Phase diagram and nuclear matter in lattice QCD at strong coupling

Akira Ohnishi (YITP, Kyoto Univ.)

in collaboration with

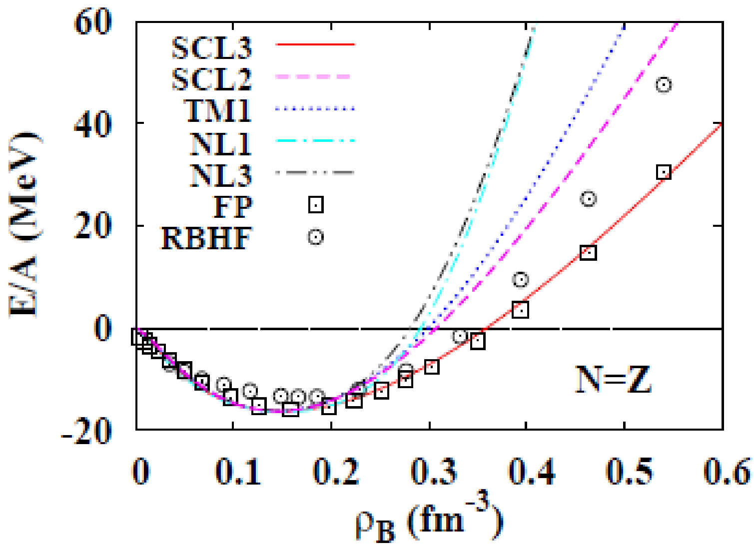
K. Miura (YITP), T. Z. Nakano (Kyoto U.), and N. Kawamoto (Hokkaido U.)

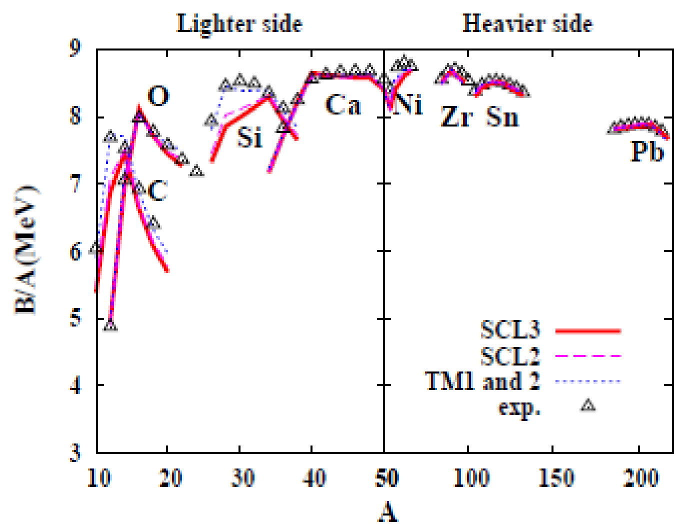
- **Introduction**
- **Effective potential in strong-coupling lattice QCD**
- **Phase diagram in SC-QCD**
- **Nuclear Matter in SC-QCD**
- **Summary**

Miura, Nakano, AO, Prog. Theor. Phys.,122(09),1045 [arXiv:0806.3357]
Miura, Nakano, AO, Kawamoto, PRD80(09),074034 [arXiv:0907.4245]
Nakano, Miura, AO, arXiv:0911.3453 [hep-lat]

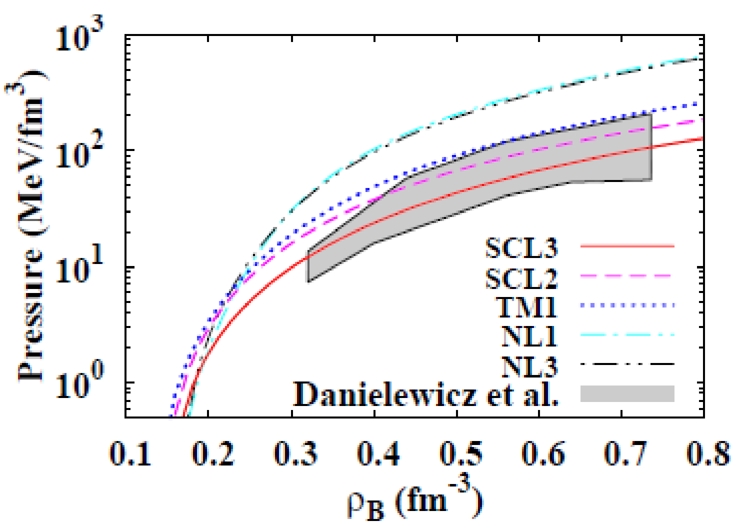
Nuclear Matter EOS

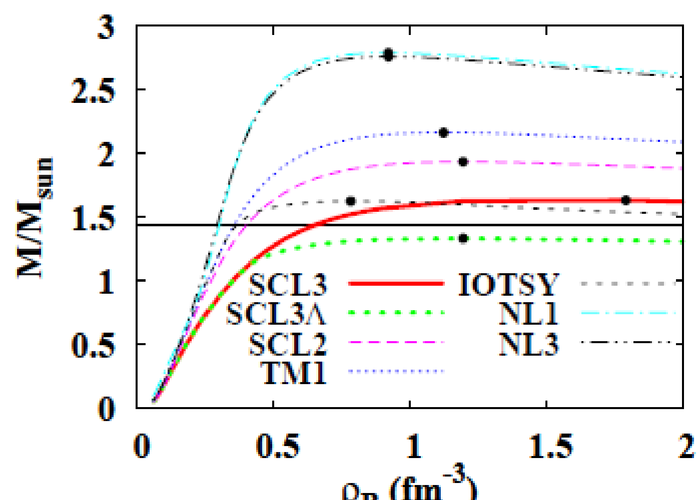
- **Nuclear matter EOS is important in**
  - **Nuclear B.E., HIC, Neutron Stars, Supernova, BH formation, ...**
    → How can we describe it from QCD ?





Incompressibility
J.P.Blaizot, Phys.Rept.64 (1980), 171.

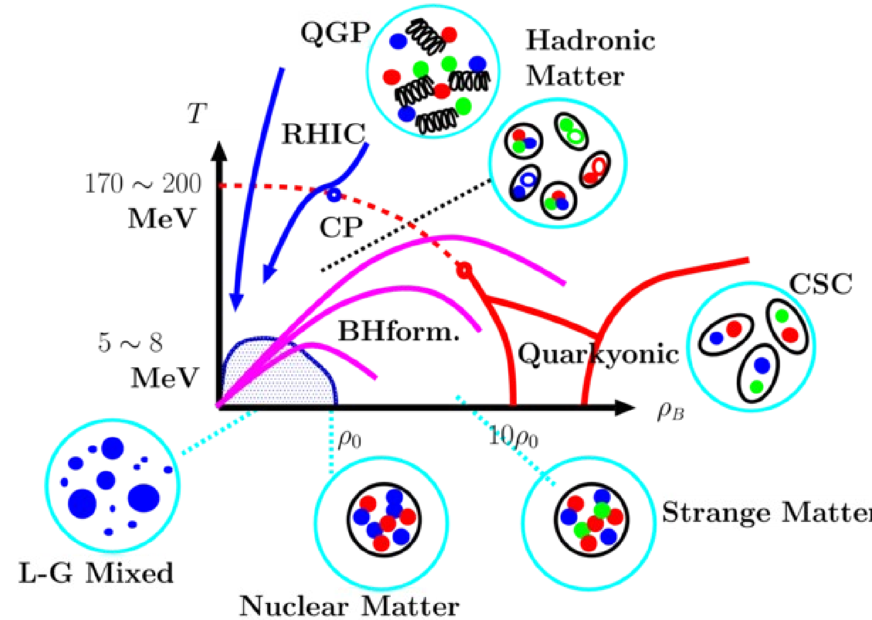


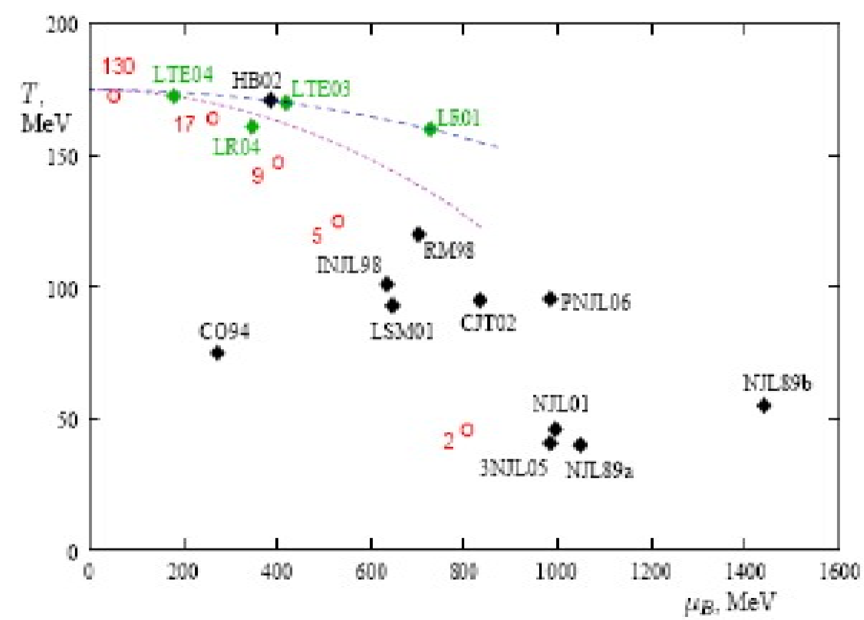


Tsubakihara et al., arXiv:0909.5058

QCD Phase diagram

- Phase transition at high T
  - Lattice MC & RHIC
- High μ transition has rich physics
  - Various phases, CEP, Astrophysical applications, ...
  - Models & Approximations are necessary !
    - Lattice MC works only for small μ (Tayler, AC, DOS, Canonical, …) or in the Strong Coupling Limit(SCL) (MDP) *Karsch, Mutter ('89), de Forcrand, Fromm ('09)*
    - Eff. Models: NJL, PNJL, PLSM, ...
    - Approximations: Large Nc, Strong Coupling, ...

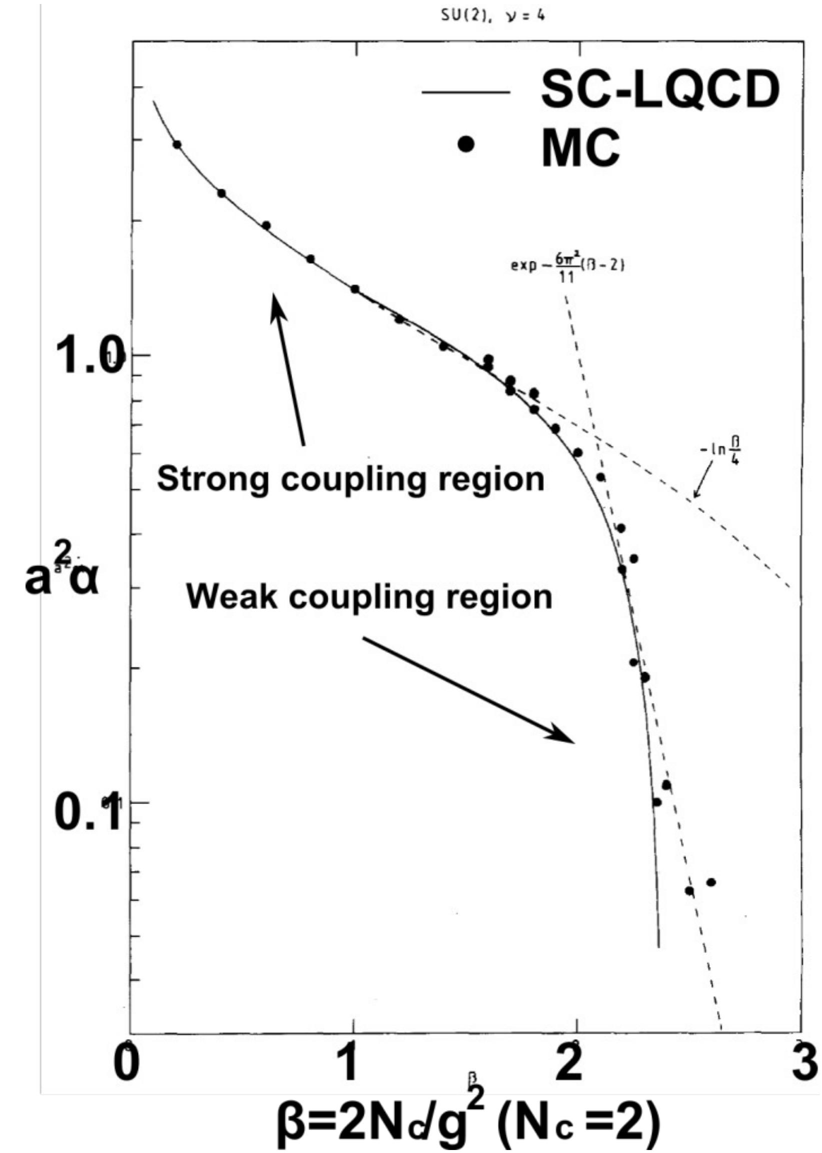




Strong Coupling Lattice QCD

- Large bare coupling → $1/g^2$ expansion
- Success in pure YM
  → Lattice MC & $1/g^2$ Expansion
  Wilson, '74; Creutz '80; Munster '81

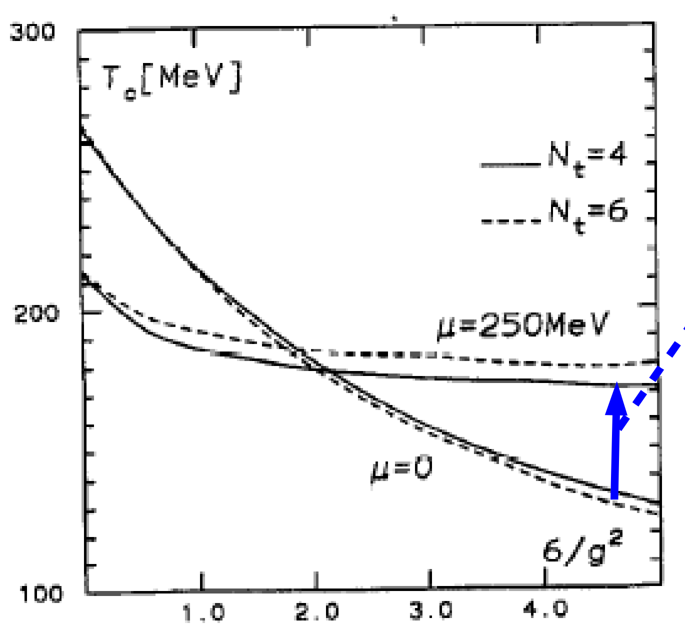
  → *Scaling region would be accessible in SC-LQCD !*
- Chiral transition at finite T and μ in Strong Coupl. Limit (SCL) & Next-to-leading order (NLO)
  Kawamoto-Smit '81, Damgaard-Kawamoto-Shigemoto, '84(U(3)), Faldt-Petersson'86 (SU(3)), Fukushima'04(SU(3)), Nishida '04 (SU(3)), Bilic-Karsch-Petersson, '92(NLO), Kawamoto-Miura-AO-Ohnuma '07 (Baryons)



Munster, '81

Strong Coupling Lattice QCD with Fermions

- **SC-LQCD with fermions**
  - **SCL: SC-LQCD (analytic) & MC (MDP) qualitatively agree**
    de Forcrand, Fromm ('09)
  - **$\beta=2N_c/g^2$ dep. of the critical point is not studied yet.**
  - **Condensates other than σ are not yet included in previous works.**
    (Faldt-Petersson '86; Bilic-Karsch-Redlich '92; Bilic-Demeterfi-Petersson '92; Bilic-Claymans '95)



Bilic-Claymans '95

$dT_c/d\mu > 0 \rightarrow \Delta s/\Delta\rho < 0$?
(Clausius-Clapeyron)

In this work, we revisit / develop NLO & NNLO SC-LQCD with fermions at finite T and μ and Study Phase diagram & nuclear matter with finite β

NLO & NNLO SC-LQCD: Setups & Disclaimer

- **We investigate the phase diagram and try to understand nuclear matter based on the strong-coupling lattice QCD (SC-LQCD).**
  - **Effective potential (free E. density) → phase boundary & EOS**
  - **Setups & Disclaimer**
    - **Effective action in SCL ($1/g^0$), NLO ($1/g^2$), NNLO ($1/g^4$) terms**
      NLO: Faldt-Petersson ('86), Bilic-Karsch-Redlich ('92)
      Conversion radius > 6 in pure YM ? Osterwalder-Seiler ('78)
    - **One species of unrooted staggered fermion (N_f=4 in continuum limit)**
      Moderate N_f deps. of phase boundary: BKR92, Nishida('04), D'Elia-Lombardo ('03)
    - **Leading order in 1/d expansion (d=3=space dim.)**
      → Min. # of quarks for a given plaquette configurations, no spatial B prop.
    - **Effective potential is obtained in mean field approximation**
    - **Polyakov loop effects are not included (No Deconfinement).**
    - **Different from "strong couling" in "large N_c"**

Still far from "Realistic", but SC-LQCD would tell us useful qualitative features of the phase diagram and EOS.

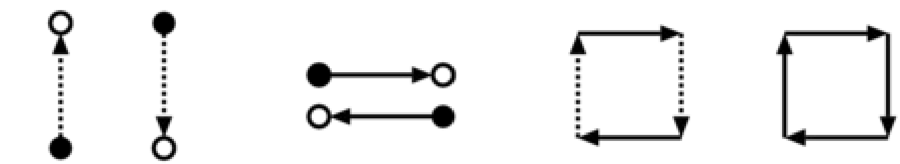
Effective Potential in NLO and NNLO Strong-Coupling Lattice QCD

SC-LQCD with Fermions (1)

- **Lattice QCD action**

$$S_{LQCD}=\frac{1}{2}\sum_x [V_x^+ - V_x^-] + m_0 \sum_x M_x + \frac{1}{2}\sum_{x,j} \eta_{j,x}(\bar{\chi}_x U_{j,x}\chi_x - \bar{\chi}_{x+j} U_{j,x}^+ \chi_x)$$

$$+\frac{1}{g^2}\sum_P (U_P + U_P^+)$$



Mesonic composites

$$M_x = \bar{\chi}_x \chi_x,\quad V_x^+ = e^{\mu}\bar{\chi}_x U_{0,x}\chi_{x+\hat{0}},\quad V_x^- = e^{-\mu}\bar{\chi}_{x+\hat{0}} U_{0,x}^+ \chi_x$$

- **Effective Action & Effective Potential**

$$Z=\int D[\chi,\bar{\chi},U_{0,}U_j]\exp(-S_{\mathrm{LQCD}})$$

$$=\int D[\chi,\bar{\chi},U_0]\exp(-S_{\mathrm{SCL}})\langle\exp(-S_G)\rangle \qquad (U_j \text{ integral})$$

$$\approx\int D[\chi,\bar{\chi},U_0]\exp(-S_{\mathrm{eff}}[\chi,\bar{\chi},U_{0,}\Phi_{\mathrm{stat.}}]) \qquad (\text{bosonization})$$

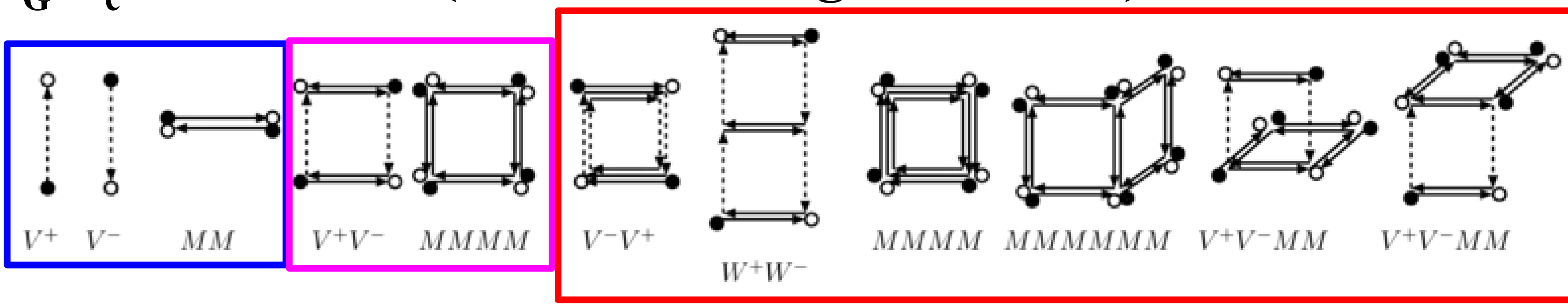
$$\approx\exp(-V F_{\mathrm{eff}}(\Phi;T,\mu)/T) \qquad (\text{fermion}+U_0 \text{ integral})$$

SC-LQCD with Fermions (2)

- **Effective Action with finite coupling corrections**
  Integral of exp(-S_G) over spatial links with exp(-S_F) weight → $S_{\rm eff}$

$$S_{\rm eff} = S_{\rm SCL} - \log\langle \exp(-S_G)\rangle = S_{\rm SCL} - \sum_{n=1} \frac{(-1)^n}{n!}\langle S_G^n\rangle_c$$

$\langle S_G^n\rangle_c$=Cumulant (connected diagram contr.) *c.f. R.Kubo('62)*



$$S_{\rm eff} = \frac{1}{2}\sum_x (V_x^+ - V_x^-) - \frac{b_\sigma}{2d}\sum_{x,j>0}[MM]_{j,x}$$
SCL (Kawamoto-Smit, '81)

$$+\frac{1}{2}\frac{\beta_\tau}{2d}\sum_{x,j>0}[V^+V^- + V^-V^+]_{j,x} - \frac{1}{2}\frac{\beta_s}{d(d-1)}\sum_{x,j>0,k>0,k\neq j}[MMMM]_{jk,x}$$
NLO (Faldt-Petersson, '86)

$$-\frac{\beta_{\tau\tau}}{2d}\sum_{x,j>0}[W^+W^- + W^-W^+]_{j,x} - \frac{\beta_{ss}}{4d(d-1)(d-2)}\sum_{\substack{x,j>0,|k|>0,|l|>0\\ |k|\neq j,|l|\neq j,|l|\neq|k|}}[MMMM]_{jk,x}[MM]_{j,x+\hat{l}}$$

$$+\frac{\beta_{\tau s}}{8d(d-1)}\sum_{x,j>0,|k|\neq j}[V^+V^- + V^-V^+]_{j,x}\left([MM]_{j,x+\hat{k}} + [MM]_{j,x+\hat{k}+\hat{0}}\right)$$
NNLO (Nakano, Miura, AO, '09)

SC-LQCD with Fermions (3)

- **Extended Hubbard-Stratonovich transformation**

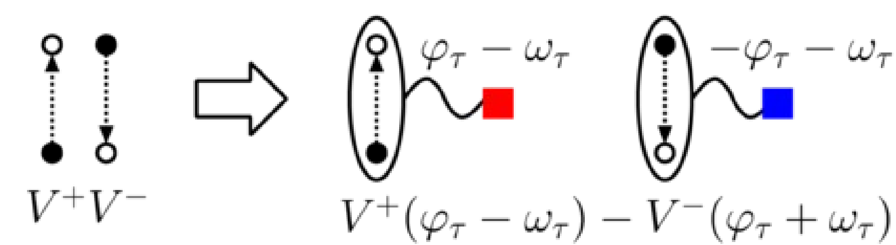
$$\exp(\alpha A B) = \int d\varphi\, d\phi \exp[-\alpha(\varphi^2 - (A+B)\varphi + \phi^2 - i(A-B)\phi)]$$
$$\approx \exp[-\alpha(\bar{\psi}\psi - A\psi - \bar{\psi}B)]_{\text{stationary}}$$

Miura, Nakano, AO (09), Miura, Nakano, AO, Kawamoto (09)

4, 8, 12 Fermion int. term → bi-linear form of quarks.

Ex.: $V^+V^- \to \varphi_\tau^2 - \omega_\tau^2 + \varphi_\tau(V^+ - V^-) - \omega_\tau(V^+ + V^-)$

- **Effective Action after bosonization (and in gluonic dressed fermion)**



$$S_{eff} = S_{eff}^{(F)} + S_{eff}^{(X)}$$
$$S_{eff}^{(F)} = \frac{1}{2}\sum_x [Z_- V_x^+(\mu) - Z_+ V_x^-(\mu)] + m_q \sum_x M_x$$
$$= Z_\chi \left\{ \frac{1}{2}\sum_x [e^{-\delta\mu} V_x^+(\mu) - e^{+\delta\mu} V_x^-(\mu)] + \tilde{m}_q \sum_x M_x \right\} = Z_\chi \sum \bar{\chi} G^{-1} \chi$$

→ **w.f. renorm. factor (Z_χ), quark mass (m_q), chem. pot. shift ($\delta\mu$)**

SC-LQCD with Fermions (3)

- **Integral over fermions and temporal links**

Damgaad, Kawamoto, Shigemoto (84), Faldt-Petersson (86), Nishida (04)

$$V_q(m,\mu,T) = -\frac{T}{L^d}\log\left\{\int D[U_0]\,det(G^{-1})\right\}$$
$$= -T\log\left[\frac{\sinh((N_c+1)E_q(m)/T)}{\sinh(E_q(m)/T)} + 2\cosh(N_c\mu/T)\right]$$
$$E_q(m) = \text{arcsinh}\, m \quad (\text{quark excitation energy})$$

- **Effective Potential in NLO/NNLO SC-LQCD**

$$F_{\text{eff}} = F_{\text{eff}}^{(X)}(\sigma,\omega_\tau) + V_q(\tilde{m}_q;\tilde{\mu},T) - N_c\log Z_\chi$$

$$\sigma \approx \langle M\rangle \ (\text{chiral condensate}),\quad \omega_\tau \approx -\partial F_{\text{eff}}/\partial\mu = \rho_q \ (\text{quark number density})$$
$$\tilde{m}_q = \frac{\tilde{b}_\sigma\sigma + m_0}{Z_\chi(1+4\beta_{\tau\tau}\varphi_\tau)} \approx \frac{d}{2\mathrm{N}_c}\sigma\times\left(1+\beta_{\sigma\sigma}^{(m)}\sigma^2 - \beta_{\sigma\omega}^{(m)}\sigma^2\omega_\tau^2 + \ldots.\right)$$
$$\delta\mu = \mu - \tilde{\mu} = \log(Z_+/Z_-) \approx \beta_\tau\omega_\tau\times\left(1+\beta_{\omega\sigma}^{(\mu)}\sigma^2 + \ldots\right)$$

NLO/NNLO SC-LQCD
≈ σω model of quarks non-linear couplings

Phase Diagram in the Strong-Coupling Lattice QCD

Stationary Condition --- Multi-Order Parameter

- **Stationary Condition** $\frac{\partial \mathcal{F}_{\text{eff}}}{\partial \Phi} = 0$

  Φ (4(NLO) / 10 (NNLO) aux. field) $\rightarrow (\sigma, \omega_\tau)$

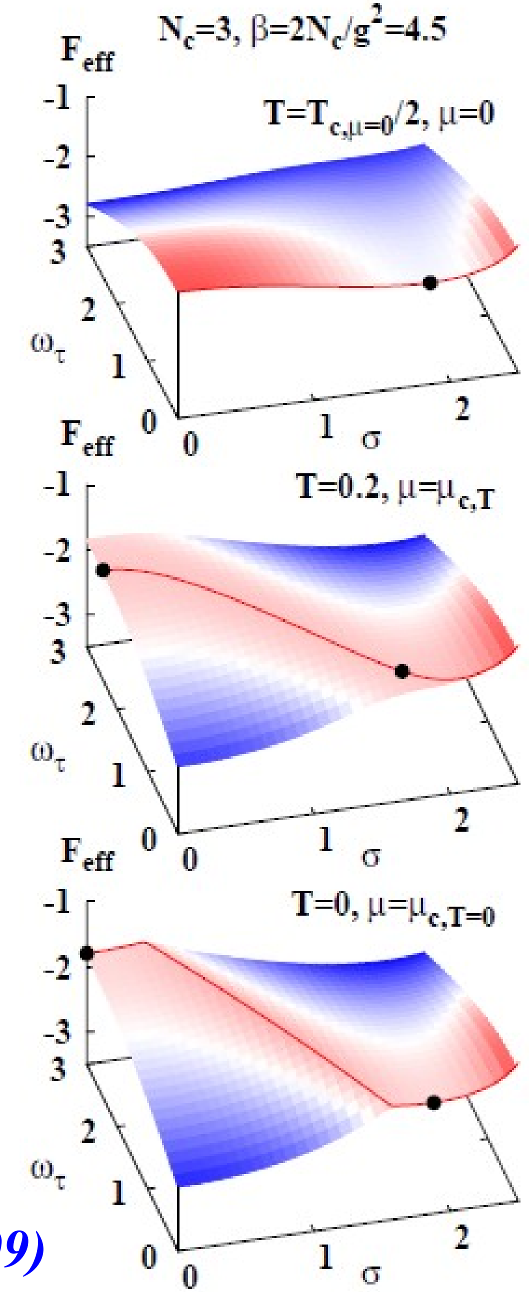
- **Multi-Order Parameter (σ, ω_τ)**

  $$\sigma \approx -\frac{\partial F_{\text{eff}}}{\partial m_0} = \text{Chiral Cond.}$$

  $$\omega \approx -\frac{\partial F_{\text{eff}}}{\partial \mu} = \text{Quark number density}$$

  - **Two indep. var. in V_q(m, μ)**
  - **Scalar (σ) and Vector (ω) potential for Quarks**

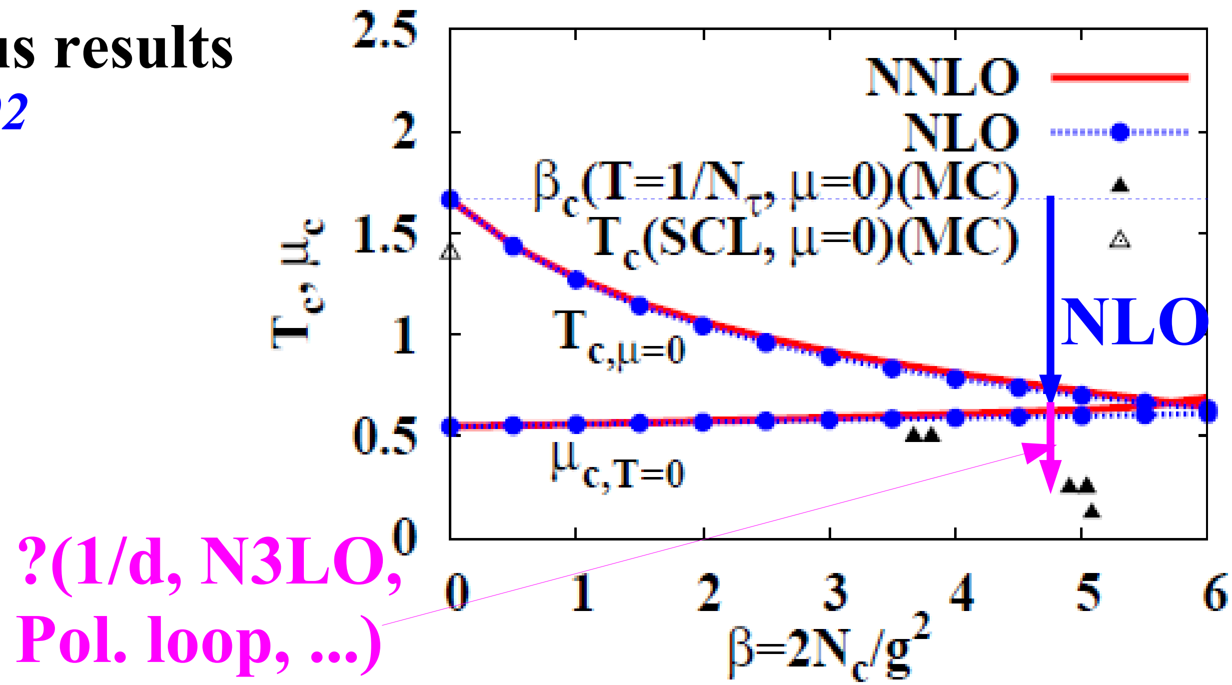
→ Saddle point in Feff(σ, ω_τ)



Miura, Nakano, AO, Kawamoto ('09)

Critical Temperature and Chemical Potential

- **Critical Temperature (μ =0) → rapid decrease with $\beta=2N_c/g^2$**
  - **W.F. Renom. factor Z_χ → suppression of mass**
  - **T_c is still larger than MC results**
    de Forcrand ('06), Gottlieb et al. ('87), Gavai et al. ('90), de Forcrand, Fromm ('09)
- **Critical Chem. Pot. (T=0) → weak deps. on β**
  - **Suppression of mass ~ Suppression of $\tilde{\mu}$**
  - **Consistent with previous results**
    Bilic-Demeterfi-Petersson, '92
- **NNLO effects are small on $T_c(\mu=0)$ and $\mu_c(T=0)$.**



Phase Diagram Evolution

- Shape of the phase diagram is compressed in T direction with β

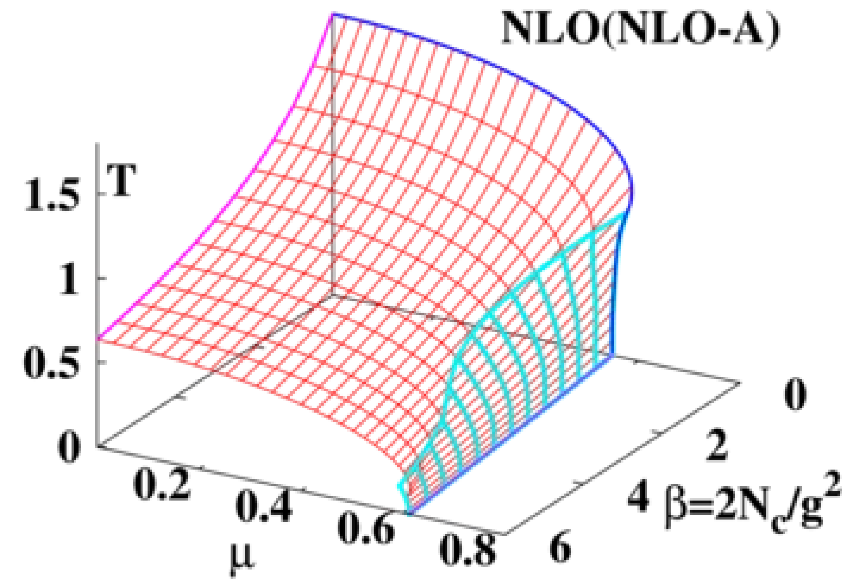
  → *Improvements in $R = \mu_c/T_c$!*

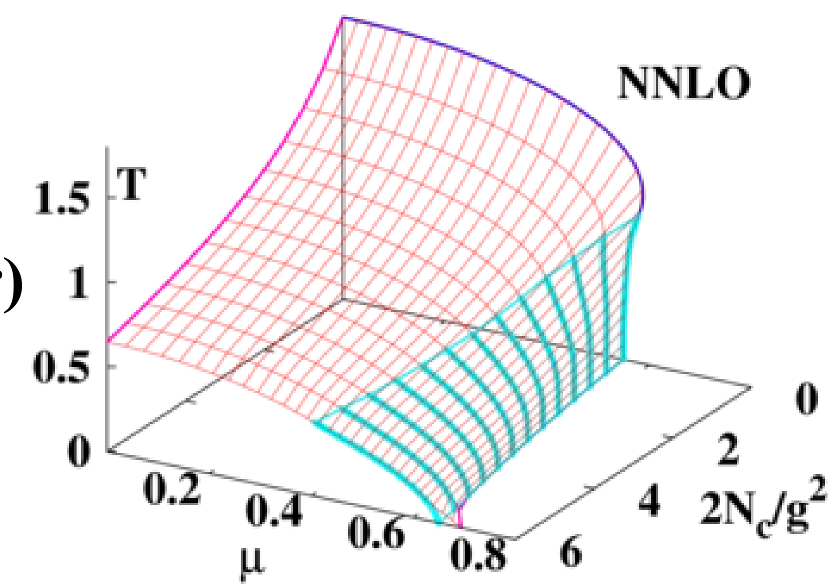
  - MC (R > 1) → SCL (R = (0.3-0.45)) → NLO/NNLO (R ~ 1) → Real World (R~(2-4))

- Critical Point
  - NLO: μ(CP) ~ Const.
  - NNLO: μ(CP) decreases with β → *Improvements !* (N_f=4 → 1st order)

    Pisarski, Wilczek ('84)
  - μ(CP)/T(CP) ~ 1 ↔ MC (μ/T > 1)

    Ejiri, ('08), Aoki et al.(WHOT,'08), Allton et al., ('03,'05)



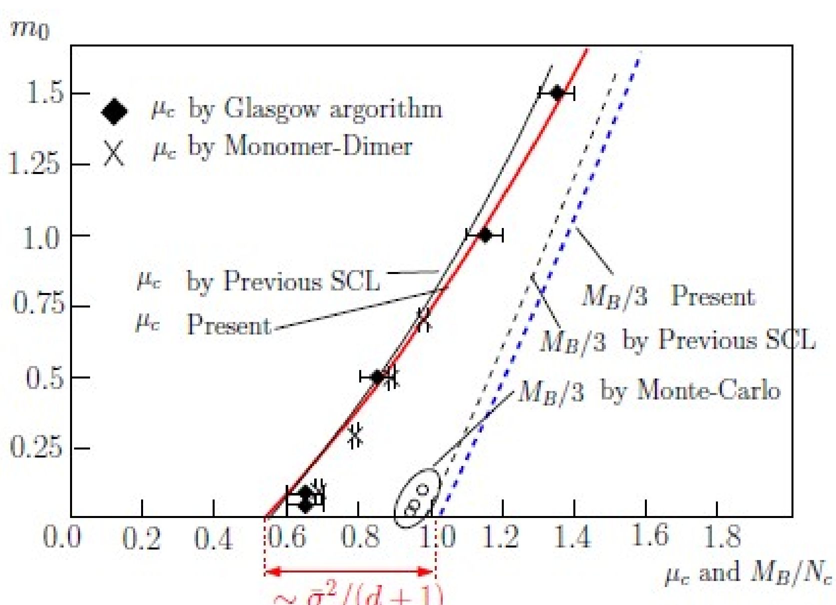


Nuclear Matter in the Strong-Coupling Lattice QCD

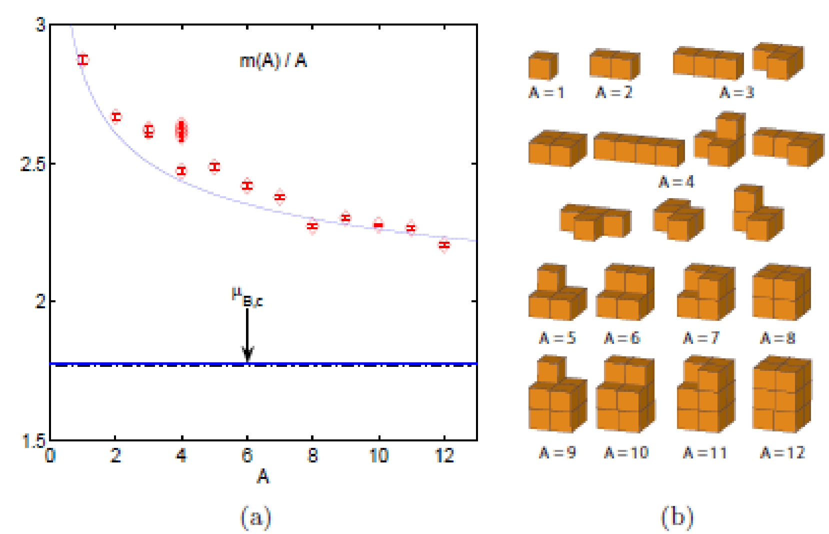
Cold Nuclear Matter in Lattice QCD

- **Baryon mass puzzle in SCL-LQCD: $N_c\mu_c < M_B$**
  → QCD phase transition takes place before baryons appear.
  Kluberg-Stern, Morel, Petersson ('83), Damgaard, Hochberg, Kawamoto ('85), Karsch, Mutter ('89), Barbour et al.('97), Bringoltz('07), Miura, Kawamoto, AO ('07)

- **Possible Solutions**
  - **Regard the matter at $\mu > \mu_c$ as nuclear matter** *de Forcrand, Fromm ('09)*
  - **Finite coupling effects: Decrease of quark mass**



Miura, Kawamoto, AO ('07)



de Forcrand, Fromm ('09)

Constituent Quark Mass in NNLO SC-LQCD

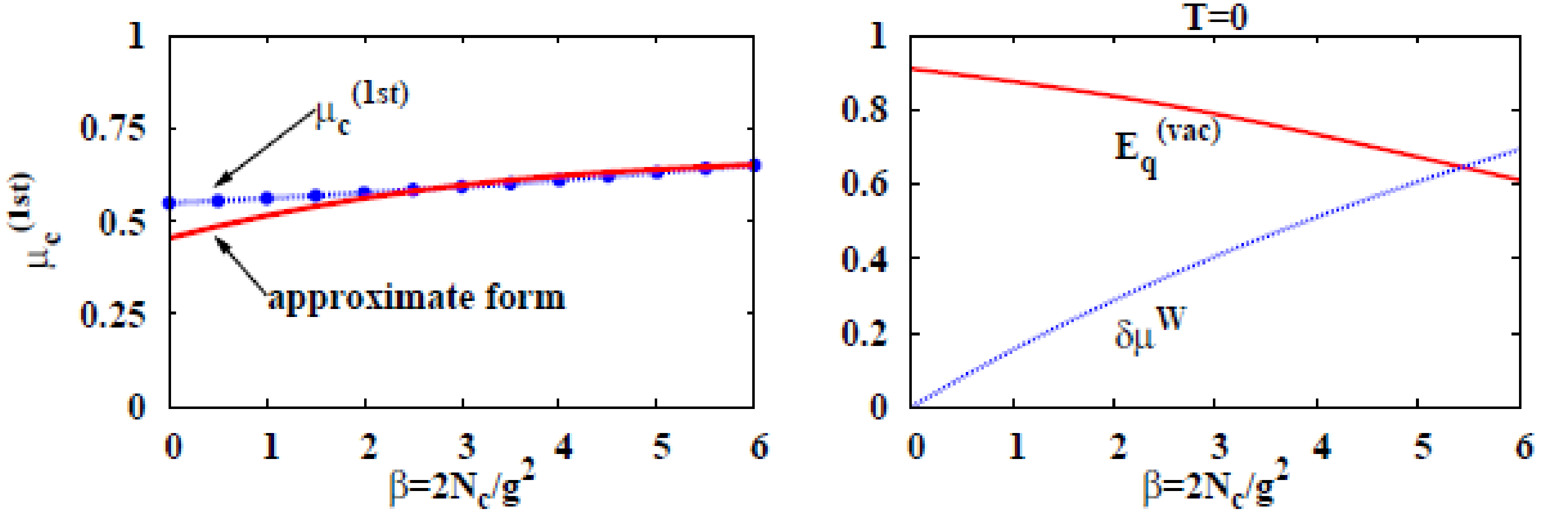
- **Mechanism of "stable" $\mu_c(T=0)$ in NLO/NNLO SC-LQCD**

  = Effects of quark mass reduction & repulsive vector pot. cancel

Transition Condition at $T=0: E_q(\tilde{m}_q)=\tilde{\mu}\simeq\mu-\beta'_\tau\omega_\tau$

$\rightarrow\mu\simeq E_q(\tilde{m}_q)+\beta'_\tau\omega_\tau$

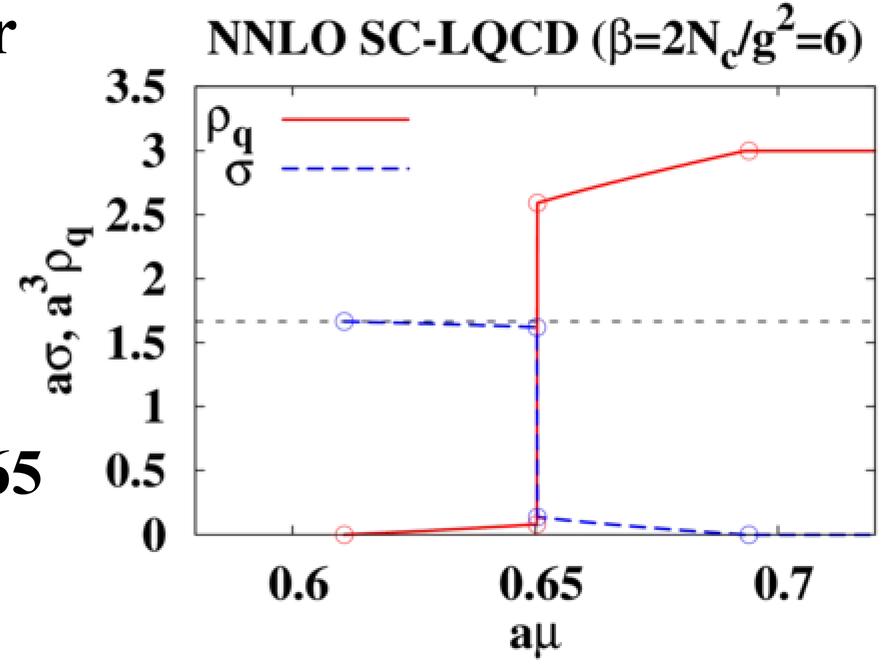
Pocket formula $\quad \mu_{c,T=0}\simeq\frac{1}{2}\left[E_q(\sigma=\sigma_{\text{vac}},\omega_\tau=0)+\delta\mu(\sigma=0,\omega=N_c)\right]$



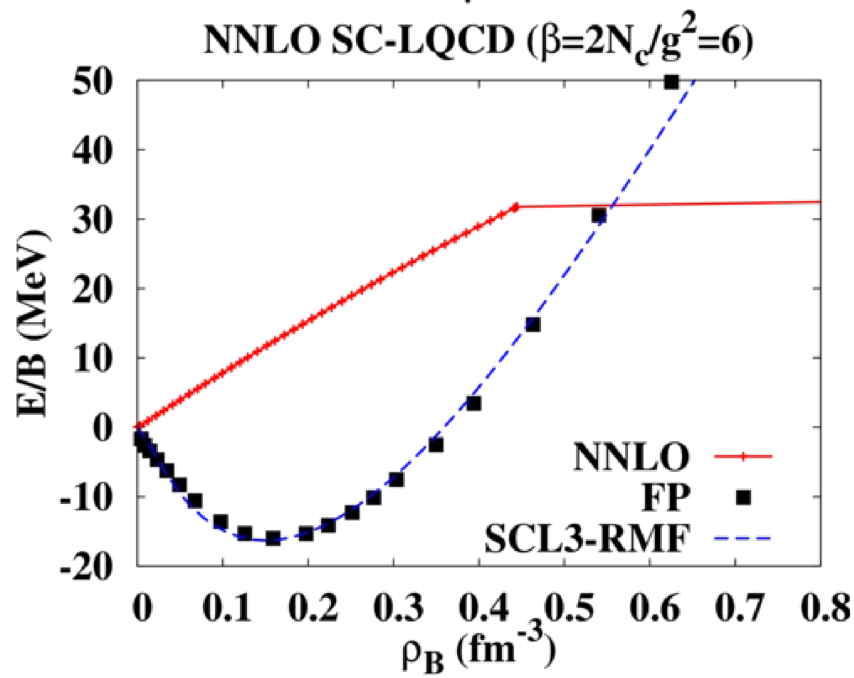
Quark mass ($\approx E_q$) is smaller than μ_c for $\beta > 5.5$.
→ "Baryon mass puzzle" may be solved !

Nuclear Matter on the Lattice at Strong Coupling

- **Do we observe finite density matter before 1st order phase transition ?**
  → Yes !
  - **$E_q(\mu=0, T=0, \beta=6)=0.61$**
    $\mu_c^{(1st)}(T=0, \beta=6)=0.65$
    → "Nuclear matter" in $0.61<\mu<0.65$



- **EOS of "Nuclear matter"**
  - **a^{-1} = 500 MeV**
    Bilic, Demeterfi, Petersson ('92)
    → Density in the order of ρ_0
  - **No saturation**
  - **1st order transition at ρ_B=0.4 fm^{-1}.**



Summary & Conclusion

- We have derived the effective potential with NLO ($1/g^2$) and NNLO($1/g^4$) effects in strong coupling lattice QCD.
  - Several techniques are developed (Extended HS transf., Multi-order parameter treatment, gluonic dressed fermion)
  - NLO & NNLO effects are found to modify the quark mass, dynamical chemical potential, and W.F. renormalization factor.
- Phase diagram is studied by using the derived effective potential.
  - Decrease of $T_c(\mu=0)$ and stable $\mu_c(T=0)$ improve the "shape" of the phase diagram, where NLO(NNLO) effects are large (small).
  - Critical Point is sensitive to NNLO effects.
- In NNLO, we observe finite density matter before the 1st order phase transition.
  - Baryon mass puzzle is solved, i.e. $M_B < N_c \mu_c$ is realized at T=0.
  - For a^{-1}=500 MeV, ρ_B is the order of ρ_0, but no saturation.

Future Works

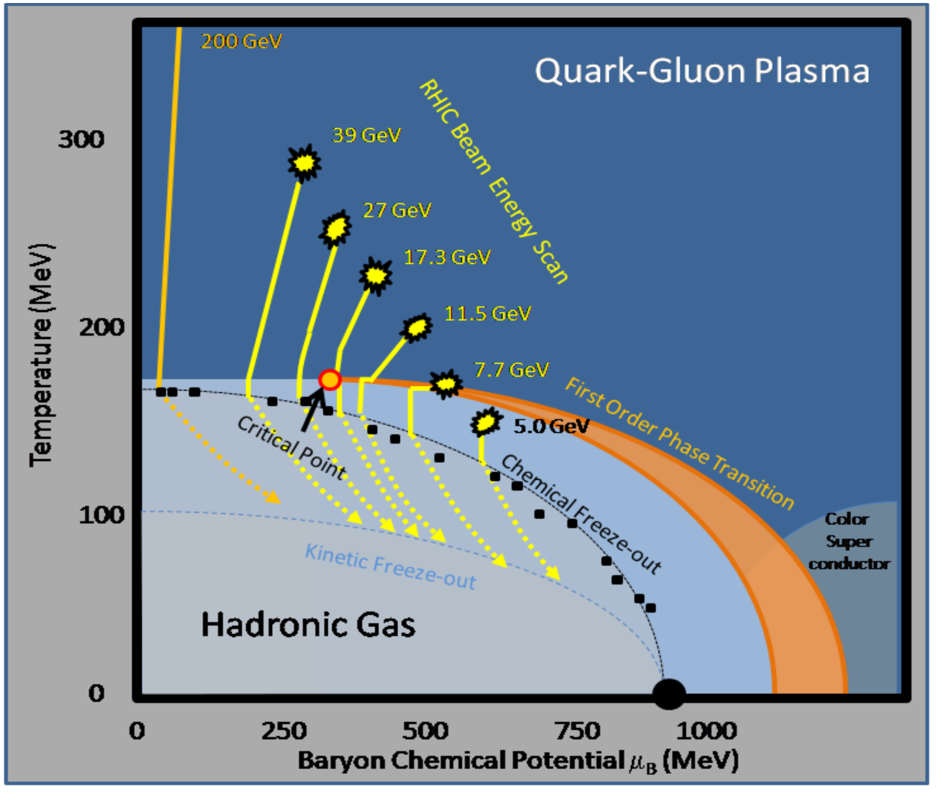
- **While it is still far from the real world, SC-LQCD seems to be a promising tool to understand the phase diagram and EOS.**
  - **Relation to lattice MC calculation is straightforward,**
  - **SC-LQCD motivated σ potential (∝ -log σ) is useful in describing nuclei and nuclear matter in chiral symmetric RMF models,**
  - **and it may be possible to describe nuclear matter directly.**
  - **But SCL/NLO/NNLO SC-LQCD show far from scaling behavior, and they do not qualitatively explain the MC results at μ=0 and at SCL($g \to \infty$).**
- **Further studies incl.**
  Polyakov loop, 1/d, meson fluctuation, higher orders in $1/g^2$
  may be necessary to describe
  the deconfinement transition, μ=0 and SCL results in MC.
- **For N_f=2+1, different types of fermion have to be considered.**

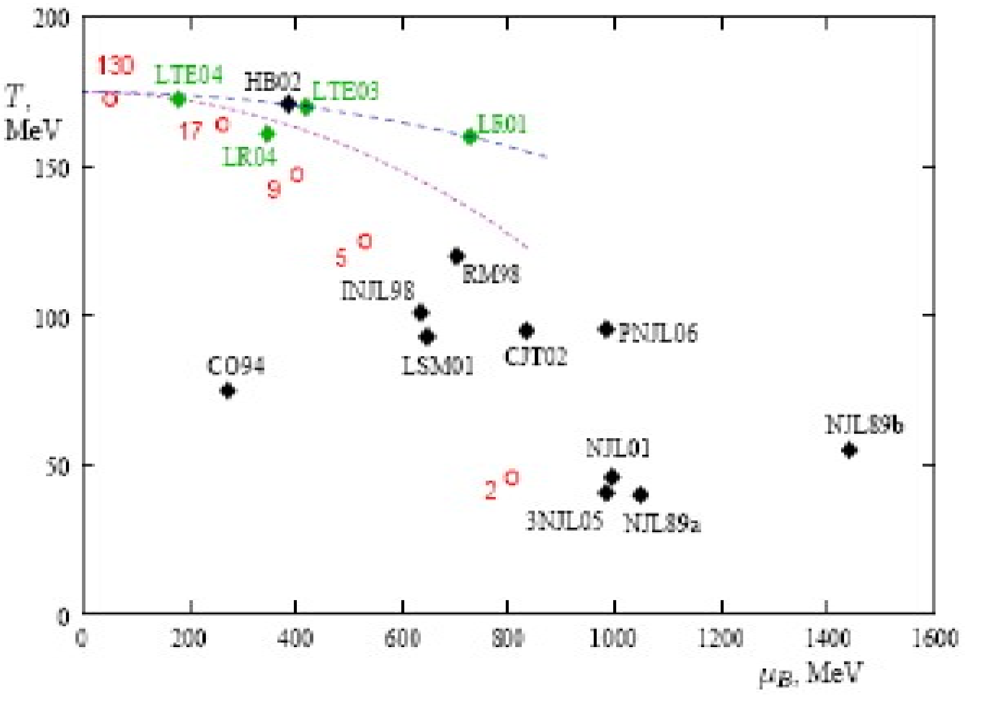
Backup

Where is the Critical Point ?

- **Critical Point Search**
  = One of the main goals in Low-E progs. at RHIC
- **Theory → No Consensus (Sign prob. at finite μ)**

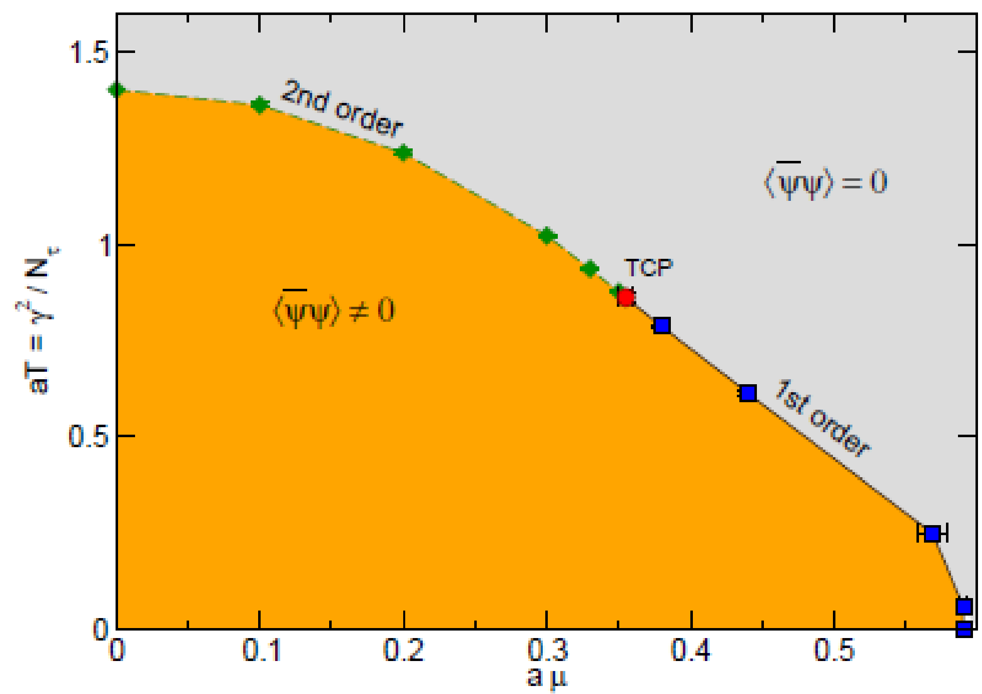
Can we attack CP in LQCD ? → Strong Coupling LQCD





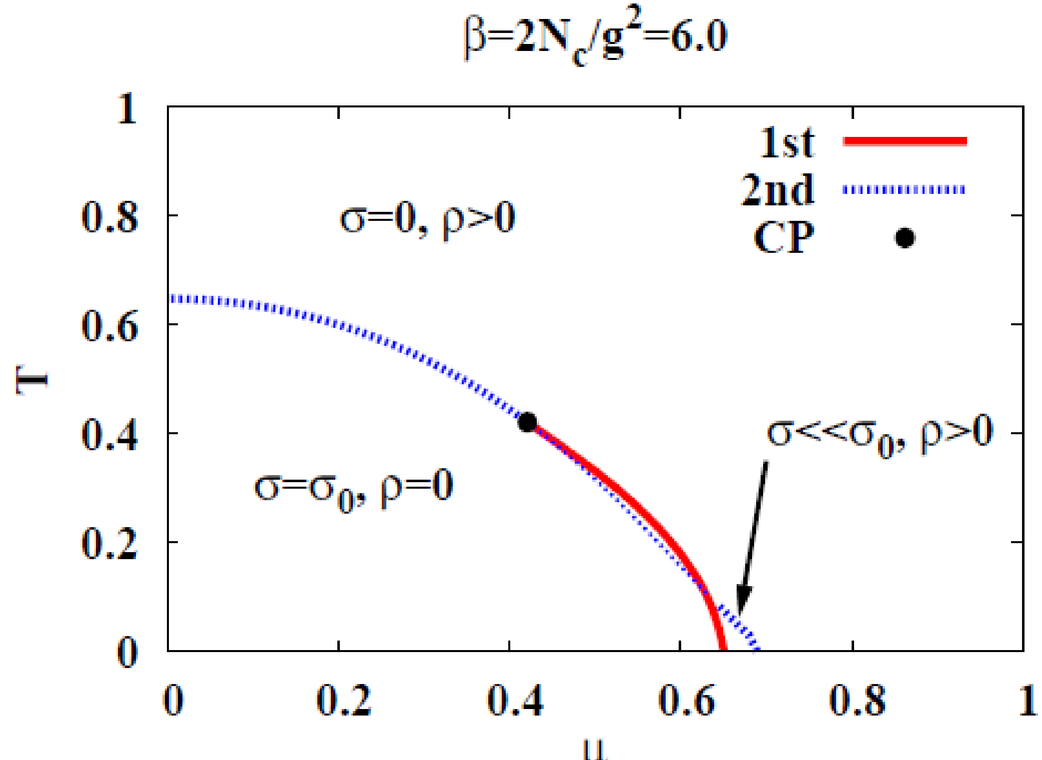
Monomer-Dimer-Polymer simulation

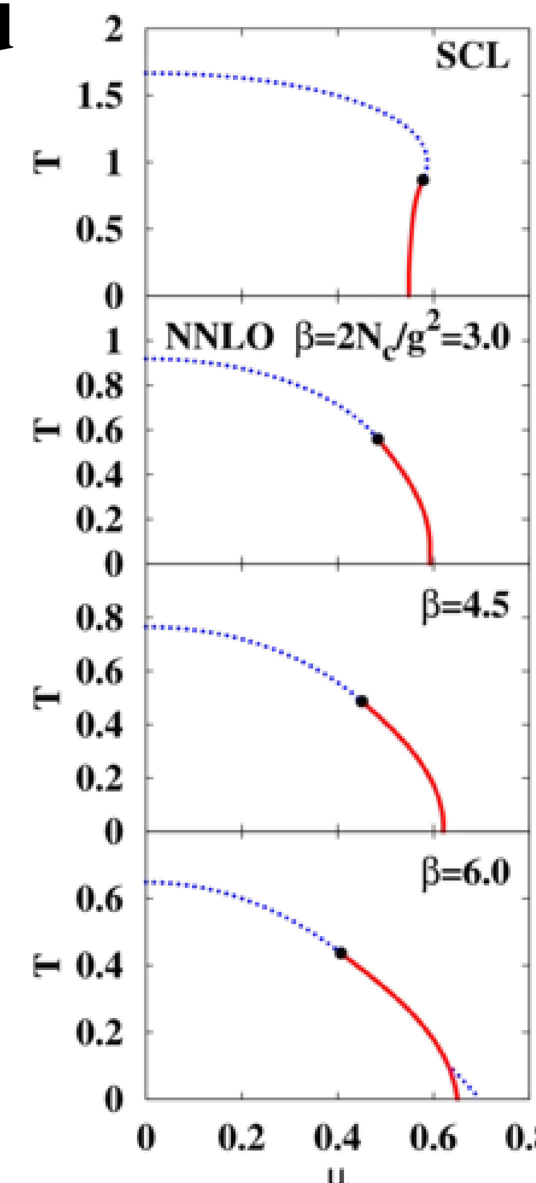
- Monomer-Dimer-Polymer simulation (MDP) *Karsch, Mutter ('89)*
  → Integrate out link variables first in the strong coupling limit
  - Sign problem is weakened (Cancellation of baryonic loops remains).
    → Phase diagram in SCL *de Forcrand, Fromm ('09)*
  - $T_c(\mu=0)$ and $\mu_c(T=0)$ qualitatively agree with SCL-LQCD (MF) results.
    $aT_c=5/3$ (MF), 1.41(3) (MDP)
    $a\mu_c=0.549$(MF), 0.593(MDP)
    $(aT_{TCP}, a\mu_{TCP})$
    =(0.867, 0.578)(MF),
    (0.86(2), 0.355(5))(MDP)



Phase diagram

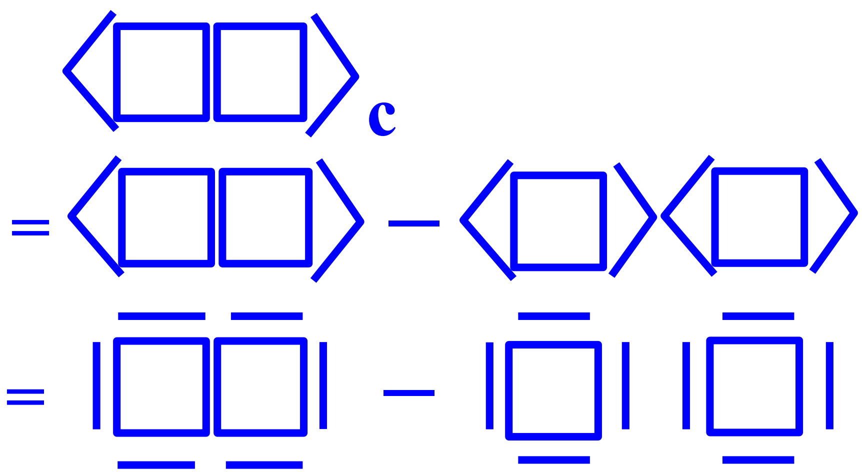
- **With increasing β, phase diagram is compressed in T direction.**
- **For finite β, 1st order boundary has a negative slope, $dT_c/d\mu<0$.** *c.f. Bilic, Demeterfi, Petersson ('92)*
- **Existence of the partially chiral restored phase in the higher μ direction of the hadron phase.**

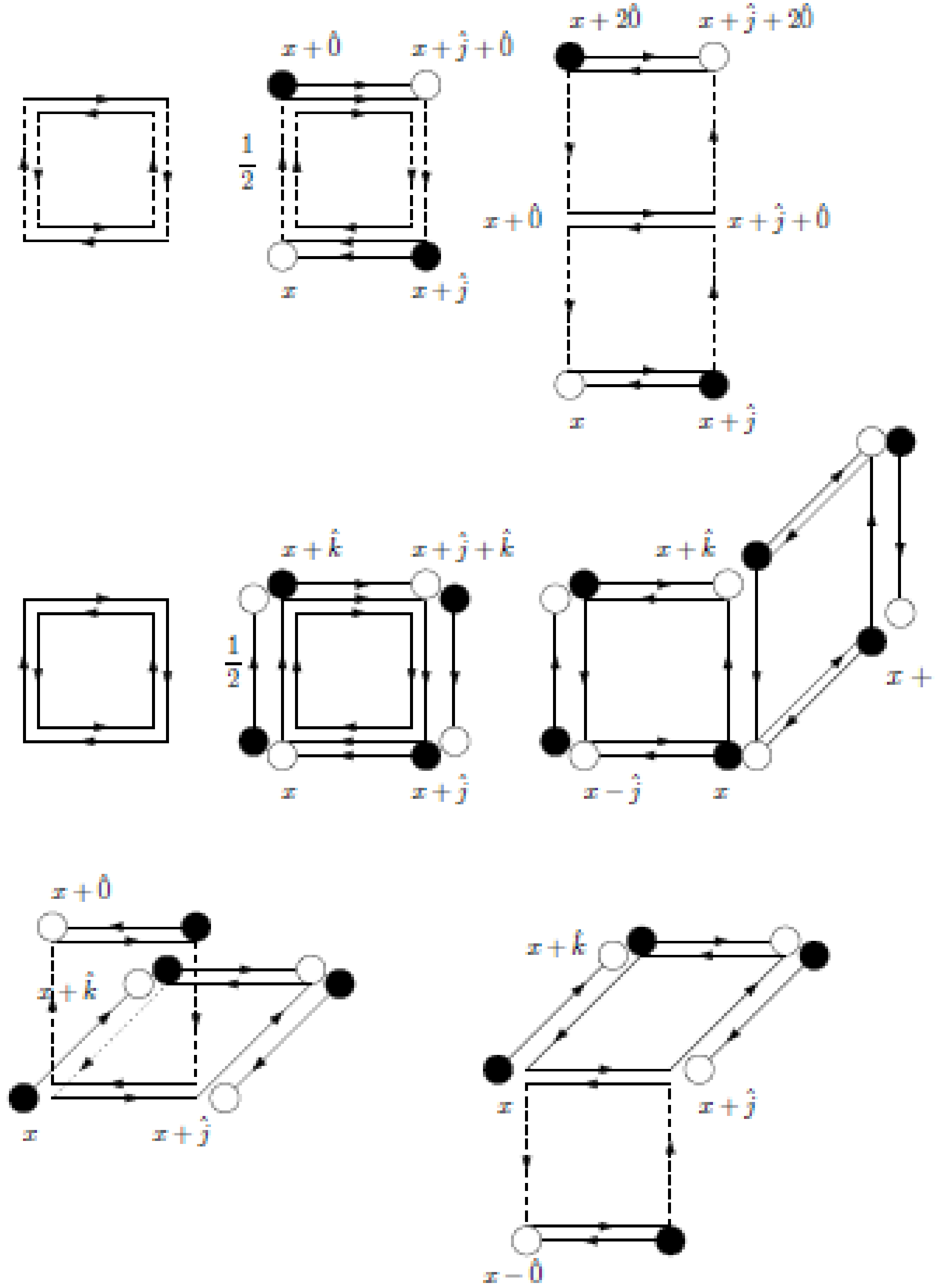




NNLO Effective Action

- **Cumulants of two plaquettes**
  = Correlation part of connected two plaquettes
- **1/d expansion: Σ_j MM ~ Const.**
  $\rightarrow \chi \sim d^{-1/4}$





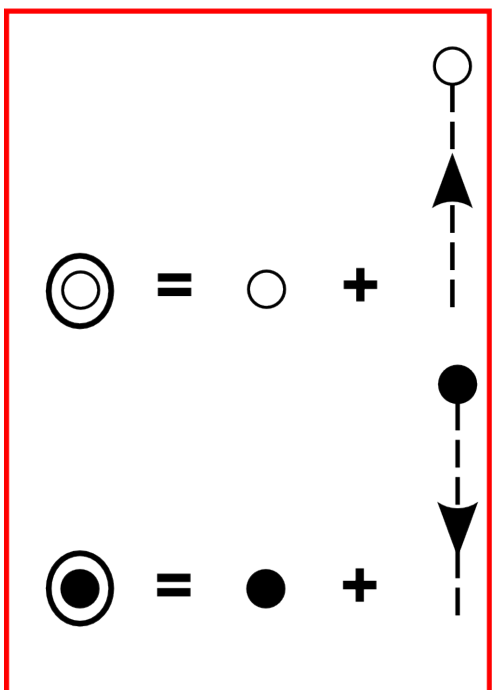
12 quarks **16 quarks**

power in d

3 – 1/4 x 12	**3 – 1/4 x 16**
=0	**= -1**

Dressed fermion

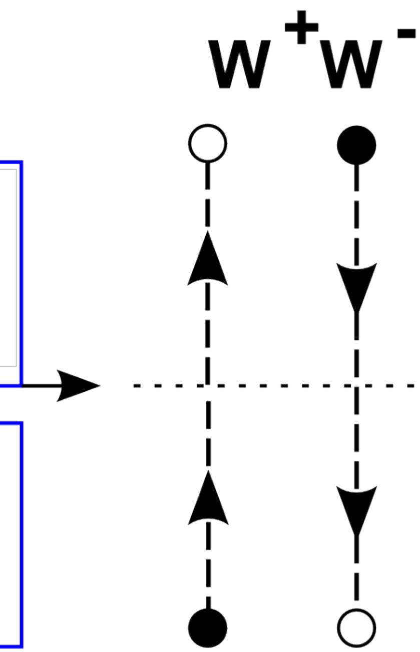
- **Next-to-nearest neighboring site interaction W±.**
  - **By introducing the “Dressed Fermion”, mixture of the quark field on the next temporal site, NNN interaction is rearranged to NN.**



$$\chi'_x = \chi_x + A e^{\mu} U_{0,x} \chi_{x+\hat{0}}$$

$$\bar{\chi}'_x = \bar{\chi}_x + \bar{A} \bar{\chi}_{x+\hat{0}} e^{-\mu} U^{\dagger}_{0,x}$$

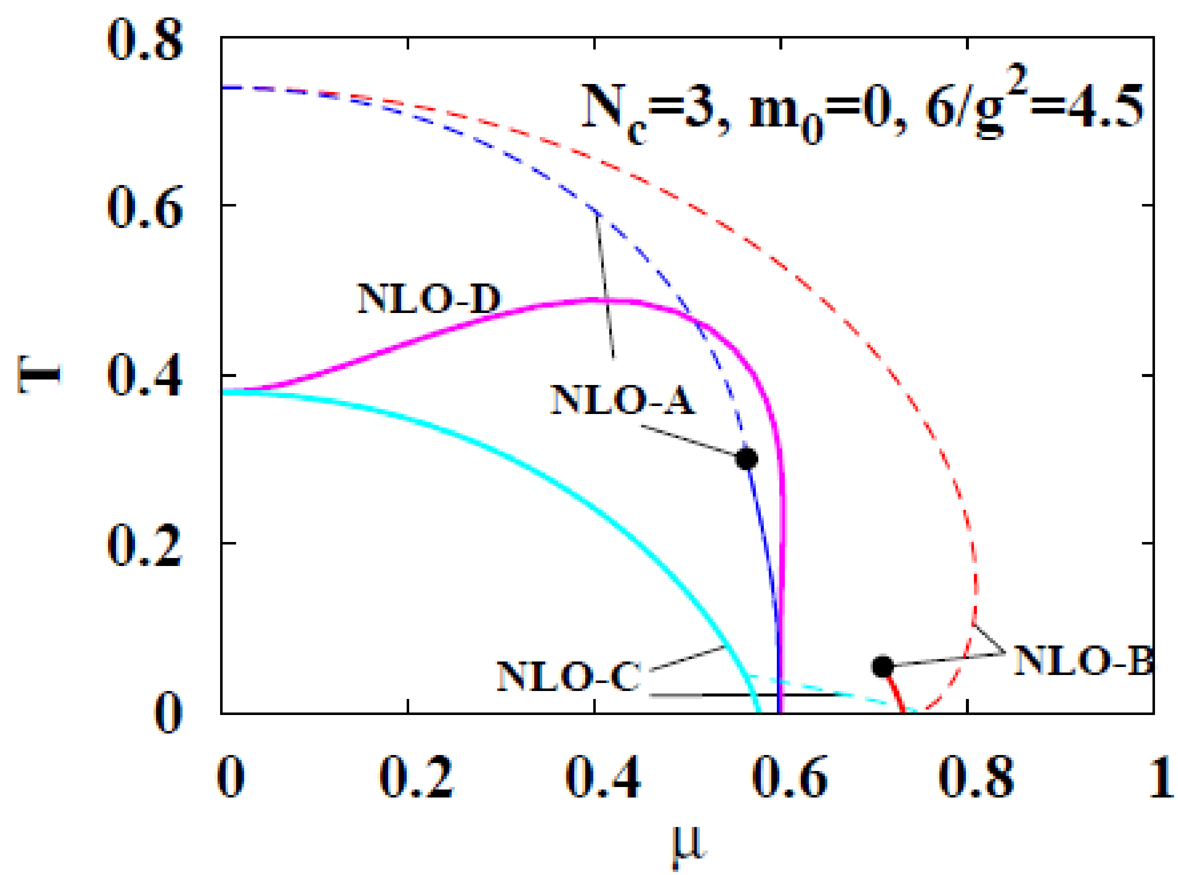
$$A = \mathcal{O}(1/g^4)$$



Truncation Deps. in NLO

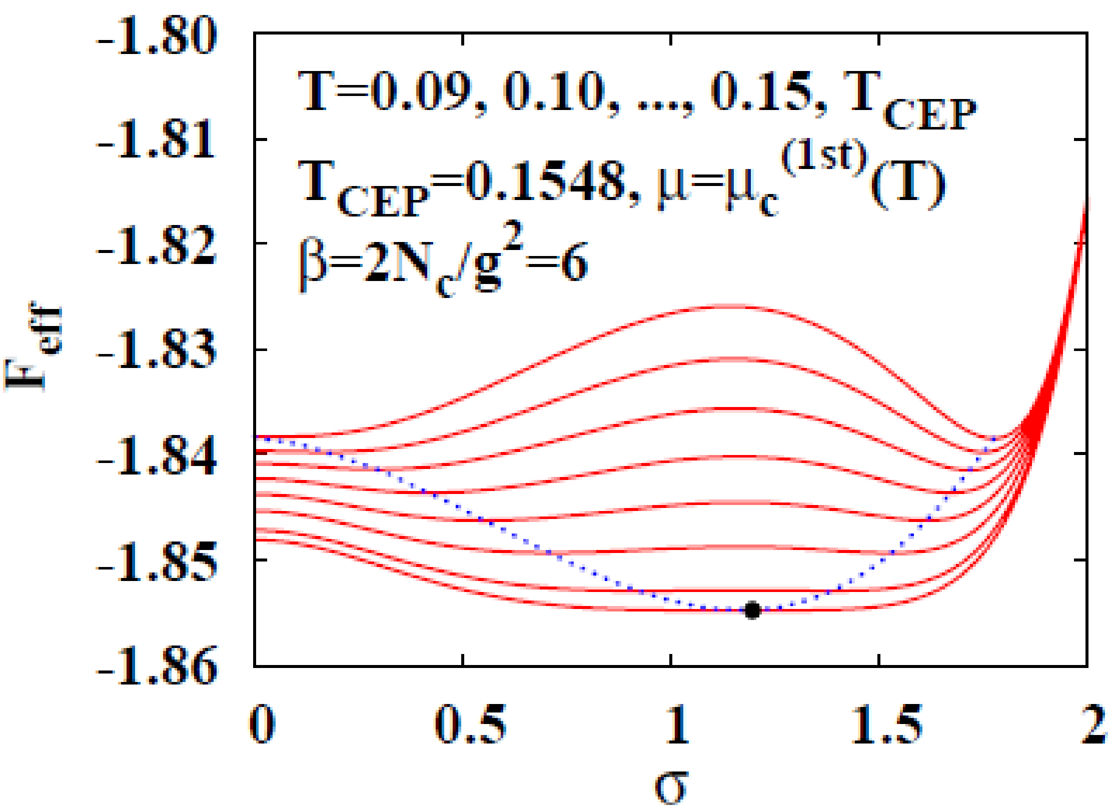
- **Phase diagram is sensitive to details, such as the truncation scheme in NLO.**

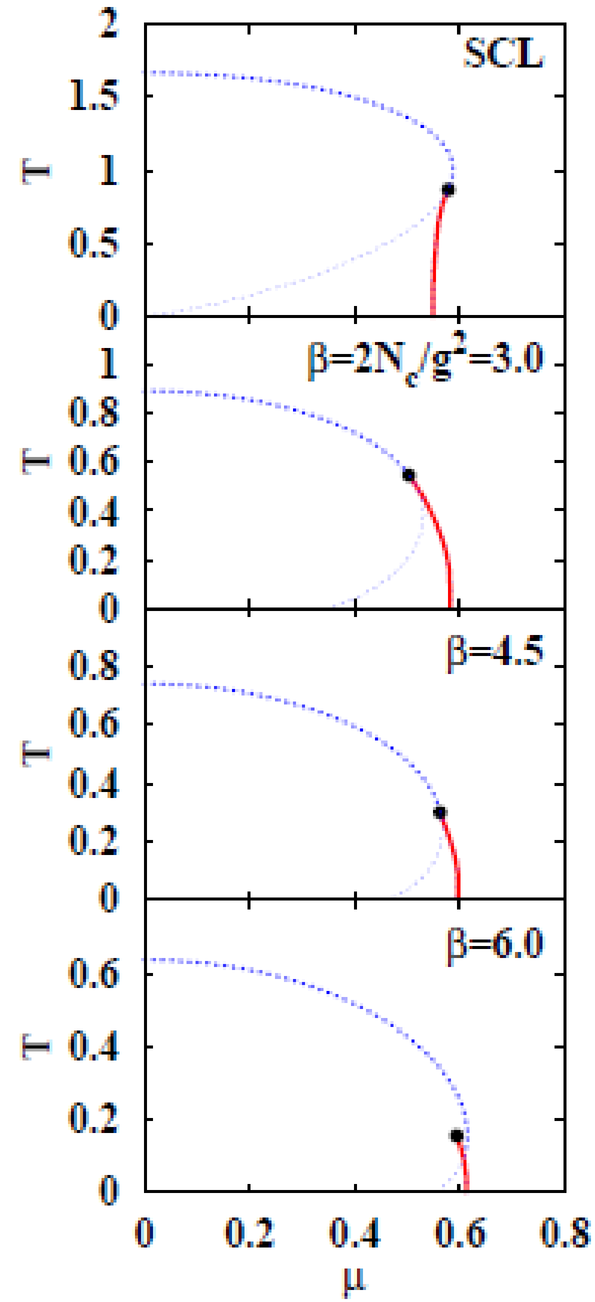
	$\delta\mu$	$\tilde{m}_q$	$\Delta\mathcal{F}_{\rm aux}$	$\mathcal{V}_q$
NLO-A	$\log\sqrt{\frac{Z_+}{Z_-}}$	$\frac{m_q}{\sqrt{Z_+Z_-}}$	$-N_c\log\sqrt{Z_+Z_-}$	$\mathcal{V}_q(\tilde{m}_q,\tilde{\mu},T)$
NLO-B	$\beta_\tau\omega_\tau$	$\frac{m_q}{1+\beta_\tau\varphi_\tau}$	$-N_c\log(1+\beta_\tau\varphi_\tau)$	$\mathcal{V}_q(\tilde{m}_q,\tilde{\mu},T)$
NLO-C	$\beta_\tau\omega_\tau$	$\tilde{m}_q^{(\rm NLO-C)}$	$-N_c\beta_\tau\varphi_\tau$	$\mathcal{V}_q(\tilde{m}_q,\tilde{\mu},T)$
NLO-D	0	$\tilde{m}_q^{(\rm NLO-D)}$	$-N_c\beta_\tau\varphi_\tau$	$\mathcal{V}_q(\tilde{m}_q,\mu,T)-\beta_\tau\omega_\tau\frac{\partial\mathcal{V}_q}{\partial\mu}$



Critical End Point in the Chiral Limit ?

- Vector field generates repulsive pot. for large ρ_q states, which may cause two local min. structure
  → Partially Chiral Restored matter
  may appear.





Strong Coupling Lattice QCD: Pure Gauge

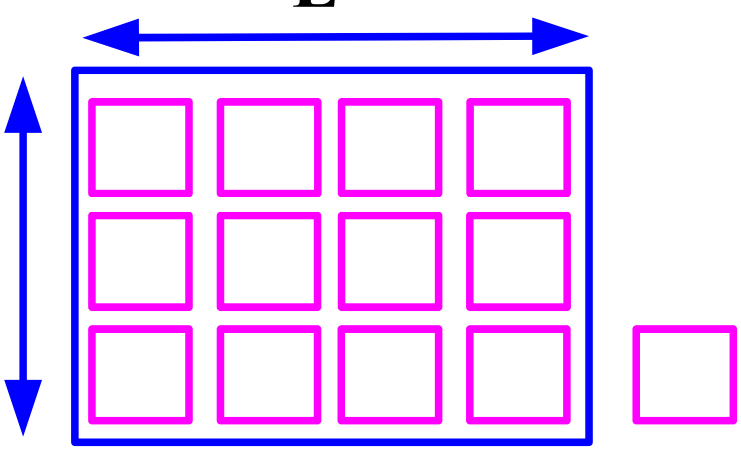
- **Quarks are confined in Strong Coupling QCD**

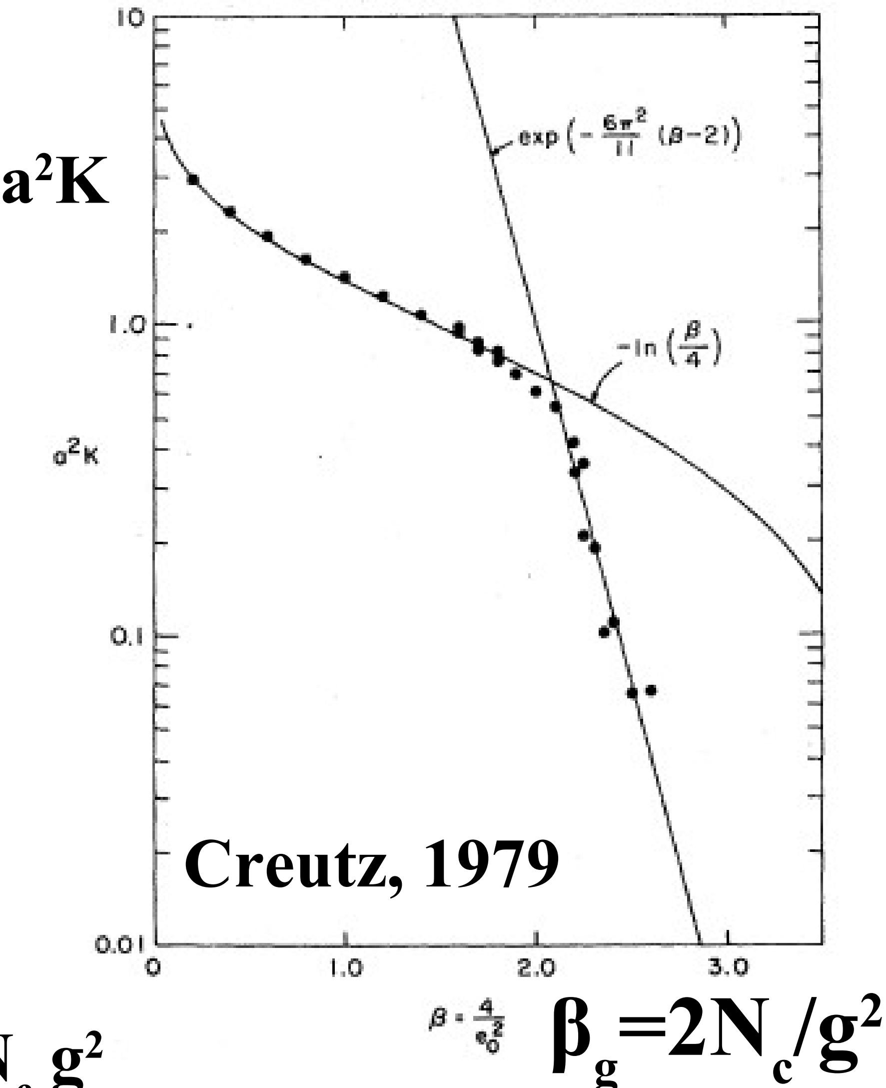
K. G. Wilson, PRD10(1974),2445
M. Creutz, PRD21(1980), 2308.
G. Munster, 1981

- **Strong Coupling Limit (SCL)**
  → Fill Wilson Loop with Min. # of Plaquettes
  → Area Law (Wilson, 1974)

$$S_{\rm LQCD} = -\frac{1}{g^2}\sum_{\Box} {\rm tr}\left[U_{\Box} + U_{\Box}^{\dagger}\right]$$

- **Smooth Transition from SCL to pQCD in MC (Creutz, 1980)**

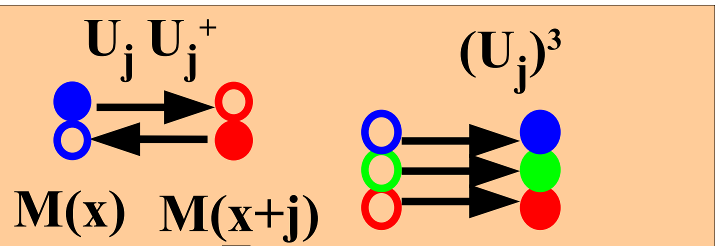




Chiral Phase Transition at Finite Density

P.H.Damgaard, D. Hochberg, N. Kawamoto, PLB158('86)239
Hasenfatz, Karsch, 1983; Azcoiti et al., 2003; Kawamoto, Miura, AO, Ohnuma, 2007

- **QCD phase transition is also expected at high density**
  - **Baryon Rich QGP and/or Color SuperConductor are expected in the Neutron Star Core**
- **Strong Coupling Limit in SU(N)**
  - **Quark Chemical Potential and Baryonic Composite → Chiral phase transition at $\mu_c = 0.66\ a^{-1}$**

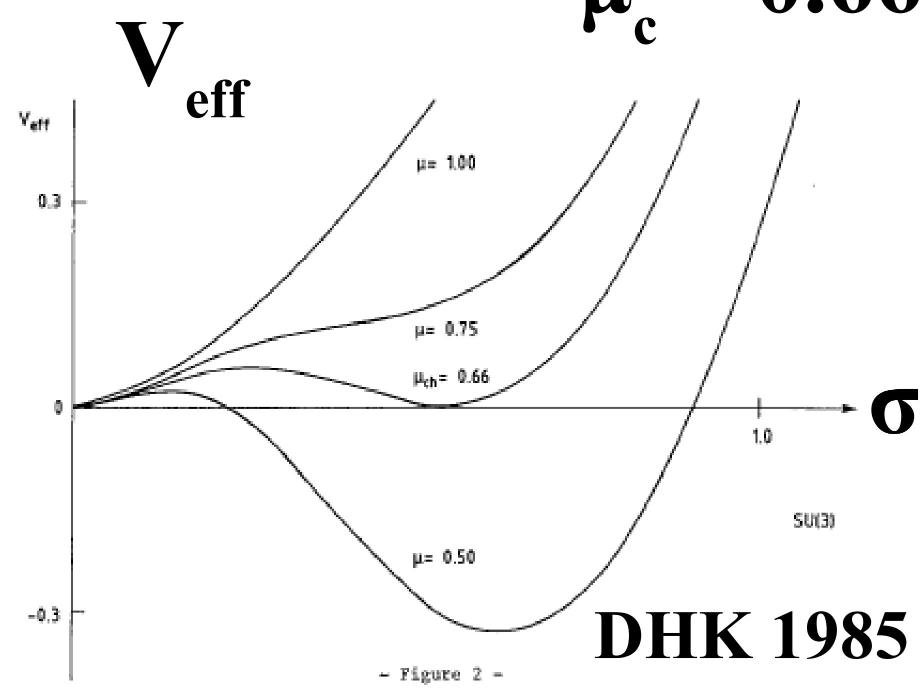


$\bar{B} = \epsilon \bar{\chi}\bar{\chi}\bar{\chi}/6 \quad B = \epsilon \chi\chi\chi/6$

$$\int dU\, U_{ab} U^{+}_{cd} = \frac{1}{N_c} \delta_{ad} \delta_{bc}$$

$$\int dU\, U_{ab} U_{cd} U_{ef} = \frac{1}{6} \epsilon_{ace} \epsilon_{bdf}$$

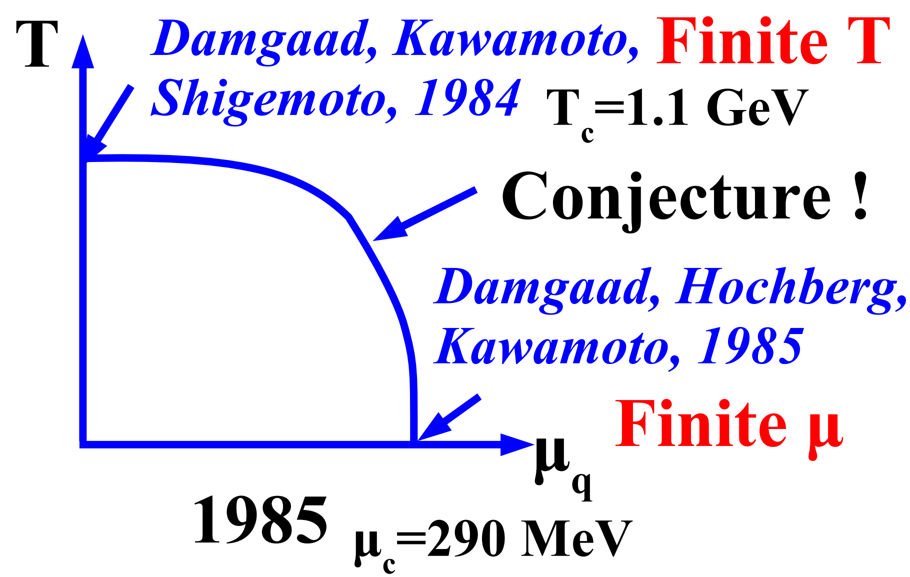
$\mu_c \sim 0.66$

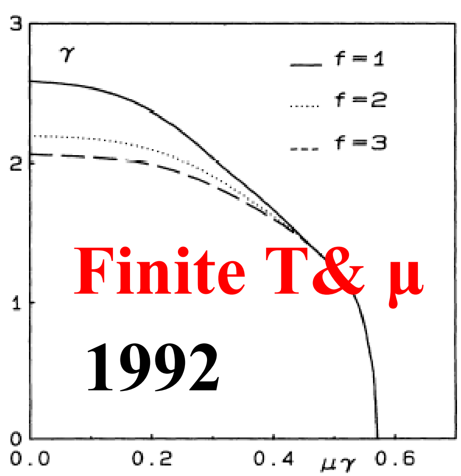


DHK 1985

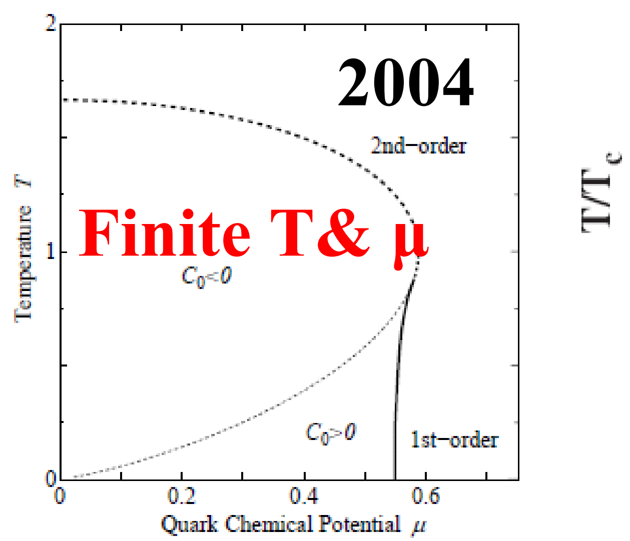
Evolution of Phase Diagram as a function of Time

- **Phase Diagram "Shape" becomes closer to that of Real World,**
  $R=\mu_c/T_c \sim (2\text{-}4)$
  - **1985 → R=0.26 (Zero T / Finite T)**
  - **1992 → R=0.28 (Finite T & μ)**
  - **2004 → R= 0.33 (Finite T& μ)**
  - **2007 → R=0.44 (Baryon)**

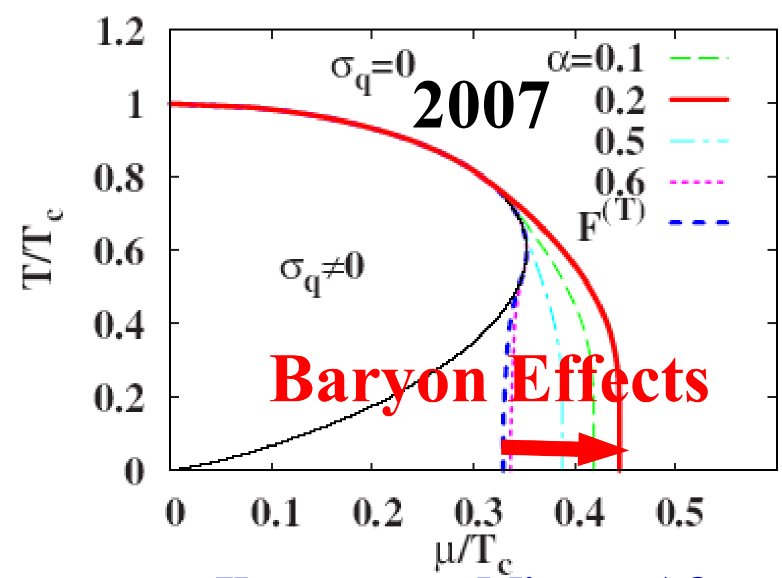




Bilic, Karsch, Redlich, 1992



Fukushima, 2004, Nishida, 2004

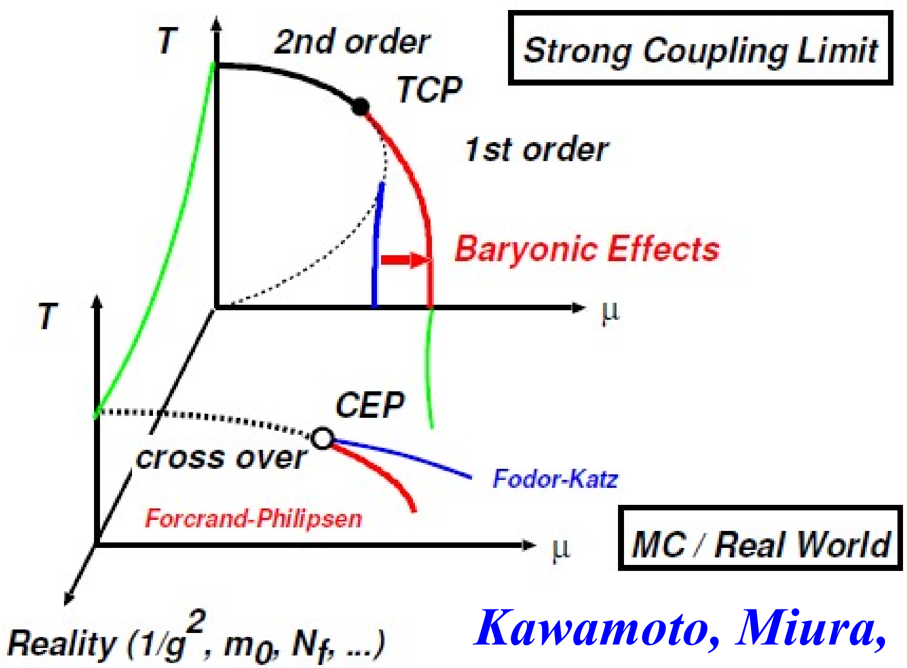


Kawamoto, Miura, AO, Ohnuma, 2007

Towards the Realistic Phase Diagram

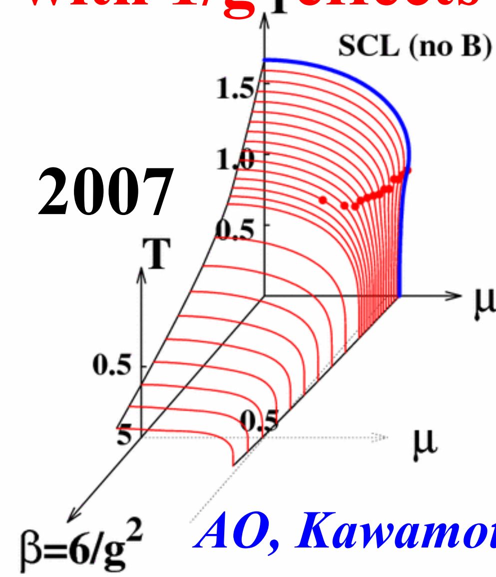
- **Why we cannot explain the phase diagram shape ?**
  → N_f (Staggered fermion) ? quark mass ? Finite Coupling ?
  - **μ_c (SCL) ~ $M_N/3$ (within a factor 2) , T_c (SCL) >> 200 MeV**
    → Larger problem should be in T_c , rather than in μ_c

Expectation before Calc.



Kawamoto, Miura, AO, Ohnuma, 2007

Preliminary Resuls with $1/g^2$ effects

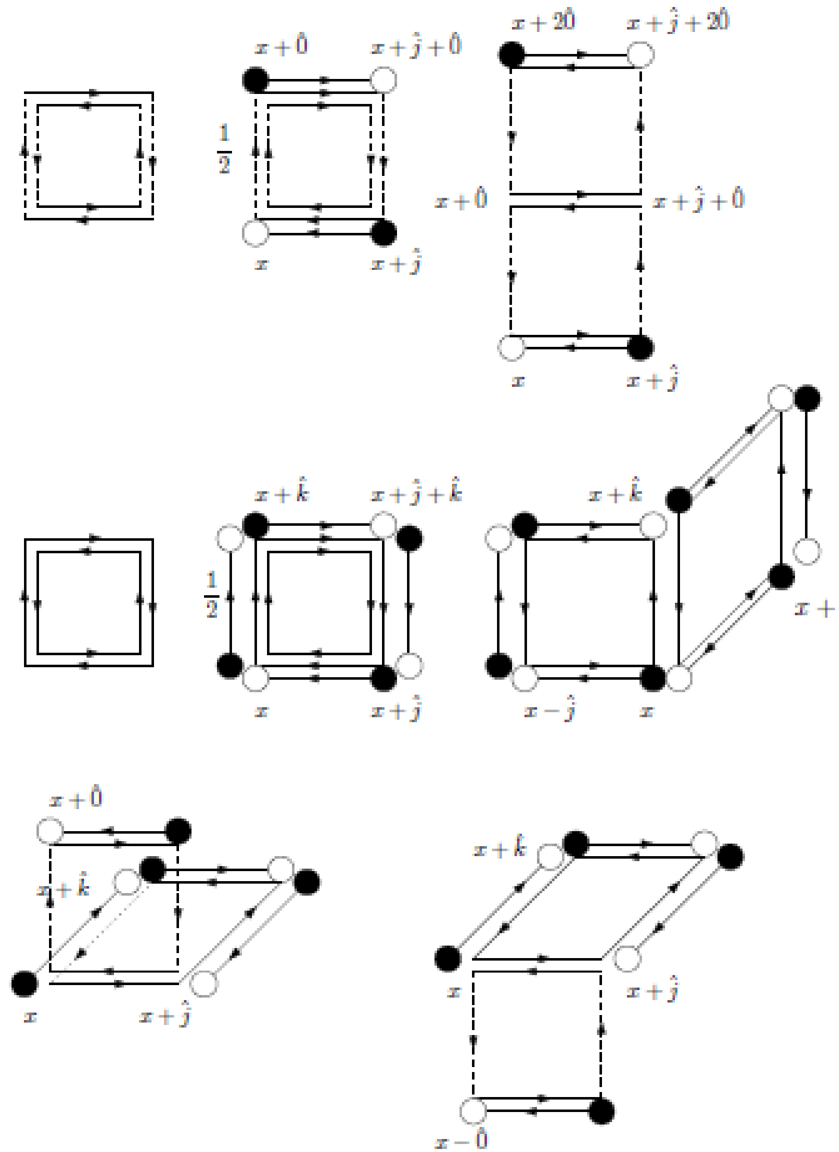


AO, Kawamoto, Miura, 2007

Gluon Contribution is important at High T

NNLO Effective Action

- **Cumulants of two plaquettes**
  = Correlation part of connected two plaquettes



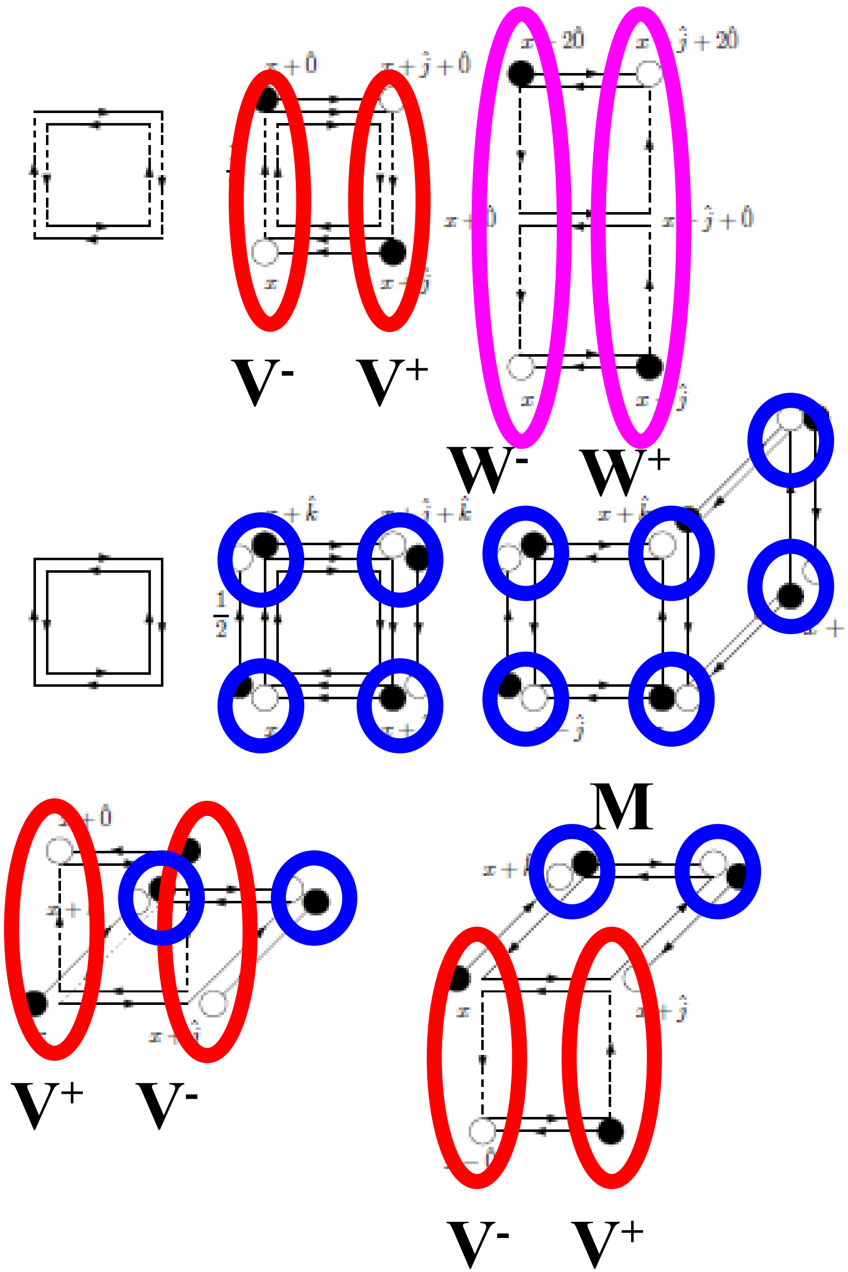
NNLO Effective Action

- **Cumulants of two plaquettes**
  = Correlation part of connected two plaquettes
  - **Uncorr. & Normalization part are suppressed in 1/d power**
  - **Effective Action consists of**

    $$V^- V^+ , W^- W^+ ,$$
    $$MMMM \; , MMMMMM \; ,$$
    $$V^- V^+ MM$$

  - **New type of Composite**
    = next-to-nearest neighboring site coupling in τ direction

$$W_x^+ = \chi_x U_{0,x} U_{0,x+\hat{0}} \bar{\chi}_{x+2\hat{0}}$$



Critical Point Evolution

- Critical Point in NLO approaches μ axis
  - Larger β
    → Stronger Vector Pot. ω_τ
  - Consistent with NJL models. (Kitazawa et al., '02; Sasaki-Friman-Redlich, '07; Fukushima'08)

    and MC suggestion
    (de Forcrand-Philipsen, '08)
- CP in NNLO→ μ(CP)/T(CP) ~ 1
  - Contradict to MC (μ/T > 1) ? (Ejiri, '08; Aoki et al.(WHOT), '08; Allton et al., '03,'05)
  - Underestimate of Tc may be the reason