Nuclear Matter Equation of State

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- **Introduction**
  Why do we study EOS ?
- **Relativistic Mean Field description of EOS**
  σω model, Non-linear terms, Chiral RMF
- **Dense Matter EOS with Hyperons**
  Hyperon potentials in nuclear matter, EOS with hyperons,
- **Collective flow and EOS in High-Energy Heavy-Ion Collisions**
  Semi-classical transport model, Collective flows at AGS & SPS
- **Summary**

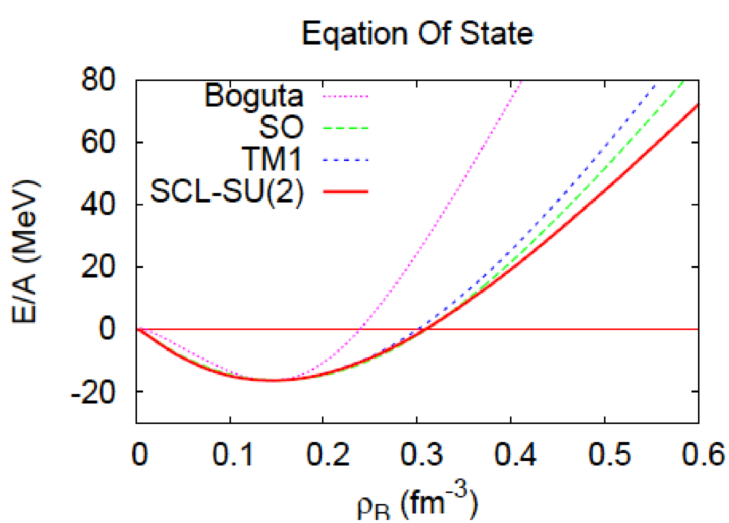
Introduction
Why do we study Nuclear Matter EOS ?

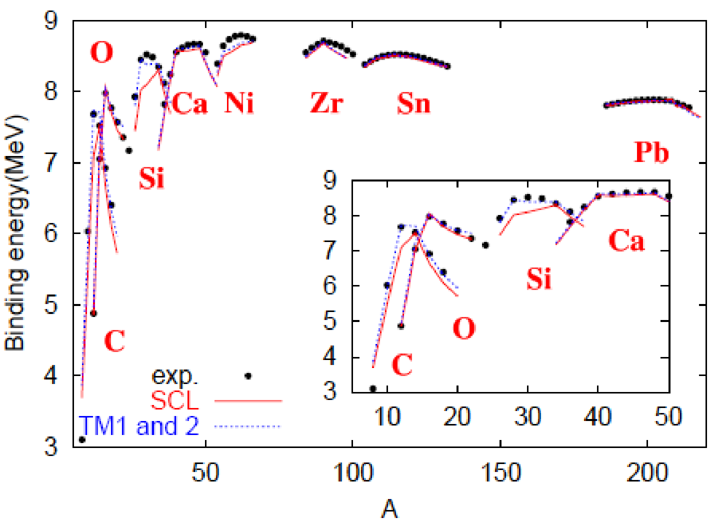
Why do we study Nuclear Matter EOS ?

- **Answer 1: Since bulk nuclear properties are mainly determined by nuclear matter EOS, it is important for nuclear physics.**
  - **Nuclear Radius → Saturation of Density**
    $R_A = r_0 A^{1/3}$ (r_0=1.2 fm)
  - **Nuclear Binding Energy (Bethe-Weizsacker Formula)**

$$B(A,Z) = a_{vol} A - a_{surf} A^{2/3} - a_{Coulomb} \frac{Z^2}{A^{1/3}} - a_{sym} \frac{(N-Z)^2}{A} + a_{pair} \delta(A,Z) A^{-3/4}$$

Nuclear Matter





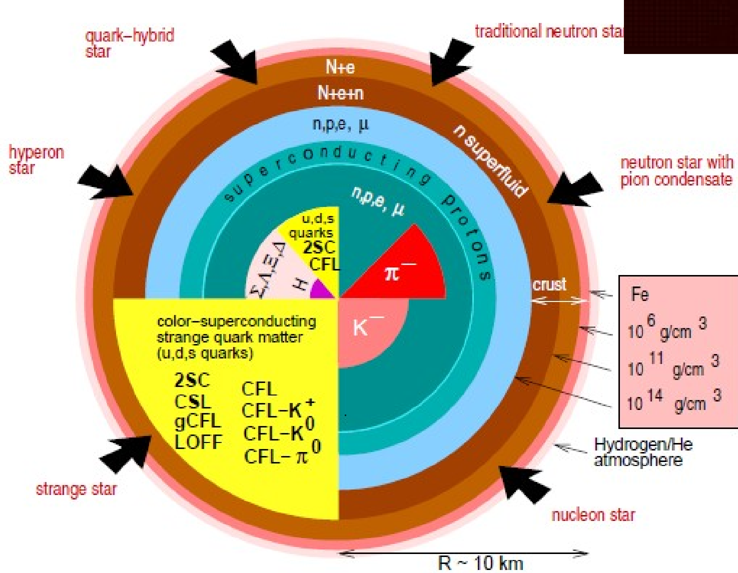
Why do we study Nuclear Matter EOS ?

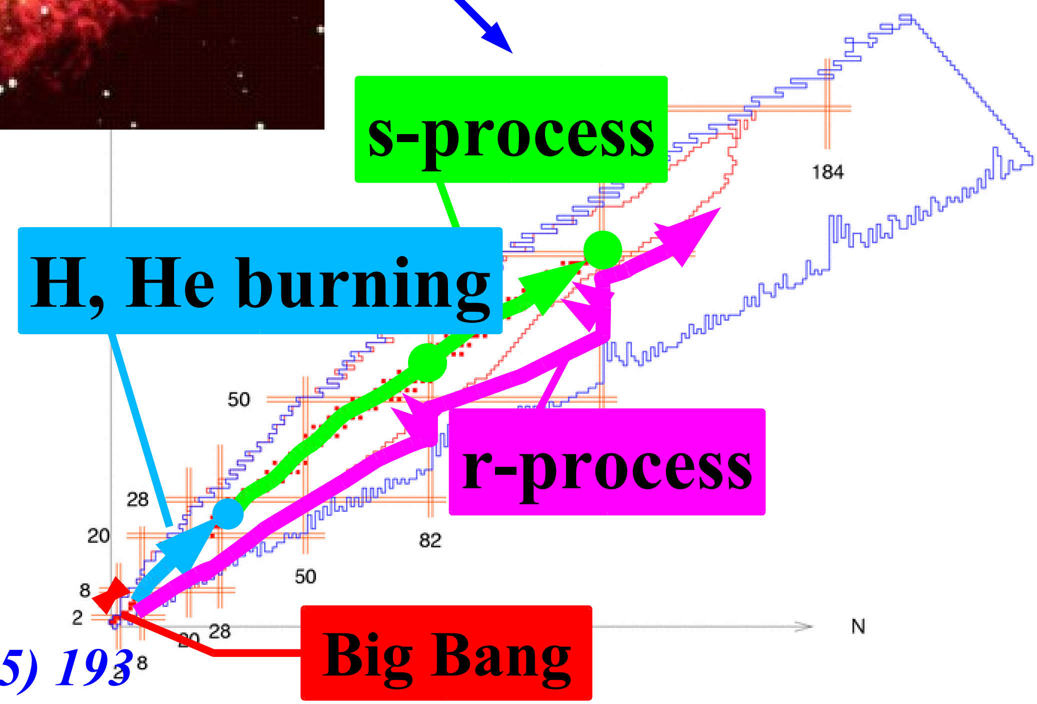
- **Answer 2: Since nuclear matter EOS is decisive in compact astrophysical objects such as neutron stars, supernovae, and black hole formation, EOS is important to understand where atomic elements are made.**

Supernova

Nucleosynthesis

Neutron Star





F. Weber, Prog. Part. Nucl. Phys. 54 (2005) 193

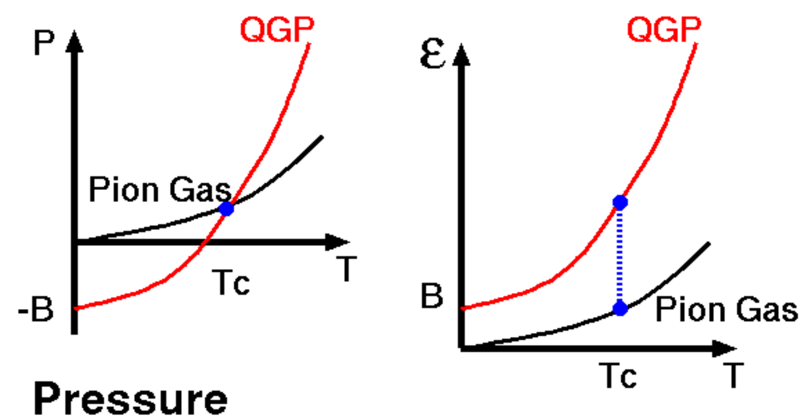
Why do we study Nuclear Matter EOS ?

- **Answer 3: Since the EOS should have singularity (or at least sudden change) at phase boundary, it would be possible to catch the signal of phase transition in nuclear collisions.**

- **Pressure and Energy Density of Free Massless Gas**

$$P=\frac{\pi^2}{90}N_B T^4 \quad , \quad \epsilon=\frac{\pi^2}{30}N_B T^4$$

N_B = Bosonic DOF (7/8 for Fermions)

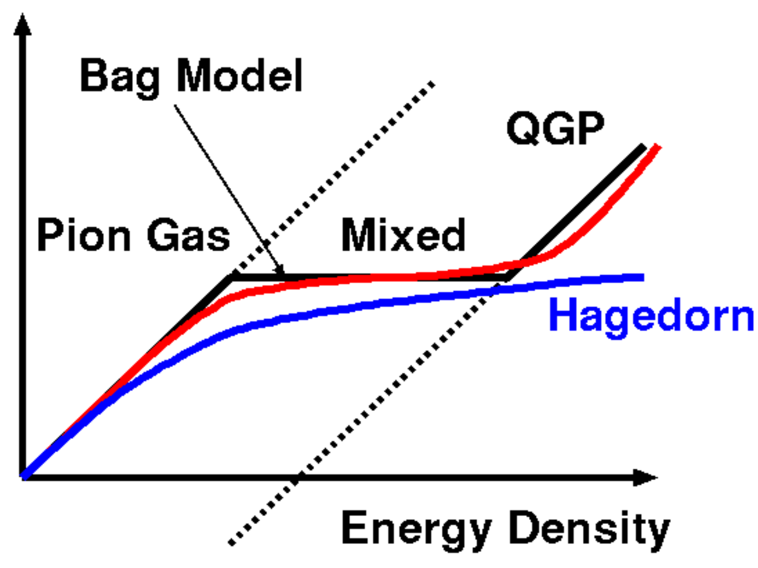


- **Hadron Gas ~ 3 pions (N_B=3)**

$$P_\pi=\frac{\pi^2}{30}T^4 \quad , \quad \epsilon_\pi=\frac{\pi^2}{10}T^4$$

- **QGP N_B=16(gluon)+24 x 7/8 (quarks) and Bag Pressure**

$$P_{QGP}=\frac{37\pi^2}{90}T^4-B \quad \epsilon_{QGP}=\frac{37\pi^2}{30}T^4+B$$



Nuclear Matter EOS

- **In this lecture, I discuss several aspects of Nuclear Matter EOS**

  Lecture 1
  (1) EOS and Mean Field in Finite Nuclei and Nuclear Matter
  → Relativistic Mean Field

  Lecture 2
  (2) Dense Matter EOS and Compact Astrophysical Objects
  → EOS with Hyperons, Neutron Stars, Supernovae, Black Hole Formation

  (3) Hot and Dense Matter EOS and High-Energy Heavy-Ion Collisions
  → Nuclear Transport Model, Collective Flows

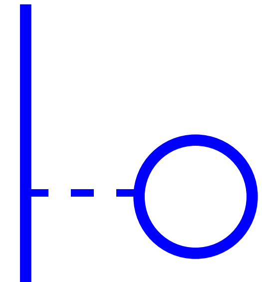
Relativistic Mean Field

Theories/Models for Nuclear Matter EOS

- **Ab initio Approach**
  - **LQCD, GFMC, Variational, DBHF, G-matrix**
    → Not easy to handle, Not satisfactory for phen. purposes
- **Mean Field from Effective Interactions ~ Nuclear Density Fuctionals**
  - **Skyrme Hartree-Fock(-Bogoliubov)**
    - **Non.-Rel.,Zero Range, Two-body + Three-body (or ρ-dep. two-body)**
    - **In HFB, Nuclear Mass is very well explained (Total B.E. ΔE ~ 0.6 MeV)**
    - **Causality is violated at very high densities.**
  - **Relativistic Mean Field**
    - **Relativistic, Meson-Baryon coupling, Meson self-energies**
    - **Successful in describing pA scatering (Dirac Phenomenology)**

Relativistic Mean Field (1)

- **Relativistic Mean Field**
  = Nuclear scalar and vector mean field generated by mesons
  → Why do we use relativistic framework ?



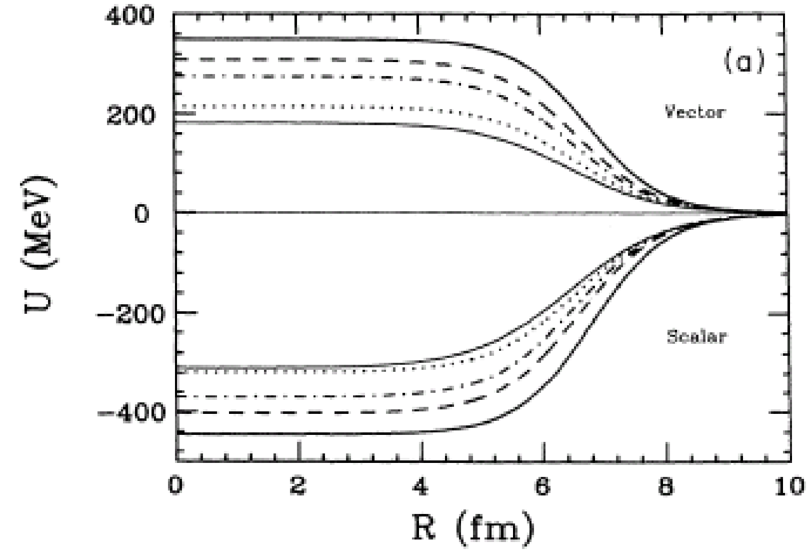
  - **Nuclear Force is mediated by mesons**
    → Let's consider meson-baryon system !
    (Entrance of Hadron Physics)
  - **We are also interested in Dense Matter EOS**
    → Sound velocity exceeds the Speed of Light (=c) with Non.-Rel. MF
  - **Success of "Dirac Phenomenology"**
    (Dirac Eq. for pA scattering → Spin Observables)
    → Strong Scalar and Vector Mean Fields are preferable to explain Spin Observables
  - **DBHF (Dirac-Brueckner-Hatree-Fock)**
    → Successful description of nuclear matter saturation point based on bare NN interactions

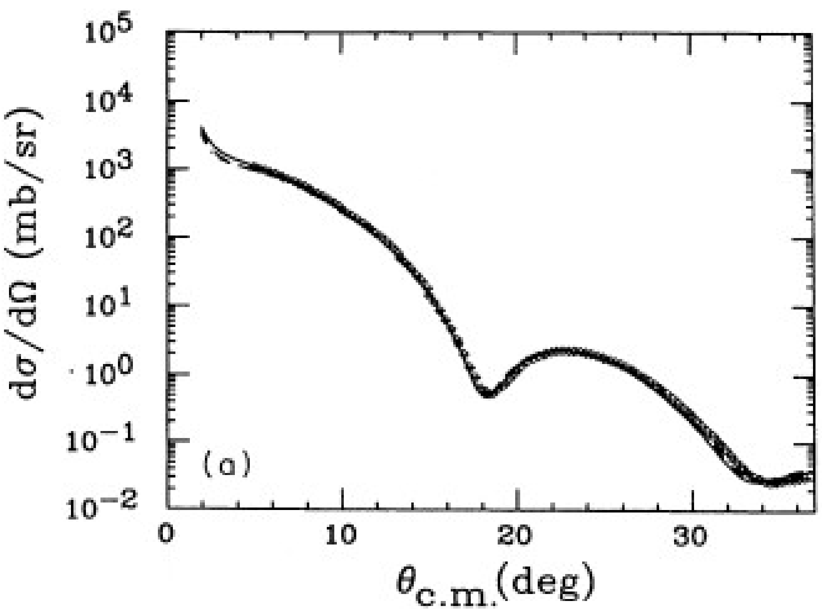
RMF is a good starting point as a framework of hadronic system including Nuclei and Nuclear Matter

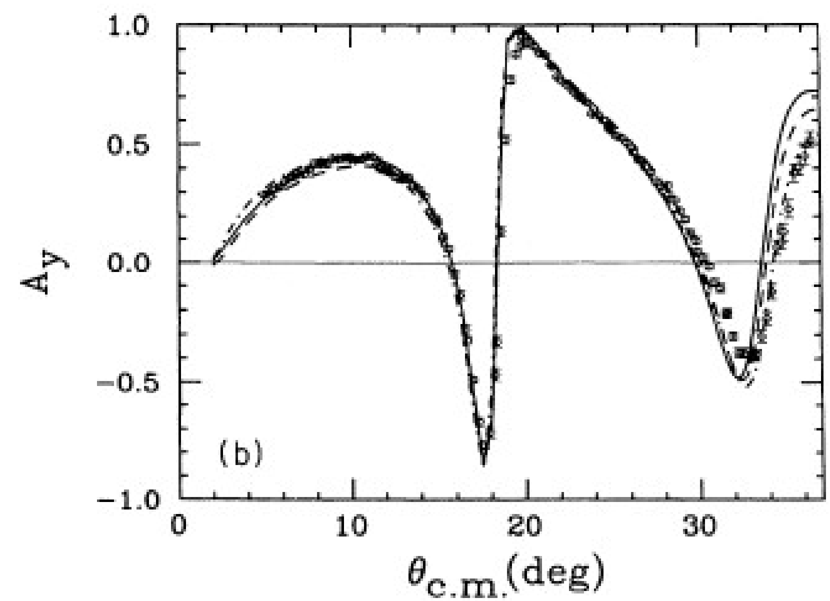
Dirac Phenomenology

E.D. Cooper, S. Hama, B.C. Clark, R.L. Mercer, PRC47('93),297

- **Dirac Eq. with Scalar + Vector pA potential (-400 MeV + 350 MeV) → Cross Section, Spin Observables**



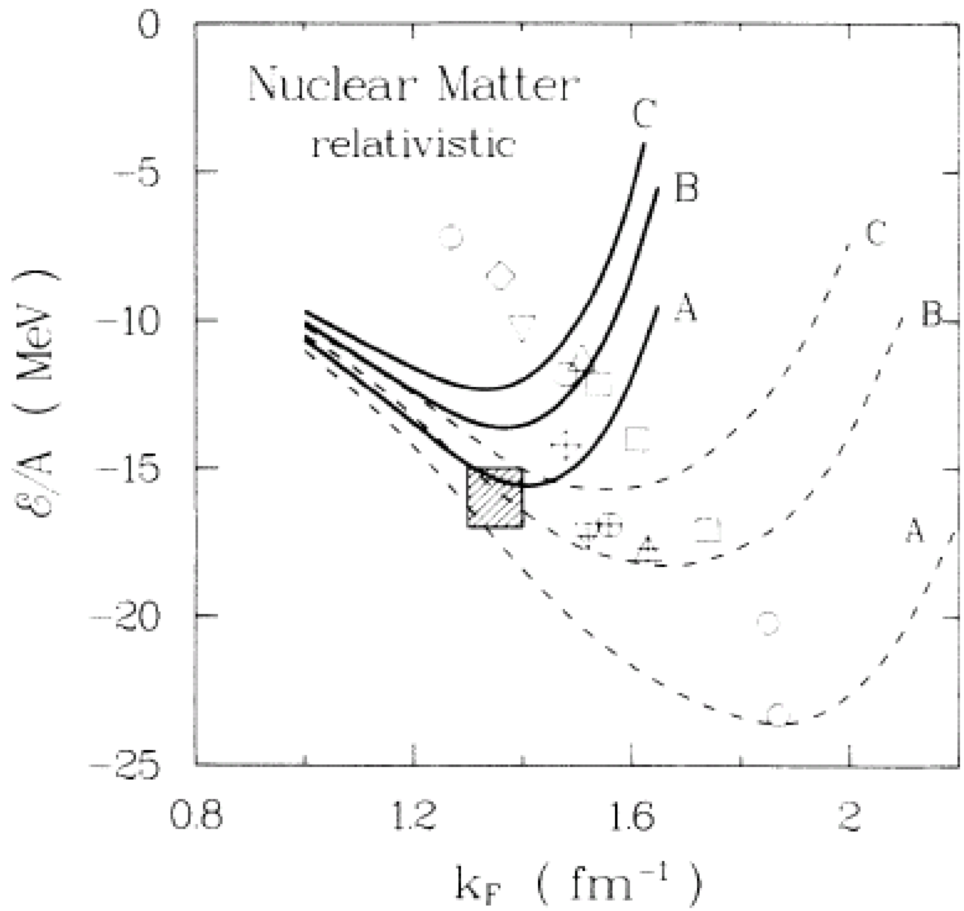




EOS in Dirac-Brueckner-Hartree-Fock

R. Brockmann, R. Machleidt, PRC42('90),1965

- **Non Relativistic Brueckner Calculation**
  → Nuclear Saturation Point cannot be reproduced (Coester Line)
- **Relativistic Approach (DBHF)**
  → Relativity gives additional repulsion, leading to successful description of the saturation point.



Relativistic Mean Field (2)

- **Mean Field treatment of meson field operator**
  = Meson ield operator is replaced with its expectation value

  $$\varphi(r) \rightarrow \langle\varphi(r)\rangle$$

  Ignoring fluctuations compared with the expectation value may be a good approximation at strong condensate.

- **Which Hadrons should be included in RMF ?**
  - **Baryons (1/2+)** — **p, n, Λ, Σ, Ξ, Δ, ...**
  - **Scalar Mesons (0+)** — **σ(600), f_0(980), a_0(980), ...**
  - **Vector Mesons (1-)** — **ω(783), ρ(770), ϕ(1020), ...**
  - **Pseuso Scalar (0-)** — **π , K, η, η', ...**
  - **Axial Vector (1+)** — **a_1, ...**

  We require that the meson field can have uniform expectation values in nuclear matter.

  → Scalar and Time-Component of Vector Mesons (σ , ω, ρ, ...)

$\sigma\omega$ Model (1)

Serot, Walecka, Adv.Nucl.Phys.16 (1986),1

- **Consider only σ and ω mesons**
- **Lagrangian**

$$L=\bar{\psi}\left(i\gamma^{\mu}\partial_{\mu}-M+g_{s}\sigma-g_{v}\omega\right)\psi$$
$$+\frac{1}{2}\partial_{\mu}\sigma\,\partial^{\mu}\sigma-\frac{1}{2}m_{s}^{2}\sigma^{2}-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2}m_{v}^{2}\omega_{\mu}\omega^{\mu}$$
$$\left(F_{\mu\nu}=\partial_{\mu}\omega_{\nu}-\partial_{\nu}\omega_{\mu}\right)$$

- **Equation of Motion**
  - **Euler-Lagrange Equation**

$$\frac{\partial}{\partial x^{\mu}}\left[\frac{\partial L}{\partial\left(\partial_{\mu}\phi_{i}\right)}\right]-\frac{\partial L}{\partial\phi_{i}}=0$$

$$\sigma:\left[\partial_{\mu}\partial^{\mu}+m_{s}^{2}\right]\sigma=g_{s}\bar{\psi}\psi$$

$$\omega:\partial_{\mu}F^{\mu\nu}+m_{v}^{2}\omega^{\nu}=g_{v}\bar{\psi}\gamma^{\nu}\psi\quad\rightarrow\quad\left[\partial_{\mu}\partial^{\mu}+m_{v}^{2}\right]\omega^{\nu}=g_{v}\bar{\psi}\gamma^{\nu}\psi$$

$$\psi:\left[\gamma^{\mu}\left(i\partial_{\mu}-g_{v}V_{\mu}\right)-\left(M-g_{s}\sigma\right)\right]\psi=0$$

EOM of ω (for beginners)

- **Euler-Lagrange Eq.**

$$\partial_\mu F^{\mu\nu} + m_v^2 \omega^\nu = g_v \bar{\psi} \gamma^\nu \psi$$

- **Divergence of LHS and RHS**

$$\partial_\nu \partial_\mu F^{\mu\nu} + m_v^2 (\partial_\nu \omega^\nu) = m_v^2 (\partial_\nu \omega^\nu) = g_v (\partial_\nu \bar{\psi} \gamma^\nu \psi) = 0$$

LHS: derivatives are sym. and $F_{\mu\nu}$ is anti-sym.

RHS: Baryon Current = Conserved Current

- **Put it in the Euler-Lagrange Eq.**

$$\partial_\mu F^{\mu\nu} = \partial_\mu (\partial^\mu \omega^\nu - \partial^\nu \omega^\mu) = \partial_\mu \partial^\mu \omega^\nu - \partial^\nu (\partial_\mu \omega^\mu) = \partial_\mu \partial^\mu \omega^\nu$$

Schroedinger Eq. for Upper Component

- **Dirac Equation for Nucleons**

$$\left(i\gamma\partial-\gamma^0 U_v - M - U_s\right)\psi=0 \quad , \quad U_v=g_\omega\omega \quad , \quad U_s=-g_\sigma\sigma$$

- **Decompose 4 spinor into Upper and Lower Components**

$$\begin{pmatrix} E-U_v-M-U_s & i\sigma\cdot\nabla \\ -i\sigma\cdot\nabla & -E+U_v-M-U_s \end{pmatrix}\begin{pmatrix} f \\ g \end{pmatrix}=0 \qquad g=\frac{-i}{E+M+U_s-U_V}(\sigma\cdot\nabla)f$$

$$(E-M-U_v-U_s)f=-i(\sigma\cdot\nabla)g$$

- **Erase Lower Component (assuming spherical sym.)**

$$-i(\sigma\cdot\nabla)g=-(\sigma\cdot\nabla)\frac{1}{X}(\sigma\cdot\nabla)f=-\frac{1}{X}\nabla^2 f-\frac{1}{r}\left[\frac{d}{dr}\frac{1}{X}\right](\sigma\cdot r)(\sigma\cdot\nabla)f=-\nabla\frac{1}{X}\nabla f+\frac{1}{r}\left[\frac{d}{dr}\frac{1}{X}\right](\sigma\cdot l)f$$

$$(\sigma\cdot r)(\sigma\cdot\nabla)=(r\cdot\nabla)+i\sigma\cdot(r\times\nabla)=r\cdot\nabla-\sigma\cdot l$$

- **"Schroedinger-like" Eq. for Upper Component**

$$-\nabla\frac{1}{E+M+U_s-U_v}\nabla f+\left(U_s+U_v+U_{LS}(\sigma\cdot l)\right)f=(E-M)f$$

$$U_{LS}=\frac{1}{r}\left[\frac{d}{dr}\frac{1}{E+M+U_s-U_v}\right]<0 \quad \text{on surface}$$

$(U_s,U_v)\sim$ (-350 MeV,280 MeV)→Small Central(U_s+U_v), Large LS (U_s-U_v)

Various Ways to Evaluate Non.-Rel. Potential

- **From Single Particle Energy**

$$\left(\gamma^0 (E - U_v) + i\gamma\cdot\nabla - (M + U_s)\right)\psi = 0 \;\rightarrow\; (E - U_v)^2 = p^2 + (M + U_s)^2$$

$$\rightarrow E = \sqrt{p^2 + (M+U_s)^2} + U_v \approx E_p + \frac{M}{E_p} U_s + U_v + \frac{p^2}{2E_p^3} U_s^2$$

$$(E_p = \sqrt{p^2 + M^2})$$

- **Schroedinger Equivalent Potential (Uniform matter)**

$$-\frac{\nabla^2}{2\mathrm{M}} f + \left[U_s + \frac{E}{M} U_v + \frac{U_s^2 - U_v^2}{2\mathrm{M}} \right] f = \frac{E+M}{2\mathrm{M}} (E - M) f$$

$$U_{\mathrm{SEP}} \approx U_s + \frac{E}{M} U_v$$

Anyway, slow baryons feel Non.-Rel. Potential,

$$U \approx U_s + U_v = -g_s \sigma + g_v \omega$$

Nuclear Matter in σω Model

Serot, Walecka, Adv.Nucl.Phys.16 (1986),1

- **Uniform Nuclear Matter**

$$E/V = \gamma_N \int^{P_F} \frac{d^3p}{(2\pi)^2} E^* + \frac{1}{2} m_s^2 \sigma^2 - \frac{1}{2} m_v^2 \omega^2 + g_v \rho_B \omega$$

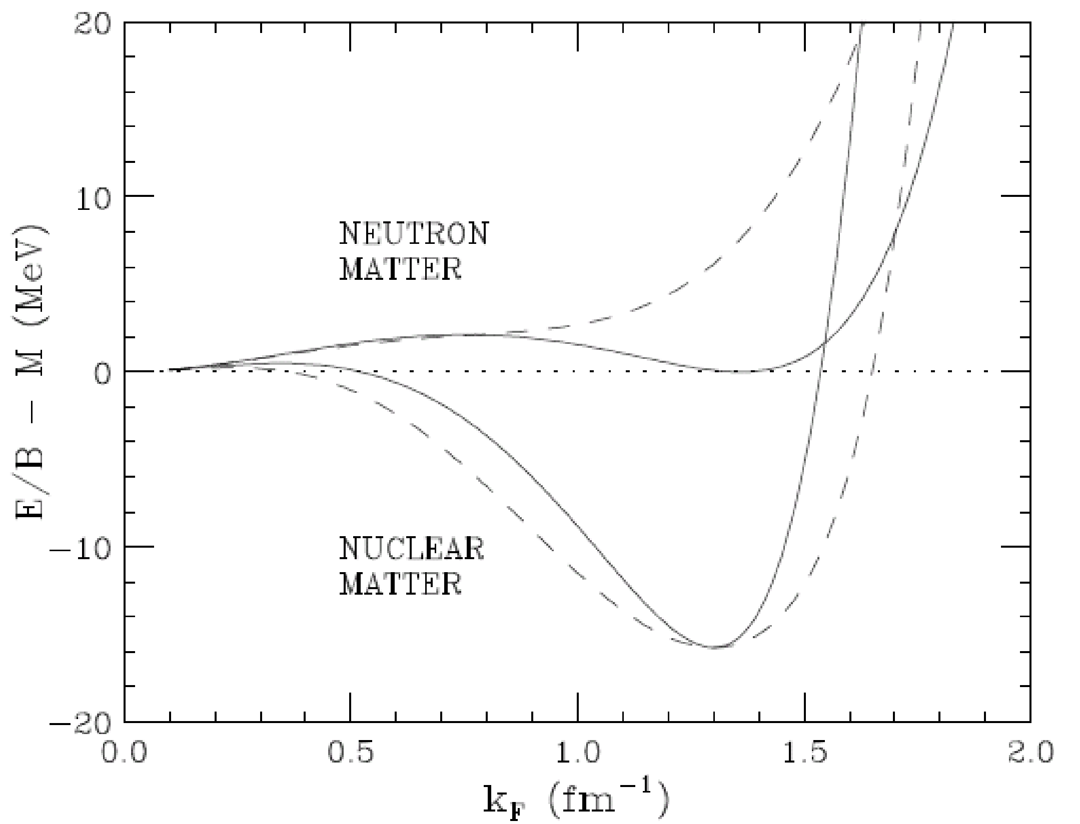
$$\sigma = \frac{g_s}{m_s^2} \rho_s = \frac{g_s}{m_s^2} \int^{P_F} \frac{\gamma_N \; d^3p}{(2\pi)^2} \frac{M^*}{E^*}$$

$$\left(M^* = M + U_s = M - g_s \sigma, \quad E^* = \sqrt{p^2 + M^{*2}}\right)$$

$$\omega = \frac{g_v}{m_v^2} \rho_B = \gamma_N \frac{g_v}{m_v^2} \int^{P_F} \frac{d^3p}{(2\pi)^3}$$

γ_N = Nucleon degeneracy
(=4 in sym. nuclear matter)

Problem: EOS is too stiff
K ~ (500-600) MeV !
→ How can we solve ?



RMF with Non-Linear Meson Int. Terms

Boguta, Bodmer ('77), NL1:Reinhardt, Rufa, Maruhn, Greiner, Friedrich ('86), NL3: Lalazissis, Konig, Ring ('97),TM1 and TM2: Sugahara, Toki ('94), Brockmann, Toki ('92)

- **Too stiff EOS in the simplest RMF (σω model) is improved by introducing non-linear terms (σ^4, ω^4)**
  - **Fit B.E. of Stable as well as Unstable (n-rich) Nuclei**
  - **Three Mesons (σ,ω,ρ) are included**
  - **Meson Self-Energy Term (σ,ω)**

$$
\begin{aligned}
\mathcal{L} =& \bar{\psi}_N \left(i\partial\!\!\!/ - M - g_\sigma \sigma - g_\omega \omega\!\!\!/ - g_\rho \tau^a \rho\!\!\!/^a \right) \psi_N \\
&+ \frac{1}{2}\partial^\mu \sigma \partial_\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4 \\
&- \frac{1}{4} W^{\mu\nu} W_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega^\mu \omega_\mu - \frac{1}{4} R^{a\mu\nu} R^a_{\mu\nu} + \frac{1}{2} m_\rho^2 \rho^{a\mu} \rho^a_\mu + \frac{1}{4} c_3 \left(\omega_\mu \omega^\mu \right)^2 \\
&+ \bar{\psi}_e \left(i\partial\!\!\!/ - m_e \right) \psi_e + \bar{\psi}_\nu i\partial\!\!\!/ \psi_\nu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} , \\
W_{\mu\nu} =& \partial_\mu \omega_\nu - \partial_\nu \omega_\mu , \\
R^a_{\mu\nu} =& \partial_\mu \rho^a_\nu - \partial_\nu \rho^a_\mu + g_\rho \epsilon^{abc} \rho^{b\mu} \rho^{c\nu} , \\
F_{\mu\nu} =& \partial_\mu A_\nu - \partial_\nu A_\mu .
\end{aligned}
\tag{2}
$$

RMF with Non-Linear Meson Int. Terms

- **Are the Lagrangian parameters are well determined ?**

$$\mathcal{L} = \mathcal{L}_{\text{free}}(\psi, \sigma, \omega, \rho, ...) + \bar{\psi}\left[g_\sigma \sigma - g_\omega \gamma^0 \omega - g_\rho \tau_z \gamma^0 \rho\right]\psi + c_\omega \omega^4/4 - V_\sigma(\sigma) \ , \quad (3)$$

$$V_\sigma = \begin{cases} \frac{1}{3} g_3 \sigma^3 + \frac{1}{4} g_4 \sigma^4 & \text{(NL1, NL3, TM1)} \\ -a_\sigma f_{\text{SCL}}(\sigma/f_\pi) & \text{(SCL)} \end{cases} , \quad (4)$$

  - **Linear terms, Meson-Nucleon Coupling → Well determined**
  - **Negative Coef. of $\sigma^4 < 0$ in some of RMF models→ Vacuum is unstable**
  - **Meson interaction terms → Different in RMF parameterization and ....**

TABLE II: RMF parameters

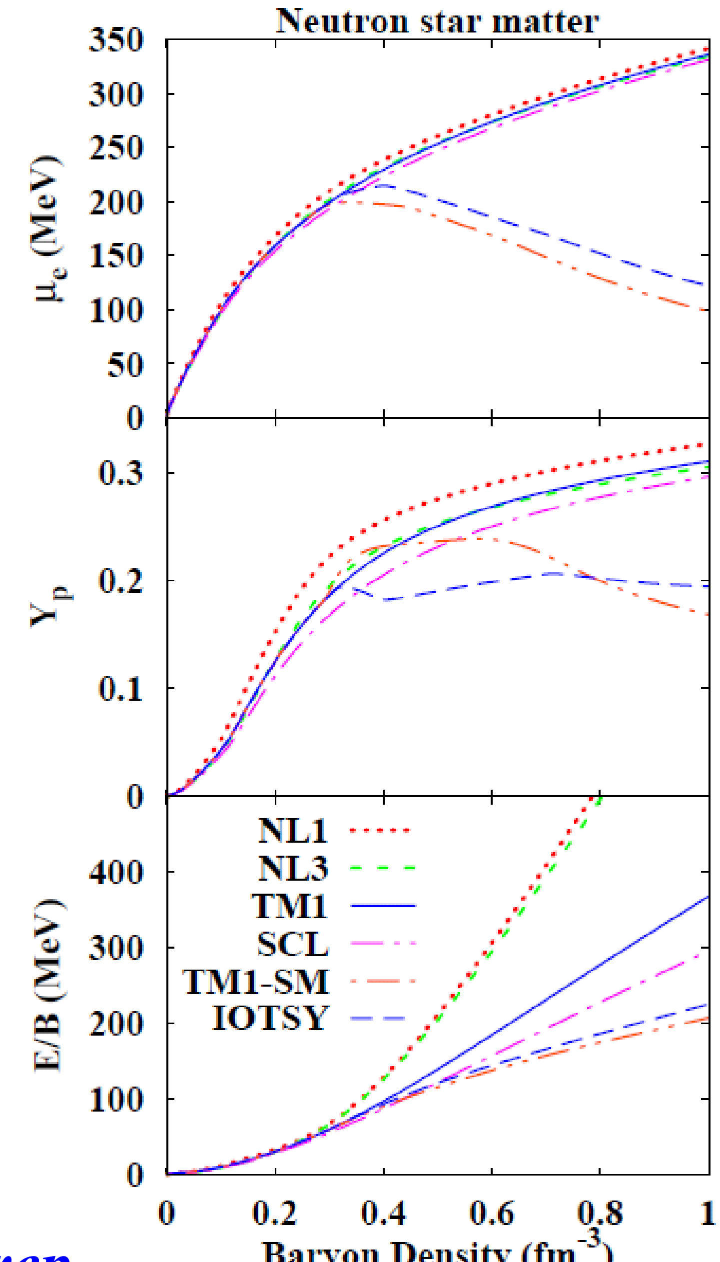
	$g_{\sigma N}$	$g_{\omega N}$	$g_{\rho N}$	g_3(MeV)	g_4	c_ω	m_σ(MeV)	m_ω(MeV)	m_ρ(MeV)
NL1[18]	10.138	13.285	4.976	2401.9	-36.265	0	492.25	795.359	763
NL3[19]	10.217	12.868	4.474	2058.35	-28.885	0	508.194	782.501	763
TM1[6]	10.0289	12.6139	4.6322	1426.466	0.6183	71.3075	511.198	783	770
SCL[20](*1)	10.08	13.02	4.40	1255.88	13.504	200	502.63	783	770

(*1): g_3 and g_4 are from the expansion of f_{SCL}.

AO, Jido, Sekihara, Tsubakihara, in prep.

RMF with Non-Linear Meson Int. Terms

- **Difference in non-linear meson terms generate different predictions of EOS at high densities**



Is there any way
to "Derive" RMF Lagrangian ?
→ Symmetry in QCD

AO, Jido, Sekihara, Tsubakihara, in prep.

Chiral RMF

Nuclear Many-Body Theory preserving Chiral Sym. ?

- **Chiral Symmetry**

$$\psi_R = \frac{1}{2}(1+\gamma_5)\psi \to U_R \psi_R, \quad \psi_L = \frac{1}{2}(1-\gamma_5)\psi \to U_L \psi_L$$

  - **Symmetry in QCD with small quark mass**

    Kinetic term = invariant: $\bar{\psi} i \gamma^\mu D_\mu \psi = \bar{\psi}_R i \gamma^\mu D_\mu \psi_R + \bar{\psi}_L i \gamma^\mu D_\mu \psi_L$

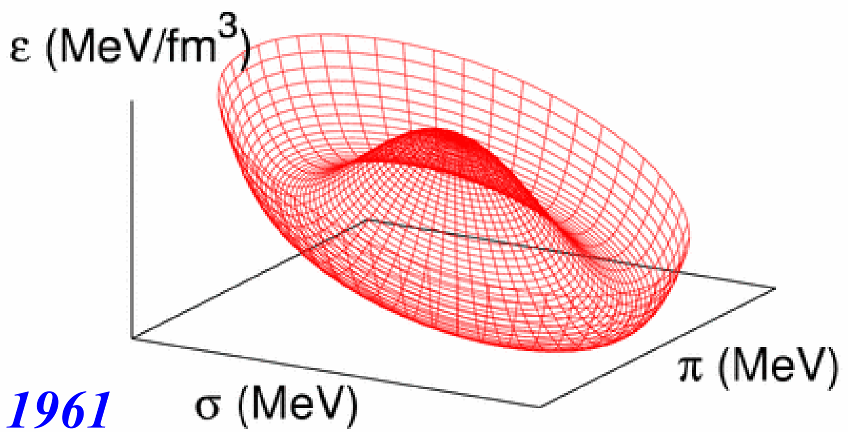
    mass term ≠ invariant: $\bar{\psi}\psi = \bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R$

  - **Should be kept in Nuclear Lagrangian**

    Problem: Nucleon cannot have mass !

    Solution: Spontaneous breaking of Chiral Sym.

$$L_{L\sigma M} = \frac{1}{2}\left(\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \pi \partial^\mu \pi\right)$$
$$-\frac{\lambda}{4}\left(\sigma^2+\pi^2\right)^2 + \frac{\mu^2}{2}\left(\sigma^2+\pi^2\right) + c\sigma$$
$$+\bar{N} i \partial_\mu \gamma^\mu N - g_\sigma \bar{N}\left(\sigma + i\pi\tau\gamma_5\right) N$$

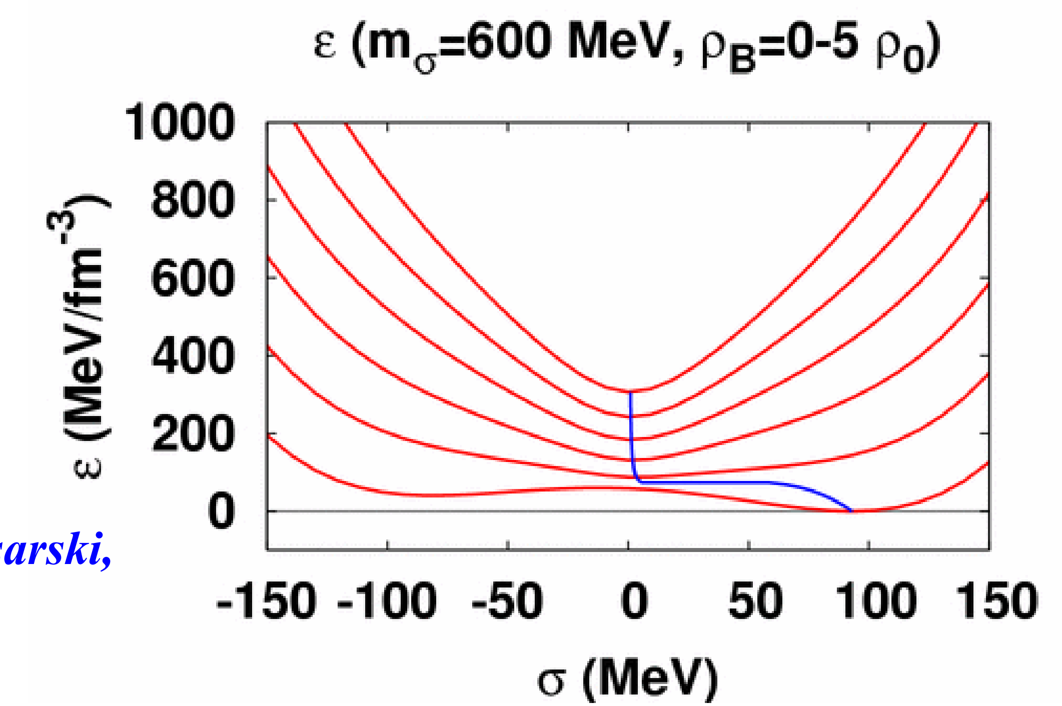


Gell-Mann, Levy, 1960; Nambu, Jona-Lasino, 1961

Chiral Collapse Problem (Lee-Wick Vacuum)

- **At finite ρ_B, Nucleon Fermi Integral favors smaller σ**
  → Chiral Sym. is restored below ρ_0 (Chiral Collapse) ***Lee, Wick, 1974***

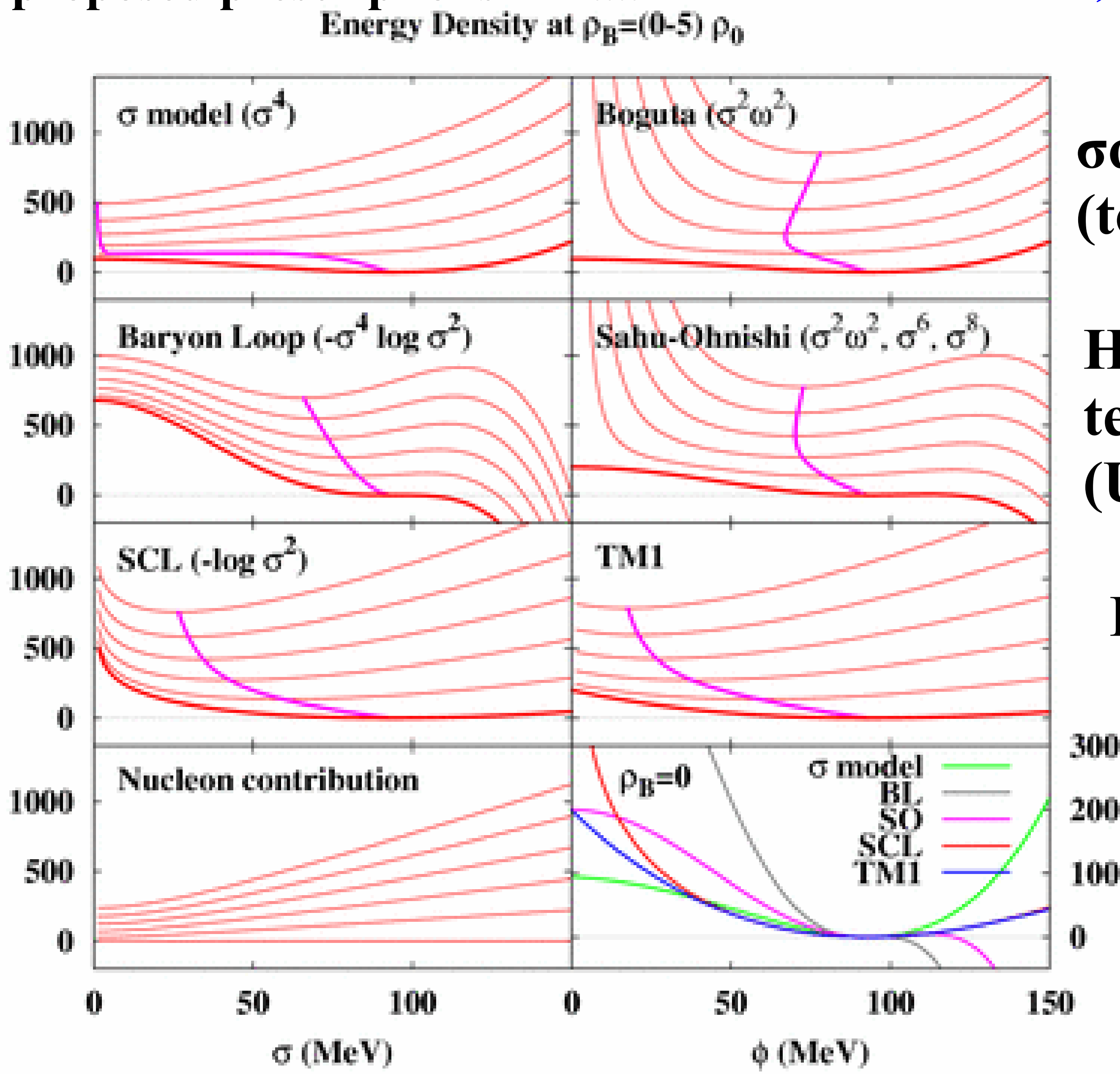
- **Prescriptions**
  - **$\sigma\omega$ coupling (too stiff EOS)** ***(Boguta 1983, Ogawa et al. 2004)***
  - **Loop effects (unstable at large σ)**
    (Matsui-Serot, 1982, Glendenning 1988, Prakash-Ainsworth 1987,Tamenaga et al. 2006)
  - **Higher order terms (unstable at large σ)** ***(Hatsuda-Prakash 1989, Sahu-Ohnishi 2000)***
  - ***Dielectric (Glueball) Field representing scale anomaly***
    (Furnstahl-Serot 1993, Heide-Rudaz-Ellis 1994, Papazoglou et al.(SU(3)) 1998)
  - **Different Chiral partner assignment**
    (DeTar-Kunihiro 1989, Hatsuda-Prakash 1989, Harada-Yamawaki 2001, Zschiesche-Tolos-Schaffner-Bielich-Pisarski, nucl-th/0608044) → $SU_f(3)$ extention ?



but .....

Chiral Collapse Problem (Lee-Wick Vacuum)

- **Many of the proposed prescriptions fail ....** *Tsubakihara, Ohnishi, 2007*



Naïve φ^4 (Unstable) *Lee-Wick*

Fermion Loop (Unstable)

SCL-LQCD

σω coupling (too stiff EOS)

Higher order terms (Unstable)

Phenom.

Chiral RMF based on SCL-LQCD

- **Relativistic Mean Field model** *Tsubakihara, Ohnishi, 2007*
  - **Effective Lagrangian consisting Baryons and Mesons**
  - **Attractive Scalar Field (σ) + Repul. Vector Field (ω) → Matter Saturation**

$$\mathcal{L}_\chi = \overline{\psi}_N \left[i\partial\!\!\!/ - g_\sigma(\sigma + i\gamma_5 \tau \cdot \pi) - g_\omega \omega\!\!\!/ - g_\rho \tau \cdot \rho\!\!\!/ \right] \psi_N + \frac{1}{2}\left(\partial^\mu \sigma \partial_\mu \sigma + \partial^\mu \pi \cdot \partial_\mu \pi\right) - V_\sigma(\sigma, \pi) - \frac{1}{4} W^{\mu\nu} W_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega^\mu \omega_\mu + \frac{c_\omega}{4}(\omega^\mu \omega_\mu)^2 - \frac{1}{4} R^{\mu\nu} \cdot R_{\mu\nu} + \frac{1}{2} m_\rho^2 \rho^\mu \cdot \rho_\mu$$

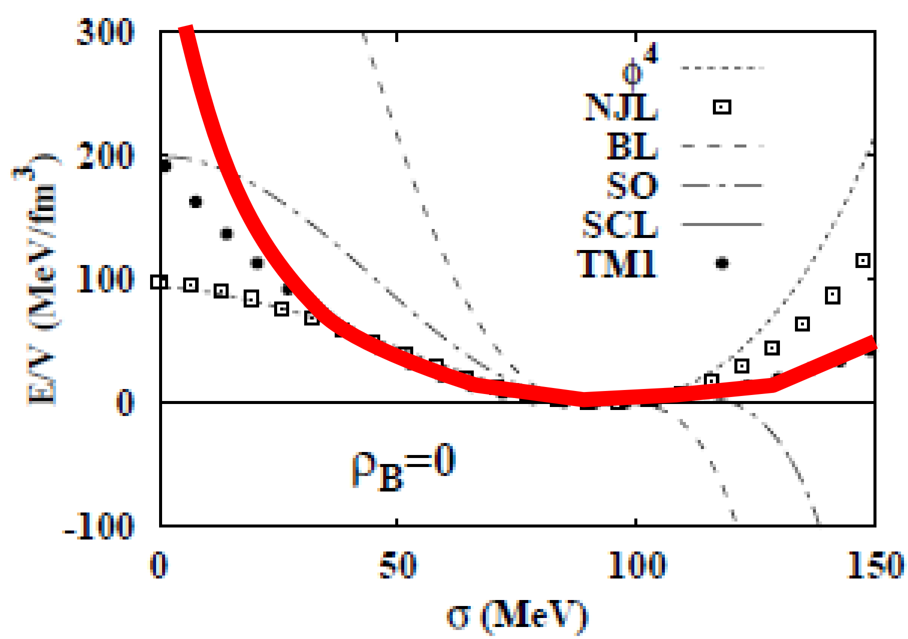
  - **Linear σ Model**

$$V_\sigma = -\frac{1}{2}\mu\sigma^2 + \frac{1}{4}\lambda\sigma^4$$

  - **SCL-LQCD (Zero T)**

$$V_\sigma = \frac{1}{2} a_\sigma \sigma^2 - b_\sigma \log \sigma$$

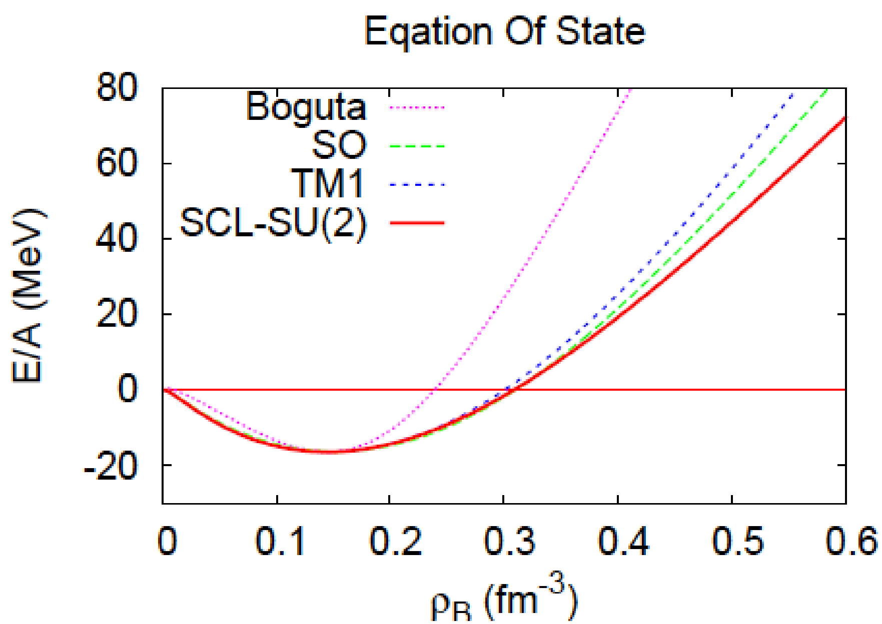
→ **Agree with Phen. RMF results at large chiral condensates.**

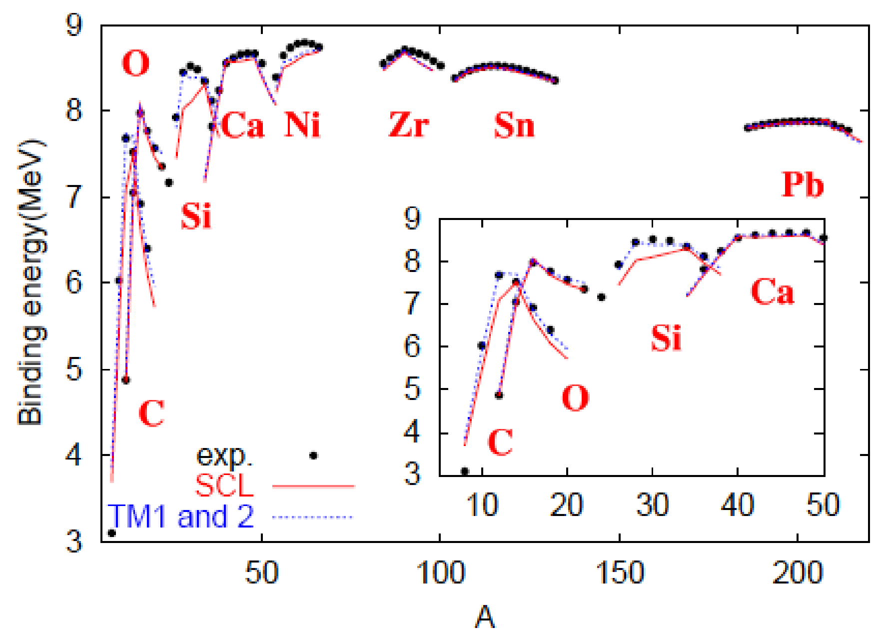


Chiral RMF based on SCL-LQCD

Tsubakihara, AO, PTP 117('07)903 [nucl-th/0607046]

- **Nuclear Matter EOS**
  - **Gives "Medium" EOS (K ~ 280 MeV), Comparable to Phen. RMF**
- **Bulk properties of nuclei**
  - **B.E./Nucleon, Charge radii → Comarable to High Quality Phen. RMF**





This would be the first step of giving Nuclear Density Functional from QCD

Binding Energies in Chiral and Non-Chiral RMF

- **Non-Chiral High Precision RMF: TM1 & 2, NL1, NL3**
  (Sugahara, Toki, 1994; Reinhard et al., 1986; Lalazissis, Koenig, Ring, 1997)
- **Log term from Scale Anomaly: I/110, IF/110, VIIIF/100**
  Chiral Symmetric, No Instability, with Glueball $V_\sigma = -\chi^4 \log \sigma^2$
  (Heide, Rudas, Ellis, 1994)
- **Quark Meson Coupling model**

	B/A (MeV)									
Nucleus	exp.	SCL	TM1	TM2	NL1	NL3	I/110	IF/110	VIIIF/100	QMC-I
^{12}C	7.68	7.09	-	7.68	-	-	-	-	-	-
^{16}O	7.98	8.06	-	7.92	7.95	8.05	7.35	7.86	7.18	5.84
^{28}Si	8.45	8.02	-	8.47	8.25	-	-	-	-	-
^{40}Ca	8.55	8.57	8.62	8.48	8.56	8.55	7.96	8.35	7.91	7.36
^{48}Ca	8.67	8.62	8.65	8.70	8.60	8.65	-	-	-	7.26
^{58}Ni	8.73	8.54	8.64	-	8.70	8.68	-	-	-	-
^{90}Zr	8.71	8.69	8.71	-	8.71	8.70	-	-	-	7.79
^{116}Sn	8.52	8.51	8.53	-	8.52	8.51	-	-	-	-
^{196}Pb	7.87	7.87	7.87	-	7.89	-	-	-	-	-
^{208}Pb	7.87	7.87	7.87	-	7.89	7.88	7.33	7.54	7.44	7.25

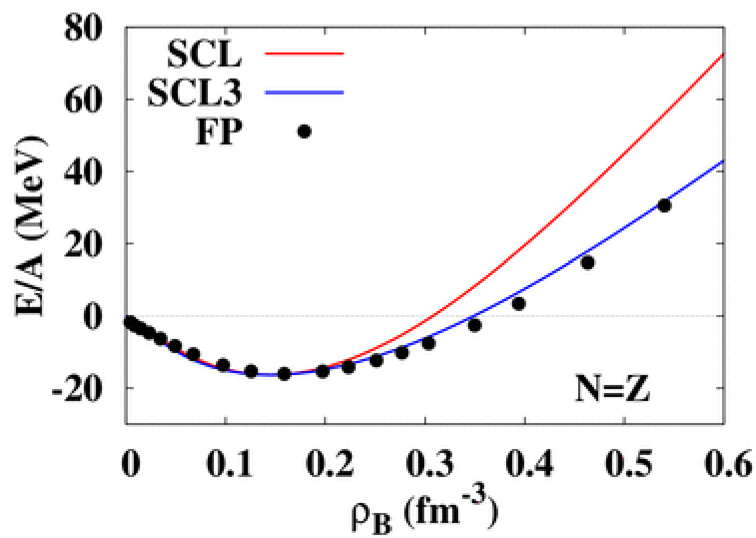
Chiral $SU_f(3)$ RMF

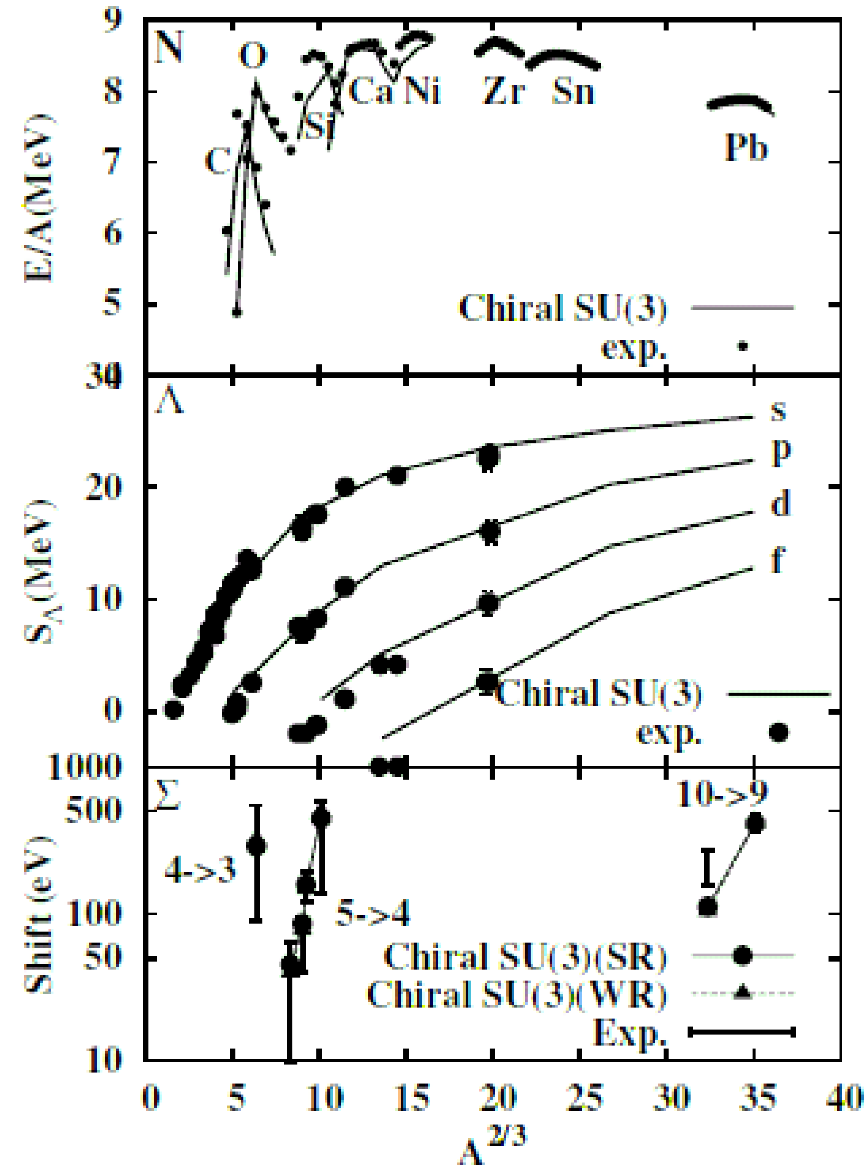
Tsubakihara, Maekawa, AO, EPJA33('07)295 [nucl-th/0702008]

- **Extention to Flavor SU(3)**
  → Chiral Potential from SCL-LQCD
  + Determinant Int. ($U_A(1)$ anomaly)
  + Explicit breaking term

$$U_{\sigma\zeta} = -\,a\,\log(\det MM^\dagger) + b\,\mathrm{tr}(MM^\dagger) + c_\sigma\sigma + c_\zeta\zeta + d\,(\det M + \det M^\dagger).$$

- **Normal, Single & Double Λ, Σ atom, EOS (~ FP), ...**





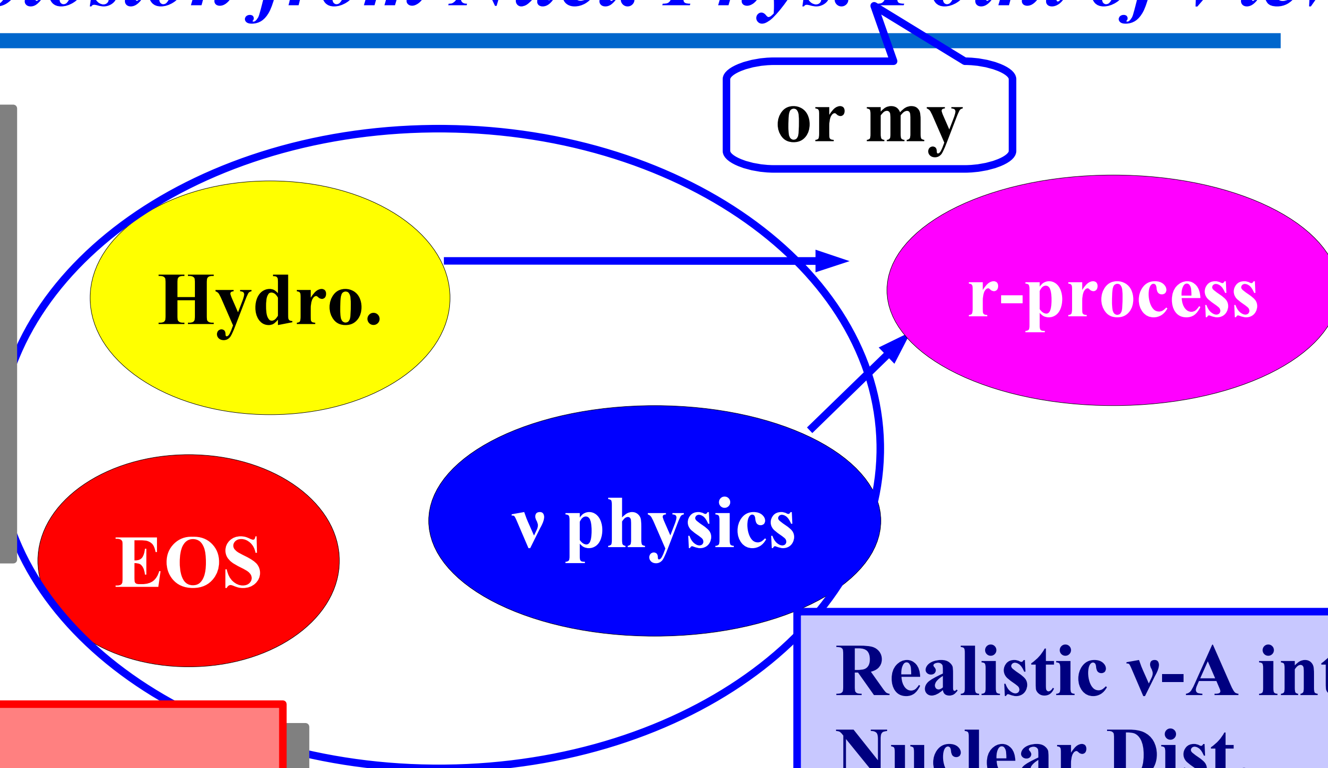
Summary of Lecture 1

- **Nuclear Matter EOS is important in various aspects of Nuclear Physics**
- **Relativistic Mean Field may be a good starting point to describe hadronic (baryon and meson) systems.**
  - **Relativistic → Saturation, Causality**
  - **Based on successes of Dirac Phenomenology and DBHF**
  - **Covariant Density Functional**
    → It is desirable to obtain E/V (energy density) in fundamental theories. (Renormalizability is not required.)
  - **We can re-write RMF equations in Schroedinger-like eqs. We may consider it as a method to parameterize DF in a transparent manner.**
  - **Higher order terms / Density dependence of the coupling constants (not mentioned) → Necessary for precise description of nuclei, but need foundations of extension.**

Relativistic EOS of Supernova Matter with Hyperons

Supernova Explosion from Nucl. Phys. Point of View

Multi Dim.
Instability
Magnetic Field
Acoustic Revival
....



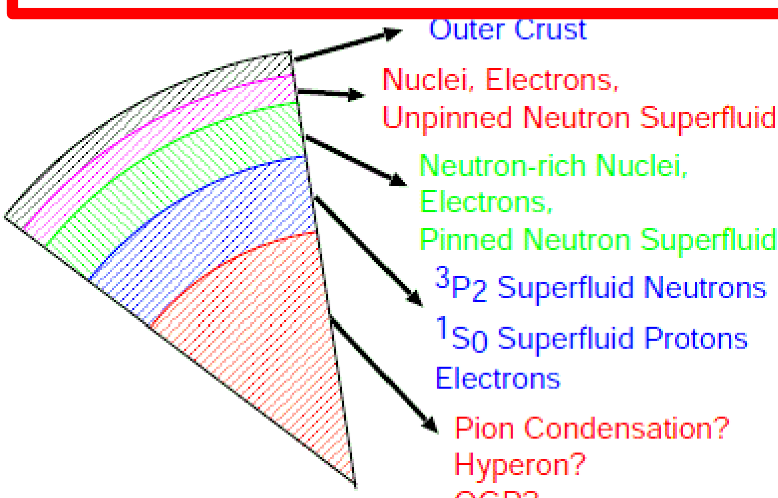
Realistic ν-A int.
Nuclear Dist.
Exact ν transfer
.....

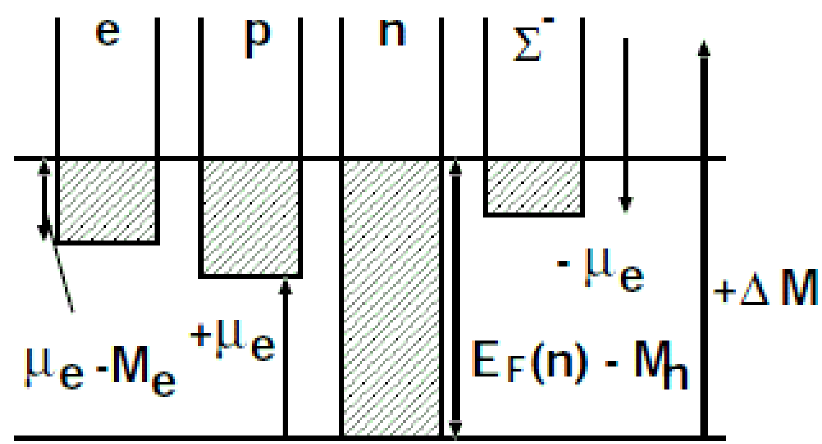
EOS tables
Lattimer-Swesty (1981)
Rel. (Shen) EOS (1998)
→ How to extend ?

- **Supernovae DO NOT EXPLODE in theor. calculation at present with realistic microphysics inputs. → How can we succeed ?**
  - **Multi-Dim. Hydro (Instability)+*Additional Energy Release* (10 %-factor 10)**

Hyperons in Dense Matter

- **What appears at high density ?**
  - **Nucleon superfluid ($^3S_1$, $^3P_2$)**
  - **Pion condensation, Kaon condensation, Baryon Rich QGP, Color SuperConductor (CSC), Quarkyonic Matter, ...**
  - **Hyperons**
    Tsuruta, Cameron (66); Langer, Rosen (70); Pandharipande (71); Itoh(75); Glendenning; Weber, Weigel; Sugahara, Toki; Schaffner, Mishustin; Balberg, Gal; Baldo et al.; Vidana et al.; Nishizaki,Yamamoto, Takatsuka; Kohno,Fujiwara et al.; Sahu,Ohnishi; Ishizuka, Ohnishi, Sumiyoshi, Yamada; ...





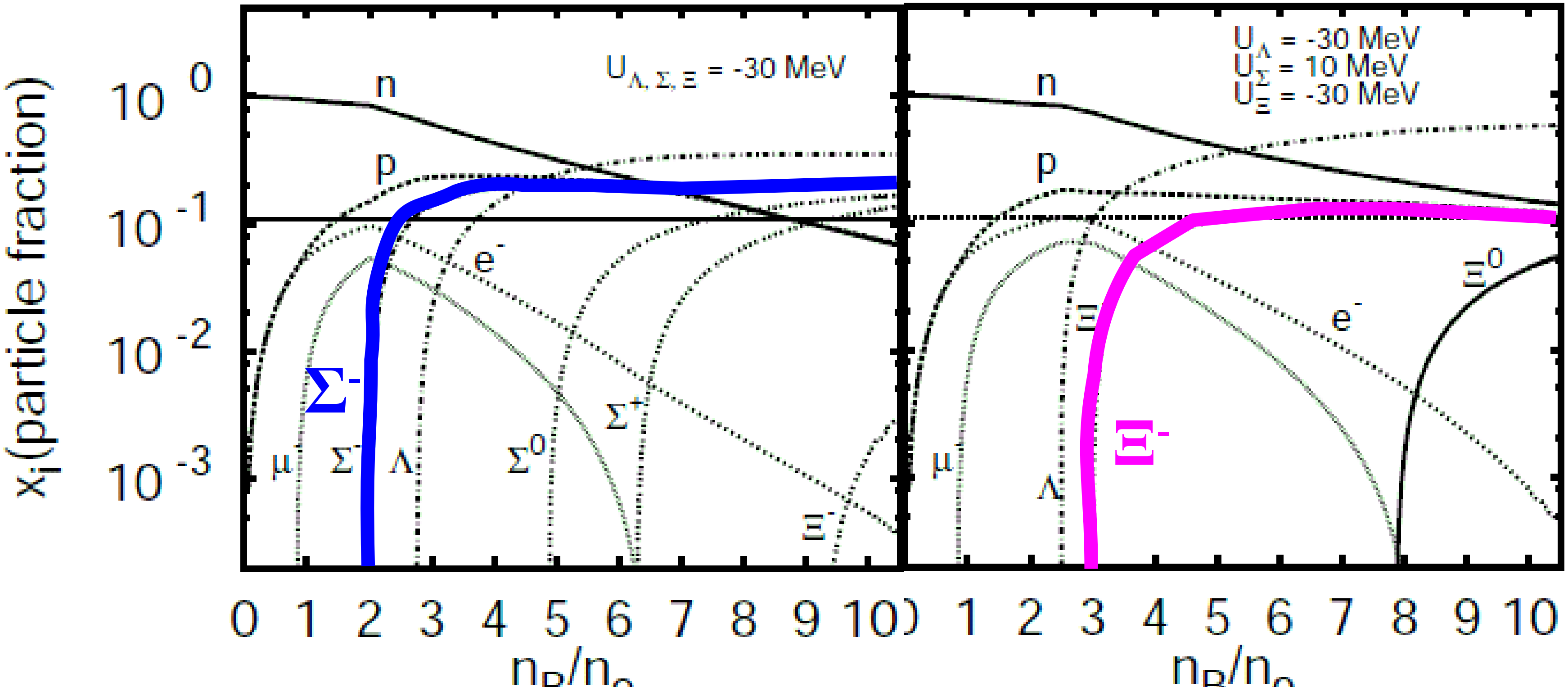
Nobody says "Hyperons do not appear in neutron star core" !

Y appears when $\mu_B = E_F(n) + U(n) \geq M(Y) + U(Y) + Q_Y \mu_e$

Hyperons in Supernova Matter

- **Problems to include hyperons in Supernova Matter EOS**
  - **Uncertainties of hyperon potentials $U_Y(\rho)$** *→ Recent Hypernuclear Phys.* **(e.g. Balberg, Gal, 1997)**
  - **Density may not be very high in supernova** *→ Needed in cooling stage*

Attractive U_Σ **Repulsive U_Σ**



Sahu, AO, 2003

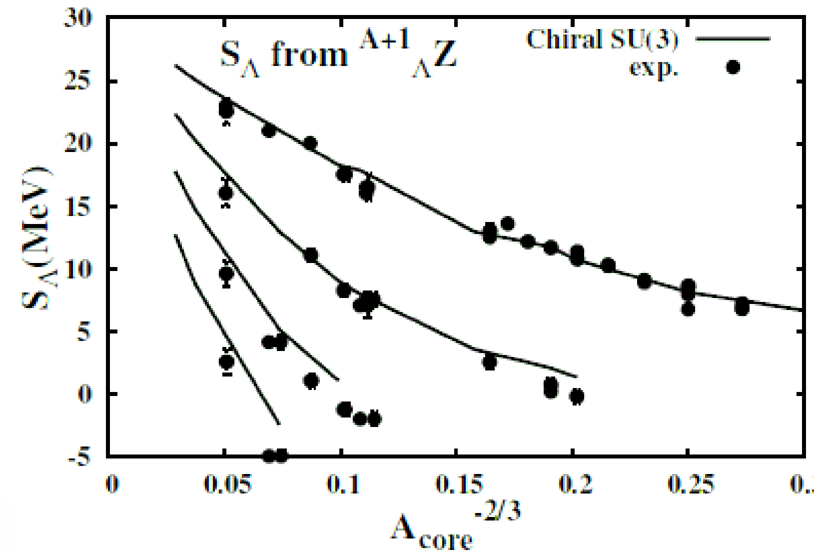
We include recent hyperon info. in supernova matter EOS

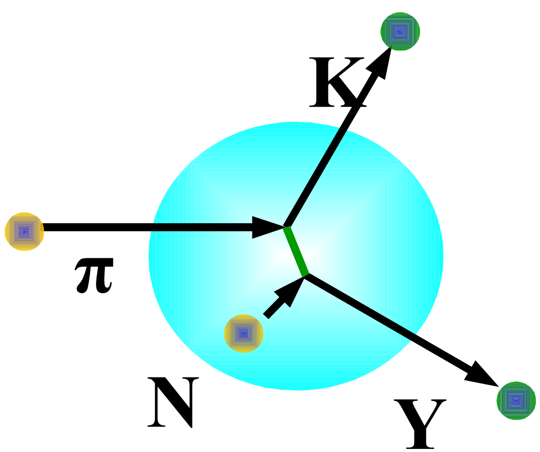
Σ Potential in Nuclear Matter

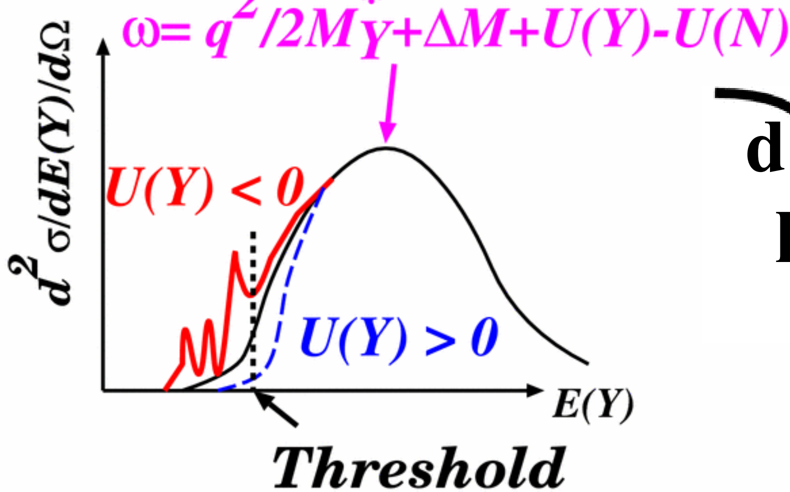
- $U_\Lambda(\rho_0) \sim -30$ MeV: Well known from single particle energies
- Naïve expectation
  = Quark Number (ud number) Scaling
  $U_\Lambda \sim 2/3\ U_N \rightarrow U_\Sigma \sim 2/3\ U_N \sim -30$ MeV
- Problems with Σ
  - Only one bound state $^4_\Sigma$He (Too light !)

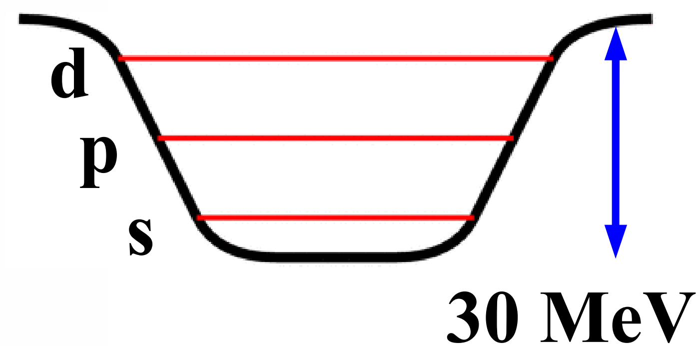
  → Continuum (Quasi-Free) Spectroscopy is necessary

Tsubakihara, Maekawa, AO, EPJA33('07),295.









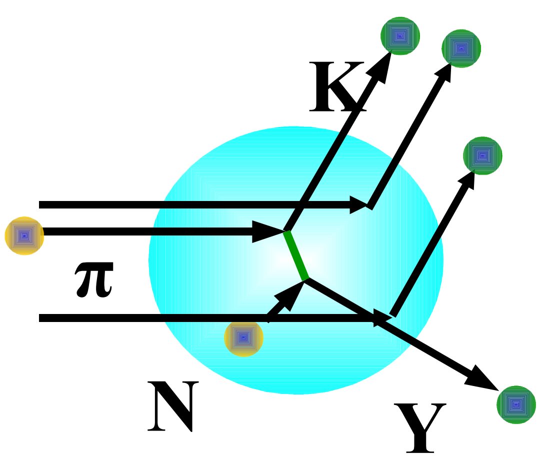
Σ Potential in Nuclear Matter

- **Cont. Spec. Theory = Distorted Wave Impulse Approx. (DWIA)**

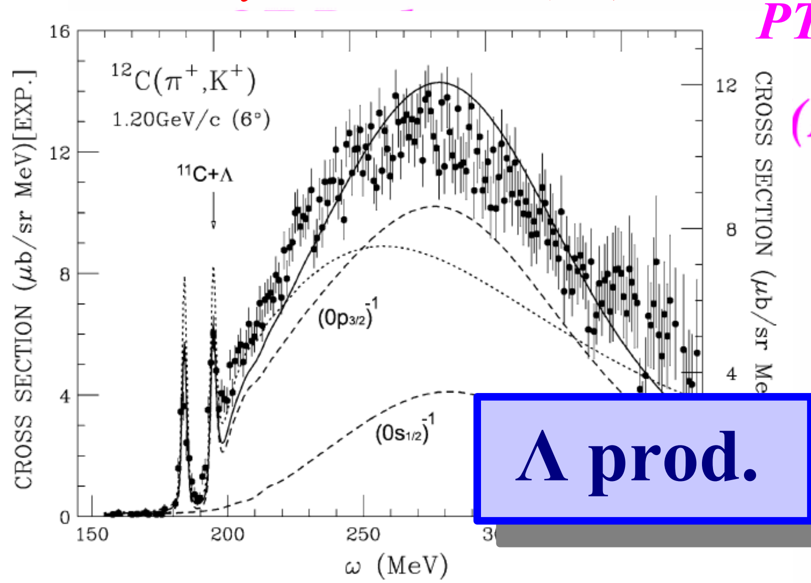
$$\frac{d^2\sigma}{dE_K\, d\Omega_K} = \boxed{\beta} \boxed{\left(\frac{d\sigma}{d\Omega}\right)^{Elem.}_{N\pi\to KY}} \boxed{S(E,\boldsymbol{q})}$$

Kinematical Factor (β) — **Elem. Cross Sec.** — **Strength Func.** ($S(E,\boldsymbol{q})$)

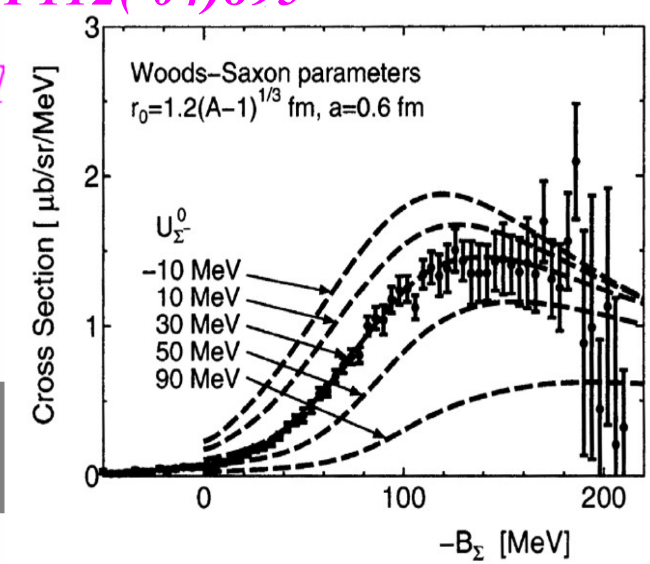
- **Large (ω, q) range → Important to respect On-Shell Kinematics**
- **Kinematics depends on Reaction Point with Hyperon Potential**



Harada, Hirabayashi, NPA744('04),323.



Kohno, Fujiwara, Kawai, et al. PTP112('04)895



Σ Potential in Nuclear Matter

Maekawa, Tsubakihara, AO, EPJA 33(2007),269.
Maekawa, Tsubakihara, Matsumiya, AO, in preparation.

- **DWIA with Local Optimal Fermi Averaging t-matrix (DWIA-LOFAt)**
  - **Green's Func. Method + Reaction Point Deps. of t-matrix**

$$\frac{d^2\sigma}{dE_K d\Omega_K} = \frac{p_K E_K}{(2\pi)^2 v_{\rm inc}} R_Y(E_Y)$$

$$R_Y(E_Y) = -\frac{1}{\pi} \mathrm{Im} \left\langle \bar{t}(\boldsymbol{r})^{+} \frac{1}{E_Y - H_Y + i\varepsilon} \bar{t}(\boldsymbol{r}') \right\rangle$$

Response Func. ($R_Y(E_Y)$) **Local t-mat.** ($\bar{t}(\boldsymbol{r})^{+}$) **Green's Func.** ($\frac{1}{E_Y - H_Y + i\varepsilon}$)

$$\bar{t}(\boldsymbol{r},\omega,\boldsymbol{q}) = \frac{\int d\boldsymbol{p}_N \; t(s,t)\rho(p_N)\delta^{(4)}(p_1(\boldsymbol{r})+p_2(\boldsymbol{r})-p_3(\boldsymbol{r})-p(\boldsymbol{r}))}{\int d\boldsymbol{p}_N \; \rho(p_N)\delta^{(4)}(p_1(\boldsymbol{r})+p_2(\boldsymbol{r})-p_3(\boldsymbol{r})-p(\boldsymbol{r}))}$$

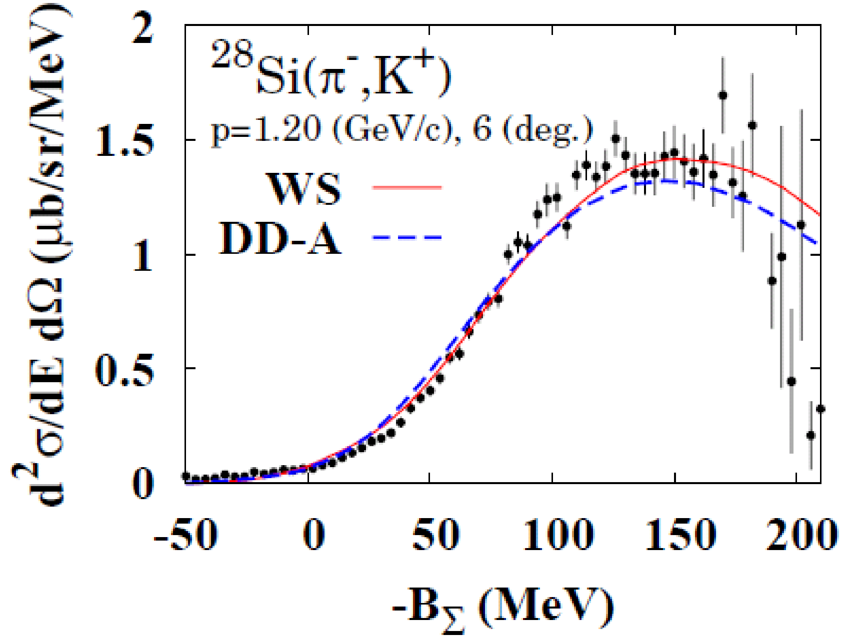
$$E_i = \sqrt{p_i^2 + m_i^*(r)^2} \simeq m_i + \frac{p_i^2}{2m_i} + V_i, \quad m_i(r)^2 = m_i^2 + 2m_i V_i(r)$$

  - **After careful treatment of**
    K+ potential, Elementary cross section, Angular distribution, ...
    we analyze the recently measured Σ^- production spectrum
    (Saha, Noumi et al. (KEK-E438), PRC70('04)044613)

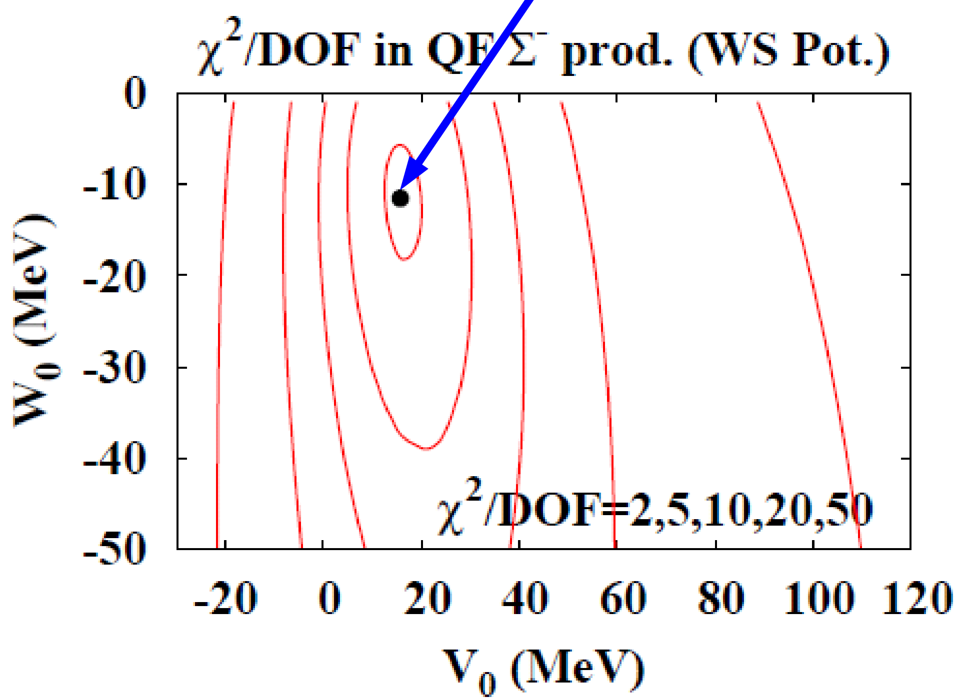
Σ Potential in Nuclear Matter

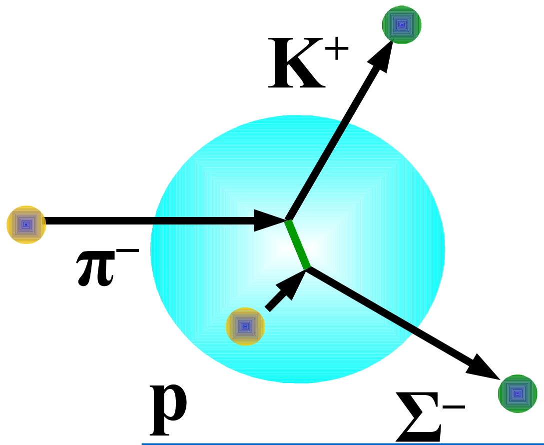
Maekawa, Tsubakihara, AO, EPJA 33(2007),269.
Maekawa, Tsubakihara, Matsumiya, AO, in preparation.

$$\frac{d^2\sigma}{d\,E_K\,d\,\Omega_K}$$



$U_\Sigma(\rho_0) \sim +15$ MeV $-i$ 10 MeV with Woods-Saxon potential, no Atomic shift fit





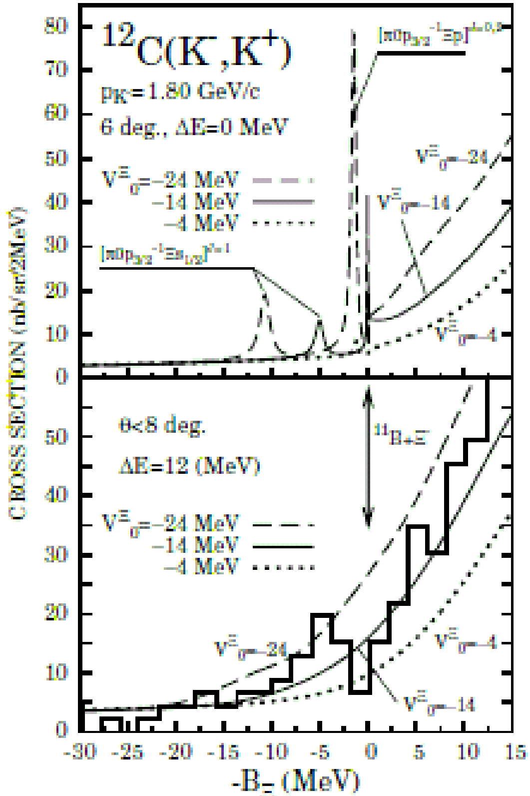
Ξ Potential in Nuclear Matter

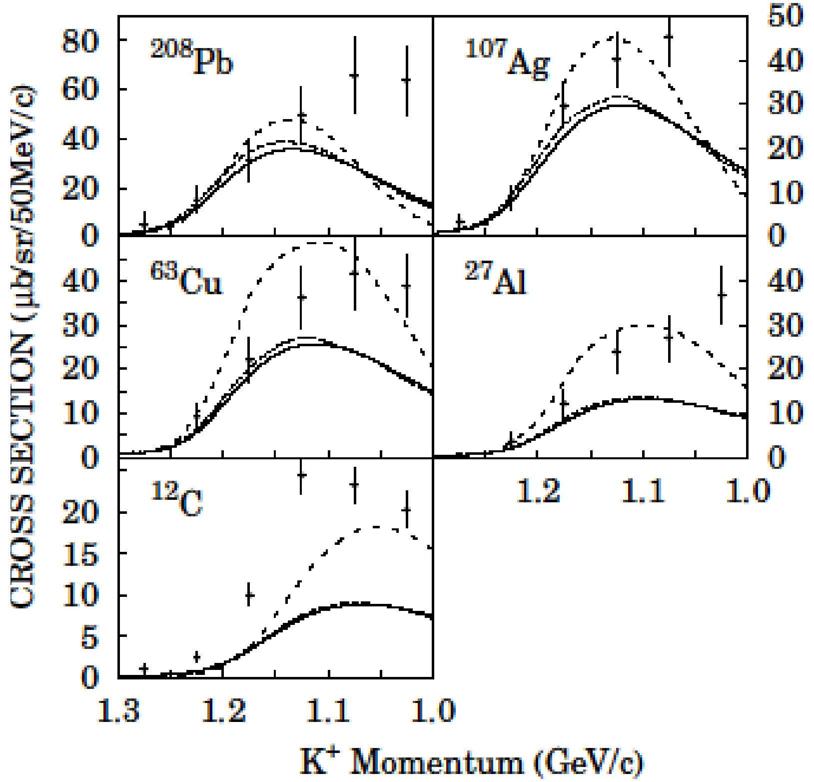
- **Currently accepted value: $U_\Xi \sim -14$ MeV**

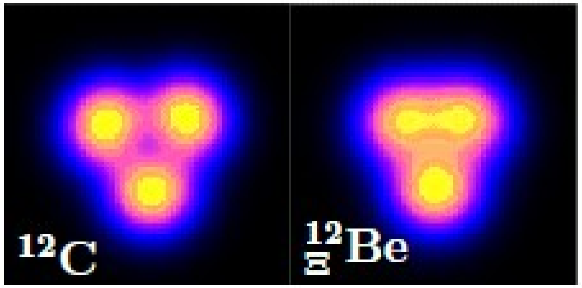
  Twin hypernuclear form., Spectrum shape in the bound state region

  (Aoki et al. PLB355('95),45; Fukuda et al. PRC58('98),1306; Khaustov et al. PRC61('00), 054603)

- **Absolute values of $^{12}C(K^-,K^+)$ spectra → Still Difficult to Understand**
- **Large q → Spectrum may depend on detailed nuclear structure**







Matsumiya, et al. (Coupled Channel AMD)

Let's wait for J-PARC results

Maekawa, Tsubakihara, Matsumiya, AO, arXiv:0704.3929.

"Stars" of Hyperon Potentials (A la Michelin)

- **$U_\Lambda(\rho_0) \sim -30$ MeV**
  - ***Bound State Spectroscopy** + **Continuum Spectroscopy***
- **$U_\Sigma(\rho_0) > +15$ MeV**
  - **Continuum (Quasi-Free) spectroscopy with *Local Optimal Fermi Averaging t-matrix* (LOFAt)**
  - **Atomic shift data (attractive at surface) should be respected.**
- **$U_\Xi(\rho_0) \sim -14$ MeV**
  - **No confirmed bound state, No atomic data, High mom. transf., ... → Small Potential Deps.**
  - **Continuum low-res. spectrum shape → – 14 MeV**
  - **Spin-Isospin deps. (π exch.) → Deformation → Spectrum shape may be modified.**

Relativistic EOS of Supernova Matter with Hyperons

- **Extention of the Relativistic (Shen) EOS to SU$_f$(3) with updated Hyperon Potentials in Nuclear Matter**
  (Ishizuka, Ohnishi, Tsubakihara, Sumiyoshi, Yamada, J. Phys. G 35 (2008), 085201)
  - **Relativistic (Shen) EOS** *(Shen, Toki, Oyamatsu, Sumiyoshi, PTP 100('98), 1013)*
    Rel. Mean Field (RMF) + Local Density Approx.(Nuclear Formation)
  - **SU$_f$(3) Extention of RMF** *(Schaffner, Mishustin, PRC53 (1996), 1416)*
    Coupling ~ Quark Number Counting

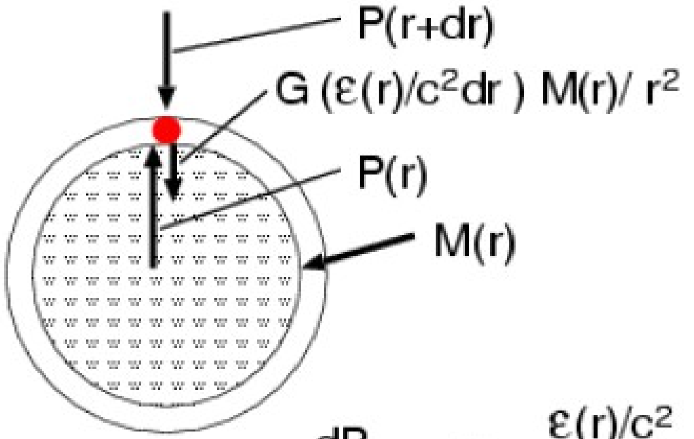
	g_{MB}	σ	ζ	ω	ρ	ϕ
	N	10.0289	0	12.6139	4.6783	0
	Λ	6.21	6.67	8.41	0	-5.95
	Σ	4.36 (6.21)	6.67	8.41	$2g_{\rho N}$	-5.95
	Ξ	3.11 (3.49)	12.35	4.20	4.63	-11.89

SM (values in parentheses)
IOTSY (values without parentheses)

- **$g_{\sigma Y}$ is tuned to fit Hyperon Potential in Nuclear Matter**
  $U_\Lambda = -30$ MeV, $U_\Sigma = +30$ MeV, $U_\Xi = -15$ MeV
- **Nuclear Formation is included using Shen EOS table**

Tolman-Oppenheimer-Volkoff (TOV) equation

- **TOV Eq. = General Relativistic Balance of pressure and gravity**



Non-Rela. $\frac{dP}{dr} = -G \frac{\varepsilon(r)/c^2 \; M(r)}{r^2}$

$$\frac{dP}{dr} = -G\frac{(\varepsilon/c^2 + P/c^2)(M + 4\pi r^3 P/c^2)}{r^2(1-2GM/rc^2)}$$

$$\frac{dM}{dr} = 4\pi r^2 \varepsilon/c^2 \;, \qquad \frac{dP}{dr} = \frac{dP}{d\varepsilon}\frac{d\varepsilon}{dr}$$

$$P = P(\varepsilon) \;, \qquad \frac{dP}{d\varepsilon} = \frac{dP}{d\varepsilon}(\varepsilon) \qquad \text{(EOS)}$$

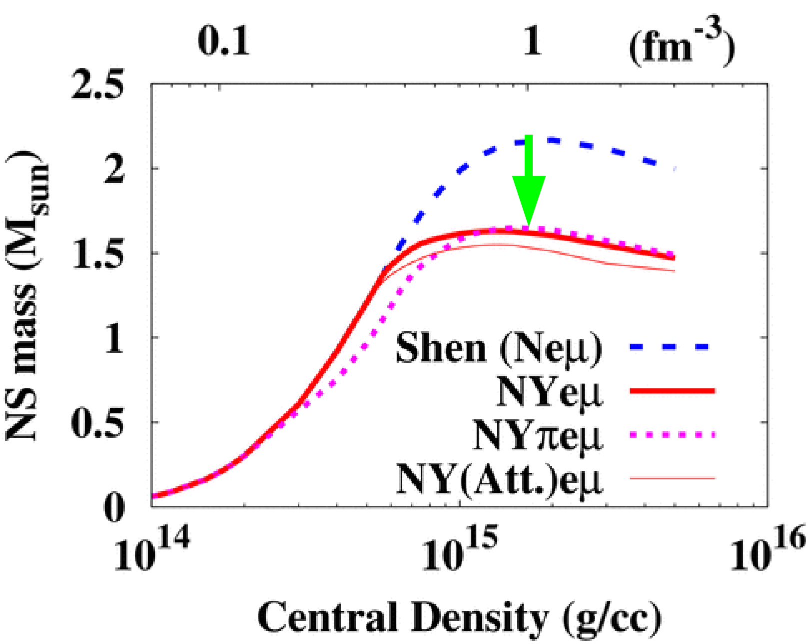
Neutron Star Mass = M(R) where P(R)=0

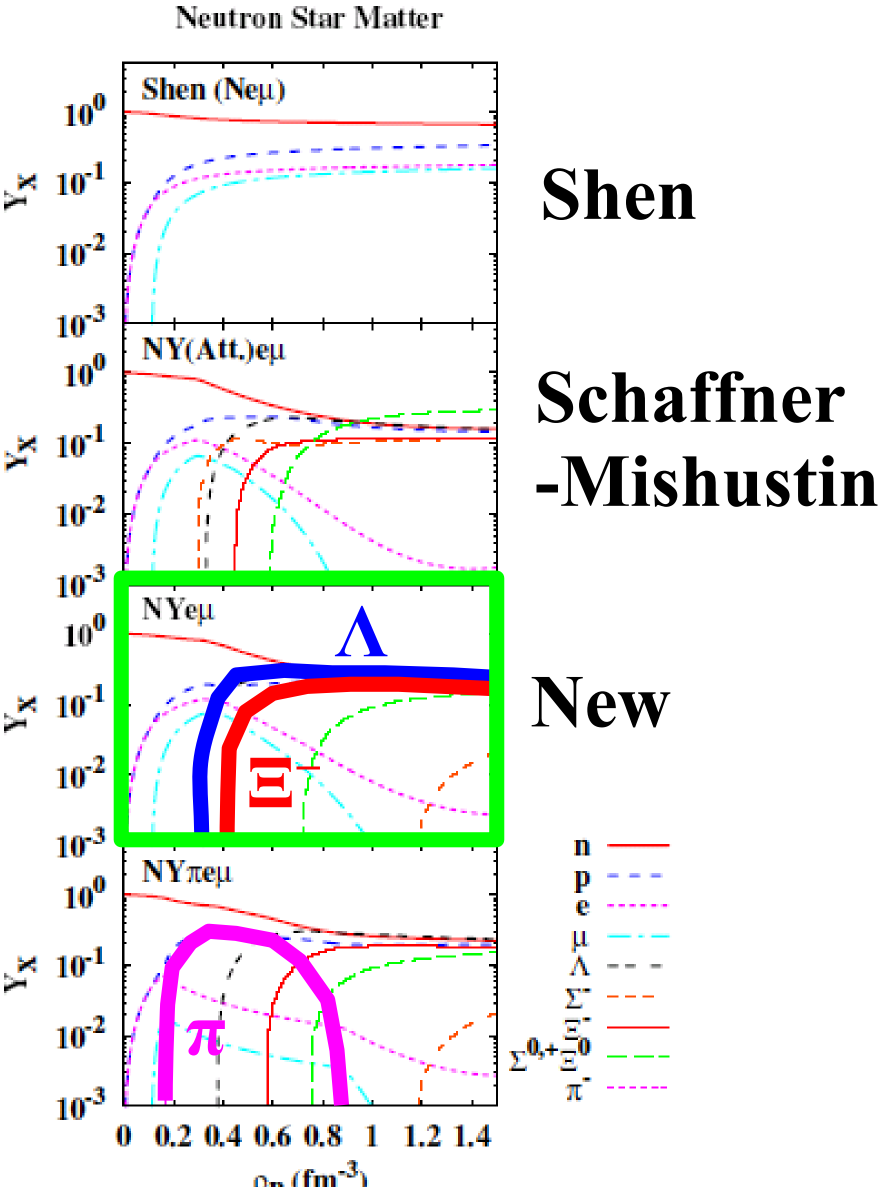
When you make a new EOS, please check the NS mass !

Neutron Star

Ishizuka, AO, Tsubakihara, Sumiyoshi, Yamada, J. Phys. G 35 (2008), 085201

- **Hyperon Effect is DRASTIC**
  - **Mmax=2.1 Msun → 1.56 Msun**
  - **Composition $Y_\Lambda \sim Y_n$**
  - **Large fraction of Ξ**
- **Thermal (free) pions can admix at $\rho > 1.5\ \rho_0$**

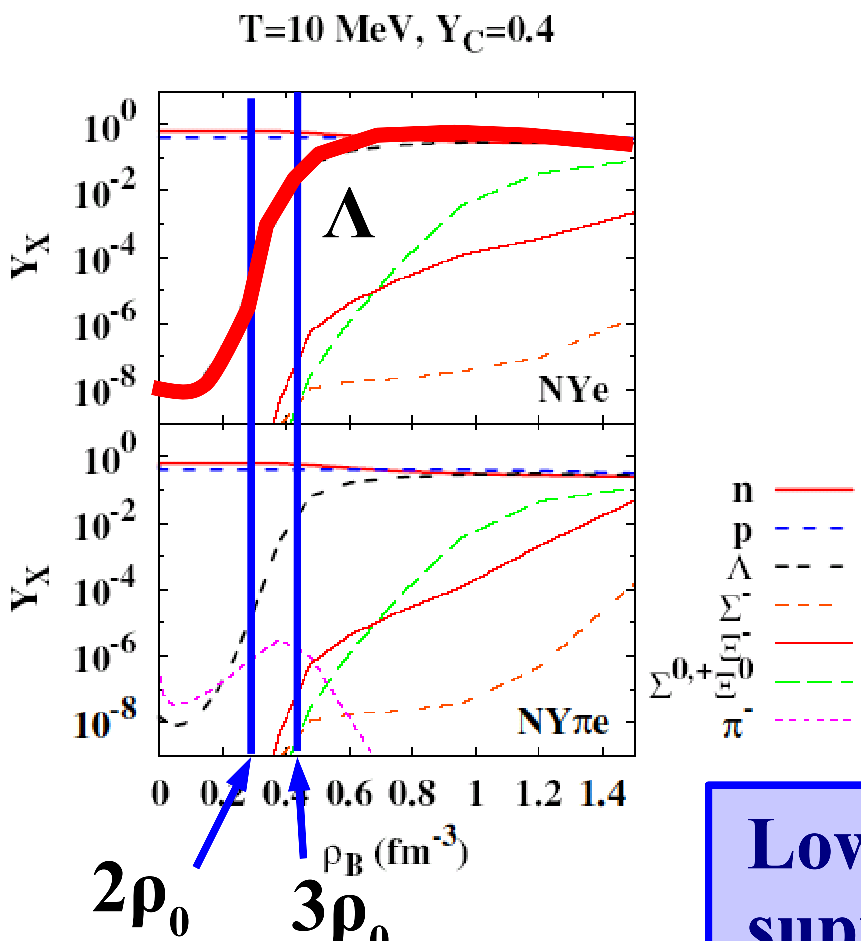




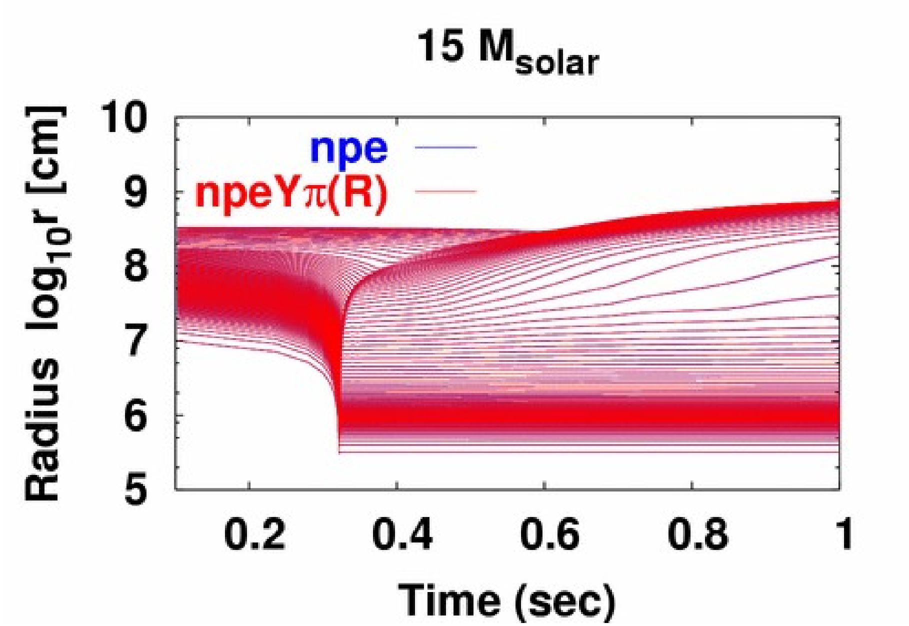
Finite Temperature and Supernova

Ishizuka, AO, Tsubakihara, Sumiyoshi, Yamada, J. Phys. G 35 (2008), 085201

- **Example: T=10 MeV, Ye = 0.4**
  - **Λ starts to increase at $\rho \sim 2\rho_0$, becomes significant at $\rho \sim 3\rho_0$.**



- **Prompt explosion (without ν transport)**
  → Almost no change
  (Expl. E. increase ~ (0.1-0.5 %))



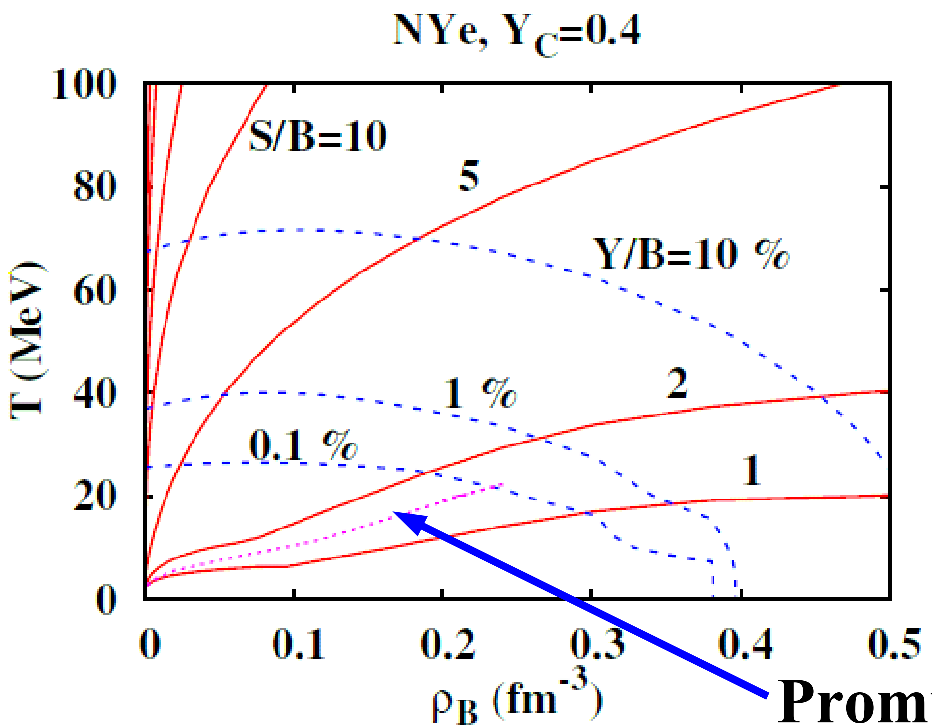
WW95 + 1 Dim. Hydro.(Sumiyoshi, Yamada)

Low density and High Ye suppresses Hyperons in the Early Stage

Where Do We See Hyperons ?

- **Hyperon Fraction is sensitive to Ye, T, and ρ_B.**
  - **Yν ~ 0 (Neutron Star) → $\rho_B > 2\ \rho_0$**
  - **Ye ~ 0.4 (Supernova, early stage) → T > 40 MeV or $\rho_B > 3\ \rho_0$**

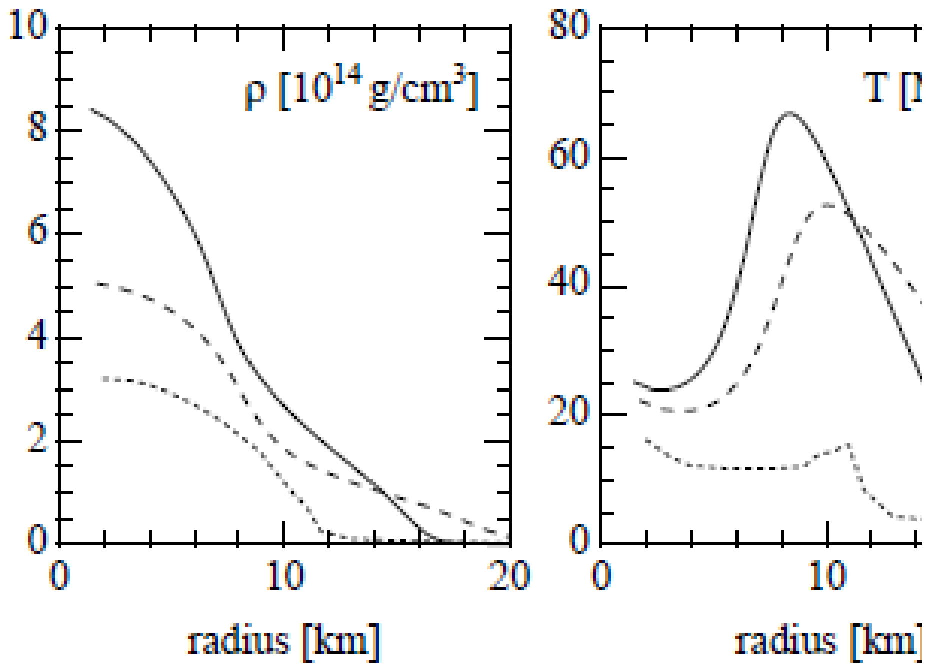
Hyperons would be important in Late Stages
Proto neutron star cooling, Black Hole Formation

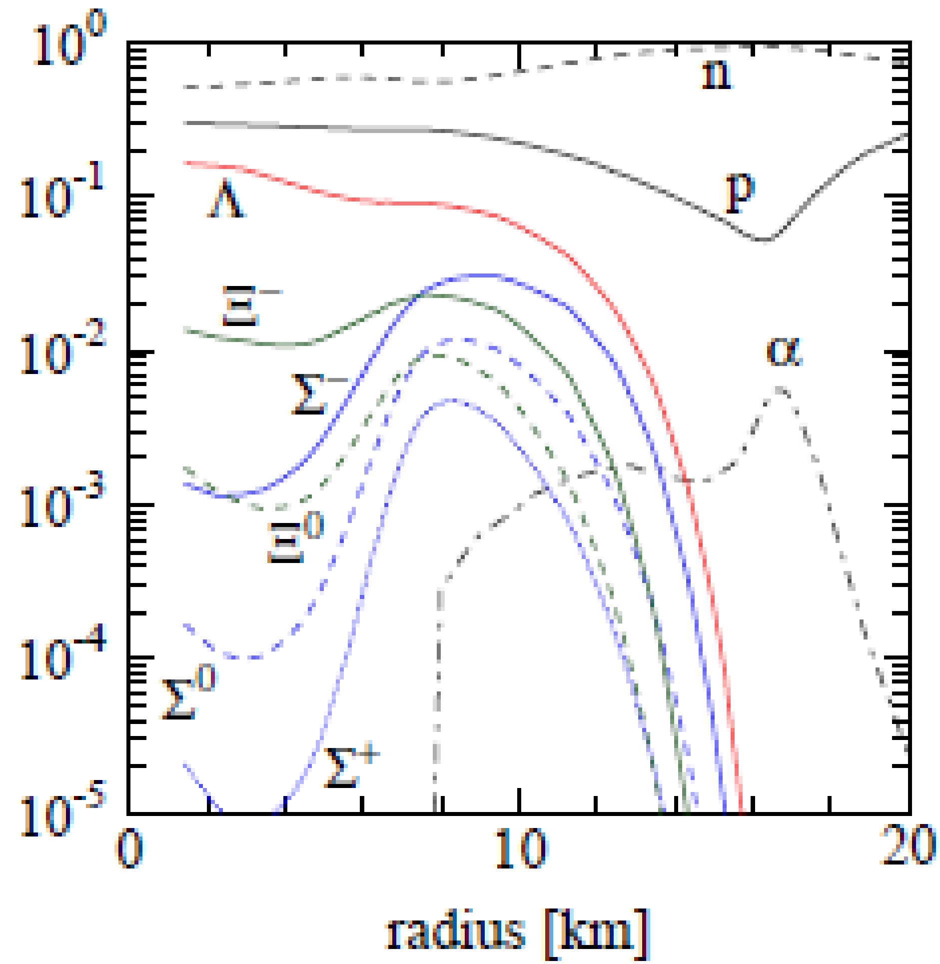


Prompt Expl. (15 Msun)

Hyperons during Black Hole Formation

- **Hyperons appears abundantly during Black Hole Formation Processes**
  - **Off-Center: Large T → Σ > Ξ**
  - **Center: Large ρ_B → Σ < Ξ**





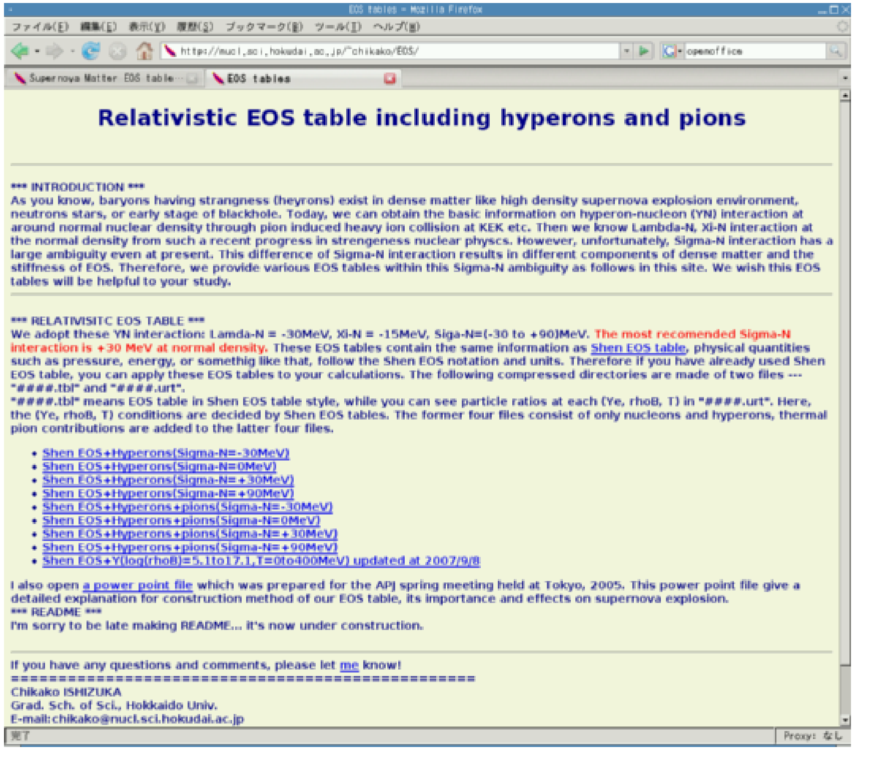
Sumiyoshi, Ishizuka, AO, Yamada, Suzuki, ApJ Lett., submitted

Summary (1) of Lecture 2

- **Hyperons are included in the Relativisitic (Shen) EOS with recently accepted Hyperon Potentials in Nuclear Matter,**
  $U_\Lambda = -30$ MeV, $U_\Sigma = +30$ MeV , $U_\Xi = -15$ MeV
  http://nucl.sci.hokudai.ac.jp/~chikako/EOS
  ρ =10(5.1-15.4) g/cc, T=0-100 MeV, Ye=0-0.56**
  (Ishizuka,AO,Tsubakihara, Sumiyoshi,Yamada, *J. Phys. G 35 (2008), 085201*)

EOSY by IOTSY

- **Hyperon effects:**
  Decisive in Nstar
  Small in SNe (early)
  Significant in BH formation.
- **Japan Proton Accelerator Research Complex (J-PARC) data will come soon. Stay Tuned !**



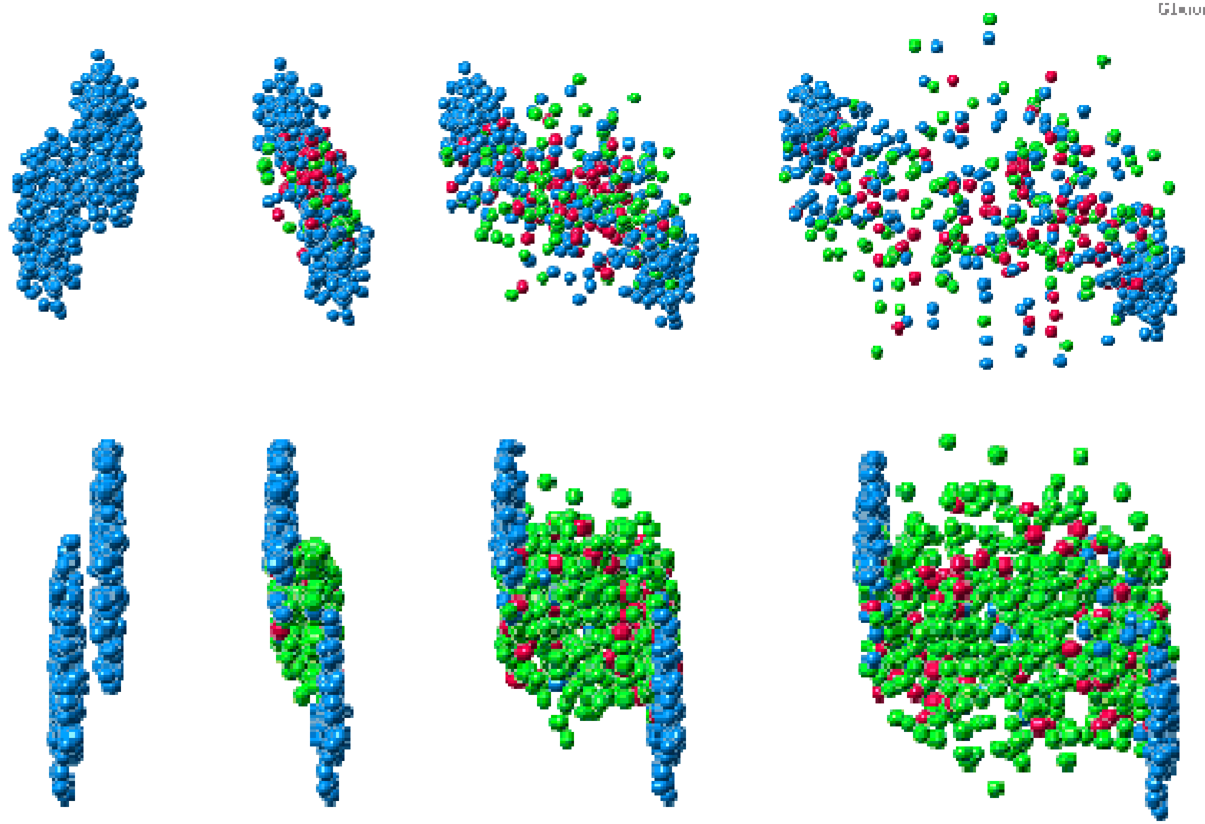
Nuclear Transport Models for Heavy-Ion Collisions and Collective Flows

Heavy-Ion Collisions at Einc ~ (1-100) A GeV

- **Study of Hot and Dense Hadronic Matter**
  → Particle Yield, Collective Dynamics (Flow), EOS, ...

AGS

SPS

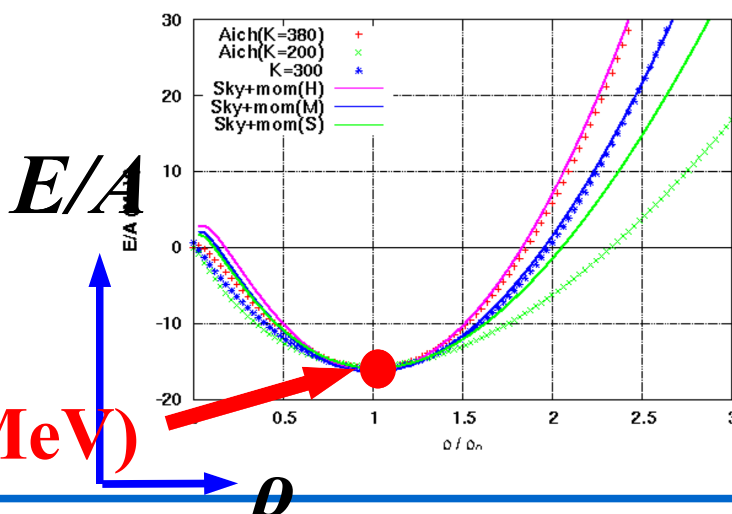


JAMming on the Web, linked from http://www.jcprg.org/

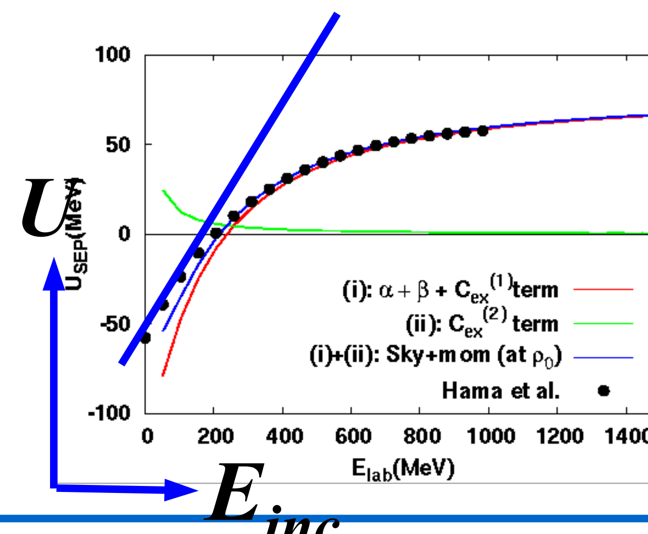
Nuclear Mean Field

- MF has on both of ρ and p-deps.
  - ρ dep.: (ρ_0, *E/A*) = (0.15 fm^{-3}, -16.3 MeV) is known
    Stiffness is not known well
  - p dep.: Global potential up to E=1 GeV is known from pA scattering
    $U(\rho_0, E) = U(\rho_0, E{=}0) + 0.3\,E$
- Ab initio Approach; LQCD, GFMC, DBHF, G-matrix, ...
  → Not easy to handle, Not satisfactory for phen. purposes
- Effective Interactions (or Energy Functionals):
  Skyrme HF, RMF, ...

$U(E)=U(0)+0.3E$



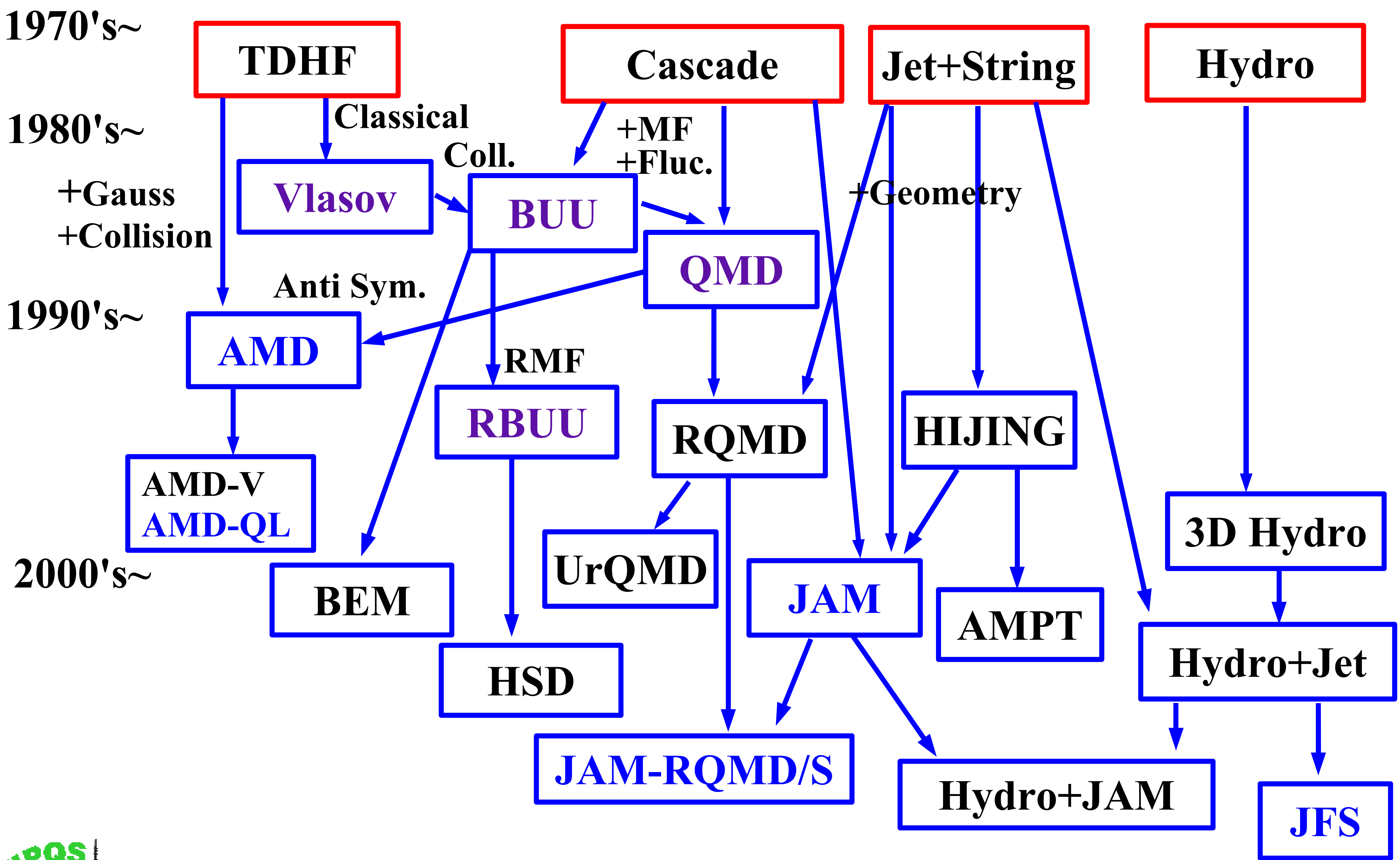
$(\rho_0, E/A)$
$=(0.15\ \mathrm{fm}^{-3}, -16.3\ \mathrm{MeV})$



HIC Transport Models: Major Four Origins

- *Nuclear Mean Field Dynamics*
  - **Basic Element of Low Energy Nuclear Physics, and Critically Determines High Density EOS / Collective Flows**
  - **TDHF → Vlasov → BUU**
- *NN two-body (residual) interaction*
  - **Main Source of Particle Production**
  - **Intranuclear Cascade Models**
- *Partonic Interaction and String Decay*
  - **Main Source of high pT Particles at Collider Energies**
  - **JETSET + (previous) PYTHIA (Lund model) → (new) PYTHIA**
- *Relativistic Hydrodynamics*
  - **Most Successful Picture at RHIC**

HIC Models: History



TDHF and Vlasov Equation

- **Time-Dependent Mean Field Theory (e.g., TDHF)** $i\hbar \frac{\partial \phi_i}{\partial t} = h\phi_i$
- **Density Matrix**

$$\rho(r, r') = \sum_i^{Occ} \phi_i(r) \phi_i^*(r') \quad \rightarrow \quad \boldsymbol{\rho_W = f} \textbf{ (phase space density)}$$

- **TDHF for Density Matrix**

$$i\hbar \frac{\partial \rho}{\partial t} = [h, \rho] \quad \rightarrow \quad \frac{\partial f}{\partial t} = \{h_W, f\}_{P.B.} + O(\hbar^2)$$

- **Wigner Transformation and Wigner-Kirkwood Expansion**

(Ref.: Ring-Schuck)

$$O_W(r, p) \equiv \int d^3s \exp(-i p \cdot s/\hbar) < r + s/2 | O | r - s/2 >$$

$$(AB)_W = A_W \exp(i\hbar\Lambda) B_W \quad \Lambda \equiv \nabla'_r \cdot \nabla_p - \nabla'_p \cdot \nabla_r \quad (\nabla' \text{ acts on the left})$$

$$[A, B]_W = 2i A_W \sin(\hbar\Lambda/2) B_W = i\hbar \{A_W, B_W\}_{P.B.} + O(\hbar^3)$$

Test Particle Method

- **Vlasov Equation**

$$\frac{\partial f}{\partial t} - \{h_W, f\}_{P.B.} = \frac{\partial f}{\partial t} + v \cdot \nabla_r f - \nabla U \cdot \nabla_p f = 0$$

- **Classical Hamiltonian**

$$h_W(r, p) = \frac{p^2}{2\mathrm{m}} + U(r, p)$$

- **Test Particle Method (C. Y. Wong, 1982)**

$$f(r, p) = \frac{1}{N_0} \sum_i^{A N_0} \delta(r - r_i)\, \delta(p - p_i) \quad \rightarrow \quad \frac{dr_i}{dt} = \nabla_p h_w, \quad \frac{dp_i}{dt} = -\nabla_r h_w,$$

Mean Field Evolution can be simulated by Classical Test Particles
→ Opened a possibility to Simulate High Energy HIC including Two-Body Collisions in Cascade

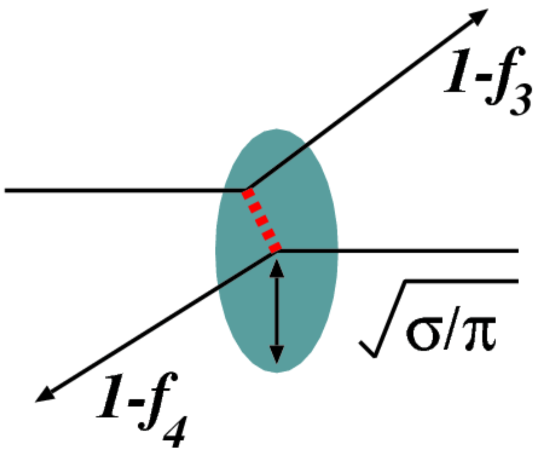
BUU (Boltzmann-Uehling-Uhlenbeck) Equation

- **BUU Equation** **(Bertsch and Das Gupta, Phys. Rept. 160(88), 190)**

$$\frac{\partial f}{\partial t} + v \cdot \nabla_r f - \nabla U \cdot \nabla_p f = I_{coll}[f]$$

$$I_{coll}[f] = -\frac{1}{2} \int \frac{d^3 p_2 \, d\Omega}{(2\pi\hbar)^3} v_{12} \frac{d\sigma}{d\Omega}$$
$$\times \quad [f f_2 (1-f_3)(1-f_4) - f_3 f_4 (1-f)(1-f_2)]$$

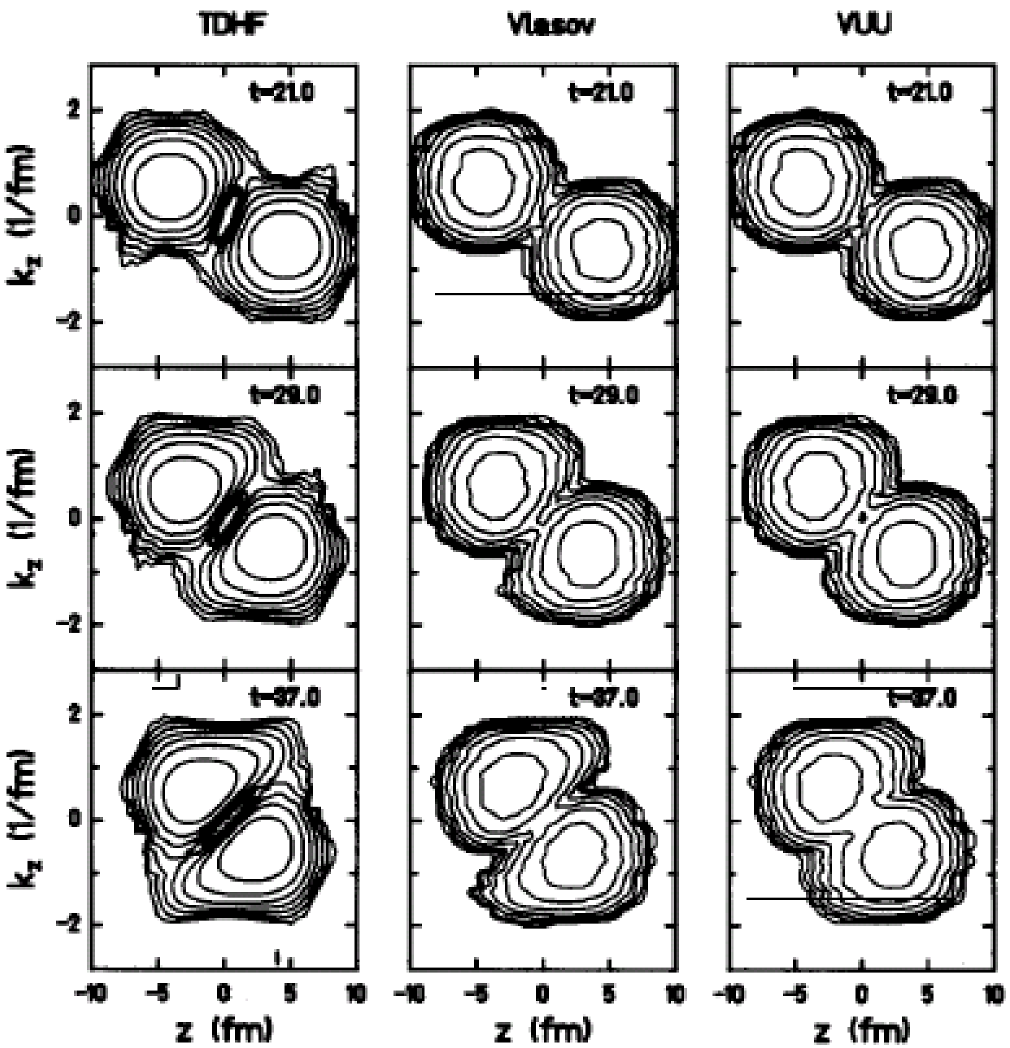
- **Incorporated Physics in BUU**
  - **Mean Field Evolution**
  - **(Incoherent) Two-Body Collisions**
  - **Pauli Blocking in Two-Body Collisions**



O One-Body Observables (Particle Spectra, Collective Flow, ..)
X Event-by-Event Fluctuation (Fragment, Intermittency, ...)

Comarison of TDHF, Vlasov and BUU(VUU)

- **Ca+Ca, 40 A MeV**
  (Cassing-Metag-Mosel-Niita, Phys. Rep. 188 (1990) 363).



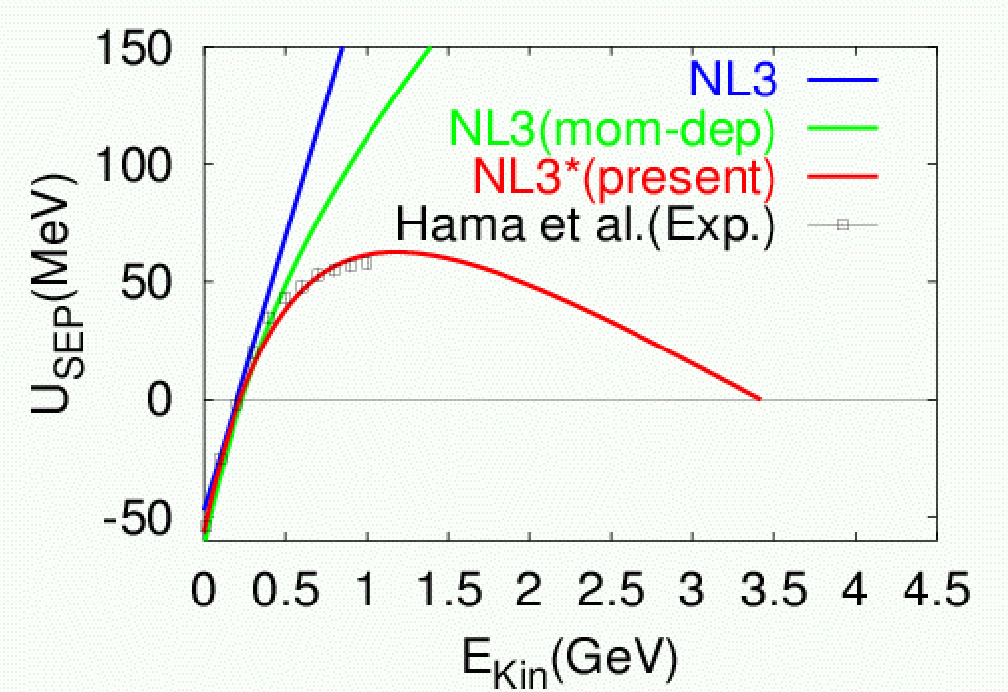
Relativistic Mean Field (II)

- **Dirac Equation** $\left(i\gamma\partial-\gamma^0 U_v - M - U_s\right)\psi=0\ ,\quad U_v=g_\omega\omega\ ,\quad U_s=-g_\sigma\sigma$
- **Schroedinger Equivalent Potential**

$$\begin{pmatrix} E-U_v-M-U_s & i\sigma\cdot\nabla \\ -i\sigma\cdot\nabla & -E+U_v-M-U_s \end{pmatrix}\begin{pmatrix} f \\ g \end{pmatrix}=0$$

$$U_{sep}\sim U_S+\frac{E}{m}U_V=-g_\sigma\sigma+\frac{E}{m}g_\omega\omega$$

$$=-\frac{g_\sigma^2}{m_\sigma^2}\rho_s+\frac{E}{m}\frac{g_\omega^2}{m_\omega^2}\rho_B$$



Saturation: -Scalar+Baryon Density
Linear Energy Dependence: Good at Low Energies,
Bad at High Energies (We need cut off !)

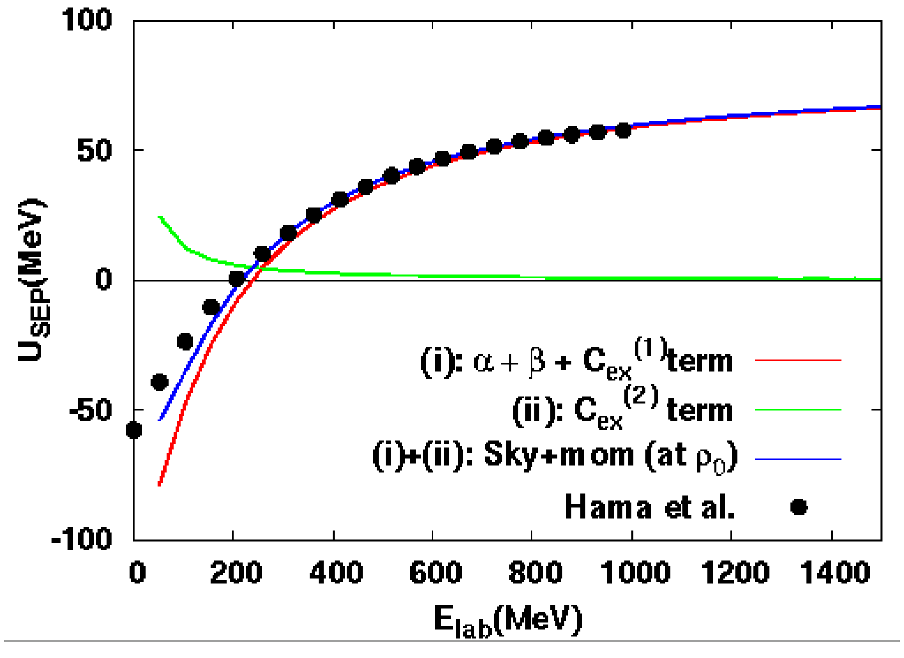
(Sahu, Cassing, Mosel, AO, Nucl. Phys. A672 (2000), 376.)

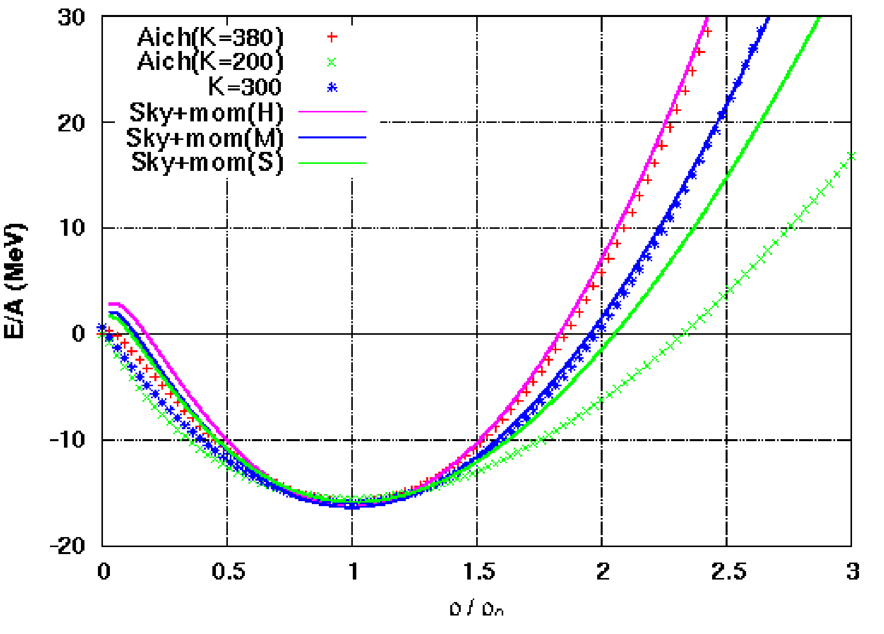
Phenomenological Mean Field

- **Skyrme type ρ-Dep. + Lorentzian p-Dep. Potential**

$$V=\sum_i V_i=\int d^3r\left[\frac{\alpha}{2}\left(\frac{\rho}{\rho_0}\right)^2+\frac{\beta}{\gamma+1}\left(\frac{\rho}{\rho_0}\right)^{\gamma+1}\right]$$

$$+\sum_k\int d^3r\,d^3p\,d^3p'\,\frac{C_{ex}^{(k)}}{2\rho_0}\frac{f(r,p)f(r,p')}{1+(p-p')^2/\mu_k^2}$$





Isse, AO, Otuka, Sahu, Nara, Phys.Rev. C 72 (2005), 064908

Exercise

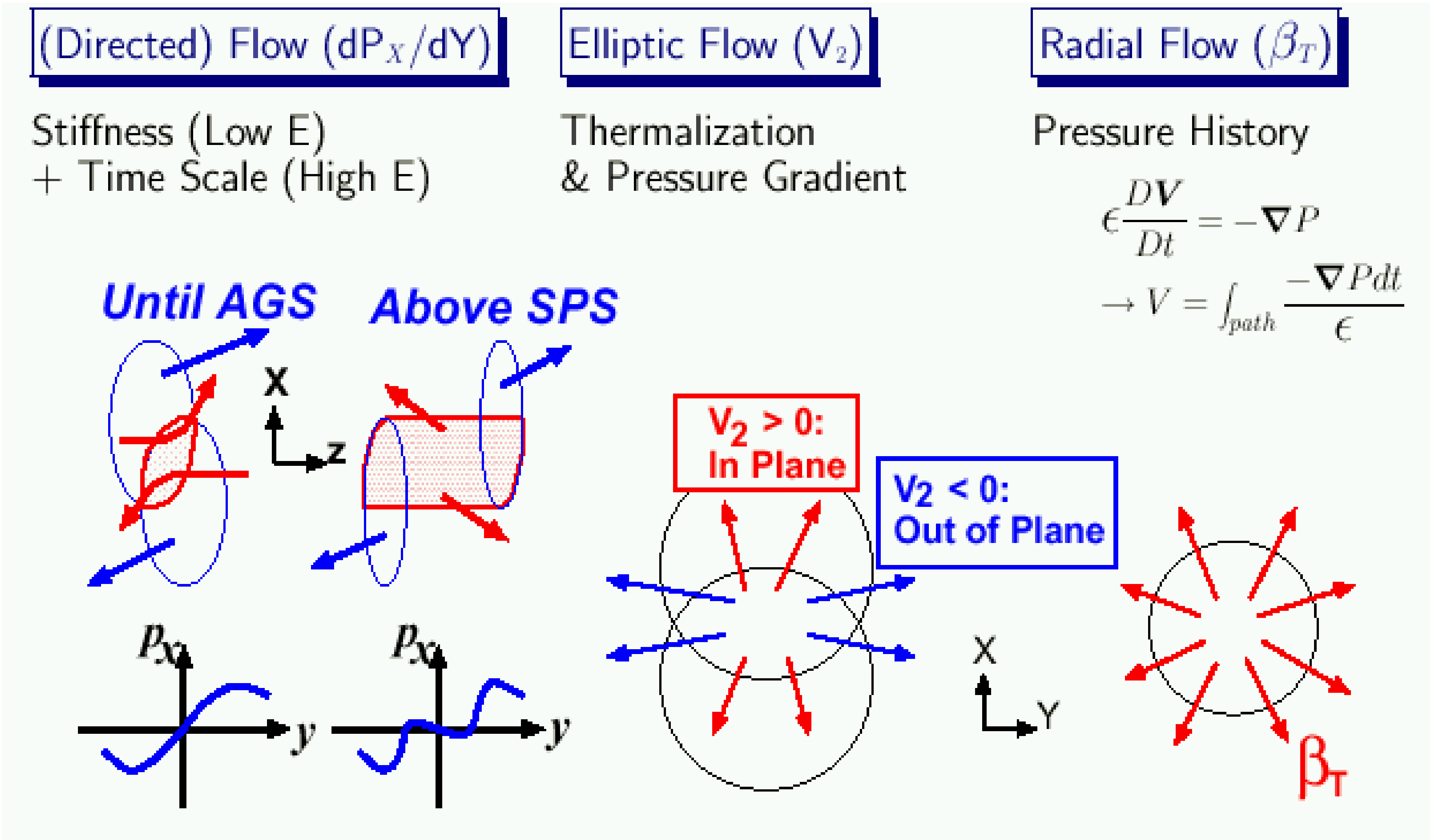
- **Prove that the spatial integral of the Wigner function *f(x,p)* gives a momentum distribution of nucleons.**
- **Prove that the Wigner function with test particles satisfy the Vlasov equation when the test particle follows the classical EOM.**
- **Prove that the collision term does not change the Wigner function in equilibrium.**

Collective Flow and EOS: Old Problem ?

- **1970's-1980's: First Suggestions and Measurement**
  - **Hydrodynamics suggested the Exsitence of Flow.**
  - **Strong Collective Flow suggests Hard EOS**
- **1980's-1990's: Deeper Discussions in Wider E_{inc} Range**
  - **Momentum Dep. Pot. can generate Strong Flows.**
  - **Einc deps. implies the importance of Momentum Deps.**
  - **Flow Measurement up to AGS Energies.**
- **2000's: Extention to SPS and RHIC Energies**
  - **EOS is determined with Mom. AND Density Dep. Pot. ?**

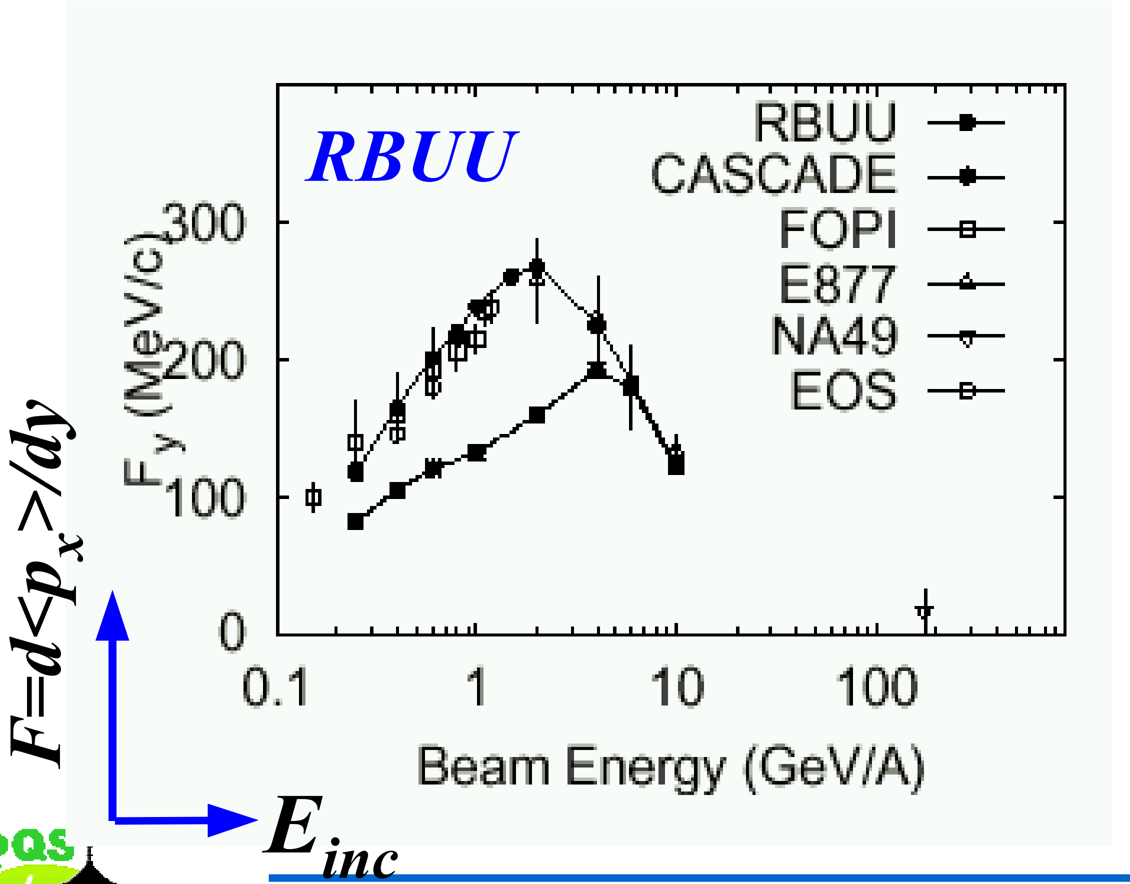
Old but New (Continuing) Problem !

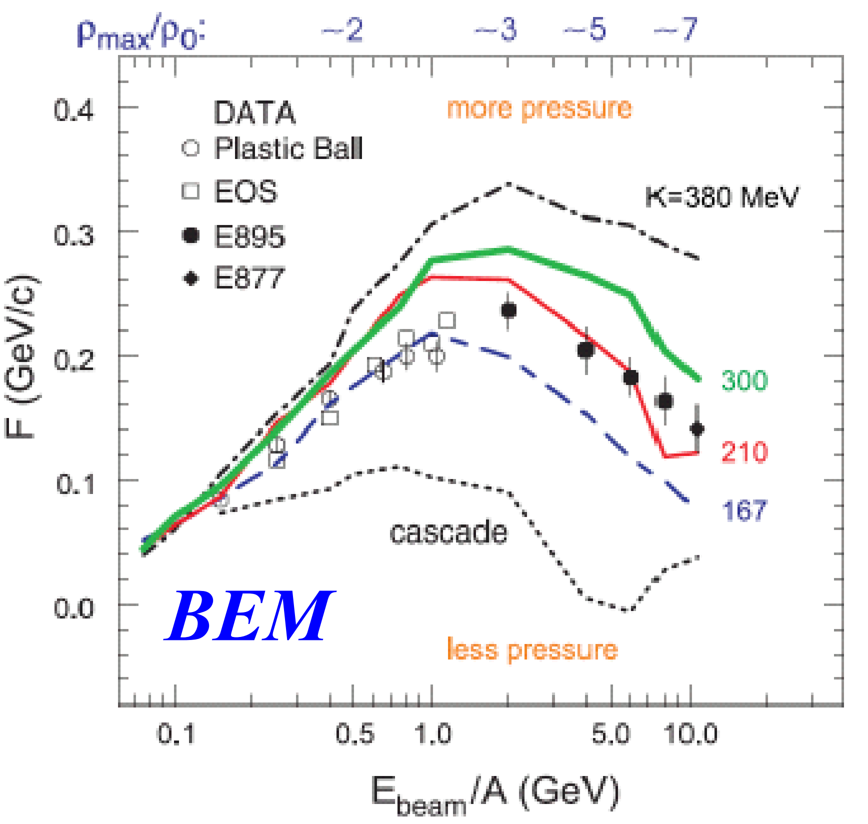
What is Collective Flow ?



Side Flow at AGS Energies

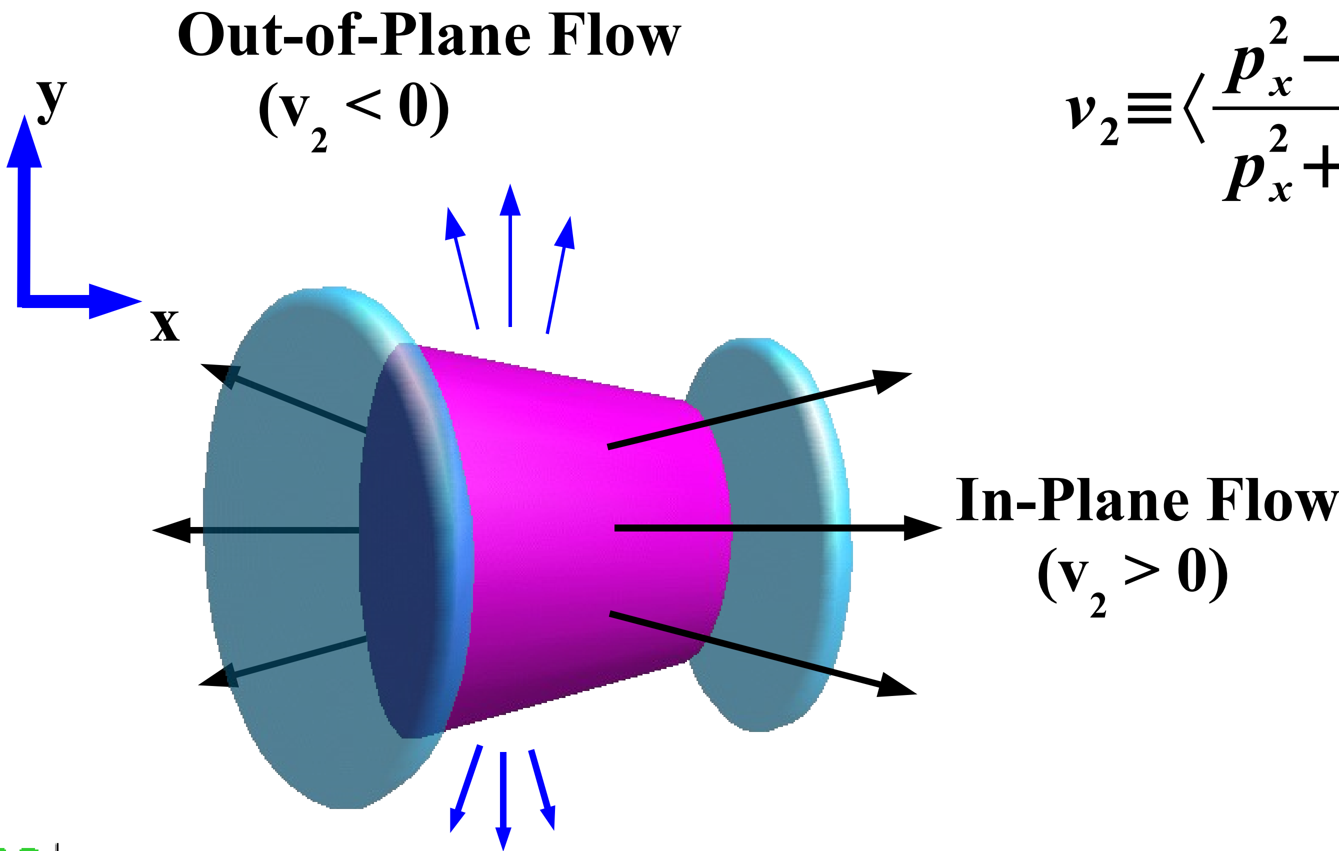
- **Relativistic BUU (RBUU) model:** ***K ~ 300 MeV***
(Sahu, Cassing, Mosel, AO, Nucl. Phys. A672 (2000), 376.)
- **Boltzmann Equation Model (BEM):** ***K=167~210 MeV***
(P. Danielewicz, R. Lacey, W.G. Lynch, Science 298(2002), 1592.)





Elliptic Flow

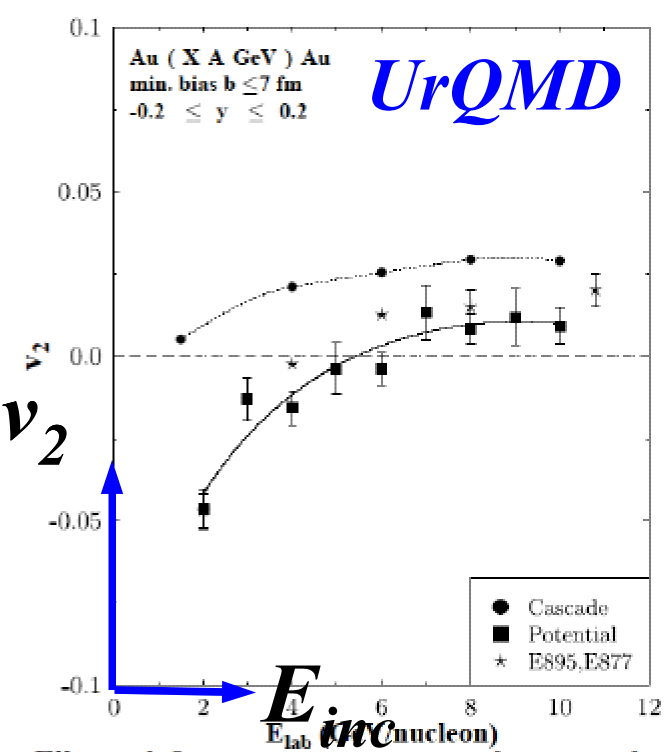
- What is Elliptic Flow ? → Anisotropy in P space
- Hydrodynamical Picture
  - Sensitive to the Pressure Anisotropy in the Early Stage
  - Early Thermalization is Required for Large V2

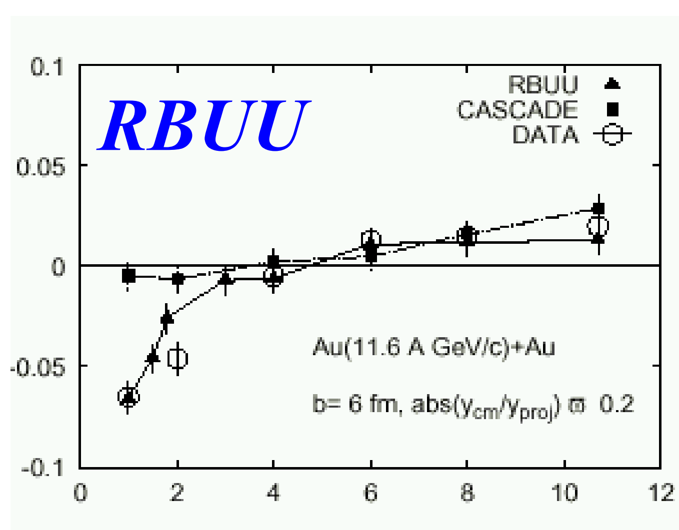


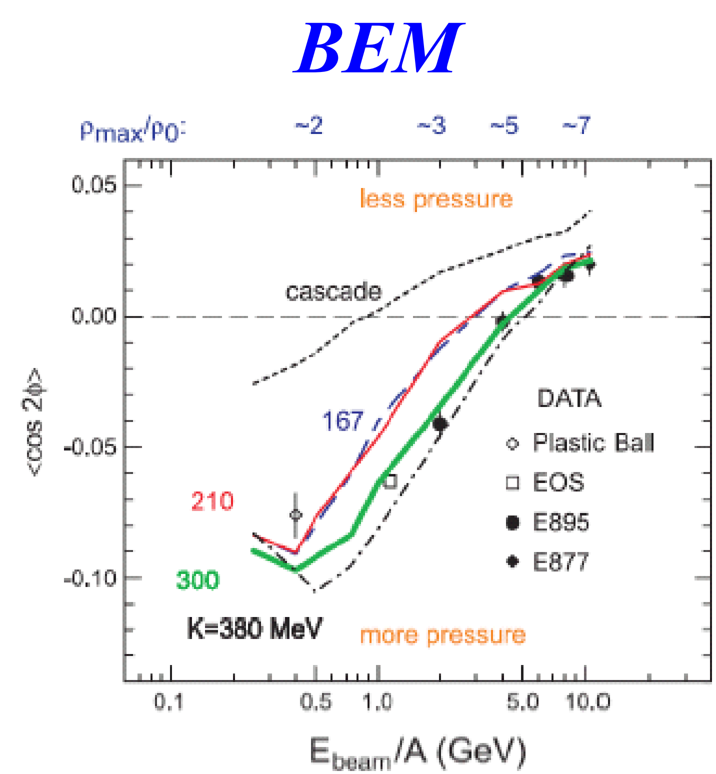
$$v_2 \equiv \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle = \langle \cos 2\phi \rangle$$

Elliptic Flow at AGS

- **Strong Squeezing Effects at low E (2-4 A GeV)**
  - **UrQMD: Hard EOS (S.Soff et al., nucl-th/9903061)**
  - **RBUU (Sahu-Cassing-Mosel-AO, 2000): K ~ 300 MeV**
  - **BEM(Danielewicz2002): K = 167 → 300 MeV**

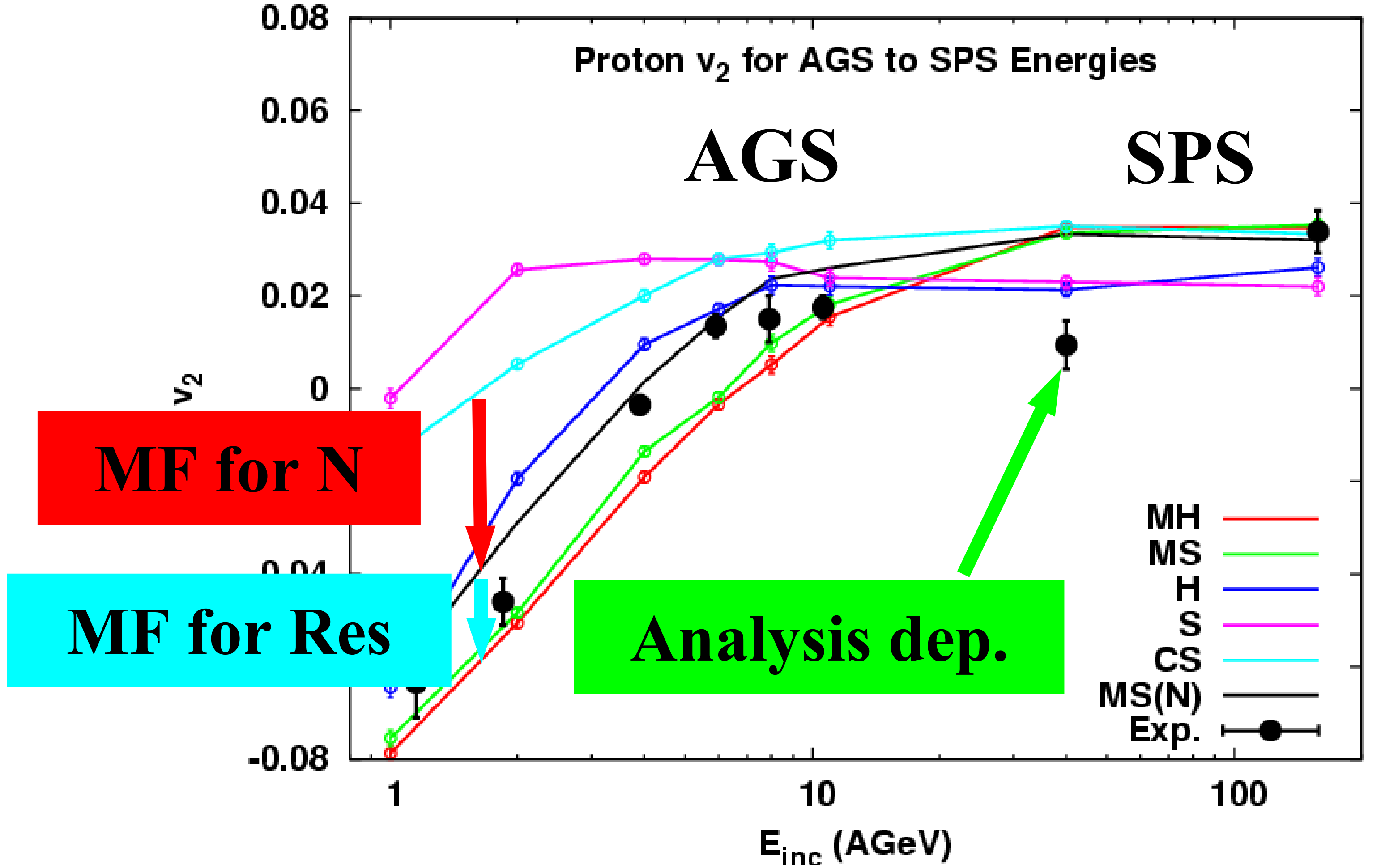






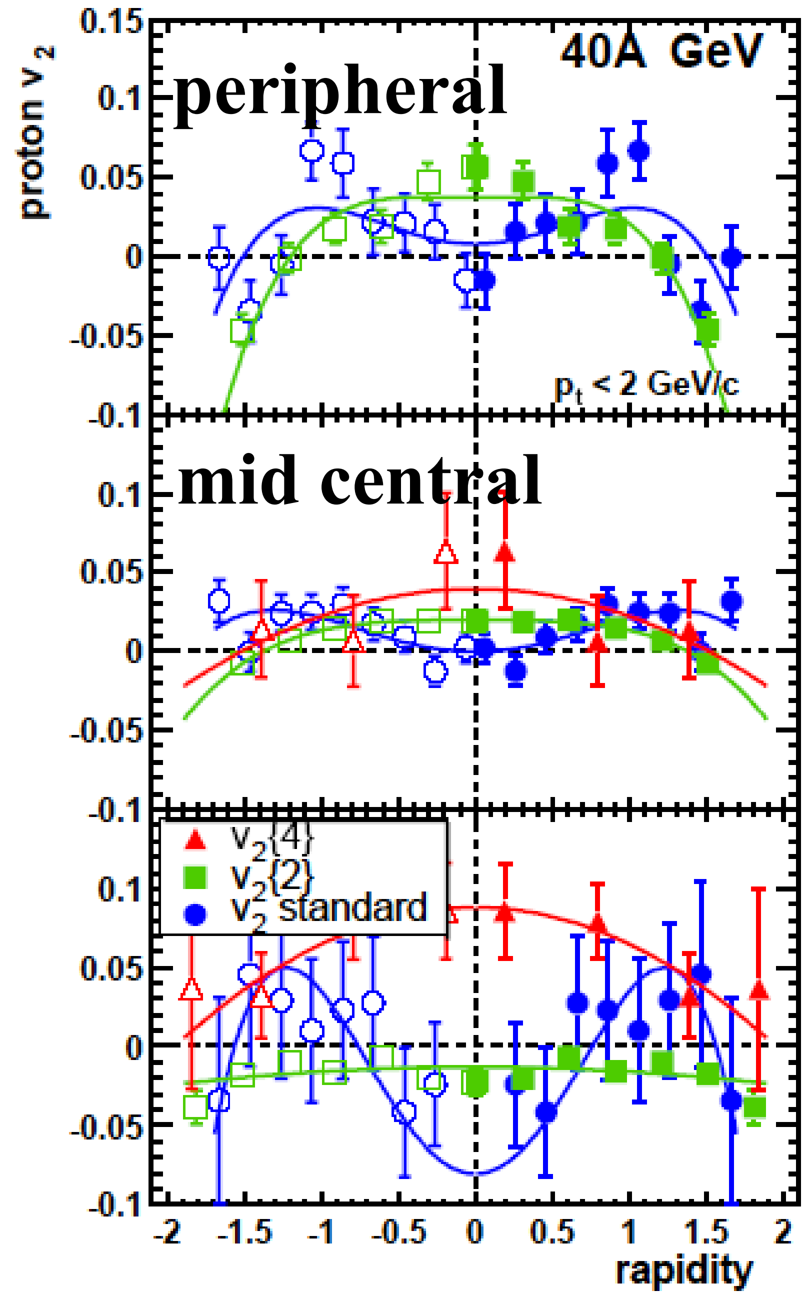
Elliptic Flow from AGS to SPS

- **JAM-MF with p dep. MF explains proton v2 at 1-158 A GeV**
  - **v2 is not very sensitive to K (incompressibility)**
  - **Data lies between MS(B) and MS(N)**



Dip of V_2 at 40 A GeV: Phase Transition ?

- **Dip of V_2 at 40 A GeV may be a signal of QCD phase transition at high baryon density.**
  (Cassing et al.)
- **However, the data is too sensitive to the way of the analysis (reaction plane/two particle correlation).**
  - **We have to wait for better data.**



Flow and EOS; to be continued

- **In addition to the ambiguities in in-medium cross sections, Res.-Res. cross sections, we have model dependence.**
  - **RBUU** *(e.g. Sahu, Cassing, Mosel, AO, Nucl. Phys. A672 (2000), 376.)*
    - **In RMF, Strong cut-off for meson-N coupling in RMF $\rightarrow$ Smaller EOS dep.**
  - **Scalar potential interpretation in BUU**
    Larionov,Cassing,Greiner,Mosel, PRC62,064611('00), Danielewicz, NPA673,375('00)

$$\varepsilon(\boldsymbol{p},\rho)=\sqrt{[m+U_s(\boldsymbol{p},\rho)]^2+\boldsymbol{p}^2}=\sqrt{m^2+\boldsymbol{p}^2}+U(\boldsymbol{p},\rho)$$

    - **Due to the Scalar potential nature, EOS dependence is smaller.**
  - **Scalar/Vector Combination** *Danielewicz, Lacey, Lynch, Science 298('02), 1592*

$$\varepsilon(p,\rho)=m+\int_0^p dp' v^*(p',\rho)+\widetilde{U}(\rho),\quad v^*(p,\rho)=\frac{p}{\sqrt{p^2+[m^*(p,\rho)]^2}}.$$

    - **Relatively Strong EOS dependence even at high energy**
  - **JAM-RQMD/S** *Isse, AO, Otuka, Sahu, Nara, PRC 72 (2005), 064908*
    - **Similar to the Scalar model BUU**

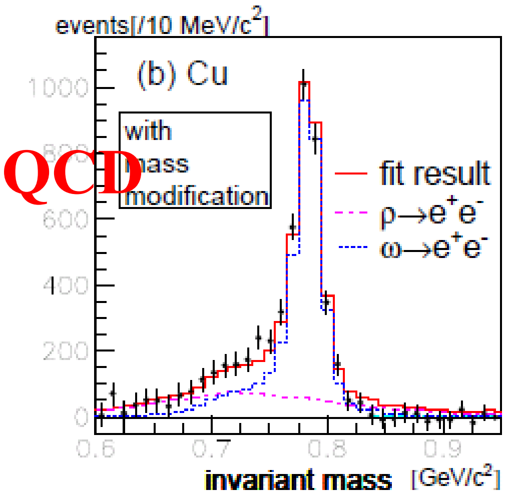
Backups

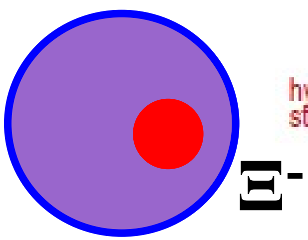
Summary

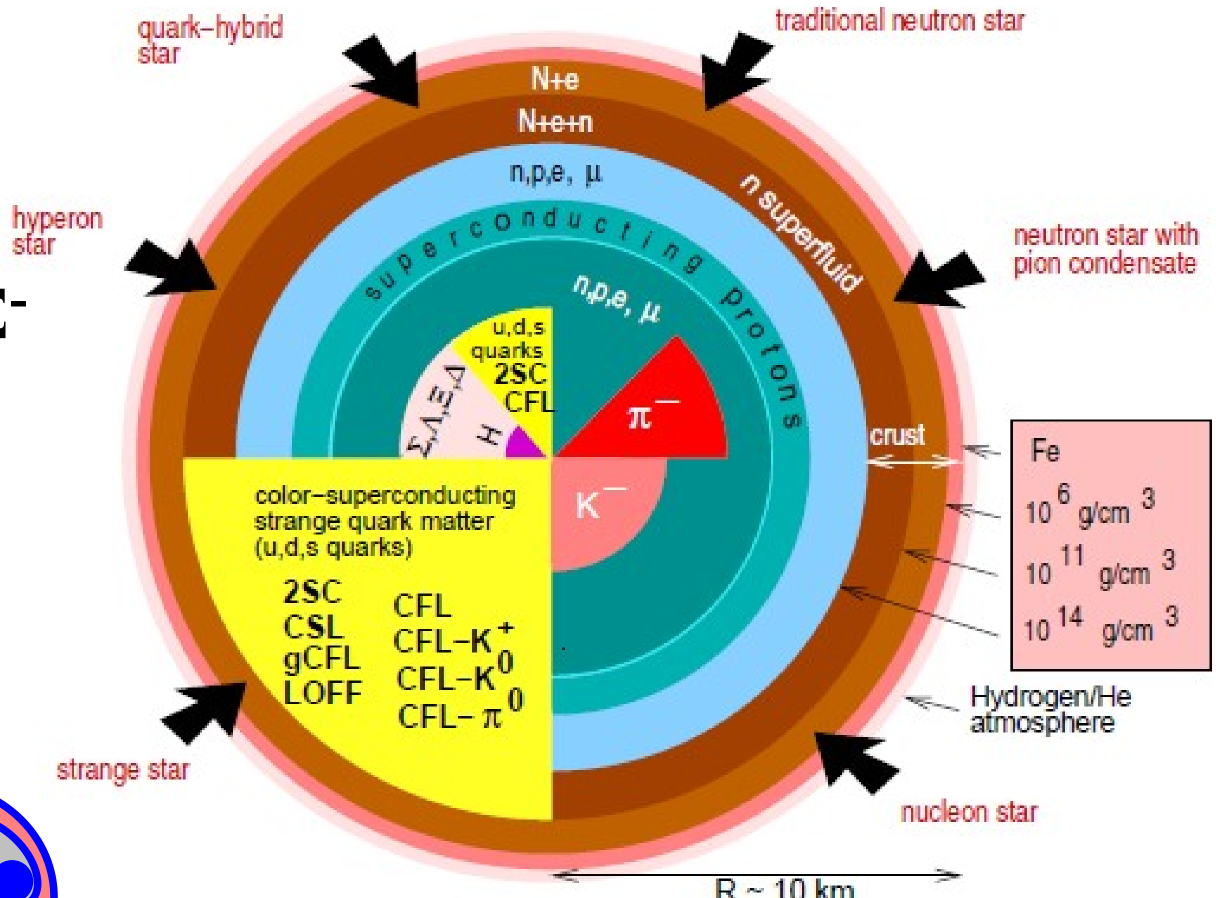
- **Nuclear Matter EOS is important in many subjects of nuclear physics.**
  - **Bulk nuclear properties (B.E., radius)**
  - **Dense Matter in Compact Astrophysical Objects**
    (Neutron Star Core or Black Hole formation)
  - **High-Energy Heavy-Ion Collisions**
- **There are many unsolved problems.**
  - **Which kind of terms should be added in RMF Lagrangian ?**
    Can we blush it up at the level of N.R. Nuclear Density Functional ?
  - **Which kind of matter (or phase) appears in dense matter ?**
    Can we access the phase transition in the high ρ_B direction through Exp't?
  - **How do hadron resonances contribute to EOS at high T ?**
    What is the final form of the transport equation ?

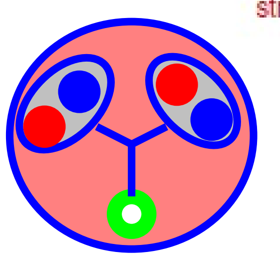
Which phase is realized in neutron stars ?

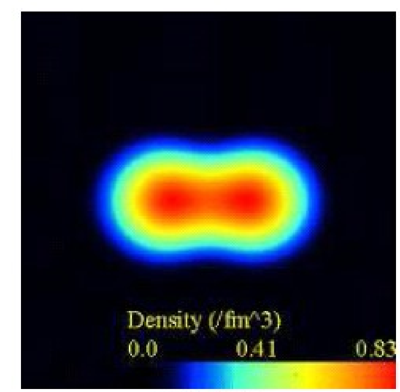
- **Physics at J-PARC**
  → Decisive Information of Neutron Star Matter
  From Quark many-body system in Lab. to Dense QCD







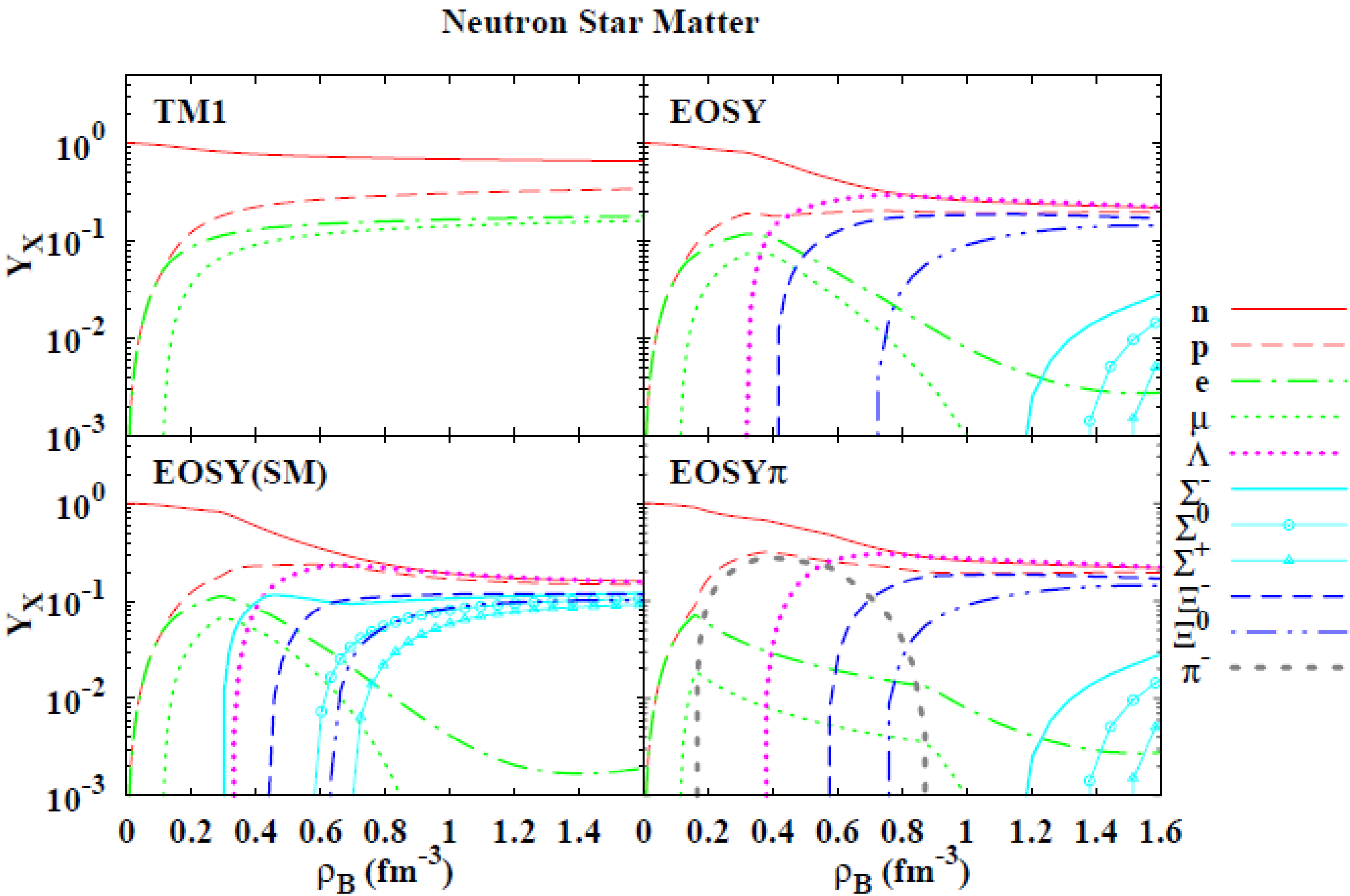




(b) ^{8}BeK^{-}

F. Weber, Prog. Part. Nucl. Phys. 54 (2005) 193

Particle Composition in Neutron Star

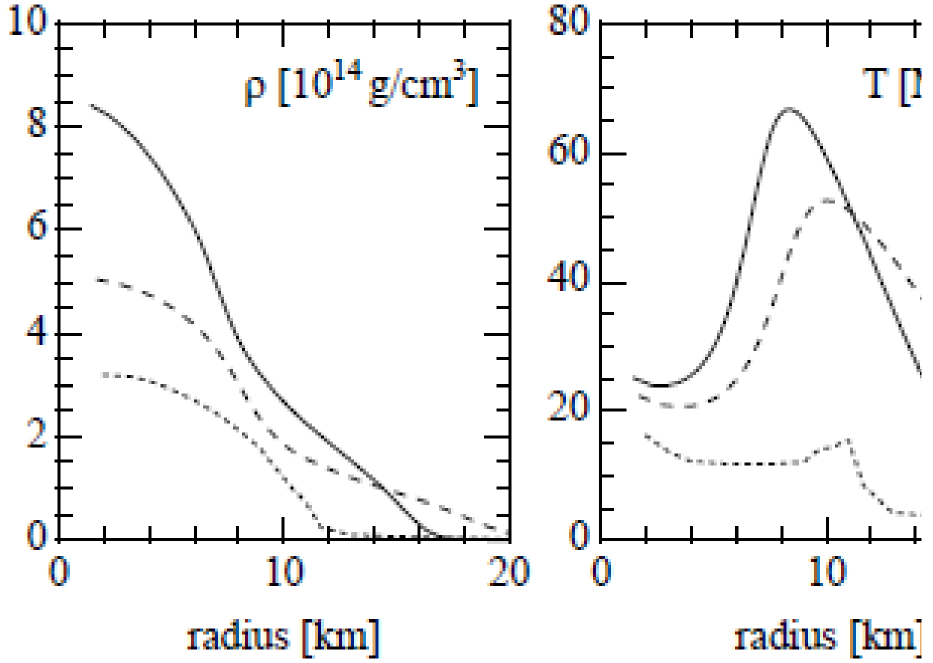


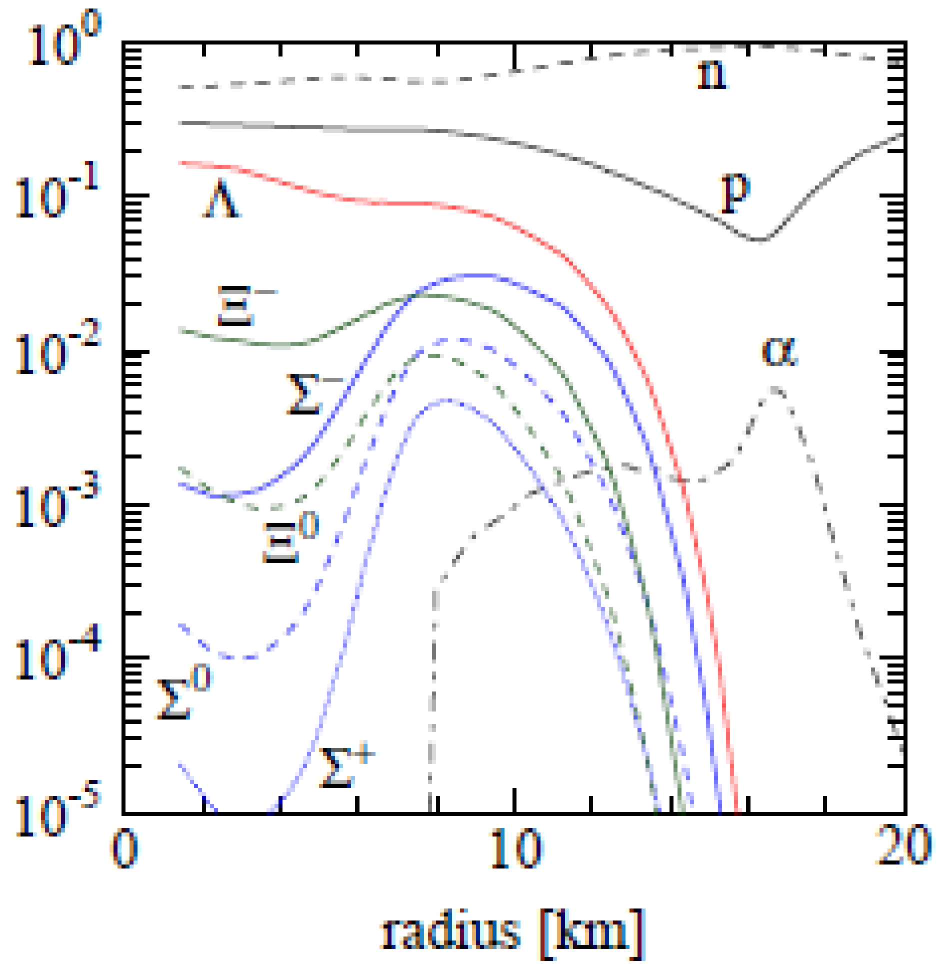
U_Ξ = -15 MeV, U_Σ = + 30 MeV

Ishizuka et al., JPG35(2008)

Hyperons during Black Hole Formation

- ブラックホール形成過程にもハイペロンは大きく寄与





Sumiyoshi et al. in prep.

Skyrme Hartree-Fock

c.f. Lecture by Nakatsukasa; See E.g. Ring-Schuck for details

- **Zero-Range Two- and Three-Body Interaction**

$$v_{ij}=t_0\delta(r_i-r_j)+\frac{1}{2}\left[\delta(r_i-r_j)k^2+k^2\delta(r_i-r_j)\right]$$
$$+t_2 k\,\delta(r_i-r_j)k+iW_0\left[\sigma_i+\sigma_j\right]\times\delta(r_i-r_j)k$$
$$k=\frac{1}{2\mathrm{i}}(\nabla_i-\nabla_j)$$
$$v_{ijk}=t_3\delta(r_i-r_j)\delta(r_j-r_k)$$

- **Energy Density (Even-Even, N=Z)**

$$H(r)=\frac{\hbar^2}{2\mathrm{m}^*(\rho)}\tau+\frac{3}{8}t_0\rho^2+\frac{1}{16}t_3\rho^3+Deriv.\ \ terms\rightarrow\rho\left[\frac{3}{5}\ \frac{\hbar^2 k_F^2}{2m^*(\rho)}+\frac{3}{8}t_0\rho+\frac{1}{16}t_3\rho^2\right]$$
$$\tau=\sum_i|\nabla\phi_i|^2\ ,\quad \frac{\hbar^2}{2\mathrm{m}^*(\rho)}=\frac{\hbar^2}{2\mathrm{m}}+\frac{1}{16}(3t_1+5t_2)\rho$$

Problems in Skyrme HF (in Dense Nuclear Matter/High Energy)
Repulsive Zero-Range 3-body Int.: → Causality Violation
Energy Dep. = Linear (m* term) → Too Repulsive at High E

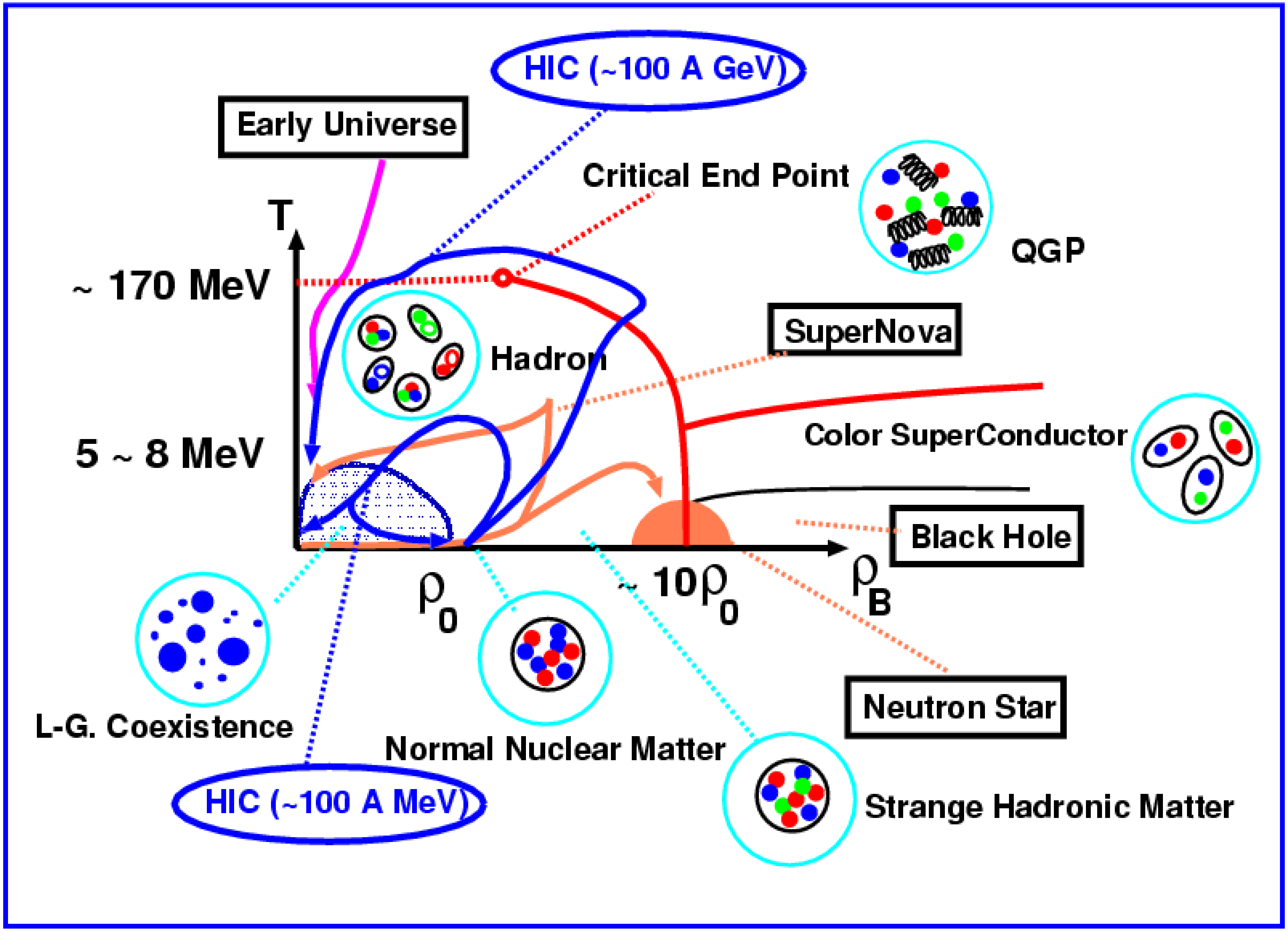
Relativistic Mean Field (I)

Serot-Walecka, Walecka text book.

- **Describe nuclear energy functional in meson and baryon fields**
  - **Fit B.E. of Stable as well as Unstable (n-rich) Nuclei**
  - **Has been successfully applied to Supernova Explosion**
  - **Three Mesons (σ,ω,ρ) are included**
  - **Meson Self-Energy Term (σ,ω)**

$$
\begin{aligned}
\mathcal{L} =& \bar{\psi}_N \left(i\partial\!\!\!/ - M - g_\sigma \sigma - g_\omega \omega\!\!\!/ - g_\rho \tau^a \rho\!\!\!/^a \right) \psi_N \\
&+ \frac{1}{2}\partial^\mu \sigma \partial_\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4 \\
&- \frac{1}{4} W^{\mu\nu} W_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega^\mu \omega_\mu - \frac{1}{4} R^{a\mu\nu} R^a_{\mu\nu} + \frac{1}{2} m_\rho^2 \rho^{a\mu} \rho^a_\mu + \frac{1}{4} c_3 \left(\omega_\mu \omega^\mu \right)^2 \\
&+ \bar{\psi}_e \left(i\partial\!\!\!/ - m_e \right) \psi_e + \bar{\psi}_\nu i\partial\!\!\!/ \psi_\nu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \ , \\
W_{\mu\nu} =& \partial_\mu \omega_\nu - \partial_\nu \omega_\mu \ , \\
R^a_{\mu\nu} =& \partial_\mu \rho^a_\nu - \partial_\nu \rho^a_\mu + g_\rho \epsilon^{abc} \rho^{b\mu} \rho^{c\nu} \ , \\
F_{\mu\nu} =& \partial_\mu A_\nu - \partial_\nu A_\mu \ . \qquad (2)
\end{aligned}
$$

Quark / Hadron / Nuclear Matter Phase Diagram



Rich Structure / Astrophysical implications / Accessible in HIC

Exercise (1)

- **Prove that the spatial integral of the Wigner function *f(x,p)* gives a momentum distribution of nucleons.**
- **Prove that the Wigner function with test particles satisfy the Vlasov equation when the test particle follows the classical EOM.**
- **Prove that the collision term becomes zero (i.e. gain and loss terms cancel) in equilibrium.**
- **Prove that the TDVP (time-dependent variational principle) gives the Schrodinger equation when the wave function is not restricted.**
- **Derive the collision term for bosons, which disappears in equilibrium.**
- ***(ADVANCED)* Prove the relation of the commutator and Poisson bracket. (It takes a long time ....)**
- ***(ADVANCED)* Prove that the Wigner function can be negative. (Therefore, the probability interpretation is not always possible.)**

Exercise (4)

- **Prove that the single particle potential with Skyrme interaction has a linear dependence on energy. From NA elastic scattering, the energy dependence is found to be**

  $$U(\rho_0, E) \sim U(\rho_0, E=0) + 0.3\,E$$

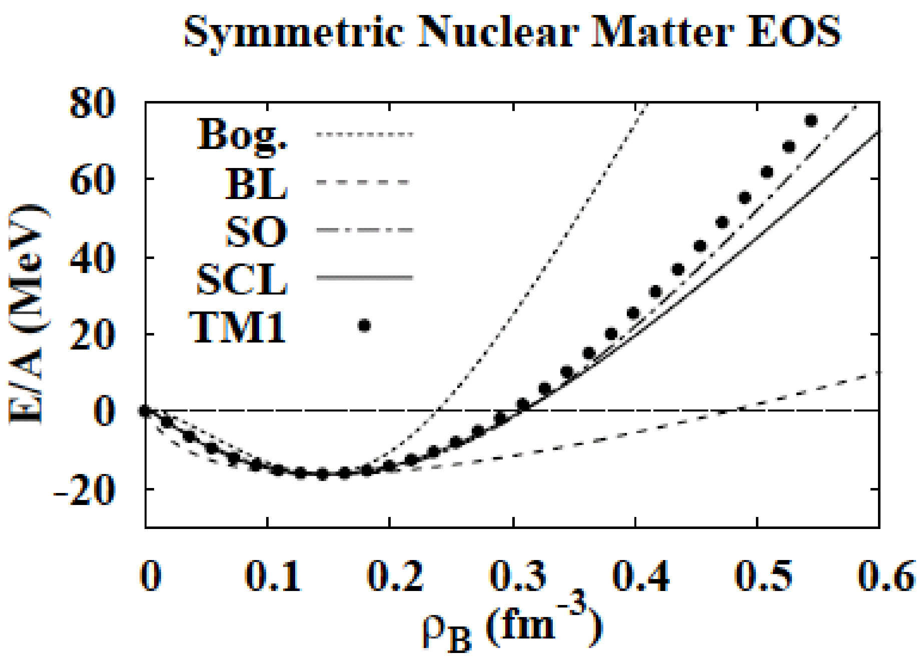
  at low energies. Obtain the value of m*/m which explains the above energy dependence.

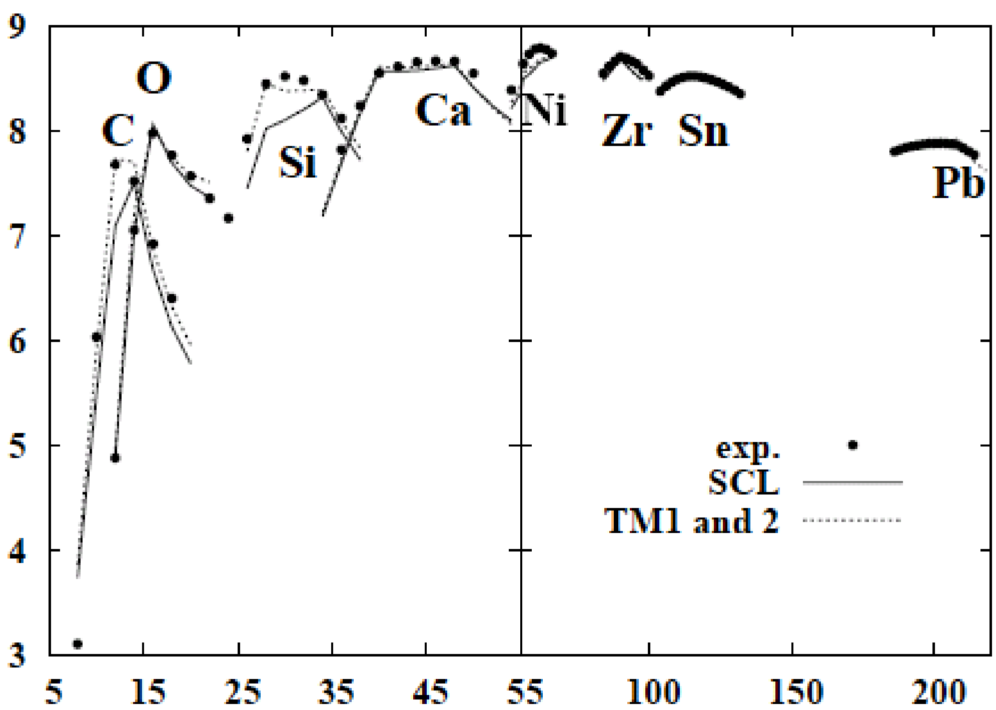
- **Obtain the form of the Schrodinger equivalent potential in RMF. You will find that the spin-orbit potential appears as a sum of scalar and vector potential.**

Nuclear Matter EOS and Nuclear Binding E in TM

Sugahara-Toki, NPA579 (1994), 557.

- **Example: TM1 parameter set**
  - **Nuclear Matter: σ4 and ω4 terms soften EOS (K ~ 280 MeV)**
  - **Finite nuclei: Explains B.E. from C to Pb isotopes**

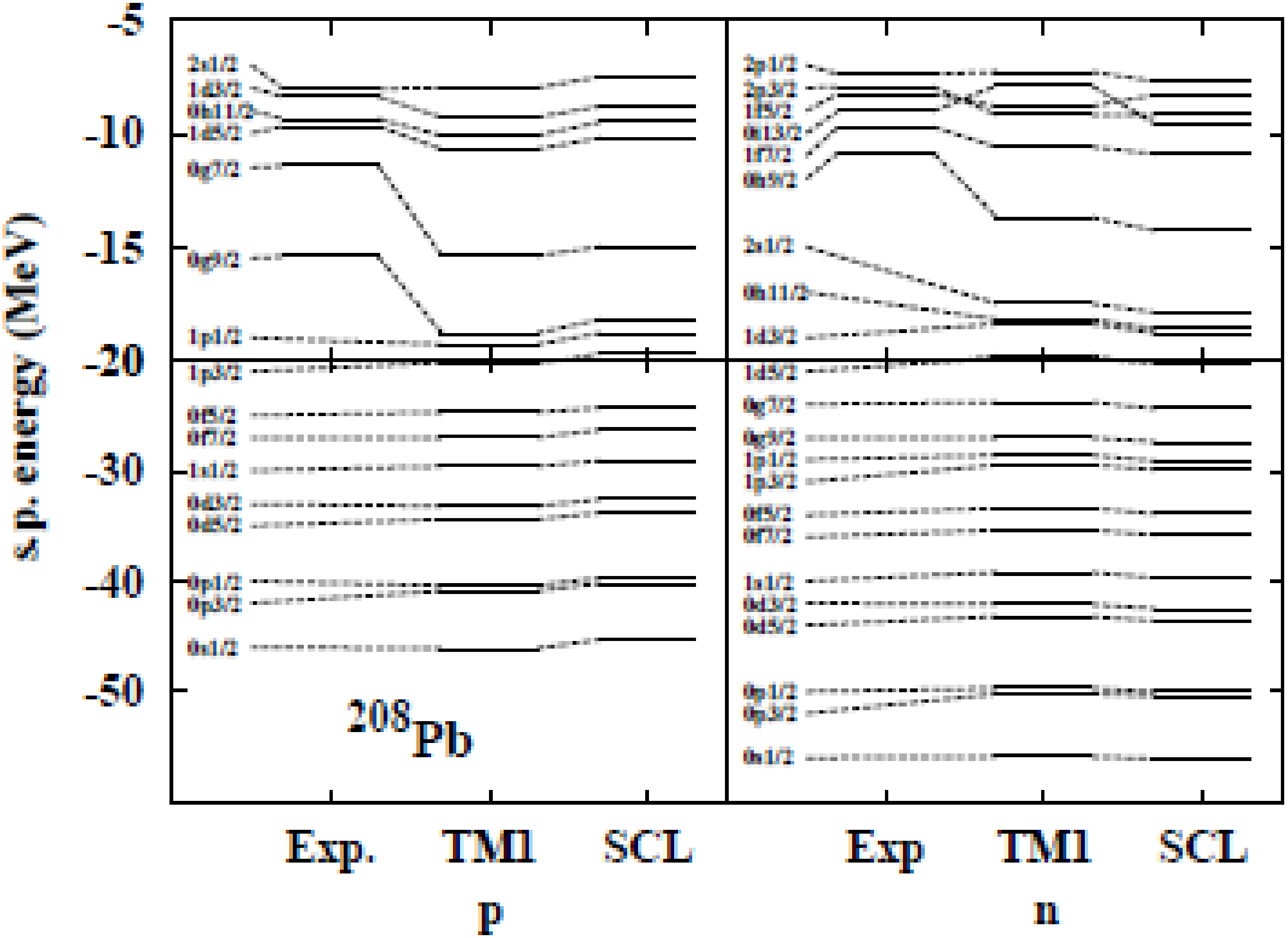




c.f. SCL=Chiral RMF with log σ term.

(K. Tsubakihara and AO, 2007)

Single Particle Energies



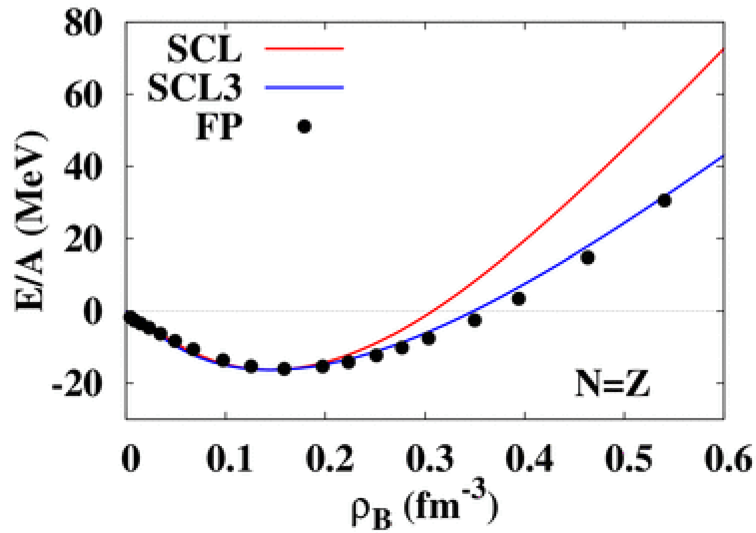
Chiral $SU_f(3)$ RMF

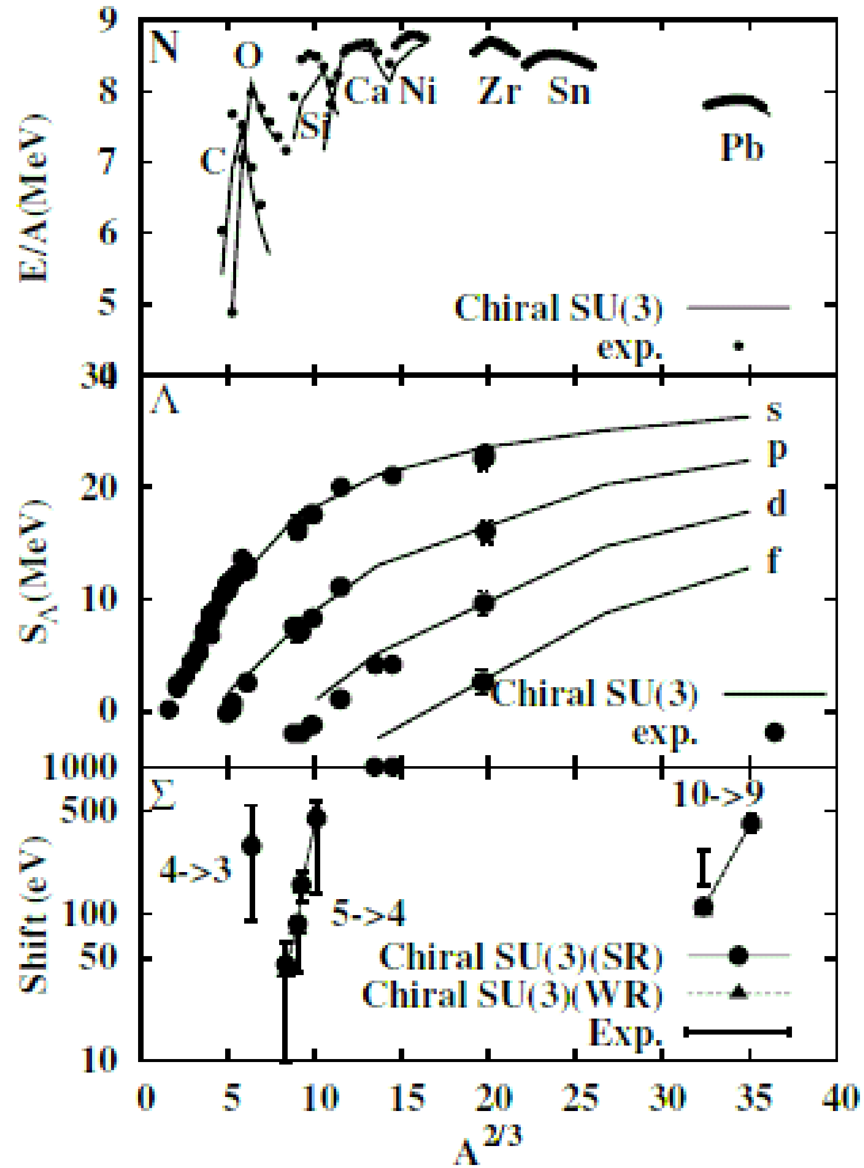
Tsubakihara, Maekawa, AO, EPJA33('07)295 [nucl-th/0702008]

- **Extention to Flavor SU(3)**
  → Chiral Potential from SCL-LQCD
  + Determinant Int. (U_A(1) anomaly)
  + Explicit breaking term

$$U_{\sigma\zeta} = -a\log(\det MM^\dagger) + b\,\mathrm{tr}(MM^\dagger) + c_\sigma\sigma + c_\zeta\zeta + d\,(\det M + \det M^\dagger).$$

- **Normal, Single & Double Λ, Σ atom, EOS (~ FP), ...**

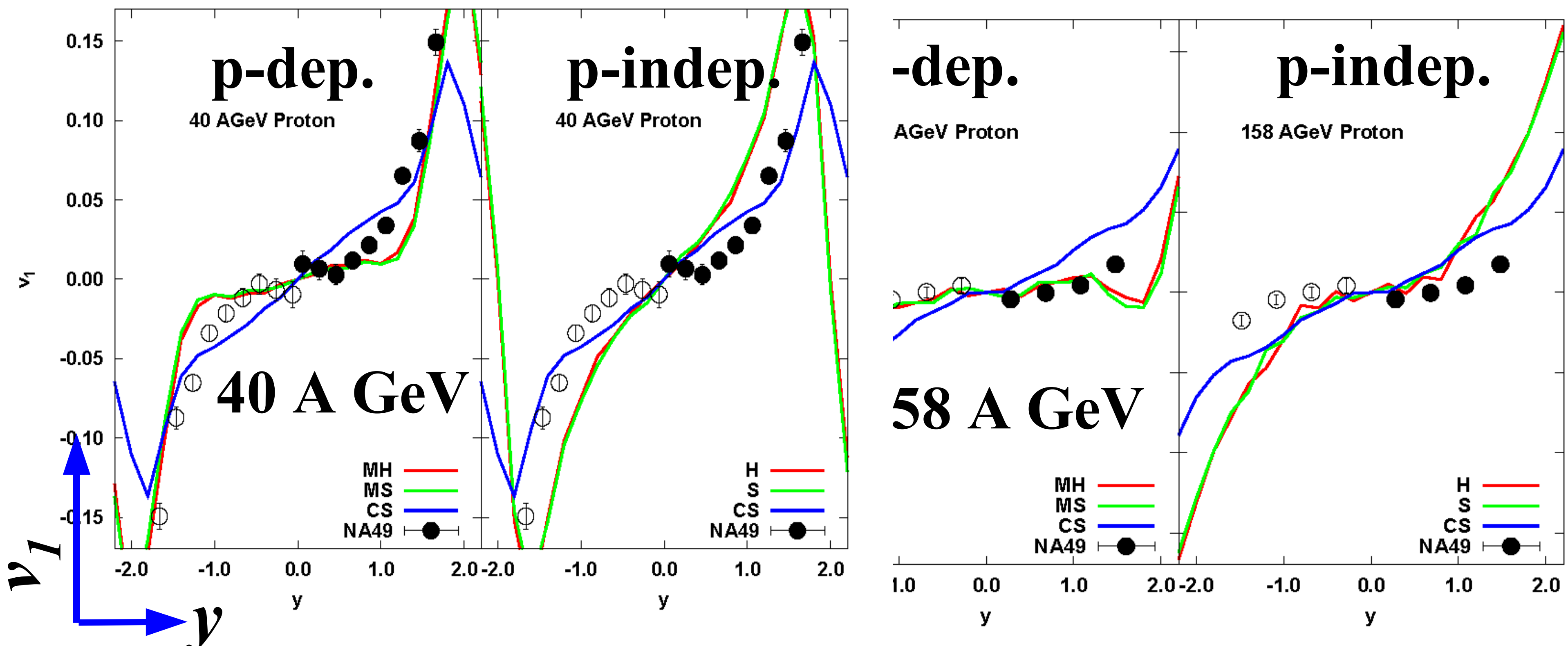




Directed flow v_1 at SPS

Isse, AO, Otuka, Sahu, Nara, PRC 72 (2005), 064908

- **JAM-RQMD/S**
  - **p-dep. (indep.) MF suppresses (enhances) v_1.** $$v_1 = \langle \cos\phi \rangle = \langle p_x / p_T \rangle$$
  - **"Wiggle" behavior appears with p-dep. MF at 158 A GeV.**



Chiral Symmetry

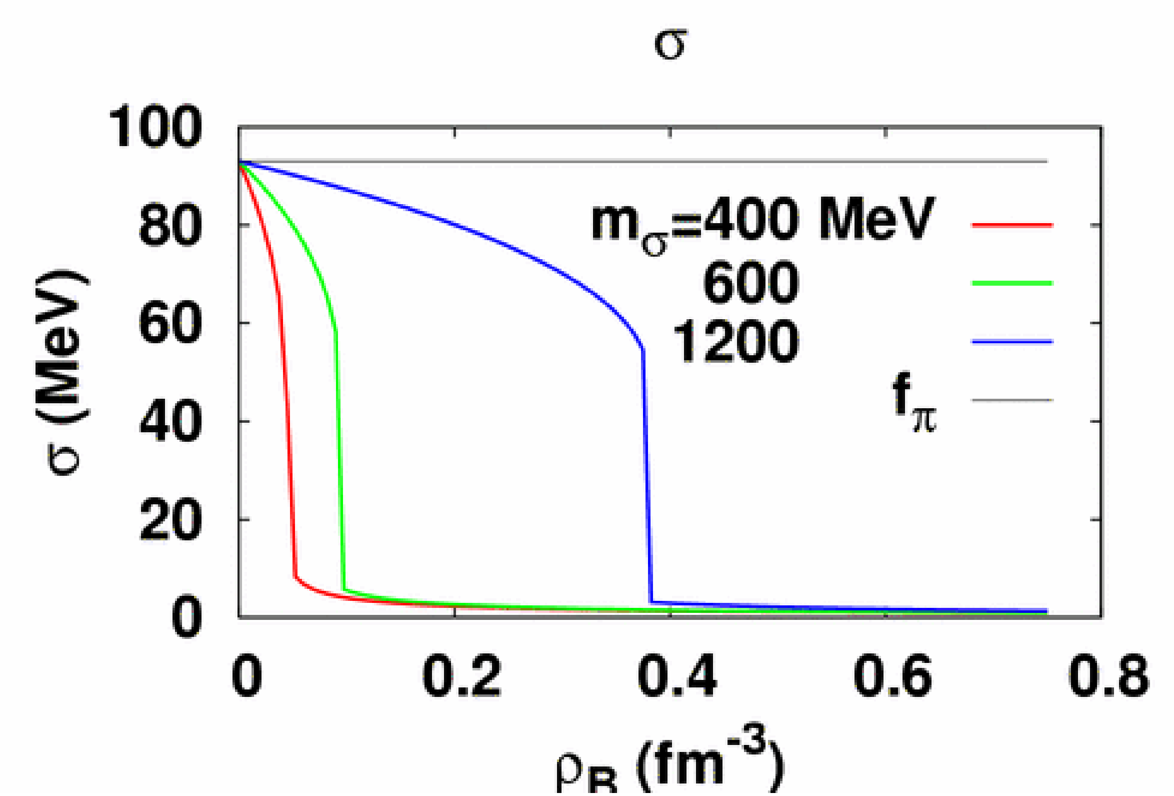
- **Good (approximate) Symmetry in QCD**
  - **In Flavor SU(2), only the small current quark mass term breaks chiral sym.**
  - **Should persist also in the hadronic world**
  - **Explains the small mass of pions, as Nambu-Goldstone particle of the chiral symmetry, and many other low energy hadronic properties.**
- **Schematic model: Linear σ model**
  - **Wine bottle shape of the effective potential → Spontaneous breaking of χ symmetry**
  - **Expectation Value of σ → Nucleon Mass**

$$L=\frac{1}{2}\left(\partial_\mu\sigma\,\partial^\mu\sigma+\partial_\mu\pi\,\partial^\mu\pi\right)-\frac{\lambda}{4}\left(\sigma^2+\pi^2\right)^2+\frac{\mu^2}{2}\left(\sigma^2+\pi^2\right)+c\,\sigma$$
$$+\bar{N}\,i\,\partial_\mu\gamma^\mu N-g_\sigma\bar{N}\left(\sigma+i\,\pi\,\tau\,\gamma_5\right)N$$

Chiral Linear σ Model at Fintite ρ_B (I)

T. D. Lee and G. C. Wick, Phys. Rev. D 9 (1974), 2291.

- **Serious problem:**
  Sudden chiral phase transition at relatively low baryon density.
  (Below ρ_0 if σ mass = 600 MeV)



Chiral Linear σ Model at Fintite ρ_B (II)

- "Vacuum" condition = Energy Minimum State

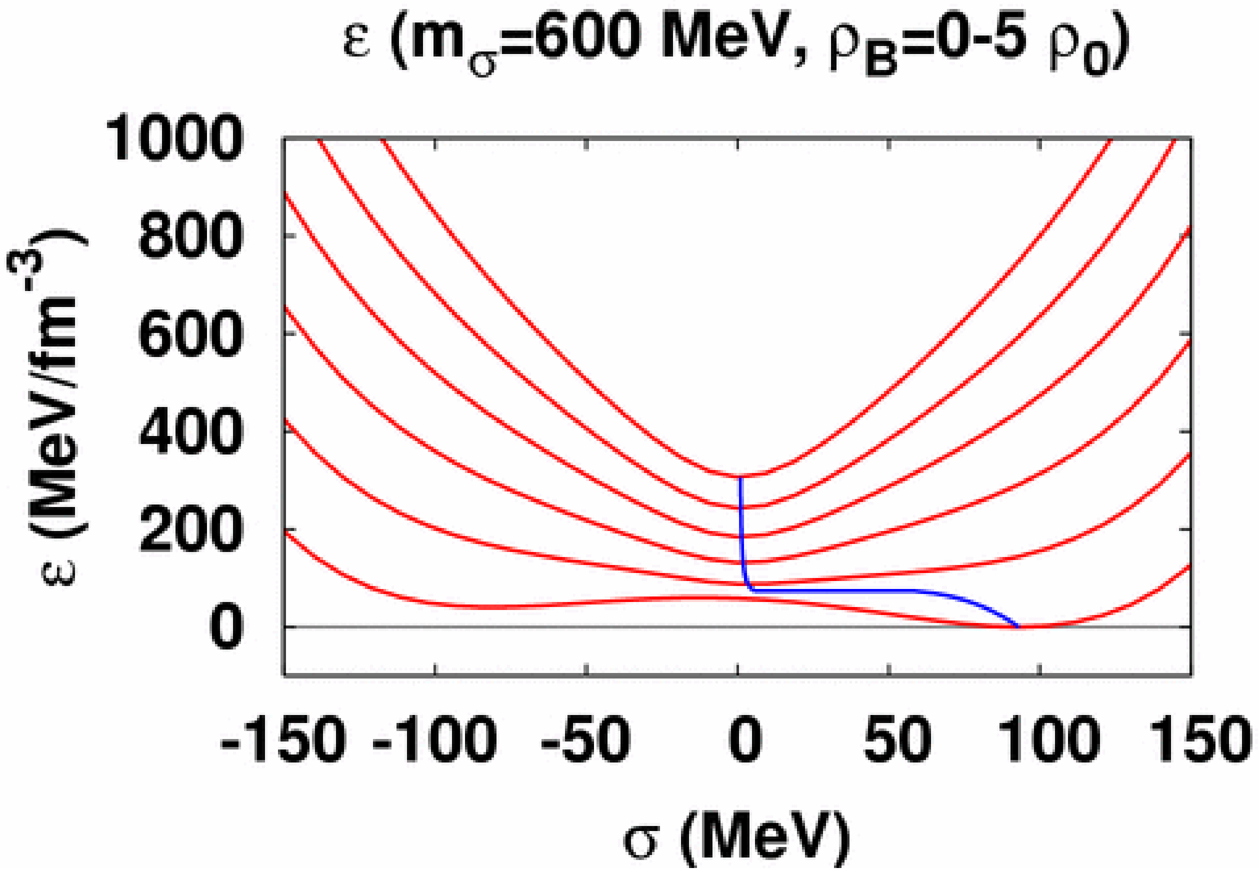
$$V = V_\sigma + E_N = \frac{\lambda}{4}\left(\sigma^2+\pi^2\right)^2 - \frac{\mu^2}{2}\left(\sigma^2+\pi^2\right) - c\,\sigma$$

$$+\int \frac{\gamma\, d^{3p}}{(2\pi)^3}\sqrt{p^2+(g_\sigma \sigma)^2}$$

$$\rightarrow \qquad \frac{\partial V}{\partial \sigma} = \frac{\partial V_\sigma}{\partial \sigma} + g_\sigma \rho_s = 0$$

- Large Nucleon Energy Gain for small <σ> due to mass decrease.

Chiral Linear σ Model at Fintite ρ_B (III)



- **We cannot avoid this sudden change even if we introduce ω meson-Nucleon coupling (indep. on <σ>)**
  - **Why do RMF models succeed ?**
  - **How about NJL model ?**

Chiral Symmetry

- **Good (approximate) Global Symmetry in QCD**

$$q \to q' = \exp(i\gamma_5 \theta_a \tau_a) q \quad \to \quad \bar{q}\gamma^\mu q = invariant$$

  - **Only the current quark mass term breaks chiral sym.**
  - **Should persist also in the hadronic world**
  - **Explains the small mass of pions, as Nambu-Goldstone particle of the chiral symmetry, and many other low energy hadronic properties.**
    (Y. Nambu and G. Jona-Lasino, Phys. Rev. 122('61),345; Phys. Rev. 124('61),246.)

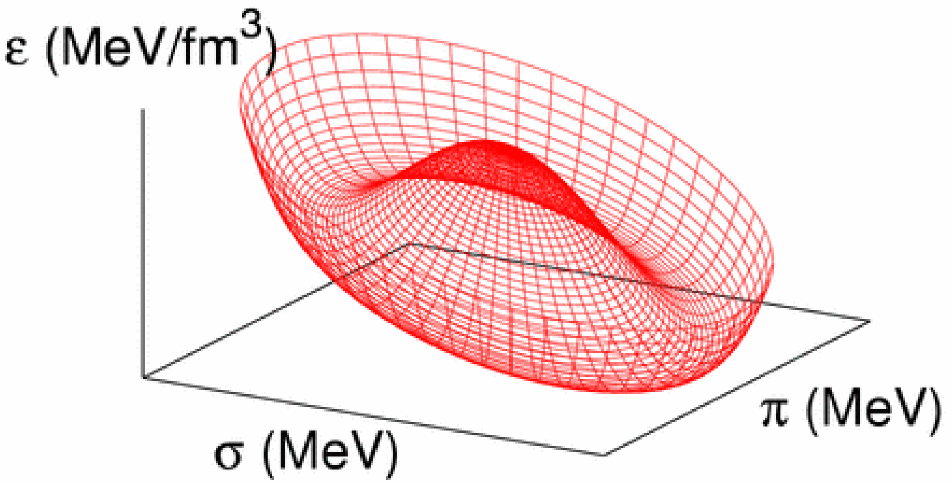
- **Schematic model: Linear σ model**
  (M. Gell-Mann and M. Levy, Nuovo Cimento 16 (1960), 705.)
  - **Wine bottle shape of the effective potential**
    → Spontaneous breaking of χ symmetry
  - **Expectation Value of σ → Nucleon Mass**

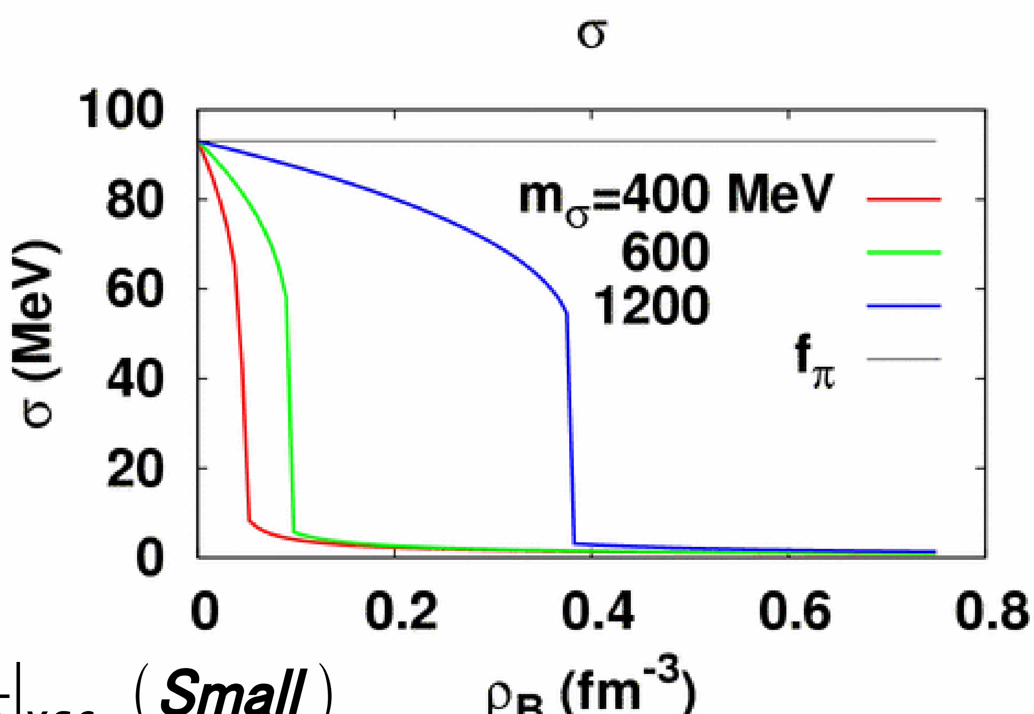
$$L = \frac{1}{2}\left(\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \pi \partial^\mu \pi\right) - \frac{\lambda}{4}\left(\sigma^2 + \pi^2\right)^2 + \frac{\mu^2}{2}\left(\sigma^2 + \pi^2\right) + c\sigma$$
$$+ \overline{N} i \partial_\mu \gamma^\mu N - g_\sigma \overline{N}\left(\sigma + i\pi\tau\gamma_5\right) N$$

RMF with Chiral Symmetry: Chiral Collapse (1)

T. D. Lee and G. C. Wick, Phys. Rev. D 9 (1974), 2291.

- **Serious problem:**
  Sudden chiral phase transition at relatively low baryon density.
  (Below ρ_0 if σ mass = 600 MeV)
  → "Chiral Collapse" or "Lee-Wick Vacuum" problem





$$m_\sigma^2 = \frac{\partial^2 \varepsilon}{\partial \sigma^2}\Big|_{vac} \quad (\textit{Large}), \qquad m_\pi^2 = \frac{\partial^2 \varepsilon}{\partial \pi^2}\Big|_{vac} \quad (\textit{Small})$$

RMF with Chiral Symmetry: Chiral Collapse (2)

- **Naïve Chiral RMF models → Chiral collapse at low ρ** *(Lee-Wick 1974)*
- **Prescriptions**
  - **σω coupling (too stiff EOS)** ***(Boguta 1983, Ogawa et al. 2004)***
  - **Loop effects (unstable at large σ)**
    (Matsui-Serot, 1982, Glendenning 1988, Prakash-Ainsworth 1987,Tamenaga et al. 2006)
  - **Higher order terms (unstable at large σ)** ***(Hatsuda-Prakash 1989, Sahu-Ohnishi 2000)***
  - ***Dielectric (Glueball) Field representing scale anomaly***
    (B.E. of nuclei are not well described)
    (Furnstahl-Serot 1993,Heide-Rudaz-Ellis 1994, Papazoglou et al.(SU(3)) 1998)
  - **Different Chiral partner assignment**
    (DeTar-Kunihiro 1989, Hatsuda-Prakash 1989, Harada-Yamawaki 2001, Zschiesche-Tolos-Schaffner-Bielich-Pisarski, nucl-th/0608044)
  - ***Nucleon Structure (Quark Meson Coupling)***
    (Saito-Thomas 1994, Bentz-Thomas 2001)

RMF with Chiral Symmetry: Chiral Collapse (3)

- **φ^4 Theory (Gell-Mann, Levy) → Collapse**

$$V_\sigma^{(\phi^4)} = \frac{\lambda}{4}(\phi^2 - f_\pi^2)^2 + \frac{1}{2}m_\pi^2\phi^2 - f_\pi m_\pi^2 \sigma\ , \quad \lambda = \frac{m_\sigma^2 - m_\pi^2}{2f_\pi^2}$$

- **NJL (Quark Loop) → Collapse**

$$V_\sigma^{\rm NJL} = \frac{m_0^2}{2}\sigma^2 + \Lambda^4 f_{\rm NJL}\left(\frac{G\sigma}{\Lambda}\right) - f_\pi m_\pi^2 \sigma \qquad f_{\rm NJL}(x) = -\frac{N_c N_f}{4\pi^2}\left[\left(1+\frac{x^2}{2}\right)\sqrt{1+x^2} - 1 - \frac{x^4}{2}\log\left(\frac{1+\sqrt{1+x^2}}{x}\right)\right]$$

- **Baryon Loop (Matsui & Serot) → Unstable at large σ**

$$V_\sigma^{\rm BL} = \frac{m_\sigma^2}{8f_\pi^2}(\phi^2 - f_\pi^2)^2 - M_N^4 f_{\rm BL}(\phi/f_\pi) \qquad f_{\rm BL}(x) = -\frac{1}{4\pi^2}\left[\frac{x^4}{2}\log x^2 - \frac{1}{4} + x^2 - \frac{3}{4}x^4\right]$$

- **Higher order terms (E.g. Sahu & Ohnishi)**

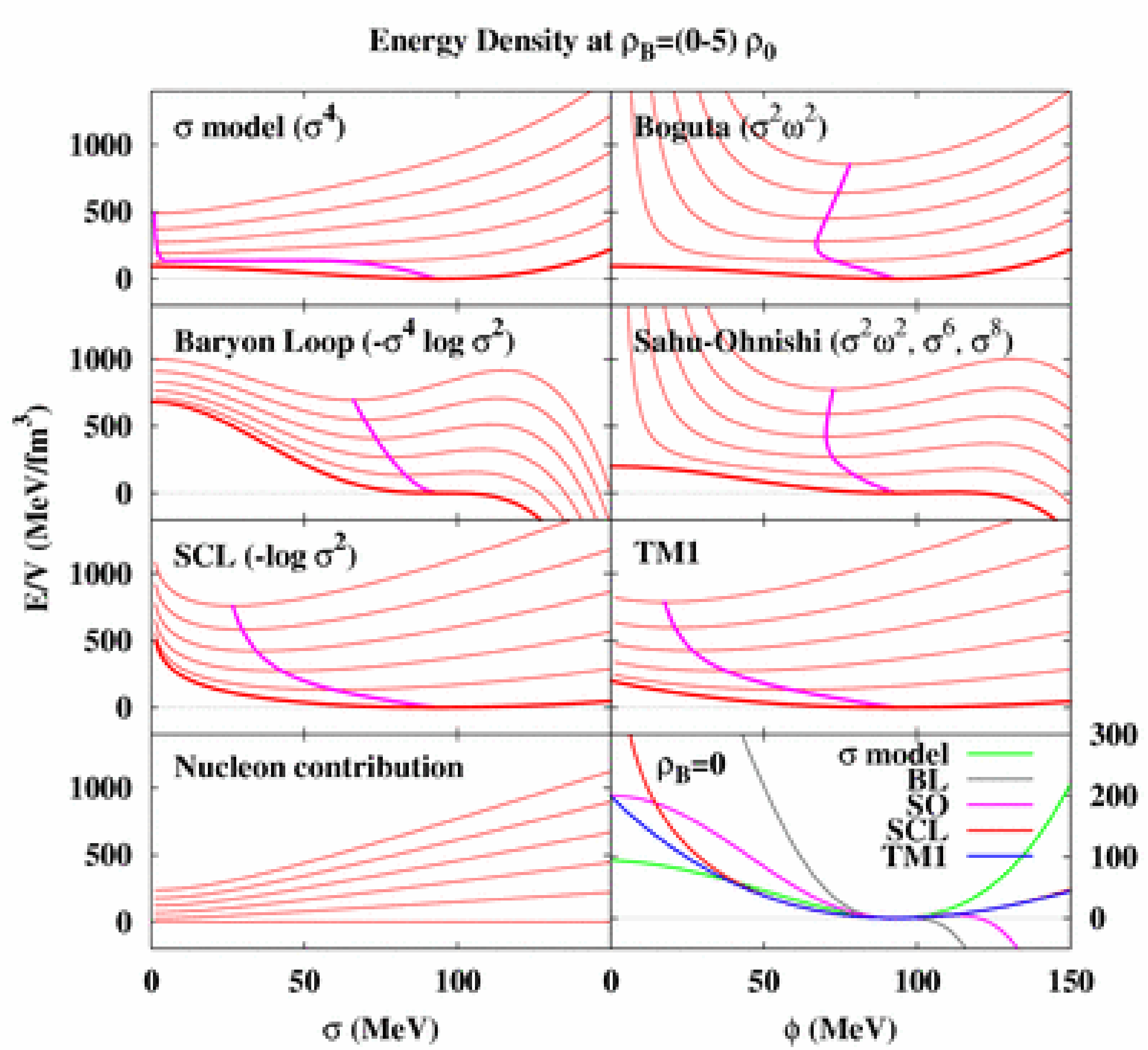
$$V_\sigma^{\rm SO} = \frac{m_\sigma^2}{8f_\pi^2}(\phi^2 - f_\pi^2)^2 + f_\pi^4 f_{\rm SO}(\phi/f_\pi) \qquad f_{\rm SO}(x) = \frac{C_6}{6}(x^2-1)^3 + \frac{C_8}{8}(x^2-1)^4$$

- **Log type term from scale anomaly (Furnstahl, Serot; Heide et al.)**
- **Log type term from SCL-LQCD (Tsubakihara & AO)**

$$V_\sigma^{\rm SCL} = V_\chi(\sigma,\pi) - c_\sigma\,\sigma = \frac{1}{2}b_\sigma\phi^2 - a_\sigma\log\phi^2 - c_\sigma\,\sigma$$

RMF with Chiral Symmetry: Chiral Collapse (3)

- **Most of the attempts do not work well.**



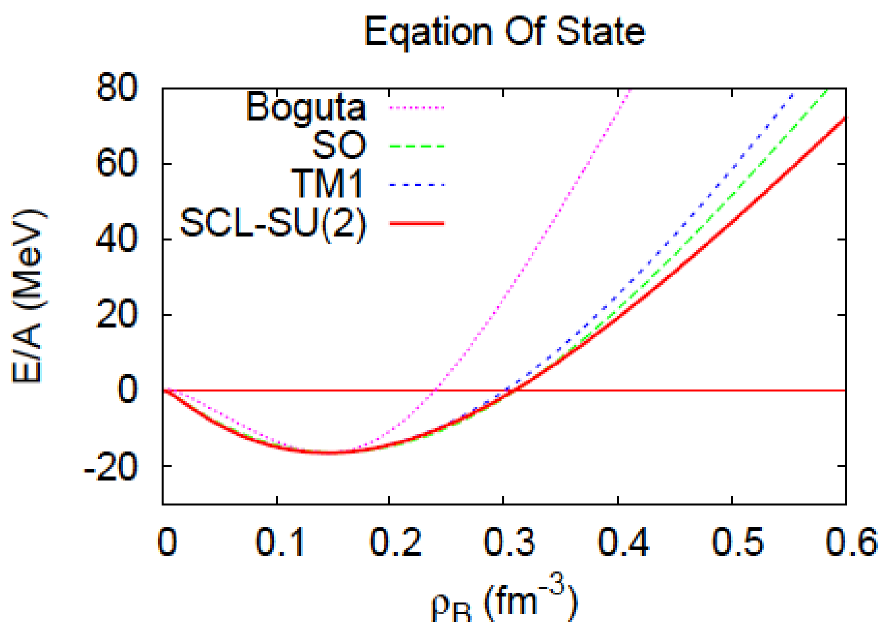
Chiral RMF based on SCL-LQCD

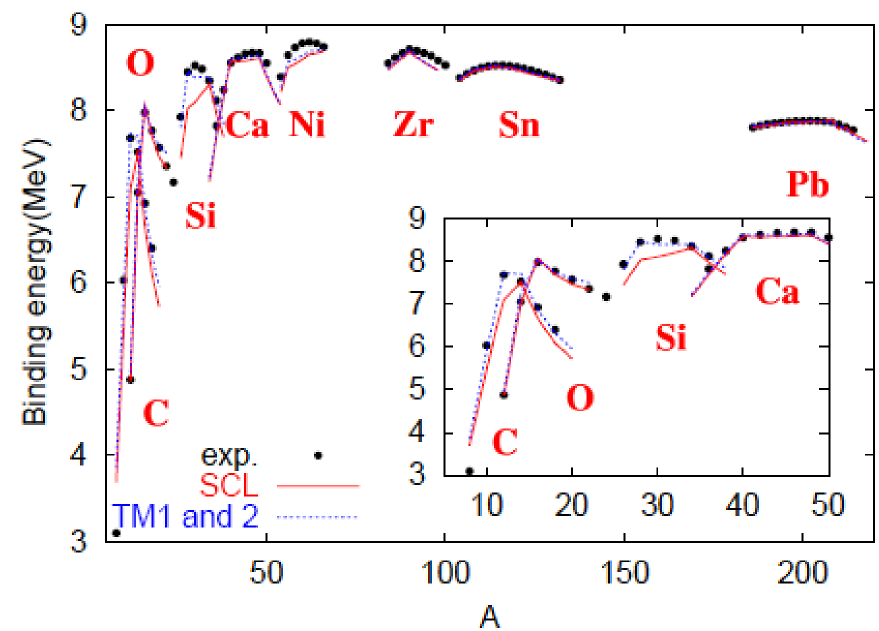
Tsubakihara, AO, PTP 117('07)903 [nucl-th/0607046]

- **強結合格子 QCD に基づく Chiral RMF 模型**

$$U_{L\sigma M}(\sigma)=-\frac{\mu^2}{2}\sigma^2+\frac{\lambda}{4}\sigma^4 \quad \longrightarrow \quad U_{\mathrm{SCL}}(\sigma)=\frac{1}{2}b_\sigma\sigma^2-a_\sigma\log\sigma$$

  - **QCD に基づき、カイラル対称性をもち、不安定性はない。**
  - **少ない数のパラメータで、核物質・原子核のバルクな性質をよく説明**





QCD から原子核の「密度汎関数」を与える第一歩！

Binding Energies in Chiral and Non-Chiral RMF

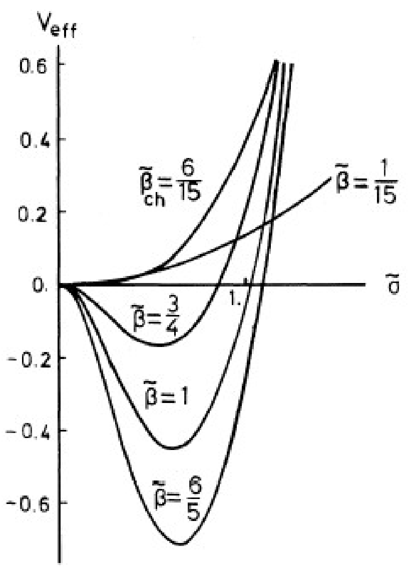
- **Non-Chiral High Precision RMF: TM1 & 2, NL1, NL3**
  (Sugahara, Toki, 1994; Reinhard et al., 1986; Lalazissis, Koenig, Ring, 1997)
- **Log term from Scale Anomaly: I/110, IF/110, VIIIF/100**
  (Heide, Rudas, Ellis, 1994)
- **Quark Meson Coupling model**
  (Saito, Tsushima, Thomas, 1997)

					B/A (MeV)					
Nucleus	exp.	SCL	TM1	TM2	NL1	NL3	I/110	IF/110	VIIIF/100	QMC-I
^{12}C	7.68	7.09	-	7.68	-	-	-	-	-	-
^{16}O	7.98	8.06	-	7.92	7.95	8.05	7.35	7.86	7.18	5.84
^{28}Si	8.45	8.02	-	8.47	8.25	-	-	-	-	-
^{40}Ca	8.55	8.57	8.62	8.48	8.56	8.55	7.96	8.35	7.91	7.36
^{48}Ca	8.67	8.62	8.65	8.70	8.60	8.65	-	-	-	7.26
^{58}Ni	8.73	8.54	8.64	-	8.70	8.68	-	-	-	-
^{90}Zr	8.71	8.69	8.71	-	8.71	8.70	-	-	-	7.79
^{116}Sn	8.52	8.51	8.53	-	8.52	8.51	-	-	-	-
^{196}Pb	7.87	7.87	7.87	-	7.89	-	-	-	-	-
^{208}Pb	7.87	7.87	7.87	-	7.89	7.88	7.33	7.54	7.44	7.25

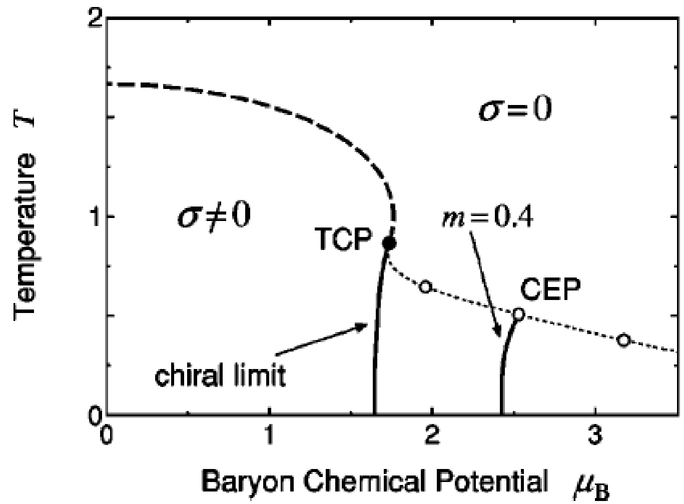
Strong Coupling Lattice QCD

Strong Coupling Limit of Lattice QCD

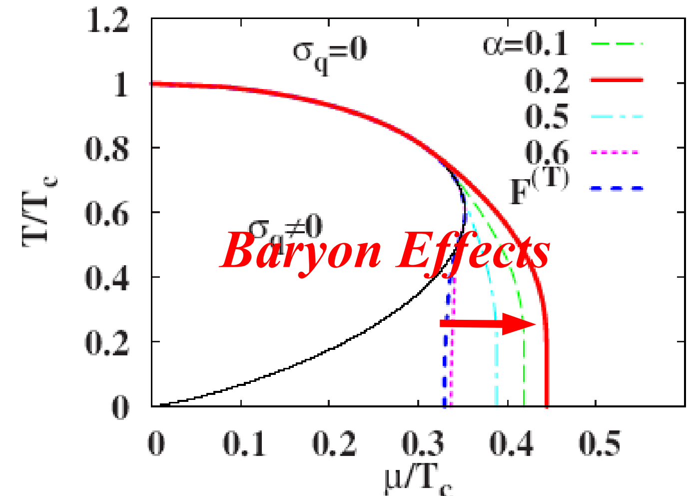
- **SCL-LQCD has been a powerful tool in "phase diagram" study !**
  - **Chiral restoration, Phase diagram, Baryon effects, Hadron masses, Finite coupling effects, ...**



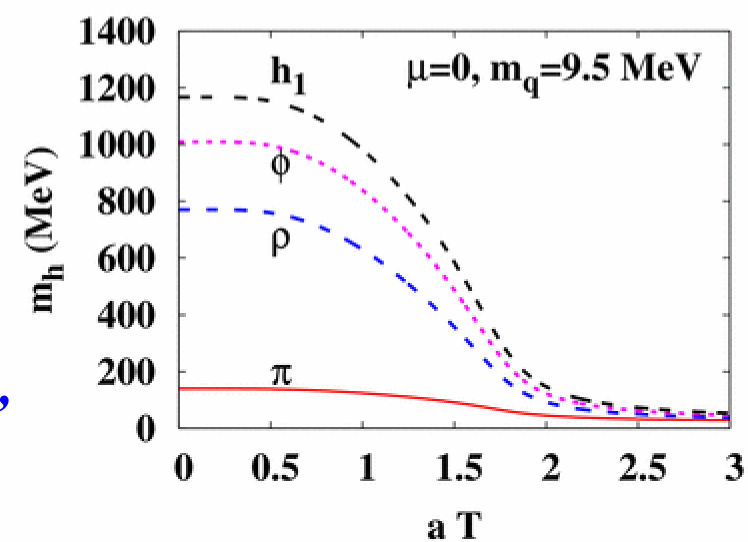
Damgaard,Kawamoto, Shigemoto, PRL53('84),2211



Nishida, PRD69, 094501 (2004)



Kawamoto,Miura,AO,Ohnuma, PRD75 (07), 014502.



AO, Kawamoto, Miura, 2008

Lattice QCD (1)

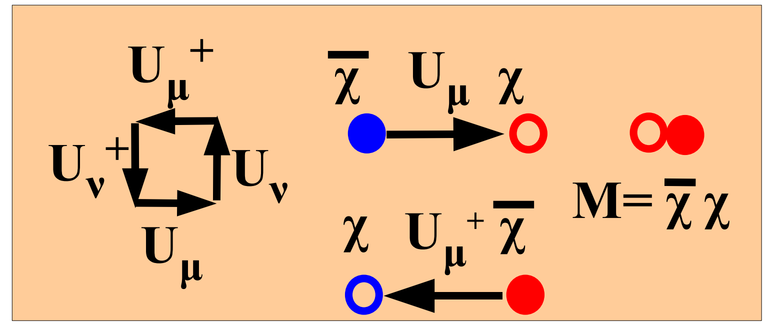
- **QCD Lagrangian**

$$L=\bar{\psi}(i\gamma^{\mu}D_{\mu}-m_0)\psi-\frac{1}{4}tr(F_{\mu\nu}F^{\mu\nu})$$

ψ = Quark, F = Gluon tensor, m_0 = (small) quark mass

- **Lattice Action**

$$S_{\rm QCD}=S_G+S_F^{(s)}+S_F^{(t)}+m_0\bar{\chi}\chi$$



$$S_G=-\frac{1}{g^2}\sum_{\rm plaq.}{\rm Tr}\,U_{ij}(x)+c.c.$$

$$S_F^{(s)}=\frac{1}{2}\sum_{x,\,j>0}\left(\bar{\chi}_x U_j(x)\chi_{x+\hat{j}}-\bar{\chi}_{x+\hat{j}}U_j^+(x)\chi_x\right)$$

$$S_F^{(t)}=\frac{1}{2}\sum_{x}\left(e^{\mu}\bar{\chi}_x U_0(x)\chi_{x+\hat{0}}-e^{-\mu}\bar{\chi}_{x+\hat{0}}U_0^+(x)\chi_x\right)$$

χ = starggered fermion (quark)
U= link variable $\in$ SU(N_c) (gluon),
μ = quark chemical potential

Lattice QCD (2)

- **Full QCD MC Simulation**
  → Monte-Carlo Integral of Det (Fermion Matrix) over link var. (U)
  - **Big Task !**
    Matrix Size= 4 (spinor) x (Color) x (Space-Time Points)
    Eigen Values are widely distributed
  - **Complex Weight with finite μ**

$$\int d\bar{\chi}\, d\chi\, dU \exp(-S_G + \bar{\chi} A \chi) = \int dU \left| \quad A \quad \right| \quad \updownarrow \mathbf{4\, N_c\, N_\tau\, N_s^3}$$

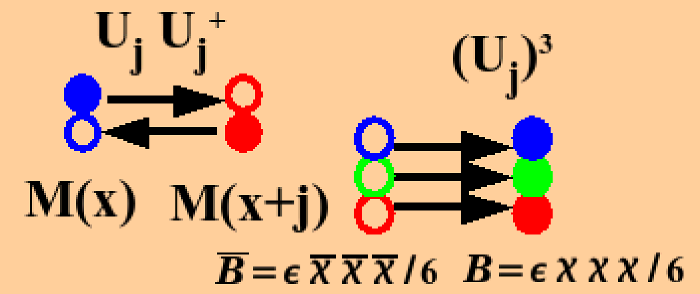
- **Quenched QCD**
  - **Assuming Det = 1 ~ Ignoring Fermion Loops**
  - **Works very well for hadron masses**
- ***Strong Coupling Limit (g → ∞)***
  - ***Pure gluonic action disappears → Analytic evaluation of Fermion Det.***

SCL-LQCD: Tools (1) --- One-Link Integral

- **Group Integral Formulae**

$$\int dU\, U_{ab} U^{+}_{cd} = \frac{1}{N_c} \delta_{ad} \delta_{bc}$$

$$\int dU\, U_{ab} U_{cd} U_{ef} = \frac{1}{6} \epsilon_{ace} \epsilon_{bdf}$$



$$\int dU\, U_{ab} U^{+}_{cd} = \frac{1}{N_c} \delta_{ad} \delta_{bc}$$

$$\int dU\, U_{ab} U_{cd} U_{ef} = \frac{1}{6} \epsilon_{ace} \epsilon_{bdf}$$

$$\int dU \exp(-a\bar{\chi}(x) U \chi(y) + b\bar{\chi}(y) U^{+} \chi(x))$$
$$= \int d\,U \left[1 - ab\, \bar{\chi}(a)^{a} U_{ab} \chi^{b}(y) \bar{\chi}^{c}(y) U^{+}_{cd} \chi^{d}(x) + \cdots \right]$$
$$= 1 + ab (\chi\bar{\chi})(x) (\chi\bar{\chi})(y) + \cdots = 1 + ab\, M(x) M(y) + \cdots$$
$$= \exp\left[ab\, M(x) M(y) + \cdots \right]$$

Quarks and Gluons → One-Link integral
→ Mesonic and Baryonic Composites

SCL-LQCD: Tools (2) --- 1/d Expansion

- **Keep mesonic action to be indep. from spatial dimension *d***
  → Higher order terms are suppressed at large *d*.

$$\sum_j (\bar{\chi} U_j \chi)(\bar{\chi} U_j^+ \chi) \to -\frac{1}{N_c} \sum_j M(x) M(x+\hat{j}) = O(1)$$

$$\to M \propto 1/\sqrt{d}\,,\ \chi \propto d^{-1/4}$$

$$\sum_j (\bar{\chi} U_j \chi)(\bar{\chi} U_j \chi)(\bar{\chi} U_j \chi) \to N_c! \sum_j B(x) B(x+\hat{j}) = O(1/\sqrt{d})$$

$$\sum_j (\bar{\chi} U_j \chi)^2 (\bar{\chi} U_j^+ \chi)^2 \to \sum_j M^2(x) M^2(x+\hat{j}) = O(1/d)$$

We can stop the expansion in U,
since higher order terms are suppressed !

SCL-LQCD: Tools (3) --- Bosonization

- **We can reduce the power in χ by introducing bosons**

$$\exp\left(\frac{1}{2}M^2\right)=\int d\sigma \exp\left(-\frac{1}{2}\sigma^2-\sigma M\right)$$

$$\text{Nuclear MFA:}\quad V=-\frac{1}{2}(\bar{\psi}\psi)(\bar{\psi}\psi)\simeq -U(\bar{\psi}\psi)+\frac{1}{2}U^2$$

$$\exp\left[-\frac{1}{2}M^2\right]=\int d\varphi \exp\left[-\frac{1}{2}\varphi^2-i\varphi M\right]$$

Reduction of the power of χ
→ Bi-Linear form in χ → Fermion Determinant

SCL-LQCD: Tools (4) --- Grassman Integral

- **Bi-linear Fermion action leads to -log(det A) effective action**

$$\int d\chi\, d\bar{\chi} \exp\left[\bar{\chi} A \chi\right] = det\, A = \exp\left[-(-\log det\, A)\right]$$

$$\int d\chi \cdot 1 = \text{anti-comm. constant} = 0 \quad , \quad \int d\chi \cdot \chi = \text{comm. constant} \equiv 1$$

$$\int d\chi\, d\bar{\chi} \exp\left[\bar{\chi} A \chi\right] = \int d\chi\, d\bar{\chi} \frac{1}{N!} (\bar{\chi} A \chi)^N = \cdots = det\, A$$

Constant σ → - log σ interaction (Chiral RMF)

- **Temporal Link Integral, Matsubara product, Staggered Fermion,**
  → I will explain next time ....

Effective Potential in SCL-LQCD (Zero T)

- **QCD Lattice Action (Zero T treatment)**

Kawamoto, Smit, 1981

$$S = \cancel{S_G} + S_F + m_0 \bar{\chi}\chi$$

Strong Coupling Limit

$$\rightarrow -\frac{1}{2}(\bar{\chi}\chi) V_M (\bar{\chi}\chi) + m_0 \bar{\chi}\chi$$

One-link integral (1/d expansion*)

$$\rightarrow \frac{1}{2}\sigma V_M^{-1}\sigma + \bar{\chi}(\sigma + m_0)\chi$$

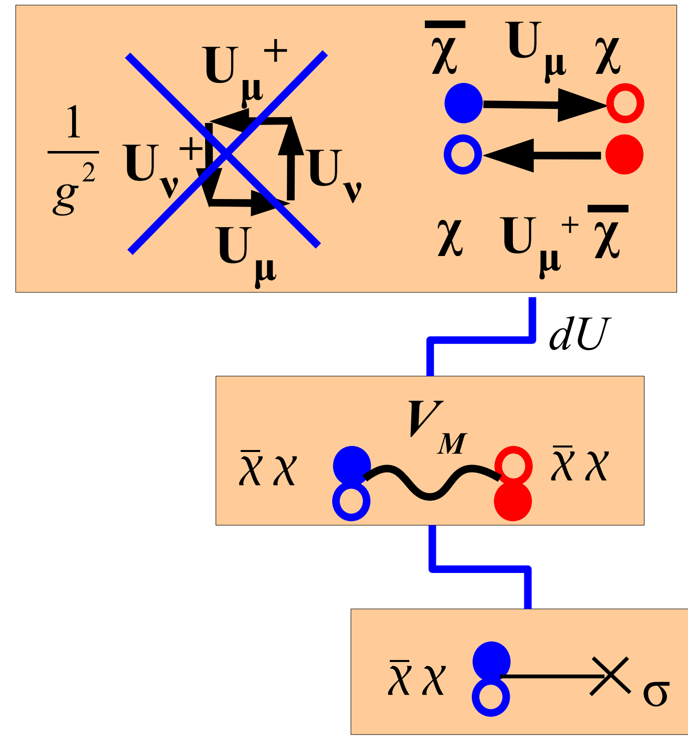
Bosonization

$$\rightarrow \frac{1}{2}\sigma V_M^{-1}\sigma - N_c \sum_x \log(\sigma(x) + m_0)$$

Fermion Integral

$$= L^d N_\tau \left[\frac{N_c}{d+1}\bar{\sigma}^2 - N_c \log(\bar{\sigma} + m_0) \right]$$

Effective Potential



*** d = Spatial dim.**

Fermion Matrix = Just a number
→ Simple Logarithmic Effective Potential for σ

$$V_\sigma = \frac{1}{2} a_\sigma \sigma^2 - b_\sigma \log \sigma$$

Effective Potential in SCL-LQCD (Zero T)

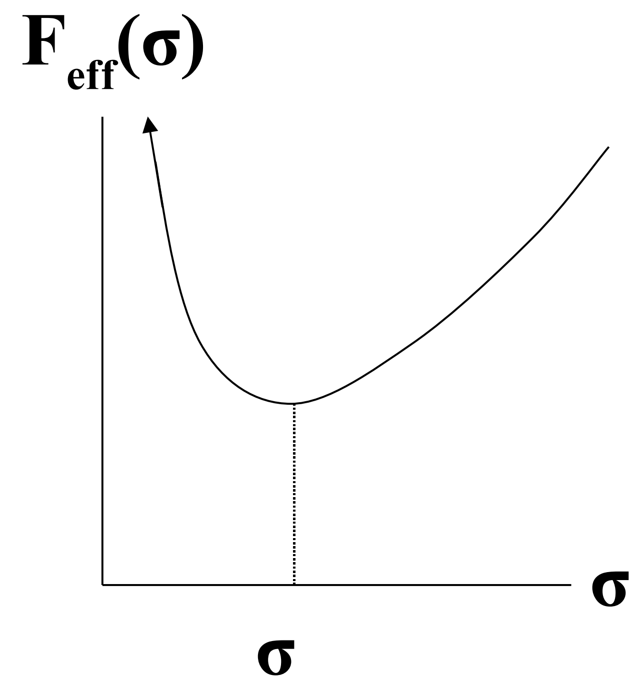
- **Effective Pot. at Zero T**

  Kawamoto, Smit, 1981
  Kluberg-Stern, Morel, Napoly, Petersson, 1981

$$F_{eff}(\sigma)=\frac{1}{2}a_\sigma\sigma^2-b_\sigma\log\sigma$$

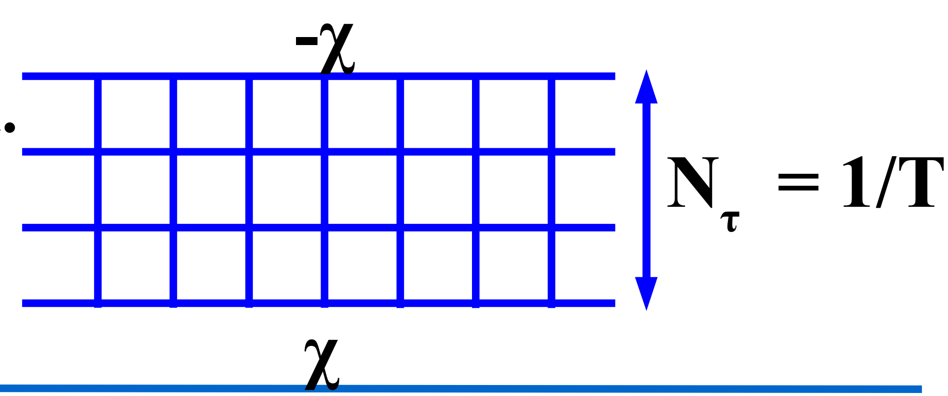
Spontaneous Chiral Symmetry breaking at T=0 is naturally explained !

No Phase Transition ?



- **Grassman integral at each space-time point in Zero T treatment**
  → "Temporal" Correlation and Anti-periodic Boundary Cond. would be important at Finite T !

Let's go to Finite T



Effective Potential in SCL-LQCD (Finite T)

- **QCD Lattice Action (Finite T treatment)**

  Damgaard, Kawamoto, Shigemoto, 1984; Bilic et al., 1992; Nishida, '04; Fukushima, '04; Kawamoto, Miura, AO, Ohnuma, '07; ...

$$S = \cancel{S_G} + S_F^{(s)} + S_F^{(t)} + m_0 \bar{\chi}\chi$$

Strong Coupling Limit

$$S_F^{(t)} = \frac{1}{2}\sum_x \left(e^{\mu} \bar{\chi}_x U_0(x) \chi_{x+\hat{0}} - e^{-\mu} \bar{\chi}_{x+\hat{0}} U_0^+(x) \chi_x \right) = \bar{\chi} V^{(t)} \chi$$

$$\rightarrow -\frac{1}{2}(\bar{\chi}\chi) V_M (\bar{\chi}\chi) + \bar{\chi}(V^{(t)} + m_0)\chi$$

Spatial-link integral (1/d expansion, no B)

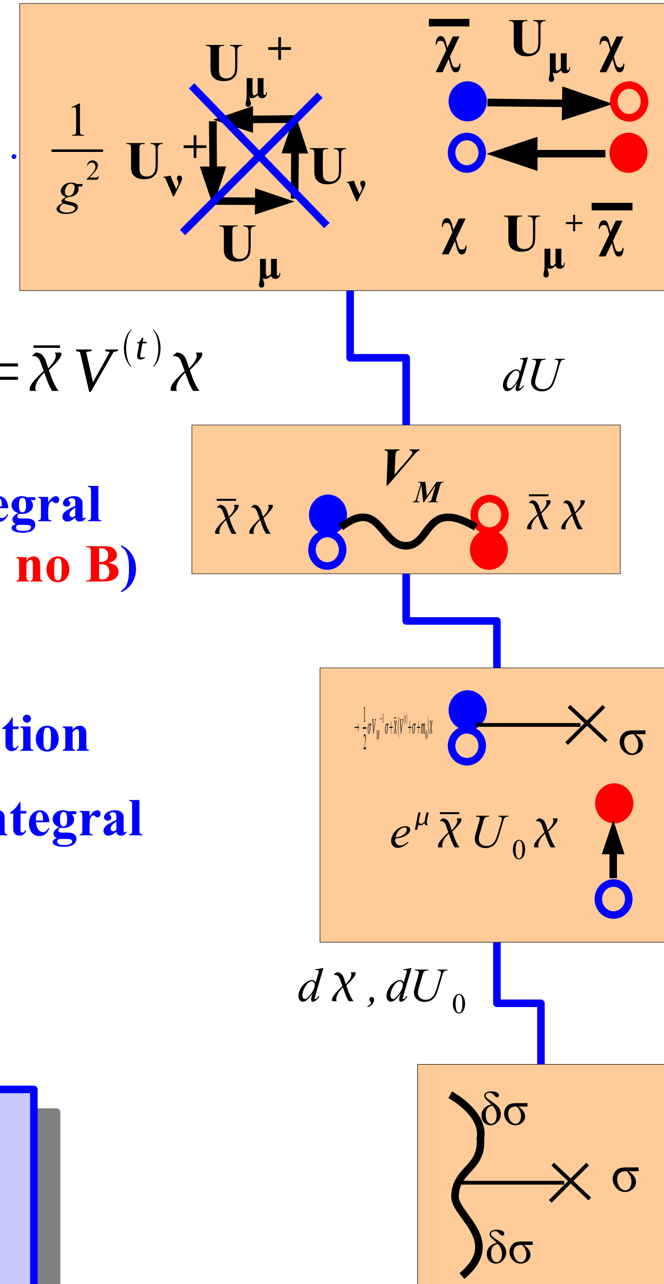
$$\rightarrow \frac{1}{2}\sigma V_M^{-1}\sigma + \bar{\chi}(V^{(t)} + \sigma + m_0)\chi$$

Bosonization

Fermion and Temporal-link Integral

$$\rightarrow L^d N_\tau \left[\frac{N_c}{d}\bar{\sigma}^2 + V_{\text{eff}}(\bar{\sigma}, T, \mu) \right]$$

Effective Potential



We need to evaluate Det. (Nc x Nτ)
→ It is POSSIBLE !

Fermion Determinant

Faldt, Petersson, 1986

- **Fermion action is separated to each spatial point and bi-linear**
  → Determinant of Nτ x Nc matrix

$$\exp(-V_{\rm eff}/T)=\int dU_0 \begin{vmatrix} I_1 & e^{\mu} & 0 & & e^{-\mu}U^+ \\ -e^{-\mu} & I_2 & e^{\mu} & & \\ 0 & -e^{-\mu} & I_3 & e^{\mu} & \\ \vdots & & & \ddots & \\ -e^{\mu}U & & & -e^{-\mu} & I_N \end{vmatrix}$$

Nc x Nτ

$$=\int dU_0 \, det\left[X_N[\sigma]\otimes \mathbf{1}_c+e^{-\mu/T}U^+ +(-1)^{N_\tau}e^{\mu/T}U\right]$$

Nc

$$I_k=2(\sigma(k)+m_0)$$

$$X_N=\begin{vmatrix} I_1 & e^{\mu} & 0 & \cdots & & e^{-\mu} \\ -e^{-\mu} & I_2 & e^{\mu} & & & 0 \\ 0 & -e^{-\mu} & I_3 & & & 0 \\ \vdots & & & \ddots & & \\ & & & & I_{N-1} & e^{\mu} \\ -e^{\mu} & 0 & 0 & \cdots & -e^{-\mu} & I_N \end{vmatrix} - \left[e^{-\mu/T}+(-1)^N e^{\mu/T}\right]$$

Effective Potential in SCL-LQCD (Time dependence...)

- **Zero T, no Baryon** *Kawamoto, Smit, 1981*

$$\mathcal{F}_{\rm eff}^{(0)} = \frac{1}{2} b_\sigma^{(0)} \sigma^2 - N_c \log(b_\sigma^{(0)}\sigma + m_0)$$

- **Zero T, with Baryon**
  Damgaad, Hochberg, Kawamoto, 1985

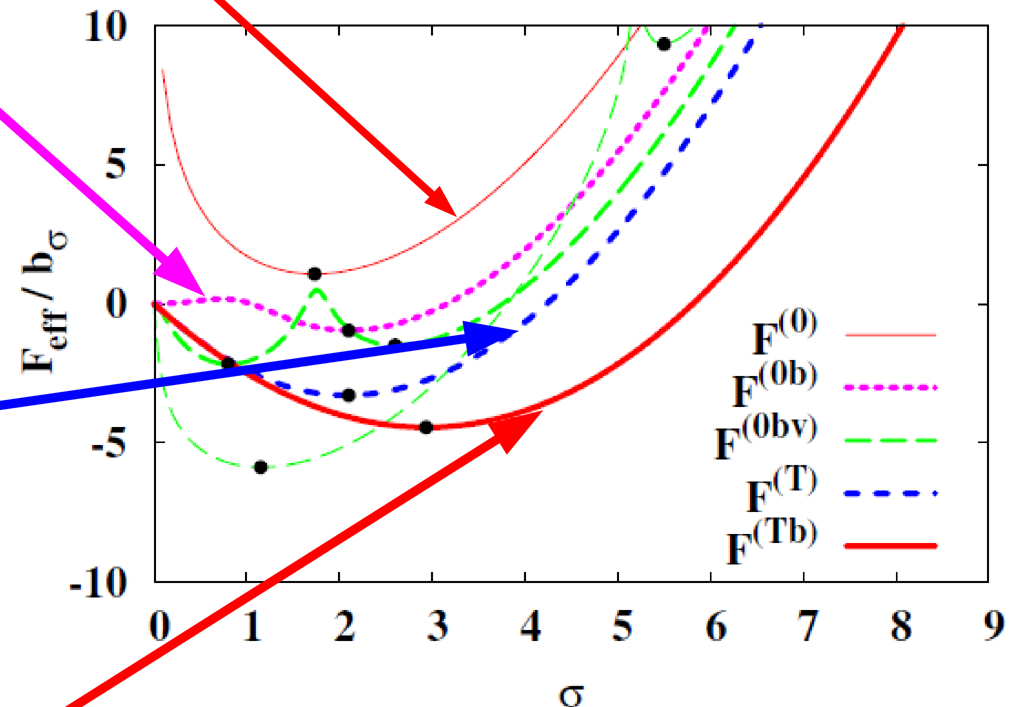
$$\mathcal{F}_{\rm eff}^{(0b)} = \frac{1}{2} b_\sigma^{(0)} \sigma^2 + F_{\rm eff}^{(b\mu)}(4{m_q}^3; T, \mu)$$

- **Finite T, no Baryon**
  Fukushima, 2004; Nishida, 2004

$$\mathcal{F}_{\rm eff}^{(T)} = \frac{1}{2} b_\sigma^{(T)} \sigma^2 + F_{\rm eff}^{(q)}(m_q)$$

- **Finite T, with Baryon**
  Kawamoto, Miura, AO, Ohnuma, 2007

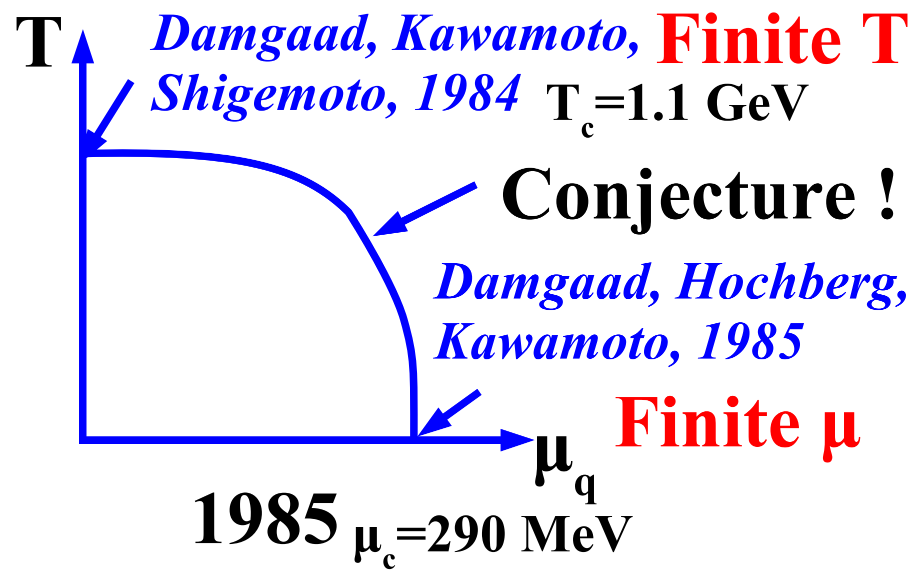
$$\mathcal{F}_{\rm eff} = \frac{b_\sigma}{2}\sigma^2 + F_{\rm eff}^{(q)}(m_q) + \Delta F_{\rm eff}^{(b)}(g_\sigma \sigma)$$

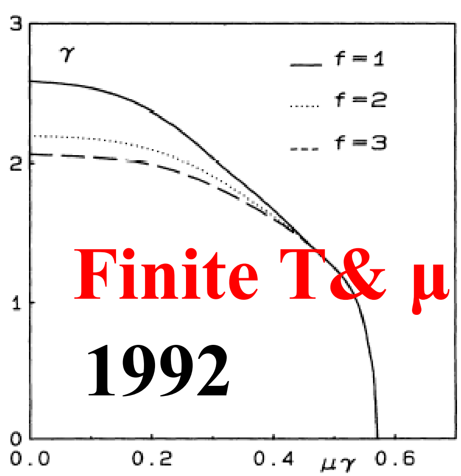


$$F_{\rm eff}^{(q)}(m_q) = -T\log\left(\frac{\sinh((N_c+1)E(m_q)/T)}{\sinh(E(m_q)/T)} + 2\cosh(N_c\mu/T)\right)$$

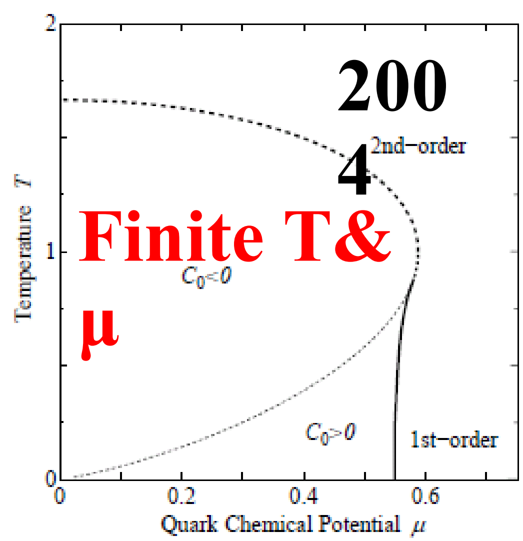
Evolution of Phase Diagram

- **Phase Diagram "Shape" becomes closer to that of Real World,**
  $R=3\,\mu_c/T_c \sim (6\text{-}12)$
  - **1985 → R=0.79 (Zero T / Finite T)**
  - **1992 → R=0.83 (Finite T & μ)**
  - **2004 → R= 0.99 (Finite T& μ)**
  - **2007 → R=1.34 (Baryon)**

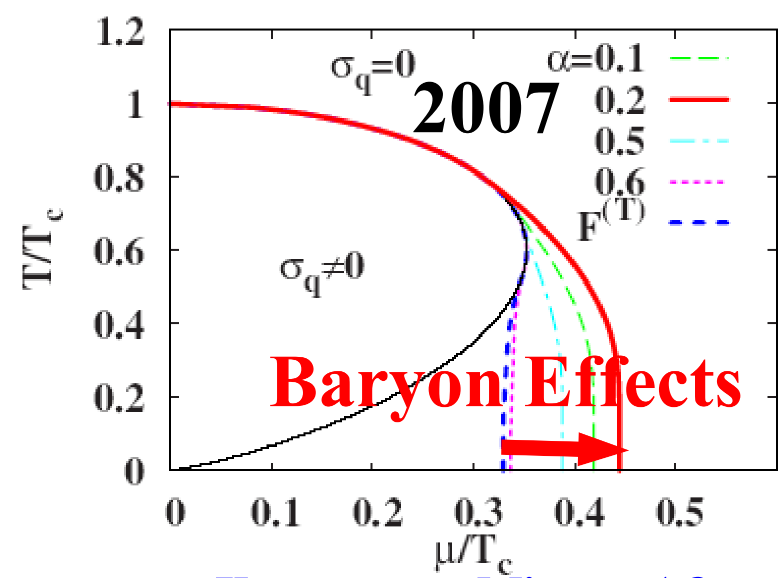




Bilic, Karsch, Redlich, 1992



Fukushima, 2004



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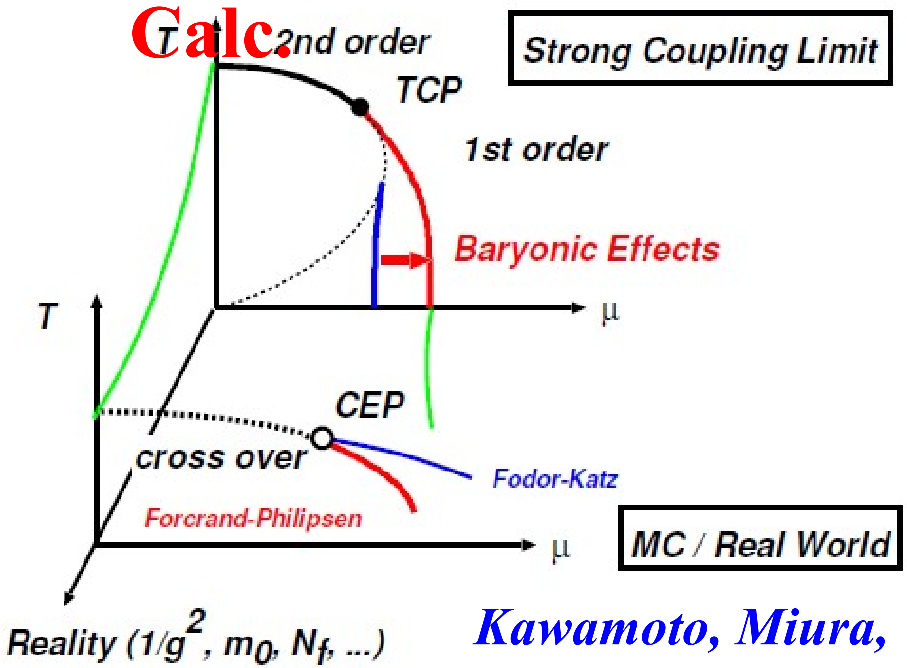
Towards the Real Phase Diagram

- **When we increase "Reality" variable,**
  Phase diagram "Shape" may be approximately explained.
  Real World: $R=3\ \mu_c/T_c \sim (6\text{-}12)$
  SCL-LQCD: R=0.79-1.34
  SC-LQCD with finite β ($=6/g^2$)~5 → R ~ 4.5

Expectation before Calc.



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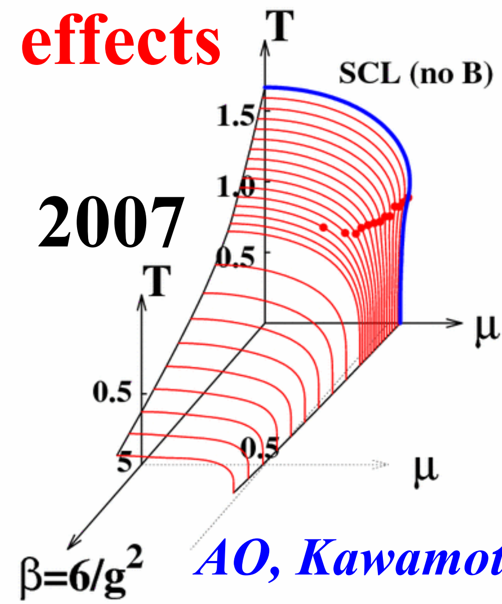
Calc. with $1/g^2$ effects

2007



AO, Kawamoto, Miura, 2007

Gluon Contribution is important at High T

As a "Nuclear" Physicist ....

- **Strong Coupling Lattice QCD may be a promising tool to understand uniform QCD matter, if "Reality" variable is enhanced**
  - **Strong Coupling Limit ($g = \infty$) → Strong Coupling Expansion (g = finite)**
  - **Baryon Effects / Higher Order terms in Fermions**
  - **Staggered Fermion → Wilson/DW/Overlap Fermion**
  - **.....**
- **But even if we can solve QCD for uniform quark matter, Nuclear Physicists are not satisfied, unless we understand NUCLEI**

How can we apply SC-LQCD results in "Nuclear" Physics ?
Effective Potential → Hadronic Lagrangian

TABLE II: RMF paramete

	$g_{\sigma N}$	$g_{\omega N}$	$g_{\rho N}$	g_3(MeV)	g_4	
NL1[18]	10.138	13.285	4.976	2401.9	-36.265	
NL3[19]	10.217	12.868	4.474	2058.35	-28.885	
TM1[6]	10.0289	12.6139	4.6322	1426.466	0.6183	71.3(
SCL[20](*1)	10.08	13.02	4.40	1255.88	13.504	2

(*1): g_3 and g_4 are from the expansion of $f_{\rm SCL}$.