Evolution of the QCD phase diagram in the Strong Coupling Region of Lattice QCD

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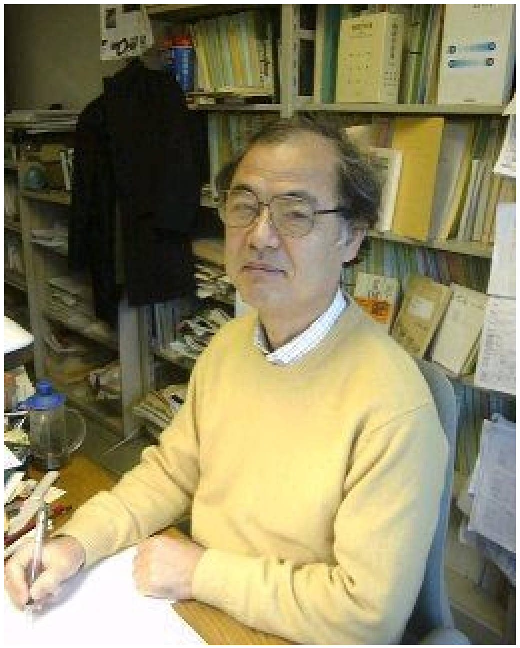
in collaboration with K. Miura, N. Kawamoto

- **Introduction**
- **Phase transition in Strong Coupling Limit of Lattice QCD along with Kawamoto's works**
- **Phase Diagram Evolution with $1/g^2$ Correction**
- **Summary**

See also, Poster by K. Miura

Congratulations, Profs. Ishikawa & Kawamoto

- For your 60 year birthday !



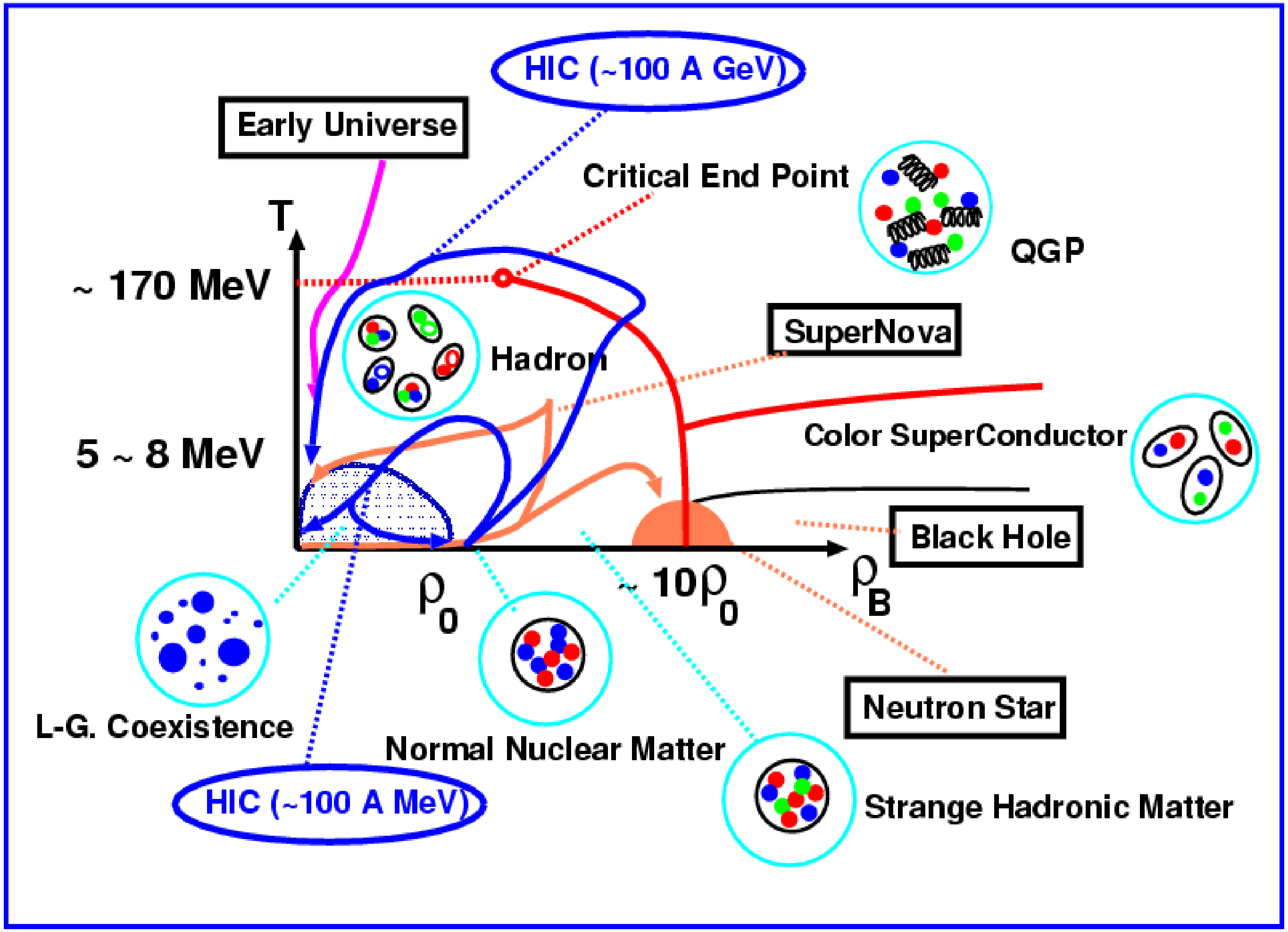
He always says,
“ バカヤロー !”
(“F*** you !”),
“ 冗談いってんじゃねえよ”
(No kidding !),
and
“Strong Coupling is much better than 4-Fermi (Nambu-Jona-Lasinio).”

- I am invited as a collaborator of Kawamoto-san, then I have to tell the truth of him.
  - I know him
    in Kyoto U. ↔ (Student)
    in Inst. for Nuclear Study ↔ (PD, staff in Hokkaido U.)
    and in Hokkaido U. ↔ (Hokkaido U./YITP)

Phase Transition in Strong Coupling Limit of Lattice QCD along with Kawamoto's works

I'm interested in ....

- **Quark / Hadron / Nuclear Matter EOS and Phase Diagram**

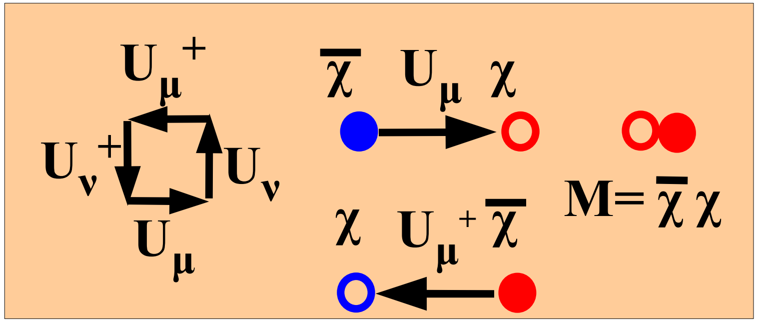


Rich Structure / Astrophysical implications / Accessible in HIC
→ Can we understand it in QCD ?

Lattice QCD

- **Lattice QCD=ab initio, non-perturbative theory (c.f. Teper's talk)**

$$S_{\rm LQCD} = \frac{1}{2}\sum_{x,j}\left[\eta_{\nu,x}\bar{\chi}_x U_{\nu,x}\chi_{x+\hat{\nu}} - \eta^{-1}_{\nu,x}\bar{\chi}_{x+\hat{\nu}} U^{\dagger}_{\nu,x}\chi_x\right] - \frac{1}{g^2}\sum_{\Box}\mathrm{tr}\left[U_{\Box} + U^{\dagger}_{\Box}\right] + m_0\sum_x \bar{\chi}_x\chi_x$$



- **Problems to overcome**
  - **DOF is too much, and MC is necessary for numerical integration**
    → Faster Computer + Faster Algorithm
  - **Doublers appear for chiral fermions → different types of fermions**
  - **Weight for gluon config. (Fermion determinant) becomes complex at finite μ**
    → Taylor expansion, Analytic Continuation, Canonical, …
    → Not Yet Applicable for Dense and Cold Matter !

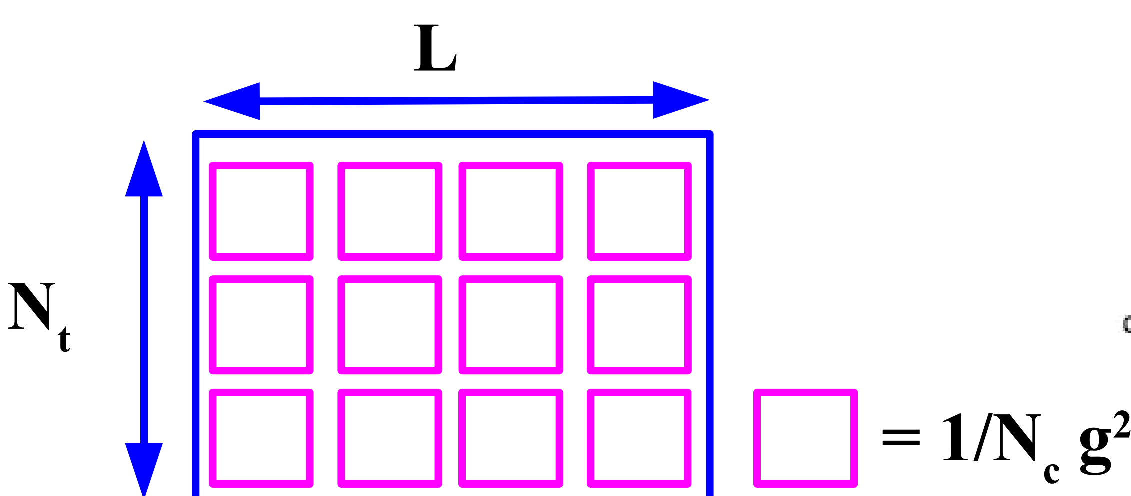
Strong Coupling Limit/Expansion makes it possible to obtain (approx.) Effective Potential analytically !

Strong Coupling Lattice QCD: Pure Gauge

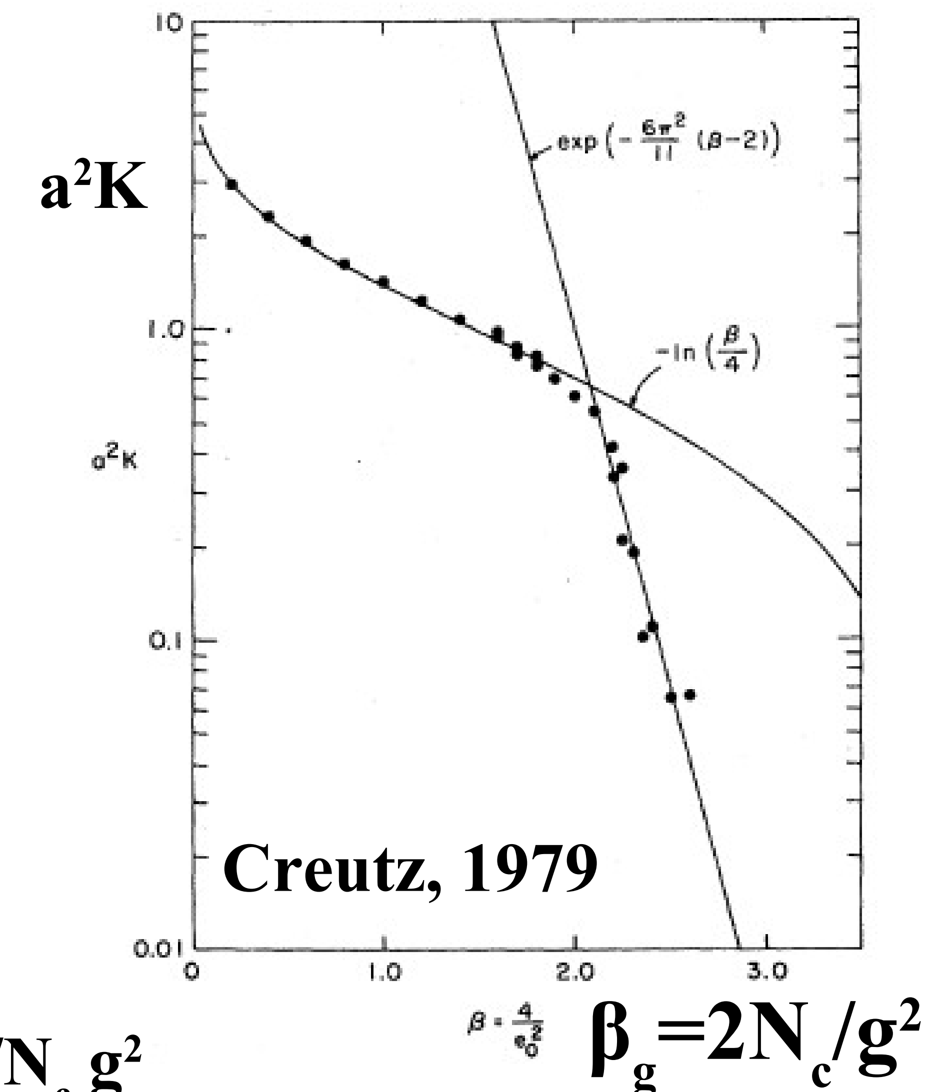
- **Quarks are confined in Strong Coupling QCD**
  - **Strong Coupling Limit (SCL)**
    → Fill Wilson Loop with Min. # of Plaquettes
    → Area Law (Wilson, 1974)

$$S_{\mathrm{LQCD}} = -\frac{1}{g^2}\sum_{\Box}\mathrm{tr}\left[U_\Box + U_\Box^\dagger\right]$$

  - **Smooth Transition from SCL to pQCD in MC (Creutz, 1980)**



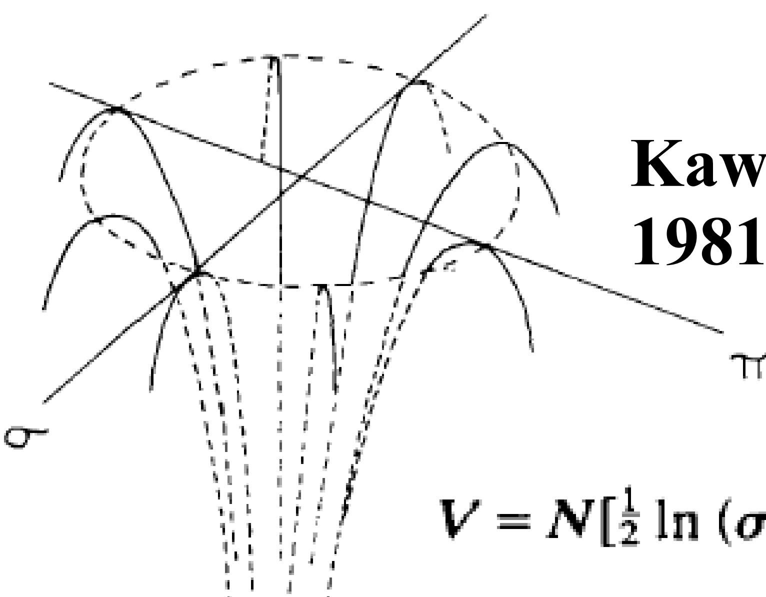
K. G. Wilson, PRD10(1974),2445
M. Creutz, PRD21(1980), 2308.
G. Munster, 1981



Strong Coupling Limit of LQCD with Quarks

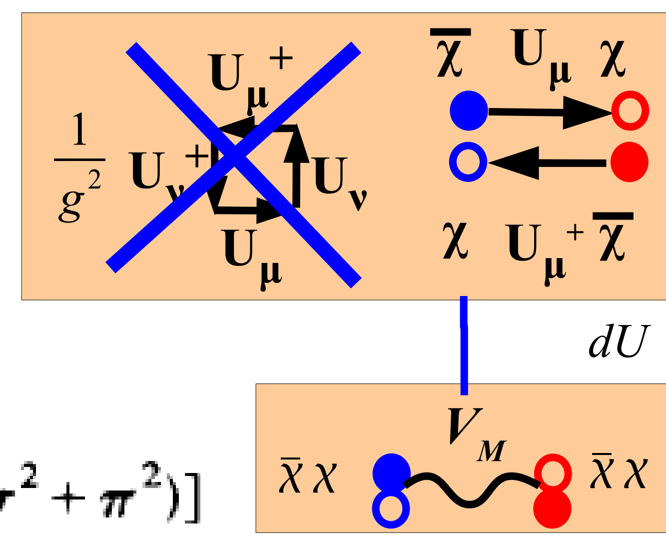
N. Kawamoto, NPB190('81),617, N. Kawamoto, J. Smit, NPB192('81)100
Kluberg-Stern, Morel, Napoly, Petersson, 1981

- **How about spontaneous chiral symmetry breaking ?**
- **Strong Coupling Limit (SCL) of Lattice QCD with Quarks**
  - **No Plaquette in SCL**
    → Mesonic Effective Action from One Link Integral
    → Effective Potential in σ and π from contour integral
    → SSB of the Chiral Sym.



Kawamoto, Smit, 1981

$$V = N[\tfrac{1}{2}\ln(\sigma^2+\pi^2) - M\sigma - dF(\sigma^2+\pi^2)]$$



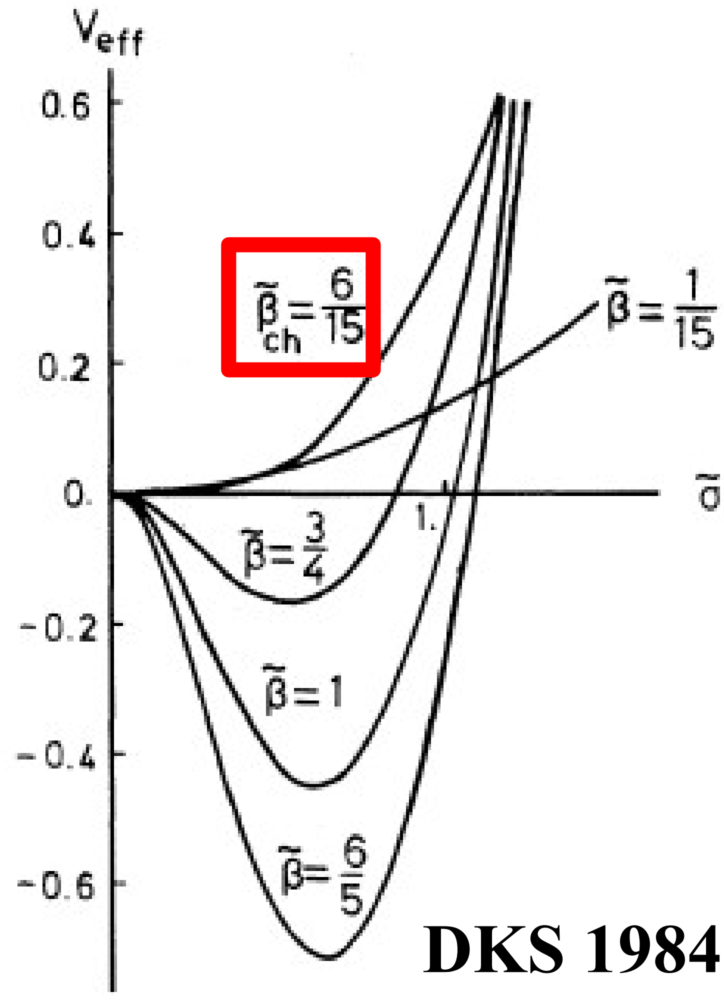
Chiral Transition at Finite Temperature

P.H.Damgaard, N. Kawamoto, K.Shigemoto, PRL53('84),2211; NPB264 ('86), 1
Faldt, Petersson, 1986; Bilic, Karsch, Redlich, 1992; Fukushima,2004, Nishida, 2004

- **Chiral Symmetry would be restored at high temperature**
  → SCL-LQCD at Finite Temperatures
  - **Staggered Fermion**
    with Anti-Periodic B.C.
    → Matsubara Product
  - **Polyakov gauge & Group integral**
    (Vandermonde determinant)
  - **Effective Potential (U(3))**

$$V_{\rm eff} = \frac{1}{4} N \beta d \sigma^2 - \ln\left\{ \frac{\sinh[(N+1)\beta s]}{\sinh(\beta s)} \right\}$$

→ Chiral Phase Transition
at $T_c = 2.5\ a^{-1}$

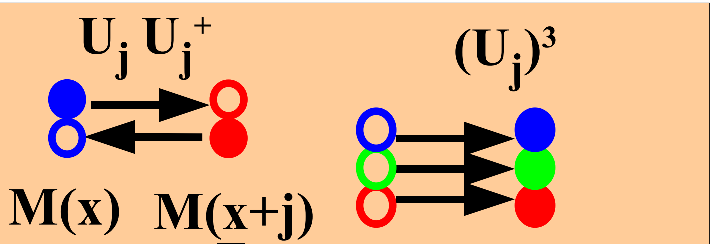


DKS 1984

Chiral Phase Transition at Finite Density

P.H.Damgaard, D. Hochberg, N. Kawamoto, PLB158('86)239
Hasenfatz, Karsch, 1983; Azcoiti et al., 2003; Kawamoto, Miura, AO, Ohnuma, 2007

- **QCD phase transition is also expected at high density**
  - **Baryon Rich QGP and/or Color SuperConductor are expected in the Neutron Star Core**
- **Strong Coupling Limit in SU(N)**
  - **Quark Chemical Potential and Baryonic Composite → Chiral phase transition at $\mu_c = 0.66\ a^{-1}$**

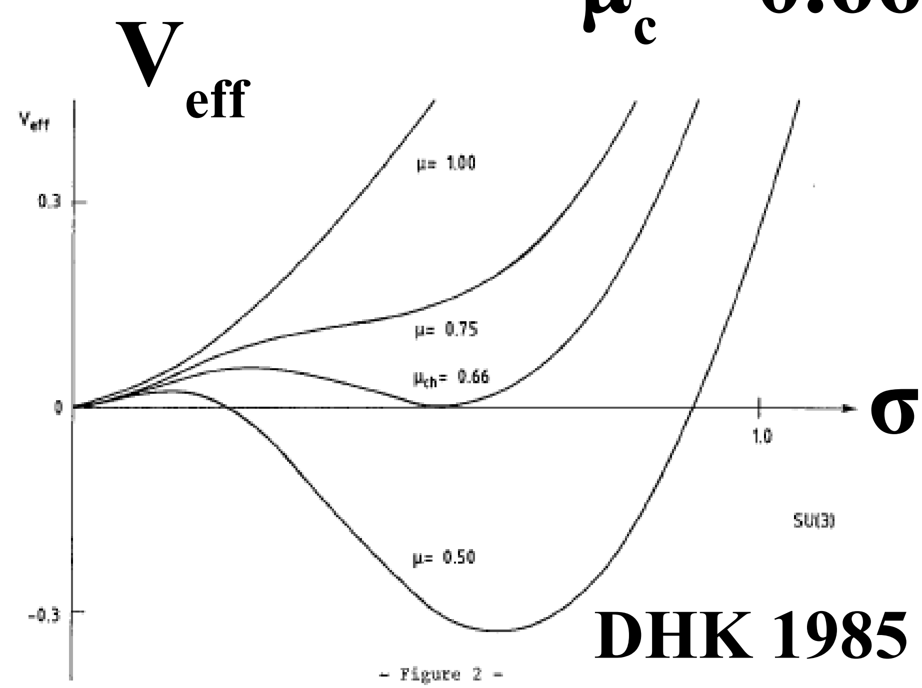


$\bar{B} = \epsilon \bar{\chi}\bar{\chi}\bar{\chi}/6 \quad B = \epsilon \chi\chi\chi/6$

$$\int dU\, U_{ab} U^+_{cd} = \frac{1}{N_c} \delta_{ad} \delta_{bc}$$

$$\int dU\, U_{ab} U_{cd} U_{ef} = \frac{1}{6} \epsilon_{ace} \epsilon_{bdf}$$

$\mu_c \sim 0.66$



DHK 1985

Is Kawamoto correct (SCL-LQCD and NJL) ?

- **Nambu-Jona-Lasinio (NJL) Model**
  - **Nobel prize in Physics, 2008 to Y. Nambu**
    Discovery of the mechanism of spontaneous broken symmetry in subatomic physics
    (Nambu and Jona-Lasinio, 1961)
  - **Chiral phase transition at finite T and μ**
    E.g. Hatsuda, Kunihiro, PRe247('94)221
  - **χ and Deconfinement transition at finite T and μ**
    E.g., Fukushima, PLB591('04)277; C. Ratti, M.A. Thaler, W. Weise, PRD73('06)014019

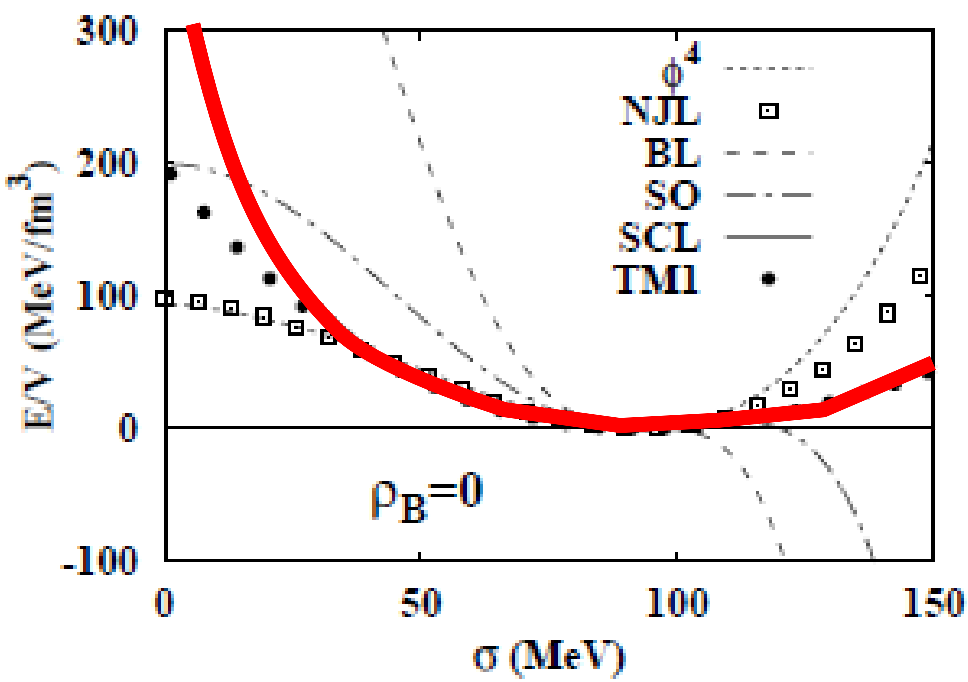
- **Strong Coupling Lattice QCD**
  - **Quark Confinement**
  - **Chiral Phase Transition at finite T and/or μ**

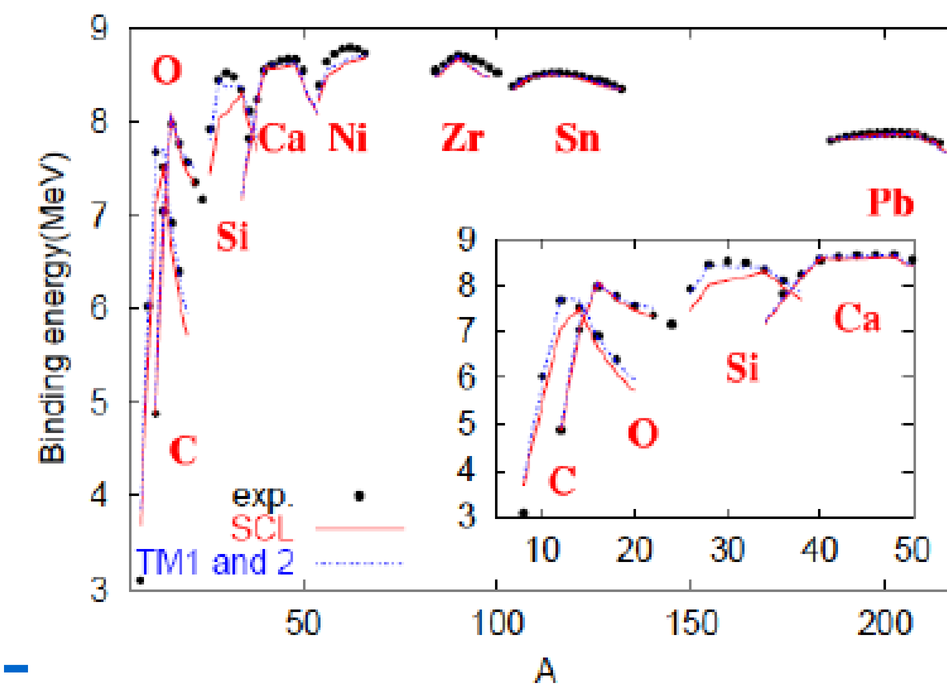
> ***We cannot say Kawamoto is correct, but at least there is one example, ...***

Chiral RMF based on SCL-LQCD

Tsubakihara, Ohnishi, PTP117('07)903

- **Chiral Collapse Problem:**
  At finite ρ_B, Nucleon Fermi Integral favors smaller σ
  $\rightarrow$Chiral Sym. is restored below ρ_0 in φ^4 theory (Lee-Wick, 1974)
  (Effective potential from NJL is similar to that in φ^4 theory)
- **Chiral nuclear many-body theory (RMF) based on the effective potential from SCL-LQCD has no chiral collapse problem, and is successful in explaining nuclear properties**

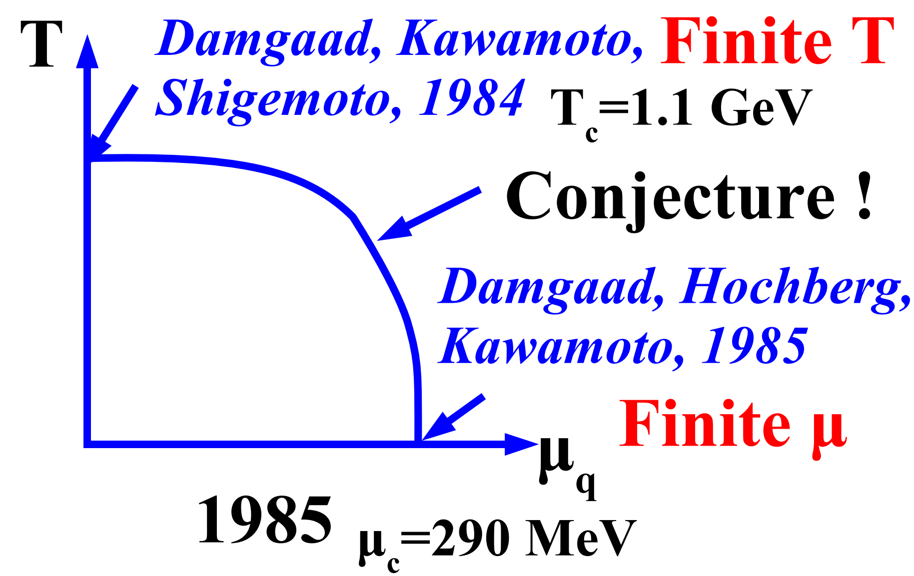


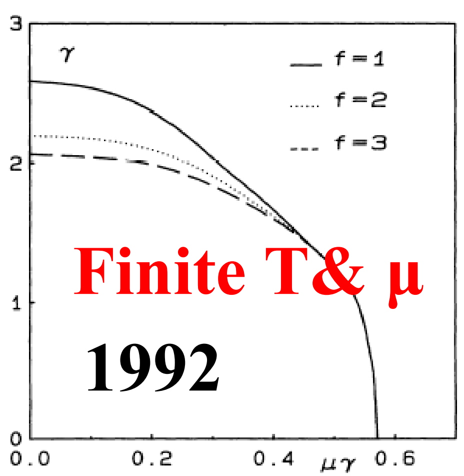


Phase Diagram Evolution with $1/g^2$ Correction

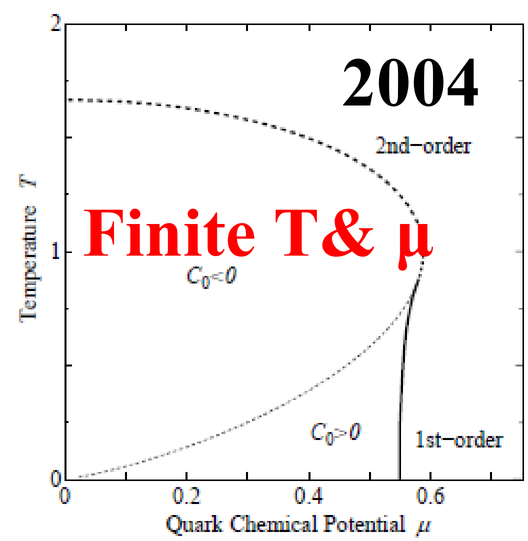
Evolution of Phase Diagram as a function of Time

- **Phase Diagram "Shape" becomes closer to that of Real World,**
  $R = 3\mu_c/T_c \sim (6\text{-}12)$
  - **1985 → R=0.79 (Zero T / Finite T)**
  - **1992 → R=0.83 (Finite T & μ)**
  - **2004 → R= 0.99 (Finite T& μ)**
  - **2007 → R=1.34 (Baryon)**

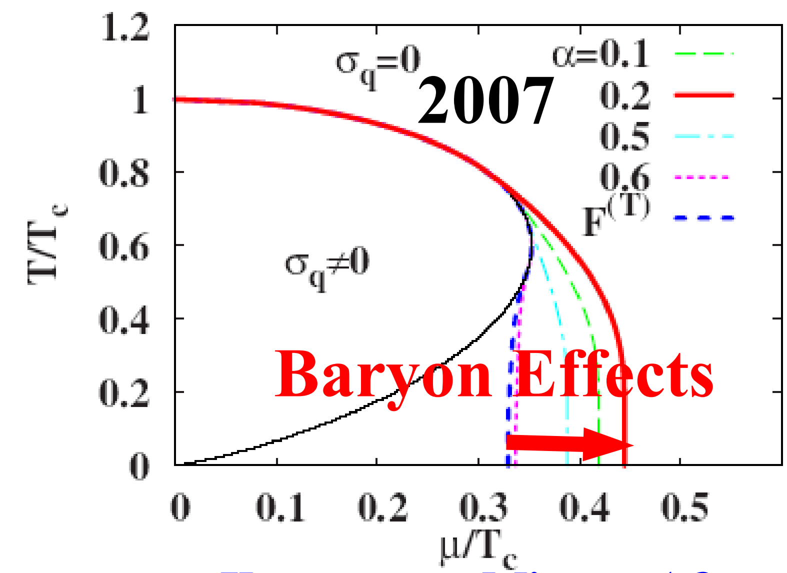




Bilic, Karsch, Redlich, 1992



Fukushima, 2004, Nishida, 2004

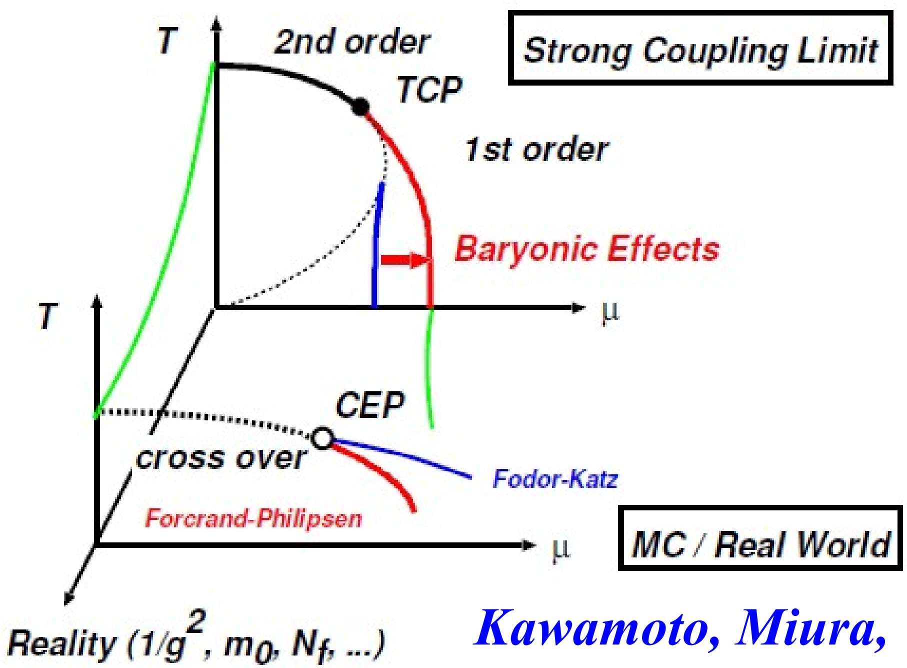


Kawamoto, Miura, AO, Ohnuma, 2007

Towards the Realistic Phase Diagram

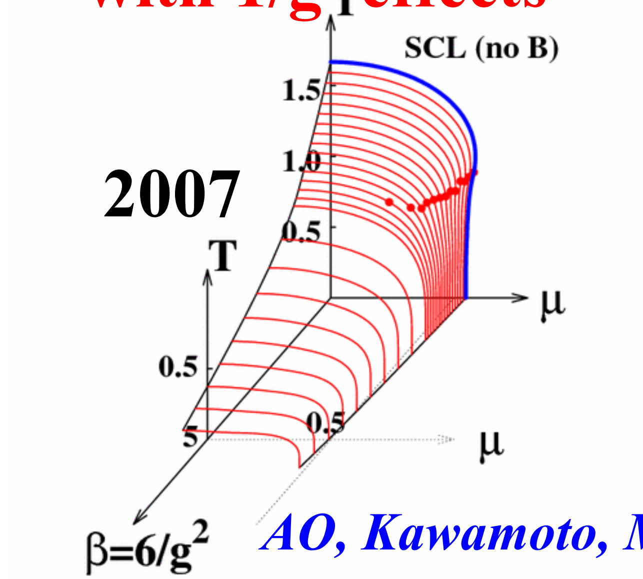
- **Why we cannot explain the phase diagram shape ?**
  → N_f (Staggered fermion) ? quark mass ? Finite Coupling ?
  - **μ_c (SCL) ~ $M_N/3$ (within a factor 2) , T_c (SCL) >> 200 MeV**
    → Larger problem should be in T_c , rather than in μ_c

Expectation before Calc.



Kawamoto, Miura, AO, Ohnuma, 2007

Preliminary Resuls with $1/g^2$ effects



AO, Kawamoto, Miura, 2007

Gluon Contribution is important at High T

Effective Potential in SCL-LQCD

- **QCD Lattice Action (Finite T treatment)**

Damgaard, Kawamoto, Shigemoto, 1984; Bilic et al., 1992; Nishida, '04; Fukushima, '04; Kawamoto, Miura, AO, Ohnuma, '07; ...

$$S = \cancel{S_G} + S_F^{(s)} + S_F^{(t)} + m_0 \bar{\chi}\chi$$

Strong Coupling Limit

$$S_F^{(t)} = \frac{1}{2}\sum_x \left(e^{\mu}\bar{\chi}_x U_0(x)\chi_{x+\hat{0}} - e^{-\mu}\bar{\chi}_{x+\hat{0}} U_0^{+}(x)\chi_x \right) = \bar{\chi} V^{(t)} \chi$$

$$\to -\frac{1}{2}(\bar{\chi}\chi) V_M (\bar{\chi}\chi) + \bar{\chi}(V^{(t)} + m_0)\chi$$

Spatial-link integral (1/d expansion)

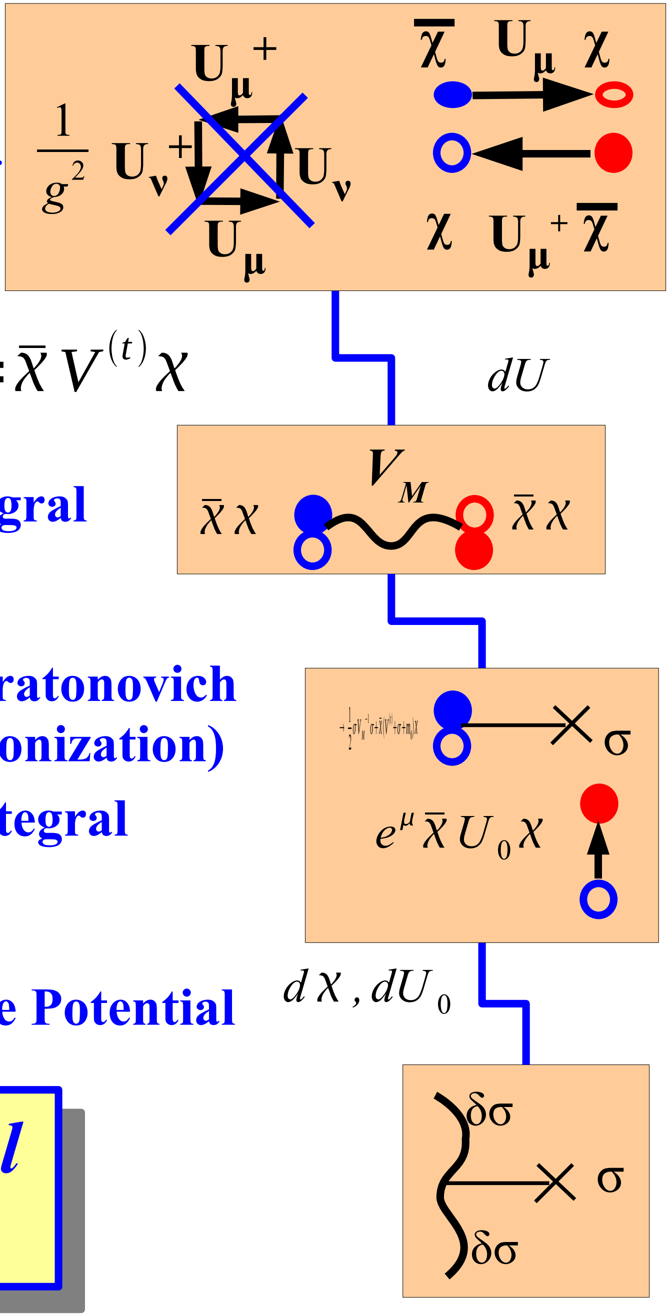
$$\to \frac{1}{2}\sigma V_M \sigma + \bar{\chi}(V^{(t)} + V_M \sigma + m_0)\chi$$

Hubbard-Stratonovich Transf. (Bosonization)

Fermion and Temporal-link Integral

$$\to L^d N_\tau \left[\frac{d}{4N_c}\bar{\sigma}^2 + V_q(b_\sigma \bar{\sigma}, T, \mu) \right]$$

SCL Effective Potential



We can obtain the Effective Potential analytically at finite T and μ

Effective Potential with $1/g^2$ (1)

- **1/d expansion of Plaquette action (Spatial One-Link Integral)**

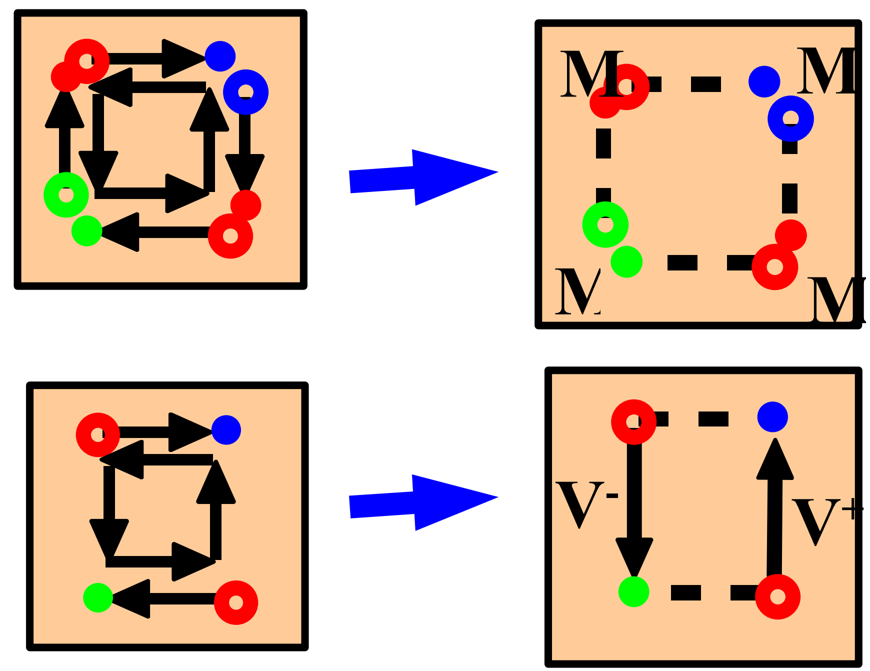
Faldt, Petersson (86); Bilic, Karsch, Redlich (92)

$$\int dU\, U_{ab} U^{+}_{cd} = \frac{1}{N_c} \delta_{ad} \delta_{bc}$$

  - **Spatial plaquett → MMMM**
  - **Temporal Link → V^+V^-**

$$V_x^+ = e^{\mu} \bar{\chi}_x U_0(x) \chi_{x+\hat{0}}$$

$$V_x^- = e^{-\mu} \bar{\chi}_{x+\hat{0}} U_0^{\dagger}(x) \chi_x$$



- **Effective Action**

$$\Delta S_\beta^{(\tau)} = \frac{1}{4N_c^2 g^2} \sum_{x,j>0} (V_x^+ V_{x+\hat{j}}^- + V_x^+ V_{x-\hat{j}}^-)$$

$$\Delta S_\beta^{(s)} = -\frac{1}{8N_c^4 g^2} \sum_{x,k>j>0} M_x M_{x+\hat{j}} M_{x+\hat{k}} M_{x+\hat{k}+\hat{j}}$$

Effective Potential with $1/g^2$ (2)

Miura, AO, arXiv:0806.3357; Proc. of LAT08

- **Extended Hubbard-Stratonovich (EHS) Transf.**

$$e^{\alpha AB} = \int d\varphi\, d\phi\, e^{-\alpha\{\varphi^2-(A+B)\varphi+\phi^2-i(A-B)\phi\}}$$
$$= \int d\psi\, d\bar{\psi}\, e^{-\alpha\{\bar{\psi}\psi-A\psi-\bar{\psi}B\}} \approx e^{-\alpha\{\bar{\psi}\psi-A\psi-\bar{\psi}B\}}\Big|_{\text{stationary}}$$

  - **Applicable to product of different composites**
  - **Keeps the scaling invariance even after stationary cond.**

$$A \to \lambda A \text{ and } B \to \lambda^{-1} B \;\Rightarrow\; \psi \to \lambda^{-1}\psi \text{ and } \bar{\psi} \to \lambda\bar{\psi}$$

  - **Real A and B → Mean field approx. φ , Saddle point approx. for ϕ**

$$e^{\alpha AB} \approx e^{-\alpha\{\varphi^2-(A+B)\varphi-\phi^2+(A-B)\phi\}}$$

Effective Potential with 1/g² (3)

- **E.g. Temporal Plaquette action becomes,**

$$\Delta S_\beta^{(\tau)} \approx \frac{1}{4N_c^2 g^2} \sum_{x,j>0} \left[\varphi_\tau^2 + (V_x^+ - V_{x+\hat{j}}^-)\varphi_\tau - \phi_\tau^2 - (V_x^+ + V_{x+\hat{j}}^-)\phi_\tau\right] + (j \leftrightarrow -j)$$

- **Effective Action becomes similar to the SCL action,**

WF Renormalization → $(1+\beta_\tau\varphi_\tau)$

μ mod. → $e^{-\beta_\tau\phi_\tau}$, $e^{\beta_\tau\phi_\tau}$

Aux. Terms → $+\beta_s\varphi_s$, $+N_\tau L^d\left[\frac{\beta_\tau}{2}(\varphi_\tau^2 - \phi_\tau^2) + \frac{d\beta_s}{2}\varphi_s^2\right]$

$$S_{\rm eff} = \frac{1}{2}(1+\beta_\tau\varphi_\tau)\sum_x \left(e^{-\beta_\tau\phi_\tau}V_x^+ - e^{\beta_\tau\phi_\tau}V_x^-\right) + m_0\sum_x M_x - \left(\frac{1}{4N_c} + \beta_s\varphi_s\right)\sum_{x,j>0} M_x M_{x+\hat{j}} + N_\tau L^d\left[\frac{\beta_\tau}{2}(\varphi_\tau^2 - \phi_\tau^2) + \frac{d\beta_s}{2}\varphi_s^2\right]$$

Effective Potential with $1/g^2$ (4)

- Effective Potential (after subst. equil. value for φ_τ and φ_s)

$$\mathcal{F}_\mathrm{eff} = \mathcal{F}_\mathrm{X}(\sigma, \phi_\tau) + \mathcal{V}_\mathrm{q}(m_q(\sigma), \tilde{\mu}(\phi_\tau), T)$$

$$\mathcal{V}_\mathrm{q} = -T \log\left[X_{N_c}(E_q/T) + 2\cosh(N_c\tilde{\mu}/T)\right]$$

Same as SCL

$$\mathcal{F}_\mathrm{X} = \frac{1}{2} b_\sigma \sigma^2 + \frac{\beta_\tau}{2}\sigma^2 (m_q^\mathrm{SCL})^2 + \frac{3d\beta_s}{2}\sigma^4 - \frac{\beta_\tau}{2}\phi_\tau^2$$

$$m_q = m_q^\mathrm{SCL}(1 - N_c\beta_\tau) + \beta_\tau\sigma(m_q^\mathrm{SCL})^2 + 2d\beta_s\sigma^3$$

$$\tilde{\mu} = \mu - \beta_\tau\phi_\tau$$

from Plaq.

- W.F.Renormalization factor $(1+\beta_\tau\varphi_\tau)$ in the Eff. Action
  → suppr. of quark mass m_q
- Higher order terms $M^4 \to \sigma^4$ (Self-energy of σ)
- Aux. Field $\phi_\tau = \rho_q$ (equil.) → "Vector" Field (μ shift, Repulsion)

Let us examine the phase diagram with this F_{eff} !

Evolution of T_c and μ_c

Miura, AO, arXiv:0806.3357; Proc. of LAT08

- **T_c (μ=0) rapidly decreases with $\beta = 6/g^2$ increases.** *(c.f. Bilic et al. '92)*
  - **MC results (N_τ =2) Quench β_c=5.097(1) (Kennedy et al, 1985)**

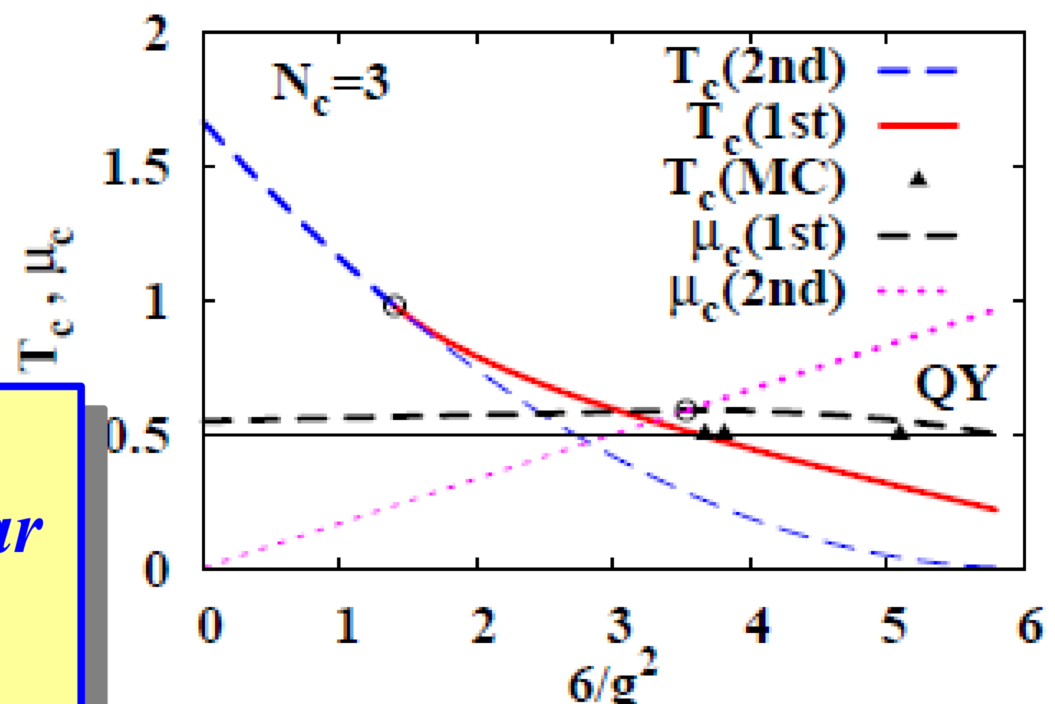
    m_0=0.05 → β_c=3.81(2), m_0=0.025 → β_c=3.67(2)

    (de Forcrand, private comm.)

MC results with small m_0 agrees with SC-LQCD !

- **$\mu_c^{(2nd)} > \mu_c^{(1st)}$ at $6/g^2 > 3.53$**
  - **Key: Effective chem. pot.**

    $$\mu_{\rm eff} = \mu - \beta_\tau \phi_\tau = \mu - \beta_\tau \rho_q$$



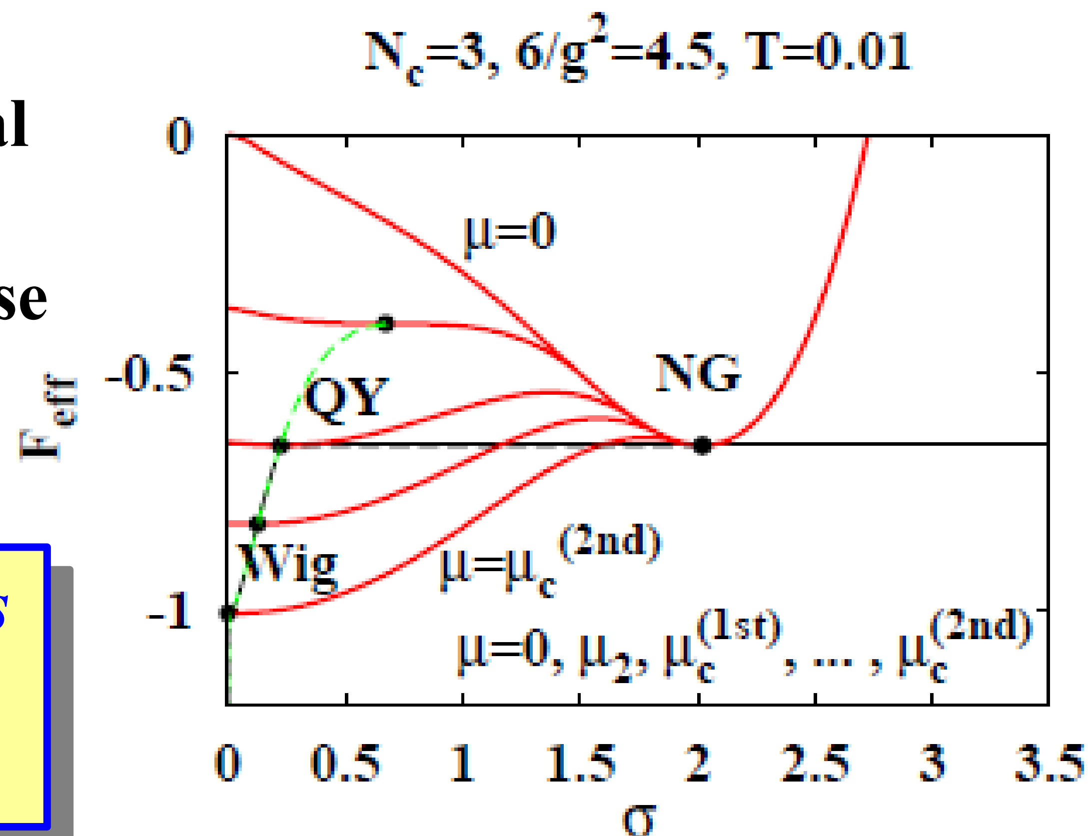
Spontaneously χ broken
high density matter may appear
(We regard it as
the Quarkyonic Phase)

Third Phase in Chiral Phase Transition

Miura, AO, arXiv:0806.3357; Proc. of LAT08

- **Vector field (ϕ_τ) acts more repulsively at smaller σ, and generates a local minimum in the region of $\sigma \ll \sigma_{vac}$**
- **Smaller σ**
  → Smaller const. quark mass
  → Larger ρ_q
  → Repulsive Vector Potential & Smaller μ_{eff}
  → Later P.T. to Wigner phase (σ=0)

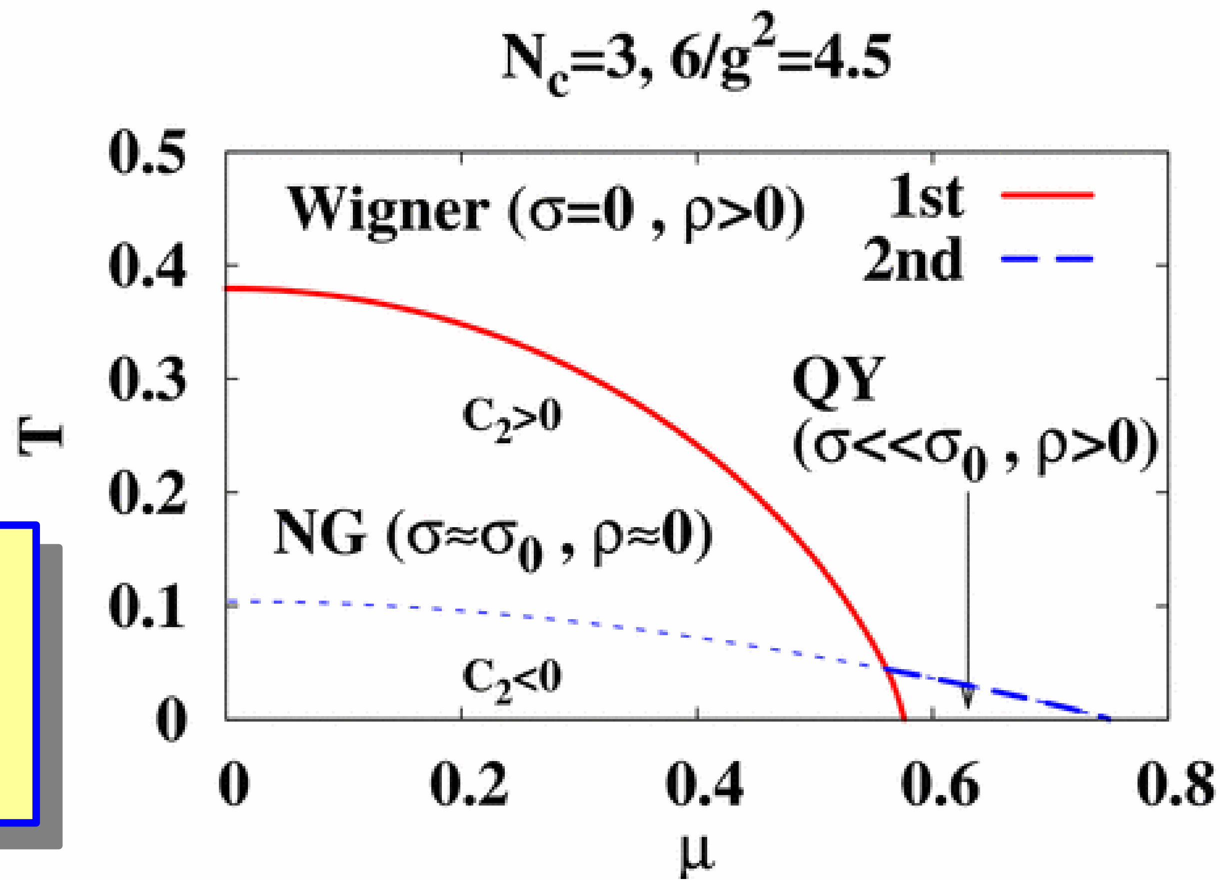
There may be THREE types of states in χSSB pattern at High Densities !



Phase Diagram

Miura, AO, arXiv:0806.3357; Proc. of LAT08

- Three phases in SC-LQCD with N_c=3, $6/g^2 > 3.53$, m_0=0 (χ limit)
  - Nambu-Goldstone (NG) phase: Large σ, Small ρ_q, Small P
  - Winger phase: σ=0, Large ρ_q, finite P
  - Quarkyonic phase:
    - $0 < \sigma << \sigma_{vac}$
    - $\rho_q(QY) \sim \rho_q(Wig.)$
    - $P(QY) < P(Wig.)$
    - Quark driven P → 0 at large N_c

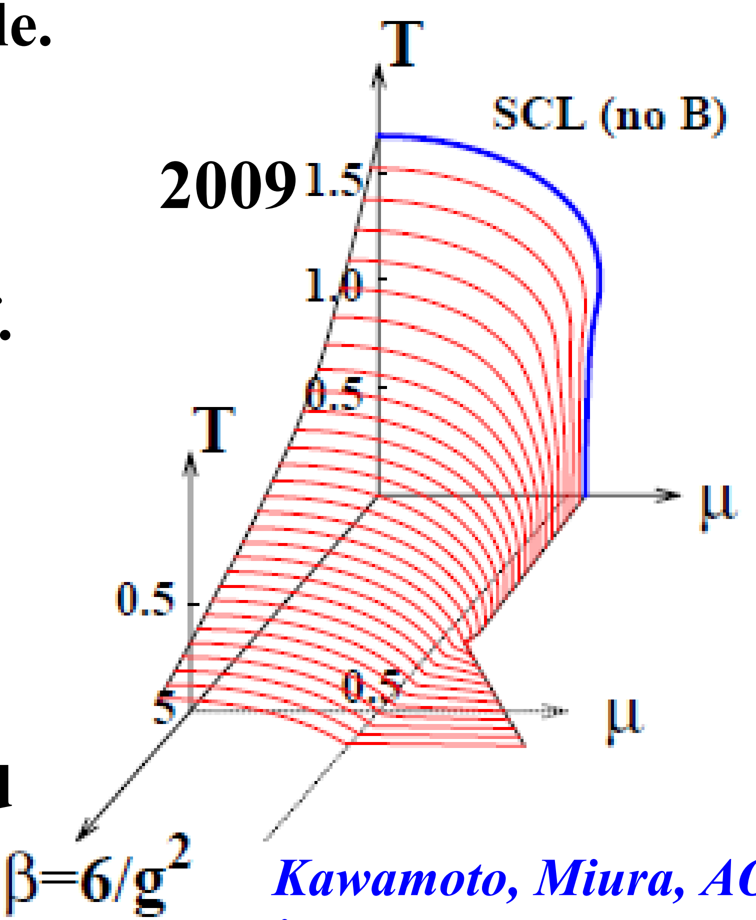


QY in SC-LQCD can be regarded as QY at large N_c

Summary and Discussions

- Strong Coupling QCD has been and still is a powerful tool to analyze the QCD phase diagram.
  - Analytic group integral → No (at least small) sign problem
  - Well-defined approximation (large g, 1/d expansion, mean field) → Systematic improvements are possible.
- Effects of finite coupling on the phase diagram are significant.
  - Extended Hubbard-Stratonovich transf. gives rise to "Vector" potential
  - Agreement of T_c (g_c) with MC is encouraging.
  - We may have the third chiral phase at finite density, which may be regarded as the Quarkyonicphase at large N_c.

Pisarski, McLerran, 2007



Kawamoto, Miura, AO, in prep.

Summary and Discussions

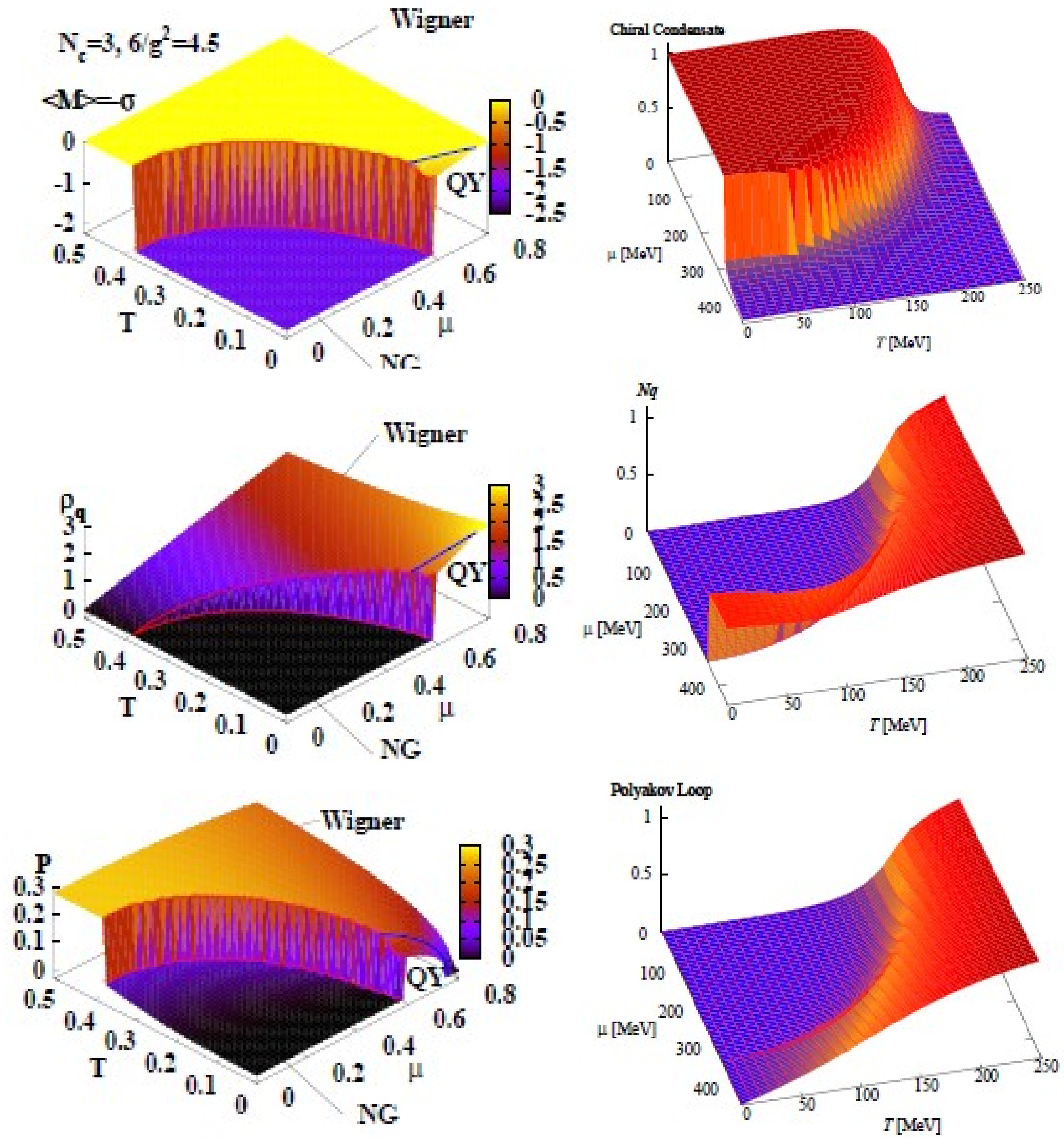
- **Problems:** ***It is not yet "The QCD"***
  - **One species of staggered fermion without quarter/square root**
    → $N_f = 4$
  - **Leading order in 1/d (d=spatial dim.)**
    → No baryon effects with $1/g^2$ corrections
  - **Mean Field treatment, No Diqaurk condensate**
  - **NLO in $1/g^2$ expansion (No Polyakov Loop dynamics) c.f. Miura**
- **Big Challenge from Nuclear Physics point of view**
  → Nuclear Matter EOS from SC-LQCD
  (MC simulation with warm MDP simulation,
  Fromm, de Forcrand, 2008)

- **Thank you Ishikawa-san and Kawamoto-san, for your contributions in Education, Research, and many other things.**

Backup

Comparison with Other Models

- **SC-LQCD results are similar to 2+1 flavor PNJL results in Chiral Cond., Baryon Density, and Polyakov Loop**
  Fukushima, PRD77(114028)08

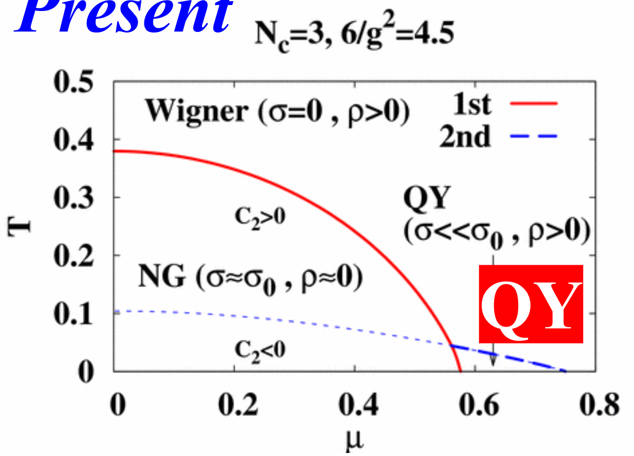


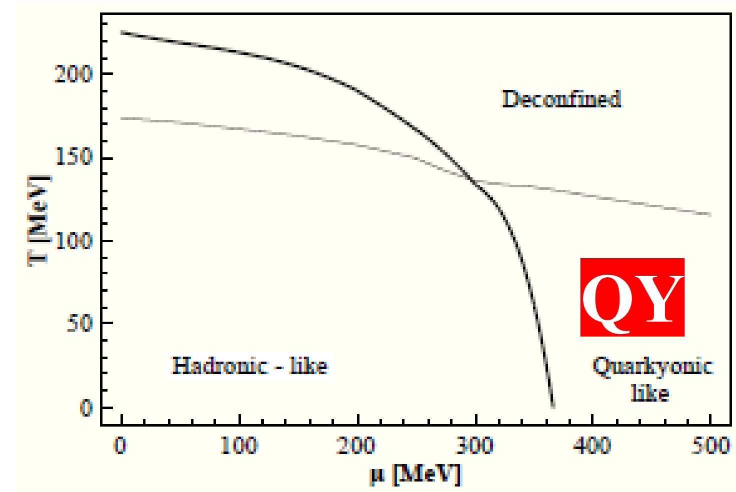
Present *Fukushima, 2008*

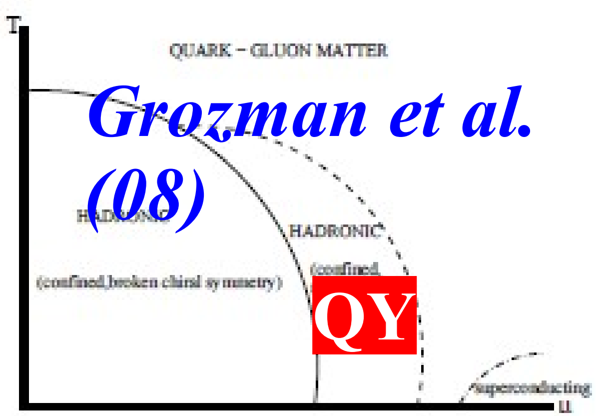
Comparison with Other Models

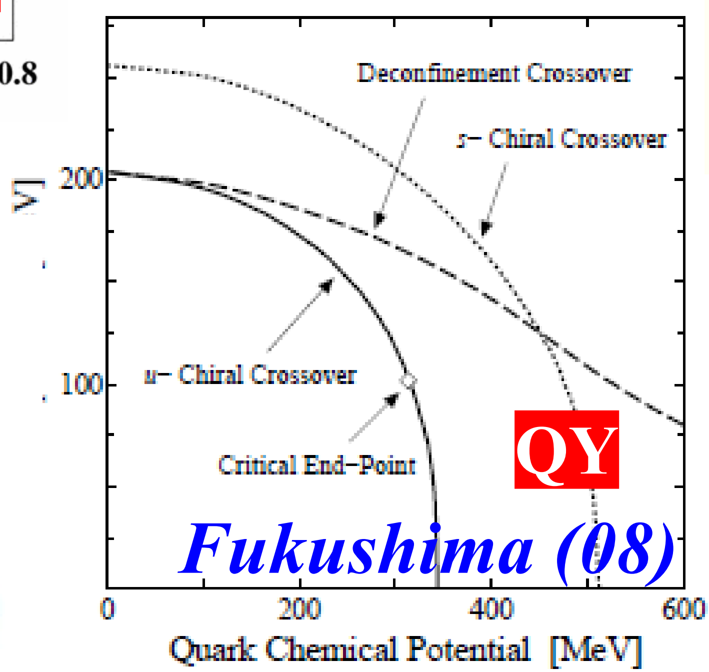
- **Quarkyonic(-like) Area in SC-LQCD is smaller than in PNJL**
  Fukushima (08)
  Abuki, Anglani Gatto, Nardulli, Ruggieril [arXiv:0805.1509]

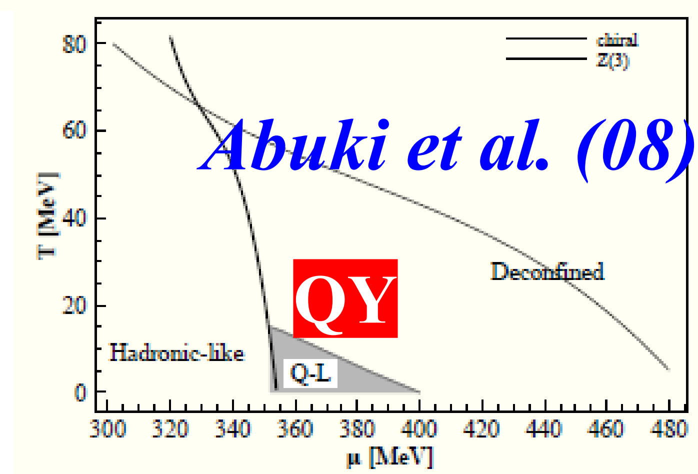
Present







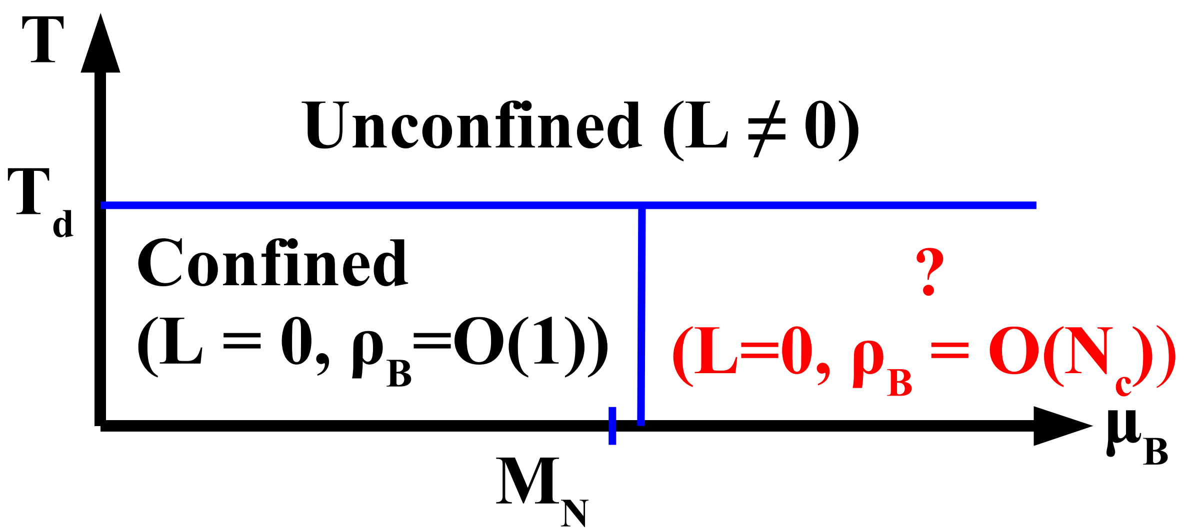




A Conjecture from Large N_c: Quarkyonic Phase

Pisarski, McLerran, 2007

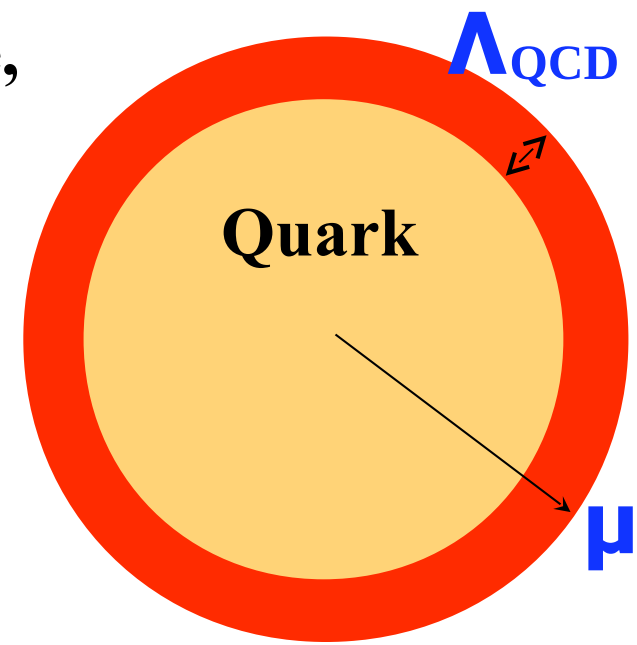
- Discussion at large N_c
  - Pressure: Gluon = $O(N_c^2)$, Quark = $O(N_c)$, Hadron = $O(1)$
    → DECONFINEMENT phase transition (order parameter = Polyakov loop) is independent from quark chemical potential μ as far as $\mu = O(1)$.
  - Large μ ($N_c \mu > M_B$) but low T ($T < T_d$)
    → Weakly interacting quark gas, but no free gluons (confined).
    = High Density *Confined* Phase



What is this ?

Pisarski, McLerran, 2007

- **Confined High Density Matter at Large N_c**
  = Quarkyonic Phase
  (Quarks deeply inside the Fermi Sphere, with baryonic excitations)



Do we really see this phase at Nc=3 ?
What happens to Chiral Symmetry ?

Confined
→ Baryonic Excitation