Quark matter phase diagram in the strong coupling region of lattice QCD

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Phase diagram at finite temperature and quark density in the strong coupling limit of lattice QCD for color SU(3)
N. Kawamoto, K. Miura, A. Ohnishi, T. Ohnuma,
Phys. Rev. D 75(2007), 014502 [arXiv: hep-lat/0512023]
Strong Coupling Limit/Region of Lattice QCD
A. Ohnishi, N. Kawamoto, K. Miura, K. Tsubakihara, H. Maekawa,
Prog. Theor. Phys. Suppl. (2007), to appear [arXiv:0704.2823]
(Proc. of YKIS06.)
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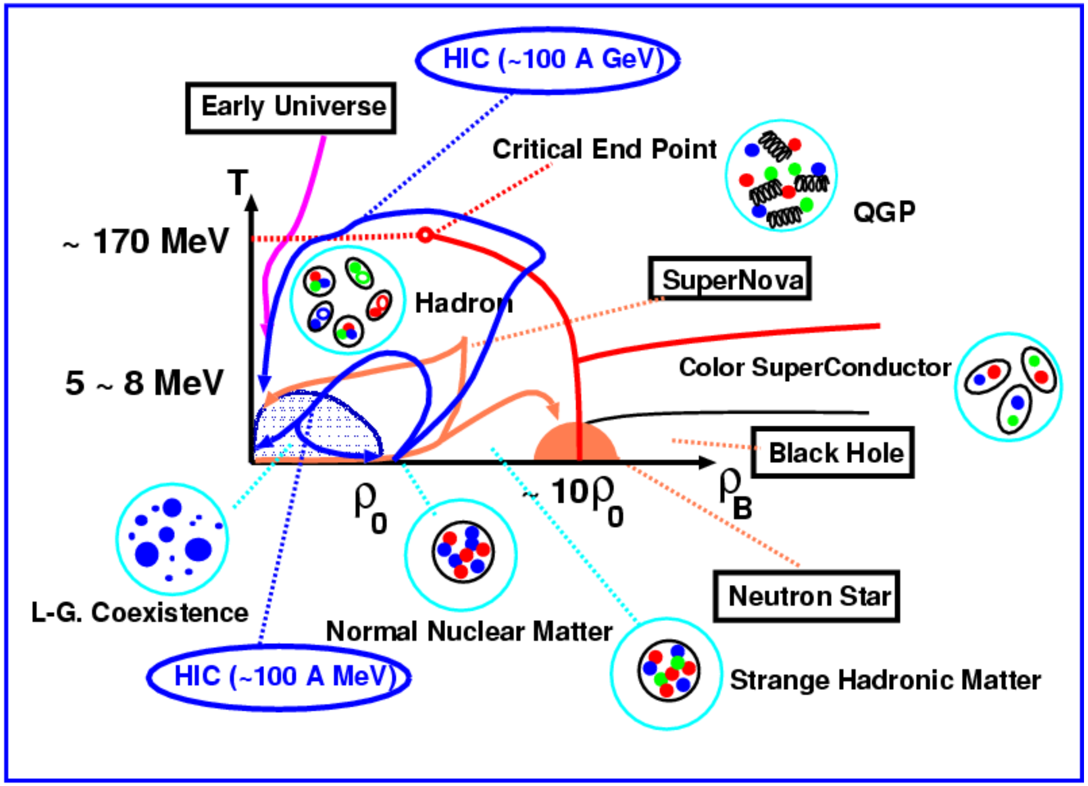
J. Phys. G (2007), to appear [arXiv:hep-lat/0701024] (Proc. of QM2006).

Quark and Hadronic Matter Phase Diagram

- **Dense quark & hadronic matter contains rich physics, but Lattice QCD simulation is not yet reliable.**
  → ***Model/Approximate approaches are necessary !***
  - **Monte-Carlo calc. of Lattice QCD:**
    Improved ReWeighting Method (Fodor-Katz)
    Taylor Expansion in μ (Bielefeld-Swansea)
    Analytic Continuation (de Forcrand-Philipssen)
  - **Model / Phen. Approaches: (P)NJL, QMC, RMF, ...**
  - ***Strong Coupling Limit of Lattice QCD***



Previous Works in Strong Coupling Limit LQCD

- **Strong Coupling Limit Lattice QCD re-attracts interests**
  c.f. Nakamura @ JHF Symp. for high density matter (2001)

Ref	T	μ	Nc	Baryon	CSC	Nf
Damgaard-Kawamoto-Shigemoto('84)	*Finite*	***0***	*U(Nc)*	***X***	*X*	*1*
Damgaard-Hochberg-Kawamoto('85)	***0***	*Finite*	*3*	*Yes*	*X*	*1*
Bilic-Karsch-Redlich('92)	*Finite*	*Finite*	*3*	***X***	*X*	*1 ~ 3*
Azcoiti-Di Carlo-Galante-Laliena('03)	***0***	*Finite*	*3*	*Yes*	*Yes*	*1*
Nishida-Fukushima-Hatsuda('04)	*Finite*	*Finite*	***2***	*Yes (*)*	*Yes (*)*	*1*
Nishida('04)	*Finite*	*Finite*	*3*	***X***	*X*	*1~2*
Kawamoto-Miura-AO-Ohnuma('07)	***Finite***	***Finite***	***3***	***Yes***	***Yes (+)***	***1***

*: bosonic baryon=diquark in SU(2)
+: analytically included, but ignored in numerical calc.

- ***Baryon effects have been ignored in finite T treatments !***
  → This work: Baryonic effects at Finite T (and μ) for $SU_c(3)$

Strong Coupling Limit Lattice QCD

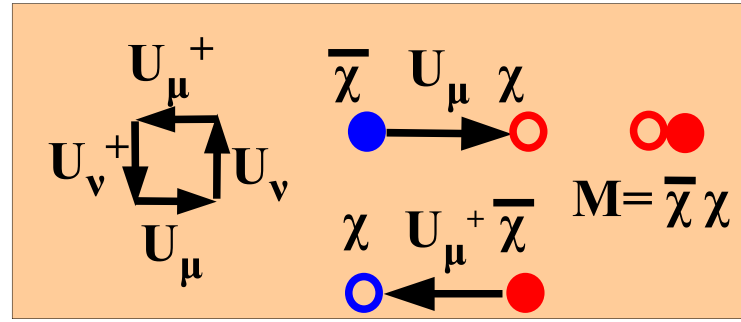
- **QCD Lattice Action**

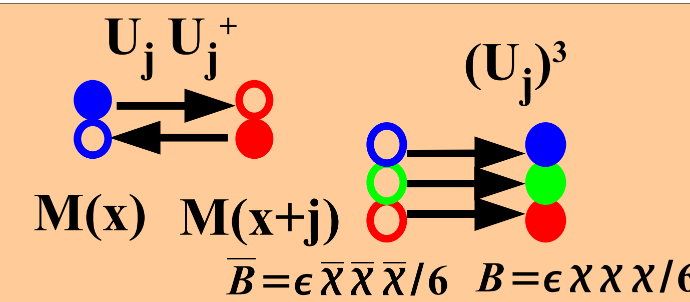
$$Z \simeq \int D[\chi,\bar{\chi},U]\exp\left[-\left(S_G+S_F^{(s)}+S_F^{(t)}+m_0 M\right)\right]$$

$$S_G=\frac{1}{g^2}\sum_{x\mu\nu}\left[\mathrm{Tr}\,U_{\mu\nu}+\mathrm{Tr}\,U_{\mu\nu}^{+}\right]$$

$$S_F^{(s)}=\frac{1}{2}\sum_{x,j}\eta_j(x)\left(\bar{\chi}_x U_j(x)\chi_{x+\hat{j}}-\bar{\chi}_{x+\hat{j}}U_j^{+}(x)\chi_x\right)$$

$$S_F^{(t)}=\frac{1}{2}\sum_{x}\left(e^{\mu}\bar{\chi}_x U_0(x)\chi_{x+\hat{0}}-e^{-\mu}\bar{\chi}_{x+\hat{0}}U_0^{+}(x)\chi_x\right)$$





$$\int dU\, U_{ab}U_{cd}^{+}=\frac{1}{N_c}\delta_{ad}\delta_{bc}$$

$$\int dU\, U_{ab}U_{cd}U_{ef}=\frac{1}{6}\epsilon_{ace}\epsilon_{bdf}$$

- **Strong Coupling Limit: g→∞**
  - **We can ignore S_G and perform one-link integral after 1/d expansion.**

$$S_F^{(s)}\to-\frac{1}{2}(M\,V_M\,M)-(\bar{B}\,V_B\,B)$$

$$=-\frac{1}{4N_c}\sum_{x,j>0}M_x M_{x+\hat{j}}+\sum_{x,j>0}\frac{\eta_j}{8}\left[\bar{B}_x B_{x+\hat{j}}-\bar{B}_{x+\hat{j}}B_x\right]$$

SCL-LQCD w/o Baryons

Damgaad-Kawamoto-Shigemoto 1984, Faldt-Petersson 1986, Bilic-Karsch-Redlich 1992, Nishida 2004, ...

- **Lattice Action (staggered fermion) in SCL**

$$Z \simeq \int D[\chi, \bar{\chi}, U] \exp\left[-S_F^{(s)} - S_F^{(t)} - m_0 \bar{\chi}\chi - S_G\right]$$

- **Spatial Link Integral**

Strong Coupling

$$\simeq \int D[\chi, \bar{\chi}, U_0] \exp\left[\frac{1}{2}(M, V_M M) + (\bar{B}, V_B B) - (\bar{\chi} G_0 \chi)\right]$$

1/d Expansion (1/√d)

- **Bosonization (Hubburd-Stratonovich transformation)**

$$\simeq \int D[\chi, \bar{\chi}, U_{0,} \sigma] \exp\left[-\frac{1}{2}(\sigma, V_M \sigma) - (\sigma, V_M M) - (\bar{\chi} G_0 \chi)\right]$$

- **Quark and U_0 Integral**

$$(\bar{\chi} G(\sigma) \chi)$$

$$\simeq \exp\left(-N_S^3 N_\tau \left[\frac{1}{2} a_\sigma \sigma^2 - T \log G_U(\sigma)\right]\right) = \exp\left(-N_S^3 F_{\mathrm{eff}}/T\right)$$

Local Bi-linear action in quarks → Effective Free Energy

SCL-LQCD with Baryons

- **Effective Action up to O(1/√d)**

$$M = \bar{\chi}_a \chi^a$$
$$B = \epsilon_{abc} \chi^a \chi^b \chi^c / 6$$

$$Z \simeq \int D[\chi, \bar{\chi}, U_0] \exp\left[\frac{1}{2}(M, V_M M) + (\bar{B}, V_B B) - (\bar{\chi} G_0 \chi)\right]$$

$$= \int D[\chi, \bar{\chi}, U_{0,} b, \bar{b}] \exp\left[\frac{1}{2}(M V_M M) - (\bar{b} V_B^{-1} b) + (\bar{b}, B) + (\bar{B}, b) - (\bar{\chi} G_0 \chi)\right]$$

- *Decomposition of bB by using diquark condensate (Azcoiti et al., 2004)*

$$\exp[(\bar{b}, B) + (\bar{B}, b)] = \exp\left[\frac{1}{6}(\bar{b}, \epsilon \chi \chi \chi) + \frac{1}{6}(\epsilon \bar{\chi} \bar{\chi} \bar{\chi}, b)\right]$$

$$= \int D[\phi_a, \phi_a^*] \exp\left[-\phi^* \phi + \phi^* \left(\frac{\gamma}{2} \epsilon \chi \chi + \frac{\bar{\chi} b}{3\gamma}\right) + \phi \left(\frac{\gamma}{2} \epsilon \bar{\chi} \bar{\chi} + \frac{\bar{b} \chi}{3\gamma}\right)\right]$$

$$\times \exp(-\gamma M^2 / 2 + M \bar{b} b / 9\gamma^2)$$

- *Decomposition of Mbb using baryon potential field ω*

$$\exp(M \bar{b} b / 9\gamma^2) = \int D[\omega] \exp\left[\frac{1}{2}\omega^2 - \omega\left(\alpha M + \frac{\bar{b} b}{9 \alpha \gamma^2}\right) - \frac{\alpha^2}{2} M^2\right]$$

  - *note:* $(\bar{b} b)^2 = 0$ *with one species of staggered fermion !*

Effective Free Energy with Baryon Effects

- **Effective Action in local bilinear form of quarks**

$$S_F = -\frac{1}{2}(M\tilde{V}_M M) + \frac{1}{2}(\omega,\omega) + (\bar{b}, \tilde{V}_B^{-1}(g_\omega \omega) b) + \alpha(\omega, M) + (\bar{\chi} G_0 \chi)$$

~~$+(\phi^*\phi)+(\phi^* D)+(D^+\phi)$~~

Bosonization + MFA
+No diquark cond.

$$= \frac{N_s^3 N_\tau}{2}\left(a_\sigma \sigma^2 + \omega^2\right) + (a_\sigma \sigma + \alpha\omega, M) + (\bar{\chi} G_0 \chi) + (\bar{b}, \tilde{V}_B^{-1}(g_\omega \omega) b)$$

quark & gluon int. *b int.*

$$F_{\mathrm{eff}}(\sigma,\omega) = \frac{1}{2} a_\sigma \sigma^2 + \frac{1}{2}\omega^2 + F_{\mathrm{eff}}^{(q)}(a_\sigma \sigma + \alpha\omega) + F_{\mathrm{eff}}^{(b)}(g_\omega \omega)$$

$O(\omega^2)$ *$O(\omega^4)$*

$$= \frac{1}{2} a_\sigma \sigma^2 + \frac{1}{2} a_\omega \omega^2 + F_{\mathrm{eff}}^{(q)}(a_\sigma \sigma + \alpha\omega) + \Delta F_{\mathrm{eff}}^{(b)}(g_\omega \omega)$$

Linear Approx. *$(\omega \sim \alpha\sigma / a_\omega)$*

$$F_{\mathrm{eff}}(\sigma) = \frac{1}{2} b_\sigma \sigma^2 + F_{\mathrm{eff}}^{(q)}(b_\sigma \sigma) + \Delta F_{\mathrm{eff}}^{(b)}(g_\sigma \sigma)$$

Effective Free Energy with Baryon Effects

(Kawamoto-Miura-AO-Ohnuma, hep-lat/0512023)

$$F_{\mathrm{eff}}(\sigma)=\frac{1}{2}b_\sigma\sigma^2+F_{\mathrm{eff}}^{(q)}(b_\sigma\sigma;T,\mu)+\Delta F_{\mathrm{eff}}^{(b)}(g_\sigma\sigma)$$

is analytically derived based on many previous works, including

- **Strong Coupling Limit** *(Kawamoto-Smit, 1981)*
- **1/d expansion** *(Kluberg-Stern-Morel-Petersson, 1983)*
- **Lattice chemical potential** *(Hasenfratz-Karsch, 1983)*
- **Quark and time-like gluon analytic integral**
  (Damgaad-Kawamoto-Shigemoto, 1984, Faldt-Petersson, 1986)

$$F_{\mathrm{eff}}^{(q)}(\sigma;T,\mu)=-T\log\left(C_\sigma^3-\frac{1}{2}C_\sigma+\frac{1}{4}C_{3\mu}\right)\quad C_\sigma=\cosh(\sinh^{-1}\sigma/T)\quad C_{3\mu}=\cosh(3\mu/T)$$

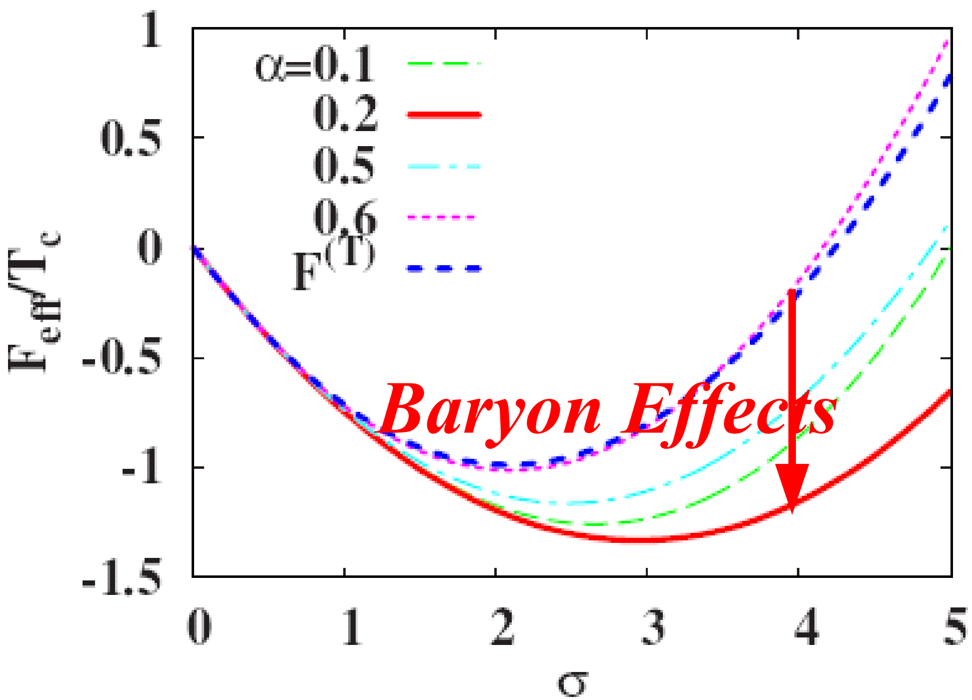
- **Decomposition of baryon-3 quark coupling**
  (Azcoiti-Di Carlo-Galante-Laliena, 2003)

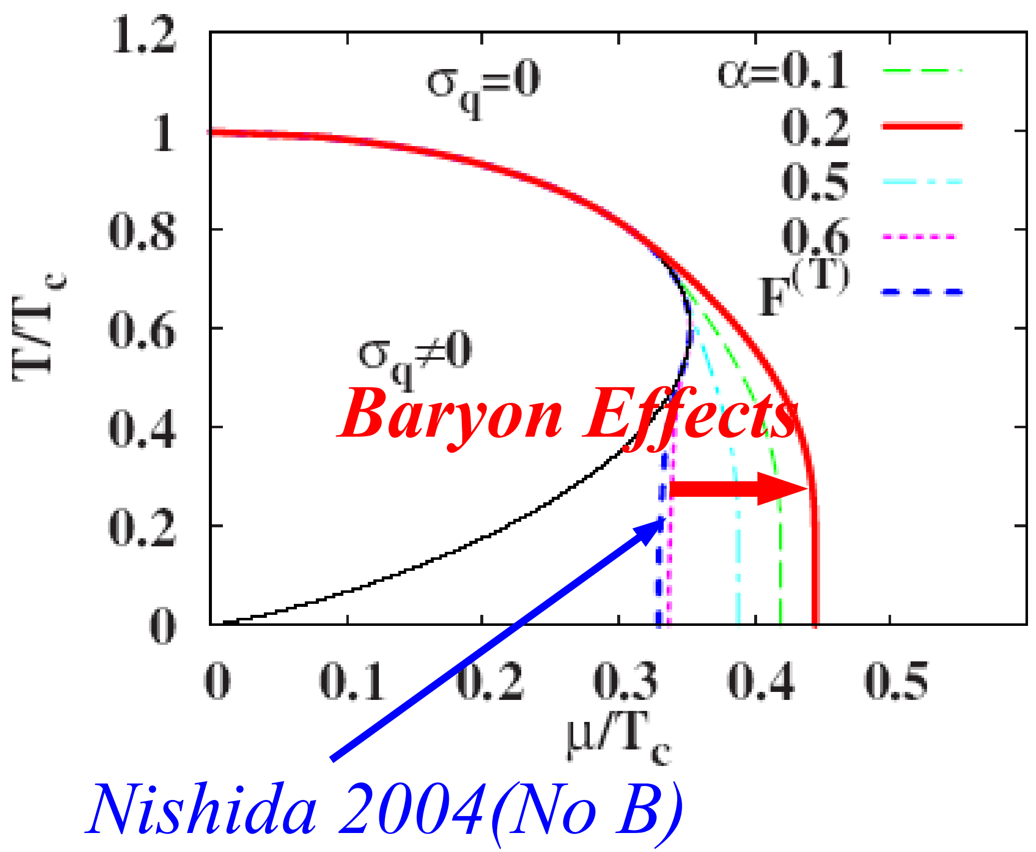
and auxiliary baryon potential and baryon integral

Phase diagram in SCL-LQCD with Baryons

(Kawamoto-Miura-AO-Ohnuma, hep-lat/0512023)

- **Baryon effects on phase diagram**
  - **Energy gain in larger condensates**
    → Extension of hadron phase to larger μ by around 30 %.





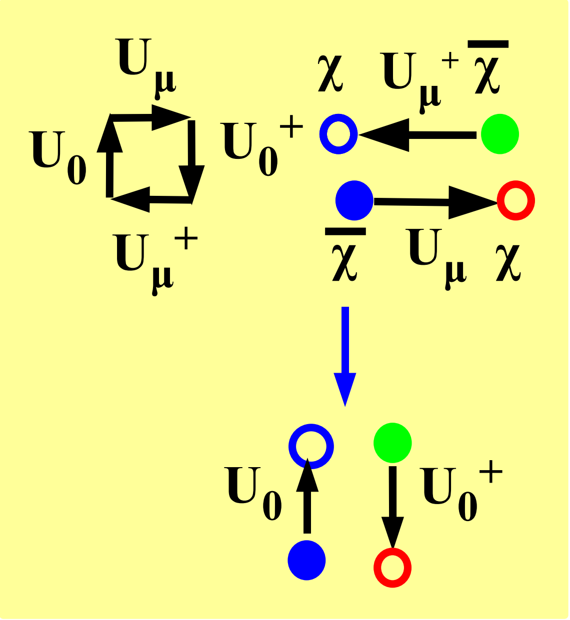
Discussions

- **Present phase diagram ↔ real phase diagram**
  - **One species of staggered fermion ~ N_f=4. Should be 1st order !**
  - **Tc seems to be too high. μ_c/T_c(present) ~ 0.45 ↔ μ_c/T_c(real)~(2-3)**
  - **No stable CSC phase** *(Azcoiti et al., 2003)*
    ↔ Stable CSC phase at large μ *(Alford, Hands, Stephanov)*
- **Two parameters are introduced through identities (HS transf.)**
  - **The results should be independent from parameter choice !**
    → MFA may break the identity...
  - **How should we fix these parameters ?**
- **Is SCL-LQCD useful ? → We would like to answer "Yes" !**
  - **Chiral RMF derived in SCL-LQCD works well in Nuclear Physics (Tsubakihara, AO, nucl-th/0607046**
    Tsubakihara,Maekawa,AO, Proc. of HYP06, to appear)
  - **$1/g^2$ expansion may connect SCL-LQCD and real world.**

$1/g^2$ expansion (w/o Baryon Effects)

- T_c (μ=0) and μ_c (T=0): Which is worse ?
  - $1/g^2$ correction reduces T_c. *(Bilic-Cleymans 1995)*
  - Hadron masses are well explained in SCL.
    (Kawamoto-Smit 1981, Kawamoto-Shigemoto 1982)

  → We expect Tc reduction with $1/g^2$ correction !



- 1/d expansion of plaquetts *(Faldt-Petersson 1986)*
  - Space-like plaquett

$$\exp\left[\frac{1}{g^2}\sum_{x,i>j>0}\mathrm{Tr}\,U_{ij}(x)\right]\to\exp\left[\quad\frac{1}{8N_c^4 g^2}\sum_{x,k>j>0}M_x M_{x+\hat{j}} M_{x+\hat{k}} M_{x+\hat{k}+\hat{j}}\right]$$

  - Time-like plaquett

$$\exp\left[\frac{1}{g^2}\sum_{x,j>0}\mathrm{Tr}\,U_{0j}(x)\right]\to\exp\left[-\frac{1}{4N_c^2 g^2}\sum_{x,j>0}\left(V_x V_{x+\hat{j}}^+ + V_x^+ V_{x+\hat{j}}\right)\right]$$

$$\left(V_x=\bar{\chi}_x U_0(x)\chi_{x+\hat{0}}\right)$$

Plaquett Bosonization

- **Bosonization of Plaquetts** *(O(1/d, 1/g⁴) and Im(V) are ignored)* **+ MFA**

$$\exp(-S_F - S_g) \to \exp\left[-\frac{1}{2}\sum_x (e^{\mu} V_x - e^{-\mu} V_x^+) + \frac{1}{4N_c}\sum_{x,j>0} M_x M_{x+\hat{j}} + m_0 \sum_x M_x\right]$$

$$\times \exp\left[-\frac{\beta_t}{2}\varphi_t \sum_x (V_x - V_x^+) + \beta_s \varphi_s \sum_{x,j>0} M_x M_{x+\hat{j}}\right]$$

$$\times \exp\left[-L^3 N_\tau \left(\frac{\beta_t}{4}\varphi_t^2 + \frac{\beta_s d}{4}\varphi_s^2\right)\right] \quad \left(\beta_t = \frac{d}{2N_c^2 g^2}\ , \quad \beta_s = \frac{d-1}{8N_c^4 g^2}\right)$$

$$= \exp\left[-\frac{L^3}{T} F_\varphi - \frac{\alpha}{2}\sum_x \left(e^{\tilde{\mu}} V_x - e^{-\tilde{\mu}} V_x^+\right) + \frac{1}{2}\sum_{x,y} M_x \tilde{V}_M(x,y) M_y\right]$$

$$\alpha = 1 + \beta_t \varphi_t \cosh\mu \quad , \quad \tilde{\mu} = \mu - \beta_t \varphi_t \sinh\mu$$

$$<\varphi_t> = <V^+ - V> \quad , \quad <\varphi_s> = 2 < M_x M_{x+\hat{j}} >$$

Time-like plaquetts modifies effective chemical potential

Effective Free Energy with $1/g^2$ Correction (w/o B)

- **After Quark and Time-like Link integral, we get *F* as**

$$F=\frac{d}{4N_c}\sigma^2(1+4N_c\beta_s\varphi_s)+\frac{\beta_t}{4}\varphi_t^2+\frac{\beta_s d}{4}\varphi_s^2-N_c\beta_t\varphi_t\cosh\mu+F_q(m_q;\tilde{\mu})$$

$$=\frac{d}{4N_c}\sigma^2+3\mathrm{d}\,\beta_s\sigma^4+\frac{\beta_t}{4}\tilde{\varphi}_t^2+F_q(m_q;\tilde{\mu})$$

$$\varphi_s=2\sigma^2\ ,\quad \varphi_t=\tilde{\varphi}_t+2N_c\cosh\mu$$

Time-like plaquetts remains finite at large μ

$$m_q=\frac{d}{2N_c}\sigma(1+4N_c\beta_s\varphi_s-\beta_t\varphi_t\cosh\mu)$$

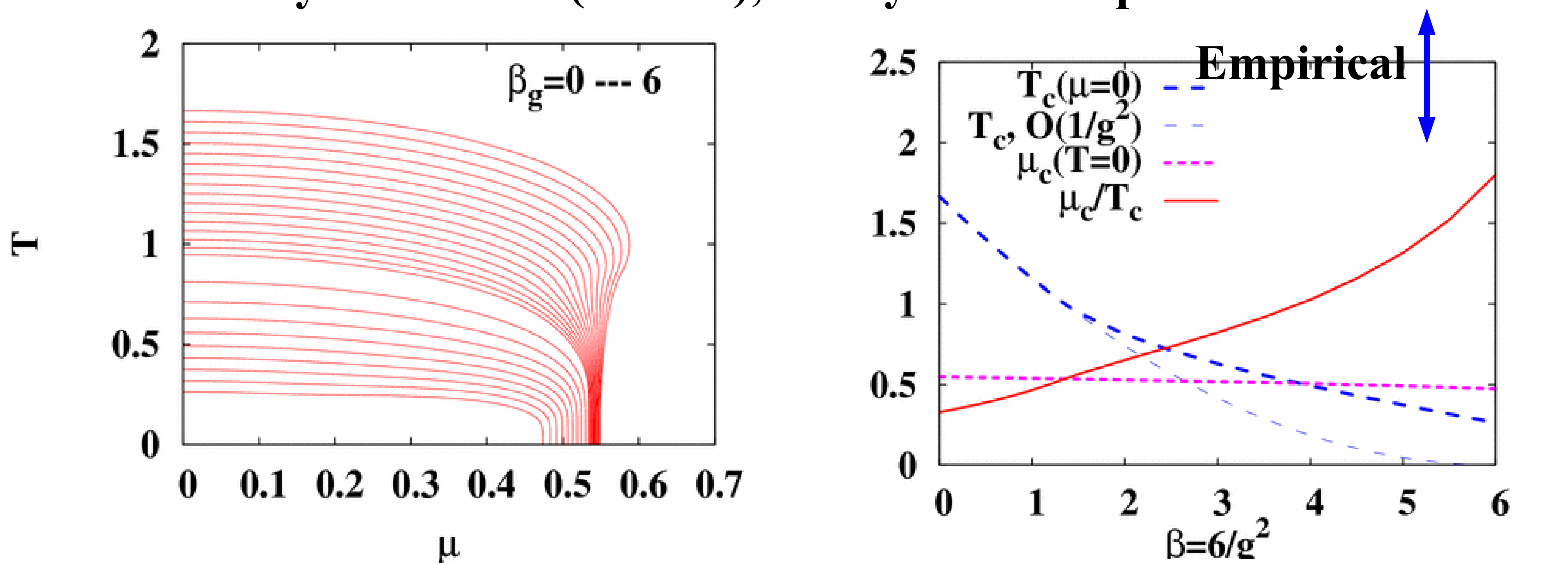
$$=\frac{d}{2N_c}\sigma(1-2N_c\beta_t\cosh^2\mu+8N_c\beta_s\sigma^4-\beta_t\tilde{\varphi}_t\cosh\mu)$$

$$\tilde{\mu}=\mu-\beta_t\varphi_t\sinh\mu=\mu-2N_c\beta_t\cosh\mu\sinh\mu-\beta_t\tilde{\varphi}_t\sinh\mu$$

- **Space-like plaquett → Repulsive pot. ∝ σ^4, Enh. σ-quark couling**
- **Time-like plaquett → Reduces μ and σ-quark coupling**
  (φ_t has to be determined to minimize F_{eff})

Phase Boundary with $1/g^2$ correction

- **Rapid decrease of $T_c(\mu=0)$, and slow decrease of $\mu_c(T=0)$.**
  - **Similar reduction of σ-quark coupling and effective μ at small condensate → can be mimicked by the scaling of T**
    (c.f. Bilic-Claymans 1995 (T_c goes down), Arai-Yoshinaga (Poster, goes up).
- **Ratio $\mu_c/T_c \sim 1.8$ @ g=1.**
  - **with baryonic effects (~ 30 %), it may reach empirical value.**



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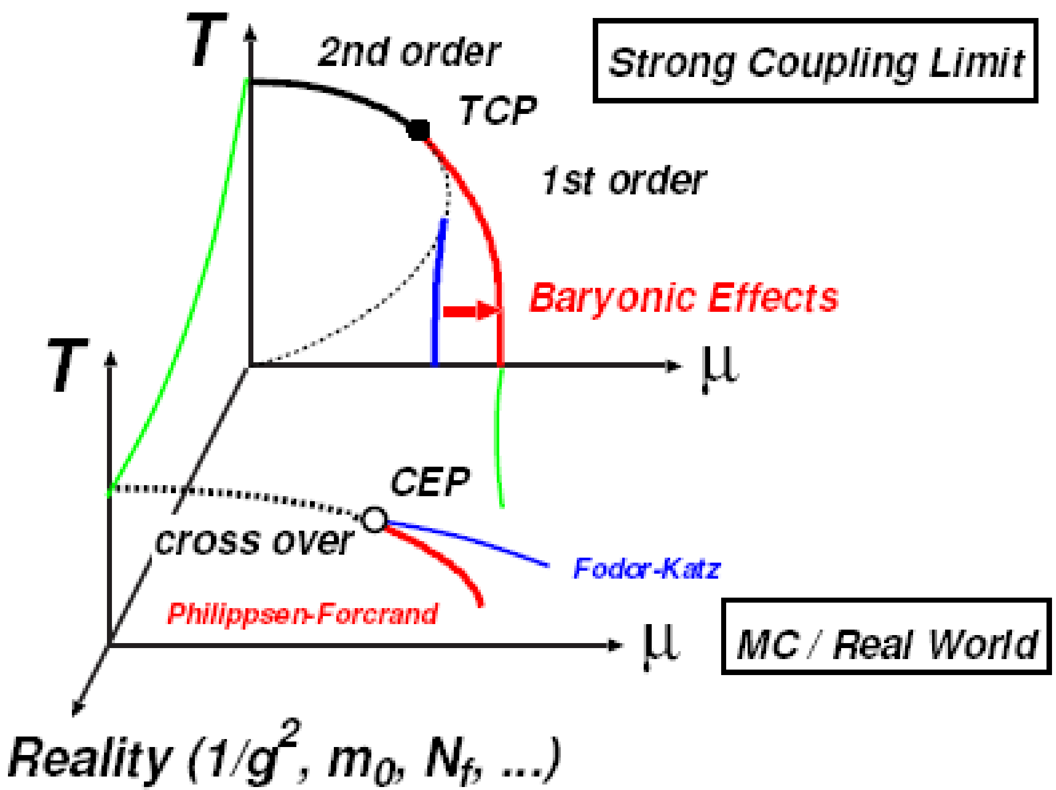
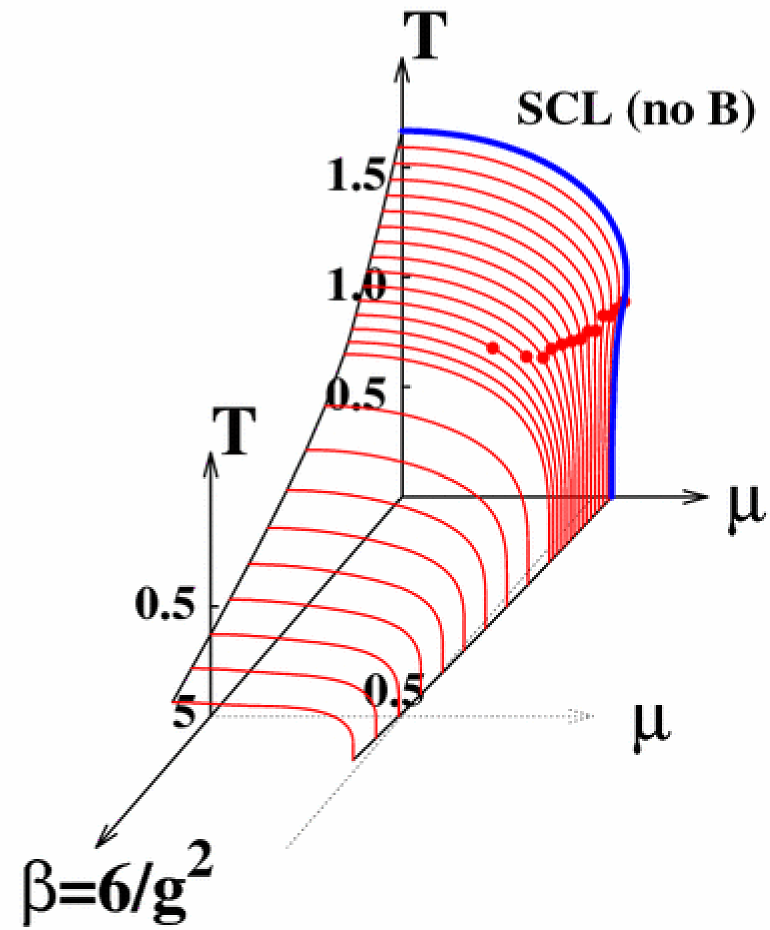
Evolution of Phase Diagram

- "Reality" Axis: $1/g^2$, n_f, m_0, ... would enhance μ_c/T_c ratio
- Example: $1/g^2$ correction enhances μ_c/T_c by a factor ~(2-3).





Summary

- **We obtain an analytical expression of effective free energy *at finite T and finite μ* with *baryonic composite* effects in the strong coupling limit of lattice QCD for color SU(3).**
  - *MFA, QG integral, 1/d expansion (NLO, O(1/√d)), bosonization with diquarks and baryon potential field using* $(\bar{b}b)^2=0$ *, Linear approx.,* ***zero diquark cond.(Color Angle Average), variational parameter choice***
- **Baryonic action is found to result in *Free Energy Gain* and *Extension of Hadron Phase to Larger μ by around 30 %.***
  - *Problem: Too small* μ_c/T_c *in the Strong Coupling Limit.*
- **Strong Coupling Limit is useful to understand Dense Matter**
  - *SCL gives a qualitative insight.*
  - *1/g² correction seems to work well (Do not believe us yet ...)*
  - *Application to chiral RMF (K. Tsubakihara, AO, nucl-th/0607046)*
    → 3rd week Poster by Tsubakihara