Phase diagram and critical point evolution in NLO and NNLO strong coupling lattice QCD

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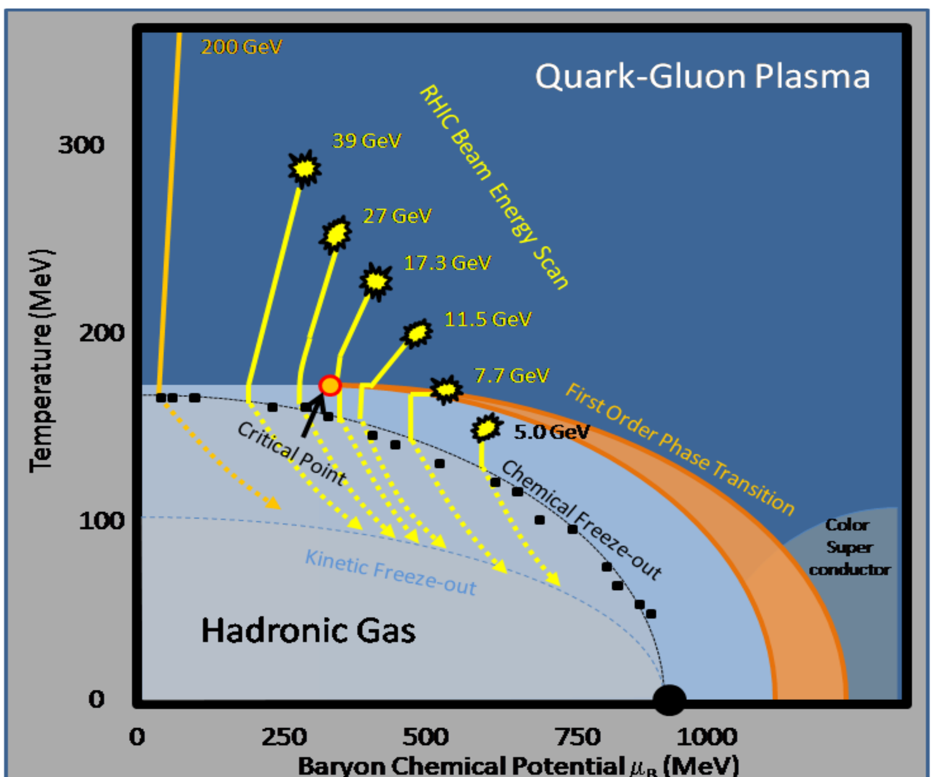
- **Introduction**
- **Effective Potential in NLO & NNLO SC-LQCD**
- **Phase Diagram and Critical Point Evolution**
- **Summary**

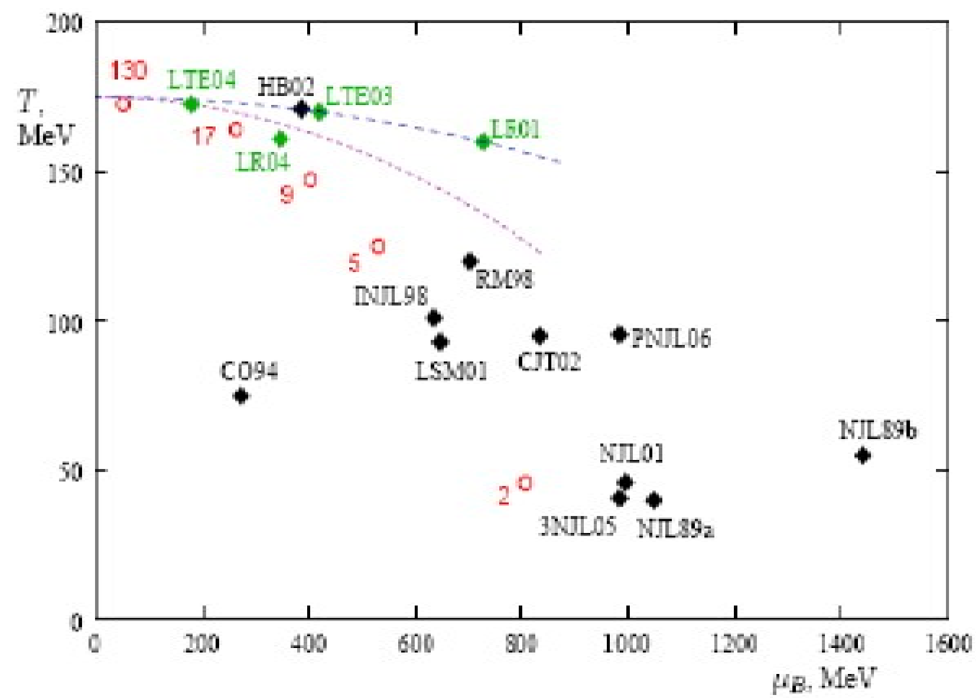
Miura, Nakano, AO, Prog. Theor. Phys., to appear [arXiv:0806.3357]
Miura, Nakano, AO, Kawamoto, arXiv:0907.4245
Nakano, Miura, AO, in prep.

Where is the Critical Point ?

- **Critical Point Search**
  = One of the main goals in Low-E progs. at RHIC
- **Theory → No Consensus (Sign prob. at finite μ)**

Can we attack CP in LQCD ? → Strong Coupling LQCD

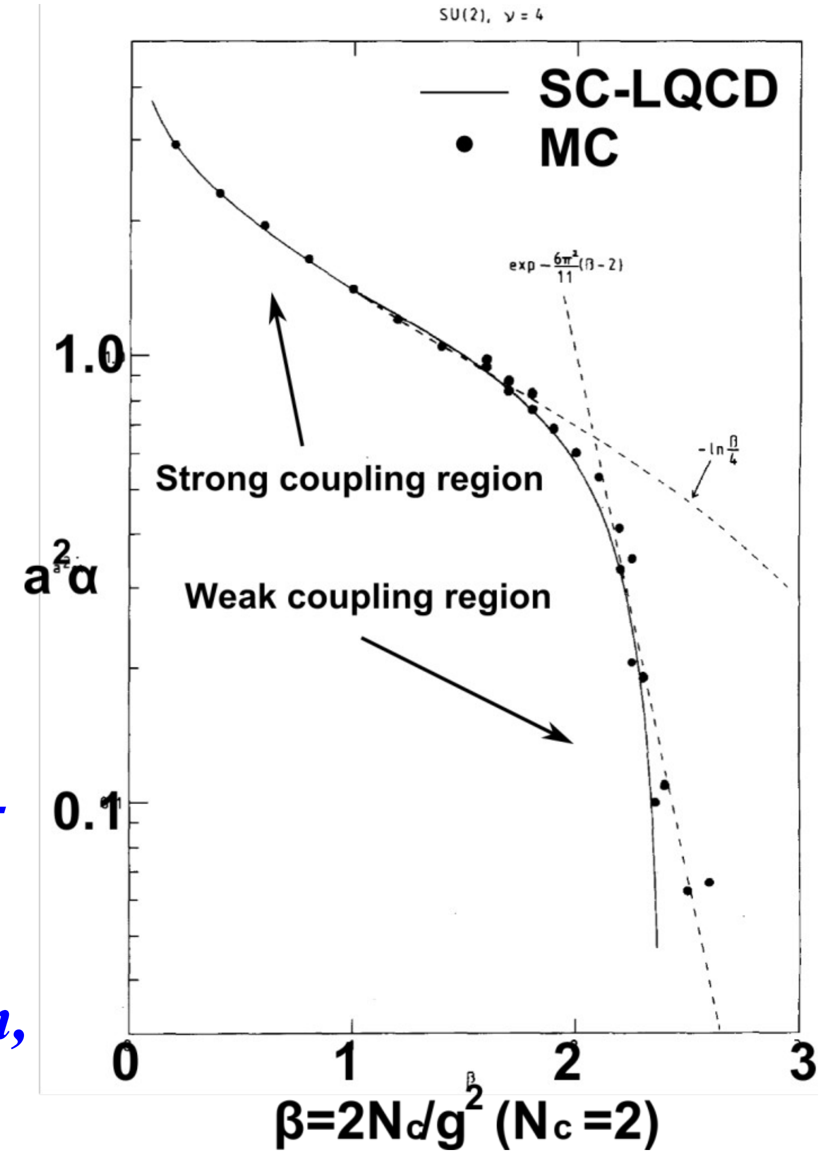




Strong Coupling Lattice QCD

- **Large bare coupling → $1/g^2$ expansion**
- **Success in pure YM**
  → Lattice MC & $1/g^2$ Expansion
  (Wilson, '74; Creutz '80; Munster '81)

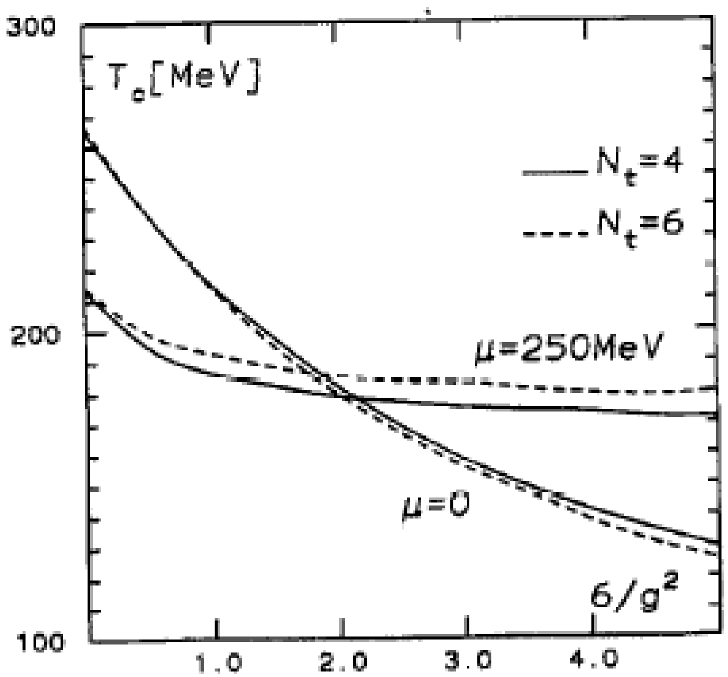
  → *Scaling region would be accessible in SC-LQCD !*
- **Chiral transition at finite T and μ in Strong Coupl. Limit (SCL) & Next-to-leading order (NLO)**
  Kawamoto-Smit '81, Damgaard-Kawamoto-Shigemoto, '84(U(3)), Faldt-Petersson'86 (SU(3)), Fukushima'04(SU(3)), Nishida '04 (SU(3)), Bilic-Karsch-Petersson, '92(NLO), Kawamoto-Miura-AO-Ohnuma '07 (Baryons)



Munster, '81

Strong Coupling Lattice QCD with Fermions

- **SC-LQCD with fermions**
  - **SCL & NLO: Far from "scaling" behavior.**
  - **$\beta=2N_c/g^2$ dep. of the critical point is not studied yet.**
  - **Condensates other than σ are not yet included in previous works.** *(Faldt-Petersson '86; Bilic-Karsch-Redlich '92; Bilic-Demeterfi-Petersson '92; Bilic-Claymans '95)*



Bilic-Claymans '95

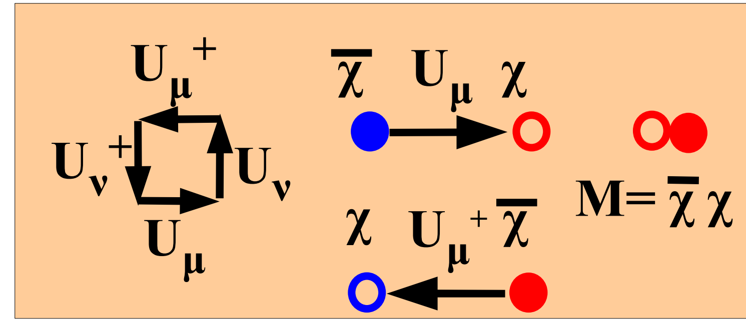
In this work, we revisit / develop NLO & NNLO SC-LQCD with fermions at finite T and μ and Study Phase diagram & critical point evolution with β

NLO & NNLO SC-LQCD: Setups & Disclaimer

- **Present setups in strong coupling LQCD**
  - **Effective action in SCL ($1/g^0$), NLO ($1/g^2$), NNLO ($1/g^4$) terms**
  - **One species of unrooted staggered fermion (N_f=4)**
  - **Leading order in 1/d expansion (d=3=space dim.)**
  - **Effective potential is obtained in mean field approximation**
- **Disclaimer**
  - **Polyakov loop effects are not included.**
    → Pure "deconfinement" transition can not be described.
  - **Different from "strong couling" in "large N_c"**
    SC-LQCD: Large bare coupling, $g >> 1$
    Large N_c: $N_c >> 1$, fixed $\lambda = N_c\, g^2$
    Strong Coupling in Large Nc: $Nc >> 1, \lambda >> 1, g << 1$

Effective Potential in NLO and NNLO Strong Coupling Lattice QCD

SC-LQCD with fermions at finite T (Outline)

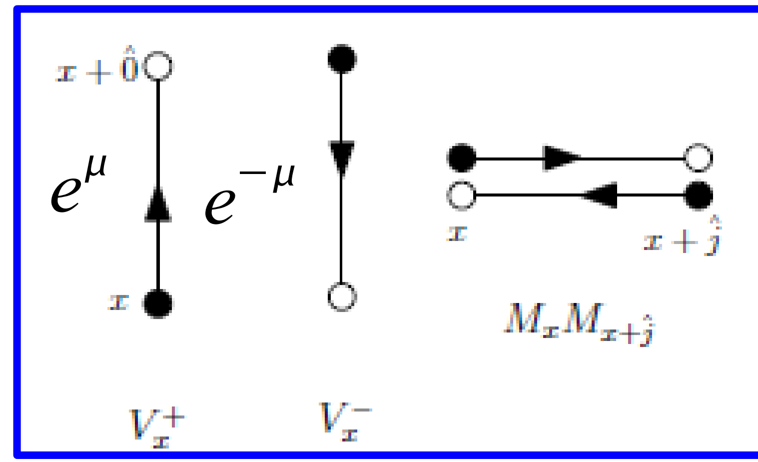


- **Lattice QCD action**

$$S_{\rm LQCD} = S_F^{(\tau)} + \sum_x m_0 M_x + S_F^{(s)} + S_G$$

- **Effective Action**
  (U_j integral + 1/d expansion)

$$S_{\rm eff} = \frac{1}{2}\sum_x \left[V_x^+(\mu) - V_x^-(\mu)\right] + m_0 \sum_x M_x - \frac{1}{4N_c}\sum_{x,j>0} M_x M_{x+\hat{j}} + \Delta S_{\rm eff}$$



- **Effective Potential**
  (Bosonization + χ & U_0 integral)

$$S_{\rm eff}^{(F)} = \sum_x \frac{1}{2}\left(V_x^+ - V_x^-\right) + m_q M_x$$

$$\mathcal{V}_q(m_q;\mu,T) = -T\log\left[X_{N_c}(E_q(m_q)/T) + 2\cosh(N_c\tilde{\mu}/T)\right]$$

$$X_N(x) = \sinh[(N+1)x]/\sinh x\ , \quad E_q(m_q) = \mathrm{arcsinh}\,(m_q)$$

Strong Coupling & Cluster Expansion

- **Cumulant (Connected Cluster) Expansion** ***(E.g., R. Kubo, 1962)***

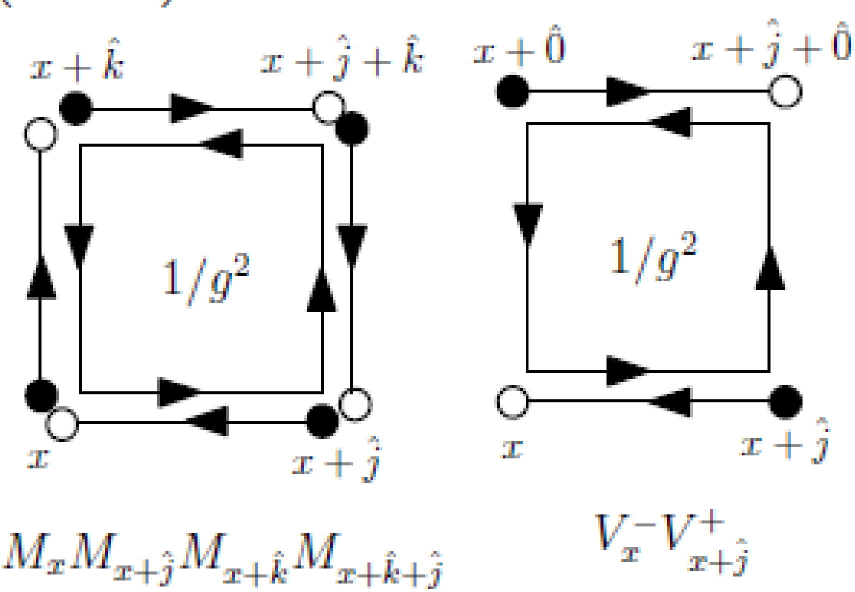
$$\langle \mathcal{O} \rangle = \frac{1}{Z_{\mathrm{SCL}}^{(s)}} \int \mathcal{D}U_j \; \mathcal{O}[U_j] \, e^{-S_F^{(s)}} \qquad Z_{\mathrm{SCL}}^{(s)} = \int \mathcal{D}U_j \; e^{-S_F^{(s)}}$$

$$\left\langle e^{-S_G} \right\rangle = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \left\langle S_G{}^n \right\rangle = \exp\left[\sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \left\langle S_G^n \right\rangle_c \right]$$

- **Next-to-leading order ($1/g^2$) = Cumulants of plaquettes**

$$\left\langle U_{jk,x} \right\rangle_c = \frac{1}{16 N_c^4} M_x M_{x+\hat{j}} M_{x+\hat{k}} M_{x+\hat{k}+\hat{j}} + \mathcal{O}\left(d^{-5/2}\right)$$

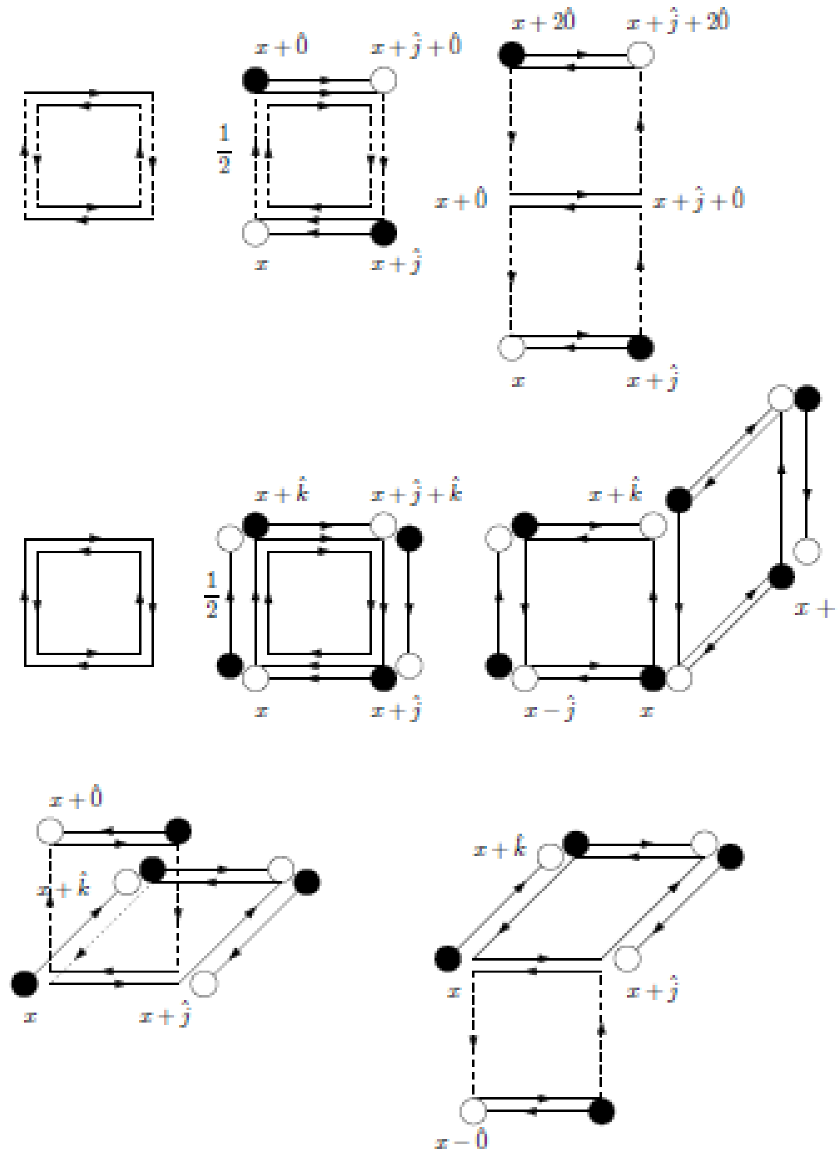
$$\left\langle U_{j0,x} \right\rangle_c = -\frac{1}{4 N_c^2} V_x^-(\mu) V_{x+\hat{j}}^+(\mu) + \mathcal{O}\left(d^{-3/2}\right)$$



Faldt-Petersson'86; Bilic-Karsch-Petersson '92; Miura, Nakano, AO '08; Miura, Nakano, AO, Kawamoto '09

NNLO Effective Action

- **Cumulants of two plaquettes**
  = Correlation part of connected two plaquettes



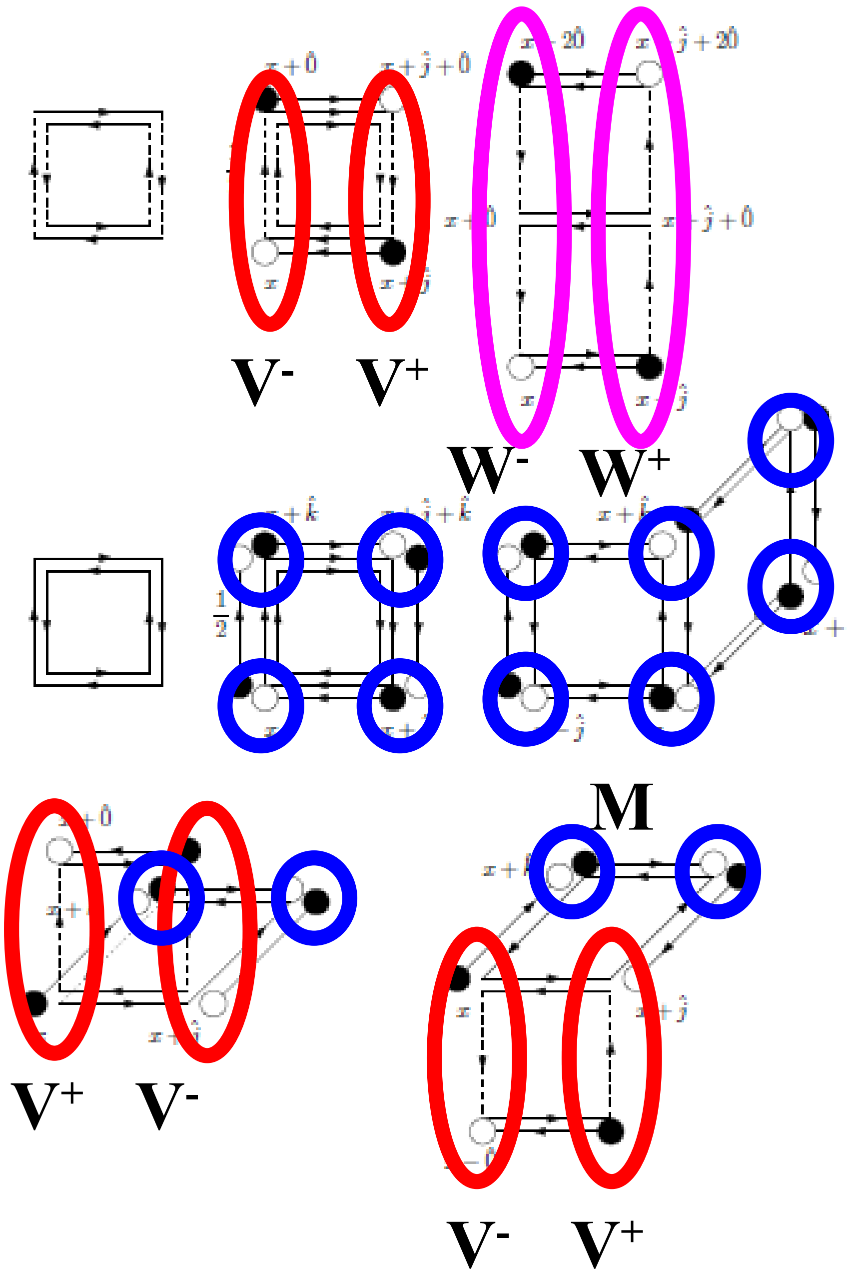
NNLO Effective Action

- **Cumulants of two plaquettes**
  = Correlation part of connected two plaquettes
  - **Uncorr. & Normalization part are suppressed in 1/d power**
  - **Effective Action consists of**

    $$V^- V^+ , W^- W^+ ,$$
    $$MMMM , MMMMMM ,$$
    $$V^- V^+ MM$$

  - **New type of Composite**
    = next-to-nearest neighboring site coupling in τ direction

$$W_x^+ = \chi_x U_{0,x} U_{0,x+\hat{0}} \bar{\chi}_{x+2\hat{0}}$$



Effective Action in NNLO SC-LQCD

Miura, Nakano, AO, Kawamoto, arXiv:0907.4245;
Nakano, Miura, AO, in prep.

$$S_{\rm NNLO} = S_{\rm SCL} + \Delta S^{\tau} + \Delta S^{s} + \Delta S^{\tau\tau} + \Delta S^{ss} + \Delta S^{\tau s}$$

$$= \frac{1}{2}\sum_{x}(V_x^+ - V_x^-) - \frac{b_\sigma}{2d}\sum_{x,j>0}[MM]_{j,x}$$

SCL

$$+\frac{1}{2}\frac{\beta_\tau}{2d}\sum_{x,j>0}[V^+V^- + V^-V^+]_{j,x}$$

$$-\frac{1}{2}\frac{\beta_s}{d(d-1)}\sum_{x,j>0,k>0,k\neq j}[MMMM]_{jk,x}$$

NLO

NNLO

$$-\frac{\beta_{\tau\tau}}{2d}\sum_{x,j>0}[W^+W^- + W^-W^+]_{j,x}$$

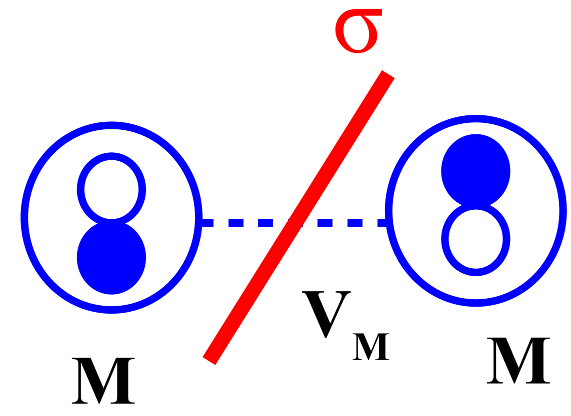
$$-\frac{\beta_{ss}}{4d(d-1)(d-2)}\sum_{x,\,j>0,\,|k|>0,\,|l|>0}[MMMM]_{jk,x}[MM]_{j,x+\hat{l}}$$

$$+\frac{\beta_{\tau s}}{8d(d-1)}\sum_{x,j>0,|k|\neq j}[V^+V^- + V^-V^+]_{j,x}\left([MM]_{j,x+\hat{k}} + [MM]_{j,x+\hat{k}+\hat{0}}\right)$$

Bosonization & Effective Potential

- **Hubbard-Stratonovich transformation**

$$\exp\left[\frac{1}{2} M\, V_M\, M\right] \approx \exp\left[-\frac{1}{2}\sigma\, V_M\, \sigma - \sigma\, V_M\, M\right]$$

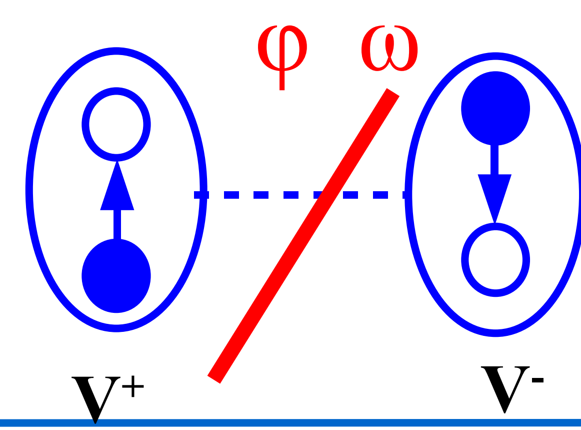


→ applies to the product of same kind

- **Extended Hubbard-Stratonovich transformation**

$$e^{\alpha AB} = \int d\varphi\, d\phi\, e^{-\alpha\left\{(\varphi-(A+B)/2)^2+(\phi-i(A-B)/2)^2\right\}+\alpha AB}$$
$$\approx\ e^{-\alpha\{\varphi^2-(A+B)\varphi-\omega^2+(A-B)\omega\}}\Big|_{\text{stationary}}$$

→ applies also to product of diff. kind



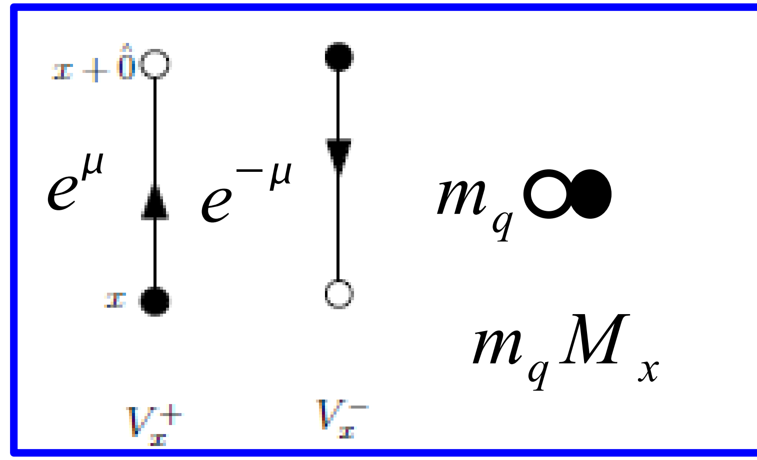
Miura, Nakano, AO, '08
Miura, Nakano, AO, Kawamoto, '09

Effective Potential

- **Bosonization + Dressed Fermion → Modification of** ***Mass, Chemical pot., and W.F. renormalization factor*** **in the Strong Coupling Limit action**

$$S_{\text{eff}}^{(F)} = Z_\chi \left[\sum_{x,y} \frac{1}{2} \left[e^{-\delta\mu} V_x^+ - e^{\delta\mu} V_x^- \right] + \sum_x m_q M_x \right]$$

$$= Z_\chi \sum_{\mathbf{x},n,m} \bar{\chi}_{\mathbf{x},n} G_{nm}^{-1}(m_q; \tilde{\mu}, T) \chi_{\mathbf{x},m}$$



- **Spatially decomposed action**
  → **Quark & Temporal Link Integral in Polyakov loop**
  → **Effective potential**

$$\mathcal{V}_q(m_q; \mu, T) \equiv -\frac{1}{N_\tau L^d} \log \left[\int dU_0 \det \left(G_{nm}^{-1}(m_q; \tilde{\mu}, T) \right) \right]$$

$$= -T \log \left[X_{N_c}(E_q(m_q)/T) + 2\cosh(N_c \tilde{\mu}/T) \right]$$

$$X_N(x) = \sinh[(N+1)x]/\sinh x \ , \quad E_q(m_q) = \text{arcsinh}\,(m_q)$$

Effective Potential

- **Effective Potential in NLO SC-LQCD**
  Miura, Nakano, AO, '08; Miura, Nakano, AO, Kawamoto, '09

$$\mathcal{F}_{\text{eff}}(\Phi;T,\mu) = \mathcal{F}_{\text{aux}}(\Phi) + \mathcal{V}_q(\tilde{m}_q(\Phi);T,\tilde{\mu})$$

$$\mathcal{F}_{\text{aux}}(\Phi) = \frac{\tilde{b}_\sigma \sigma^2}{2} + \frac{\beta_s \varphi_s^2}{2} + \frac{\beta_\tau}{2}(\varphi_\tau^2 - \omega_\tau^2) - N_c \log Z_\chi$$

- **Feff in NNLO SC-LQCD** *(Nakano,Miura,AO, in prep.)*

$$\begin{aligned}
\mathcal{F}_{\text{eff}} =& \mathcal{F}_{\text{eff}}^{(X)} + \mathcal{V}_q(m_q;\tilde{\mu},T) - N_c \log Z_\chi \ , \\
\mathcal{F}_{\text{eff}}^{(X)} =& \frac{1}{2}\tilde{b}_\sigma\,\sigma^2 + \frac{1}{2}\beta_s\sigma^4 + 2\beta_{ss}\sigma^6 + \frac{1}{2}(\beta_\tau + 2\beta_{\tau s}\sigma^2)(\varphi_\tau^2 - \omega_\tau^2) \\
& + \beta_{\tau\tau}(4(Z_\chi m_q)^2(\varphi_\tau^2 - \omega_\tau^2) - 4Z_\chi m_q \varphi_\tau \sigma + \sigma^2) + \beta_{\tau s}\sigma^2(\varphi_\tau^2 - \omega_\tau^2) \ , \\
m_q =& \frac{\tilde{b}_\sigma \sigma + m_0 - 2\beta_{\tau\tau}(2m'\varphi_\tau - \sigma)}{Z_\chi} = \frac{\tilde{b}_\sigma\sigma + m_0 + 2\beta_{\tau\tau}\sigma}{(1 + 4\beta_{\tau\tau}\varphi_\tau)Z_\chi} \ , \\
Z_\pm =& 1 + (\beta_\tau + 2\beta_{\tau s}\sigma^2)(\varphi_\tau \pm \omega_\tau) + 4\beta_{\tau\tau} Z_\chi m_q (2Z_\chi m_q(\varphi_\tau \pm \omega_\tau) - \sigma) \ , \\
\tilde{b}_\sigma =& b_\sigma + 2\beta_s\sigma^2 + 6\beta_{ss}\sigma^4 + 2\beta_{\tau s}(\varphi_\tau^2 - \omega_\tau^2) \ .
\end{aligned}$$

Phase Diagram and Critical Point Evolution in NLO and NNLO SC-LQCD

Stationary Condition --- Multi-Order Parameter

- **Stationary Condition** $\frac{\partial \mathcal{F}_{\mathrm{eff}}}{\partial \Phi} = 0$

  Φ (4(NLO) / 10 (NNLO) aux. field) $\to (\sigma, \omega_\tau)$

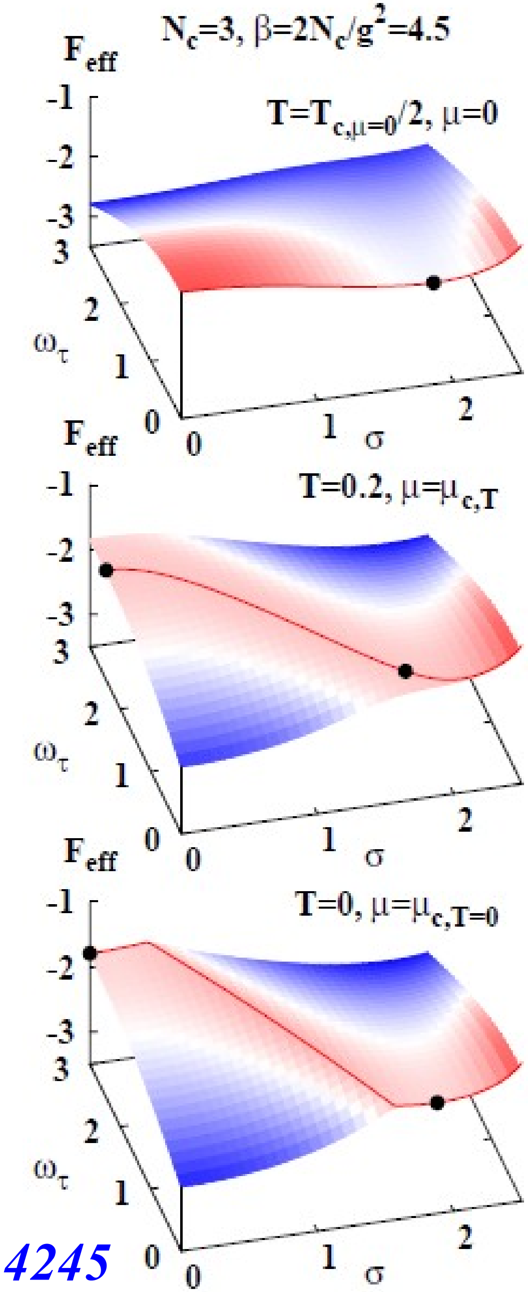
- **Multi-Order Parameter (σ, ω_τ)**

  $$\sigma \approx -\frac{\partial F_{\mathrm{eff}}}{\partial m_0} = \text{Chiral Cond.}$$

  $$\omega \approx -\frac{\partial F_{\mathrm{eff}}}{\partial \mu} = \text{Quark number density}$$

  - **Two indep. var. in V_q(m, μ)**
  - **Scalar (σ) and Vector (ω) potential for Quarks**

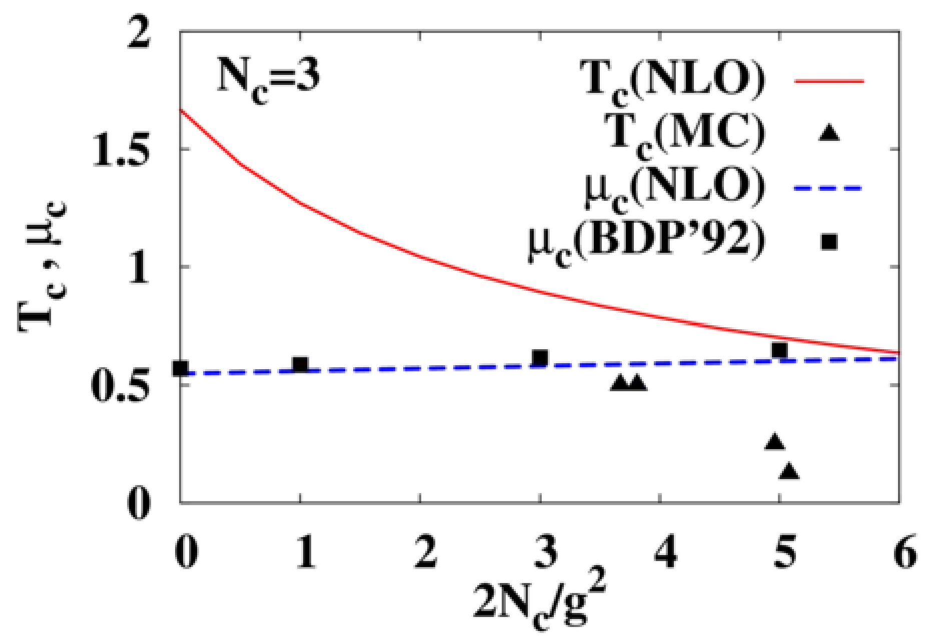
→ Saddle point in Feff(σ, ω_τ)



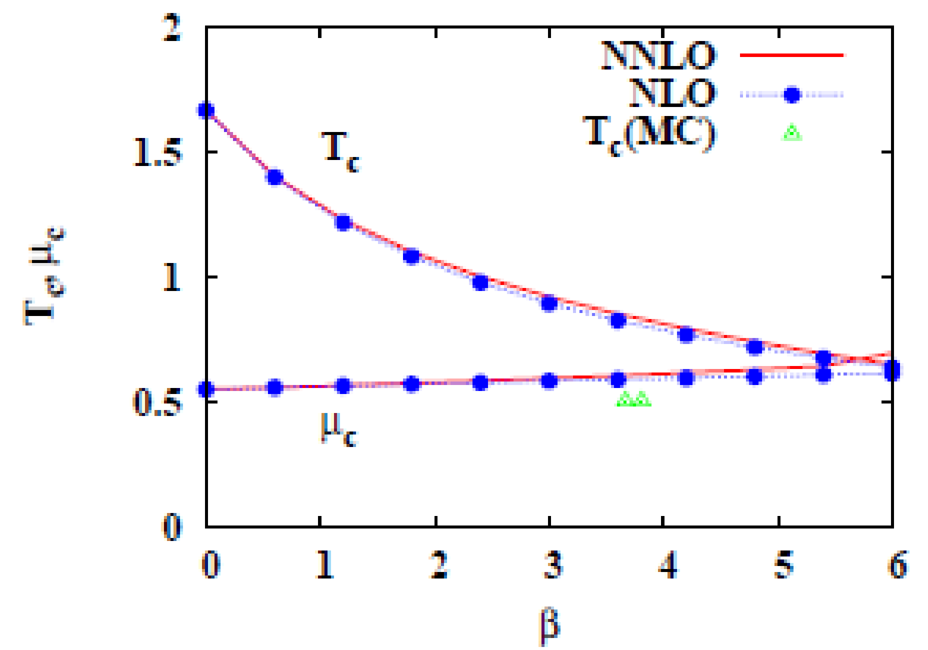
Miura, Nakano, AO, Kawamoto, arXiv:0907.4245

Critical Temperature and Chemical Potential

- $T_c(\mu = 0)$
  → rapid decrease with $\beta = 2N_c/g^2$
  - W.F. Renom. factor Z_χ
    → suppression of mass
  - Larger than MC results
    (de Forcrand '06; Gottlieb et al. '87; Gavai et al. '90)



- μ_c (T=0)
  → small deps. on β
  - Suppression of mass
    ~ Suppression of μ
  - Consistent with previous results
    Bilic-Demeterfi-Petersson, '92
- NNLO ~ NLO



Phase Diagram Evolution

- **Shape of the phase diagram is suppressed in T direction with β**

  → Improvements !

  - **Real world value:**

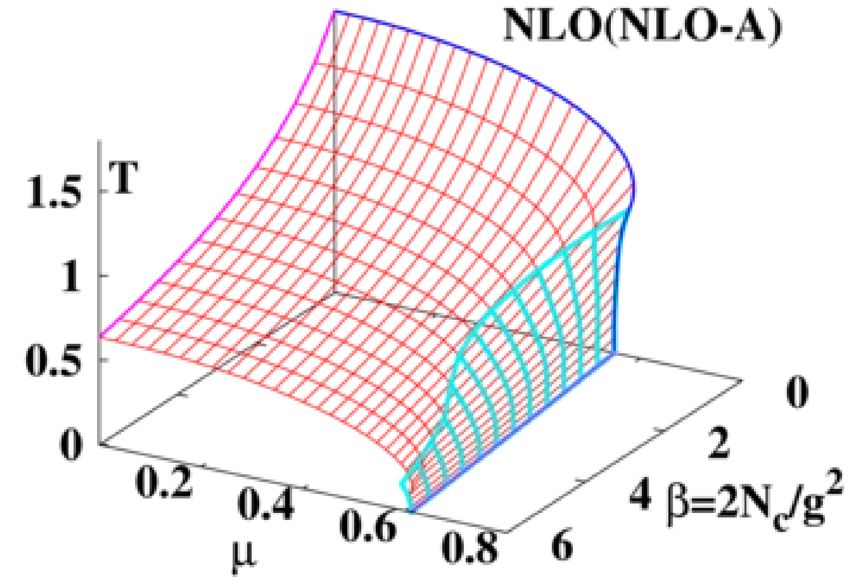
    T_c ~ (160-200) MeV

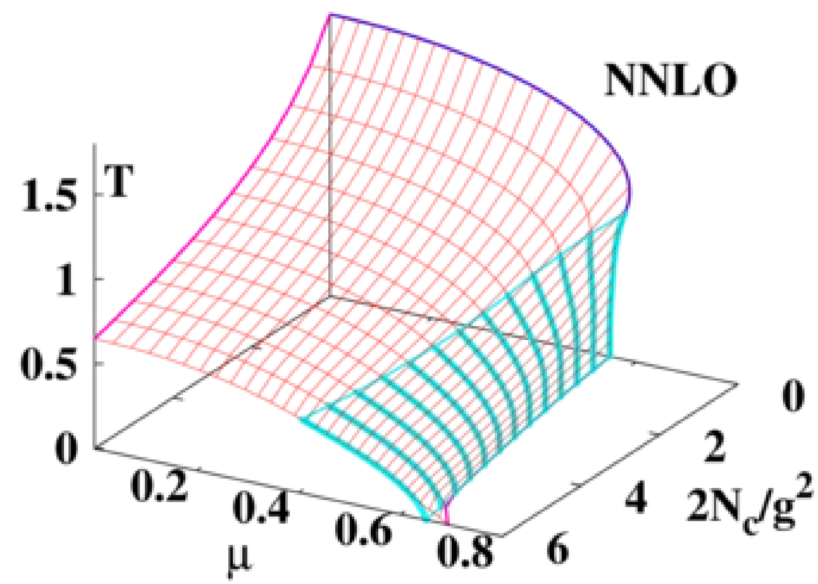
    μ_c > 350 MeV (nuclear matter)

    $R = \mu_c / T_c$ ~ (1.5-3)

  - **MC → R > 1**
  - **SCL → R ~ (0.3-0.45)**
  - **N(N)LO → R ~ 1**

- **First order P.T. boundary**
  - **NLO: μ(CP) ~ Const.**
  - **NNLO: μ(CP) decreases with β**

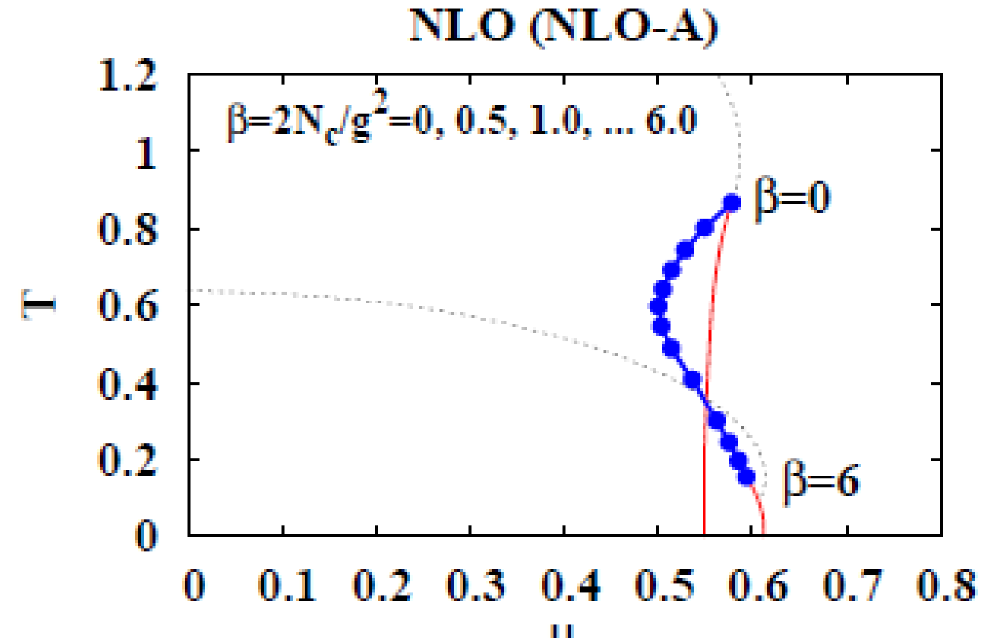




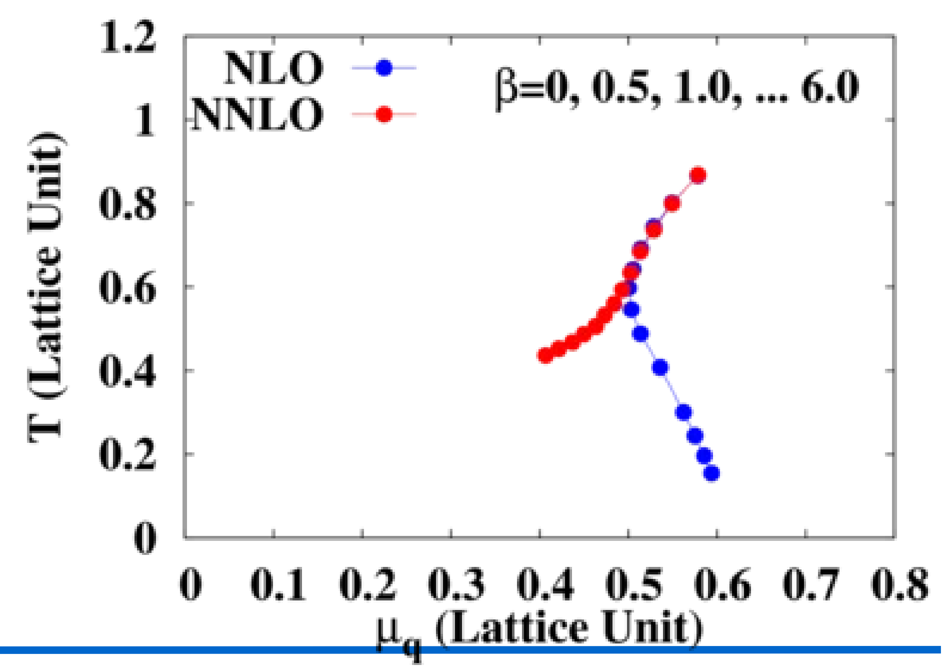
Critical Point Evolution

- **Critical Point in NLO approaches μ axis**
  - Larger β → Stronger Vector Pot. ω_τ
  - Consistent with NJL models. (Kitazawa et al., '02; Sasaki-Friman-Redlich, '07; Fukushima'08)

    and MC suggestion (de Forcrand-Philipsen, '08)
- **CP in NNLO→ μ(CP)/T(CP) ~ 1**
  - Contradict to MC (μ/T > 1) ? (Ejiri, '08; Aoki et al.(WHOT), '08; Allton et al., '03,'05)
  - Underestimate of Tc may be the reason



Critical Point in SC-LQCD



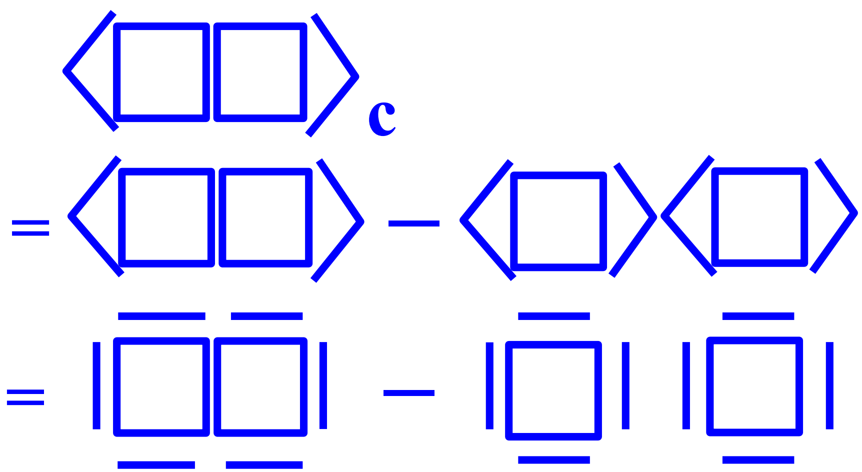
Summary & Conclusion

- **We have derived the effective potential with *next-to-leading order (NLO, $1/g^2$)* and *next-to-next-to-leading order (NNLO, $1/g^4$)* effects in strong coupling lattice QCD.**
  - **Several auxiliary fields including chiral condensate (σ) and quark number density (ρ_q) are introduced on the same footing.**
    [scalar (σ) and vector ($\rho_q \to \omega_\tau$) potential for quarks]
  - **NLO & NNLO effects are found to modify the quark mass, dynamical chemical potential, and W.F. renormalization factor.**
- **NLO and NNLO effects seems to be favorable.**
  - **$T_c(\mu=0)$ decreases from 1.6 (SCL) to around 0.5 (NLO, NNLO), and give closer value to MC, but it is still larger.**
  - **μ_c (T=0) is rather stable, showing smaller effects of gluons at low T.**
  - **Critical point moves in the lower T direction.**
- **Further studies incl. Polyakov loop, 1/d, meson fluc. are necessary**

Backup

NNLO Effective Action

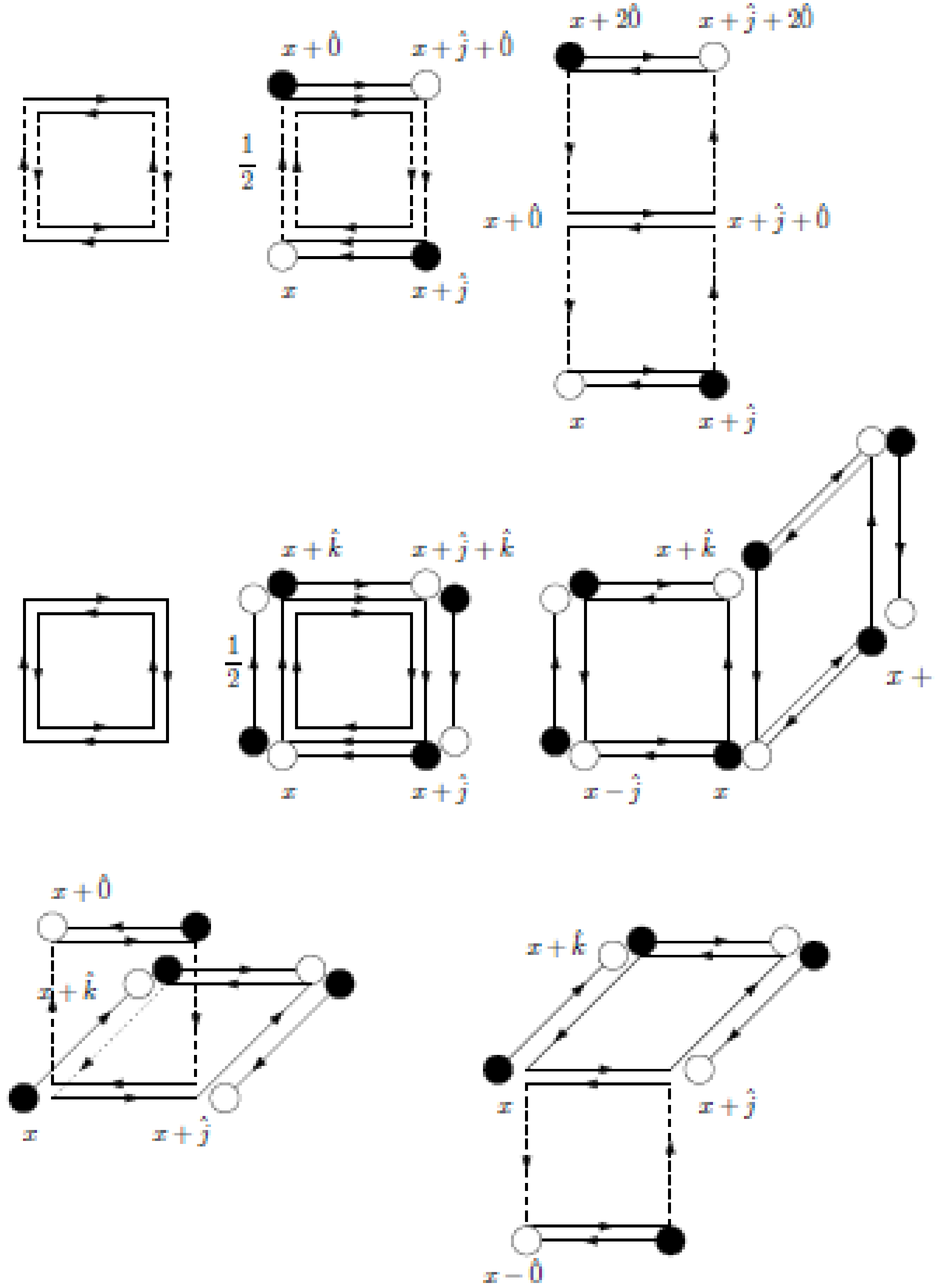
- Cumulants of two plaquettes
  = Correlation part of connected two plaquettes
- 1/d expansion: Σ_j MM ~ Const.
  $\rightarrow \chi \sim d^{-1/4}$



12 quarks　　16 quarks

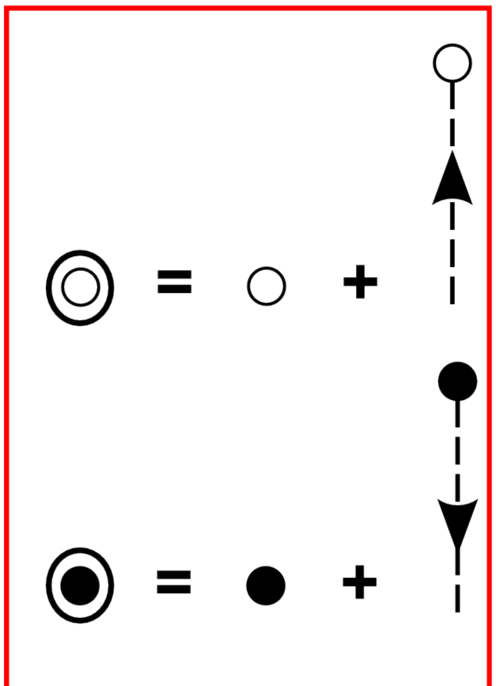
power in d

$3 - 1/4 \times 12 = 0$　　$3 - 1/4 \times 16 = -1$



Dressed fermion

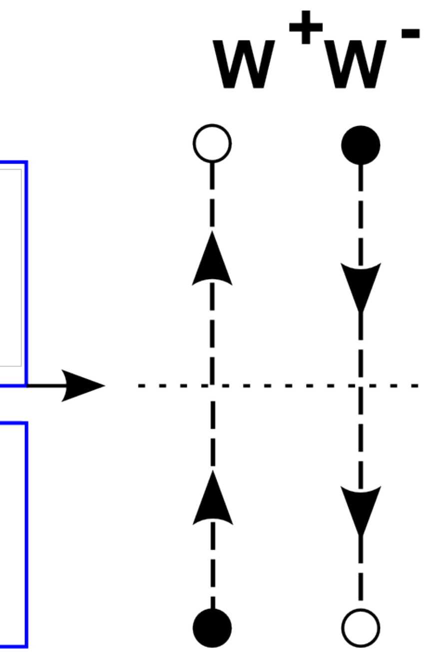
- **Next-to-nearest neighboring site interaction $W^{\pm}$.**
  - **By introducing the "Dressed Fermion", mixture of the quark field on the next temporal site, NNN interaction is rearranged to NN.**



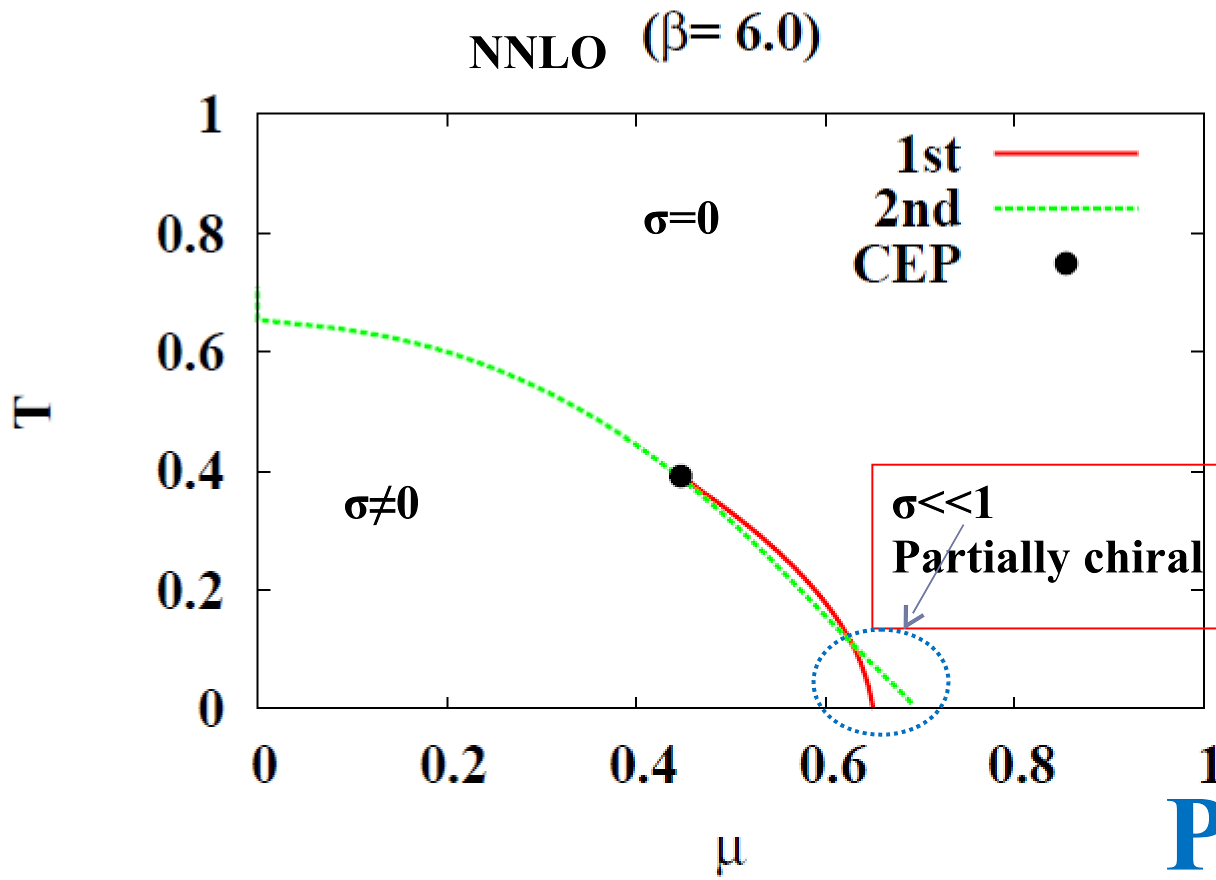
$$\chi'_x = \chi_x + A e^{\mu} U_{0,x} \chi_{x+\hat{0}}$$

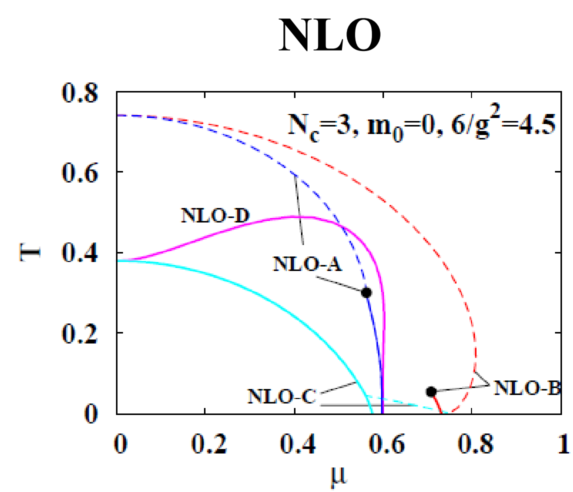
$$\bar{\chi}'_x = \bar{\chi}_x + \bar{A} \bar{\chi}_{x+\hat{0}} e^{-\mu} U^{\dagger}_{0,x}$$

$$A = \mathcal{O}(1/g^4)$$



Phase diagram



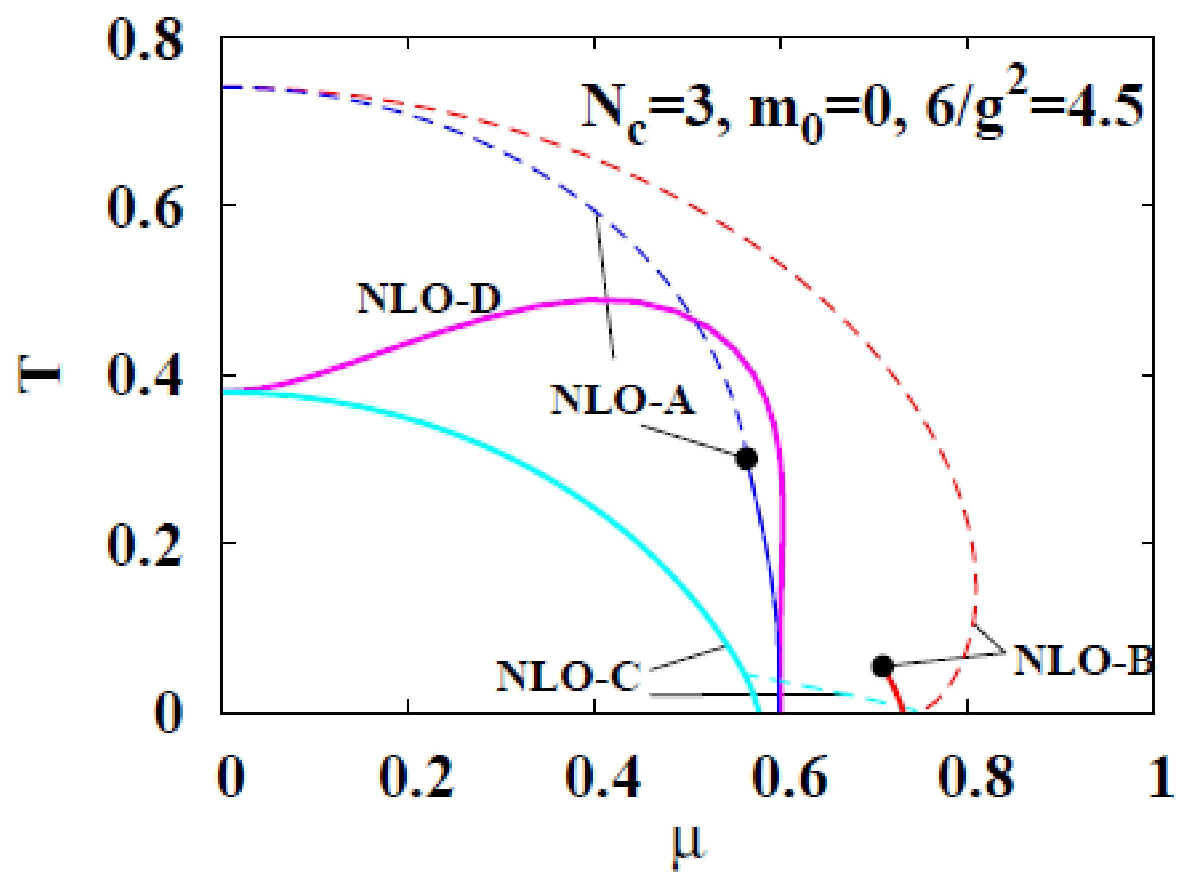


Preliminary

Truncation Deps. in NLO

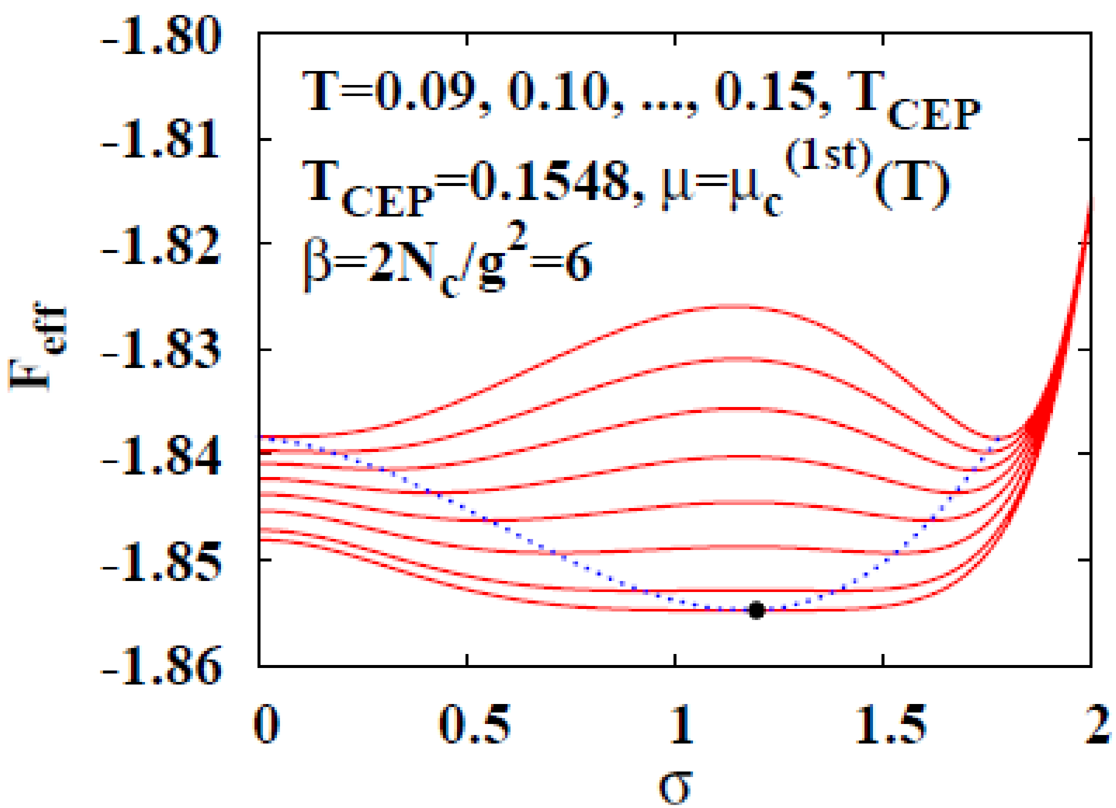
- **Phase diagram is sensitive to details, such as the truncation scheme in NLO.**

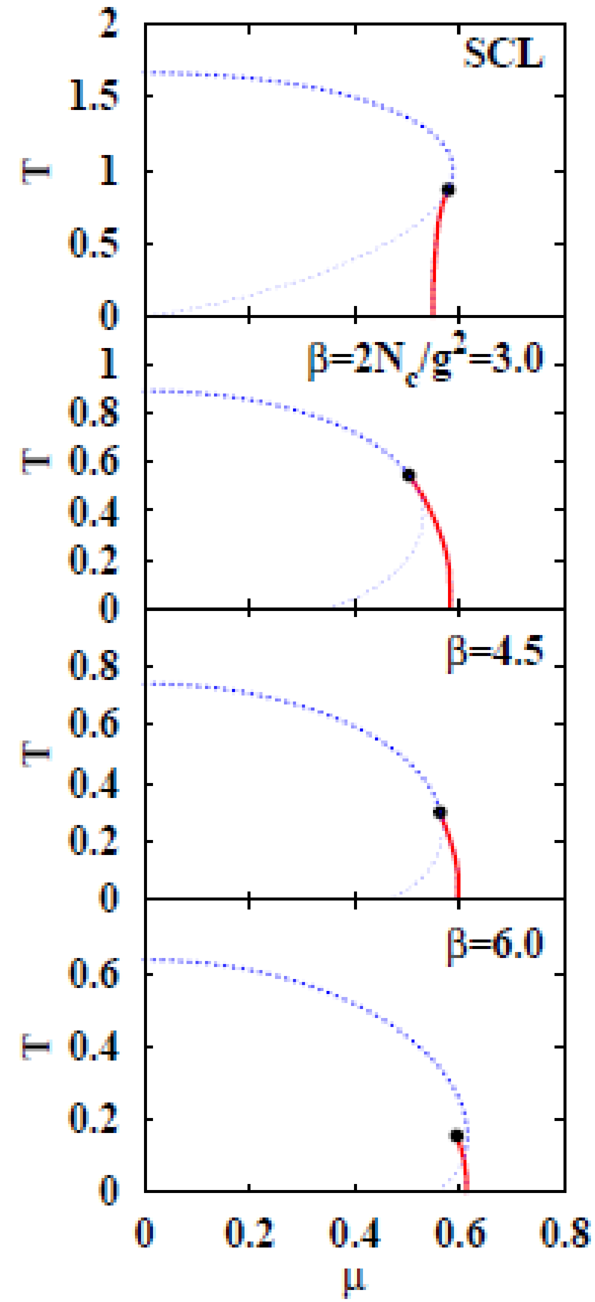
	$\delta\mu$	$\bar{m}_q$	$\Delta\mathcal{F}_{\rm aux}$	$\mathcal{V}_q$
NLO-A	$\log\sqrt{\frac{Z_+}{Z_-}}$	$\frac{m_q}{\sqrt{Z_+Z_-}}$	$-N_c\log\sqrt{Z_+Z_-}$	$\mathcal{V}_q(\bar{m}_q,\bar{\mu},T)$
NLO-B	$\beta_\tau\omega_\tau$	$\frac{m_q}{1+\beta_\tau\varphi_\tau}$	$-N_c\log(1+\beta_\tau\varphi_\tau)$	$\mathcal{V}_q(\bar{m}_q,\bar{\mu},T)$
NLO-C	$\beta_\tau\omega_\tau$	$\bar{m}_q^{(\rm NLO-C)}$	$-N_c\beta_\tau\varphi_\tau$	$\mathcal{V}_q(\bar{m}_q,\bar{\mu},T)$
NLO-D	0	$\bar{m}_q^{(\rm NLO-D)}$	$-N_c\beta_\tau\varphi_\tau$	$\mathcal{V}_q(\bar{m}_q,\mu,T)-\beta_\tau\omega_\tau\frac{\partial\mathcal{V}_q}{\partial\mu}$



Critical End Point in the Chiral Limit ?

- **Vector field generates repulsive pot. for large ρ_q states, which may cause two local min. structure**
  → Partially Chiral Restored matter may appear.





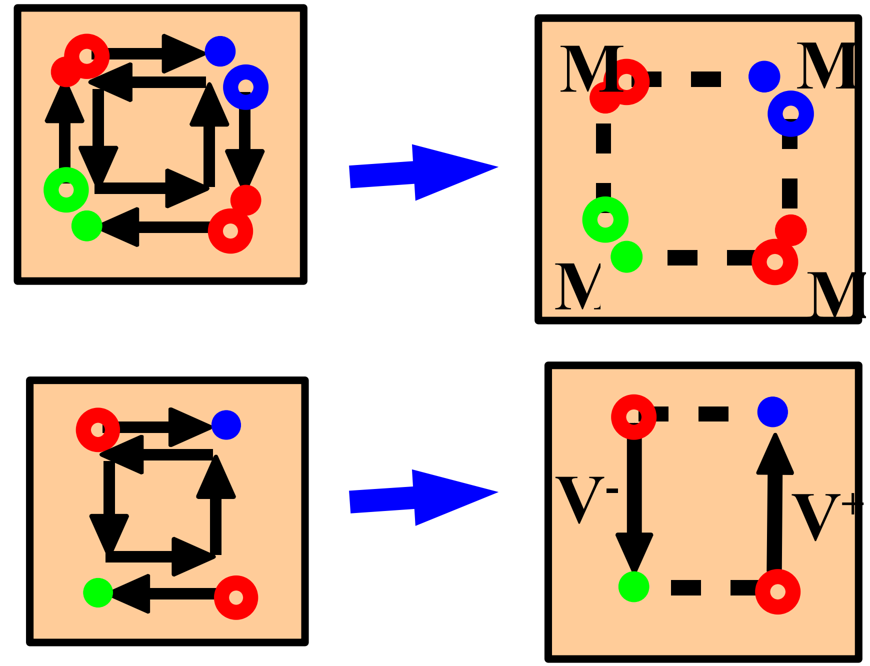
Effective Potential with $1/g^2$ (1)

- **1/d expansion of Plaquette action (Spatial One-Link Integral)**

Faldt, Petersson (86); Bilic, Karsch, Redlich (92)

$$\int dU\, U_{ab} U^{+}_{cd} = \frac{1}{N_c} \delta_{ad} \delta_{bc}$$

  - **Spatial plaquett → MMMM**
  - **Temporal Link → V^+V^-**



$$V_x^+ = e^{\mu} \bar{\chi}_x U_0(x) \chi_{x+\hat{0}}$$

$$V_x^- = e^{-\mu} \bar{\chi}_{x+\hat{0}} U_0^\dagger(x) \chi_x$$

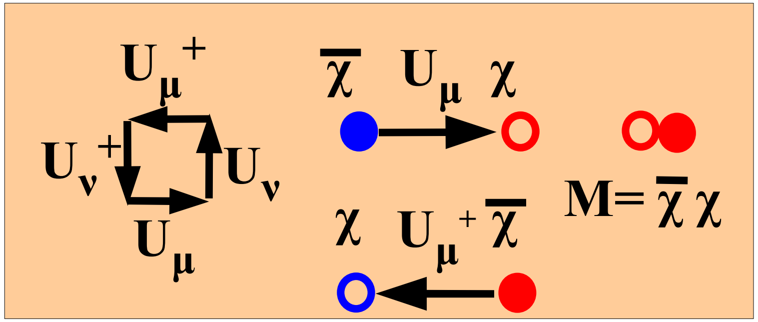
- **Effective Action**

$$\Delta S_\beta^{(\tau)} = \frac{1}{4N_c^2 g^2} \sum_{x,j>0} \left(V_x^+ V_{x+\hat{j}}^- + V_x^+ V_{x-\hat{j}}^- \right)$$

$$\Delta S_\beta^{(s)} = -\frac{1}{8N_c^4 g^2} \sum_{x,k>j>0} M_x M_{x+\hat{j}} M_{x+\hat{k}} M_{x+\hat{k}+\hat{j}}$$

- Lattice QCD=ab initio, non-perturbative theory (c.f. Teper's talk)

$$S_{\mathrm{LQCD}} = \frac{1}{2}\sum_{x,j}\left[\eta_{\nu,x}\bar{\chi}_x U_{\nu,x}\chi_{x+\hat{\nu}} - \eta^{-1}_{\nu,x}\bar{\chi}_{x+\hat{\nu}} U^{\dagger}_{\nu,x}\chi_x\right] - \frac{1}{g^2}\sum_{\square}\mathrm{tr}\left[U_{\square} + U^{\dagger}_{\square}\right] + m_0\sum_x \bar{\chi}_x\chi_x$$



- Problems to overcome
  - DOF is too much, and MC is necessary for numerical integration
    → Faster Computer + Faster Algorithm
  - Doublers appear for chiral fermions → different types of fermions
  - Weight for gluon config. (Fermion determinant) becomes complex at finite μ
    → Taylor expansion, Analytic Continuation, Canonical, …
    → Not Yet Applicable for Dense and Cold Matter !

Strong Coupling Limit/Expansion makes it possible to obtain (approx.) Effective Potential analytically !

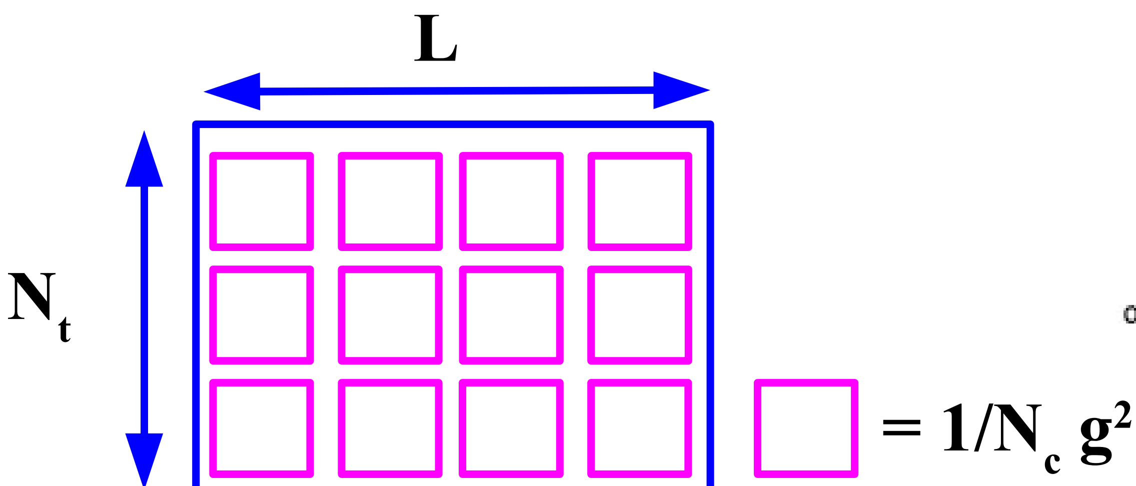
Strong Coupling Lattice QCD: Pure Gauge

K. G. Wilson, PRD10(1974),2445
M. Creutz, PRD21(1980), 2308.
G. Munster, 1981

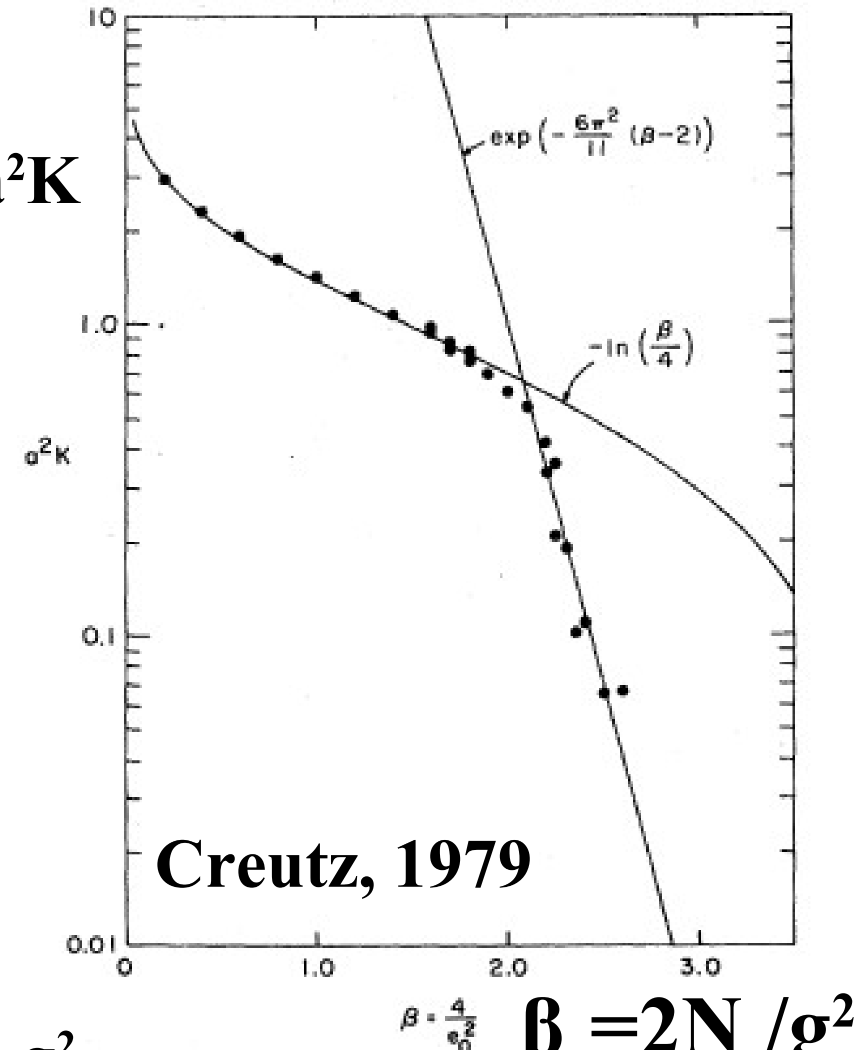
- **Quarks are confined in Strong Coupling QCD**
  - **Strong Coupling Limit (SCL)**
    → Fill Wilson Loop with Min. # of Plaquettes
    → Area Law (Wilson, 1974)

$$S_{\rm LQCD} = -\frac{1}{g^2}\sum_{\Box} {\rm tr}\left[U_{\Box} + U_{\Box}^{\dagger}\right]$$

  - **Smooth Transition from SCL to pQCD in MC (Creutz, 1980)**



a^2K

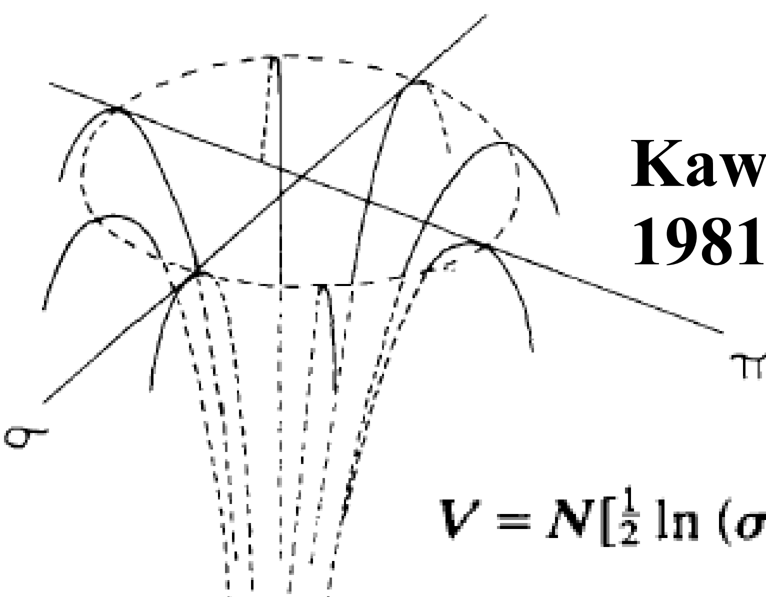


$\beta_g = 2N_c/g^2$

Strong Coupling Limit of LQCD with Quarks

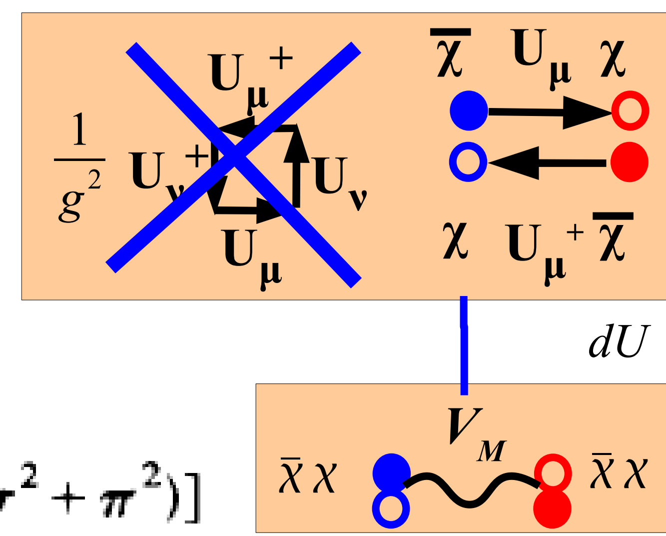
N. Kawamoto, NPB190('81),617, N. Kawamoto, J. Smit, NPB192('81)100
Kluberg-Stern, Morel, Napoly, Petersson, 1981

- **How about spontaneous chiral symmetry breaking ?**
- **Strong Coupling Limit (SCL) of Lattice QCD with Quarks**
  - **No Plaquette in SCL**
    → Mesonic Effective Action from One Link Integral
    → Effective Potential in σ and π from contour integral
    → SSB of the Chiral Sym.



Kawamoto, Smit, 1981

$$V = N[\tfrac{1}{2}\ln(\sigma^2+\pi^2) - M\sigma - dF(\sigma^2+\pi^2)]$$



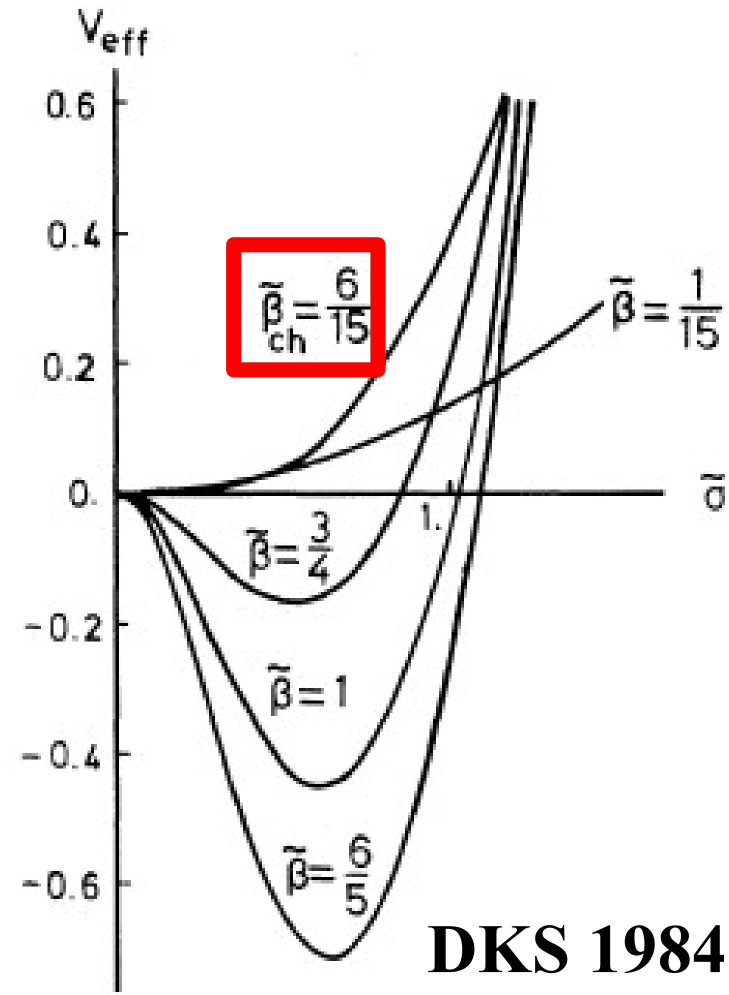
Chiral Transition at Finite Temperature

P.H.Damgaard, N. Kawamoto, K.Shigemoto, PRL53('84),2211; NPB264 ('86), 1
Faldt, Petersson, 1986; Bilic, Karsch, Redlich, 1992; Fukushima,2004, Nishida, 2004

- **Chiral Symmetry would be restored at high temperature → SCL-LQCD at Finite Temperatures**
  - **Staggered Fermion with Anti-Periodic B.C. → Matsubara Product**
  - **Polyakov gauge & Group integral (Vandermonde determinant)**
  - **Effective Potential (U(3))**

$$V_{\text{eff}} = \frac{1}{4} N \beta d \sigma^2 - \ln\left\{ \frac{\sinh[(N+1)\beta s]}{\sinh(\beta s)} \right\}$$

→ Chiral Phase Transition at $T_c = 2.5\ a^{-1}$

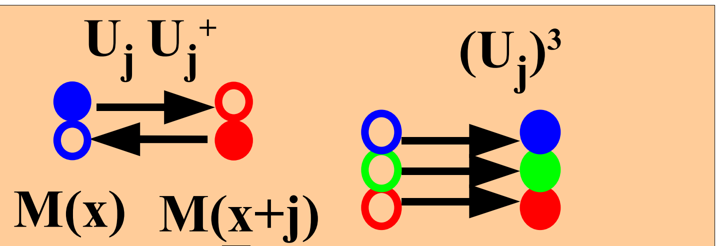


DKS 1984

Chiral Phase Transition at Finite Density

P.H.Damgaard, D. Hochberg, N. Kawamoto, PLB158('86)239
Hasenfatz, Karsch, 1983; Azcoiti et al., 2003; Kawamoto, Miura, AO, Ohnuma, 2007

- **QCD phase transition is also expected at high density**
  - **Baryon Rich QGP and/or Color SuperConductor are expected in the Neutron Star Core**
- **Strong Coupling Limit in SU(N)**
  - **Quark Chemical Potential and Baryonic Composite → Chiral phase transition at $\mu_c = 0.66\ a^{-1}$**

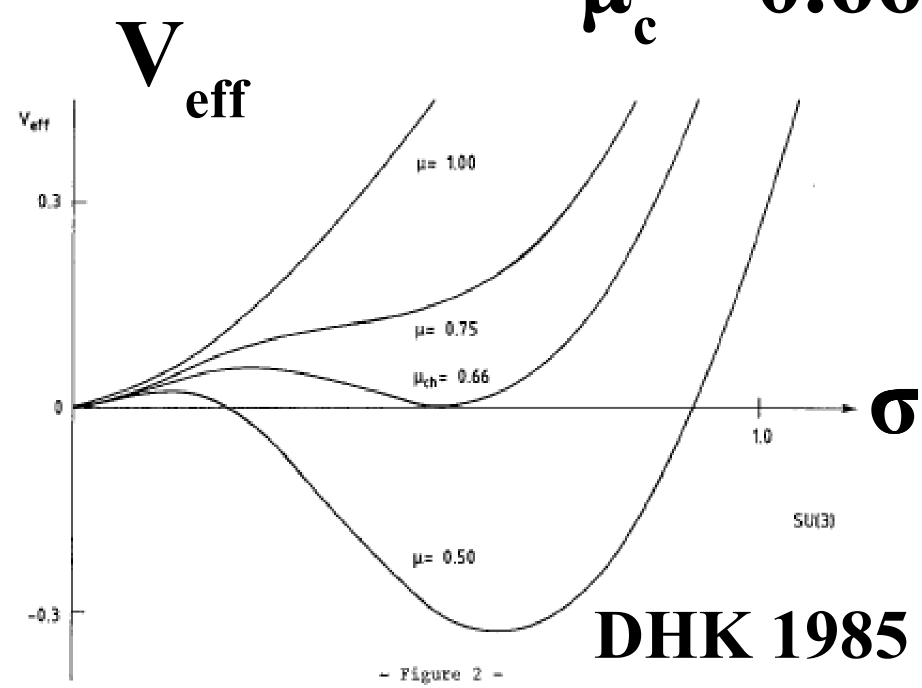


$\bar{B} = \epsilon \bar{\chi}\bar{\chi}\bar{\chi}/6 \quad B = \epsilon \chi\chi\chi/6$

$$\int dU\, U_{ab} U^{+}_{cd} = \frac{1}{N_c} \delta_{ad} \delta_{bc}$$

$$\int dU\, U_{ab} U_{cd} U_{ef} = \frac{1}{6} \epsilon_{ace} \epsilon_{bdf}$$

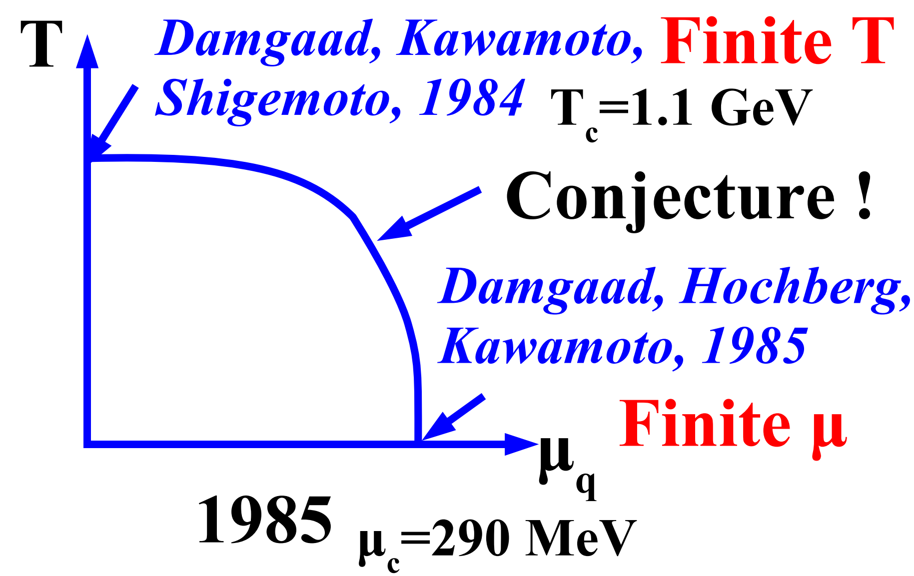
$\mu_c \sim 0.66$

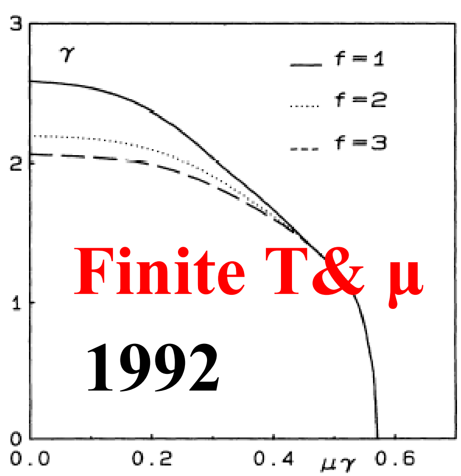


DHK 1985

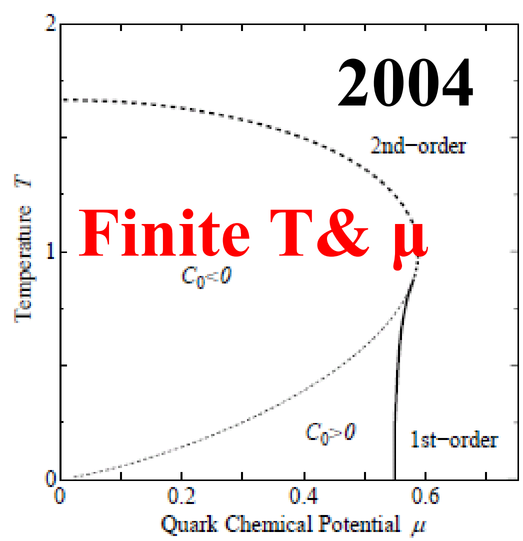
Evolution of Phase Diagram as a function of Time

- **Phase Diagram "Shape" becomes closer to that of Real World,**
  $R=3\ \mu_c/T_c \sim (6\text{-}12)$
  - **1985 → R=0.79 (Zero T / Finite T)**
  - **1992 → R=0.83 (Finite T & μ)**
  - **2004 → R= 0.99 (Finite T& μ)**
  - **2007 → R=1.34 (Baryon)**

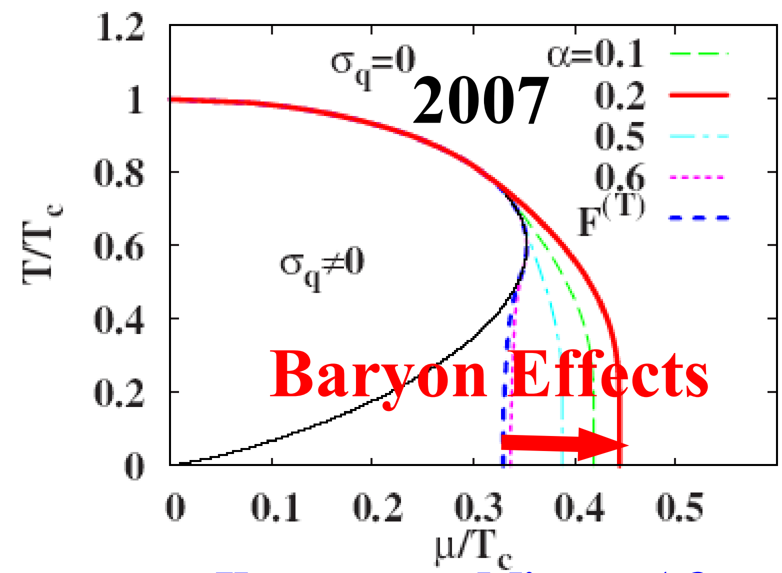




Bilic, Karsch, Redlich, 1992



Fukushima, 2004, Nishida, 2004

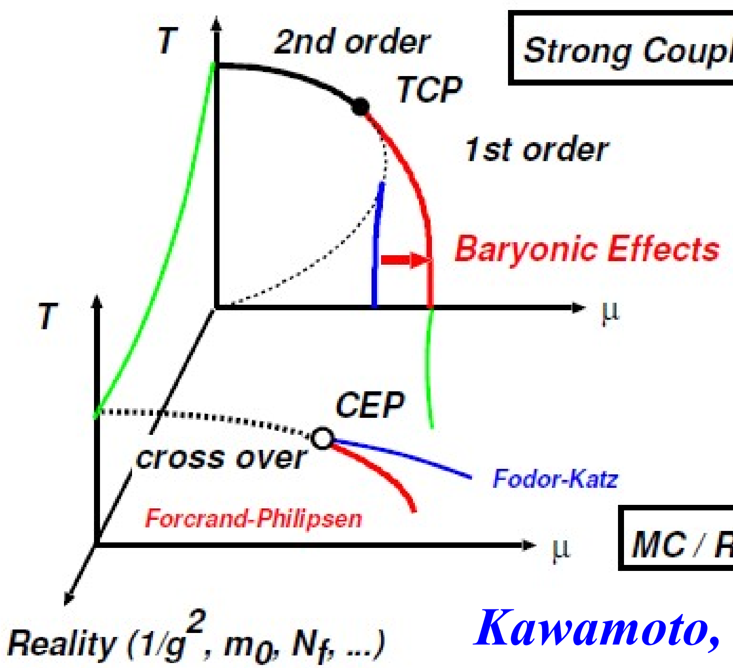


Kawamoto, Miura, AO, Ohnuma, 2007

Towards the Realistic Phase Diagram

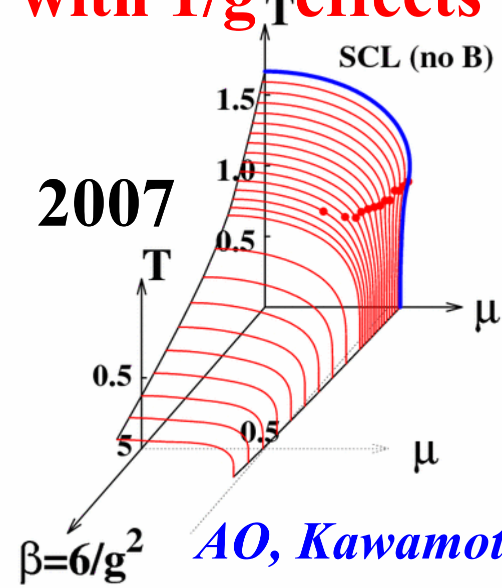
- **Why we cannot explain the phase diagram shape ?**
  → N_f (Staggered fermion) ? quark mass ? Finite Coupling ?
  - **μ_c (SCL) ~ $M_N/3$ (within a factor 2) , T_c (SCL) >> 200 MeV**
    → Larger problem should be in T_c , rather than in μ_c

Expectation before Calc.



Kawamoto, Miura, AO, Ohnuma, 2007

Preliminary Resuls with 1/g² effects



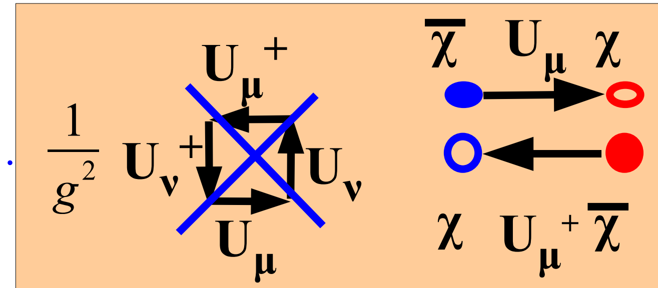
AO, Kawamoto, Miura, 2007

Gluon Contribution is important at High T

Effective Potential in SCL-LQCD

- **QCD Lattice Action (Finite T treatment)**

Damgaard, Kawamoto, Shigemoto, 1984; Bilic et al., 1992; Nishida, '04; Fukushima, '04; Kawamoto, Miura, AO, Ohnuma, '07; ...



$$S = \cancel{S_G} + S_F^{(s)} + S_F^{(t)} + m_0 \bar{\chi}\chi$$

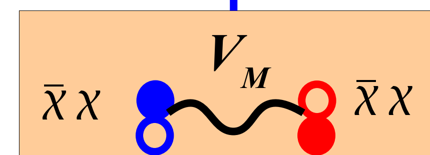
Strong Coupling Limit

$$S_F^{(t)} = \frac{1}{2}\sum_x \left(e^{\mu}\bar{\chi}_x U_0(x)\chi_{x+\hat{0}} - e^{-\mu}\bar{\chi}_{x+\hat{0}} U_0^+(x)\chi_x \right) = \bar{\chi} V^{(t)} \chi$$

dU

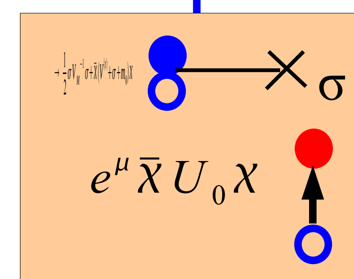
$$\rightarrow -\frac{1}{2}(\bar{\chi}\chi) V_M (\bar{\chi}\chi) + \bar{\chi}(V^{(t)} + m_0)\chi$$

Spatial-link integral (1/d expansion)



$$\rightarrow \frac{1}{2}\sigma V_M \sigma + \bar{\chi}(V^{(t)} + V_M \sigma + m_0)\chi$$

Hubbard-Stratonovich Transf. (Bosonization)

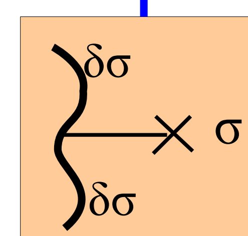


Fermion and Temporal-link Integral

$$\rightarrow L^d N_\tau \left[\frac{d}{4N_c}\bar{\sigma}^2 + V_q(b_\sigma \bar{\sigma}, T, \mu) \right]$$

SCL Effective Potential

$d\chi, dU_0$



We can obtain the Effective Potential analytically at finite T and μ

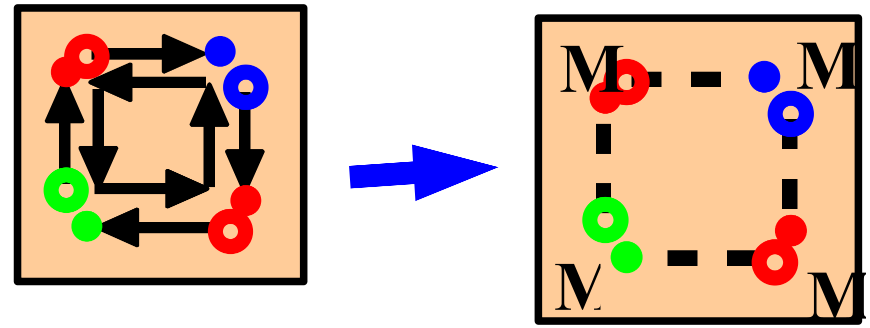
Effective Potential with $1/g^2$ (1)

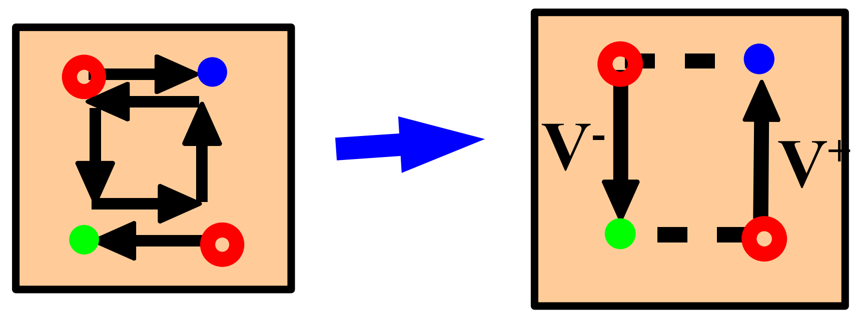
- **1/d expansion of Plaquette action (Spatial One-Link Integral)**

Faldt, Petersson (86); Bilic, Karsch, Redlich (92)

$$\int dU\, U_{ab} U^{+}_{cd} = \frac{1}{N_c} \delta_{ad} \delta_{bc}$$

- **Spatial plaquett → MMMM**
- **Temporal Link → $V^{+}V^{-}$**





$$V_x^{+} = e^{\mu} \bar{\chi}_x U_0(x) \chi_{x+\hat{0}}$$

$$V_x^{-} = e^{-\mu} \bar{\chi}_{x+\hat{0}} U_0^{\dagger}(x) \chi_x$$

- **Effective Action**

$$\Delta S_\beta^{(\tau)} = \frac{1}{4N_c^2 g^2} \sum_{x, j>0} \left(V_x^{+} V_{x+\hat{j}}^{-} + V_x^{+} V_{x-\hat{j}}^{-} \right)$$

$$\Delta S_\beta^{(s)} = -\frac{1}{8N_c^4 g^2} \sum_{x, k>j>0} M_x M_{x+\hat{j}} M_{x+\hat{k}} M_{x+\hat{k}+\hat{j}}$$