強結合格子QCDにおけるQuarkyonic相
Quarkyonic Phase in Lattice QCD at Strong Coupling

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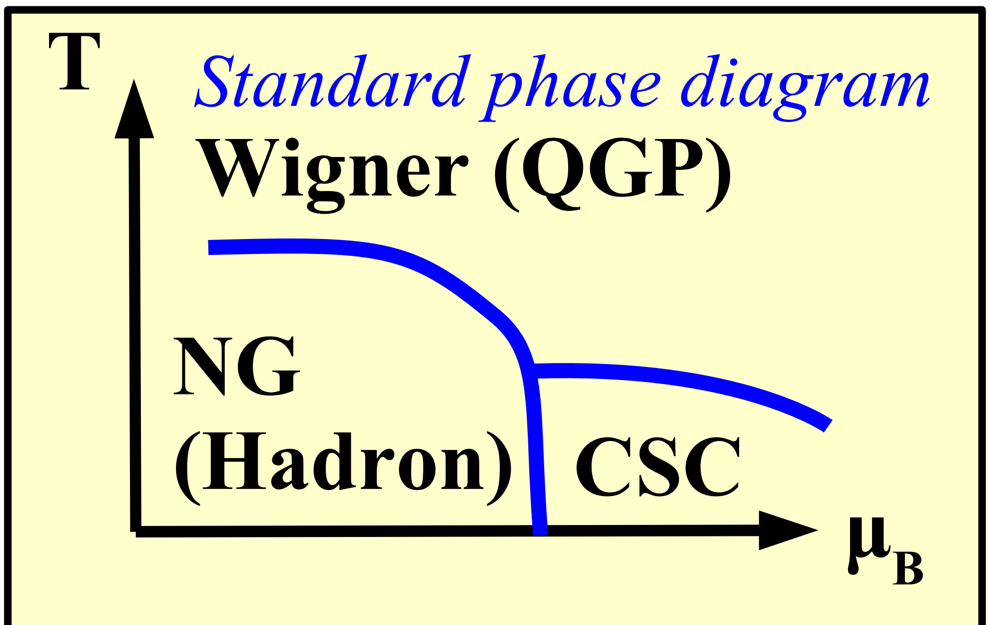
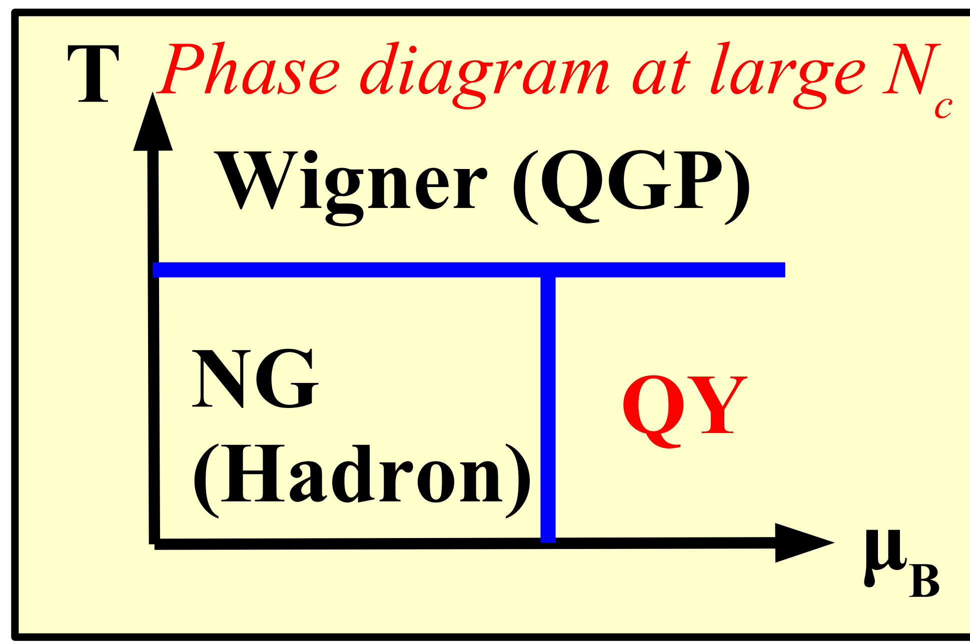
- **Introduction**
- **Strong Coupling Lattice QCD with $1/g^2$ Correction**
- **Phase diagram in SC-LQCD and Quarkyonic Phase**
- **Summary**

Miura and AO, arXiv:0806.3357
Our previous refs. on $1/g^2$ corr.
AO et al. PTP Suppl. 168(07) 261 [arXiv:0704.2823]
AO, Kawamoto, Miura, J.Phys.G34(07)S655 [hep-lat/0701024]

Introduction: Quark Matter Phase Diagram

- **What is the NEXT to the hadron phase ?**
  - **High T direction → (Strongly correlated) Quark Gluon Plasma**
  - **High μ direction ; Important for Dense Matter Physics**
    Baryon rich QGP
    or Color SuperConductor (CSC)
    or Quarkyonic (QY) matter *McLerran, Pisarski (07)*





Quarkyonic Phase & Strong Coupling Lattice QCD

- **Quarkyonic matter at large N_c** *McLerran, Pisarski (07)*
  - **Gluon contribution $O(N_c^2)$ >> Quark $O(N_c)$, Hadron O(1)**
    → Gluonic (deconf.) P.T. is independent of μ (as far as μ = O(1))
  - **At $N_c\mu > M_B$, baryon density rapidly grows, and soon reaches $O(N_c)$**
    → Existence of "Confined High Baryon Density Matter"
    made of *quark*s but with bar*yonic* excitation (Quarkyonic Matter)

> ***Do we have Quarkyonic phase in QCD with N_c=3 ?***
> ***→ QCD at Finite (but not very large) μ***
> ***→ Strong Coupling Lattice QCD***

Quarkyonic phase in SC-LQCD

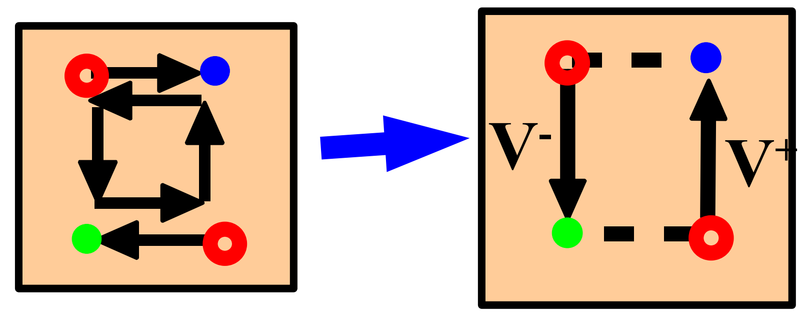
- **We study the phase diagram**
  in Strong Coupling Lattice QCD (SC-LQCD)
  with 1/g^2 correction,
  and examine the existence of the Quarkyonic (QY) phase.

- **Reservations:** ***It is still a "Toy"***
  - **One species of staggered fermion without quarter/square root**
    → $N_f = 4$
  - **Leading order in 1/d (d=spatial dim.)**
    → No baryon effects (cf. *Par-Tue, Miura***)**
  - **Mean Field treatment**
  - **No Diqaurk condensate**
  - **NLO in 1/g^2 expansion, ...**

Effective Action with $1/g^2$ (1)

- **Strong Coupling Limit → No Plaquette Contribution**
- **$1/g^2$ → Single plaquette contribution**
  - **Spatial One-Link Integral (1/d expansion)**
    → MMMM (Spatial Plaq.), V^+V^- (Temporal Plaq.)
    Faldt, Petersson (86); Bilic, Karsch, Redlich (92)

$$V_x^+ = e^{\mu}\bar{\chi}_x U_0(x)\chi_{x+\hat{0}}$$
$$V_x^- = e^{-\mu}\bar{\chi}_{x+\hat{0}} U_0^\dagger(x)\chi_x$$



  - **Product of Different Composites**
    → Extended Hubbard-Stratonovich Transf.
    (Mean field approx. φ (Scalar), Saddle point approx. for ϕ (Vector))

$$\begin{aligned} e^{\alpha AB} &= \int d\varphi\, d\phi\, e^{-\alpha\{\varphi^2-(A+B)\varphi+\phi^2-i(A-B)\phi\}} \\ &\approx e^{-\alpha\{\varphi^2-(A+B)\varphi-\phi^2+(A-B)\phi\}}\,. \end{aligned}$$

Effective Action with 1/g² (2)

- **Temporal Plaquette action**

$$\Delta S_\beta^{(\tau)} \approx \frac{1}{4N_c^2 g^2} \sum_{x,j>0} \left[\varphi_\tau^2 + (V_x^+ - V_{x+\hat{j}}^-)\varphi_\tau - \phi_\tau^2 - (V_x^+ + V_{x+\hat{j}}^-)\phi_\tau\right] + (j \leftrightarrow -j)$$

- **Effective Action with $1/g^2$**

Scale of Temp. Spacing

μ mod.

Aux. Terms

$$S_{\mathrm{eff}} = \frac{1}{2}(1+\beta_\tau\varphi_\tau)\sum_x \left(e^{-\beta_\tau\phi_\tau}V_x^+ - e^{\beta_\tau\phi_\tau}V_x^-\right) + m_0 \sum_x M_x$$
$$- \left(\frac{1}{4N_c} + \beta_s\varphi_s\right)\sum_{x,j>0} M_x M_{x+\hat{j}} + N_\tau L^d \left[\frac{\beta_\tau}{2}(\varphi_\tau^2 - \phi_\tau^2) + \frac{d\beta_s}{2}\varphi_s^2\right]$$

Effective Potential with $1/g^2$

- **Effective Potential (after subst. equil. value for φ_τ and φ_s)**

$$\mathcal{F}_\mathrm{eff} = \mathcal{F}_\mathrm{X}(\sigma, \phi_\tau) + \mathcal{V}_\mathrm{q}(m_q(\sigma), \tilde{\mu}(\phi_\tau), T)$$

$$\mathcal{V}_\mathrm{q} = -T \log\left[X_{N_c}(E_q/T) + 2\cosh(N_c\tilde{\mu}/T)\right]$$

Same as SCL

$$\mathcal{F}_\mathrm{X} = \frac{1}{2} b_\sigma \sigma^2 + \frac{\beta_\tau}{2}\sigma^2 (m_q^\mathrm{SCL})^2 + \frac{3d\beta_s}{2}\sigma^4 - \frac{\beta_\tau}{2}\phi_\tau^2$$

$$m_q = m_q^\mathrm{SCL}(1 - N_c\beta_\tau) + \beta_\tau\sigma (m_q^\mathrm{SCL})^2 + 2d\beta_s\sigma^3$$

$$\tilde{\mu} = \mu - \beta_\tau\phi_\tau$$

from Plaq.

- **Scaling of temporal spacing $(1+\beta_\tau\varphi_\tau)$ in the Eff. Action**
  → suppr. of quark mass m_q
- **Higher order terms $M^4 \to \sigma^4$ (Self-energy of σ)**
- **Aux. Field $\phi_\tau = \rho_q$ (equil.) → μ is shifted by baryon density**

Let us examine the phase diagram with this F_{eff} !

Evolution of T_c and μ_c

- **T_c (μ=0) rapidly decreases with $\beta = 6/g^2$ increases.**
  - **MC results (N_τ =2) Quench β_c=5.097(1) (Kennedy et al, 1985)**

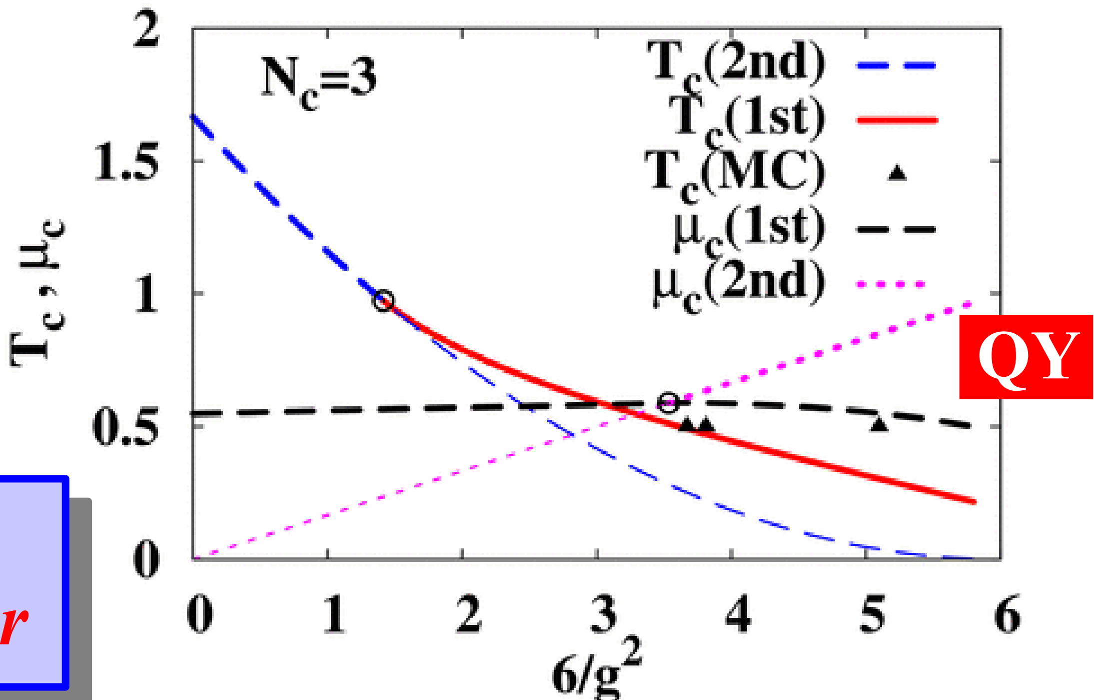
    m_0=0.05 → β_c=3.81(2), m_0=0.025 → β_c=3.67(2)
    (de Forcrand, private comm.)

MC results with small m_0 agrees with SC-LQCD !

- **$\mu_c^{(2nd)} > \mu_c^{(1st)}$ at $6/g^2 > 3.53$**
  - **Key: Effective chem. pot.**

$$\mu_{\rm eff} = \mu - \beta_\tau \phi_\tau = \mu - \beta_\tau \rho_q$$

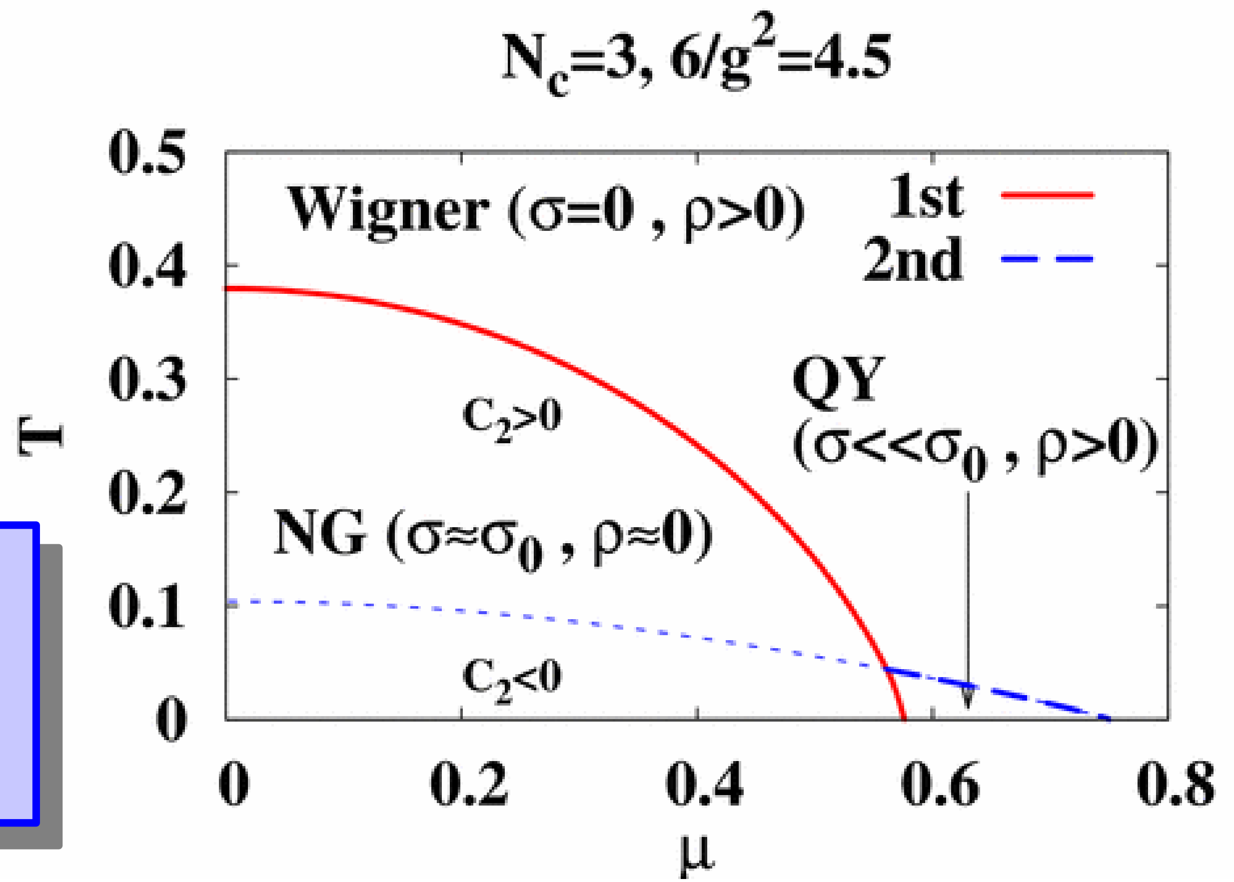
Spontaneously χ broken
high density matter may appear



Phase Diagram

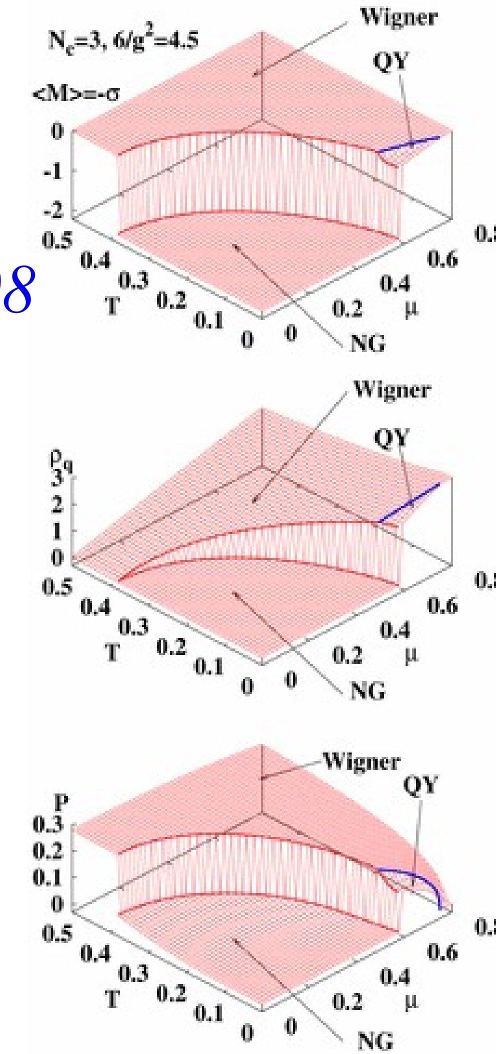
- **Three phases in SC-LQCD with N_c=3, $6/g^2$ > 3.53, m_0=0 (χ limit)**
  - **Nambu-Goldstone (NG) phase: Large σ, Small ρ_q, Small P**
  - **Winger phase: σ=0, Large ρ_q, finite P**
  - **Quarkyonic phase:**
    $0 < \sigma << \sigma_{vac}$
    $\rho_q(QY) \sim \rho_q(Wig.)$
    P(QY) < P(Wig.)
    Quark driven P → 0 at large N_c

QY in SC-LQCD can be regarded as QY at large N_c

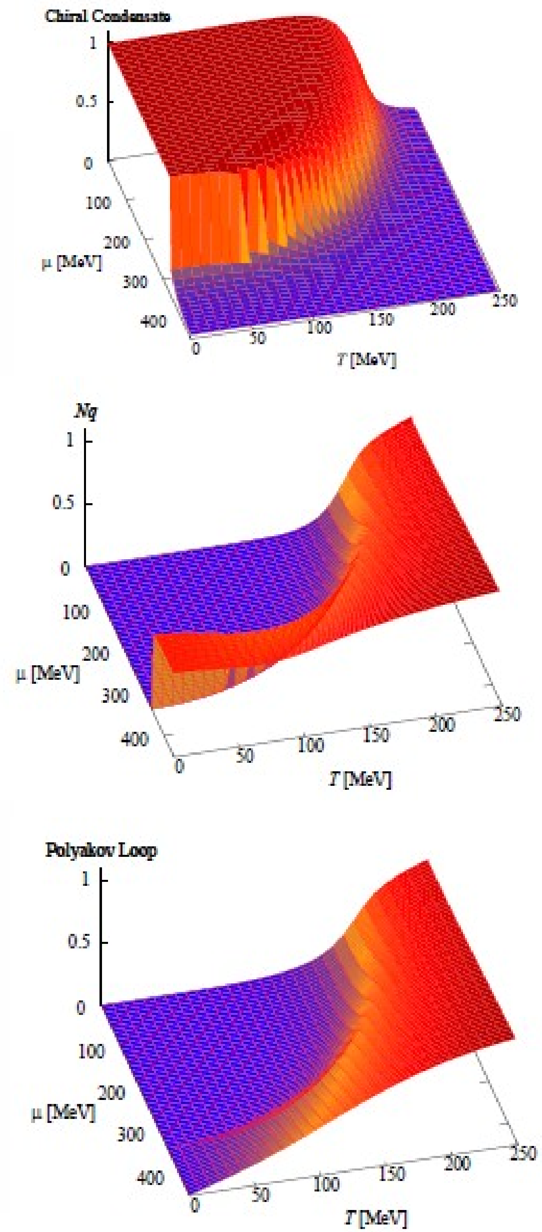


Comparison with Other Models

- **SC-LQCD results are qualitatively similar to 2+1 flavor PNJL Model in Chiral Cond., Baryon Density, and Polyakov Loop**
  Fukushima, PRD77(114028)08



Present

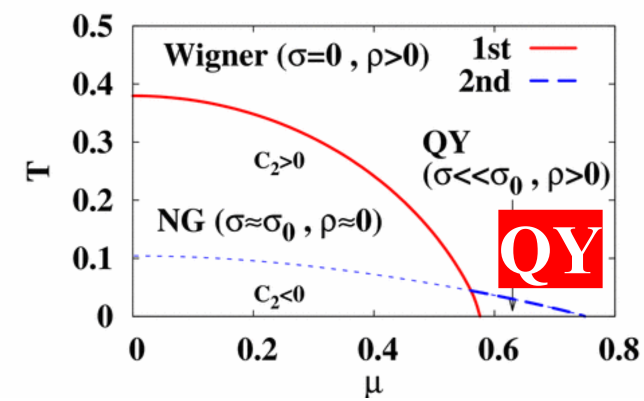


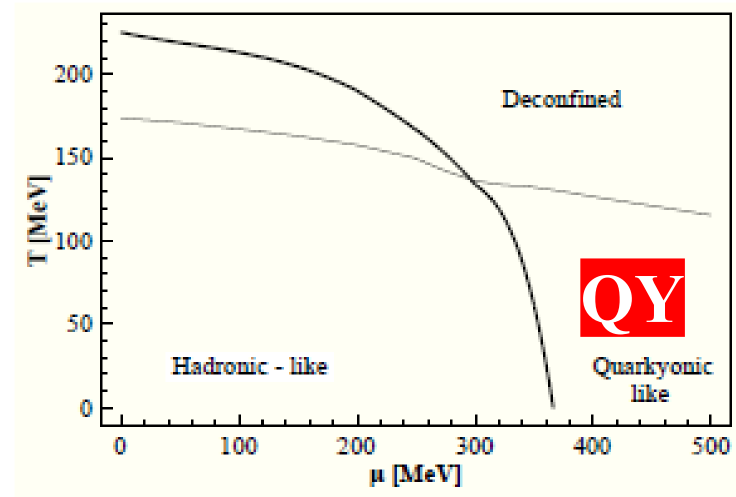
Fukushima, 2008

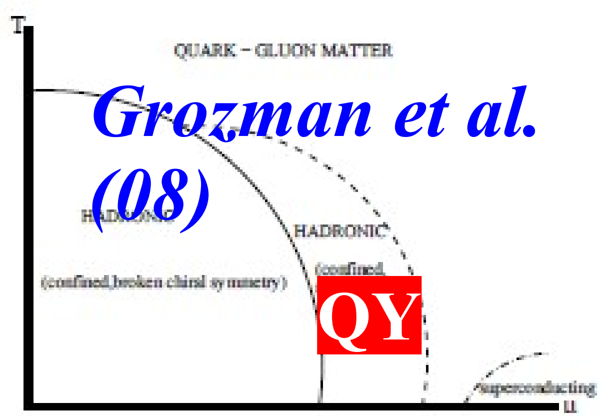
Comparison with Other Models

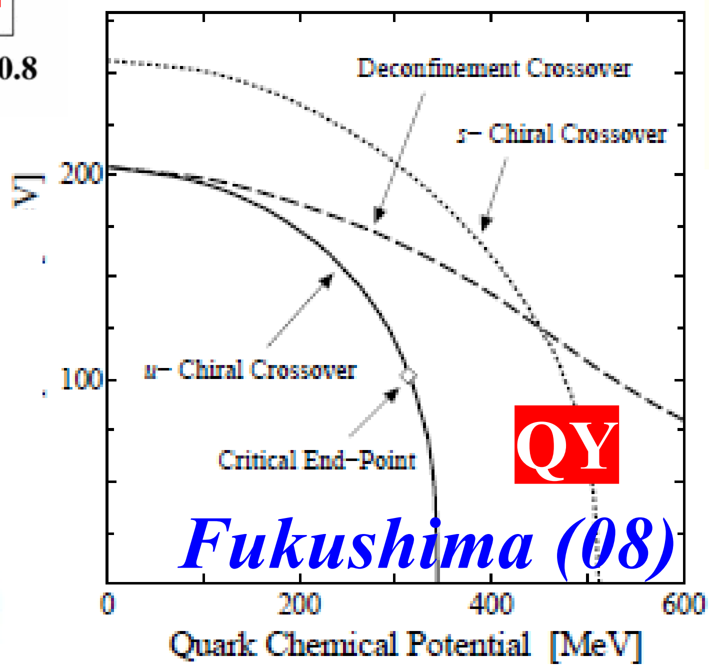
- **Quarkyonic(-like) Area in SC-LQCD is smaller than in PNJL**
  Fukushima (08)
  Abuki, Anglani Gatto, Nardulli, Ruggieril [arXiv:0805.1509]

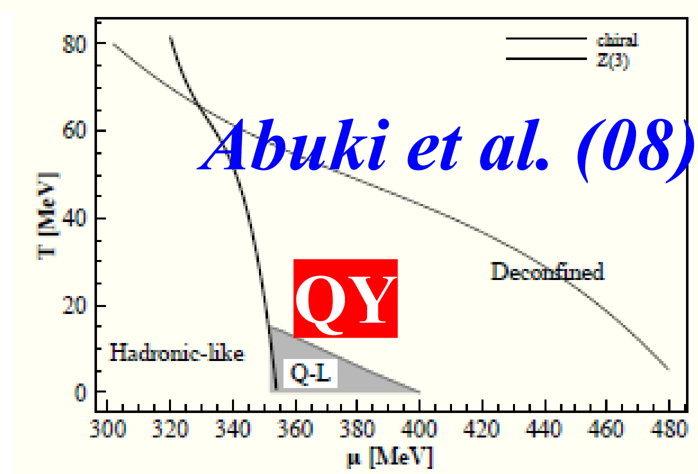
Present











Conclusions

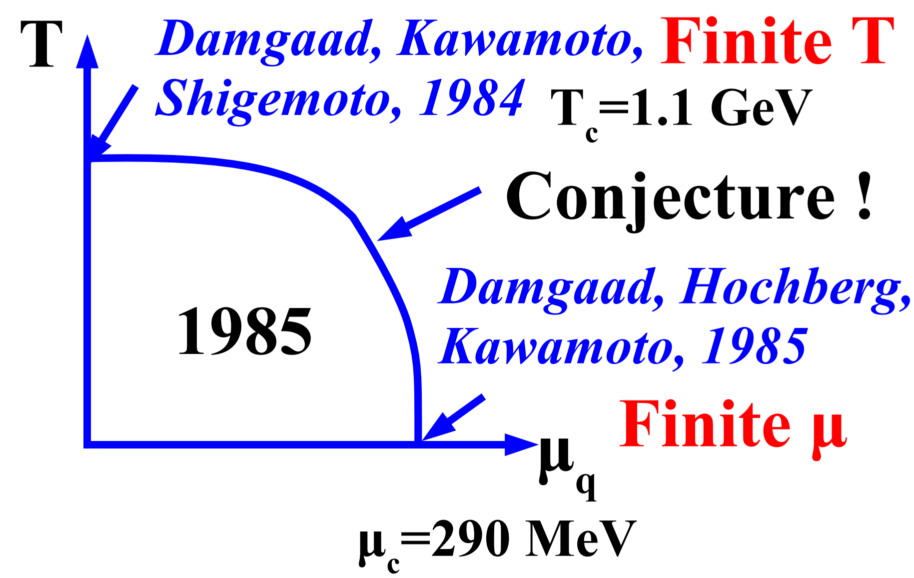
Miura and AO, arXiv:0806.3357

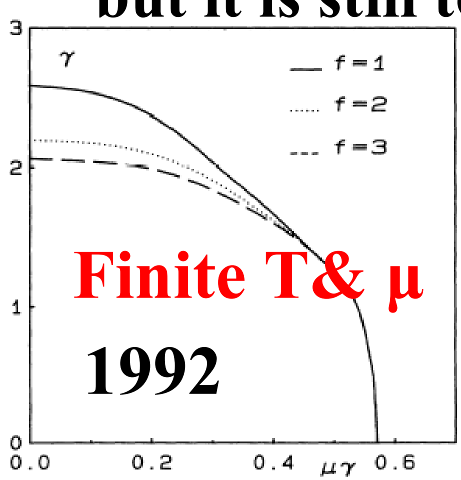
- **We have investigated the phase diagram in Strong Coupling Lattice QCD with $1/g^2$ corrections. Through the extended Hubbard-Stratonovich transf., we find that vector field for quarks is introduced.**
- **Critical Temperature at $\mu=0$ is found to be consistent with MC results by P. de Forcrand, $T_c=1/2$ ($N_\tau=2$) at $6/g_c^2 \sim 3.6$**
- **We find that the Quarkyonic (QY) phase at large Nc proposed by McLerran & Pisarski appears also at $N_c=3$ in SC-LQCD with $6/g^2 > 3.53$, where the chiral symmetry is weakly but spontaneously broken, and the baryon density is high.**
  QY may be the "NEXT" to the hadron phase even at $N_c=3$.
- **Do we really have QY in nature ?**
  → $N_c=2$, Imaginary μ, $1/g^4$ (to be studied)

Backups

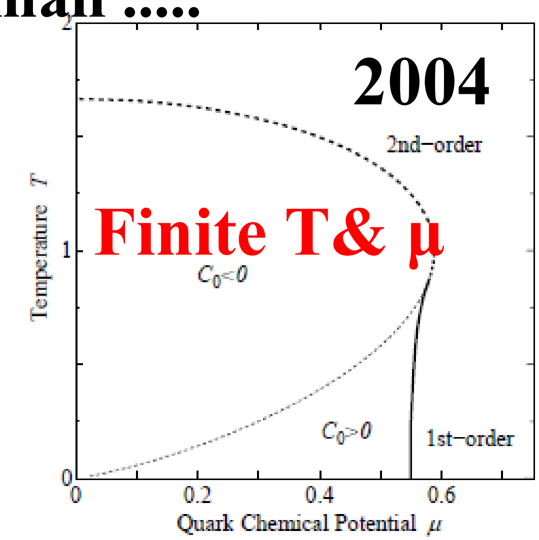
"Evolution" of Phase Diagram

- **Phase Diagram "Shape" becomes closer to that of Real World.**
  - **Real world R=3 μ_c/T_c ~ (6-12)**
  - **1985 → R=0.79 (Zero T / Finite T)**
  - **1992 → R=0.83 (Finite T & μ)**
  - **2004 → R= 0.99 (Finite T& μ)**
  - **2007 → R=1.34 (Baryon)**
    but it is still too small .....

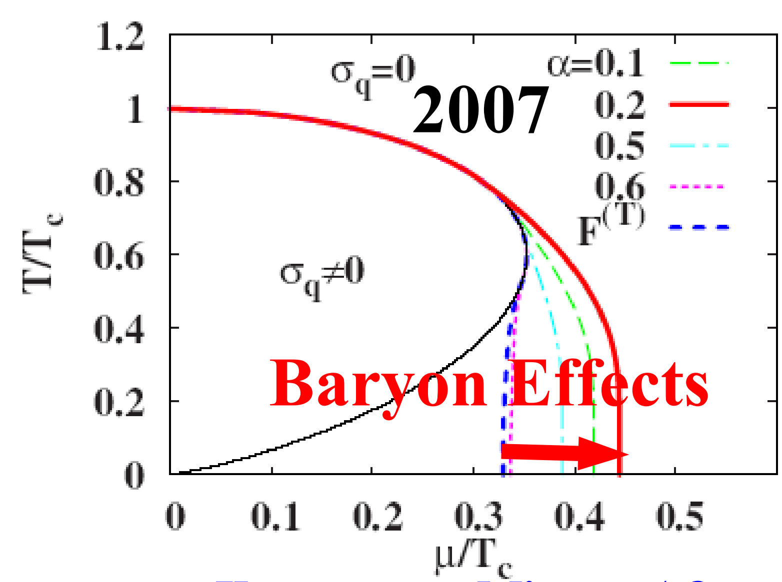




Bilic, Karsch, Redlich, 1992



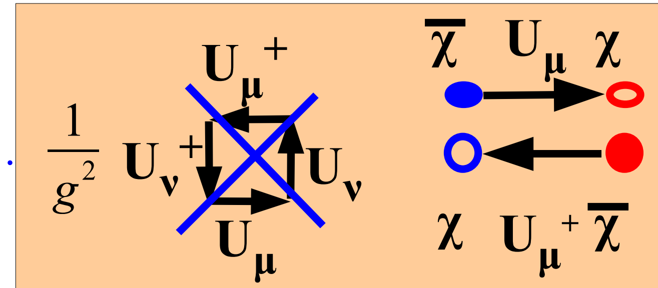
Fukushima, 2004



Kawamoto, Miura, AO, Ohnuma, 2007

Effective Potential in SCL-LQCD

- **QCD Lattice Action (Finite T treatment)**
  Damgaard, Kawamoto, Shigemoto, 1984; Bilic et al., 1992; Nishida, '04; Fukushima, '04; Kawamoto, Miura, AO, Ohnuma, '07; ...

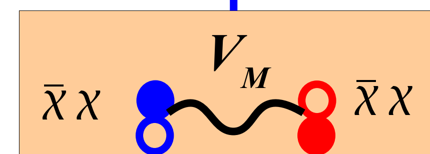


$$S = \cancel{S_G} + S_F^{(s)} + S_F^{(t)} + m_0 \bar{\chi}\chi$$

Strong Coupling Limit

$$S_F^{(t)} = \frac{1}{2}\sum_x \left(e^{\mu}\bar{\chi}_x U_0(x)\chi_{x+\hat{0}} - e^{-\mu}\bar{\chi}_{x+\hat{0}} U_0^{+}(x)\chi_x \right) = \bar{\chi} V^{(t)} \chi$$

dU

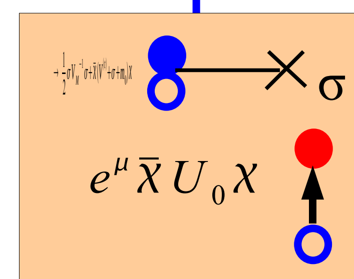


$$\to -\frac{1}{2}(\bar{\chi}\chi) V_M (\bar{\chi}\chi) + \bar{\chi}(V^{(t)} + m_0)\chi$$

Spatial-link integral (1/d expansion)

$$\to \frac{1}{2}\sigma V_M \sigma + \bar{\chi}(V^{(t)} + V_M\sigma + m_0)\chi$$

Hubbard-Stratonovich Transf. (Bosonization)

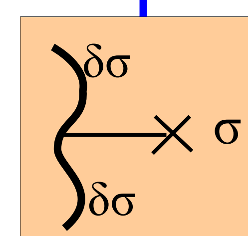


Fermion and Temporal-link Integral

$d\chi, dU_0$

$$\to L^d N_\tau \left[\frac{d}{4N_c}\bar{\sigma}^2 + V_q(b_\sigma \bar{\sigma}, T, \mu) \right]$$

SCL Effective Potential



We can obtain the Effective Potential analytically at finite T and μ

Effective Potential with $1/g^2$ (1)

- **1/d expansion of Plaquette action (Spatial One-Link Integral)**

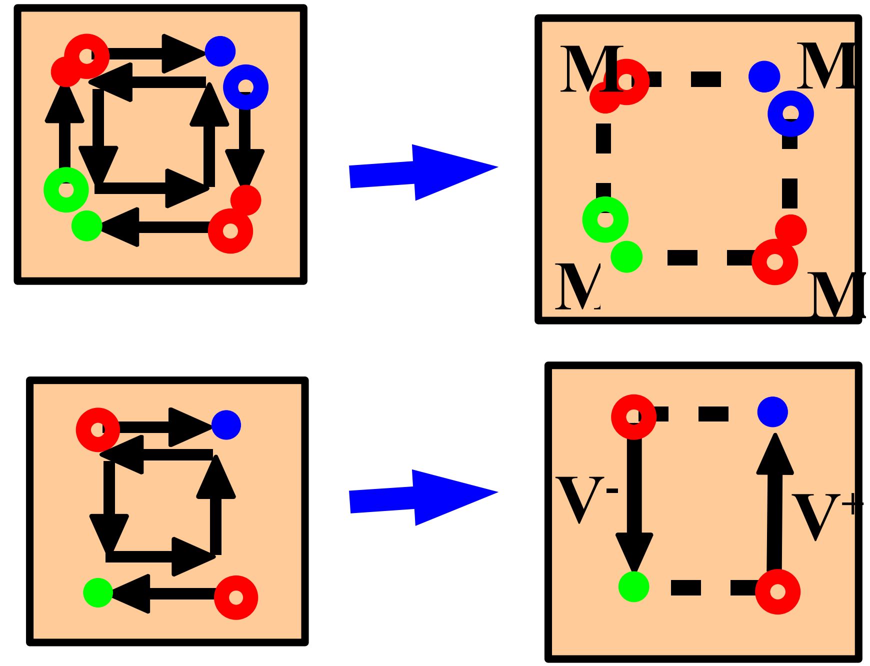
Faldt, Petersson (86); Bilic, Karsch, Redlich (92)

$$\int dU\, U_{ab} U^{+}_{cd} = \frac{1}{N_c} \delta_{ad} \delta_{bc}$$

  - **Spatial plaquett → MMMM**
  - **Temporal Link → V^+V^-**

$$V_x^+ = e^{\mu} \bar{\chi}_x U_0(x) \chi_{x+\hat{0}}$$

$$V_x^- = e^{-\mu} \bar{\chi}_{x+\hat{0}} U_0^\dagger(x) \chi_x$$



- **Effective Action**

$$\Delta S_\beta^{(\tau)} = \frac{1}{4N_c^2 g^2} \sum_{x, j>0} \left(V_x^+ V_{x+\hat{j}}^- + V_x^+ V_{x-\hat{j}}^- \right)$$

$$\Delta S_\beta^{(s)} = -\frac{1}{8N_c^4 g^2} \sum_{x, k>j>0} M_x M_{x+\hat{j}} M_{x+\hat{k}} M_{x+\hat{k}+\hat{j}}$$

Effective Potential with 1/g² (2)

- **Extended Hubbard-Stratonovich Transf.**

$$e^{\alpha AB} = \int d\varphi\, d\phi\, e^{-\alpha\{\varphi^2-(A+B)\varphi+\phi^2-i(A-B)\phi\}}$$
$$\approx e^{-\alpha\{\varphi^2-(A+B)\varphi-\phi^2+(A-B)\phi\}} .$$

  - **Mean field approx. φ , Saddle point approx. for ϕ**
  - **E.g. Temporal Plaquette action becomes,**

$$\Delta S_\beta^{(\tau)} \approx \frac{1}{4N_c^2 g^2} \sum_{x,j>0} \left[\varphi_\tau^2 + (V_x^+ - V_{x+\hat{j}}^-)\varphi_\tau - \phi_\tau^2 - (V_x^+ + V_{x+\hat{j}}^-)\phi_\tau\right] + (j \leftrightarrow -j)$$

  - **Effective Action becomes similar to the SCL action,**

$$S_{\text{eff}} = \frac{1}{2}(1+\beta_\tau\varphi_\tau)\sum_x \left(e^{-\beta_\tau\phi_\tau}V_x^+ - e^{\beta_\tau\phi_\tau}V_x^-\right) + m_0 \sum_x M_x$$
$$-\left(\frac{1}{4N_c} + \beta_s\varphi_s\right)\sum_{x,j>0} M_x M_{x+\hat{j}} + N_\tau L^d \left[\frac{\beta_\tau}{2}(\varphi_\tau^2 - \phi_\tau^2) + \frac{d\beta_s}{2}\varphi_s^2\right]$$

1/g²

Effective Potential with $1/g^2$ (2)

- **Extended Hubbard-Stratonovich Transf.**

$$e^{\alpha AB} = \int d\varphi\, d\phi\, e^{-\alpha\{\varphi^2-(A+B)\varphi+\phi^2-i(A-B)\phi\}} \approx e^{-\alpha\{\varphi^2-(A+B)\varphi-\phi^2+(A-B)\phi\}} .$$

  - **Mean field approx. φ , Saddle point approx. for ϕ**
  - **E.g. Temporal Plaquette action becomes,**

$$\Delta S_\beta^{(\tau)} \approx \frac{1}{4N_c^2 g^2}\sum_{x,j>0}\left[\varphi_\tau^2 + (V_x^+ - V_{x+\hat{j}}^-)\varphi_\tau - \phi_\tau^2 - (V_x^+ + V_{x+\hat{j}}^-)\phi_\tau\right] + (j \leftrightarrow -j)$$

  - **Effective Action becomes similar to the SCL action,**

WF Renormalization

μ mod.

Aux. Terms

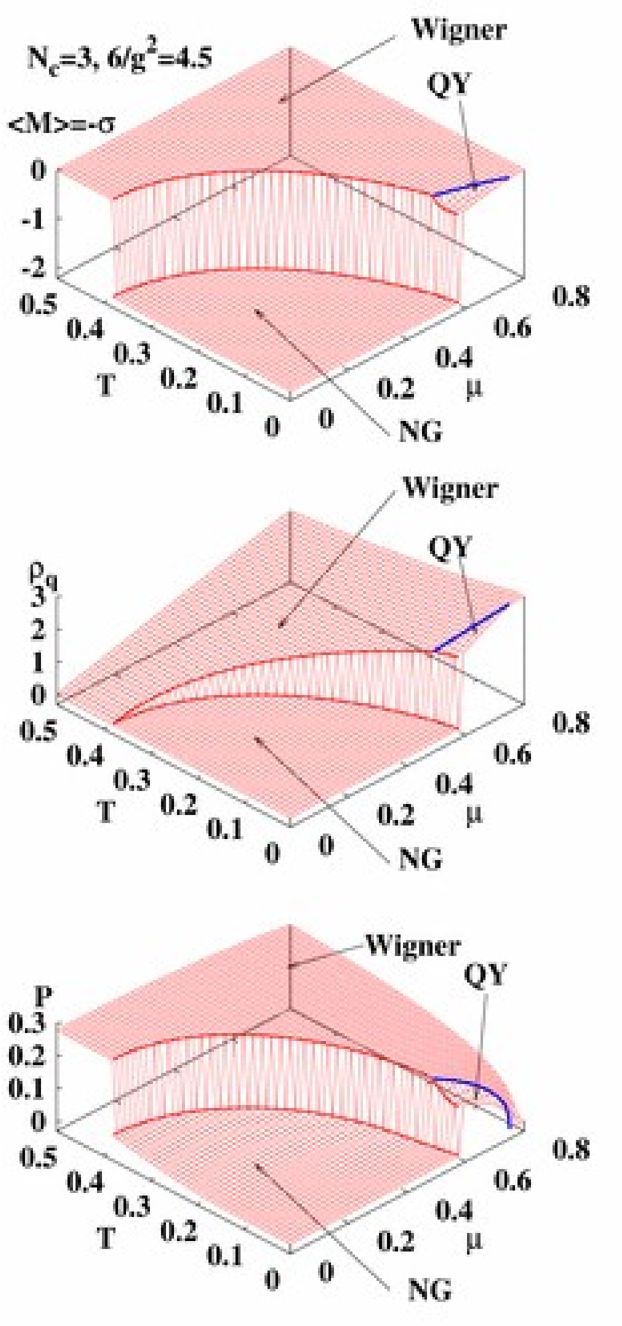
$$S_{\rm eff} = \frac{1}{2}(1+\beta_\tau\varphi_\tau)\sum_x\left(e^{-\beta_\tau\phi_\tau}V_x^+ - e^{\beta_\tau\phi_\tau}V_x^-\right) + m_0\sum_x M_x - \left(\frac{1}{4N_c} + \beta_s\varphi_s\right)\sum_{x,j>0} M_x M_{x+\hat{j}} + N_\tau L^d\left[\frac{\beta_\tau}{2}(\varphi_\tau^2 - \phi_\tau^2) + \frac{d\beta_s}{2}\varphi_s^2\right]$$

Baryon Density and Polyakov Loop in QY

- **Example: $N_c=3$, $6/g^2=4.5$, $m_0=0$ (χ limit)**
- **Baryon Density (= $\rho_q/3$)**
  - **$\rho_q \sim 0$ in Nambu-Goldstone (NG) phase**
    $\rho_q > 0$ in Wigner phase
  - **ρ_q in QY ~ ρ_q in Wigner phase**
- **(Quark Driven) Polyakov Loop (=P)**
  - **Quark driven P ~ O(N_c)**
  - **P(QY) < P (Wig.)**

$$P \equiv \frac{1}{2N_c}\left\langle \mathrm{tr}\left[\prod_\tau U_0 + \prod_\tau U_0^\dagger\right]\right\rangle$$

$$= \frac{X_{N_c-1}\ \cosh\left[\tilde{\mu}/T\right] + X_1\ \cosh\left[(N_c-1)\tilde{\mu}/T\right]}{N_c\left(X_{N_c} + 2\cosh\left[N_c\tilde{\mu}/T\right]\right)}$$



Why do we have QY ? Which Explanation do you like ?

- **Vector field (ϕ_τ) acts more repulsively at smaller σ, and generates a local minimum in the region of $\sigma \ll \sigma_{vac}$**
- **2nd order P.T. condition (C_2 =0) leads to the relation of effective μ and T.**

$$F_{\text{eff}} = F(\sigma=0) + C_2\sigma^2 + C_4\sigma^4 + \ldots$$

$$C_2(T, \mu - \beta_\tau \rho_q(\sigma=0)) = 0$$

$$\rightarrow \mu_c^{(\text{2nd})} = f(T) - \beta_\tau \rho_q$$

- **Smaller σ**
  - **→ Smaller const. quark mass**
  - **→ Larger ρ_q**
  - **→ Smaller μ_{eff}**
  - **→ Later P.T. to Wigner phase (σ=0)**