Phase diagram and hadron properties in the strong coupling lattice QCD

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Introduction

Hadron Mass in a Finite T treatment
of Strong Coupling Limit for Lattice QCD

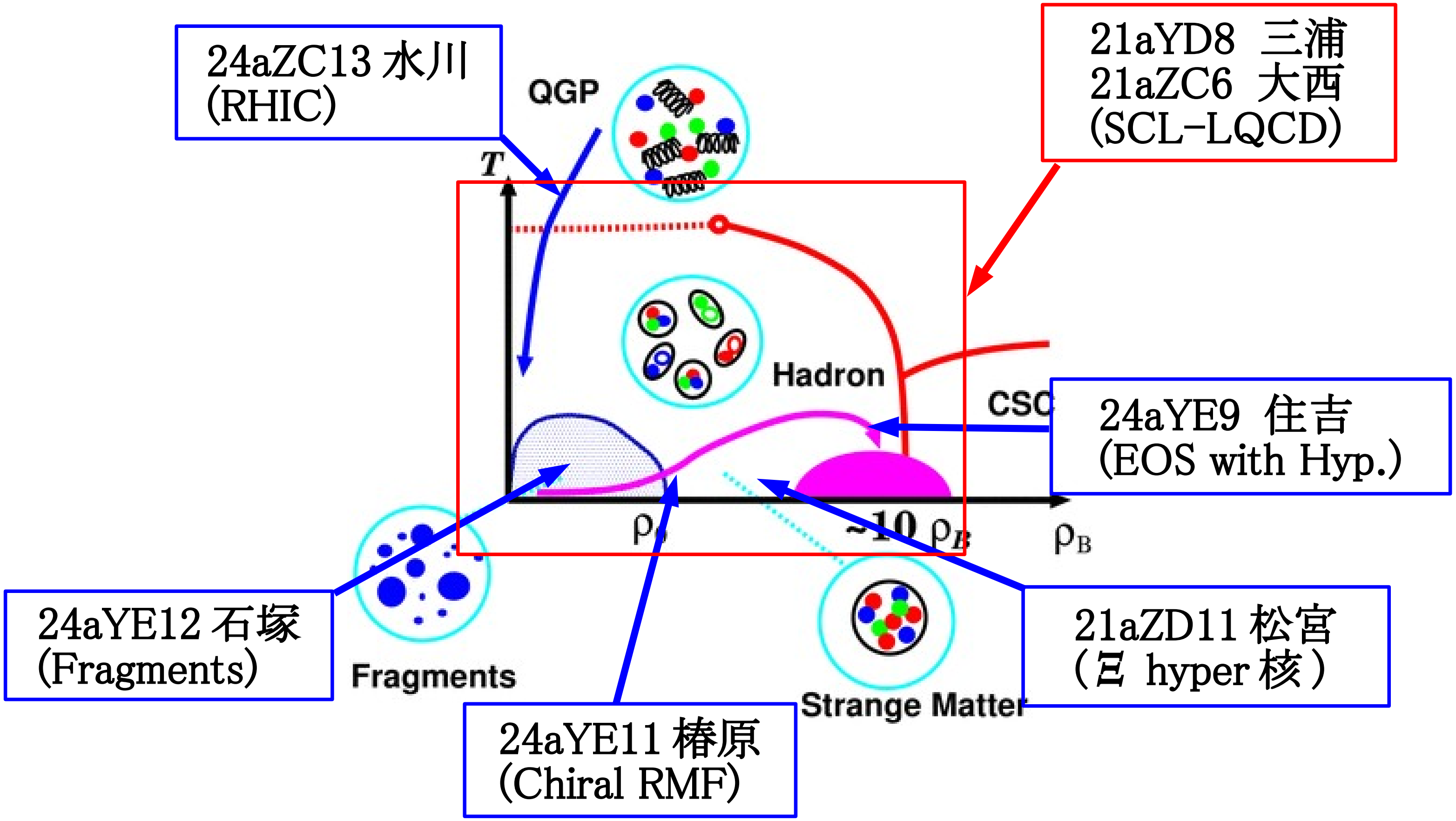
Finite Coupling Effects
in the Strong Coupling Lattice QCD

Summary

Quark and Hadronic Matter Phase Diagram

原子核・ハドロン・クォークの3階層状態方程式とコンパクト天体現象
(科研費基盤研究 (C), 大西、河本、住吉)

クォーク、ハドロン、原子核の 3 階層をつなぐ EOS を作りたい！



Hadron Mass in Nuclear Matter

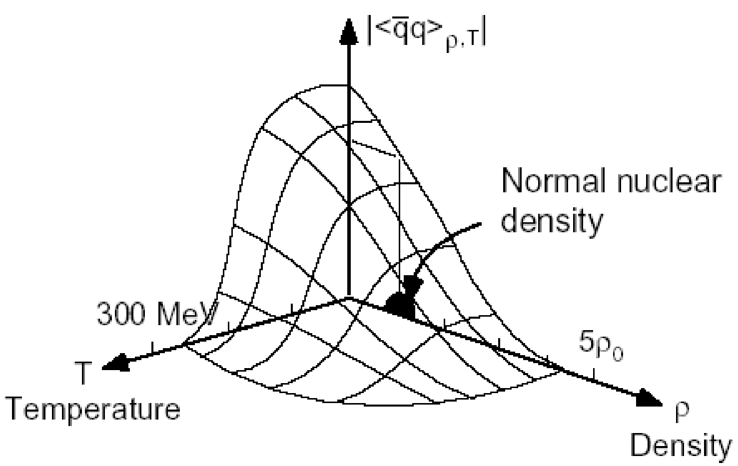
Medium meson mass modification

may be the signal of partial restoration of chiral sym.

Kunihiro,Hatsuda, PRep 247('94),221; Brown, Rho, PRL66('91)2720;
Hatsuda, Lee, PRC46('92)R34.

and is suggested experimentally.

CERES Collab., PRL75('95),1272;
KEK-E325 Collab.(Ozawa et al.), PRL86('01),5019.
Also at RHIC (PHENIX Collab.)



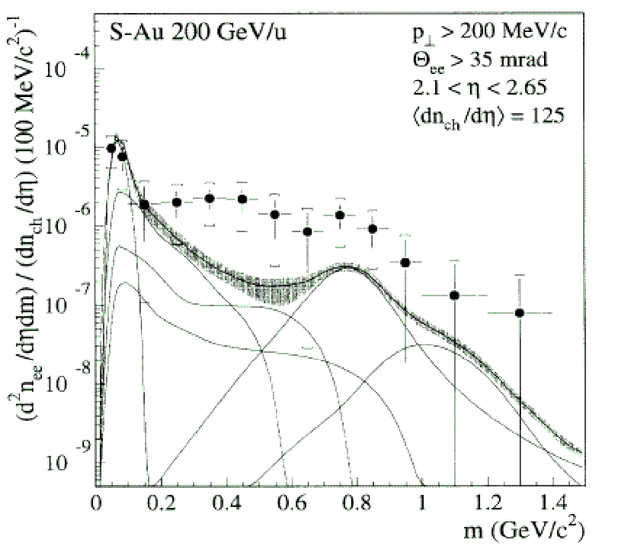
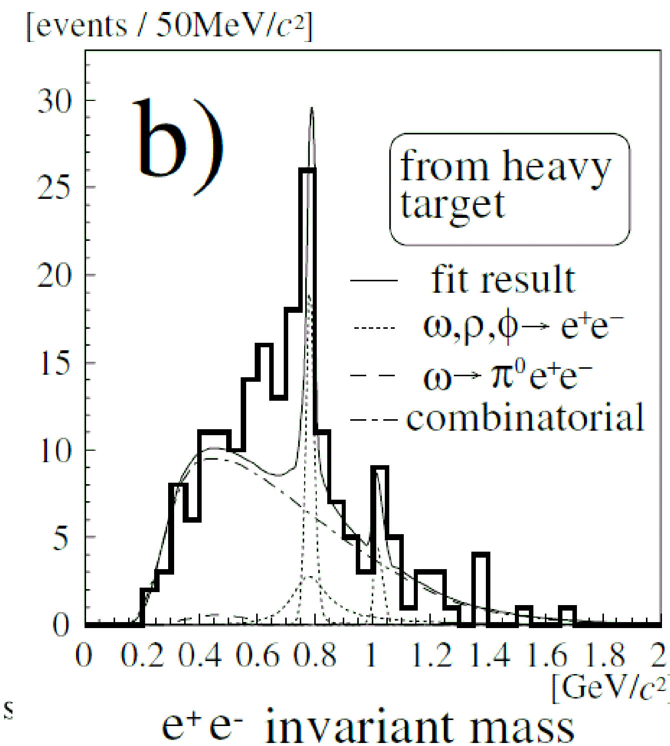


FIG. 4. Inclusive e^+e^- mass spectra in 200 GeV/nucleon S Au collisions. For explanations see Fig. 2.



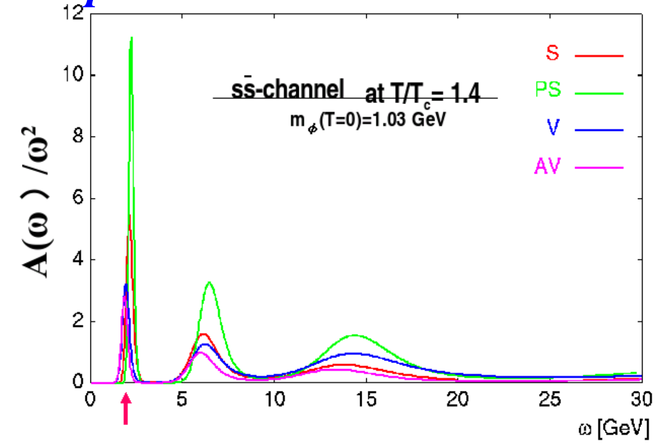
Hadron Mass in Nuclear Matter

Can we understand it in Lattice QCD ?

Finite T: It is possible !

Finite μ: Difficult due to the sign problem.

Asakawa, Nakahara, Hatsuda, hep-lat/0208059.



前回の学会シンポでの中村さん

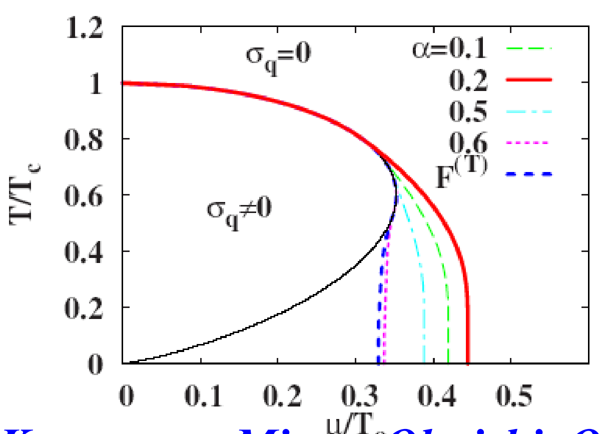
Strong Coupling Limit of Lattice QCD
→ We can study finite (T, μ) !

Hadron masses in the Zero T treatment

Kluberg-Stern, Morel, Petersson, '83; Kawamoto, Shigemoto, '82.

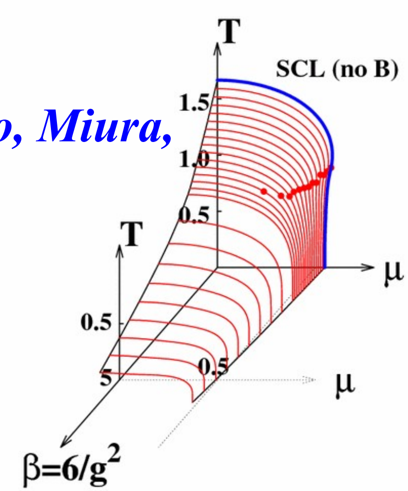
To do: Finite T, Baryons with finite T, $1/g^2$ corr., ...

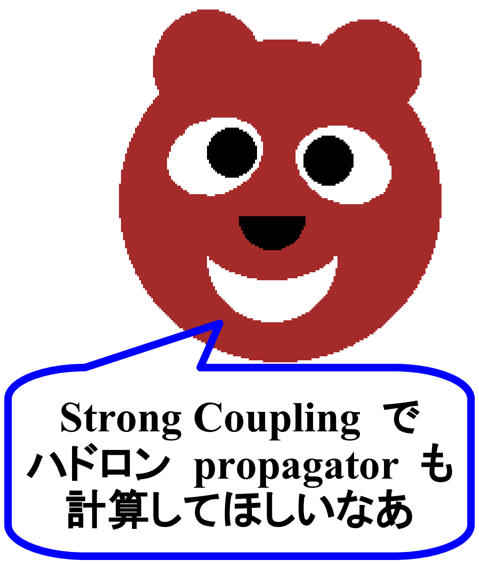
This Talk



Kawamoto, Miura, Ohnishi, Ohnuma, PRD75('07)014502

Ohnishi, Kawamoto, Miura, hep-lat/0701024



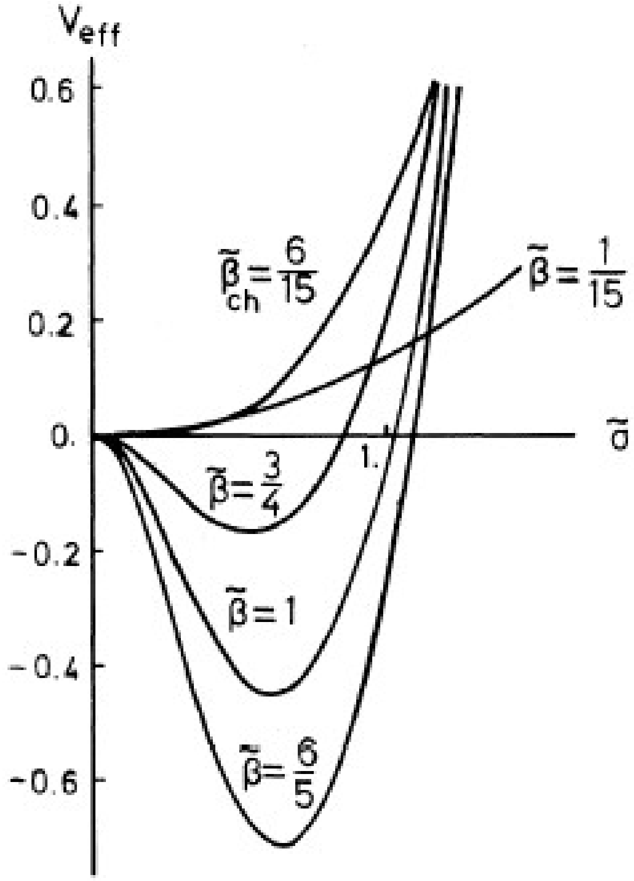


Strong Coupling Limit of Lattice QCD

Strong Coupling Limit of Lattice QCD

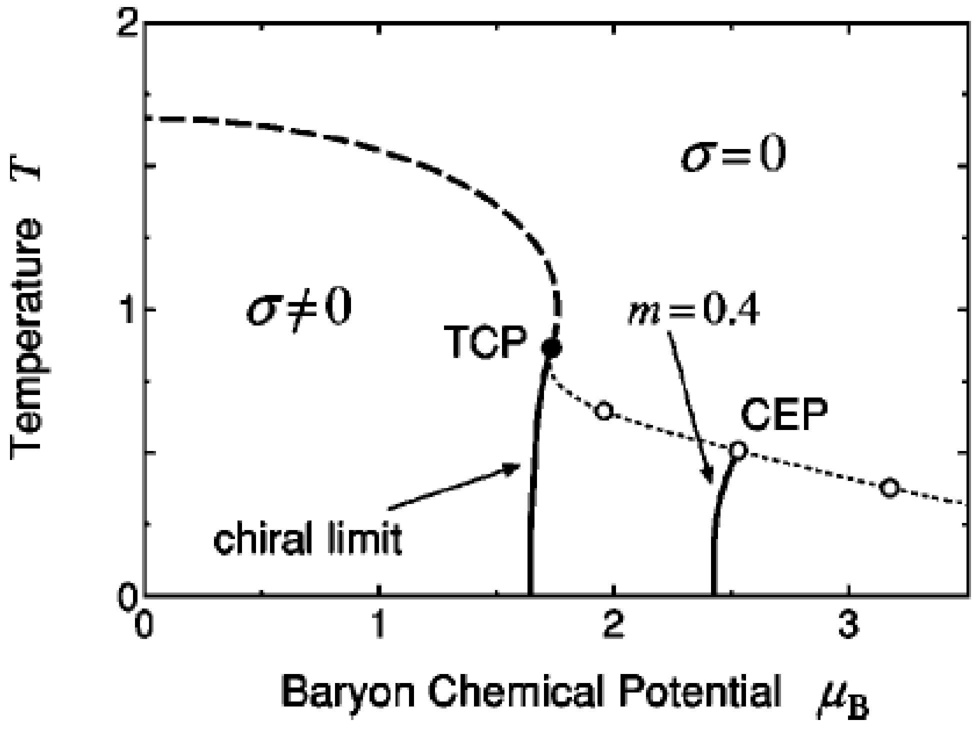
Chiral Restoration at μ=0.

Damgaard, Kawamoto, Shigemoto, PRL53(1984),2211



Phase Diagram with Nc=3

Nishida, PRD69, 094501 (2004)



Previous Works in Strong Coupling Limit LQCD

Strong Coupling Limit Lattice QCD re-attracts interests

c.f. Nakamura @ JHF Symp. for high density matter (2001)

Ref	T	μ	Nc	Baryon	CSC	Nf
Damgaard-Kawamoto-Shigemoto('84)	Finite	**0**	U(Nc)	**X**	X	1
Damgaard-Hochberg-Kawamoto('85)	**0**	Finite	3	Yes	X	1
Bilic-Karsch-Redlich('92)	Finite	Finite	3	**X**	X	1 ~ 3
Azcoiti-Di Carlo-Galante-Laliena('03)	**0**	Finite	3	Yes	Yes	1
Nishida-Fukushima-Hatsuda('04)	Finite	Finite	**2**	Yes (*)	Yes (*)	1
Nishida('04)	Finite	Finite	3	**X**	X	1~2

*: bosonic baryon=diquark in SU(2)

+: analytically included, but ignored in numerical calc.

Baryon effects have been ignored in finite T treatments !

→ This work: Baryonic effects at Finite T (and μ) for $SU_c(3)$

Strong Coupling Limit Lattice QCD

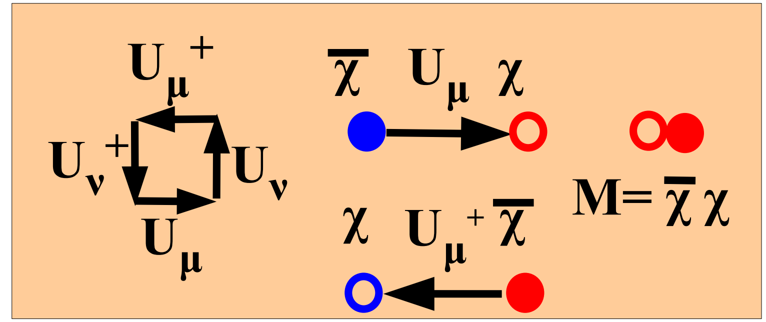
QCD Lattice Action

$$Z \simeq \int D[\chi, \bar{\chi}, U] \exp\left[-\left(S_G + S_F^{(s)} + S_F^{(t)} + m_0 M\right)\right]$$

$$S_G = \frac{1}{g^2} \sum_{x\mu\nu} \left[\mathrm{Tr}\, U_{\mu\nu} + \mathrm{Tr}\, U_{\mu\nu}^{+}\right]$$

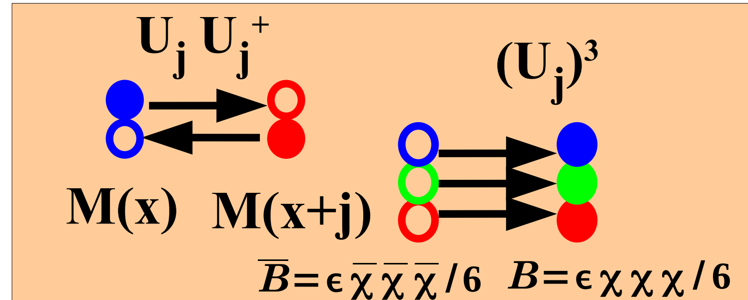
$$S_F^{(s)} = \frac{1}{2} \sum_{x,j} \eta_j(x) \left(\bar{\chi}_x U_j(x) \chi_{x+\hat{j}} - \bar{\chi}_{x+\hat{j}} U_j^{+}(x) \chi_x\right)$$

$$S_F^{(t)} = \frac{1}{2} \sum_{x} \left(e^{\mu} \bar{\chi}_x U_0(x) \chi_{x+\hat{0}} - e^{-\mu} \bar{\chi}_{x+\hat{0}} U_0^{+}(x) \chi_x\right)$$



Strong Coupling Limit: g→∞

We can ignore S_G and perform one-link integral after 1/d expansion.



$$\int dU\, U_{ab} U_{cd}^{+} = \frac{1}{N_c} \delta_{ad} \delta_{bc}$$

$$\int dU\, U_{ab} U_{cd} U_{ef} = \frac{1}{6} \epsilon_{ace} \epsilon_{bdf}$$

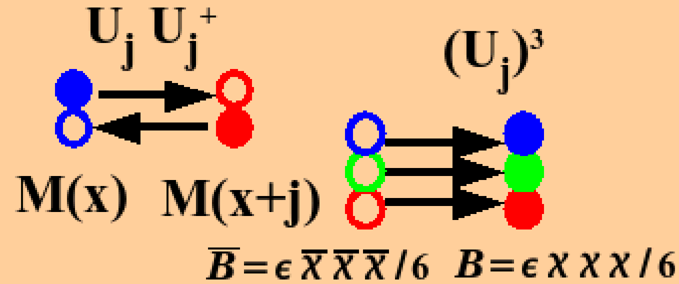
$$S_F^{(s)} \to -\frac{1}{2}(M V_M M) - (\bar{B} V_B B)$$

$$= -\frac{1}{4N_c} \sum_{x,j>0} M_x M_{x+\hat{j}} + \sum_{x,j>0} \frac{\eta_j}{8} \left[\bar{B}_x B_{x+\hat{j}} - \bar{B}_{x+\hat{j}} B_x\right]$$

Group Integral Formulae

$$\int dU\, U_{ab} U^{+}_{cd} = \frac{1}{N_c} \delta_{ad} \delta_{bc}$$

$$\int dU\, U_{ab} U_{cd} U_{ef} = \frac{1}{6} \epsilon_{ace} \epsilon_{bdf}$$



$$\int dU\, U_{ab} U^{+}_{cd} = \frac{1}{N_c} \delta_{ad} \delta_{bc}$$

$$\int dU\, U_{ab} U_{cd} U_{ef} = \frac{1}{6} \epsilon_{ace} \epsilon_{bdf}$$

$$\int dU \exp(-a\bar{\chi}(x) U \chi(y) + b\bar{\chi}(y) U^{+} \chi(x))$$
$$= \int dU \left[1 - ab\bar{\chi}(a)^a U_{ab} \chi^b(y) \bar{\chi}^c(y) U^{+}_{cd} \chi^d(x) + \cdots \right]$$
$$= 1 + ab \left(\chi\bar{\chi}\right)(x) \left(\chi\bar{\chi}\right)(y) + \cdots = 1 + ab\, M(x) M(y) + \cdots$$
$$= \exp\left[ab\, M(x) M(y) + \cdots \right]$$

Quarks and Gluons → One-Link integral → Mesonic and Baryonic Composites

SCL-LQCD: Tools (2) --- 1/d Expansion

Keep mesonic action to be indep. from spatial dimension *d*
→ Higher order terms are suppressed at large *d*.

$$\sum_j (\bar\chi U_j \chi)(\bar\chi U_j^+ \chi) \to -\frac{1}{N_c}\sum_j M(x) M(x+\hat{j}) = O(1)$$

$$\to M \propto 1/\sqrt{d}\,,\ \chi \propto d^{-1/4}$$

$$\sum_j (\bar\chi U_j \chi)(\bar\chi U_j \chi)(\bar\chi U_j \chi) \to N_c! \sum_j B(x) B(x+\hat{j}) = O(1/\sqrt{d})$$

$$\sum_j (\bar\chi U_j \chi)^2 (\bar\chi U_j^+ \chi)^2 \to \sum_j M^2(x) M^2(x+\hat{j}) = O(1/d)$$

We can stop the expansion in U,
since higher order terms are suppressed !

SCL-LQCD: Tools (3) --- Bosonization

We can reduce the power in χ by introducing bosons

$$\exp\left(\frac{1}{2}M^2\right)=\int d\sigma\exp\left(-\frac{1}{2}\sigma^2-\sigma M\right)$$

Nuclear MFA: $V=-\frac{1}{2}(\bar{\psi}\psi)(\bar{\psi}\psi)\simeq-U(\bar{\psi}\psi)+\frac{1}{2}U^2$

$$\exp\left[-\frac{1}{2}M^2\right]=\int d\varphi\exp\left[-\frac{1}{2}\varphi^2-i\varphi M\right]$$

Reduction of the power of χ
$\rightarrow$ Bi-Linear form in χ $\rightarrow$ Fermion Determinant

SCL-LQCD: Tools (4) --- Grassman Integral

Bi-linear Fermion action leads to -log(det A) effective action

$$\int d\chi\, d\bar{\chi} \exp\left[\bar{\chi} A \chi\right] = det\ A = \exp\left[-(-\log det\ A)\right]$$

$$\int d\chi \cdot 1 = \text{anti-comm. constant} = 0 \quad , \quad \int d\chi \cdot \chi = \text{comm. constant} \equiv 1$$

$$\int d\chi\, d\bar{\chi} \exp\left[\bar{\chi} A \chi\right] = \int d\chi\, d\bar{\chi} \frac{1}{N!} (\bar{\chi} A \chi)^N = \cdots = det\ A$$

Constant σ → - log σ interaction (Chiral RMF)

Temporal Link Integral, Matsubara product, Staggered Fermion,
→ I will explain next time ....

SCL-LQCD w/o Baryons

Damgaad-Kawamoto-Shigemoto 1984, Faldt-Petersson 1986, Bilic-Karsch-Redlich 1992, Nishida 2004, ...

Lattice Action (staggered fermion) in SCL

$$Z \simeq \int D[\chi, \bar{\chi}, U] \exp\left[-S_F^{(s)} - S_F^{(t)} - m_0 \bar{\chi}\chi - S_G\right]$$

Strong Coupling

Spatial Link Integral

$$\simeq \int D[\chi, \bar{\chi}, U_0] \exp\left[\frac{1}{2}(M, V_M M) + (\bar{B}, V_B B) - (\bar{\chi} G_0 \chi)\right]$$

1/d Expansion (1/√d)

Bosonization (Hubburd-Stratonovich transformation)

$$\simeq \int D[\chi, \bar{\chi}, U_{0,} \sigma] \exp\left[-\frac{1}{2}(\sigma, V_M \sigma) - (\sigma, V_M M) - (\bar{\chi} G_0 \chi)\right]$$

Quark and U_0 Integral

$$(\bar{\chi} G(\sigma) \chi)$$

$$\simeq \exp\left(-N_S^3 N_\tau \left[\frac{1}{2} a_\sigma \sigma^2 - T \log G_U(\sigma)\right]\right) = \exp(-N_S^3 F_{\mathrm{eff}}/T)$$

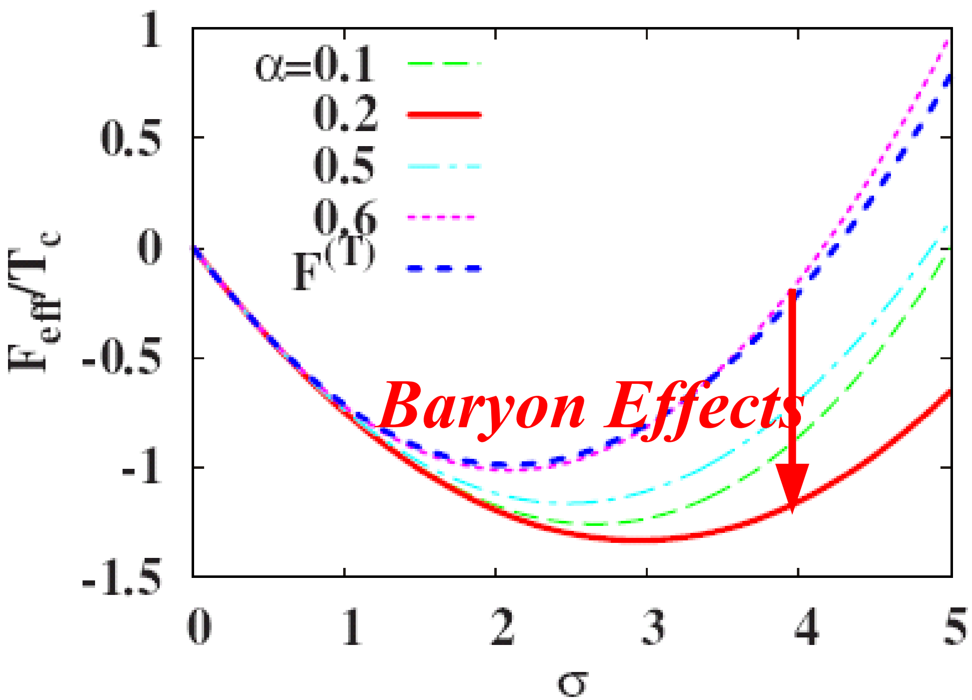
Local Bi-linear action in quarks → Effective Free Energy

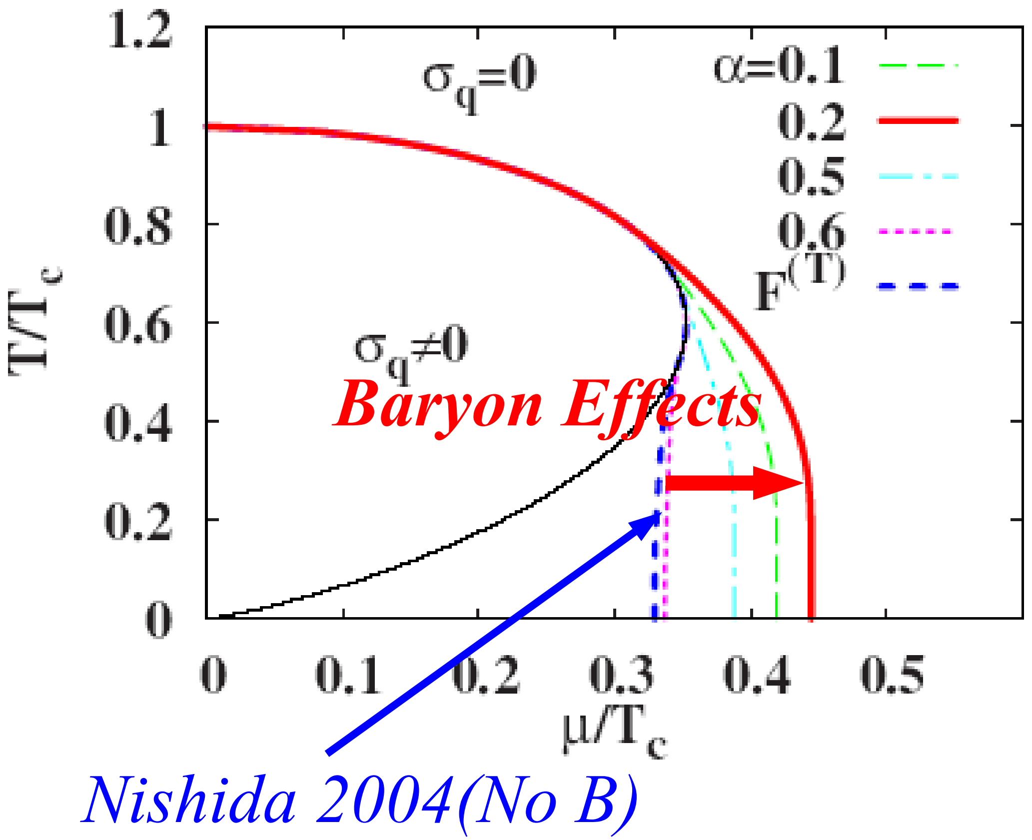
Phase diagram in SCL-LQCD with Baryons

(Kawamoto-Miura-AO-Ohnuma, hep-lat/0512023)

Baryon effects on phase diagram

Energy gain in larger condensates
→ Extension of hadron phase to larger μ by around 30 %.





Hadron Mass Spectrum in the Strong Coupling Limit of Lattice QCD

Hadron Mass in SCL-LQCD (Zero T)

SCL Effective Action (Zero T treatment, staggered fermion)

$$S_{\rm eff} = \frac{1}{2}\sum_{x,y} \sigma(x) V_M^{-1}(x,y)\sigma(y) - N_c \sum_x \log(\sigma(x)+m_q)$$

Kawamoto, Smit, '81

$$= L^d N_\tau \left[\frac{N_c}{d+1}\bar{\sigma}^2 - N_c \log(\bar{\sigma}+m_q)\right] + \frac{1}{2}\sum_k G(k)^{-1}(\delta\sigma(k))^2$$

Effective Potential **Meson Propagation**

Meson Mass in SCL-LQCD

Kawamoto, Shigemoto, '82; Kluberg-Stern, Morel, Petersson, '83; Fukushima, '04

Pole of $G(k)$ at "zero" momentum: $k_i \to 0$ or π, $\omega \to i\,m +$ "0 or π"

$$G(k)^{-1} = F.T.\frac{\delta^2 S_{\rm eff}}{\delta\sigma(x)\delta\sigma(y)} = 2N_c\left[\sum_\mu \cos k_\mu\right]^{-1} + \frac{N_c}{(\bar{\sigma}+m_q)^2} \to 2N_c[\kappa \pm \cosh m]^{-1} + \frac{N_c}{(\bar{\sigma}+m_q)^2} = 0$$

$$\cosh m = 2(\bar{\sigma}+m_q)^2 + \kappa \quad \to \quad (d+1)(\lambda^2-1)+\kappa+d+1$$ **Equilibrium σ**

$\kappa = -d, -d+2, \dots d$ (diff. meson species), $\lambda = \bar{m}_q + \sqrt{\bar{m}_q^{\,2}+1}$, $\bar{m}_q = m_q/\sqrt{2(d+1)}$ $(d=3)$

Well explains data, Funny σ dep., No (T, μ) dep.,

Hadron Mass in SCL-LQCD (Zero T)

Meson Mass in SCL-LQCD

Kluberg-Stern, Morel, Petersson, 1982; Kawamoto, Shigemoto, 1982

Pole of the propagator at zero momentum → Meson Mass

Doubler DOF: $k_\mu \to 0$ or π, Euclidian: $\omega \to i\, m$ + "0 or π"

$$G^{-1}(k) = N_c \left[\sum_{i=1}^{d} \cos\pi\,\delta_i \pm \cosh m \right]^{-1} + \frac{N_c}{(\bar{\sigma}+m_q)^2} = 0$$

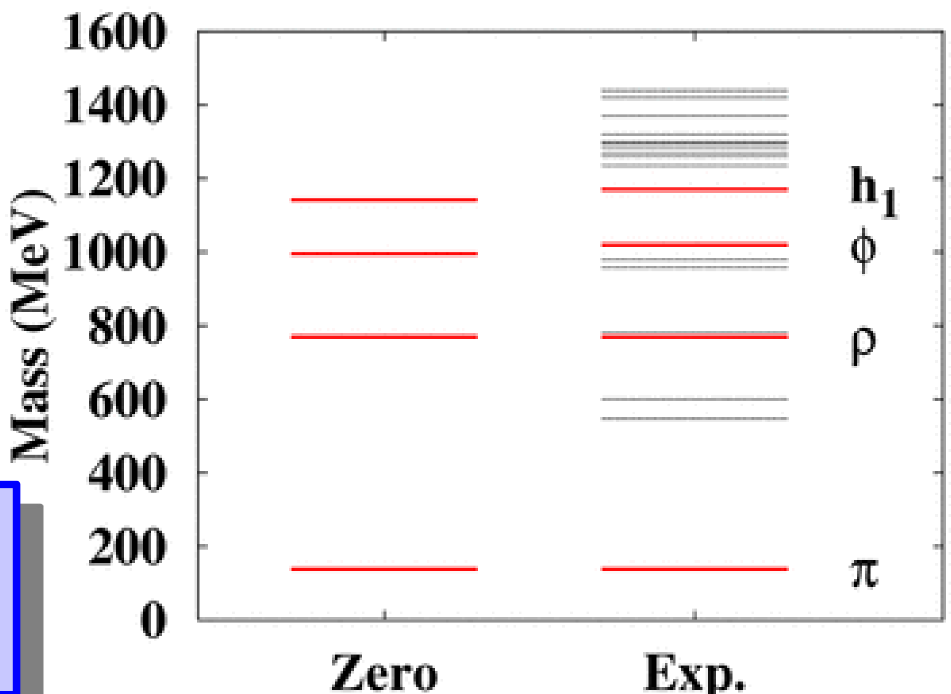
$$\to \quad \cosh m = 2(\bar{\sigma}+m_q)^2 + \kappa$$

$$= (d+1)(\lambda^2-1) + 2n + 1$$

Equilibrium Condition

$$n = 0, 1, \ldots d \quad (\text{diff. meson species})$$

$$\lambda = \bar{m}_q + \sqrt{\bar{m}_q^{\,2}+1}, \quad \bar{m}_q = m_q / \sqrt{2(d+1)}$$

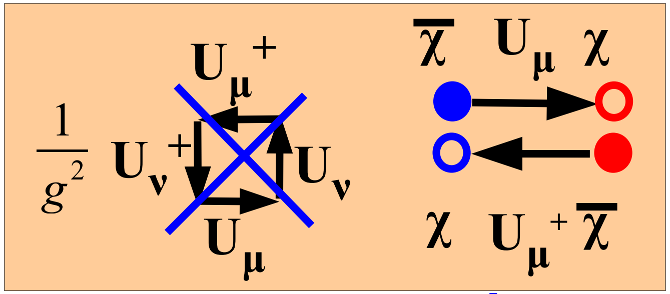


Explains Meson Mass Spectrum
No (T, μ) dependence

Hadron Mass in SCL-LQCD (Finite T)

QCD Lattice Action (Finite T treatment)

Damgaard, Kawamoto, Shigemoto, 1984; Bilic et al., 1992; Nishida, 2003; Kawamoto, Miura, Ohnishi, Ohnuma, 2007; ...

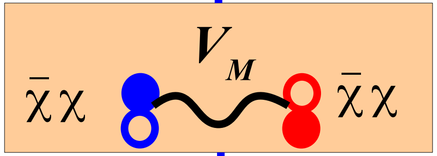


$$S = \cancel{S_G} + S_F^{(s)} + S_F^{(t)} + m_0\bar\chi\chi$$

Strong Coupling Limit

$$S_F^{(t)} = \frac{1}{2}\sum_x \left(e^{\mu}\bar\chi_x U_0(x)\chi_{x+\hat0} - e^{-\mu}\bar\chi_{x+\hat0}U_0^+(x)\chi_x\right) = \bar\chi V^{(t)}\chi$$

dU

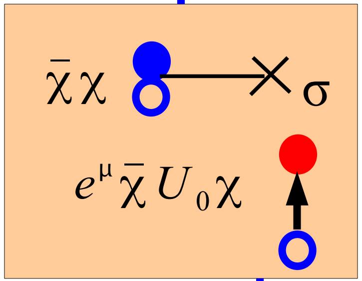


$$\rightarrow -\frac{1}{2}(\bar\chi\chi)V_M(\bar\chi\chi) + \bar\chi(V^{(t)}+m_q)\chi$$

Spatial-link integral (1/d expansion, no B)

$$\rightarrow \frac{1}{2}\sigma V_M^{-1}\sigma + \bar\chi(V^{(t)}+\sigma+m_q)\chi$$

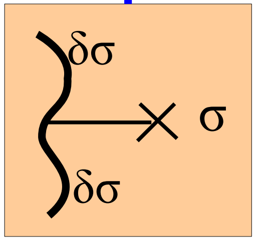
Bosonization



$$\rightarrow L^d N_\tau\left[\frac{N_c}{d}\bar\sigma^2 + F_{\rm eff}^{(q)}(\bar\sigma, T, \mu)\right] + \frac{1}{2}\sum_k G(k)^{-1}(\delta\sigma(k))^2$$

$d\chi, dU_0$

Fermion and Temporal-link Integral



Effective Potential

$$F_{\rm eff}^{(q)}(m; T, \mu) = -T\log\int dU_0\, det(V^{(t)}+m)$$

$$= -T\log\left[\frac{\sinh((N_c+1)E_q/T)}{\sinh(E_q/T)} + 2\cosh(N_c\mu/T)\right]$$

$$E_q(m) = \text{arcsinh}\, m$$

Hadron Mass in SCL-LQCD (Finite T)

Meson propagator at Finite T *Faldt, Petersson, '86*

U_0 integrated quark determinant = Function of X_N

X_N = Functional of m(τ)

$$F_{\mathrm{eff}}^{(q)}(m(x);T,\mu)=-T\log\int dU_0\, det_{\tau\tau'}(V^{(t)}+m(\boldsymbol{x},\tau))=F_{\mathrm{eff}}^{(q)}(X_N[m])$$

$$X_N[I]=B_N(I_1,..,I_N)+B_{N-2}(I_2,..,I_{N-1})$$

$$(I_k=2m(k)=2(\sigma(k)+m_q))$$

$$B_N=\begin{vmatrix} I_1 & e^{\mu} & & & 0 \\ -e^{-\mu} & I_2 & e^{\mu} & & \\ 0 & -e^{-\mu} & I_3 & e^{\mu} & \\ \vdots & & & \ddots & \\ 0 & & & -e^{-\mu} & I_N \end{vmatrix}$$

Derivatives

$$\frac{\delta X_N}{\delta I_N}=B_{N-1}(I_1,..,I_{N-1})$$

$$\frac{\delta B_N}{\delta I_k}=B_{k-1}(I_1,..,I_{k-1})B_{N-k}(I_{k+1},\dots,I_N)$$

Equilibrium Value

$$B_N(I_k=\mathrm{const.})=\begin{cases}\cosh((N+1)E_q)/\cosh E_q & (\mathrm{even}\,N)\\ \sinh((N+1)E_q)/\cosh E_q & (\mathrm{odd}\,N)\end{cases}$$

Hadron Mass in SCL-LQCD (Finite T)

Meson Mass

$$G^{-1}(\boldsymbol{k},\omega)=\frac{2N_c}{\kappa(\boldsymbol{k})}+\frac{4N_c}{d}\frac{\bar{\sigma}(\bar{\sigma}+m_q)}{\cos\omega+\cosh 2E_q}$$

$$\kappa(\boldsymbol{k})=\sum_{i=1}^{d}\cos k_i \quad \rightarrow \quad \kappa=-d,-d+2,\ldots d$$

$$\cosh M=2(\bar{\sigma}+m_q)\left(\frac{d+\kappa}{d}\bar{\sigma}+m_q\right)+1$$

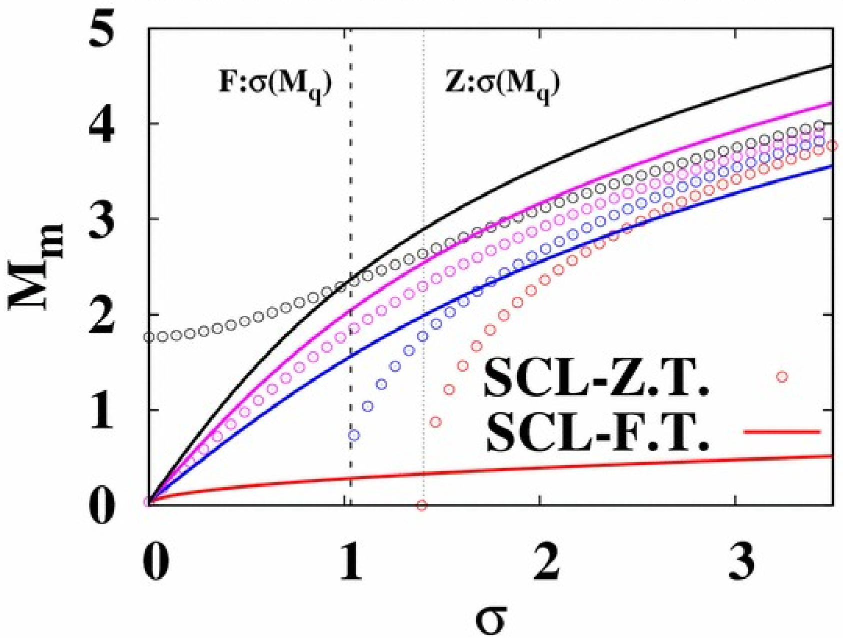
or $M=2\,\mathrm{arcsinh}\sqrt{(\bar{\sigma}+m_q)\left(\frac{d+\kappa}{d}\bar{\sigma}+m_q\right)}$

Meson masses are determined by the chiral condensate, σ.

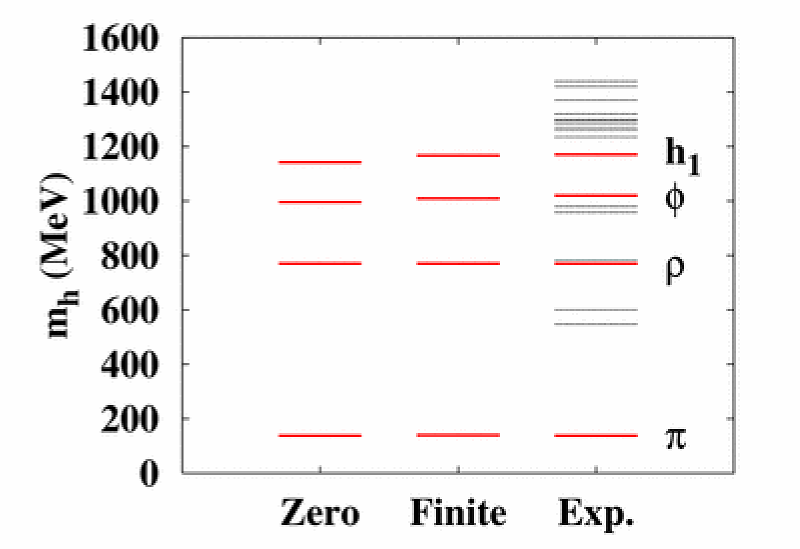
Chiral condensate is determined by the equilibrium condition, and given as a function of (T, μ).

→ ***Approximate Brown-Rho scaling is proven in SCL-LQCD***

Meson Mass as a funciton of σ



Medium Modification of Meson Masses

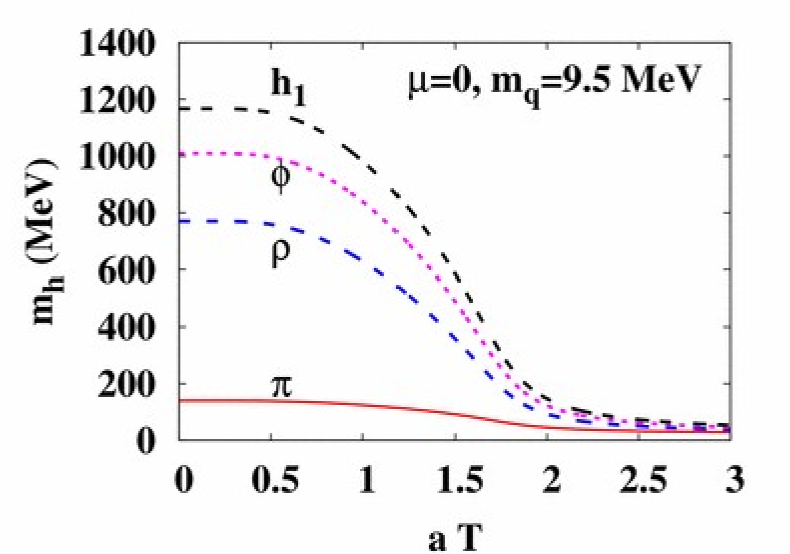


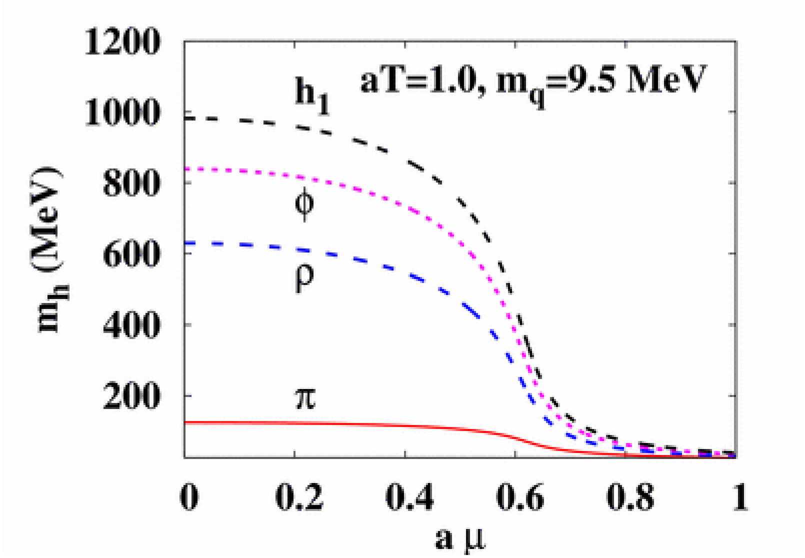
Scale fixing

- Search for σ_{vac} to minimize free E.
- Assign κ=-3, -1 as π and ρ
- Determine m_q and a^{-1} (lattice unit) to fit m_π / m_ρ

Medium modification

- Search for σ(T, μ) → Meson mass





Discussion

SCL ではは小さな μ で σ は変化しない

π, ρ mass fit の結果

a^{-1} = 497 MeV, m_q=9.5 MeV → T_c= 5/3a = 828 MeV Too large !

(SCL での昔からの問題点)

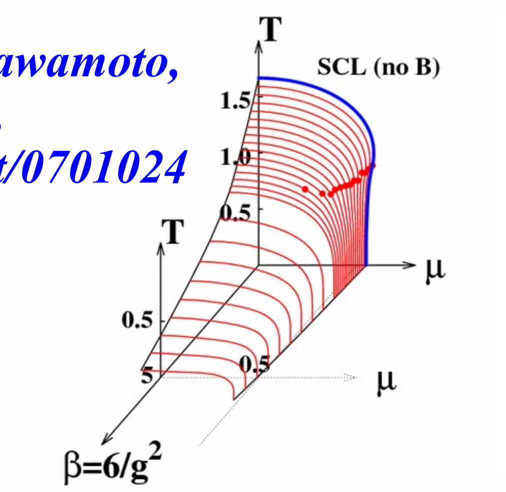
有限結合効果 **(1/g^2 correction)** により **T_c** は小さくなる

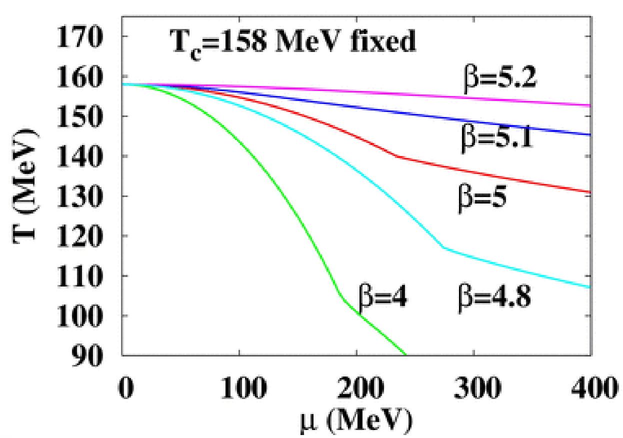
Bilic, Claymans, '95; AO, Kawamoto, Miura, '07

複数の補助場の導入が必要 → $\sigma=-\langle \bar{q} q\rangle$ と $\varphi=\langle \bar{q}\, g\, q\rangle$ **の対角化が必要**

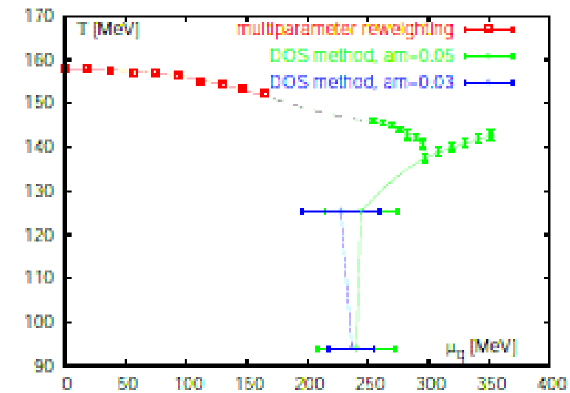
→ 間に合いませんでした .....

AO, Kawamoto, Miura, hep-lat/0701024





Kawamoto,Miura,AO, in prep.



Fodor,Katz,Schmit, 2007

Finite Coupling Correction and the Shape of the Phase Diagram

Discussions

Present phase diagram ↔ real phase diagram

- One species of staggered fermion ~ N_f=4. Should be 1st order !
- Tc seems to be too high. μ_c/T_c(present) ~ 0.45 ↔ μ_c/T_c(real)~(2-3)
- No stable CSC phase *(Azcoiti et al., 2003)*
  ↔ Stable CSC phase at large μ *(Alford, Hands, Stephanov)*

Two parameters are introduced through identities (HS transf.)

- The results should be independent from parameter choice !
  → MFA may break the identity...
- How should we fix these parameters ?

Is SCL-LQCD useful ? → We would like to answer "Yes" !

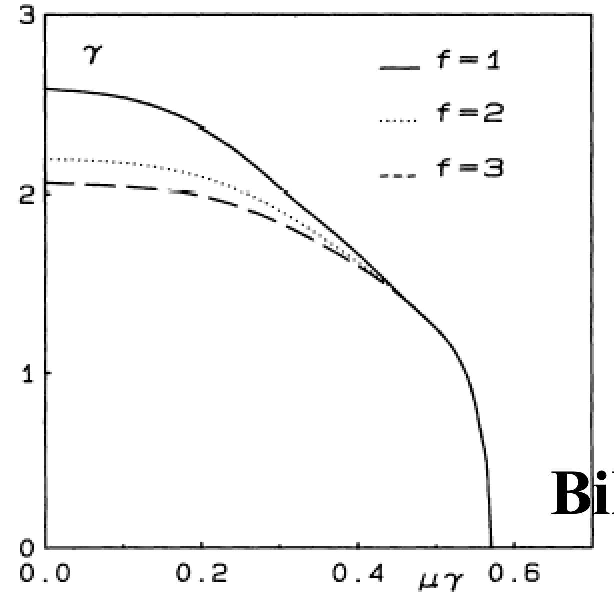
- Chiral RMF derived in SCL-LQCD works well in Nuclear Physics
  (Tsubakihara, AO, nucl-th/0607046
  Tsubakihara,Maekawa,AO, Proc. of HYP06, to appear)
- $1/g^2$ expansion may connect SCL-LQCD and real world.

Small Critical μ : Common in SCL-LQCD ?

Finite *T* SCL-LQCD

No B: $\mu_c(0)/T_c(0) \sim (0.2\text{-}0.35)$
(Nishida2004,
Bilic-Karsch-Redlich 1992, ...)

Present: $\mu_c(0)/T_c(0) < 0.44$
(Parameter dep.)



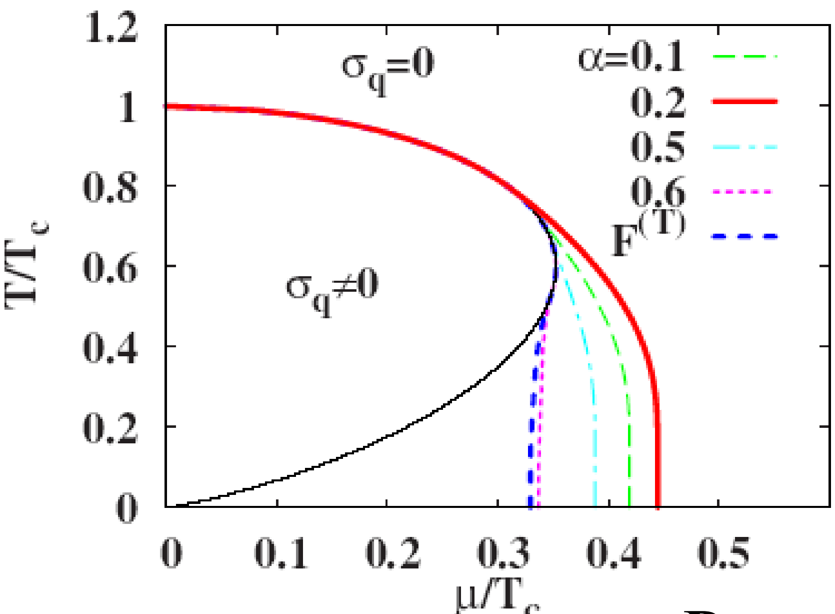
Bilic et al.

Monte-Carlo: $\mu_c(0)/T_c(0) > 1$

Fodor-Katz (Improved Reweighting)
Bielefeld (Taylor expansion),
de Forcrand-Philipsen (AC), ...

Real World: $\mu_c(0)/T_c(0) > 2$

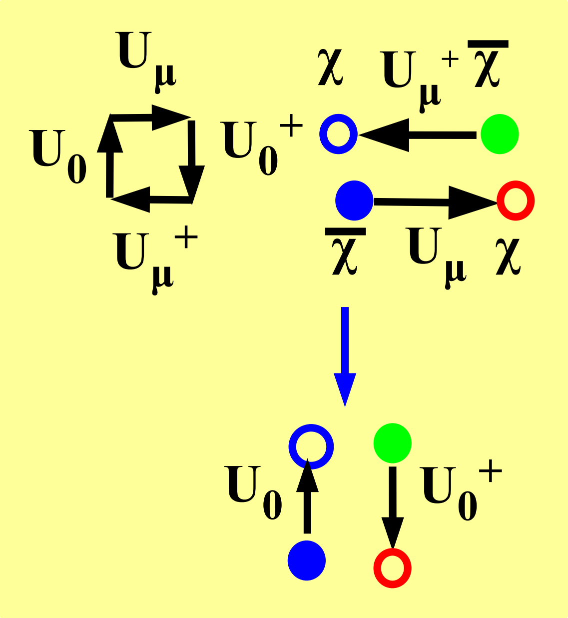
$T_c(0) \sim 170$ *MeV*, $\mu_c(0) > 330$ *MeV*



Present

$1/g^2$ expansion (w/o Baryon Effects)

T_c (μ=0) and μ_c (T=0): Which is worse ?



$1/g^2$ correction reduces T_c. *(Bilic-Cleymans 1995)*

Hadron masses are well explained in SCL.
(Kawamoto-Smit 1981, Kawamoto-Shigemoto 1982)

→ We expect Tc reduction with $1/g^2$ correction !

1/d expansion of plaquetts *(Faldt-Petersson 1986)*

Space-like plaquett

$$\exp\left[\frac{1}{g^2}\sum_{x,i>j>0}\mathrm{Tr}\,U_{ij}(x)\right]\to\exp\left[\frac{1}{8N_c^4 g^2}\sum_{x,k>j>0}M_x M_{x+\hat{j}} M_{x+\hat{k}} M_{x+\hat{k}+\hat{j}}\right]$$

Time-like plaquett

$$\exp\left[\frac{1}{g^2}\sum_{x,j>0}\mathrm{Tr}\,U_{0j}(x)\right]\to\exp\left[-\frac{1}{4N_c^2 g^2}\sum_{x,j>0}\left(V_x V_{x+\hat{j}}^+ + V_x^+ V_{x+\hat{j}}\right)\right]$$

$$\left(V_x=\overline{\chi}_x U_0(x)\chi_{x+\hat{0}}\right)$$

Plaquett Bosonization

Bosonization of Plaquetts *(O(1/d, 1/g⁴) and Im(V) are ignored)* **+ MFA**

$$\exp(-S_F - S_g) \to \exp\left[-\frac{1}{2}\sum_x (e^{\mu} V_x - e^{-\mu} V_x^+) + \frac{1}{4N_c}\sum_{x,j>0} M_x M_{x+\hat{j}} - m_0 \sum_x M_x\right]$$

$$\times\exp\left[-\frac{\beta_t}{2}\varphi_t \sum_x (V_x - V_x^+) + \beta_s \varphi_s \sum_{x,j>0} M_x M_{x+\hat{j}}\right]$$

$$\times\exp\left[-L^3 N_\tau \left(\frac{\beta_t}{4}\varphi_t^2 + \frac{\beta_s d}{4}\varphi_s^2\right)\right] \quad \left(\beta_t = \frac{d}{2N_c^2 g^2}\ ,\quad \beta_s = \frac{d-1}{8N_c^4 g^2}\right)$$

$$= \exp\left[-\frac{L^3}{T} F_\varphi - \frac{\alpha}{2}\sum_x \left(e^{\tilde{\mu}} V_x - e^{-\tilde{\mu}} V_x^+\right) + \frac{1}{2}\sum_{x,y} M_x \tilde{V}_M(x,y) M_y\right]$$

$$\alpha = 1 + \beta_t \varphi_t \cosh\mu \quad , \quad \tilde{\mu} = \mu - \beta_t \varphi_t \sinh\mu$$

$$<\varphi_t> = <V^+ - V> \quad , \quad <\varphi_s> = 2 <M_x M_{x+\hat{j}}>$$

Time-like plaquetts modifies effective chemical potential

Effective Free Energy with $1/g^2$ Correction (w/o B)

After Quark and Time-like Link integral, we get *F* as

$$F = \frac{d}{4N_c}\sigma^2(1+4N_c\beta_s\varphi_s) + \frac{\beta_t}{4}\varphi_t^2 + \frac{\beta_s d}{4}\varphi_s^2 - N_c\beta_t\varphi_t\cosh\mu + F_q(m_q;\tilde{\mu})$$

$$= \frac{d}{4N_c}\sigma^2 + 3\mathrm{d}\,\beta_s\sigma^4 + \frac{\beta_t}{4}\tilde{\varphi}_t^2 + F_q(m_q;\tilde{\mu})$$

$$\varphi_s = 2\sigma^2 \quad , \quad \varphi_t = \tilde{\varphi}_t + 2N_c\cosh\mu$$

Time-like plaquetts remains finite at large μ (c.f., S. Hands' talk)

$$m_q = \frac{d}{2N_c}\sigma(1+4N_c\beta_s\varphi_s - \beta_t\varphi_t\cosh\mu)$$

$$= \frac{d}{2N_c}\sigma(1-2N_c\beta_t\cosh^2\mu + 8N_c\beta_s\sigma^4 - \beta_t\tilde{\varphi}_t\cosh\mu)$$

$$\tilde{\mu} = \mu - \beta_t\varphi_t\sinh\mu = \mu - 2N_c\beta_t\cosh\mu\sinh\mu - \beta_t\tilde{\varphi}_t\sinh\mu$$

Space-like plaquett → Repulsive pot. ∝ σ^4, Enh. σ-quark couling

Time-like plaquett → Reduces μ and σ-quark coupling
(φ_t has to be determined to minimize F_{eff})

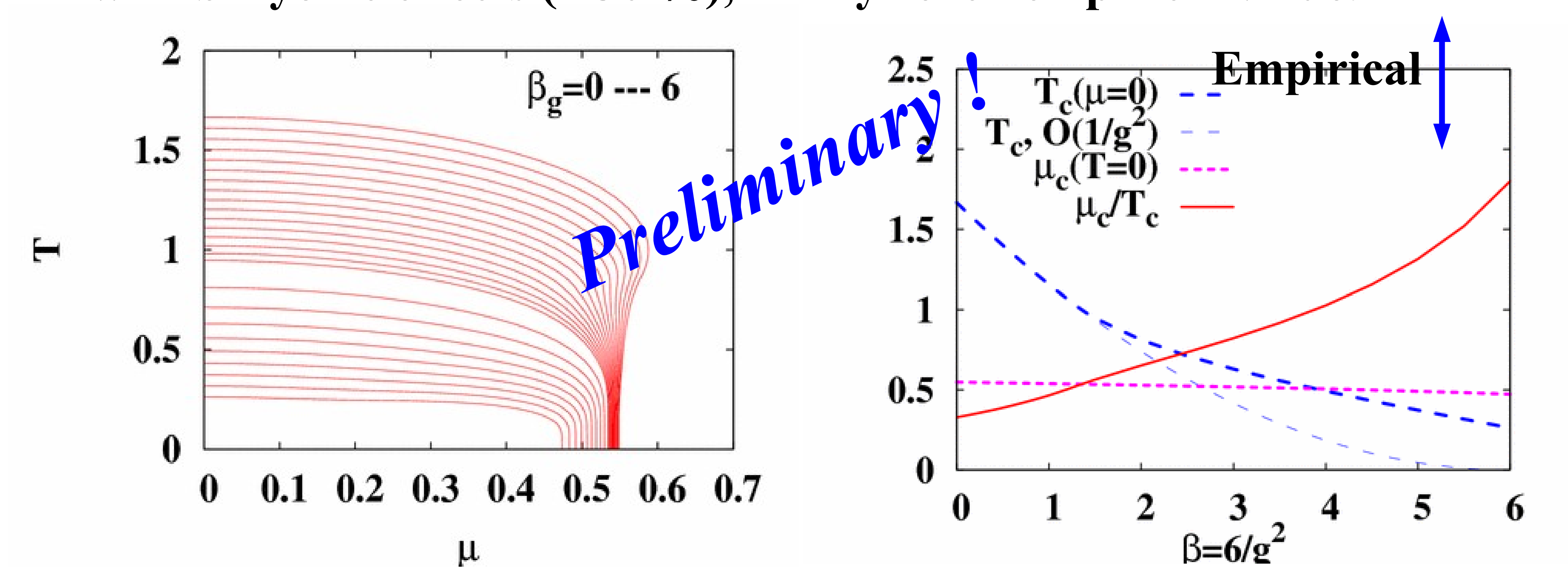
Phase Boundary with $1/g^2$ correction

Rapid decrease of $T_c(\mu=0)$, and slow decrease of $\mu_c(T=0)$.

Similar reduction of σ-quark coupling and effective μ at small condensate → can be mimicked by the scaling of T
(c.f. Bilic-Claymans 1995 (T_c goes down), Arai-Yoshinaga (Poster, goes up).

Ratio $\mu_c/T_c \sim 1.8$ @ g=1.

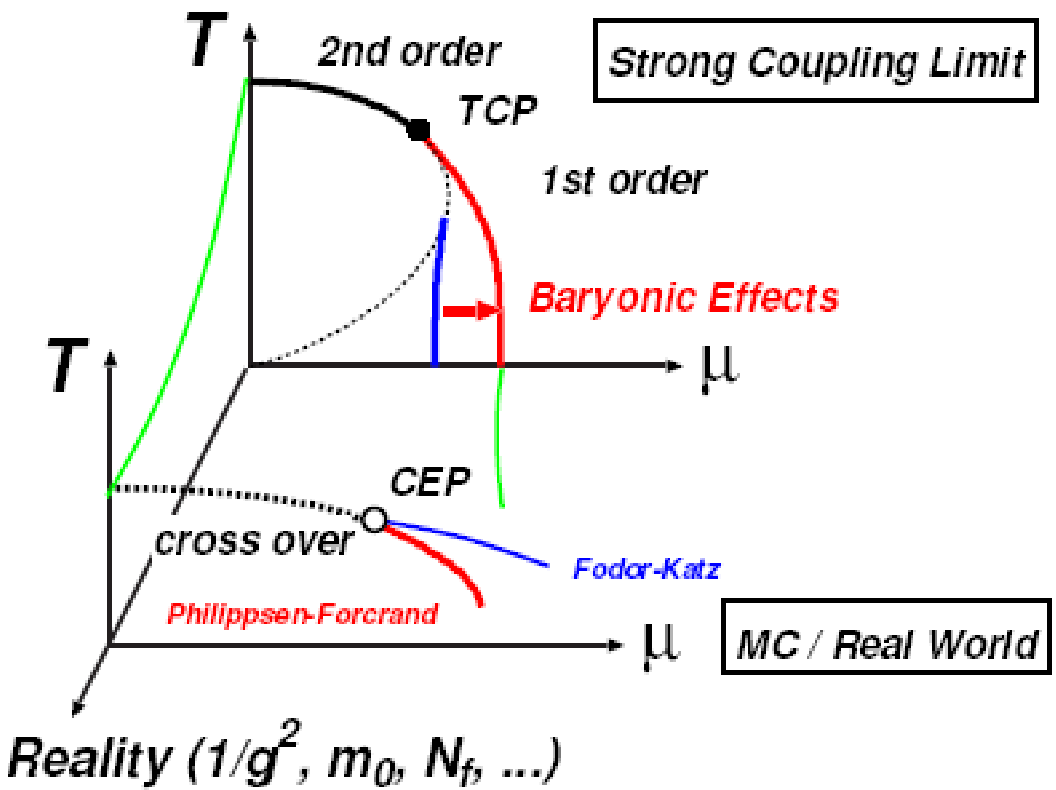
with baryonic effects (~ 30 %), it may reach empirical value.

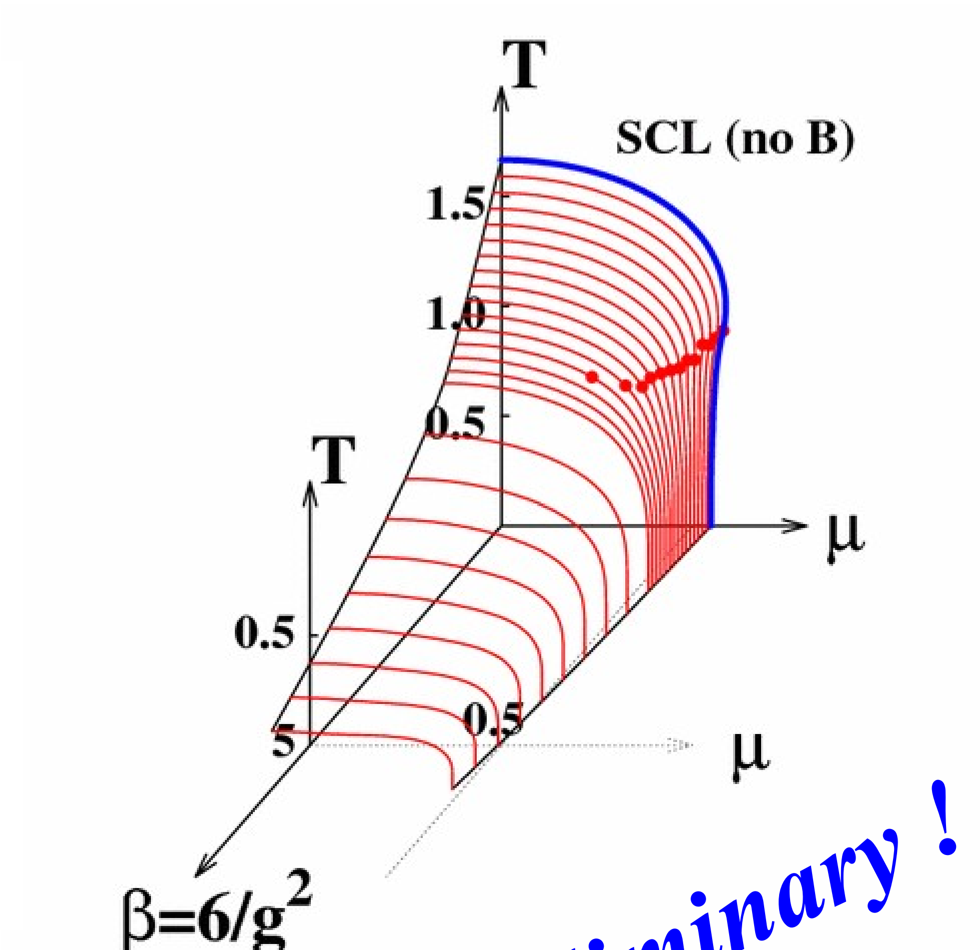


Evolution of Phase Diagram

"Reality" Axis: $1/g^2$, n_f, m_0, ... would enhance μ_c/T_c ratio

Example: $1/g^2$ correction enhances μ_c/T_c by a factor ~(2-3).





Preliminary !

Summary

Hadron (Meson) masses are evaluated in the Strong Coupling Limit of Lattice QCD at Finite T and μ.

Meson masses are determined by the chiral condensate, and they are approximately linear functions of σ, while m_π is always 0 in the chiral limit.

For high T or μ, meson masses decrease as σ decreases.
→ *Approximate Brown-Rho(-Hatsuda) scaling is supported*.

When we fit π and ρ masses, lattice unit (a^{-1}) is found to be around 500 MeV, suggesting T_c ~ 800 MeV in the Strong Coupling Limit. (Longstanding problem in the strong coupling limit....)

Finite coupling effects are found to decrease T_c (in the lattice unit), while approximately keeping μ_c.
→ Meson mass with $1/g^2$ correction has to be calculated.

Baryon mass → Miura's talk

Backups

Strong Coupling Limit Lattice QCD

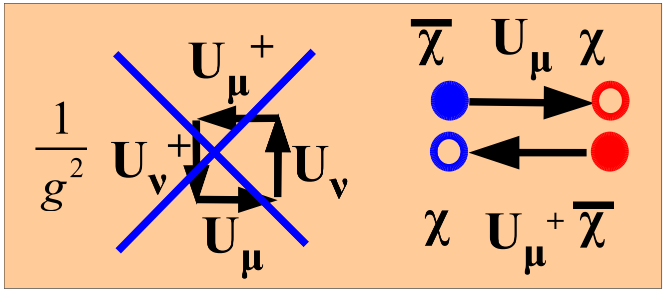
QCD Lattice Action

$$Z \simeq \int D[\chi, \bar{\chi}, U] \exp\left[-\left(S_G + S_F^{(s)} + S_F^{(t)} + m_0 M\right)\right]$$

$$S_G = \frac{1}{g^2} \sum_{x\mu\nu} \left[\mathrm{Tr}\, U_{\mu\nu} + \mathrm{Tr}\, U^+_{\mu\nu}\right]$$

$$S_F^{(s)} = \frac{1}{2} \sum_{x,j} \eta_j(x) \left(\bar{\chi}_x U_j(x) \chi_{x+\hat{j}} - \bar{\chi}_{x+\hat{j}} U_j^+(x) \chi_x\right)$$

$$S_F^{(t)} = \frac{1}{2} \sum_x \left(e^{\mu} \bar{\chi}_x U_0(x) \chi_{x+\hat{0}} - e^{-\mu} \bar{\chi}_{x+\hat{0}} U_0^+(x) \chi_x\right)$$

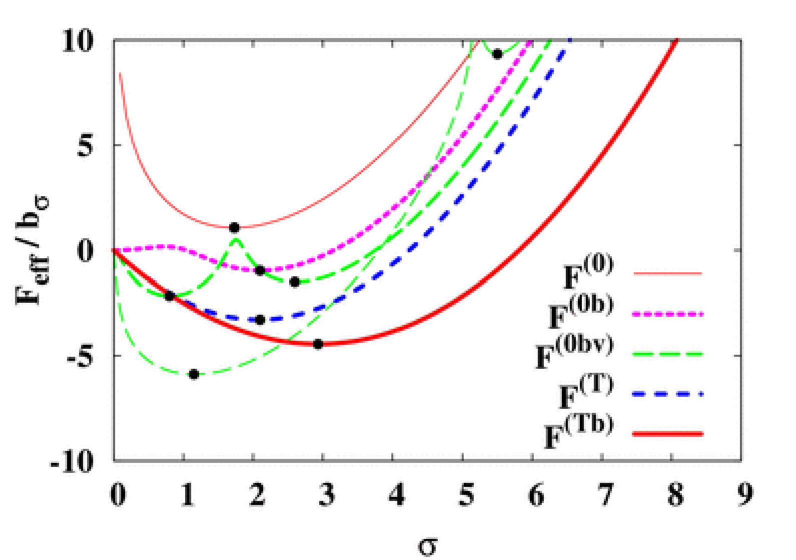


Strong Coupling Limit: g→∞

Ignore S_G → Link integral

Zero T treatment
→ All Links are integrated first

Finite T treatment
→ Temporal Links are integrated later exactly.



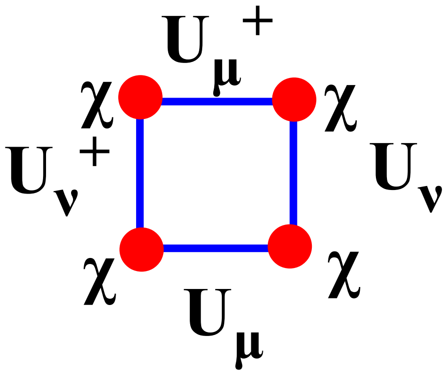
$$F_{\mathrm{eff}} = \frac{N_c}{d+1} \bar{\sigma}^2 - N_c \log(\bar{\sigma} + m_q)$$

$$F^{(q)}_{\mathrm{eff}}(m; T, \mu) = \frac{N_c \bar{\sigma}^2}{d} - T \log\left[\frac{\sinh((N_c+1) E_q/T)}{\sinh(E_q/T)} + 2\cosh(N_c \mu/T)\right]$$

$$E_q(m) = \mathrm{arcsinh}\, m$$

Lattice Action in SCL-LQCD

Lattice Action with staggered Fermions



$$S[U,\chi,\bar{\chi}] = S_G[U] + S_F[U,\chi,\bar{\chi}] \;,$$

$$S_G[U] = \frac{2N_c}{g^2} \sum_{x,\mu,\nu} \left\{ 1 - \frac{1}{N_c} \mathrm{Re}\mathrm{Tr} U_{\mu\nu}(x) \right\}$$

$g \to \infty$ → **0**

$$U_{\mu\nu}(x) = U_\nu^\dagger(x) U_\mu^\dagger(x+\hat{\nu}) U_\nu(x+\hat{\mu}) U_\mu(x) \;,$$

Chem. Pot.

$$\begin{aligned}
S_F[U,\chi,\bar{\chi}] &= S_F^{(m)}[\chi,\bar{\chi}] + S_F^{(j)}[U_j,\chi,\bar{\chi}] + S_F^{(U0)}[U_0,\chi,\bar{\chi}] \;, \\
S_F^{(m)}[\chi,\bar{\chi}] &= m \sum_x \bar{\chi}^a(x)\chi^a(x) \;, \\
S_F^{(j)}[U_j,\chi,\bar{\chi}] &= \frac{1}{2} \sum_x \sum_{j=1}^{d} \eta_j(x) \left\{ \bar{\chi}(x) U_j(x) \chi(x+\hat{j}) - \bar{\chi}(x+\hat{j}) U_j^\dagger(x) \chi(x) \right\} \;, \\
S_F^{(U0)}[U_0,\chi,\bar{\chi}] &= \frac{1}{2} \sum_x \eta_0(x) \left\{ \bar{\chi}(x) e^{\mu} U_0(x) \chi(x+\hat{0}) - \bar{\chi}(x+\hat{0}) U_0^\dagger(x) e^{-\mu} \chi(x) \right\} \;.
\end{aligned}$$

In the Strong Coupling Limit ($g \to \infty$), we can ignore S_G, and semi-analytic calculation becomes possible.

details

Lattice QCD action

$$S_F^{(U_j)} = \frac{1}{2}\sum_x \eta_j(x)\left[\bar{\chi}(x)U_\mu(x)\chi(x+\hat{\mu}) - \bar{\chi}(x+\hat{\mu})U_\mu^\dagger(x)\chi(x)\right]$$

$$S_F^{(U_0)} = \frac{1}{2}\sum_x \left[\bar{\chi}(x)e^{\mu}U_\mu(x)\chi(x+\hat{\mu}) - \bar{\chi}(x+\hat{\mu})e^{-\mu}U_\mu^\dagger(x)\chi(x)\right]$$

$$S_F^{(m)} = m_0\sum_x \bar{\chi}^a(x)\chi^a(x) \ ,$$

Mesonic and Baryonic Composites

$$M(x) = \delta_{ab}\bar{\chi}^a(x)\chi^b(x) \ ,$$

$$B(x) = \frac{1}{6}\varepsilon_{abc}\chi^a(x)\chi^b(x)\chi^c(x) \ , \quad \bar{B}(x) = \frac{1}{N_c!}\varepsilon_{abc}\bar{\chi}^c(x)\bar{\chi}^b(x)\bar{\chi}^a(x)$$

Fermion Integral

$$\int \mathcal{D}[U_0,\chi,\bar{\chi}]\exp\left[-\sum_t \sigma M - S_F^{(U_0)}\right] = \int \mathcal{D}[U_0,\chi,\bar{\chi}]\prod_k \exp\left[-\bar{\chi}_k G(k)\chi_k/2\right]$$

$$= \cdots = C_\sigma^3 - \frac{1}{2}C_\sigma + \frac{1}{4}\cosh(3\beta\mu)$$

$$F_{\rm eff}^{(q)}(\sigma_q) = -T\log\left[\frac{4}{3}\left(C_\sigma^3 - \frac{1}{2}C_\sigma + \frac{1}{4}C_{3\mu}\right)\right] \qquad C_\sigma = \cosh\left[\beta \mathrm{arcsinh}\,\tilde{\sigma}\right]$$

SCL-LQCD with Baryons

Effective Action up to O(1/√d)

$$M = \bar{\chi}_a \chi^a$$
$$B = \epsilon_{abc} \chi^a \chi^b \chi^c / 6$$

$$Z \simeq \int D[\chi, \bar{\chi}, U_0] \exp\left[\frac{1}{2}(M, V_M M) + (\bar{B}, V_B B) - (\bar{\chi} G_0 \chi)\right]$$

$$= \int D[\chi, \bar{\chi}, U_{0,} b, \bar{b}] \exp\left[\frac{1}{2}(M V_M M) - (\bar{b} V_B^{-1} b) + (\bar{b}, B) + (\bar{B}, b) - (\bar{\chi} G_0 \chi)\right]$$

Decomposition of bB by using diquark condensate (Azcoiti et al., 2004)

$$\exp\left[(\bar{b}, B) + (\bar{B}, b)\right] = \exp\left[\frac{1}{6}(\bar{b}, \epsilon\chi\chi\chi) + \frac{1}{6}(\epsilon\bar{\chi}\bar{\chi}\bar{\chi}, b)\right]$$

$$= \int D[\phi_a, \phi_a^*] \exp\left[-\phi^* \phi + \phi^* \left(\frac{\gamma}{2}\epsilon\chi\chi + \frac{\bar{\chi} b}{3\gamma}\right) + \phi\left(\frac{\gamma}{2}\epsilon\bar{\chi}\bar{\chi} + \frac{\bar{b}\chi}{3\gamma}\right)\right]$$

$$\times \exp(-\gamma M^2/2 + M \bar{b} b / 9\gamma^2)$$

Decomposition of Mbb using baryon potential field ω

$$\exp(M \bar{b} b / 9\gamma^2) = \int D[\omega] \exp\left[\frac{1}{2}\omega^2 - \omega\left(\alpha M + \frac{\bar{b} b}{9\alpha\gamma^2}\right) - \frac{\alpha^2}{2} M^2\right]$$

note: $(\bar{b} b)^2 = 0$ *with one species of staggered fermion !*

Effective Free Energy with Baryon Effects

Effective Action in local bilinear form of quarks

$$S_F = -\frac{1}{2}(M\tilde{V}_M M) + \frac{1}{2}(\omega,\omega) + (\bar{b},\tilde{V}_B^{-1}(g_\omega \omega)b) + \alpha(\omega, M) + (\bar{\chi}G_0\chi)$$

~~$+(\phi^*\phi)+(\phi^* D)+(D^+\phi)$~~

Bosonization + MFA +No diquark cond.

$$= \frac{N_s^3 N_\tau}{2}\left(a_\sigma \sigma^2 + \omega^2\right) + (a_\sigma\sigma + \alpha\omega, M) + (\bar{\chi}G_0\chi) + (\bar{b},\tilde{V}_B^{-1}(g_\omega\omega)b)$$

quark & gluon int. *b int.*

$$F_{\rm eff}(\sigma,\omega) = \frac{1}{2}a_\sigma\sigma^2 + \frac{1}{2}\omega^2 + F_{\rm eff}^{(q)}(a_\sigma\sigma + \alpha\omega) + F_{\rm eff}^{(b)}(g_\omega\omega)$$

$O(\omega^2)$ *$O(\omega^4)$*

$$= \frac{1}{2}a_\sigma\sigma^2 + \frac{1}{2}a_\omega\omega^2 + F_{\rm eff}^{(q)}(a_\sigma\sigma + \alpha\omega) + \Delta F_{\rm eff}^{(b)}(g_\omega\omega)$$

Linear Approx. $(\omega \sim \alpha\sigma/a_\omega)$

$$F_{\rm eff}(\sigma) = \frac{1}{2}b_\sigma\sigma^2 + F_{\rm eff}^{(q)}(b_\sigma\sigma) + \Delta F_{\rm eff}^{(b)}(g_\sigma\sigma)$$

Color Angle Average

Problem: Diquark Condensates induce quark-baryon coupling, and Baryon integral becomes difficult.
→ Solution: *Color Angle Average*

$$D = \frac{\gamma}{2}\epsilon\chi\chi + \frac{\bar{\chi}b}{3\gamma}$$

Integral of "Color Angle Variables"

$$\int \mathcal{D}[\phi_a, \phi_a^\dagger] \exp\left\{\phi_a^\dagger D_a + D_a^\dagger \phi_a\right\} = \int \mathcal{D}[v] \exp\left\{\frac{v^2}{3} D_a^\dagger D_a + \frac{v^4}{162} M^3 \bar{b} b\right\}$$

Three-Quark and Baryon Coupling is ReBorn !

$$D_a^\dagger D_a = Y + \bar{b}B + \bar{B}b \ , \quad Y = \frac{\gamma^2}{2} M^2 - \frac{1}{9\gamma^2} M \bar{b} b$$

Solve "Self-Consistent" Equaton

$$\exp(\bar{b}B + \bar{B}b) \simeq \exp\left[-v^2 - Y + \frac{v^2}{3}\left(\bar{b}B + \bar{B}b + Y\right) + \frac{v^4}{162} M^3 \bar{b} b\right]$$

$$\simeq \exp\left[-\frac{v^2}{R_v} + \frac{v^4 M^3 \bar{b} b}{162 R_v} - Y\right] \quad (R_v = 1 - v^2/3)$$

Effective Free Energy with Diquark Condensate

Bosonization of $M^k \bar{b} b$ → **Introduce k bosons**

$$\exp M^k \bar{b} b = \int d\omega_k \exp[-\frac{1}{2}(\omega_k + \alpha_k M + 1/\alpha_k M^{k-1} \bar{b} b)^2 + M^k \bar{b} b]$$

$$= \int d\omega_k \exp[-\omega_k^2/2 - \omega(\alpha_k M + 1/\alpha_k M^{k-1} \bar{b} b) - \alpha_k^2 M^2/2]$$

Effective Free Energy

$$\mathcal{F}_{\rm eff}^{(Tbv)} = F_X(\sigma, v, \omega_i) + F_{\rm eff}^{(b)}(g_\omega \omega) + F_{\rm eff}^{(q)}(m_q)$$

$$F_X = \frac{1}{2}(a_\sigma \sigma^2 + \omega^2 + \omega_1^2 + \omega_2^2) + \frac{v^2}{R_v} \qquad m_q = a_\sigma \sigma + \alpha\omega + \alpha_1\omega_1 + \alpha_2\omega_2 + m_0$$

$$a_\sigma = \frac{1}{2} - \gamma^2 - \alpha^2 - \alpha_1^2 - \alpha_2^2 \qquad g_\omega = \frac{1}{9\alpha\gamma^2}\left[1 + \frac{\gamma^2 v^4 \omega_1 \omega_2}{18\alpha_1\alpha_2 R_v}\right]$$

The same F_{eff} ***is obtained at v=0.***
Diquark Effects in interaction start from v^4.
(No Stable CSC phase appears at g= ∞)

c.f. Ipp, Yamamoto

Effective Free Energy with Baryon Effects

(Kawamoto-Miura-AO-Ohnuma, hep-lat/0512023)

$$F_{\text{eff}}(\sigma)=\frac{1}{2}b_\sigma \sigma^2+F_{\text{eff}}^{(q)}(b_\sigma \sigma;T,\mu)+\Delta F_{\text{eff}}^{(b)}(g_\sigma \sigma)$$

is analytically derived based on many previous works, including

Strong Coupling Limit *(Kawamoto-Smit, 1981)*

1/d expansion *(Kluberg-Stern-Morel-Petersson, 1983)*

Lattice chemical potential *(Hasenfratz-Karsch, 1983)*

Quark and time-like gluon analytic integral
(Damgaad-Kawamoto-Shigemoto, 1984, Faldt-Petersson, 1986)

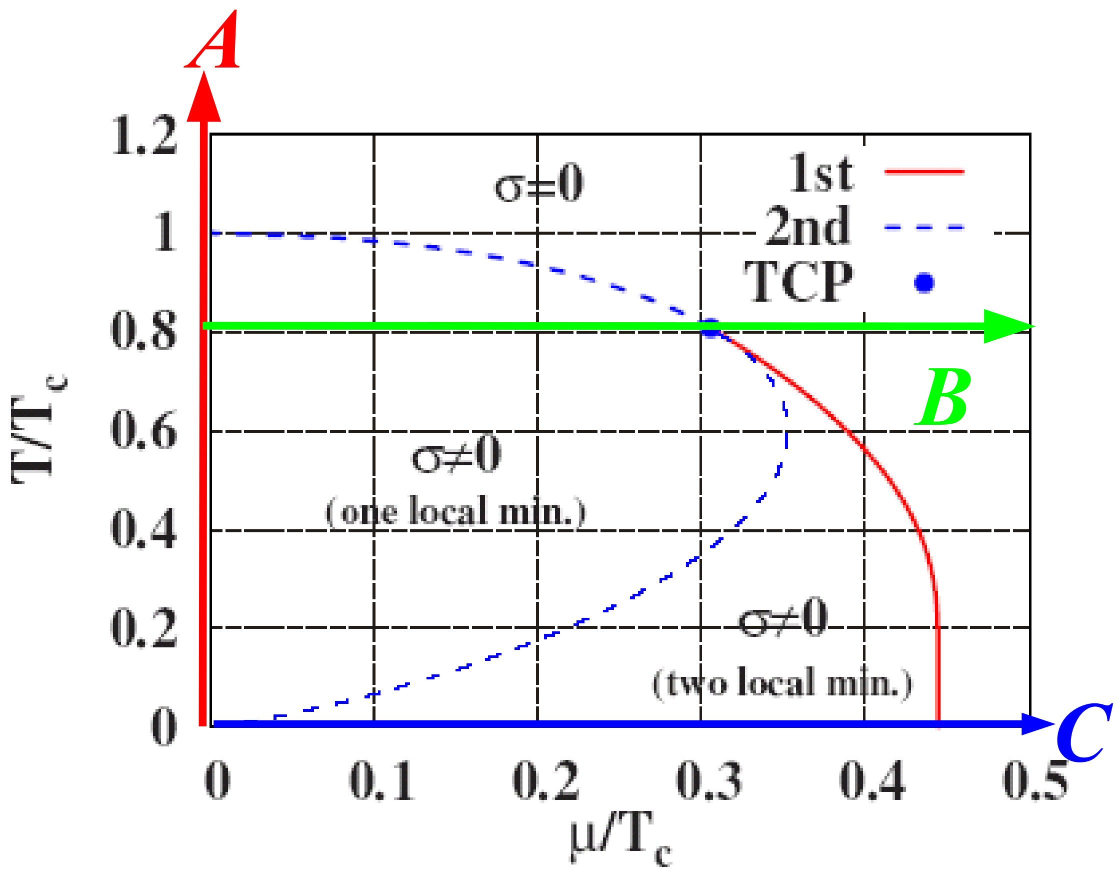
$$F_{\text{eff}}^{(q)}(\sigma;T,\mu)=-T\log\left(C_\sigma^3-\frac{1}{2}C_\sigma+\frac{1}{4}C_{3\mu}\right)\quad C_\sigma=\cosh(\sinh^{-1}\sigma/T)\quad C_{3\mu}=\cosh(3\mu/T)$$

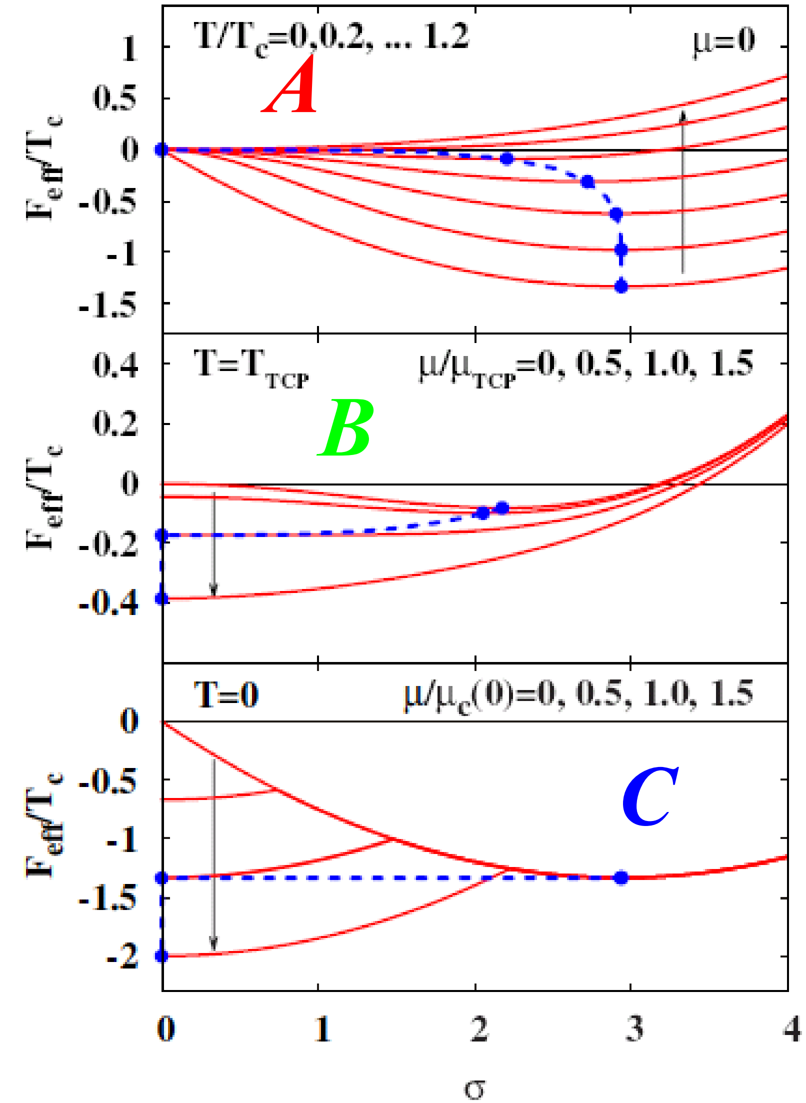
Decomposition of baryon-3 quark coupling
(Azcoiti-Di Carlo-Galante-Laliena, 2003)

and auxiliary baryon potential and baryon integral

Free Energy Surface and Phase Diagram

At μ ≠ 0, quark can gain Free Energy even at σ =0
→ Two Min. Structure
→ First Order





α =0.2