集中講義題目：

高密度核物質の探索
―― ストレンジネス・格子・重イオン ――

大西　明　（京都大学・基礎物理学研究所）

- Introduction: J-PARC で展開される物理の概観
- ハイペロン生成反応
  - 直接反応理論とグリーン関数法
- 強結合領域における格子 QCD
  - 格子場の理論の基礎、強結合領域での有効ポテンシャル
  - 強結合格子 QCD で探るクォーク物質の相 (コロキウム)
- 高エネルギー重イオン反応における輸送理論
  - 流体模型 (完全流体) 、輸送方程式、エントロピー生成
- まとめ

物理教室コロキウム：

強結合格子 QCD で探るクォーク物質の相
Phase Diagram of Quark Matter from Strong Coupling Lattice QCD

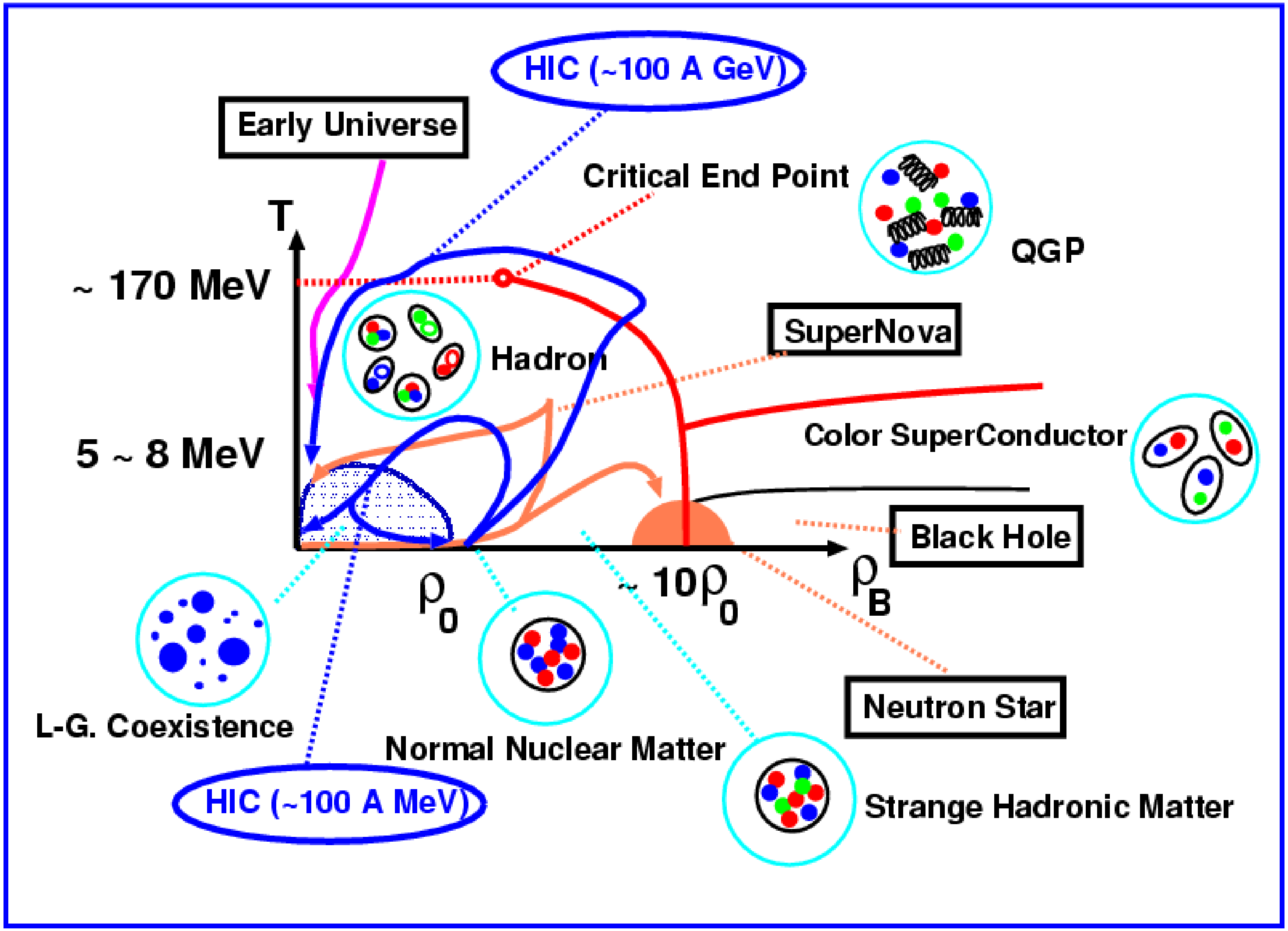
大西　明（京都大学・基礎物理学研究所）

- **Introduction**
- **Strong Coupling Lattice QCD with $1/g^2$ Correction**
- **Phase diagram in SC-LQCD and Quarkyonic Phase**
- **Summary**

Miura and AO, arXiv:0806.3357
Kawamoto, Miura, AO, Ohnuma,
Phys. Rev. D 75 (2007), 014502 [arXiv:hep-lat/0512023]

I'm interested in ....

- **Quark / Hadron / Nuclear Matter EOS and Phase Diagram**



Rich Structure / Astrophysical implications / Accessible in HIC

2008年ノーベル物理学賞

南部さん、小林さん、益川さん、おめでとうございます！



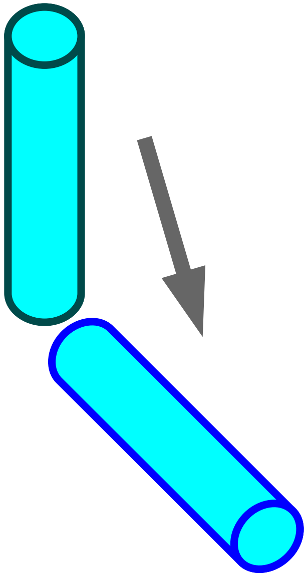
- 南部陽一郎（シカゴ大学教授）
- 小林誠（高エネルギー加速器研究機構教授）
  益川敏英（京都産業大学教授・京都大学基礎物理学研究所教授）

- ともに「クォーク」の不思議な性質の解明に寄与
  → クォークとは何か？どのような性質が解明されたのか？

南部理論

- ノーベル賞受賞理由
for the discovery of the mechanism of spontaneous broken symmetry in subatomic physics

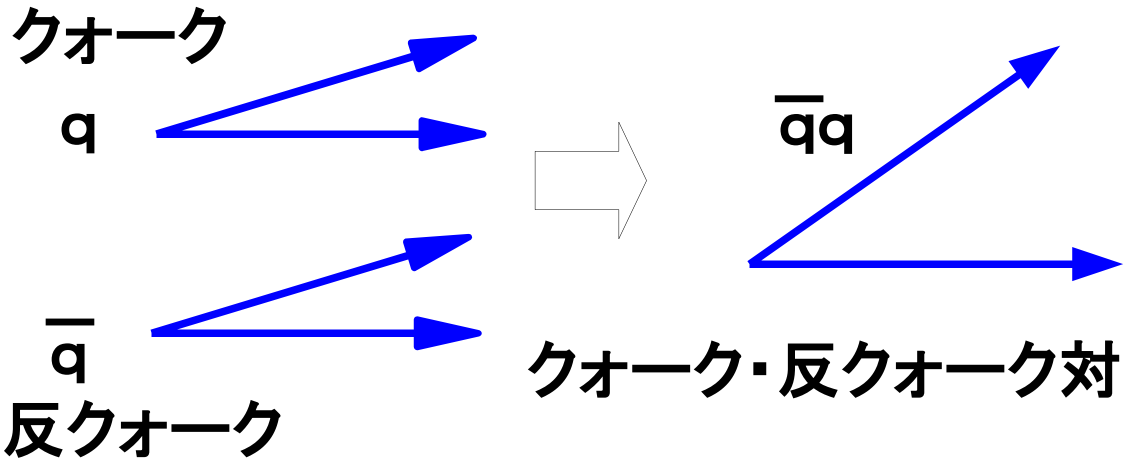
  素粒子･原子核物理において対称性が自発的に破れて粒子が質量を獲得する機構の発見に対して
- 対称性の自発的破れとは？
「まっすぐ立てた鉛筆は、どの方向に倒れる確率も同じ(等方的)だが、少しの揺らぎである方向に倒れ、元に戻ることはない。」
- 南部理論では、
カイラル対称性が自発的に破れる機構を発見し、生の質量が小さい(5 MeV)クォークが大きな質量を得ることを示した。

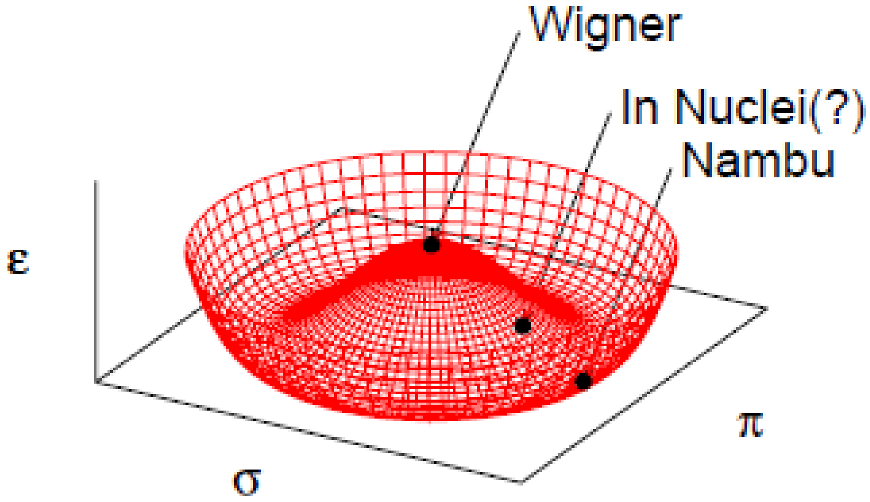


南部理論:カイラル対称性の自発的破れ

- カイラル対称性
  クォークと反クォークを複素平面で同じ方向にまわしても
  エネルギーは変わらない
  → クォーク・反クォーク対を複素平面でまわしても
  エネルギーは変わらない

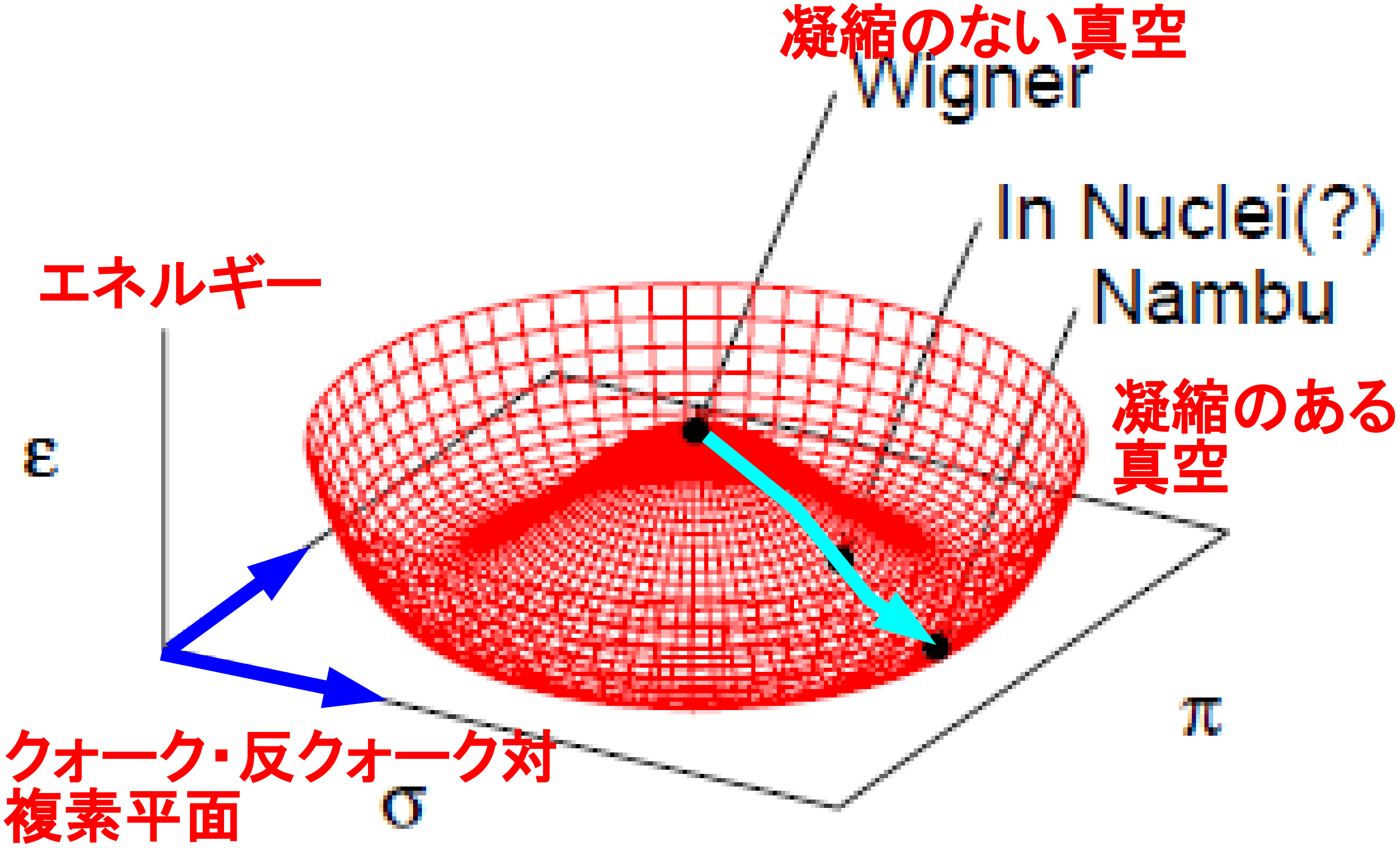
- クォークと反クォークの間には強い引力が働くので、
  対を作って同じ方向に凝縮する(空間を埋め尽くす)。





南部理論：カイラル対称性の自発的破れ

- 最も安定な状態(真空)ではクォーク・反クォーク対が凝縮
  → ある方向が選ばれる(真空での自発的対称性の破れ)
  → クォークは凝縮体にぶつかって動きにくくなる
  → 質量の増加



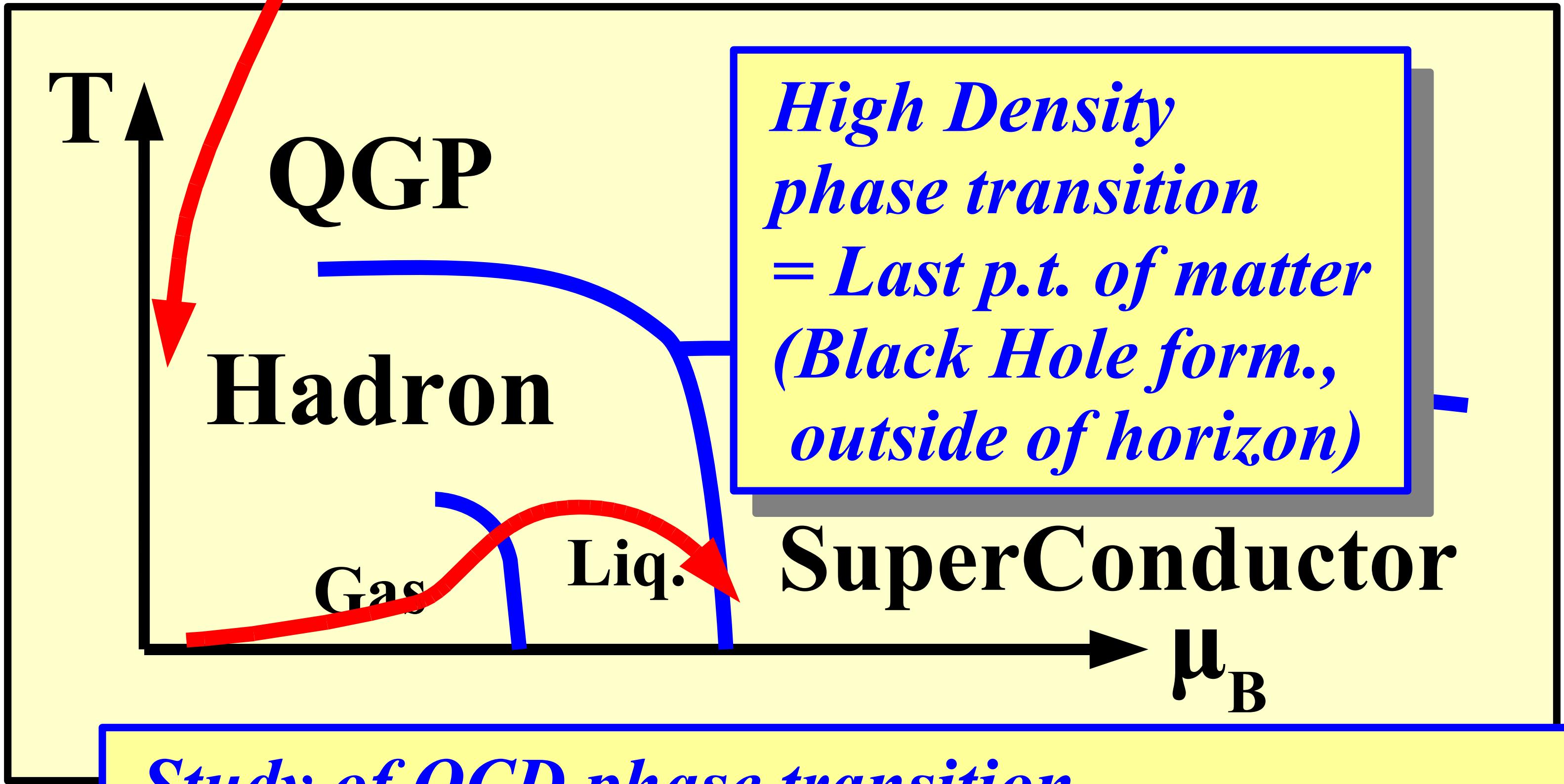
南部理論からの帰結

- カイラル対称性のオーダーパラメータ
  = カイラル凝縮（クォーク・反クォーク対の凝縮）
  → 2 つの異なる相が存在
  - カイラル対称性が自発的に破れた相 (Nambu-Goldstone 相)
  - カイラル対称性が回復した相 (Wigner 相)
- 他のオーダーパラメータは？
  - Polyakov Loop (閉じ込め)
    格子 QCD シミュレーション (バリオン密度 = 0) では、
    カイラル対称性の回復とほぼ同時に非閉じ込め相転移
  - クォーク対凝縮 (カラー超伝導)
    低温・超高密度ではクォークの超伝導状態 (摂動論的 QCD)
  - バリオン密度
    低温・高密度では、格子 QCD シミュレーションが困難

高密度でのカイラル相転移は？

Why do we want to study QCD phase diagram ?

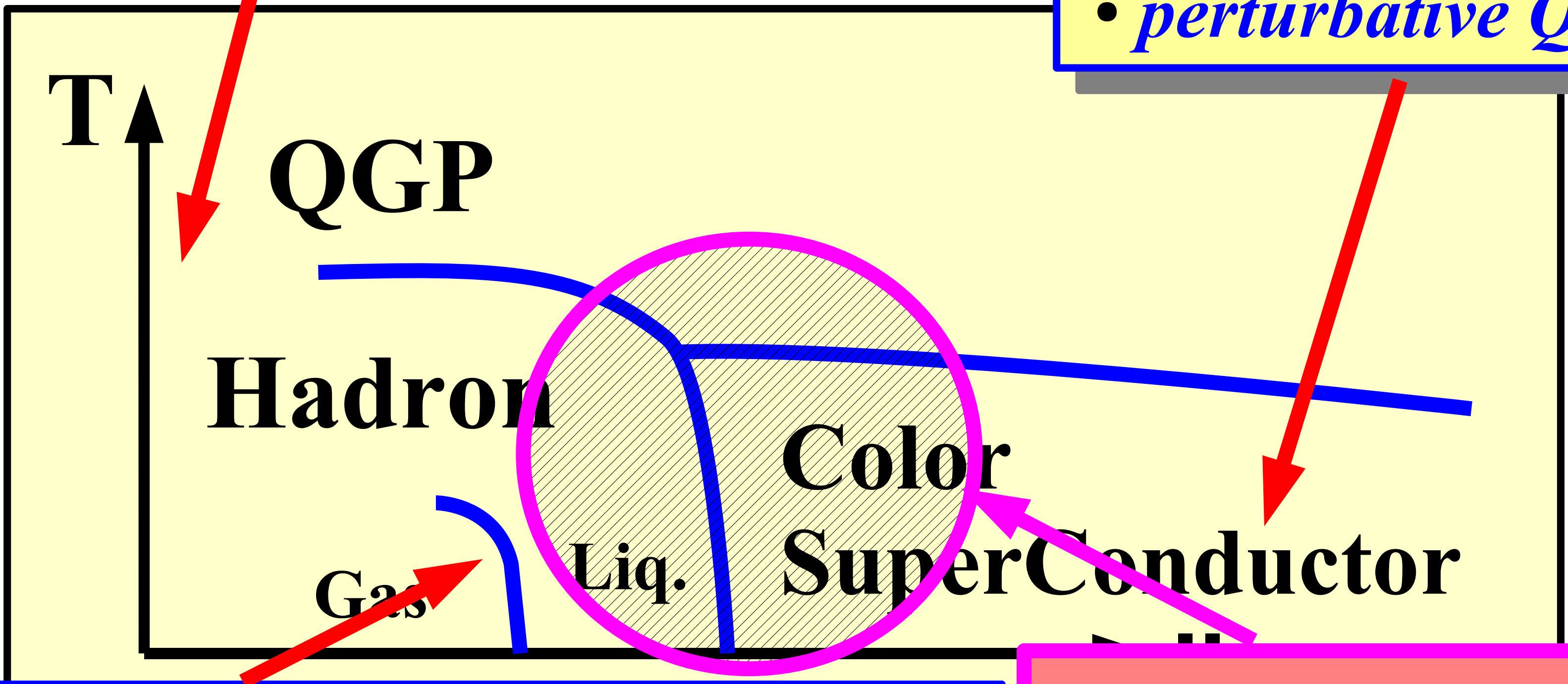
High T phase transition
= Latest vacuum p.t.
of our universe (Big Bang)



High Density
phase transition
= Last p.t. of matter
(Black Hole form.,
outside of horizon)

Study of QCD phase transition
→ Where do we come from, where do we go ?

How Far Do We Know ?



High T P.T. is observed
- *RHIC Experiment*
- *Lattice QCD MC simulation*

High Density Limit is proven to be CSC (Color SuperConductor)
- *perturbative QCD*

Liquid Gas P. T. is
- *expected in Mean Field*
- *and Observed in Caloric Curve*

Little is known for High Density Phase Transition Region !

A Conjecture from Large N_c: Quarkyonic Phase

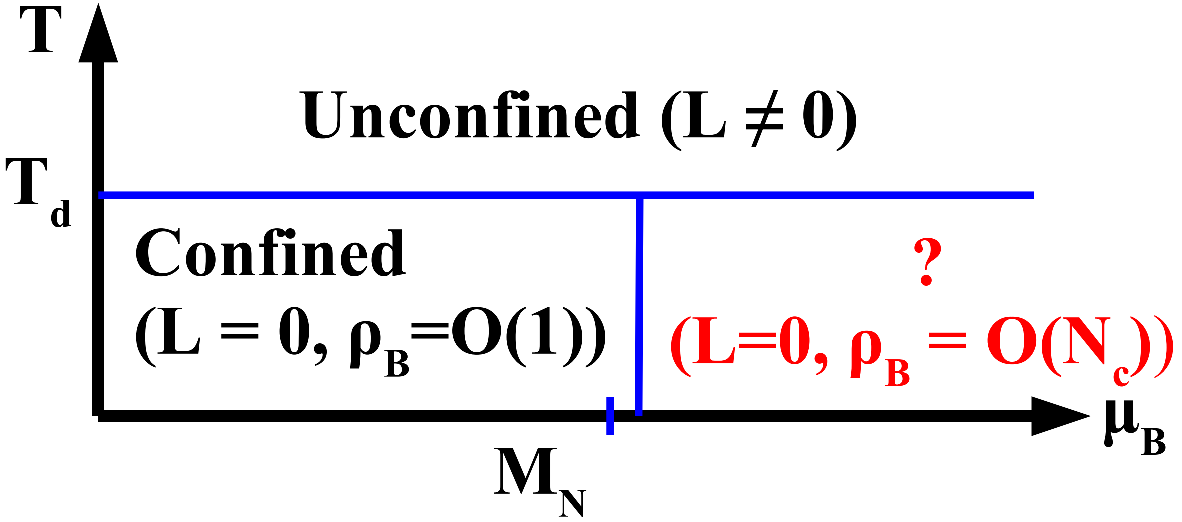
Pisarski, McLerran, 2007

- **Discussion at large N_c**
  - **Pressure:** **Gluon = $O(N_c^2)$, Quark = $O(N_c)$, Hadron = $O(1)$**

    → DECONFINEMENT phase transition (order parameter = Polyakov loop) is independent from quark chemical potential μ as far as μ = O(1).
  - **Large μ ($N_c\,\mu > M_B$) but low T ($T < T_d$)**

    → Weakly interacting quark gas, but no free gluons (confined).

    = High Density *Confined* Phase



T

Unconfined ($L \neq 0$)

T_d

Confined

($L = 0$, $\rho_B = O(1)$)

?

($L=0$, $\rho_B = O(N_c)$)

M_N

μ_B

What is this ?

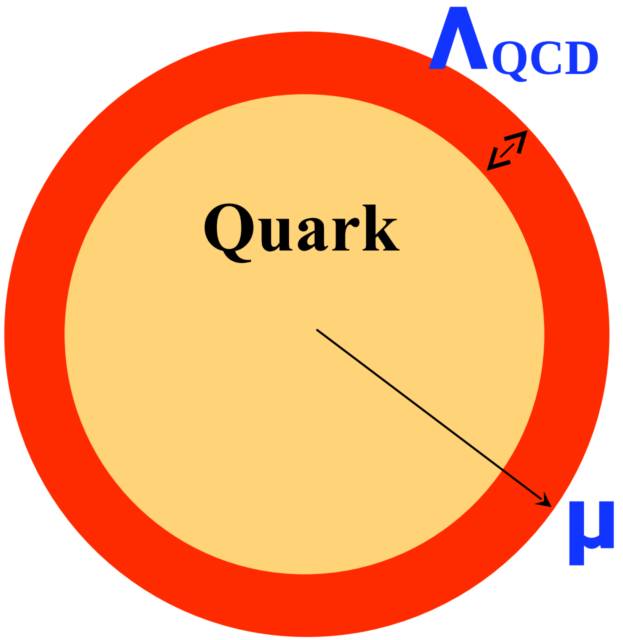
A Conjecture from Large N_c: Quarkyonic Phase

Pisarski, McLerran, 2007

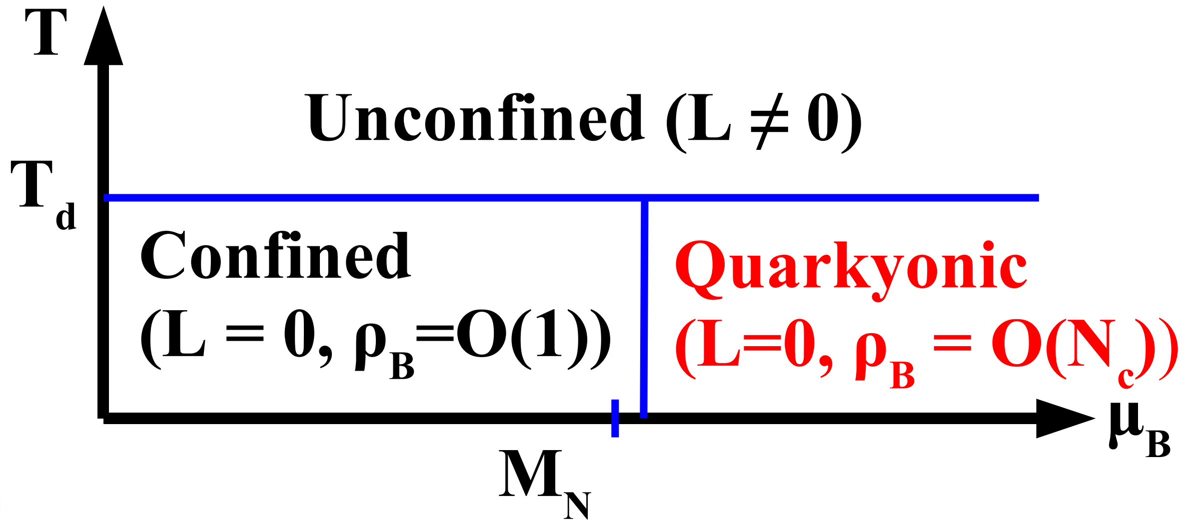
- **Confined High Density Matter at Large N_c**

  = Quarkyonic Phase
  (Quarks deeply inside the Fermi Sphere,
  with baryonic excitations)

Do we really see this phase at Nc=3 ?
What happens to Chiral Symmetry ?



Confined
→ Baryonic Excitation



T

T_d

Unconfined ($L \neq 0$)

Confined
($L = 0$, ρ_B=O(1))

Quarkyonic
(L=0, ρ_B = O(N_c))

μ_B

M_N

Quarkyonic phase in SC-LQCD

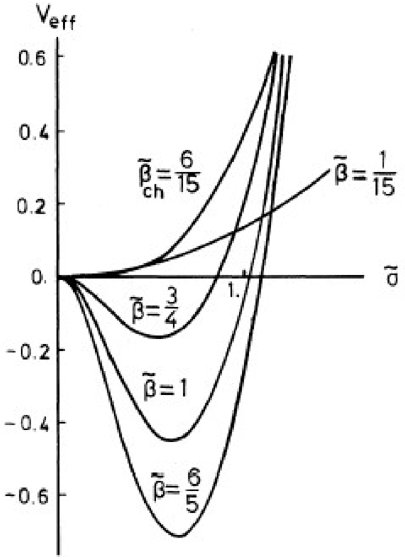
- **We study the phase diagram**
  in Strong Coupling Lattice QCD (SC-LQCD)
  with $1/g^2$ correction,
  and examine the existence of the Quarkyonic (QY) phase.

- **Reservations:** ***It is still a "Toy"***
  - **One species of staggered fermion without quarter/square root**
    → $N_f = 4$
  - **Leading order in 1/d (d=spatial dim.)**
    → No baryon effects (cf. *Par-Tue, Miura***)**
  - **Mean Field treatment**
  - **No Diqaurk condensate**
  - **NLO in $1/g^2$ expansion, ...**

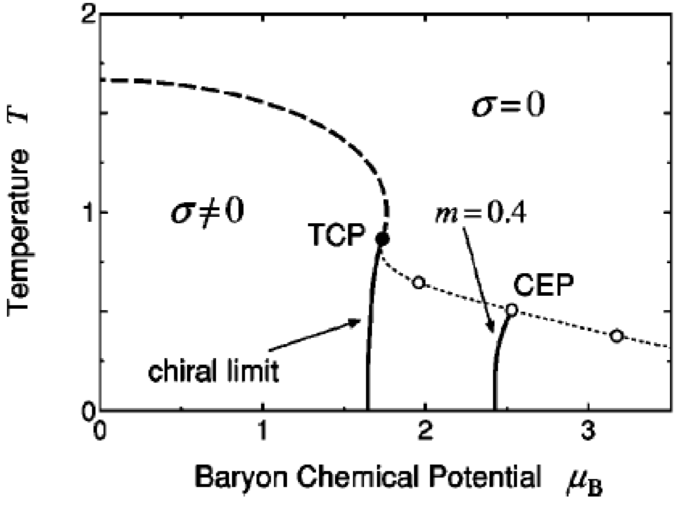
Strong Coupling Lattice QCD

Strong Coupling Limit of Lattice QCD

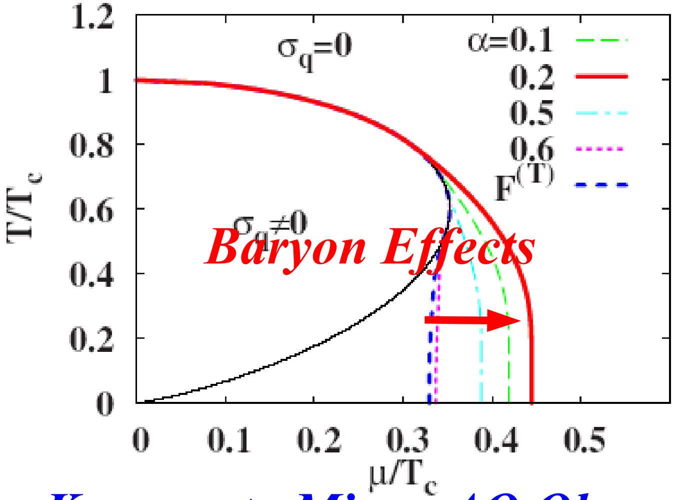
- **SCL-LQCD has been a powerful tool in “phase diagram” study !**
  - **Chiral restoration, Phase diagram, Baryon effects, Hadron masses, Finite coupling effects, ...**



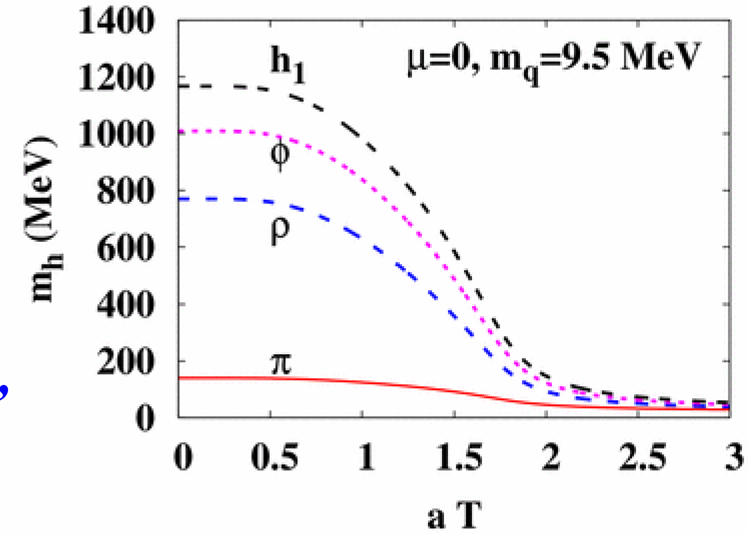
Damgaard,Kawamoto, Shigemoto, PRL53('84),2211



Nishida, PRD69, 094501 (2004



Kawamoto,Miura,AO,Ohnuma, PRD75 (07), 014502.



AO, Kawamoto, Miura, 2008

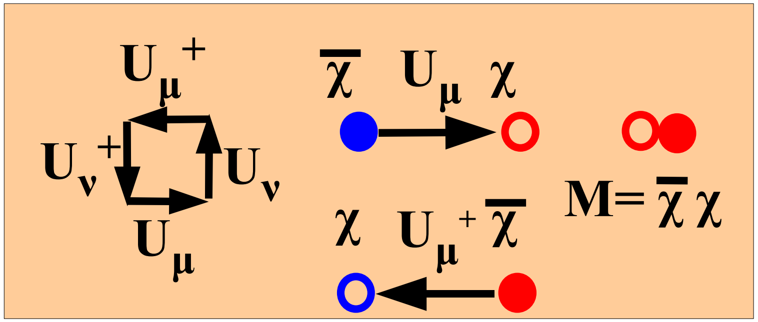
Lattice QCD (1)

- **QCD Lagrangian**

$$L=\bar{\psi}(i\gamma^{\mu}D_{\mu}-m_0)\psi-\frac{1}{2}tr(F_{\mu\nu}F^{\mu\nu})$$

ψ = Quark, F = Gluon tensor, m_0 = (small) quark mass

- **Lattice Action**



$$S_{\mathrm{QCD}}=S_G+S_F^{(s)}+S_F^{(t)}+m_0\bar{\chi}\chi$$

$$S_G=-\frac{1}{g^2}\sum_{\mathrm{plaq.}}\mathrm{Tr}\,U_{ij}(x)+c.c.$$

$$S_F^{(s)}=\frac{1}{2}\sum_{x,\,j>0}\left(\bar{\chi}_x U_j(x)\chi_{x+\hat{j}}-\bar{\chi}_{x+\hat{j}}U_j^{+}(x)\chi_x\right)$$

$$S_F^{(t)}=\frac{1}{2}\sum_{x}\left(e^{\mu}\bar{\chi}_x U_0(x)\chi_{x+\hat{0}}-e^{-\mu}\bar{\chi}_{x+\hat{0}}U_0^{+}(x)\chi_x\right)$$

χ = starggered fermion (quark)
U= link variable ∈ SU(N_c) (gluon),
μ = quark chemical potential

Lattice QCD (2)

- **Full QCD MC Simulation**
  → Monte-Carlo Integral of Det (Fermion Matrix) over link var. (U)
  - **Big Task !**
    Matrix Size= 4 (spinor) x (Color) x (Space-Time Points)
    Eigen Values are widely distributed
  - **Complex Weight with finite μ**

$$\int d\bar{\chi}\, d\chi\, dU \exp\left(-S_G+\bar{\chi} A \chi\right)=\int dU \left| \quad A \quad \right| \quad \updownarrow \mathbf{4\ N_c\ N_\tau\ N_s^3}$$

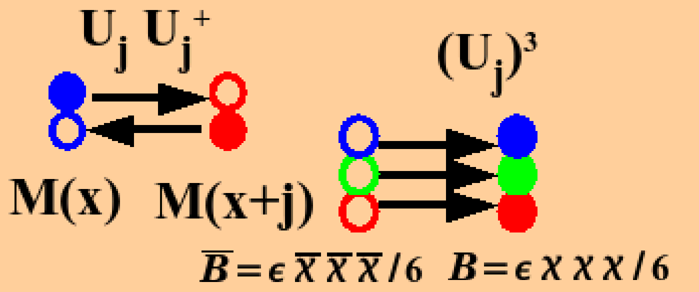
- **Quenched QCD**
  - **Assuming Det = 1 ~ Ignoring Fermion Loops**
  - **Works very well for hadron masses**
- ***Strong Coupling Limit (g → ∞)***
  - ***Pure gluonic action disappears → Analytic evaluation of Fermion Det.***

SCL-LQCD: Tools (1) --- One-Link Integral

- **Group Integral Formulae**

$$\int dU\, U_{ab} U^{+}_{cd} = \frac{1}{N_c} \delta_{ad} \delta_{bc}$$

$$\int dU\, U_{ab} U_{cd} U_{ef} = \frac{1}{6} \epsilon_{ace} \epsilon_{bdf}$$



$$\int dU\, U_{ab} U^{+}_{cd} = \frac{1}{N_c} \delta_{ad} \delta_{bc}$$

$$\int dU\, U_{ab} U_{cd} U_{ef} = \frac{1}{6} \epsilon_{ace} \epsilon_{bdf}$$

$$\begin{aligned}
&\int dU \exp\left(-a \bar{\chi}(x) U \chi(y) + b \bar{\chi}(y) U^{+} \chi(x)\right) \\
&= \int dU \left[1 - ab\, \bar{\chi}(a)^{a} U_{ab} \chi^{b}(y) \bar{\chi}^{c}(y) U^{+}_{cd} \chi^{d}(x) + \cdots\right] \\
&= 1 + ab \left(\chi \bar{\chi}\right)(x) \left(\chi \bar{\chi}\right)(y) + \cdots = 1 + ab\, M(x) M(y) + \cdots \\
&= \exp\left[ab\, M(x) M(y) + \cdots\right]
\end{aligned}$$

Quarks and Gluons → One-Link integral
→ Mesonic and Baryonic Composites

SCL-LQCD: Tools (2) --- 1/d Expansion

- **Keep mesonic action to be indep. from spatial dimension *d***
  → Higher order terms are suppressed at large *d*.

$$\sum_j (\bar{\chi} U_j \chi)(\bar{\chi} U_j^+ \chi) \to -\frac{1}{N_c} \sum_j M(x) M(x+\hat{j}) = O(1)$$

$$\to M \propto 1/\sqrt{d}\,, \chi \propto d^{-1/4}$$

$$\sum_j (\bar{\chi} U_j \chi)(\bar{\chi} U_j \chi)(\bar{\chi} U_j \chi) \to N_c! \sum_j B(x) B(x+\hat{j}) = O(1/\sqrt{d})$$

$$\sum_j (\bar{\chi} U_j \chi)^2 (\bar{\chi} U_j^+ \chi)^2 \to \sum_j M^2(x) M^2(x+\hat{j}) = O(1/d)$$

We can stop the expansion in U,
since higher order terms are suppressed !

SCL-LQCD: Tools (3) --- Bosonization

- **We can reduce the power in χ by introducing bosons**

$$\exp\left(\frac{1}{2}M^2\right)=\int d\sigma \exp\left(-\frac{1}{2}\sigma^2-\sigma M\right)$$

Nuclear MFA: $V=-\frac{1}{2}(\bar{\psi}\psi)(\bar{\psi}\psi)\simeq-U(\bar{\psi}\psi)+\frac{1}{2}U^2$

$$\exp\left[-\frac{1}{2}M^2\right]=\int d\varphi \exp\left[-\frac{1}{2}\varphi^2-i\varphi M\right]$$

Reduction of the power of χ
→ Bi-Linear form in χ → Fermion Determinant

SCL-LQCD: Tools (4) --- Grassman Integral

- **Bi-linear Fermion action leads to -log(det A) effective action**

$$\int d\chi\, d\bar{\chi} \exp\left[\bar{\chi} A \chi\right] = det\ A = \exp\left[-(-\log det\ A)\right]$$

$$\int d\chi \cdot 1 = \text{anti-comm. constant} = 0 \quad , \quad \int d\chi \cdot \chi = \text{comm. constant} \equiv 1$$

$$\int d\chi\, d\bar{\chi} \exp\left[\bar{\chi} A \chi\right] = \int d\chi\, d\bar{\chi} \frac{1}{N!} (\bar{\chi} A \chi)^N = \cdots = det\ A$$

Constant σ → - log σ interaction (Chiral RMF)

- **Temporal Link Integral, Matsubara product, Staggered Fermion,**
  → I will explain next time ....

Effective Potential in SCL-LQCD (Zero T)

- **QCD Lattice Action (Zero T treatment)**

Kawamoto, Smit, 1981

$$S = \cancel{S_G} + S_F + m_0 \bar{\chi}\chi$$

Strong Coupling Limit

$$\to -\frac{1}{2}(\bar{\chi}\chi) V_M (\bar{\chi}\chi) + m_0 \bar{\chi}\chi$$

One-link integral (1/d expansion*)

$$\to \frac{1}{2}\sigma V_M^{-1} \sigma + \bar{\chi}(\sigma + m_0)\chi$$

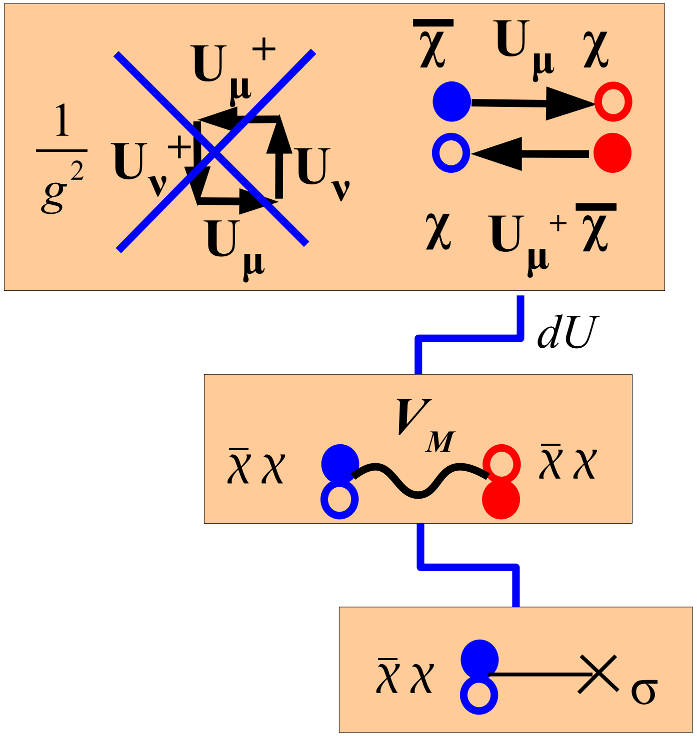
Bosonization

$$\to \frac{1}{2}\sigma V_M^{-1} \sigma - N_c \sum_x \log(\sigma(x) + m_0)$$

Fermion Integral

$$= L^d N_\tau \left[\frac{N_c}{d+1} \bar{\sigma}^2 - N_c \log(\bar{\sigma} + m_0) \right]$$

Effective Potential



*** d = Spatial dim.**

Fermion Matrix = Just a number
→ Simple Logarithmic Effective Potential for σ

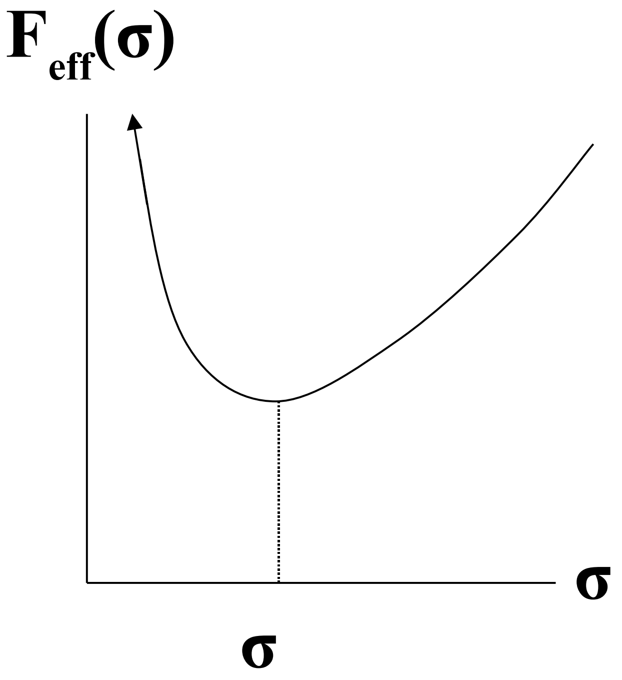
$$V_\sigma = \frac{1}{2} a_\sigma \sigma^2 - b_\sigma \log \sigma$$

Effective Potential in SCL-LQCD (Zero T)

- **Effective Pot. at Zero T**

  Kawamoto, Smit, 1981
  Kluberg-Stern, Morel, Napoly, Petersson, 1981

$$F_{eff}(\sigma)=\frac{1}{2}a_\sigma\sigma^2-b_\sigma\log\sigma$$

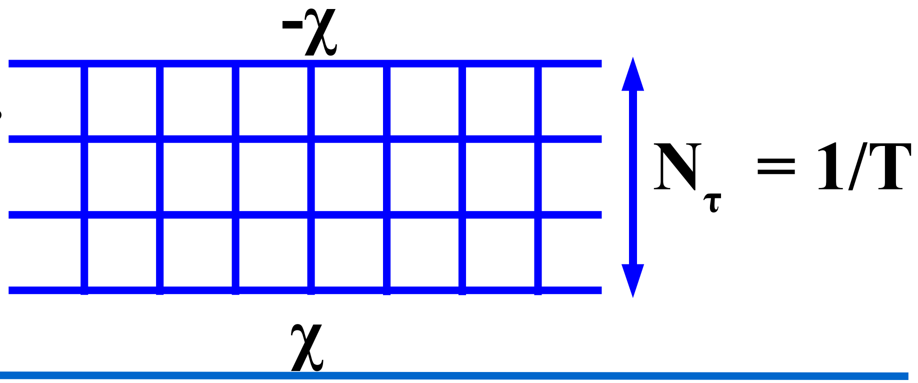


Spontaneous Chiral Symmetry breaking at T=0 is naturally explained !

No Phase Transition ?

- **Grassman integral at each space-time point in Zero T treatment**

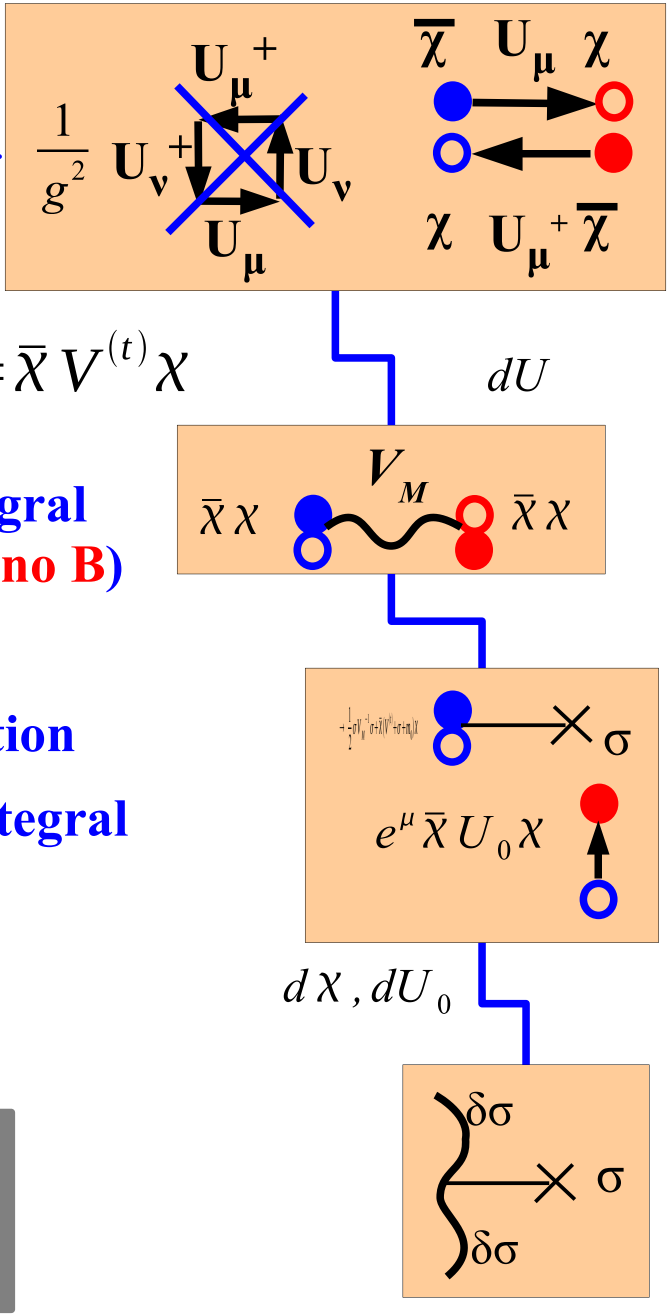
  → "Temporal" Correlation
  and Anti-periodic Boundary Cond.
  would be important at Finite T !



Let's go to Finite T

Effective Potential in SCL-LQCD (Finite T)

- **QCD Lattice Action (Finite T treatment)**
  Damgaard, Kawamoto, Shigemoto, 1984; Bilic et al., 1992; Nishida, '04; Fukushima, '04; Kawamoto, Miura, AO, Ohnuma, '07; .



$$S = \cancel{S_G} + S_F^{(s)} + S_F^{(t)} + m_0 \bar\chi \chi$$

Strong Coupling Limit

$$S_F^{(t)} = \frac{1}{2} \sum_x \left(e^\mu \bar\chi_x U_0(x) \chi_{x+\hat 0} - e^{-\mu} \bar\chi_{x+\hat 0} U_0^+(x) \chi_x \right) = \bar\chi V^{(t)} \chi$$

$$\to -\frac{1}{2} (\bar\chi \chi) V_M (\bar\chi \chi) + \bar\chi (V^{(t)} + m_0) \chi$$

Spatial-link integral (1/d expansion, no B)

$$\to \frac{1}{2} \sigma V_M{}^{-1} \sigma + \bar\chi (V^{(t)} + \sigma + m_0) \chi$$

Bosonization

Fermion and Temporal-link Integral

$$\to L^d N_\tau \boxed{\left[\frac{N_c}{d} \bar\sigma^2 + V_{\mathrm{eff}}(\bar\sigma, T, \mu) \right]}$$

Effective Potential

We need to evaluate Det. (Nc x Nτ)
→ It is POSSIBLE !

Fermion Determinant

Faldt, Petersson, 1986

- **Fermion action is separated to each spatial point and bi-linear → Determinant of Nτ x Nc matrix**

$$\exp(-V_{\mathrm{eff}}/T)=\int dU_0 \begin{vmatrix} I_1 & e^{\mu} & 0 & & e^{-\mu}U^{+} \\ -e^{-\mu} & I_2 & e^{\mu} & & \\ 0 & -e^{-\mu} & I_3 & e^{\mu} & \\ \vdots & & & \ddots & \\ -e^{\mu}U & & & -e^{-\mu} & I_N \end{vmatrix}$$

Nc x Nτ

$$=\int dU_0 \, det\left[\underline{X_N[\sigma]\otimes\mathbf{1}_c}+\underline{e^{-\mu/T}U^{+}}+\underline{(-1)^{N_\tau}e^{\mu/T}U}\right]$$

Nc

$$I_k=2(\sigma(k)+m_0)$$

$$X_N=\begin{vmatrix} I_1 & e^{\mu} & 0 & \cdots & & e^{-\mu} \\ -e^{-\mu} & I_2 & e^{\mu} & & & 0 \\ 0 & -e^{-\mu} & I_3 & & & 0 \\ \vdots & & & \ddots & & \\ & & & & I_{N-1} & e^{\mu} \\ -e^{\mu} & 0 & 0 & \cdots & -e^{-\mu} & I_N \end{vmatrix}-\left[e^{-\mu/T}+(-1)^N e^{\mu/T}\right]$$

Effective Potential in SCL-LQCD (Time dependence...)

- **Zero T, no Baryon** *Kawamoto, Smit, 1981*

$$\mathcal{F}_{\text{eff}}^{(0)} = \frac{1}{2} b_\sigma^{(0)} \sigma^2 - N_c \log(b_\sigma^{(0)} \sigma + m_0)$$

- **Zero T, with Baryon**
  Damgaad, Hochberg, Kawamoto, 1985

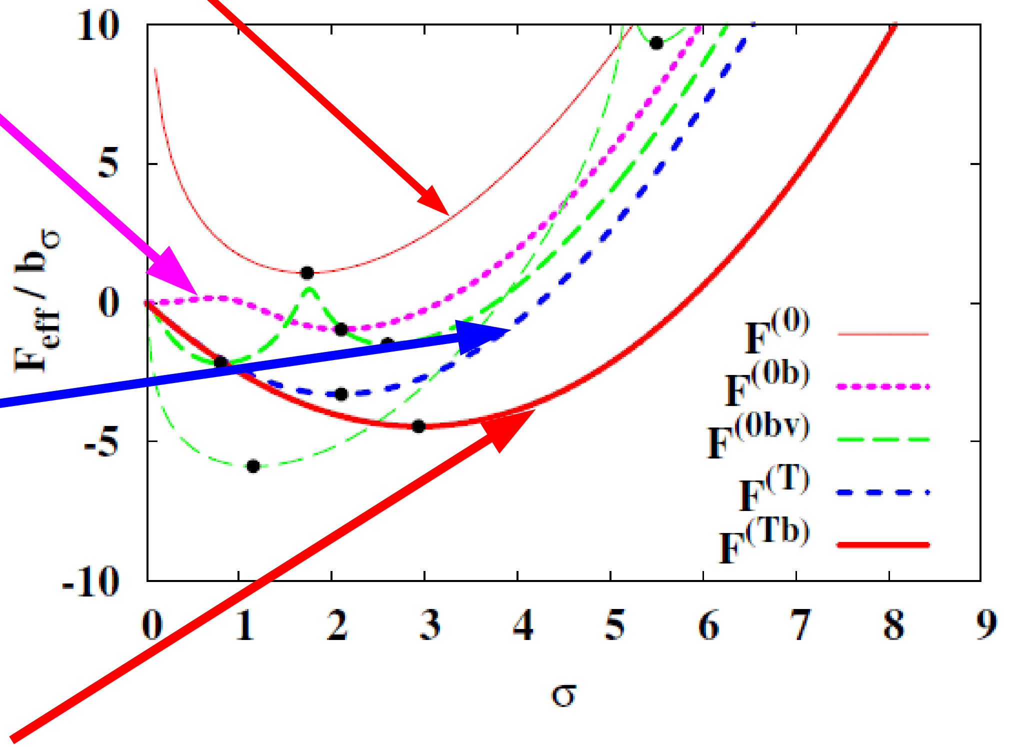
$$\mathcal{F}_{\text{eff}}^{(0\text{b})} = \frac{1}{2} b_\sigma^{(0)} \sigma^2 + F_{\text{eff}}^{(b\mu)}(4{m_q}^3; T, \mu)$$

- **Finite T, no Baryon**
  Fukushima, 2004; Nishida, 2004

$$\mathcal{F}_{\text{eff}}^{(T)} = \frac{1}{2} b_\sigma^{(T)} \sigma^2 + F_{\text{eff}}^{(q)}(m_q)$$

- **Finite T, with Baryon**
  Kawamoto, Miura, AO, Ohnuma, 2007

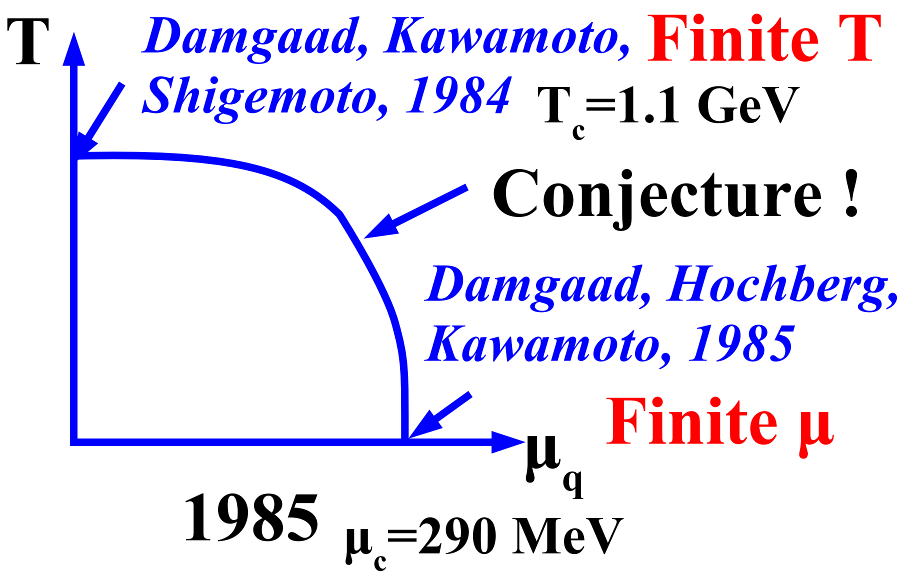
$$\mathcal{F}_{\text{eff}} = \frac{b_\sigma}{2} \sigma^2 + F_{\text{eff}}^{(q)}(m_q) + \Delta F_{\text{eff}}^{(b)}(g_\sigma \sigma)$$



$$F_{\text{eff}}^{(q)}(m_q) = -T \log\left(\frac{\sinh((N_c+1)E(m_q)/T)}{\sinh(E(m_q)/T)} + 2\cosh(N_c \mu/T) \right)$$

Evolution of Phase Diagram

- **Phase Diagram "Shape" becomes closer to that of Real World,**
  $R=3\ \mu_c/T_c \sim (6\text{-}12)$
  - **1985 → R=0.79 (Zero T / Finite T)**
  - **1992 → R=0.83 (Finite T & μ)**
  - **2004 → R= 0.99 (Finite T& μ)**
  - **2007 → R=1.34 (Baryon)**



Damgaad, Kawamoto, Shigemoto, 1984

Finite T

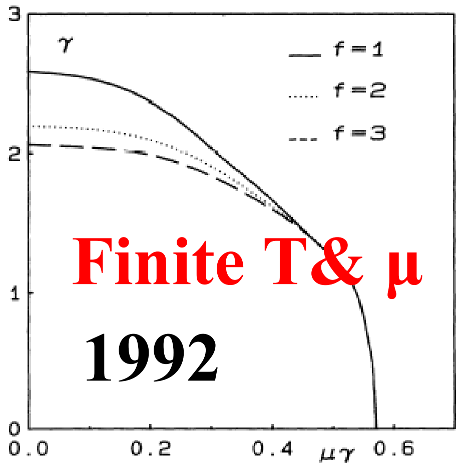
T_c=1.1 GeV

Conjecture !

Damgaad, Hochberg, Kawamoto, 1985

Finite μ

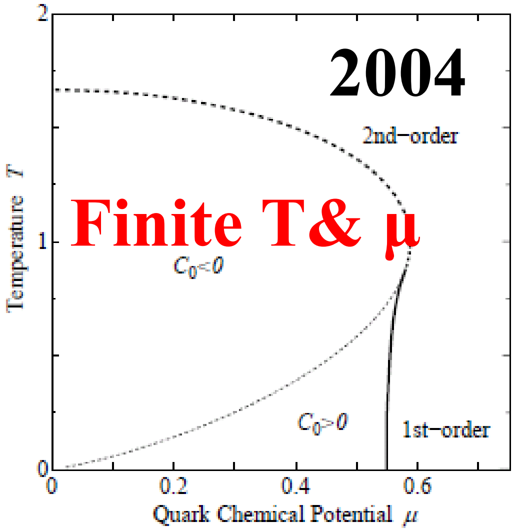
1985 **μ_c=290 MeV**



Finite T& μ

1992

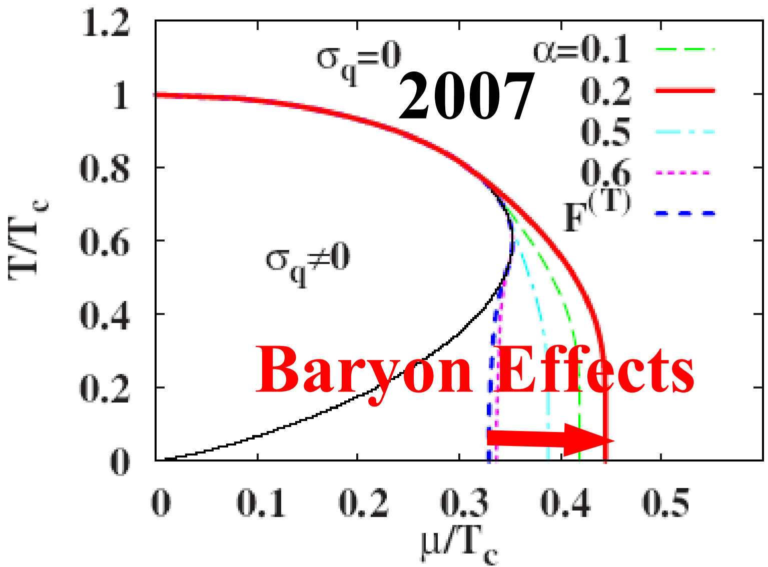
Bilic, Karsch, Redlich, 1992



2004

Finite T& μ

Fukushima, 2004



2007

Baryon Effects

Kawamoto, Miura, AO, Ohnuma, 2007

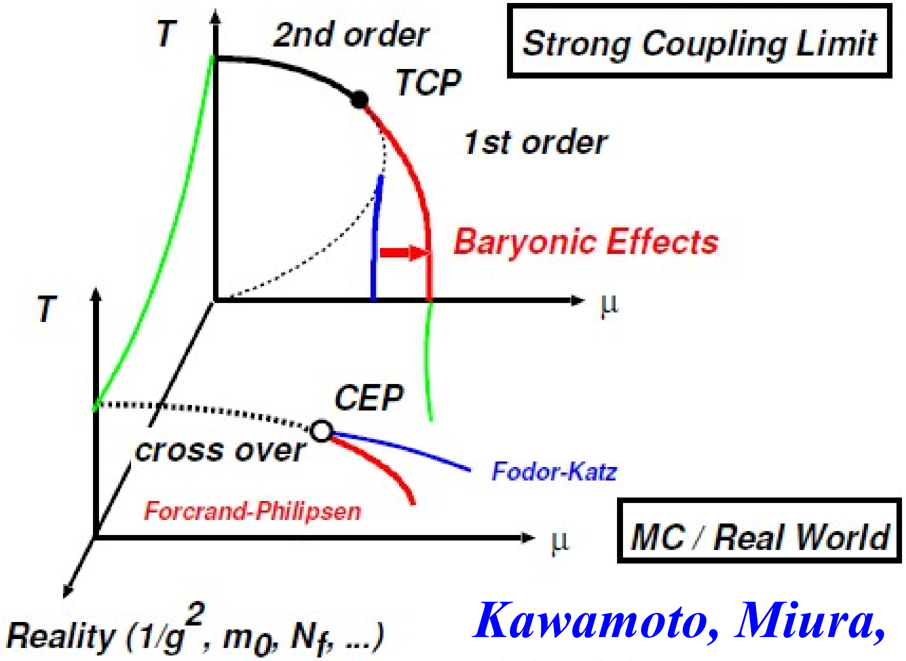
Finite Coupling Effects on the Phase Diagram and the Quarkyonic Phase

Towards the Real Phase Diagram

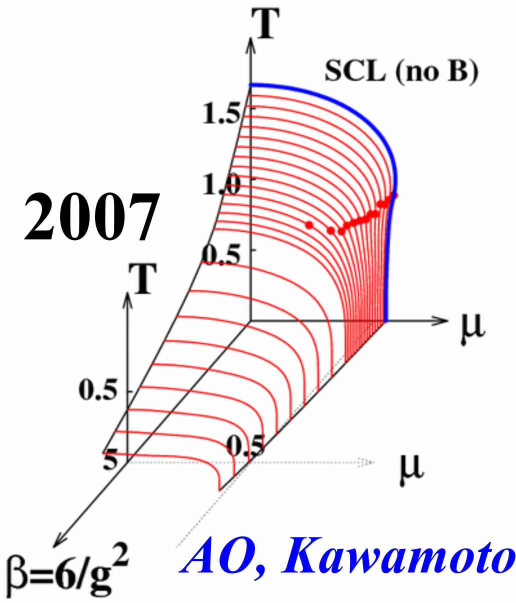
- **When we increase “Reality” variable,**
  Phase diagram “Shape” may be approximately explained.
  Real World: R=3 μ_c/T_c ~ (6-12)
  SCL-LQCD: R=0.79-1.34
  SC-LQCD with finite β (=6/g^2)~5 → R ~ 4.5

Expectation before Calc.

Calc. with 1/g^2 effects



Kawamoto, Miura, AO, Ohnuma, 2007



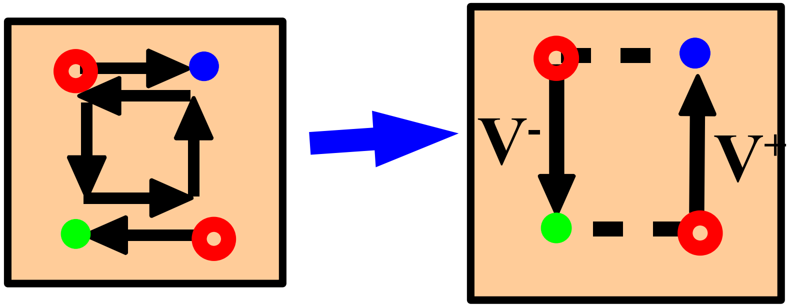
AO, Kawamoto, Miura, 2007

Gluon Contribution is important at High T

Effective Action with $1/g^2$ (1)

- Strong Coupling Limit → No Plaquette Contribution
- $1/g^2$ → Single plaquette contribution
  - Spatial One-Link Integral (1/d expansion)
    → MMMM (Spatial Plaq.), V^+V^- (Temporal Plaq.)
    Faldt, Petersson (86); Bilic, Karsch, Redlich (92)

$$V_x^+ = e^{\mu}\bar{\chi}_x U_0(x)\chi_{x+\hat{0}}$$
$$V_x^- = e^{-\mu}\bar{\chi}_{x+\hat{0}} U_0^\dagger(x)\chi_x$$



  - Product of Different Composites
    → Extended Hubbard-Stratonovich Transf.
    (Mean field approx. φ (Scalar), Saddle point approx. for ϕ (Vector))

$$\boxed{\begin{aligned} e^{\alpha AB} &= \int d\varphi\, d\phi\, e^{-\alpha\{\varphi^2-(A+B)\varphi+\phi^2-i(A-B)\phi\}} \\ &\approx e^{-\alpha\{\varphi^2-(A+B)\varphi-\phi^2+(A-B)\phi\}}\ . \end{aligned}}$$

Effective Action with 1/g² (2)

- **Temporal Plaquette action**

$$\Delta S_\beta^{(\tau)} \approx \frac{1}{4N_c^2 g^2}\sum_{x,j>0}\left[\varphi_\tau^2 + (V_x^+ - V_{x+\hat{j}}^-)\varphi_\tau - \phi_\tau^2 - (V_x^+ + V_{x+\hat{j}}^-)\phi_\tau\right] + (j \leftrightarrow -j)$$

- **Effective Action with 1/g²**

Scale of Temp. Spacing

μ mod.

Aux. Terms

$$S_{\rm eff} = \frac{1}{2}\boxed{(1+\beta_\tau\varphi_\tau)}\sum_x \left(\boxed{e^{-\beta_\tau\phi_\tau}}V_x^+ - \boxed{e^{\beta_\tau\phi_\tau}}V_x^-\right) + m_0\sum_x M_x$$

$$-\left(\frac{1}{4N_c}\boxed{+\beta_s\varphi_s}\right)\sum_{x,j>0} M_x M_{x+\hat{j}} \boxed{+N_\tau L^d\left[\frac{\beta_\tau}{2}(\varphi_\tau^2-\phi_\tau^2)+\frac{d\beta_s}{2}\varphi_s^2\right]}$$

Effective Potential with 1/g^2

- **Effective Potential (after subst. equil. value for φ_τ and φ_s)**

$$\mathcal{F}_{\rm eff} = \mathcal{F}_{\rm X}(\sigma,\phi_\tau) + \mathcal{V}_{\rm q}(m_q(\sigma),\tilde{\mu}(\phi_\tau),T)$$

$$\mathcal{V}_{\rm q} = -T\log\left[X_{N_c}(E_q/T) + 2\cosh(N_c\tilde{\mu}/T)\right]$$

Same as SCL

$$\mathcal{F}_{\rm X} = \frac{1}{2}b_\sigma\sigma^2 \boxed{+\frac{\beta_\tau}{2}\sigma^2(m_q^{\rm SCL})^2 + \frac{3d\beta_s}{2}\sigma^4 - \frac{\beta_\tau}{2}\phi_\tau^2}$$

$$m_q = m_q^{\rm SCL}\boxed{(1-N_c\beta_\tau) + \beta_\tau\sigma(m_q^{\rm SCL})^2 + 2d\beta_s\sigma^3}$$

$$\tilde{\mu} = \mu \boxed{-\beta_\tau\phi_\tau}$$

from Plaq.

- **Scaling of temporal spacing (1+ $\beta_\tau\varphi_\tau$) in the Eff. Action**
  → suppr. of quark mass m_q
- **Higher order terms M^4 → σ^4 (Self-energy of σ)**
- **Aux. Field $\phi_\tau = \rho_q$ (equil.) → μ is shifted by baryon density**

Let us examine the phase diagram with this F_{eff} !

Evolution of T_c and μ_c

- T_c (μ=0) rapidly decreases with $\beta = 6/g^2$ increases.
  - MC results (N_τ =2) Quench β_c=5.097(1) (Kennedy et al, 1985)

    m_0=0.05 → β_c=3.81(2), m_0=0.025 → β_c=3.67(2)

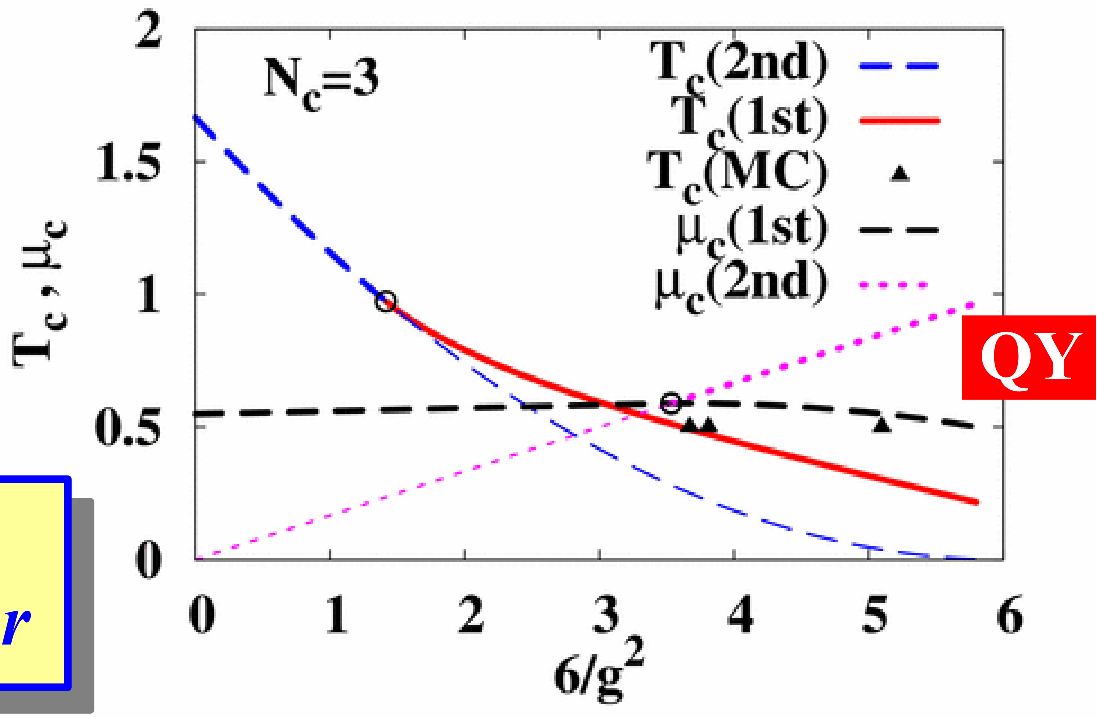
    (de Forcrand, private comm.)

MC results with small m_0 agrees with SC-LQCD !

- $\mu_c^{(2nd)} > \mu_c^{(1st)}$ at $6/g^2 > 3.53$
  - Key: Effective chem. pot.

$$\mu_{\mathrm{eff}} = \mu - \beta_\tau \phi_\tau = \mu - \beta_\tau \rho_q$$

Spontaneously χ broken high density matter may appear



Phase Diagram

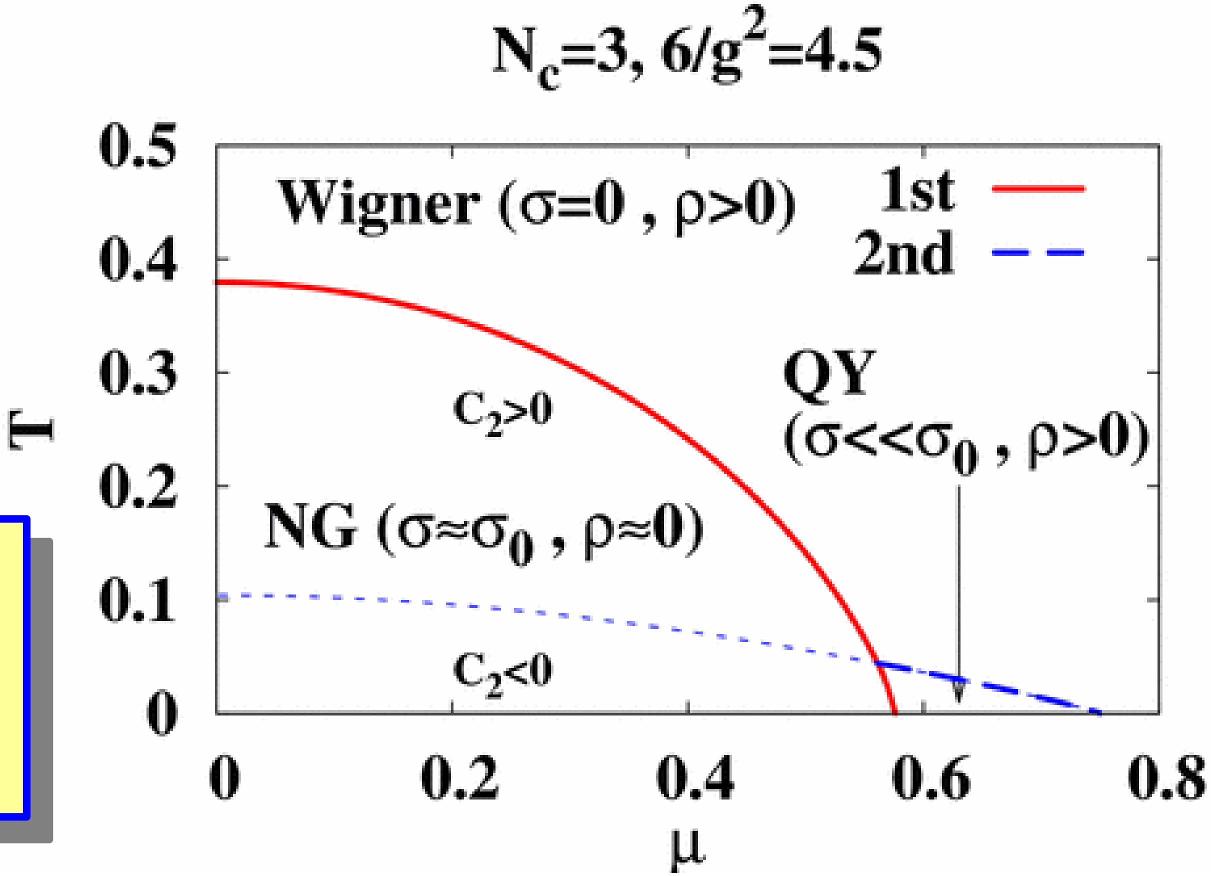
- **Three phases in SC-LQCD with N_c=3, 6/g^2 > 3.53, m_0=0 (χ limit)**
  - **Nambu-Goldstone (NG) phase: Large σ, Small ρ_q, Small P**
  - **Winger phase: σ=0, Large ρ_q, finite P**
  - **Quarkyonic phase:**

    $0 < \sigma << \sigma_{vac}$

    ρ_q(QY) ~ ρ_q(Wig.)

    P(QY) < P(Wig.)

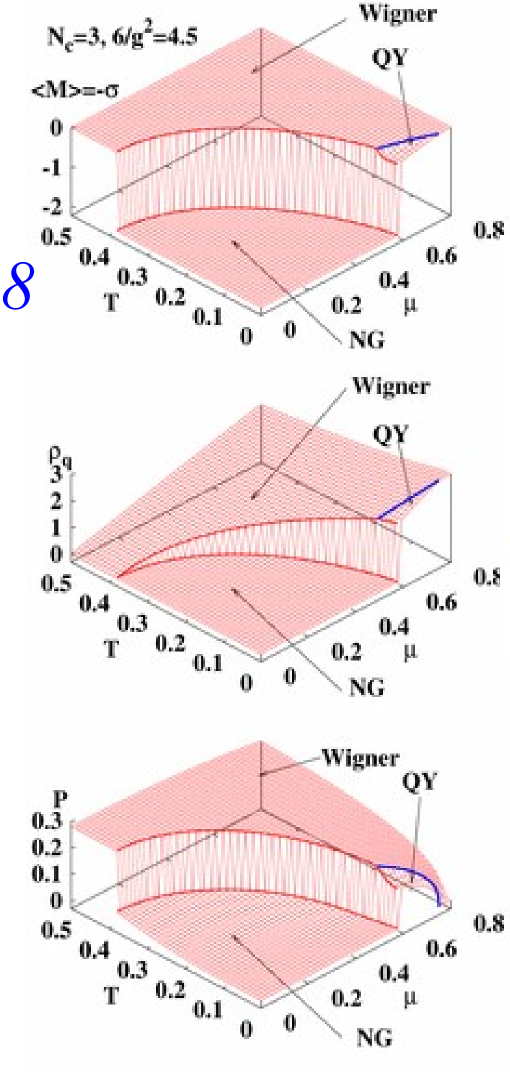
    Quark driven P → 0 at large N_c



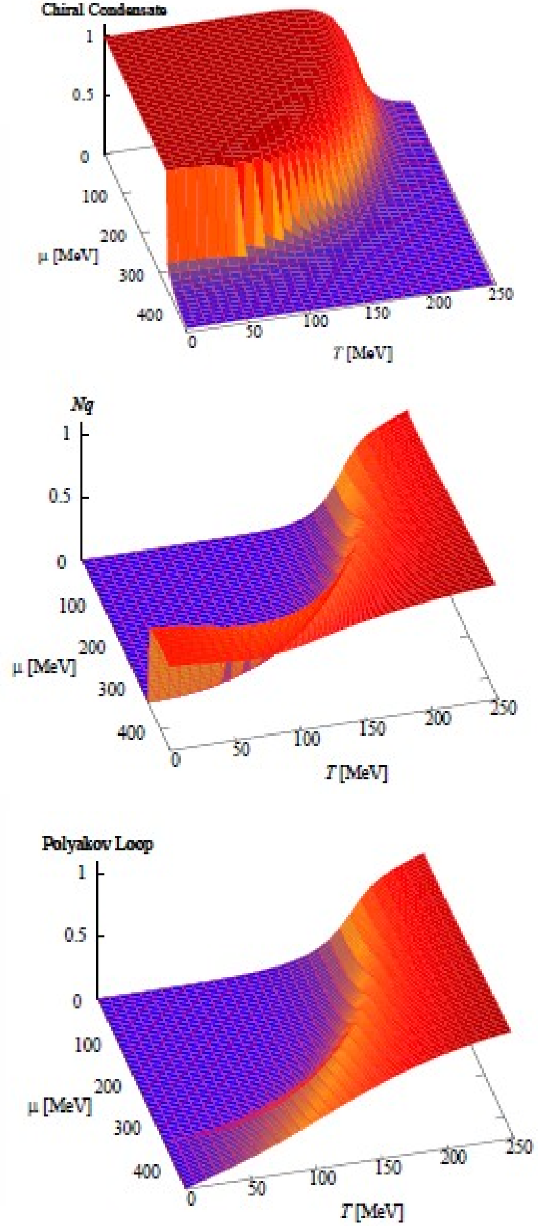
QY in SC-LQCD
can be regarded
as QY at large N_c

Comparison with Other Models

- **SC-LQCD results are qualitatively similar to 2+1 flavor PNJL Model in Chiral Cond., Baryon Density, and Polyakov Loop**
  Fukushima, PRD77(114028)08



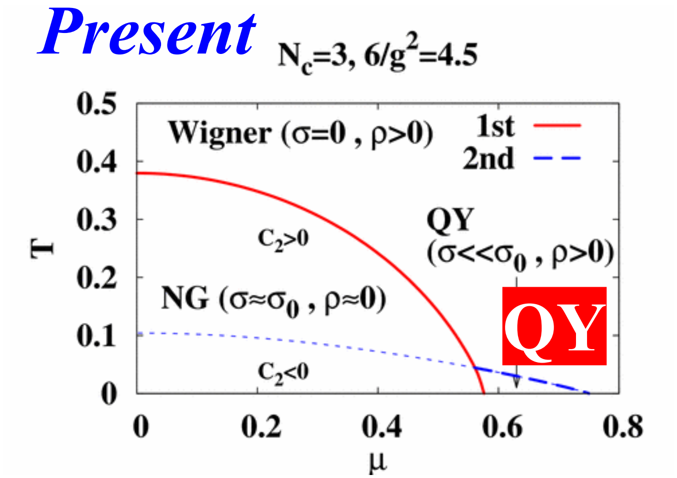
Present

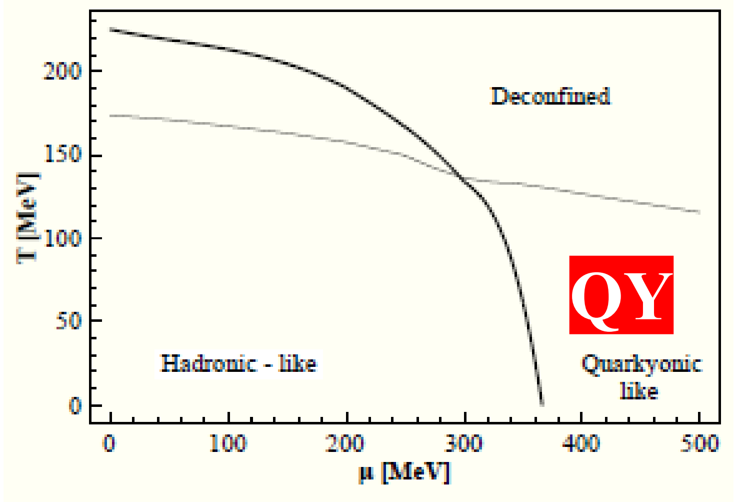


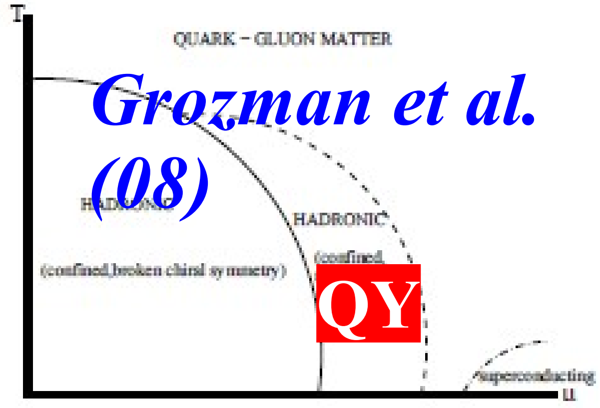
Fukushima, 2008

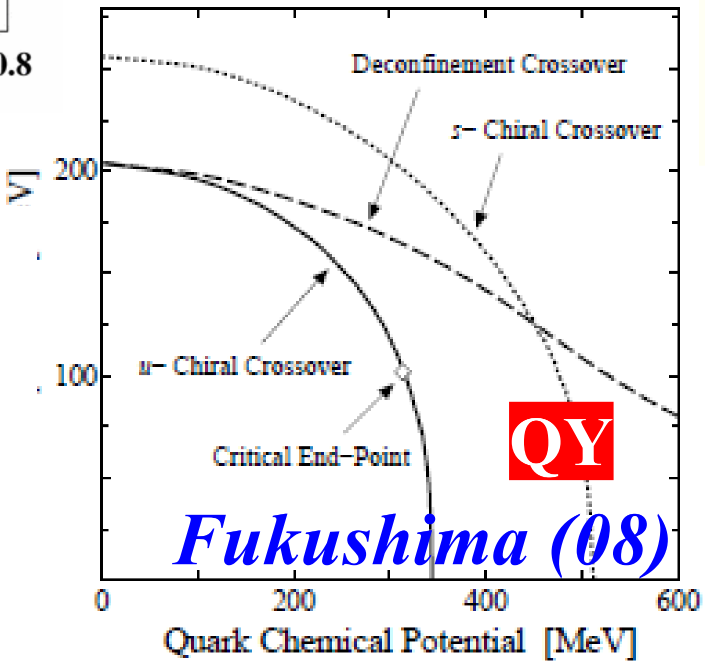
Comparison with Other Models

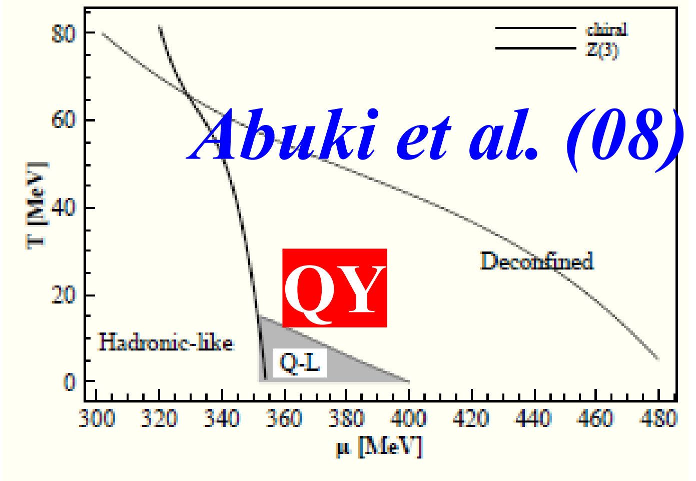
- **Quarkyonic(-like) Area in SC-LQCD is smaller than in PNJL**
  Fukushima (08)
  Abuki, Anglani Gatto, Nardulli, Ruggieril [arXiv:0805.1509]











まとめ

- **格子 QCD における強結合展開によるクォーク物質の相の研究**
  - **低温・高密度物質の研究が可能**
  - **拡張されたハバード変換 (Extended Hubbard-Stratonovich transf.) により、有限結合効果 ($1/g^2$ 効果) の評価が可能**
  - **ゼロバリオン密度 (μ=0) での臨界結合定数は、格子 QCD シミュレーションの結果と consistent (P. de Forcrand, T_c=1/2 (N_τ =2) at $6/g_c^2 \sim 3.6$)**
- **有限バリオン密度では、ベクトルポテンシャル斥力により、「カイラル対称性が部分的に回復した高密度相」が存在する可能性がある。**
  → McLerran & Pisarski の提案した Quarkyonic 相に対応
  QY may be the "NEXT" to the hadron phase even at N_c=3.
- **より realistic な取り扱いへ向けて**
  $1/g^4$, 他の Fermion, フレーバー効果 .....
  → to be continued !