Finite Couling Effects on the QCD Phase Diagram at Strong Coupling

Akira Ohnishi (YITP, Kyoto Univ.)

- **How I started to work in strong coupling lattice QCD**
- **SC-LQCD with finite coupling corrections**
- **Nuclear Matter in SC-LQCD**
- **Summary**

Miura, Nakano, AO, Prog. Theor. Phys.,122(09),1045 [arXiv:0806.3357]
Miura, Nakano, AO, Kawamoto, PRD80(09),074034 [arXiv:0907.4245]
Nakano, Miura, AO, arXiv:0911.3453 [hep-lat]

First of All ...

- **Congratulations for your KANREKI (60th birthday)**

Congratulations !

I became 60 !

Congratulations !

Congratulations !

Congratulations !

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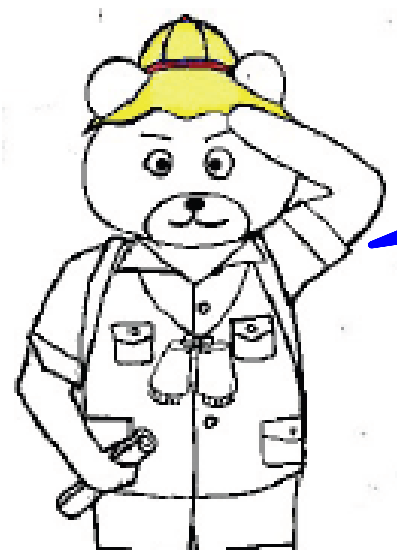
2002 Autumn JPS Meeting

- **In a symposium session at 2002 JPS meeting,**
  "Towards global understanding of dense QCD"
  Hatsuda, Tatsumi, Harada, Muto, Iida, Tachibana,
  Nakamura, Ohnishi, Kunihiro



We can study finite density
QCD in the strong coupling limit !

MDP algorithm: Karsch, Mutter ('89)
Finite μ review
Muroya, Nonaka, Nakamura, Takaishi,
PTP 2003
Karsch, PTPS 2004

Nakamura-san initiated me to think about
Strong coupling lattice QCD

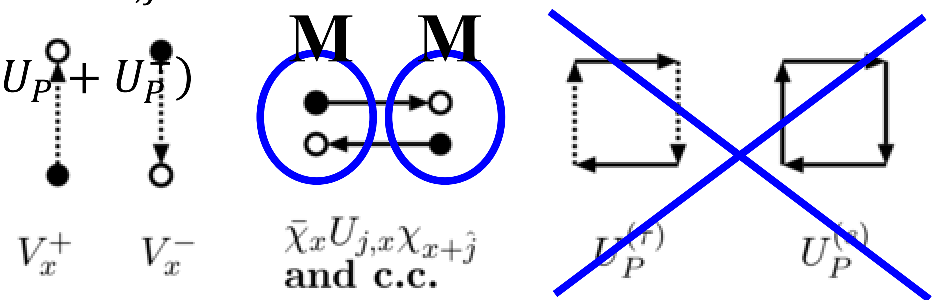
Strong Coupling Limit of Lattice QCD (1)

a la Kawamoto-Smit('81), Damgaad-Kawamoto-Shigemoto ('84)

Lattice QCD action (unrooted staggered fermion)

$$S_{LQCD} = \frac{1}{2}\sum_x [V_x^+ - V_x^-] + m_0 \sum_x M_x + \frac{1}{2}\sum_{x,j} \eta_{j,x} \left(\bar{\chi}_x U_{j,x} \chi_x - \bar{\chi}_{x+j} U_{j,x}^+ \chi_x\right)$$

$$+ \frac{1}{g^2}\sum_P (U_P + U_P^\dagger)$$



Strong Coupling Limit (U_j integral + 1/d expansion)

$$\to \frac{1}{2}\sum_x [V_x^+ - V_x^-] + \frac{1}{4N_c}\sum_{x,j} M_x M_{x+\hat{j}} + m_0 \sum_x M_x + O\left(1/\sqrt{d}\right)$$

Mesonic Composites

$$M_x = \bar{\chi}_x \chi_x, V_x^+ = e^{\mu} \bar{\chi}_x U_{0,x} \chi_{x+\hat{0}}, V_x^- = e^{-\mu} \bar{\chi}_{x+\hat{0}} U_{0,x}^+ \chi_x$$

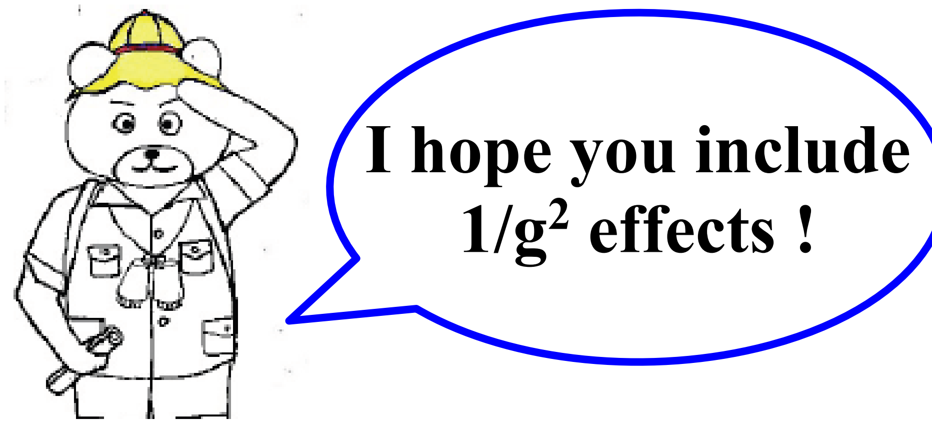
Quark-Gluon Dynamics → Hadronic Composites(+ U_0)

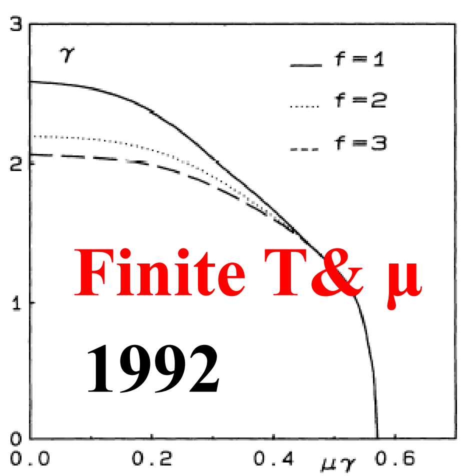
(U_0)

Strong Coupling Limit of Lattice QCD (2)

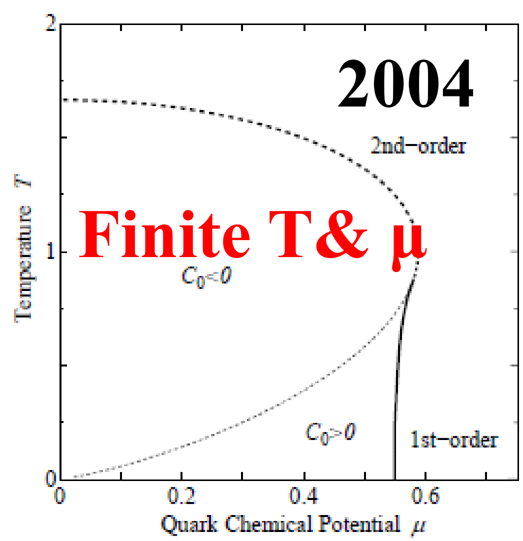
- Bosonization + Mean Field Approx. → Effective Potential
- Analytic, but the phase diagram is far from that in real world.
- Transition at μ=0 is 2nd order.
  (SCL ~ 3d O(2))
- Shape is very different.

Real world	$R = \mu_c / T_c = (2\text{-}4)$
SCL	$R = 0.3\text{-}0.45$

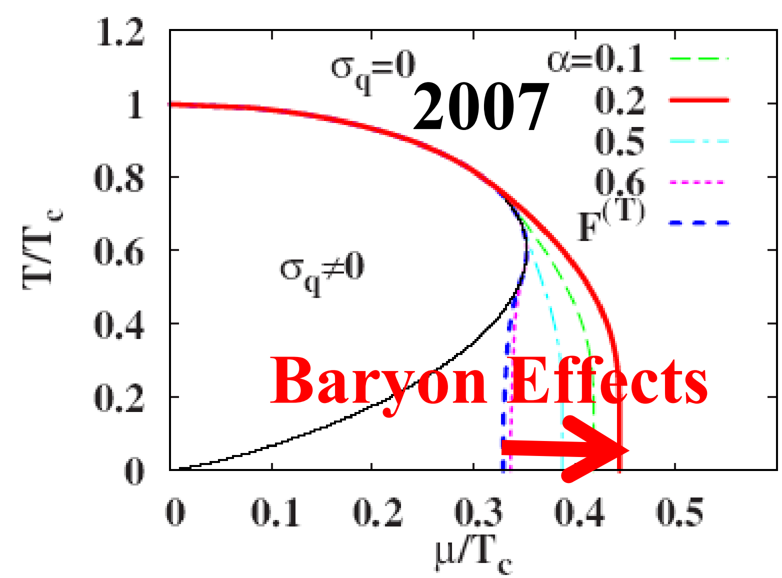




Bilic, Karsch, Redlich, 1992



Fukushima, 2004



Kawamoto, Miura, AO, Ohnuma, 2007

Effective Potential and Phase Diaram Strong-Coupling Lattice QCD with NLO and NNLO Corrections

Critical Temperature and Chemical Potential

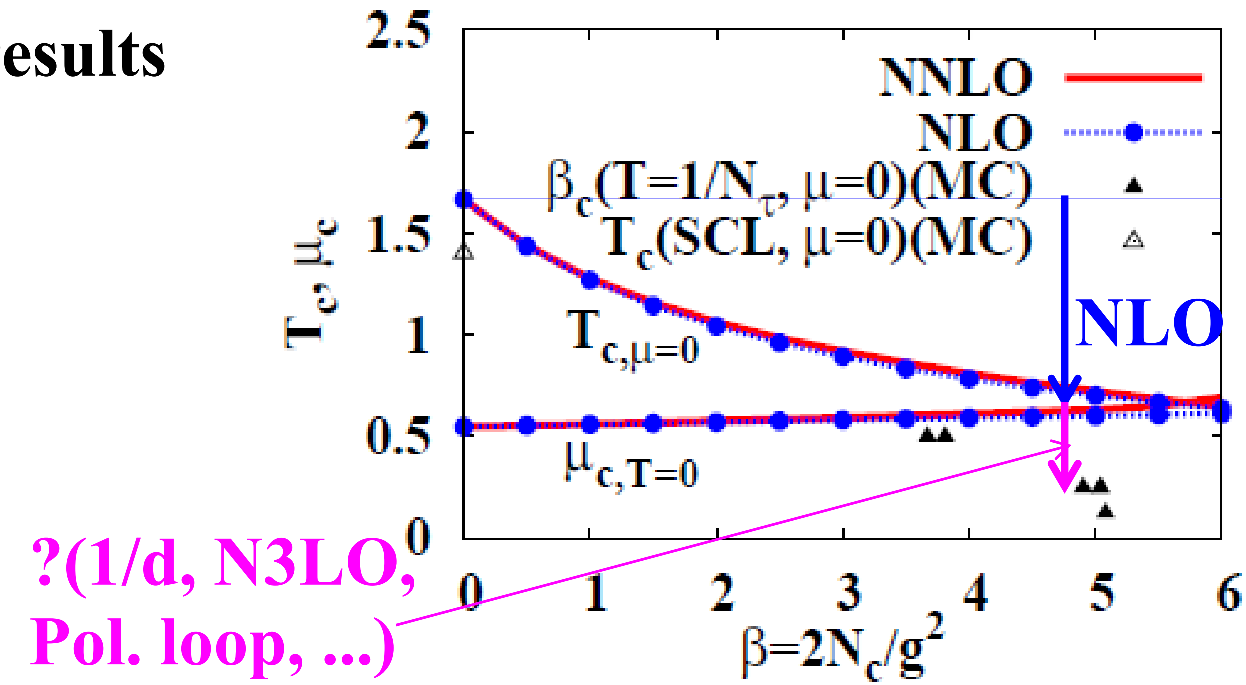
- Critical Temperature (μ =0) → rapid decrease with $\beta=2N_c/g^2$
- W.F. Renom. factor Z_χ → suppression of mass
- T_c is still larger than MC results

de Forcrand ('06), Gottlieb et al. ('87), Gavai et al. ('90), de Forcrand, Fromm ('09)

- Critical Chem. Pot. (T=0) → weak deps. on β
- Suppression of mass ~ Suppression of μ
- Consistent with previous results

Bilic-Demeterfi-Petersson, '92

- NNLO effects are small on $T_c(\mu=0)$ and $\mu_c(T=0)$.



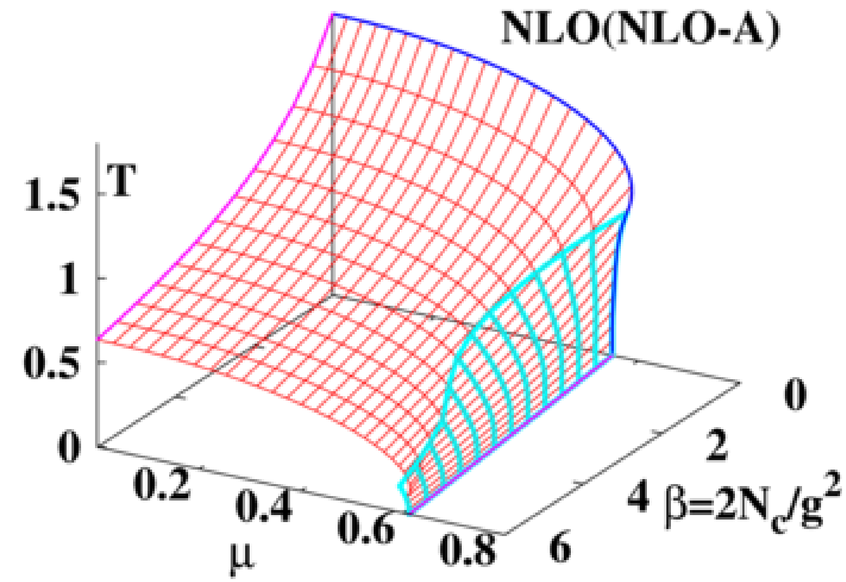
Nakano, Miura, AO ('09)

Phase Diagram Evolution

- **Shape of the phase diagram is compressed in T direction with β**

→ Improvements in $R = \mu_c / T_c$!

- **MC (R > 1) → SCL (R = (0.3-0.45))**
  → NLO/NNLO (R ~ 1)
  → Real World (R~(2-4))

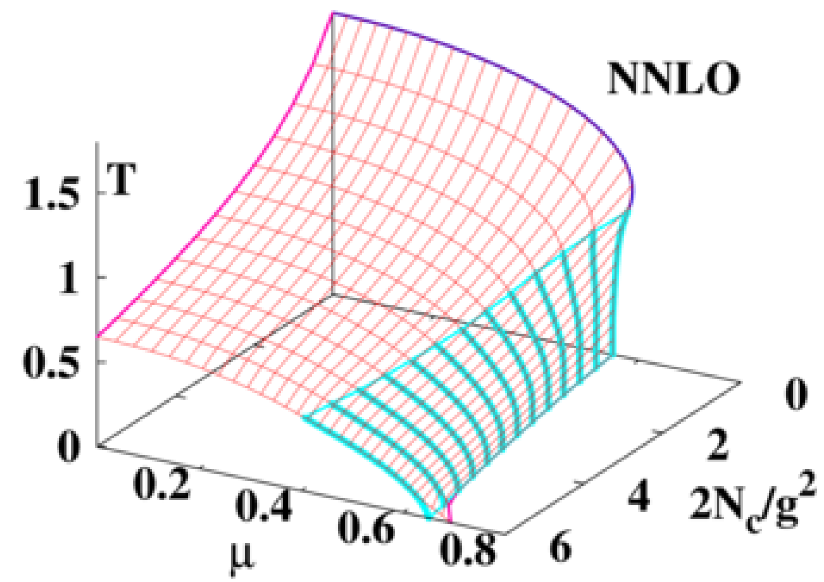


- **Critical Point**

- **NLO: μ(CP) ~ Const.**

- **NNLO: μ(CP) decreases with β**
  → Improvements ! **(N_f=4 → 1st order)**

Kronfeld ('07), Pisarski, Wilczek ('84)



- **μ(CP)/T(CP) ~ 1 ↔ MC (μ/T > 1)**

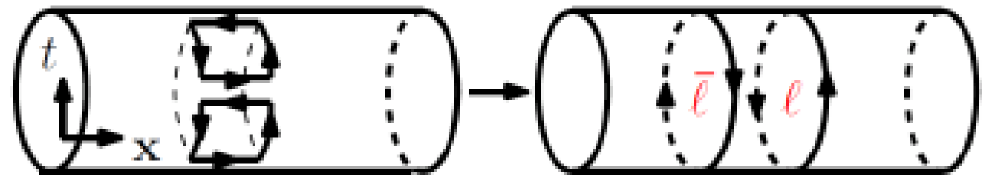
Ejiri, ('08), Aoki et al.(WHOT,'08), Allton et al., ('03,'05)

Miura, Nakano, AO, Kawamoto ('09)
Nakano, Miura, AO ('09)

Polyakov Loop Effects

- **T_c(NLO) ~ T_c(NNLO) > T_c(MC)**

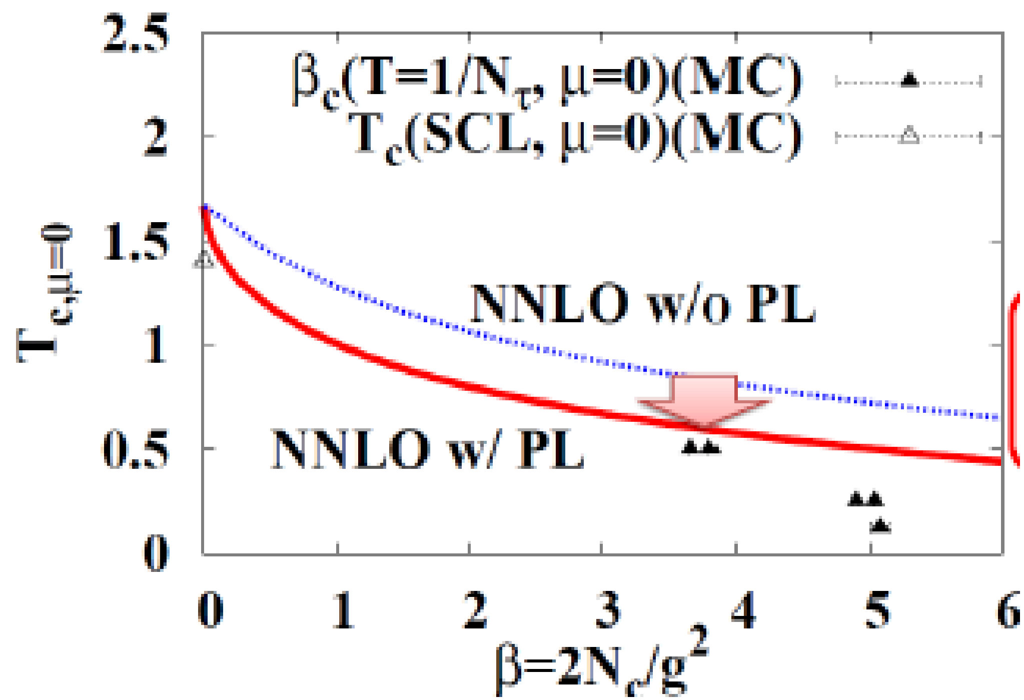
→ Slow convergence ?
Deconfinement ?
→ Resummation is necessary !



- **NNLO SC-LQCD**

with Polyakov loop effects

Nakano, Miura, AO, in prep.
c.f. PNJL (Fukushima /
Ratti-Weise et al. / Kyushu group)



- **Pros**

Chiral & Deconf. transition
Large effects on T_c

- **Cons**

Expansion is not systematic in $1/g^2$
Does not improve at SCL

Boyd, Karsch et al. / de Forcrand & Fromm ('09)

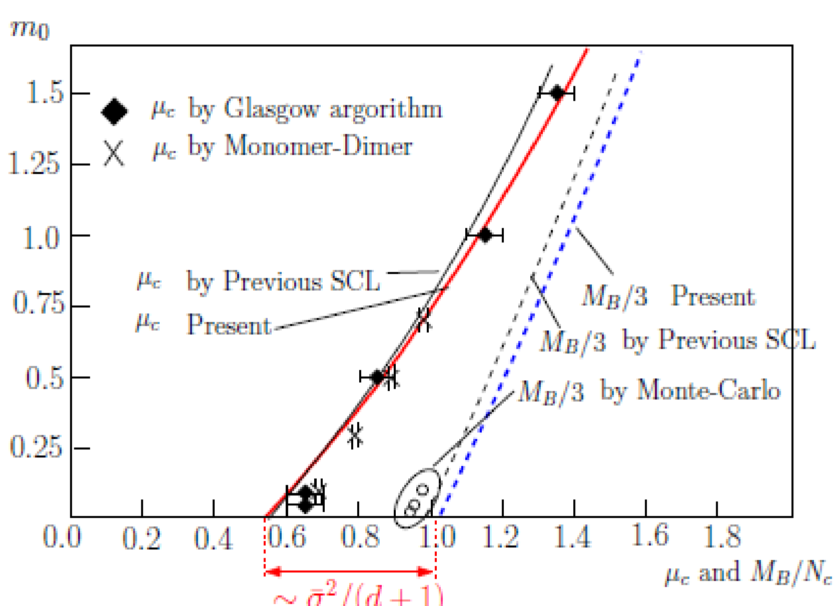
Nuclear Matter in the Strong-Coupling Lattice QCD

Cold Nuclear Matter in Lattice QCD

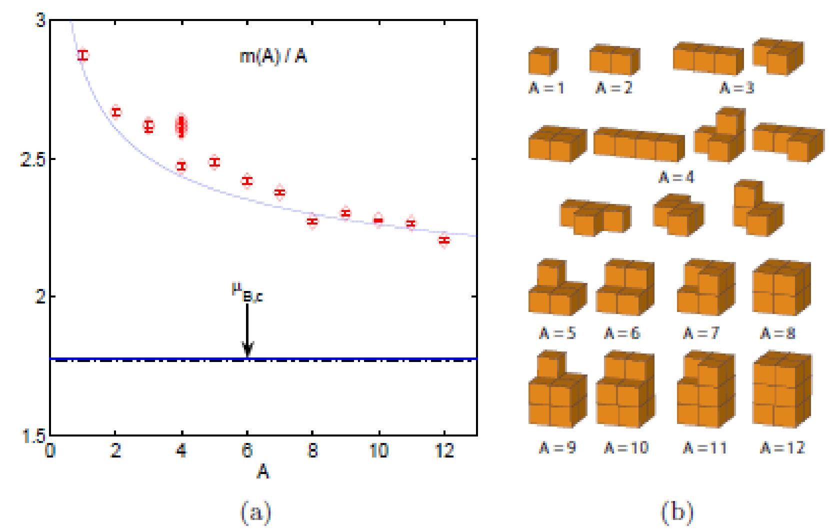
- **Baryon mass puzzle in SCL-LQCD: $N_c\mu_c < M_B$**
  → QCD phase transition takes place before baryons appear.
  Kluberg-Stern, Morel, Petersson ('83), Damgaard, Hochberg,Kawamoto ('85), Karsch, Mutter ('89), Barbour et al.('97), Bringoltz('07), Miura, Kawamoto, AO ('07)

- **Possible Solutions**

  - **Regard the matter at $\mu > \mu_c$ as nuclear matter** *de Forcrand, Fromm ('09)*
  - **Finite coupling effects: Decrease of quark mass**



Miura, Kawamoto, AO ('07)

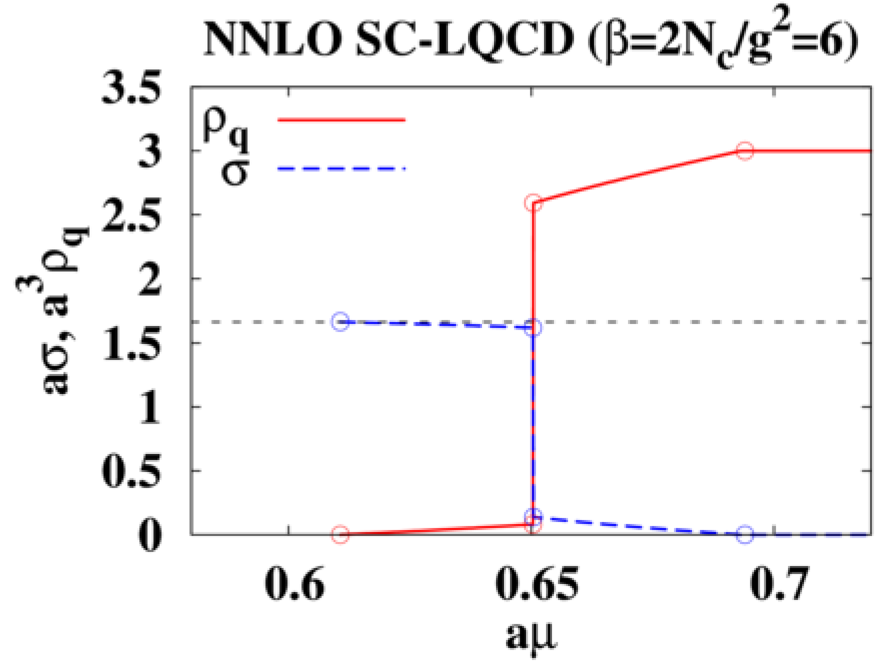


de Forcrand, Fromm ('09)

Nuclear Matter on the Lattice at Strong Coupling

- **Do we observe finite density matter before 1st order phase transition ?**
  → Yes !

- **$E_q(\mu=0, T=0, \beta=6)=0.61$**
  $\mu_c^{(1st)}(T=0, \beta=6)=0.65$
  → "Nuclear matter" in $0.61<\mu<0.65$



- **EOS of "Nuclear matter"**

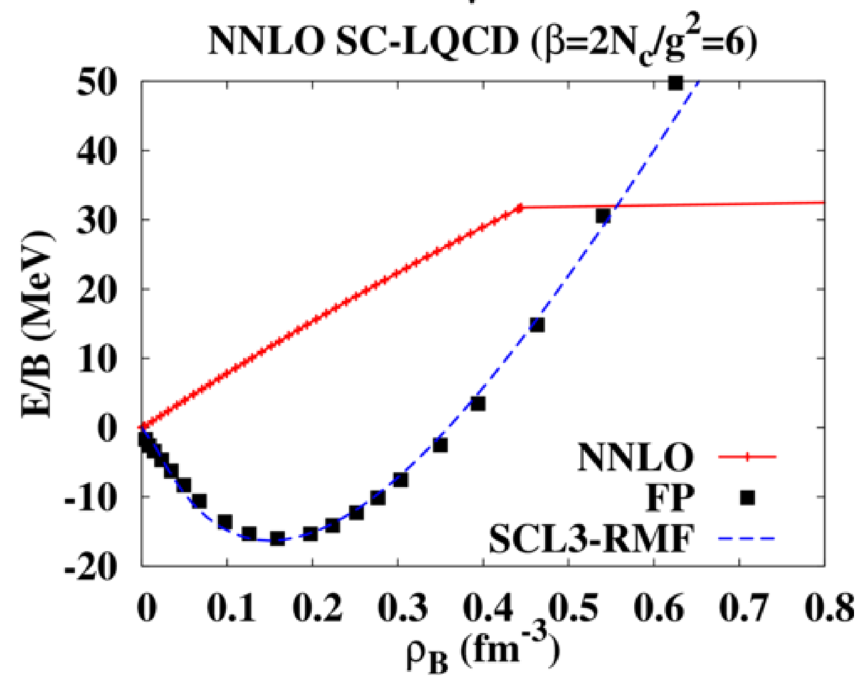
- **a^{-1} = 500 MeV**

Bilic, Demeterfi, Petersson ('92)

→ Density in the order of ρ_0

- **No saturation**

- **1st order transition at ρ_c =0.4 fm^{-1}...**



> *Nuclear matter on the lattice.*
> *Can we attack it soon ?*

Can we attack it soon ?

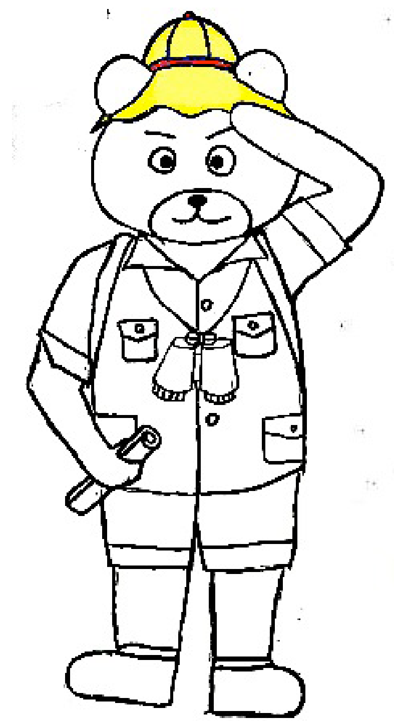
Objection !

I do not think SC-LQCD would be the starting point of lattice MC study at finite density !
(Jan. 2010)

- **Mean Field approach**
  → Not directly connected with MC
- **MDP approach**
  → Algorithm with finite coupling correction is not proposed yet.

Summary

- Strong coupling lattice QCD may be a *good* starting point to study finite density QCD matter.
- NLO, NNLO, Polyakov loop .....
→ Chiral & Deconf. transition
(c.f. Langelage, Munster, Philipsen, YM+Heavy Quark)
- Resummation of higher order terms of plaquettes seems to be necessary.
- Items to be solved: 1/d expansion, Mean Field, Higher-order terms
→ Each component should be verified where there is no sign problem,
imag. μ (Lombardo / XQCD),
SUc(2)(Kim), SCL(de Forcrand), ...



Nakamura-san, thank you for giving me the motivation, and for continuous encourangements. And we need your further works.

Backup

Summary & Conclusion

- We have derived the effective potential with NLO ($1/g^2$) and NNLO($1/g^4$) effects in strong coupling lattice QCD.
- Several techniques are developed (Extended HS transf., Multi-order parameter treatment, gluonic dressed fermion)
- NLO & NNLO effects are found to modify the quark mass, dynamical chemical potential, and W.F. renormalization factor.
- Phase diagram is studied by using the derived effective potential.
- Decrease of $T_c(\mu=0)$ and stable $\mu_c(T=0)$ improve the "shape" of the phase diagram, where NLO(NNLO) effects are large (small).
- Critical Point is sensitive to NNLO effects.
- In NNLO, we observe finite density matter before the 1st order phase transition.
- Baryon mass puzzle is solved, i.e. $M_B < N_c\mu_c$ is realized at T=0.
- For a^{-1}=500 MeV, ρ_B is the order of ρ_0, but no saturation.

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Future Works

- While it is still far from the real world, SC-LQCD seems to be a promising tool to understand the phase diagram and EOS.
- Relation to lattice MC calculation is straightforward,
- SC-LQCD motivated σ potential (∝ -log σ) is useful in describing nuclei and nuclear matter in chiral symmetric RMF models,
- and it may be possible to describe nuclear matter directly.
- But SCL/NLO/NNLO SC-LQCD show far from scaling behavior, and they do not qualitatively explain the MC results at $\mu=0$ and at SCL($g \to \infty$).
- Further studies incl.

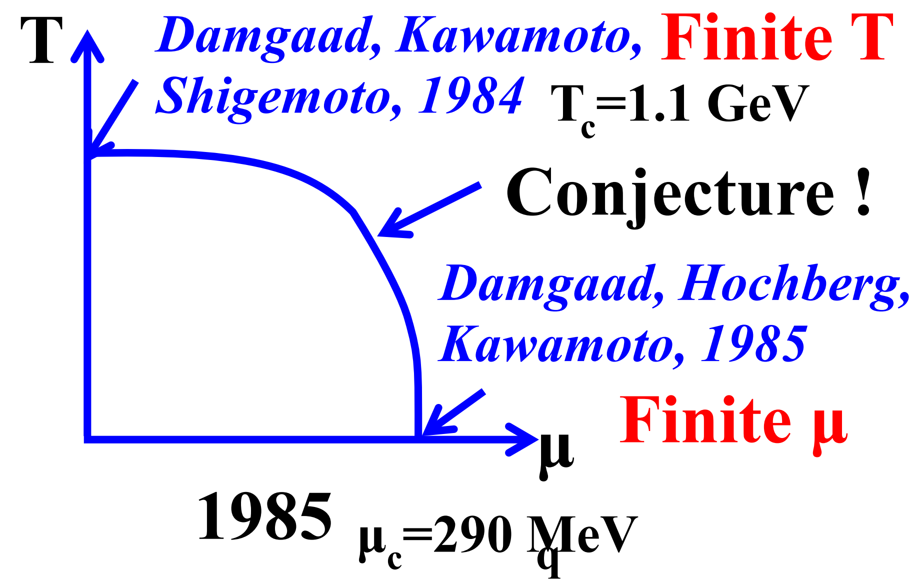
  Polyakov loop, 1/d, meson fluctuation, higher orders in $1/g^2$

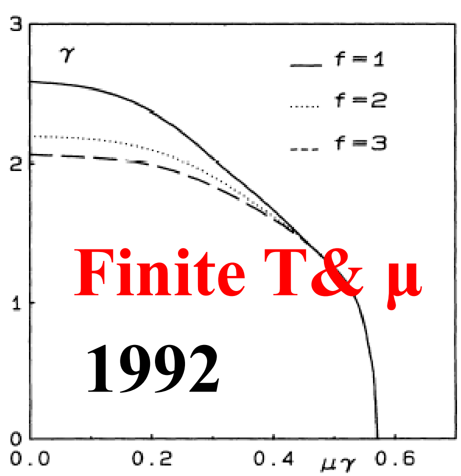
  may be necessary to describe

  the deconfinement transition, $\mu=0$ and SCL results in MC.
- For $N_f=2+1$, different types of fermion have to be considered.

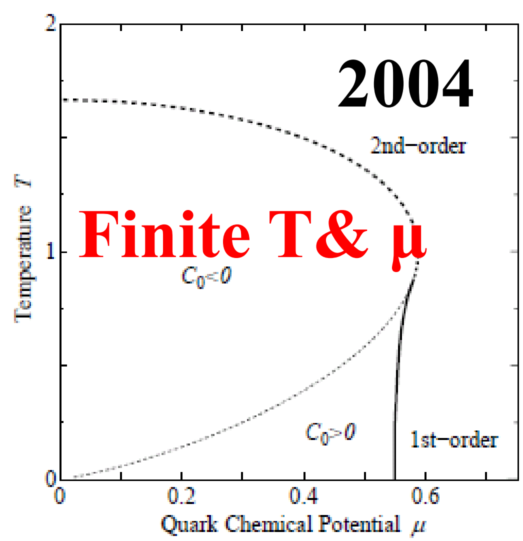
Evolution of Phase Diagram

- Phase Diagram "Shape" becomes closer to that of Real World,
  $R=\mu_c/T_c \sim (2\text{-}4)$
- 1985 → R=0.26 (Zero T / Finite T)
- 1992 → R=0.28 (Finite T & μ)
- 2004 → R= 0.33 (Finite T& μ)
- 2007 → R=0.45 (Baryon)

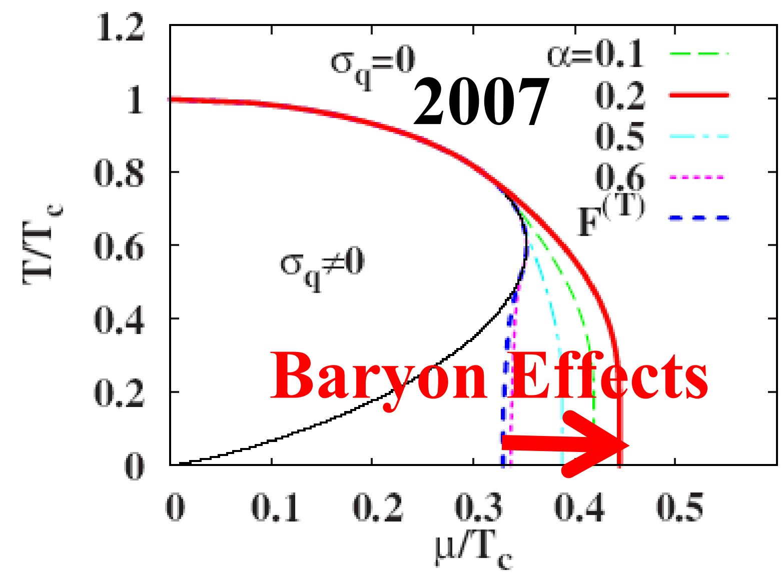




Bilic, Karsch, Redlich, 1992



Fukushima, 2004



Kawamoto, Miura, AO, Ohnuma, 2007

Phase Diagram in the Strong-Coupling Lattice QCD

Nakamura-san (2)

Nakamura-san (3)

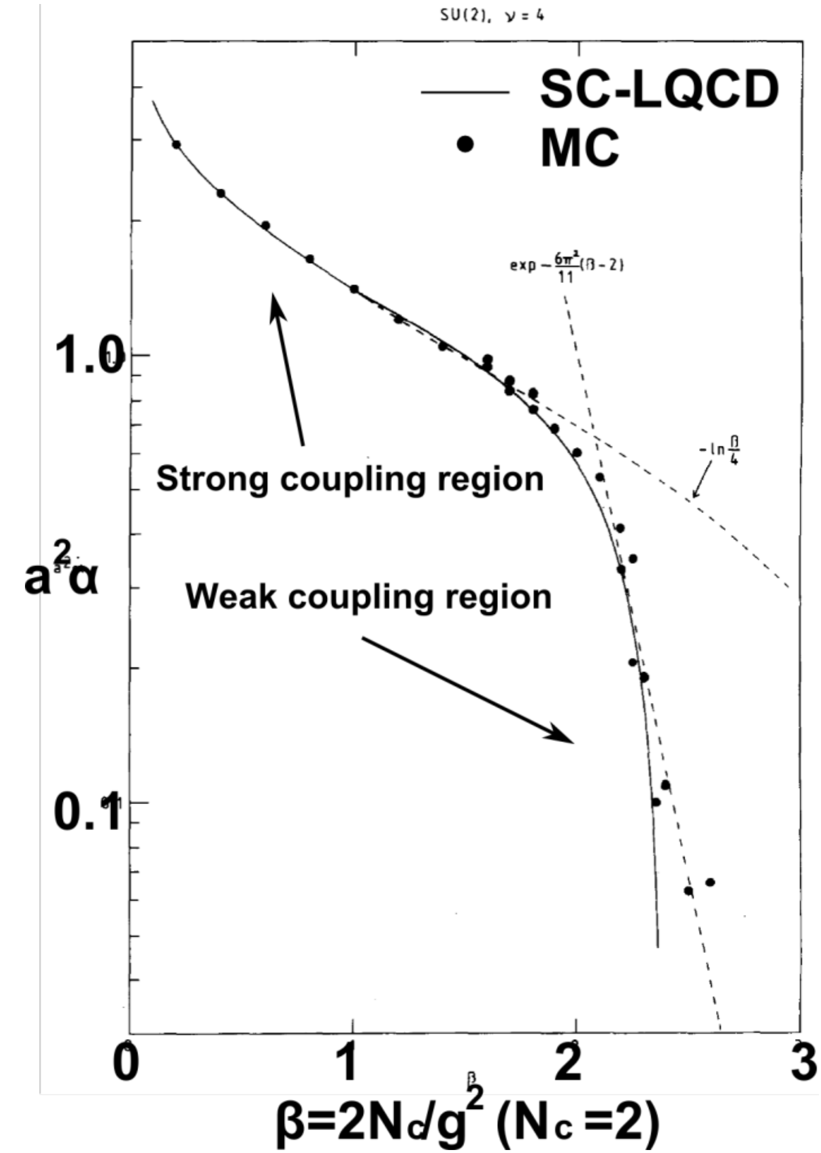
- In a molecule-type workshop on
  "Lattice QCD at Finite Densities" (Jan. 2010)

Nakamura-san told us,

"We need to have a firm starting point to attack finite density in QCD. Naively, we expect the strong coupling lattice QCD is would be

Strong Coupling Lattice QCD

- Large bare coupling → $1/g^2$ expansion
- Success in pure YM
→ Lattice MC & $1/g^2$ Expansion
Wilson, '74; Creutz '80; Munster '81
- → ***Scaling region would be accessible in SC-LQCD !***
- Chiral transition at finite T and μ in Strong Coupl. Limit (SCL) & Next-to-leading order (NLO)
Kawamoto-Smit '81, Damgaard-Kawamoto-Shigemoto, '84(U(3)), Faldt-Petersson'86 (SU(3)), Fukushima'04(SU(3)), Nishida '04 (SU(3)), Bilic-Karsch-Petersson, '92(NLO), Kawamoto-Miura-AO-Ohnuma '07 (Baryons)



Munster, '81

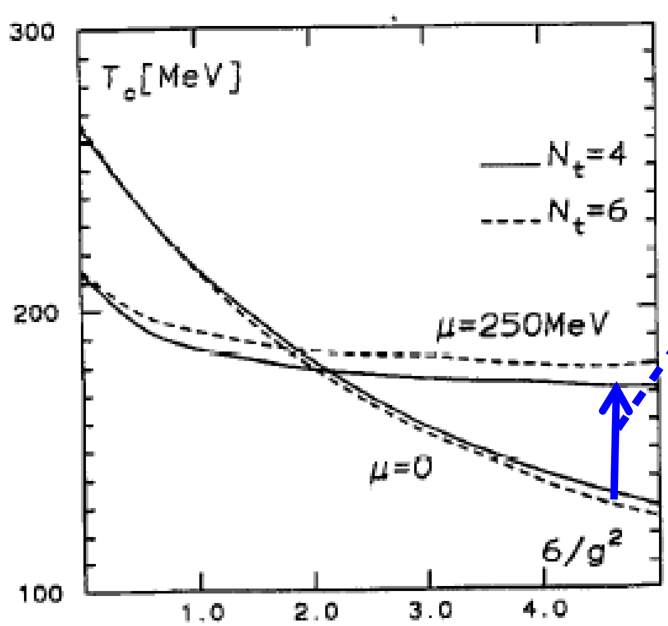
Strong Coupling Lattice QCD with Fermions

- **SC-LQCD with fermions**
- **SCL: SC-LQCD (analytic) & MC (MDP) qualitatively agree**

de Forcrand, Fromm ('09)

- **$\beta=2N_c/g^2$ dep. of the critical point is not studied yet.**
- **Condensates other than σ are not yet included in previous works.**

(Faldt-Petersson '86; Bilic-Karsch-Redlich '92; Bilic-Demeterfi-Petersson '92; Bilic-Claymans '95)



Bilic-Claymans '95

$dT_c/d\mu > 0 \rightarrow \Delta s/\Delta\rho < 0$?
(Clausius-Clapeyron)

In this work, we revisit / develop
NLO & NNLO SC-LQCD
with fermions at finite T
and μ
and Study Phase diagram
& nuclear matter with finite β

NLO & NNLO SC-LQCD: Setups & Disclaimer

- **We investigate the phase diagram and try to understand nuclear matter based on the strong-coupling lattice QCD (SC-LQCD).**
- **Effective potential (free E. density) → phase boundary & EOS**
- **Setups & Disclaimer**
- **Effective action in SCL ($1/g^0$), NLO ($1/g^2$), NNLO ($1/g^4$) terms**

  NLO: Faldt-Petersson ('86), Bilic-Karsch-Redlich ('92)
  Conversion radius > 6 in pure YM ? Osterwalder-Seiler ('78)

- **One species of unrooted staggered fermion (N_f=4 in continuum limit)**

  Moderate N_f deps. of phase boundary: BKR92, Nishida('04), D'Elia-Lombardo ('03)

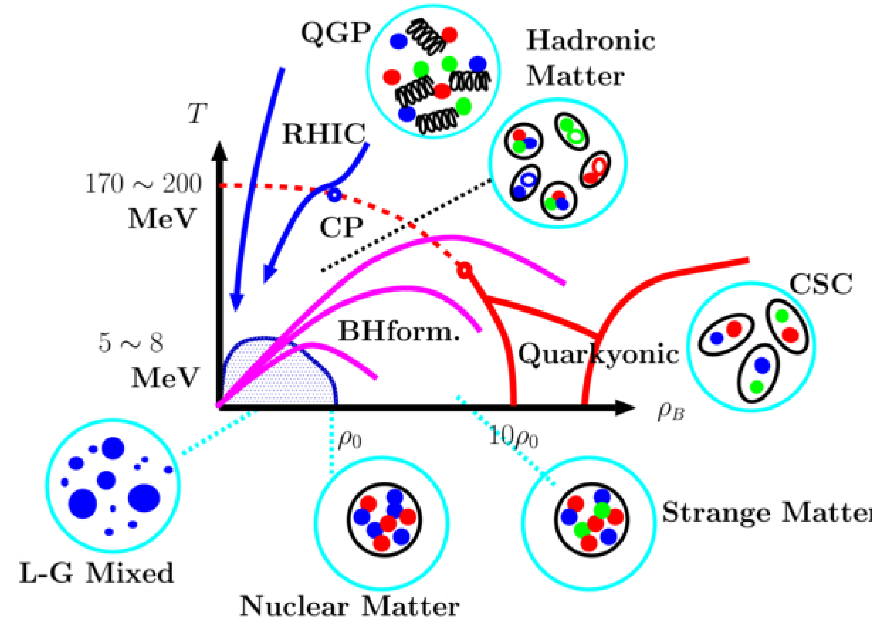
- **Leading order in 1/d expansion (d=3=space dim.)**
  → Min. # of quarks for a given plaquette configurations, no spatial B prop
- **Effe**

Still far from "Realistic", but SC-LQCD would tell us useful qualitative features of the phase diagram and EOS.

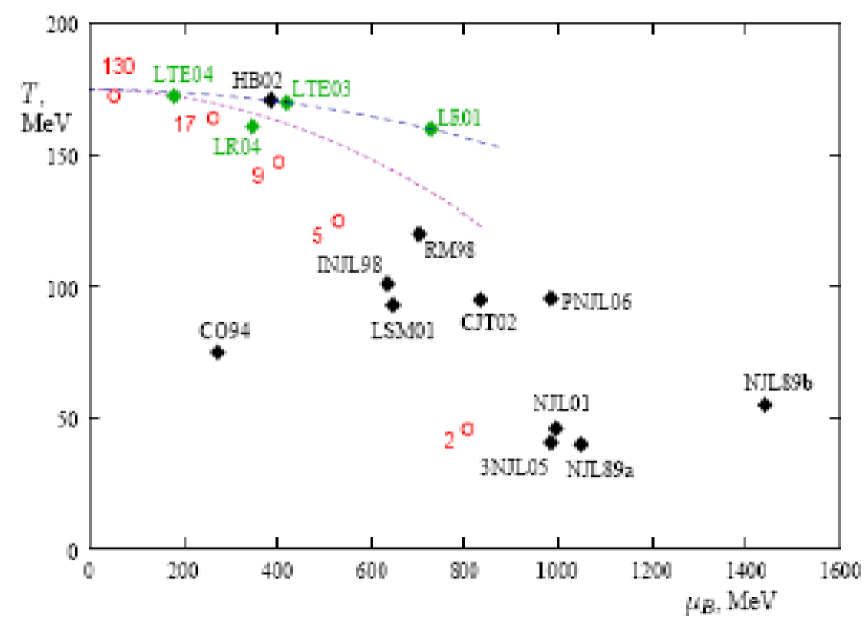
- **Polyakov loop effects are not included (No Deconfinement)**

QCD Phase diagram

- **Phase transition at high T**
- **Lattice MC & RHIC**
- **High μ transition has rich physics**
- **Various phases, CEP, Astrophysical applications, ...**
- **Models & Approximations are necessary !**



- **Lattice MC works only for small μ (Tayler, AC, DOS, Canonical, …) or in the Strong Coupling Limit(SCL) (MDP)** ***Karsch, Mutter ('89), de Forcrand, Fromm ('09)***



- **Eff. Models: NJL, PNJL, PLSM, ...**
- **Approximations:**

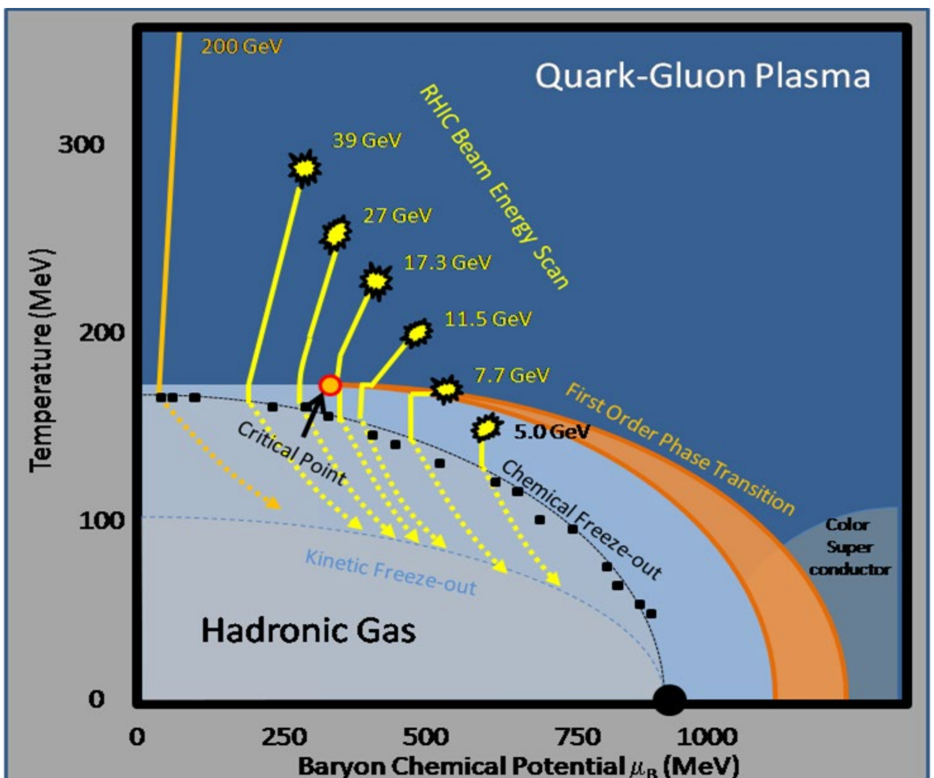
Where is the Critical Point ?

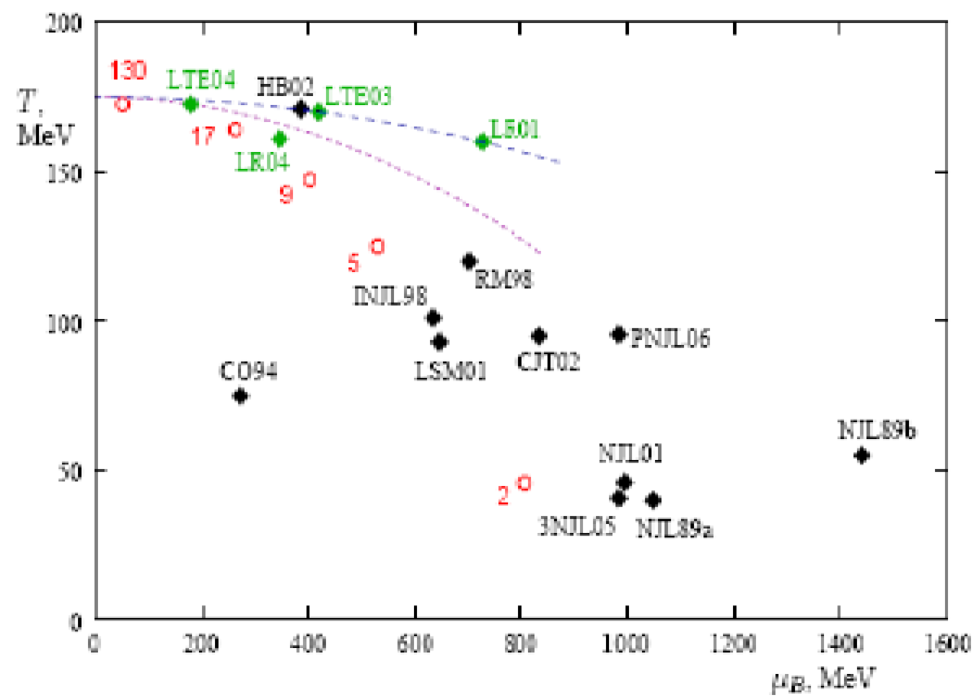
- **Critical Point Search**

= One of the main goals in Low-E progs. at RHIC

- **Theory → No Consensus (Sign prob. at finite μ)**

Can we attack CP in LQCD ? → Strong Coupling LQCD





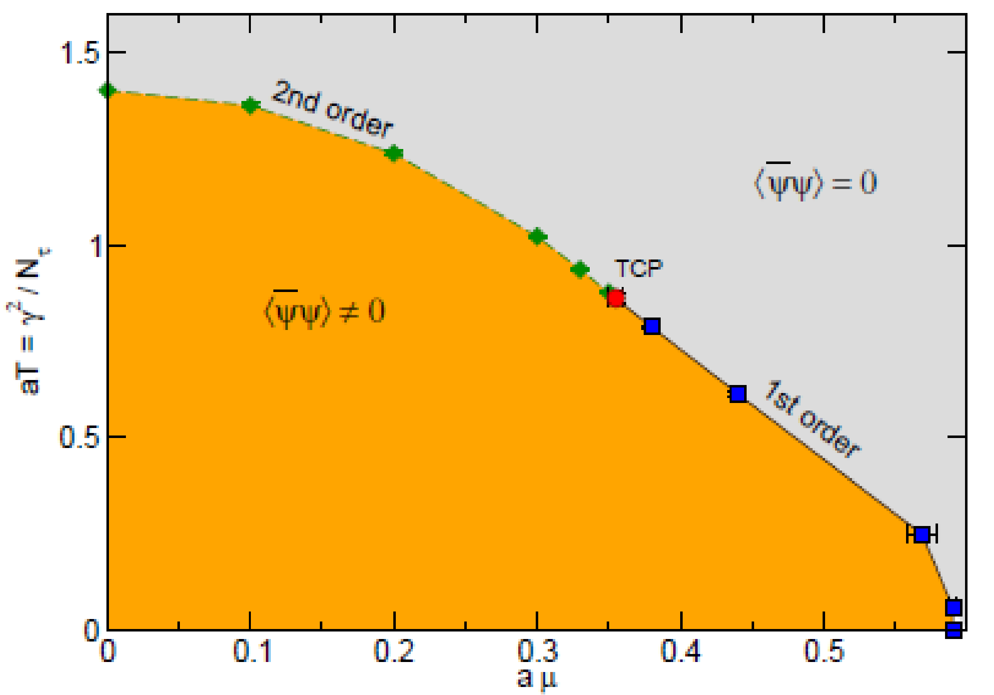
Monomer-Dimer-Polymer simulation

- Monomer-Dimer-Polymer simulation (MDP) *Karsch, Mutter ('89)*
→ Integrate out link variables first in the strong coupling limit

- Sign problem is weakened (Cancellation of baryonic loops remains).
→ Phase diagram in SCL *de Forcrand, Fromm ('09)*

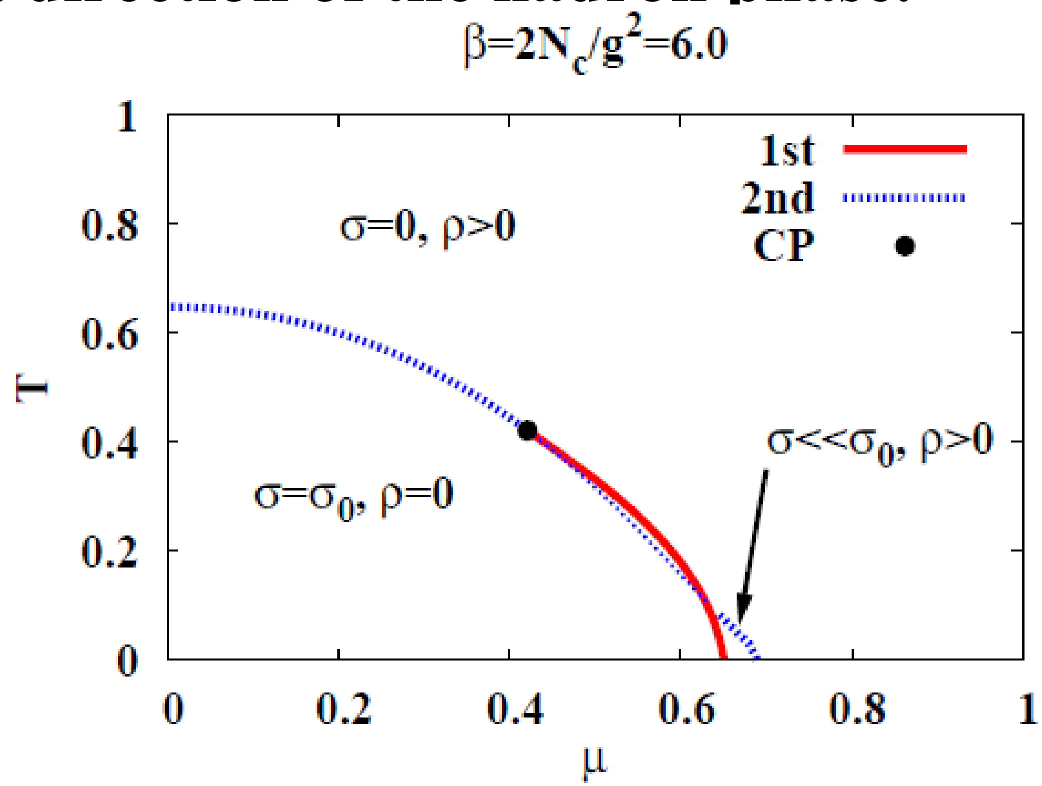
- $T_c(\mu=0)$ and $\mu_c(T=0)$ qualitatively agree with SCL-LQCD (MF) results.

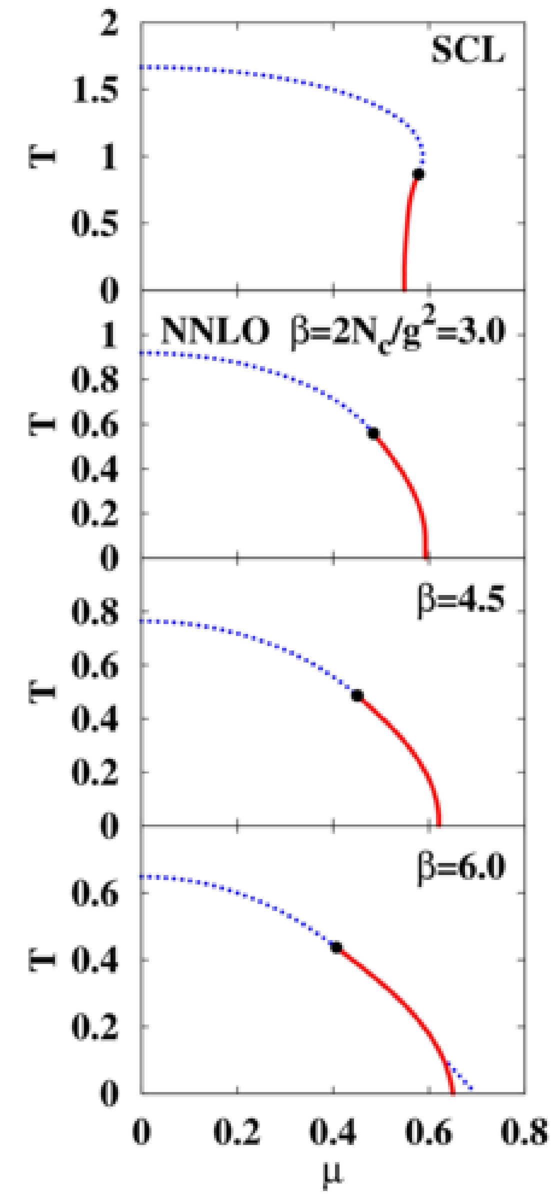
  aT_c=5/3 (MF), 1.41(3) (MDP)
  $a\mu_c$=0.549(MF), 0.593(MDP)
  (aT_{TCP} , $a\mu_{TCP}$)
=(0.867, 0.578)(MF),
  (0.86(2), 0.355(5))(MDP)



Phase diagram

- **With increasing β, phase diagram is compressed in T direction.**
- **For finite β, 1st order boundary has a negative slope, $dT_c/d\mu<0$.** *c.f. Bilic, Demeterfi, Petersson ('92)*
- **Existence of the partially chiral restored phase in the higher μ direction of the hadron phase.**

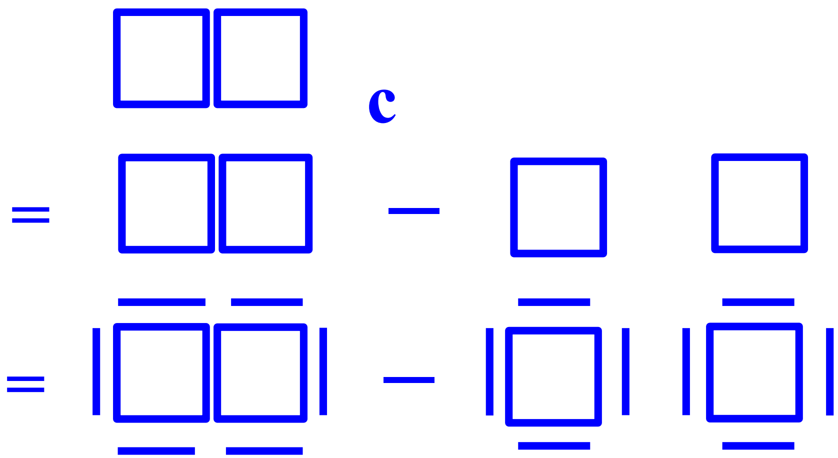




NNLO Effective Action

- Cumulants of two plaquettes
  = Correlation part of connected two plaquettes

- 1/d expansion: Σ_j MM ~ Const.
  $\rightarrow \chi \sim d^{-1/4}$

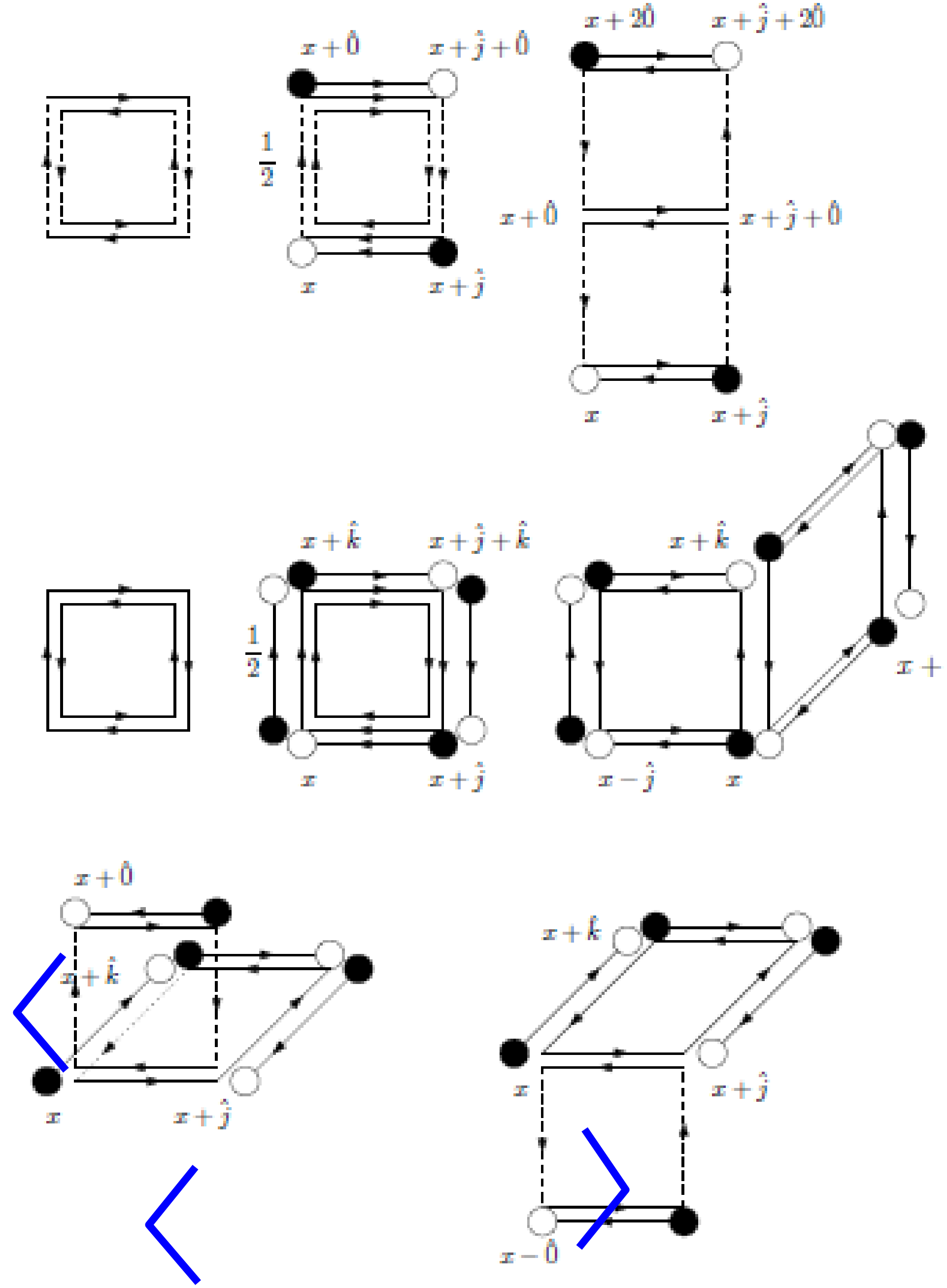


12 quarks 16 quarks

power in d

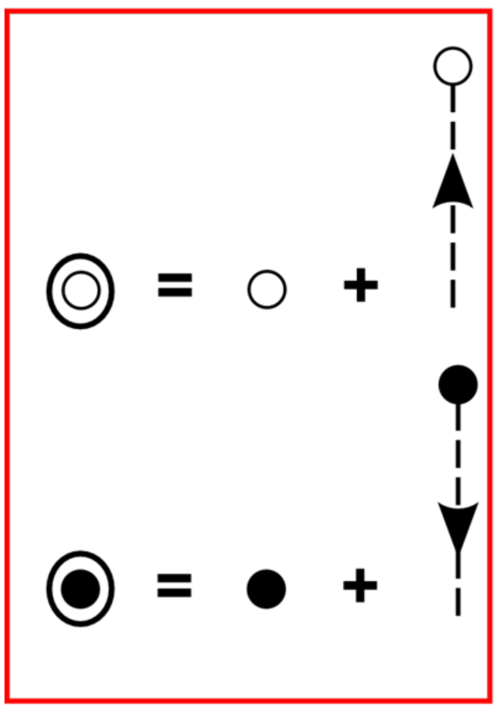
3 – 1/4 x 12 3 – 1/4 x 16

=0 = -1



Dressed fermion

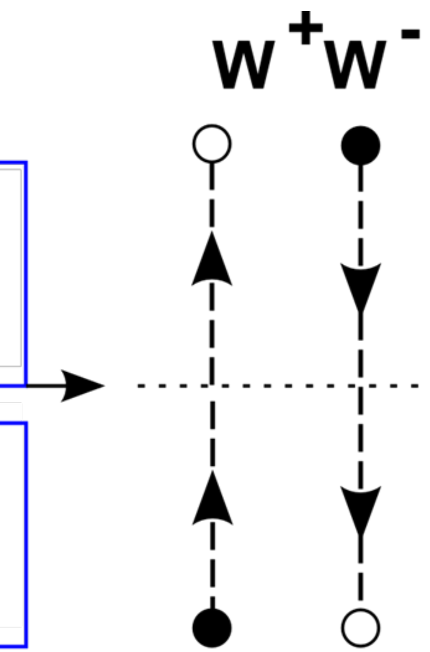
- Next-to-nearest neighboring site interaction $W^{\pm}$.
- By introducing the "Dressed Fermion", mixture of the quark field on the next temporal site, NNN interaction is rearranged to NN.



$$\chi'_x = \chi_x + A e^{\mu} U_{0,x} \chi_{x+\hat{0}}$$

$$\bar{\chi}'_x = \bar{\chi}_x + \bar{A} \bar{\chi}_{x+\hat{0}} e^{-\mu} U^{\dagger}_{0,x}$$

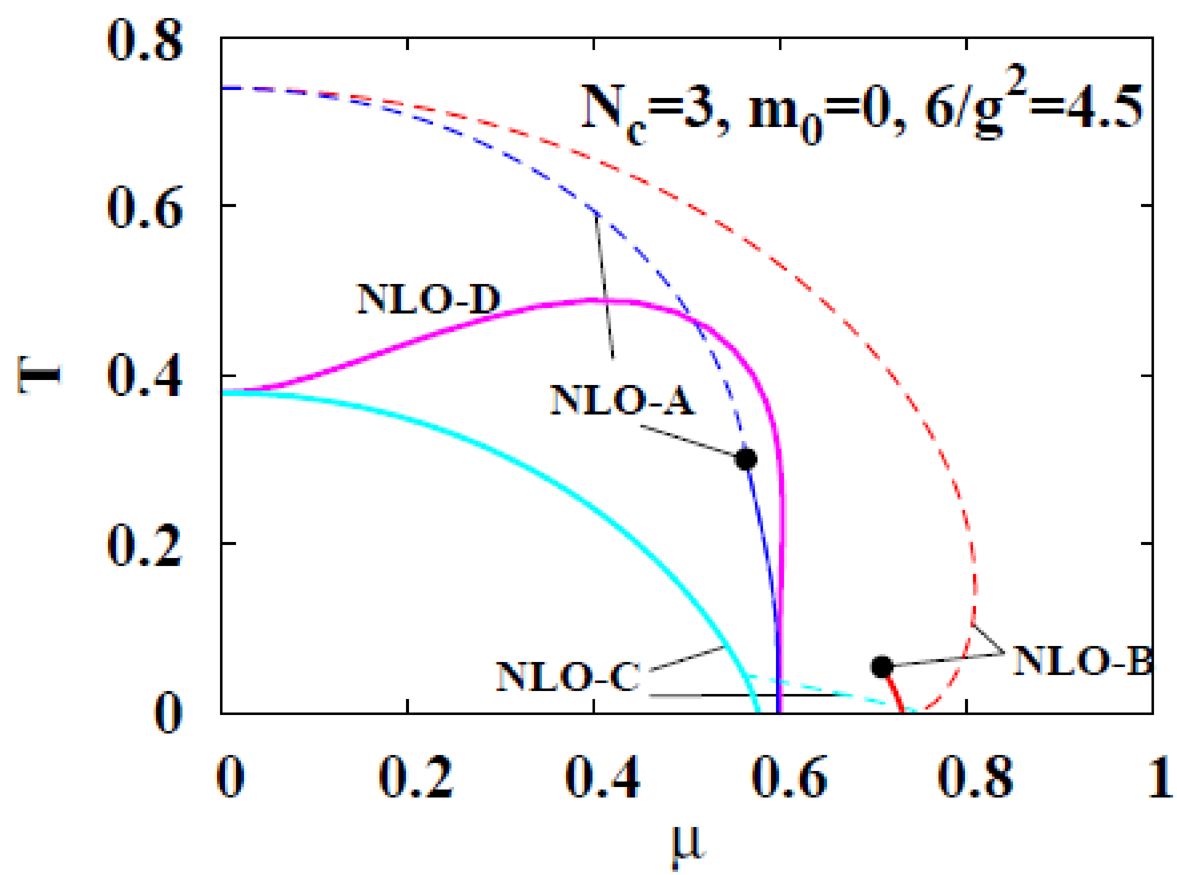
$$A = \mathcal{O}(1/g^4)$$



Truncation Deps. in NLO

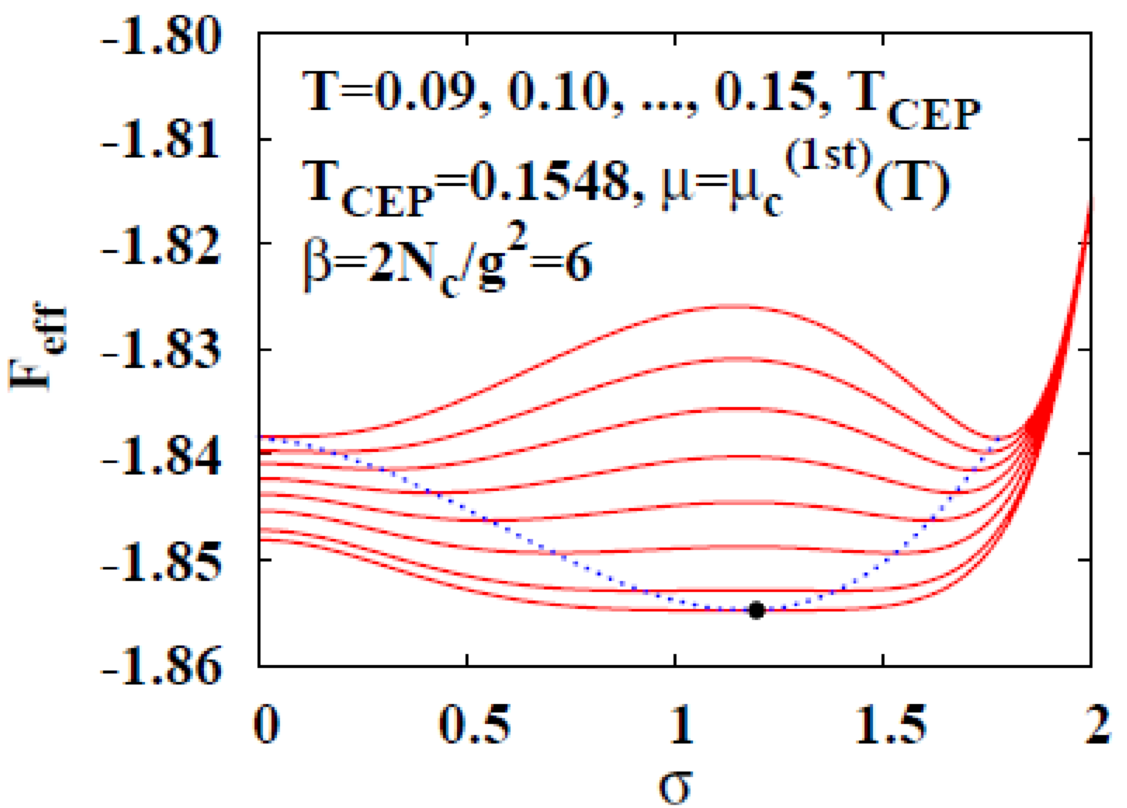
- Phase diagram is sensitive to details, such as the truncation scheme in NLO.

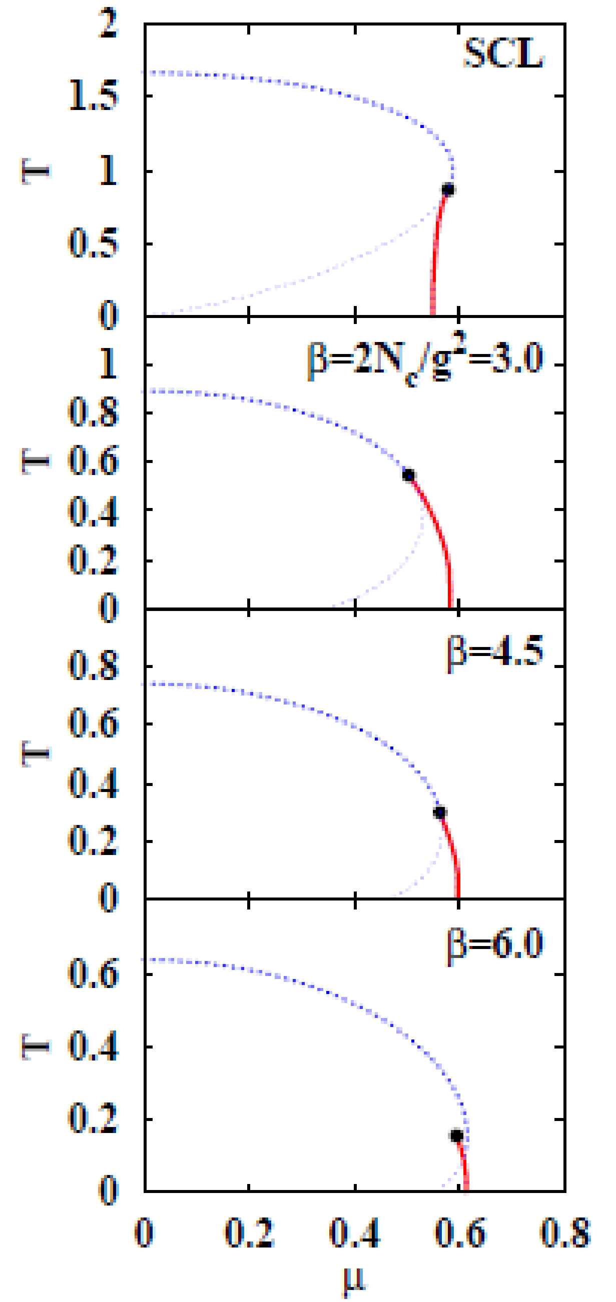
	$\delta\mu$	$\bar{m}_q$	$\Delta\mathcal{F}_{\mathrm{aux}}$	$\mathcal{V}_q$
NLO-A	$\log\sqrt{\frac{Z_+}{Z_-}}$	$\frac{m_q}{\sqrt{Z_+Z_-}}$	$-N_c \log\sqrt{Z_+Z_-}$	$\mathcal{V}_q(\bar{m}_q, \bar{\mu}, T)$
NLO-B	$\beta_\tau\omega_\tau$	$\frac{m_q}{1+\beta_\tau\varphi_\tau}$	$-N_c \log(1+\beta_\tau\varphi_\tau)$	$\mathcal{V}_q(\bar{m}_q, \bar{\mu}, T)$
NLO-C	$\beta_\tau\omega_\tau$	$\bar{m}_q^{(\mathrm{NLO-C})}$	$-N_c\beta_\tau\varphi_\tau$	$\mathcal{V}_q(\bar{m}_q, \bar{\mu}, T)$
NLO-D	0	$\bar{m}_q^{(\mathrm{NLO-D})}$	$-N_c\beta_\tau\varphi_\tau$	$\mathcal{V}_q(\bar{m}_q, \mu, T) - \beta_\tau\omega_\tau\frac{\partial\mathcal{V}_q}{\partial\mu}$



Critical End Point in the Chiral Limit ?

- Vector field generates repulsive pot. for large ρ_q states, which may cause two local min. structure
  → Partially Chiral Restored matter may appear.





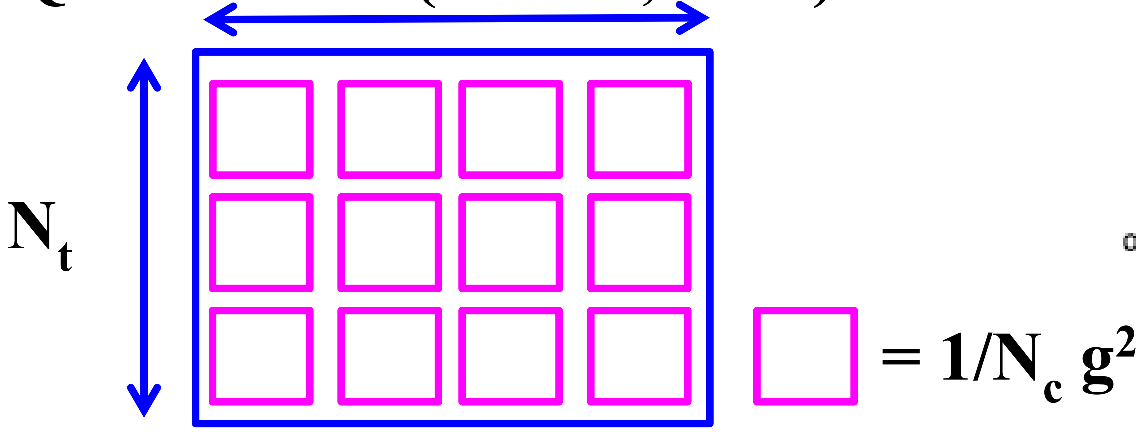
Strong Coupling Lattice QCD: Pure Gauge

- Quarks are confined in Strong Coupling QCD

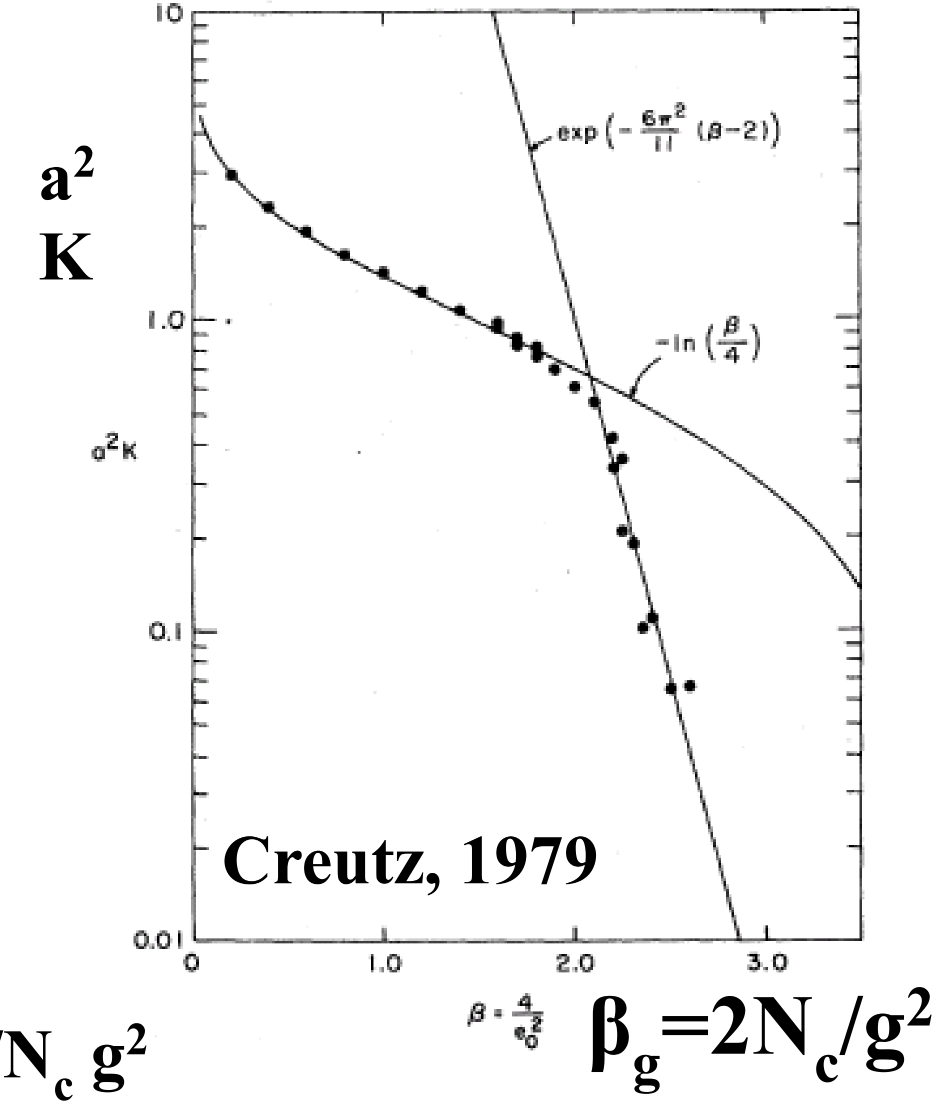
- Strong Coupling Limit (SCL)
  → Fill Wilson Loop with Min. # of Plaquettes
  → Area Law (Wilson, 1974)

$$S_{\rm LQCD} = -\frac{1}{g^2}\sum_{\square} {\rm tr}\left[U_\square + U_\square^\dagger\right]$$

- Smooth Transition from SCL to pQCD in MC (Creutz, 1980)

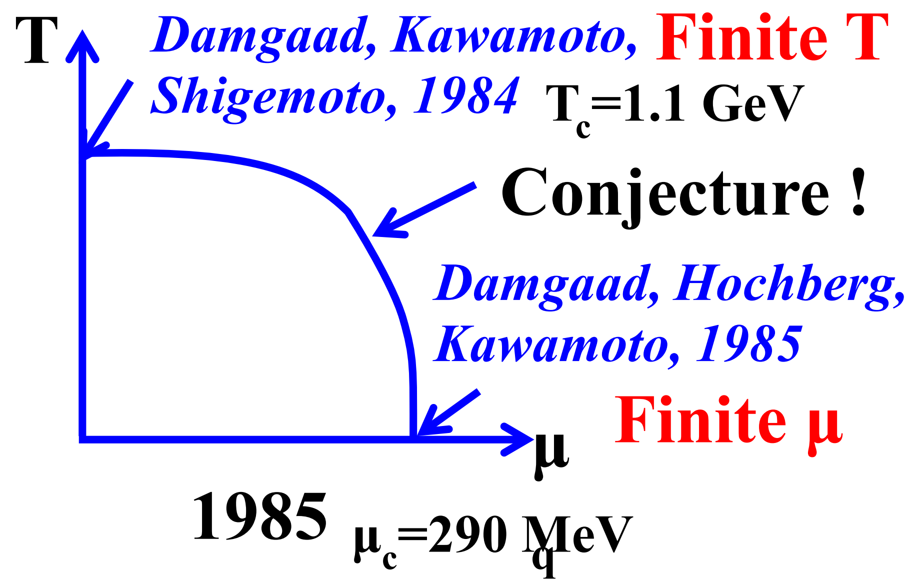


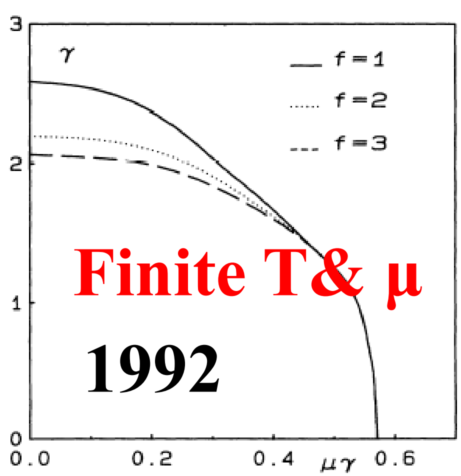
K. G. Wilson, PRD10(1974),2445
M. Creutz, PRD21(1980), 2308.
G. Munster, 1981



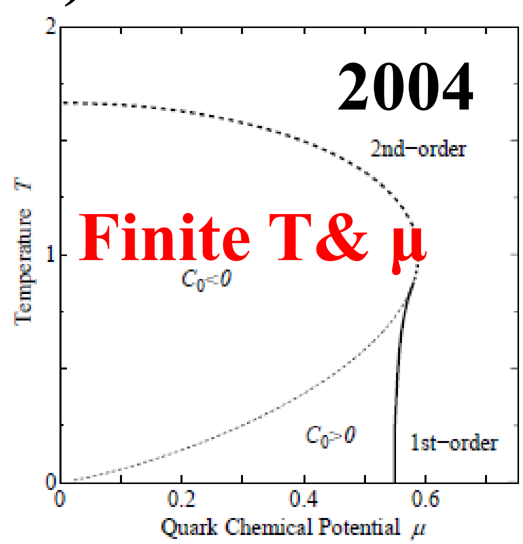
Evolution of Phase Diagram as a function of Time

- **Phase Diagram "Shape" becomes closer to that of Real World,**
  $R=\mu_c/T_c \sim (2\text{-}4)$
- **1985 → R=0.26 (Zero T / Finite T)**
- **1992 → R=0.28 (Finite T & μ)**
- **2004 → R= 0.33 (Finite T& μ)**
- **2007 → R=0.44 (Baryon)**

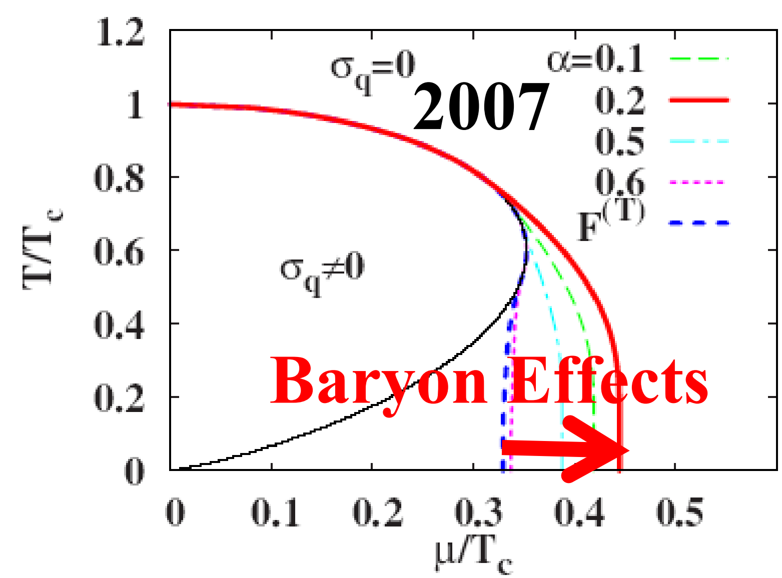




Bilic, Karsch, Redlich, 1992



Fukushima, 2004, Nishida, 2004



Kawamoto, Miura, AO, Ohnuma, 2007

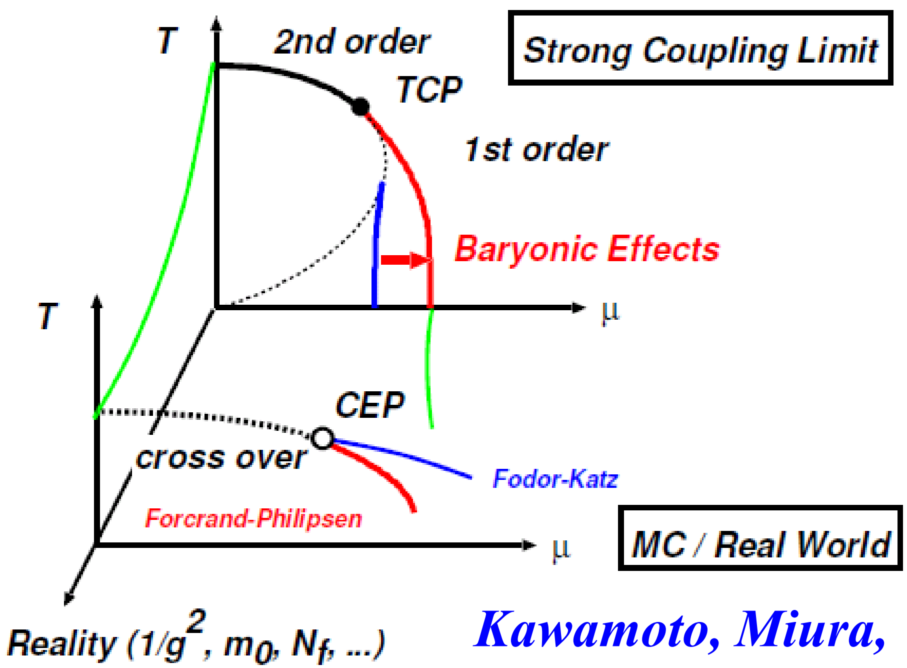
- Why we cannot explain the phase diagram shape ?

→ N_f (Staggered fermion) ? quark mass ? Finite Coupling ?

- μ_c (SCL) ~ $M_N/3$ (within a factor 2) , T_c (SCL) >> 200 MeV

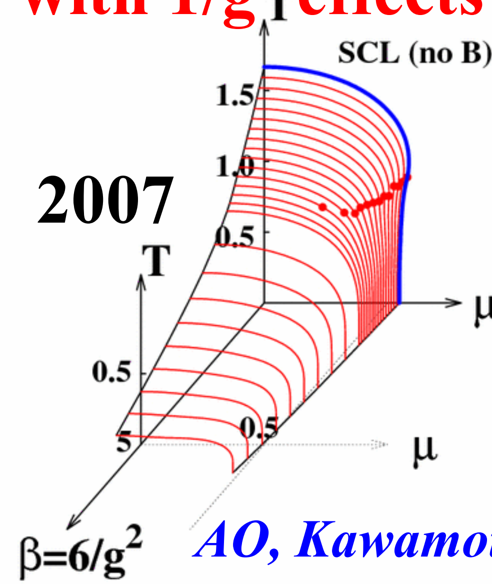
→ Larger problem should be in T_c , rather than in μ_c

Expectation before Calc.



Kawamoto, Miura, AO, Ohnuma, 2007

Preliminary Resuls with $1/g^2$ effects

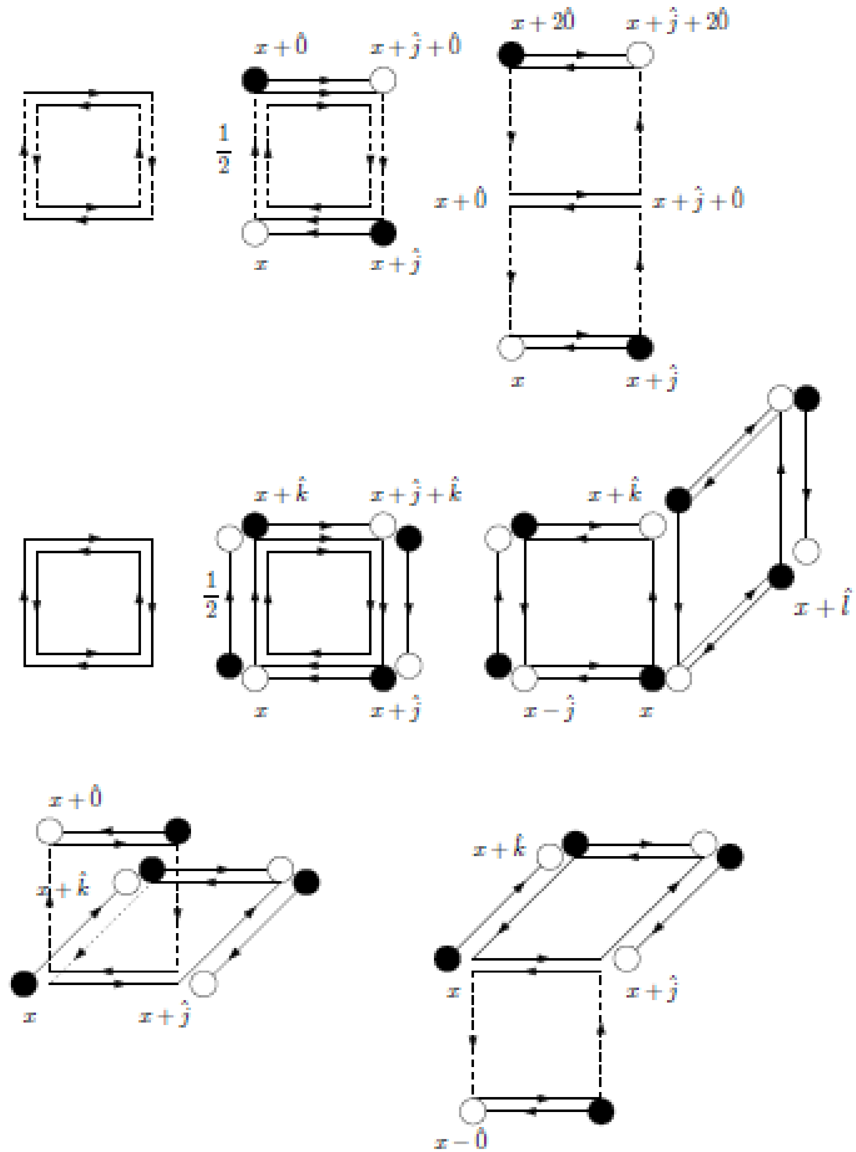


AO, Kawamoto, Miura, 2007

Gluon Contribution is important at High T

NNLO Effective Action

- Cumulants of two plaquettes
  = Correlation part of connected two plaquettes



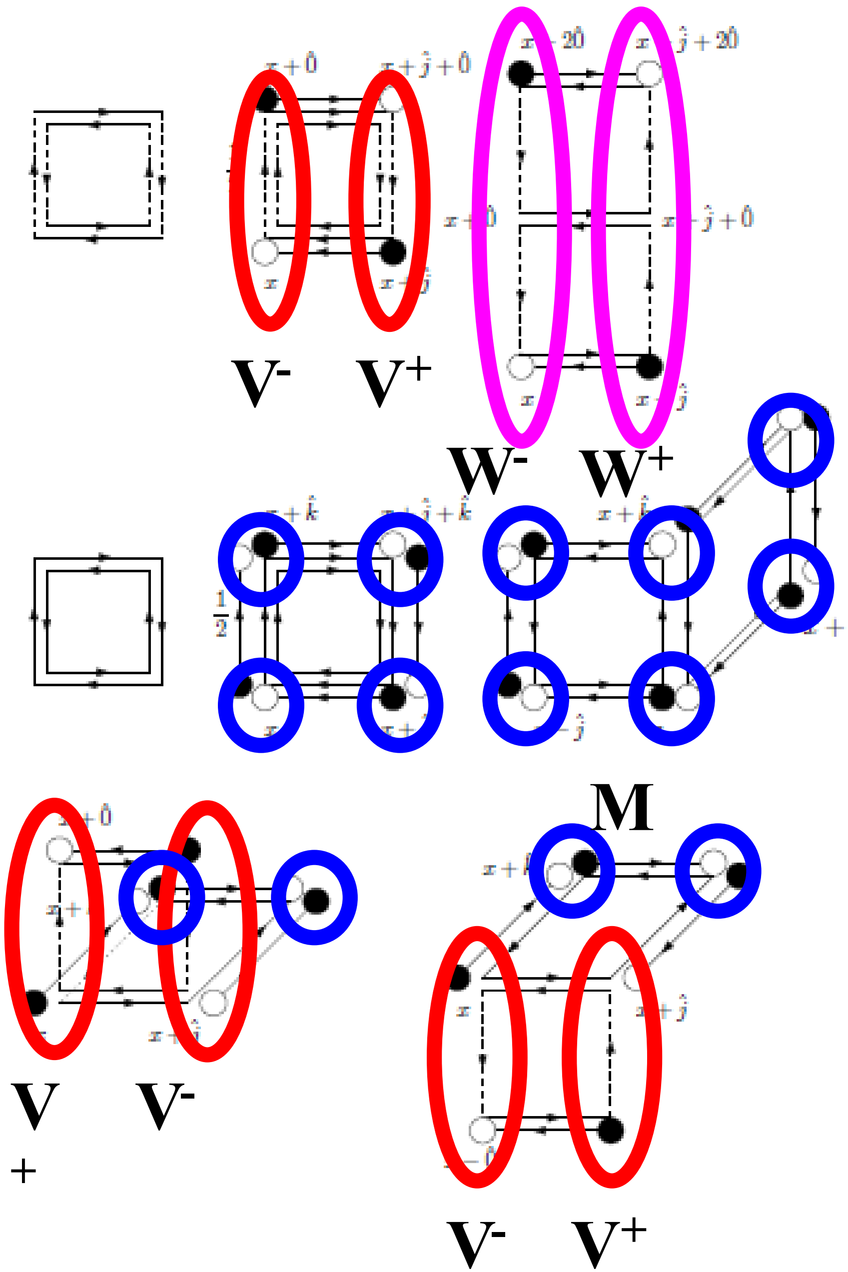
NNLO Effective Action

- Cumulants of two plaquettes
= Correlation part of connected two plaquettes

- Uncorr. & Normalization part are suppressed in 1/d power

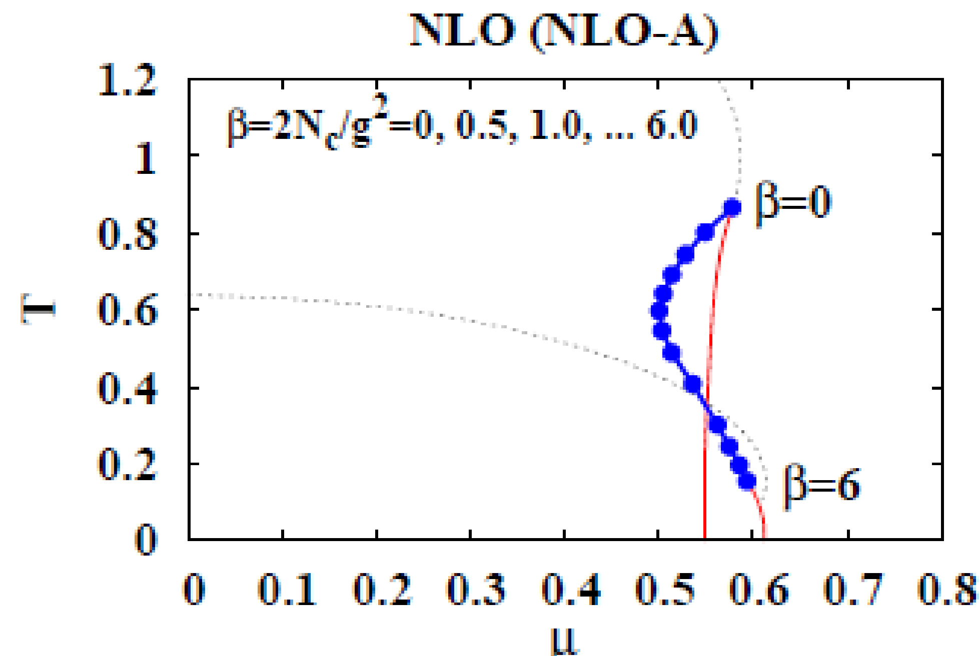
- Effective Action consists of
$V^-V^+, W^-W^+,$
$MMMM, MMMMMM,$
V^-V^+MM

- New type of Composite
= next-to-nearest neighboring site coupling in τ direction
$W^+ = \chi_x U_{0,x+0} U_{x+20}$

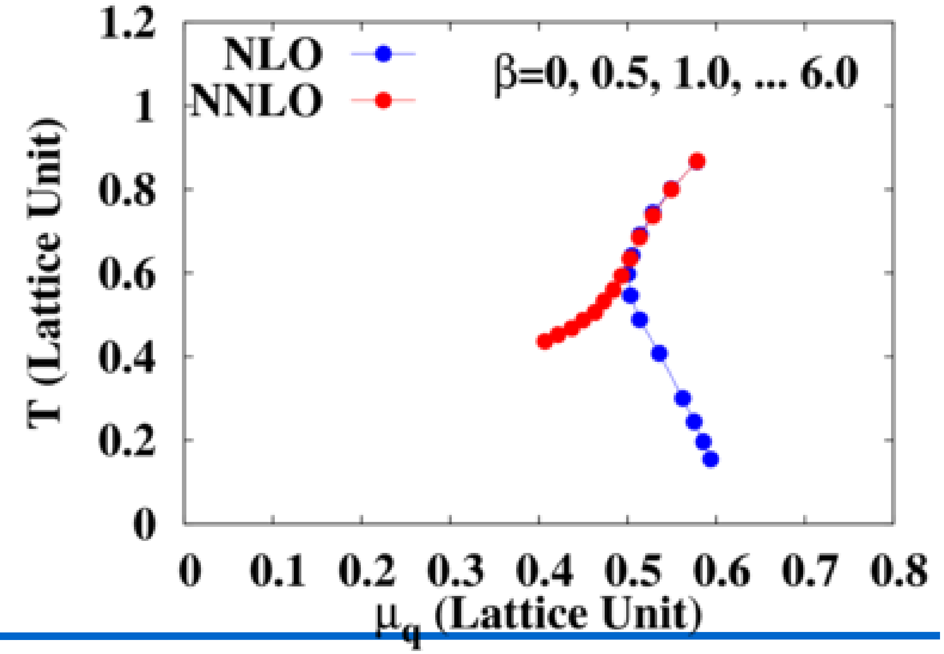


Critical Point Evolution

- Critical Point in NLO approaches μ axis
- Larger β → Stronger Vector Pot. ω_τ
- Consistent with NJL models. (Kitazawa et al., '02; Sasaki-Friman-Redlich, '07; Fukushima'08)
- and MC suggestion (de Forcrand-Philipsen, '08)
- CP in NNLO→ μ(CP)/T(CP) ~ 1
- Contradict to MC (μ/T > 1) ? (Ejiri, '08; Aoki et al.(WHOT), '08; Allton et al., '03,'05)
- Underestimate of Tc may be



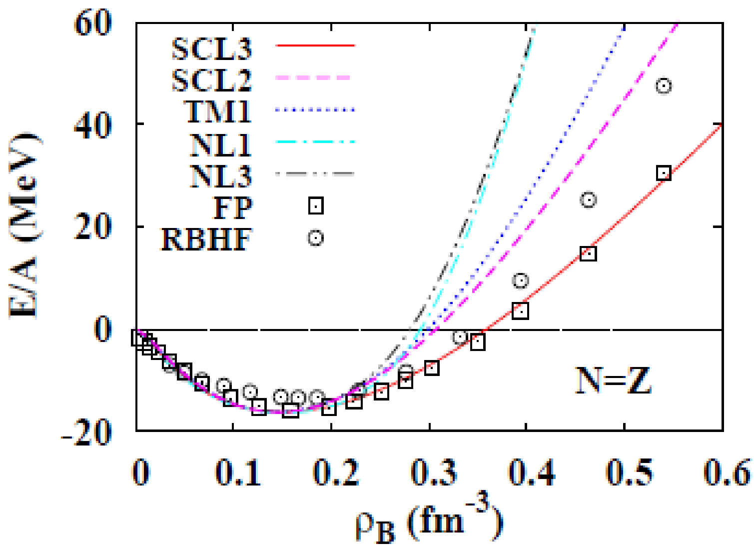
Critical Point in SC-LQCD

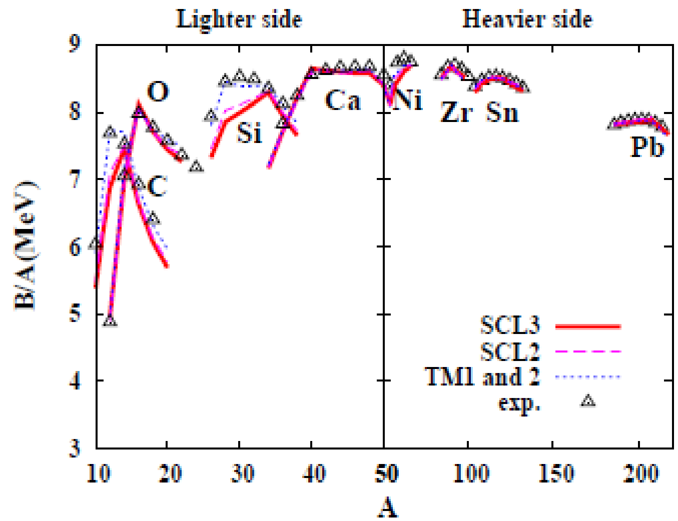


Nuclear Matter EOS

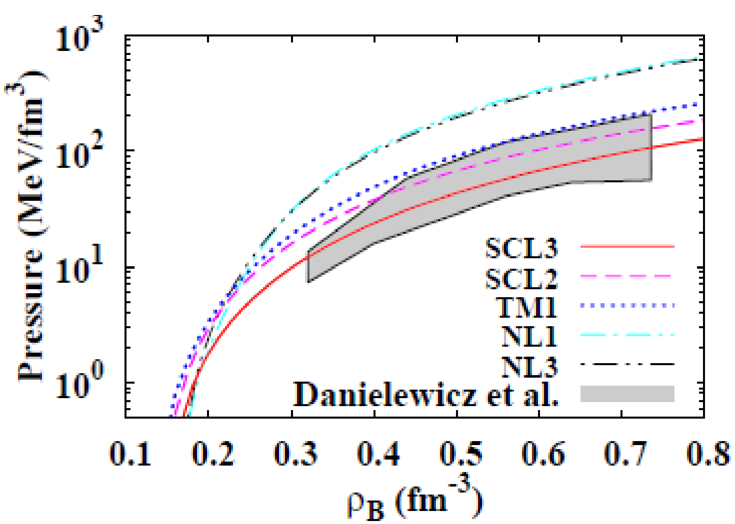
- **Nuclear matter EOS is important in**
- **Nuclear B.E., HIC, Neutron Stars, Supernova, BH formation, ...**

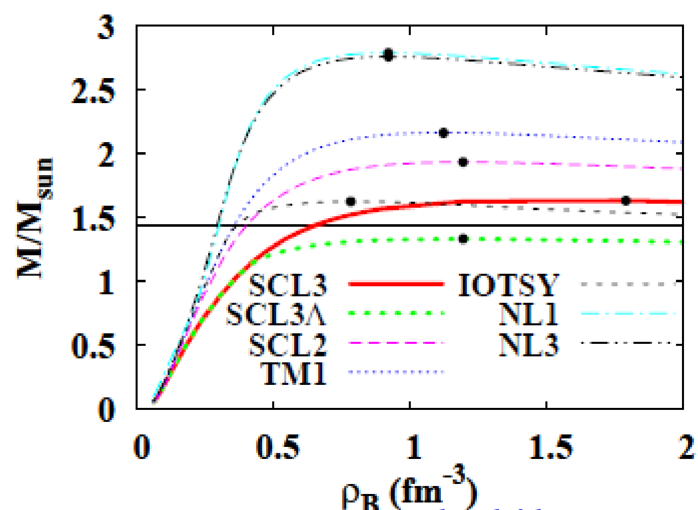
→ How can we describe it from QCD ?





Incompressibility
J.P.Blaizot,
Phys.Rept.64 (1980), 171.





Tsubakihara et al., arXiv:0909.5058