有限温度強 結合格子 QCD におけるハドロ ン質量
Hadron Masses at Finite T in the Strong Coupling QCD

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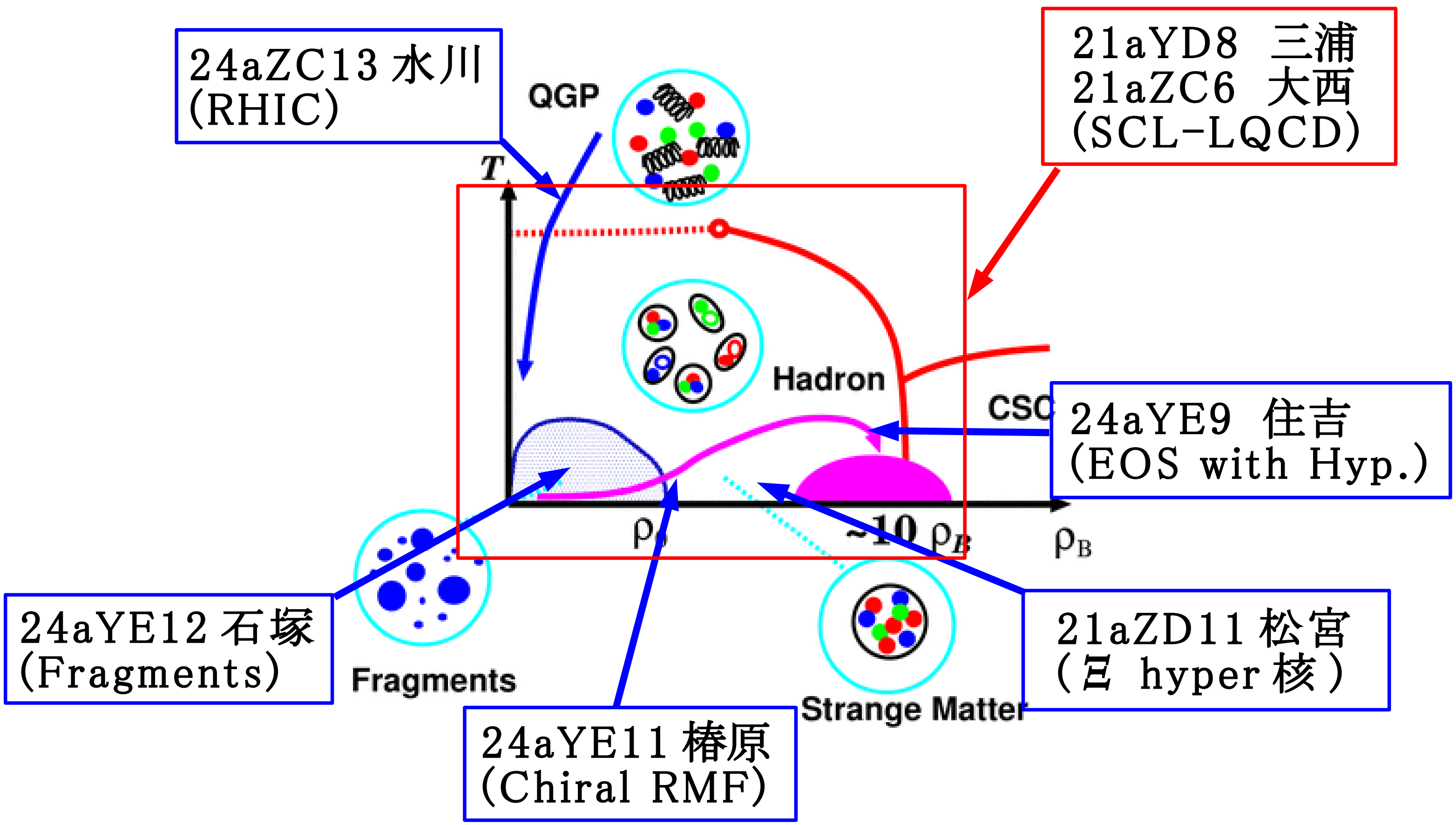
- **Introduction**
- **Hadron Mass in a Finite T treatment of Strong Coupling Limit for Lattice QCD**
- **Summary**

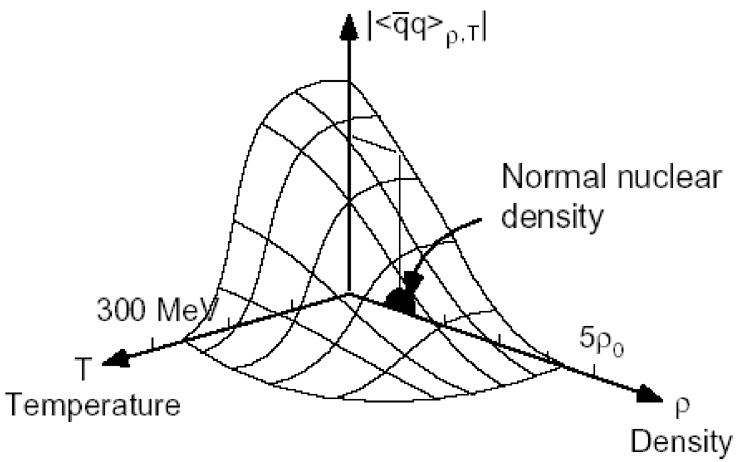
Quark and Hadronic Matter Phase Diagram

- 原子核・ハドロン・クォークの3階層状態方程式とコンパクト天体現象 (科研費基盤研究 (C), 大西、河本、住吉)
  - **クォーク、ハドロン、原子核の 3 階層をつなぐ EOS を作りたい！**



Hadron Mass in Nuclear Matter

- **Medium meson mass modification**
  - **may be the signal of partial restoration of chiral sym.**
    Kunihiro,Hatsuda, PRep 247('94),221; Brown, Rho, PRL66('91)2720; Hatsuda, Lee, PRC46('92)R34.
  - **and is suggested experimentally.**
    CERES Collab., PRL75('95),1272;
    KEK-E325 Collab.(Ozawa et al.), PRL86('01),5019.
    Also at RHIC (PHENIX Collab.)



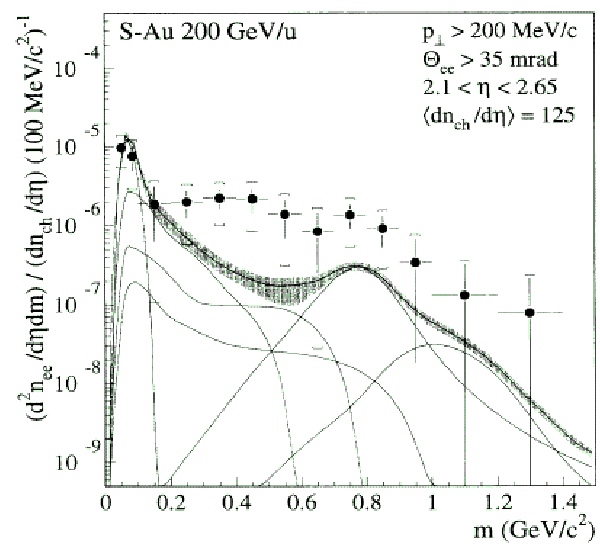
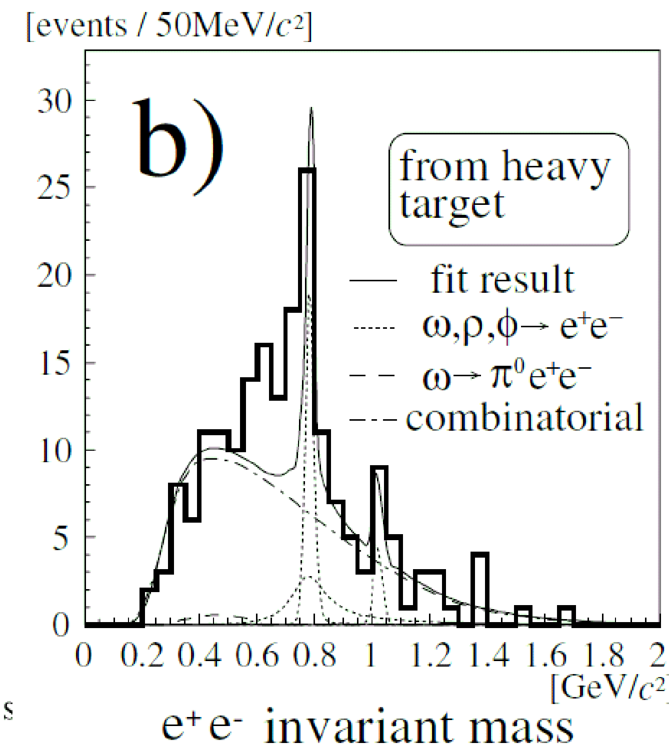


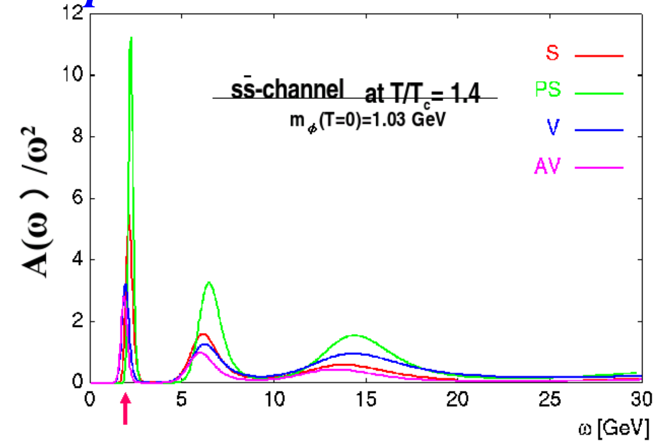
FIG. 4. Inclusive e^+e^- mass spectra in 200 GeV/nucleon S Au collisions. For explanations see Fig. 2.



Hadron Mass in Nuclear Matter

- Can we understand it in Lattice QCD ?
  - Finite T: It is possible !
  - Finite μ: Difficult due to the sign problem.
- Strong Coupling Limit of Lattice QCD
  → We can study finite (T, μ) !
  - Hadron masses in the Zero T treatment
    Kluberg-Stern, Morel, Petersson, '83; Kawamoto, Shigemoto, '82.
  - To do: Finite T, Baryons with finite T, $1/g^2$ corr., ...

Asakawa, Nakahara, Hatsuda, hep-lat/0208059.



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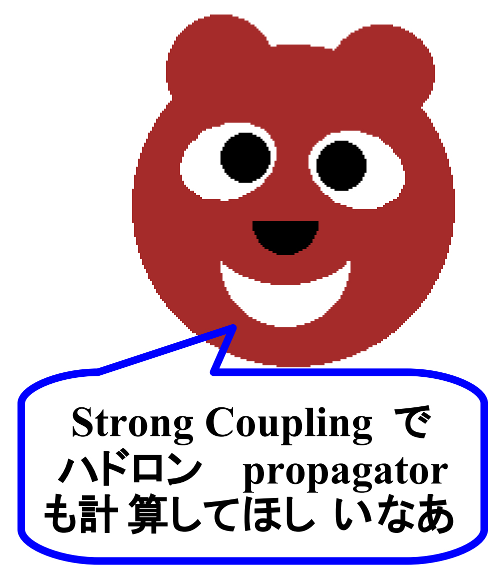
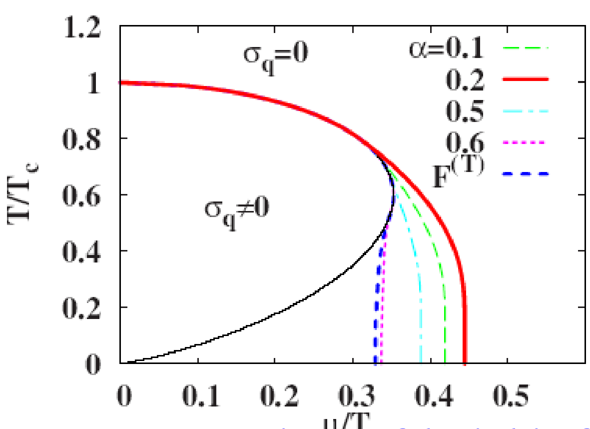
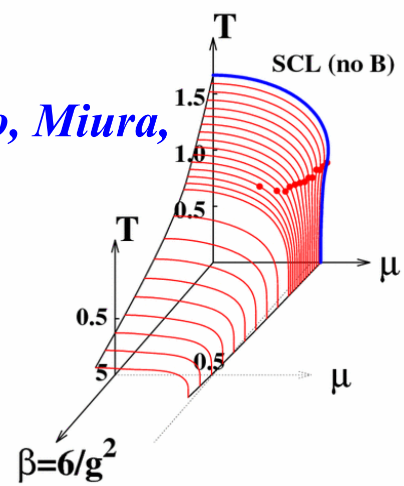


This Talk



Kawamoto, Miura, Ohnishi, Ohnuma, PRD75('07)014502

Ohnishi, Kawamoto, Miura, hep-lat/0701024



Hadron Mass in SCL-LQCD (Zero T)

- **SCL Effective Action (Zero T treatment, staggered fermion)**

$$S_{\rm eff}=\frac{1}{2}\sum_{x,y}\sigma(x)V_M^{-1}(x,y)\sigma(y)-N_c\sum_x\log(\sigma(x)+m_q)$$ *Kawamoto, Smit, '81*

$$= L^d N_\tau\left[\frac{N_c}{d+1}\bar{\sigma}^2-N_c\log(\bar{\sigma}+m_q)\right]+\frac{1}{2}\sum_k G(k)^{-1}(\delta\sigma(k))^2$$

Effective Potential **Meson Propagation**

- **Meson Mass in SCL-LQCD**

Kawamoto, Shigemoto, '82; Kluberg-Stern, Morel, Petersson, '83; Fukushima, '04

  - **Pole of $G(k)$ at "zero" momentum: $k_i \to 0$ or π, $\omega \to i\,m$ + "0 or π"**

$$G(k)^{-1}=F.T.\frac{\delta^2 S_{\rm eff}}{\delta\sigma(x)\delta\sigma(y)}=2N_c\left[\sum_\mu\cos k_\mu\right]^{-1}+\frac{N_c}{(\bar{\sigma}+m_q)^2}\ \to\ 2N_c\left[\kappa\pm\cosh m\right]^{-1}+\frac{N_c}{(\bar{\sigma}+m_q)^2}\ =\ 0$$

$$\cosh m=2(\bar{\sigma}+m_q)^2+\kappa\ \to\ (d+1)(\lambda^2-1)+\kappa+d+1$$ **Equilibrium σ**

$$\kappa=-d,-d+2,\ldots d\quad(\text{diff. meson species}),\quad \lambda=\bar{m}_q+\sqrt{\bar{m}_q^{\,2}+1},\quad \bar{m}_q=m_q/\sqrt{2(d+1)}\quad(d=3)$$

Well explains data, Funny σ dep., No (T, μ) dep.,

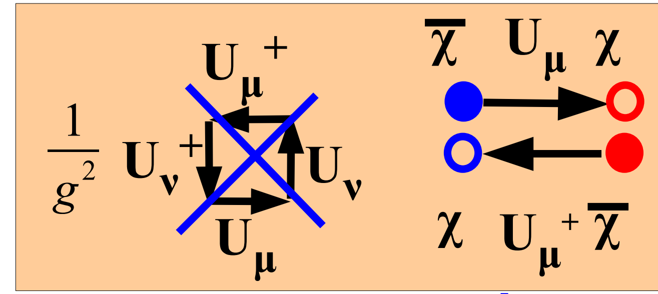
Hadron Mass in SCL-LQCD (Finite T)

- **QCD Lattice Action (Finite T treatment)**

Damgaard, Kawamoto, Shigemoto, 1984; Bilic et al., 1992; Nishida, 2003; Kawamoto, Miura, Ohnishi, Ohnuma, 2007; ...

$$S = \cancel{S_G} + S_F^{(s)} + S_F^{(t)} + m_0 \bar{\chi}\chi$$

Strong Coupling Limit

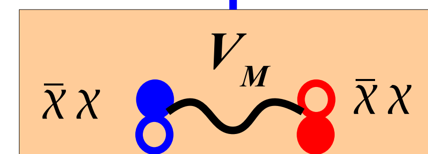


$$S_F^{(t)} = \frac{1}{2}\sum_x \left(e^{\mu}\bar{\chi}_x U_0(x)\chi_{x+\hat{0}} - e^{-\mu}\bar{\chi}_{x+\hat{0}} U_0^+(x)\chi_x \right) = \bar{\chi} V^{(t)} \chi$$

dU

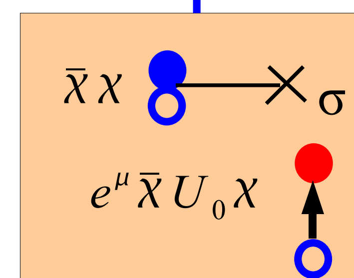
$$\to -\frac{1}{2}(\bar{\chi}\chi) V_M (\bar{\chi}\chi) + \bar{\chi}(V^{(t)} + m_q)\chi$$

Spatial-link integral (1/d expansion, no B)



$$\to \frac{1}{2}\sigma V_M^{-1}\sigma + \bar{\chi}(V^{(t)} + \sigma + m_q)\chi$$

Bosonization

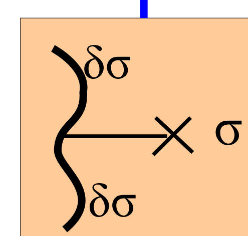


$$\to L^d N_\tau \left[\frac{N_c}{d}\bar{\sigma}^2 + F_{\rm eff}^{(q)}(\bar{\sigma}, T, \mu) \right] + \frac{1}{2}\sum_k G(k)^{-1}(\delta\sigma(k))^2$$

$d\chi, dU_0$

- **Effective Potential**

Fermion and Temporal-link Integral



$$F_{\rm eff}^{(q)}(m; T, \mu) = -T \log \int dU_0 \, det(V^{(t)} + m)$$

$$= -T \log \left[\frac{\sinh((N_c+1)E_q/T)}{\sinh(E_q/T)} + 2\cosh(N_c \mu/T) \right]$$

$$E_q(m) = \mathrm{arcsinh}\, m$$

Hadron Mass in SCL-LQCD (Finite T)

- **Meson propagator at Finite T** *Faldt, Petersson, '86*
  - **U_0 integrated quark determinant = Function of X_N**
    X_N = Functional of m(τ)

$$F^{(q)}_{\text{eff}}(m(x);T,\mu) = -T\log\int dU_0 \det_{\tau\tau'}(V^{(t)}+m(\boldsymbol{x},\tau)) = F^{(q)}_{\text{eff}}(X_N[m])$$

$$X_N[I] = B_N(I_1,..,I_N) + B_{N-2}(I_2,..,I_{N-1})$$
$$(I_k = 2m(k) = 2(\sigma(k)+m_q))$$

$$B_N = \begin{vmatrix} I_1 & e^{\mu} & & & 0 \\ -e^{-\mu} & I_2 & e^{\mu} & & \\ 0 & -e^{-\mu} & I_3 & e^{\mu} & \\ \vdots & & & \ddots & \\ 0 & & & -e^{-\mu} & I_N \end{vmatrix}$$

  - **Derivatives**

$$\frac{\delta X_N}{\delta I_N} = B_{N-1}(I_1,..,I_{N-1})$$

$$\frac{\delta B_N}{\delta I_k} = B_{k-1}(I_1,..,I_{k-1})B_{N-k}(I_{k+1},\ldots,I_N)$$

  - **Equilibrium Value**

$$B_N(I_k=\text{const.}) = \begin{cases} \cosh((N+1)E_q)/\cosh E_q & (\text{even } N) \\ \sinh((N+1)E_q)/\cosh E_q & (\text{odd } N) \end{cases}$$

Hadron Mass in SCL-LQCD (Finite T)

- **Meson Mass**

$$G^{-1}(\boldsymbol{k},\omega)=\frac{2N_c}{\kappa(\boldsymbol{k})}+\frac{4N_c}{d}\frac{\bar{\sigma}(\bar{\sigma}+m_q)}{\cos\omega+\cosh 2E_q}$$

$$\kappa(\boldsymbol{k})=\sum_{i=1}^{d}\cos k_i \quad\rightarrow\quad \kappa=-d,-d+2,\ldots d$$

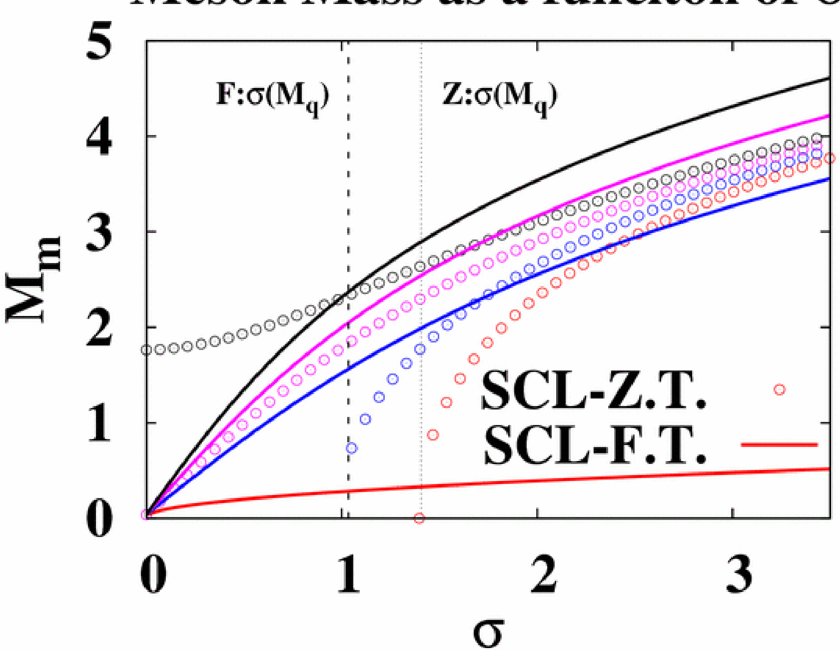
$$\cosh M=2(\bar{\sigma}+m_q)\left(\frac{d+\kappa}{d}\bar{\sigma}+m_q\right)+1$$

or $M=2\,\mathrm{arcsinh}\sqrt{(\bar{\sigma}+m_q)\left(\frac{d+\kappa}{d}\bar{\sigma}+m_q\right)}$

- **Meson masses are determined by the chiral condensate, σ.**
- **Chiral condensate is determined by the equilibrium condition, and given as a function of (T, μ).**

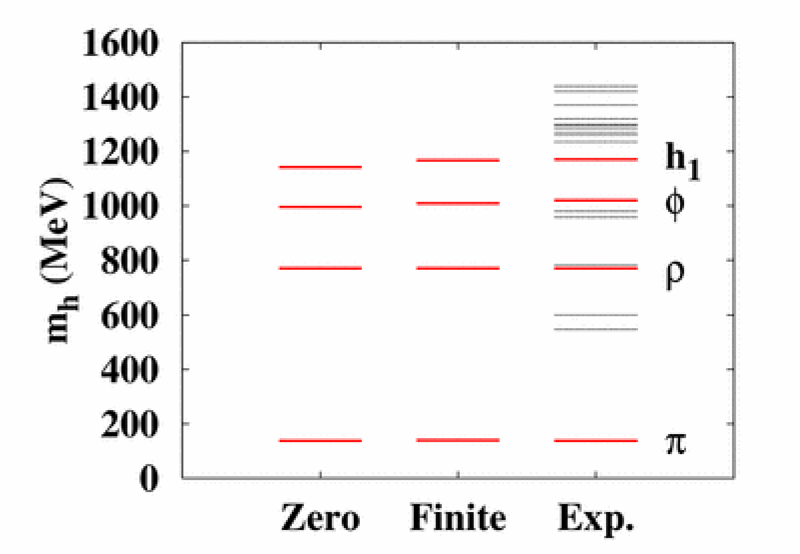
→ ***Approximate Brown-Rho scaling is proven in SCL-LQCD***

Meson Mass as a funciton of σ

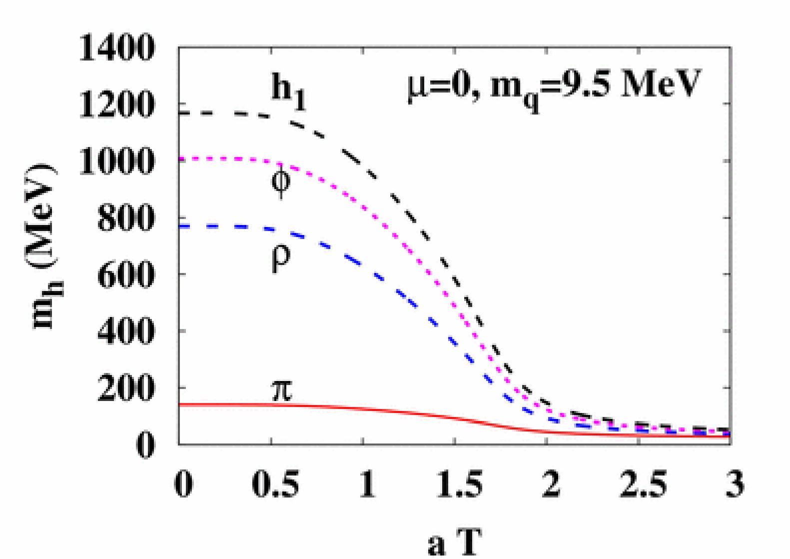


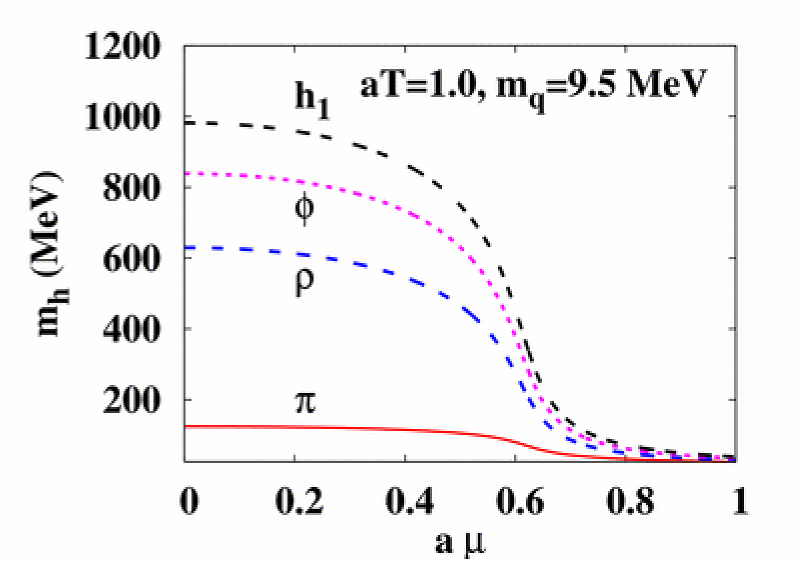
Medium Modification of Meson Masses

- **Scale fixing**
  - **Search for σ_{vac} to minimize free E.**
  - **Assign κ=-3, -1 as π and ρ**
  - **Determine m_q and a^{-1} (lattice unit) to fit m_π / m_ρ**



- **Medium modification**
  - **Search for σ(T, μ) → Meson mass**





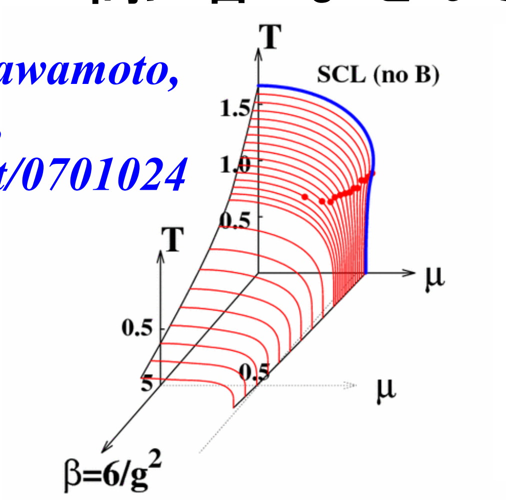
Discussion

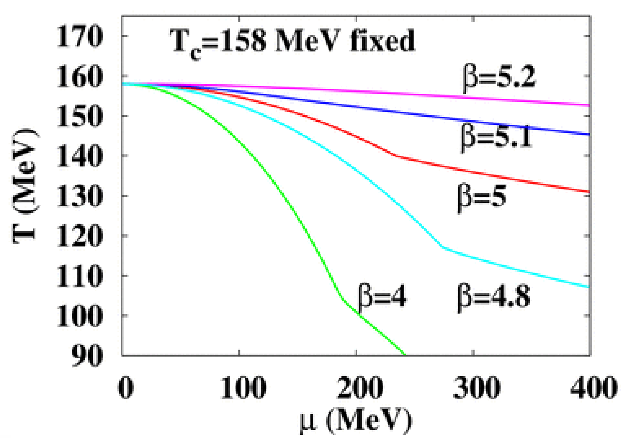
- SCL では小さな μ で σ は変化しない
- π, ρ mass fit の結果
  a^{-1} = 497 MeV, m_q=9.5 MeV → T_c= 5/3a = 828 MeV Too large !
  (SCL での昔からの問題点)
- 有限結合効果 (1/g^2 correction) により T_c は小さくなる

  Bilic, Claymans, '95; AO, Kawamoto, Miura, '07

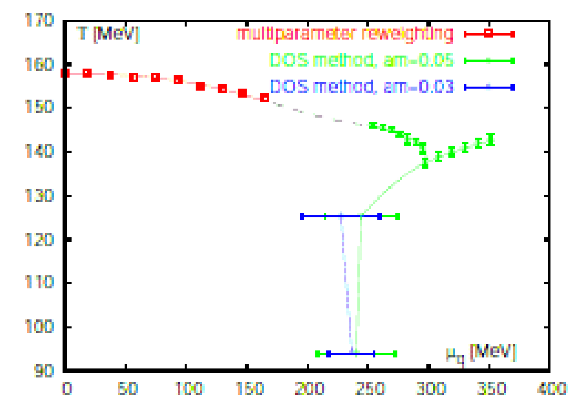
  - **複数の 補助場の 導入が必 要 →** $\sigma=-\langle\bar{q}\,q\rangle$ **と** $\varphi=\langle\bar{q}\,gq\rangle$ **の対角化が 必要**
    → 間に 合いま せんでした .....

AO, Kawamoto, Miura, hep-lat/0701024





Kawamoto,Miura,AO, in prep.



Fodor,Katz,Schmit, 2007

Summary

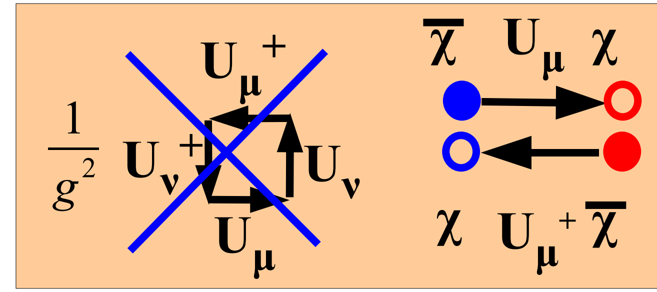
- **Hadron (Meson) masses are evaluated in the Strong Coupling Limit of Lattice QCD at Finite T and μ.**
  - **Meson masses are determined by the chiral condensate, and they are approximately linear functions of σ, while m_π is always 0 in the chiral limit.**
  - **For high T or μ, meson masses decrease as σ decreases. → *Approximate Brown-Rho(-Hatsuda) scaling is supported*.**
  - **When we fit π and ρ masses, lattice unit (a^{-1}) is found to be around 500 MeV, suggesting $T_c \sim 800$ MeV in the Strong Coupling Limit. (Longstanding problem in the strong coupling limit....)**
- **Finite coupling effects are found to decrease T_c (in the lattice unit), while approximately keeping μ_c. → Meson mass with $1/g^2$ correction has to be calculated.**
- **Baryon mass → Miura's talk**

Backups

Strong Coupling Limit Lattice QCD

- **QCD Lattice Action**

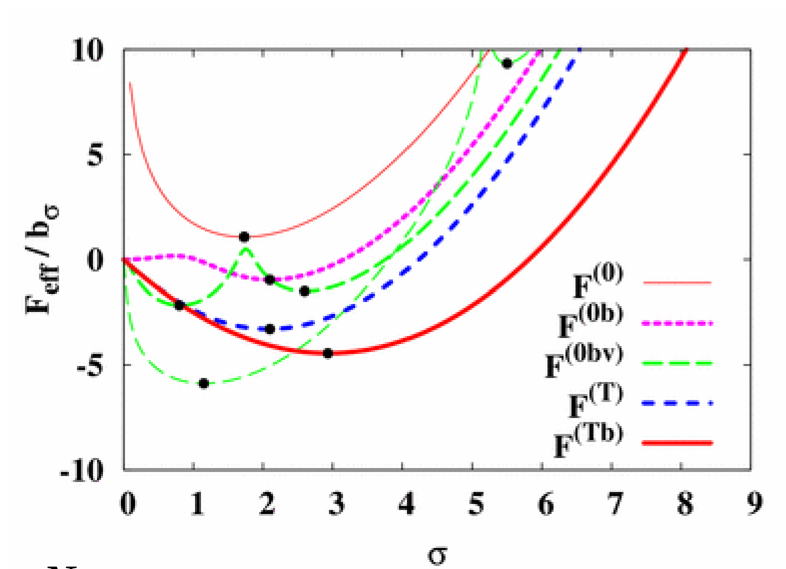
$$F_{\mathrm{eff}} = \frac{N_c}{d+1}\bar{\sigma}^2 - N_c \log(\bar{\sigma} + m_q)$$



$$S_G = \frac{1}{g^2}\sum_{x\mu\nu}\left[\mathrm{Tr}\,U_{\mu\nu} + \mathrm{Tr}\,U_{\mu\nu}^+\right]$$

$$S_F^{(s)} = \frac{1}{2}\sum_{x,j}\eta_j(x)\left(\bar{\chi}_x U_j(x)\chi_{x+\hat{j}} - \bar{\chi}_{x+\hat{j}} U_j^+(x)\chi_x\right)$$

$$S_F^{(t)} = \frac{1}{2}\sum_{x}\left(e^{\mu}\bar{\chi}_x U_0(x)\chi_{x+\hat{0}} - e^{-\mu}\bar{\chi}_{x+\hat{0}} U_0^+(x)\chi_x\right)$$



- **Strong Coupling Limit: g→∞**
  - **Ignore S_G → Link integral**
  - **Zero T treatment**
    → All Links are integrated first

$$F_{\mathrm{eff}} = \frac{N_c}{d+1}\bar{\sigma}^2 - N_c \log(\bar{\sigma} + m_q)$$

  - **Finite T treatment**
    → Temporal Links are integrated later exactly.

$$F_{\mathrm{eff}}^{(q)}(m;T,\mu) = \frac{N_c\bar{\sigma}^2}{d} - T\log\left[\frac{\sinh((N_c+1)E_q/T)}{\sinh(E_q/T)} + 2\cosh(N_c\mu/T)\right]$$

$$E_q(m) = \mathrm{arcsinh}\, m$$

Hadron Mass in SCL-LQCD (Zero T)

- **Meson Mass in SCL-LQCD**
  Kluberg-Stern, Morel, Petersson, 1982; Kawamoto, Shigemoto, 1982
  - **Pole of the propagator at zero momentum → Meson Mass**
  - **Doubler DOF: $k_\mu \to 0$ or π, Euclidian: $\omega \to i\,m$ + "0 or π"**

$$G^{-1}(k) = N_c \left[\sum_{i=1}^{d} \cos\pi\delta_i \pm \cosh m \right]^{-1} + \frac{N_c}{(\bar\sigma + m_q)^2} = 0$$

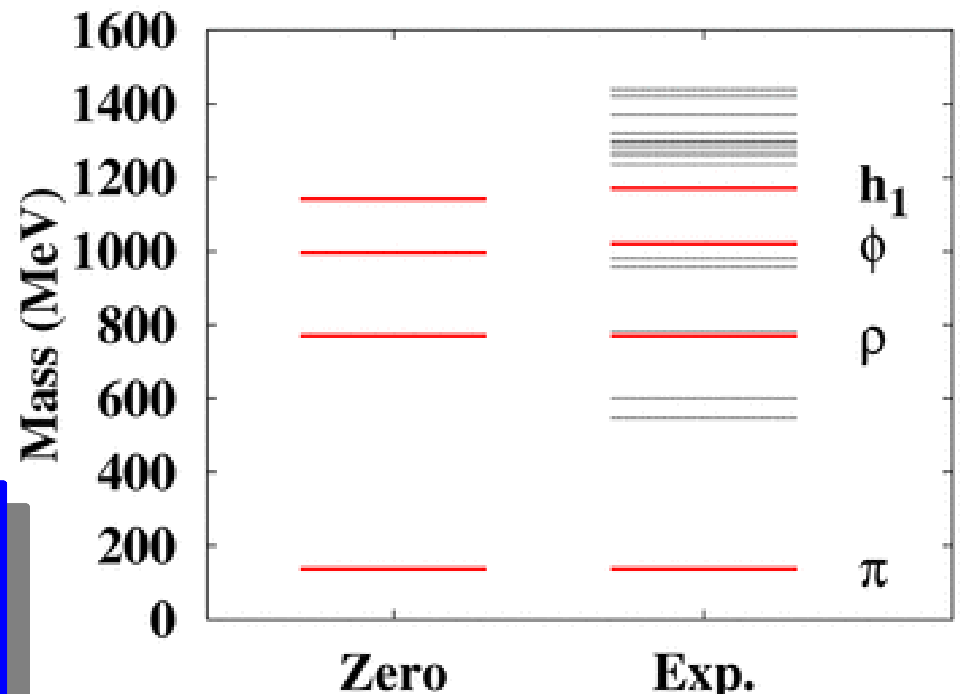
$$\to \cosh m = 2(\bar\sigma + m_q)^2 + \kappa$$

$$= (d+1)(\lambda^2 - 1) + 2n + 1$$

Equilibrium Condition

$$n = 0, 1, \ldots d \quad \text{(diff. meson species)}$$

$$\lambda = \bar m_q + \sqrt{\bar m_q^{\,2} + 1}, \quad \bar m_q = m_q / \sqrt{2(d+1)}$$



Explains Meson Mass Spectrum
No (T, μ) dependence