Strong Coupling QCD
→ Strong Coupling Limit/Region of Lattice QCD

Akira Ohnishi *Hokkaido University, Sapporo, Japan*

This talk is based on following Eprints

(1) Phase diagram at finite temperature and quark density in the strong coupling limit of lattice QCD for color SU(3)
N. Kawamoto, K. Miura, A. Ohnishi, T. Ohnuma, hep-lat/0512023

(2) A chiral symmetric relativistic mean field model with logarithmic sigma potential
K. Tsubakihara and A. Ohnishi, nucl-th/0607046

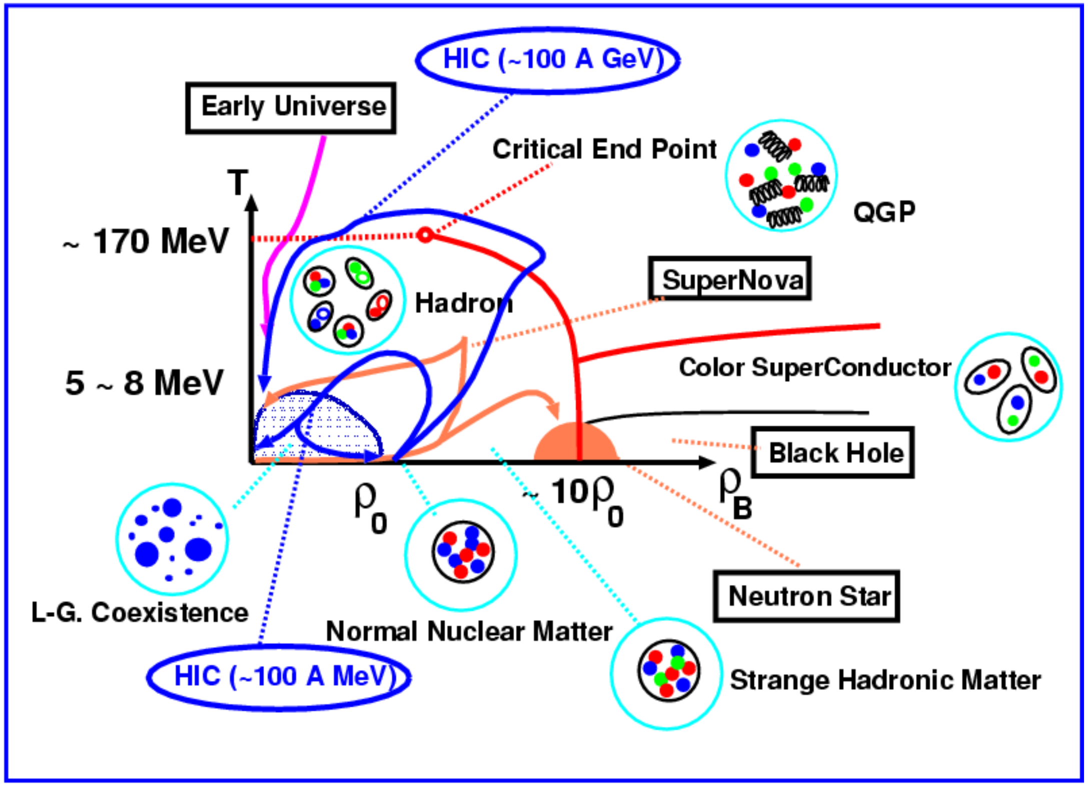
and I would like to add some preliminary results to get some ideas from YOU.

Outline

- Introduction
- Strong coupling limit lattice QCD with baryon effects
- $1/g^2$ correction of Phase Diagram
- Chiral RMF with logarithmic σ potential
  (→ 3rd week Poster by Tsubakihara)
- Summary

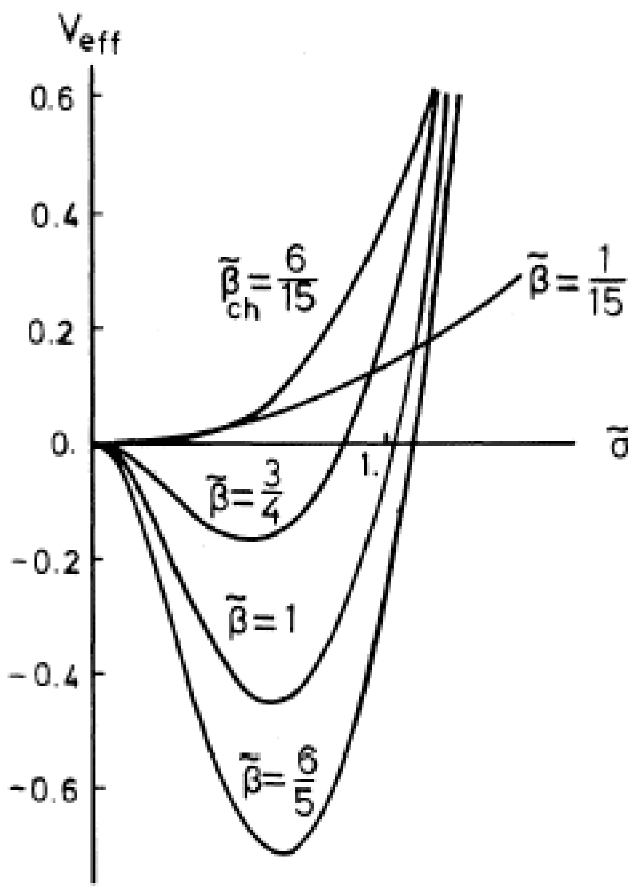
Quark and Hadronic Matter Phase Diagram

- **Dense quark & hadronic matter contains rich physics, but Lattice QCD simulation is not yet reliable.**
  → *Model/Approximate approaches are necessary !*
  - **Monte-Carlo calc. of Lattice QCD:**
    Improved ReWeighting Method (Fodor-Katz)
    Taylor Expansion in μ (Bielefeld-Swansea)
    Analytic Continuation (de Forcrand-Philipssen)
  - **Model / Phen. Approaches: (P)NJL, QMC, RMF, ...**
  - ***Strong Coupling Limit of Lattice QCD***

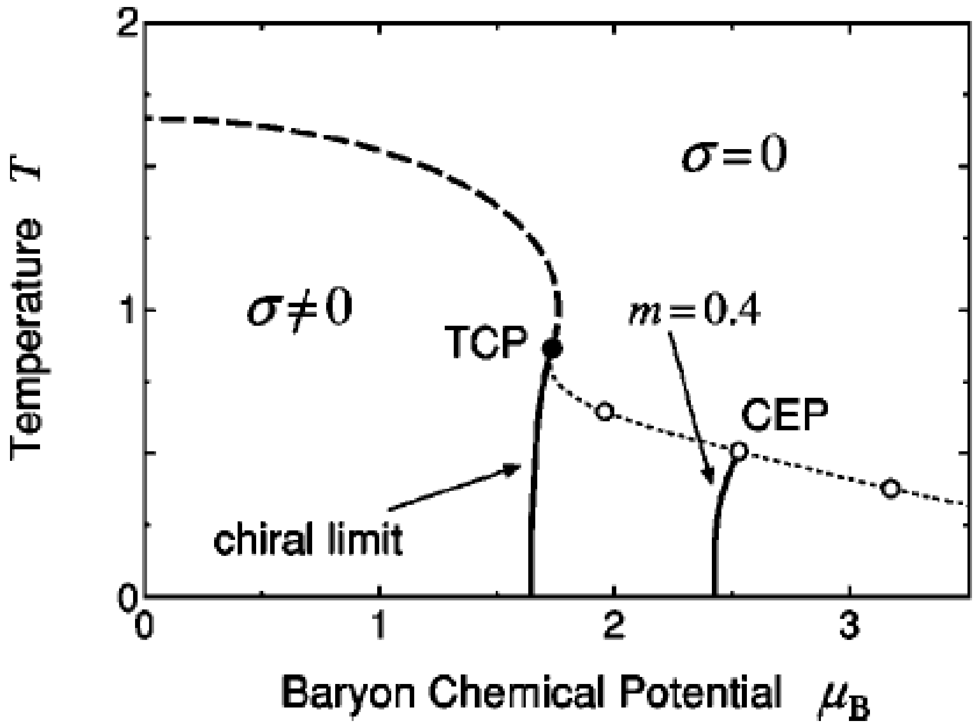


Strong Coupling Limit of Lattice QCD

- **Chiral Restoration at μ=0.**
  - **Damgaard, Kawamoto, Shigemoto, PRL53(1984),2211**



- **Phase Diagram with Nc=3**
  - **Nishida, PRD69, 094501 (2004)**



Previous Works in Strong Coupling Limit LQCD

- **Strong Coupling Limit Lattice QCD re-attracts interests**
  c.f. Nakamura @ JHF Symp. for high density matter (2001)

Ref	T	μ	Nc	Baryon	CSC	Nf
Damgaard-Kawamoto-Shigemoto('84)	*Finite*	***0***	*U(Nc)*	***X***	*X*	*1*
Damgaard-Hochberg-Kawamoto('85)	***0***	*Finite*	*3*	*Yes*	*X*	*1*
Bilic-Karsch-Redlich('92)	*Finite*	*Finite*	*3*	***X***	*X*	*1 ~ 3*
Azcoiti-Di Carlo-Galante-Laliena('03)	***0***	*Finite*	*3*	*Yes*	*Yes*	*1*
Nishida-Fukushima-Hatsuda('04)	*Finite*	*Finite*	***2***	*Yes (*)*	*Yes (*)*	*1*
Nishida('04)	*Finite*	*Finite*	*3*	***X***	*X*	*1~2*
Kawamoto-Miura-AO-Ohnuma('05)	***Finite***	***Finite***	***3***	***Yes***	***Yes (+)***	***1***

*: bosonic baryon=diquark in SU(2)
+: analytically included, but ignored in numerical calc.

- ***Baryon effects have been ignored in finite T treatments !***
  → This work: Baryonic effects at Finite T (and μ) for $SU_c(3)$

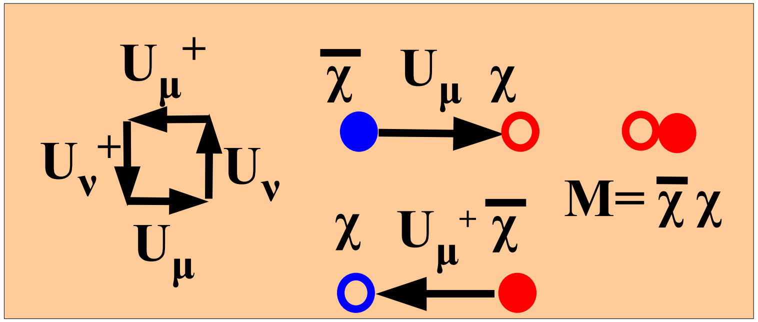
- **QCD Lattice Action**

$$Z \simeq \int D[\chi,\bar{\chi},U] \exp\left[-\left(S_G + S_F^{(s)} + S_F^{(t)} + m_0 M\right)\right]$$

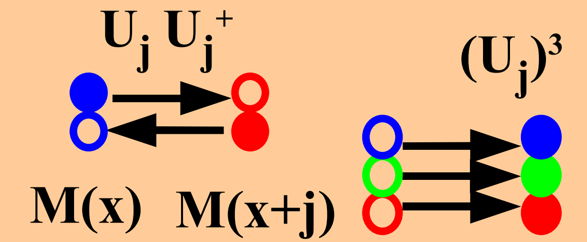
$$S_G = \frac{1}{g^2} \sum_{x\mu\nu} \left[\mathrm{Tr}\, U_{\mu\nu} + \mathrm{Tr}\, U_{\mu\nu}^{+}\right]$$

$$S_F^{(s)} = \frac{1}{2} \sum_{x,j} \eta_j(x) \left(\bar{\chi}_x U_j(x) \chi_{x+\hat{j}} - \bar{\chi}_{x+\hat{j}} U_j^{+}(x) \chi_x\right)$$

$$S_F^{(t)} = \frac{1}{2} \sum_{x} \left(e^{\mu} \bar{\chi}_x U_0(x) \chi_{x+\hat{0}} - e^{-\mu} \bar{\chi}_{x+\hat{0}} U_0^{+}(x) \chi_x\right)$$



- **Strong Coupling Limit: g→∞**
  - **We can ignore S_G and perform one-link integral after 1/d expansion.**



$$\bar{B} = \epsilon \bar{\chi}\bar{\chi}\bar{\chi}/6 \qquad B = \epsilon \chi\chi\chi/6$$

$$\int dU\, U_{ab} U_{cd}^{+} = \frac{1}{N_c} \delta_{ad} \delta_{bc}$$

$$\int dU\, U_{ab} U_{cd} U_{ef} = \frac{1}{6} \epsilon_{ace} \epsilon_{bdf}$$

$$S_F^{(s)} \to -\frac{1}{2} (M V_M M) - (\bar{B} V_B B)$$

$$= -\frac{1}{4N_c} \sum_{x,j>0} M_x M_{x+\hat{j}} + \sum_{x,j>0} \frac{\eta_j}{8} \left[\bar{B}_x B_{x+\hat{j}} - \bar{B}_{x+\hat{j}} B_x\right]$$

SCL-LQCD w/o Baryons

Damgaad-Kawamoto-Shigemoto 1984, Faldt-Petersson 1986, Bilic-Karsch-Redlich 1992, Nishida 2004, ...

- **Lattice Action (staggered fermion) in SCL**

$$Z \simeq \int D[\chi,\bar{\chi},U] \exp\left[-S_F^{(s)} - S_F^{(t)} - m_0 \bar{\chi}\chi - S_G\right]$$

- **Spatial Link Integral**

Strong Coupling

$$\simeq \int D[\chi,\bar{\chi},U_0] \exp\left[\frac{1}{2}(M, V_M M) + (\bar{B}, V_B B) - (\bar{\chi} G_0 \chi)\right]$$

1/d Expansion (1/√d)

- **Bosonization (Hubburd-Stratonovich transformation)**

$$\simeq \int D[\chi,\bar{\chi},U_0,\sigma] \exp\left[-\frac{1}{2}(\sigma, V_M \sigma) - (\sigma, V_M M) - (\bar{\chi} G_0 \chi)\right]$$

- **Quark and U_0 Integral**

$(\bar{\chi} G(\sigma) \chi)$

$$\simeq \exp\left(-N_S^3 N_\tau \left[\frac{1}{2} a_\sigma \sigma^2 - T \log G_U(\sigma)\right]\right) = \exp(-N_S^3 F_{\mathrm{eff}}/T)$$

Local Bi-linear action in quarks → Effective Free Energy

SCL-LQCD with Baryons

$$M = \bar{\chi}_a \chi^a$$
$$B = \epsilon_{abc} \chi^a \chi^b \chi^c / 6$$

- **Effective Action up to O(1/√d)**

$$Z \simeq \int D[\chi, \bar{\chi}, U_0] \exp\left[\frac{1}{2}(M, V_M M) + (\bar{B}, V_B B) - (\bar{\chi} G_0 \chi)\right]$$
$$= \int D[\chi, \bar{\chi}, U_{0,} b, \bar{b}] \exp\left[\frac{1}{2}(M V_M M) - (\bar{b} V_B^{-1} b) + (\bar{b}, B) + (\bar{B}, b) - (\bar{\chi} G_0 \chi)\right]$$

- *Decomposition of bB by using diquark condensate (Azcoiti et al., 2004)*

$$\exp[(\bar{b}, B) + (\bar{B}, b)] = \exp\left[\frac{1}{6}(\bar{b}, \epsilon \chi \chi \chi) + \frac{1}{6}(\epsilon \bar{\chi} \bar{\chi} \bar{\chi}, b)\right]$$
$$= \int D[\phi_a, \phi_a^*] \exp\left[-\phi^* \phi + \phi^* \left(\frac{\gamma}{2} \epsilon \chi \chi + \frac{\bar{\chi} b}{3\gamma}\right) + \phi \left(\frac{\gamma}{2} \epsilon \bar{\chi} \bar{\chi} + \frac{\bar{b} \chi}{3\gamma}\right)\right]$$
$$\times \exp(-\gamma M^2/2 + M \bar{b} b / 9\gamma^2)$$

- *Decomposition of Mbb using baryon potential field ω*

$$\exp(M \bar{b} b / 9\gamma^2) = \int D[\omega] \exp\left[\frac{1}{2}\omega^2 - \omega\left(\alpha M + \frac{\bar{b} b}{9\alpha\gamma^2}\right) - \frac{\alpha^2}{2} M^2\right]$$

  - *note:* $(\bar{b} b)^2 = 0$ *with one species of staggered fermion !*

Effective Free Energy with Baryon Effects

- **Effective Action in local bilinear form of quarks**

$$S_F = -\frac{1}{2}(M \tilde{V}_M M) + \frac{1}{2}(\omega,\omega) + (\bar{b}, \tilde{V}_B^{-1}(g_\omega \omega) b) + \alpha(\omega, M) + (\bar{\chi} G_0 \chi)$$

~~$+(\phi^* \phi) + (\phi^* D) + (D^+ \phi)$~~

Bosonization + MFA
+No diquark cond.

$$= \frac{N_s^3 N_\tau}{2}\left(a_\sigma \sigma^2 + \omega^2\right) + (a_\sigma \sigma + \alpha\omega, M) + (\bar{\chi} G_0 \chi) + (\bar{b}, \tilde{V}_B^{-1}(g_\omega \omega) b)$$

quark & gluon int. b int.

$$F_{\text{eff}}(\sigma,\omega) = \frac{1}{2} a_\sigma \sigma^2 + \frac{1}{2}\omega^2 + F_{\text{eff}}^{(q)}(a_\sigma \sigma + \alpha\omega) + F_{\text{eff}}^{(b)}(g_\omega \omega)$$

$O(\omega^2)$ $O(\omega^4)$

$$= \frac{1}{2} a_\sigma \sigma^2 + \frac{1}{2} a_\omega \omega^2 + F_{\text{eff}}^{(q)}(a_\sigma \sigma + \alpha\omega) + \Delta F_{\text{eff}}^{(b)}(g_\omega \omega)$$

Linear Approx. $(\omega \sim \alpha\sigma/a_\omega)$

$$F_{\text{eff}}(\sigma) = \frac{1}{2} b_\sigma \sigma^2 + F_{\text{eff}}^{(q)}(b_\sigma \sigma) + \Delta F_{\text{eff}}^{(b)}(g_\sigma \sigma)$$

Color Angle Average

- **Problem: Diquark Condensates induce quark-baryon coupling, and Baryon integral becomes difficult.**
  → Solution: *Color Angle Average*

$$D = \frac{\gamma}{2}\epsilon\chi\chi + \frac{\bar{\chi} b}{3\gamma}$$

  - **Integral of "Color Angle Variables"**

$$\int \mathcal{D}[\phi_a, \phi_a^\dagger] \exp\left\{\phi_a^\dagger D_a + D_a^\dagger \phi_a\right\} = \int \mathcal{D}[v] \exp\left\{\frac{v^2}{3} D_a^\dagger D_a + \frac{v^4}{162} M^3 \bar{b} b\right\}$$

  - **Three-Quark and Baryon Coupling is ReBorn !**

$$D_a^\dagger D_a = Y + \bar{b}B + \bar{B}b\ , \quad Y = \frac{\gamma^2}{2} M^2 - \frac{1}{9\gamma^2} M \bar{b} b$$

  - **Solve "Self-Consistent" Equaton**

$$\exp(\bar{b}B + \bar{B}b) \simeq \exp\left[-v^2 - Y + \frac{v^2}{3}(\bar{b}B + \bar{B}b + Y) + \frac{v^4}{162} M^3 \bar{b} b\right]$$

$$\simeq \exp\left[-\frac{v^2}{R_v} + \frac{v^4 M^3 \bar{b} b}{162 R_v} - Y\right] \quad (R_v = 1 - v^2/3)$$

Effective Free Energy with Diquark Condensate

- **Bosonization of $M^k \bar{b} b$ → Introduce k bosons**

$$\exp M^k \bar{b} b = \int d\omega_k \exp[-\frac{1}{2}(\omega_k + \alpha_k M + 1/\alpha_k M^{k-1} \bar{b} b)^2 + M^k \bar{b} b]$$

$$= \int d\omega_k \exp[-\omega_k^2/2 - \omega(\alpha_k M + 1/\alpha_k M^{k-1} \bar{b} b) - \alpha_k^2 M^2/2]$$

- **Effective Free Energy**

$$\mathcal{F}_{\mathrm{eff}}^{(Tbv)} = F_X(\sigma, v, \omega_i) + F_{\mathrm{eff}}^{(b)}(g_\omega \omega) + F_{\mathrm{eff}}^{(q)}(m_q)$$

$$F_X = \frac{1}{2}(a_\sigma \sigma^2 + \omega^2 + \omega_1^2 + \omega_2^2) + \frac{v^2}{R_v} \qquad m_q = a_\sigma \sigma + \alpha\omega + \alpha_1\omega_1 + \alpha_2\omega_2 + m_0$$

$$a_\sigma = \frac{1}{2} - \gamma^2 - \alpha^2 - \alpha_1^2 - \alpha_2^2 \qquad g_\omega = \frac{1}{9\alpha\gamma^2}\left[1 + \frac{\gamma^2 v^4 \omega_1 \omega_2}{18\alpha_1\alpha_2 R_v}\right]$$

The same F_{eff} is obtained at v=0.
Diquark Effects in interaction start from v^4.
(No Stable CSC phase appears at g= ∞)

c.f. Ipp, Yamamoto

Effective Free Energy with Baryon Effects

(Kawamoto-Miura-AO-Ohnuma, hep-lat/0512023)

$$F_{\mathrm{eff}}(\sigma)=\frac{1}{2}b_\sigma\sigma^2+F_{\mathrm{eff}}^{(q)}(b_\sigma\sigma;T,\mu)+\Delta F_{\mathrm{eff}}^{(b)}(g_\sigma\sigma)$$

is analytically derived based on many previous works, including

- **Strong Coupling Limit** *(Kawamoto-Smit, 1981)*
- **1/d expansion** *(Kluberg-Stern-Morel-Petersson, 1983)*
- **Lattice chemical potential** *(Hasenfratz-Karsch, 1983)*
- **Quark and time-like gluon analytic integral**
  (Damgaad-Kawamoto-Shigemoto, 1984, Faldt-Petersson, 1986)

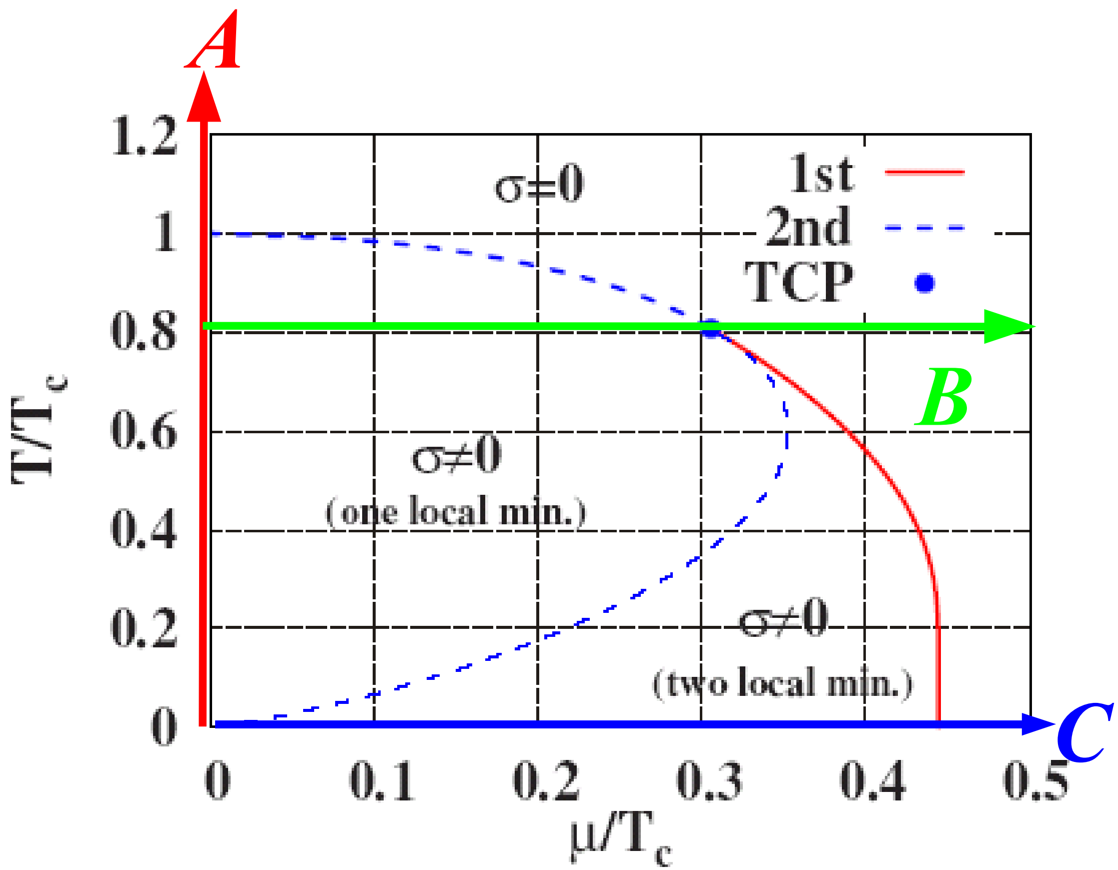
$$F_{\mathrm{eff}}^{(q)}(\sigma;T,\mu)=-T\log\left(C_\sigma^3-\frac{1}{2}C_\sigma+\frac{1}{4}C_{3\mu}\right)\quad C_\sigma=\cosh(\sinh^{-1}\sigma/T)\quad C_{3\mu}=\cosh(3\mu/T)$$

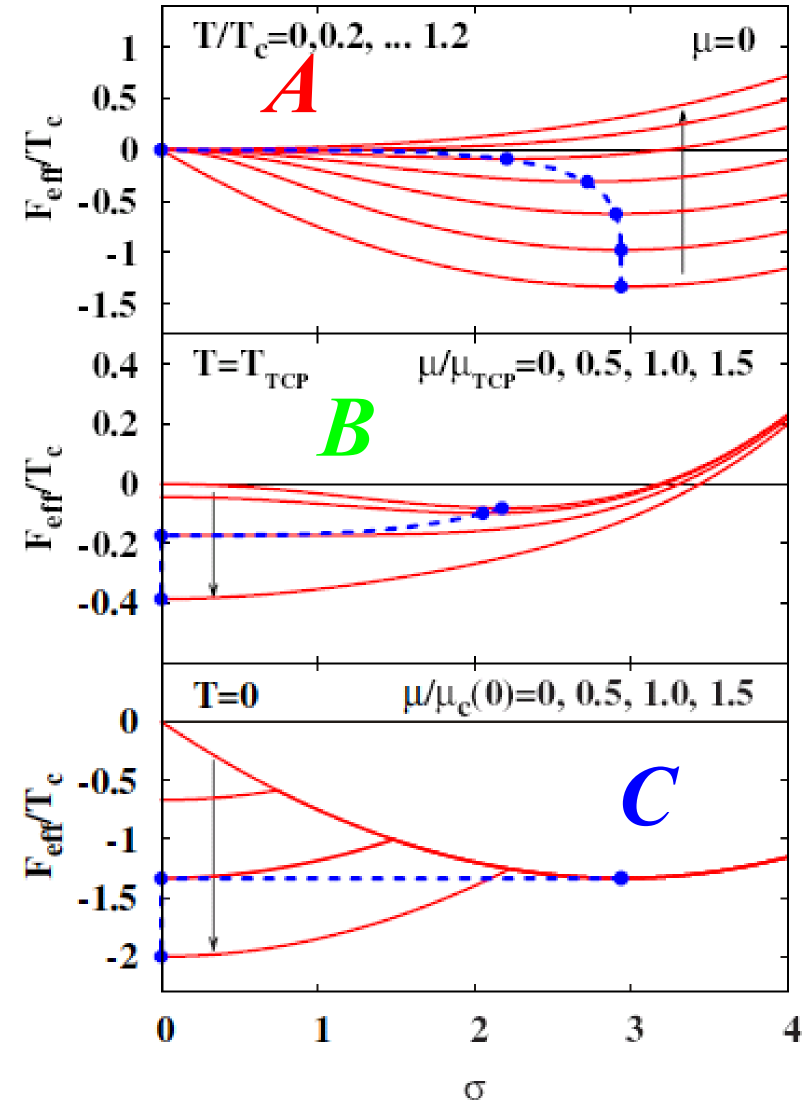
- **Decomposition of baryon-3 quark coupling**
  (Azcoiti-Di Carlo-Galante-Laliena, 2003)

and auxiliary baryon potential and baryon integral

Free Energy Surface and Phase Diagram

- **At $\mu \neq 0$, quark can gain Free Energy even at $\sigma = 0$**
  - **→ Two Min. Structure**
  - **→ First Order**



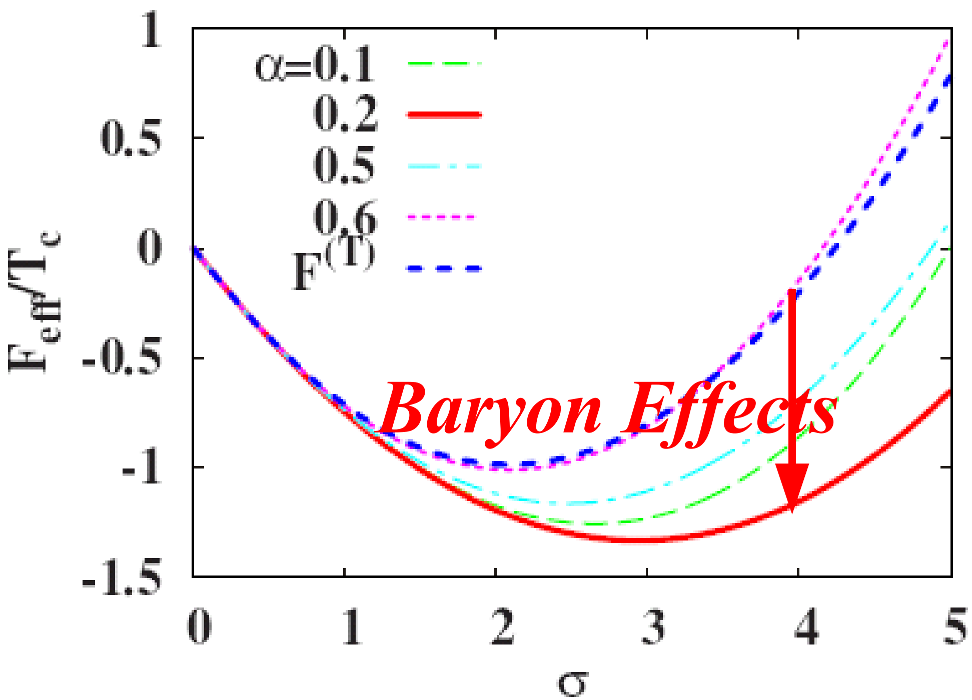


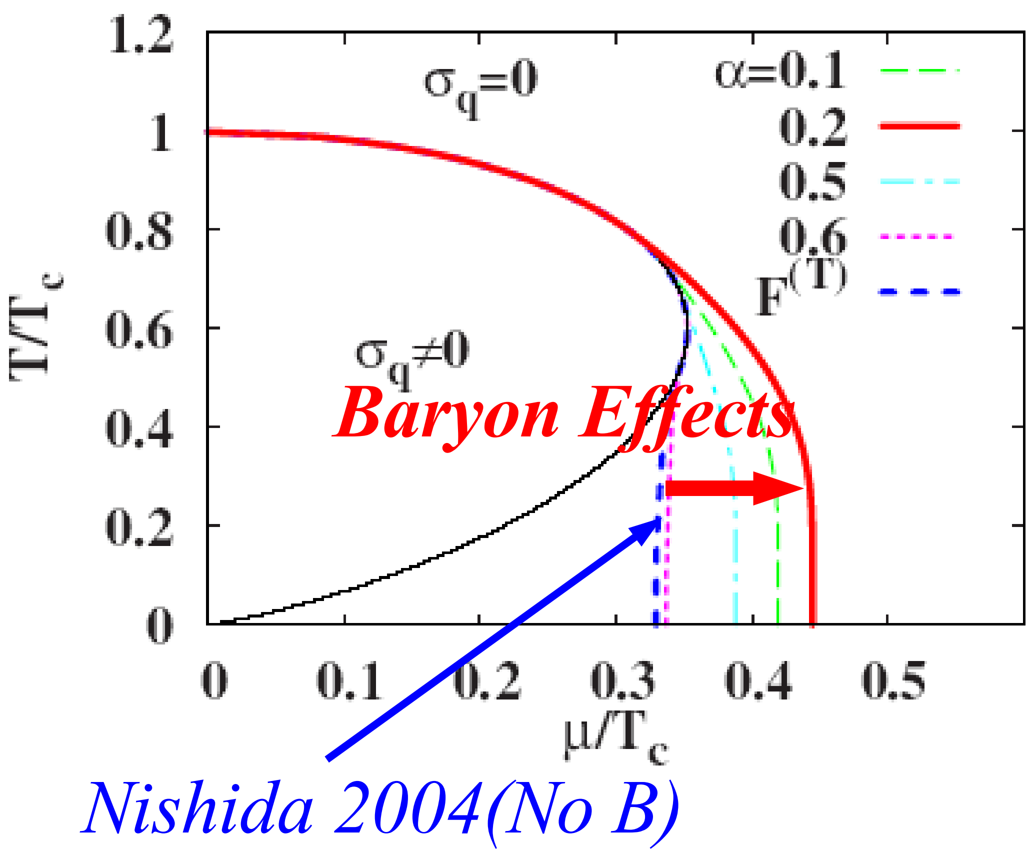
$\alpha = 0.2$

Phase diagram in SCL-LQCD with Baryons

(Kawamoto-Miura-AO-Ohnuma, hep-lat/0512023)

- **Baryon effects on phase diagram**
  - **Energy gain in larger condensates**
    → Extension of hadron phase to larger μ by around 30 %.



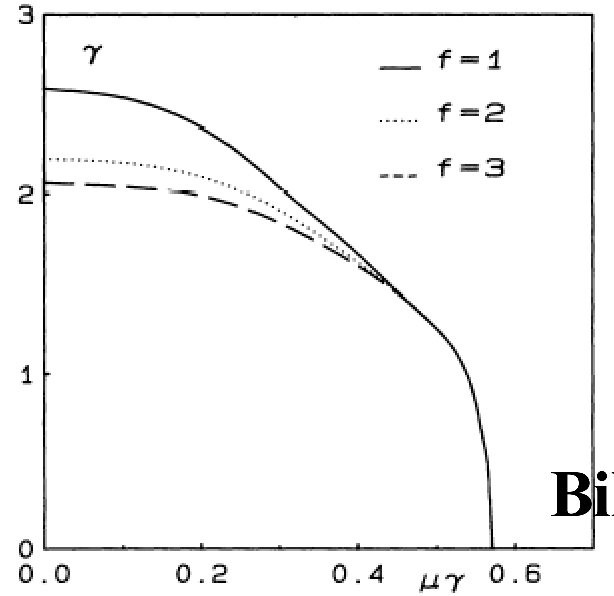


Discussions

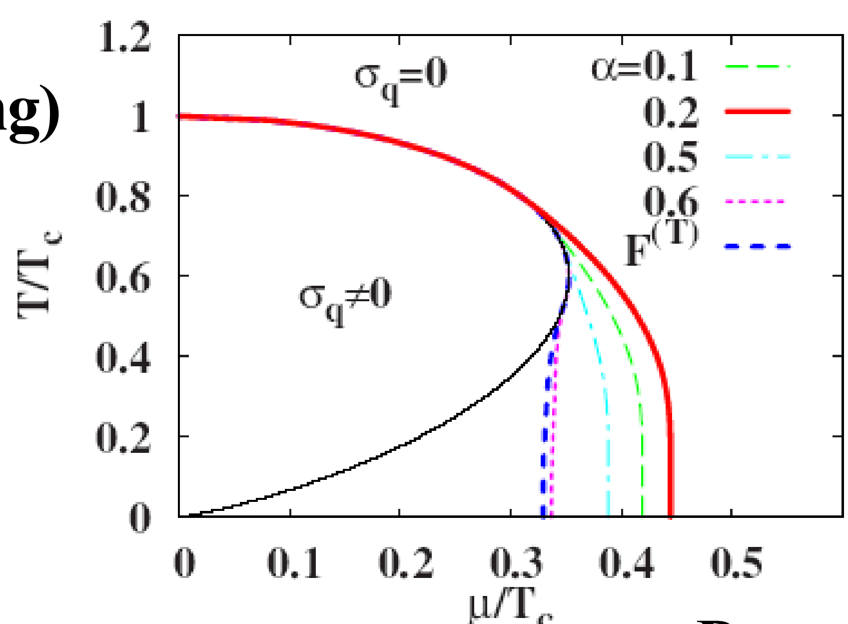
- **Present phase diagram ↔ real phase diagram**
  - **One species of staggered fermion ~ N_f=4. Should be 1st order !**
  - **Tc seems to be too high. μ_c/T_c(present) ~ 0.45 ↔ μ_c/T_c(real)~(2-3)**
  - **No stable CSC phase** *(Azcoiti et al., 2003)*
    ↔ Stable CSC phase at large μ *(Alford, Hands, Stephanov)*
- **Two parameters are introduced through identities (HS transf.)**
  - **The results should be independent from parameter choice !**
    → MFA may break the identity...
  - **How should we fix these parameters ?**
- **Is SCL-LQCD useful ? → We would like to answer "Yes" !**
  - **Chiral RMF derived in SCL-LQCD works well in Nuclear Physics (Tsubakihara, AO, nucl-th/0607046**
    Tsubakihara,Maekawa,AO, Proc. of HYP06, to appear)
  - **$1/g^2$ expansion may connect SCL-LQCD and real world.**

Small Critical μ : Common in SCL-LQCD ?

- Finite T SCL-LQCD
  - No B: $\mu_c(0)/T_c(0) \sim (0.2\text{-}0.35)$ (Nishida2004, Bilic-Karsch-Redlich 1992, ...)
  - Present: $\mu_c(0)/T_c(0) < 0.44$ (Parameter dep.)
- Monte-Carlo: $\mu_c(0)/T_c(0) > 1$
  - Fodor-Katz (Improved Reweighting) Bielefeld (Taylor expansion), de Forcrand-Philipsen (AC), ...
- Real World: $\mu_c(0)/T_c(0) > 2$
  - $T_c(0) \sim 170$ MeV, $\mu_c(0) > 330$ MeV



Bilic et al.

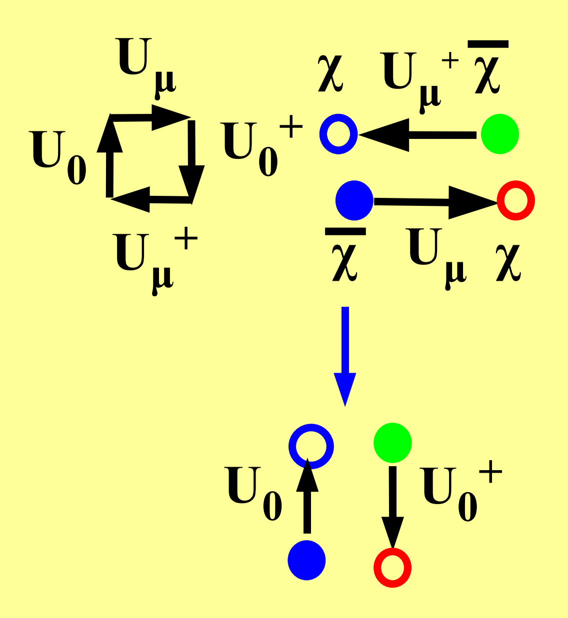


Present

$1/g^2$ expansion (w/o Baryon Effects)

- **T_c (μ=0) and μ_c (T=0): Which is worse ?**
  - **$1/g^2$ correction reduces T_c.** *(Bilic-Cleymans 1995)*
  - **Hadron masses are well explained in SCL.**
    (Kawamoto-Smit 1981, Kawamoto-Shigemoto 1982)

  → We expect Tc reduction with $1/g^2$ correction !



- **1/d expansion of plaquetts** *(Faldt-Petersson 1986)*
  - **Space-like plaquett**

$$\exp\left[\frac{1}{g^2}\sum_{x,i>j>0}\mathrm{Tr}\,U_{ij}(x)\right]\to\exp\left[\quad\frac{1}{8N_c^4g^2}\sum_{x,k>j>0}M_xM_{x+\hat{j}}M_{x+\hat{k}}M_{x+\hat{k}+\hat{j}}\right]$$

  - **Time-like plaquett**

$$\exp\left[\frac{1}{g^2}\sum_{x,j>0}\mathrm{Tr}\,U_{0j}(x)\right]\to\exp\left[-\frac{1}{4N_c^2g^2}\sum_{x,j>0}\left(V_xV_{x+\hat{j}}^{+}+V_x^{+}V_{x+\hat{j}}\right)\right]$$

$$\left(V_x=\bar{\chi}_xU_0(x)\chi_{x+\hat{0}}\right)$$

Plaquett Bosonization

- **Bosonization of Plaquetts** *(O(1/d, 1/g⁴) and Im(V) are ignored)* **+ MFA**

$$\exp(-S_F - S_g) \to \exp\left[-\frac{1}{2}\sum_x (e^\mu V_x - e^{-\mu} V_x^+) + \frac{1}{4N_c}\sum_{x,j>0} M_x M_{x+\hat{j}} + m_0 \sum_x M_x\right]$$

$$\times \exp\left[-\frac{\beta_t}{2}\varphi_t \sum_x (V_x - V_x^+) + \beta_s \varphi_s \sum_{x,j>0} M_x M_{x+\hat{j}}\right]$$

$$\times \exp\left[-L^3 N_\tau \left(\frac{\beta_t}{4}\varphi_t^2 + \frac{\beta_s d}{4}\varphi_s^2\right)\right] \quad \left(\beta_t = \frac{d}{2N_c^2 g^2}\ ,\quad \beta_s = \frac{d-1}{8N_c^4 g^2}\right)$$

$$= \exp\left[-\frac{L^3}{T}F_\varphi - \frac{\alpha}{2}\sum_x \left(e^{\tilde{\mu}} V_x - e^{-\tilde{\mu}} V_x^+\right) + \frac{1}{2}\sum_{x,y} M_x \tilde{V}_M(x,y) M_y\right]$$

$$\alpha = 1 + \beta_t \varphi_t \cosh\mu \ ,\quad \tilde{\mu} = \mu - \beta_t \varphi_t \sinh\mu$$

$$<\varphi_t> = <V^+ - V> \ ,\quad <\varphi_s> = 2 <M_x M_{x+\hat{j}}>$$

Time-like plaquetts modifies effective chemical potential

Effective Free Energy with $1/g^2$ Correction (w/o B)

- **After Quark and Time-like Link integral, we get *F* as**

$$F = \frac{d}{4N_c}\sigma^2(1+4N_c\beta_s\varphi_s) + \frac{\beta_t}{4}\varphi_t^2 + \frac{\beta_s d}{4}\varphi_s^2 - N_c\beta_t\varphi_t\cosh\mu + F_q(m_q;\tilde{\mu})$$

$$= \frac{d}{4N_c}\sigma^2 + 3\mathrm{d}\,\beta_s\sigma^4 + \frac{\beta_t}{4}\tilde{\varphi}_t^2 + F_q(m_q;\tilde{\mu})$$

$$\varphi_s = 2\sigma^2 \quad , \quad \varphi_t = \tilde{\varphi}_t + 2N_c\cosh\mu$$

Time-like plaquetts remains finite at large μ (c.f., S. Hands' talk)

$$m_q = \frac{d}{2N_c}\sigma(1+4N_c\beta_s\varphi_s - \beta_t\varphi_t\cosh\mu)$$

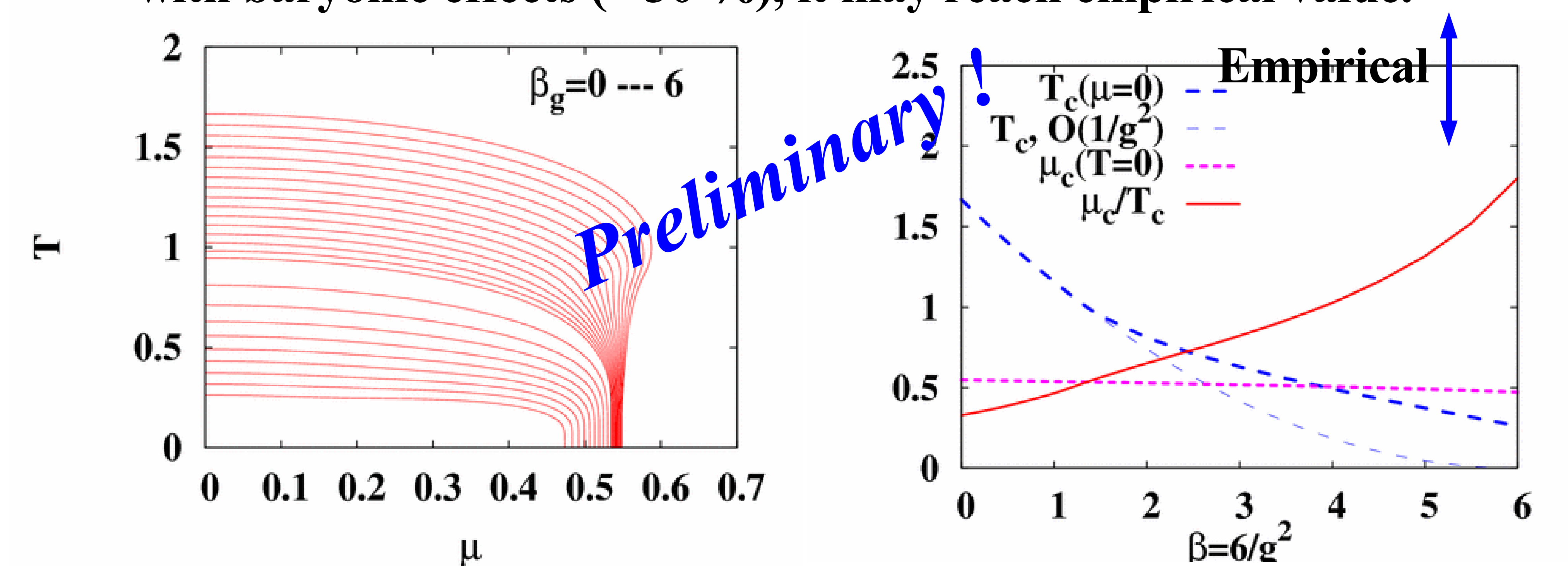
$$= \frac{d}{2N_c}\sigma(1-2N_c\beta_t\cosh^2\mu + 8N_c\beta_s\sigma^4 - \beta_t\tilde{\varphi}_t\cosh\mu)$$

$$\tilde{\mu} = \mu - \beta_t\varphi_t\sinh\mu = \mu - 2N_c\beta_t\cosh\mu\sinh\mu - \beta_t\tilde{\varphi}_t\sinh\mu$$

- **Space-like plaquett → Repulsive pot. ∝ σ^4, Enh. σ-quark couling**
- **Time-like plaquett → Reduces μ and σ-quark coupling**
  (φ_t has to be determined to minimize F_{eff})

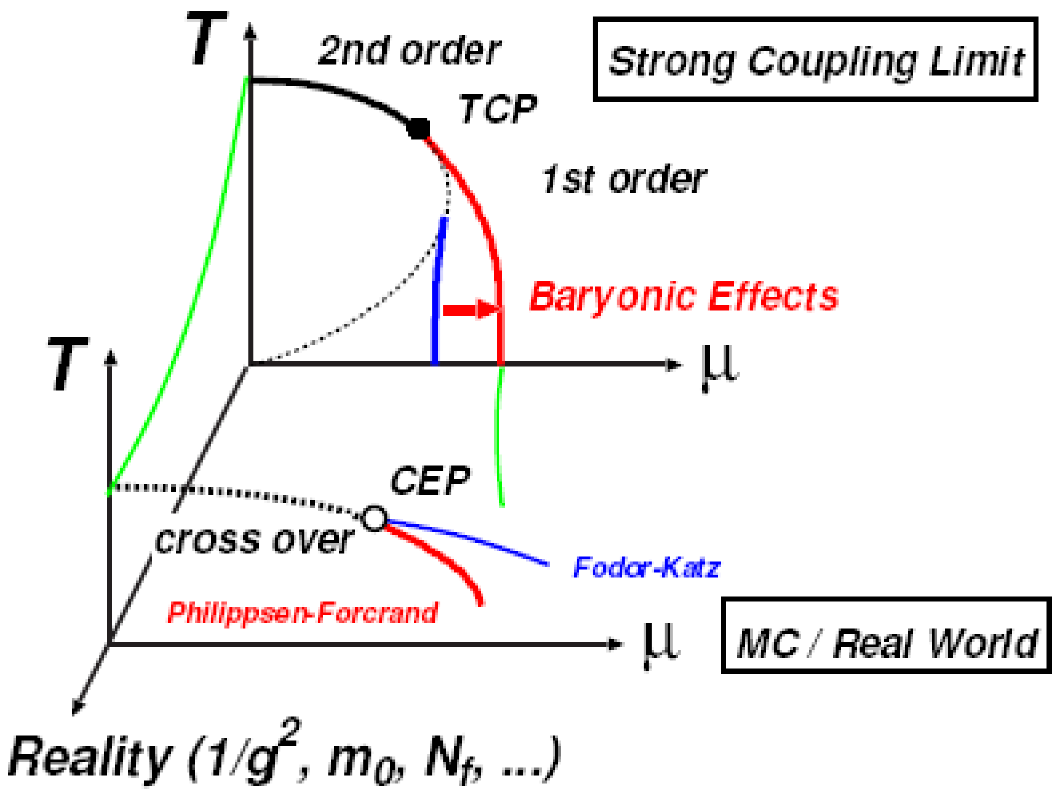
Phase Boundary with $1/g^2$ correction

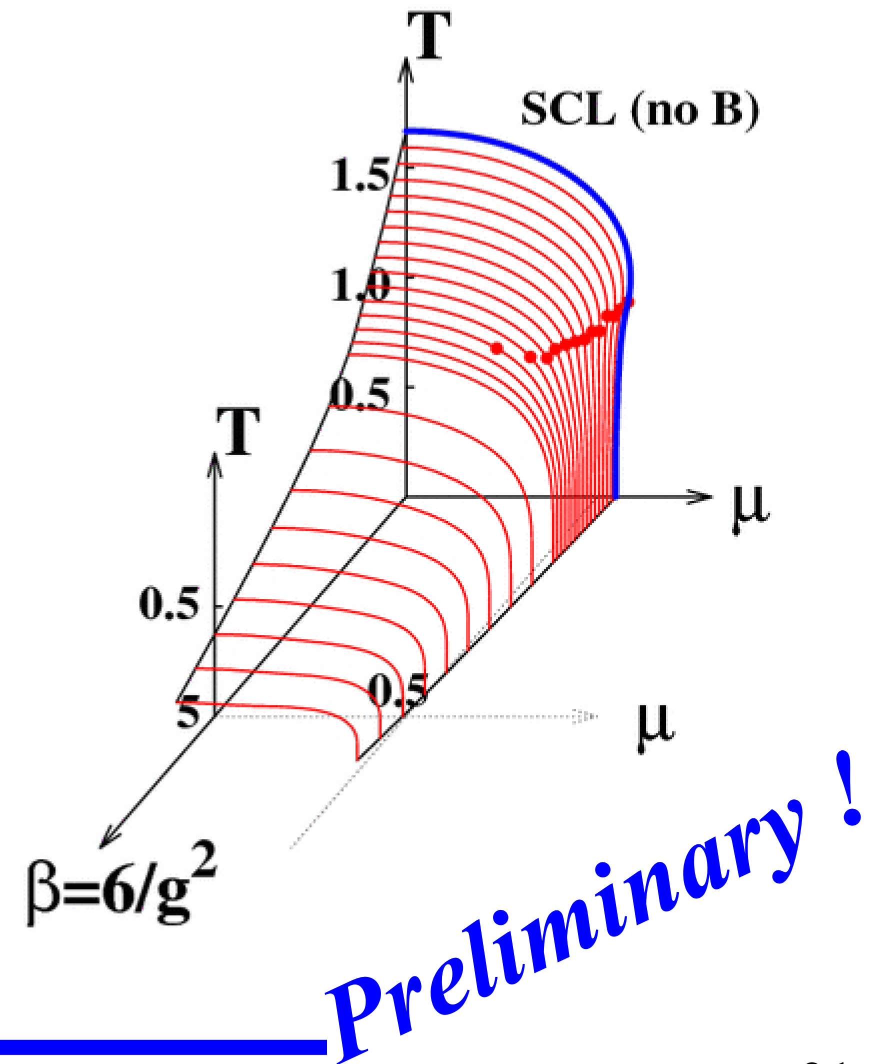
- **Rapid decrease of $T_c(\mu=0)$, and slow decrease of $\mu_c(T=0)$.**
  - **Similar reduction of σ-quark coupling and effective μ at small condensate → can be mimicked by the scaling of T**
    (c.f. Bilic-Claymans 1995 (T_c goes down), Arai-Yoshinaga (Poster, goes up).
- **Ratio $\mu_c/T_c \sim 1.8$ @ g=1.**
  - **with baryonic effects (~ 30 %), it may reach empirical value.**



Evolution of Phase Diagram

- "Reality" Axis: $1/g^2$, n_f, m_0, ... would enhance μ_c/T_c ratio
- Example: $1/g^2$ correction enhances μ_c/T_c by a factor ~(2-3).





Chiral symmetric RMF with logrithmic σ potential

K. Tsubakihara, AO, nucl-th/0607046
K. Tsubakihara, H. Maekawa, AO, Proc. of HYP06, to appear.
T.Tsubakihara shows a poster in the 3rd week.

RMF with Chiral Symmetry: Chiral Collapse

- **Naïve Chiral RMF models → Chiral collapse at low ρ** *(Lee-Wick 1974)*

$$L=\frac{1}{2}\left(\partial_\mu\sigma\,\partial^\mu\sigma+\partial_\mu\pi\,\partial^\mu\pi\right)-\frac{\lambda}{4}\left(\sigma^2+\pi^2\right)^2+\frac{\mu^2}{2}\left(\sigma^2+\pi^2\right)+c\,\sigma$$
$$+\overline{N}\,i\,\partial_\mu\gamma^\mu N-g_\sigma\overline{N}\left(\sigma+i\,\pi\,\tau\,\gamma_5\right)N$$

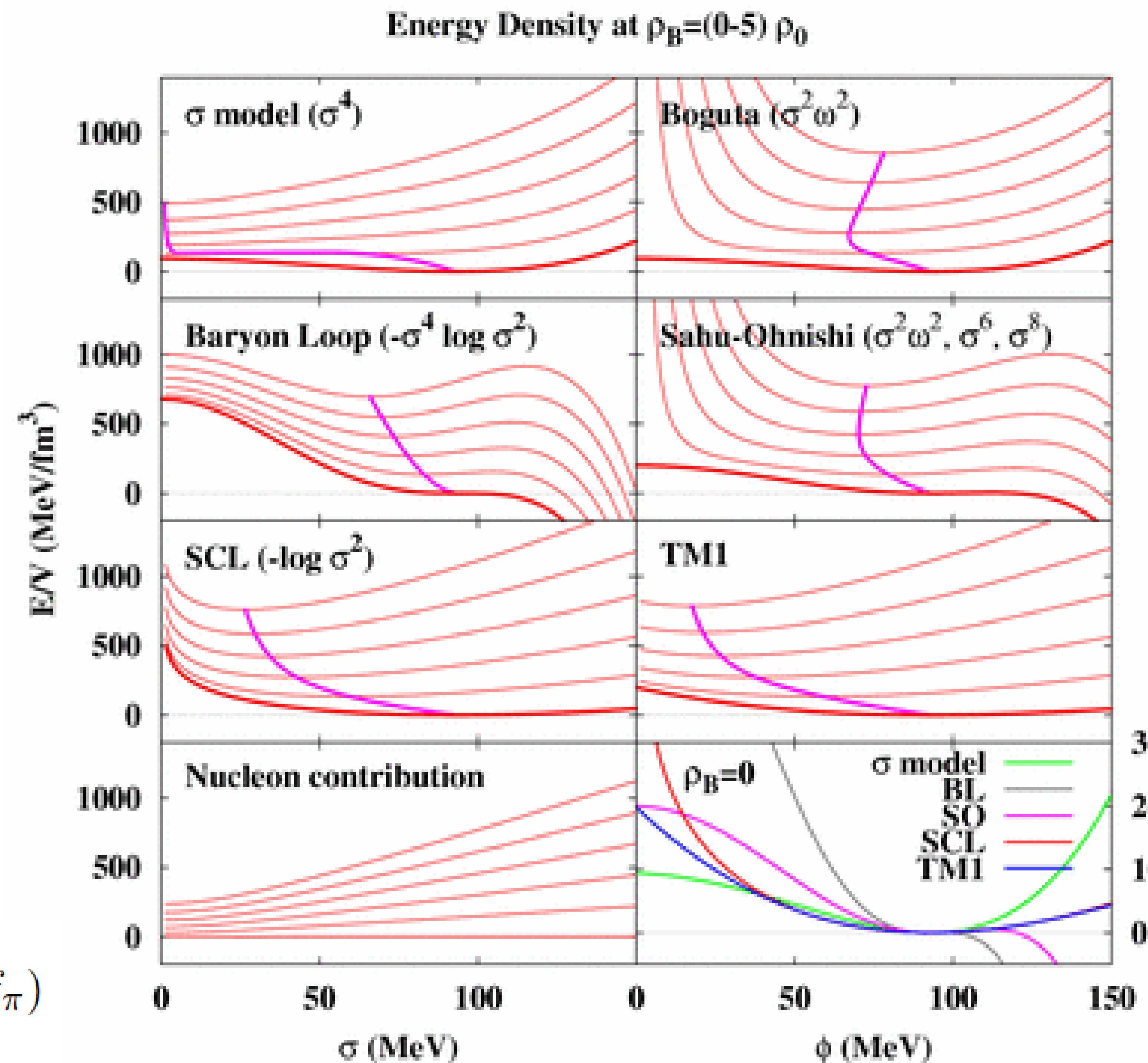
- **Prescriptions**
  - **σω coupling (too stiff EOS)** *(Boguta 1983, Ogawa et al. 2004)*
  - **Loop effects (unstable at large σ)** *(Matsui-Serot, 1982, Glendenning 1988, Prakash-Ainsworth 1987, Tamenaga et al. 2006)*
  - **Higher order terms (unstable at large σ)** *(Hatsuda-Prakash 1989, Sahu-Ohnishi 2000)*
  - ***Dielectric (Glueball) Field representing scale anomaly*** *(Furnstahl-Serot 1993, Heide-Rudaz-Ellis 1994, Papazoglou et al.(SU(3)) 1998)*
  - **Different Chiral partner assignment** *(DeTar-Kunihiro 1989, Hatsuda-Prakash 1989, Harada-Yamawaki 2001, Zschiesche-Tolos-Schaffner-Bielich-Pisarski, nucl-th/0608044) → $SU_f(3)$ extention ?*
  - ***Nucleon Structure*** *(Saito-Thomas 1994, Bentz-Thomas 2001)*

Instability in Chiral Models

- Linear σ Model
  → Chiral restor.
  Below ρ_0.
- Baryon Loop
  & Sahu-Ohnishig
  models
  → Unstable
  at large σ
- Boguta model
  → Too Stiff EOS

$$V_\sigma^{\rm BL} = \frac{m_\sigma^2}{2f_\pi^2}(\phi^2 - f_\pi^2)^2 - M_N^4 f_{\rm BL}(\phi/f_\pi)$$

$$f_{\rm BL} = -\frac{1}{4\pi^2}\left[\frac{x^4}{2}\log x^2 - \frac{1}{4} + x^2 - \frac{3}{4}x^4\right]$$



RMF with σ Self Energy from SCL-LQCD

- σ Self Energy from simple Strong Coupling Limit LQCD

$$
\begin{aligned}
S &\rightarrow -\tfrac{1}{2}(M, V_M M) && (1/d \text{ expansion}) \\
&\rightarrow b\sigma^2 + (\bar{\chi}\ \sigma\chi) && (\text{auxiliary field}) \\
&\rightarrow b\sigma^2 \boxed{- a \log \sigma^2} && (\text{Fermion Integral})
\end{aligned}
$$

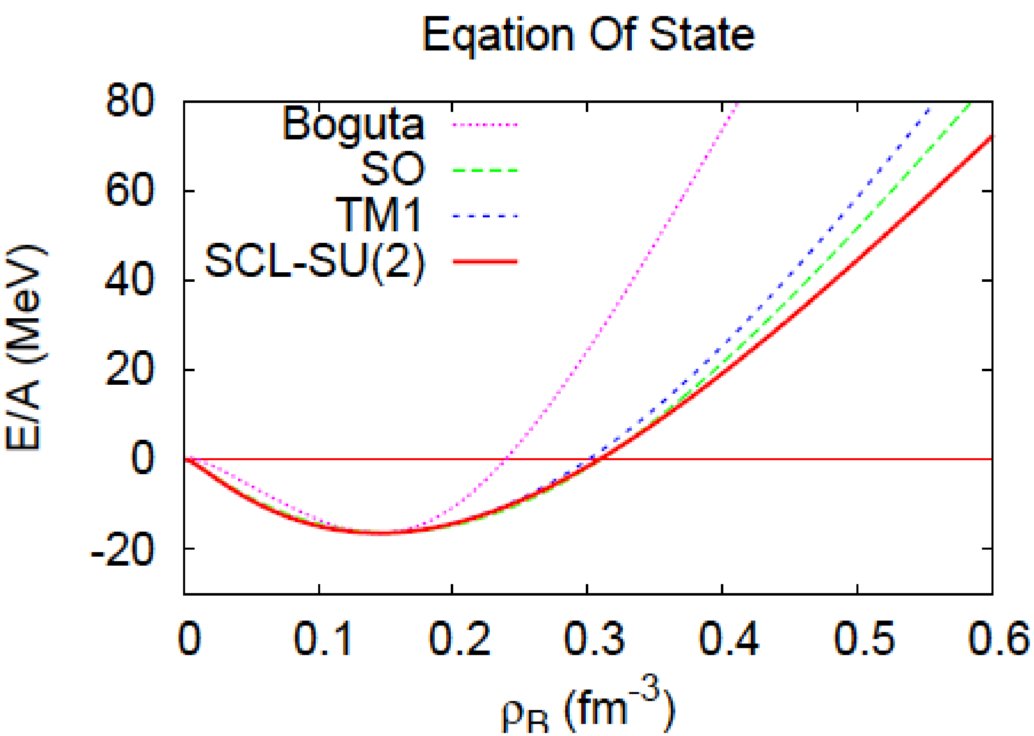
- RMF Lagrangian   Non-Analytic Type σ Self Energy
  - σ is shifted by f_π , and small explicit χ breaking term is added.

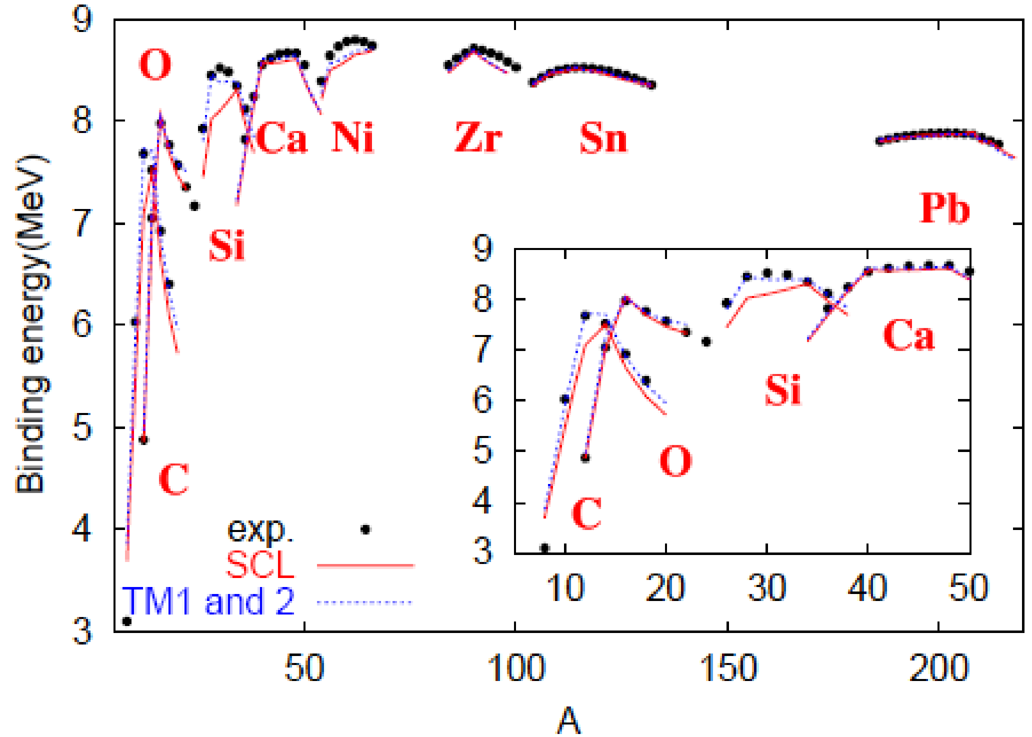
$$
\begin{aligned}
\mathcal{L} = \bar{\psi}\left(i\gamma^\mu\partial_\mu - \gamma^\mu V_\mu - M + g_\sigma\sigma\right)\psi + \mathcal{L}_\sigma^{(0)} + \mathcal{L}_\omega^{(0)} + \mathcal{L}_\rho^{(0)} \\
-U_\sigma + \frac{\lambda}{4}(\omega_\mu\omega^\mu)^2
\end{aligned}
$$

$$
U_\sigma(\sigma) = 2a\, f(\sigma/f_\pi), \quad f(x) = \frac{1}{2}\left[-\log(1+x) + x - \frac{x^2}{2}\right], \quad a = \frac{f_\pi^{\,2}}{2}\left(m_\sigma^2 - m_\pi^2\right)
$$

Nuclear Matter and Finite Nuclei

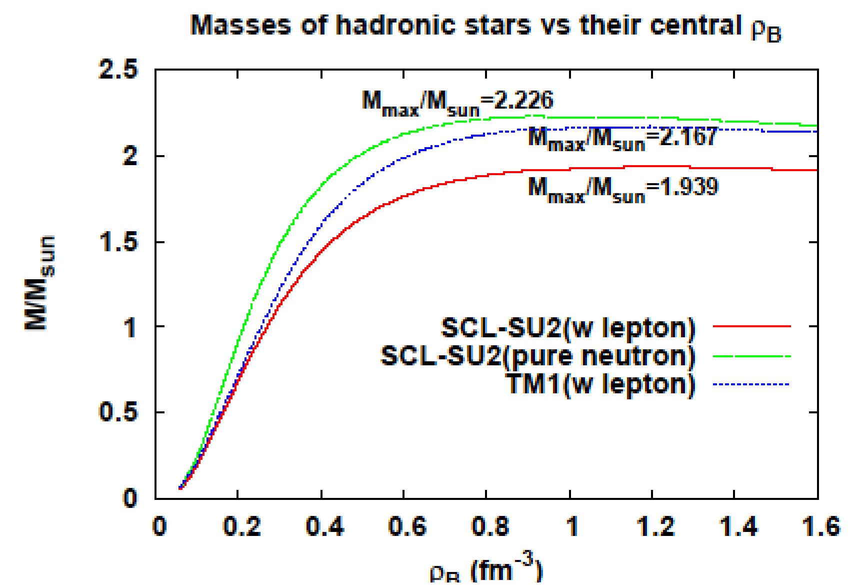
- **Nuclear Matter: By tuning λ, $g_{\omega N}$, m_σ, *EOS can be Soft !***
- **Finite Nuclei: By tuning $g_{\rho N}$, Global behavior of B.E. is reproduced, except for j-j closed nuclei (C, Si, Ni).**

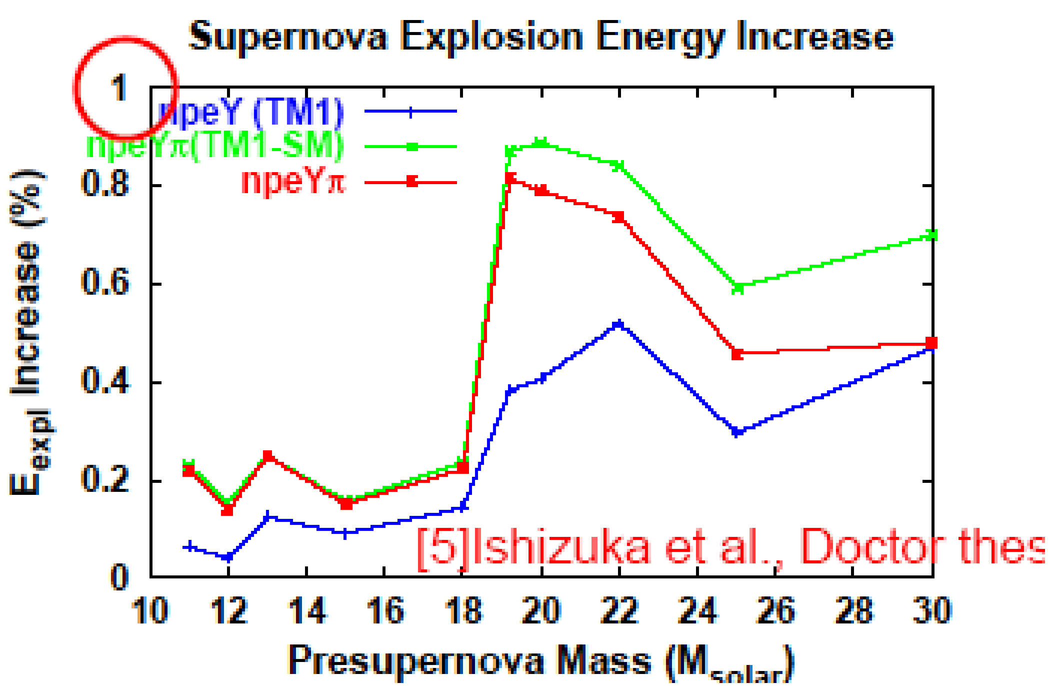


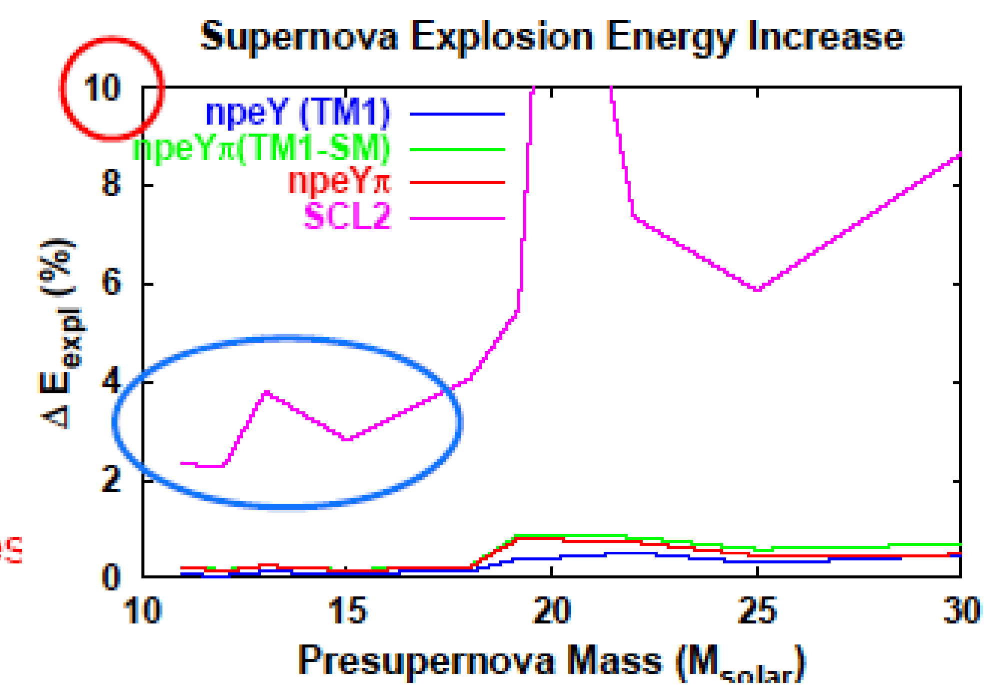


Astrophysical Applications

- **Neutron Stars**
  → Supported up to 1.9 Msolar
- **Supernova**
  → Explision E. Enhancement of around 2-4 % compared to TM1







Extention to Chiral SU(3)

- **Strong Coupling Limit LQCD guess**

$$F_{eff} = b\, Tr(M^+ M) - a \log det(M^+ M) - c_\sigma \sigma - c_\zeta \zeta + d(det\, M^+ + det\, M)$$

Bosonization + Quark integral + Explicit breaking **+ $U_A(1)$ anomaly**

$$M = \Sigma + i\Pi = diag(\sigma/\sqrt{2}, \sigma/\sqrt{2}, \zeta)(in\ \ MFA)$$

$$= a\left[2f(\sigma/f_\pi) + \frac{1}{2} f(\zeta/f'_\zeta)\right] + \frac{m_\sigma^2}{2}\sigma^2 + \frac{m_\zeta^2}{2}\zeta^2 + \xi\sigma\zeta + const.$$

(after shifting $\sigma \to f_\pi + \sigma$, $\zeta \to f_\zeta + \zeta$)

$$f(x) = \frac{1}{2}\left[-\log(1+x) + x + \frac{x^2}{2}\right], \quad a = \frac{f_\pi^2}{2}\left(m_\sigma^2 - m_\pi^2\right)$$

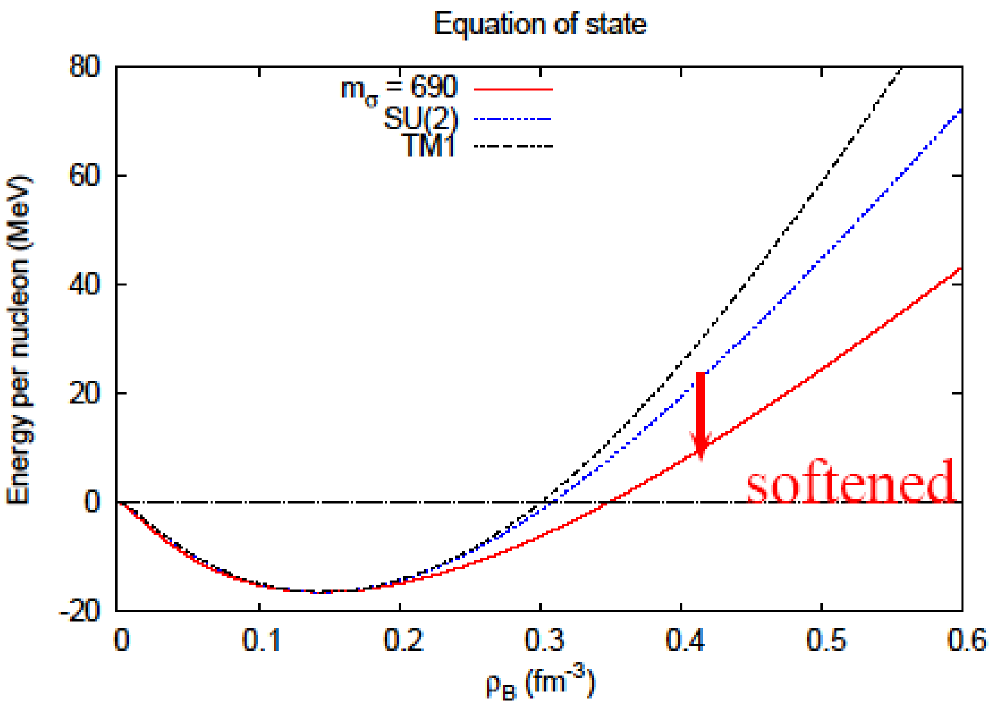
most of the parameters are determined to fit meson masses !

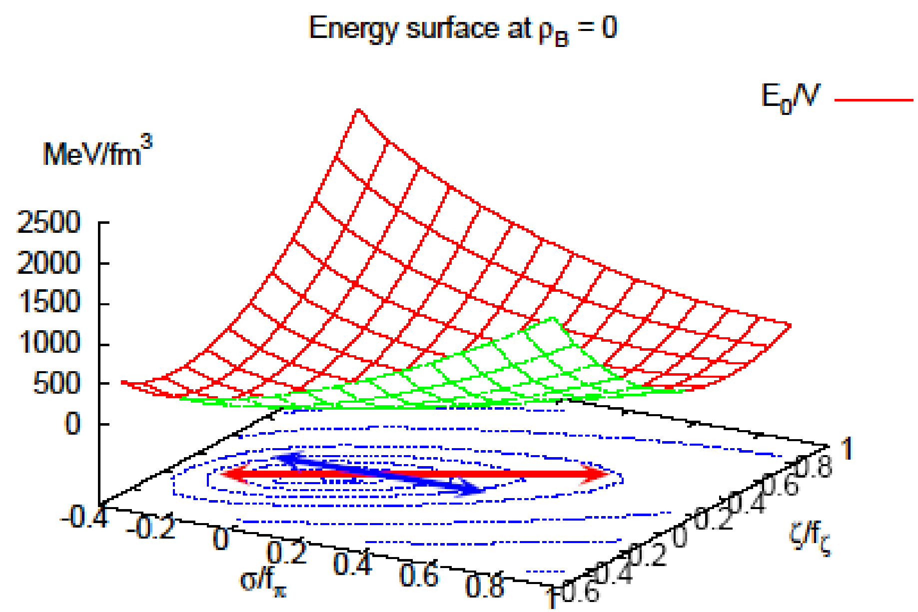
→ One parameter m_σ

Is it consistent with Nuclear Matter and Finite Nuclei ?

Symmetric Nuclear Matter in Chiral SU(3) RMF

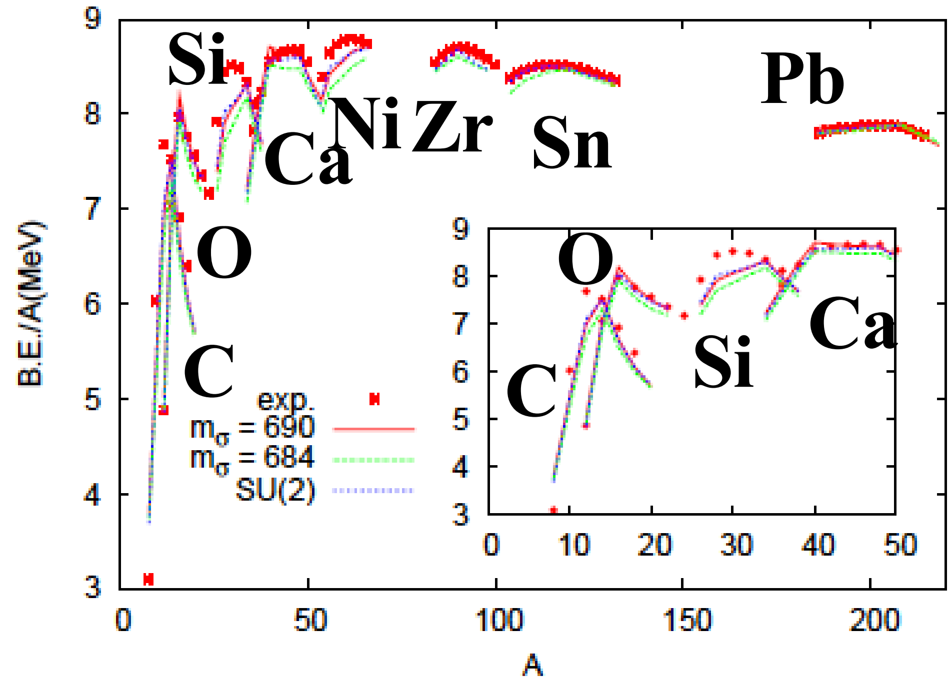
- **Soft EOS in Chiral SU(3) RMF**
  - **σ-ζ mixing → Evolution along σ-ζ valley**
  - **K= 216 MeV @ m_σ = 690 MeV**
    → Consistent with K=210 ± 30 MeV

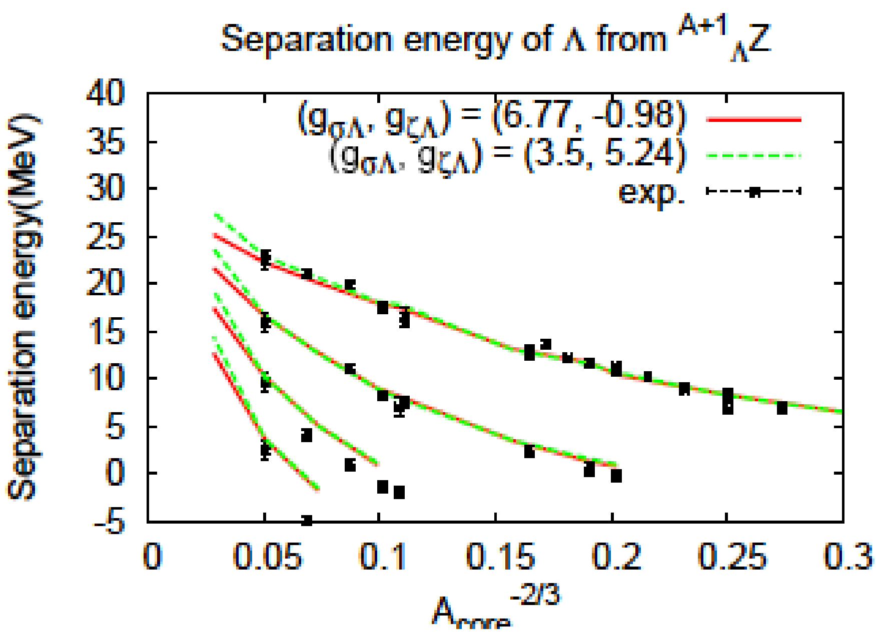


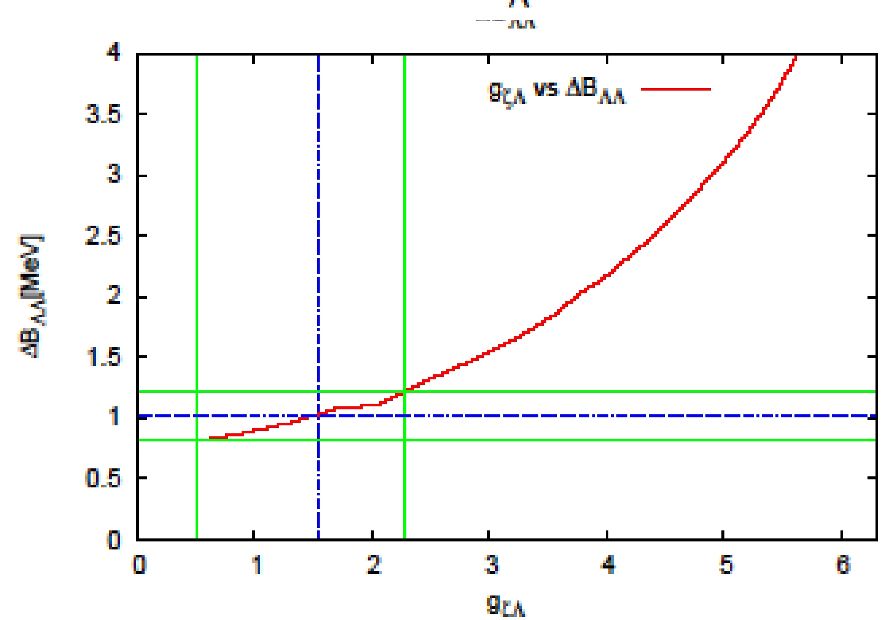


Finite Nuclei

- **Other Model Parameters**
  - $g_{\rho N} \rightarrow$ Normal Nuclei
  - $(g_{\sigma\Lambda}, g_{\zeta\Lambda}) \rightarrow$ Single Λ Nuclei
  - $g_{\zeta\Lambda} \rightarrow {}^{6}_{\Lambda\Lambda}\mathrm{He}$
    ($SU_V(3)$ is assumed for $g_{V\Lambda}$)







Summary

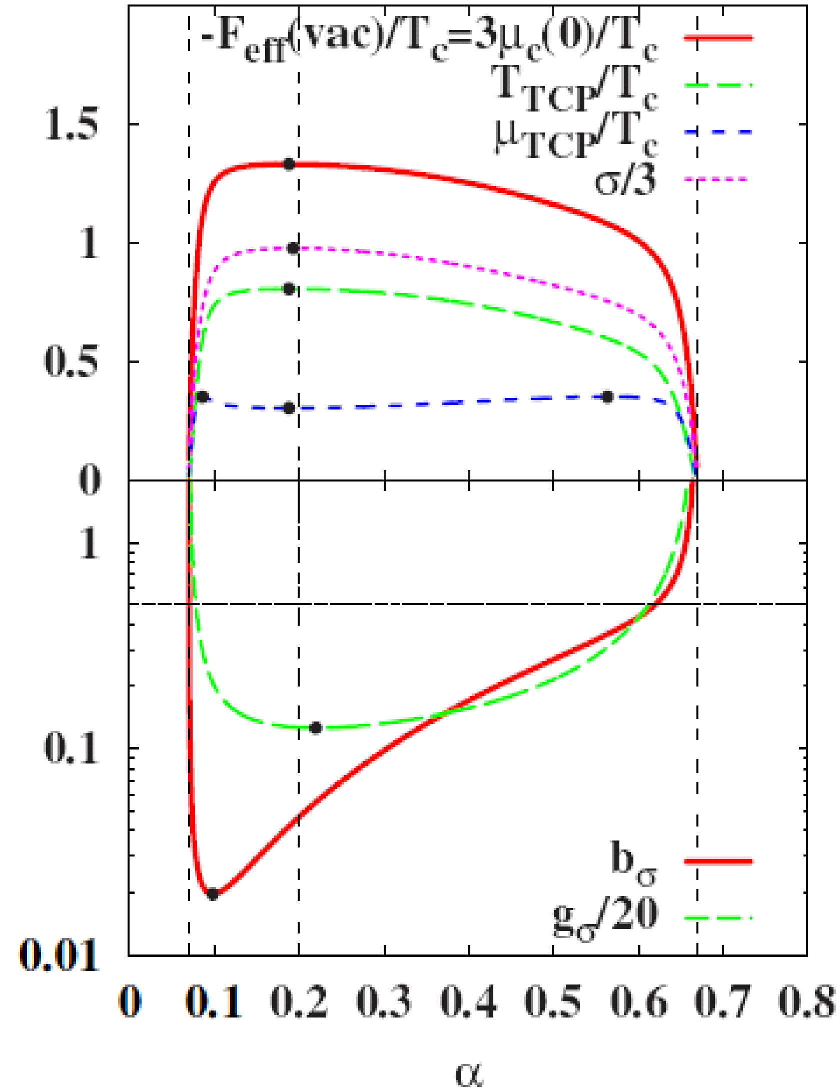
- **We obtain an analytical expression of effective free energy *at finite T and finite μ* with *baryonic composite* effects in the strong coupling limit of lattice QCD for color SU(3).**
  - *MFA, QG integral, 1/d expansion (NLO, O(1/√d)), bosonization with diquarks and baryon potential field using* $(\bar{b}b)^2=0$ *, Linear approx.,* ***zero diquark cond.(Color Angle Average), variational parameter choice***
- **Baryonic action is found to result in *Free Energy Gain* and *Extension of Hadron Phase to Larger μ by around 30 %.***
  - *Problem: Too small* μ_c/T_c *in the Strong Coupling Limit.*
- **Strong Coupling Limit is useful to understand Dense Matter**
  - *SCL gives a qualitative insight.*
  - *1/g² correction seems to work well (Do not believe us yet ...)*
  - *Application to chiral RMF (K. Tsubakihara, AO, nucl-th/0607046)*
    → 3rd week Poster by Tsubakihara

Backups

Parameter Choice

- **In bosonization, two parameters (γ and α) are introduced through identities.**
  - **Major effects**
    → Modify the energy scale
  - **Minor effects**
    → Controls the higher order potential terms

→ *We have fixed them to minimize F_{eff}/T_c at vacuum*



Baryon Integral

- **Baryon integral can be evaluated in an almost analytic way !**

$$F_{\rm eff}^{(b)}(g_\omega\omega) = \frac{1}{\beta L^3}\log \mathrm{Det}\left[1+g_\omega\omega V_B\right]$$

$$\simeq \frac{-a_0^{(b)}/2}{(4\pi\Lambda^3/3)}\int_0^\Lambda 4\pi k^2 dk \log\left[1+\frac{g_\omega^2\omega^2 k^2}{16}\right]$$

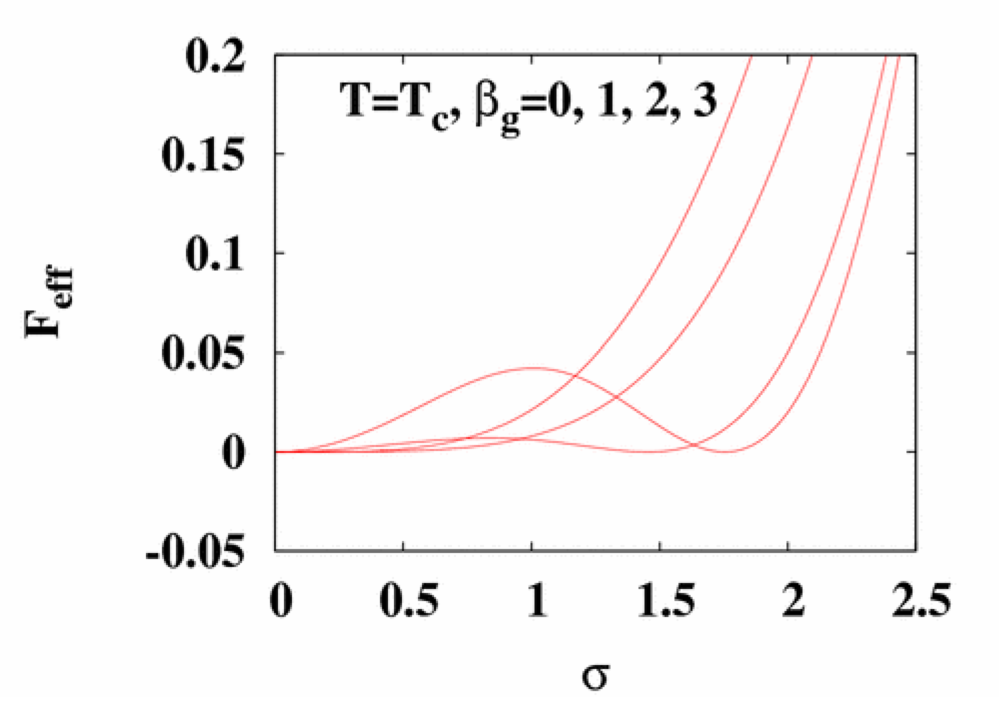
$$= -a_0^{(b)} f^{(b)}\left(\frac{g_\omega\omega\Lambda}{4}\right)$$

$$f^{(b)}(x) = \frac{1}{2}\log(1+x^2) - \frac{1}{x^3}\left[\arctan x - x + \frac{x^3}{3}\right]$$

$$a_0^{(b)} = 1.0055\ ,\ \ \Lambda = 1.01502\times\pi/2$$

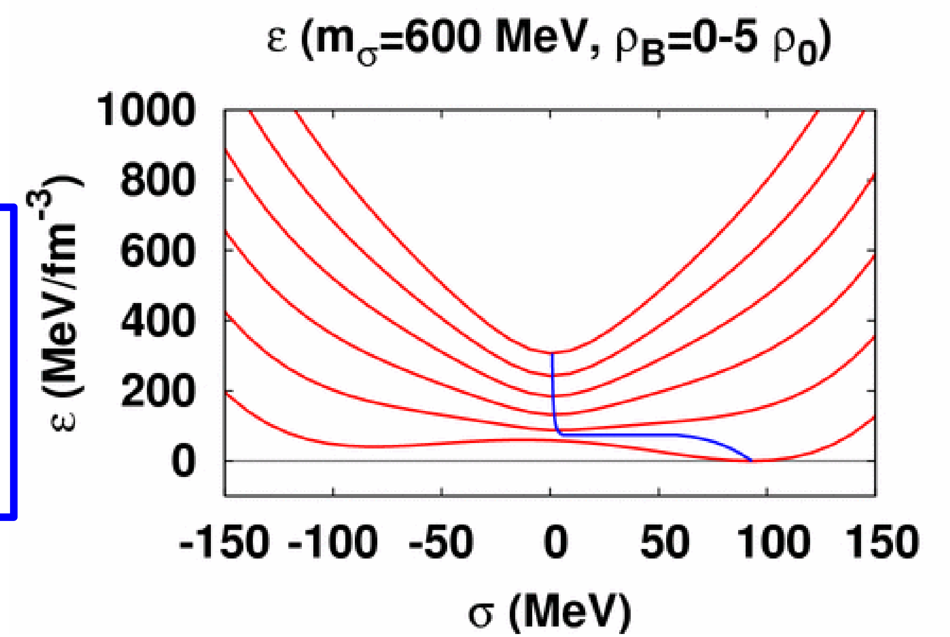
Disappearance of TCP

- **Tri-Critical point disappears at around $\beta_g \sim 1.4$**
  $\rightarrow$ 1st order phase transition even at μ=0.
  - **One species of staggered fermion in the chiral limit ~ mass less quark flavor N_f=4**
  - **Need quarter-root treatment or Wilson fermion with finite s-quark mass**
  - **Reason: Space-like plaquett enhances σ-quark coupling at large condensate ???**



Problems in RMF with Chiral Symmetry

- **Sudden Change of <σ>**

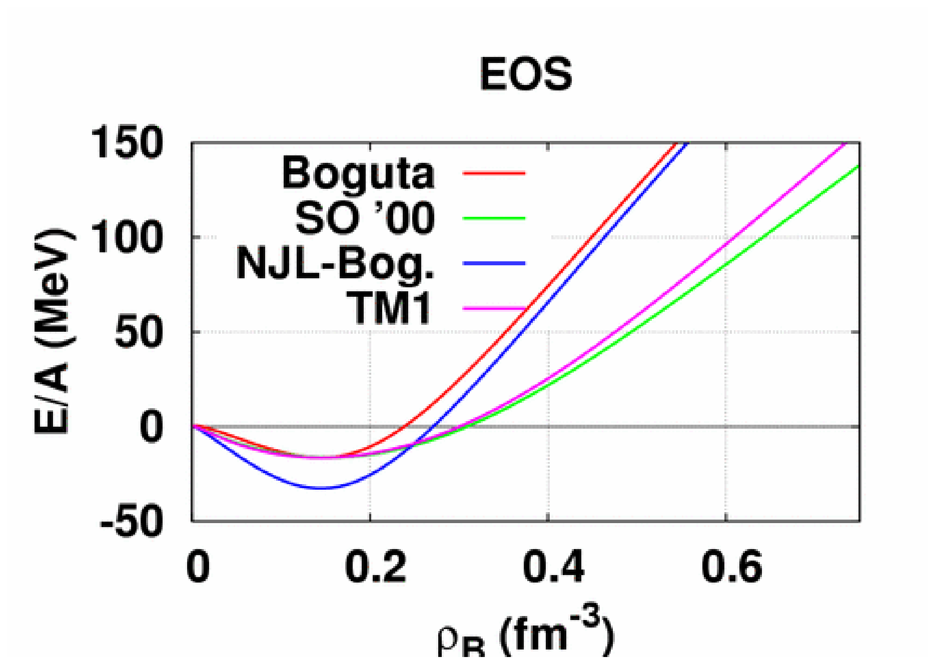


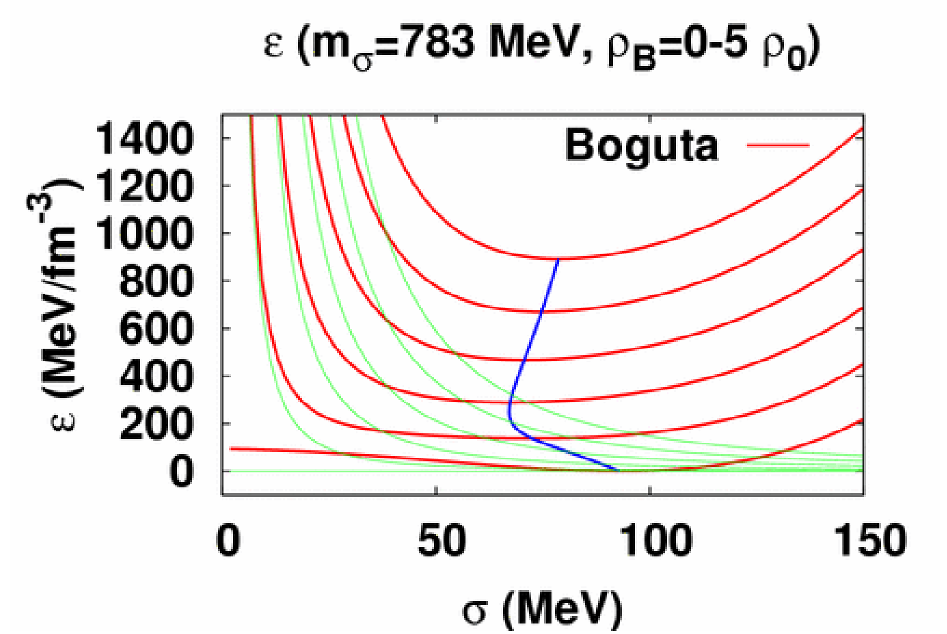
- **σ ω Coupling**

$$L_{\omega\sigma} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} C_{\sigma\omega} \sigma^2 \omega^2 - g_\omega \overline{N} \gamma_\mu \omega^\mu N$$

$$\omega = g_\omega \rho_B / C_{\sigma\omega} \sigma^2 \quad \rightarrow \quad V_{\sigma\omega} = \frac{g_\omega^2 \rho_B^2}{2 C_{\sigma\omega} \sigma^2}$$

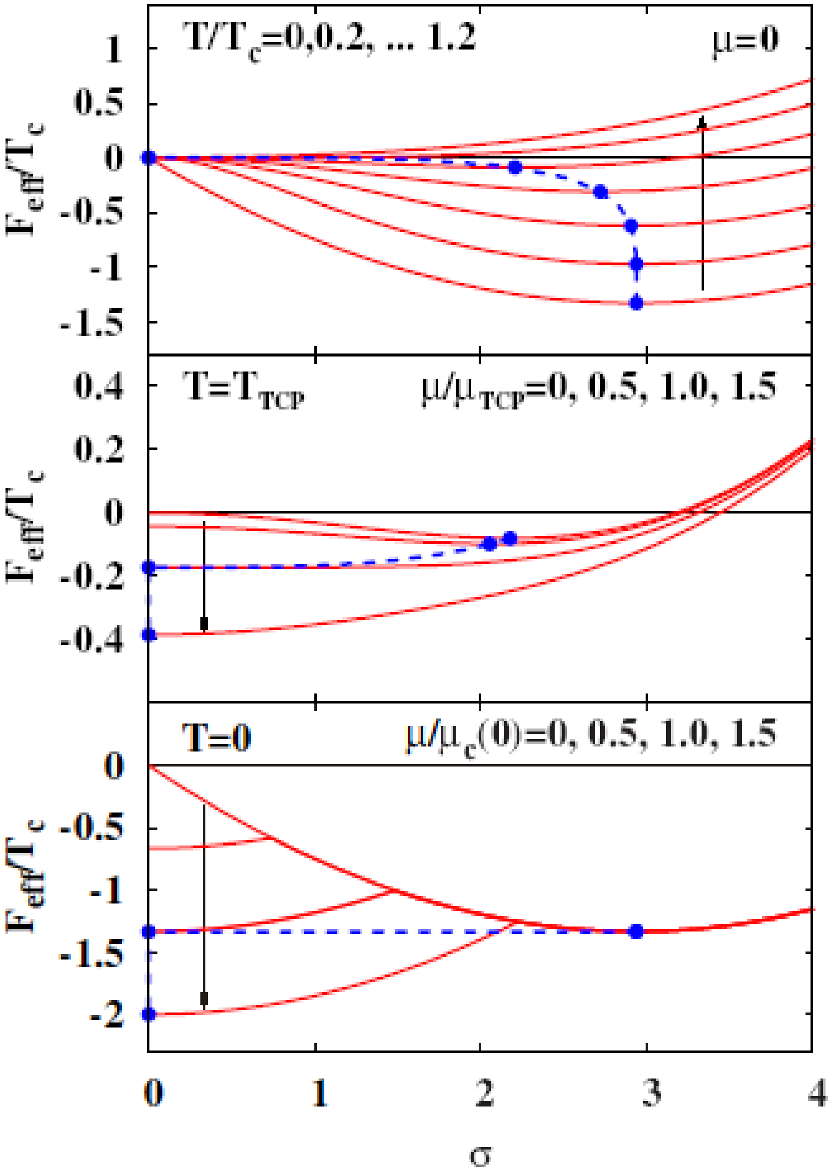
- **Stiff EOS**



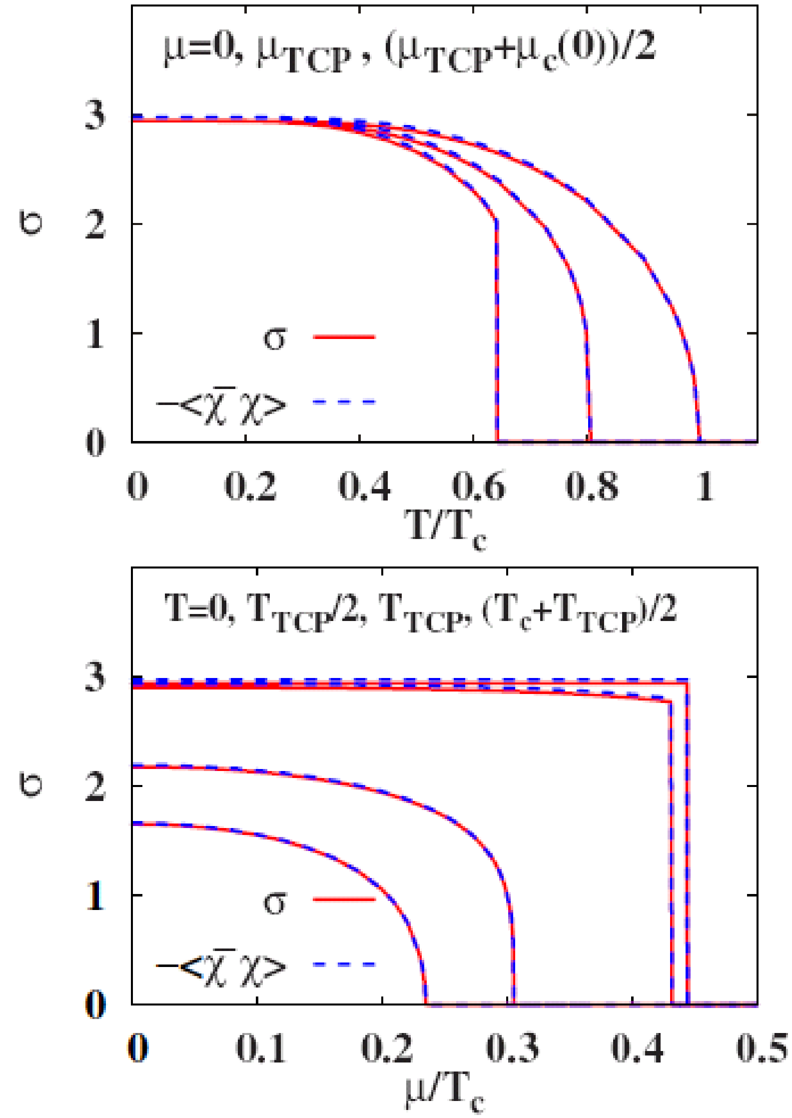


Figures

Energy surface



Validity of "Linear" Approx.

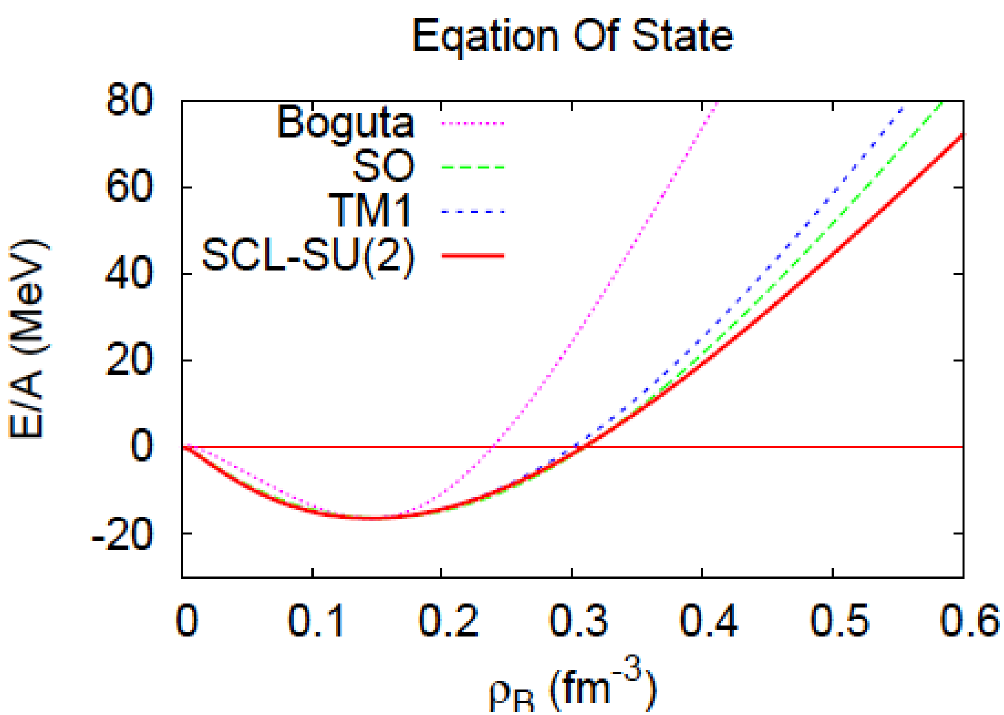


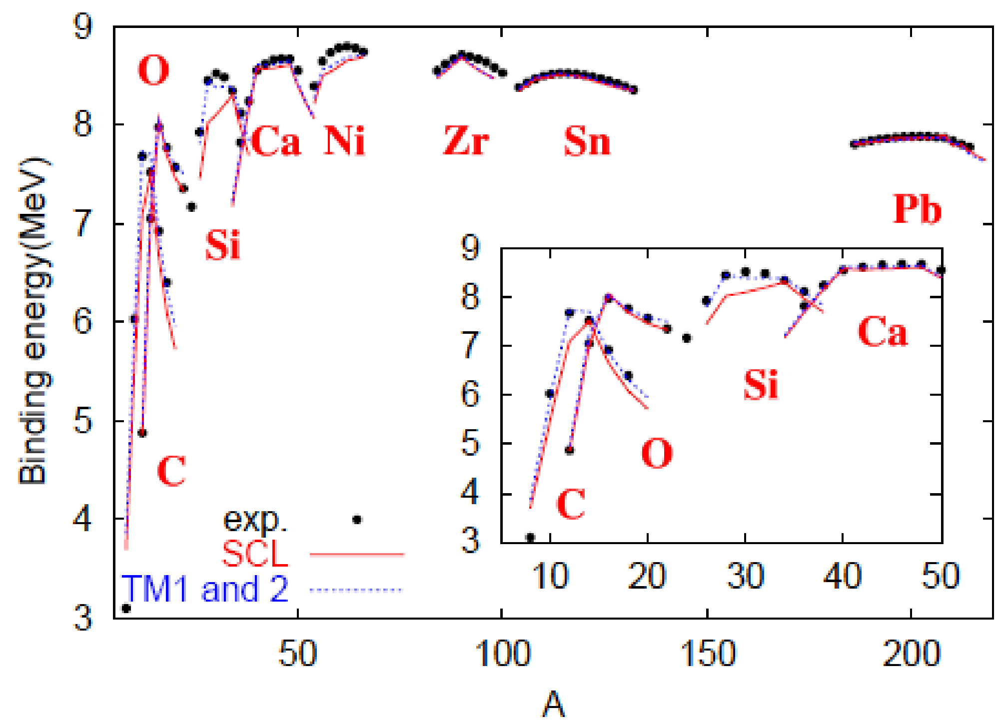
RMF with σ Self Energy from SCL-LQCD

- **σ Self Energy from simple SCL-LQCD**

$$S \to -\frac{1}{2}(M V_M M) \to \frac{1}{2}(\sigma V_M \sigma) + (\bar{\chi} V_M \sigma \chi) \to U_\sigma \simeq \frac{1}{2} b \sigma^2 - N_c \log \sigma^2$$

- **Chiral RMF with logarithmic σ potential**
  (Tsubakihara-AO, nucl-th/0607046)





RMF with σ Self Energy from SCL-LQCD

- **σ Self Energy from simple Strong Coupling Limit LQCD**

$$
\begin{aligned}
S \quad &\to \quad -\tfrac{1}{2}(M, V_M M) &&(1/d \text{ expansion})\\
&\to \quad b\sigma^2 + (\bar{\chi}\,\sigma\chi) &&(\text{auxiliary field})\\
&\to \quad b\sigma^2 \boxed{-\,a\log\sigma^2} &&(\text{Fermion Integral})
\end{aligned}
$$

- **RMF Lagrangian** **Non-Analytic Type σ Self Energy**
  - **σ is shifted by f_π, and small explicit χ breaking term is added.**

$$
\mathcal{L} = \bar{\psi}\left(i\gamma^\mu\partial_\mu - \gamma^\mu V_\mu - M + g_\sigma\sigma\right)\psi + \mathcal{L}_\sigma^{(0)} + \mathcal{L}_\omega^{(0)} + \mathcal{L}_\rho^{(0)}
$$
$$
-U_\sigma + \frac{\lambda}{4}(\omega_\mu\omega^\mu)^2
$$

$$
U_\sigma(\sigma) = 2a\, f(\sigma/f_\pi),\quad f(x) = \frac{1}{2}\left[-\log(1+x) + x - \frac{x^2}{2}\right],\quad a = \frac{f_\pi^{\ 2}}{2}\left(m_\sigma^2 - m_\pi^2\right)
$$

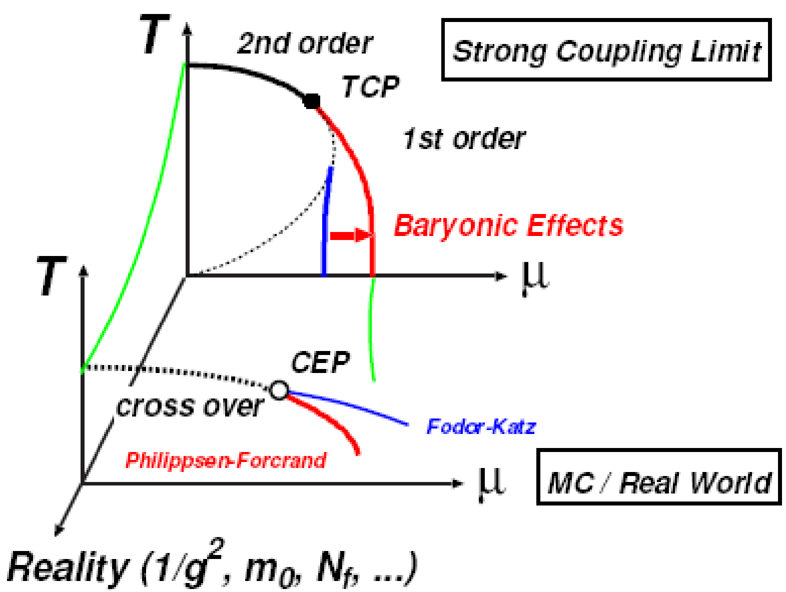
Towards Realistic Understanding

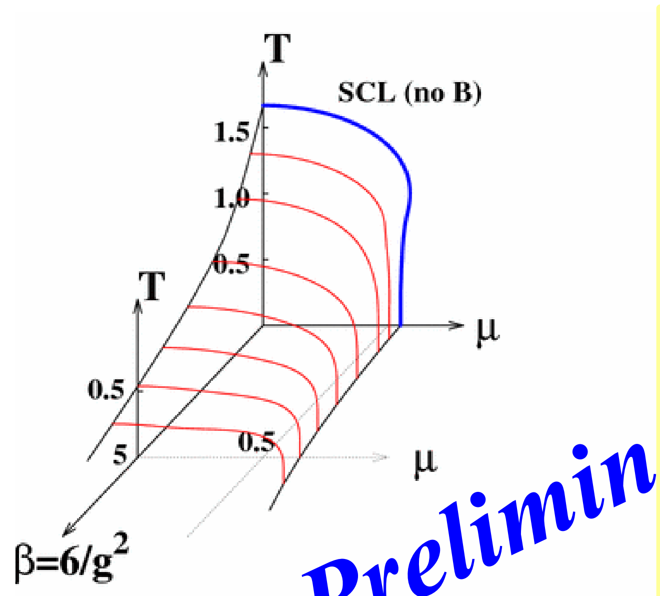
- "Reality" Axis: $1/g^2$, n_f, m_0, ... would enhance μ_c/T_c ratio
- Example: $1/g^2$ correction enhances μ_c/T_c by a factor ~(2-3).

$$\exp\left[\frac{1}{g^2}\mathrm{Tr}\,U_{0j}(x)\right]\sim\exp\left[-V_x V^+_{x+\hat{j}}/4N_c^2g^2\right]\to\exp\left[-\left(\varphi^2+\boxed{2\varphi\left(V_x-V^+_{x+\hat{j}}\right)}\right)/16N_c^2g^2\right]$$

$$S_F^{(t)}=\frac{1}{2}\left(e^{\mu}V_x-e^{-\mu}V_x^+\right)\to\boxed{\frac{\alpha}{2}\left(\exp\tilde{\mu}\,V_x-\exp(-\tilde{\mu})\,V_x^+\right)}\qquad\left(V_x=\bar{\chi}_x U_0(x)\chi_{x+\hat{0}}\right)$$

Time-like plaquetts can modify effective chemical potential





Preliminary !