
QCD phase diagram and dense matter EOS

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- **QCD phase diagram --- How can we observe it ?**
- **Phase diagram and in Strong coupling lattice QCD**

Miura, Nakano, AO, Prog. Theor. Phys., 122(09), 1045 [arXiv:0806.3357]

Miura, Nakano, AO, Kawamoto, PRD80(09), 074034 [arXiv:0907.4245]

Nakano, Miura, AO, arXiv:0911.3453 [hep-lat]

- **EOS of dense hyperonic matter
and its application to black hole formation**

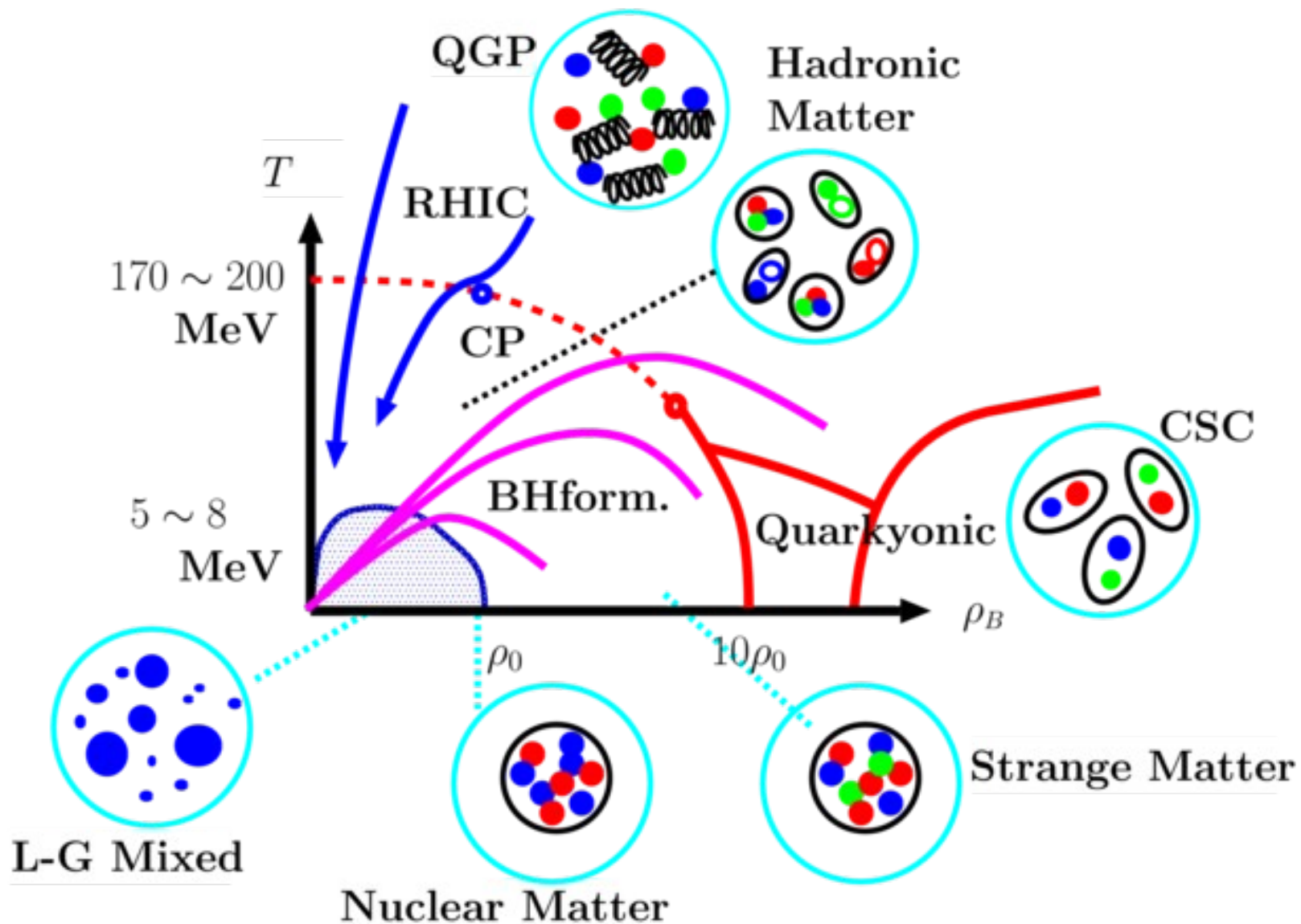
Ishizuka, AO, Tsubakihara, Sumiyoshi, Yamada, JPG35(08)085201.

Sumiyoshi, Ishizuka, AO, Yamada, Suzuki, ApJ690(09)L43.

Tsubakihara, Maekawa, Matsumiya, AO, arXiv:0909.5058.

- **Summary**

QCD Phase diagram



Critical Point Search at RHIC

■ Critical Point Search

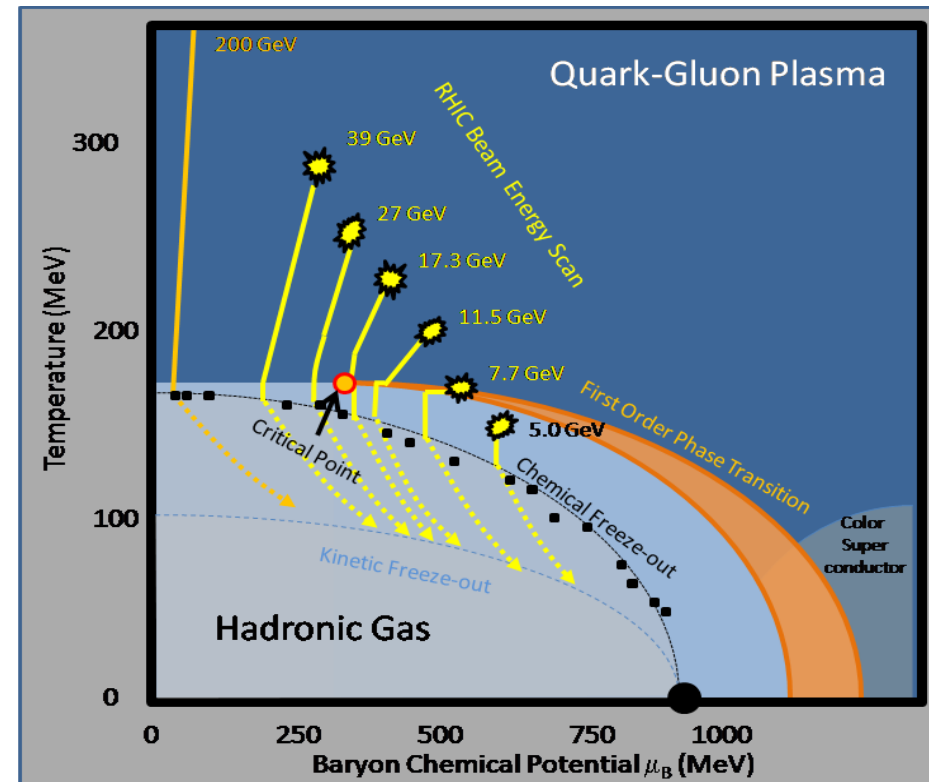
= One of the main goals in Low-E progs. at RHIC

■ $\sqrt{s} = 200$ A GeV (RHIC top energy): Cross Over

→ $\sqrt{s} = 5$ A GeV (Injector (AGS) energy): Hadron phase

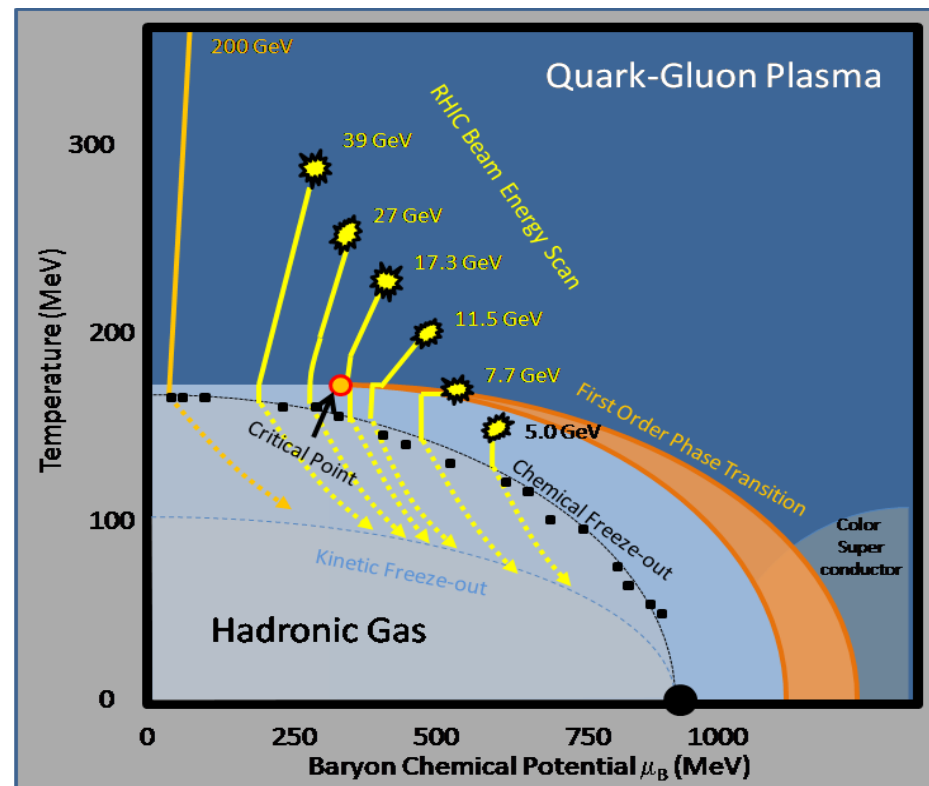
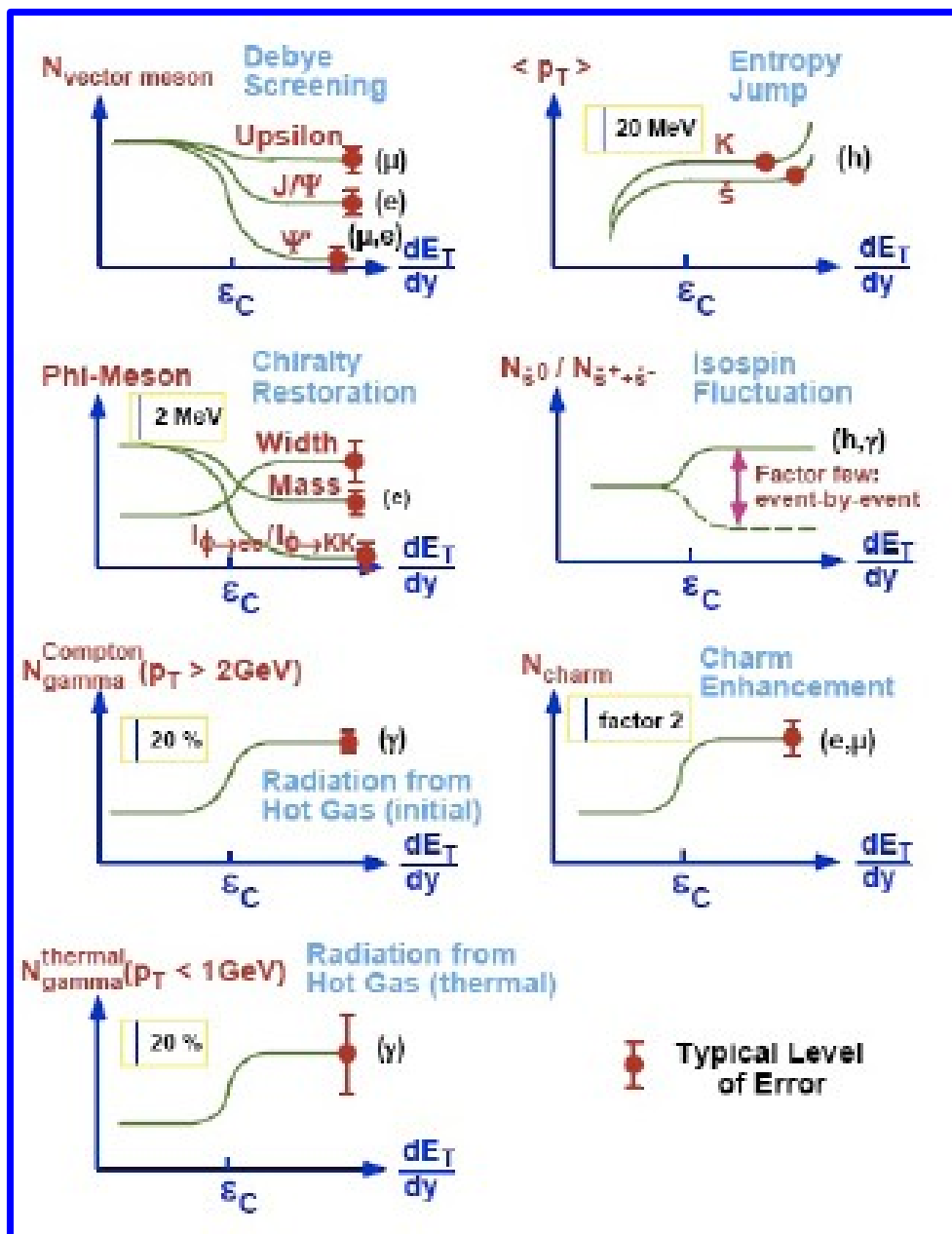
We may observe “first order phase transition”

and the QCD critical point in the range, $\sqrt{s} = 5$ -200 A GeV



「相転移」をどうやって見つけるか？

- 永宮 Method → 様々な観測量の変化を同時測定

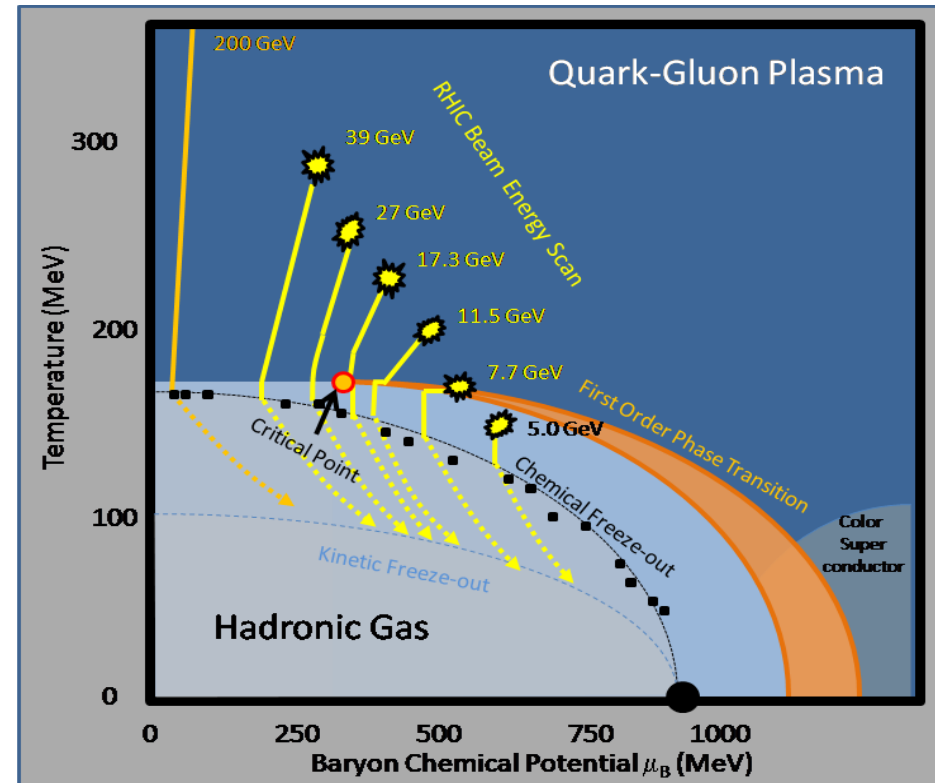


「ストレンジネスから新ハドロンへ」, RCNP, 12/11-12, 2009.

「相転移」をどうやって見つけるか？

- 永宮 Method → 様々な観測量の変化を同時測定
- Critical Point (CP) or 1st order P.T.
 - Critical Point 近辺で期待されること
 - ◆ Fluctuation の増大, Non-Gaussian 揺らぎ (Stephanov, Rajagopal, Shuryak)
 - ◆ 軌道の focusing とそれにとまなう粒子分布の変化 (Ejiri, Karsch, Laermann, Schmidt; Asakawa, Bass, Muller, Nonaka)
 - ◆ 音波の消失 → Mach Cone の消失 (Minami, Kunihiro)
 - 1st order phase transition での期待
 - ◆ 密度揺らぎをもつ相転移後の状態からのハドロン化 (Mishustin; Randrup; Koch, Majumber, ...)

まだまだシグナル候補は未確定
→ 多くのアイデアが必要



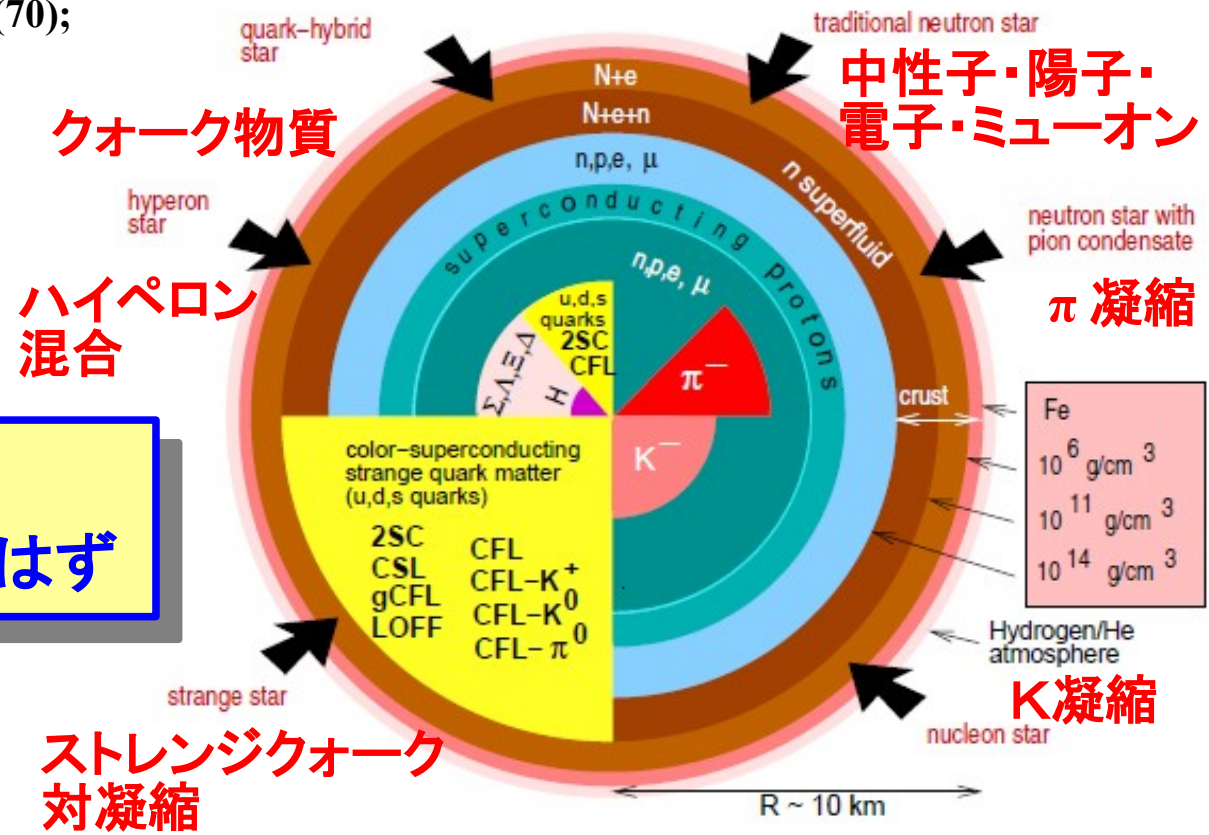
Neutron Stars

高密度物質はどのような状態か？

- 核子の超流動状態 ($^3S_1, ^3P_2$)
- π 凝縮、K 凝縮、QGP、カラー超伝導、Quarkyonic 物質、...
- ハイペロン混合

Tsuruta, Cameron (66); Langer, Rosen (70);
 Pandharipande (71); Itoh(75);
 Glendenning; Weber, Weigel;
 Sugahara, Toki; Schaffner, Mishustin;
 Balberg, Gal; Baldo et al.; Vidana et al.;
 Nishizaki, Yamamoto, Takatsuka;
 Kohno, Fujiwara et al.; Sahu, AO;
 Ishizuka, AO, Tsubakihara,
 Sumiyoshi, Yamada; ...

重い *Neutron star* では、
strangeness が現れているはず



F. Weber, *Prog. Part. Nucl. Phys.* 54 (2005) 193

Core Collapse processes

■ Numerical Simulation = ν radiation hydrodynamics

- Baryons, Electrons, Photons (Hydro) + neutrinos (Boltzmann)

■ Supernova

- 1-dim. (Spherical Sym.) → Exact ν transport, Explosion fails
Sumiyoshi, Yamada, Suzuki, Shen, Chiba, Toki, 2005

- 2-dim. Hydrodynamics

→ Some successful explosion results,
but not yet conclusive

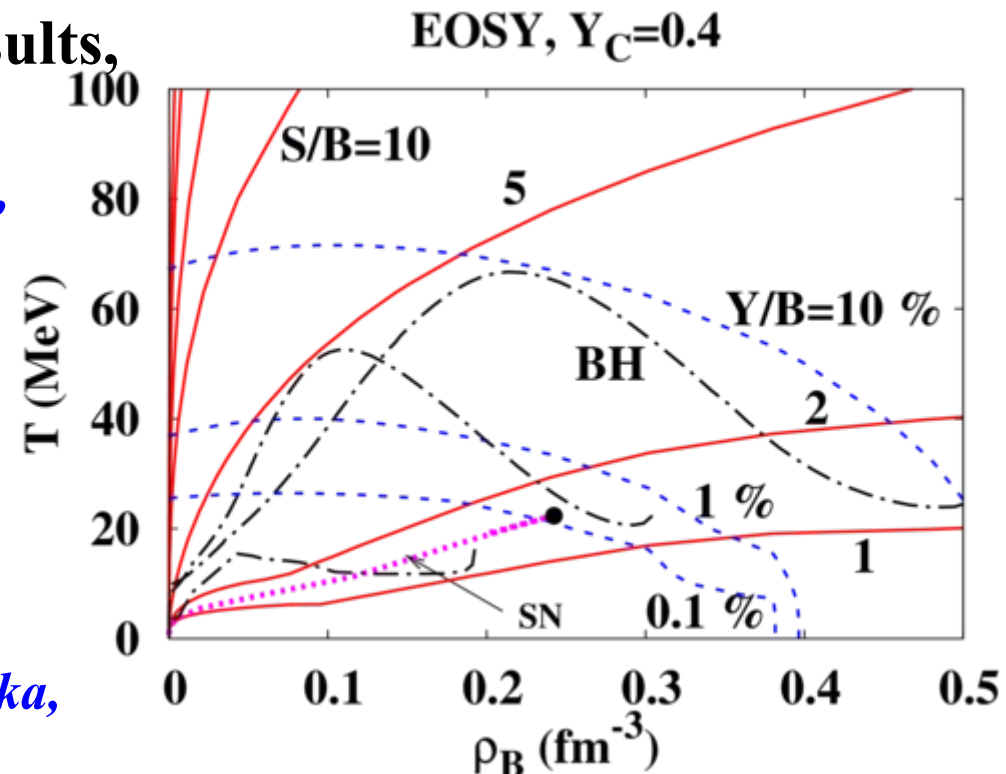
*Janka et al. (02); Kitaura et al. (06),
Marek, Janka (08)*

- Second Shock due to QCD p.t. ?
(Hatsuda, 1987; Sagert et al., 2009)

■ Black hole formation

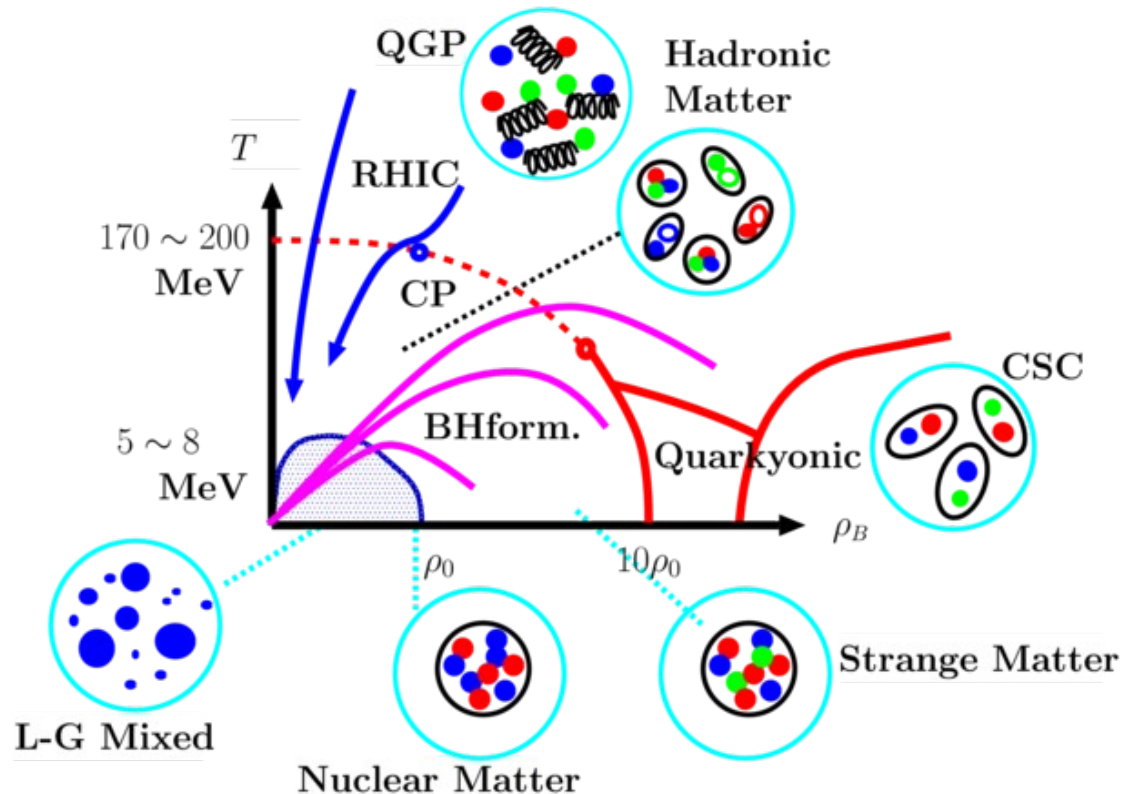
- Hot and dense matter is formed

*Sumiyoshi et al. (05), Sumiyoshi, Ishizuka,
AO, Yamada, Suzuki (09)*



QCD Phase diagram

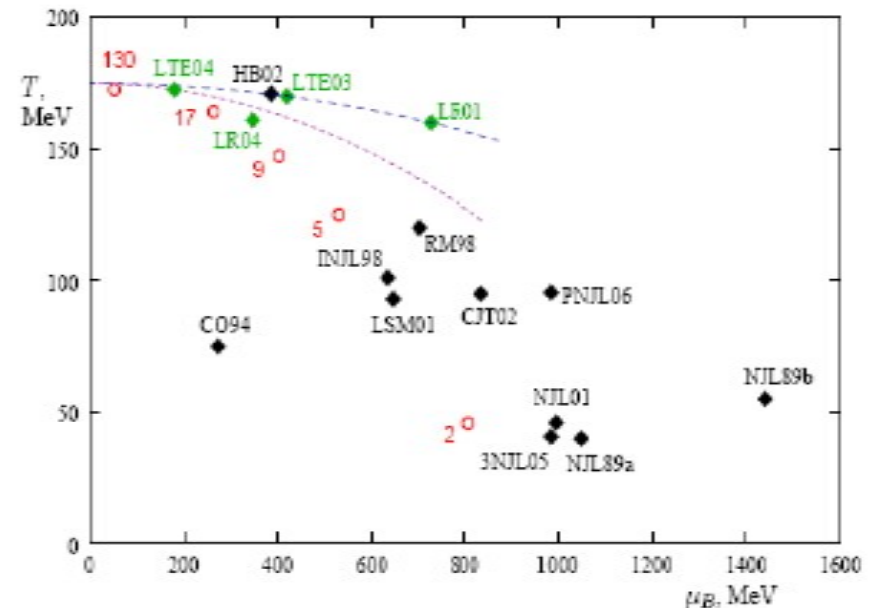
- Phase transition at high T → Lattice MC & RHIC
- High μ transition has rich physics
→ Various phases, CEP, Astrophysical applications, ...



QCD phase diagram is not only an academic subject, but also to be observed in near future !

Theoretical Approaches

- Sign problem
 - Important sampling in Lattice MC does not work precisely.
- Models & Approximate Approaches
 - Lattice MC for small μ (Tayler, AC, DOS, Canonical, ...)
 - Consistent in small μ ($\mu/T < 1$)
 - Monomer-Dimer-Polymer sim. in the Strong Coupling Limit
Karsch, Mutter ('89), de Forcrand, Fromm ('09)
 - Eff. Models:
 - ◆ NJL, PNJL (c.f. Sakai's talk)
Nambu, Jona-Lasino ('61), Hatsuda, Kunihiro ('94), Fukushima ('03)
 - ◆ PLSM (Linear σ model with Pol.)
 - Approximations:
 - ◆ Large N_c
E.g. Hidaka, McLerran, Pisarski (08)
 - ◆ **Strong coupling lattice QCD**



*Phase diagram
in Strong-Coupling Lattice QCD*

Strong Coupling Lattice QCD: Pure Gauge

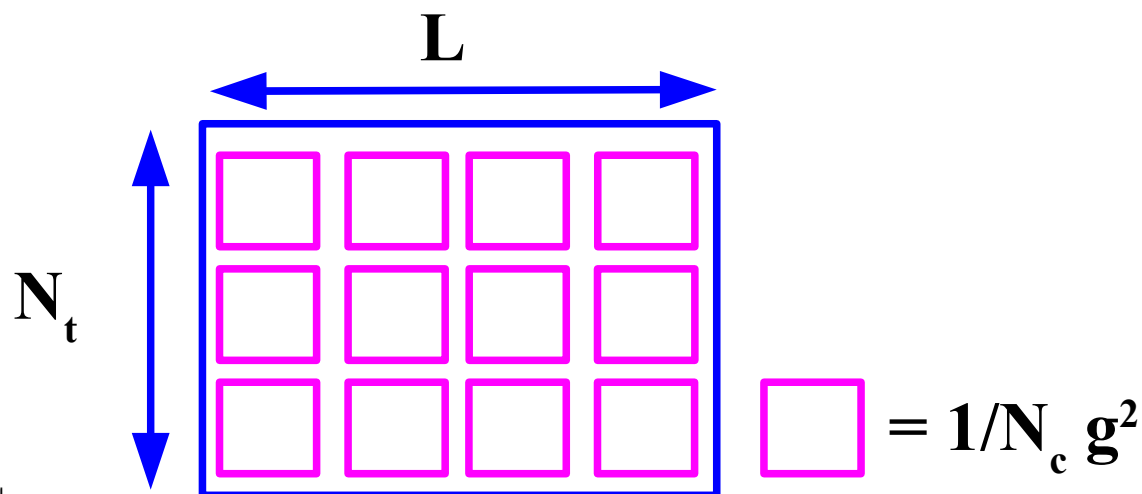
- Quarks are confined in Strong Coupling QCD

- Strong Coupling Limit (SCL)

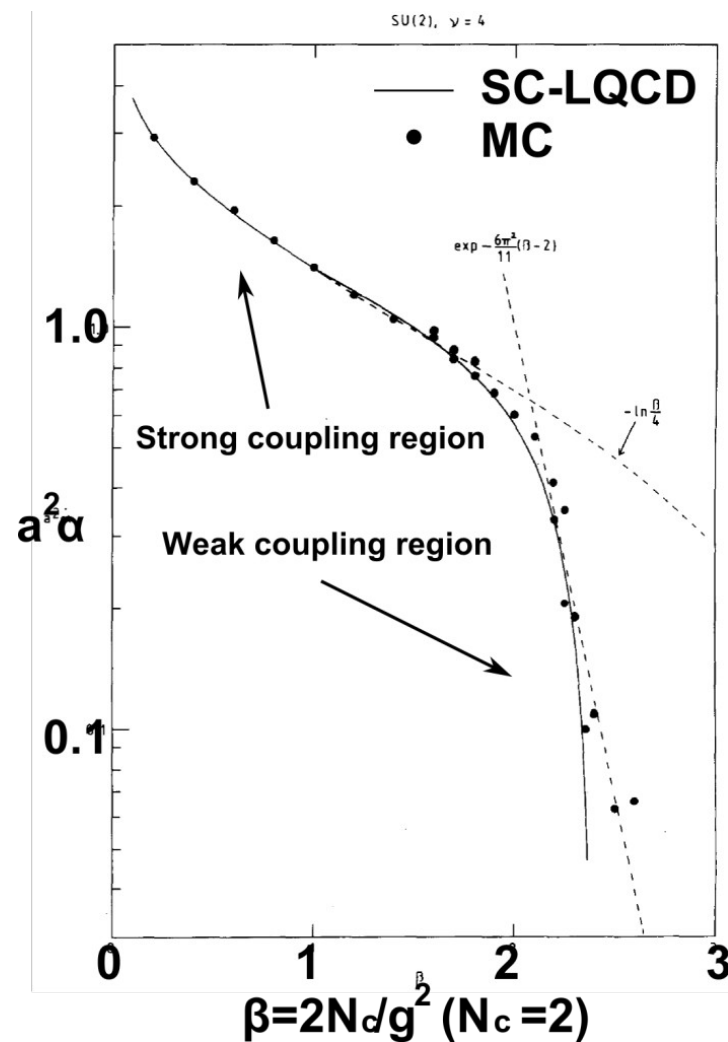
- Fill Wilson Loop with Min. # of Plaquettes
 - Area Law (Wilson, 1974)

$$S_{\text{LQCD}} = -\frac{1}{g^2} \sum_{\square} \text{tr} [U_{\square} + U_{\square}^{\dagger}]$$

- Smooth Transition from SCL to pQCD in MC (Creutz, 1980)



K. G. Wilson, PRD10(1974),2445
M. Creutz, PRD21(1980), 2308.
G. Munster, 1981



Munster, '81

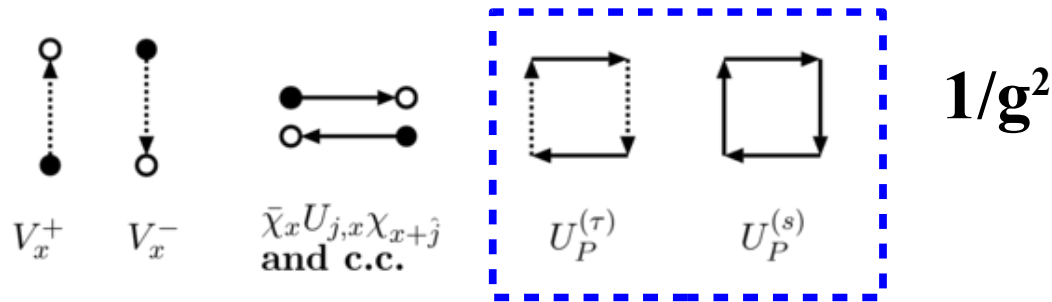
NLO & NNLO SC-LQCD: Setups & Disclaimer

- We investigate the phase diagram and try to understand nuclear matter based on the strong-coupling lattice QCD (SC-LQCD).
 - Effective potential (free E. density) → phase boundary & EOS
 - Setups & Disclaimer
 - ◆ Effective action in SCL ($1/g^0$), NLO ($1/g^2$), **NNLO ($1/g^4$)** terms
NLO: Faldt-Petersson ('86), Bilic-Karsch-Redlich ('92)
Conversion radius > 6 in pure YM? Osterwalder-Seiler ('78)
 - ◆ **One species of unrooted staggered fermion** ($N_f=4$ in continuum limit)
Moderate N_f deps. of phase boundary: BKR92, Nishida('04), D'Elia-Lombardo ('03)
 - ◆ Leading order in $1/d$ expansion ($d=3$ =space dim.)
→ Min. # of quarks for a given plaquette configurations, no spatial B prop.
 - ◆ Effective potential is obtained in mean field approximation
 - ◆ Polyakov loop effects are not included (**No Deconfinement**).
 - ◆ Different from “strong coupling” in “large N_c ”

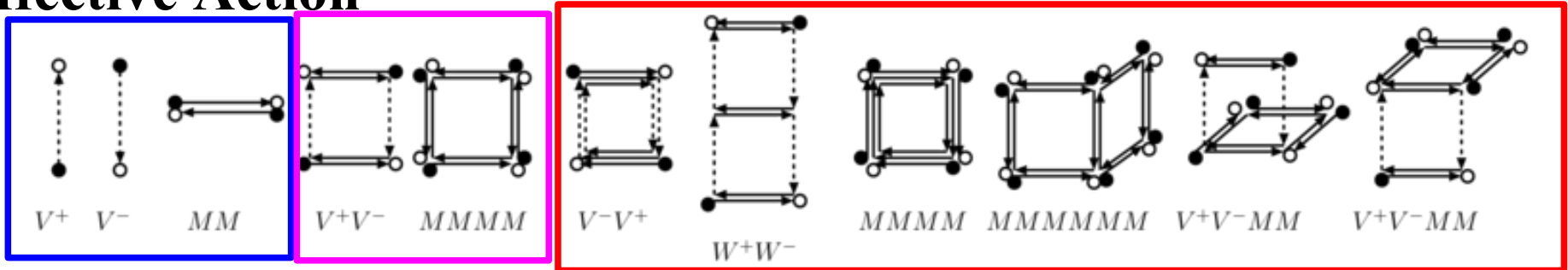
Still far from “Realistic”, but SC-LQCD would tell us useful qualitative features of the phase diagram and EOS.

Effective Potential in SC-LQCD with Fermions

- Lattice QCD partition function
 - Effective action of fermion and U_0 (U_j integral)
 - Linearized eff. Action (Bosonization)
 - Eff. potential (fermion & U_0 integral)
- Lattice Action = fermion x link + plaq.



Effective Action



SCL (Kawamoto-Smit, '81) NLO (Faldt-Petersson, '86) NNLO (Nakano, Miura, AO, '09)

NNLO Effective Action

- Cumulants of two plaquettes
= Correlation part of connected two plaquettes

- Uncorr. & Normalization part are suppressed in $1/d$ power

- Effective Action consists of

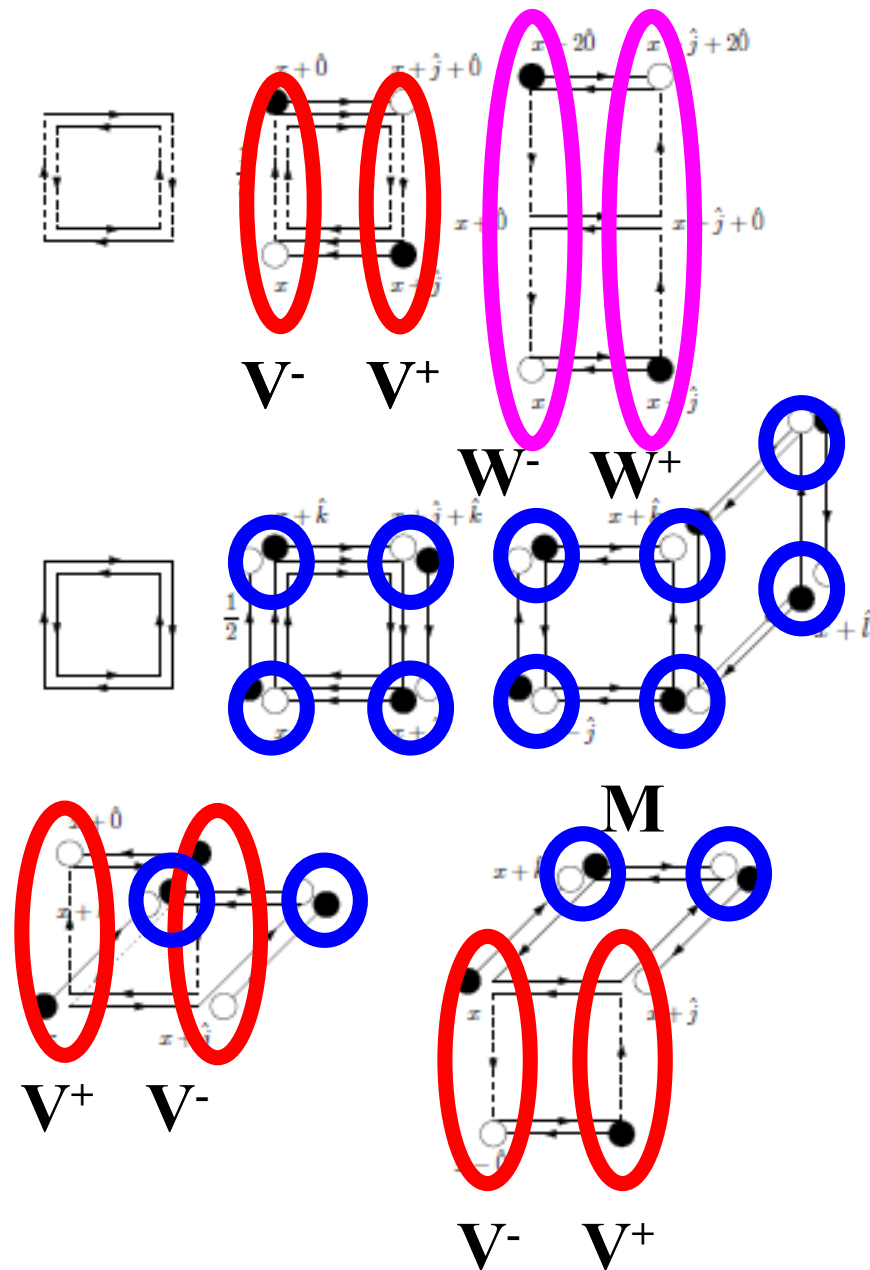
$$V^- V^+, W^- W^+,$$

$$MMMM, MMMMMM,$$

$$V^- V^+ MM$$

- New type of Composite
= next-to-nearest neighboring site coupling in τ direction

$$W_x^+ = \chi_x U_{0,x} U_{0,x+\hat{0}} \bar{\chi}_{x+2\hat{0}}$$



Effective Potential and Phase Diagram

■ Effective Potential in NLO/NNLO SC-LQCD

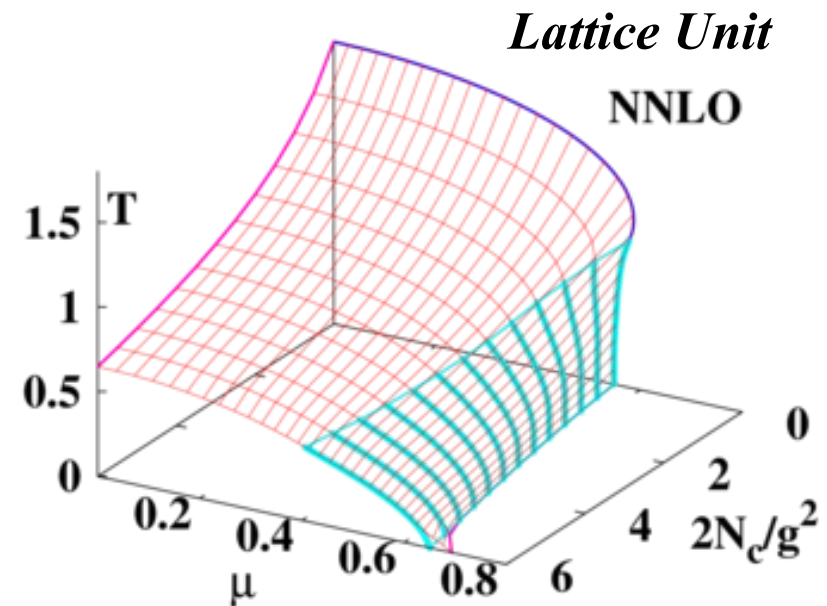
- Suppression of const. quark mass → Smaller T_c
 - Vector potential from temporal plaq. → Chem. Pot. is shifted
- $\sigma\omega$ model of quarks with non-linear couplings !

■ Phase diagram evolution with β

→ Miura's talk

- Shape of the phase diagram is compressed in T direction with β
- Critical Point moves in lower μ direction in NNLO.

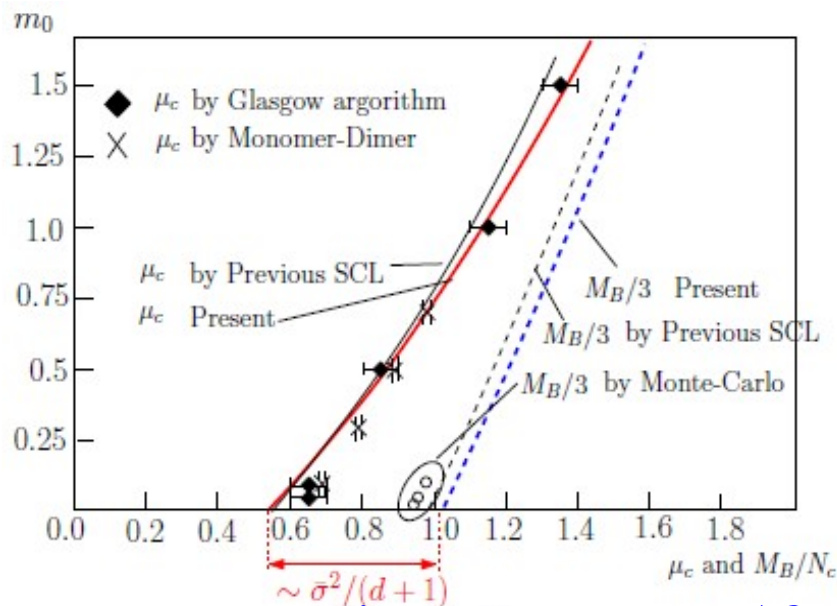
*EPisarski-Wilczek '84), Ejiri, ('08),
Aoki et al.(WHOT,'08), Allton et al., ('03,'05)*



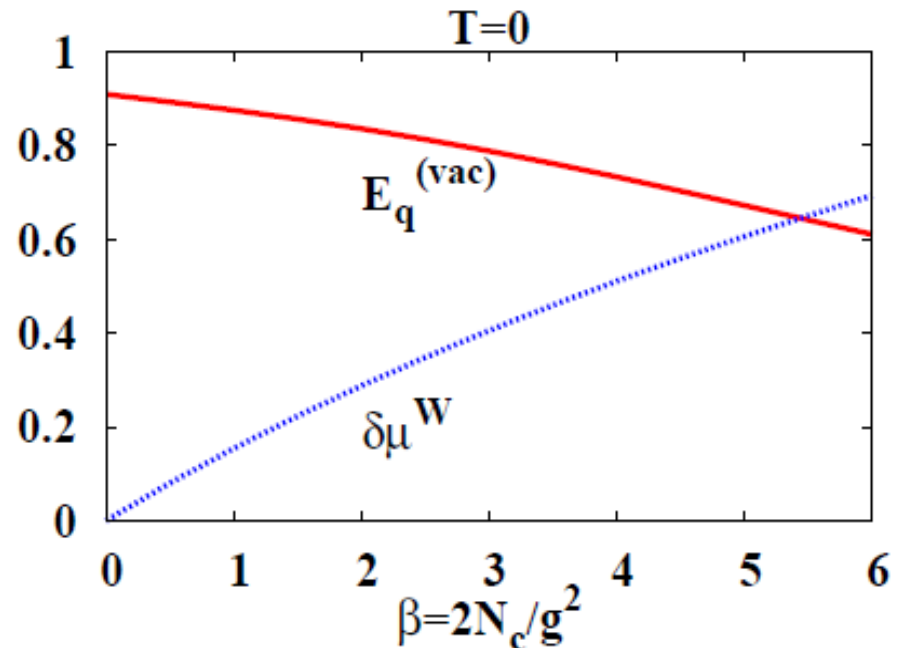
*NLO/NNLO improves the shape of the phase diagram,
but effects are not enough to explain the real world.*

Cold Nuclear Matter on the Lattice

- **Baryon mass puzzle in SCL-LQCD: $N_c \mu_c < M_B$**
 → **QCD phase transition takes place before baryons appear.**
Kluberg-Stern, Morel, Petersson ('83), Damgaard, Hochberg, Kawamoto ('85), Karsch, Mutter ('89), Barbour et al. ('97), Bringoltz ('07), Miura, Kawamoto, AO ('07)
- **Finite coupling effects: Decrease of quark mass.**
 → **Const. quark mass ($\sim E_q$) becomes smaller than μ_c for $\beta > 5.5$ in NNLO SC-LQCD.**



Miura, Kawamoto, AO ('07)



Cold Nuclear Matter on the Lattice

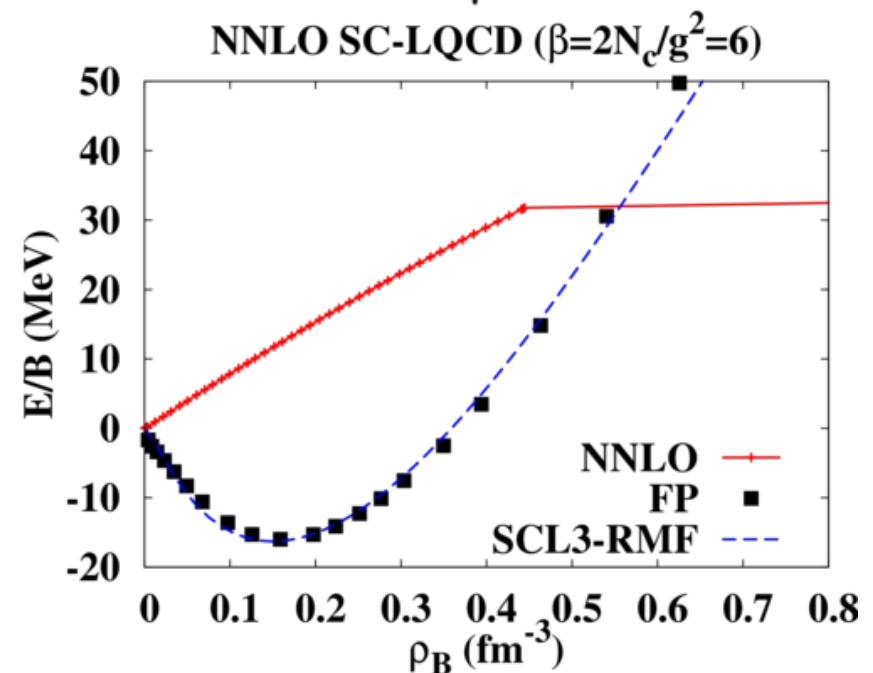
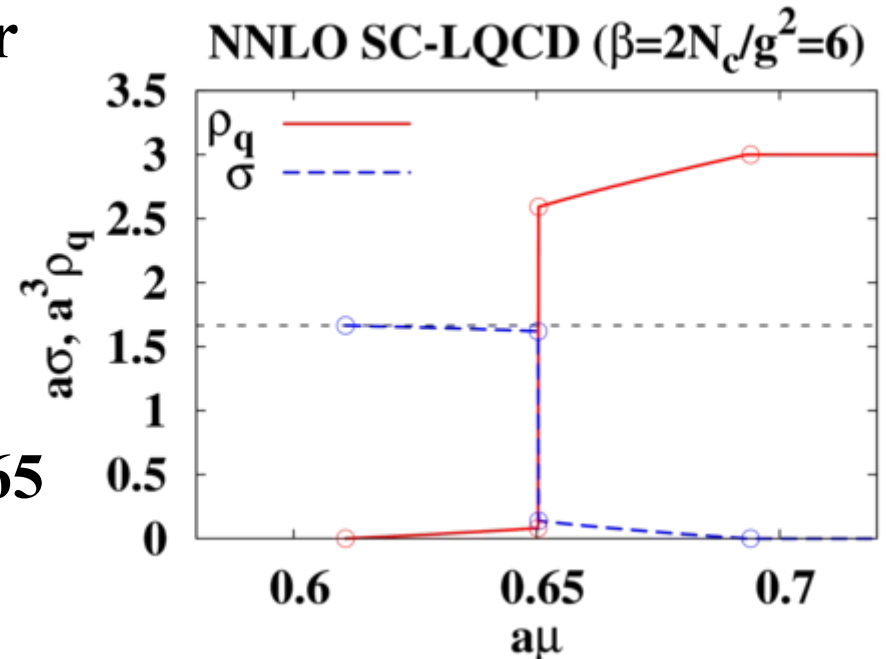
- Do we observe finite density matter before 1st order phase transition ?

→ Yes !

- $E_q(\mu=0, T=0, \beta=6)=0.61$
 $\mu_c^{(1st)}(T=0, \beta=6)=0.65$
 → “Nuclear matter” in $0.61 < \mu < 0.65$

- EOS of “Nuclear matter”

- $a^{-1} \approx 500$ MeV
Bilic, Demeterfi, Petersson ('92)
 → Density in the order of ρ_0
- No saturation
- 1st order transition at $\rho_B \approx 0.4$ fm⁻³.

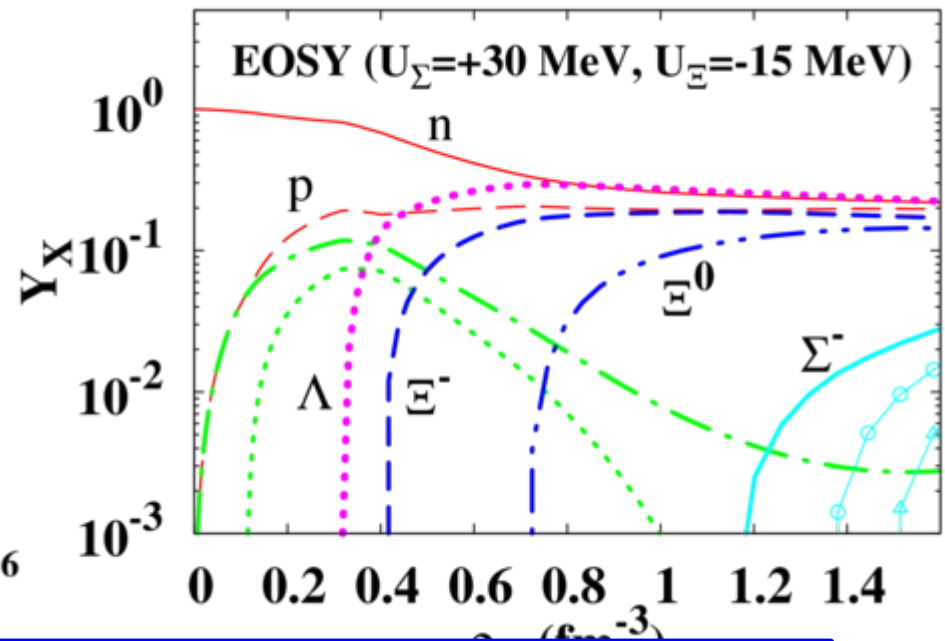
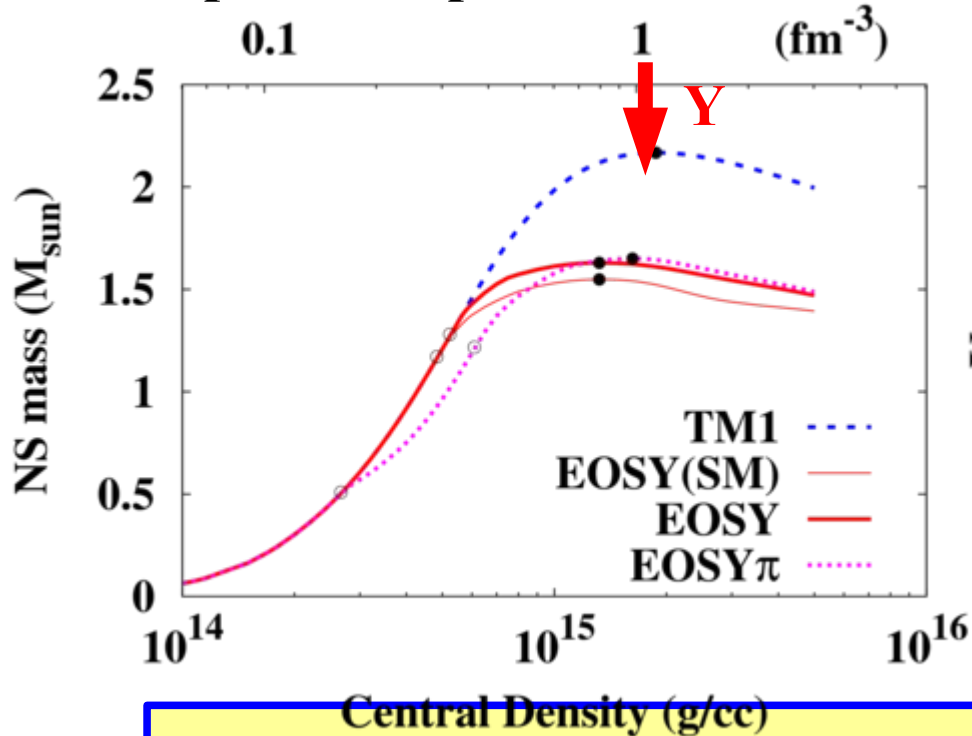


*EOS of Dense Hyperonic Matter
and Its Application to Black Hole Formation*

Neutron Stars

■ RMF with Hyperons

- **Hyperon Potential:** $U_{\Lambda} = -30$ MeV, $U_{\Sigma} = +30$ MeV, $U_{\Xi} = -15$ MeV
- **Neutron Star:** $(\rho_B, T, Y_e) \sim (5\rho_0, 0 \text{ MeV}, 0.2) \rightarrow$ Hyperon fraction $\sim 50 \%$
- Reduction of max. mass of NS
- Repulsive Σ pot. $\rightarrow \Xi$ will be the next hyperon to Λ !



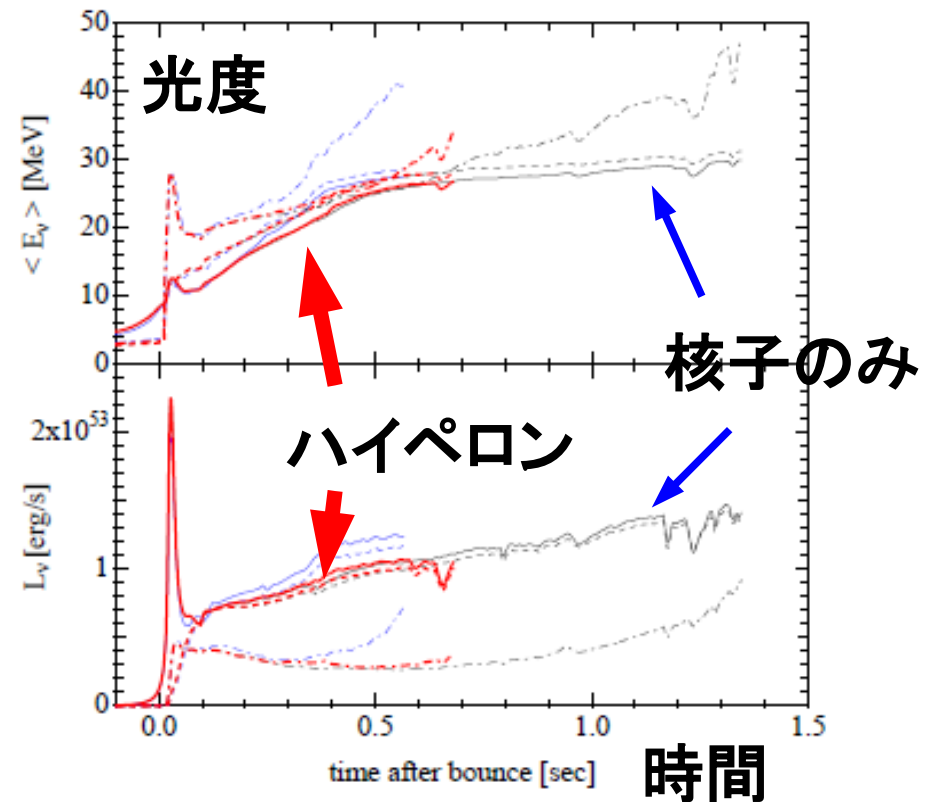
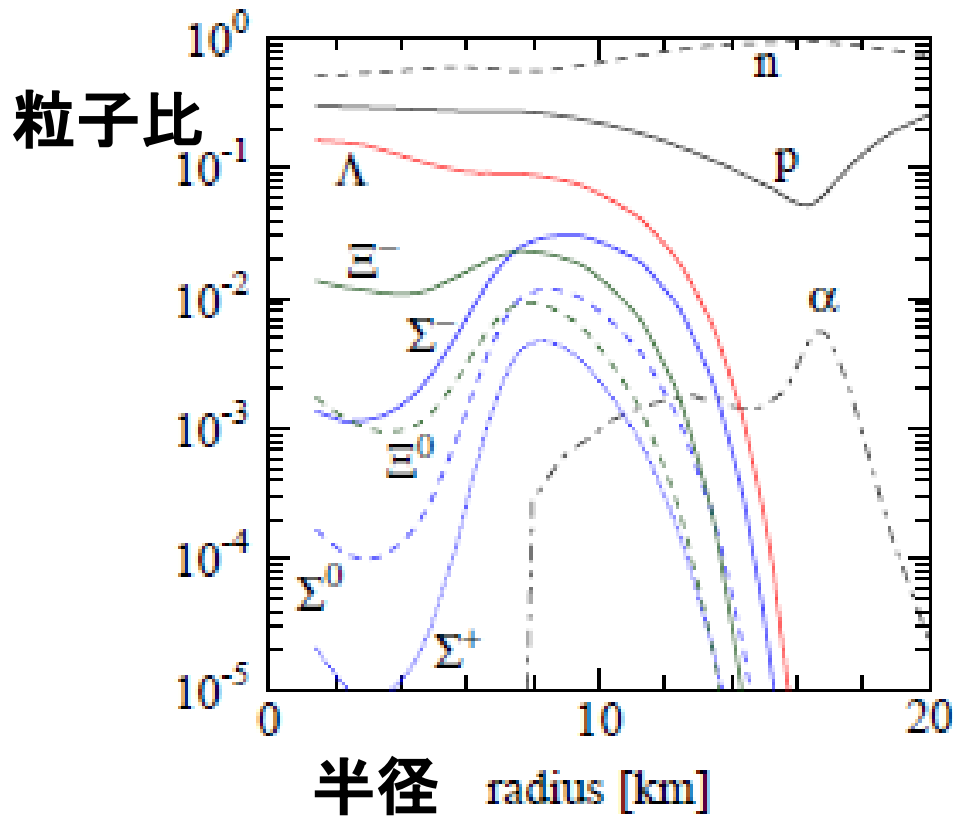
Neutron Star can be understood as Hyperon Star !

Black Hole Formation (Failed Supernova)

- High T during BH formation

→ Abundant hyperons → Soft EOS → Earlier Collapse to BH

Short ν emission may be the signal of Hyperon Admixture at high density and/or temperature



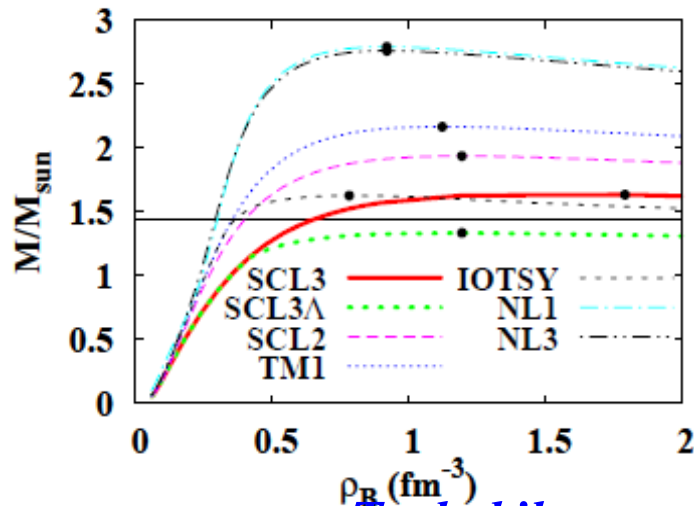
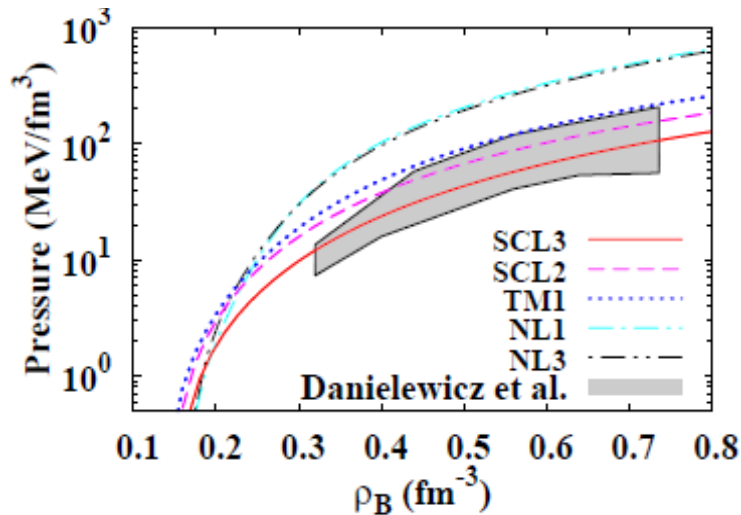
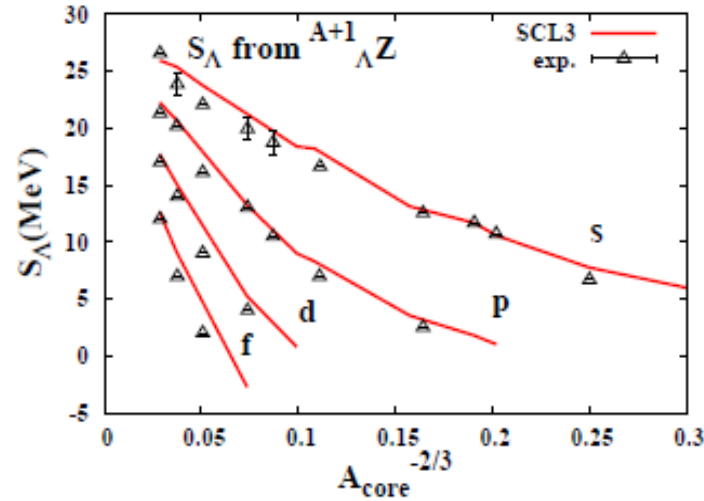
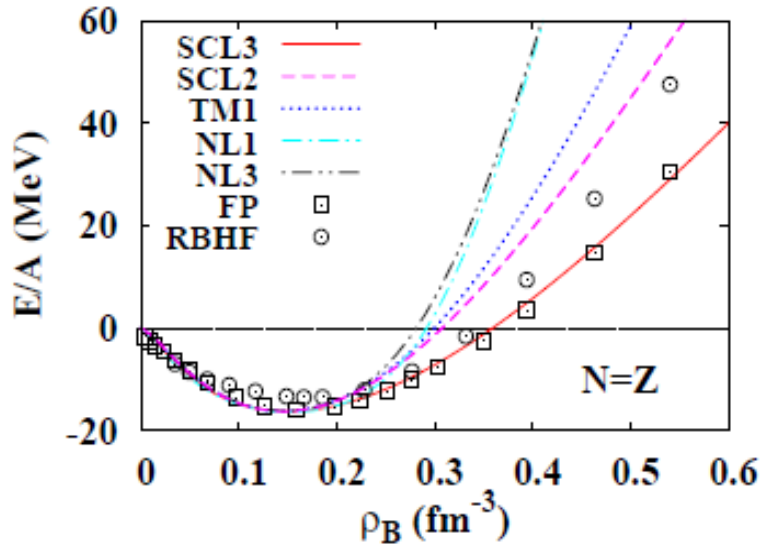
Sumiyoshi, Ishizuka, AO, Yamada, Suzuki, 2009



Chiral $SU(3)$ RMF with a Logarithmic σ potential

■ Nuclear matter EOS is important in

- Nuclear B.E., HIC, Neutron Stars, Supernova, BH formation, ...
→ Chiral symmetry may be as important as flavor $SU(3)$ sym.



Tsubakihara et al., arXiv:0909.5058

Summary

- **QCD phase diagram at finite μ or ρ_B** may be probed in near future experiments (RHIC/FAIR) and observations (v/GW), in addition to the current study in the $\mu=0$ region.
- Due to the sign problem, **approximate/effective model studies** are needed to understand finite density matter.
- We have derived the **effective potential** with NLO ($1/g^2$) and NNLO($1/g^4$) effects in strong coupling lattice QCD.
 - finite coupling \rightarrow smaller quark mass, repulsive vector pot.
 - Phase diagram shape is improved, while the effects are not enough.
 - Baryon mass puzzle is solved (at least weakened) with finite coupling.

Nuclear matter on the lattice may not be too far (?)

Thank you !

■ Collaborators

- **Strong coupling lattice QCD:** K. Miura (YITP), T.Z. Nakano (Kyoto U.), N.Kawamoto (Hokkaido U.)
- **Hyperonic EOS for Supernovae:** K. Sumiyoshi (Numazu), K. Tsubakihara (Hokkaido), C. Ishizuka (Keele), S. Yamada (Waseda), H. Suzuki (Tokyo Sci. U.)
- **Chiral RMF / Hyperon production reactions:** K. Tsubakihara, H. Maekawa, H. Matsumiya (Hokkaido), P.K. Sahu (India)

Not mentioned....

- **Ξ and Λ hypernuclei in AMD:** M. Isaka, H. Matsumiya, M. Kimura (Hokkaido), A. Dote (KEK)
- **pions in neutron star:** D. Jido (YITP), T. Sekihara (Kyoto U.), K. Tsubakihara (Hokkaido)
- **Heavy-Ion Collisions:** Isse, Nara, Hirano, Kunihiro, Schafer, Muller