Meson masses at finite density on the lattice

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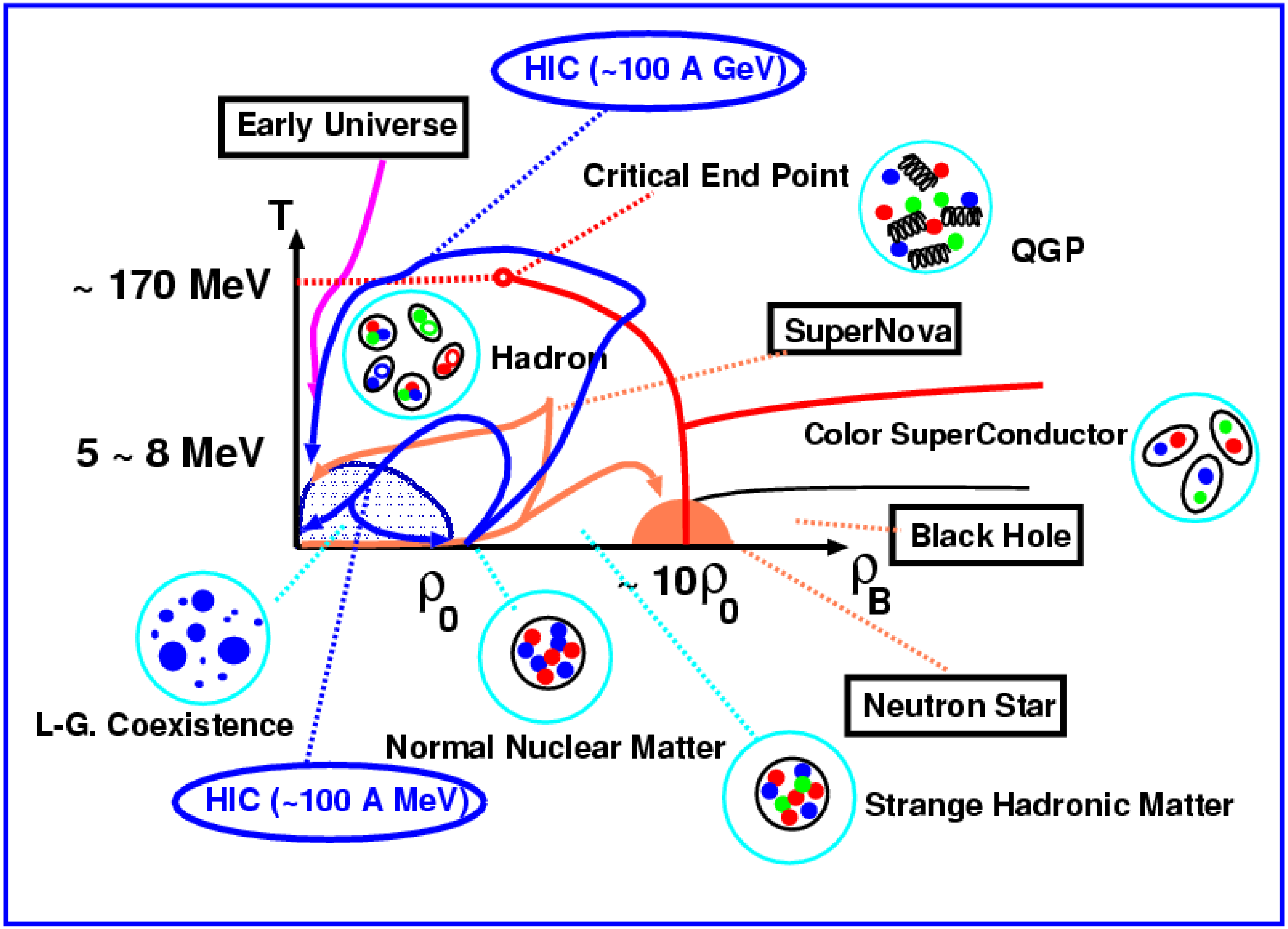
- **Introduction**
- **Brown-Rho Scaling for Meson masses in the Strong Coupling Limit of Lattice QCD**
  Kawamoto, Miura, AO, PoS(LAT07) (2007), 209 [arXiv:0710.1720]*
  AO, Kawamoto, Miura, arXiv:0803.0255 (Chiral07 Proc.)
- **Summary**
- **(If I have time....) Quarkyonic phase in Strong Coupling Expansion of LQCD ($1/g^2$ effects)**
  (Last night work)

Quark / Hadron / Nuclear Matter Phase Diagram

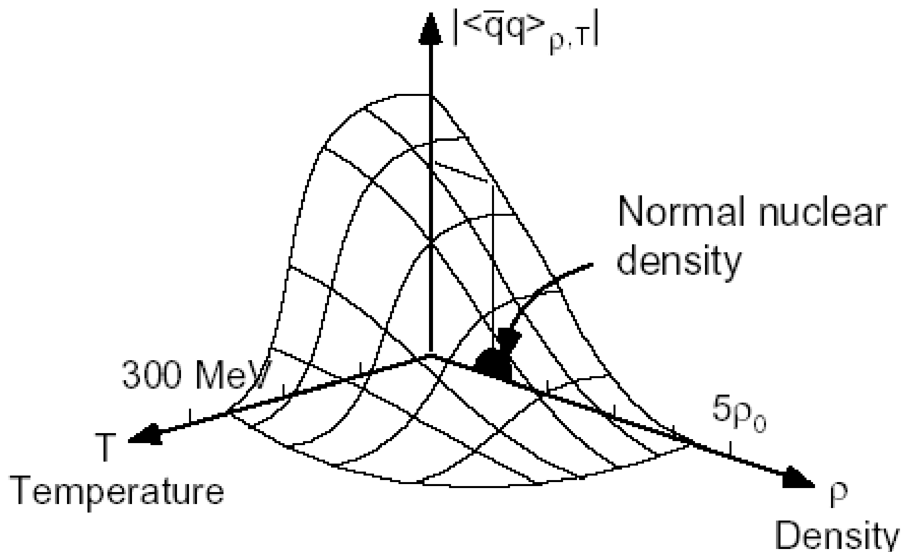


How can we probe the "phase" properties ?

Hadron Mass Modification

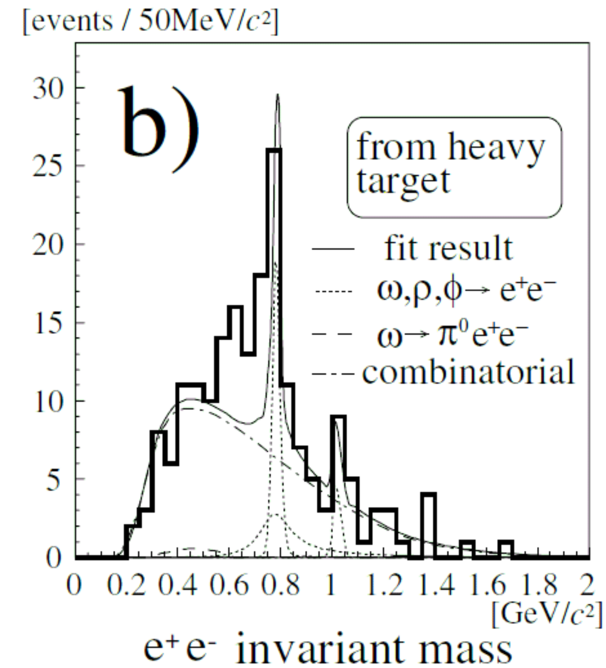
- **Medium meson mass modification may be the signal of partial restoration of chiral sym.**
  - *Brown, Rho, PRL66('91)2720;*
  - *Kunihiro,Hatsuda, PRep 247('94),221; Hatsuda, Lee, PRC46('92)R34.*
- **Brown-Rho Scaling**

$$M^*_N / M_N = M^*_\sigma / M_\sigma = M^*_\rho / M_\rho = M^*_\omega / M_\omega = f^*_\pi / f_\pi$$



Hadron Mass Modification

- **Medium meson mass modification**
  is suggested experimentally.
  CERES Collab., PRL75('95),1272;
  KEK-E325 Collab.(Ozawa et al.), PRL86('01),5019;
  PHENIX Collab., arXiv:0706.3034



Interpretation is model dependent
→ Investigation in
non-perturbative QCD is desired !

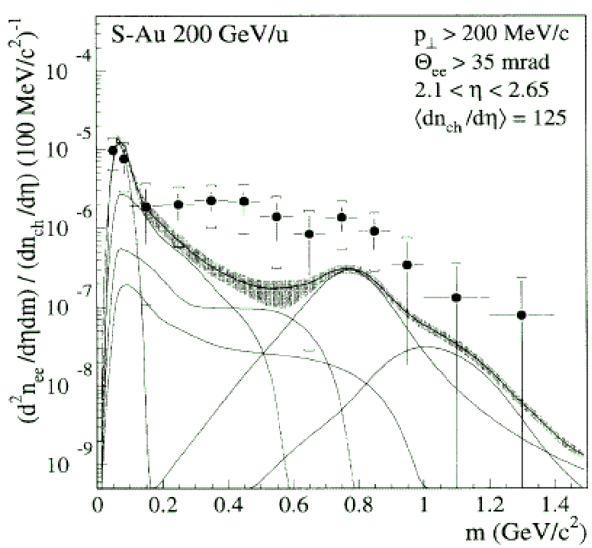
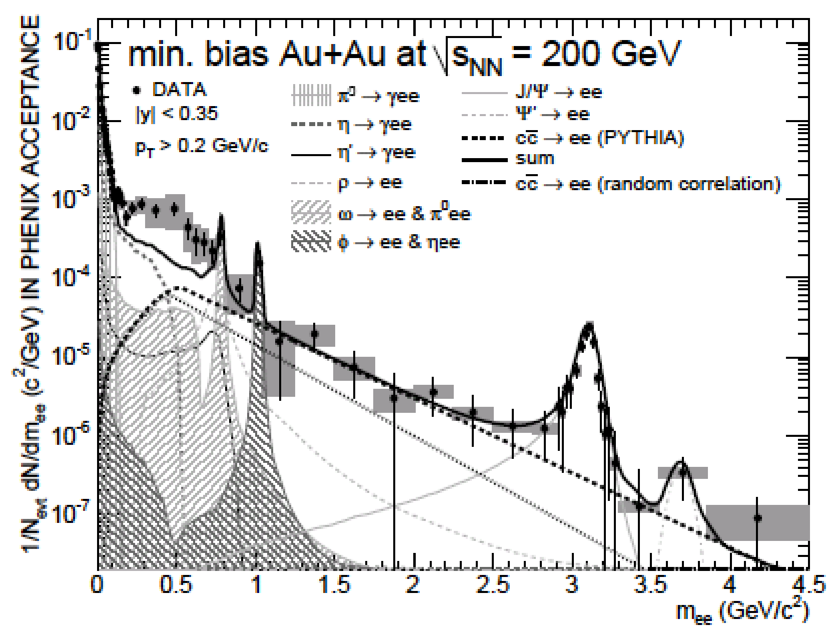


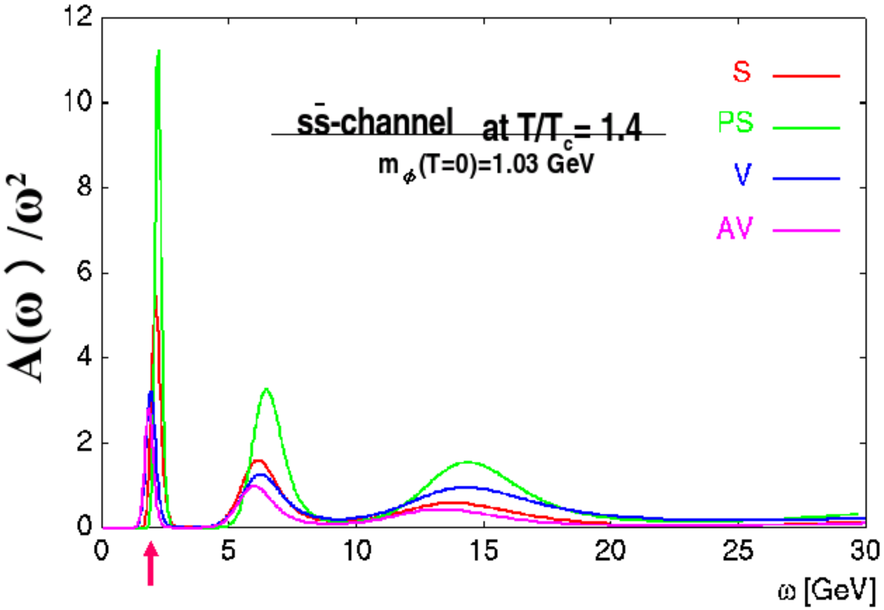
FIG. 4. Inclusive e^+e^- mass spectra in 200 GeV/nucleon S-Au collisions. For explanations see Fig. 2.



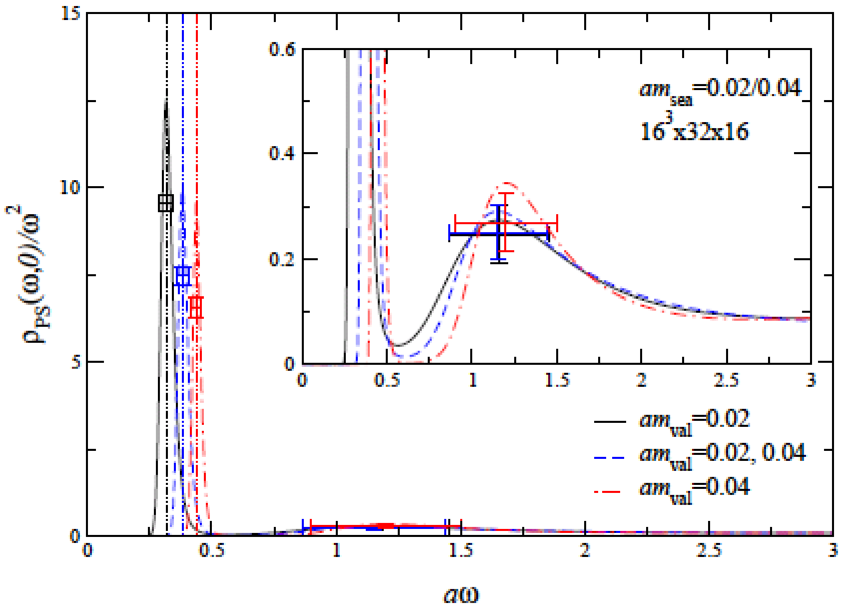
Hadron Mass Modification in Lattice QCD

- Can we understand it in Lattice QCD ?
  - Finite T: It is possible even for light quarks ! (Talk by G. Aarts in the Symp.)
  - Finite μ (and low T): Difficult due to the sign problem.

Asakawa, Nakahara, Hatsuda, hep-lat/0208059.



G. Aarts, Foley, 2007



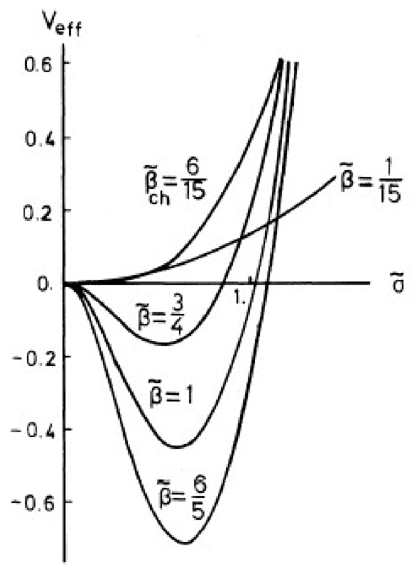
Domain-Wall QCD, PS channel

Strong Coupling Limit of Lattice QCD

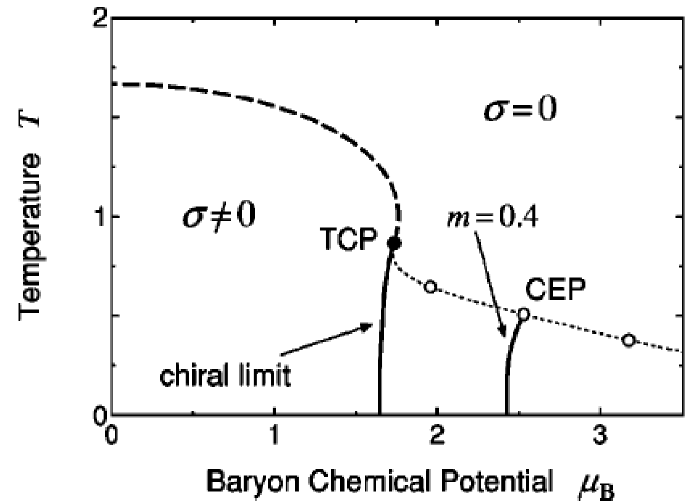
- **SCL-LQCD has been a powerful tool in "phase diagram" study !**
  - **Analytical integral over link variables → No Sign Problem**
  - **Chiral restoration, Phase diagram, Baryon effects**
- **Hadron masses are also studied, but only at zero T treatment.**

  Kluberg-Stern, Morel, Petersson, '83; Kawamoto, Shigemoto, '82; Fukushima, 2004

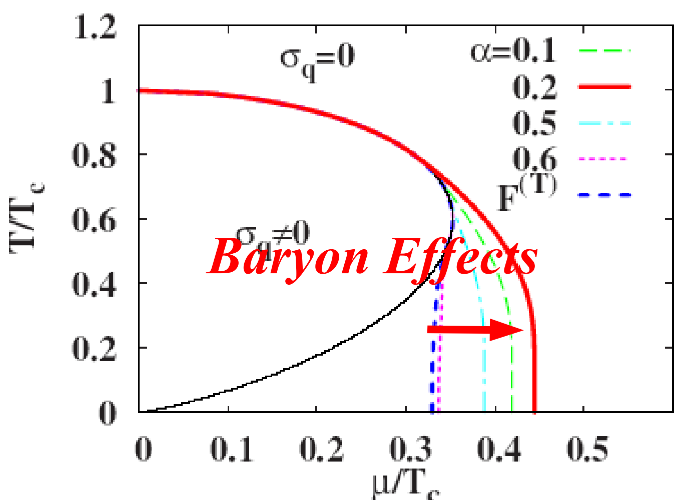
  - **To do: Finite T and μ, Baryon Effects at Finite T, $1/g^2$ corrections, ...**



Damgaard,Kawamoto,Shigemoto, PRL53('84),2211



Nishida, PRD69, 094501 (2004)



Kawamoto,Miura,AO,Ohnuma, PRD75 (07), 014502.

Meson Masses at Finite T and μ in the Strong Coupling Limit of Lattice QCD

Strong Coupling Limit of Lattice QCD

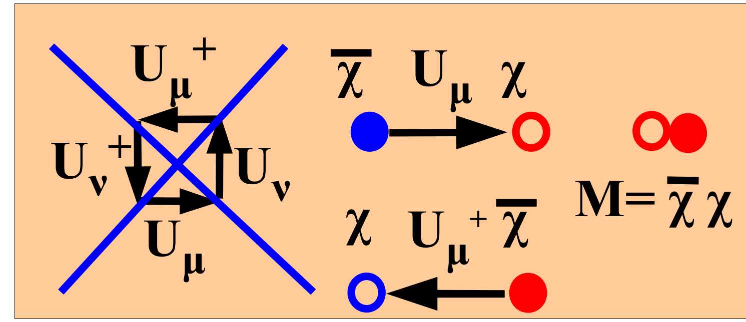
- **Strong Coupling Limit: Pure gluonic action disappers at g → ∞**

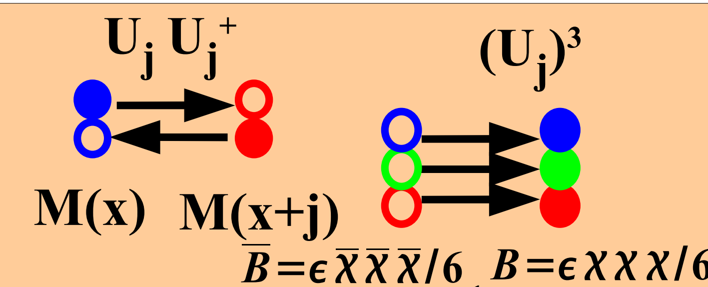
$$S_{\rm QCD} = \cancel{S_G} + S_F^{(s)} + S_F^{(t)} + m_0 \bar{\chi}\chi$$

$$\cancel{S_G = -\frac{1}{g^2}\sum_{\rm plaq.} {\rm Tr}\, U_{ij}(x) + c.c.}$$

$$S_F^{(s)} = \frac{1}{2}\sum_{x,j>0}\left(\bar{\chi}_x U_j(x)\chi_{x+\hat{j}} - \bar{\chi}_{x+\hat{j}} U_j^+(x)\chi_x\right)$$

$$S_F^{(t)} = \frac{1}{2}\sum_{x}\left(e^{\mu}\bar{\chi}_x U_0(x)\chi_{x+\hat{0}} - e^{-\mu}\bar{\chi}_{x+\hat{0}} U_0^+(x)\chi_x\right)$$





$$\int dU\, U_{ab} U_{cd}^+ = \frac{1}{N_c}\delta_{ad}\delta_{bc}$$

$$\int dU\, U_{ab} U_{cd} U_{ef} = \frac{1}{6}\epsilon_{ace}\epsilon_{bdf}$$

- **One-link integral leaves mesonic and baryonic action.**

$$S_F^{(s)} \to -\frac{1}{2}(M V_M M) - (\bar{B} V_B B)$$

$$= -\frac{1}{4N_c}\sum_{x,j>0} M_x M_{x+\hat{j}} + \sum_{x,j>0}\frac{\eta_j}{8}\left[\bar{B}_x B_{x+\hat{j}} - \bar{B}_{x+\hat{j}} B_x\right]$$

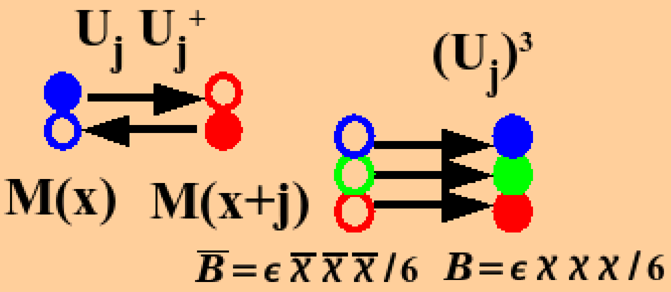
- **Analytic Link Integral → No Sign Problem at finite μ.**

SCL-LQCD: Tools (1) --- One-Link Integral

- **Group Integral Formulae**

$$\int dU\, U_{ab} U^{+}_{cd} = \frac{1}{N_c} \delta_{ad} \delta_{bc}$$

$$\int dU\, U_{ab} U_{cd} U_{ef} = \frac{1}{6} \epsilon_{ace} \epsilon_{bdf}$$



$$\int dU\, U_{ab} U^{+}_{cd} = \frac{1}{N_c} \delta_{ad} \delta_{bc}$$

$$\int dU\, U_{ab} U_{cd} U_{ef} = \frac{1}{6} \epsilon_{ace} \epsilon_{bdf}$$

$$\int dU \exp(-a\bar{\chi}(x) U \chi(y) + b\bar{\chi}(y) U^{+} \chi(x))$$
$$= \int dU \left[1 - ab\,\bar{\chi}(a)^a U_{ab} \chi^b(y) \bar{\chi}^c(y) U^{+}_{cd} \chi^d(x) + \cdots\right]$$
$$= 1 + ab(\chi\bar{\chi})(x)(\chi\bar{\chi})(y) + \cdots = 1 + ab\, M(x) M(y) + \cdots$$
$$= \exp\left[ab\, M(x) M(y) + \cdots\right]$$

Quarks and Gluons → One-Link integral
→ Mesonic and Baryonic Composites

SCL-LQCD: Tools (2) --- 1/d Expansion

- **Keep mesonic action to be indep. from spatial dimension *d***
  → Higher order terms are suppressed at large *d*.

$$\sum_j (\bar{\chi} U_j \chi)(\bar{\chi} U_j^+ \chi) \rightarrow -\frac{1}{N_c} \sum_j M(x) M(x+\hat{j}) = O(1)$$

$$\rightarrow M \propto 1/\sqrt{d},\ \chi \propto d^{-1/4}$$

$$\sum_j (\bar{\chi} U_j \chi)(\bar{\chi} U_j \chi)(\bar{\chi} U_j \chi) \rightarrow N_c! \sum_j B(x) B(x+\hat{j}) = O(1/\sqrt{d})$$

$$\sum_j (\bar{\chi} U_j \chi)^2 (\bar{\chi} U_j^+ \chi)^2 \rightarrow \sum_j M^2(x) M^2(x+\hat{j}) = O(1/d)$$

We can stop the expansion in U,
since higher order terms are suppressed !

Meson Mass in SCL-LQCD (Zero T)

- **QCD Lattice Action (Zero T treatment)**

Kawamoto, Smit, 1981

$$S = \not{S_G} + S_F + m_0 \bar{\chi}\chi$$
Strong Coupling Limit

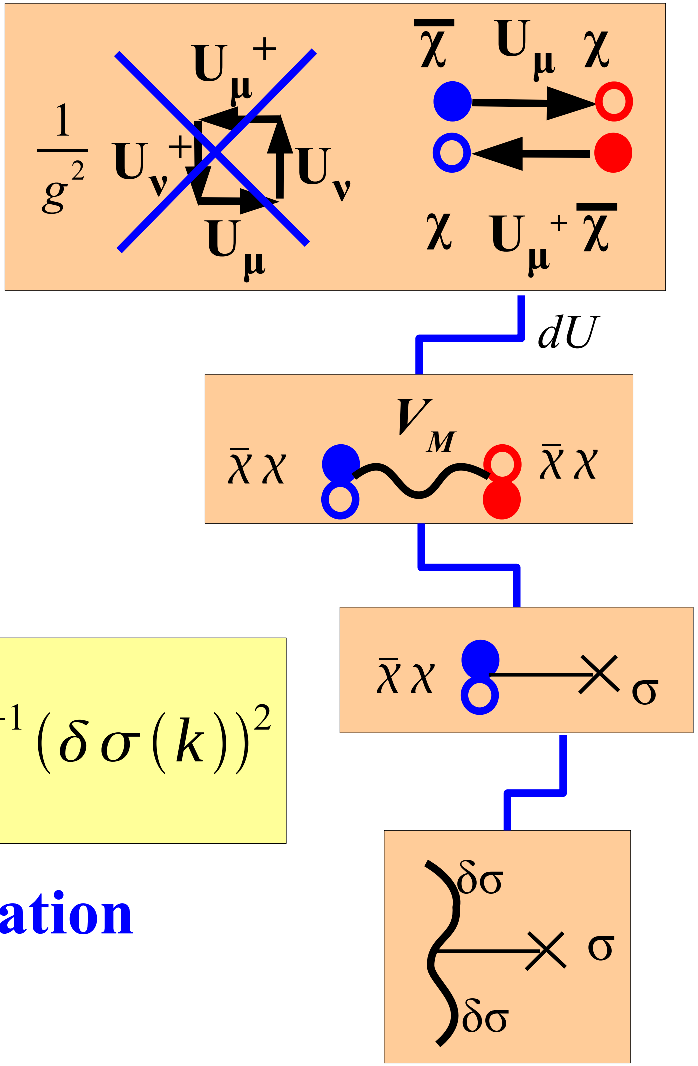
$$\rightarrow -\frac{1}{2}(\bar{\chi}\chi) V_M (\bar{\chi}\chi) + m_0 \bar{\chi}\chi$$
One-link integral (1/d expansion)

$$\rightarrow \frac{1}{2}\sigma V_M{}^{-1}\sigma + \bar{\chi}(\sigma + m_0)\chi$$
Bosonization

$$\rightarrow \frac{1}{2}\sigma V_M{}^{-1}\sigma - N_c \sum_x \log(\sigma(x) + m_0)$$
Fermion Integral

$$= L^d N_\tau \left[\frac{N_c}{d+1}\bar{\sigma}^2 - N_c \log(\bar{\sigma} + m_0)\right] + \frac{1}{2}\sum_k G(k)^{-1}(\delta\sigma(k))^2$$

Effective Potential **Fluctuation**



- **Meson Propagator**

$$G(k)^{-1} = F.T.\frac{\delta^2 S}{\delta\sigma(x)\delta\sigma(y)} = 2N_c\left[\sum_\mu \cos k_\mu\right]^{-1} + \frac{N_c}{(\bar{\sigma}+m_0)^2}$$

Meson Mass in SCL-LQCD (Zero T)

- **Meson Mass in SCL-LQCD**
  Kluberg-Stern, Morel, Petersson, '82; Kawamoto, Shigemoto, '82; K. Fukushima, 2004
  - **Pole of the propagator at zero momentum → Meson Mass**
  - **Doubler DOF: $k_\mu \to 0$ or π, Euclidian: $\omega \to i\,m$ + "0 or π"**

$$G^{-1}(k) = N_c \left[\sum_{i=1}^{d} \cos \pi \delta_i \pm \cosh m \right]^{-1} + \frac{N_c}{(\bar{\sigma} + m_q)^2} = 0$$

$$\to \quad \cosh m = 2(\bar{\sigma} + m_q)^2 + \kappa$$

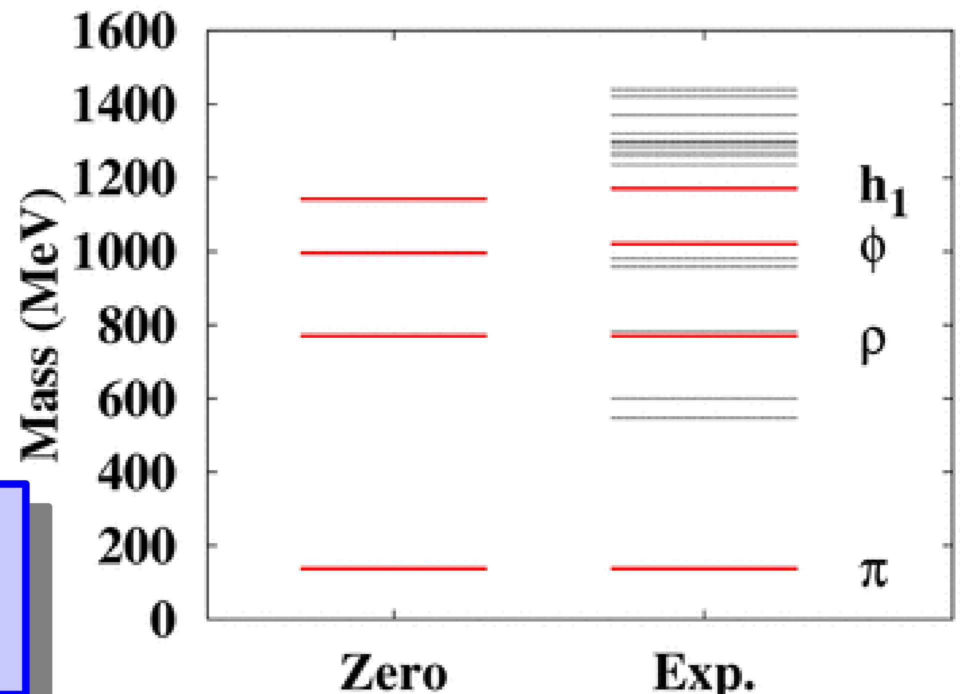
$$= (d+1)(\lambda^2 - 1) + 2n + 1$$

Equilibrium Condition

$$n = 0, 1, \ldots d \quad (\text{diff. meson species})$$

$$\lambda = \bar{m}_q + \sqrt{\bar{m}_q^{\,2} + 1}, \quad \bar{m}_q = m_q / \sqrt{2(d+1)}$$

Explains Meson Mass Spectrum
No (T, μ) dependence



Meson Mass in SCL-LQCD (Finite T)

- **QCD Lattice Action (Finite T treatment)**
  Damgaard, Kawamoto, Shigemoto, 1984; Bilic et al., 1992; Nishida, '03; Fukushima, '03; Kawamoto, Miura, AO, Ohnuma, '07; ...

$$S = \cancel{S_G} + S_F^{(s)} + S_F^{(t)} + m_0 \bar{\chi}\chi$$

Strong Coupling Limit

$$S_F^{(t)} = \frac{1}{2}\sum_x \left(e^{\mu} \bar{\chi}_x U_0(x) \chi_{x+\hat{0}} - e^{-\mu} \bar{\chi}_{x+\hat{0}} U_0^{+}(x) \chi_x \right) = \bar{\chi} V^{(t)} \chi$$

$$\to -\frac{1}{2}(\bar{\chi}\chi) V_M (\bar{\chi}\chi) + \bar{\chi}(V^{(t)} + m_0)\chi$$

Spatial-link integral (1/d expansion, no B)

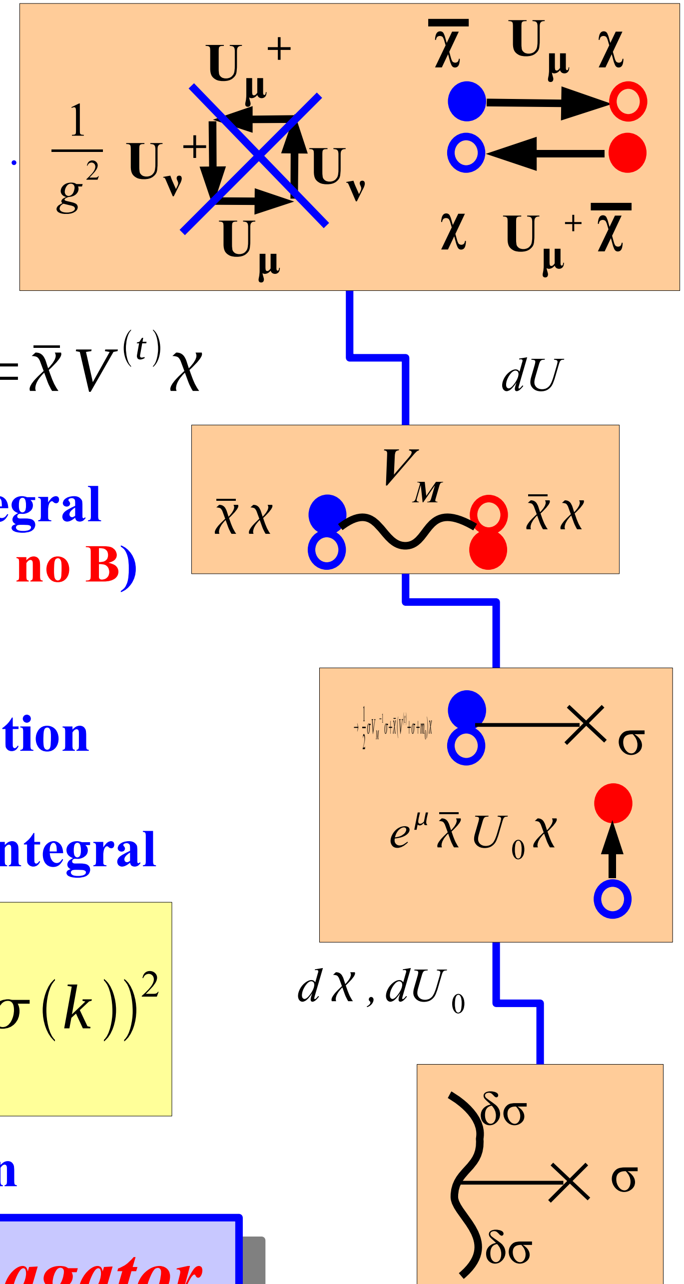
$$\to \frac{1}{2}\sigma V_M^{-1}\sigma + \bar{\chi}(V^{(t)} + \sigma + m_0)\chi$$

Bosonization

Fermion and Temporal-link Integral

$$\to L^d N_\tau \left[\frac{N_c}{d}\bar{\sigma}^2 + V_{\text{eff}}(\bar{\sigma}, T, \mu) \right] + \frac{1}{2}\sum_k G(k)^{-1} (\delta\sigma(k))^2$$

Effective Potential **Fluctuation**



V_{eff} as a functional of σ → Meson propagator

Fermion Determinant

Faldt, Petersson, 1986

- **Fermion action is separated to each spatial point and bi-linear → Determinant of Nτ x Nc matrix**

$$\exp(-V_{\text{eff}}/T) = \int dU_0 \begin{vmatrix} I_1 & e^{\mu} & 0 & & e^{-\mu}U^{+} \\ -e^{-\mu} & I_2 & e^{\mu} & & \\ 0 & -e^{-\mu} & I_3 & e^{\mu} & \\ \vdots & & & \ddots & \\ -e^{\mu}U & & & -e^{-\mu} & I_N \end{vmatrix}$$

$$= \int dU_0 \, det\left[X_N[\sigma] \otimes \mathbf{1}_c + e^{-\mu/T} U^{+} + (-1)^{N_\tau} e^{\mu/T} U \right]$$

$$I_k = 2(\sigma(k) + m_0)$$

$$X_N = \begin{vmatrix} I_1 & e^{\mu} & 0 & \cdots & & e^{-\mu} \\ -e^{-\mu} & I_2 & e^{\mu} & & & 0 \\ 0 & -e^{-\mu} & I_3 & & & 0 \\ \vdots & & & \ddots & & \\ & & & & I_{N-1} & e^{\mu} \\ -e^{\mu} & 0 & 0 & \cdots & -e^{-\mu} & I_N \end{vmatrix} - \left[e^{-\mu/T} + (-1)^N e^{\mu/T} \right]$$

Interaction Term Veff

- **Interaction term Veff = Function of X_N**

  X_N = Functional of σ , given in the determinant form.

  → It is possible to obtain derivatives and equilibrium values

  Kawamoto, Miura, AO, PoS(LAT07) (2007), 209 [arXiv:0710.1720]*
  AO, Kawamoto, Miura, arXiv:0803.0255 (Chiral07 Proc.)

$$V_{\rm eff} = -T\log\left[2\cosh(N_c\mu/T) + \frac{\sinh[(N_c+1)E/T]}{\sinh[E/T]}\right]$$

$$\left.\frac{\partial^2 V_{\rm eff}}{\partial\sigma_n\partial\sigma_{n+k}}\right|_{\sigma=\bar\sigma} = \left[\frac{dV_{\rm eff}}{dX_N}\frac{\partial^2 X_N}{\partial\sigma_n\partial\sigma_{n+k}} + \frac{d^2V_{\rm eff}}{dX_N^2}\frac{1}{N^2}\left[\frac{dX_N^{(0)}}{d\bar\sigma}\right]^2\right]_{\sigma=\bar\sigma}$$

$$\left.\frac{\partial^2 X_N}{\partial\sigma_n\partial\sigma_{n+k}}\right|_{\sigma=\bar\sigma} = \frac{2}{\cosh^2 E}\left[\cosh NE - e^{i\pi k}\cosh[(N-2k)E]\right] ,$$

$$E = \text{arcsinh}\left[\bar\sigma + m_0\right]$$

Let's check meson masses !

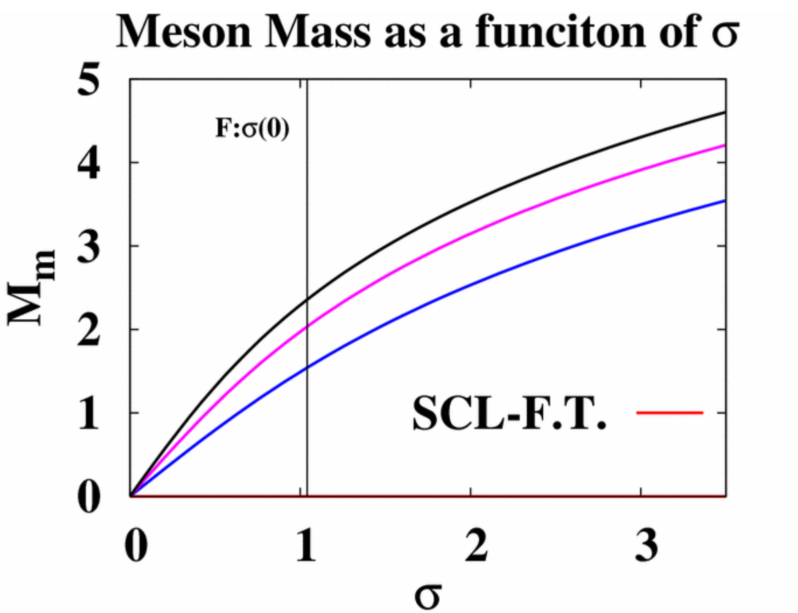
Hadron Mass in SCL-LQCD

- **Meson Mass**

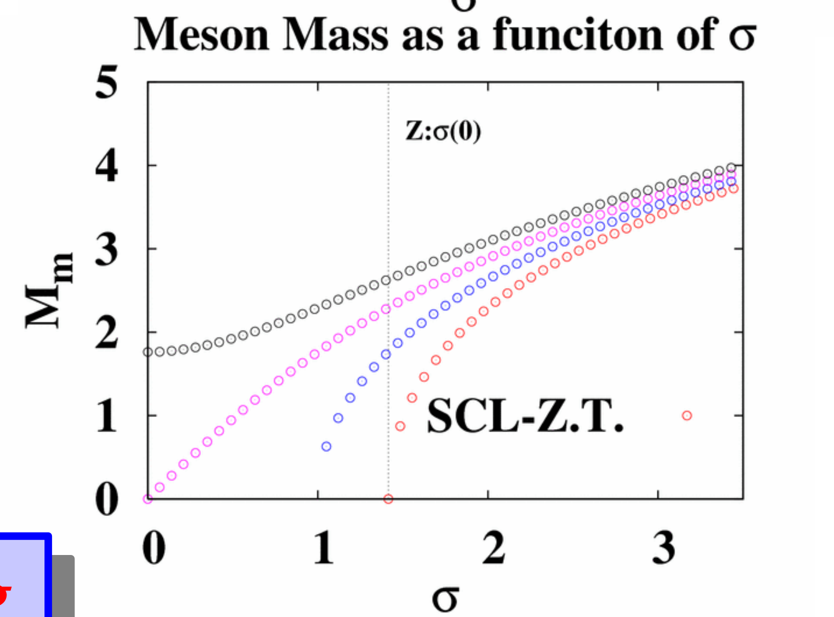
$$G^{-1}(\boldsymbol{k},\omega)=\frac{2N_c}{\kappa(\boldsymbol{k})}+\frac{4N_c}{d}\frac{\bar{\sigma}(\bar{\sigma}+m_0)}{\cos\omega+2(\bar{\sigma}+m_0)^2+1}$$

$$\kappa(\boldsymbol{k})=\sum_{i=1}^{d}\cos k_i \quad\rightarrow\quad \kappa=-d,-d+2,\ldots d$$

$$M=2\,\mathrm{arcsinh}\sqrt{(\bar{\sigma}+m_0)\left(\frac{d+\kappa}{d}\bar{\sigma}+m_0\right)}$$



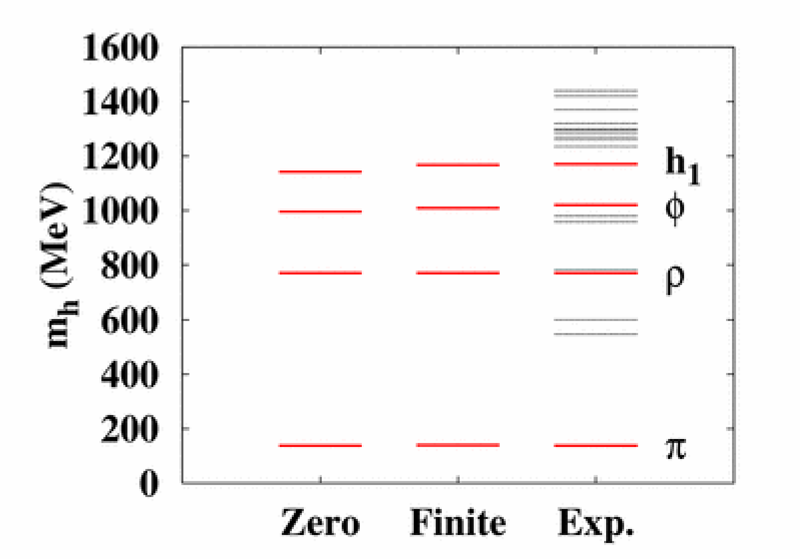
- **Meson masses are determined by the equilibrium chiral condensate, σ.**
- **Chiral condensate is determined by the equilibrium condition, and given as a function of (T, μ).**



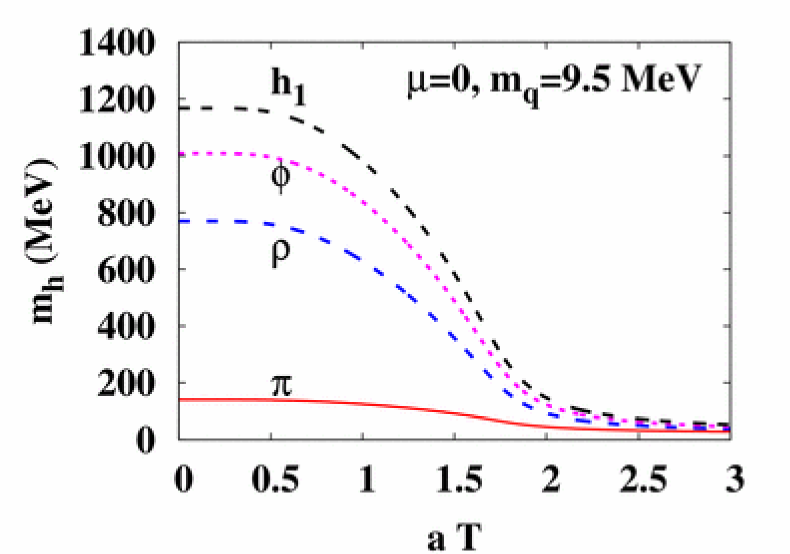
Approximate Brown-Rho Scaling is proven in SCL-LQCD

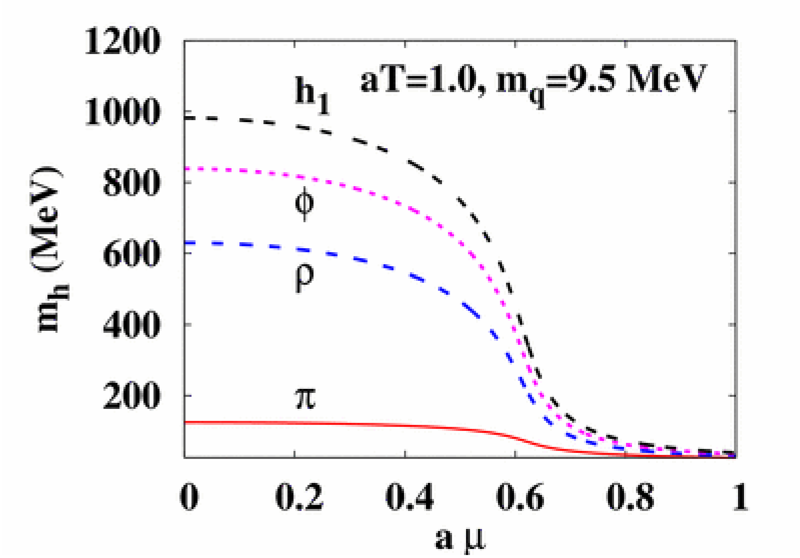
Medium Modification of Meson Masses

- Scale fixing
  - Search for σ_{vac} to minimize free E.
  - Assign κ=-3, -1 as π and ρ
  - Determine m_0 and a^{-1} (lattice unit) to fit m_π / m_ρ



- Medium modification
  - Search for σ(T, μ) → Meson mass





Summary

- **Meson masses are evaluated in the Strong Coupling Limit of Lattice QCD with one species of staggered fermion at Finite T and μ.**

$$M = 2\,\mathrm{arcsinh}\sqrt{(\bar{\sigma}+m_0)\left(\frac{d+\kappa}{d}\bar{\sigma}+m_0\right)} \qquad \kappa = -d, -d+2, \ldots d$$

  - **Mass of NG boson (κ=-d) is always zero in the chiral limit.**
  - **Different from Zero T treatment:** $\cosh M = 2(\bar{\sigma}+m_0)^2+\kappa$

    Kluberg-Stern, Morel, Petersson, '83; Kawamoto, Shigemoto, '82.

  - **Equilibrium Condition removes the explicit dependence on (T, μ), and meson masses depends on (T, μ) only via σ.**
    → *Approximate Brown-Rho scaling holds in SCL-LQCD.*

Problems....

- **Physical Scale Problem in SCL-LQCD**
  $T_c \sim 800$ MeV, $\mu_c \sim 300$ MeV
  $\rightarrow$ Finite coupling ($1/g^2$) correction may help
- **Meson assignment is not seriously considered**
  Golterman, Smit, '84
- **Why does the pion mass keep decreasing around Tc ?**
  $\leftrightarrow$ NJL results (*Hatsuda, Kunihiro, '94*)
- **Where did "σ" pole go ?**
- **Bosonization with negative eigen values of V_M**
  Miura, Thesis, March 2008, Hokkaido Univ.
- **.....**

Quarkyonic Phase in SC-LQCD ?

Problems: Physical Scale

- **Are the SCL-LQCD results reliable ?**
  - **π, ρ mass fit → Physical Scale (a^{-1}) is fixed**
    a^{-1} = 497 MeV, m_q=9.5 MeV
    → T_c= 5/3a = 828 MeV (Too large !)
    (Long standing problem in SCL-LQCD)
  - **Brown-Rho scaling may not be realized in the real world**
    → Finite condensate other than σ would be necessary
- **Finite coupling effects may help !**
  - **$1/g^2$ correction reduces T_c**
    Bilic, Claymans, '95; AO, Kawamoto, Miura, '07
  - **Other condensate than σ will appear from the plaquett term.**

$$\langle \bar{q} g q \rangle, \quad \langle M(x) M(x+\hat{j}) \rangle, , ,$$

$1/g^2$ expansion at Finite T

- **1/d expansion of plaquetts** *(Faldt, Petersson 1986)*

$$\Delta S_\beta = \frac{\beta_t}{2d} \sum_{x, j>0} \left(V_x^{(+)} V_{x+\hat{j}}^{(-)} + V_x^{(-)} V_{x+\hat{j}}^{(+)} \right) - \frac{\beta_s}{2(d-1)} \sum_{x, k>j>0} M_x M_{x+\hat{j}} M_{x+\hat{j}+\hat{k}} M_{x+\hat{k}}$$

$$\left(V_x^{(+)} = e^{\mu} \bar{\chi}_x U_0(x) \chi_{x+\hat{0}}, \quad V_x^{(-)} = e^{-\mu} \bar{\chi}_{x+\hat{0}} U_0^{+}(x) \chi_x \right)$$

- **Bosonization & Mean Field Approximation**
  (Kawamoto, Miura, AO, in preparation)

$$\begin{aligned} S_{\mathrm{SCL}} + \Delta S_\beta &= \frac{1}{2} \sum_x V_x^{(+)} \times (1 + \beta_t \varphi_t + \beta_t \phi_t) \\ &- \frac{1}{2} \sum_x V_x^{(-)} \times (1 + \beta_t \varphi_t - \beta_t \phi_t) \\ &- \frac{1}{4} N_c \sum_{x, x>0} M_x M_{x+\hat{j}} \times \left(1 + 4 N_c \beta_s (\varphi_s - \phi_s)\right) \\ &+ m_0 \sum_x \bar{\chi}_x \chi_x + N_\tau N_s^d \left[\frac{\beta_t}{4} (\varphi_t^2 - \phi_t^2) + \frac{\beta_s d}{4} (\varphi_s^2 - 2\phi_s^2) \right] \end{aligned}$$

Correction terms are absorbed in the SCL action terms.

Effective Potential with $1/g^2$

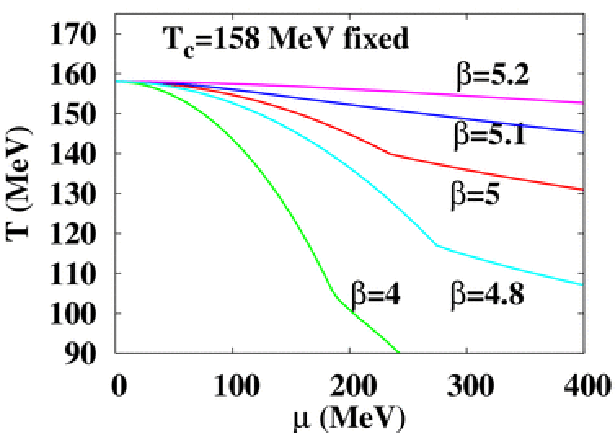
- **Quark and Time-like Link integral → Effective Potential**

$$F=\frac{d}{4N_c}\sigma^2+F_q(m_q;\tilde{\mu})$$

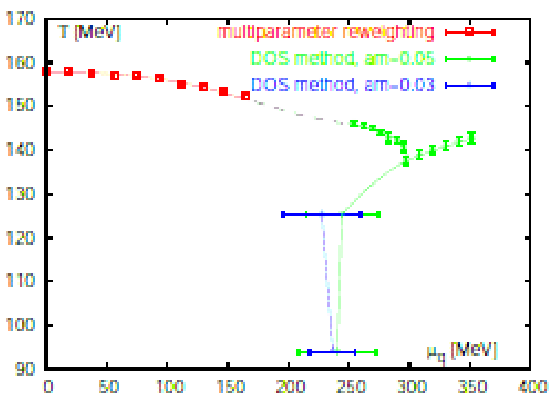
$$+\beta_s d\sigma^2(\varphi_s-\phi_s)+\frac{\beta_t}{4}(\varphi_t^2-\phi_t^2)+\frac{\beta_s d}{4}(\varphi_s^2-2\phi_s^2)-N_c\beta_t\varphi_t$$

$$m_q=\frac{d}{2N_c}\sigma(1+4N_c\beta_s(\varphi_s-\phi_s)-\beta_t\varphi_t),\quad \tilde{\mu}=\mu-\beta_t\phi_t$$

  - **At β ~ 5, results with $1/g^2$ correction would be comparable with MC results (Density of States method)**



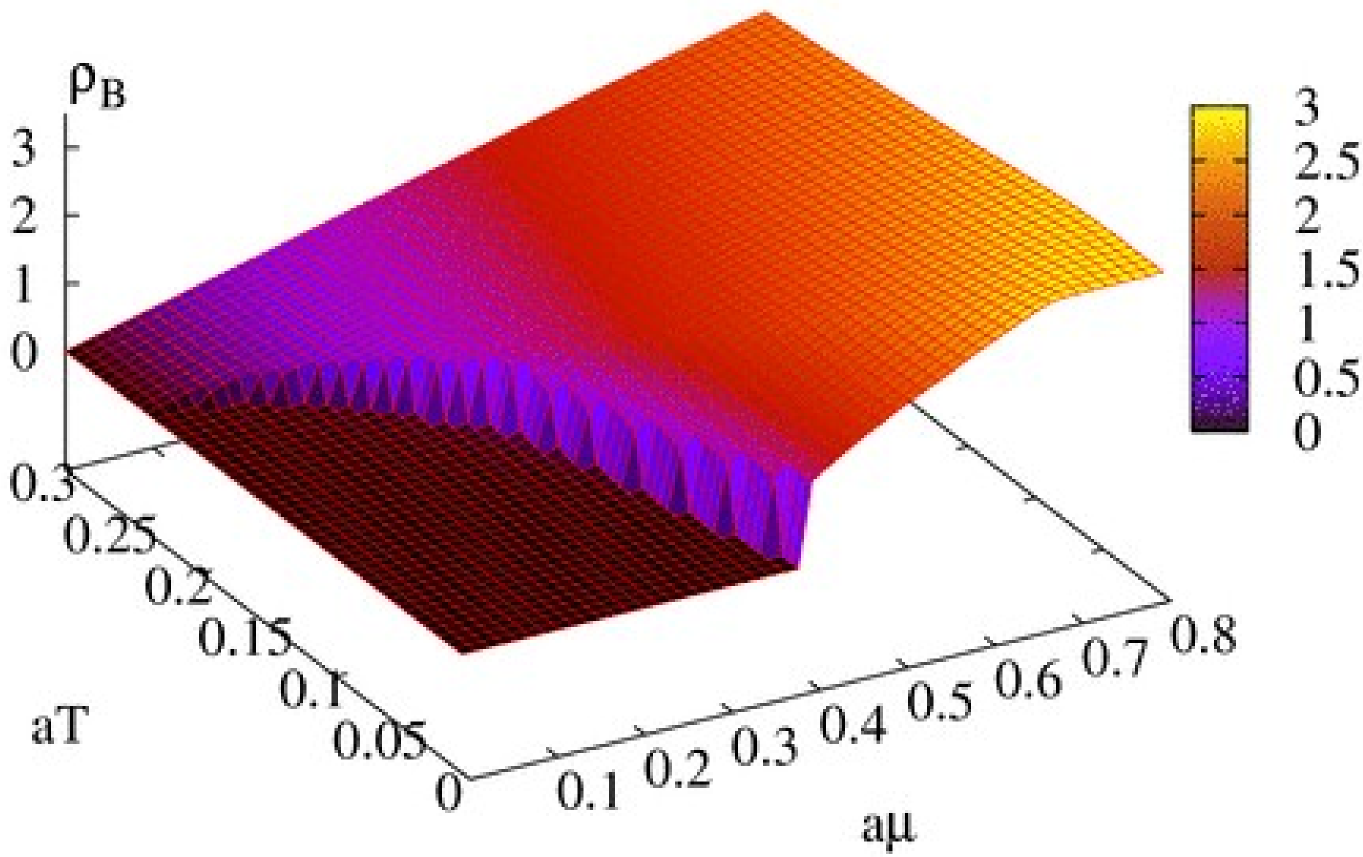
Kawamoto,Miura,AO, in prep.



Fodor,Katz,Schmit, 2007

Quarkyonic Phase in SC-LQCD ?

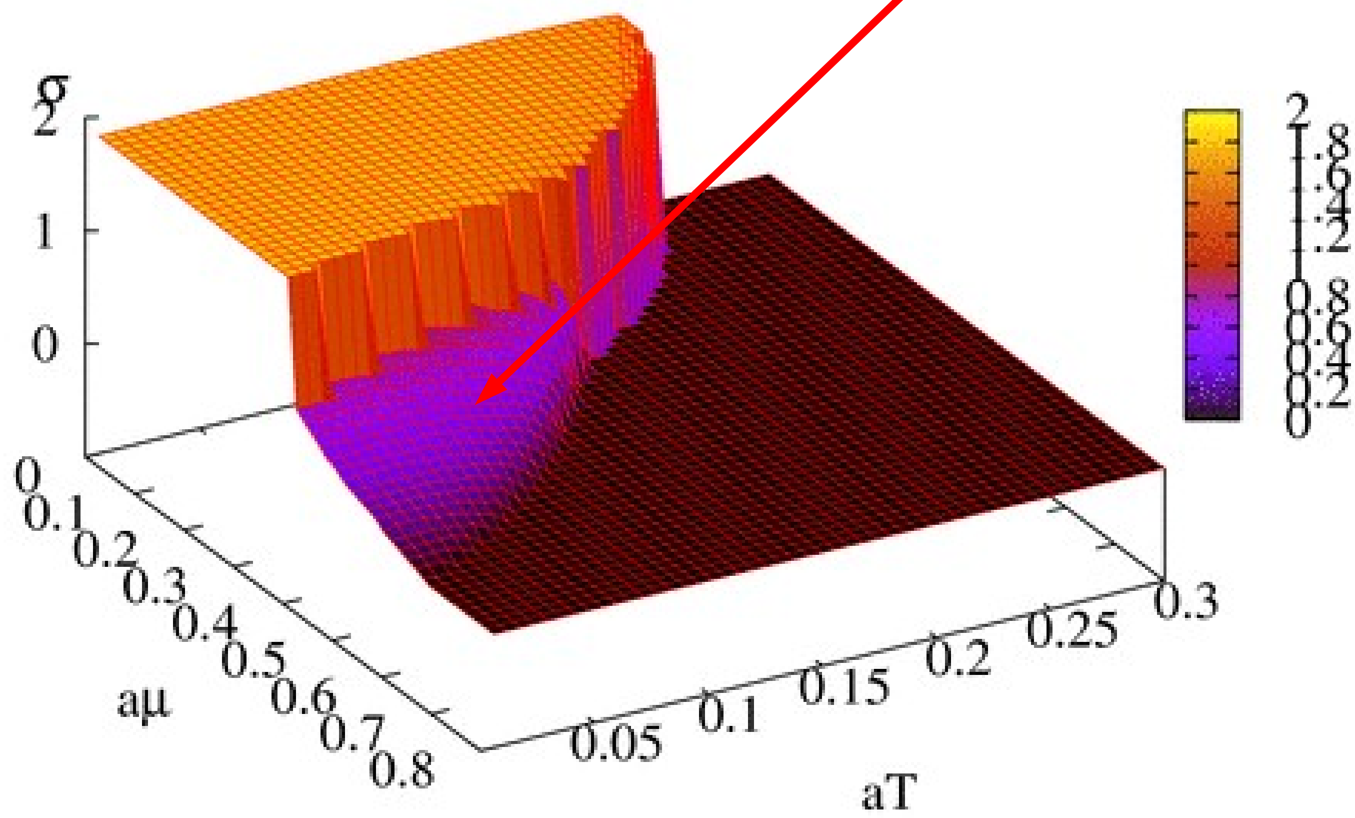
- **Baryon Density ($\beta_g = 4$)**



Quarkyonic Phase in SC-LQCD ?

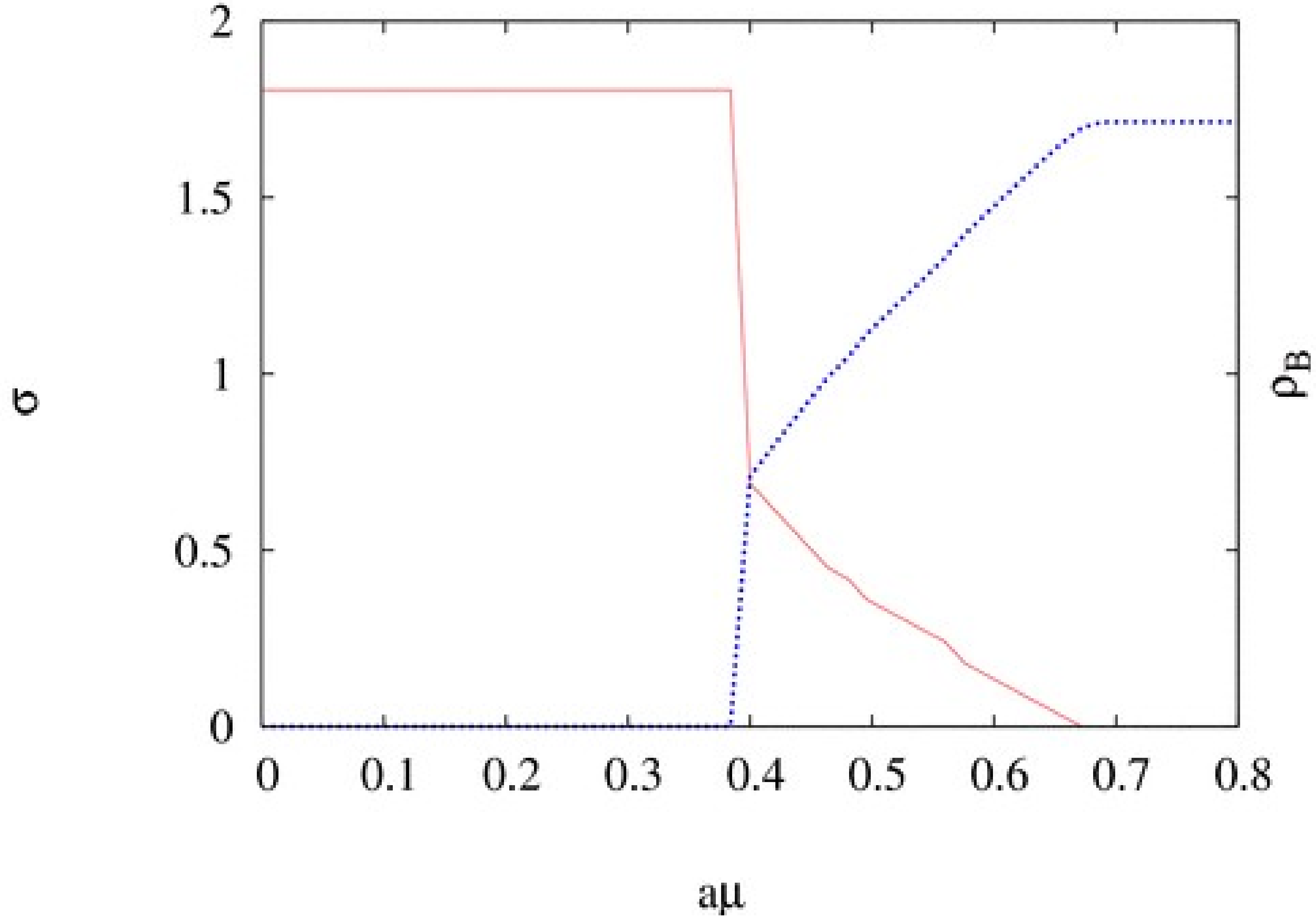
- **Chiral Condensate ($\beta_g = 4$)**

Chiral Sym. is partially restored and baryon density is high !



Quarkyonic Phase in SC-LQCD ?

- **Red: Chiral Condensate, Blue: Baryon Density**

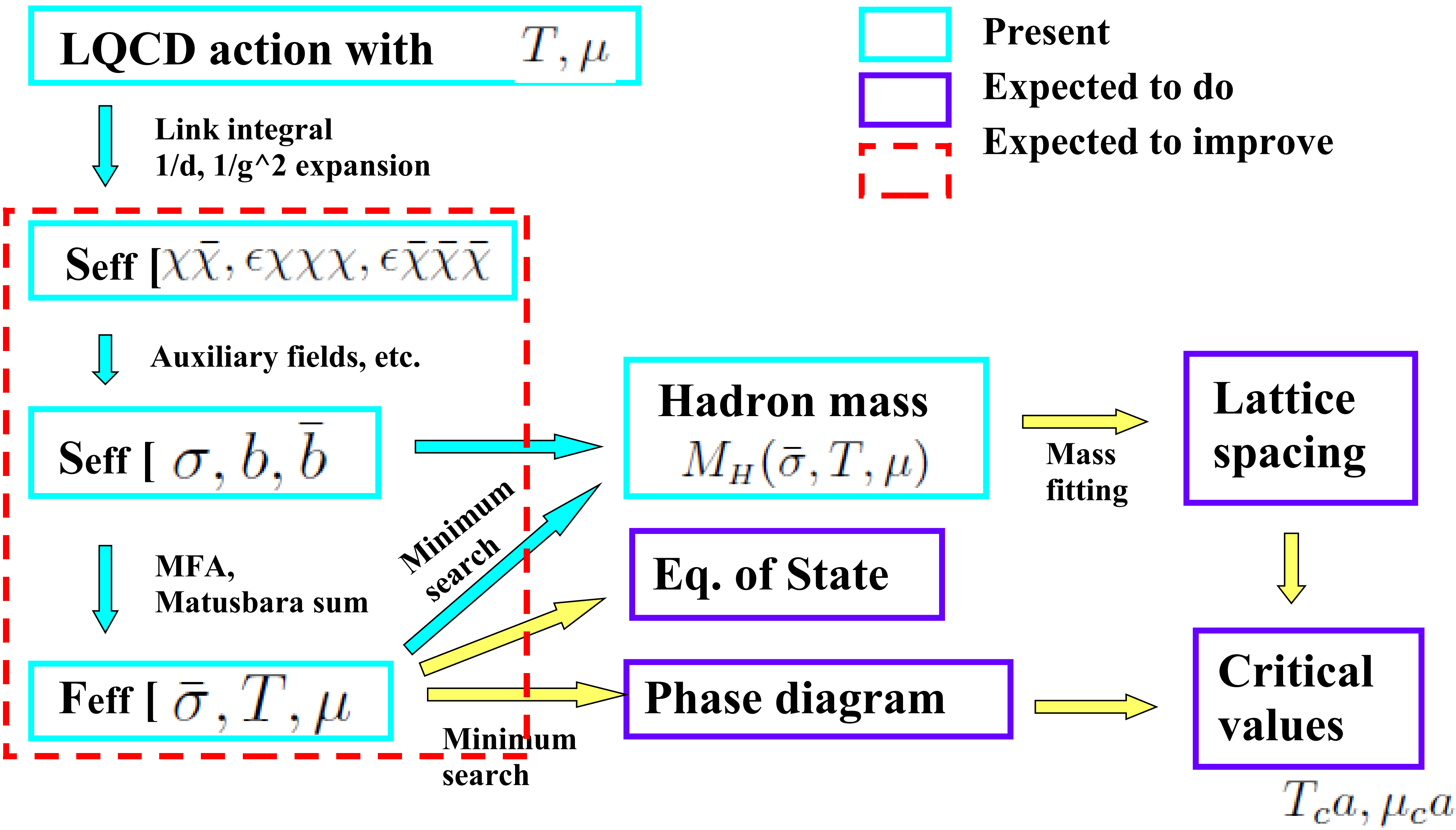


Quarkyonic Phase in SC-LQCD ?

- **Effective potential in the strong coupling lattice QCD including $1/g^2$ correction is evaluated.**
- **We have applied the saddle point approximation for auxiliary fields whose "equilibrium" value lies off the real axis.**
- **At finite coupling, Tc decreases rapidly while μ_c stays almost constant.**
- **At small T and large μ, we find a "Quarkyonic" phase, where the baryon density is high and the chiral condensate is significantly smaller than that in vacuum.**
  This phase appears mainly from the ϕ_t field, which behaves like a vector field in Rel. Mean Field (RMF) and modifies the effective chemical potential.

Backups

Strong coupling LQCD



Kohtaroh Miura, Talk in Lattice '07 on 070803

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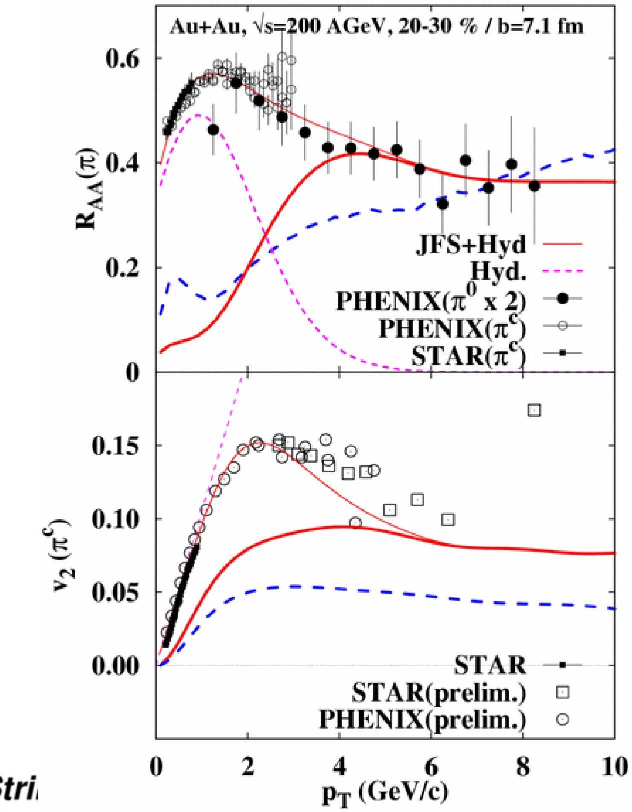
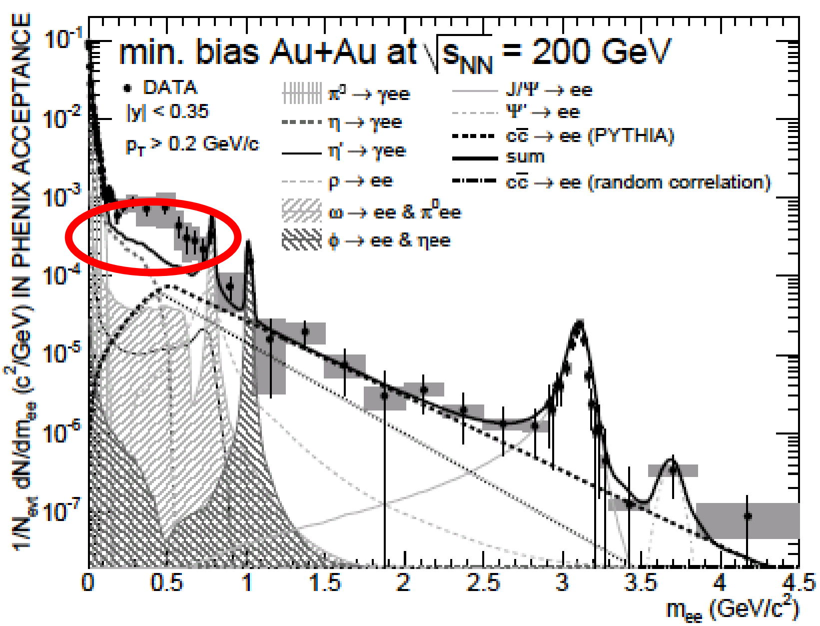
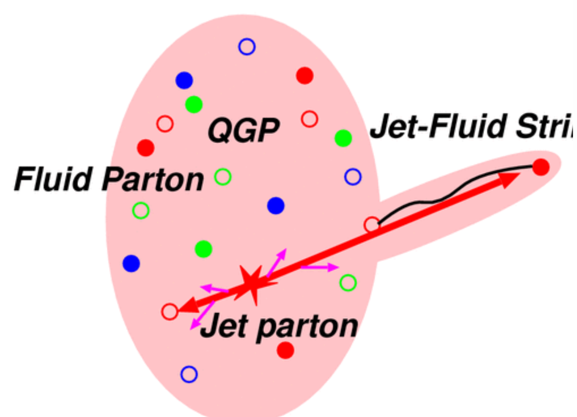
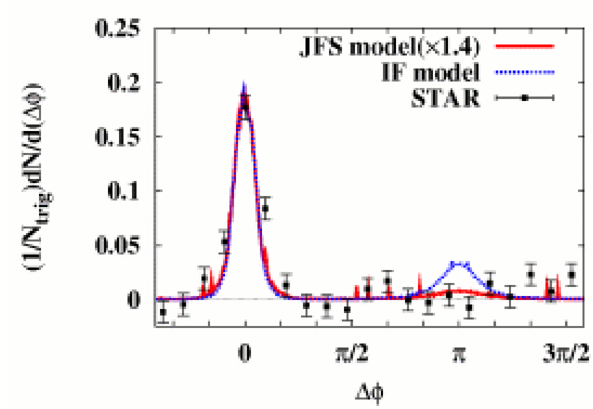
Hadron Mass in Nuclear Matter

- What kind of matter is created at RHIC ?
  - *Deconfined* quark and gluon matter
    → Jet quenching, Large v2,
    No backward h-h corr.
    Isse, Hirano, Nara, AO, Yoshino, nucl-th/0702068.
  - Chiral Restored Matter ?
    PHENIX Collab., arXiv:0706.3034





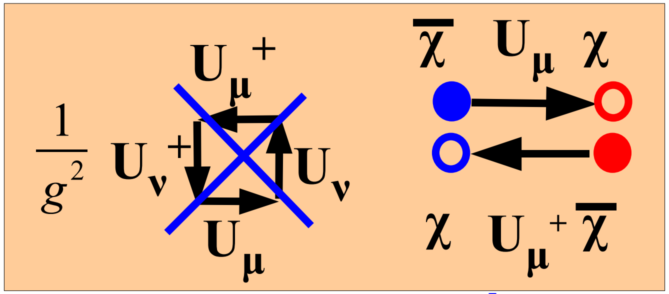




Hadron Mass in SCL-LQCD (Finite T)

- **QCD Lattice Action (Finite T treatment)**

Damgaard, Kawamoto, Shigemoto, 1984; Bilic et al., 1992; Nishida, 2003; Kawamoto, Miura, Ohnishi, Ohnuma, 2007; ...



$$S = \cancel{S_G} + S_F^{(s)} + S_F^{(t)} + m_0 \bar{\chi}\chi$$ **Strong Coupling Limit**

$$S_F^{(t)} = \frac{1}{2}\sum_x \left(e^{\mu}\bar{\chi}_x U_0(x)\chi_{x+\hat{0}} - e^{-\mu}\bar{\chi}_{x+\hat{0}} U_0^+(x)\chi_x\right) = \bar{\chi} V^{(t)} \chi$$

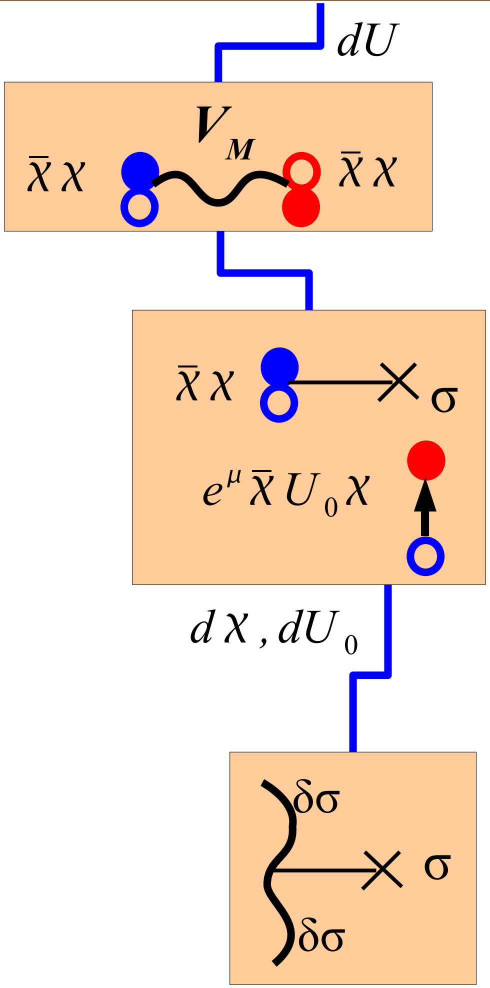
$$\to -\frac{1}{2}(\bar{\chi}\chi) V_M (\bar{\chi}\chi) + \bar{\chi}(V^{(t)} + m_q)\chi$$

Spatial-link integral (1/d expansion, no B)

$$\to \frac{1}{2}\sigma V_M^{-1}\sigma + \bar{\chi}(V^{(t)} + \sigma + m_q)\chi$$

Bosonization

$$\to L^d N_\tau \left[\frac{N_c}{d}\bar{\sigma}^2 + V_{\text{eff}}(\bar{\sigma}, T, \mu)\right] + \frac{1}{2}\sum_k G(k)^{-1}(\delta\sigma(k))^2$$



- **Effective Potential**

Fermion and Temporal-link Integral

$$F_{\text{eff}}^{(q)}(m; T, \mu) = -T \log \int dU_0 \det(V^{(t)} + m)$$

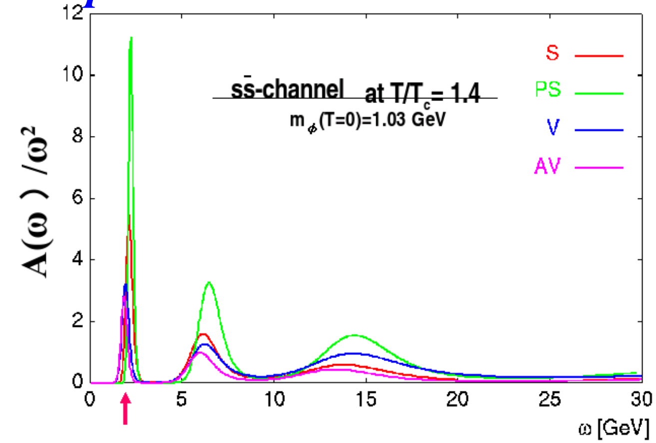
$$= -T \log \left[\frac{\sinh((N_c+1)E_q/T)}{\sinh(E_q/T)} + 2\cosh(N_c \mu/T)\right]$$

$$E_q(m) = \text{arcsinh}\, m$$

Hadron Mass Modification in Lattice QCD

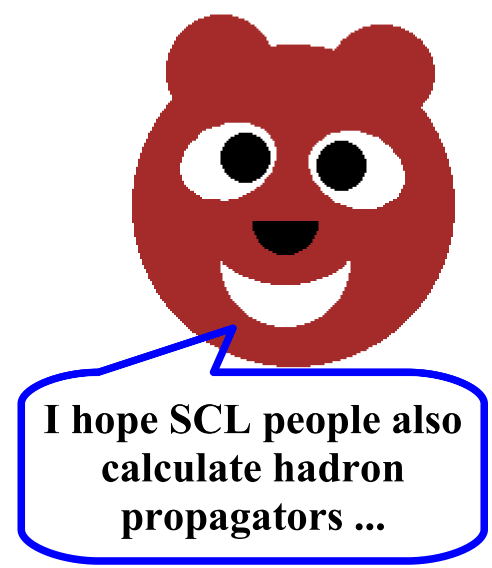
- **Can we understand it in Lattice QCD ?**
  - **Finite T: It is possible !**
  - **Finite μ: Difficult due to the sign problem.**

Asakawa, Nakahara, Hatsuda, hep-lat/0208059.

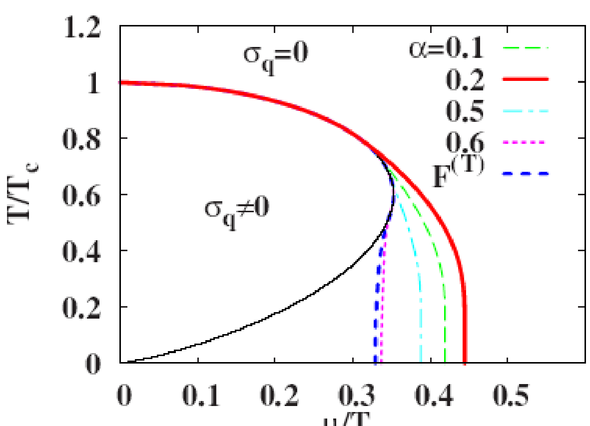


- **Strong Coupling Limit of Lattice QCD**
  → We can study finite (T, μ) !
  - **Hadron masses in the Zero T treatment**
    Kluberg-Stern, Morel, Petersson, '83; Kawamoto, Shigemoto, '82.
  - **To do: Finite T, Baryons with finite T, $1/g^2$ corr., ...**

JPS Symp., 2007/03
A. Nakamura said,

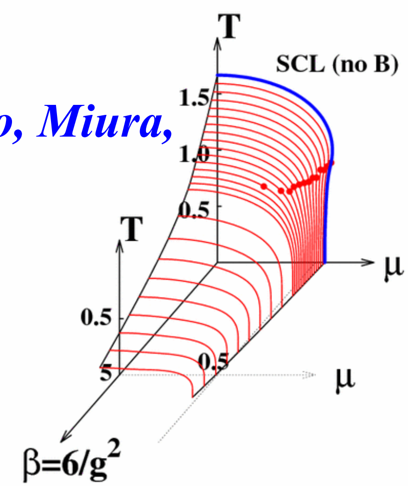


This Talk



Kawamoto, Miura, Ohnishi, Ohnuma, PRD75('07)014502

Ohnishi, Kawamoto, Miura, hep-lat/0701024



Hadron Mass in SCL-LQCD (Finite T)

- **Meson propagator at Finite T** *Faldt, Petersson, '86*
  - **U_0 integrated quark determinant = Function of X_N**
    X_N = Functional of m(τ)

$$F_{\rm eff}^{(q)}(m(x);T,\mu) = -T\log\int dU_0\, det_{\tau\tau'}(V^{(t)}+m(\boldsymbol{x},\tau)) = F_{\rm eff}^{(q)}(X_N[m])$$

$$X_N[I] = B_N(I_{1,..},\dots,I_N) + B_{N-2}(I_{2,..},\dots,I_{N-1})$$
$$(I_k = 2m(k) = 2(\sigma(k)+m_q))$$

$$B_N = \begin{vmatrix} I_1 & e^{\mu} & & & 0 \\ -e^{-\mu} & I_2 & e^{\mu} & & \\ 0 & -e^{-\mu} & I_3 & e^{\mu} & \\ \vdots & & & \ddots & \\ 0 & & & -e^{-\mu} & I_N \end{vmatrix}$$

  - **Derivatives**

$$\frac{\delta X_N}{\delta I_N} = B_{N-1}(I_{1,..},\dots,I_{N-1})$$

$$\frac{\delta B_N}{\delta I_k} = B_{k-1}(I_{1,..},\dots,I_{k-1})\,B_{N-k}(I_{k+1},\dots,I_N)$$

  - **Equilibrium Value**

$$B_N(I_k=\text{const.}) = \begin{cases} \cosh((N+1)E_q)/\cosh E_q & (\text{even } N) \\ \sinh((N+1)E_q)/\cosh E_q & (\text{odd } N) \end{cases}$$

SCL-LQCD: Tools (3) --- Bosonization

- **We can reduce the power in χ by introducing bosons**

$$\exp\left(\frac{1}{2}M^2\right)=\int d\sigma \exp\left(-\frac{1}{2}\sigma^2-\sigma M\right)$$

Nuclear MFA: $V=-\frac{1}{2}(\bar{\psi}\psi)(\bar{\psi}\psi)\simeq -U(\bar{\psi}\psi)+\frac{1}{2}U^2$

$$\exp\left[-\frac{1}{2}M^2\right]=\int d\varphi \exp\left[-\frac{1}{2}\varphi^2-i\varphi M\right]$$

Reduction of the power of χ
→ Bi-Linear form in χ → Fermion Determinant

SCL-LQCD: Tools (4) --- Grassman Integral

- **Bi-linear Fermion action leads to -log(det A) effective action**

$$\int d\chi\, d\bar{\chi} \exp\left[\bar{\chi} A \chi\right] = det\ A = \exp\left[-(-\log det\ A)\right]$$

$$\int d\chi \cdot 1 = \text{anti-comm. constant} = 0 \ , \quad \int d\chi \cdot \chi = \text{comm. constant} \equiv 1$$

$$\int d\chi\, d\bar{\chi} \exp\left[\bar{\chi} A \chi\right] = \int d\chi\, d\bar{\chi} \frac{1}{N!} (\bar{\chi} A \chi)^N = \cdots = det\ A$$

Constant σ → - log σ interaction (Chiral RMF)

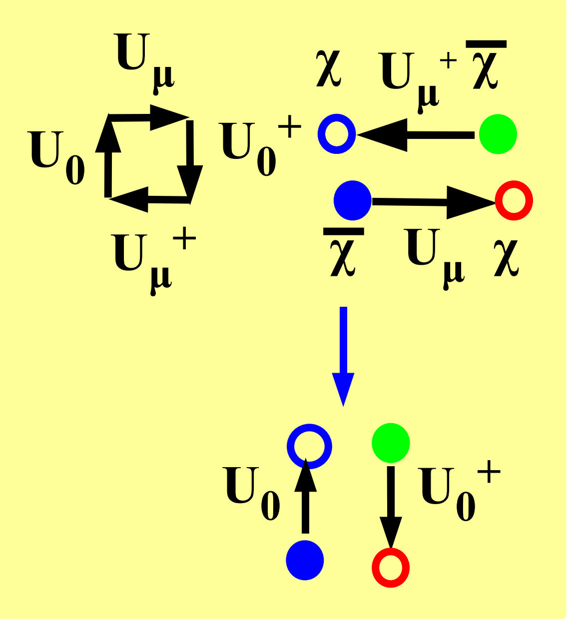
- **Temporal Link Integral, Matsubara product, Staggered Fermion,**

→ I will explain next time ....

$1/g^2$ expansion (w/o Baryon Effects)

- **T_c (μ=0) and μ_c (T=0): Which is worse ?**
  - **$1/g^2$ correction reduces T_c.** *(Bilic-Cleymans 1995)*
  - **Hadron masses are well explained in SCL.** *(Kawamoto-Smit 1981, Kawamoto-Shigemoto 1982)*

  → We expect Tc reduction with $1/g^2$ correction !



- **1/d expansion of plaquetts** *(Faldt-Petersson 1986)*
  - **Space-like plaquett**

$$\exp\left[\frac{1}{g^2}\sum_{x,i>j>0}\mathrm{Tr}\,U_{ij}(x)\right]\rightarrow\exp\left[\frac{1}{8N_c^4 g^2}\sum_{x,k>j>0}M_x M_{x+\hat{j}} M_{x+\hat{k}} M_{x+\hat{k}+\hat{j}}\right]$$

  - **Time-like plaquett**

$$\exp\left[\frac{1}{g^2}\sum_{x,j>0}\mathrm{Tr}\,U_{0j}(x)\right]\rightarrow\exp\left[-\frac{1}{4N_c^2 g^2}\sum_{x,j>0}\left(V_x V_{x+\hat{j}}^+ + V_x^+ V_{x+\hat{j}}\right)\right]$$

$$\left(V_x = \bar{\chi}_x U_0(x)\chi_{x+\hat{0}}\right)$$