Hot and dense hadrons in strong coupling lattice QCD

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- **Introduction**
- **Chiral Condensate in SC-LQCD**
  - **Strong coupling limit (SCL), next-to-leading order (NLO), and next-to-next-leading order (NNLO) results**
- **Meson masses in SCL-LQCD**
  - **Mean Field Treatment and Brown-Rho scaling**
- **Summary**

Hadron Mass Modification

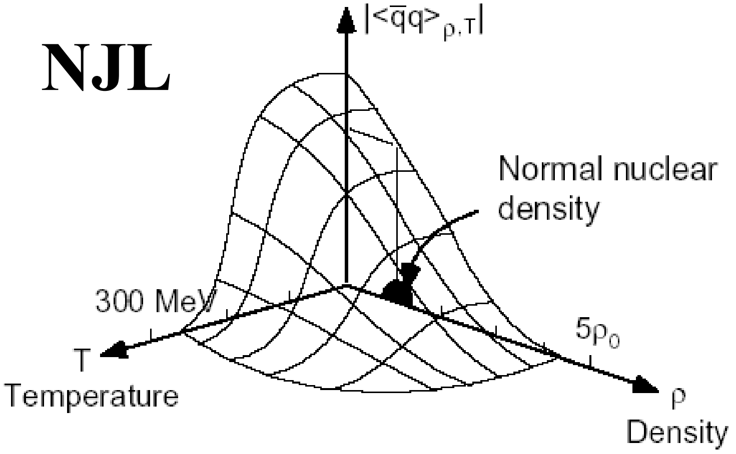
- **Medium meson mass modification**
  may be the signal of partial restoration of chiral sym.

  Brown, Rho, PRL66('91)2720;
  Kunihiro,Hatsuda, PRep 247('94),221; Hatsuda, Lee, PRC46('92)R34.

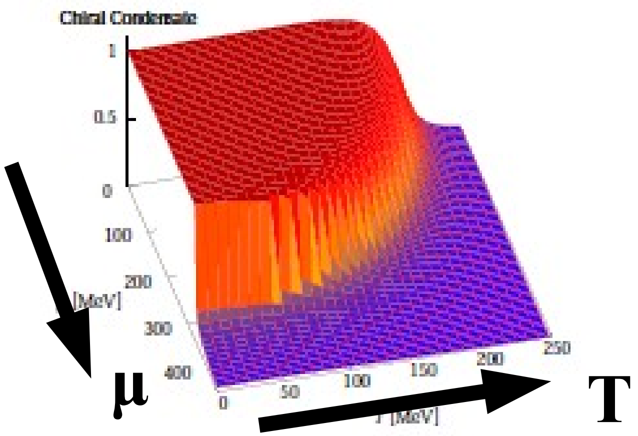
- **Brown-Rho Scaling**

$$M_N^*/M_N = M_\sigma^{¿}/M_\sigma = M_\rho^{¿}/M_\rho = M_\omega^{¿}/M_\omega = f_\pi^*/f_\pi$$

NJL



W. Weise, Nucl. Phys. A 553 (1993) 59c

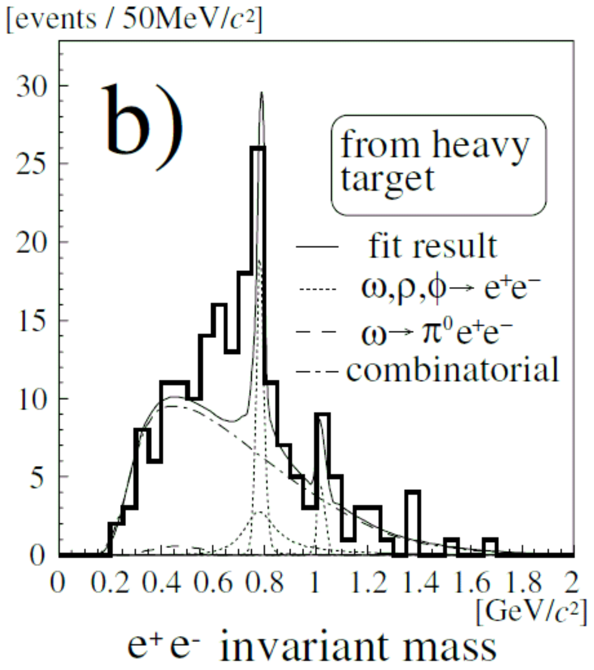


PNJL (Fukushima, 2008)

Hadron Mass Modification

- **Medium meson mass modification**
  is suggested experimentally.
  CERES Collab., PRL75('95),1272;
  KEK-E325 Collab.(Ozawa et al.), PRL86('01),5019;
  PHENIX Collab., arXiv:0706.3034

Interpretation is model dependent
→ Investigation in
non-perturbative QCD is desired !



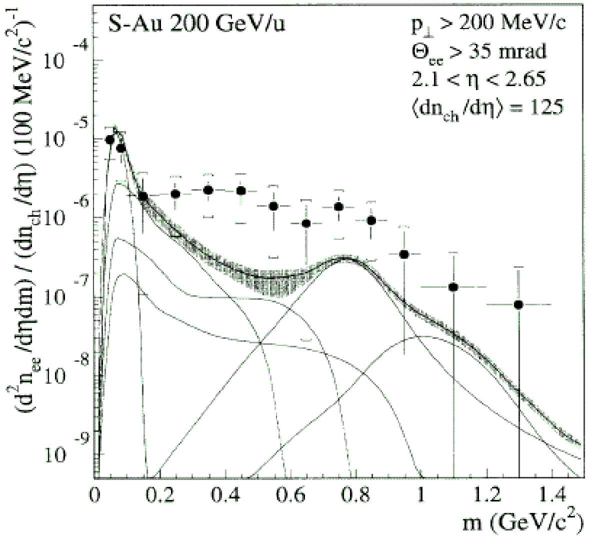
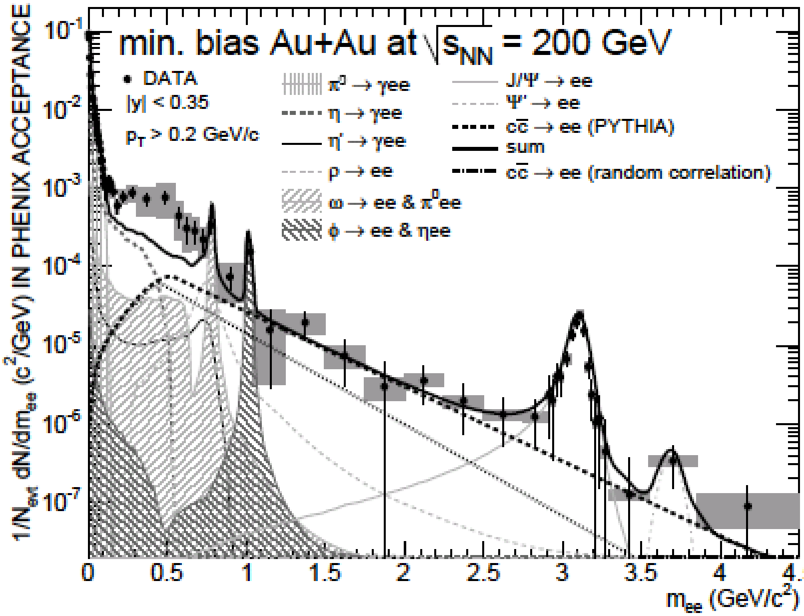


FIG. 4. Inclusive e^+e^- mass spectra in 200 GeV/nucleon S-Au collisions. For explanations see Fig. 2.



Hot and Dense Hadrons

- **Hadron masses in medium: Two facets**
  - **How does the *chiral condensate* behave at finite T and μ ?**
  - **How are *meson masses* related to the *chiral condensates* ?**
- **Can we understand in-medium meson masses in lattice QCD ?**
  - **Finite T (Zero μ) → Possible !**
    Maximum Entropy Method
    Asakawa, Hatsuda, Nakahara; Aarts, Foley
  - **Finite μ → Not yet possible (sign problem)**
- **We investigate *chiral condensate* and *meson masses***
  at finite T and μ in strong coupling lattice QCD (SC-LQCD).
  - **strong coupling (g >> 1) → Expansion in # of plaquetts.**

Disclaimer

- **Our work is based on**
  - **the *Strong Coupling Limit (SCL)* of Lattice QCD**
  - **with n_f (=1, 2, 3) staggered fermions,**
    which corresponds to *$N_f=4n_f$ flavors*.
    → The effective potential may be different in continuum theory with 2+1 flavors, and vacuum hadron masses are not very well explained.
    [16 doublers (4 components) → 16 doublers (1 component) ~ 4 flavors (tates)]

- **Notice**
  - **"Strong Coupling" in this work means , g >> 1,**
    and is different from the large N_c,

    $$N_c \gg 1,\ \lambda = N_c\, g^2 = \text{const.},\quad g^2 \ll 1,$$

    and "Strong Coupling" in AdS/CFT,

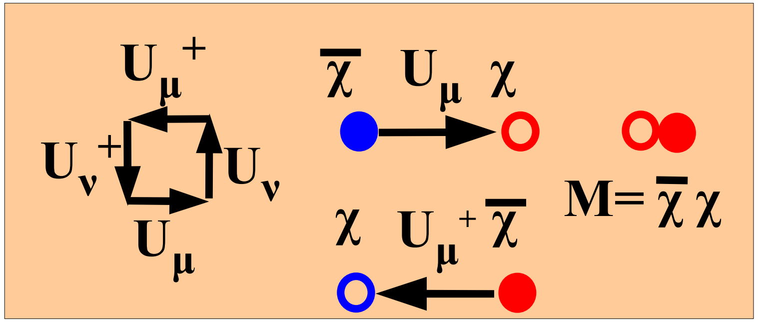
    $$N_c \gg 1,\ \lambda = N_c\, g^2 \gg 1.$$

Chiral Condensate in Strong Coupling Lattice QCD

Strong Coupling Lattice QCD

- Lattice QCD=ab initio, non-perturbative theory

$$S_{\rm LQCD} = \frac{1}{2}\sum_{x,j}\left[\eta_{\nu,x}\bar{\chi}_x U_{\nu,x}\chi_{x+\hat{\nu}} - \eta^{-1}_{\nu,x}\bar{\chi}_{x+\hat{\nu}}U^\dagger_{\nu,x}\chi_x\right] - \frac{1}{g^2}\sum_{\Box}{\rm tr}\left[U_\Box + U^\dagger_\Box\right] + m_0\sum_x \bar{\chi}_x\chi_x$$



- Strong Coupling Lattice QCD
  - $1/g^2 << 1$ → perturbative treatment of plaquetts
    - Effective action of color singlet objects (Mesons, Baryons, Loops)
  - Great successes in pure YM
    - Area law (Wilson), Strong and weak coupling (Creutz), Character expansion to higher orders (Munster), …
  - Chiral transition at finite T and μ: → mainly discussed in the Strong Coupling Limit (g → ∞)
    - *Kawamoto, Damgaard, Shigemoto; Bilic, Karsch, Redlich; Fukushima; Nishida, Fukushima, Hatsuda; …*

→ NLO and NNLO in Strong Coupling Expansion

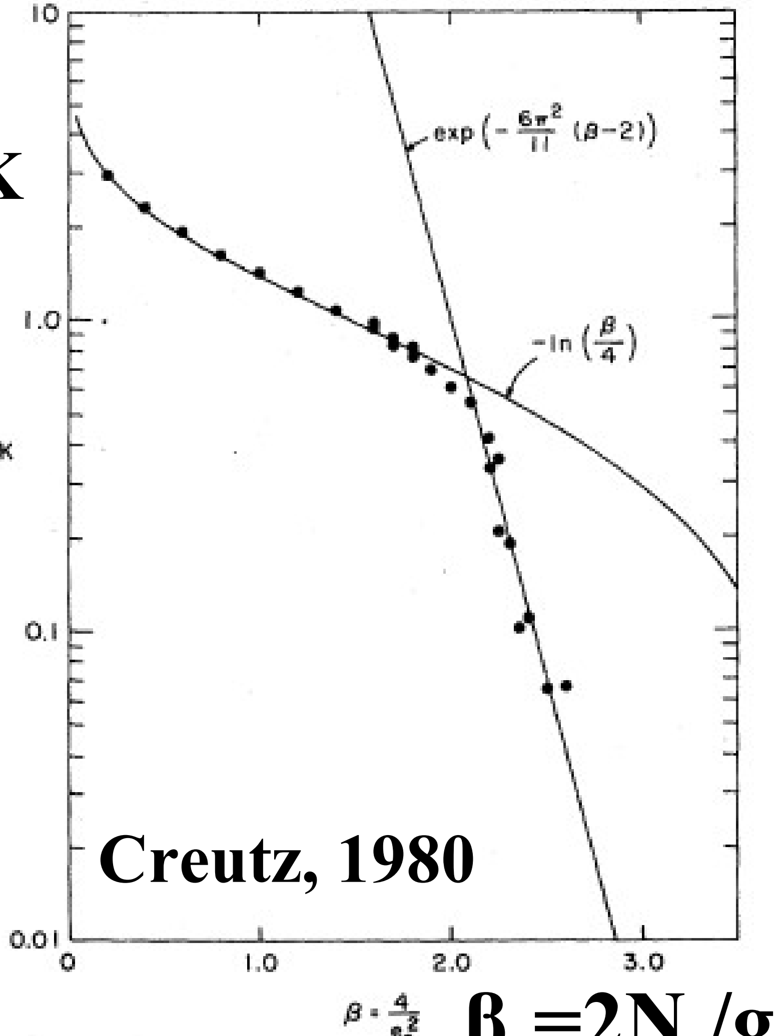
Strong Coupling Lattice QCD: Pure Gauge

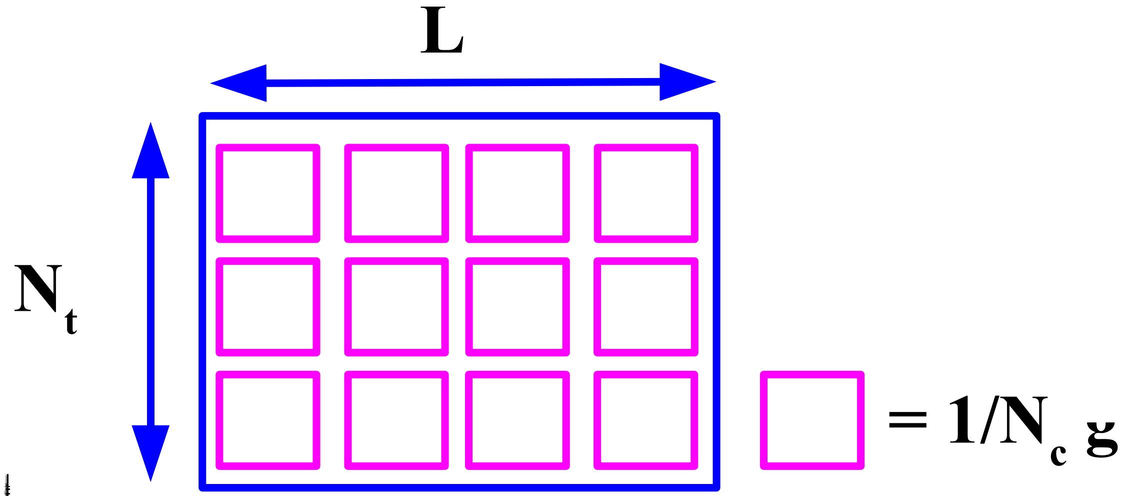
K. G. Wilson, PRD10(1974),2445
M. Creutz, PRD21(1980), 2308.
G. Munster, 1981

- **Quarks are confined in Strong Coupling QCD**
  - **Strong Coupling Limit (SCL)**
    → Fill Wilson Loop with Min. # of Plaquettes
    → Area Law (Wilson, 1974)

$$S_{\rm LQCD} = -\frac{1}{g^2}\sum_{\Box} \mathrm{tr}\left[U_{\Box} + U_{\Box}^{\dagger}\right]$$

  - **Smooth Transition from SCL to pQCD in MC (Creutz, 1980)**

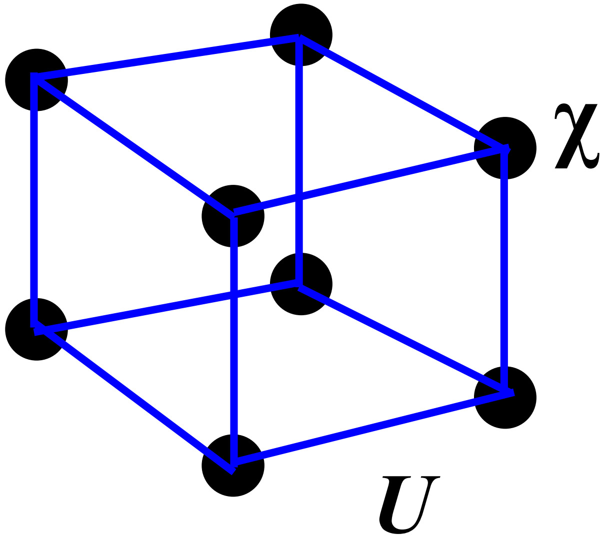




Strong Coupling Lattice QCD with Quarks

- **Standard Lattice QCD simulation**
  → Monte-Carlo integral of Fermion det.

$$Z=\int DU\,D\chi\,D\bar{\chi}\exp\left(-S_G-\bar{\chi}A\chi\right)$$
$$=\int DU\,\det(A)\exp\left(-S_G\right)$$



- **Strong Coupling Lattice QCD with Quarks**
  → Small # of plaquettes
  → Link integral of exp(-S)
  → Effective Action of quark composites
  → Effective potential in mean field approximation

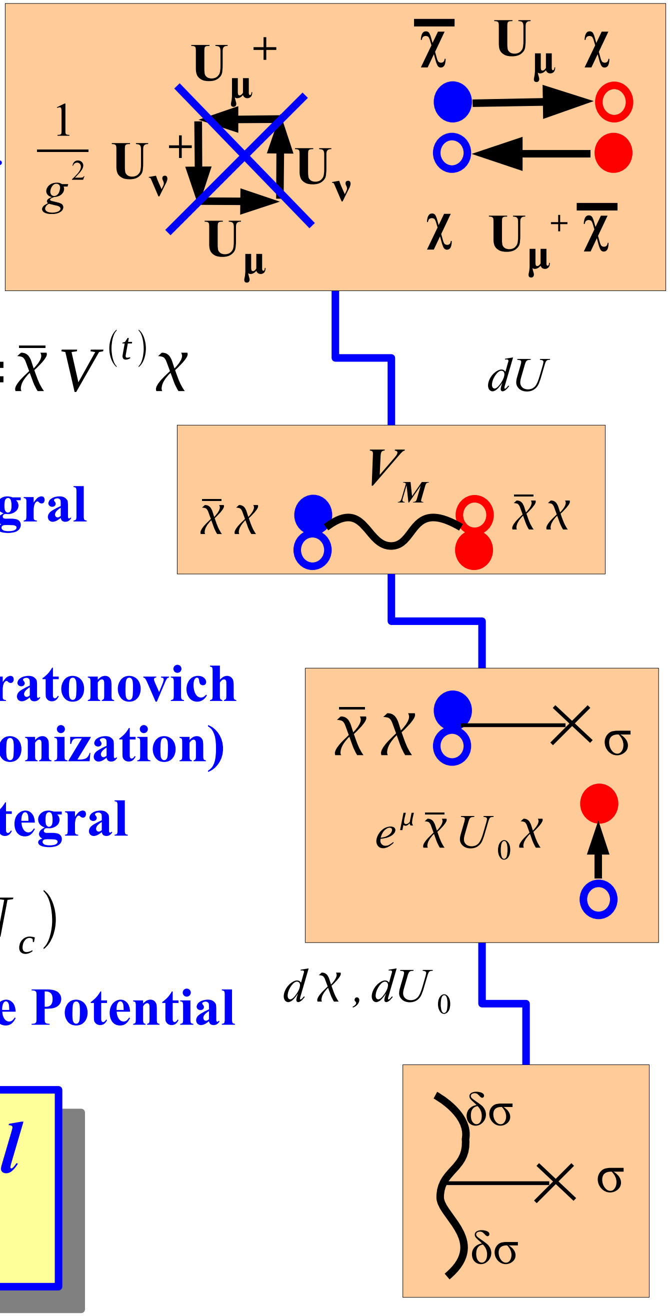
$$\int dU\,U_{ab}U^{+}_{cd}=\frac{1}{N_c}\delta_{ad}\delta_{bc}$$

$$Z=\int DU\,D\chi\,D\bar{\chi}\exp\left(-S_G-S_F\right)$$
$$=\int D\chi\,D\bar{\chi}\int DU\,\exp\left(-S_F\right)\left(1-S_G+\frac{1}{2}S_G^2+\ldots\right)$$
$$=\int D\chi\,D\bar{\chi}\exp\left(-S_{\mathrm{eff}}\left(M=\bar{\chi}\chi\,,B=\epsilon\chi\chi\chi\,,\ldots\right)\right)$$

Effective Potential in SCL-LQCD

- **QCD Lattice Action (Finite T treatment)**

Damgaard, Kawamoto, Shigemoto, 1984; Bilic et al., 1992;
Nishida, '04; Fukushima, '04; Kawamoto, Miura, AO, Ohnuma, '07; .



$$S = \cancel{S_G} + S_F^{(s)} + S_F^{(t)} + m_0 \bar\chi\chi$$

Strong Coupling Limit

$$S_F^{(t)} = \frac{1}{2}\sum_x \left(e^\mu \bar\chi_x U_0(x)\chi_{x+\hat{0}} - e^{-\mu}\bar\chi_{x+\hat{0}} U_0^+(x)\chi_x\right) = \bar\chi V^{(t)}\chi$$

$$\to -\frac{1}{2}(\bar\chi\chi)V_M(\bar\chi\chi) + \bar\chi(V^{(t)}+m_0)\chi$$

Spatial-link integral
(1/d expansion)

$$\to \frac{1}{2}\sigma V_M \sigma + \bar\chi(V^{(t)} + V_M\sigma + m_0)\chi$$

Hubbard-Stratonovich
Transf. (Bosonization)

Fermion and Temporal-link Integral

$$\to L^d N_\tau \boxed{\left[\frac{b_\sigma}{2}\bar\sigma^2 + V_{\mathrm{q}}(b_\sigma\bar\sigma, T, \mu)\right]}$$

$(b_\sigma = d/2N_c)$

SCL Effective Potential

We can obtain the Effective Potential analytically at finite T and μ

NLO ($1/g^2$) Effective Potential (1)

- **1/d expansion of Plaquette action (Spatial One-Link Integral)**

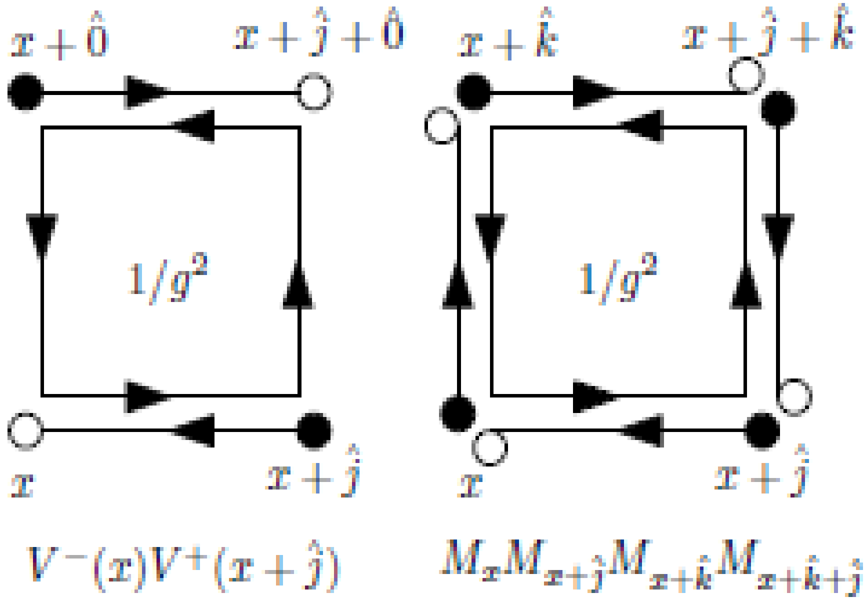
Faldt, Petersson (86); Bilic, Karsch, Redlich (92)

$$\int dU\, U_{ab} U^{+}_{cd} = \frac{1}{N_c} \delta_{ad} \delta_{bc}$$

  - **Spatial plaquett → MMMM**
  - **Temporal Link → $V^{+}V^{-}$**

$$V_x^{+} = e^{\mu} \bar{\chi}_x U_0(x) \chi_{x+\hat{0}}$$

$$V_x^{-} = e^{-\mu} \bar{\chi}_{x+\hat{0}} U_0^{\dagger}(x) \chi_x$$



- **Effective Action**

$$\Delta S_{\beta} = \frac{1}{4 N_c^2 g^2} \sum_{x,\, j>0} \left(V_x^{(+)} V_{x+\hat{j}}^{(-)} + V_x^{(-)} V_{x+\hat{j}}^{(+)} \right) - \frac{1}{8 N_c^4 g^2} \sum_{x,\, k>j>0} M_x M_{x+\hat{j}} M_{x+\hat{j}+\hat{k}} M_{x+\hat{k}}$$

NLO ($1/g^2$) Effective Potential (2)

K. Miura, AO, arXiv:0806.3357; Miura, Kawamoto, AO, PoS(Lat2008),

- **Extended Hubbard-Stratonovich Transf.**
  → Auxiliary fields for *MM*, V_+, V_-

$$-MMMM \rightarrow \varphi_s^2 - 2\varphi_s MM \quad (\varphi_s \sim MM)$$

$$V_+ V_- \rightarrow \varphi_\tau^2 + (V_+ - V_-)\varphi_\tau - \omega_\tau^2 - (V_+ + V_-)\omega_\tau \quad (\text{EHS})$$

$$(\varphi_\tau \sim (V_+ - V_-)/2, \quad \omega_\tau \sim -(V_+ + V_-)/2 = \rho_q)$$

- **NLO Effective Action: Modification of coefficients**

$$V_\pm \rightarrow (1 + \beta_\tau(\varphi_\tau \pm \omega_\tau)) V_\pm$$

$$b_\sigma M_x M_{x+\hat{j}} \rightarrow (b_\sigma + 2\beta_s \varphi_s) M_x M_{x+\hat{j}}$$

- **NLO Effective Potential**

$$F_{\text{eff}} = \frac{1}{2} b_\sigma \sigma^2 + \beta_s \varphi_s \sigma^2 + \frac{\beta_\tau}{2}(\varphi_\tau^2 - \omega_\tau^2) + \frac{\beta_s}{2}\varphi_s^2$$

$$+ V_q(\tilde{m}_q, \tilde{\mu}, T) - N_c \log Z_\chi$$

$$\tilde{m}_q = (b_\sigma + 2\beta_s \varphi_s \sigma + m_0)/Z_\chi, \quad \tilde{\mu} = \mu - \log(Z_+/Z_-)/2$$

$$Z_\chi = \sqrt{Z_+ Z_-}, \quad Z_\pm = 1 + \beta_\tau(\varphi_\tau \pm \omega_\tau)$$

W.F. ren.

μ shift

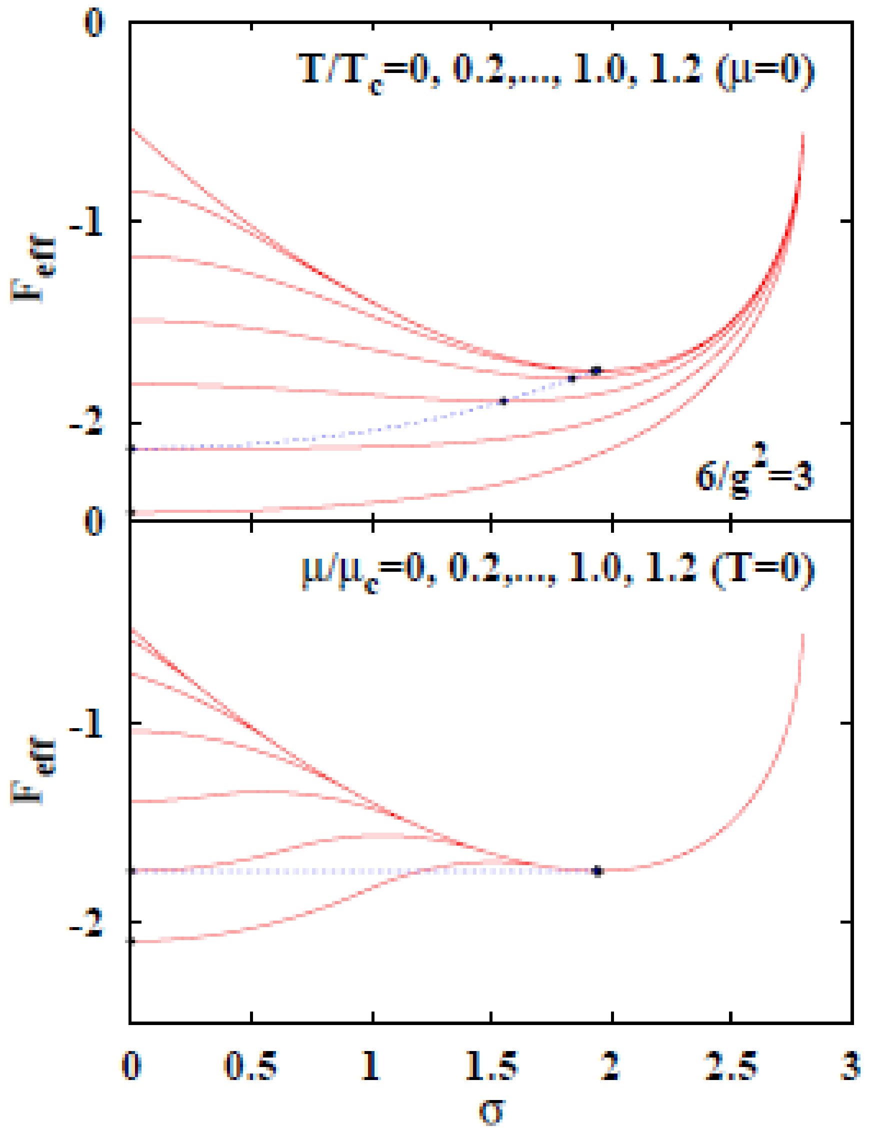
Mass mod.

NLO ($1/g^2$) Effective Potential (3)

- **Phase transition at finite T**
  → 2nd order in χ limit
- **Phase transition at finite μ**

  → 1st order with moderate $6/g^2$.

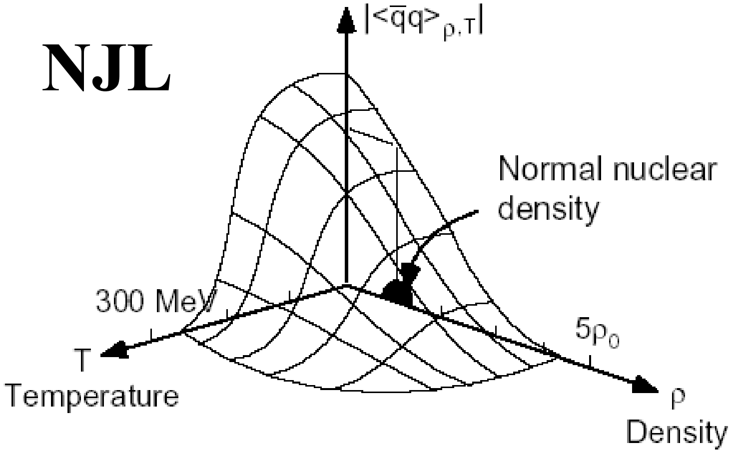
Density Jumps from almost 0 to a large value.
~ Nuclear matter is NOT described.



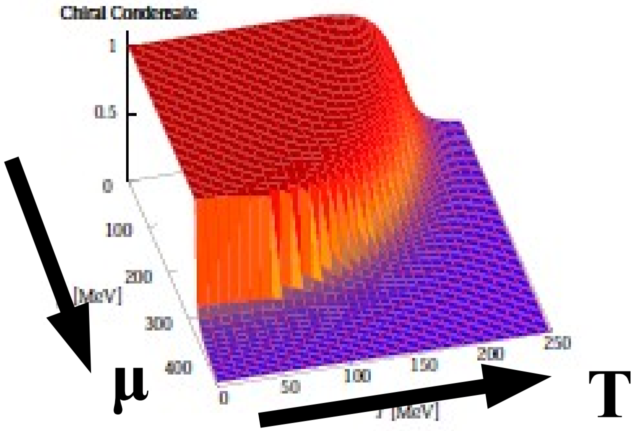
Chiral Condensate in SC-LQCD

- **Comparison of NJL, PNJL, SCL-LQCD, and NLO-SC-LQCD**

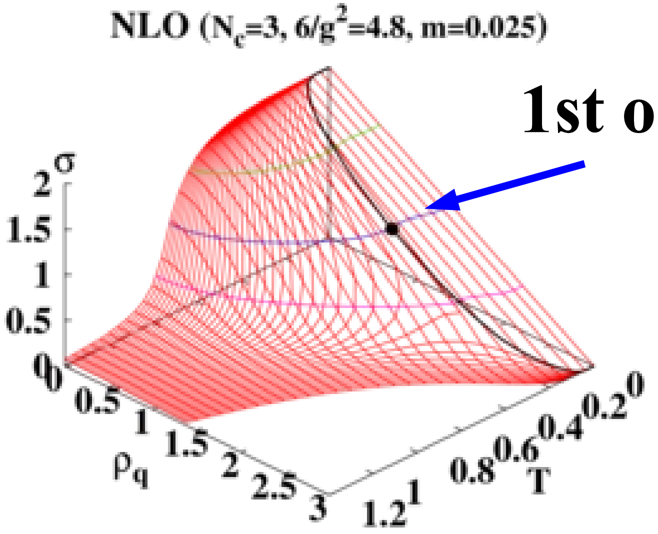
NJL



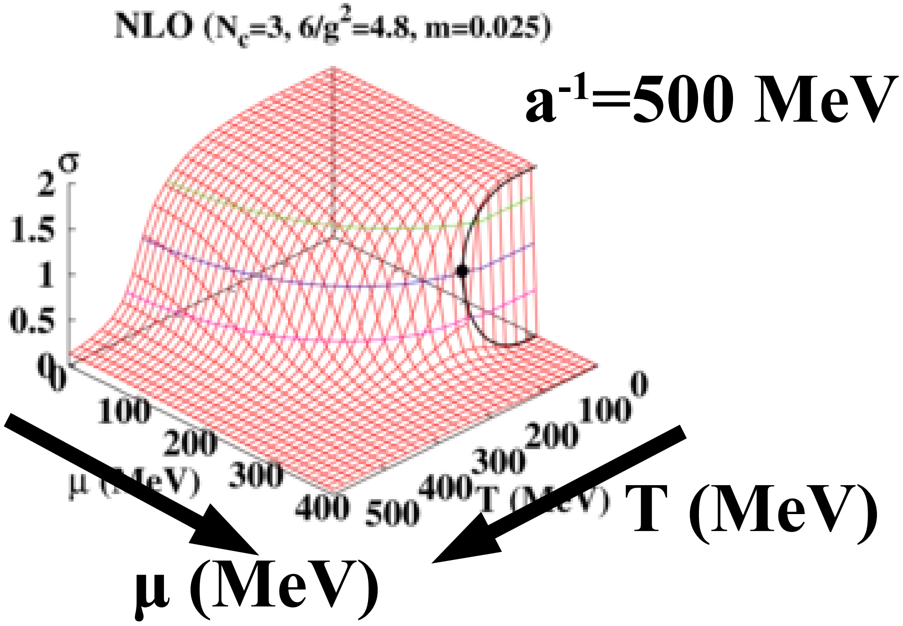
W. Weise, Nucl. Phys. A 553 (1993) 59c



PNJL (Fukushima, 2008)



1st order p.t.

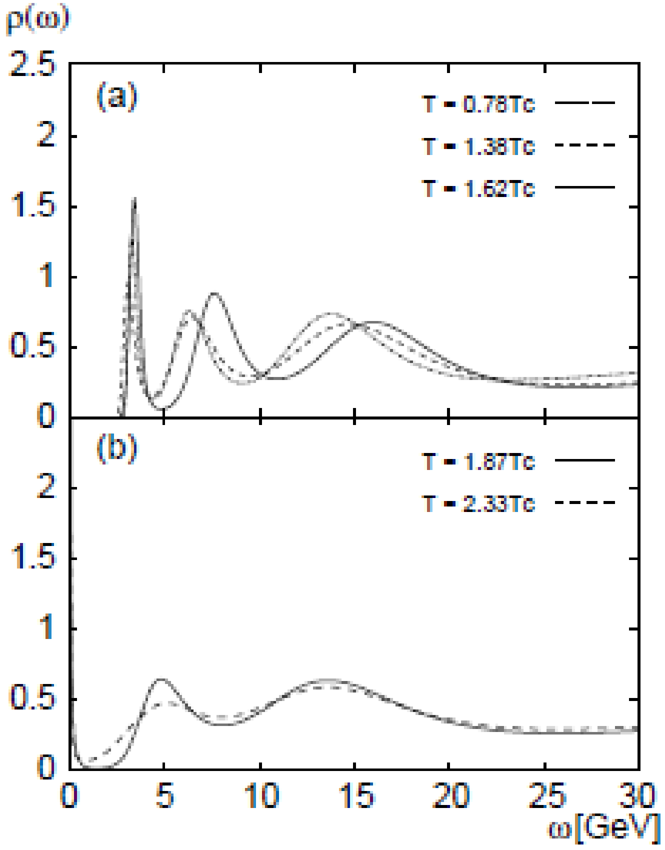


a^{-1}=500 MeV

Meson Masses in Strong Coupling Limit of Lattice QCD

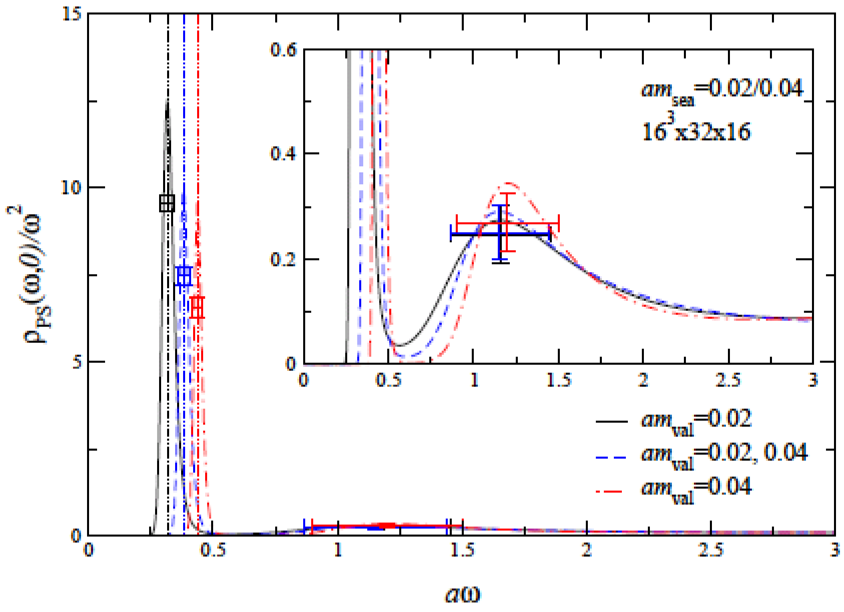
Hadron Mass Modification in Lattice QCD

- **Can we understand it in Lattice QCD ?**
  - **Finite T: It is possible even for light quarks !**
    (But it is not an easy task...) → Hatsuda-san's talk
  - **Finite μ (and low T): Difficult due to the sign problem.**



Asakawa, Hatsuda, PRL92(2004),012001

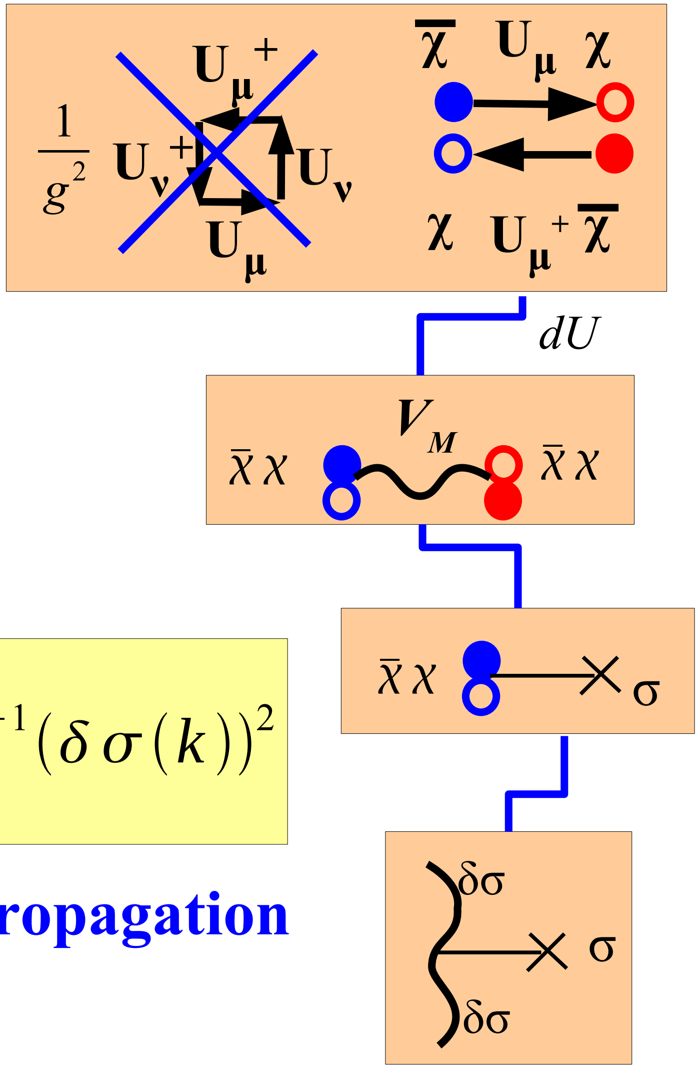
G. Aarts, Foley, 2007



Domain-Wall QCD
PS channel (T=0)

Hadron Mass in SCL-LQCD (Zero T)

- **QCD Lattice Action (Zero T treatment)**



$$S = \cancel{S_G} + S_F + m_0 \bar\chi\chi$$

Strong Coupling Limit

$$\rightarrow -\frac{1}{2}(\bar\chi\chi)V_M(\bar\chi\chi) + m_q\bar\chi\chi$$

One-link integral (1/d expansion)

$$\rightarrow \frac{1}{2}\sigma V_M{}^{-1}\sigma + \bar\chi(\sigma + m_q)\chi$$

Bosonization

$$\rightarrow \frac{1}{2}\sigma V_M{}^{-1}\sigma - N_c\sum_x \log(\sigma(x) + m_q)$$

Fermion Integral

$$= L^d N_\tau\left[\frac{N_c}{d+1}\bar\sigma^2 - N_c\log(\bar\sigma + m_q)\right] + \frac{1}{2}\sum_k G(k)^{-1}(\delta\sigma(k))^2$$

Effective Potential **Meson Propagation**

- **Meson Propagator**

$$G(k)^{-1} = F.T.\frac{\delta^2 S}{\delta\sigma(x)\delta\sigma(y)} = 2N_c\left[\sum_\mu \cos k_\mu\right]^{-1} + \frac{N_c}{(\bar\sigma + m_q)^2}$$

Hadron Mass in SCL-LQCD (Zero T)

- **Meson Mass in SCL-LQCD**

  Kluberg-Stern, Morel, Petersson, 1982; Kawamoto, Shigemoto, 1982

  - **Pole of the propagator at zero momentum → Meson Mass**
  - **Doubler DOF: $k_\mu \to 0$ or π, Euclidian: $\omega \to i\, m +$ "0 or π"**

$$G^{-1}(k)=N_c\left[\sum_{i=1}^{d}\cos\pi\delta_i\pm\cosh m\right]^{-1}+\frac{N_c}{(\bar{\sigma}+m_q)^2}=0$$

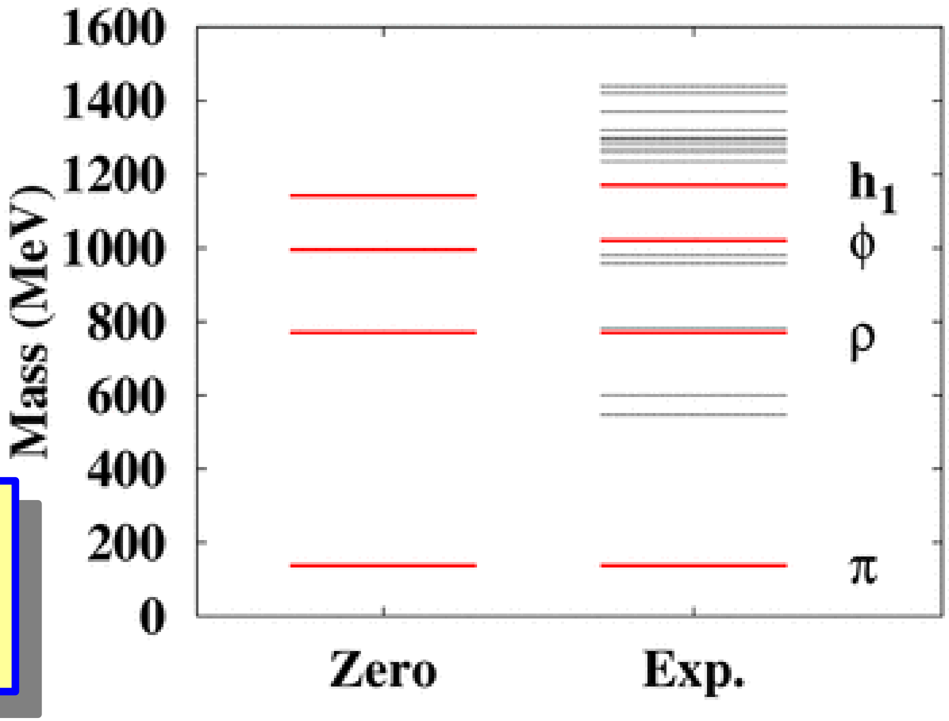
$$\to \boxed{\cosh m=2(\bar{\sigma}+m_q)^2+\kappa}$$

$$=(d+1)(\lambda^2-1)+2n+1$$

Equilibrium Condition

$$n=0,1,\ldots d \quad (\text{diff. meson species})$$

$$\lambda=\bar{m}_q+\sqrt{\bar{m}_q^{\,2}+1},\quad \bar{m}_q=m_q/\sqrt{2(d+1)}$$



Explains Meson Mass Spectrum
No (T, μ) dependence

Meson Masses in SCL-LQCD

- **Separation of $S_{\rm eff}$ into average ($F_{\rm eff}$) and fluctuations**

$$S_{\rm eff}(\sigma) = \frac{1}{2}\sum_{x,y}\sigma_x V_M^{-1}(x,y)\sigma_y - N_\tau \sum_{\boldsymbol{x}} V_q(X_N[\sigma],\mu,T)$$

$$\approx L^d N_\tau F_{\rm eff}(\bar{\sigma},\mu,T) + \frac{1}{2}\sum_{\boldsymbol{k},\omega} G_\sigma^{-1}(\boldsymbol{k},\omega;\bar{\sigma},\mu,T)\,\delta\sigma_k^2$$

  - **Quark determinant & temporal link integral → V_q**
  - **V_q is a *function* of X_N, which is a *functional* of σ.**
    (Faldt, Petersson, 1986)

- **Meson Propagater in the MF treatment = F.T. ($\partial^2 S/\partial\sigma\partial\sigma$)**
  → (T, μ) dependent

$$G_\sigma^{-1} = \frac{2N_c}{\kappa(\boldsymbol{k})} - \frac{\partial V_q(\sigma,T,\mu)}{\partial\sigma}\,\frac{2\sinh E_q(\sigma)}{\cos\omega + \cosh 2E_q(\sigma)}$$

- **Equilibrium Condition of σ ($\partial V/\partial\sigma = -2N_c\sigma/d$)**

$$G_\sigma^{-1} = \frac{2N_c}{\kappa(\boldsymbol{k})} + \frac{2N_c\bar{\sigma}}{d}\,\frac{2\sinh E_q(\bar{\sigma})}{\cos\omega + \cosh 2E_q(\bar{\sigma})}, \quad E_q(\bar{\sigma}) = \operatorname{arcsinh}(\bar{\sigma} + m_0)$$

Fermion Determinant

Faldt, Petersson, 1986

- **Fermion action is separated to each spatial point and bi-linear → Determinant of Nτ x Nc matrix**

$$\exp(-V_{\rm eff}/T)=\int dU_0 \begin{vmatrix} I_1 & e^{\mu} & 0 & & e^{-\mu}U^{+} \\ -e^{-\mu} & I_2 & e^{\mu} & & \\ 0 & -e^{-\mu} & I_3 & e^{\mu} & \\ \vdots & & & \ddots & \\ -e^{\mu}U & & & -e^{-\mu} & I_N \end{vmatrix}$$

Nc x Nτ

$$=\int dU_0 \det\left[\underline{X_N[\sigma]\otimes\mathbf{1}_c}+\underline{e^{-\mu/T}U^{+}}+\underline{(-1)^{N_\tau}e^{\mu/T}U}\right]$$

Nc

$$I_k=2(\sigma(k)+m_0)$$

$$X_N=\begin{vmatrix} I_1 & e^{\mu} & 0 & \cdots & & e^{-\mu} \\ -e^{-\mu} & I_2 & e^{\mu} & & & 0 \\ 0 & -e^{-\mu} & I_3 & & & 0 \\ \vdots & & & \ddots & & \\ & & & & I_{N-1} & e^{\mu} \\ -e^{\mu} & 0 & 0 & \cdots & -e^{-\mu} & I_N \end{vmatrix}-\left[e^{-\mu/T}+(-1)^{N}e^{\mu/T}\right]$$

Prescriptions related to lattice staggered fermions

- **Mass = Pole energy of G at "zero" momentum**
  - **"Zero" momentum: $\underline{k}$ = -$\underline{k}$ (vector) → $\underline{k}$ = (0,0,0), (0,0,π), (0, π, 0)**

$$\kappa(\boldsymbol{k}) = \sum_{j=1}^{d} \cos k_j = -3, -1, 1, 3 \quad \text{for zero momentum} (\boldsymbol{k} = -\boldsymbol{k})$$

Four different types of meson appear !
(Bound state with doubler)

  - **"Zero" Euclidean energy: ω= -ω → ω = 0 or π**

→ Search for the pole with ($\underline{k}$, ω)= (δ_π, δ_π, δ_π, i*M* +δ_π) (δ_π =0 or π)

$$G^{-1}(\boldsymbol{k} = '\boldsymbol{0}', \omega = iM + \delta\pi) = \frac{2N_c}{\kappa} + \frac{4N_c}{d} \frac{\bar{\sigma}(\bar{\sigma} + m_q)}{\pm \cosh M + \cosh 2E_q} = 0$$

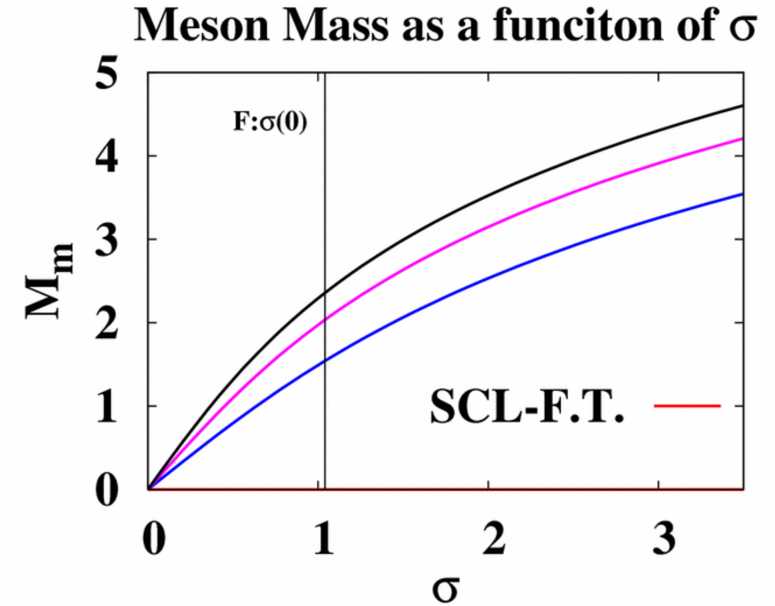
Hadron Mass in SCL-LQCD (Finite T)

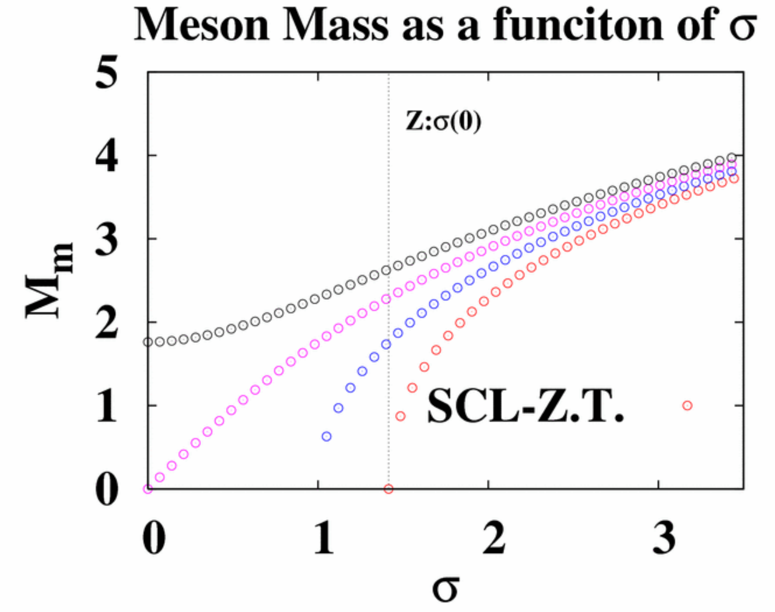
Kawamoto,Miura,AO,PoS(LATTICE 2007) (2007), 209
AO, Kawamoto, Miura, Mod. Phys. Lett. A 23 (2008), 2459.

- **Meson Mass**

$$M = 2\,\mathrm{arcsinh}\sqrt{(\bar{\sigma}+m_q)\left(\frac{d+\kappa}{d}\bar{\sigma}+m_q\right)}$$

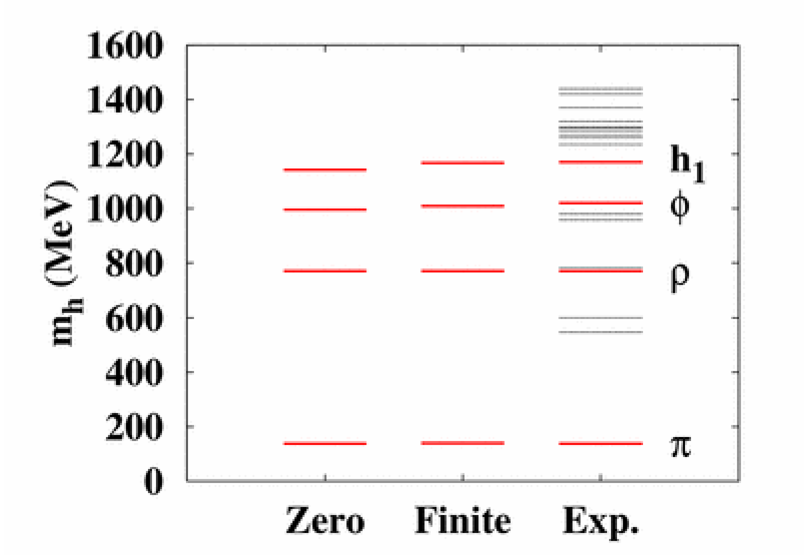
- **Meson masses are determined by the chiral condensate σ.**
- **Chiral condensate is determined by the equilibrium condition, and given as a function of (*T*, *μ*).**
  → ***Approximate Brown-Rho scaling is realized in SCL-LQCD***
- **Different from zero *T* treatments,**
  Kluberg-Stern, Morel, Petersson, 1982;
  Kawamoto, Shigemoto, 1982



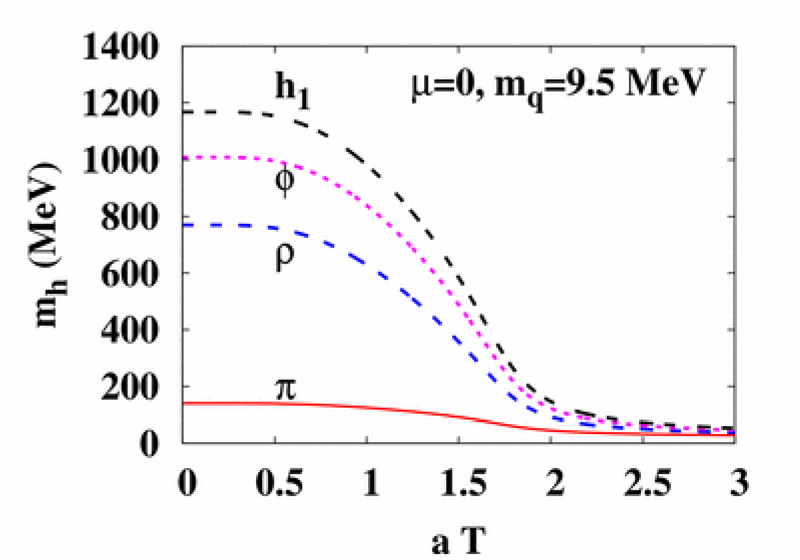


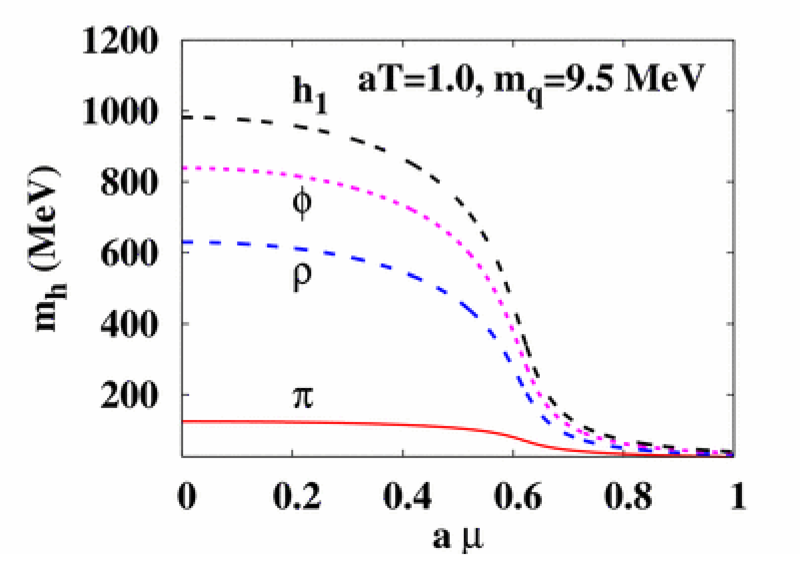
Medium Modification of Meson Masses

- Scale fixing
  - Search for σ_{vac} to minimize free E.
  - Assign κ=-3, -1 as π and ρ
  - Determine m_q and a^{-1} (lattice unit) to fit m_π / m_ρ



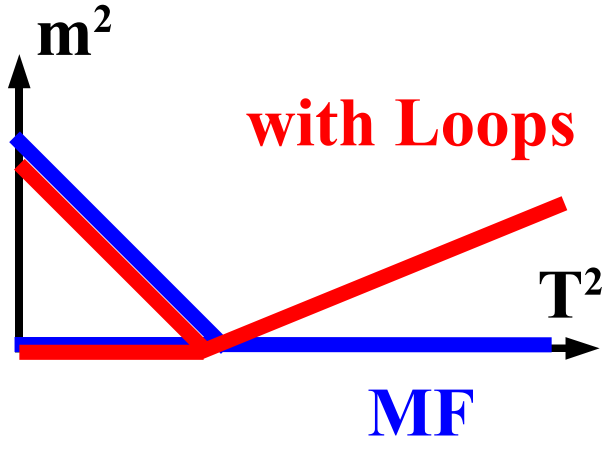
- Medium modification
  - Search for σ(T, μ) → Meson mass





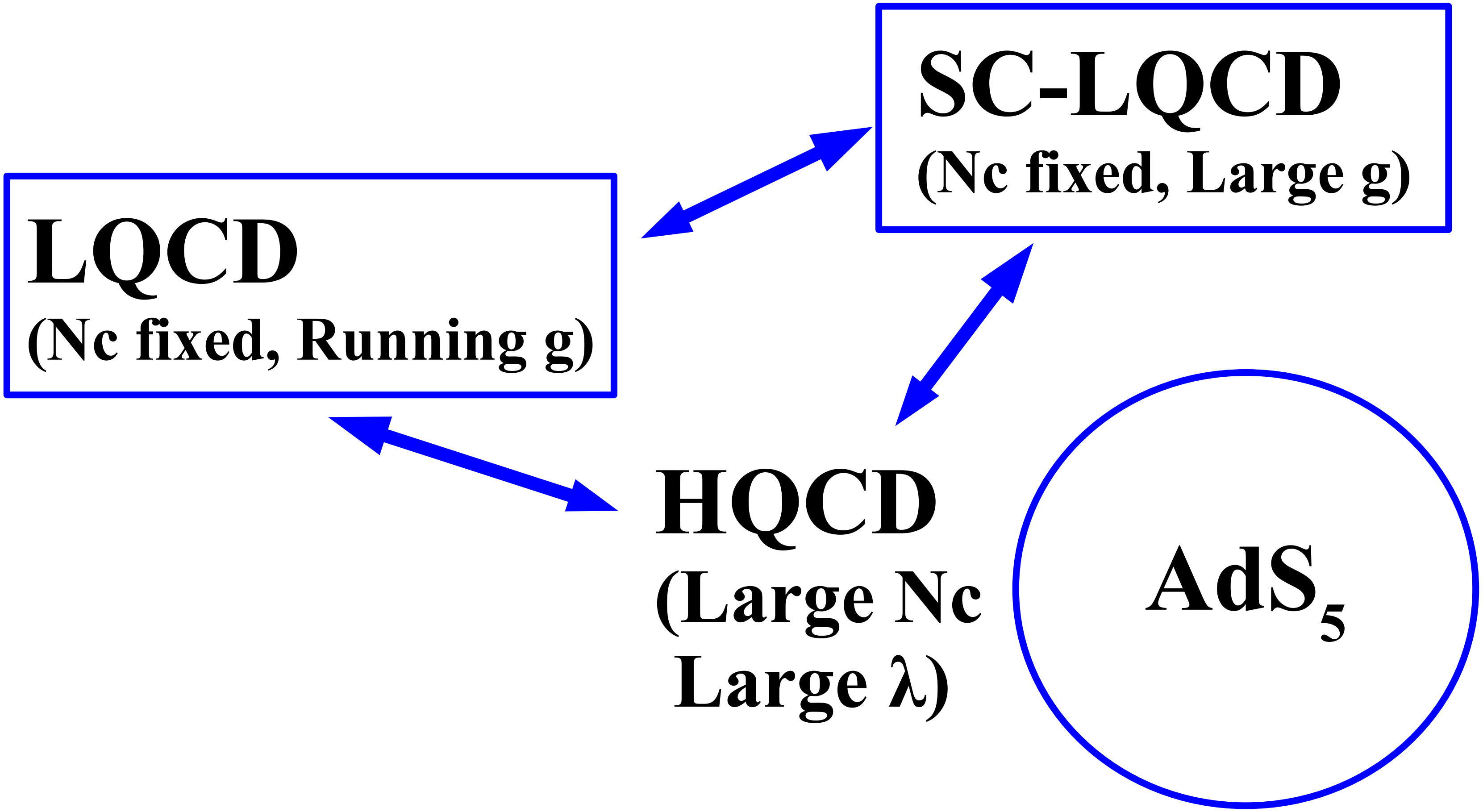
Summary

- Chiral condensates at finite T and μ are investigated with strong coupling lattice QCD (SC-LQCD).
  - Partial restoration of χ sym. is expected at finite T and/or μ in SC-LQCD (SCL, NLO, NNLO).
  - Qualitative behavior is similar to NJL and PNJL results.
  - Quantitative differences to be further discussed → T_c and μ_c, Density gap at finite μ,Critical point, Deconfinement,..
- Meson masses at finite T and μ are studied in SCL-LQCD.
  - Results with mean field approx. shows Brown-Rho scaling behavior.
  - Loop effects of mesons are expected to enhance meson masses after χ restoration (cf. Hatsuda, Kunihiro / Kapusta)
  - Meson assignment is correct ? (cf. Golterman, Smit; Bazabov et al., 2009)



Challenge

- **There should be some relation btw SC-LQCD and HQCD**
  - **HQCD: Large Nc, Large λ=g^2 N_c**
  - **SC-LQCD: Fixed N_c, Large g^2**



SC-LQCD
(Nc fixed, Large g)

LQCD
(Nc fixed, Running g)

HQCD
(Large Nc
Large λ)

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