Lambda hypernuclei and neutron star matter in chiral SU(3) RMF

A. Ohnishi (YITP, Kyoto Univ.)

- **Introduction**
- **Chiral Collapse and Effective Potential in SCL-LQCD**
- **RMF with a logarithmic σ potential (SUf(2))**
  Tsubakihara, AO, PTP 117('07)903.
- **Hypernuclei and Hyperonic Matter in Chiral SU(3) RMF**
  Tsubakihara, Maekawa, AO, EPJA33('07)295 [nucl-th/0702008]
  Tsubakihara, Matsumiya, Maekawa, AO, AIP Conf. Proc.,1016 (2008), 288.
  Tsubakihara, Matsumiya, Maekawa, AO, to be submited.
- **Summary**

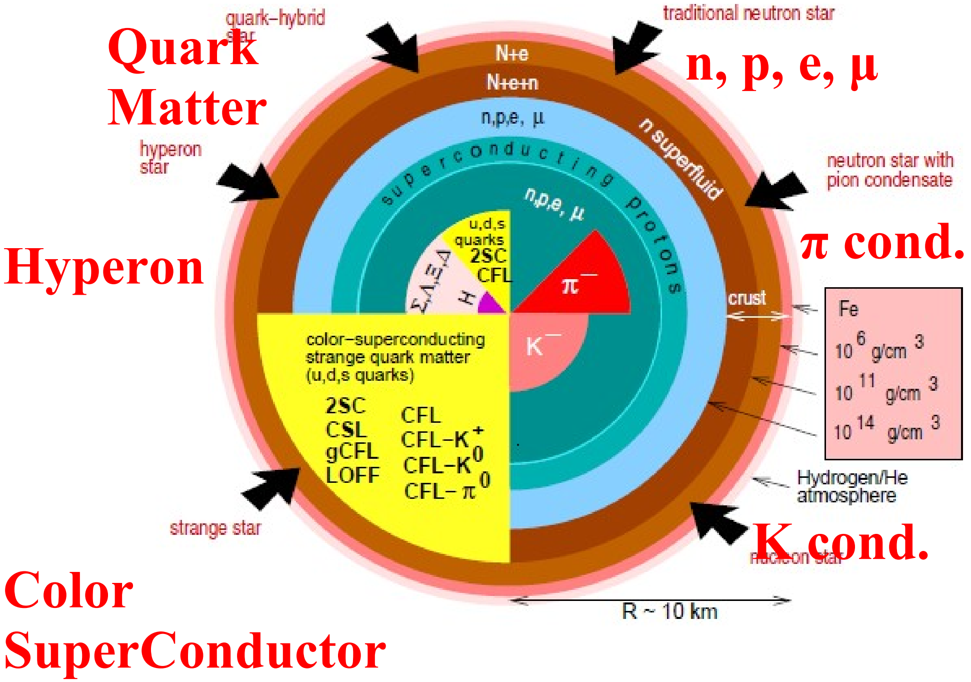
What happens at High Densities ?

- **Particle composition in Neutron Star Core**

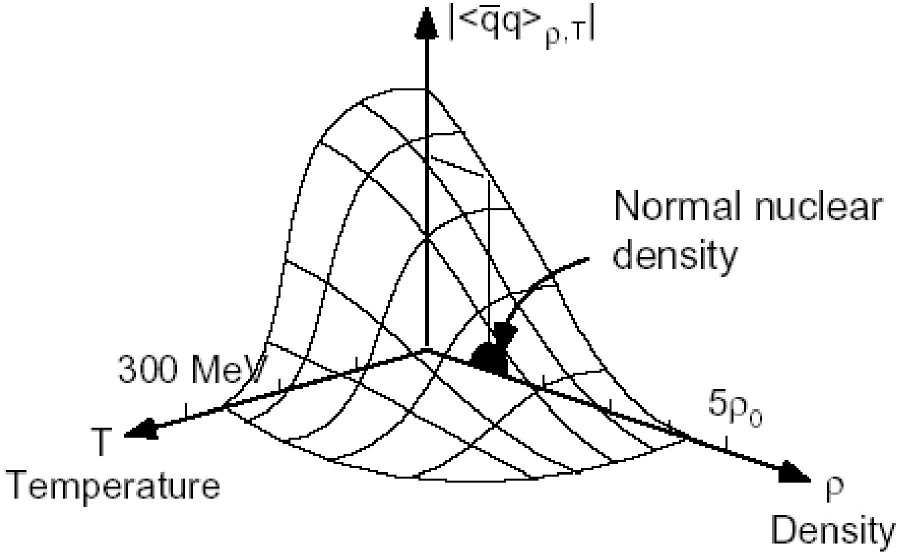
  → n, p, e, μ, Y(Hyperons), mesons (π , K), quarks, quark pair (CFL, ..)
  → *Strangeness* is the key ingredient !

- **Particle property at High Density**

  → Partial restoration of the chiral symmetry
  → Hadron mass modification (CERES, KEK-E325, PHENIX,....)



F. Weber, Prog. Part. Nucl. Phys. 54 (2005) 193

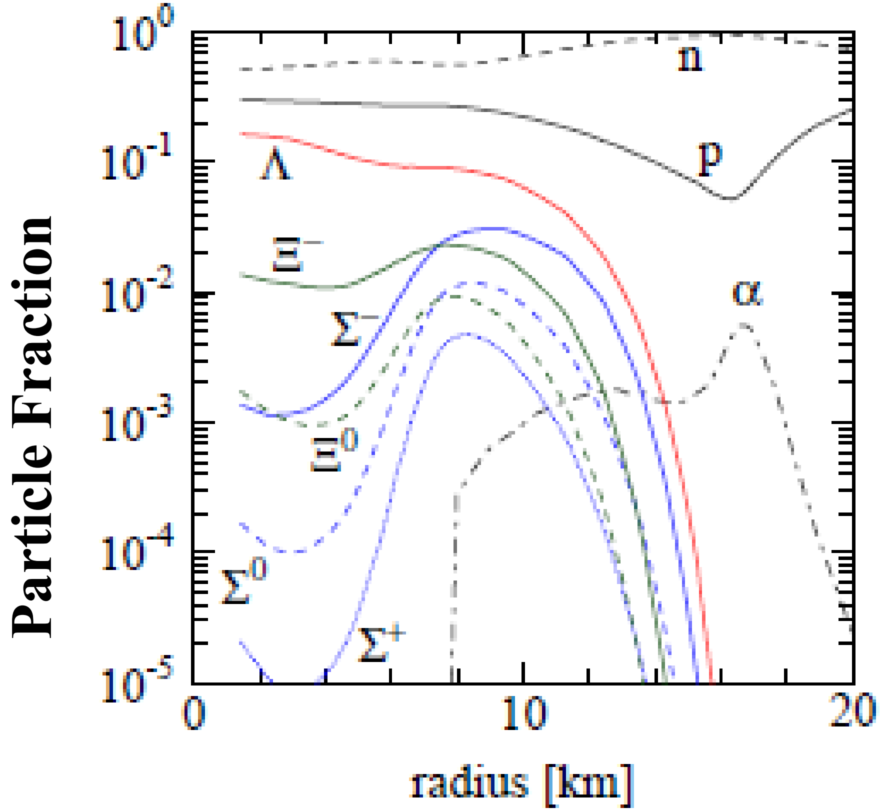


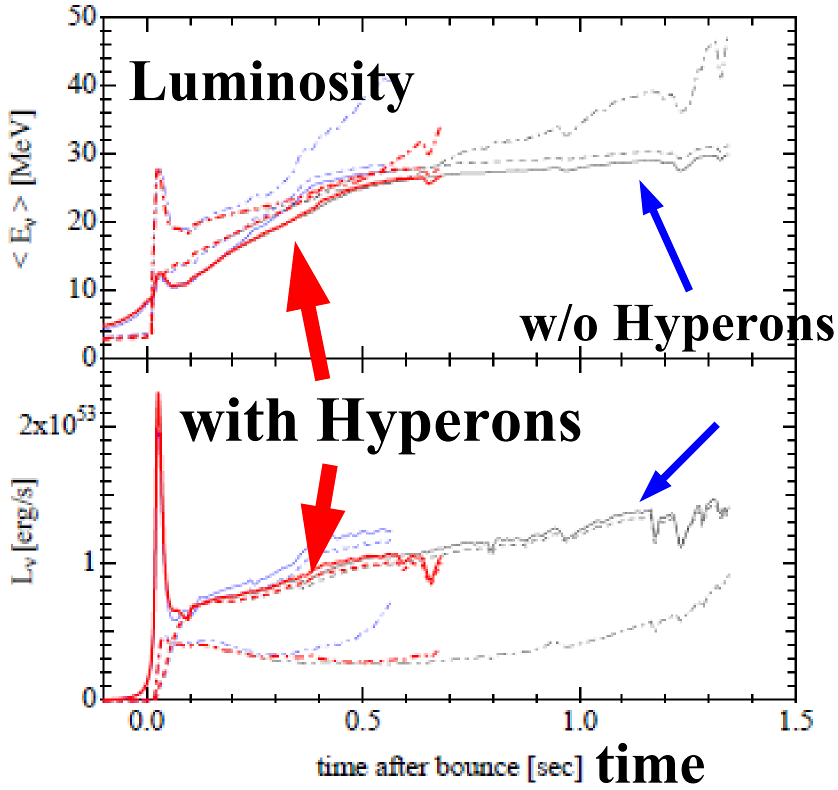
W. Weise, Nucl. Phys. A 553 (1993) 59c

Hyperons during Black Hole Formation

- **Hot matter (~ 70 MeV) is formed during Black Hole formation**
  → Many hyperons appear → EOS is softened
  → Earlier formation of BH
  → *Shorter ν emission time*

High density EOS may be probed by ν (not only with NS/GW).





Luminosity

w/o Hyperons

with Hyperons

time

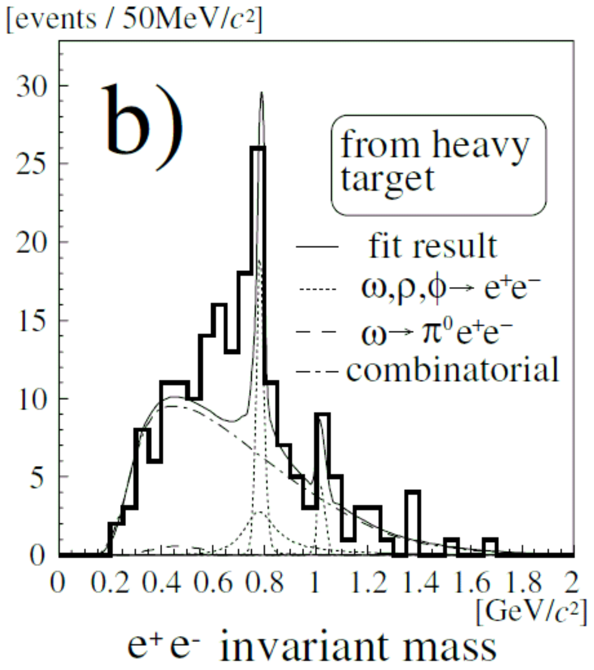
Sumiyoshi, Ishizuka, AO, Yamada, Suzuki, 2009

Hadron Mass Modification

- **Medium meson mass modification** is suggested experimentally.

  CERES Collab., PRL75('95),1272;
  KEK-E325 Collab.(Ozawa et al.), PRL86('01),5019;
  PHENIX Collab., arXiv:0706.3034

> ***Interpretation is model dependent***
> ***→ Investigation in***
> ***non-perturbative QCD is desired !***



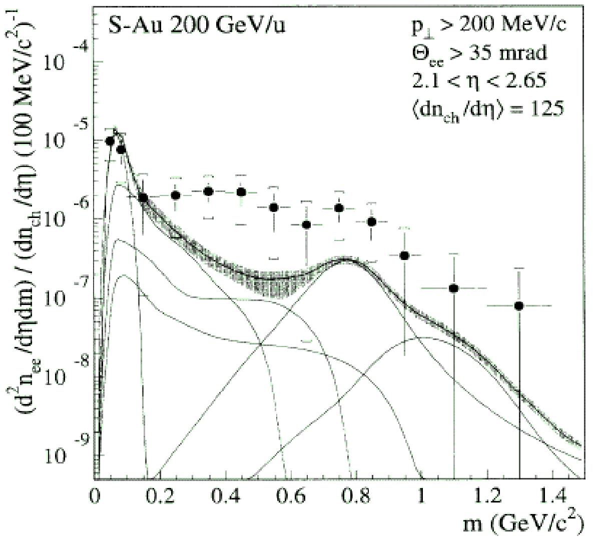
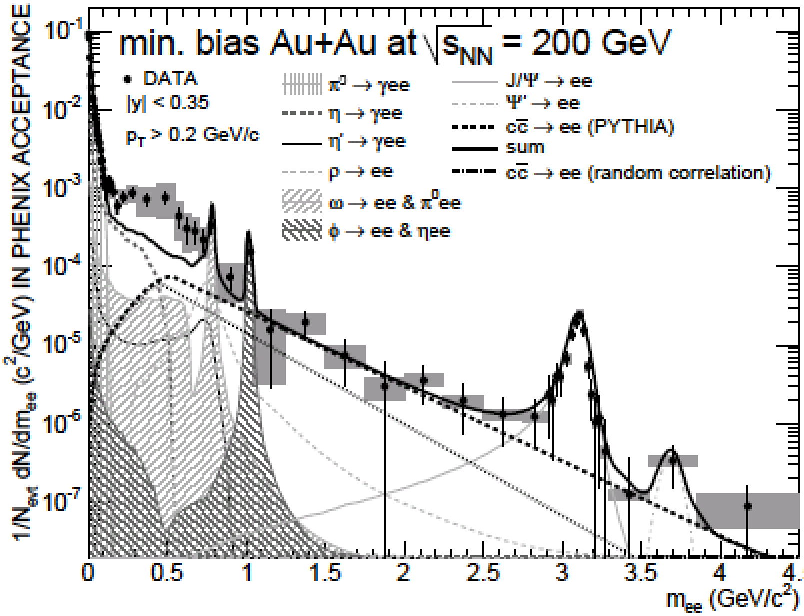


FIG. 4. Inclusive e^+e^- mass spectra in 200 GeV/nucleon S-Au collisions. For explanations see Fig. 2.



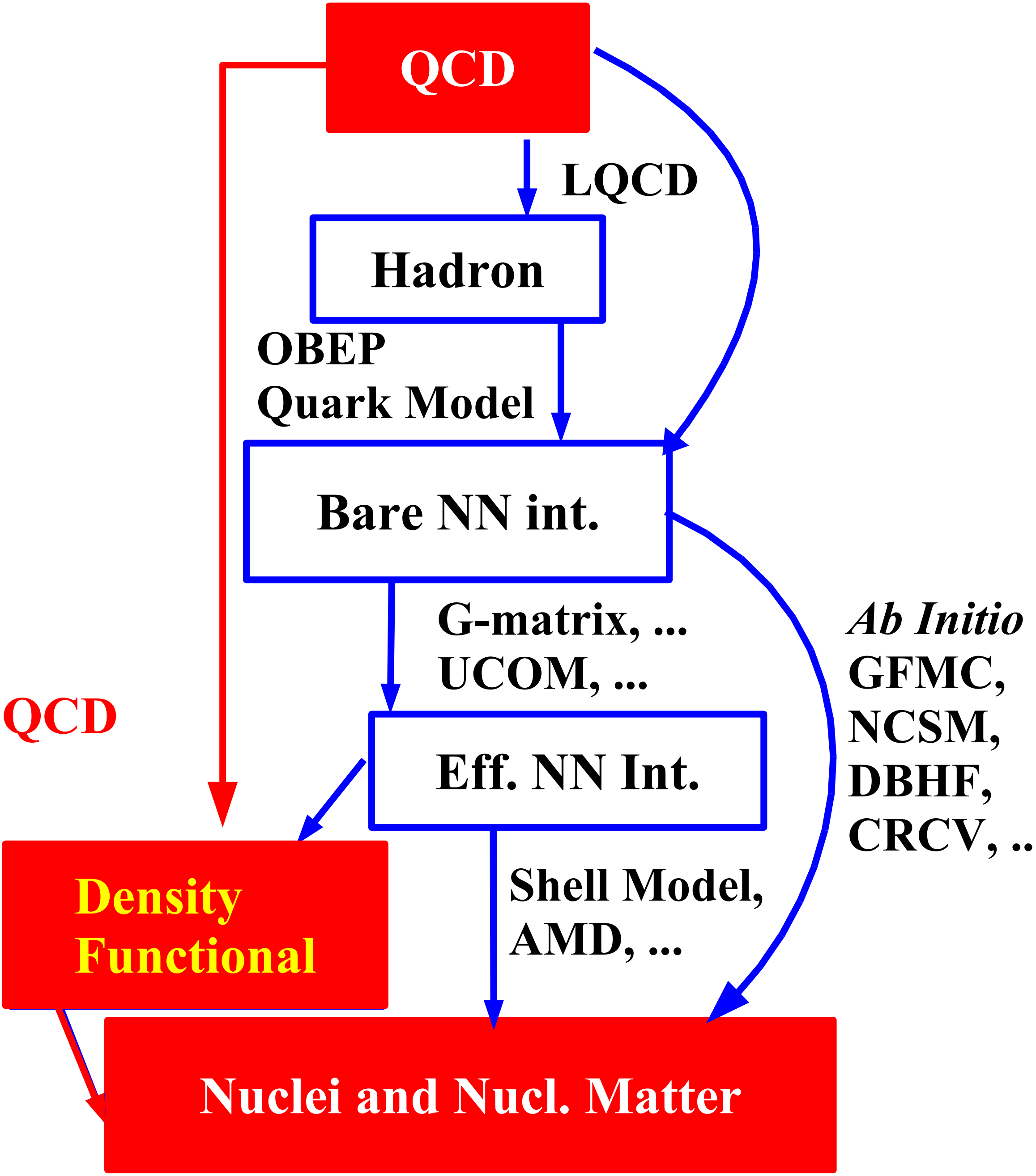
Purpose of This Work

- **Strangeness & Chiral Sym. are the keys in understanding dense matter**
- **We are aiming at developing a nuclear many-body theory,**
  - **(1) which explains nuclear matter saturation property,**
  - **(2) does not contradict to nuclear ab initio calculations,**
  - **(3) well describes bulk properties (B.E., rmsr) of normal nuclei**
  - **(4) and hypernuclei,**
  - **(5) possesses the chiral symmetry,**
  - **(6) explains known properties of neutron stars,**
  - **(7) explodes supernovae,**
  - **(8) and has clear relation to QCD.**
- **There are many problems to be solved.**
  - **Chiral Collapse problem (Lee-Wick vacuum)**
  - **Lattice QCD at finite density (sign problem)**

We try to develop an RMF model
with the free energy density (effective potential)
derived from SCL-LQCD.

Long ways from QCD to Nuclei

- **Royal Way**
  - **QCD (→ Hadron)**
    → Bare NN Int.
    (→ Effective NN Int.)
    → Nuclei
- **Another Way ?**
  - **QCD**
    → Energy Density
    → Density Functional
    → Nuclei
  - **Problem**
    MC is not yet reliable
    for cold dense matter **→ SC-LQCD**



QCD

LQCD

Hadron

OBEP
Quark Model

Bare NN int.

G-matrix, ...
UCOM, ...

Eff. NN Int.

Ab Initio
GFMC,
NCSM,
DBHF,
CRCV, ..

Density
Functional

Shell Model,
AMD, ...

+ Chiral RMF

Skyrme HF, RMF

Nuclei and Nucl. Matter

Disclaimer

- We work is based on
  - the *Strong Coupling Limit (SCL)* of Lattice QCD
  - in the *zero T treatment*,
  - with n_f (=1, 2, 3) staggered fermions, which corresponds to *N_f=4n_f flavors*.

  → The effective potential may be different in continuum theory with 2+1 flavors.
- We have adopted the SCL-LQCD results of the scalar meson self-interaction. Baryons, vector mesons, and their couplings are introduced and determined phenomenologically.
- Nevertheless, the present results show that we can improve RMF by using the idea from QCD !

Chiral Collapse & Effective Potential in SCL-LQCD

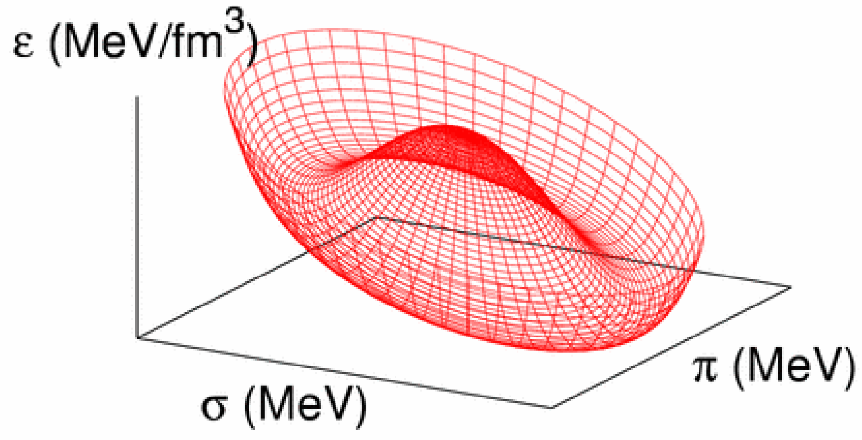
Chiral Collapse Problem

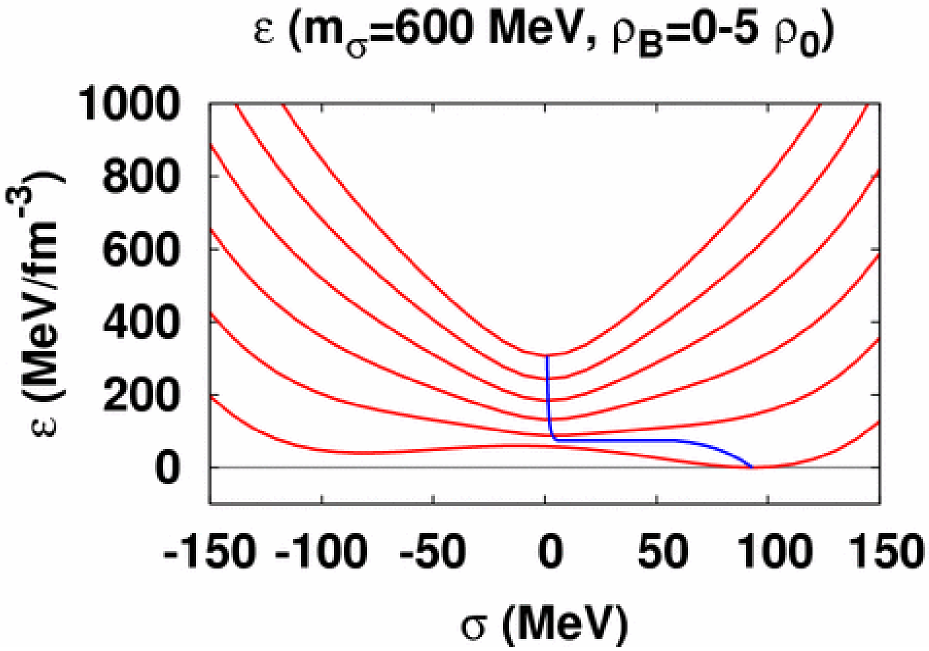
- **Chiral symmetry = Fundamental Symmetry of QCD**
  - **SSB of χ sym. → Hadron masses + NG boson (π)**
  - **χ sym. should also persist in hadron many-body problems**
- **Schematic model : Linear σ model → χ sym. is restored below ρ_0**

Lee, Wick, 1974

$$L = \frac{1}{2}\left(\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \pi \partial^\mu \pi\right) - \frac{\lambda}{4}\left(\sigma^2 + \pi^2\right)^2 + \frac{\mu^2}{2}\left(\sigma^2 + \pi^2\right) + c\sigma$$

$$+ \overline{N}\, i\partial_\mu \gamma^\mu N - g_\sigma \overline{N}\left(\sigma + i\pi\tau\gamma_5\right) N$$



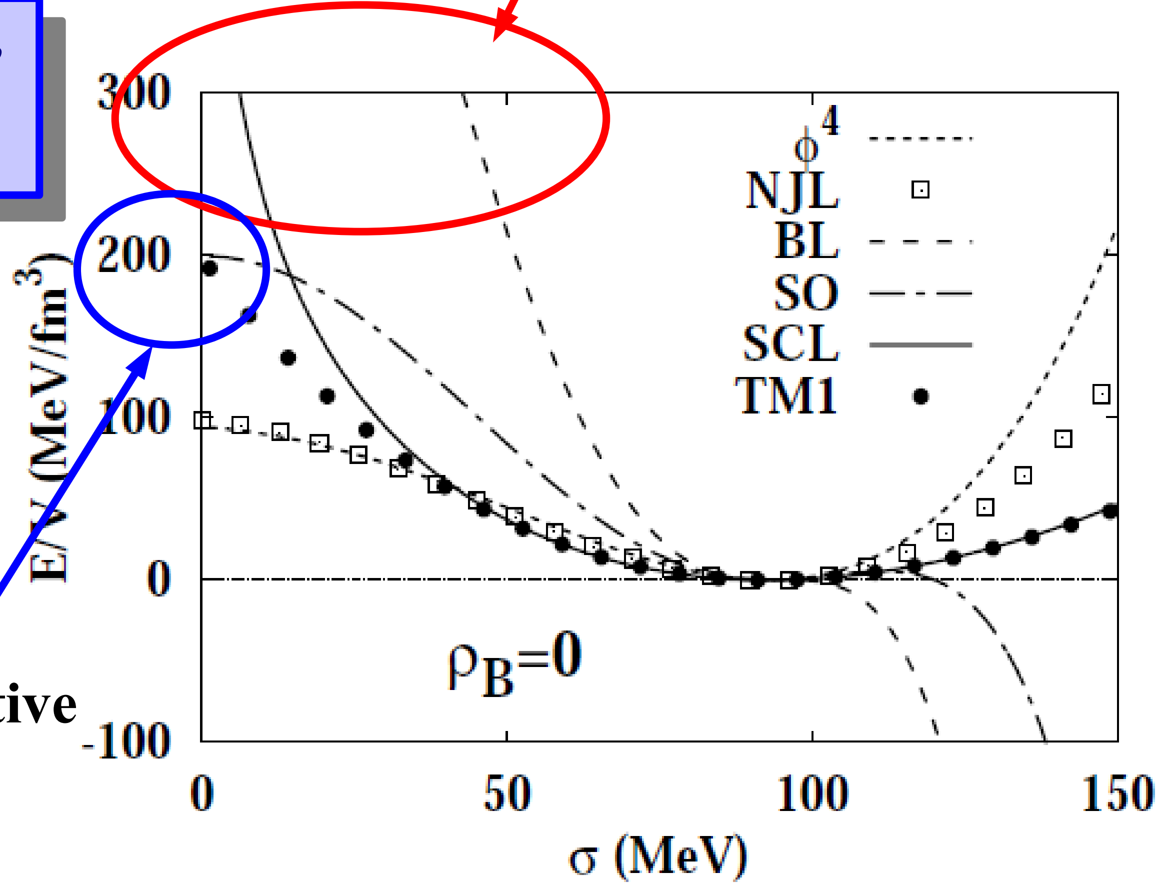


How can we avoid Chiral Collapse ?

- **We need**
  - **Very Strong Repulsion at small σ**

    or
  - **Finite Negative Derivative at σ = 0**

Is there any way to derive these "Repulsion" ?



Very Large Repulsion at small σ

Finite Derivative at σ = 0

Various Attempts to Cure Chiral Collapse

- **φ^4 Theory (Gell-Mann, Levy) → Collapse**

$$V_\sigma^{(\phi^4)} = \frac{\lambda}{4}(\phi^2 - f_\pi^2)^2 + \frac{1}{2}m_\pi^2\phi^2 - f_\pi m_\pi^2 \sigma\ , \quad \lambda = \frac{m_\sigma^2 - m_\pi^2}{2f_\pi^2}$$

- **NJL (Quark Loop, e.g. Hatsuda, Kunihiro) → Collapse**

$$V_\sigma^{\mathrm{NJL}} = \frac{m_0^2}{2}\sigma^2 + \Lambda^4 f_{\mathrm{NJL}}\left(\frac{G\sigma}{\Lambda}\right) - f_\pi m_\pi^2 \sigma \qquad f_{\mathrm{NJL}}(x) = -\frac{N_c N_f}{4\pi^2}\left[\left(1+\frac{x^2}{2}\right)\sqrt{1+x^2} - 1 - \frac{x^4}{2}\log\left(\frac{1+\sqrt{1+x^2}}{x}\right)\right]$$

- **Baryon Loop (Matsui, Serot) → Unstable at large σ**

$$V_\sigma^{\mathrm{BL}} = \frac{m_\sigma^2}{8f_\pi^2}(\phi^2 - f_\pi^2)^2 - M_N^4 f_{\mathrm{BL}}(\phi/f_\pi) \qquad f_{\mathrm{BL}}(x) = -\frac{1}{4\pi^2}\left[\frac{x^4}{2}\log x^2 - \frac{1}{4} + x^2 - \frac{3}{4}x^4\right]$$

- **Higher order terms (E.g. Sahu, AO) → Unstable at large σ**

$$V_\sigma^{\mathrm{SO}} = \frac{m_\sigma^2}{8f_\pi^2}(\phi^2 - f_\pi^2)^2 + f_\pi^4 f_{\mathrm{SO}}(\phi/f_\pi) \qquad f_{\mathrm{SO}}(x) = \frac{C_6}{6}(x^2-1)^3 + \frac{C_8}{8}(x^2-1)^4$$

- **Log type term from scale anomaly (Furnstahl, Serot; Heide et al.)**
- **Log type term from SCL-LQCD (Tsubakihara, AO)**

$$V_\sigma^{\mathrm{SCL}} = V_\chi(\sigma, \pi) - c_\sigma\,\sigma = \frac{1}{2}\,b_\sigma\phi^2 - a_\sigma\log\phi^2 - c_\sigma\,\sigma$$

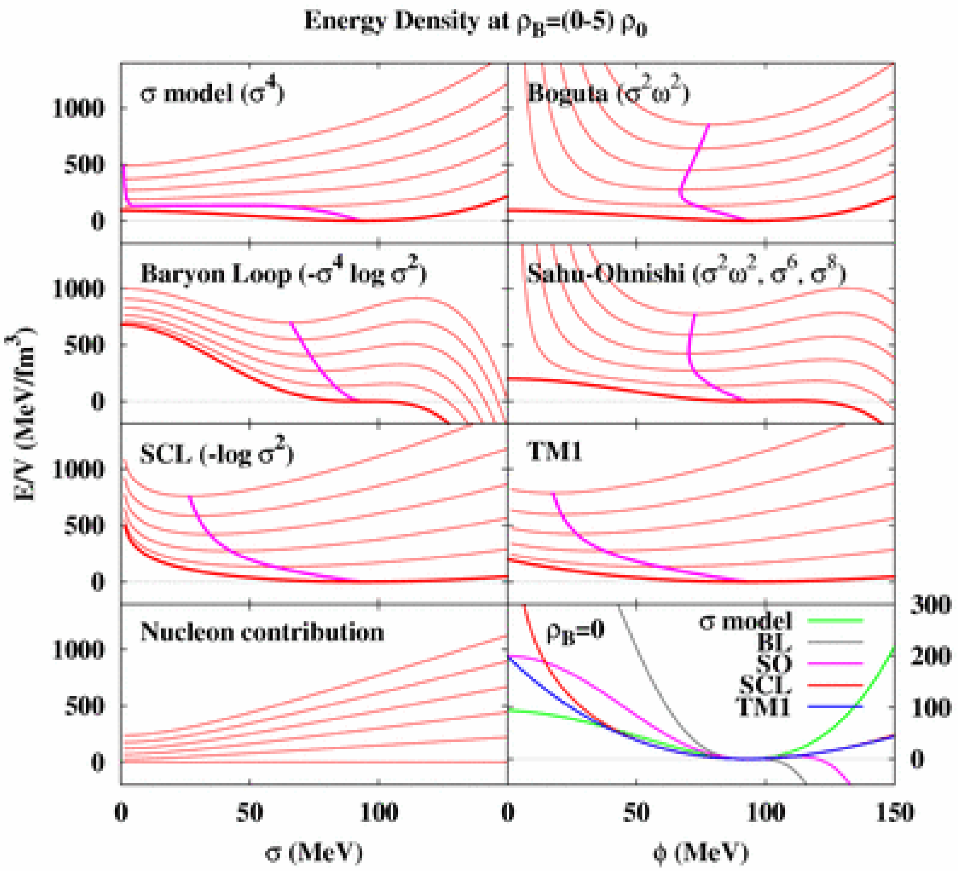
- **Non-Linear σN coupling (Saito, Tsushima, Thomas ; Bentz, Thomas)**

RMF with Chiral Symmetry: Chiral Collapse

- **Naïve Chiral RMF models → Chiral collapse at low ρ** *(Lee-Wick 1974)*
- **Prescriptions**

$$L = \frac{1}{2}\left(\partial_\mu\sigma\,\partial^\mu\sigma + \partial_\mu\pi\,\partial^\mu\pi\right) - \frac{\lambda}{4}\left(\sigma^2+\pi^2\right)^2 + \frac{\mu^2}{2}\left(\sigma^2+\pi^2\right) + c\,\sigma + \bar{N}\,i\,\partial_\mu\gamma^\mu N - g_\sigma\bar{N}\left(\sigma + i\,\pi\,\tau\,\gamma_5\right)N$$

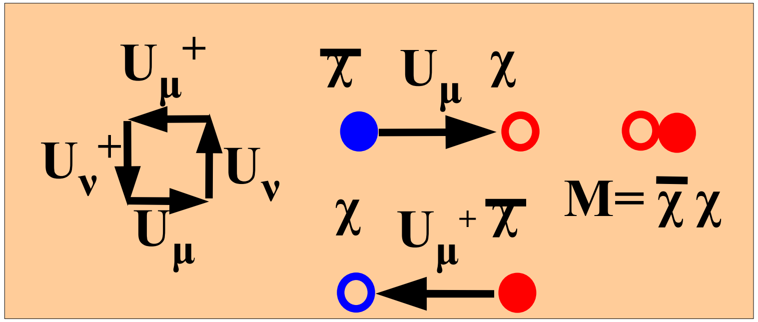
  - **σω coupling (too stiff EOS)**
    (Boguta 1983, Ogawa et al. 2004)
  - **Loop effects (unstable at large σ)**
    (Matsui-Serot, 1982, Glendenning 1988, Prakash-Ainsworth 1987,Tamenaga et al. 2006)
  - **Higher order terms (unstable at large σ)**
    (Hatsuda-Prakash 1989, Sahu-Ohnishi 2000)
  - ***Dielectric (Glueball) Field representing scale anomaly***
    (Furnstahl-Serot 1993,Heide-Rudaz-Ellis 1994, Papazoglou et al.(SU(3)) 1998)
  - **Different Chiral partner assignment**
    (DeTar-Kunihiro 1989, Hatsuda-Prakash 1989 Harada-Yamawaki 2001, Zschiesche-Tolos-Schaffner-Bielich-Pisarski, nucl-th/0608044)
  - ***Nucleon Structure***
    (Saito-Thomas 1994, Bentz-Thomas 2001)



Lattice QCD

- **Lattice QCD=ab initio, non-perturbative theory (c.f. Teper's talk)**

$$S_{\rm LQCD} = \frac{1}{2}\sum_{x,j}\left[\eta_{\nu,x}\bar{\chi}_x U_{\nu,x}\chi_{x+\hat{\nu}} - \eta^{-1}_{\nu,x}\bar{\chi}_{x+\hat{\nu}} U^\dagger_{\nu,x}\chi_x\right] - \frac{1}{g^2}\sum_{\Box}\mathrm{tr}\left[U_\Box + U^\dagger_\Box\right] + m_0\sum_x \bar{\chi}_x\chi_x$$



- **Problems to overcome**
  - **DOF is too much, and MC is necessary for numerical integration**
    → Faster Computer + Faster Algorithm
  - **Doublers appear for chiral fermions → different types of fermions**
  - **Weight for gluon config. (Fermion determinant) becomes complex at finite μ**
    → Taylor expansion, Analytic Continuation, Canonical, …
    → Not Yet Applicable for Dense and Cold Matter !

Strong Coupling Limit/Expansion makes it possible to obtain (approx.) Effective Potential analytically !

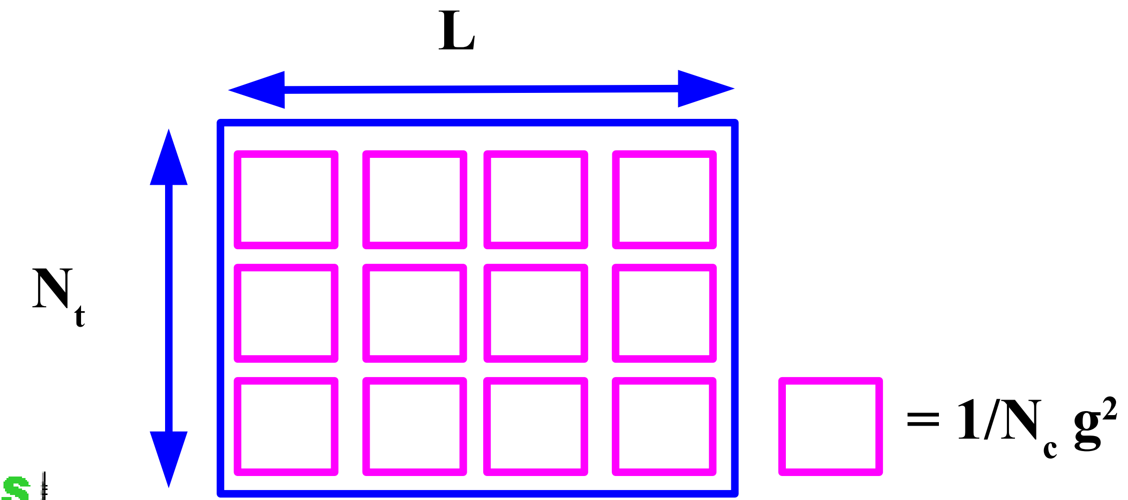
Strong Coupling Lattice QCD: Pure Gauge

K. G. Wilson, PRD10(1974),2445
M. Creutz, PRD21(1980), 2308.
G. Munster, 1981

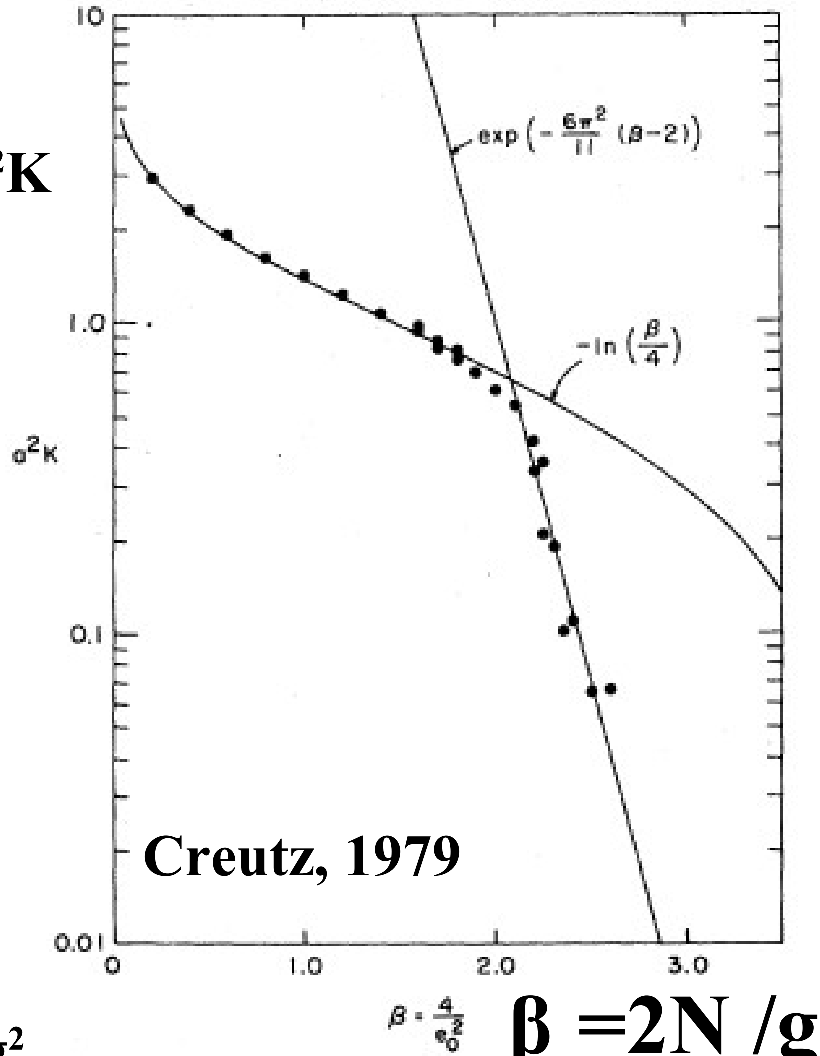
- **Quarks are confined in Strong Coupling QCD**
  - **Strong Coupling Limit (SCL)**
    → Fill Wilson Loop with Min. # of Plaquettes
    → Area Law (Wilson, 1974)

$$S_{\rm LQCD} = -\frac{1}{g^2}\sum_{\Box} \mathrm{tr}\left[U_{\Box} + U_{\Box}^{\dagger}\right]$$

  - **Smooth Transition from SCL to pQCD in MC (Creutz, 1980)**



a^2K



Creutz, 1979

$\beta_g = 2N_c/g^2$

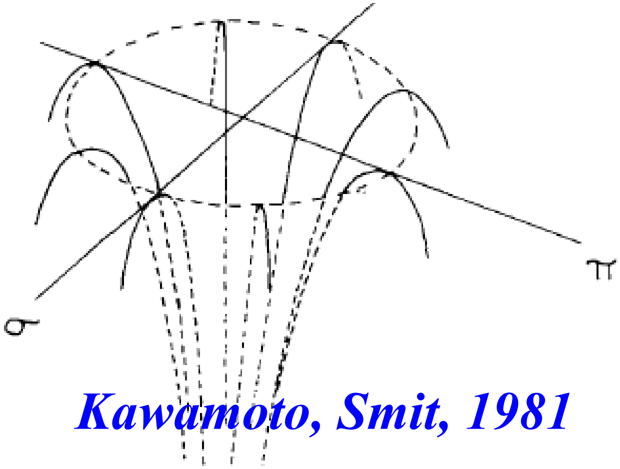
Strong Coupling Lattice QCD with Quarks (1)

- **No Plaquette in SCL→ One Link Integral → Mesonic Eff. Action → Eff. Pot. → SSB of χ Sym.**
- **Strong Coupling Limit (Zero T treatment) → log type potential of σ**
  N. Kawamoto, NPB190('81),617, N. Kawamoto, J. Smit, NPB192('81)100
  Kluberg-Stern, Morel, Napoly, Petersson, 1981

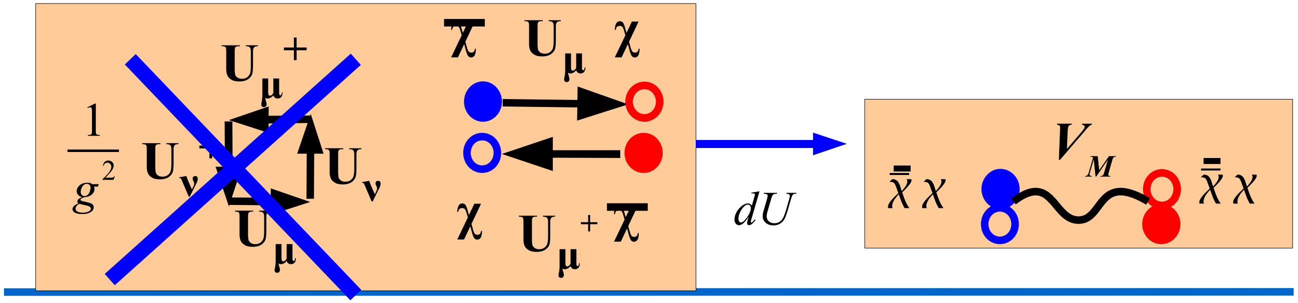
$$V = N[\tfrac{1}{2}\ln(\sigma^2+\pi^2) - M\sigma - dF(\sigma^2+\pi^2)]$$

- **SCL (Finite T Treatment) → arcsinh type potential of σ**
  P.H.Damgaard, N. Kawamoto, K.Shigemoto, PRL53('84),2211; NPB264 ('86), 1
  Faldt, Petersson, 1986; Bilic, Karsch, Redlich, 1992; Fukushima,2004, Nishida, 2004

$$\mathcal{F}_{\rm eff} = \frac{d}{4N_c}\sigma^2 + \mathcal{V}_{\rm q}\left(\frac{d\sigma}{2N_c} + m_0; \mu, T\right) \quad \mathcal{V}_{\rm q}(m_q;\mu,T) = -T\log\left[\frac{\sinh[(N_c+1){\rm arcsinh}(m_q)/T]}{\sinh[{\rm arcsinh}(m_q)/T]} + 2\cosh(N_c\mu/T)\right]$$



Kawamoto, Smit, 1981



Strong Coupling Lattice QCD with Quarks (2)

- **With Baryons**
  P.H.Damgaard, D. Hochberg, N. Kawamoto, PLB158('86)239; Hasenfatz, Karsch, 1983; Azcoiti et al., 2003; Kawamoto, Miura, AO, Ohnuma, 2007.

$$\mathcal{F}_{\rm eff} = \frac{b_\sigma}{2}\sigma^2 + \mathcal{V}_{\rm q}\left(m_q;\mu,T\right) + \Delta\mathcal{V}_{\rm b}(g_\sigma\sigma) \quad \Delta\mathcal{V}_{\rm b}(x) = -f^{(b)}\left(\frac{\pi x}{8}\right) \quad f^{(b)}(x) = \frac{1}{2}\log(1+x^2) - \frac{1}{x^3}\left[\arctan x - x + \frac{x^3}{3}\right] - \frac{3}{5}x^2$$

- **Next-to-Leading Order correction → σ ω model of quarks**
  N. Bilic, F. Karsch, K. Redlich, 1992, AO, N. Kawamoto, K. Miura, 2007.

$$\mathcal{F}_{\rm eff} = \frac{1}{2}b_\sigma\sigma^2 + \beta_s\varphi_s\sigma^2 + \frac{\beta_\tau}{2}(\varphi_\tau^2 - \omega_\tau^2) + \frac{\beta_s}{2}\varphi_s^2 + \mathcal{V}_{\rm q}(m_q;\tilde{\mu},T) - N_c\log Z$$

$$\varphi_s = \sigma^2\,, \quad \varphi_\tau = \frac{2\varphi_0}{1+\sqrt{1+4\beta_\tau\varphi_0}} \qquad \sigma = -\frac{1}{Z}\frac{\partial\mathcal{V}_{\rm q}}{\partial m_q} \qquad \omega_\tau = -\frac{\partial\mathcal{V}_{\rm q}}{\partial\tilde{\mu}} = \rho_q$$

σ : chiral condensate, ω: vector potential

- **Next-to-Next-Leading Order corrections**
  → σ ω model with σ ω coupling
  T.Z.Nakano, K. Miura, AO, in preparation.

Higher order terms of 1/g^2 → Non-linear terms of mesons

Effective Potential in SCL-LQCD (Zero T)

- **QCD Lattice Action (Zero T treatment)**

$$S = \cancel{S_G} + S_F + m_0 \bar{\chi}\chi$$

Strong Coupling Limit

$$\rightarrow -\frac{1}{2}(\bar{\chi}\chi) V_M (\bar{\chi}\chi) + m_0 \bar{\chi}\chi$$

One-link integral (1/d expansion)

$$\rightarrow \frac{1}{2}\sigma V_M \sigma + \bar{\chi}(V_M \sigma + m_0)\chi$$

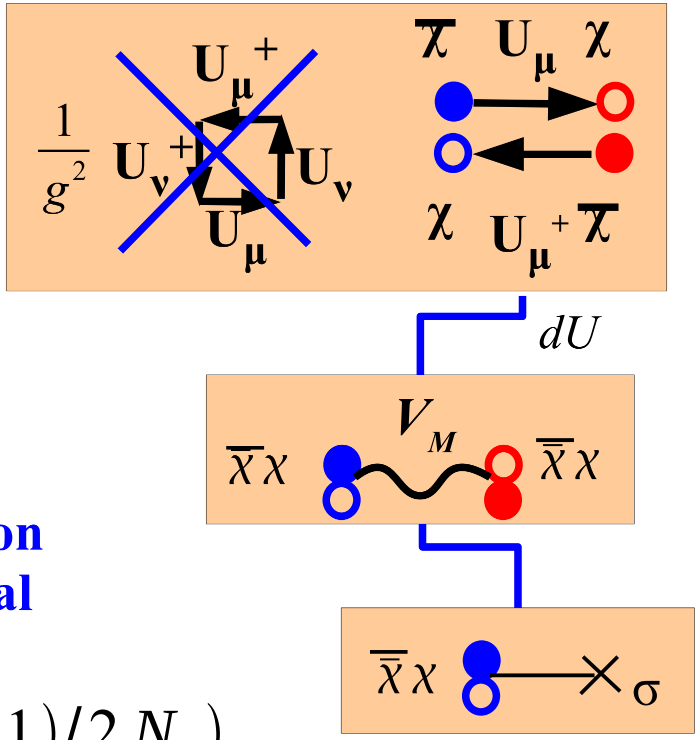
Bosonization

$$\rightarrow \frac{1}{2}\sigma V_M \sigma - N_c \sum_x \log(V_M \sigma(x) + m_q)$$

Fermion Integral

$$= L^d N_\tau \boxed{\left[\frac{1}{2} b_\sigma \bar{\sigma}^2 - N_c \log(b_\sigma \bar{\sigma} + m_q)\right]} \quad (b_\sigma = (d+1)/2N_c)$$

Effective Potential



- **Effective Potential in SCL-LQCD**

$$\boxed{U(\sigma) = \frac{1}{2} b_\sigma \sigma^2 - N_c \log \sigma \quad (b_\sigma = (d+1)/2N_c)}$$

n_f species of staggered fermion

- **σ in one species of staggered fermions**

→ $\sigma_{\alpha\beta}$ in n_f species of staggered fermions

$$\mathcal{Z} = \int \mathcal{D}[\chi,\bar{\chi},U] \exp\left(-S_F[\chi,\bar{\chi},U]\right) \simeq \int \mathcal{D}[\chi,\bar{\chi}] \exp\left[\frac{1}{2}\sum_{x,y,\alpha,\beta} \mathcal{M}_{\alpha\beta}(x) V_M(x,y) \mathcal{M}(y)_{\beta\alpha}\right]$$

$$= \int \mathcal{D}[\chi,\bar{\chi},\sigma] \exp\left(-S_\sigma[\chi,\bar{\chi},\sigma]\right)$$

$$S_\sigma = \frac{1}{2}\sum_{x,y,\alpha,\beta} \sigma(x)_{\alpha\beta} V_M(x,y) \sigma(y)_{\beta\alpha} + \sum_{x,y,\alpha,\beta} \sigma(y)_{\alpha\beta} V_M(y,x) \mathcal{M}(x)_{\beta\alpha}$$

$$\mathcal{M}_{\alpha\beta}(x) = \bar{\chi}^a_\alpha(x)\chi^a_\beta(x)$$

$$V_M(x,y) \;= \textstyle\sum_\mu \left(\delta_{y,x+\hat{\mu}} + \delta_{y,x-\hat{\mu}}\right)/4N_c$$

- **Mean field ansatz of the meson field**

$$\sigma_{\alpha\beta}(x) = \Sigma_{\alpha\beta} + i\epsilon(x)\Pi_{\alpha\beta}$$

ε = 1 (even site), -1 (odd site) → σ = M (even) , M^+(odd)

- **Effective Potential (Free Energy Density) in the Lattice Unit**

$$\boxed{V_\chi(\sigma,\pi) = \frac{1}{2}\langle \mathrm{tr}\left[\sigma V_M \sigma\right]\rangle - N_c \langle \log\det(V_M\sigma)\rangle \;= \frac{1}{2} b_\sigma \mathrm{tr}\left[M^\dagger M\right] - \frac{a_\sigma}{2}\log\det\left[M^\dagger M\right]}$$

E.g. SU(2) : tr(MM^+) = 2 det (MM^+) = $\sigma^2 + \pi^2$

Chiral $SU_f(2)$ Relativistic Mean Field with a Logarithmic σ Potential

Chiral RMF

- **RMF Lagrangian**

$$\begin{aligned}\mathcal{L}_\chi = {} & \overline{\psi}_N \left[i\partial\!\!\!/ - g_\sigma(\sigma + i\gamma_5 \boldsymbol{\tau}\cdot\boldsymbol{\pi}) - g_\omega \omega\!\!\!/ - g_\rho \boldsymbol{\tau}\cdot\boldsymbol{\rho}\!\!\!/ \right]\psi_N \\ & + \frac{1}{2}(\partial^\mu\sigma\partial_\mu\sigma + \partial^\mu\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\pi}) - \boxed{V_\sigma(\sigma,\boldsymbol{\pi})} \\ & - \frac{1}{4}W^{\mu\nu}W_{\mu\nu} + \frac{1}{2}m_\omega^2\omega^\mu\omega_\mu + \boxed{\frac{c_\omega}{4}(\omega^\mu\omega_\mu)^2} - \frac{1}{4}\boldsymbol{R}^{\mu\nu}\cdot\boldsymbol{R}_{\mu\nu} + \frac{1}{2}m_\rho^2\boldsymbol{\rho}^\mu\cdot\boldsymbol{\rho}_\mu\end{aligned}$$

  - **Mesons: σ , ω , ρ (π is omitted in MFA)**
  - **ω^4 term is added phenomenologically to simulate RBHF results**
  - **V_σ(σ , π) : chiral potential + explicit χ breaking term**

    (Coef. are chosen to fit meson masses → One parameter m_σ is left)

$$V_\sigma^{\mathrm{SCL}} = V_\chi(\sigma,\boldsymbol{\pi}) - c_\sigma\,\sigma = \frac{1}{2}b_\sigma\phi^2 - a_\sigma\log\phi^2 - c_\sigma\,\sigma$$

$$\phi^2 = \sigma^2 + \boldsymbol{\pi}^2 \quad a_\sigma = \frac{f_\pi^2}{4}(m_\sigma^2 - m_\pi^2)\ , \quad b_\sigma = \frac{1}{2}(m_\sigma^2 + m_\pi^2)\ , \quad c_\sigma = f_\pi m_\pi^2$$

  - **Remaining parameters: g_ω, g_ρ, c_ω**

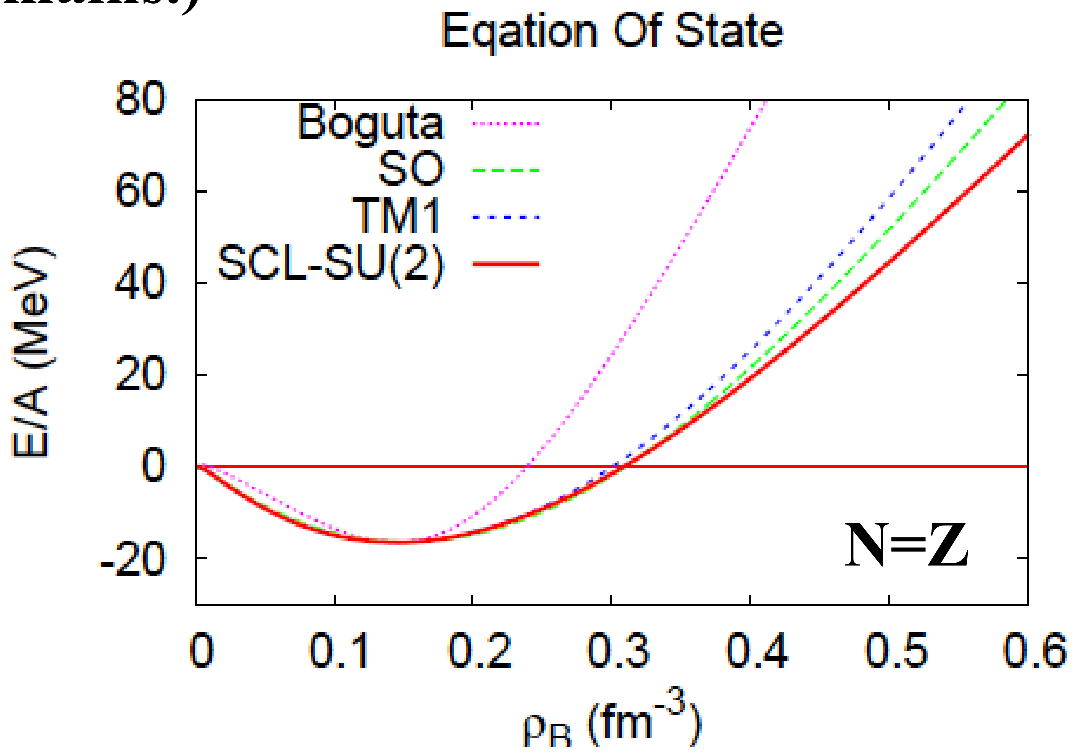
Nuclear Matter EOS

- **Energy Density**

$$E/V = g_N \int^{p_F} \frac{dp}{(2\pi)^3}\sqrt{p^2 + {M_N^*}^2(\sigma)} + g_\omega \omega \rho_B - \frac{m_\omega^2}{2}\omega^2 - \frac{c_\omega}{4}\omega^4 + V_\sigma(\sigma)$$

- **Relevant par. = g_ω, c_ω , m_σ**
  → Fit saturation point (ρ_0, E/A)=(0.145 fm^{-3}, -16.3 MeV)
  (One pamameter (e.g. m_σ) remains.)

- **EOS is as soft as TM1**
  (Non-chiral RMF)



Bog.: Boguta 1983, Ogawa et al. 2004
SO: Sahu, AO, 2000
TM1: Sugahara, Toki, 1994

Finite Nuclei (1)

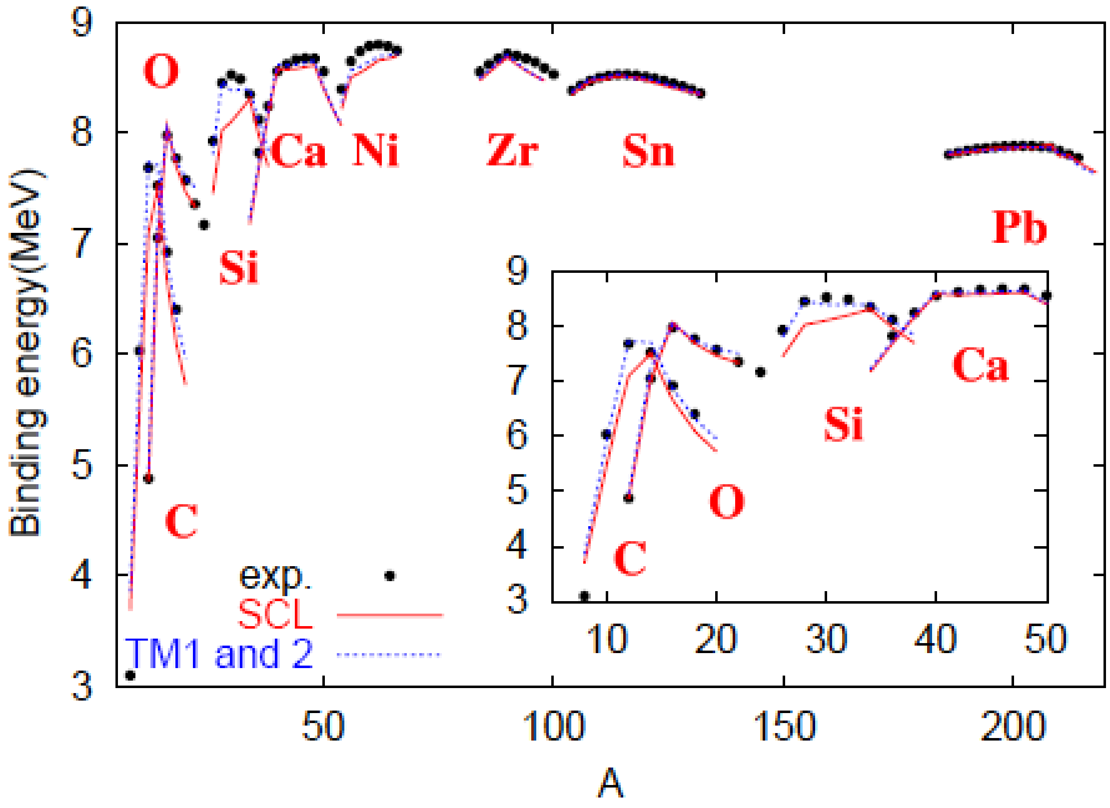
- Total Energy

$$E = \sum_{i,\kappa,\alpha} n_{i\kappa\alpha}^{\rm occ}\, \varepsilon_{i\kappa\alpha} - \frac{1}{2}\int \left\{ -g_\sigma \varphi \rho_S + g_\omega \omega \rho_B + g_\rho R \rho_\tau + e^2 A \rho_B^p \right\} dr + \int \left(V_\varphi - \frac{1}{2}\varphi \frac{dV_\varphi}{d\varphi} + \frac{c_\omega}{4}\omega^4 \right) dr$$

- Free par. = m_σ , g_ρ → Fit B.E. of Sn and Pb isotopes
  m_σ =503 MeV, g_ρ =4.40

- Problem
  - Underestimate of B.E. of light jj-closed nuclei
    → π effects ?
  - Underestimate of Zn isotopes
    → Deformation
    (Nuclei are assumed to be spherical here.)

D. Hirata, et al., 1997
Sugahara, Toki, 1994



Finite Nuclei (2)

- **Detailed Comparison of B.E. with other models**
  - **SCL is comparable with “High precision” non-chiral RMF models (TM1/2, NL1, NL3)**
  - **much better than another chiral models (glueball model)**

B/A (MeV)										
Nucleus	exp.	SCL	TM1	TM2	NL1	NL3	I/110	IF/110	VIIIF/100	QMC-I
^{12}C	7.68	7.09	-	7.68	-	-	-	-	-	-
^{16}O	7.98	8.06	-	7.92	7.95	8.05	7.35	7.86	7.18	5.84
^{28}Si	8.45	8.02	-	8.47	8.25	-	-	-	-	-
^{40}Ca	8.55	8.57	8.62	8.48	8.56	8.55	7.96	8.35	7.91	7.36
^{48}Ca	8.67	8.62	8.65	8.70	8.60	8.65	-	-	-	7.26
^{58}Ni	8.73	8.54	8.64	-	8.70	8.68	-	-	-	-
^{90}Zr	8.71	8.69	8.71	-	8.71	8.70	-	-	-	7.79
^{116}Sn	8.52	8.51	8.53	-	8.52	8.51	-	-	-	-
^{196}Pb	7.87	7.87	7.87	-	7.89	-	-	-	-	-
^{208}Pb	7.87	7.87	7.87	-	7.89	7.88	7.33	7.54	7.44	7.25

NL1: Reinhard et al. (Frankfurt group), 1986
NL3: Lalazissis, Konig, Ring, 1997
Glueball model: Heide, Rudaz, Ellis, 1994
QMC: Saito, Tsushima, Thomas, 1997

Short Summary of Chiral $SU_f(2)$ RMF

- We have developed an $SU_f(2)$ chiral symmetric RMF with a logarithmic σ potential derived from SCL-LQCD.
- Judge
  - (1) which explains nuclear matter saturation property → O
  - (2) does not contradict to nuclear ab initio calculations → △ (Vector pot. is similar to RBHF, no pions)
  - (3) well describes bulk properties (B.E., rmsr) of normal nuclei → O (at least for heavy nuclei)
  - (4) and hypernuclei → X
  - (5) possesses the chiral symmetry → O
  - (6) explains known properties of neutron stars → ?(not yet studied)
  - (7) explodes supernovae → ? (not yet studied)
  - (8) and has clear relation to QCD. → △ (see, Disclaimer)
- It is promising. We should go to $SU_f(3)$.

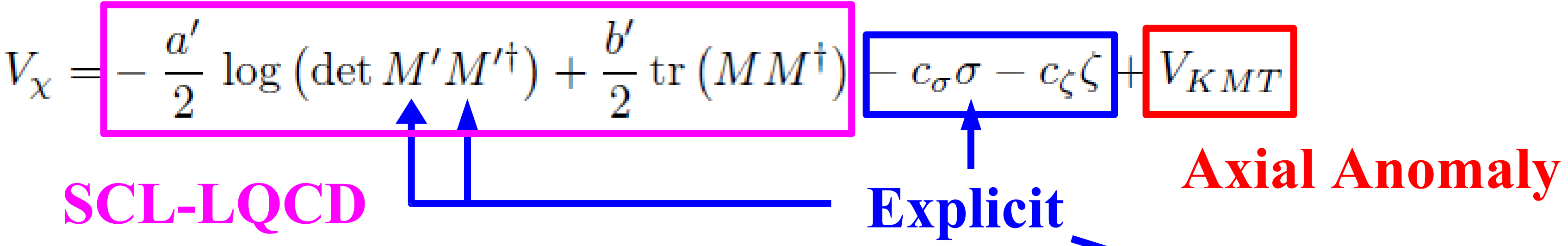
Hypernuclei and Nuclear Matter EOS in Chiral $SU_f(3)$ Relativistic Mean Field

Chiral $SU_f(3)$ Potential (1)

- **Characteristic features in $SU_f(3)$**
  - **Hidden strangeness (s sbar) mesons can have expectation values.**
    → scalar (ς) and vector (ϕ) mesons *(Glendenning, Schaffner, Gal, ...)*
  - **s quark mass is not small**
  - **Axial ($U_A(1)$) anomaly has to be included.**

Kobayashi, Maskawa, 1970,
't Hooft, 1976

- **Chiral SUf(3) Potential**
  SCL-LQCD (Zero T treatment)+ Explicit χ breaking
  + Kobayashi-Maskawa-'t Hooft term ($U_A(1)$ anomaly)



$$V_\chi = \boxed{-\frac{a'}{2}\log\left(\det M'M'^\dagger\right) + \frac{b'}{2}\,\mathrm{tr}\left(MM^\dagger\right)}\ \boxed{-c_\sigma\sigma - c_\zeta\zeta} + \boxed{V_{KMT}}$$

- **M = Σ + i Π = Meson Matrix**

$$M = \begin{pmatrix} M_{11} & a_0^+ + i\pi^+ & \kappa^0 + iK^+ \\ a_0^- + i\pi^- & M_{22} & \kappa^0 + iK^0 \\ \kappa^- + iK^- & \bar{\kappa}^0 + i\bar{K}^0 & M_{33} \end{pmatrix}$$

$$M_{11} \equiv \left(\frac{a_0^0}{\sqrt{2}} + \frac{\sigma}{\sqrt{2}}\right) + i\left(\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta'}{\sqrt{3}}\right)$$

$$M_{22} \equiv \left(-\frac{a_0^0}{\sqrt{2}} + \frac{\sigma}{\sqrt{2}}\right) + i\left(-\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta'}{\sqrt{3}}\right)$$

$$M_{33} \equiv \zeta + i\left(-\frac{2\eta}{\sqrt{6}} + \frac{\eta'}{\sqrt{3}}\right)$$

$$M' = M + \mathrm{diag}(0, 0, \delta_\zeta)$$

Chiral SU$_f$(3) Potential (2)

- **Decomposition into mass and interaction terms**

$$V_\chi = \frac{1}{2} m_\sigma \varphi_\sigma^2 + \frac{1}{2} m_\zeta \varphi_\zeta^2 + V_{\sigma\zeta}(\varphi_\sigma, \varphi_\zeta) + \frac{1}{2} \sum_\alpha m_\alpha^2 \phi_\alpha^2 + \delta V(\varphi_\sigma, \varphi_\zeta, \{\phi_\alpha\})$$

$\varphi_\sigma = f_\pi - \sigma$

Deviation from vacuum values

$\varphi_\zeta = f_\zeta - \zeta$

π , K , … interaction terms

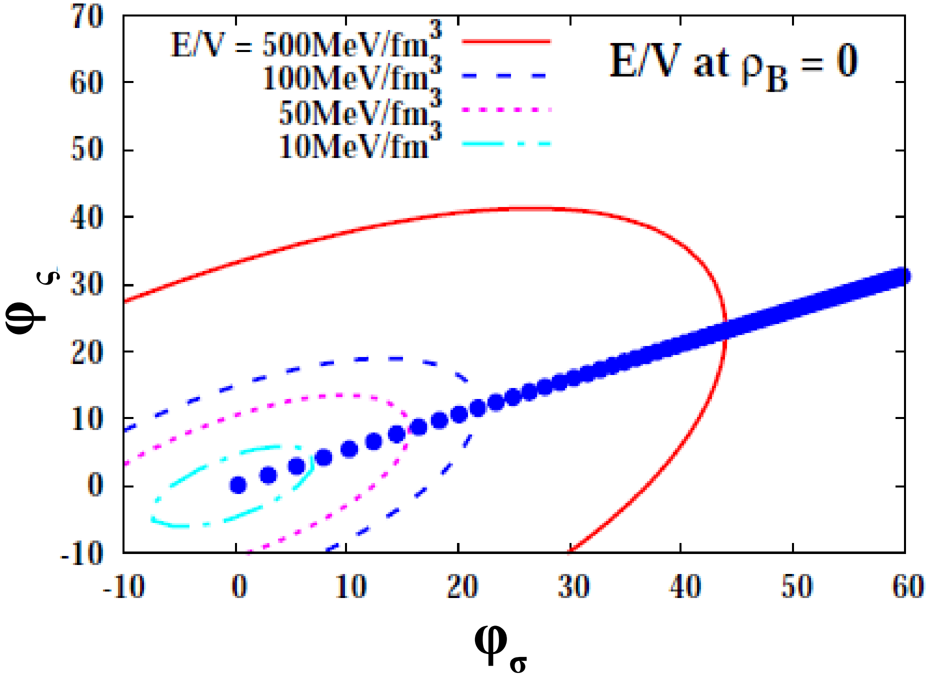
π, K, ...

- **Interaction term of (σ , ς)**

$$V_{\sigma\zeta} = -a' \left[2 f_{\rm SCL}\left(\frac{\varphi_\sigma}{f_\pi}\right) + f_{\rm SCL}\left(\frac{\varphi_\zeta}{f'_\zeta}\right) \right] + \xi_{\sigma\zeta}\varphi_\sigma\varphi_\zeta$$

$$f_{\rm SCL}(x) = \log(1-x) + x + \frac{x^2}{2}$$

  - **σ and ς mixes through KMT ($\varphi_\sigma^2 \varphi_\varsigma$ term is omitted)**



Chiral $SU_f(3)$ RMF (for Normal and Λ nuclei)

- **Chiral $SU_f(3)$ RMF Lagrangian (SCL3)**

$$\mathcal{L} = \sum_i \bar{\psi}_i \left[i\partial\!\!\!/ - M_i^* - \gamma_\mu U_i^\mu \right] \psi_i$$

Baryons (N, Λ)

Mesons(free)

$$\boxed{-\frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{m_\omega^2}{2}\omega_\mu\omega^\mu - \frac{1}{4}R_{\mu\nu}R^{\mu\nu} + \frac{m_\rho^2}{2}R_\mu R^\mu - \frac{1}{4}\phi_{\mu\nu}\phi^{\mu\nu} + \frac{m_\phi^2}{2}\phi_\mu\phi^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}}$$

$$\boxed{+\frac{1}{2}\partial_\mu\varphi_\sigma\partial^\mu\varphi_\sigma - \frac{1}{2}m_\sigma^2\varphi_\sigma^2 + \frac{1}{2}\partial_\mu\varphi_\zeta\partial^\mu\varphi_\zeta - \frac{1}{2}m_\zeta^2\varphi_\zeta^2} \boxed{+\frac{c_\omega}{4}\left(\omega_\nu\omega^\nu\right)^2} \boxed{-V_{\sigma\zeta}\left(\varphi_\sigma, \varphi_\zeta\right)}$$

$$M_i^* = M_i - g_{\sigma i}\varphi_\sigma - g_{\zeta i}\varphi_\zeta \qquad U_i^\mu = g_{\omega i}\,\omega^\mu + g_{\rho i}\,R^\mu + \frac{1+\tau_3}{2}eA^\mu$$

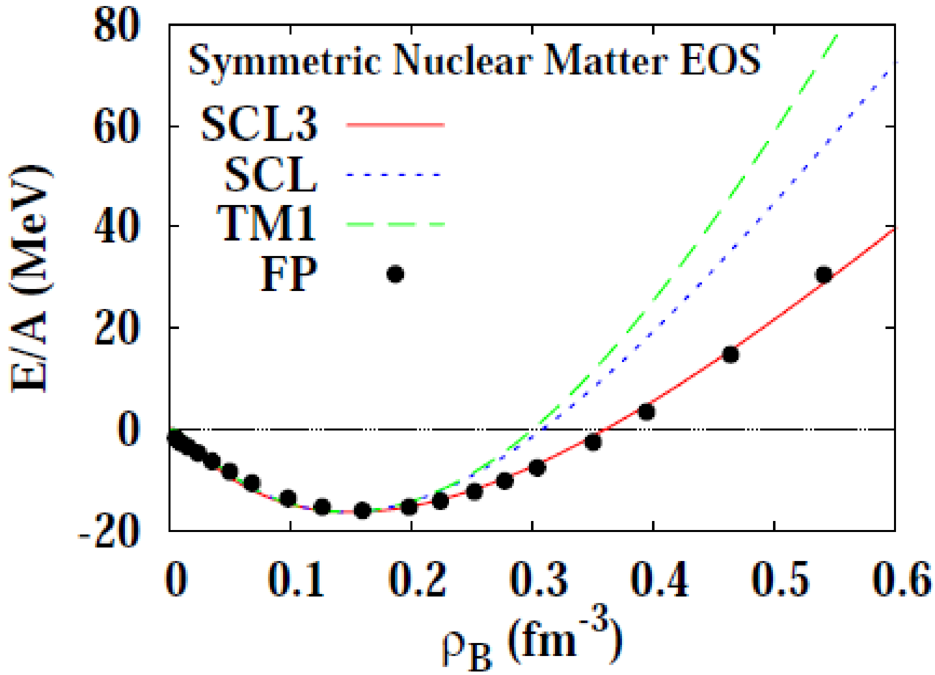
ω int. **χ int.**

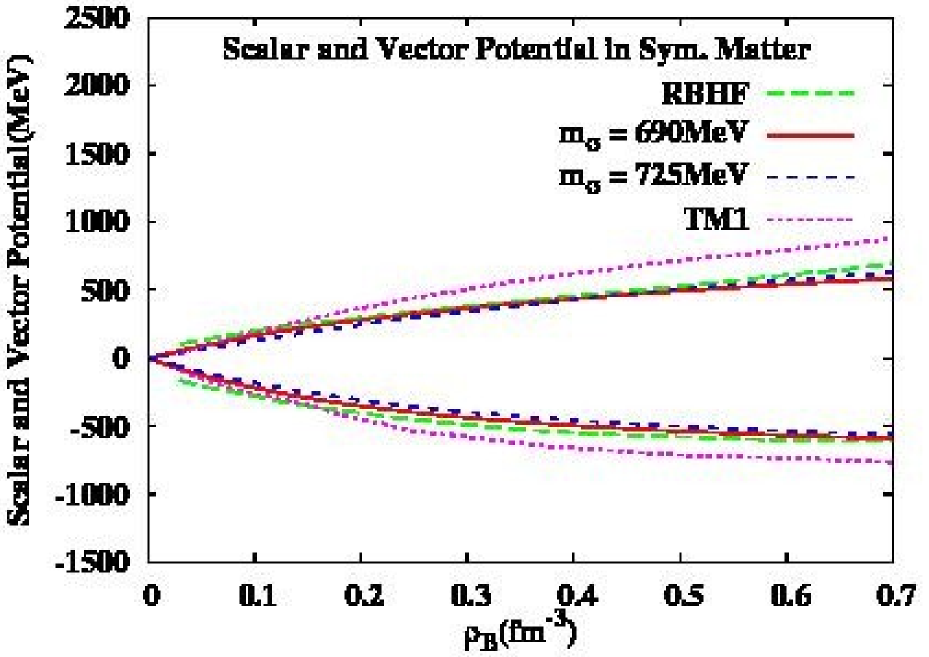
- **Parameter Fitting**
  - **Vacuum part ($V_{\sigma\varsigma}$)**
    6 pars. → Fit f_π, f_K, m_π, m_K, M_ς (f0(980)) → 1 par. (e.g. m_σ)
  - **Nucleon part (assumed not to couple with $s^{\mathrm{bar}}s$)**
    1 + 3 pars. (m_σ, g_ω, c_ω, g_ρ) → Saturation Point (2) + Finite Nuclei (2)
  - **Λ part**
    4 pars. (g_σ , g_ς , g_ω, g_ϕ) → $SU_f(3)$ relation for vector (2)

Nuclear Matter

- **Nuclear Matter EOS**
  - **Softer than other RMF models incl. SCL2 (K ~ 210 MeV).**
  - **Agrees with Friedman-Pandharipande (FP) EOS at around ρ_0**

    (Softer than FP EOS at higher densities) *Friedman, Pandharipande, 1981*
- **Scalar and Vector Potentials**
  - **Agree with RBHF results.** *Brockmann, Toki, 1992*



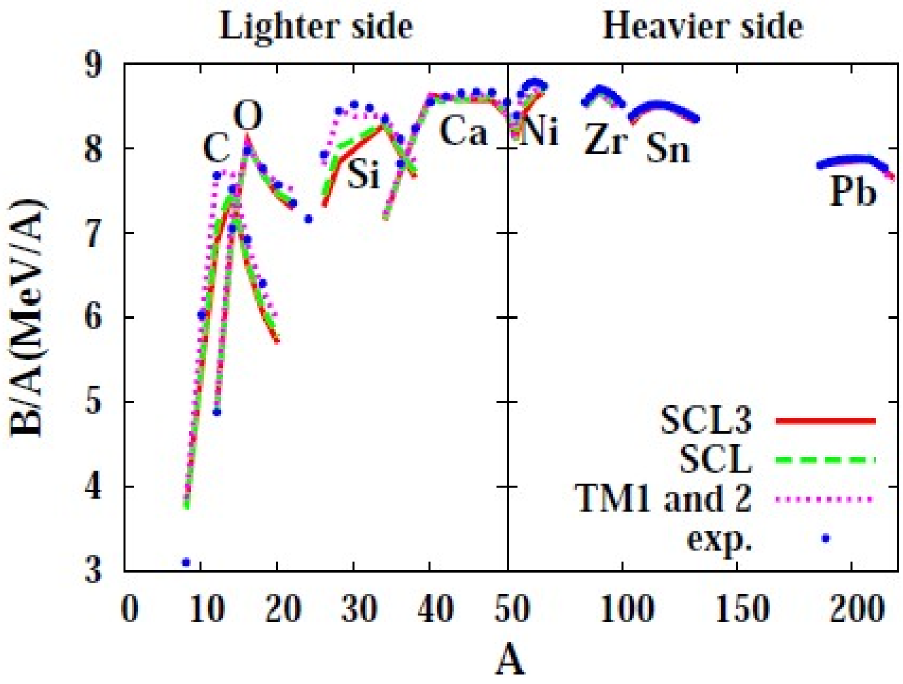


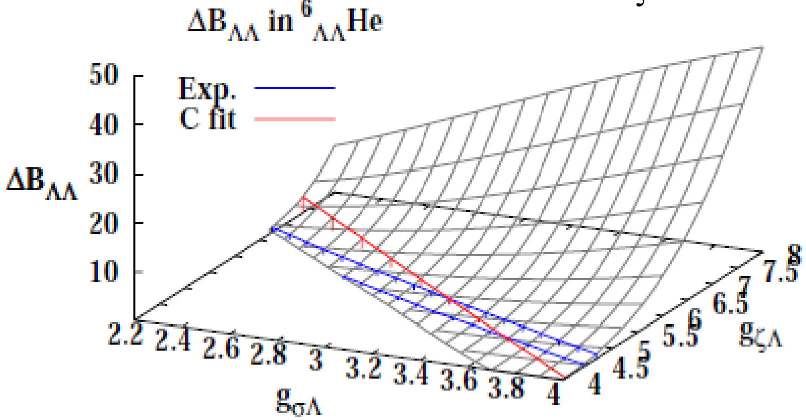
Finite Normal Nuclei and Hypernuclei

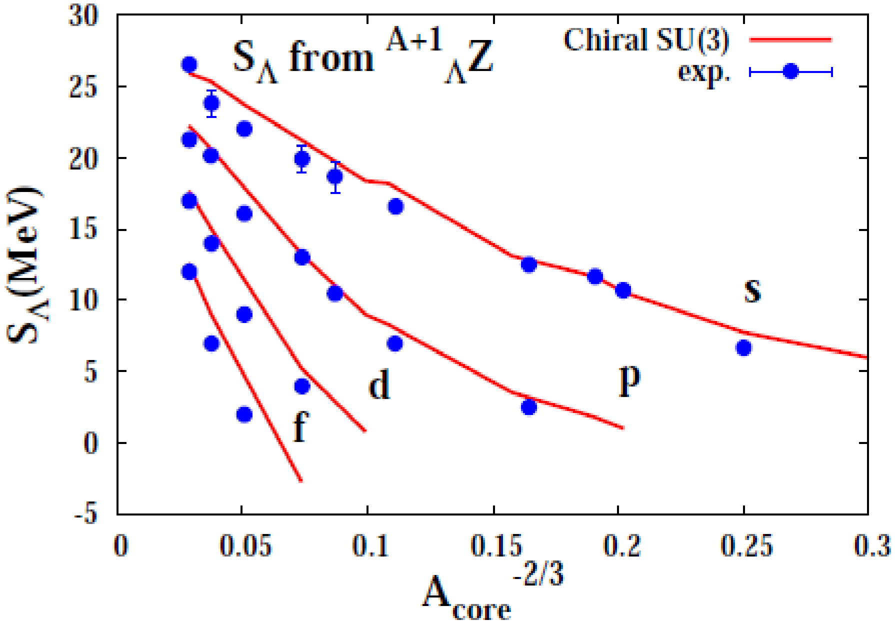
- **Normal Nuclei**
  - **B.E. and Charge RMSR are well described with $c_\omega = 295$, $g_{\rho N}=4.54$, $m_\sigma = 690$ MeV**
- **Single Hypernuclei**
  - **Similar sep. E. (S_Λ) are obtained if the scalar potential depth is fixed.**

$$U_\Lambda^{(S)}(\rho) = g_{\sigma\Lambda}\sigma(\rho) + g_{\zeta\Lambda}\zeta(\rho)$$

- **Double Hypernuclei**
  - **$\Delta B_{\Lambda\Lambda}$ is more sensitive to $g_{\varsigma\Lambda}$**

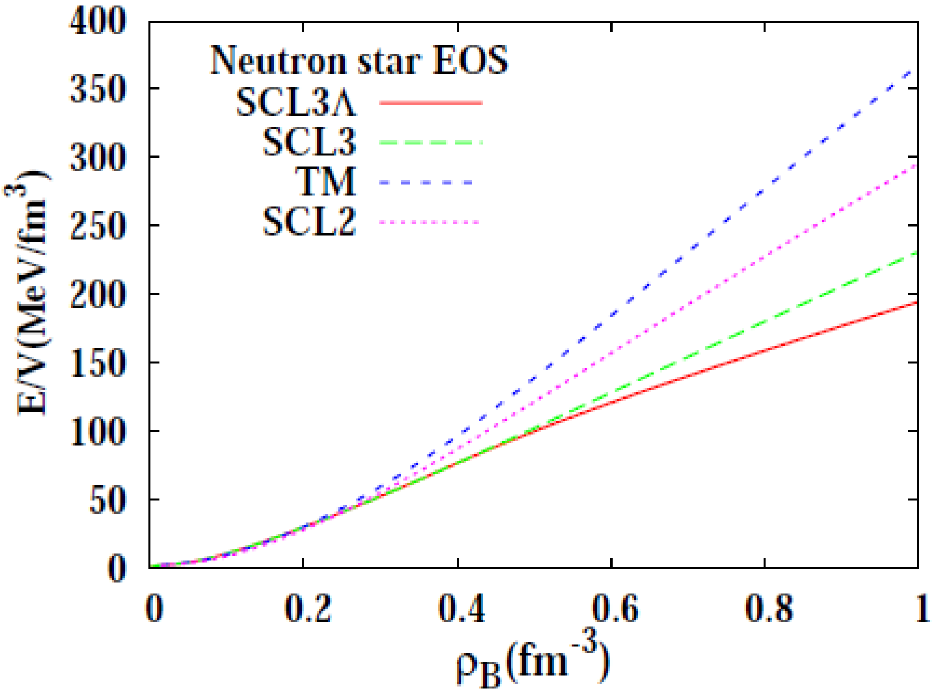


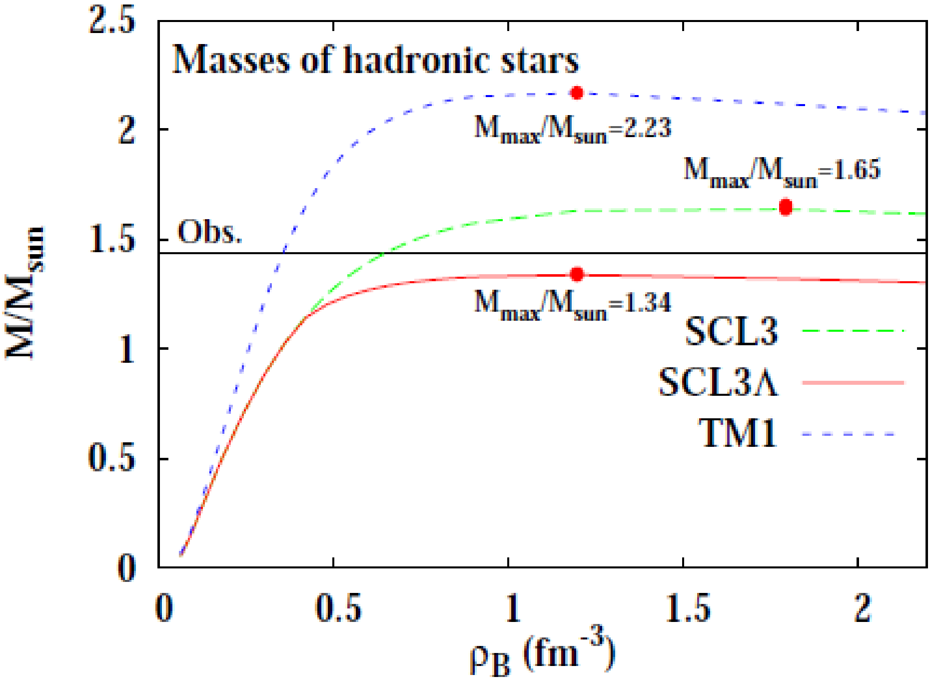




Neutron Stars

- Neutron star matter
  = Cold and dense charge neutral matter under ν-less β equilibrium.
- EOS in SCL3-RMF is much softer than others at high densities.
  - small $g_{\omega N}$, large c_ω → suppression of vector potential at high ρ_B
- SCL3-RMF with Λ (SCL3Λ) → Max. mass of NS < 1.44 M_{sun}





Summary

- We have developed an $SU_f(3)$ "chiral symmetric" RMF with a logarithmic potential of scalar condensates derived from SCL-LQCD.
  - Hidden strangeness scalar meson (ζ) is found to soften the EOS through the coupling with σ (KMT interaction).
- Judge

  (1) which explains nuclear matter saturation property → O

  (2) does not contradict to nuclear ab initio calculations
  → O' (Vector pot. is similar to RBHF, EOS ~ FP)

  (3) well describes bulk properties (B.E., rmsr)
  of normal nuclei
  → O' (at least for heavy nuclei, worse than SCL2)

  (4) and hypernuclei → O

  (5) possesses the chiral symmetry → O

  (6) explains known properties of neutron stars → X

  (7) explodes supernovae → ? (not yet studied)

  (8) and has clear relation to QCD. → △ (see, Disclaimer)

Problems and Future Works

- Problems to be solved
  - How can we support NS with mass 1.44 M_{sun}?
    EOS around ρ_0 seems to be good.
    → Extra repulsion at high density is necessary.
    Vector meson mass reduction does not help.
    (Tsubakihara et al., AIP Conf. Proc. 1016 (2008), 156.)
  - Spin-orbit interaction is too week in normal nuclei,
    too big in Λ hypernuclei.
    Scalar and vector potentials are smaller than other RMF models.
    → Explicit effect of π ?
    (E.g. Ikeda, Sugimoto, Toki, 2004; Isshiki, AO, Naito, 2005)
  - Log. type potential from SCL-LQCD (zero T treatment) may be too simple.
    → Finite T (arcsinh), NLO- (Miura) or NNLO-SC-LQCD (T.Z. Nakano)

From QCD to Supernovae: Underway