Phase diagram and hadron properties in the strong coupling lattice QCD

Brown-Rho Scaling in the strong coupling lattice QCD

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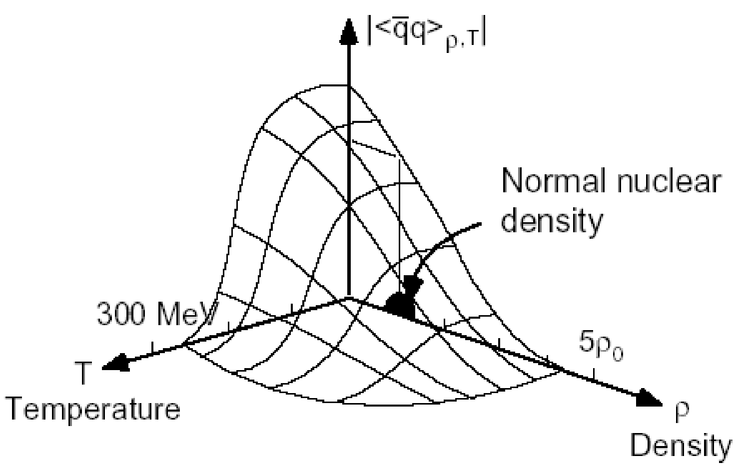
- **Introduction**
- **Hadron Mass in a Finite T treatment of Strong Coupling Limit for Lattice QCD**
  Kawamoto, Miura, AO, PoS(LAT07) (2007), 209 [arXiv:0710.1720]*
- **Finite Coupling Effects in the Strong Coupling Lattice QCD**
  AO,Kawamoto,Miura,Tsubakihara,Maekawa, PTP Supp. 168(2007),261 [arXiv:0704.2823].
  AO,Kawamoto, Miura, J. Phys. G 34 (2007), S655 [arXiv:hep-lat/0701024].
- **Summary**

Hadron Mass in Nuclear Matter

- **Medium meson mass modification**
  - **may be the signal of partial restoration of chiral sym.**
    Kunihiro,Hatsuda, PRep 247('94),221; Brown, Rho, PRL66('91)2720; Hatsuda, Lee, PRC46('92)R34.
  - **and is suggested experimentally.**
    CERES Collab., PRL75('95),1272;
    KEK-E325 Collab.(Ozawa et al.), PRL86('01),5019.



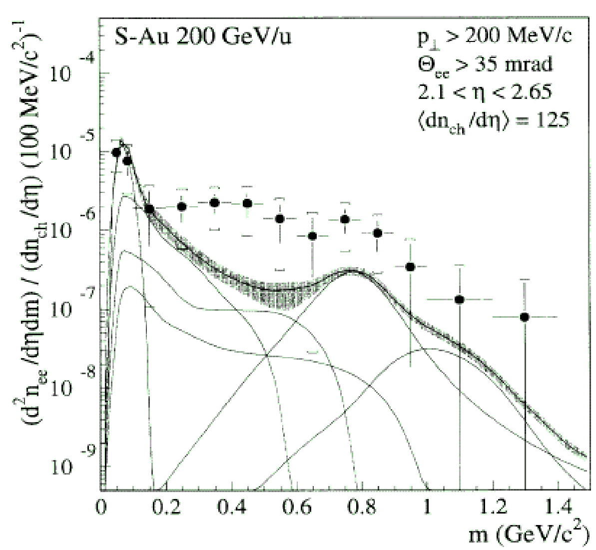
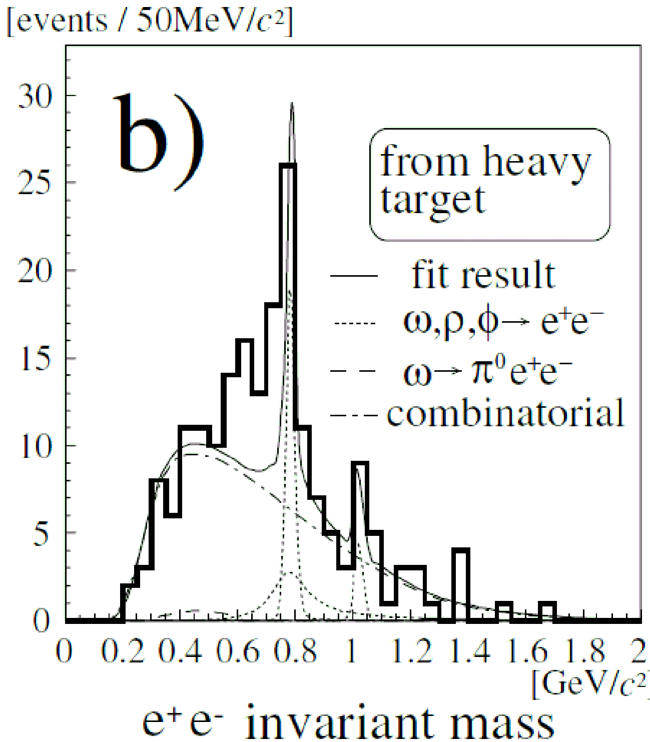
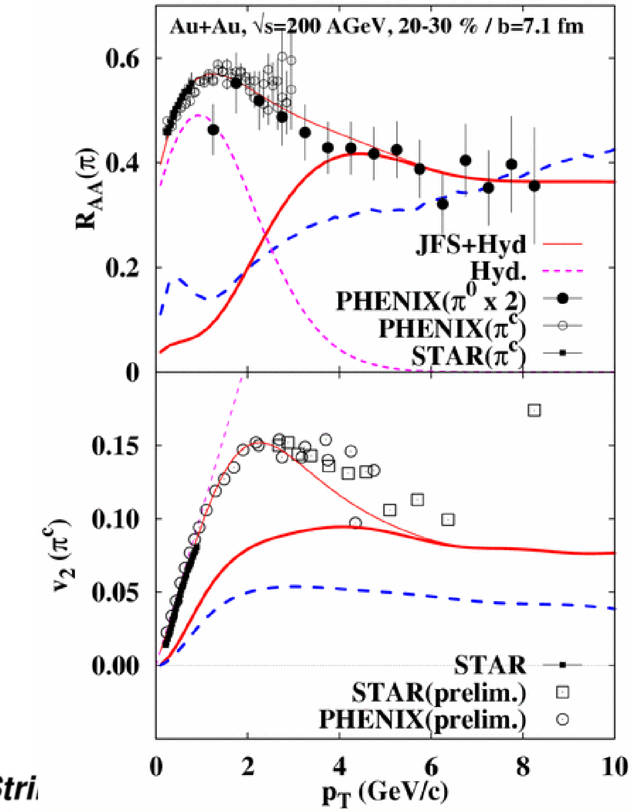


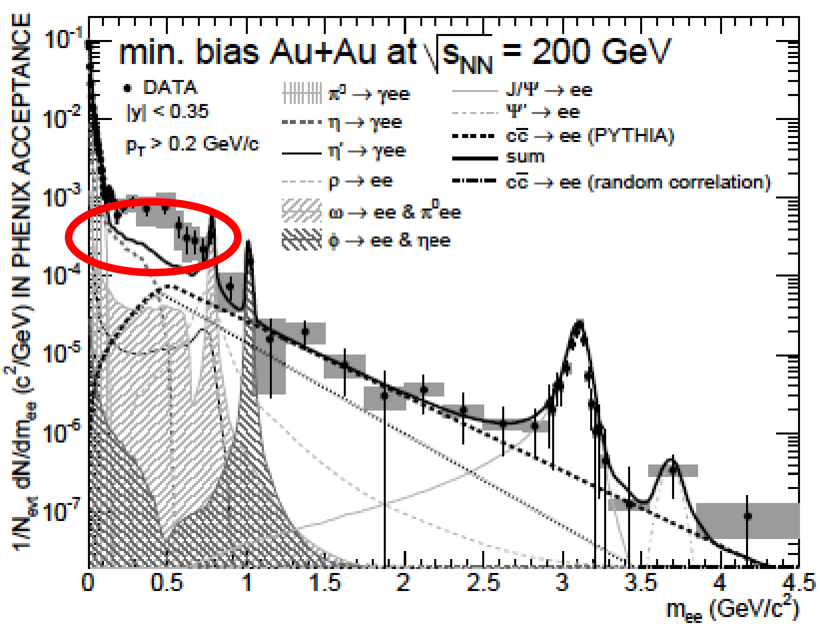
FIG. 4. Inclusive e^+e^- mass spectra in 200 GeV/nucleon S Au collisions. For explanations see Fig. 2.

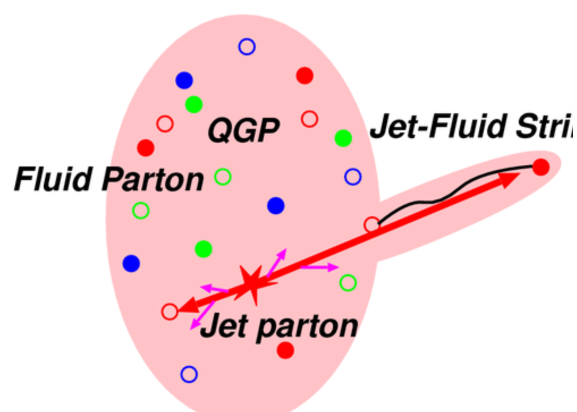


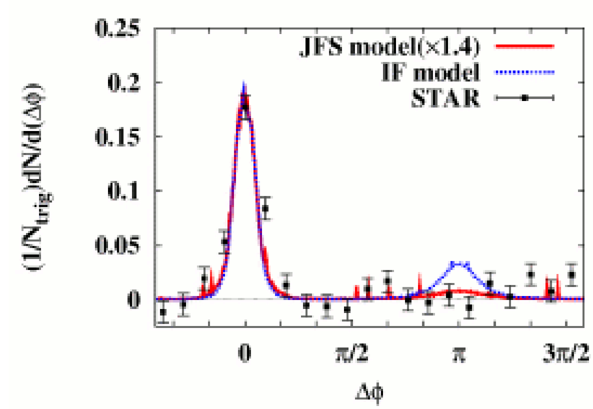
Hadron Mass in Nuclear Matter

- **What kind of matter is created at RHIC ?**
  - ***Deconfined*** **quark and gluon matter**
    → Jet quenching, Large v2,
    No backward h-h corr.
    Isse, Hirano, Nara, AO, Yoshino, nucl-th/0702068.
  - **Chiral Restored Matter ?**
    PHENIX Collab., arXiv:0706.3034







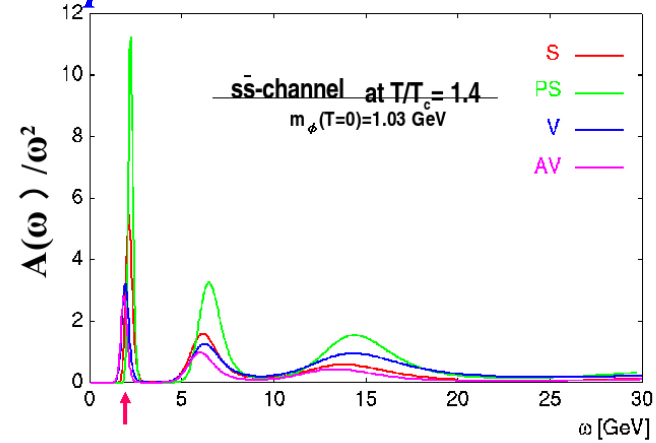


Hadron Mass in Nuclear Matter

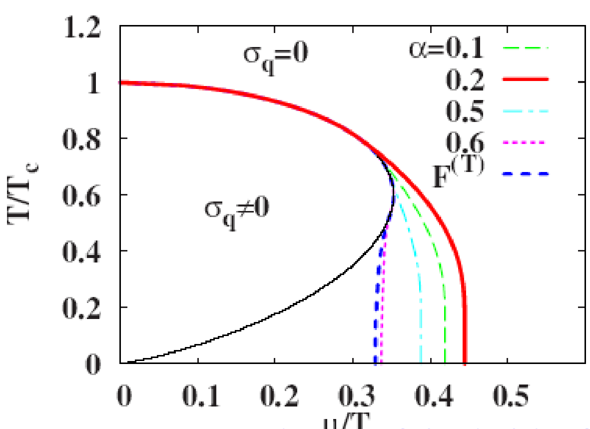
- **Can we understand it in Lattice QCD ?**
  - **Finite T: It is possible !**
  - **Finite μ: Difficult due to the sign problem.**
- **Strong Coupling Limit of Lattice QCD**
  → We can study finite (T, μ) !
  - **Hadron masses in the Zero T treatment**
    Kluberg-Stern, Morel, Petersson, '83; Kawamoto, Shigemoto, '82.
  - **To do: Finite T, Baryons with finite T, $1/g^2$ corr., ...**

This Talk

Asakawa, Nakahara, Hatsuda, hep-lat/0208059.

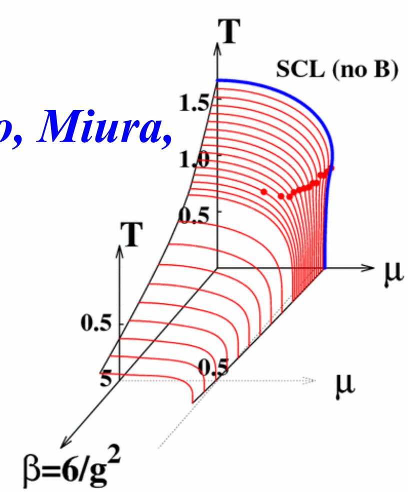


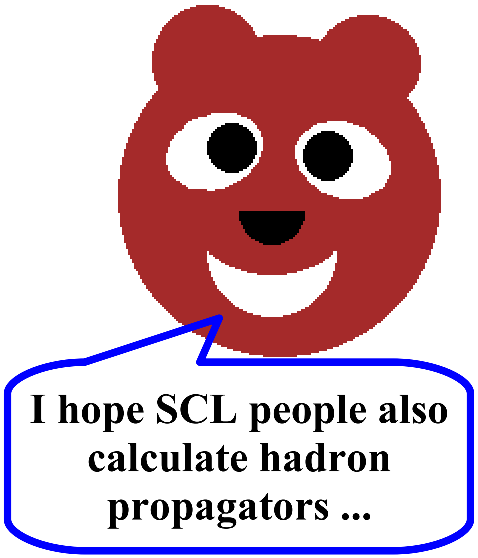
JPS Symp., 2007/03
A. Nakamura said,



Kawamoto, Miura, Ohnishi, Ohnuma, PRD75('07)014502

Ohnishi, Kawamoto, Miura, hep-lat/0701024

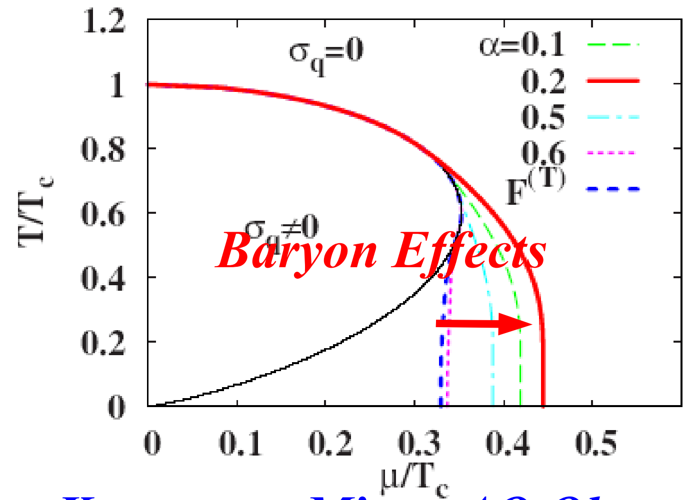




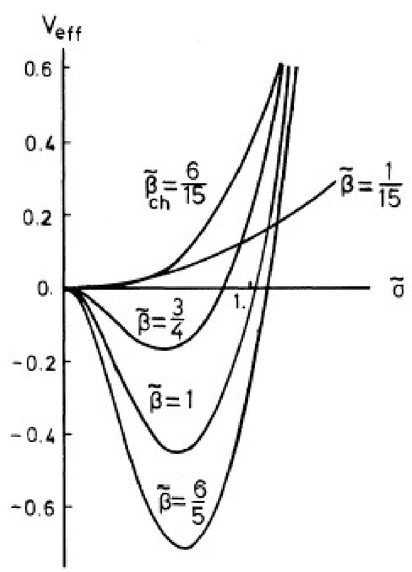
Hadron Mass Spectrum in the Strong Coupling Limit of Lattice QCD

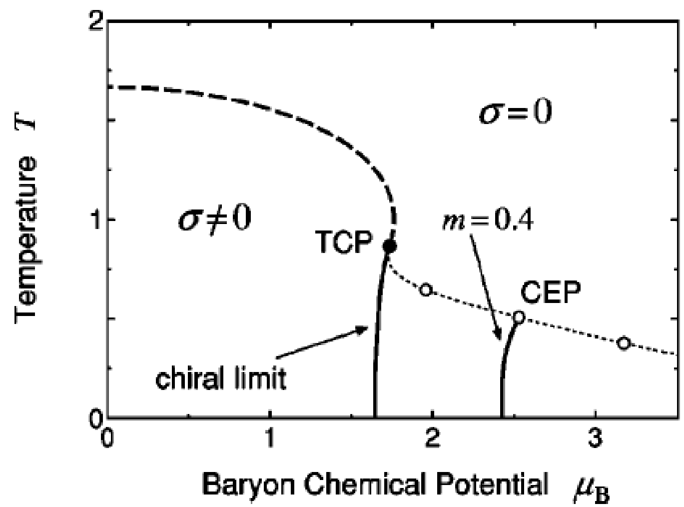
Strong Coupling Limit of Lattice QCD

- **SCL-LQCD has been a powerful tool in "phase diagram" study !**
  - **Chiral restoration, Phase diagram, Baryon effects, Hadron masses, Finite coupling effects, ...**



Kawamoto,Miura,AO,Ohnuma, PRD75 (07), 014502.





Nishida, PRD69, 094501 (2004)

Damgaard,Kawamoto,Shigemoto, PRL53('84),2211

Strong Coupling Limit of Lattice QCD

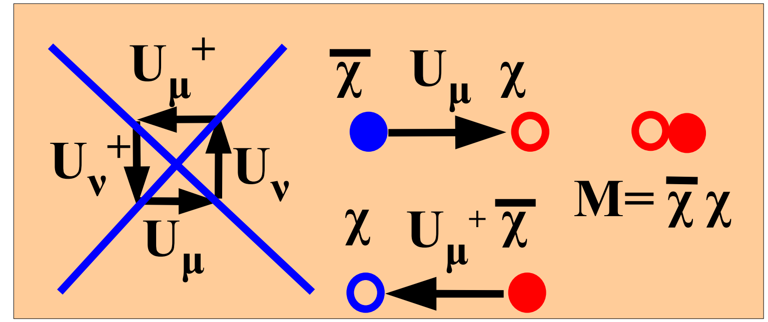
- **Strong Coupling Limit: Pure gluonic action disappers at g → ∞**

$$S_{\rm QCD} = \cancel{S_G} + S_F^{(s)} + S_F^{(t)} + m_0 \bar{\chi}\chi$$

$$\cancel{S_G = -\frac{1}{g^2}\sum_{\rm plaq.} {\rm Tr}\, U_{ij}(x) + c.c.}$$

$$S_F^{(s)} = \frac{1}{2}\sum_{x,j>0}\left(\bar{\chi}_x U_j(x)\chi_{x+\hat{j}} - \bar{\chi}_{x+\hat{j}} U_j^+(x)\chi_x\right)$$

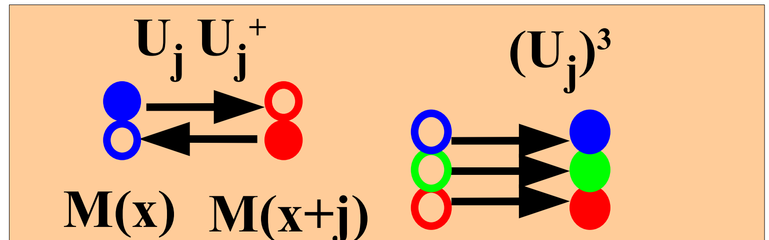
$$S_F^{(t)} = \frac{1}{2}\sum_{x}\left(e^{\mu}\bar{\chi}_x U_0(x)\chi_{x+\hat{0}} - e^{-\mu}\bar{\chi}_{x+\hat{0}} U_0^+(x)\chi_x\right)$$



- **One-link integral leaves mesonic and baryonic action.**

$$S_F^{(s)} \to -\frac{1}{2}(M V_M M) - (\bar{B} V_B B)$$

$$= -\frac{1}{4N_c}\sum_{x,j>0} M_x M_{x+\hat{j}} + \sum_{x,j>0}\frac{\eta_j}{8}\left[\bar{B}_x B_{x+\hat{j}} - \bar{B}_{x+\hat{j}} B_x\right]$$



$\bar{B} = \epsilon\bar{\chi}\bar{\chi}\bar{\chi}/6$ $\quad B = \epsilon\chi\chi\chi/6$

$$\int dU\, U_{ab} U_{cd}^+ = \frac{1}{N_c}\delta_{ad}\delta_{bc}$$

$$\int dU\, U_{ab} U_{cd} U_{ef} = \frac{1}{6}\epsilon_{ace}\epsilon_{bdf}$$

- **Analytic Link Integral → No Sign Problem at finite μ.**

QCD Lattice Action (Zero T treatment)

$$S = \cancel{S_G} + S_F + m_0 \bar{\chi}\chi$$
Strong Coupling Limit

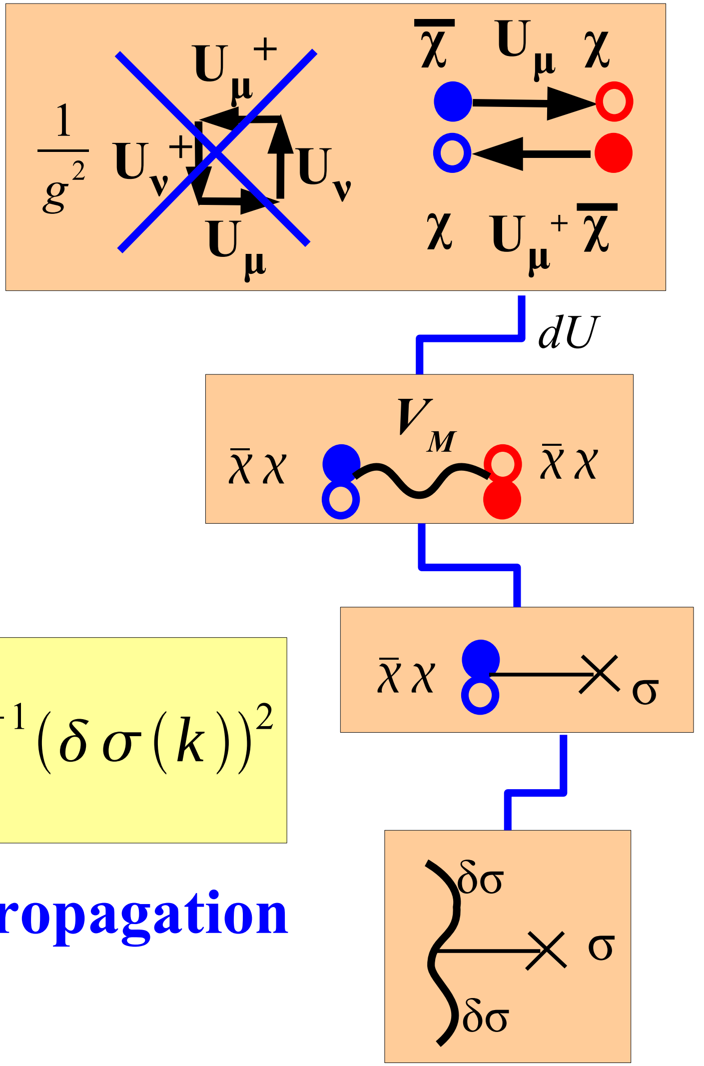
$$\rightarrow -\frac{1}{2}(\bar{\chi}\chi) V_M (\bar{\chi}\chi) + m_q \bar{\chi}\chi$$
One-link integral (1/d expansion)

$$\rightarrow \frac{1}{2}\sigma V_M^{-1}\sigma + \bar{\chi}(\sigma + m_q)\chi$$
Bosonization

$$\rightarrow \frac{1}{2}\sigma V_M^{-1}\sigma - N_c \sum_x \log(\sigma(x) + m_q)$$
Fermion Integral

$$= L^d N_\tau \left[\frac{N_c}{d+1}\bar{\sigma}^2 - N_c \log(\bar{\sigma} + m_q) \right] + \frac{1}{2}\sum_k G(k)^{-1} (\delta\sigma(k))^2$$

Effective Potential **Meson Propagation**



Meson Propagator

$$G(k)^{-1} = F.T.\frac{\delta^2 S}{\delta\sigma(x)\delta\sigma(y)} = 2N_c\left[\sum_\mu \cos k_\mu\right]^{-1} + \frac{N_c}{(\bar{\sigma} + m_q)^2}$$

Hadron Mass in SCL-LQCD (Zero T)

- **Meson Mass in SCL-LQCD**
  Kluberg-Stern, Morel, Petersson, 1982; Kawamoto, Shigemoto, 1982
  - **Pole of the propagator at zero momentum → Meson Mass**
  - **Doubler DOF: $k_\mu \to 0$ or π, Euclidian: $\omega \to i\, m$ + "0 or π"**

$$G^{-1}(k) = N_c \left[\sum_{i=1}^{d} \cos \pi \delta_i \pm \cosh m \right]^{-1} + \frac{N_c}{(\bar{\sigma} + m_q)^2} = 0$$

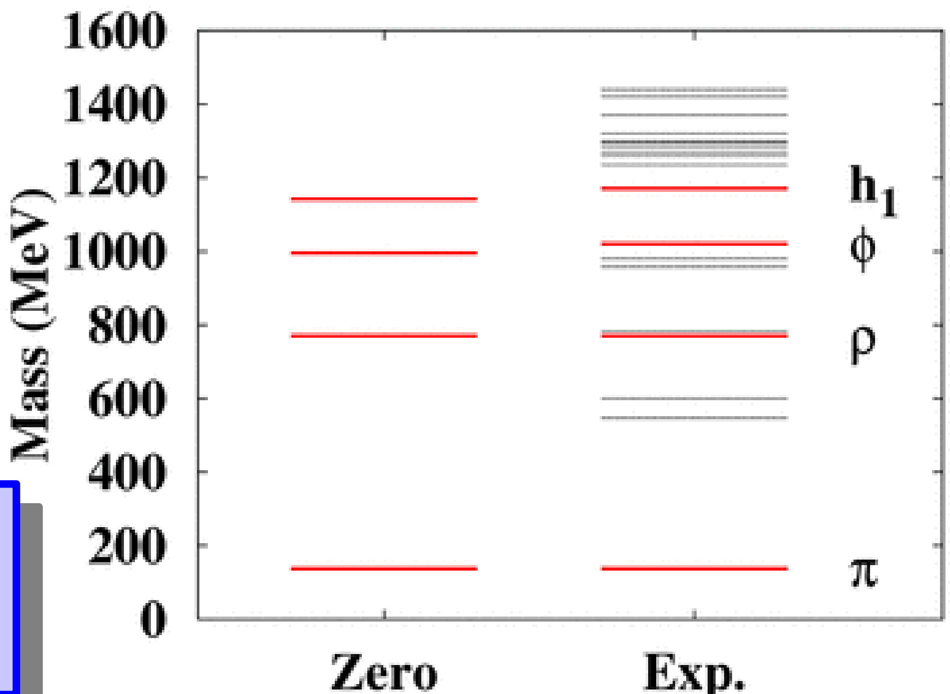
$$\to \cosh m = 2(\bar{\sigma} + m_q)^2 + \kappa$$

$$= (d+1)(\lambda^2 - 1) + 2n + 1$$

Equilibrium Condition

$$n = 0, 1, \ldots d \quad (\text{diff. meson species})$$

$$\lambda = \bar{m}_q + \sqrt{\bar{m}_q^{\,2} + 1}, \quad \bar{m}_q = m_q / \sqrt{2(d+1)}$$



Explains Meson Mass Spectrum
No (T, μ) dependence

Hadron Mass in SCL-LQCD (Finite T)

- **QCD Lattice Action (Finite T treatment)**

  Damgaard, Kawamoto, Shigemoto, 1984; Bilic et al., 1992; Nishida, 2003; Kawamoto, Miura, Ohnishi, Ohnuma, 2007; ...

$$S = \cancel{S_G} + S_F^{(s)} + S_F^{(t)} + m_0 \bar{\chi}\chi$$

Strong Coupling Limit

$$S_F^{(t)} = \frac{1}{2}\sum_x \left(e^{\mu}\bar{\chi}_x U_0(x)\chi_{x+\hat{0}} - e^{-\mu}\bar{\chi}_{x+\hat{0}} U_0^{+}(x)\chi_x \right) = \bar{\chi} V^{(t)} \chi$$

$$\to -\frac{1}{2}(\bar{\chi}\chi) V_M (\bar{\chi}\chi) + \bar{\chi}(V^{(t)} + m_q)\chi$$

Spatial-link integral (1/d expansion, no B)

$$\to \frac{1}{2}\sigma V_M^{-1}\sigma + \bar{\chi}(V^{(t)} + \sigma + m_q)\chi$$

Bosonization

$$\to L^d N_\tau \left[\frac{N_c}{d}\bar{\sigma}^2 + F_{\mathrm{eff}}^{(q)}(\bar{\sigma}, T, \mu) \right] + \frac{1}{2}\sum_k G(k)^{-1}(\delta\sigma(k))^2$$

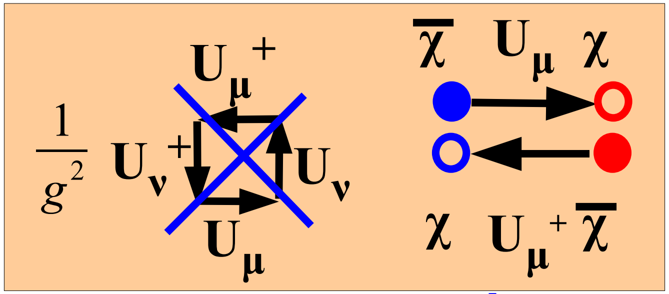
- **Effective Potential**

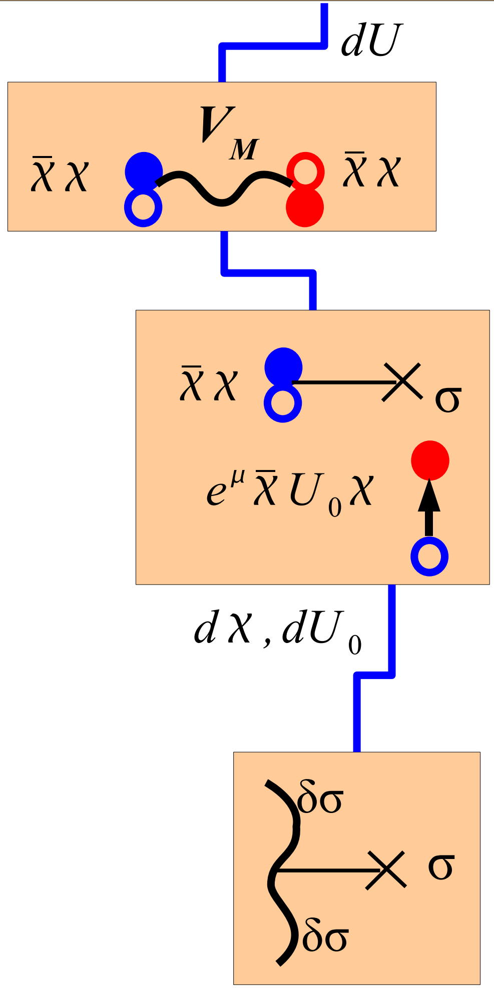
$$F_{\mathrm{eff}}^{(q)}(m; T, \mu) = -T \log \int dU_0 \, det(V^{(t)} + m)$$

$$= -T \log \left[\frac{\sinh((N_c+1)E_q/T)}{\sinh(E_q/T)} + 2\cosh(N_c \mu/T) \right]$$

$$E_q(m) = \mathrm{arcsinh}\, m$$

Fermion and Temporal-link Integral





- **Meson propagator at Finite T** *Faldt, Petersson, '86*
  - **U_0 integrated quark determinant = Function of X_N**
    X_N = Functional of m(τ)

$$F_{\text{eff}}^{(q)}(m(x);T,\mu)=-T\log\int dU_0 \det_{\tau\tau'}(V^{(t)}+m(\boldsymbol{x},\tau))=F_{\text{eff}}^{(q)}(X_N[m])$$

$$X_N[I]=B_N(I_{1,..},..,I_N)+B_{N-2}(I_{2,..},..,I_{N-1})$$
$$(I_k=2m(k)=2(\sigma(k)+m_q))$$

$$B_N=\begin{vmatrix} I_1 & e^{\mu} & & & 0 \\ -e^{-\mu} & I_2 & e^{\mu} & & \\ 0 & -e^{-\mu} & I_3 & e^{\mu} & \\ \vdots & & & \ddots & \\ 0 & & & -e^{-\mu} & I_N \end{vmatrix}$$

  - **Derivatives**

$$\frac{\delta X_N}{\delta I_N}=B_{N-1}(I_{1,..},..,I_{N-1})$$

$$\frac{\delta B_N}{\delta I_k}=B_{k-1}(I_{1,..},..,I_{k-1})B_{N-k}(I_{k+1},\dots,I_N)$$

  - **Equilibrium Value**

$$B_N(I_k=\text{const.})=\begin{cases}\cosh((N+1)E_q)/\cosh E_q & (\text{even } N)\\ \sinh((N+1)E_q)/\cosh E_q & (\text{odd } N)\end{cases}$$

Hadron Mass in SCL-LQCD (Finite T)

- **Meson Mass**

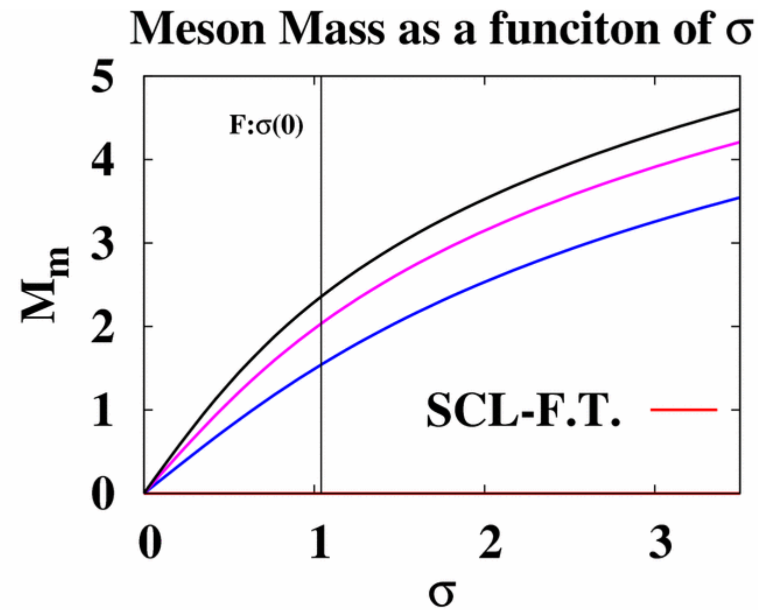
$$G^{-1}(\boldsymbol{k},\omega)=\frac{2N_c}{\kappa(\boldsymbol{k})}+\frac{4N_c}{d}\frac{\bar{\sigma}(\bar{\sigma}+m_q)}{\cos\omega+\cosh 2E_q}$$

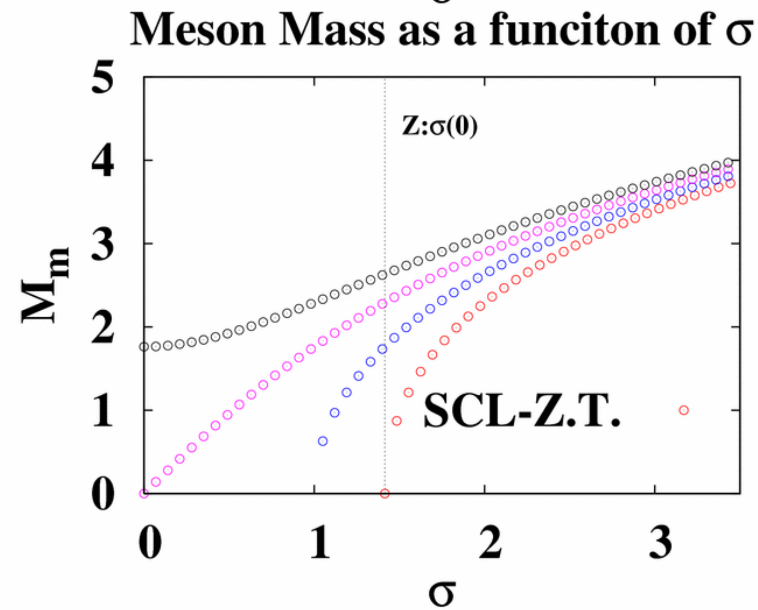
$$\kappa(\boldsymbol{k})=\sum_{i=1}^{d}\cos k_i \quad\rightarrow\quad \kappa=-d,-d+2,\ldots d$$

$$M=2\,\mathrm{arcsinh}\sqrt{(\bar{\sigma}+m_q)\left(\frac{d+\kappa}{d}\bar{\sigma}+m_q\right)}$$

- **Meson masses are determined by the chiral condensate, σ.**
- **Chiral condensate is determined by the equilibrium condition, and given as a function of (T, μ).**

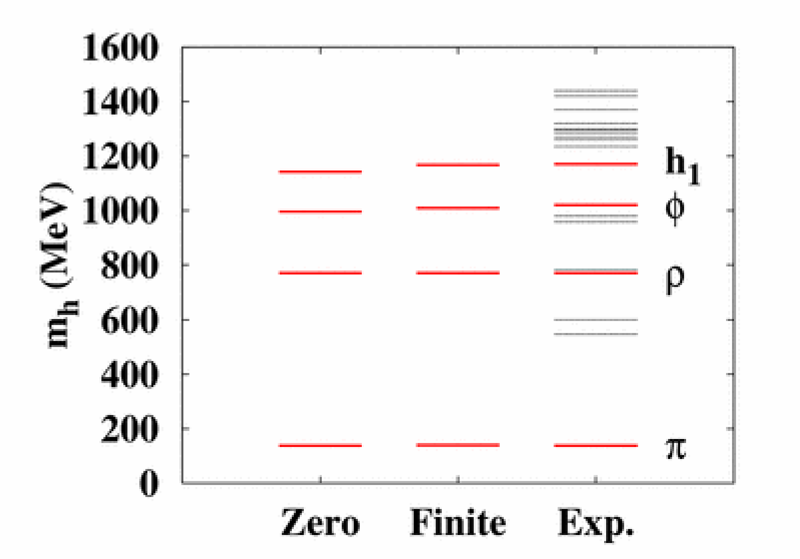
→ ***Approximate Brown-Rho scaling is proven in SCL-LQCD***



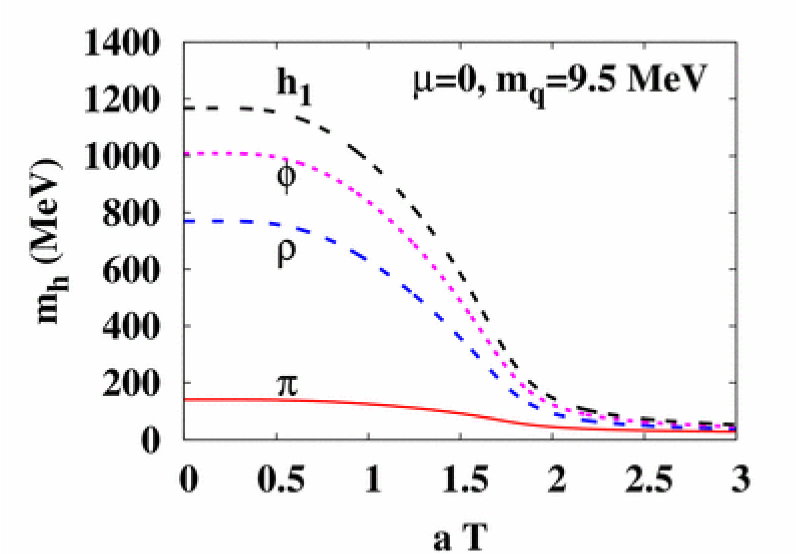


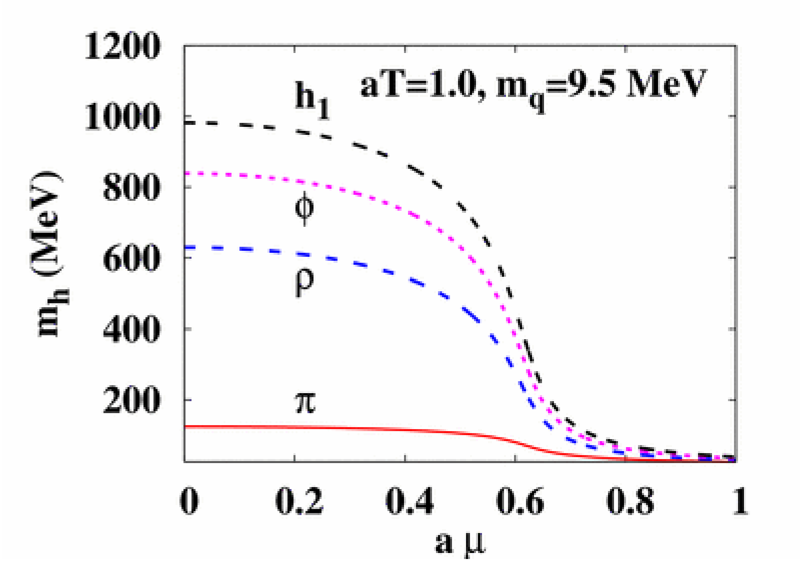
Medium Modification of Meson Masses

- **Scale fixing**
  - Search for σ_{vac} to minimize free E.
  - Assign κ=-3, -1 as π and ρ
  - Determine m_q and a^{-1} (lattice unit) to fit m_π / m_ρ



- **Medium modification**
  - Search for σ(T, μ) → Meson mass





Why ?

- **Conjecture:** **Hadron mass is proportional to σ^n,**

$$\frac{m^*}{m_{\text{vac}}} = \left(\frac{\overline{\sigma}}{\sigma_{vac}}\right)^{|Q|/2} \text{(mesons)}, \qquad \frac{m^*}{m_{\text{vac}}} = \left(\frac{\overline{\sigma}}{\sigma_{vac}}\right)^{|Q|} \text{(baryons)}$$

where Q is the chiral charge, under the following conditions:

  - **Chiral limit, Only one finite condensate (σ),Mean field is dominant.**

- **Reason: Effective action should be chiral invariant**

  - **Quark → Q=1** $\quad \chi \to \exp(i\theta\varepsilon(x))\chi, \quad \overline{\chi} \to \exp(i\theta\varepsilon(x))\overline{\chi}$ ¿
  - **q-qbar mesons → Q=2**

$$\delta\sigma \to \exp(2i\theta\varepsilon(x))\delta\sigma, \quad \text{mass term} \propto \left(\overline{\sigma}(x+\hat{j})\right)^2 (\delta\sigma(x))^2$$

  - **baryons → Q=3**

Kluberg-Stern, Morel, Petersson, '82

Kawamoto, Miura, AO, YITP workshop "Thermo Field Dynamics"

$$b \to \exp(-3i\theta\varepsilon(x))b, \qquad \text{mass term} \propto \bar{b}(x)\left(\overline{\sigma}(x)\right)^3 b(x)$$

Finite Coupling Correction and the Shape of the Phase Diagram

Problems

- **Are the SCL-LQCD results reliable ?**
  - **π, ρ mass fit → Physical Scale (a^{-1}) is fixed**
    a^{-1} = 497 MeV, m_q=9.5 MeV
    → T_c= 5/3a = 828 MeV (Too large !)
    (Long standing problem in SCL-LQCD)
  - **Brown-Rho scaling may not be realized in the real world**
    → Finite condensate other than σ would be necessary
- **Finite coupling effects may help !**
  - **$1/g^2$ correction reduces T_c**
    Bilic, Claymans, '95; AO, Kawamoto, Miura, '07
  - **Other condensate than σ will appear from the plaquett term.**

$$\langle \bar{q}\, g q \rangle, \quad \langle M(x) M(x+\hat{j}) \rangle,,,$$

$1/g^2$ expansion at Finite T

- **1/d expansion of plaquetts** *(Faldt-Petersson 1986)*

$$\Delta S_\beta = \frac{\beta_t}{2d} \sum_{x,j>0} \left(V_x^{(+)} V_{x+\hat{j}}^{(-)} + V_x^{(-)} V_{x+\hat{j}}^{(+)} \right) - \frac{\beta_s}{2(d-1)} \sum_{x,k>j>0} M_x M_{x+\hat{j}} M_{x+\hat{j}+\hat{k}} M_{x+\hat{k}}$$

$$\left(V_x^{(+)} = e^{\mu} \bar{\chi}_x U_0(x) \chi_{x+\hat{0}}, \quad V_x^{(-)} = e^{-\mu} \bar{\chi}_{x+\hat{0}} U_0^{+}(x) \chi_x \right)$$

- **Bosonization & Mean Field Approximation**

$$\begin{aligned} S_{\mathrm{SCL}} + \Delta S_\beta &= \frac{1}{2} \sum_x V_x^{(+)} \times (1 + \beta_t \varphi_t + \beta_t \phi_t) \\ &- \frac{1}{2} \sum_x V_x^{(-)} \times (1 + \beta_t \varphi_t - \beta_t \phi_t) \\ &- \frac{1}{4} N_c \sum_{x,x>0} M_x M_{x+\hat{j}} \times \left(1 + 4 N_c \beta_s (\varphi_s - \phi_s)\right) \\ &+ m_0 \sum_x \bar{\chi}_x \chi_x + N_\tau N_s^d \left[\frac{\beta_t}{4} (\varphi_t^2 - \phi_t^2) + \frac{\beta_s d}{4} (\varphi_s^2 - 2 \phi_s^2) \right] \end{aligned}$$

Correction terms are absorbed in the SCL action terms.

Effective Potential with $1/g^2$

- **Quark and Time-like Link integral → Effective Potential**

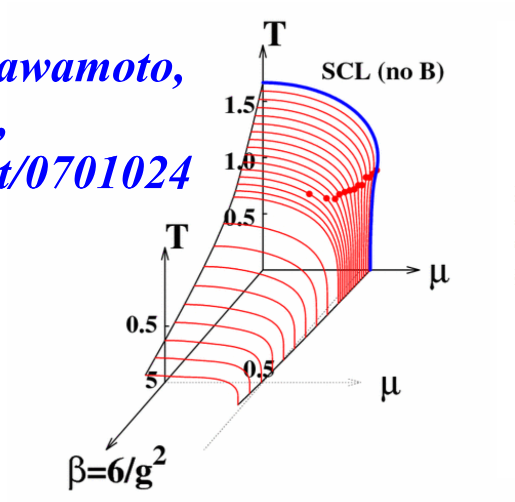
$$F=\frac{d}{4N_c}\sigma^2+F_q(m_q;\tilde{\mu})$$

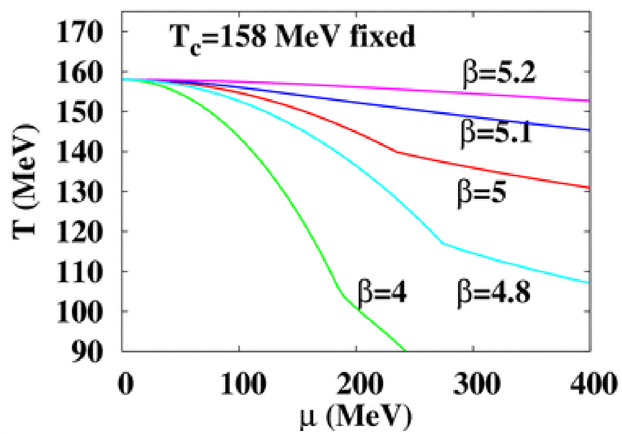
$$+\beta_s d\,\sigma^2(\varphi_s-\phi_s)+\frac{\beta_t}{4}(\varphi_t^2-\phi_t^2)+\frac{\beta_s d}{4}(\varphi_s^2-2\phi_s^2)-N_c\beta_t\varphi_t$$

$$m_q=\frac{d}{2N_c}\sigma\left(1+4N_c\beta_s(\varphi_s-\phi_s)-\beta_t\varphi_t\right),\quad \tilde{\mu}=\mu-\beta_t\phi_t$$

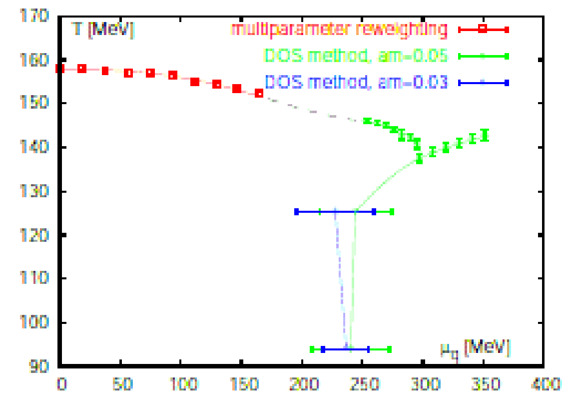
  - **At β ~ 5, results with $1/g^2$ correction would be comparable with MC results (Density of States method)**

AO, Kawamoto, Miura, hep-lat/0701024





Kawamoto,Miura,AO, in prep.



Fodor,Katz,Schmit, 2007

Summary

- **Hadron (Meson) masses are evaluated in the Strong Coupling Limit of Lattice QCD at Finite T and μ.**
  - ***Brown-Rho scaling is proven in SCL-LQCD.***
  - **Conjecture:**

$$\frac{m^*}{m_{\text{vac}}} = \left(\frac{\overline{\sigma}}{\sigma_{vac}}\right)^{|Q|/2} \text{(mesons)}, \qquad \frac{m^*}{m_{\text{vac}}} = \left(\frac{\overline{\sigma}}{\sigma_{vac}}\right)^{|Q|} \text{(baryons)}$$

Staggered Fermion → $Q_B = 3$, Real World $Q_B = 1$

- **Finite coupling effects are found to decrease T_c , and other condensates than σ appears.**
  → Brown-Rho scaling would be violated at finite couplings.