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# *AMD and Statistical Model Study of $\Xi$ Hypernuclear Physics*

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in Collaboration with*

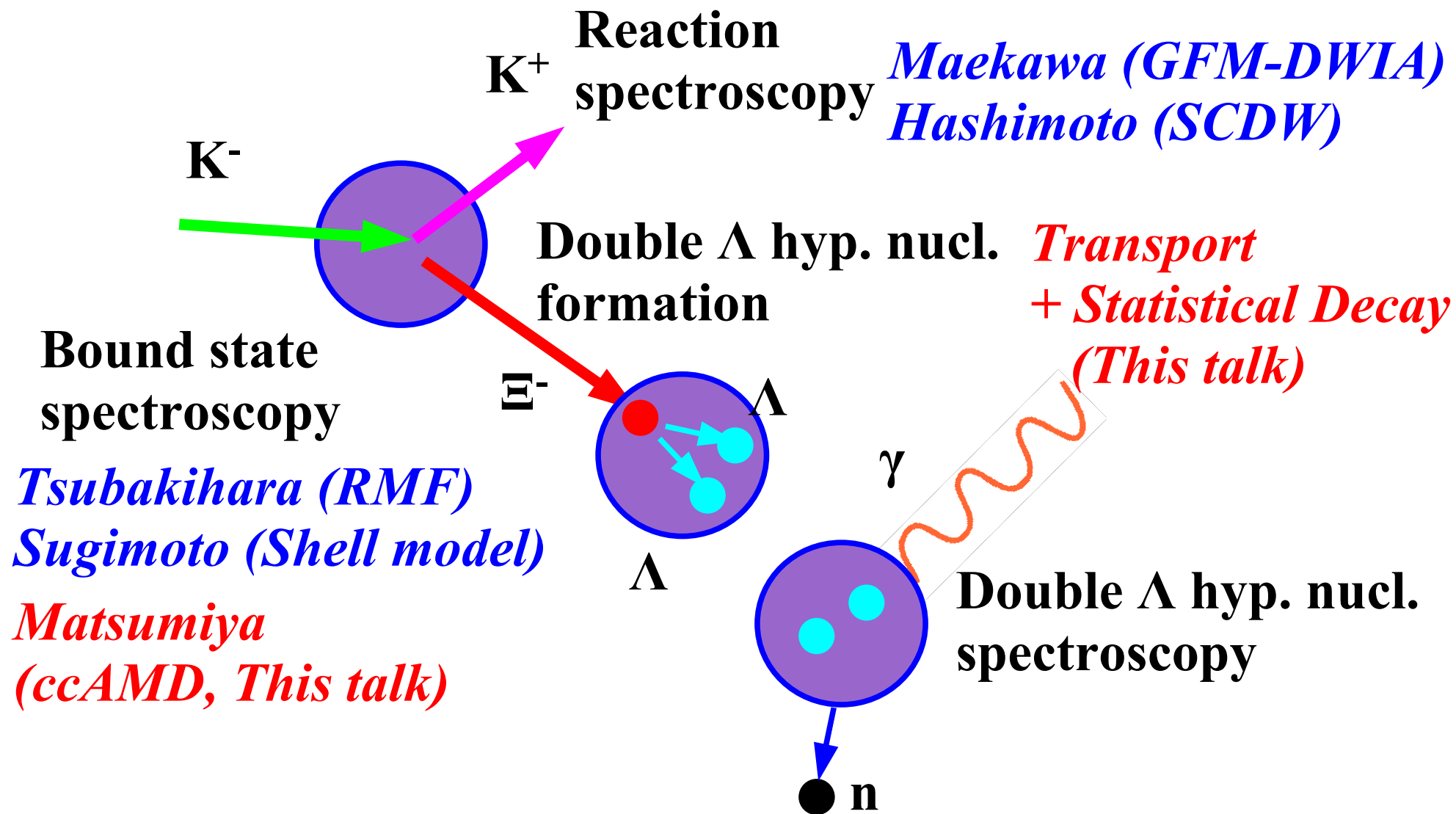
*H. Matsumiya, K. Tsubakihara, H. Maekawa (Hokkaido U.)  
A. Dote (KEK-IPNS)*

- **Introduction**
- **Spectroscopic Study of  $\Xi$  Hypernuclei  
in Coupled-Channel Antisymmetrized Molecular Dynamics**
- **Statistical Model Study of  $\Lambda\Lambda$  Hypernuclear Formation  
from  $\Xi$ - Absorption at Rest**
- **Summary**

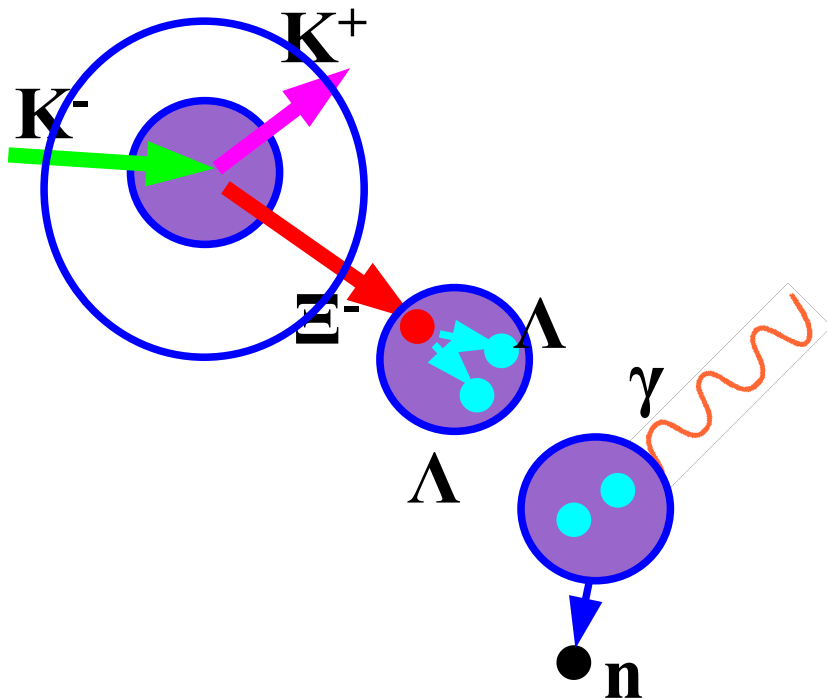


# $\Xi$ Hypernuclear Physics

- $\Xi$  hypernuclei  $\rightarrow$  Doorway to Multi Strangeness Systems



# $\Xi$ hypernuclear structure study in coupled channel AMD



In producing  $\Xi$  hypernuclei,  
we need to know  
the excited levels  
and the transition rate

# Antisymmetrized Molecular Dynamics (AMD)

## ■ Microscopic Model for *Structure and Reaction* Studies

*Ono, Horiuchi, Maruyama, AO, 1992 / Kanada-En'yo, Horiuchi, 1995*

- Slater determinant of Gaussian wave packets
- **Capable of describing Shell / Cluster states**
- Variation of parameters determines the shape of nuclei
- Good description of transition matrix element (B(E2), ..)
- **Problems in structure studies:**
  - High CPU cost for heavy nuclei → **Wait for Faster CPU**
  - Nodes are generated by Antisym. → No Node for one particle
    - **Multigauss AMD** (*Dote, Akaishi, Horiuchi, Yamazaki, 2004*)
  - Inverse matrix elements of s.p. overlap → No particle mixing
    - **CoFactor rather than Inverse Matrix** (*This work*)

# Coupled Channel AMD (ccAMD)

- Wave function = superposition of Channel AMD w.f.

$$|\Psi\rangle = \sum_a x_a |\Phi^a\rangle \quad (a : \text{channel}) \quad |\Phi^a\rangle = \frac{1}{\sqrt{A!}} \det [|\varphi_j^a(i)\rangle]$$

$\left( \frac{A}{\Xi} Z \right) = x_1 \left( (A-1) Z \otimes \Xi^0 \right) + x_2 \left( (A-1) (Z+1) \otimes \Xi^- \right)$

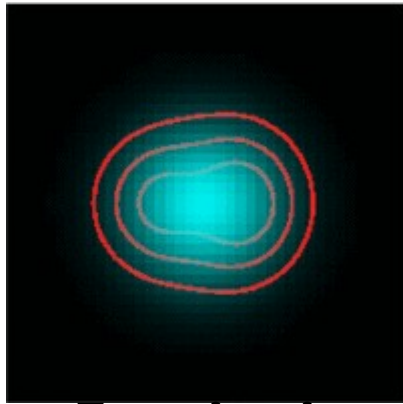
- Hamiltonian =  $T + V_{\text{NN}} + V_{\text{YN}} + \text{Mass diff.}$

$$\hat{H} = \hat{T} - \hat{T}_{\text{cm}} + \hat{V} + \Delta mc^2$$

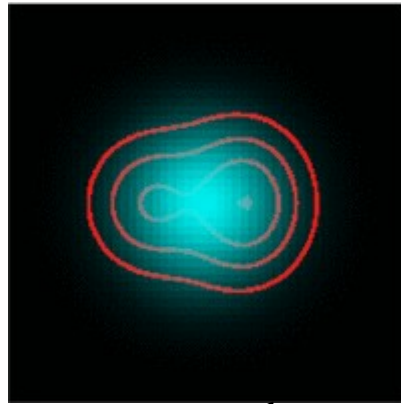
- $V_{\text{NN}}$  : Brink-Boeker-Okabe (BBO1)
  - $V_{\text{YN}}$  : G-matrix of Nijmegen Extended Soft Core (ESC04d, *Rijken, Yamamoto, 2006*)
- Consistency  $V_{\text{YN}}(\rho) \leftrightarrow \rho = \langle \rho \rangle$  (Dote-Akaishi prescription)

# *Density Distribution*

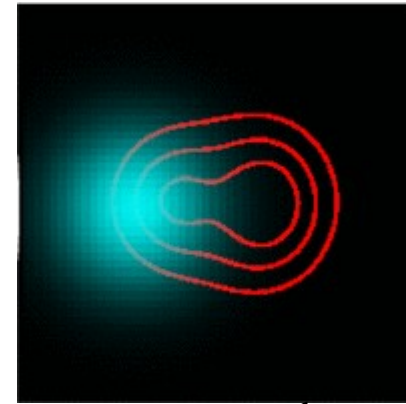
■  $^{12}_{\Xi}\text{Be}$



**Intrinsic**

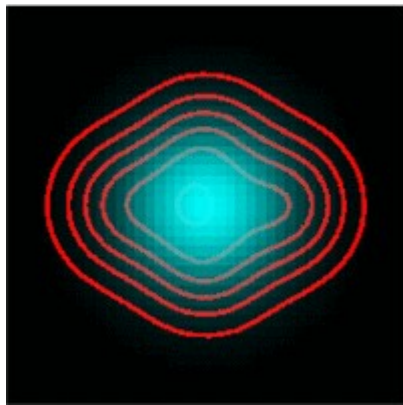


**$\pi = -1$**

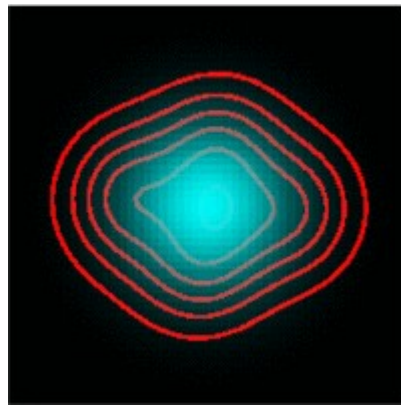


**$\pi = +1$**

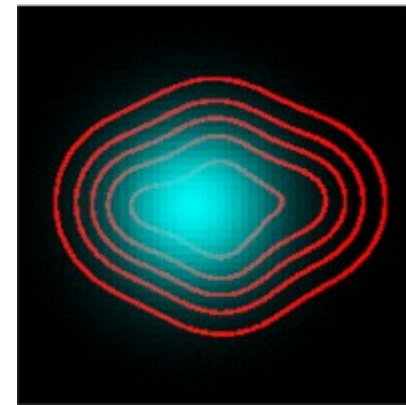
■  $^{28}_{\Xi}\text{Mg}$



**Intrinsic**



**$\pi = +1$**



**$\pi = -1$**

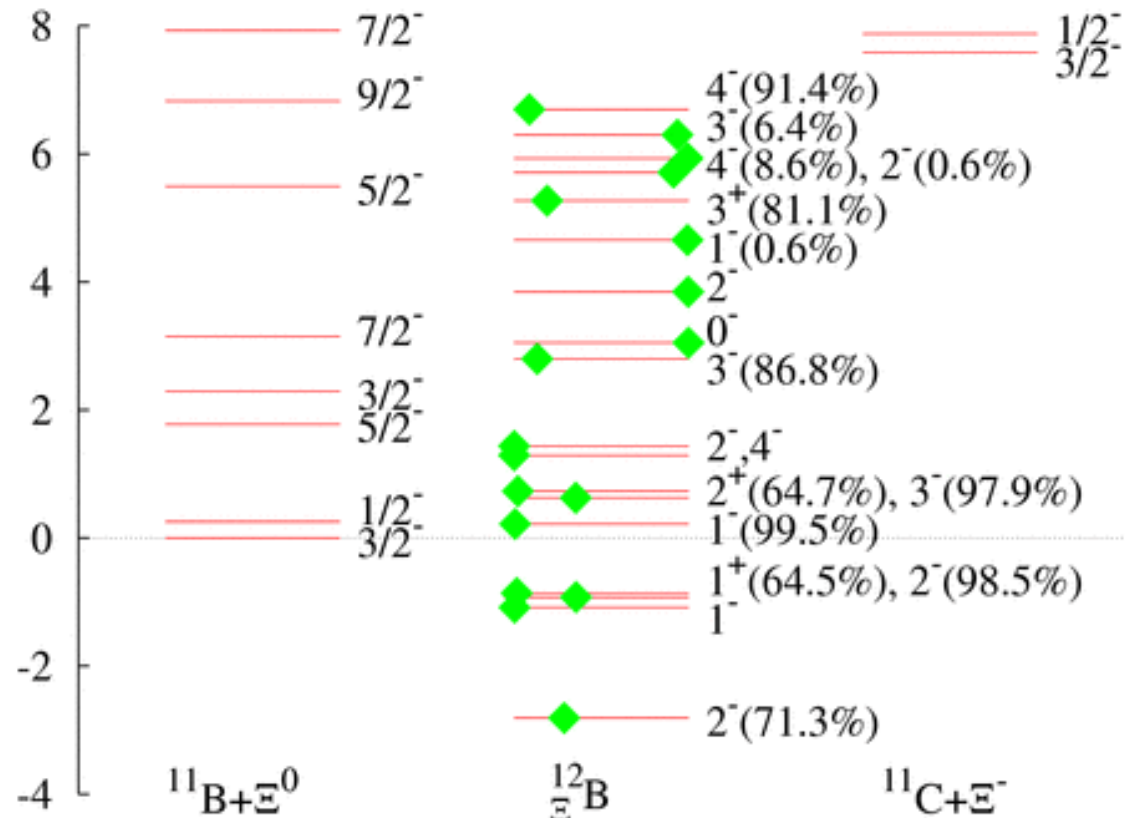
■ **Nuclei are not necessarily spherical !**

■ **Core and  $\Xi$  Parities are mixed !**

# Level Structure: $^{12}_{\Xi}B$ (Coherent mixing ?)

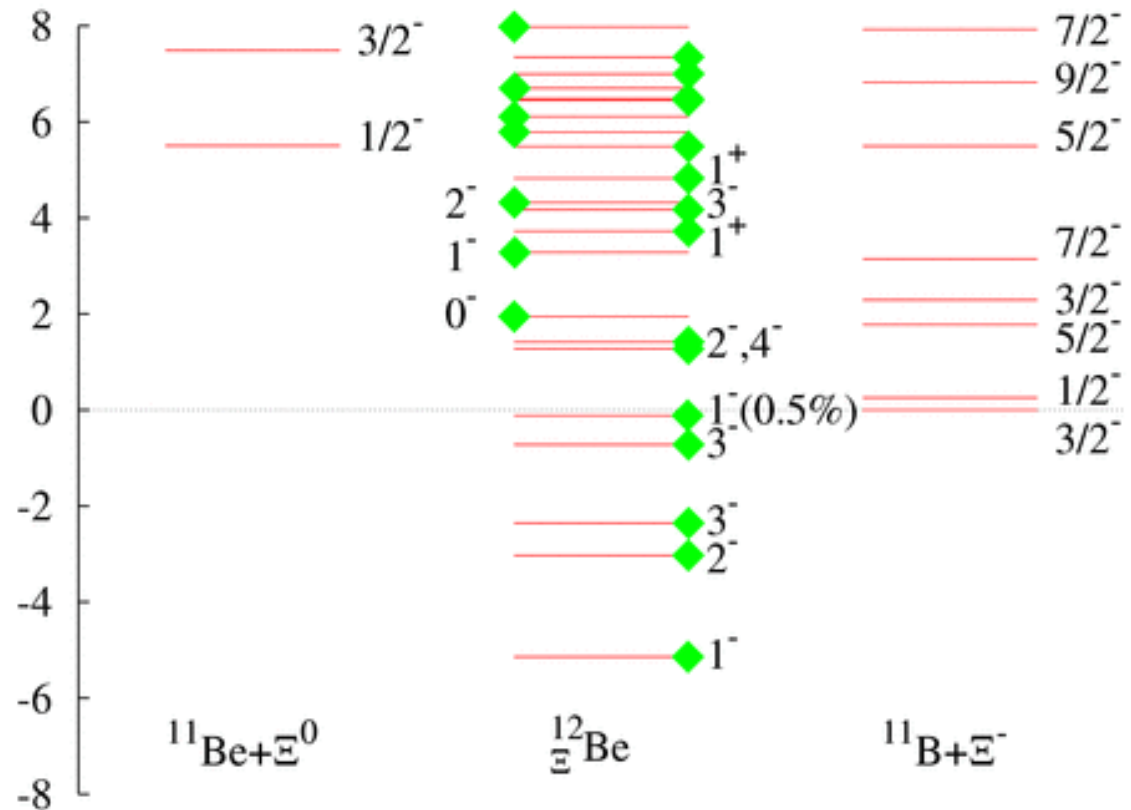
■  $12C(K^-, K^0)^{12}_{\Xi}B, \quad ^{12}_{\Xi}B = (^{11}B + \Xi^0) + (^{11}C + \Xi^-)$

- (Mirror Core)  $\otimes \Xi, T=0, 1 \rightarrow \Xi^0 : \Xi^- = 1:1$  without isospin breaking
- Mass diff. ( $M(\Xi^-) \sim M(\Xi^0) + 7 \text{ MeV}$ ) & Coulomb break isospin sym.  $\rightarrow$  We need Charge base !



# Level Structure: $^{12}_{\Xi}\text{Be}$ (Day-One Experiment Nucleus)

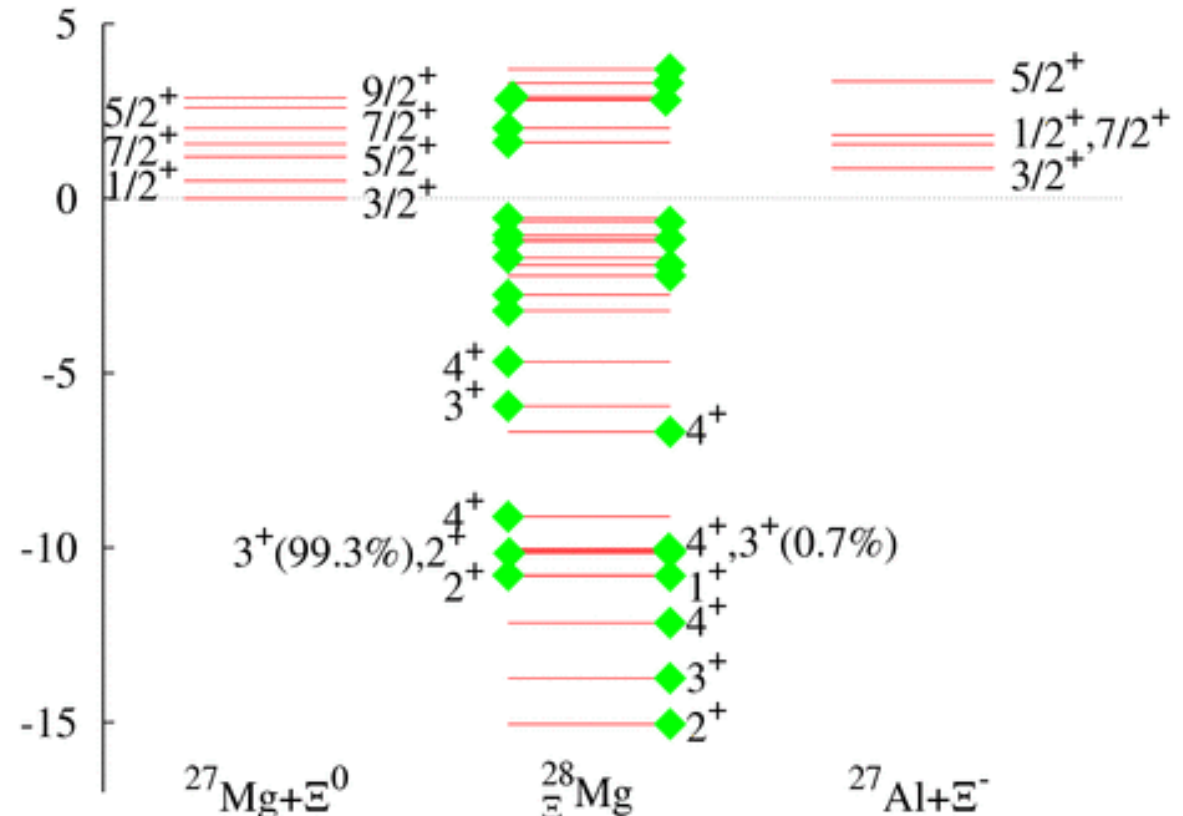
- $12\text{C}(\text{K}^-, \text{K}^+)^{12}_{\Xi}\text{Be}$ ,  $^{12}_{\Xi}\text{Be} = ({}^{11}\text{Be}(\text{T}=3/2) + \Xi^0) + ({}^{11}\text{B}(\text{T}=1/2) + \Xi^-)$ 
  - “Mass diff. of Core > Mass diff. of  $\Xi$ ” + “Core T diff.”
    - Almost No Coupling Effects
    - Single Channel (potential) description would be good enough !



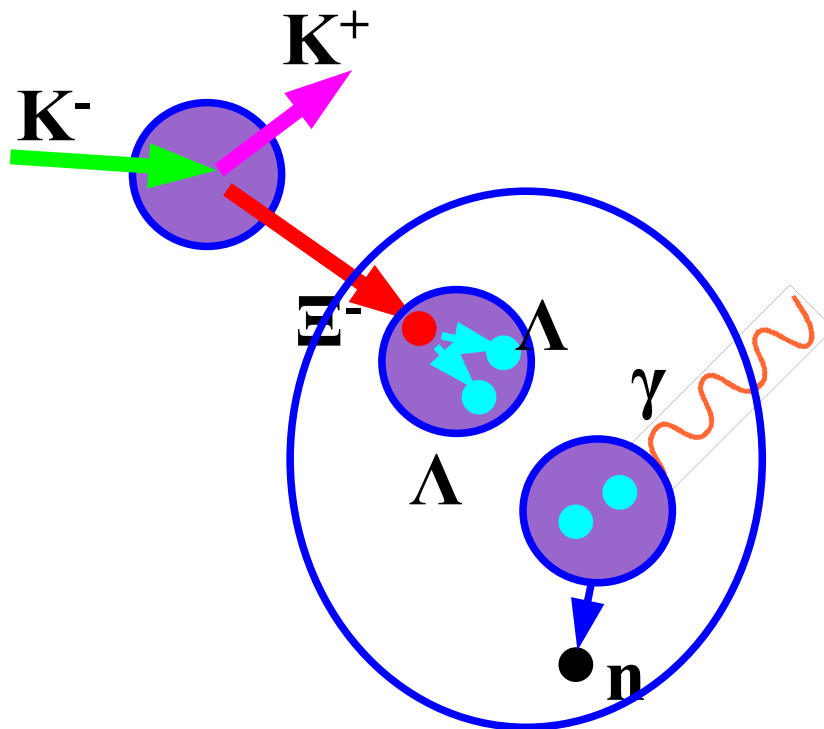


# Level Structure: $^{28}_{\Xi}\text{Mg}$ (Day-Two Experiment ?)

- $^{28}\text{Si}(\text{K}^-, \text{K}^+)^{28}_{\Xi}\text{Mg}$ ,  $^{28}_{\Xi}\text{Mg} = ({}^{27}\text{Mg}(\text{T}=3/2) + \Xi^0) + ({}^{27}\text{Al}(\text{T}=1/2) + \Xi^-)$ 
  - “Masses of Core+ $\Xi$ ” are Comparable, but Core T are different.
  - Almost no mixture of  $\Xi^-$  and  $\Xi^0$  channels
    - $\Xi^-$  states will be selectively populated in (K<sup>-</sup>,K<sup>+</sup>) reaction



# Statistical decay model study of $\Xi$ hypernuclei

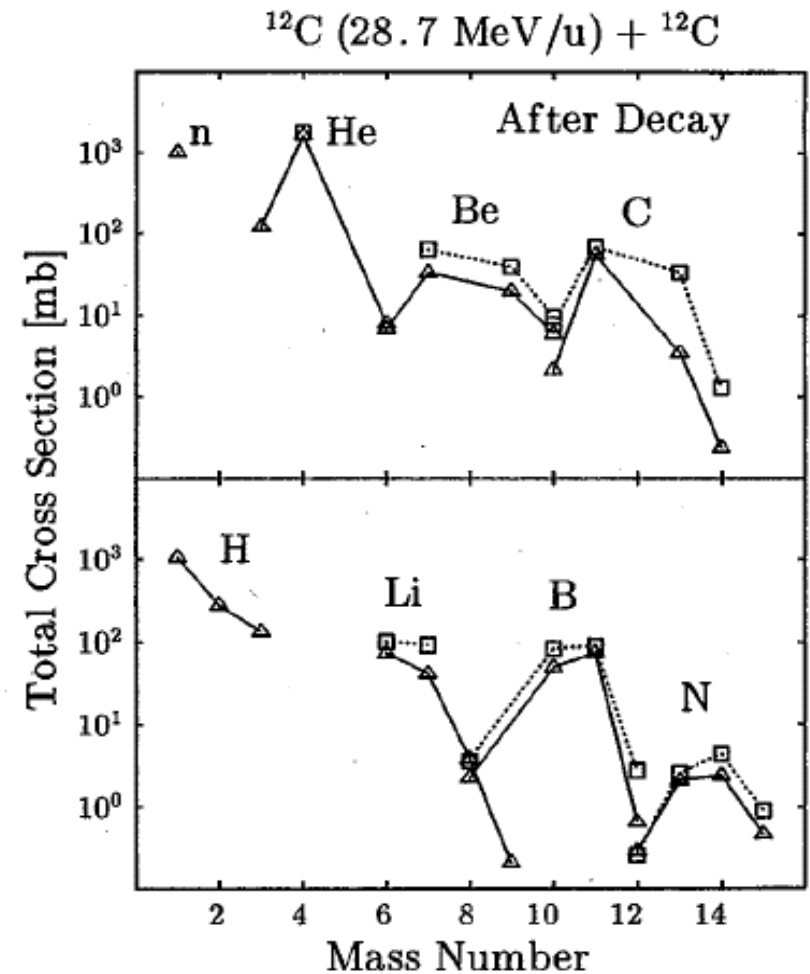


For double  $\Lambda$  hyp. nucl. (D $\Lambda$ HN)  
 $\gamma$  ray spectroscopy,  
it is necessary to know  
which D $\Lambda$ HN are produced !

# Transport + Statistical Decay Model (1)

## ■ Multistep evap. of nucleons and $\alpha$ from Excited Nuclei (Statistical Cascade Decay Process)

- Established decay process
- Combined with transport model, Cascade gives reliable results !
- AMD + Cascade ( $\Delta$ )  
→ Describes frag. form. data ( $\square$ ) within a factor of two !



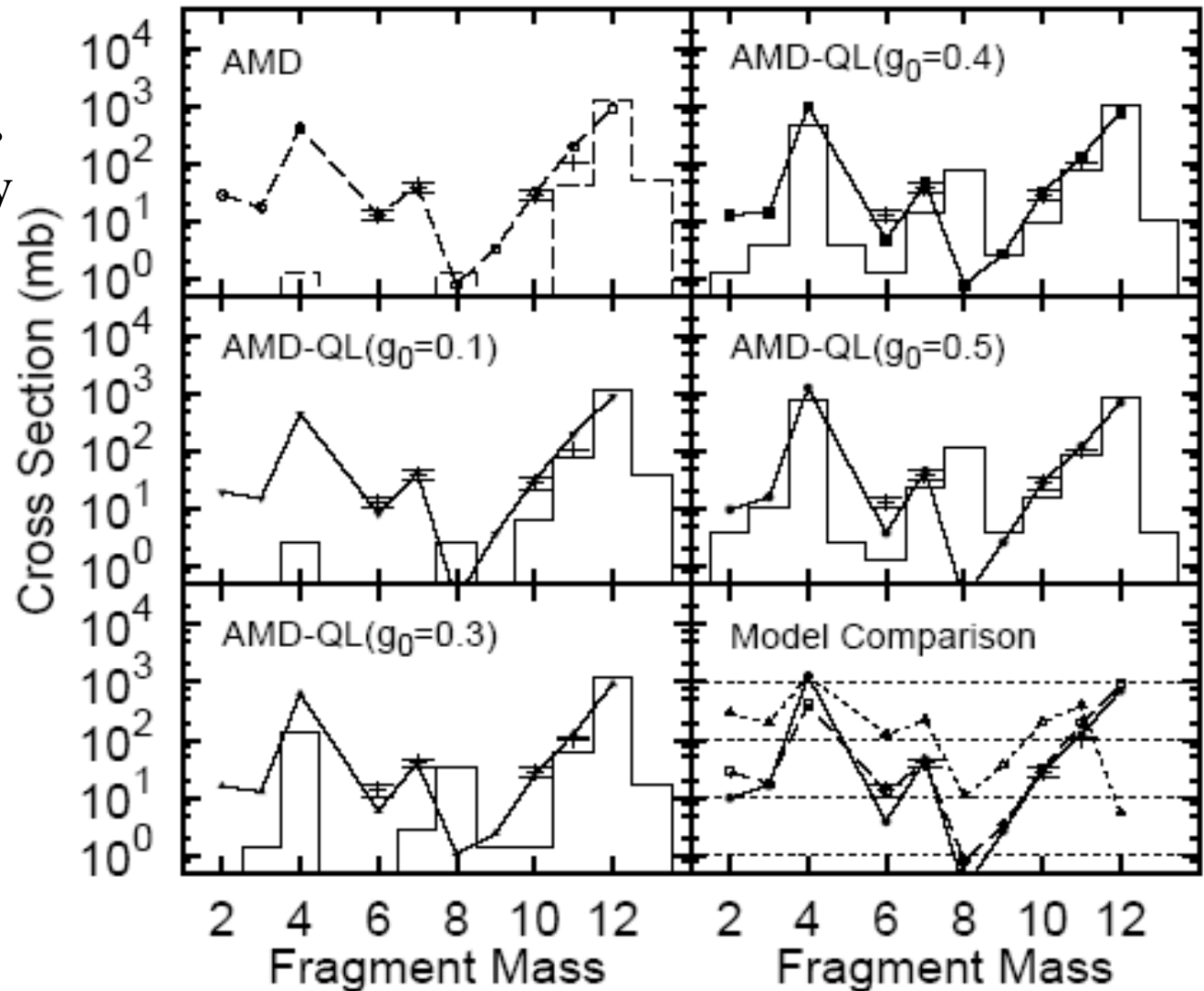
*Ono, Horiuchi, Maruyama, AO, PRL and PTP, 1992.*



# Transport + Statistical Decay Model (2)

## ■ AMD-QL (with Quantum Statistical Fluctuation) + Cascade decay

- Incl. quantum fluc. simulats stat. decay dynamically.



$p(45 \text{ MeV})+^{12}\text{C}$

*Hirata, Nara, AO, Harada, Randrup, 1999*



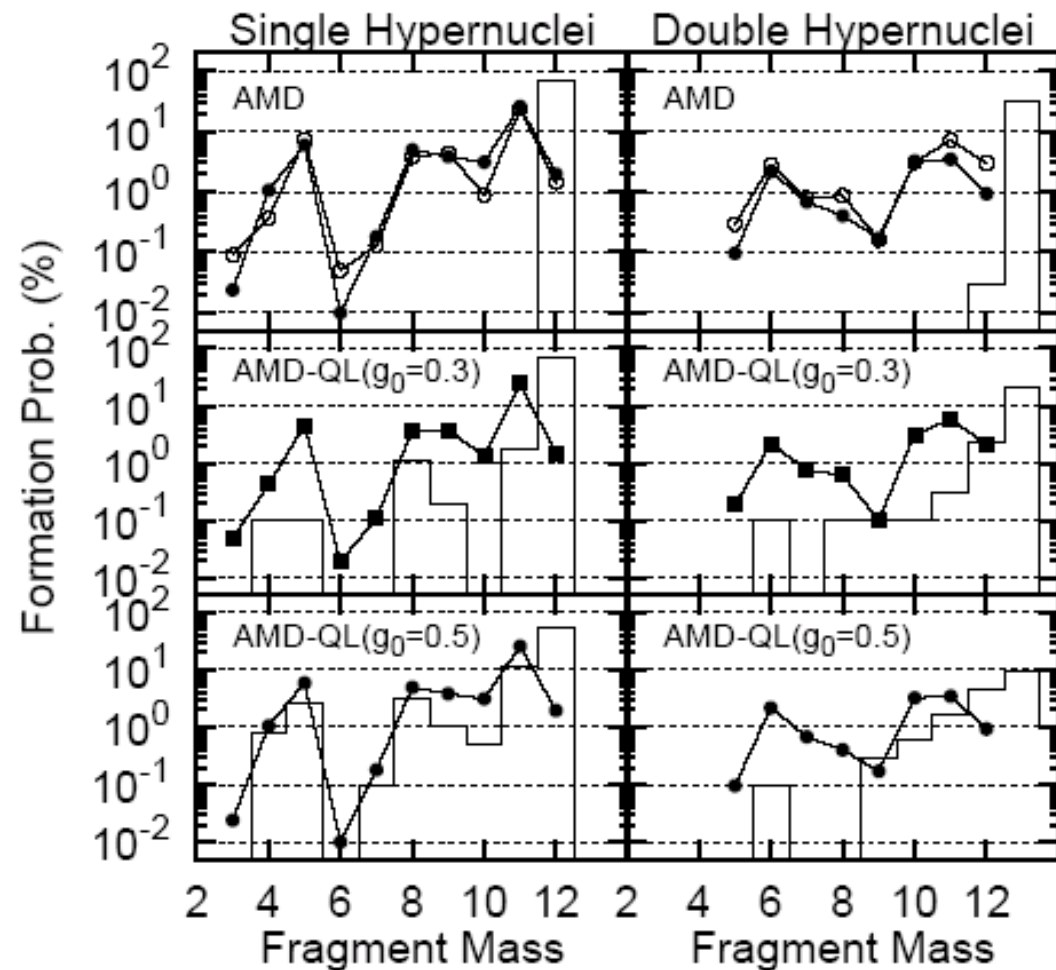
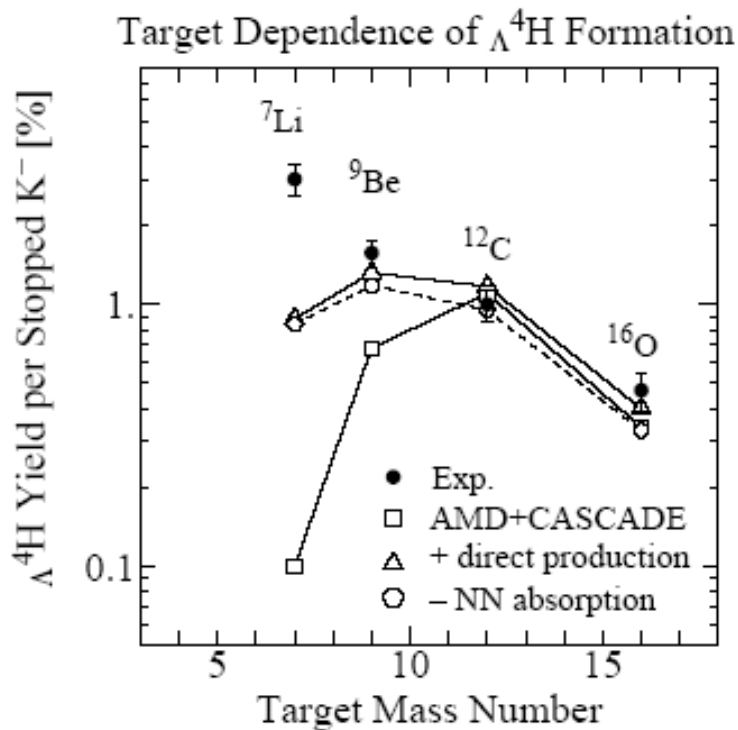
# Hyperfragment Formation

- Stopped  $K^- + (Li, C, O) \rightarrow {}^4_{\Lambda}H$

*Tamura et al. 1989 / Nara, AO, Harada, 1995*

- Stopped  $\Xi^- + {}^{12}C \rightarrow$  Double, Twin, Single hypernuclei

*KEK-E176 / Hirata et al. 1999*



# Double Hypernuclear Formation from Stopped $\Xi^-$

## ■ Theoretical Models

- AMD/AMD-QL + Cascade (*Hirata et al., 1999*)
- Direct Reaction (*Yamada, Ikeda, 1997*)  
Two-Cluster Res. dominance in Twin hypernuclear form.
- Statistical Decay (*Yamamoto, Sano, Wakai, 1994*)  
Canonical dist. model ( $\sim$  Copenhagen model, *c.f. Botvina's talk*)

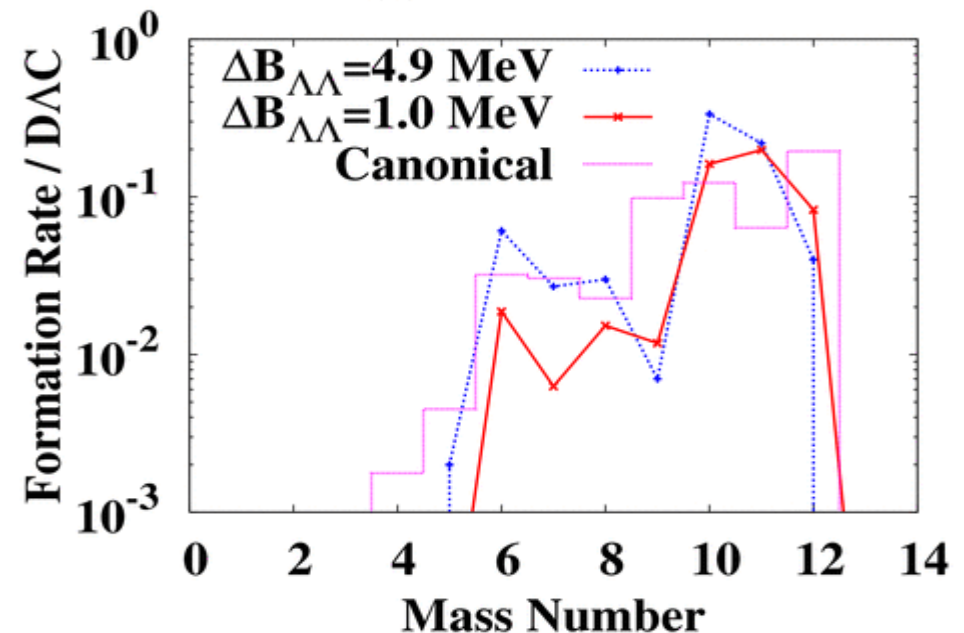
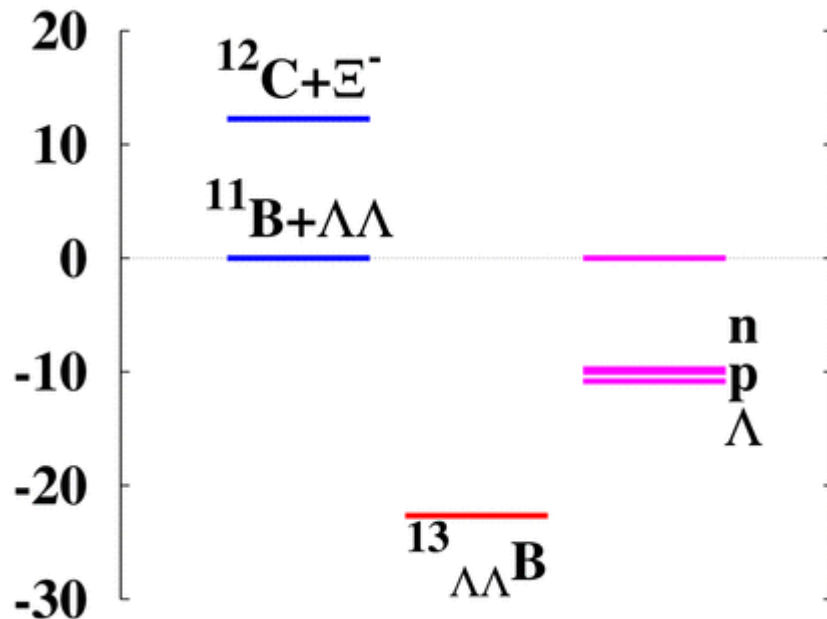
→ *Dominant Double  $\Lambda$  hypernuclear formation Prob.*  
*= (Double  $\Lambda$  Compound Nucleus (DAC) formation prob.*  
*w/o quantum fluc.)*  
 *$\times$  (Double  $\Lambda$  hypernucleus ( $D\Lambda HN$ ) survival prob.)*

*Let's evaluate  $D\Lambda HN$  survival prob.*  
*in Cascade Decay Model*  
*in Stopped  $\Xi^-$  on  $^{56}\text{Fe}$*

# Stopped $\Xi^-$ on $^{12}\text{C}$

- Double  $\Lambda$  Compound Nucleus ( $\text{D}\Lambda\text{C}$ ) Formation Prob.  
 $\sim 30\%$  (AMD), 16%, (AMD-QL (with quantum fluc.))
- Double  $\Lambda$  Hyp. Nucl. ( $\text{D}\Lambda\text{HN}$ ) survival prob.  
 $\sim 72\%$  ( $\Delta B_{\Lambda\Lambda} = 4.9\text{ MeV}$ ), 49% ( $\Delta B_{\Lambda\Lambda} = 1.0\text{ MeV}$ )
- Main products ( $\Delta B_{\Lambda\Lambda} = 1.0\text{ MeV}$ )  
 $^{11}_{\Lambda\Lambda}\text{Be}$  (15%),  $^{10}_{\Lambda\Lambda}\text{Be}$  (16%),  $^{12}_{\Lambda\Lambda}\text{B}$  (6%),  $^6_{\Lambda\Lambda}\text{He}$  (1.8%)

Stopped  $\Xi^-$  on  $^{12}\text{C}$  (L=1)



# Stopped $\Xi^-$ on $^{12}\text{C}$

## ■ Our Previous Estimate

- Mass Table based on old data,  $\Delta B_{\Lambda\Lambda} \sim 4.9 \text{ MeV}$
- $\text{D}\Lambda\text{C}$  form. prob.  $\sim 30 \%$  (AMD),  $16 \%$  (AMD-QL)
- $\text{D}\Lambda\text{HN}$  ( ${}^6_{\Lambda\Lambda}\text{He}$ ) form. prob.  
 $\sim 18$  (**2.7**) % (AMD+Casc),  $11$  (**1.9**) % (AMD-QL+Casc.)

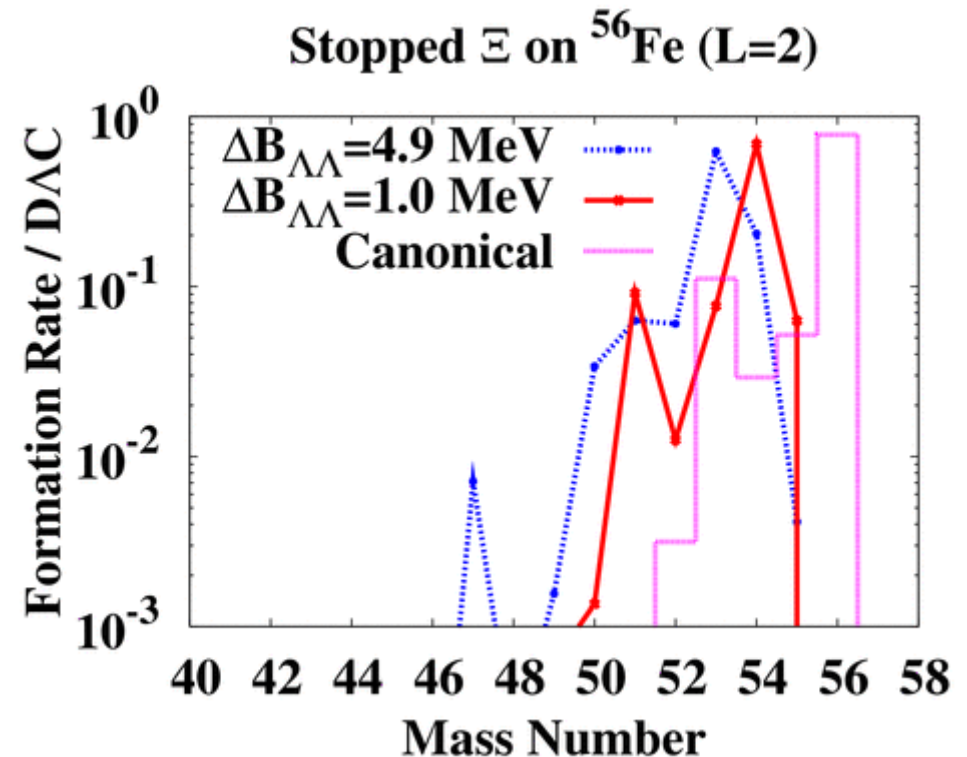
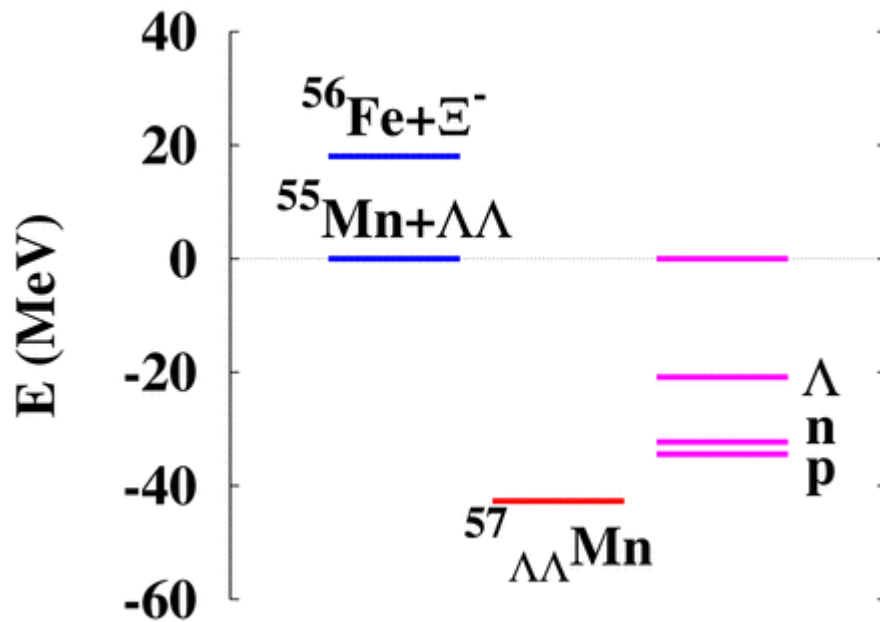
## ■ Present Estimate

- Mass Table based on new data (Nagara),  $\Delta B_{\Lambda\Lambda} \sim 1.0 \text{ MeV}$
- $\text{D}\Lambda\text{HN}$  ( ${}^6_{\Lambda\Lambda}\text{He}$ ) *survival* prob.  $\sim 49$  (1.8) %
- $\text{D}\Lambda\text{HN}$  ( ${}^6_{\Lambda\Lambda}\text{He}$ ) *formation* prob.  
 $\sim (\text{D}\Lambda\text{C form.} \sim 30 \%) \times (\text{D}\Lambda\text{HN surv. prob.}) \sim 15$  (**0.5**) %  
→ *Explains why we cannot observe  ${}^6_{\Lambda\Lambda}\text{He}$  frequently !*  
(E176, E373)



# Stopped $\Xi^-$ on $^{56}\text{Fe}$

- $\text{D}\Delta\text{C}$  formation prob.  $\sim (5-30) \%$  ?
- Double  $\Lambda$  hyp. nucl. survival prob.  
 $\sim 99 \%$  ( $\Delta B_{\Lambda\Lambda} = 4.9 \text{ MeV}$ ),  $93 \%$  ( $\Delta B_{\Lambda\Lambda} = 1.0 \text{ MeV}$ )
- Main products  
 $^{54}_{\Lambda\Lambda}\text{Mn}$  (3n emission, 52 %),  $^{54}_{\Lambda\Lambda}\text{Cr}$  (p2n emission, 16 %)



# *Stopped $\Xi^-$ on $^{56}\text{Fe}$*

- **Why do we have high  $\Delta\text{HN}$  survival prob. ?**
  - **Total Excitation  $E \sim 60 \text{ MeV} \rightarrow E^*/A \sim 1 \text{ MeV}$**
  - **$E^*/A = T^2/8 \rightarrow T \sim 3 \text{ MeV}$**
  - **$S_p \sim 8 \text{ MeV}, S_n \sim 10 \text{ MeV}, S_\Lambda \sim 22 \text{ MeV}$**   
**Emission prob.  $\sim \exp(-S/T)$**   
 **$\rightarrow$  One  $\Lambda$  emission / One n emission  $\sim 1.4 \%$**



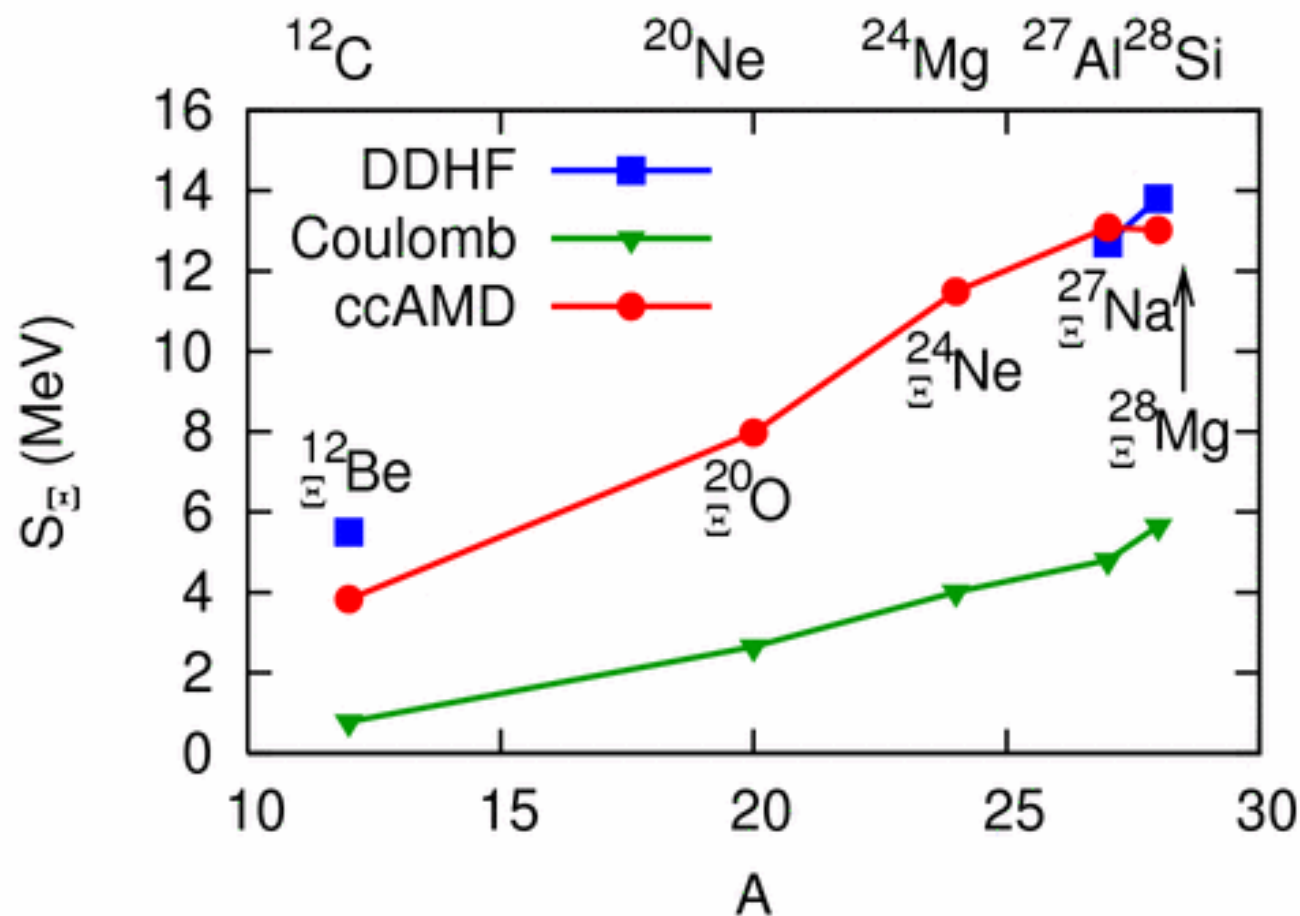
# Summary

- *Structure study of  $\Xi$  hypernuclei in ccAMD*
  - Coupled Channel AMD is also applicable to NKbar- $Y\pi$  and N $\pi$ /N coupling system.  
c.f. AMD with neutral  $\pi$  coherent state, Isshiki, Naito, AO, 2005
  - Core Isospin & Core/ $\Xi$  mass diff. & Coulomb pot. suppresses coupling of  $\Xi^-$  and  $\Xi^0$  states.
  - *To do: Effective number / Transition density / Multigauss AMD*
- *Statistical model study of stopped  $\Xi^-$  reactions*
  - Double  $\Lambda$  form. prob.  
 $\sim$  (Double  $\Lambda$  comp. form.)  $\times$  (Double  $\Lambda$  surv.)  $\sim$  (5-40 %)  $\times$  (90 %)
  - $^{56}\text{Fe}$  target  $\rightarrow$   $^{54}_{\Lambda\Lambda}\text{Mn}$  will be abundantly formed.
  - *To do: Estimate of L-dependent  $D\Lambda C$  form. prob.*
- *$\Xi$  physics requires us to understand all of Production, Structure and Decay.*



# Binding Energies

- Binding Energies in ccAMD  
~ DDHF Results in sd-shell nuclei



■ w.f.

$$|\Phi\rangle = \frac{1}{\sqrt{A!}} \det [|\varphi_j(i)\rangle] \quad |\varphi_j\rangle = |z_j\rangle |\chi_j^\sigma\rangle |\chi_j^\tau\rangle |f_j\rangle$$

isospin, flavor  
spin

$$|\chi_j^\sigma\rangle = \begin{cases} |\uparrow\rangle \\ |\downarrow\rangle \end{cases}, \quad |\chi_j^\tau\rangle \otimes |f_j\rangle = |T, T_z\rangle \otimes \begin{Bmatrix} |N\rangle \\ |\Xi\rangle \\ \vdots \end{Bmatrix}$$

$$= |n\rangle, |p\rangle, |\Xi^0\rangle, |\Xi^-\rangle, \dots$$

$$|\Phi\rangle = \frac{1}{\sqrt{A!}} \det [|\varphi_j(i)\rangle] \quad |\varphi_j\rangle = |z_j\rangle |\chi_j^\sigma\rangle |\chi_j^\tau\rangle |f_j\rangle$$

空間部分

$$z_j = \sqrt{\nu} d_j + \frac{i k_j}{2\hbar\sqrt{\nu}}$$

$$\langle \mathbf{r} | z_j \rangle = \left( \frac{2\nu}{\pi} \right)^{3/4} \exp \left[ -\nu \left( \mathbf{r} - \frac{z_j}{\sqrt{\nu}} \right)^2 + \frac{z_j^2}{2} \right]$$

$$\propto \exp \left[ -\nu (\mathbf{r} - \mathbf{d}_j)^2 + i \mathbf{k}_j \cdot \mathbf{r} / \hbar \right]$$

