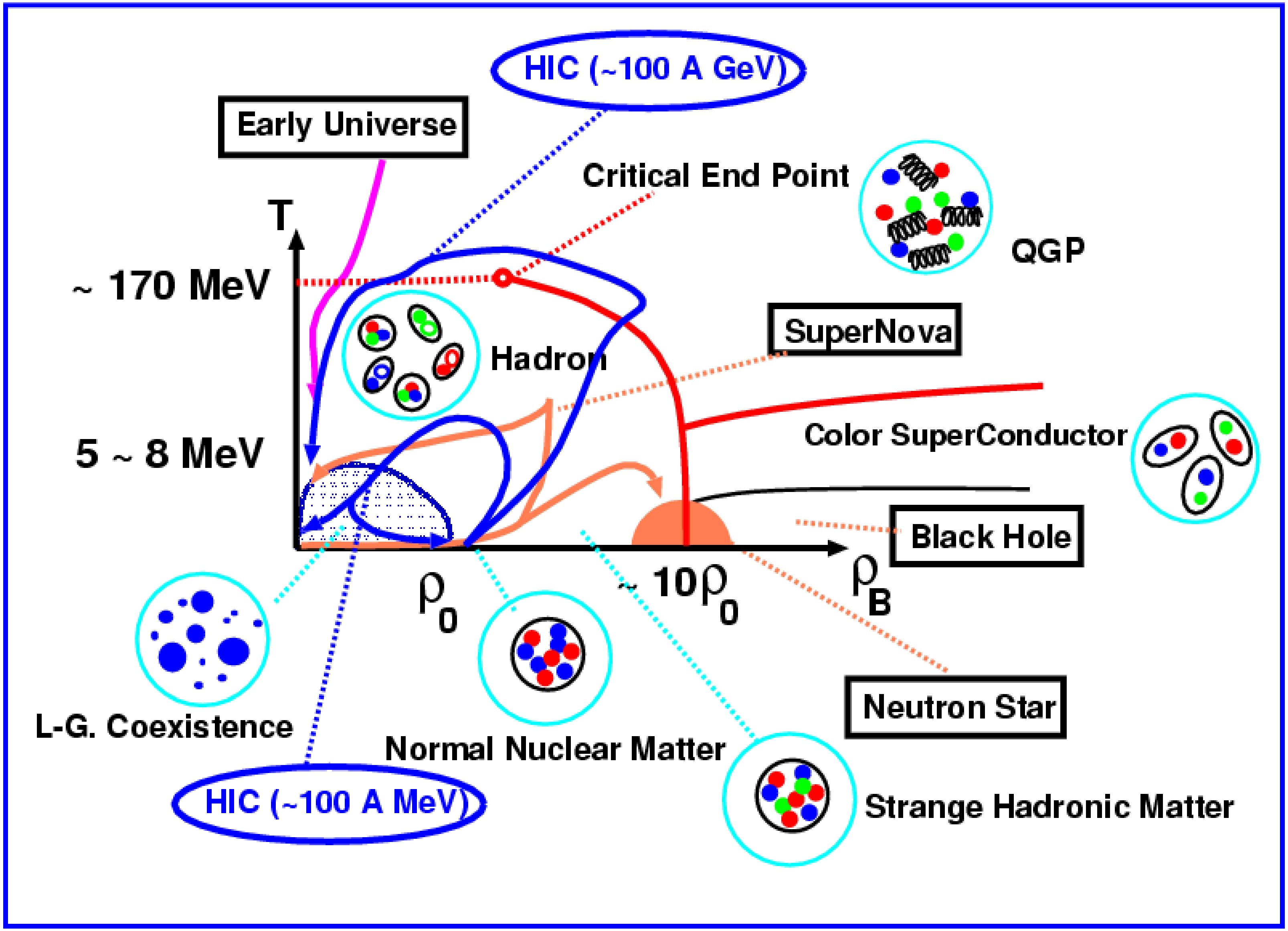
ハイペロン・ポテンシャルと高密度物質の状態方程式

A. Ohnishi (YITP, Kyoto Univ.)

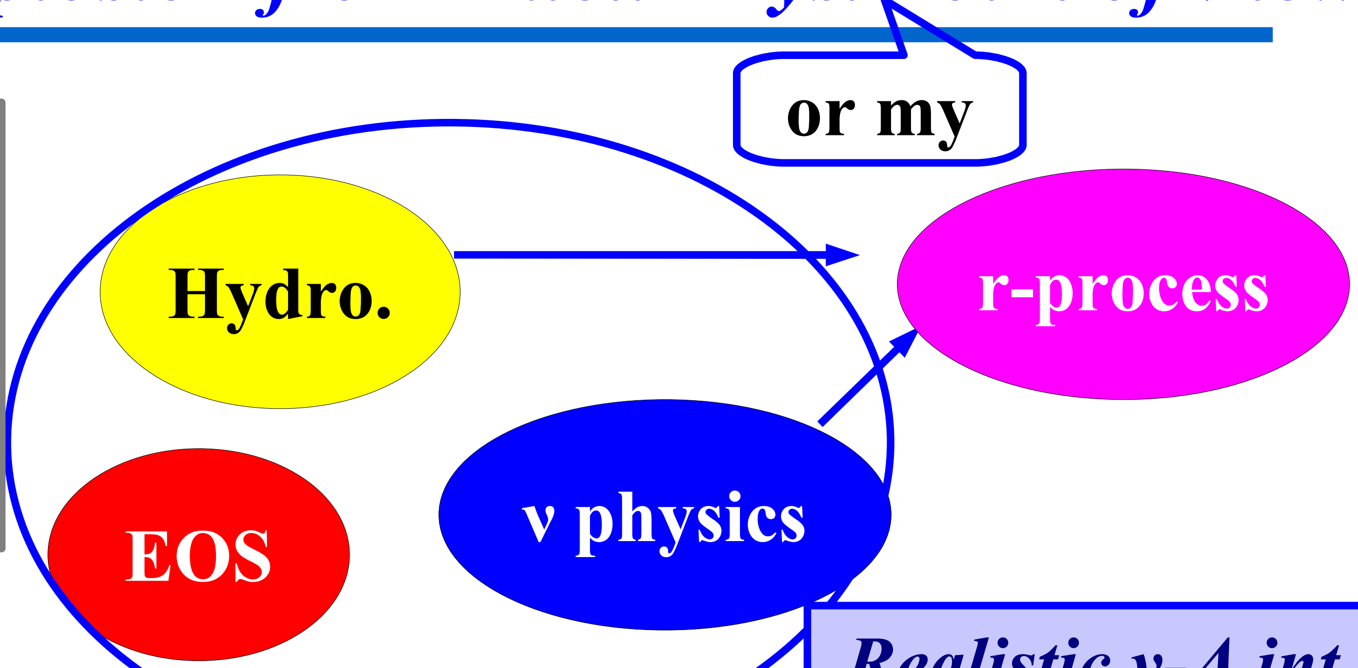
- **Introduction**
- **Σ potential in nuclear matter (Maekawa (PhD, March 2008))**
  Maekawa, Tsubakihara, AO, EPJA33('07),269 [nucl-th/0701066]
  Maekawa, Tsubakihara, Matsumiya, AO, arXiv:0704.3929
  Maekawa, Thesis, March 2008, Hokkaido Univ.
- **Coupled Channel AMD Study of Ξ hypernuclear structure**
  Matsumiya, Maekawa, Tsubakihara, AO, Dote, Kimura, in prep.
- **Relativistic EOS of Supernova Matter with Hyperons**
  Ishizuka, AO, Tsubakihara, Sumiyoshi, Yamada, arXiv:0802.2318
  AO et al., AIP Conf. Proc., to appear.
- **Hypernuclei and Hyperonic Matter in Chiral SU(3) RMF**
  Tsubakihara, AO, PTP 117('07)903 [nucl-th/0607046]
  Tsubakihara, Maekawa, AO, EPJA33('07)295 [nucl-th/0702008]
  Tsubakihara, Matsumiya, Maekawa, AO, AIP conf. proc., to appear
- **Summary**

Quark / Hadron / Nuclear Matter Phase Diagram



Rich Structure / Astrophysical implications / Accessible in HIC

Supernova Explosion from Nucl. Phys. Point of View



Multi Dim.
Instability
Magnetic Field
Acoustic Revival
....

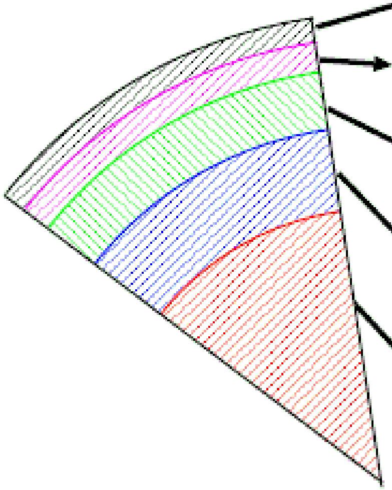
EOS tables
Lattimer-Swesty (1981)
Rel. (Shen) EOS (1998)
→ How to extend ?

Realistic ν-A int.
Nuclear Dist.
Exact ν transfer
.....

- **Supernovae DO NOT EXPLODE in theor. calculation at present with realistic microphysics inputs. → How can we succeed ?**
  - **Multi-Dim. Hydro (Instability)+*Additional Energy Release* (10 %-factor 10)**

Hyperons in Dense Matter

- **What appears at high density ?**
  - **Nucleon superfluid ($^3S_1$, $^3P_2$)**
  - **Pion condensation, Kaon condensation, Baryon Rich QGP**
  - **Hyperons**
    Tsuruta, Cameron (66); Langer, Rosen (70); Pandharipande (71); Itoh(75); Glendenning; Weber, Weigel; Sugahara, Toki; Schaffner, Mishustin; Balberg, Gal; Baldo et al.; Vidana et al.; Nishizaki,Yamamoto, Takatsuka; Kohno,Fujiwara et al.; Sahu,Ohnishi; Ishizuka, Ohnishi, Sumiyoshi, Yamada; ...



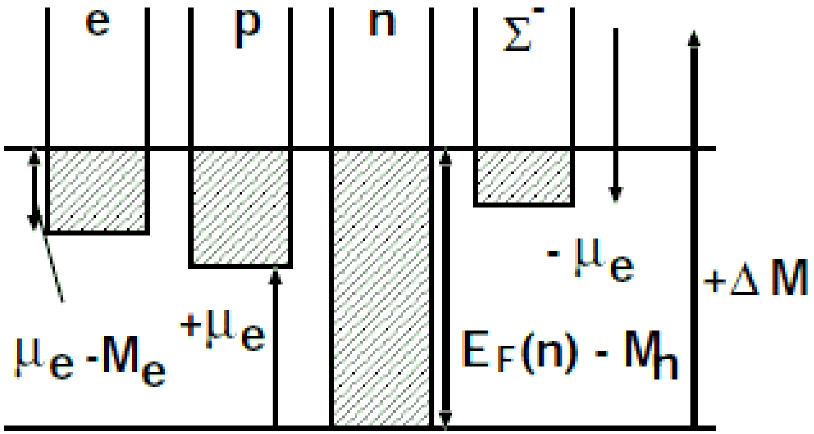
Outer Crust

Nuclei, Electrons,
Unpinned Neutron Superfluid

Neutron-rich Nuclei,
Electrons,
Pinned Neutron Superfluid

$^3P_2$ Superfluid Neutrons
$^1S_0$ Superfluid Protons
Electrons

Pion Condensation?
Hyperon?
QGP?

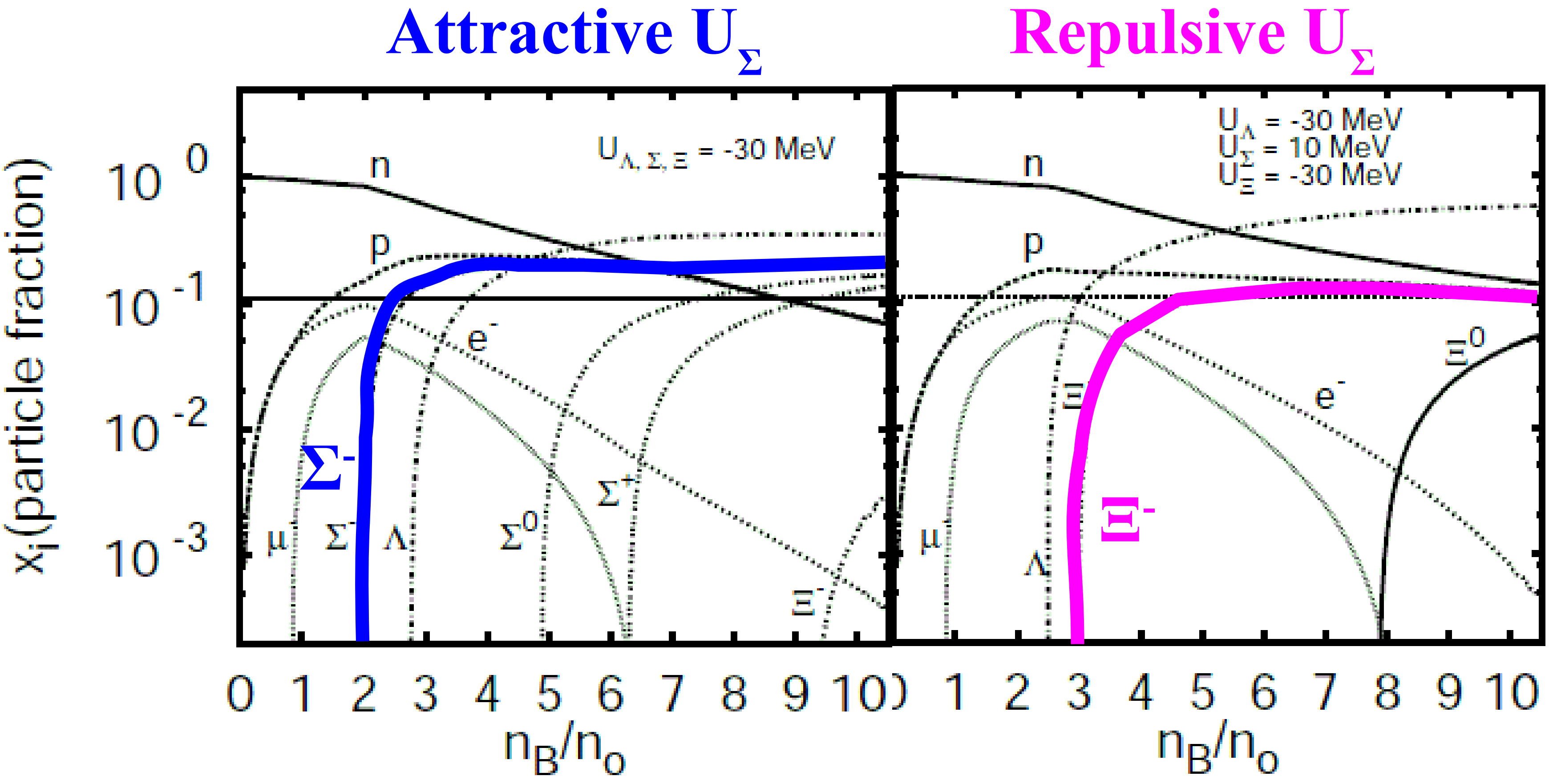


Nobody says "Hyperons do not appear in neutron star core" !

Y appears when $\mu_B = E_F(n) + U(n) \geq M(Y) + U(Y) + Q_Y \mu_e$

Hyperons in Supernova Matter

- **Problems to include hyperons in Supernova Matter EOS**
  - **Uncertainties of hyperon potentials $U_Y(\rho)$ (e.g. Balberg, Gal, 1997)** → ***Recent Hypernuclear Phys.***
  - **Density may not be very high in supernova** → ***Needed in cooling stage***



Sahu, AO, 2003

My (Personal) Interest in Hypernuclear Physics

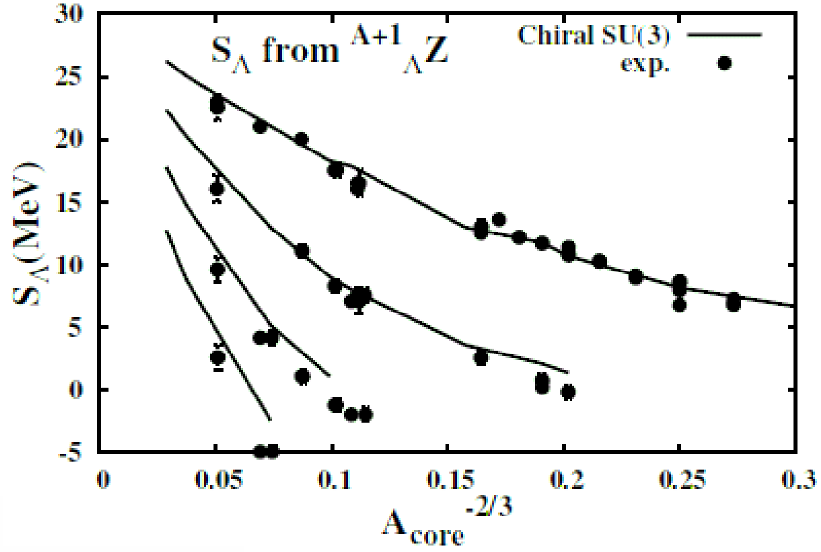
- **How much attraction/repulsion hyperon feels ?**
  - **Bound state / continuum state spectroscopy**
- **When and where hyperons appear in Compact Astrophysical Object ?**
  - **Supernova matter EOS table with hyperons**
- **Can we obtain Extended Nuclear Density Functional from QCD?**
  - **Density Functional as a functional of**
    Baryon / Charge / Strangeness Density and Chiral Condensate

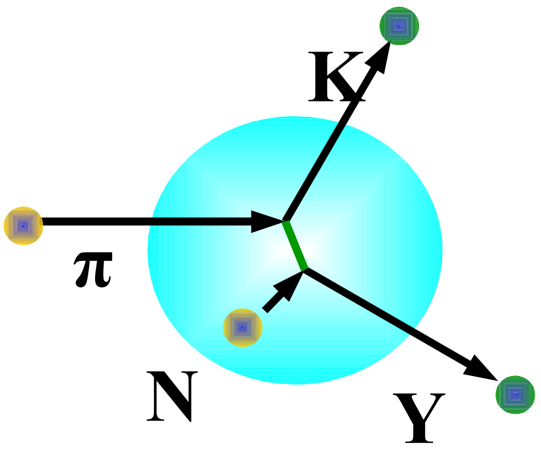
Σ potential in nuclear matter

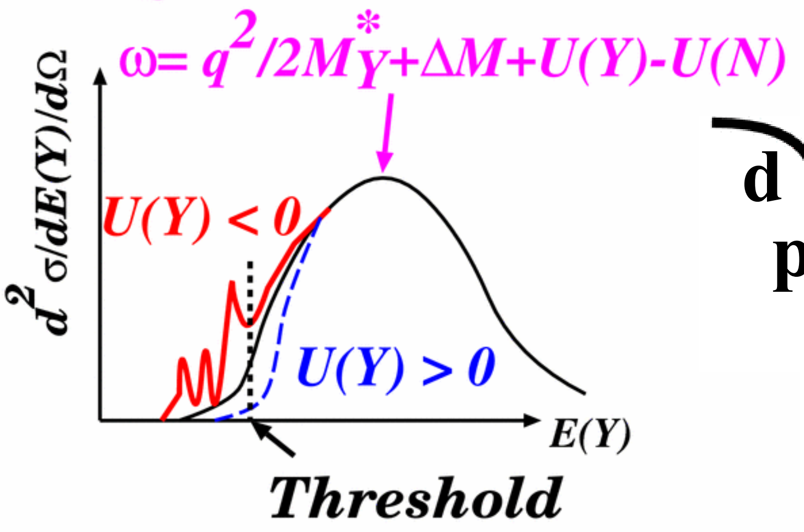
Σ Potential in Nuclear Matter

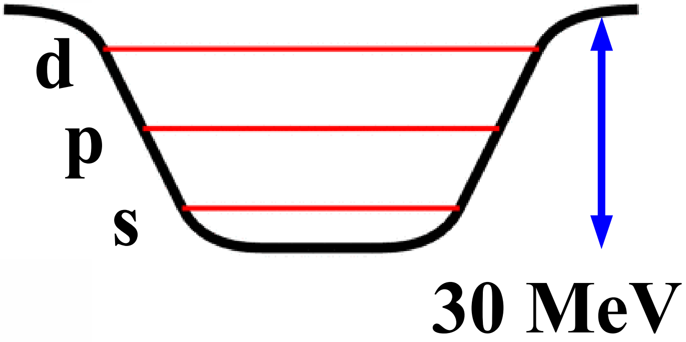
- $U_\Lambda(\rho_0) \sim -30$ MeV: Well known from single particle energies
- Naïve expectation
  = Quark Number (ud number) Scaling
  $U_\Lambda \sim 2/3\ U_N \to U_\Sigma \sim 2/3\ U_N \sim -30$ MeV
- Problems with Σ
  - Only one bound state ${}^4_\Sigma$He (Too light !)
  - → Continuum (Quasi-Free) Spectroscopy is necessary

Tsubakihara, Maekawa, AO, EPJA33('07),295.









Σ Potential in Nuclear Matter

- **Cont. Spec. Theory = Distorted Wave Impulse Approx. (DWIA)**

$$\frac{d^2\sigma}{dE_K\, d\Omega_K} = \boxed{\beta}\ \boxed{\left(\frac{d\sigma}{d\Omega}\right)^{Elem.}_{N\pi\to KY}}\ \boxed{S(E,\boldsymbol{q})}$$

Strength Func.

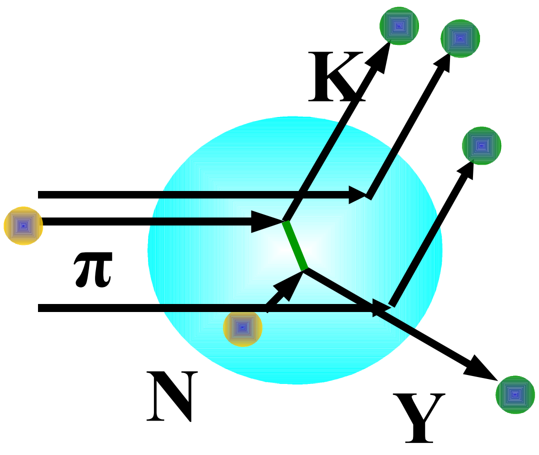
Kinematical Factor

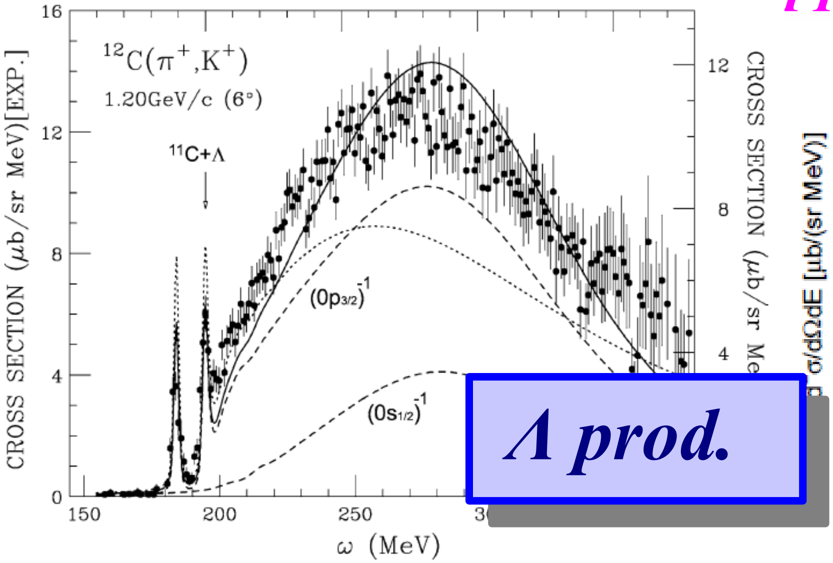
Elem. Cross Sec.

- **Large (ω, q) range → Important to respect On-Shell Kinematics**
- **Kinematics depends on Reaction Point with Hyperon Potential**

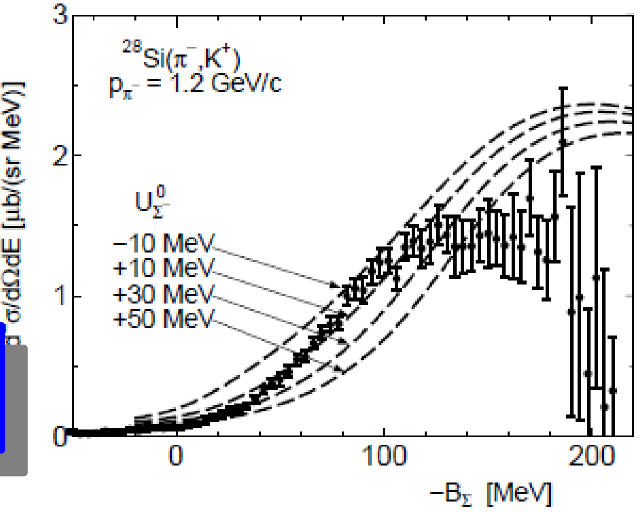
Harada, Hirabayashi, NPA744('04),323.

Kohno, Fujiwara, Kawai, et al. PRC74('06)064613





Λ prod.



Σ Potential in Nuclear Matter

Maekawa, Tsubakihara, AO, EPJA 33(2007),269.
Maekawa, Tsubakihara, Matsumiya, AO, in preparation.

- **DWIA with Local Optimal Fermi Averaging t-matrix (DWIA-LOFAt)**
  - **Green's Func. Method + Reaction Point Deps. of t-matrix**

$$\frac{d^2\sigma}{dE_K\, d\Omega_K} = \frac{p_K E_K}{(2\pi)^2 v_{\mathrm{inc}}} R_Y(E_Y) \qquad \boxed{R_Y(E_Y)} = -\frac{1}{\pi}\mathrm{Im}\left\langle \boxed{\bar{t}(\boldsymbol{r})^{+}}\, \boxed{\frac{1}{E_Y - H_Y + i\varepsilon}}\, \bar{t}(\boldsymbol{r}')\right\rangle$$

Response Func. **Local t-mat.** **Green's Func.**

$$\bar{t}(\boldsymbol{r},\omega,\boldsymbol{q}) = \frac{\int d\boldsymbol{p}_N\ t(s,t)\rho(p_N)\delta^{(4)}(p_1(\boldsymbol{r})+p_2(\boldsymbol{r})-p_3(\boldsymbol{r})-p(\boldsymbol{r}))}{\int d\boldsymbol{p}_N\ \rho(p_N)\delta^{(4)}(p_1(\boldsymbol{r})+p_2(\boldsymbol{r})-p_3(\boldsymbol{r})-p(\boldsymbol{r}))}$$

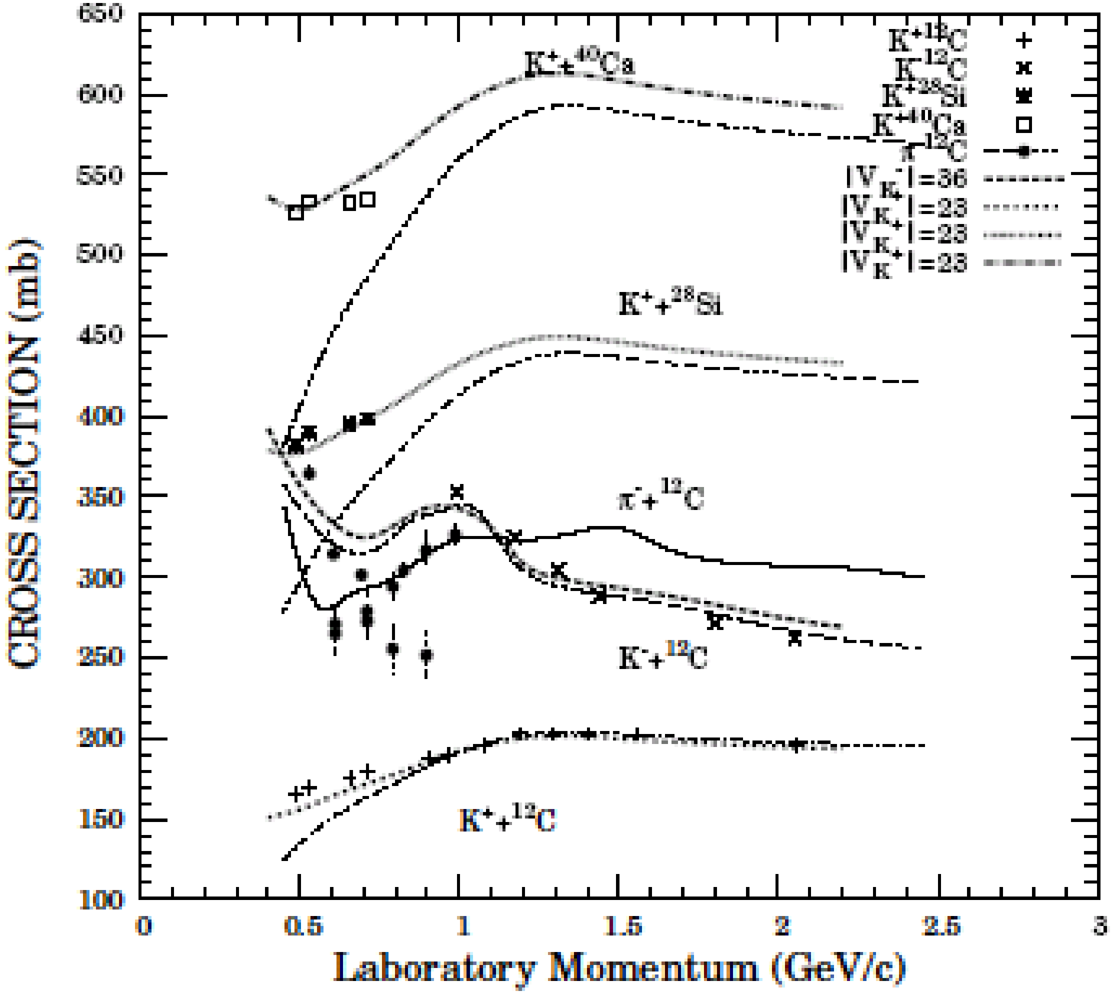
$$E_i = \sqrt{p_i^2 + m_i^*(r)^2} \simeq m_i + \frac{p_i^2}{2\mathrm{m}_i} + V_i, \quad m_i(r)^2 = m_i^2 + 2m_i V_i(r)$$

- **After careful treatment of**
  K+ potential, Elementary cross section, Angular distribution, ...
  we analyze the recently measured Σ^- production spectrum
  (Saha, Noumi et al. (KEK-E438), PRC70('04)044613)

K^+ nucleus potential

Maekawa, Tsubakihara, Matsumiya, AO, in preparation.

- **K^+-nucleus potential**
  ... determined by fitting K^+-nucleus cross sections
  - **$p_{K,\pi} > 1$ GeV/c**
    → small effects
  - **$p_K \sim 0.6$ GeV/c**
    → 30-50 % effects



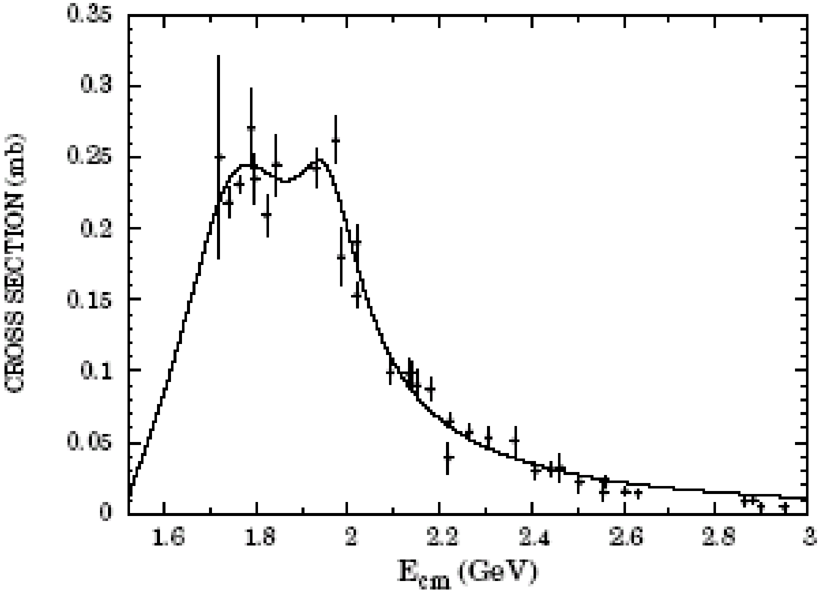
$V(K^+) \sim 23$ MeV

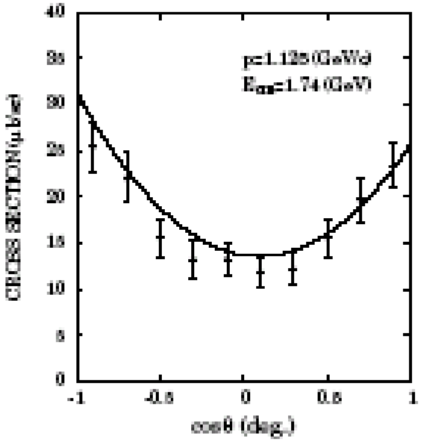
Elementary cross section: $p(\pi^-,K^+)\,\Sigma^-$

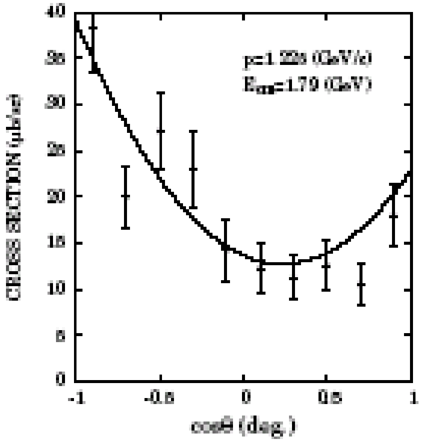
- **Elementary differential cross section**

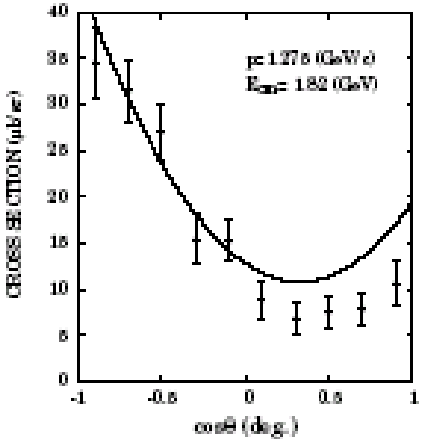
Maekawa

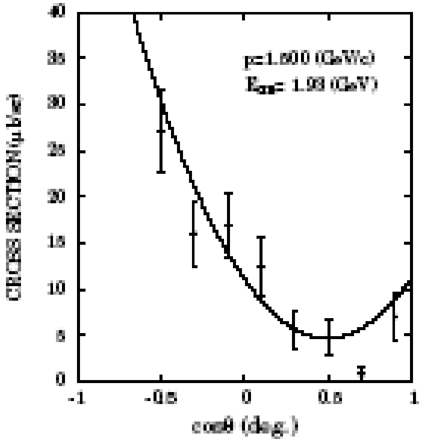
  - Determines the shape of QF spectra
  - High energy
    → Backward peaked due to u-channel dominance
  - Low energies
    → Forward-Backward peaked (Res.)







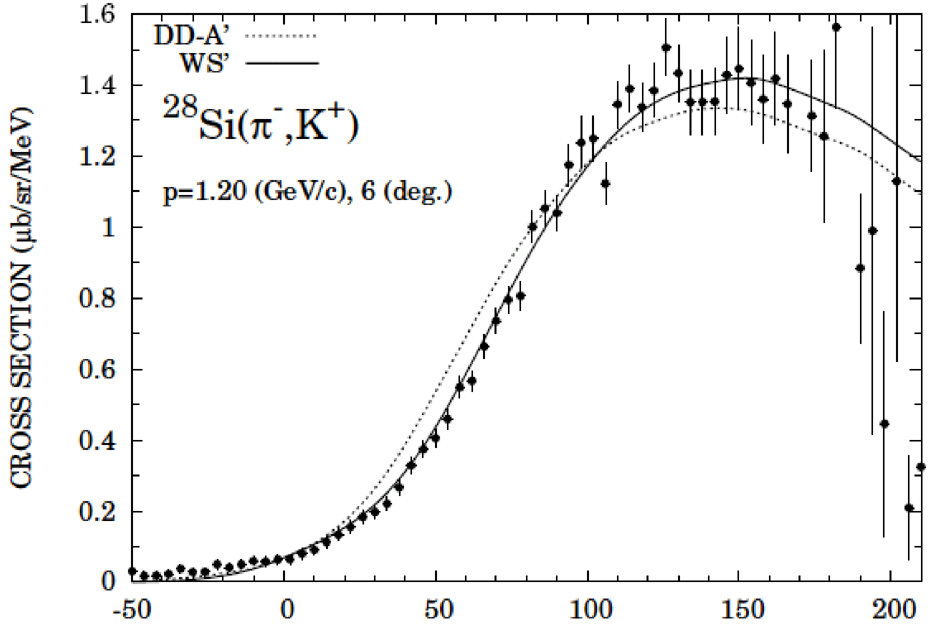




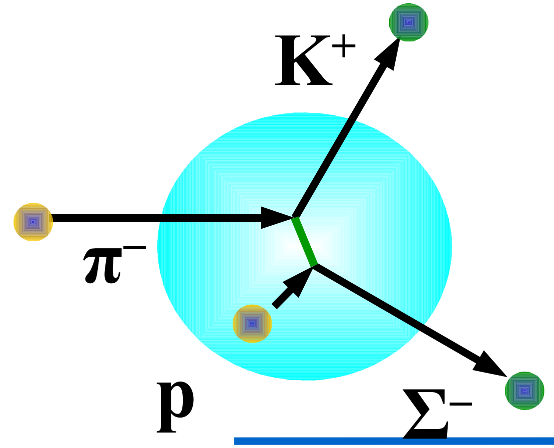
Σ Potential in Nuclear Matter

Maekawa, Tsubakihara, AO, EPJA 33(2007),269.
Maekawa, Tsubakihara, Matsumiya, AO, in preparation.

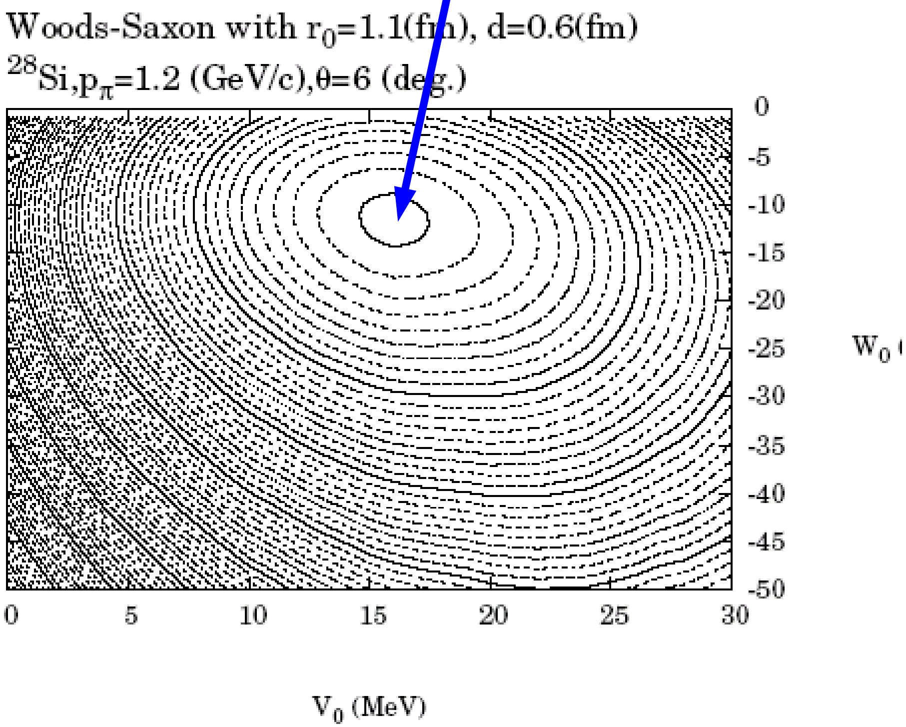
$$\frac{d^2\sigma}{dE_K\, d\Omega_K}$$



E_Σ(MeV)



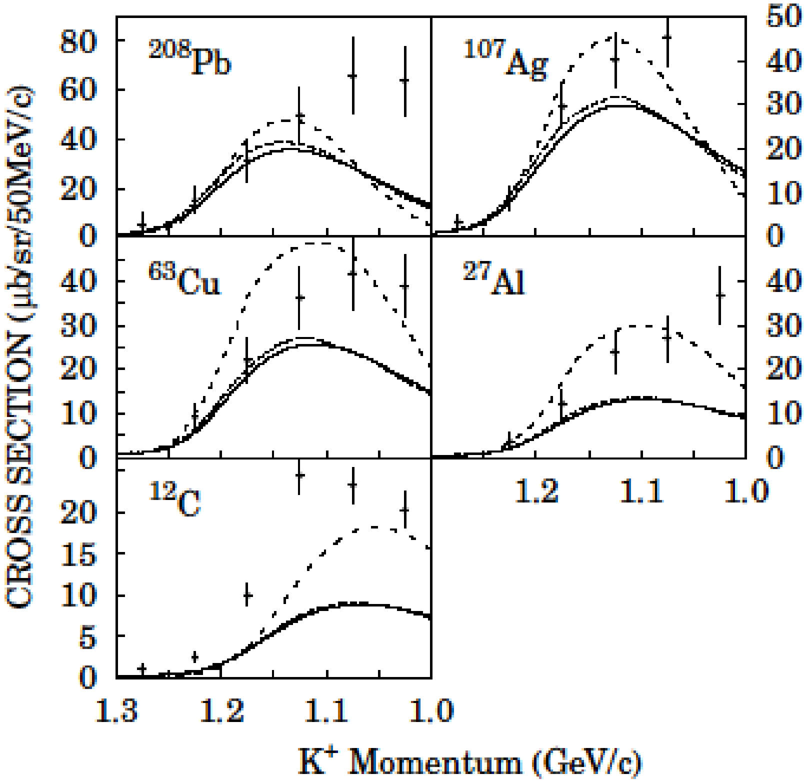
$U_\Sigma(\rho_0) \sim +15$ MeV –i 10 MeV
with Woods-Saxon potential,
no Atomic shift fit



Comment on Ξ-nucleus potential

Maekawa, Tsubakihara, AO, EPJA33('07), 269 [nucl-th/0701066]
Maekawa, Tsubakihara, Matsumiya, AO, arXiv:0704.3929.

- **Spectrum shape in the bound state region → V(Ξ) ~ -14 MeV**
  (Fukuda et al. PRC58('98),1306; Khaustov et al. PRC61('00), 054603)
- **It is difficult to understand absolute values of ^{12}C(K$^-$,K$^+$) spectra.**
  - **Rough fit of elem. t-matrix + Larger eikonal σ**
    → Rough target dependence and absolute values (EPJA)
  - **Careful fit of elem. t-matrix**
    → Underestimate for ^{12}C target.
    (Maekawa et al.(arXiv), Hashimoto et al., Tadokoro et al.(0 deg.), Nara et al. (Cascade), ...)

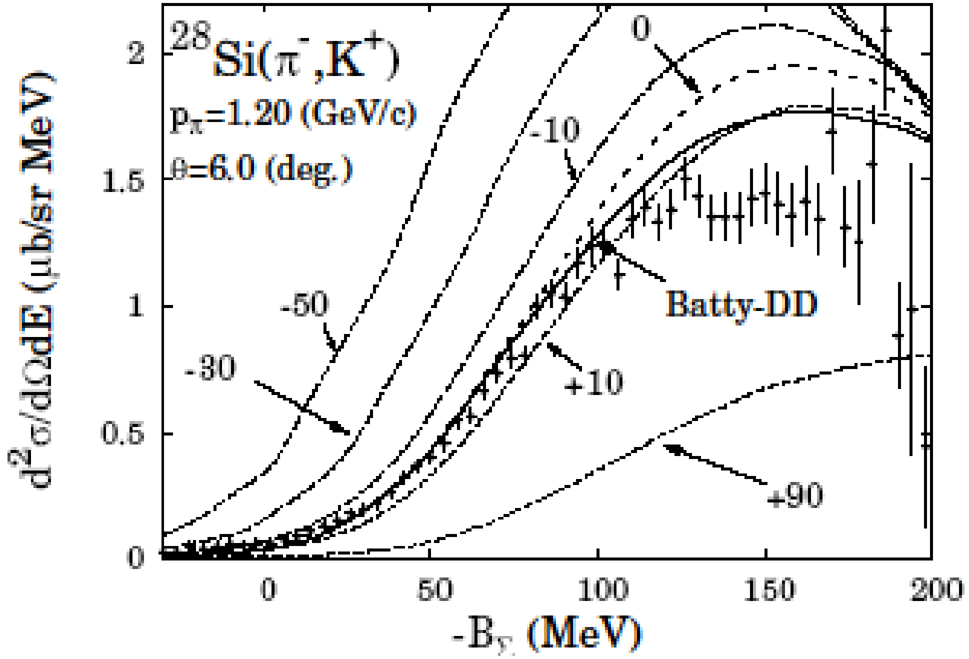


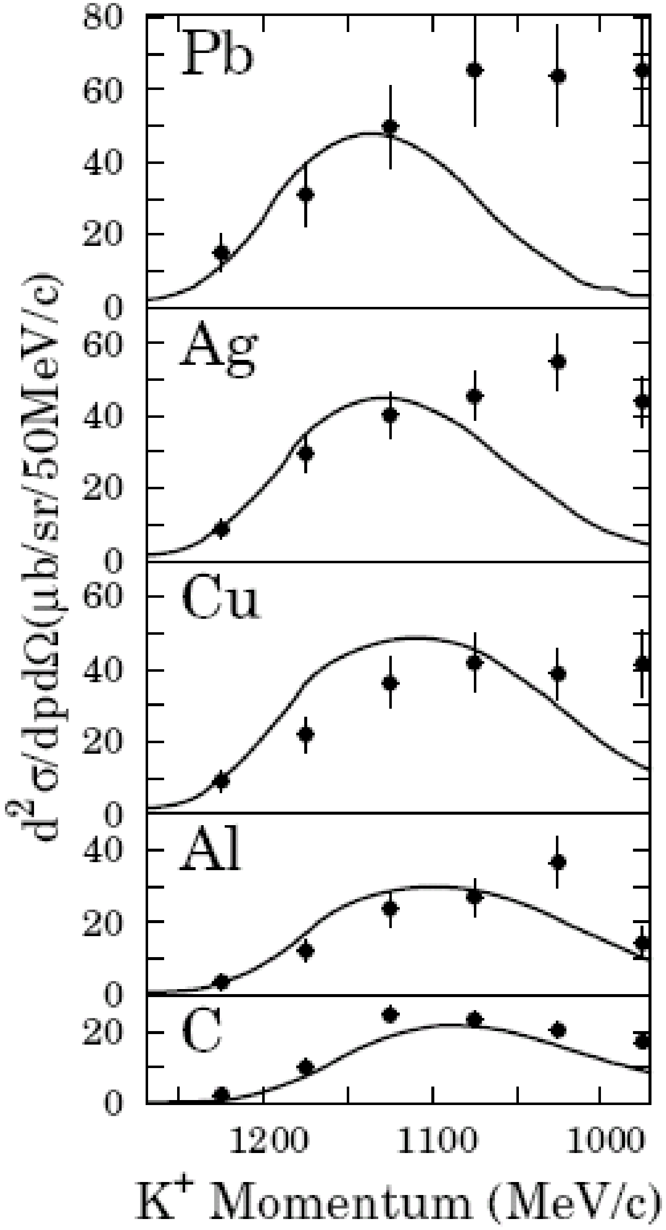
- **Reason**
  - **Center-of-mass correction ?**
  - **Short range correlation in ^{12}C ?**
  - **K$^+$-nucleus potential ?**

Hyp06 で示した結果

Maekawa, Tsubakihara, AO, EPJA33('07), 269 [nucl-th/0701066]

- **Σ : Simple t-matrix (Regge type),**
  No K$^+$ optical potential.
- **Ξ : Simple t-matrix,**
  No K$^+$ optical potential,
  Larger Eikonal σ.

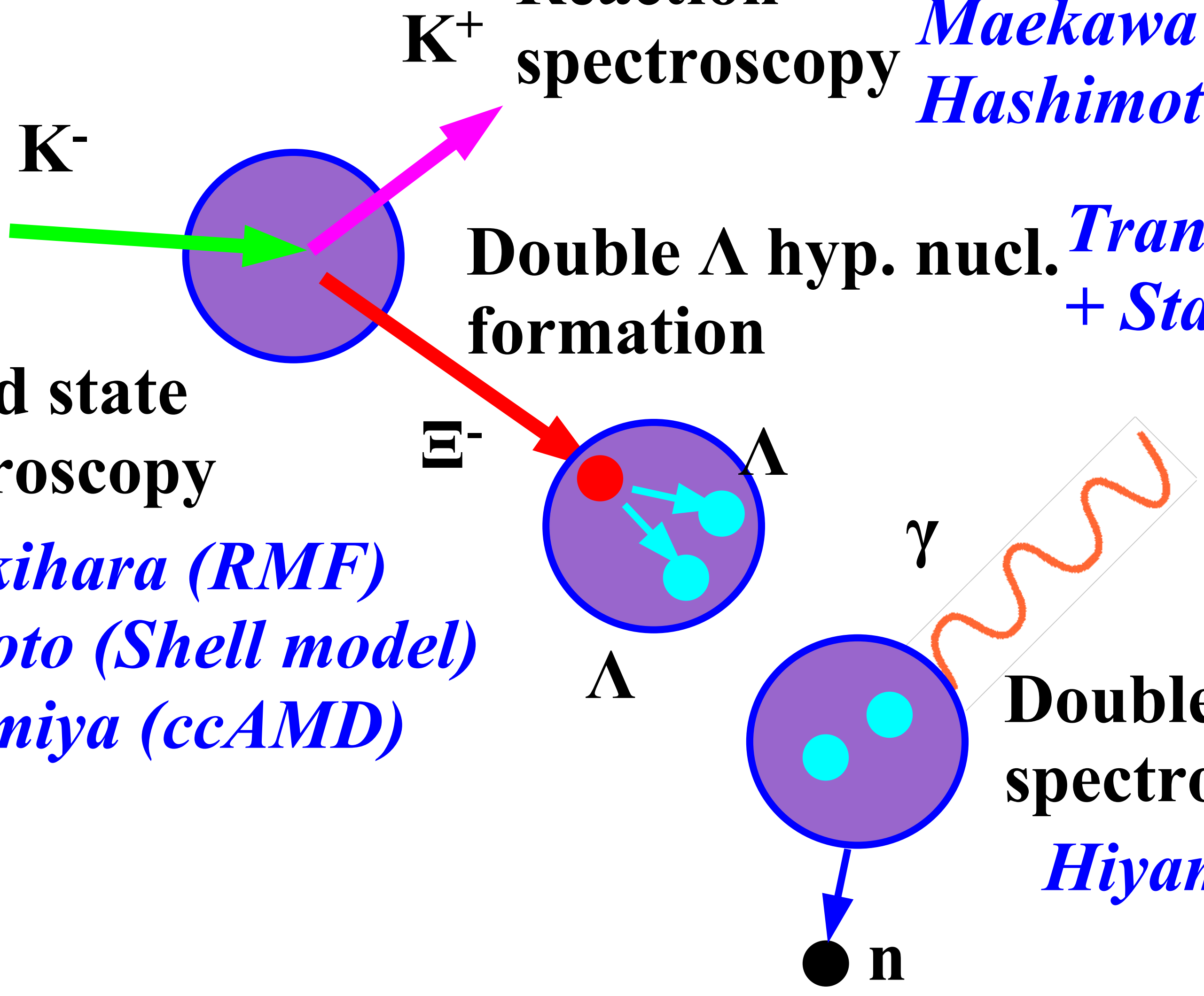




Ξ hypernuclear structure study in coupled channel AMD

Ξ Hypernuclear Physics

- **Ξ hypernuclei → Doorway to Multi Strangeness Systems**



Reaction spectroscopy

Maekawa (GFM-DWIA)
Hashimoto (SCDW)

Double Λ hyp. nucl. formation

Transport
+ Statistical Decay

Bound state spectroscopy

Tsubakihara (RMF)
Sugimoto (Shell model)
Matsumiya (ccAMD)

Double Λ hyp. nucl. spectroscopy

Hiyama

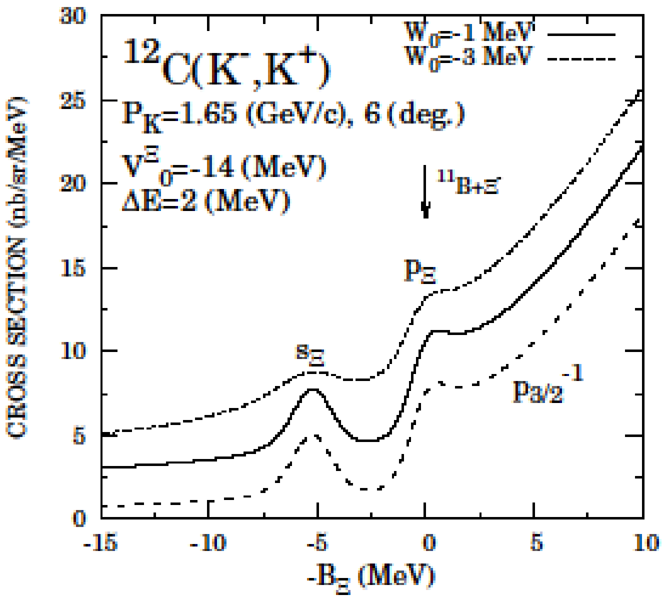
Ξ hypernuclear physics

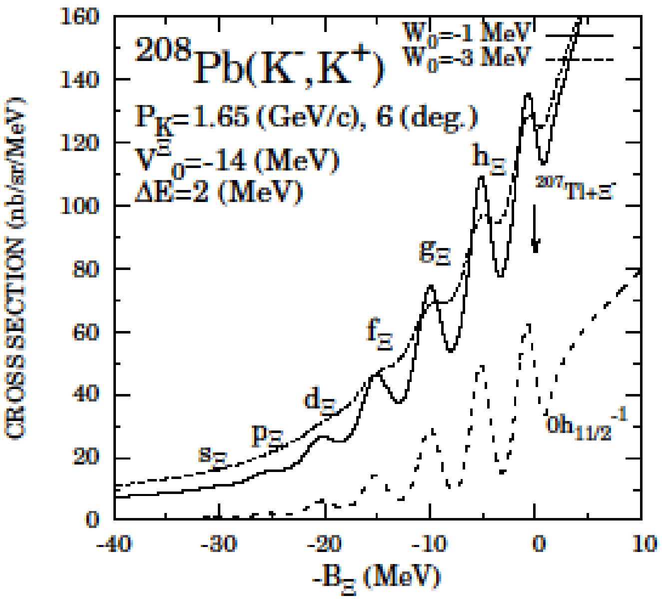
- **Interest**
  - **Laboratory of BB interaction models**
    QCD → Hadrons → Bare BB interaction → Eff. interaction → Nuclei
    Nemura, Fujiwara Yamamoto, Kohno
  - **Admixture in neutron star core**
    $V(\Sigma) > 0$ → Ξ^- may be the first hyperon which appear at high ρ.
  - **Structure change due to Ξ**
    Tensor interaction from $N\Xi$ OPEP would generate rich structure.
- **Difficulty**
  - **Small production cross section, Conversion width to $\Lambda\Lambda$, Shallow pot. ...**
    → Need to obtain spectroscopic info. from low stat. production spectra.

Ξ hypernuclear reaction spectroscopy

- **平均場的取り扱い**

  Ikeda, Fukuda, Motoba, Takahashi, Yamamoto, PTP91('94),747;
  Tadokoro, Kobayashi, Akaishi, PRC51('95),2656;
  Hashimoto, Kohno, Ogata, Kawai, nucl-th/0610126;
  Maekawa, Tsubakihara, AO, EPJA 33 (2007), 269 [arXiv:nucl-th/0701066]
  Maekawa, Tsubakihara, Matsumiaya, AO, arXiv:0704.3929.
  Sugimoto, Motoba, Yamamoto



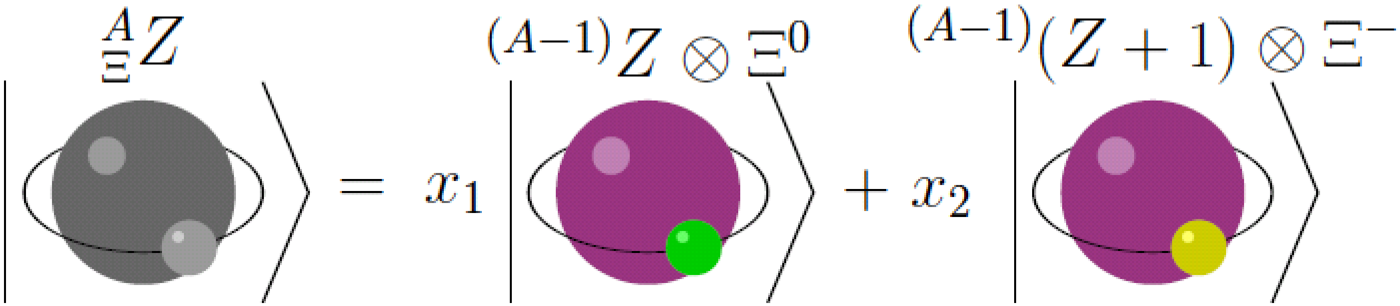


- **軽い核は必ずしも丸くない。**
  → クラスター構造がある / 変化するとき、スペクトルはどうなるか？

Coupled Channel AMD (ccAMD)

- **Wave function = superposition of Channel AMD w.f.**
  → Use co-factor rather than inv. matrix in transition matrix elements.

$$|\Psi\rangle = \sum_a x_a |\Phi^a\rangle \qquad (a : \text{channel}) \qquad |\Phi^a\rangle = \frac{1}{\sqrt{A!}}\det\left[|\varphi_j^a(i)\rangle\right]$$



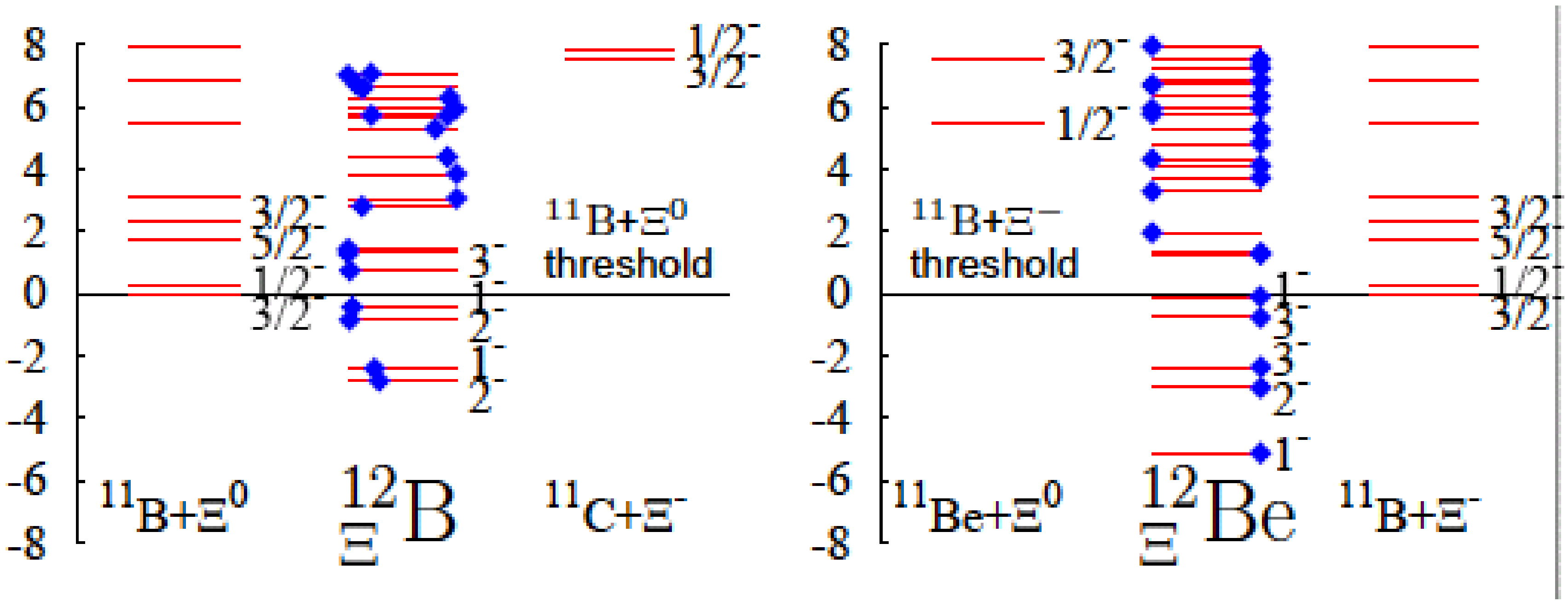
- **Hamiltonian =T + V$_{NN}$ + V$_{YN}$ + Mass diff.**

$$\hat{H} = \hat{T} - \hat{T}_{\rm cm} + \hat{V} + \Delta mc^2$$

  - **VNN : Brink-Boeker-Okabe (BBO1)**
  - **VYN : G-matrix of Nijmegen Extended Soft Core**
    (ESC04d, *Rijken, Yamamoto, 2006*)
    Consistency VYN (ρ) ↔ ρ =< ρ> (Dote-Akaishi prescription)

Ξ hypernuclear level structure

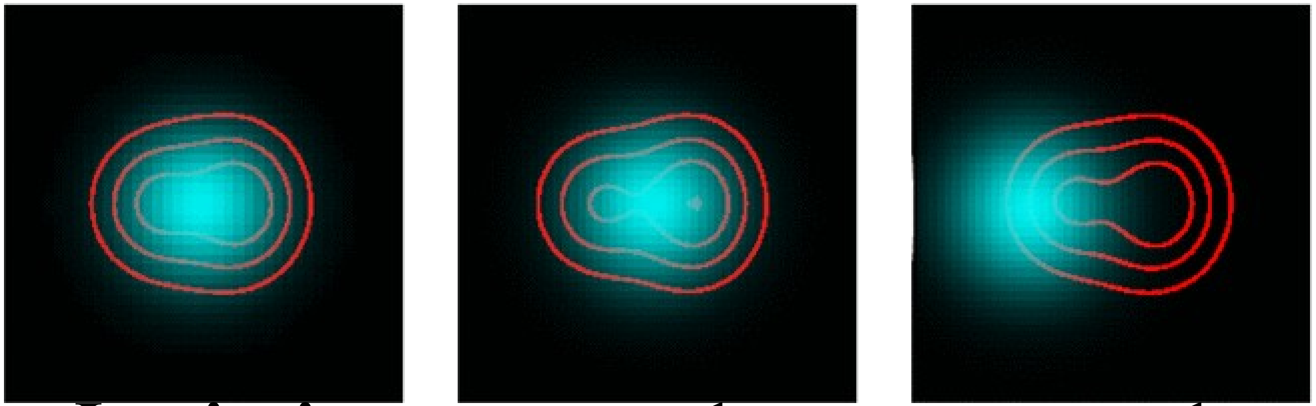
- **$^{12}C(K^-,K^0)^{12}{}_{\Xi}B$, $^{12}{}_{\Xi}B = (^{11}B+\Xi^0)+(^{11}C+\Xi^-)$**
  (Mirror Core)⊗ Ξ → Coherent coupling, but not perfect
- **$^{12}C(K^-,K^+)^{12}{}_{\Xi}Be$, $^{12}{}_{\Xi}Be = (^{11}Be(T=3/2)+\Xi^0)+(^{11}B(T=1/2)+\Xi^-)$**
  Different T of core → Almost no coupling in low lying levels



(Preliminary: No spin rotation, No GCM, Single Gauss AMD, ...)

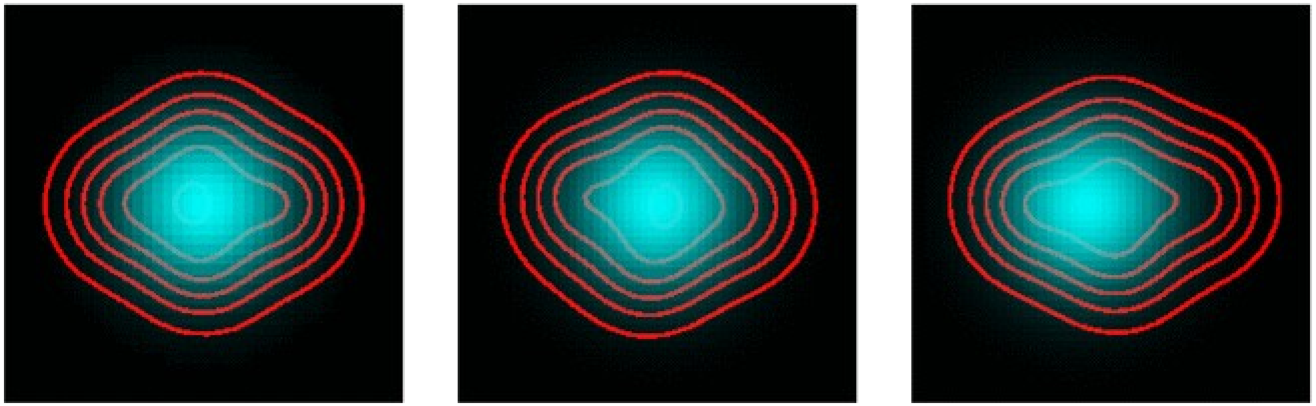
Density Distribution

- ${}^{12}_{\Xi}$Be



Intrinsic π=-1 π =+1

- ${}^{28}_{\Xi}$Mg



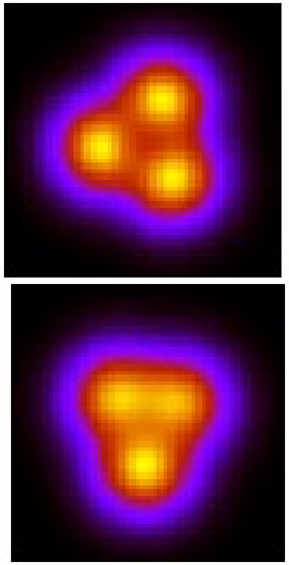
Intrinsic π=+1 π =-1

- **Nuclei are not necessarily spherical !**
- **Core and Ξ Parities are mixed !**

Density and Effective Number ($^{12}C \to {}^{12}_{\Xi}Be$)

- **Brink-Boeker type effective NN interaction**
  → Developed clustering structure, Small Effective Number
- **Volkov (m=0.56)**
  → Smaller deformation, Larger Effective Number

$Z_{\text{eff}} \times 10^3$	AMD		WS14	WS24[10]
	BBO1	Volkov		
$(0_1^+ \to 1_1^-)$	0.101	1.43	2.30	27.5

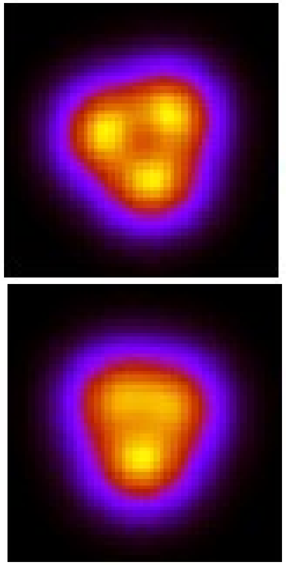


BBO1

^{12}C $\beta = 0.44, \gamma = 59^\circ$

$\Downarrow$

$^{12}_{\Xi}$Be $\beta = 0.39, \gamma = 44^\circ$



Volkov

$\beta = 0.35, \gamma = 59^\circ$

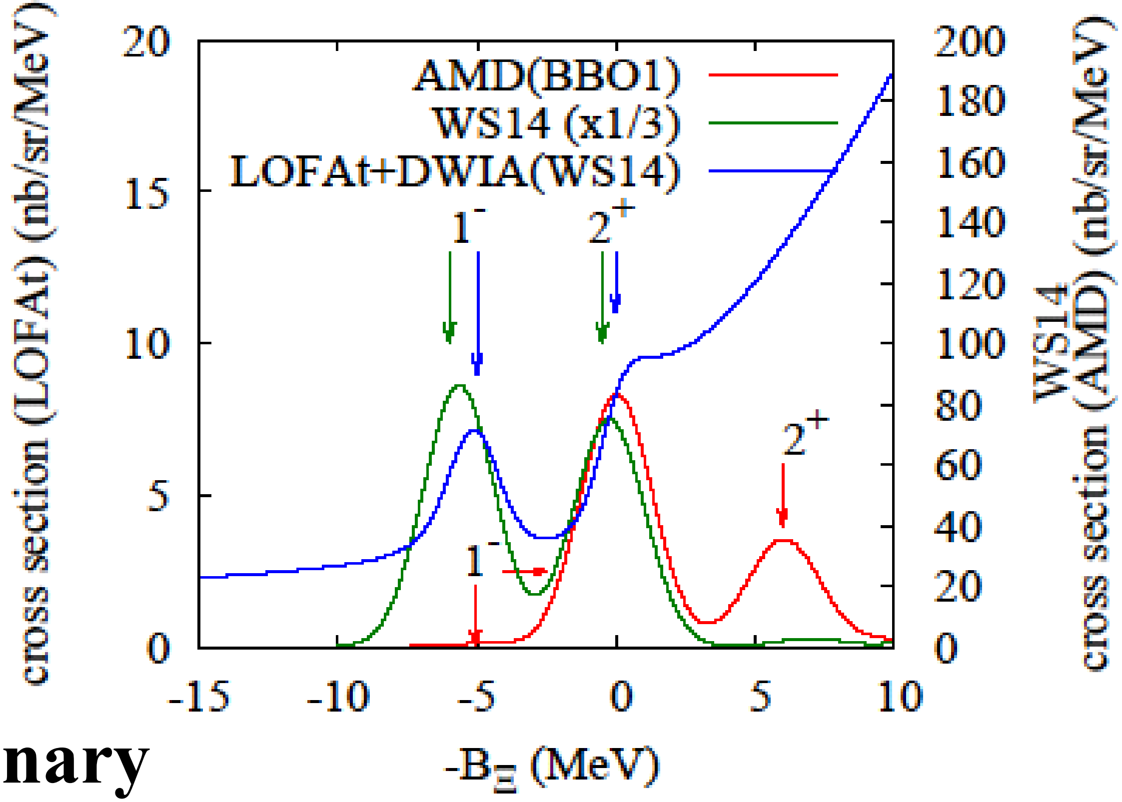
$\Downarrow$

$\beta = 0.29, \gamma = 53^\circ$

Comparison of $^{12}C(K^-,K^+)^{12}_{\Xi}Be$ Sectrum

- **Effective Number 法による束縛近似でのΞ核生成スペクトル**
  → 標的・Ξ核の変形 (クラスター構造の発達度合い) により、
  生成スペクトルが大きく変化する可能性あり。
- **ccAMD による結果 (Preliminary)**
  - **1_1^- から 1_2^- へ Zeff が流れている。**
    (一つの intrinsic state からの
    projection の影響か？)
  - **2^+ は高く出る。**
    (single Gauss AMD のため、
    Ξ の p-wave がうまく
    作れていない。)

> ***Need more studies !***
> ***E.g. Application***
> ***to Λ hypernuclei***



Preliminary

Summary (1), A la Michelin

- **$U_\Lambda(\rho_0) \sim -30$ MeV** ✿✿✿
  - ***Bound State Spectroscopy + Continuum Spectroscopy***
- **$U_\Sigma(\rho_0) > +15$ MeV** ✿✿
  - **Continuum (Quasi-Free) spectroscopy with *Local Optimal Fermi Averaging t-matrix* (LOFAt)**
  - **Atomic shift data (attractive at surface) should be respected.**
- **$U_\Xi(\rho_0) \sim -14$ MeV** ✿
  - **No confirmed bound state, No atomic data, High mom. transf., ... → Small Potential Deps.**
  - **Continuum low-res. spectrum shape → – 14 MeV**
  - **Spin-Isospin deps. (π exch.) → Deformation → Spectrum shape may be modified.**

There is no “No Star” Restaurant in Michelin Tokyo

Relativistic EOS of Supernova Matter with Hyperons

Relativistic EOS of Supernova Matter with Hyperons

- **Extention of the Relativistic (Shen) EOS to SU$_f$(3) with updated Hyperon Potentials in Nuclear Matter**
  (Ishizuka, Ohnishi, Tsubakihara, Sumiyoshi, Yamada, in preparation)
  - **Relativistic (Shen) EOS** *(Shen, Toki, Oyamatsu, Sumiyoshi, PTP 100('98), 1013)*
    Rel. Mean Field (RMF) + Local Density Approx.(Nuclear Formation)
  - **SU$_f$(3) Extention of RMF** *(Schaffner, Mishustin, PRC53 (1996), 1416)*
    Coupling ~ Quark Number Counting

g_{MB}	σ	ζ	ω	ρ	ϕ
N	10.0289	0	12.6139	4.6783	0
Λ	6.21	6.67	8.41	0	-5.95
Σ	4.36 (6.21)	6.67	8.41	$2g_{\rho N}$	-5.95
Ξ	3.11 (3.49)	12.35	4.20	4.63	-11.89

SM
IOTSY

- ***$g_{\sigma Y}$ is tuned to fit Hyperon Potential in Nuclear Matter***
  $U_\Lambda = -30$ MeV, $U_\Sigma = +30$ MeV, $U_\Xi = -15$ MeV
- ***Nuclear Formation is included using Shen EOS table***

Schroedinger Equivalent Potential

- **RMF Lagrangian**

$$
\begin{aligned}
\mathcal{L} = &\sum_B \bar{\Psi}_B \left(i\partial\!\!\!/ - M_B\right) \Psi_B + \frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - U_\sigma(\sigma) \\
&- \frac{1}{4}\omega^{\mu\nu}\omega_{\mu\nu} + \frac{1}{2}m_\omega^2\omega^\mu\omega_\mu - \frac{1}{4}\vec{R}^{\mu\nu}\cdot\vec{R}_{\mu\nu} + \frac{1}{2}m_\rho^2\vec{R}^\mu\cdot\vec{R}_\mu \\
&- \sum_B \bar{\Psi}_B \left(g_{\sigma B}\sigma + g_{\omega B}\omega\!\!\!/ + g_{\rho B}\vec{R}\!\!\!/\cdot\vec{t}_B\right)\Psi_B + \frac{1}{4}c_\omega(\omega^\mu\omega_\mu)^2 + \mathcal{L}^{YY}
\end{aligned}
$$

- **Schroedinger Equvalent Potential**
  - **Dirac spinor の上２成分を Non-Rel. w.f. として Schroedinger Eq. を書いた時に現れるポテンシャル (微分項は無視)**

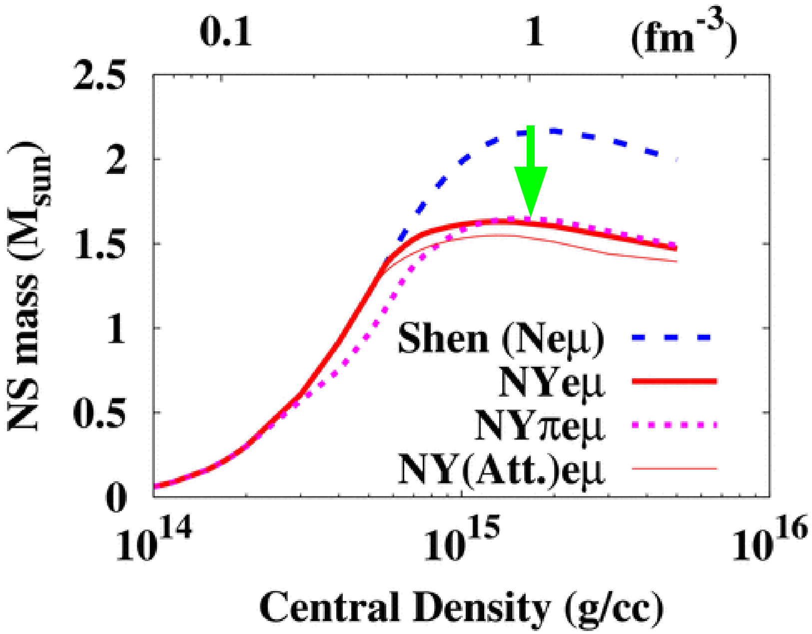
$$
\begin{aligned}
U_B(\rho, E(\mathbf{p})) &= U_s(\rho) + \frac{E(\mathbf{p})}{M_B}U_v(\rho) \\
&= g_{\sigma B}\sigma + g_{\zeta B}\zeta + \frac{E}{M}\left(g_{\omega B}\omega + g_{\rho B}R + g_{\phi B}\phi\right)
\end{aligned}
$$

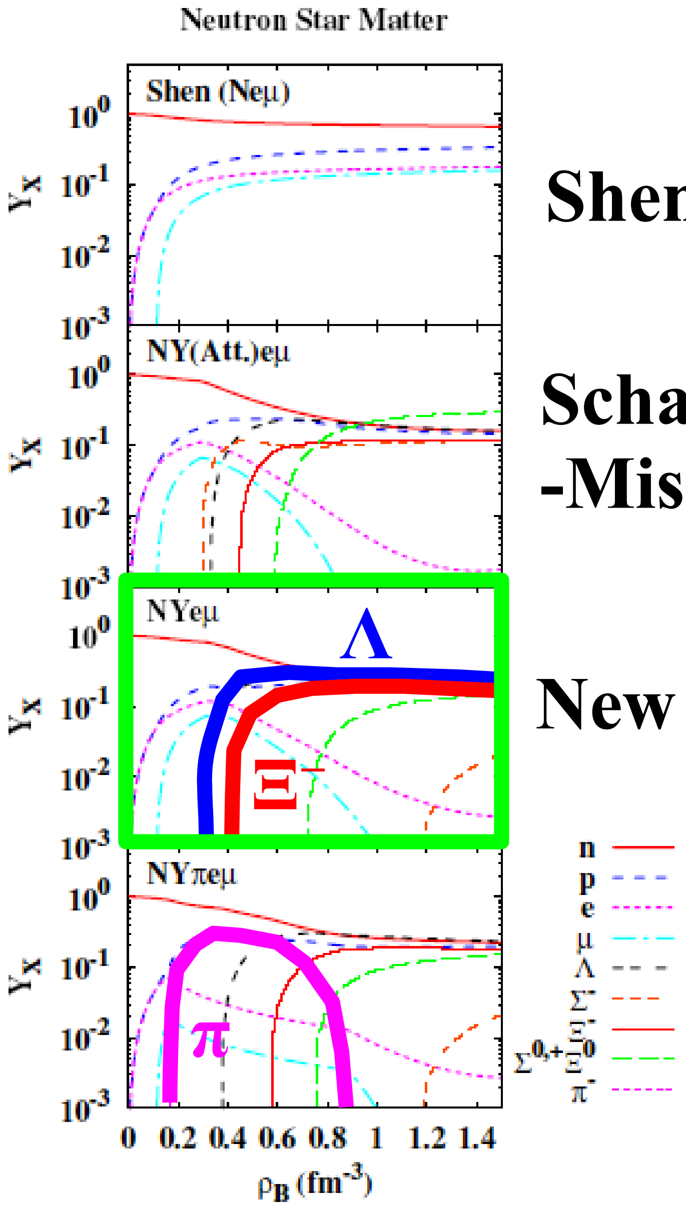
SEP=U_Y → RMF coupling → EOS

Neutron Star

Ishizuka, AO, Tsubakihara, Sumiyoshi, Yamada, arXiv:0802.2318

- **Hyperon Effect is DRASTIC**
  - **Mmax=2.1 Msun → 1.56 Msun**
  - **Composition $Y_\Lambda \sim Y_n$**
  - **Large fraction of Ξ**
- **Thermal (free) pions can admix at $\rho > 1.5\,\rho_0$**





Shen

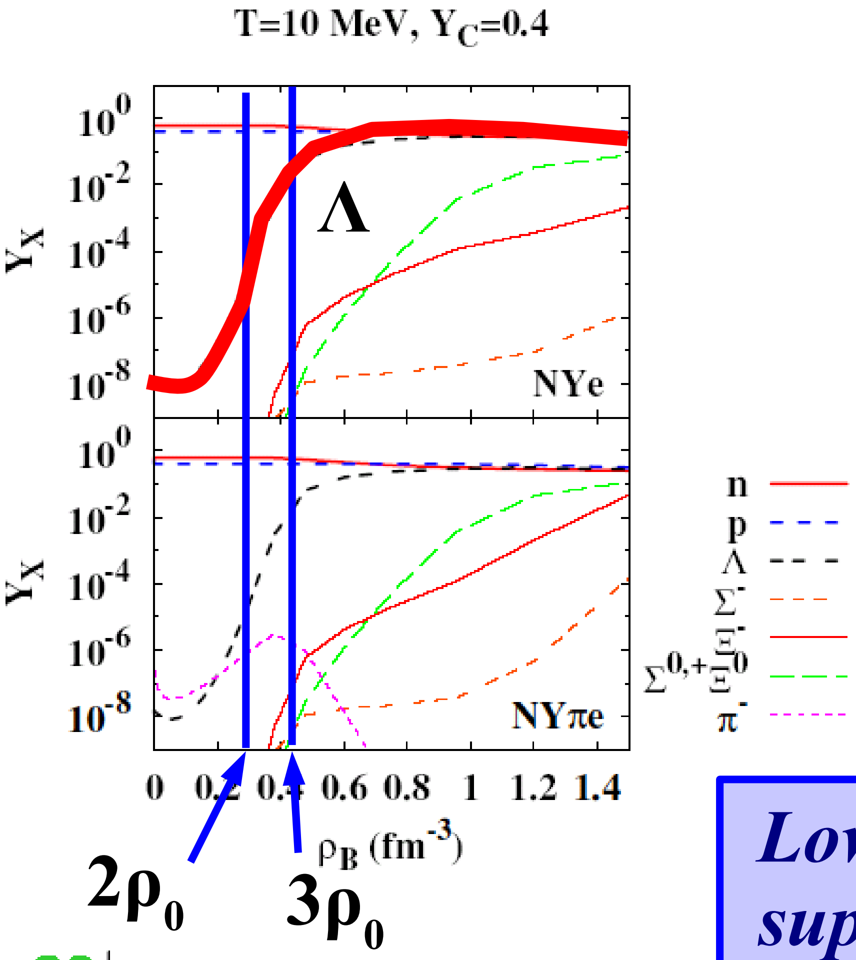
Schaffner -Mishustin

New

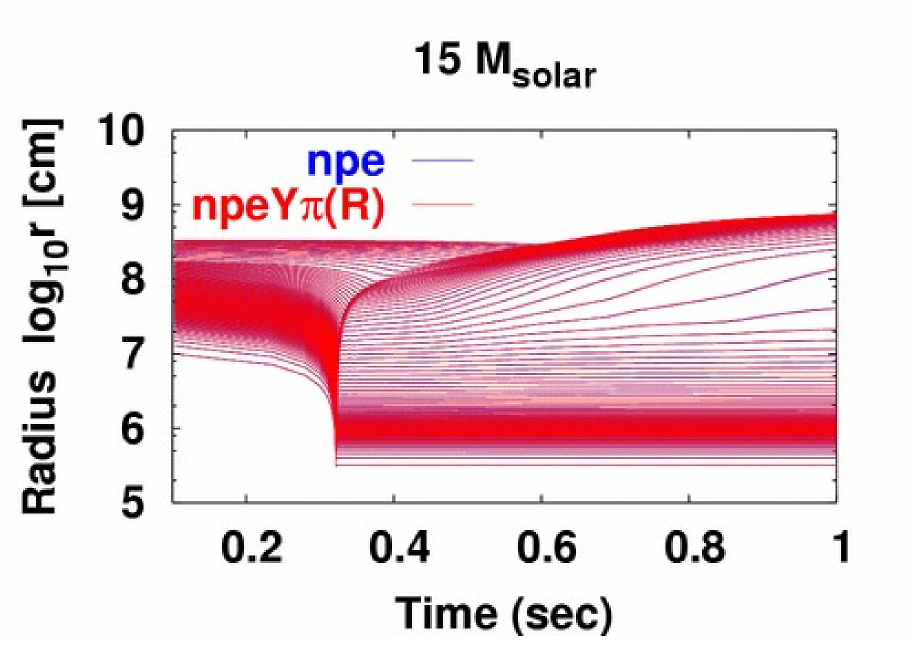
Finite Temperature and Supernova

Ishizuka, AO, Tsubakihara, Sumiyoshi, Yamada, arXiv:0802.2318

- **Example: T=10 MeV, Ye = 0.4**
  - **Λ starts to increase at ρ ~ 2 ρ_0 , becomes significant at ρ ~ 3ρ_0 .**



- **Prompt explosion (without ν transport) → Almost no change (Expl. E. increase ~ (0.1-0.5 %))**



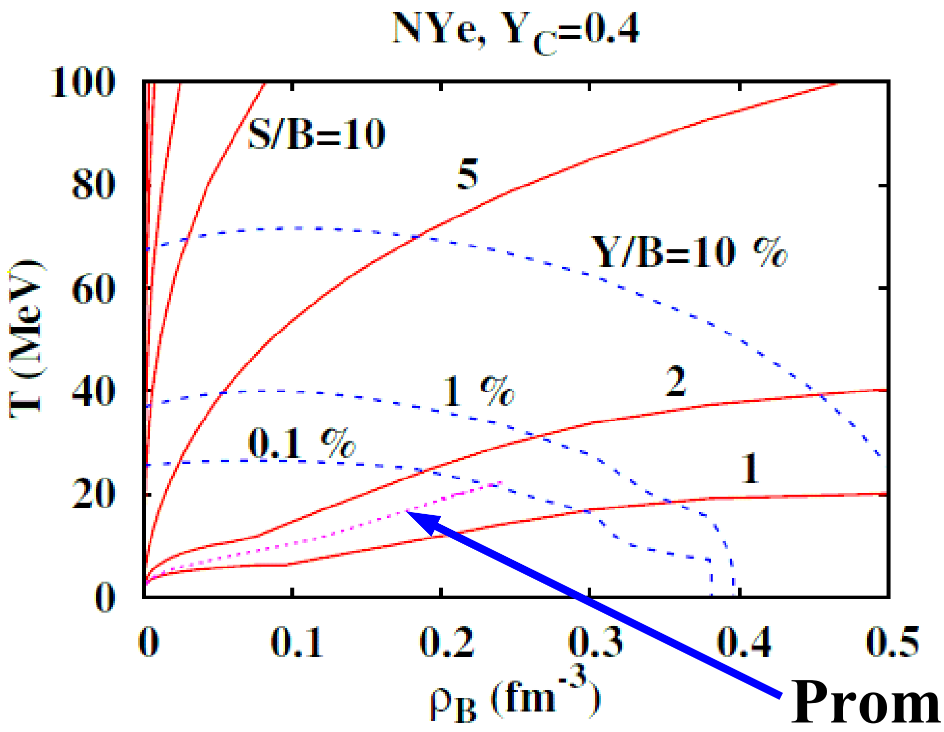
WW95 + 1 Dim. Hydro.(Sumiyoshi, Yamada)

> ***Low density and High Ye suppresses Hyperons in the Early Stage***

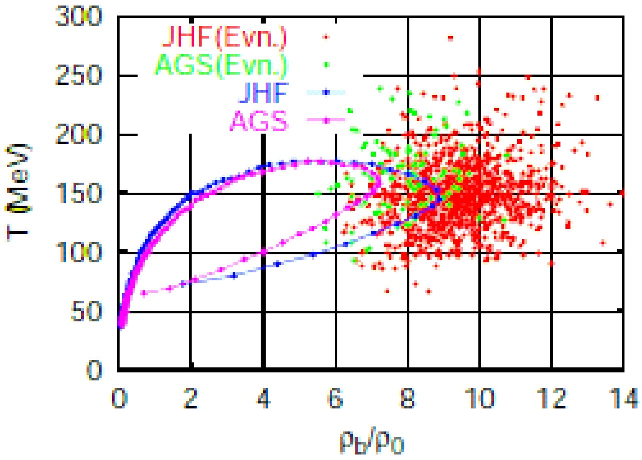
Where Do We See Hyperons ?

- **Hyperon Fraction is sensitive to Ye, T, and ρ_B.**
  - **Yν ~ 0 (Neutron Star) → $\rho_B > 2\ \rho_0$**
  - **Ye ~ 0.4 (Supernova, early stage) → T > 40 MeV or $\rho_B > 3\ \rho_0$**

Hyperons would be important in Late Stage(Nstar cooling), BH formation, and Heavy-Ion Collisions



Prompt Expl. (15 Msun)



Heavy-Ion Collision Simulation by using JAM (Nara et al.)

Summary (2)

- **Hyperons are included in the Relativisitic (Shen) EOS with recently accepted Hyperon Potentials in Nuclear Matter,**

  $U_\Lambda = -30$ MeV, $U_\Sigma = +30$ MeV , $U_\Xi = -15$ MeV

  http://nucl.sci.hokudai.ac.jp/~chikako/EOS

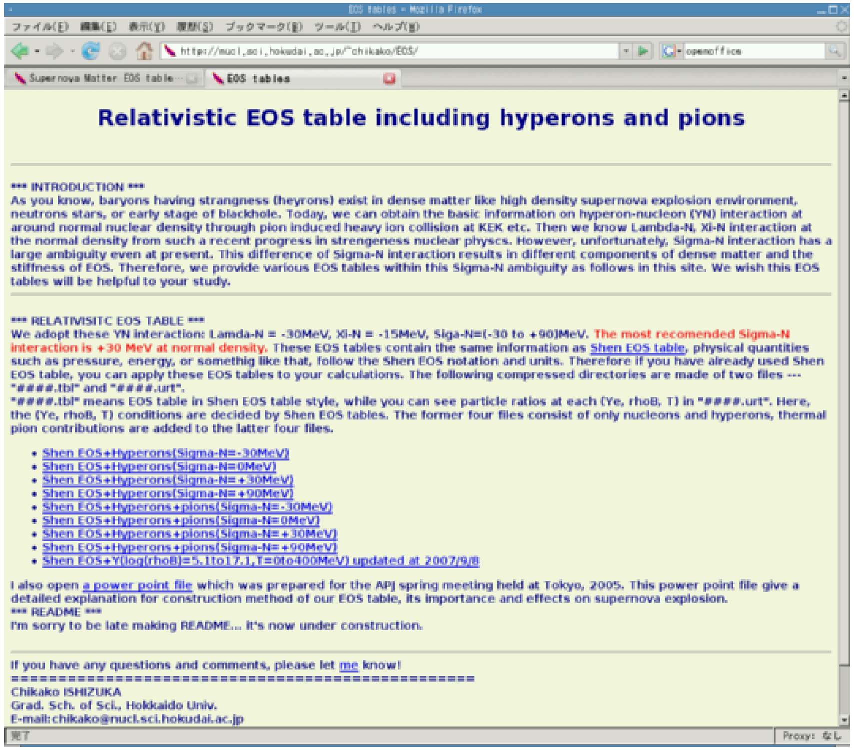
  ρ =10(5.1-15.4) g/cc, T=0-100 MeV, Ye=0-0.56**

  (Ishizuka,AO,Tsubakihara, Sumiyoshi,Yamada, arXiv:0802.2318)

EOSY by IOTSY

- **Hyperon effects:**
  Decisive in Nstar
  Small in SNe (early)
  Significant in BH formation.
  (Sumiyoshi's Talk !)

- **Japan Proton Accelerator Research Complex (J-PARC) data will come soon.**
  Stay Tuned !



Relativistic EOS table including hyperons and pions

*** INTRODUCTION ***
As you know, baryons having strangness (heyrons) exist in dense matter like high density supernova explosion environment, neutrons stars, or early stage of blackhole. Today, we can obtain the basic information on hyperon-nucleon (YN) interaction at around normal nuclear density through pion induced heavy ion collision at KEK etc. Then we know Lambda-N, Xi-N interaction at the normal density from such a recent progress in strengeness nuclear physcs. However, unfortunately, Sigma-N interaction has a large ambiguity even at present. This difference of Sigma-N interaction results in different components of dense matter and the stiffness of EOS. Therefore, we provide various EOS tables within this Sigma-N ambiguity as follows in this site. We wish this EOS tables will be helpful to your study.

*** RELATIVISITC EOS TABLE ***
We adopt these YN interaction: Lamda-N = -30MeV, Xi-N = -15MeV, Siga-N=(-30 to +90)MeV. The most recomended Sigma-N interaction is +30 MeV at normal density. These EOS tables contain the same information as Shen EOS table, physical quantities such as pressure, energy, or somethig like that, follow the Shen EOS notation and units. Therefore if you have already used Shen EOS table, you can apply these EOS tables to your calculations. The following compressed directories are made of two files --- "####.tbl" and "####.urt".
"####.tbl" means EOS table in Shen EOS table style, while you can see particle ratios at each (Ye, rhoB, T) in "####.urt". Here, the (Ye, rhoB, T) conditions are decided by Shen EOS tables. The former four files consist of only nucleons and hyperons, thermal pion contributions are added to the latter four files.

- Shen EOS+Hyperons(Sigma-N=-30MeV)
- Shen EOS+Hyperons(Sigma-N=0MeV)
- Shen EOS+Hyperons(Sigma-N=+30MeV)
- Shen EOS+Hyperons(Sigma-N=+90MeV)
- Shen EOS+Hyperons+pions(Sigma-N=-30MeV)
- Shen EOS+Hyperons+pions(Sigma-N=0MeV)
- Shen EOS+Hyperons+pions(Sigma-N=+30MeV)
- Shen EOS+Hyperons+pions(Sigma-N=+90MeV)
- Shen EOS+Y(log(rhoB)=5.1to17.1,T=0to400MeV) updated at 2007/9/8

I also open a power point file which was prepared for the APJ spring meeting held at Tokyo, 2005. This power point file give a detailed explanation for construction method of our EOS table, its importance and effects on supernova explosion.
*** README ***
I'm sorry to be late making README... it's now under construction.

If you have any questions and comments, please let me know!
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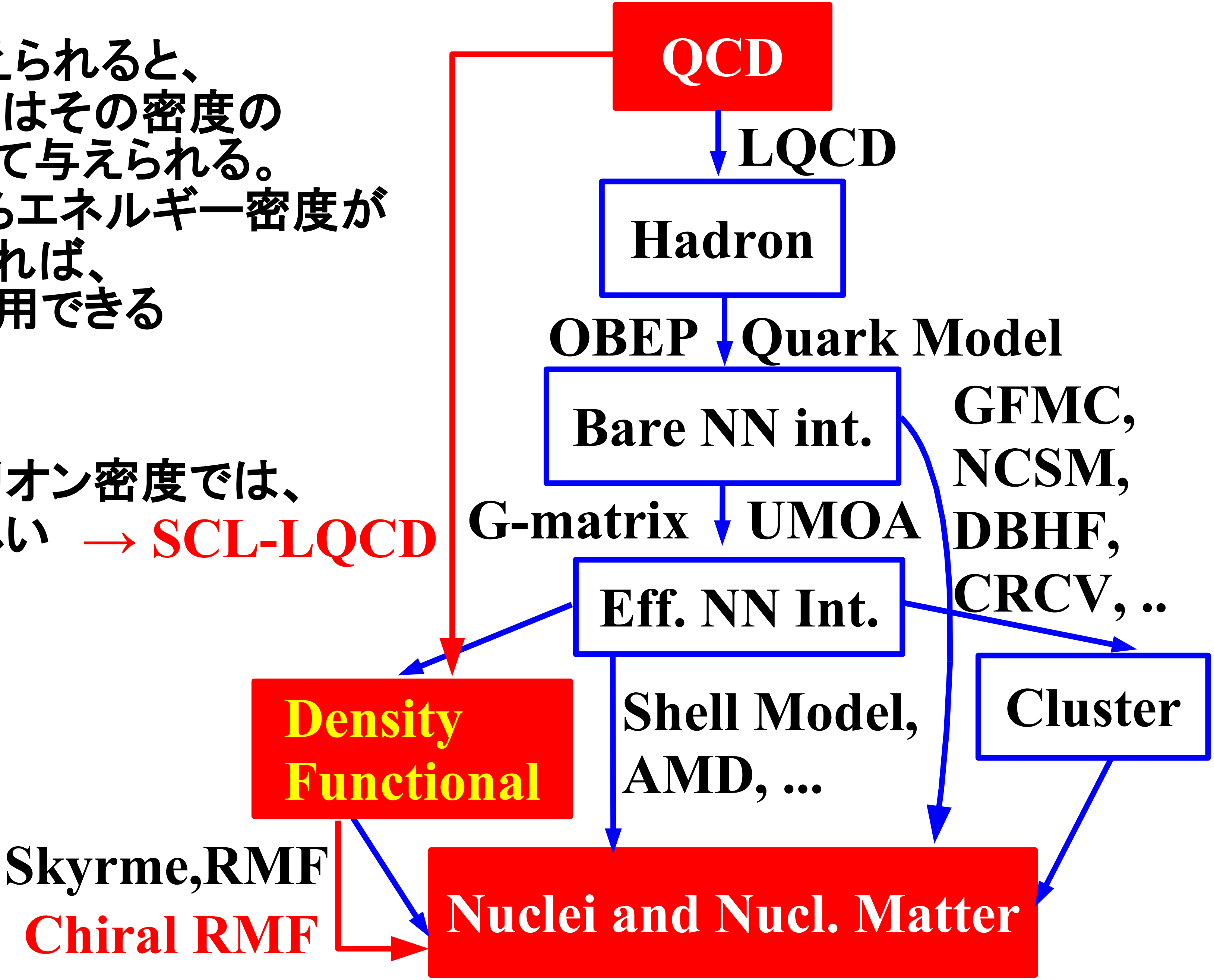
Hypernuclei and Hyperonic Matter in Chiral SU(3) RMF

QCD から原子核密度汎関数へ

- 密度汎関数
  = 密度が与えられると、
  エネルギーはその密度の
  汎関数として与えられる。
  → QCD からエネルギー密度が
  与えられれば、
  それを採用できる

- 問題点
  = 有限のバリオン密度では、
  MC が難しい → SCL-LQCD



QCD

LQCD

Hadron

OBEP

Quark Model

Bare NN int.

GFMC,
NCSM,
DBHF,
CRCV, ..

G-matrix

UMOA

Eff. NN Int.

Density
Functional

Shell Model,
AMD, ...

Cluster

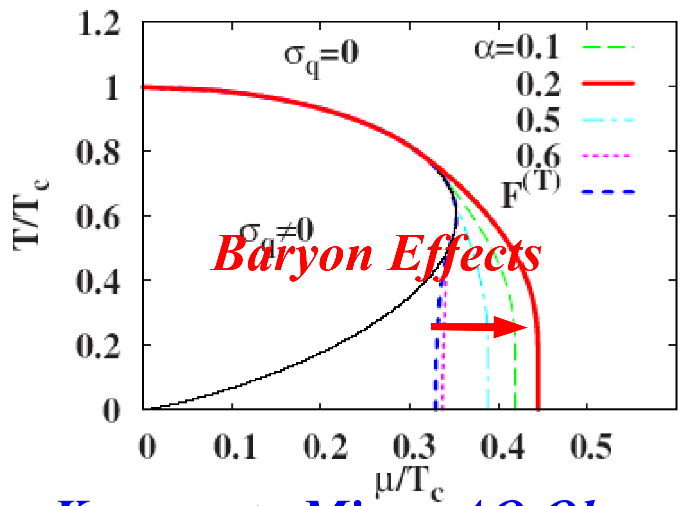
Skyrme,RMF

Chiral RMF

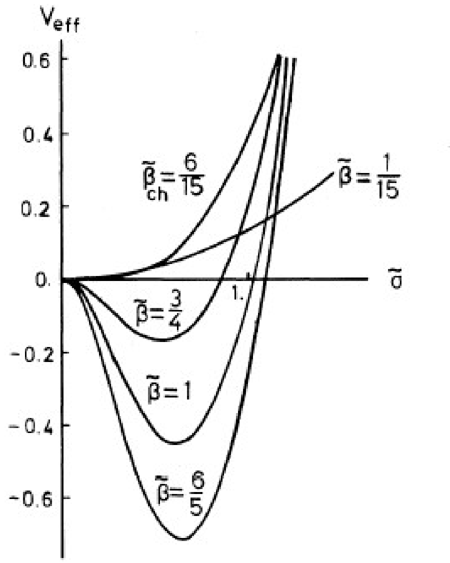
Nuclei and Nucl. Matter

Strong Coupling Limit of Lattice QCD

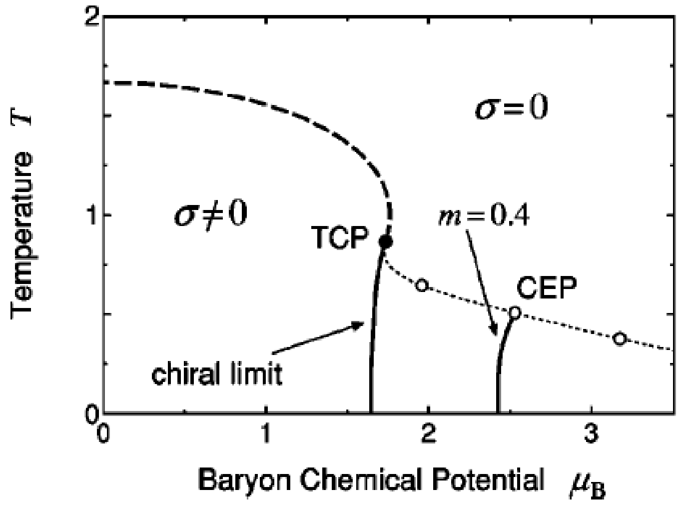
- **SCL-LQCD has been a powerful tool in “phase diagram” study !**
  - **Chiral restoration, Phase diagram, Baryon effects, Hadron masses, Finite coupling effects, ...**



Kawamoto,Miura,AO,Ohnuma, PRD75 (07), 014502.



Damgaard,Kawamoto,Shigemoto, PRL53('84),2211



Nishida, PRD69, 094501 (2004)

Strong Coupling Limit of Lattice QCD

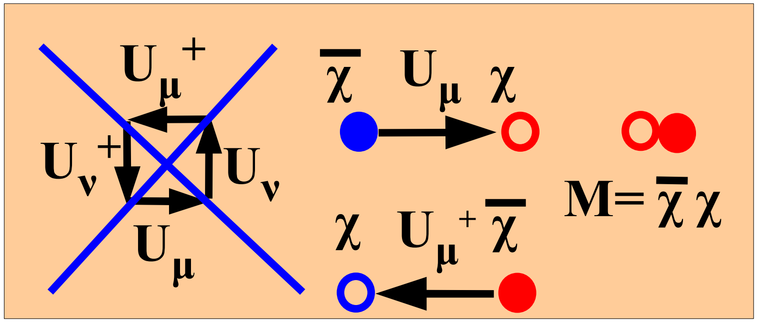
- **Strong Coupling Limit: Pure gluonic action disappers at g → ∞**

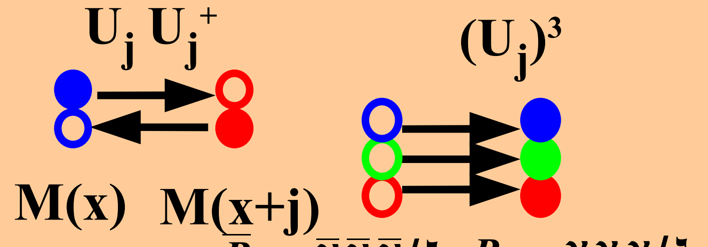
$$\boxed{S_{\mathrm{QCD}} = \cancel{S_G} + S_F^{(s)} + S_F^{(t)} + m_{٠}\,\bar{\chi}\chi}$$

$$\cancel{S_G = -\frac{١}{g^{٢}}\sum_{\text{plaq.}} \mathrm{Tr}\, U_{ij}(x) + c.c.}$$

$$S_F^{(s)} = \frac{١}{٢}\sum_{x,\, j>٠}\left(\bar{\chi}_x U_j(x)\chi_{x+\hat{j}} - \bar{\chi}_{x+\hat{j}} U_j^{+}(x)\chi_x\right)$$

$$S_F^{(t)} = \frac{١}{٢}\sum_{x}\left(e^{\mu}\bar{\chi}_x U_{٠}(x)\chi_{x+\hat{٠}} - e^{-\mu}\bar{\chi}_{x+\hat{٠}} U_{٠}^{+}(x)\chi_x\right)$$





$\bar{B} = \epsilon\bar{\chi}\bar{\chi}\bar{\chi}/٦$, $B = \epsilon\chi\chi\chi/٦$

$$\int dU\, U_{ab} U_{cd}^{+} = \frac{١}{N_c}\delta_{ad}\delta_{bc}$$

$$\int dU\, U_{ab} U_{cd} U_{ef} = \frac{١}{٦}\epsilon_{ace}\epsilon_{bdf}$$

- **One-link integral leaves mesonic and baryonic action.**

$$\boxed{S_F^{(s)} \to -\frac{١}{٢}(M\, V_M\, M) - (\bar{B}\, V_B\, B)}$$

$$= -\frac{١}{٤N_c}\sum_{x,\, j>٠} M_x M_{x+\hat{j}} + \sum_{x,\, j>٠}\frac{\eta_j}{٨}\left[\bar{B}_x B_{x+\hat{j}} - \bar{B}_{x+\hat{j}} B_x\right]$$

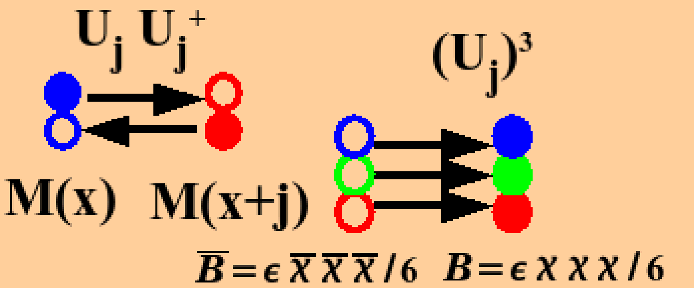
- **Analytic Link Integral → No Sign Problem at finite μ.**

SCL-LQCD: Tools (1) --- One-Link Integral

- **Group Integral Formulae**

$$\int dU\, U_{ab} U^{+}_{cd} = \frac{1}{N_c}\delta_{ad}\delta_{bc}$$

$$\int dU\, U_{ab} U_{cd} U_{ef} = \frac{1}{6}\epsilon_{ace}\epsilon_{bdf}$$



$\bar{B} = \epsilon\bar{\chi}\bar{\chi}\bar{\chi}/6$ $\quad B = \epsilon\chi\chi\chi/6$

$$\int dU\, U_{ab} U^{+}_{cd} = \frac{1}{N_c}\delta_{ad}\delta_{bc}$$

$$\int dU\, U_{ab} U_{cd} U_{ef} = \frac{1}{6}\epsilon_{ace}\epsilon_{bdf}$$

$$\begin{aligned}
&\int dU \exp(-a\bar{\chi}(x)U\chi(y) + b\bar{\chi}(y)U^{+}\chi(x)) \\
&= \int dU\left[1 - ab\,\bar{\chi}(a)^{a} U_{ab}\chi^{b}(y)\bar{\chi}^{c}(y)U^{+}_{cd}\chi^{d}(x) + \cdots\right] \\
&= 1 + ab(\chi\bar{\chi})(x)(\chi\bar{\chi})(y) + \cdots = 1 + ab\,M(x)M(y) + \cdots \\
&= \exp\left[ab\,M(x)M(y) + \cdots\right]
\end{aligned}$$

Quarks and Gluons → One-Link integral
→ Mesonic and Baryonic Composites

SCL-LQCD: Tools (2) --- 1/d Expansion

- **Keep mesonic action to be indep. from spatial dimension *d*
  → Higher order terms are suppressed at large *d*.**

$$\sum_j (\bar{\chi} U_j \chi)(\bar{\chi} U_j^{+} \chi) \to -\frac{1}{N_c} \sum_j M(x) M(x+\hat{j}) = O(1)$$

$$\to M \propto 1/\sqrt{d}\,,\ \chi \propto d^{-1/4}$$

$$\sum_j (\bar{\chi} U_j \chi)(\bar{\chi} U_j \chi)(\bar{\chi} U_j \chi) \to N_c! \sum_j B(x) B(x+\hat{j}) = O(1/\sqrt{d})$$

$$\sum_j (\bar{\chi} U_j \chi)^2 (\bar{\chi} U_j^{+} \chi)^2 \to \sum_j M^2(x) M^2(x+\hat{j}) = O(1/d)$$

***We can stop the expansion in U,
since higher order terms are suppressed !***

SCL-LQCD: Tools (3) --- Bosonization

- **We can reduce the power in χ by introducing bosons**

$$\exp\left(\frac{1}{2}M^2\right)=\int d\sigma \exp\left(-\frac{1}{2}\sigma^2-\sigma M\right)$$

Nuclear MFA: $V=-\frac{1}{2}(\bar{\psi}\psi)(\bar{\psi}\psi)\simeq -U(\bar{\psi}\psi)+\frac{1}{2}U^2$

$$\exp\left[-\frac{1}{2}M^2\right]=\int d\varphi \exp\left[-\frac{1}{2}\varphi^2-i\varphi M\right]$$

Reduction of the power of χ
→ Bi-Linear form in χ → Fermion Determinant

SCL-LQCD: Tools (4) --- Grassman Integral

- **Bi-linear Fermion action leads to -log(det A) effective action**

$$\int d\chi\, d\bar{\chi} \exp\left[\bar{\chi} A \chi\right] = \det A = \exp\left[-(-\log \det A)\right]$$

$$\int d\chi \cdot 1 = \text{anti-comm. constant} = 0 \quad , \quad \int d\chi \cdot \chi = \text{comm. constant} \equiv 1$$

$$\int d\chi\, d\bar{\chi} \exp\left[\bar{\chi} A \chi\right] = \int d\chi\, d\bar{\chi} \frac{1}{N!} (\bar{\chi} A \chi)^N = \cdots = \det A$$

Constant σ → - log σ interaction (Chiral RMF)

- **Temporal Link Integral, Matsubara product, Staggered Fermion,**
  → I will explain next time ....

Effective Potential in SCL-LQCD (Zero T)

- **QCD Lattice Action (Zero T treatment)**

$$S = \cancel{S_G} + S_F + m_0 \bar{\chi}\chi$$

Strong Coupling Limit

$$\to -\frac{1}{2}(\bar{\chi}\chi)V_M(\bar{\chi}\chi) + m_q \bar{\chi}\chi$$

One-link integral (1/d expansion)

$$\to \frac{1}{2}\sigma V_M^{-1}\sigma + \bar{\chi}(\sigma + m_q)\chi$$

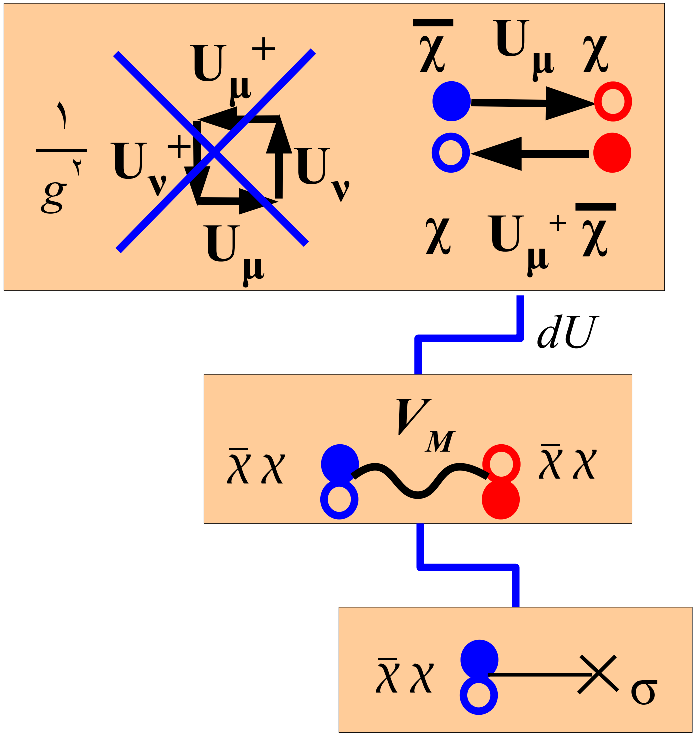
Bosonization

$$\to \frac{1}{2}\sigma V_M^{-1}\sigma - N_c \sum_x \log(\sigma(x) + m_q)$$

Fermion Integral

$$= L^d N_\tau \boxed{\left[\frac{N_c}{d+1}\bar{\sigma}^2 - N_c \log(\bar{\sigma} + m_q)\right]}$$

Effective Potential



- **Effective Potential in SCL-LQCD**

$$\boxed{U(\sigma) = \frac{N_c}{d+1}\sigma^2 - N_c \log\sigma = \frac{1}{2} b_\sigma \sigma^2 - a_\sigma \log\sigma}$$

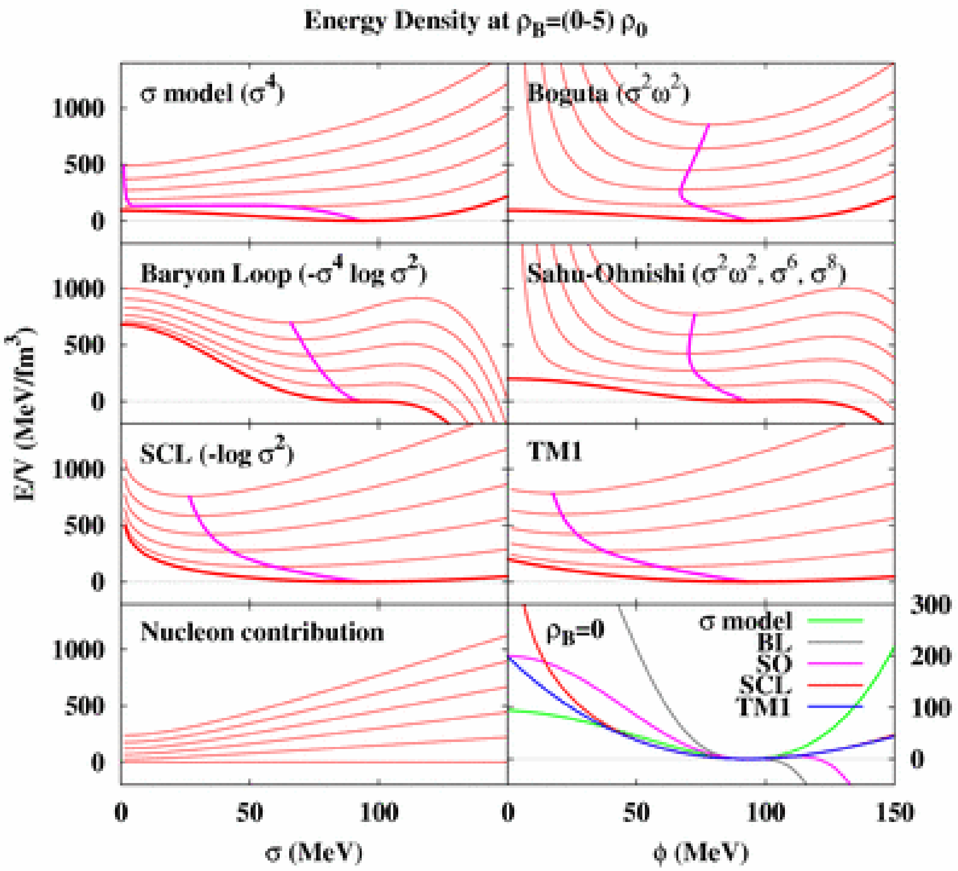
RMF with Chiral Symmetry: Chiral Collapse

- **Naïve Chiral RMF models → Chiral collapse at low ρ** *(Lee-Wick 1974)*
- **Prescriptions**
  - **σω coupling (too stiff EOS)**
    (Boguta 1983, Ogawa et al. 2004)
  - **Loop effects (unstable at large σ)**
    (Matsui-Serot, 1982, Glendenning 1988, Prakash-Ainsworth 1987,Tamenaga et al. 2006)
  - **Higher order terms (unstable at large σ)**
    (Hatsuda-Prakash 1989, Sahu-Ohnishi 2000)
  - ***Dielectric (Glueball) Field representing scale anomaly***
    (Furnstahl-Serot 1993,Heide-Rudaz-Ellis 1994, Papazoglou et al.(SU(3)) 1998)
  - **Different Chiral partner assignment**
    (DeTar-Kunihiro 1989, Hatsuda-Prakash 1989, Harada-Yamawaki 2001, Zschiesche-Tolos-Schaffner-Bielich-Pisarski, nucl-th/0608044) → $SU_f(3)$ extention ?
  - ***Nucleon Structure***
    (Saito-Thomas 1994, Bentz-Thomas 2001)

$$L=\frac{1}{2}\left(\partial_\mu\sigma\,\partial^\mu\sigma+\partial_\mu\pi\,\partial^\mu\pi\right)$$

$$-\frac{\lambda}{4}\left(\sigma^2+\pi^2\right)^2+\frac{\mu^2}{2}\left(\sigma^2+\pi^2\right)+c\,\sigma$$

$$+\overline{N}\,i\partial_\mu\gamma^\mu N-g_\sigma\overline{N}\left(\sigma+i\pi\tau\gamma_5\right)N$$



Various Attempts to Cure Chiral Collapse

- **φ^4 Theory (Gell-Mann, Levy) → Collapse**

$$V_\sigma^{(\phi^4)} = \frac{\lambda}{4}(\phi^2 - f_\pi^2)^2 + \frac{1}{2}m_\pi^2\phi^2 - f_\pi m_\pi^2\sigma\ , \quad \lambda = \frac{m_\sigma^2 - m_\pi^2}{2f_\pi^2}$$

- **NJL (Quark Loop) → Collapse**

$$V_\sigma^{\mathrm{NJL}} = \frac{m_0^2}{2}\sigma^2 + \Lambda^4 f_{\mathrm{NJL}}\left(\frac{G\sigma}{\Lambda}\right) - f_\pi m_\pi^2\sigma \qquad f_{\mathrm{NJL}}(x) = -\frac{N_c N_f}{4\pi^2}\left[\left(1+\frac{x^2}{2}\right)\sqrt{1+x^2} - 1 - \frac{x^4}{2}\log\left(\frac{1+\sqrt{1+x^2}}{x}\right)\right]$$

- **Baryon Loop (Matsui & Serot) → Unstable at large σ**

$$V_\sigma^{\mathrm{BL}} = \frac{m_\sigma^2}{8f_\pi^2}(\phi^2 - f_\pi^2)^2 - M_N^4 f_{\mathrm{BL}}(\phi/f_\pi) \qquad f_{\mathrm{BL}}(x) = -\frac{1}{4\pi^2}\left[\frac{x^4}{2}\log x^2 - \frac{1}{4} + x^2 - \frac{3}{4}x^4\right]$$

- **Higher order terms (E.g. Sahu & Ohnishi)**

$$V_\sigma^{\mathrm{SO}} = \frac{m_\sigma^2}{8f_\pi^2}(\phi^2 - f_\pi^2)^2 + f_\pi^4 f_{\mathrm{SO}}(\phi/f_\pi) \qquad f_{\mathrm{SO}}(x) = \frac{C_6}{6}(x^2-1)^3 + \frac{C_8}{8}(x^2-1)^4$$

- **Log type term from scale anomaly (Furnstahl, Serot; Heide et al.)**
- **Log type term from SCL-LQCD (Tsubakihara & AO)**

$$V_\sigma^{\mathrm{SCL}} = V_\chi(\sigma,\pi) - c_\sigma\,\sigma = \frac{1}{2}b_\sigma\phi^2 - a_\sigma\log\phi^2 - c_\sigma\,\sigma$$

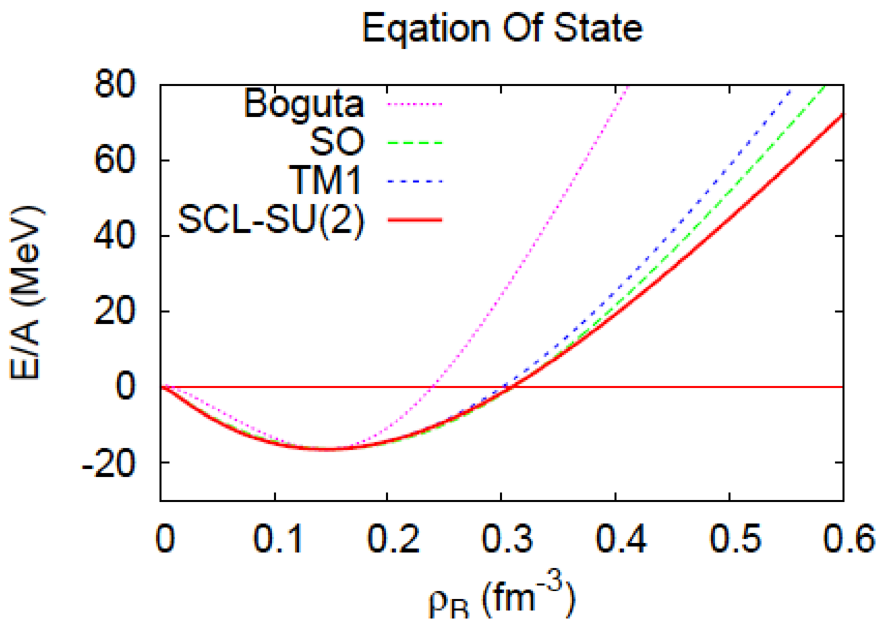
Chiral RMF based on SCL-LQCD

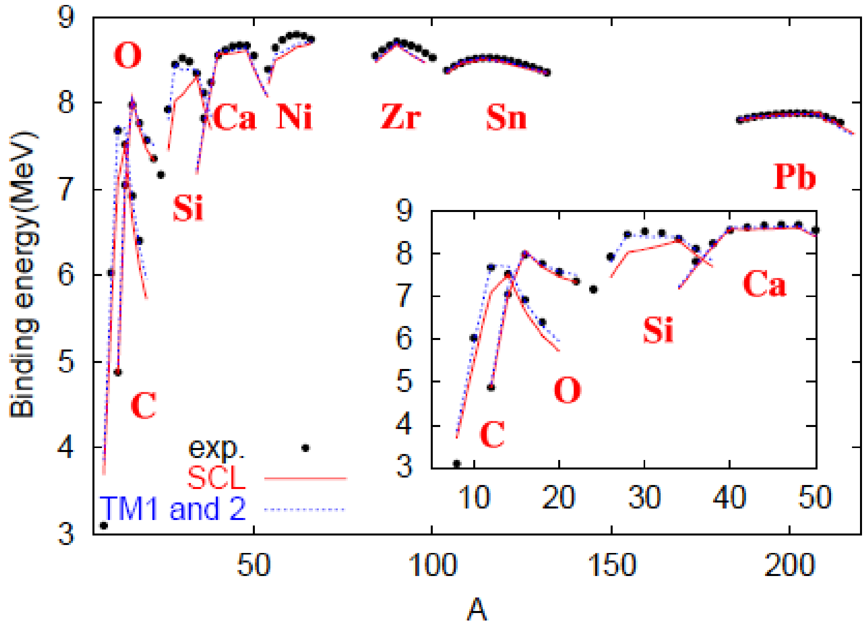
Tsubakihara, AO, PTP 117('07)903 [nucl-th/0607046]

- **強結合格子 QCD に基づく Chiral RMF 模型**

$$U_{\mathrm{Linear}\sigma\mathrm{model}}(\sigma)=-\frac{\mu^2}{2}\sigma^2+\frac{\lambda}{4}\sigma^4 \longrightarrow U_{\mathrm{SCL}}(\sigma)=\frac{1}{2}b_\sigma\sigma^2-a_\sigma\log\sigma$$

  - **QCD に基づき、カイラル対称性をもち、不安定性はない。**
  - **少ない数のパラメータで、核物質・原子核のバルクな性質をよく説明**

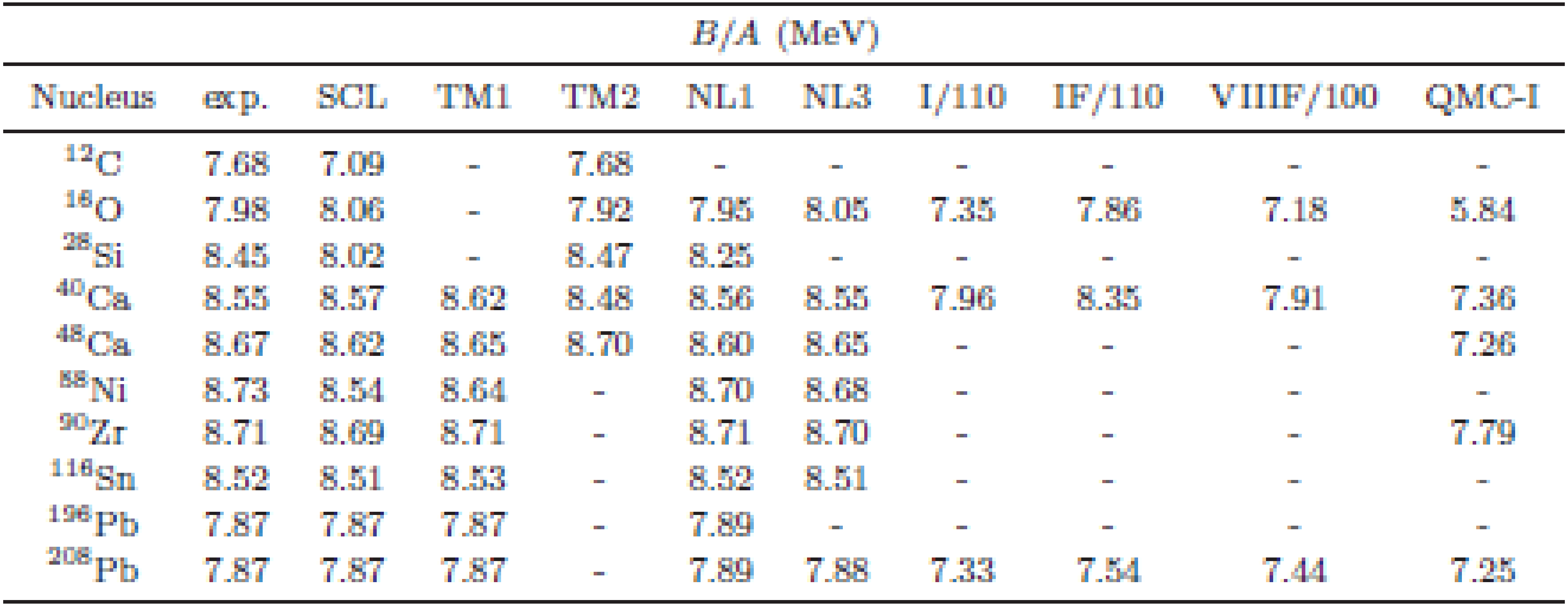




QCD から原子核の「密度汎関数」を与える第一歩！

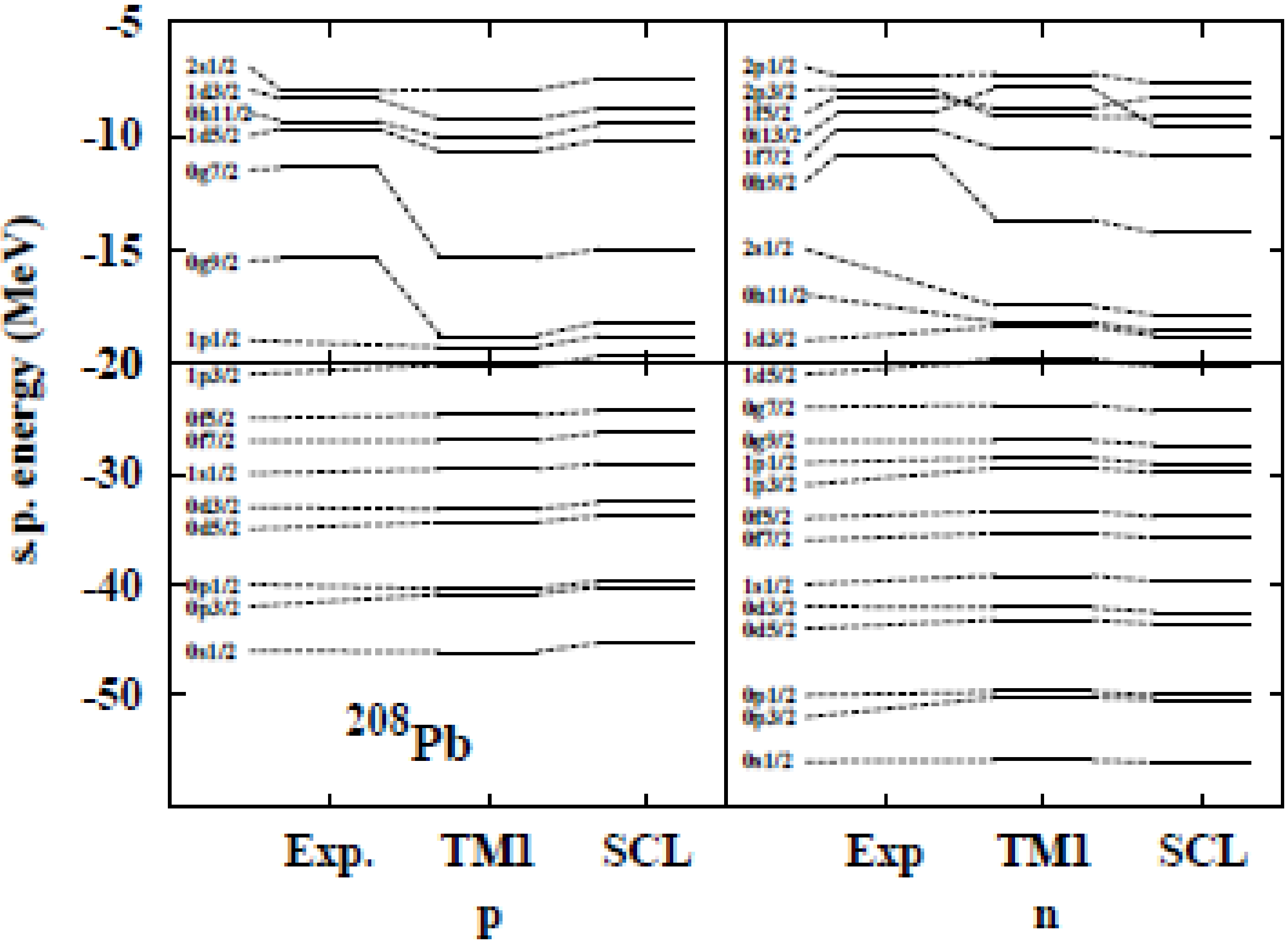
Binding Energies in Chiral and Non-Chiral RMF

- **Non-Chiral High Precision RMF: TM1 & 2, NL1, NL3**
  (Sugahara, Toki, 1994; Reinhard et al., 1986; Lalazissis, Koenig, Ring, 1997)
- **Log term from Scale Anomaly: I/110, IF/110, VIIIF/100**
  (Heide, Rudas, Ellis, 1994)
- **Quark Meson Coupling model**
  (Saito, Tsushima, Thomas, 1997)



B/A (MeV)										
Nucleus	exp.	SCL	TM1	TM2	NL1	NL3	I/110	IF/110	VIIIF/100	QMC-I
^{12}C	7.68	7.09	-	7.68	-	-	-	-	-	-
^{16}O	7.98	8.06	-	7.92	7.95	8.05	7.35	7.86	7.18	5.84
^{28}Si	8.45	8.02	-	8.47	8.25	-	-	-	-	-
^{40}Ca	8.55	8.57	8.62	8.48	8.56	8.55	7.96	8.35	7.91	7.36
^{48}Ca	8.67	8.62	8.65	8.70	8.60	8.65	-	-	-	7.26
^{58}Ni	8.73	8.54	8.64	-	8.70	8.68	-	-	-	-
^{90}Zr	8.71	8.69	8.71	-	8.71	8.70	-	-	-	7.79
^{116}Sn	8.52	8.51	8.53	-	8.52	8.51	-	-	-	-
^{196}Pb	7.87	7.87	7.87	-	7.89	-	-	-	-	-
^{208}Pb	7.87	7.87	7.87	-	7.89	7.88	7.33	7.54	7.44	7.25

Single Particle Energies



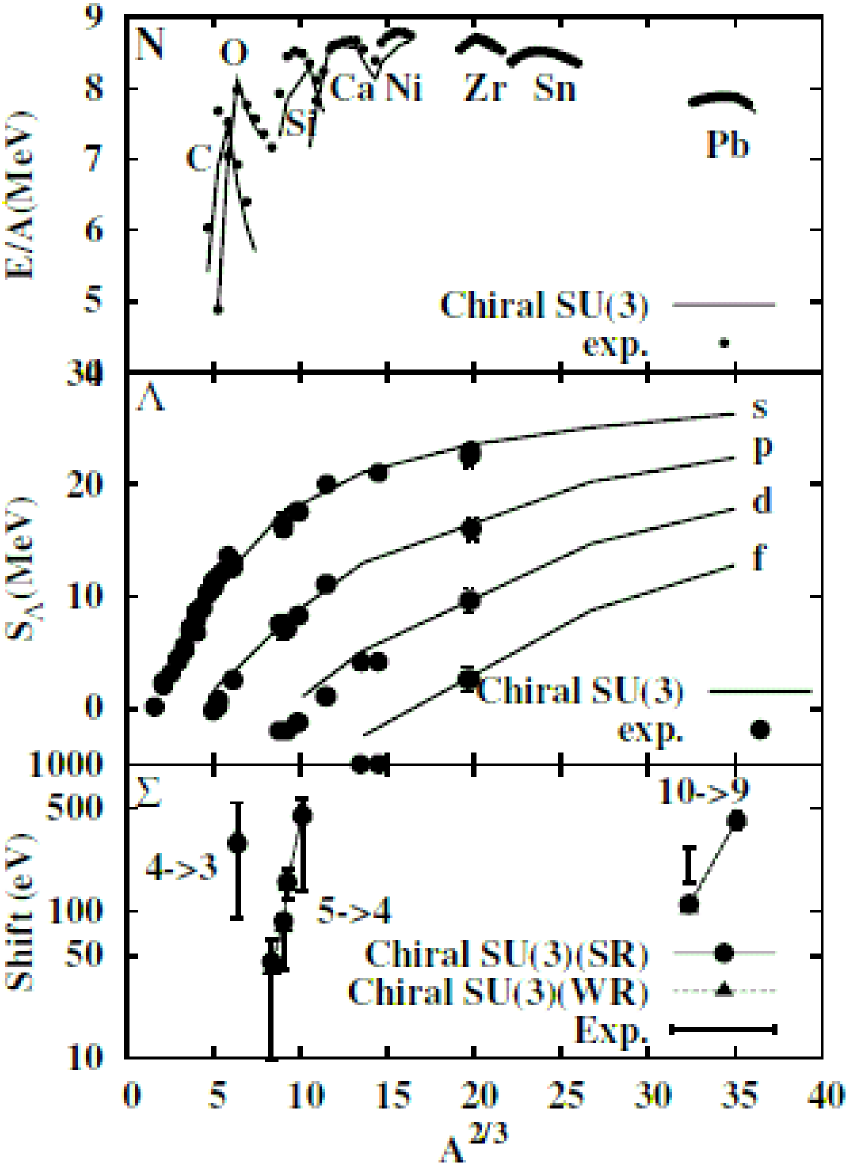
Chiral $SU_f(3)$ RMF

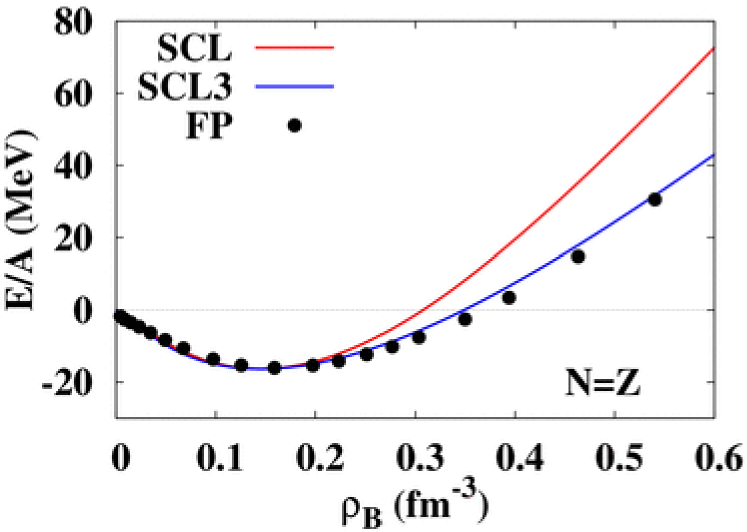
Tsubakihara, Maekawa, AO, EPJA33('07)295 [nucl-th/0702008]

- **Extention to Flavor SU(3)**
  → Chiral Potential from SCL-LQCD
  + Determinant Int. ($U_A(1)$ anomaly)
  + Explicit breaking term

$$U_{\sigma\zeta} = -\,a\,\log(\det MM^\dagger) + b\,\mathrm{tr}(MM^\dagger) + c_\sigma\sigma + c_\zeta\zeta + d\,(\det M + \det M^\dagger).$$

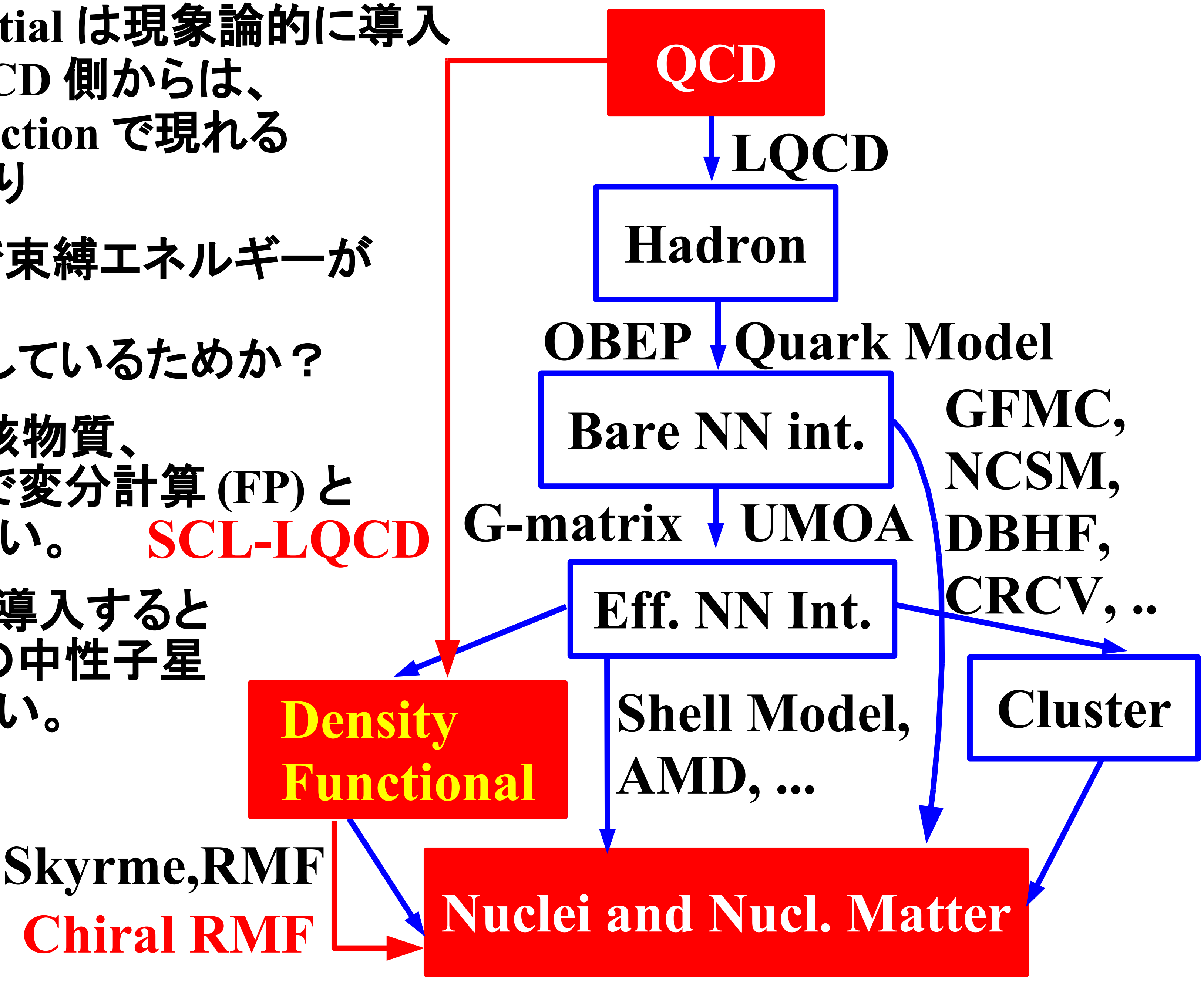
- **Normal, Single & Double Λ, Σ atom, EOS (~ FP), ...**





Problems

- Vector potential は現象論的に導入
  → SCL-LQCD 側からは、
  $1/g^2$ correction で現れる
  可能性あり
- jj closed 核で束縛エネルギーが
  足りない！
  → π を無視しているためか？
- 高密度対称核物質、
  中性子物質で変分計算 (FP) と
  一致していない。
- ハイペロンを導入すると
  1.44 Msun の中性子星
  を支えられない。



Summary

- **Strangeness Nuclear Physics**
  - **Discovery Science, Probe of Nuclei, BB interaction, Dense Matter**
  - **J-PARC の目的の一つは、様々な「バリオン、電荷、ストレンジ密度」での密度汎関数を与えることである！**
- **ハイペロン・ポテンシャルと高密度物質の状態方程式**
  - **ハイペロン生成反応：連続状態 Spectroscopy → ポテンシャル**
    (はっきりとは決まらないが、制限を与える)
  - **Ξ 核生成反応 : 「ハイペロンは原子核を穏やかにかえる」のは正しいか？**
    π と直接結合しない Λ の特殊性のためかも知れない
  - **ハイペロンを含む超新星物質状態方程式 :**
    ハイペロンは密度 0.4 fm^{-3} 以上、または温度 40 MeV 以上で現れる
    → 超新星爆発の初期段階 (prompt) では大きな効果はないだろう
  - **Chiral RMF: 強結合極限格子 QCD から得た potential を含む RMF**
    不安定性はなく、現象論的にも有望。SU(3) では too soft
- **やるべきことは多くある。J-PARC に期待！**

backups

核物理におけるストレンジネスの役割

- New DOF in nuclei (Discovery Science)
  - New type of hadrons and nuclei (Θ^+, deeply bound kaonic nuclei)
  - Ξ hypernuclei, Double Λ hypernuclei, ...
- Hyperon as a probe of nuclei (不純物効果、外場としての役割)
  - New type of excited states (genuine hypernuclear state)
  - Change of Nuclear Size & Shape (shrinkage)
- Laboratory to examine baryon-baryon interaction model
  - Very small ΛN LS (Antisymmetric LS from quark model interaction)
  - Repulsive Core between BB (Quark Pauli effect, OGE)
- Modification of Dense Matter EOS
  - Softening of EOS with hyperon admixture (Universal 3B repulsion ?)
  - Modification of cooling rate (Hyperon superfluidity)

Summary

- Hyperon の存在と、その核物質中でのポテンシャルの深さは、高密度核物質の性質とコンパクト天体現象に大きな役割を果たしうる。
- Σ - 核ポテンシャルの分析と影響
  - QF スペクトルの「形」と「絶対値」の再現 → $V(\Sigma) > 15$ MeV
- ccAMD による Ξ 核構造と生成スペクトルの分析 (preliminary)
  - 軽い核では、Ξ によって変形度が大きく変化しうる。
    → 生成スペクトルの形を変化させる可能性あり。
- Σ, Ξ のポテンシャルを取り入れた RMF による EOS table
  - 低密度領域では Shen EOS と結合
  - Σ とベクトル中間子の結合に単純なクォーク数 counting を用いると、ρ_0 で斥力であれば Σ はなかなか現れない。
  - 中性子星では大きな影響。超新星爆発では限定的。
    Black Hole 形成では有意の影響 (住吉) 。
  - 重い中性子星が確定すれば、EOS の硬化が必要。→ ?