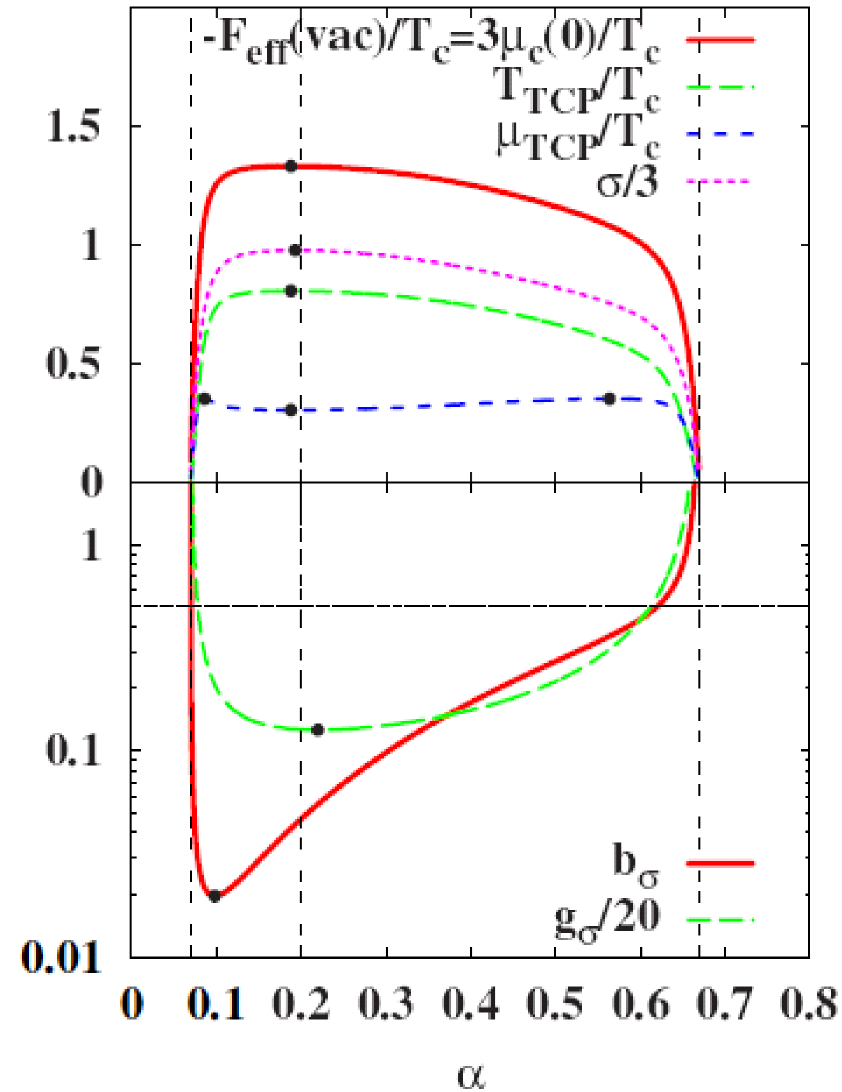
c.f. Ipp, Yamamoto

Backups

Parameter Choice

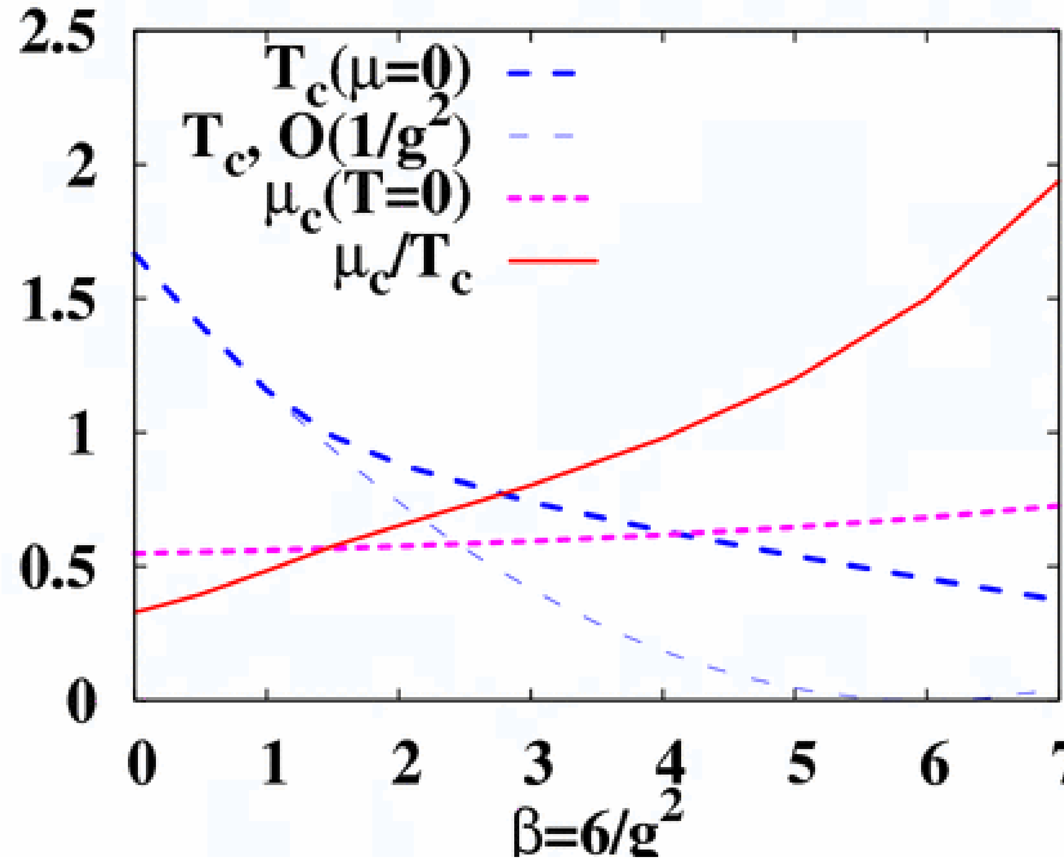
- **In bosonization, two parameters (γ and α) are introduced through identities.**
  - **Major effects**
    → Modify the energy scale
  - **Minor effects**
    → Controls the higher order potential terms

→ *We have fixed them to minimize F_{eff}/T_c at vacuum*



$1/g_2$ correction

- **Gluons tend to break hadrons, then $1/g^2$ correction is expected to reduce T_c.** *(Bilic-Cleymans 1995)*
- **Naive extapolation of 1/g2 correction seems to give $\mu_c/T_c \sim 1.3$ @ $6/g^2=5$**



Baryon Integral

- **Baryon integral can be evaluated in an almost analytic way !**

$$F_{\rm eff}^{(b)}(g_\omega\omega) = \frac{1}{\beta L^3}\log{\rm Det}\left[1+g_\omega\omega V_B\right]$$

$$\simeq \frac{-a_0^{(b)}/2}{(4\pi\Lambda^3/3)}\int_0^\Lambda 4\pi k^2 dk\log\left[1+\frac{g_\omega^2\omega^2k^2}{16}\right]$$

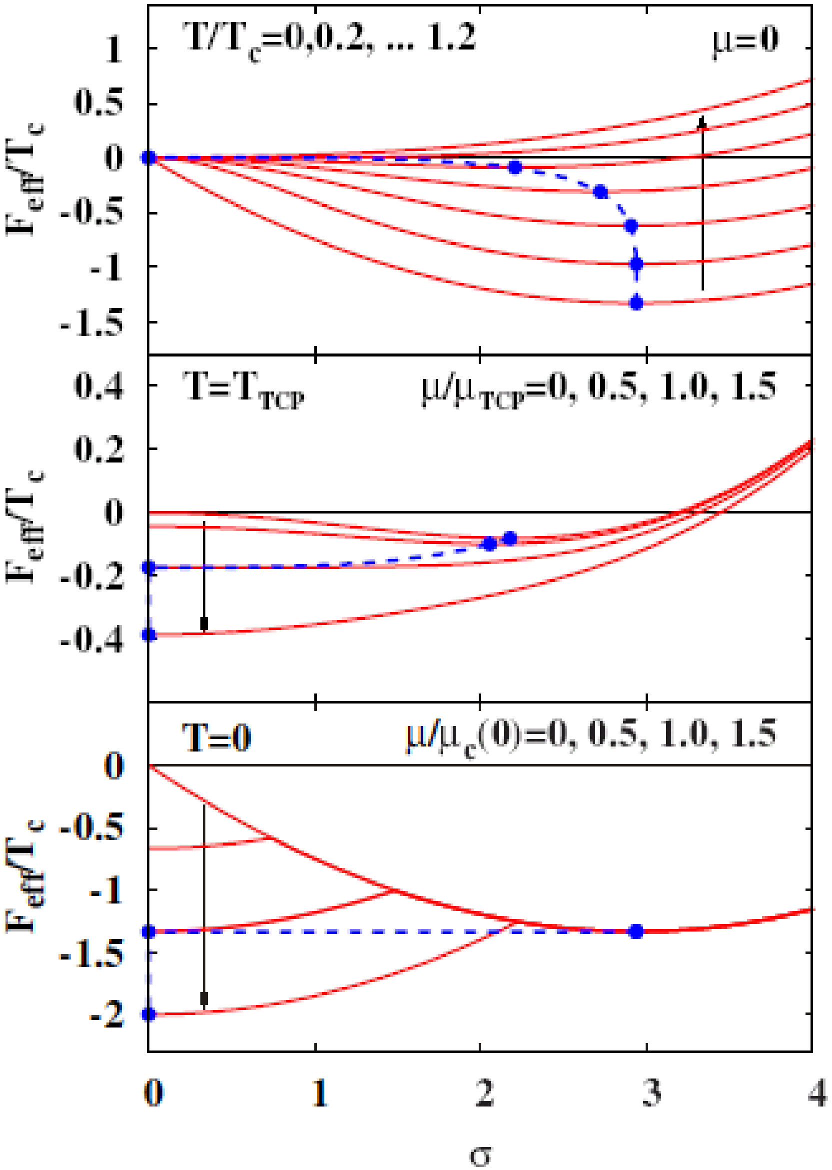
$$= -a_0^{(b)} f^{(b)}\left(\frac{g_\omega\omega\Lambda}{4}\right)$$

$$f^{(b)}(x) = \frac{1}{2}\log(1+x^2) - \frac{1}{x^3}\left[\arctan x - x + \frac{x^3}{3}\right]$$

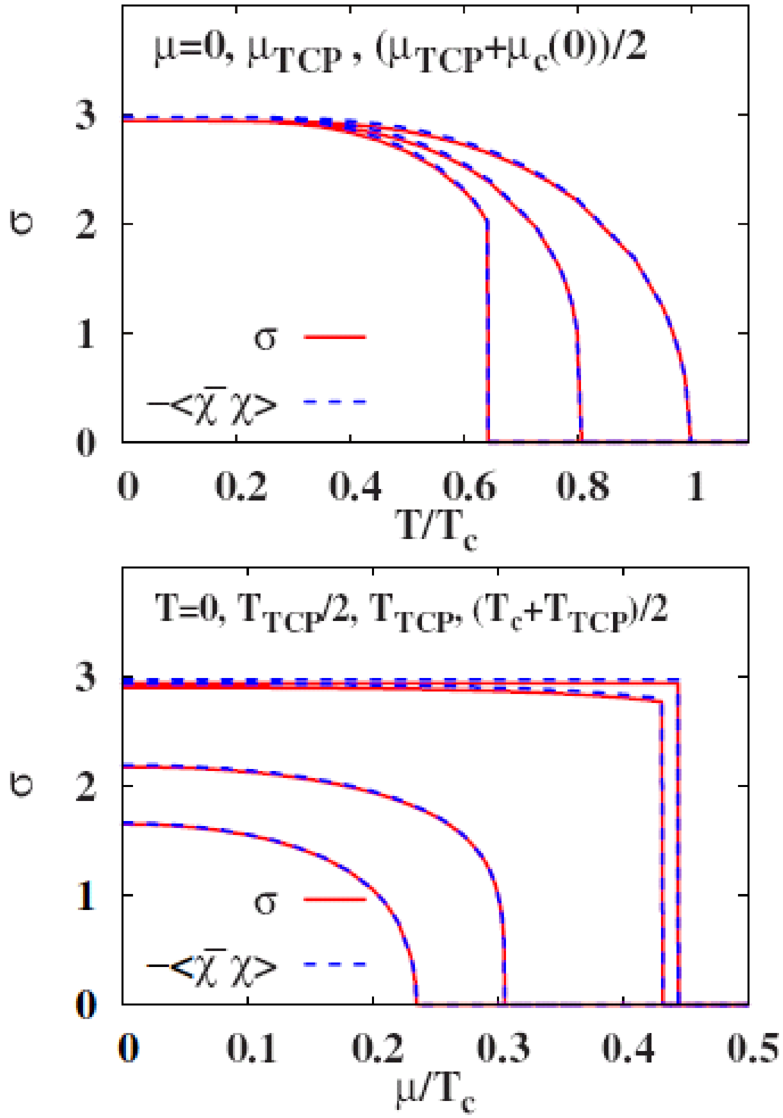
$$a_0^{(b)} = 1.0055\ ,\ \ \Lambda = 1.01502\times\pi/2$$

Figures

- **Energy surface**



- **Validity of "Linear" Approx.**



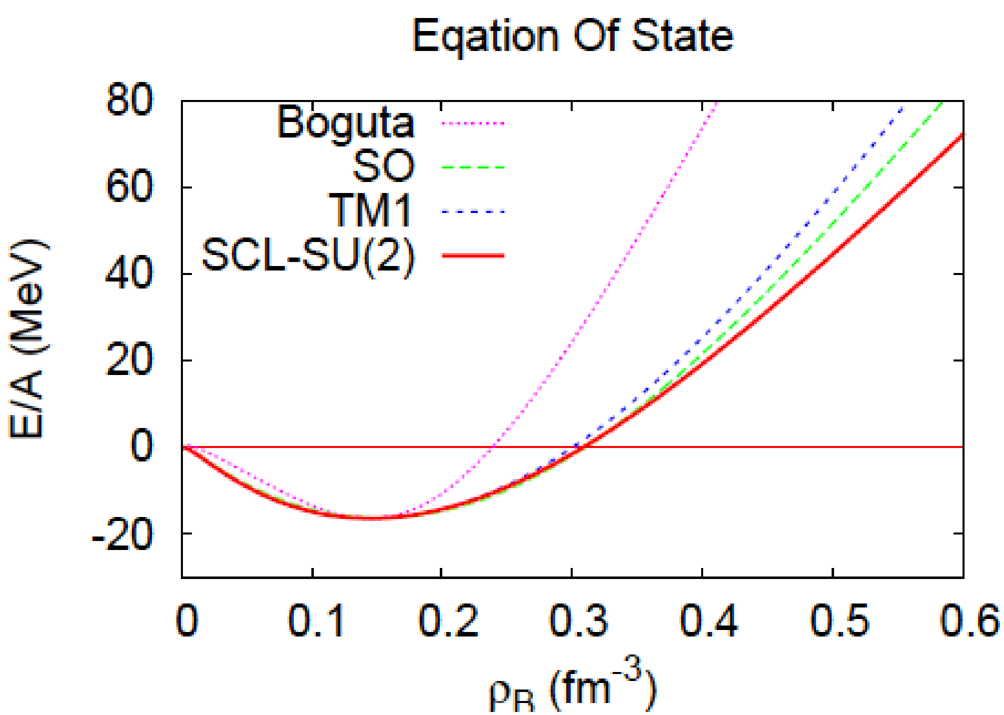
RMF with σ Self Energy from SCL-LQCD

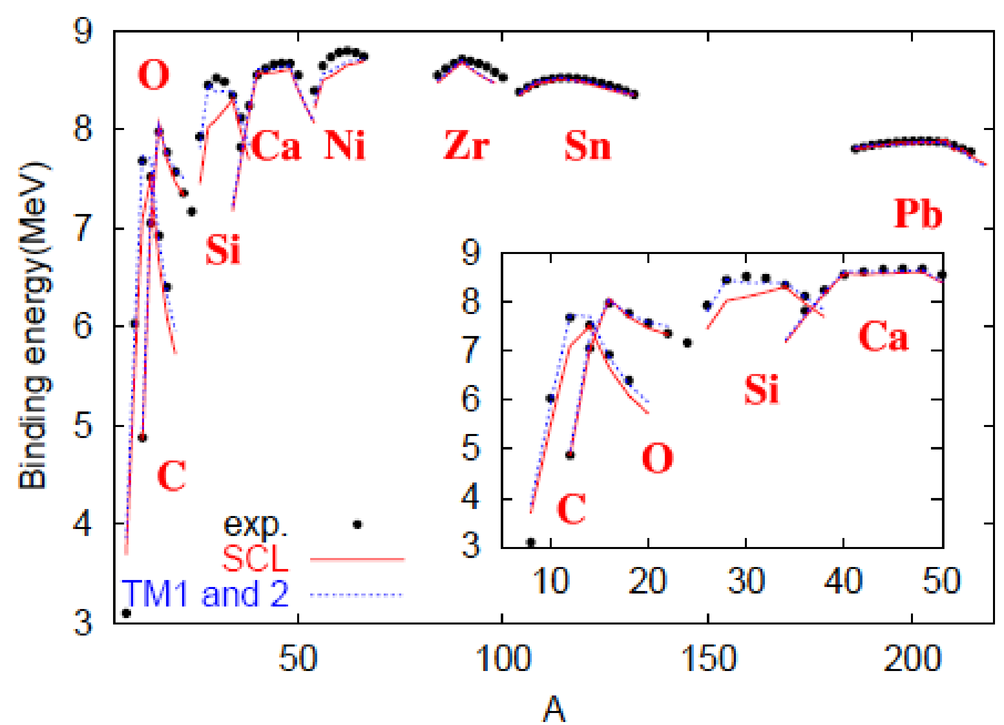
- **σ Self Energy from simple SCL-LQCD**

$$S \to -\frac{1}{2}(M V_M M) \to \frac{1}{2}(\sigma V_M \sigma) + (\bar{\chi} V_M \sigma \chi) \to U_\sigma \simeq \frac{1}{2} b\sigma^2 - N_c \log \sigma^2$$

- **Chiral RMF with logarithmic σ potential**

(Tsubakihara-AO, nucl-th/0607046)





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Chirally Symmetric Relativistic Mean Field and Its Application

K. Tsubakihara and AO, nucl-th/0607046

RMF with Chiral Symmetry

- Good Sym. in QCD, and Spontaneous breaking generates hadron masses.
- Schematic model: Linear σ model

$$L=\frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma+\partial_\mu\pi\partial^\mu\pi\right)-\frac{\lambda}{4}\left(\sigma^2+\pi^2\right)^2+\frac{\mu^2}{2}\left(\sigma^2+\pi^2\right)+c\sigma$$
$$+\bar{N}i\partial_\mu\gamma^\mu N-g_\sigma\bar{N}\left(\sigma+i\pi\tau\gamma_5\right)N$$

- Problems and Prescriptions
  - χ Sym. is restored at a very small density. σω Coupling stabilizes normal vacuum, but gives too stiff EOS. *(Boguta PLB120,34, Ogawa et al. PTP111(2004)75)*
  - Loop effects *(N.K. Gledenning, NPA480,597; M. Prakash and T. L. Ainsworth, PRC36, 346; Tamenaga et al.)*
  - Higher order terms *(Hatsuda-Prakashi 1989, Sahu-Ohnishi, 2000)*
  - Dielectric Field *(Papazoglou et al. (Frankfurt), 1998)*
  - Different Chiral partner assignment *(Kunihiro et al., Hatsuda et al. Harada et al.)*

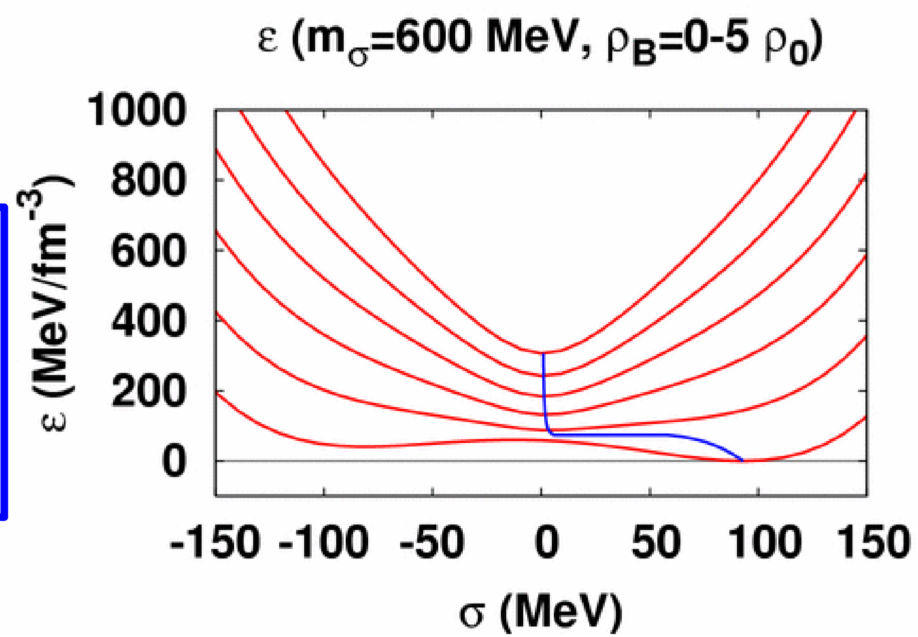
Problems in RMF with Chiral Symmetry

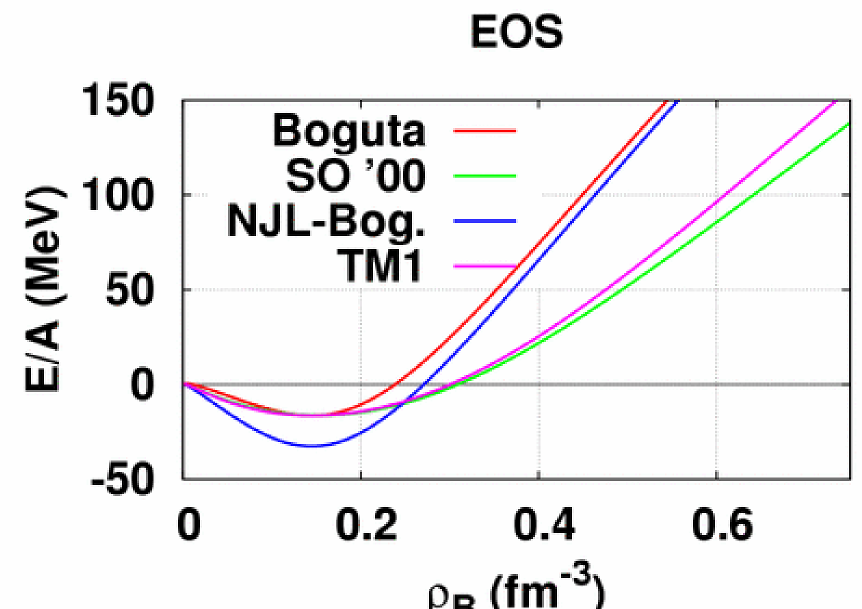
- Sudden Change of <σ>
- σ ω Coupling

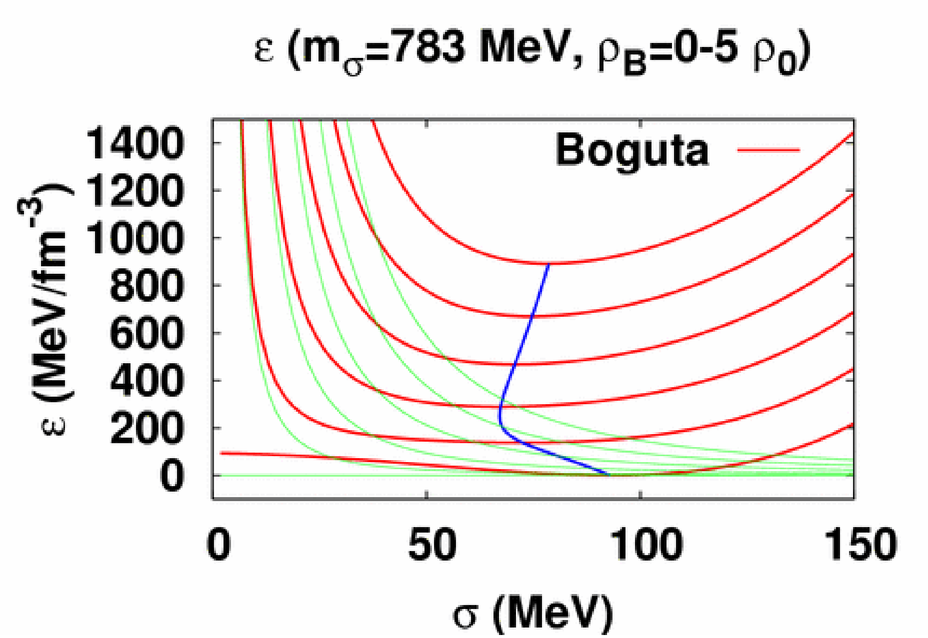
$$L_{\omega\sigma} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} C_{\sigma\omega} \sigma^2 \omega^2 - g_\omega \overline{N} \gamma_\mu \omega^\mu N$$

$$\omega = g_\omega \rho_B / C_{\sigma\omega} \sigma^2 \quad \rightarrow \quad V_{\sigma\omega} = \frac{g_\omega^2 \rho_B^2}{2C_{\sigma\omega} \sigma^2}$$

- Stiff EOS

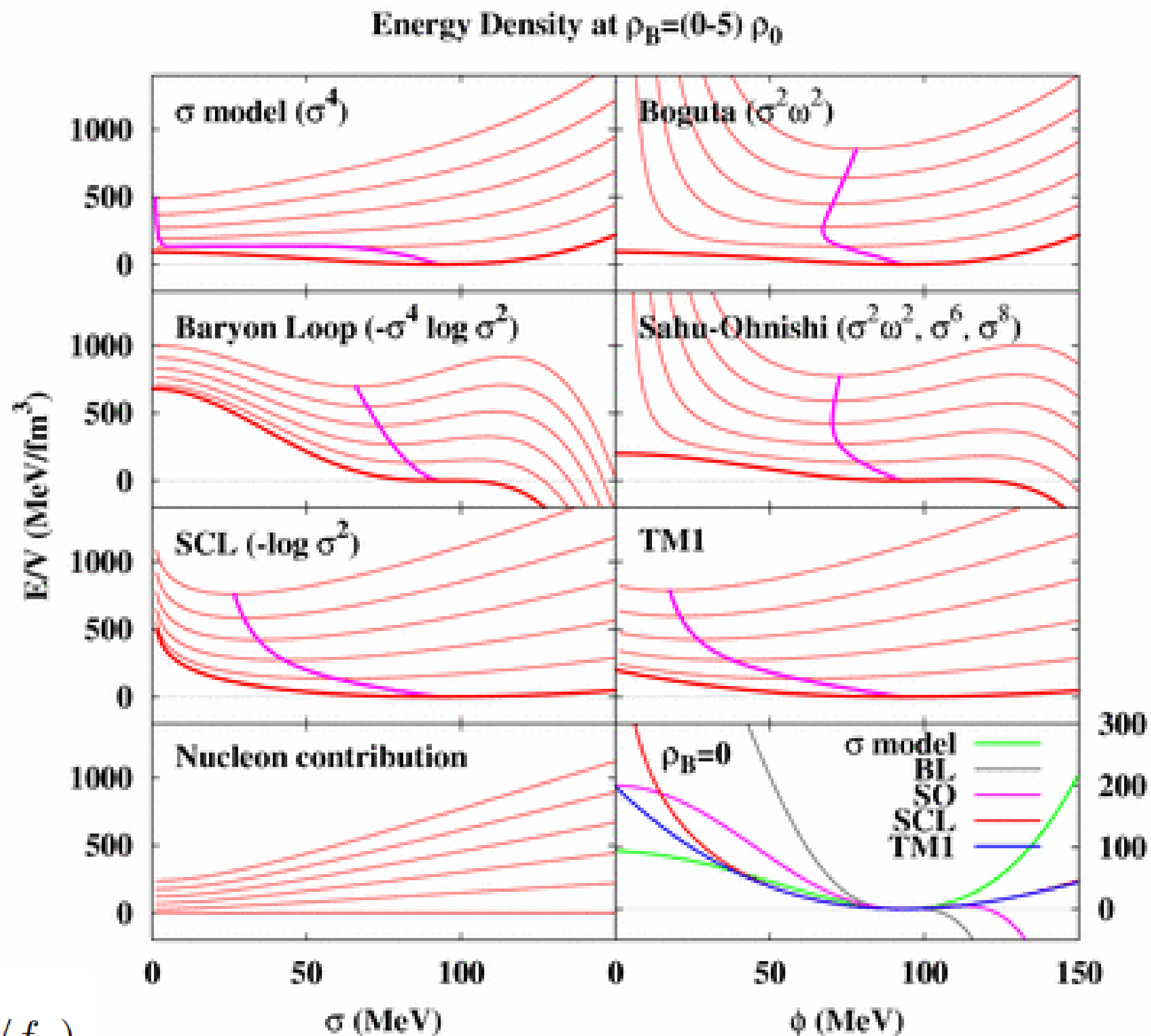






Instability in Chiral Models

- **Linear σ Model**
  → Chiral restor.
  Below ρ_0.
- **Baryon Loop**
  & Sahu-Ohnishig
  models
  → Unstable
  at large σ
- **Boguta model**
  → Too Stiff EOS



$$V_\sigma^{\rm BL} = \frac{m_\sigma^2}{2f_\pi^2}(\phi^2 - f_\pi^2)^2 - M_N^4 f_{\rm BL}(\phi/f_\pi)$$

$$f_{\rm BL} = -\frac{1}{4\pi^2}\left[\frac{x^4}{2}\log x^2 - \frac{1}{4} + x^2 - \frac{3}{4}x^4\right]$$

RMF with σ Self Energy from SCL-LQCD

- **σ Self Energy from simple Strong Coupling Limit LQCD**

$$
\begin{aligned}
S &\rightarrow -\tfrac{1}{2}(M, V_M M) && (1/d \text{ expansion})\\
&\rightarrow b\sigma^2 + (\bar{\chi}\ \sigma\chi) && (\text{auxiliary field})\\
&\rightarrow b\sigma^2 \boxed{- a\log\sigma^2} && (\text{Fermion Integral})
\end{aligned}
$$

- **RMF Lagrangian** **Non-Analytic Type σ Self Energy**
  - **σ is shifted by f_π , and small explicit χ breaking term is added.**

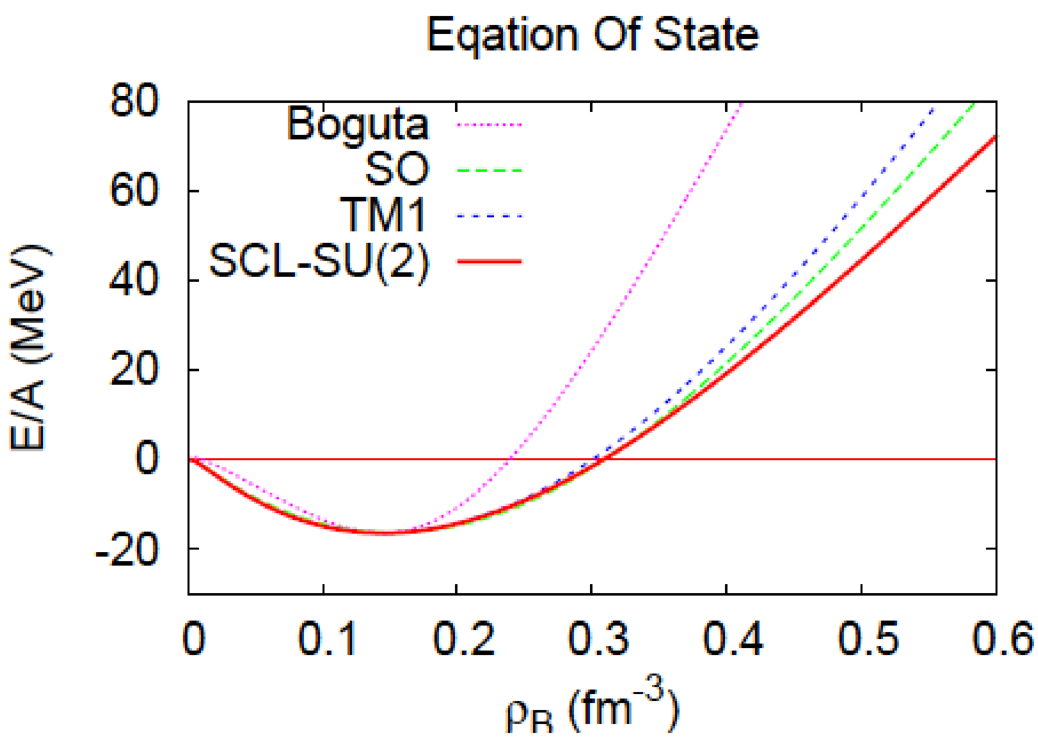
$$
\mathcal{L} = \bar{\psi}\left(i\gamma^\mu\partial_\mu - \gamma^\mu V_\mu - M + g_\sigma\sigma\right)\psi + \mathcal{L}_\sigma^{(0)} + \mathcal{L}_\omega^{(0)} + \mathcal{L}_\rho^{(0)}
$$

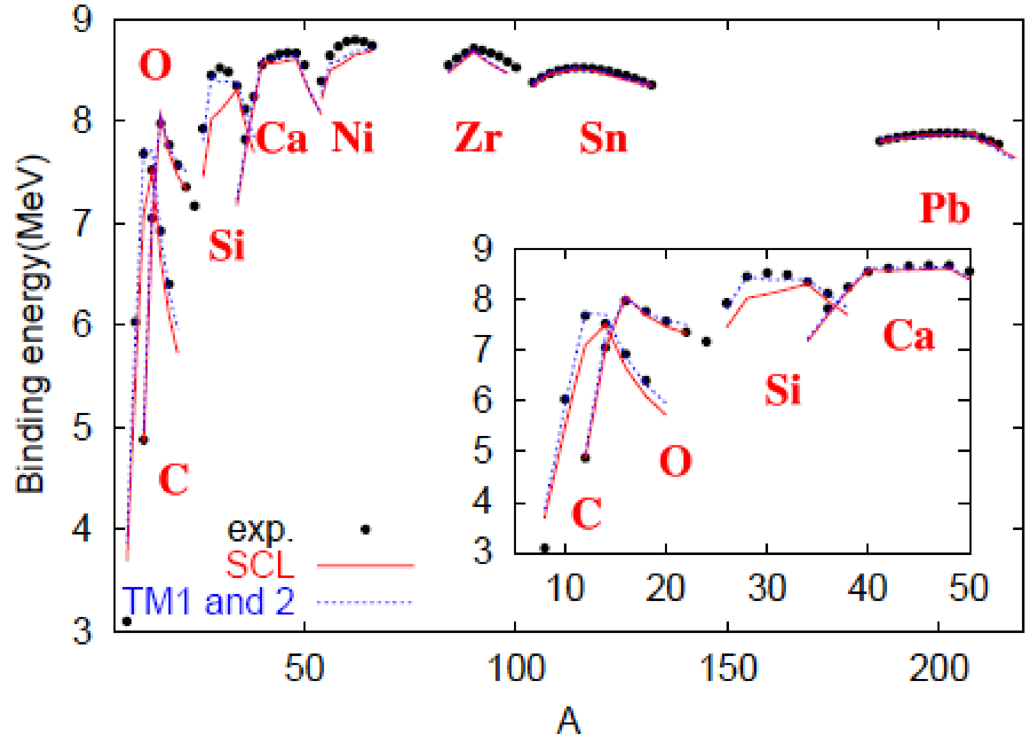
$$
-U_\sigma + \frac{\lambda}{4}(\omega_\mu\omega^\mu)^2
$$

$$
U_\sigma(\sigma) = 2a\, f(\sigma/f_\pi),\quad f(x) = \frac{1}{2}\left[-\log(1+x) + x - \frac{x^2}{2}\right],\quad a = \frac{f_\pi^2}{2}\left(m_\sigma^2 - m_\pi^2\right)
$$

Nuclear Matter and Finite Nuclei

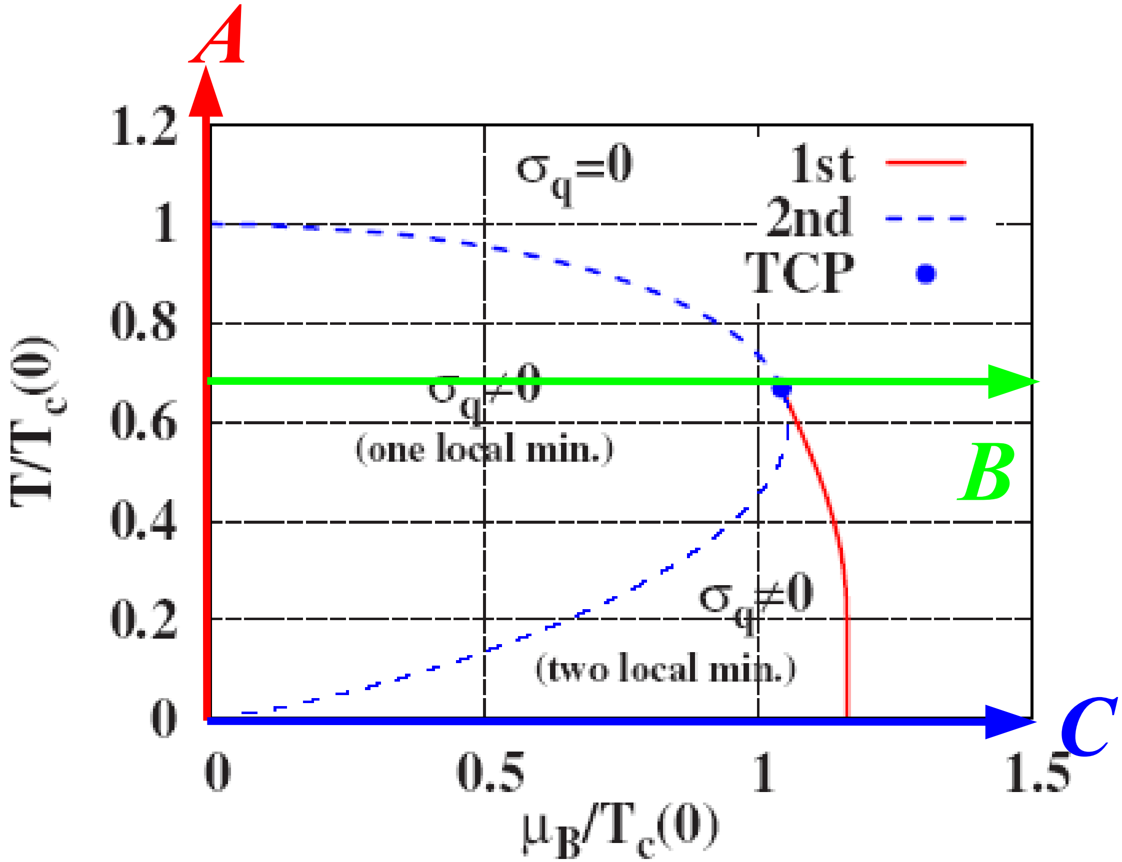
- **Nuclear Matter: By tuning λ, $g_{\omega N}$, m_σ, *EOS can be Soft !***
- **Finite Nuclei: By tuning $g_{\rho N}$, Global behavior of B.E. is reproduced, except for j-j closed nuclei (C, Si, Ni).**

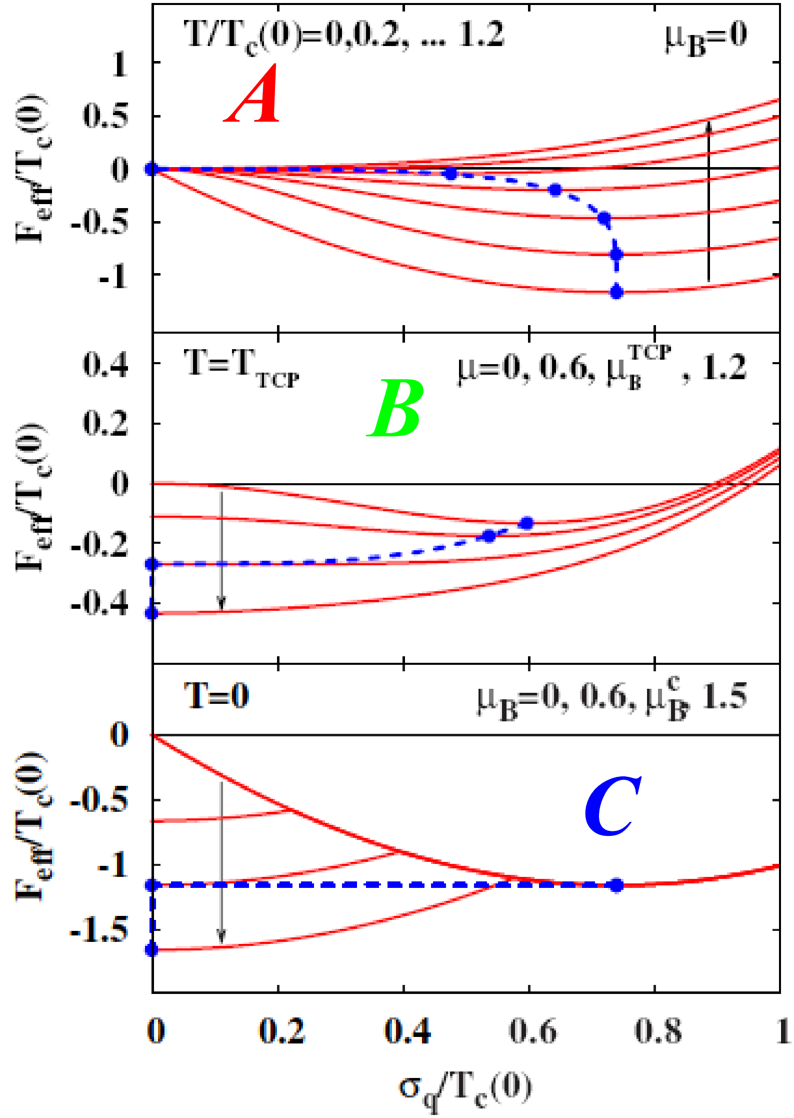




Free Energy Surface and Phase Diagram

- **At μ ≠ 0, quark can gain Free Energy even at σ =0**
  → Two Min. Structure
  → First Order





α =1/2