Another Mean Field Treatment in the Strong Coupling Limit of Lattice QCD

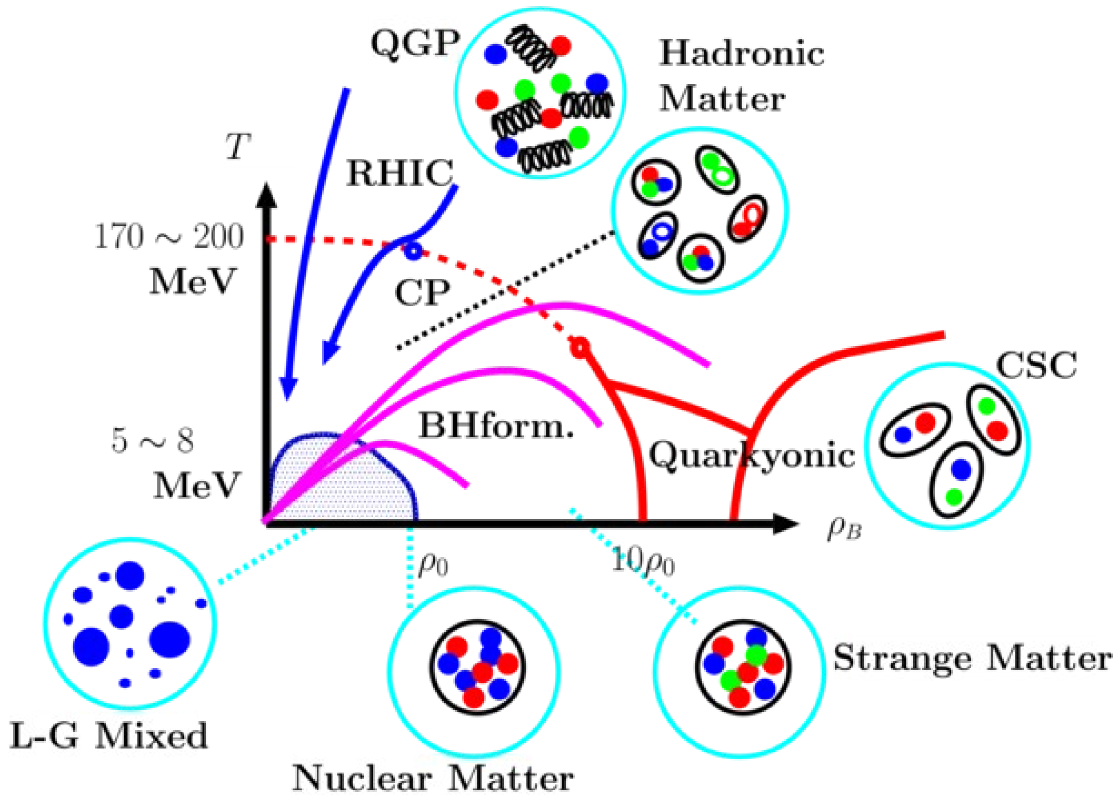
Akira Ohnishi
(YITP, Kyoto Univ.)
in collaboration with
K. Miura (Frascati), T.Z.Nakano (YITP & Kyoto U.)

- **Introduction --- Homeworks in SCL-LQCD**
- **Chiral phase diagram in SCL-LQCD (σ as a mean field)**
- **Phase transitions with another mean field than σ**
- **Conclusion**

Related talks: Miura (Tue), Nakano (Tue)

QCD Phase diagram

- **Phase transition at high T → Lattice MC, RHIC, LHC**
- **High μ transition has rich physics**
  → Various phases, CEP, Astrophysical applications, ...



Sign problem in Lattice MC at finite density
→ We need approximations and/or eff. models

Strong Coupling Lattice QCD

- **SC-LQCD is a powerful tool to investigate QCD phase diagram including finite density region !**
- **Pure Yang-Mills theory**

  Wilson ('74), Munster ('81), ..., Langelage, Münster, Philipsen ('08, Finite T), Langelage, Lottini, Philipsen ('10, Finite T, Tue)

- **Spontaneous breaking of the chiral symmetry & Phase diagram**

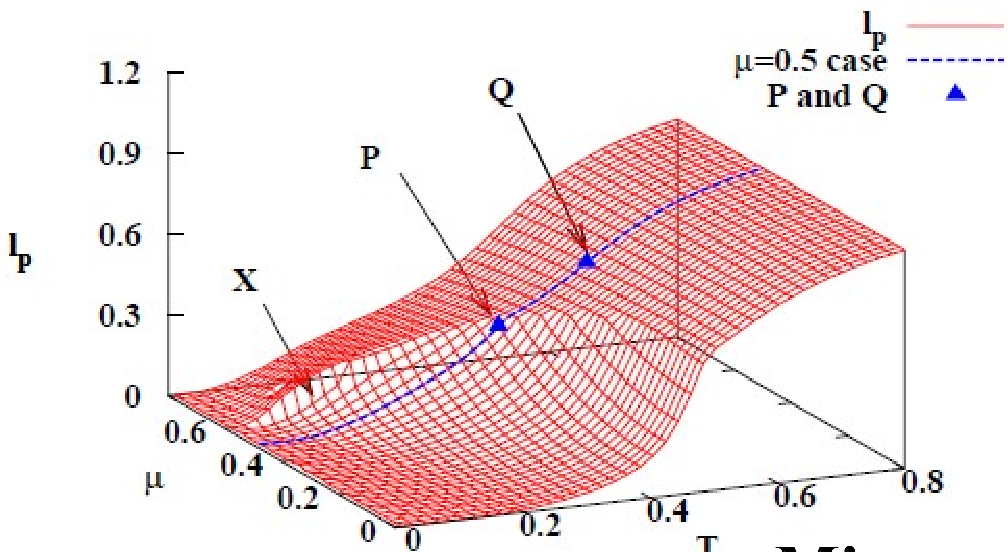
  Kawamoto, Smit ('81), Kluberg-Stern, Morel, Petersson ('83), Damgaard, Kawamoto, Shigemoto('84), Rossi, Wolff ('84), … Nishida, Fukushima, Hatsuda ('04), Fukushima ('04), Kawamoto, Miura, AO, Ohnuma ('07, SCL, *LAT07*), de Forcrand, Fromm ('10)

- **Finite coupling & Polyakov loop**

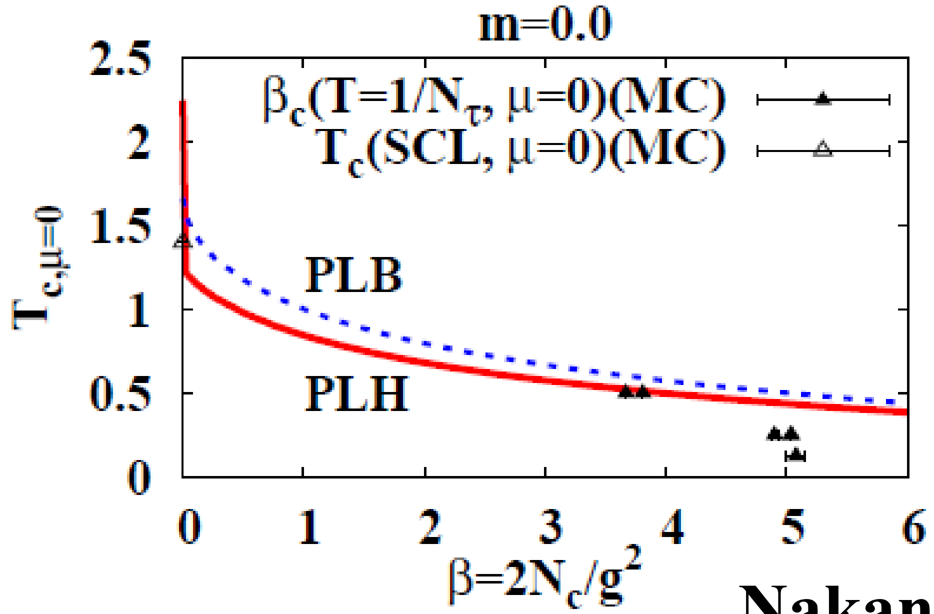
  Kogut, Snow, Stone ('82), Ilgenfritz, Kripfganz ('85), Gocksch, Ogilvie ('85), Faldt, Petersson ('86), Bilic,Karsch, Redlich('92), Fukkushima('03), Miura,Nakano,AO,Kawamoto('09, NLO, *LAT08*), Nakano,Miura,AO('10, NNLO, *LAT09*; PL, *LAT10*)

SC-LQCD with Polyakov Loop Effects

- **Phase diagram with NLO ($1/g^2$) & Polyakov loop effects (PNLO)**
  - **Phase boundary of chiral & deconfinement** *Miura (Tue)*
- **Phase transition at μ=0 with NNLO ($1/g^4$) & Polyakov loop effects**
  - **Non-trivial Polyakov-chiral coupling via U_0 integral** *Nakano (Tue)*



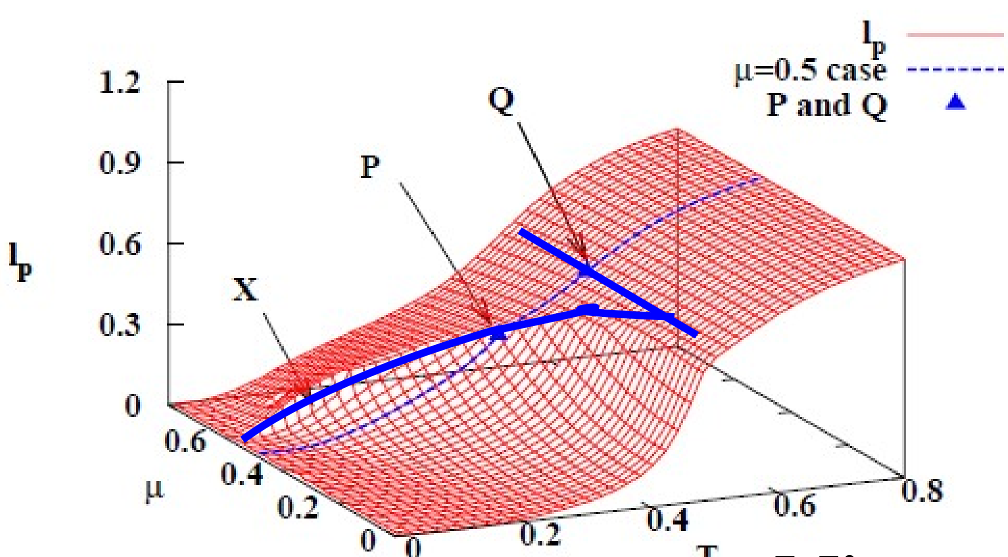
Miura



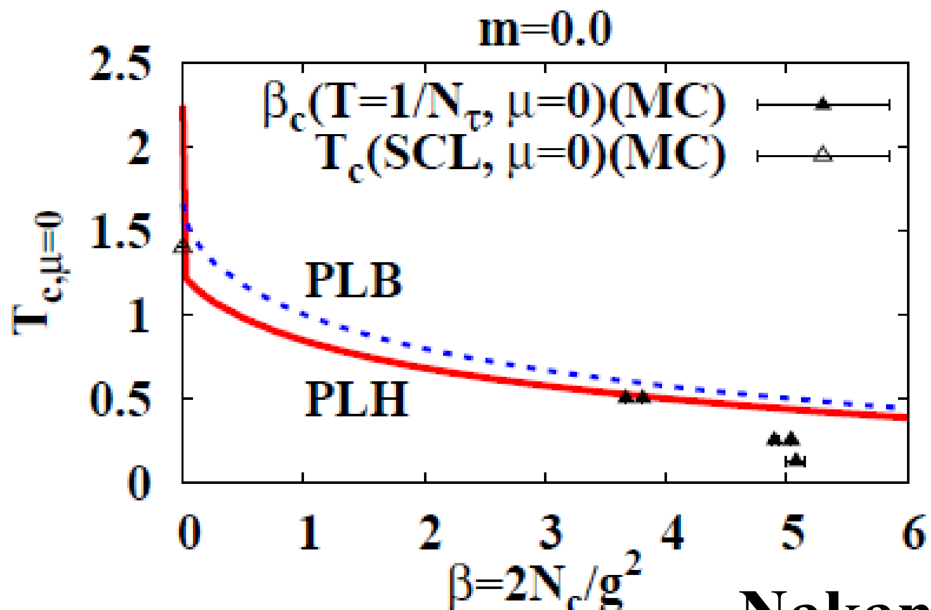
Nakano

SC-LQCD with Polyakov Loop Effects

- **Phase diagram with NLO ($1/g^2$) & Polyakov loop effects (PNLO)**
  - **Phase boundary of chiral & deconfinement** *Miura (Tue)*
- **Phase transition at μ=0 with NNLO ($1/g^4$) & Polyakov loop effects**
  - **Non-trivial Polyakov-chiral coupling via U_0 integral** *Nakano (Tue)*



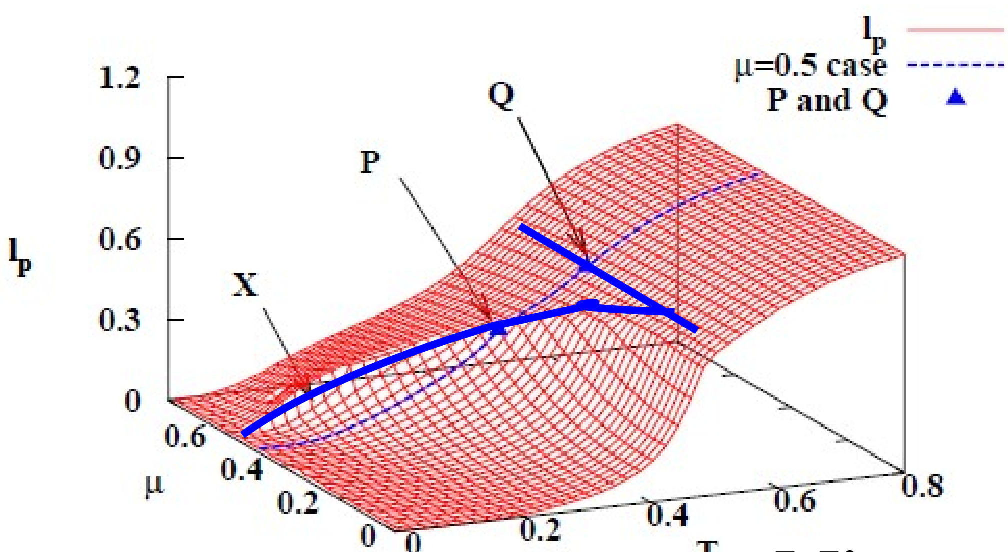
Miura



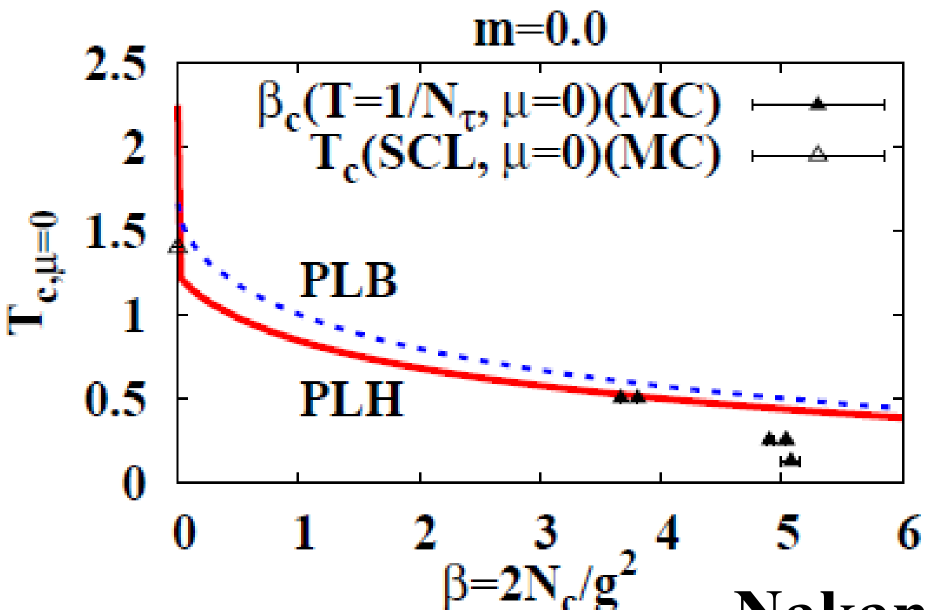
Nakano

SC-LQCD with Polyakov Loop Effects

- **Phase diagram with NLO ($1/g^2$) & Polyakov loop effects (PNLO)**
  - **Phase boundary of chiral & deconfinement** *Miura (Tue)*
- **Phase transition at μ=0 with NNLO ($1/g^4$) & Polyakov loop effects**
  - **Non-trivial Polyakov-chiral coupling via U_0 integral** *Nakano (Tue)*



Miura



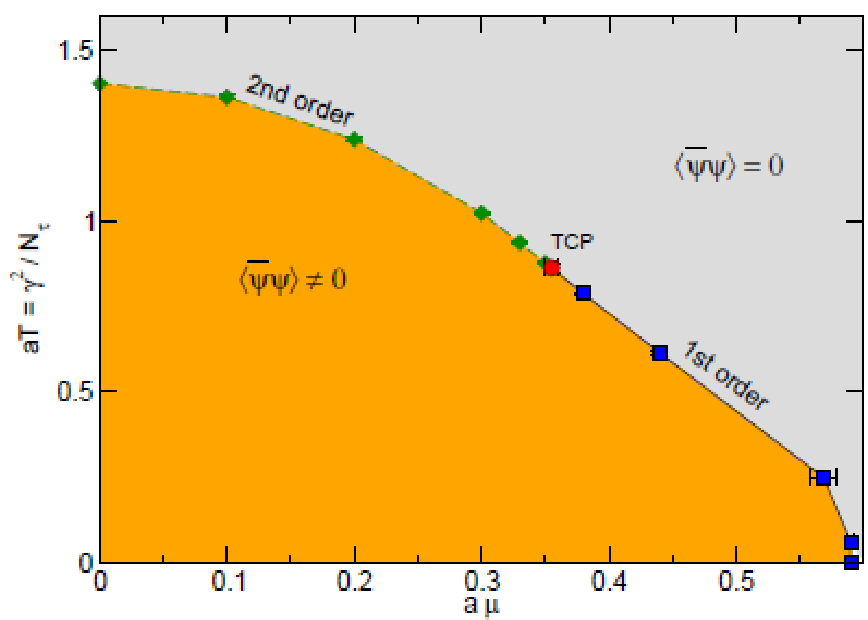
Nakano

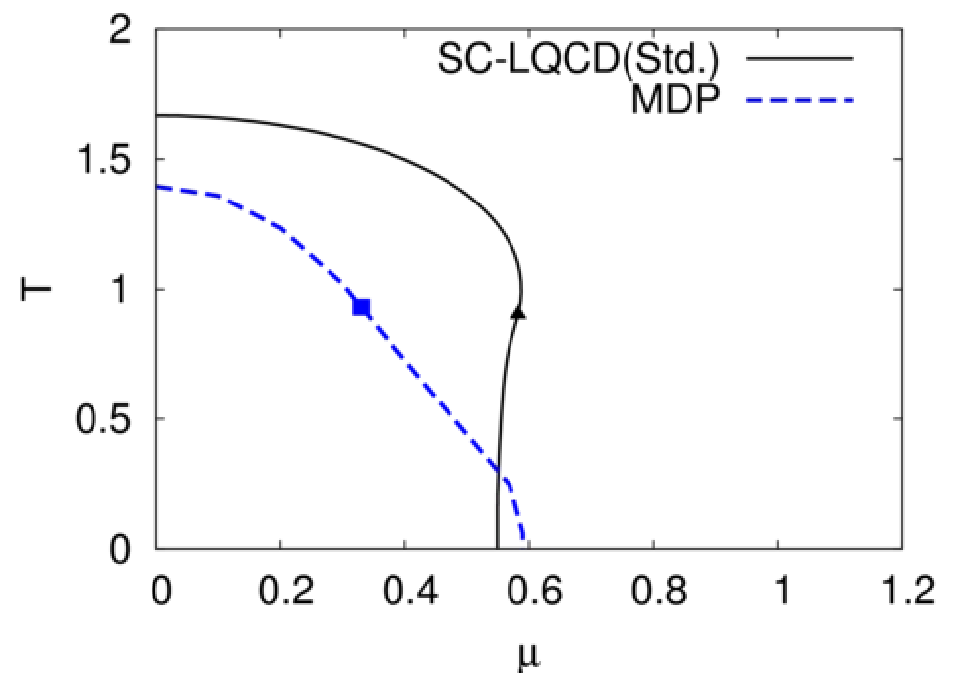
With improved PL effective action (*) and higher order terms, it may be possible to understand QCD phase diagram

**Langelage,Lottini, Philipsen(Tue)*

Homeworks in SC-LQCD

- **Higher orders in $1/g^2$, Roughening transition, Convergence, Fermi sphere of quarks, Fluctuations of aux. fields, ...**
- **Homeworks in the Strong Coupling Limit**
  - **Mean field results do not reproduce MC Results (MDP)** *de Forcrand, Fromm (PRL, '10)*
  - **Zero T (U_0 integral first) & Finite T (U_0 integral later) treatments give different results at low T.**





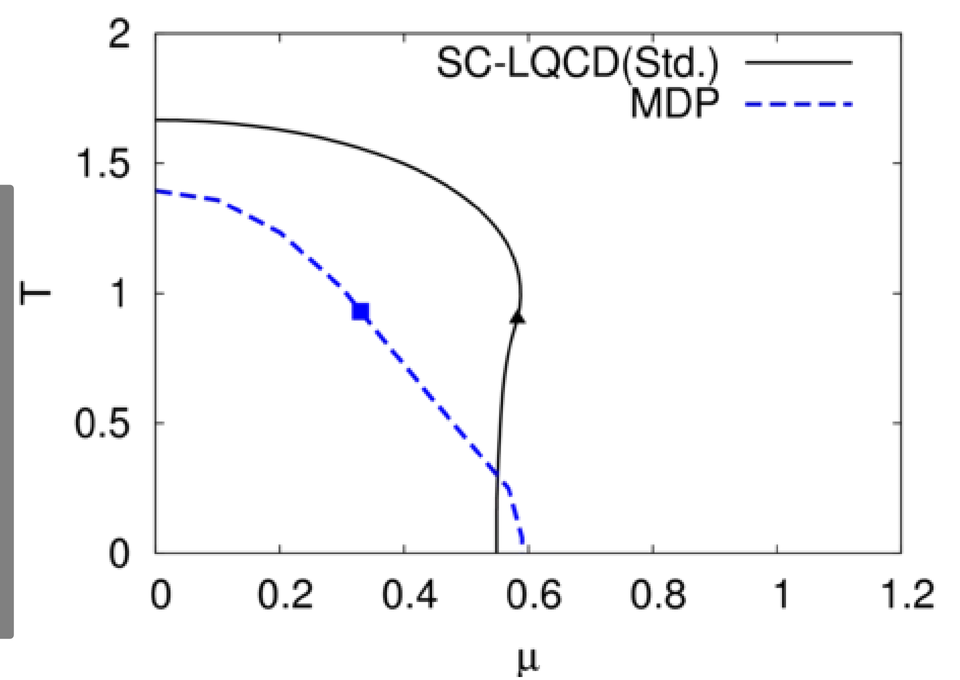
Homeworks in SC-LQCD

- **Higher orders in $1/g^2$, Roughening transition, Convergence, Fermi sphere of quarks, Fluctuations of aux. fields, ...**
- **Homeworks in the Strong Coupling Limit**
  - **Mean field results do not reproduce MC Results (MDP)** *de Forcrand, Fromm (PRL, '10)*
  - **Zero T (U_0 integral first) & Finite T (U_0 integral later) treatments give different results at low T.**

We examine these problems by including MF

$$V_{\pm\nu,x} = \eta_{\nu,x} \bar{\chi}_x \chi_{x+\hat{\nu}}$$

in Zero T treatment



Strong Coupling Limit of Lattice QCD

Damgaad-Kawamoto-Shigemoto ('84), Fukushima ('04)

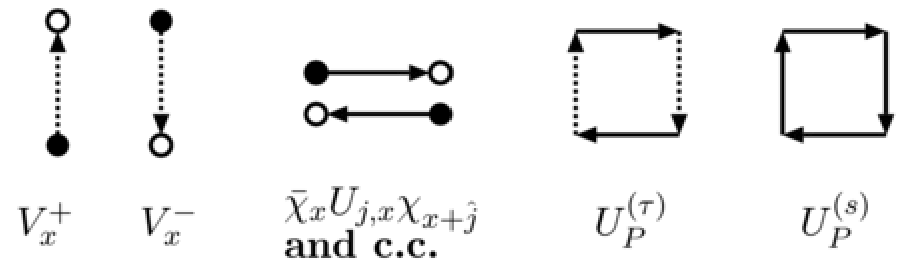
- **Lattice QCD action (unrooted staggered fermion)**

$$S_{LQCD}=\frac{1}{2}\sum_{x}\left[V_x^{+}-V_x^{-}\right]+m_0\sum_{x}M_x+\frac{1}{2}\sum_{x,j}\eta_{j,x}\left(\bar{\chi}_x U_{j,x}\chi_x-\bar{\chi}_{x+j}U_{j,x}^{+}\chi_x\right)$$

$$+\frac{1}{g^2}\sum_{P}\left(U_P+U_P^{+}\right)$$

$$M_x=\bar{\chi}_x\chi_x,$$

$$V_x^{+}=e^{\mu}\bar{\chi}_x U_{0,x}\chi_{x+\hat{0}},\quad V_x^{-}=e^{-\mu}\bar{\chi}_{x+\hat{0}}U_{0,x}^{+}\chi_x$$



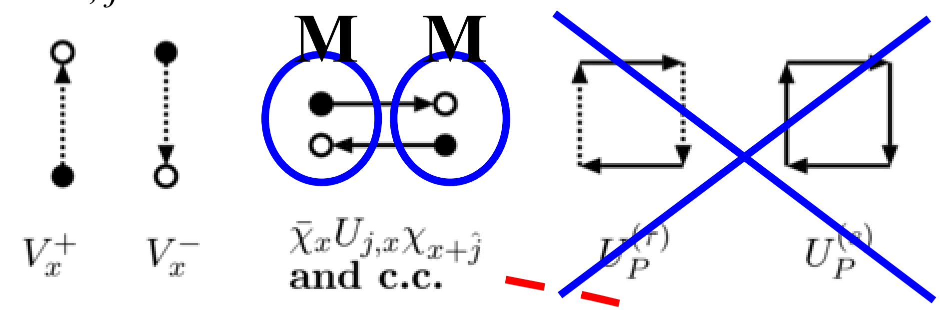
Strong Coupling Limit of Lattice QCD

Damgaad-Kawamoto-Shigemoto ('84), Fukushima ('04)

- **Lattice QCD action (unrooted staggered fermion)**

$$S_{LQCD}=\frac{1}{2}\sum_x\left[V_x^+-V_x^-\right]+m_0\sum_x M_x+\frac{1}{2}\sum_{x,j}\eta_{j,x}\left(\bar{\chi}_x U_{j,x}\chi_x-\bar{\chi}_{x+j}U_{j,x}^+\chi_x\right)$$

$$+\frac{1}{g^2}\sum_P\left(U_P+U_P^+\right)$$



$$M_x=\bar{\chi}_x\chi_x,$$

$$V_x^+=e^{\mu}\bar{\chi}_x U_{0,x}\chi_{x+\hat{0}},\quad V_x^-=e^{-\mu}\bar{\chi}_{x+\hat{0}}U_{0,x}^+\chi_x$$

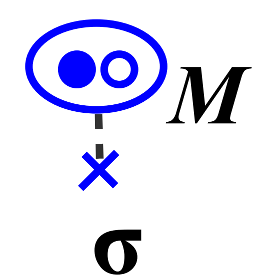
- **Strong Coupling Limit (U_j integral + 1/d expansion)**

$$S_{\rm eff}=\frac{1}{2}\sum_x\left[V_x^+-V_x^-\right]-\frac{1}{4N_c}\sum_{x,j}M_x M_{x+\hat{j}}+m_0\sum_x M_x+O(1/\sqrt{d})$$

- **Bosonization+ quark & U_0 integral → Effective Potential**

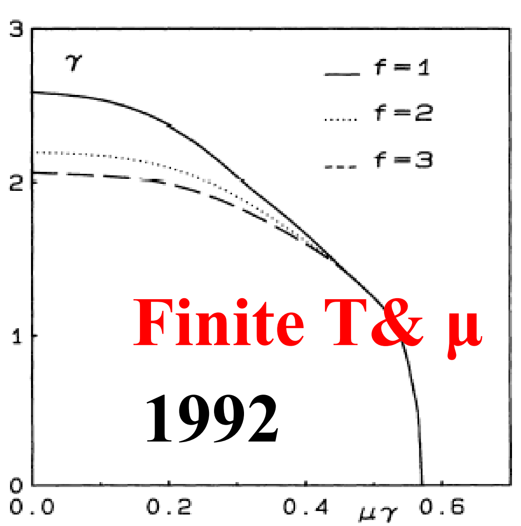
$$F_{\rm eff}=\frac{d}{4N_c}\sigma^2-T\log\left[\frac{\sinh\left((N_c+1)E_q/T\right)}{\sinh\left(E_q/T\right)}+2\cosh\left(N_c\mu/T\right)\right],$$

$$E_q=\mathrm{arcsinh}\left(d\,\sigma/2N_c+m_0\right)$$

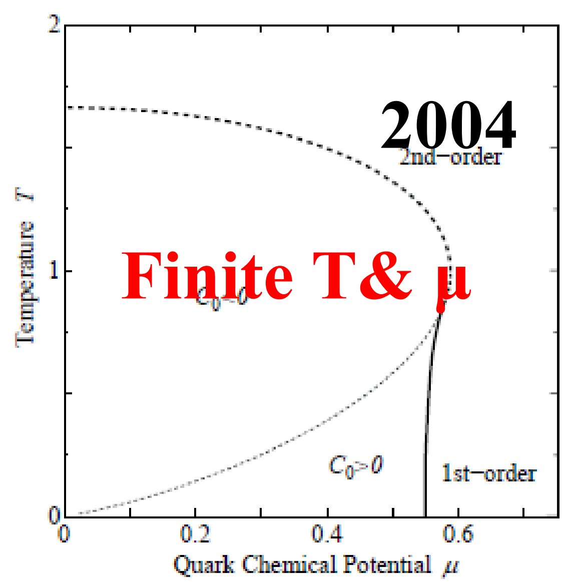


Phase diagram in SCL-LQCD

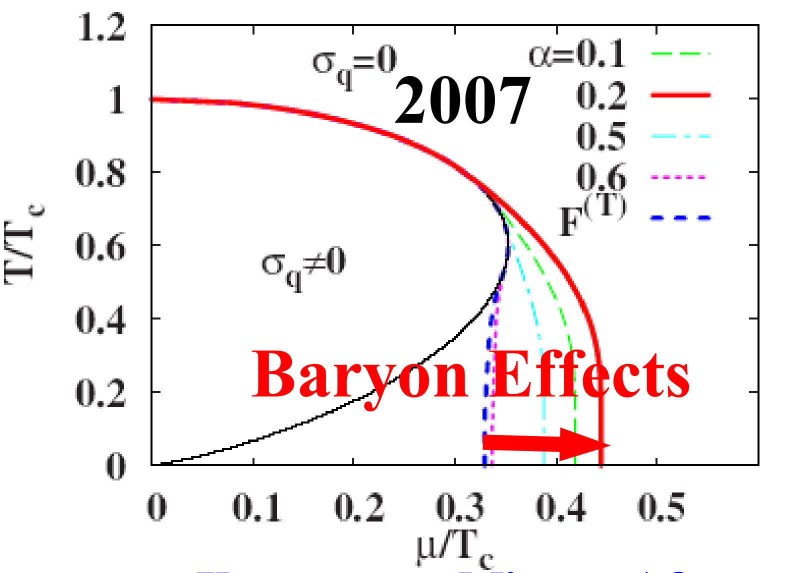
- **Bilic, Karsch, Redlich ('92): M → σ**
  - **Shape looks good. 2nd → 1st @ β ~ 1, but β ~ 4 in MC (de Forcrand)**
- **Fukushima ('04), Nishida ('04): Bosonization (Weiss MF app.)**
  - **T_c=5/3 > 1.4 (MDP), Region with $d\mu_c/dT>0$ (Lattice artefact)**
- **Kawamoto, Miura, AO, Ohnuma ('07): Bosonization+Baryon**
  - **Better shape but bosonization breaks chiral sym.**



Bilic, Karsch, Redlich, 1992



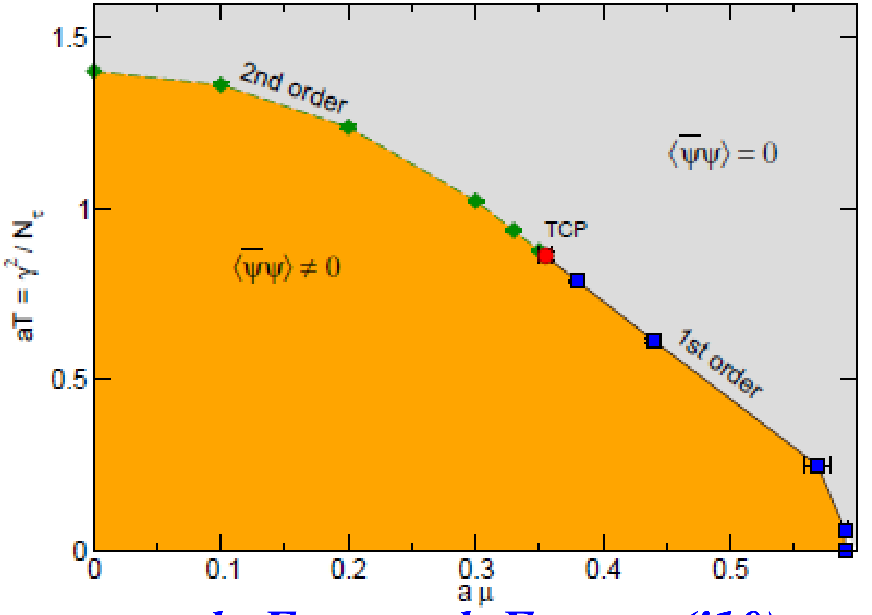
Fukushima, 2004



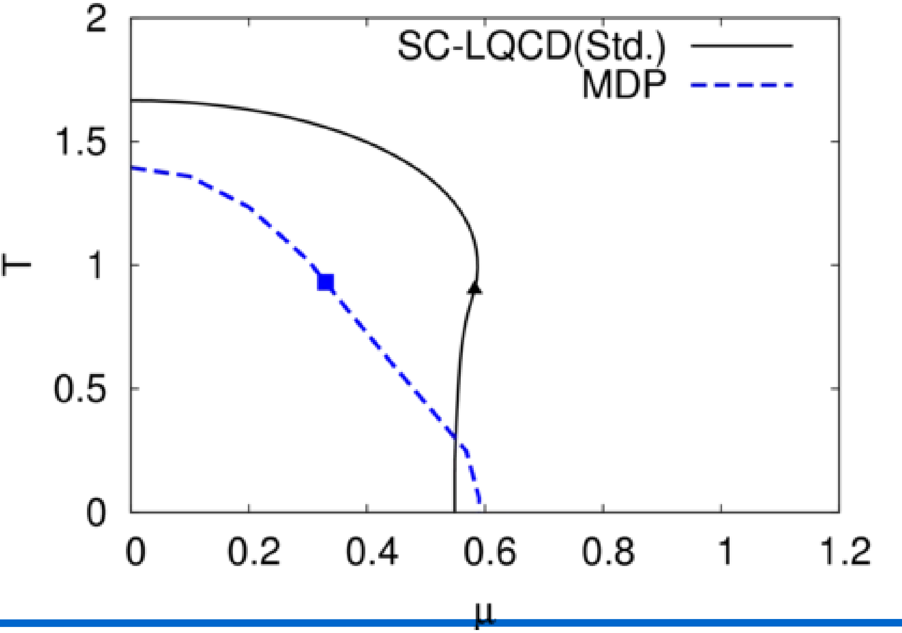
Kawamoto, Miura, AO, Ohnuma, 2007

Phase diagram in Monomer-Dimer-Polymer simulation

- **MDP simulation** *Karsch, Mutter ('89), Rossi, Wolff ('84, U(3))*
  - **Integrate out link variables first in the strong coupling limit (Zero T treatment)**
  - **Integral over quarks are replaced with loop config. sum → Sign problem is weakened**
  - **Phase diagram on anisotropic lattice** *de Forcrand, Fromm ('10)*
  - **Critical Point at a lower μ than SCL-LQCD**



de Forcrand, Fromm ('10)



What's wrong ? Polyakov loop effects in SCL ?

- **SC-LQCD with Polyakov loop effects** *Miura (Tue), Nakano(Tue)*

$$Z^{(F)} \simeq \prod_{\mathbf{x}} \int d\mathcal{U}(x) e^{N_c E_q/T + 2\beta_p \bar{\ell}\ell} \mathrm{det}_c \left[(1 + \mathcal{U} e^{-(E_q - \tilde{\mu})/T})(1 + \mathcal{U}^\dagger e^{-(E_q + \tilde{\mu})/T}) \right]$$

$$\left[d\mathcal{U} = d\ell d\bar{\ell} H(\ell, \bar{\ell}),\ H = 1 - 6\ell\bar{\ell} + 4(\ell^3 + \bar{\ell}^3) - 3(\ell\bar{\ell})^3,\ \mathrm{det}_c(...) = D(E_q, \tilde{\mu}, \ell, \bar{\ell}) \right]$$

$$= \prod_{\mathbf{x}} \int d\ell d\bar{\ell} \exp\left[-\frac{1}{T} \left(N_c E_q - 2T\beta_p \bar{\ell}\ell - T \log D - T \log H \right) \right]$$

What's wrong ? Polyakov loop effects in SCL ?

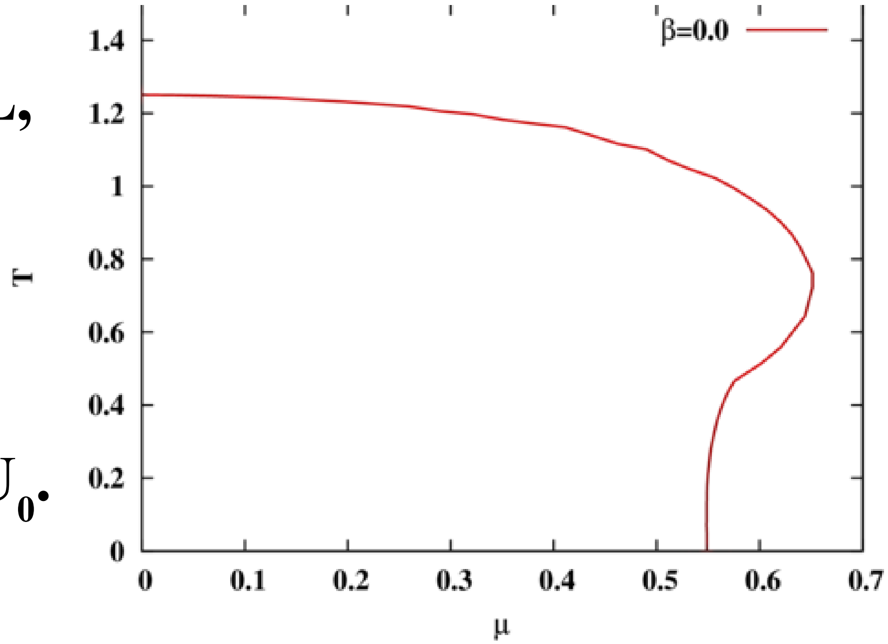
- **SC-LQCD with Polyakov loop effects** *Miura (Tue), Nakano(Tue)*

$$Z^{(F)} \simeq \prod_{\mathbf{x}} \int d\mathcal{U}(x) e^{N_c E_q/T + 2\beta_p \bar{\ell}\ell} \mathrm{det}_c \left[(1+\mathcal{U}e^{-(E_q-\tilde{\mu})/T})(1+\mathcal{U}^\dagger e^{-(E_q+\tilde{\mu})/T}) \right]$$

$$\left[d\mathcal{U} = d\ell d\bar{\ell} H(\ell,\bar{\ell}),\ H = 1 - 6\ell\bar{\ell} + 4(\ell^3+\bar{\ell}^3) - 3(\ell\bar{\ell})^3,\ \mathrm{det}_c(\ldots) = D(E_q,\tilde{\mu},\ell,\bar{\ell}) \right]$$

$$= \prod_{\mathbf{x}} \int d\ell d\bar{\ell} \exp\left[-\frac{1}{T} \boxed{\left(N_c E_q - 2T\beta_p \bar{\ell}\ell - T\log D - T\log H \right)} \right] \equiv F_{\mathrm{eff}}^{(q)}$$

- **In the Haar Measure method, Polyakov loop affect F_{eff} even in SCL, and reduces T_c.**
- **Phase diagram shape is not improved in SCL.**
- **No effects on T_c if we integrate out U_0. (Danzer(Thu), PL dist. is spread.)**



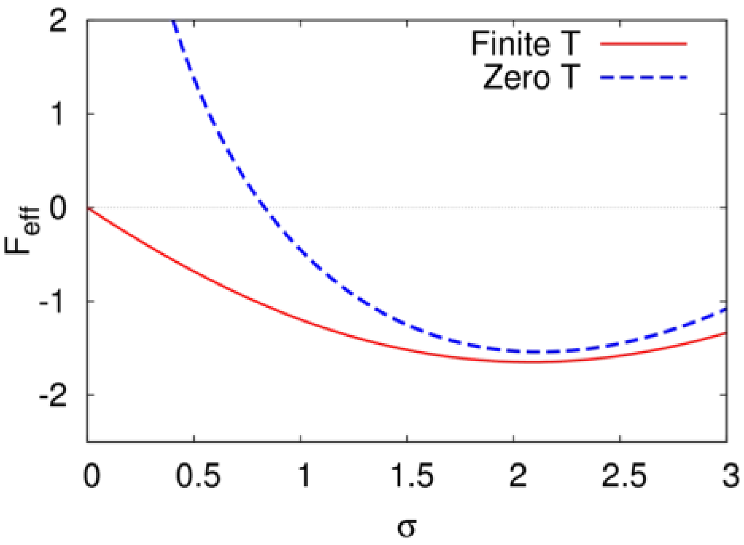
What's wrong ? Zero T treatment in SCL-LQCD ?

- **Diff. in Finite T (U_0 int. later) & Zero T (U_0 int. first) treatments**
  → No. Naïve Zero T treatment leads to divergent potential(-log σ)

$$S_{\rm eff} = -\frac{1}{4N_c}\sum_{\nu,x} M_x M_{x+\hat\nu} + m_0 \sum_x M_x + O(1/\sqrt{d+1}) \simeq \frac{b'_\sigma}{2}\sum_x \sigma^2 + \sum_x m_q M_x$$

$$\to F_{\rm eff} = \frac{b'_\sigma}{2}\sigma^2 - N_c \log(m_q) \qquad \left[b'_\sigma = (d+1)/2N_c, m_q = b'_\sigma \sigma + m_0\right]$$

- **Higher order in 1/d expansion helps ?**
  → No, in the previously works.
  NLO in 1/d (baryonic term) gives rise to σ^6 potential, and no effects on 2nd ord. p.t.
  Damgaard, Hochberg, Kawamoto ('85)



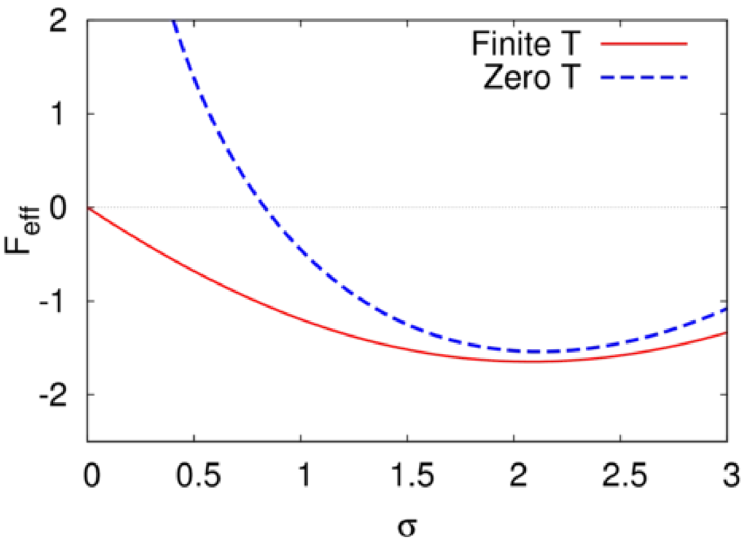
What's wrong ? Zero T treatment in SCL-LQCD ?

- **Diff. in Finite T (U_0 int. later) & Zero T (U_0 int. first) treatments**
  → No. Naïve Zero T treatment leads to divergent potential(-log σ)

$$S_{\rm eff}=-\frac{1}{4N_c}\sum_{\nu,x} M_x M_{x+\hat\nu}+m_0\sum_x M_x+O(1/\sqrt{d+1})\simeq\frac{b'_\sigma}{2}\sum_x\sigma^2+\sum_x m_q M_x$$

$$\to F_{\rm eff}=\frac{b'_\sigma}{2}\sigma^2-N_c\log(m_q)\qquad\left[b'_\sigma=(d+1)/2N_c, m_q=b'_\sigma\sigma+m_0\right]$$

- **Higher order in 1/d expansion helps ?**
  → No, in the previously works.
  NLO in 1/d (baryonic term) gives rise to σ^6 potential, and no effects on 2nd ord. p.t.
  Damgaard, Hochberg, Kawamoto ('85)



These problems come from "diagonal" mean field.
→ What happens if we introduce "temporally non-local" MF ?

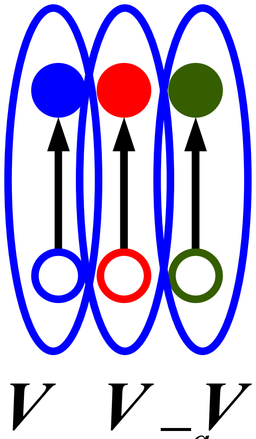
Another treatment of baryonic composites

- **bosonize V^3 into V and introduce mean field for V**

$$\exp(\mp\alpha V^3) \simeq \exp[-\alpha(\bar{\psi}^{(1)}\psi^{(1)} - V^2\psi^{(1)} \pm \bar{\psi}^{(1)} V)]$$

$$\exp(\alpha\psi^{(1)} V^2) \simeq \exp[-\alpha(\bar{\psi}^{(2)}\psi^{(2)} - V\psi^{(2)} - \bar{\psi}^{(2)} V\psi^{(1)})]$$

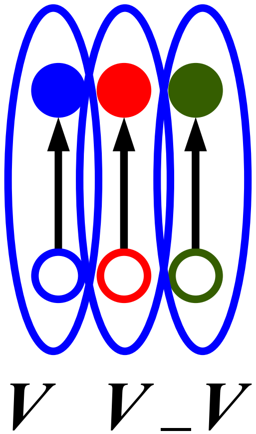
via Extended HS transf. *Miura, Nakano, AO('09)*



$$V_{+\mu,x} = \eta_{\nu,x}\bar{\chi}^a_x\chi^a_{x+\hat{\mu}}$$

Another treatment of baryonic composites

- **bosonize V^3 into V and introduce mean field for V**



$$\exp(\mp\alpha V^3) \simeq \exp[-\alpha(\bar{\psi}^{(1)}\psi^{(1)} - V^2\psi^{(1)} \pm \bar{\psi}^{(1)}V)]$$
$$\exp(\alpha\psi^{(1)}V^2) \simeq \exp[-\alpha(\bar{\psi}^{(2)}\psi^{(2)} - V\psi^{(2)} - \bar{\psi}^{(2)}V\psi^{(1)})]$$

via Extended HS transf. ***Miura, Nakano, AO('09)***

$$V_{+\mu,x} = \eta_{\nu,x}\bar{\chi}^a_x \chi^a_{x+\hat{\mu}}$$

- **Effective action for Quarks**
  → quarks on different sites are connected via V

$$S_{\text{eff}} = N_\tau L^d F^{(X)}_{\text{eff}}\left(\sigma, \psi^{(k)}_{\pm\nu}, \bar{\psi}^{(k)}_{\pm\nu}\right) + \frac{1}{2}\sum_{\nu,x}\left[Z_{+\nu}V_{+\nu,x} - Z_{-\nu}V_{-\nu,x}\right] + m_q \sum_x M_x$$

$$Z_{+\nu} = 2\alpha\left(\bar{\psi}^{(1)}_{+\nu} - \psi^{(2)}_{+\nu} - \bar{\psi}^{(2)}_{+\nu}\psi^{(1)}_{+\nu}\right), \quad Z_{-\nu} = 2\alpha\left(\bar{\psi}^{(1)}_{+\nu} + \psi^{(2)}_{+\nu} + \bar{\psi}^{(2)}_{+\nu}\psi^{(1)}_{+\nu}\right)$$

- **Assuming constant aux. fields (ψ and σ)**
  → Free quarks couples with aux. fields through
  Constituent quark mass (m_q)
  and W.F. Renormalization Factors ($Z_{\pm\nu}$)

Effective potential

- **Fourier transf.+Anti-periodic temporal B.C.+ Matsubara product → Effective potential**

$$S_{F,\mathrm{eff}}=\frac{1}{2}\sum_{\nu,x}\left[Z_{+\nu}V_{+\nu,x}-Z_{-\nu}V_{-\nu,x}\right]+m_q\sum_x M_x$$

$$F_{\mathrm{eff}}=\frac{b_\sigma}{2}\sigma^2+2\alpha(\varphi_+^3+\varphi_-^3)+4\alpha d\,\varphi_s^3+V_q$$

$$V_q=-N_c T\frac{1}{L^d}\sum_{\boldsymbol{k}}\left[\frac{E_k}{T}+\log\left(1+e^{-(E_k-\tilde{\mu})/T}\right)+\log\left(1+e^{-(E_k+\tilde{\mu})/T}\right)\right]-N_c\log Z_\chi$$

$$Z_\pm=6\alpha\varphi_\pm^2,\ \ Z_\chi=\sqrt{Z_+Z_-},\ \ \tilde{\mu}=\mu+\log(Z_+/Z_-),\ \ Z_s=6\alpha\varphi_s^2,\ \ E_k=\mathrm{arcsinh}(\varepsilon_k/Z_\chi),\ \ \varepsilon_k=\sqrt{m_q^2+Z_s^2\sin^2\boldsymbol{k}}$$

Effective potential

- **Fourier transf.+Anti-periodic temporal B.C.+ Matsubara product → Effective potential**

$$S_{F,\mathrm{eff}}=\frac{1}{2}\sum_{\nu,x}\left[Z_{+\nu}V_{+\nu,x}-Z_{-\nu}V_{-\nu,x}\right]+m_q\sum_x M_x$$

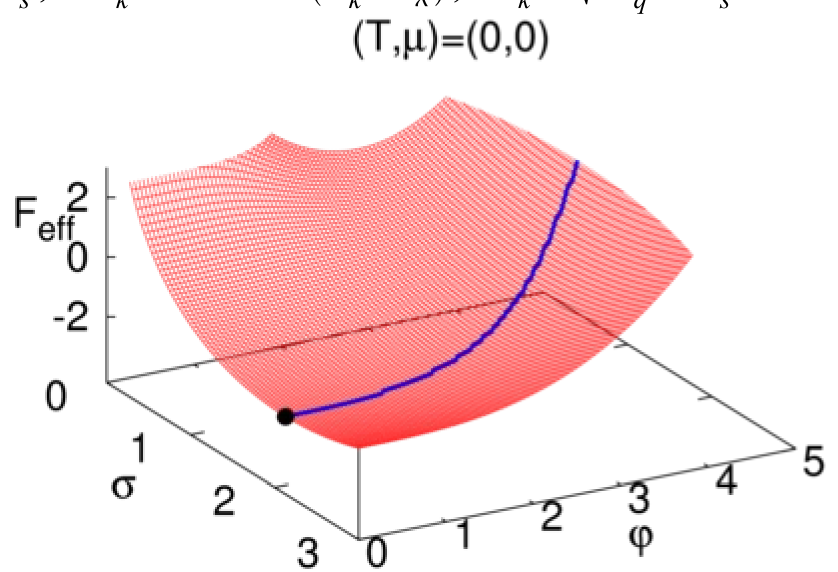
$$F_{\mathrm{eff}}=\frac{b_\sigma}{2}\sigma^2+2\alpha(\varphi_+^3+\varphi_-^3)+4\alpha d\,\varphi_s^3+V_q$$

Quarks & Anti-quarks

$$V_q=-N_c T\frac{1}{L^d}\sum_{\boldsymbol{k}}\left[\frac{E_k}{T}+\log\left(1+e^{-(E_k-\tilde{\mu})/T}\right)+\log\left(1+e^{-(E_k+\tilde{\mu})/T}\right)\right]-N_c\log Z_\chi$$

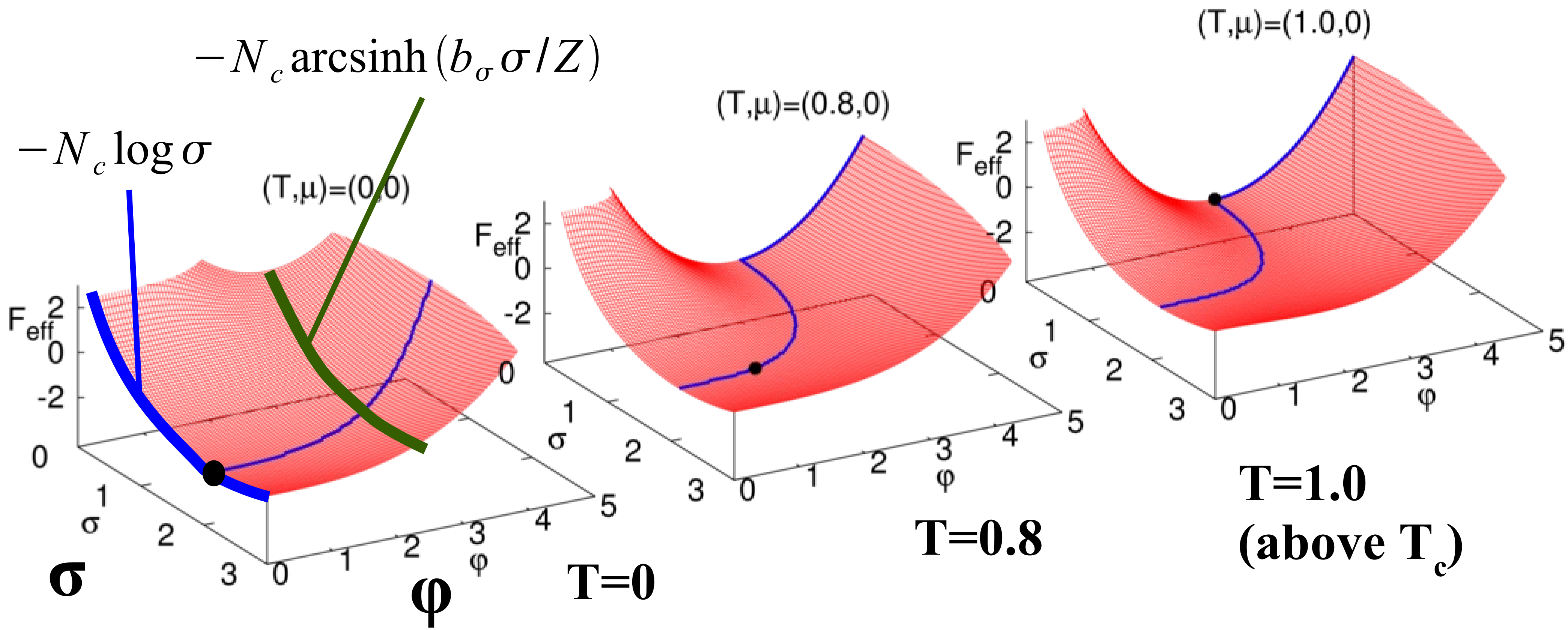
$$Z_\pm=6\alpha\varphi_\pm^2,\quad Z_\chi=\sqrt{Z_+Z_-},\quad \tilde{\mu}=\mu+\log(Z_+/Z_-),\quad Z_s=6\alpha\varphi_s^2,\quad E_k=\mathrm{arcsinh}(\varepsilon_k/Z_\chi),\quad \varepsilon_k=\sqrt{m_q^2+Z_s^2\sin^2\boldsymbol{k}}$$

- **This F_{eff} has interesting featues as,**
  - **NJL type eff. pot. with variable wave func. renormalization factor.**
  - **Momentum integral with *k* → sin *k* (k integral is omitted later.)**



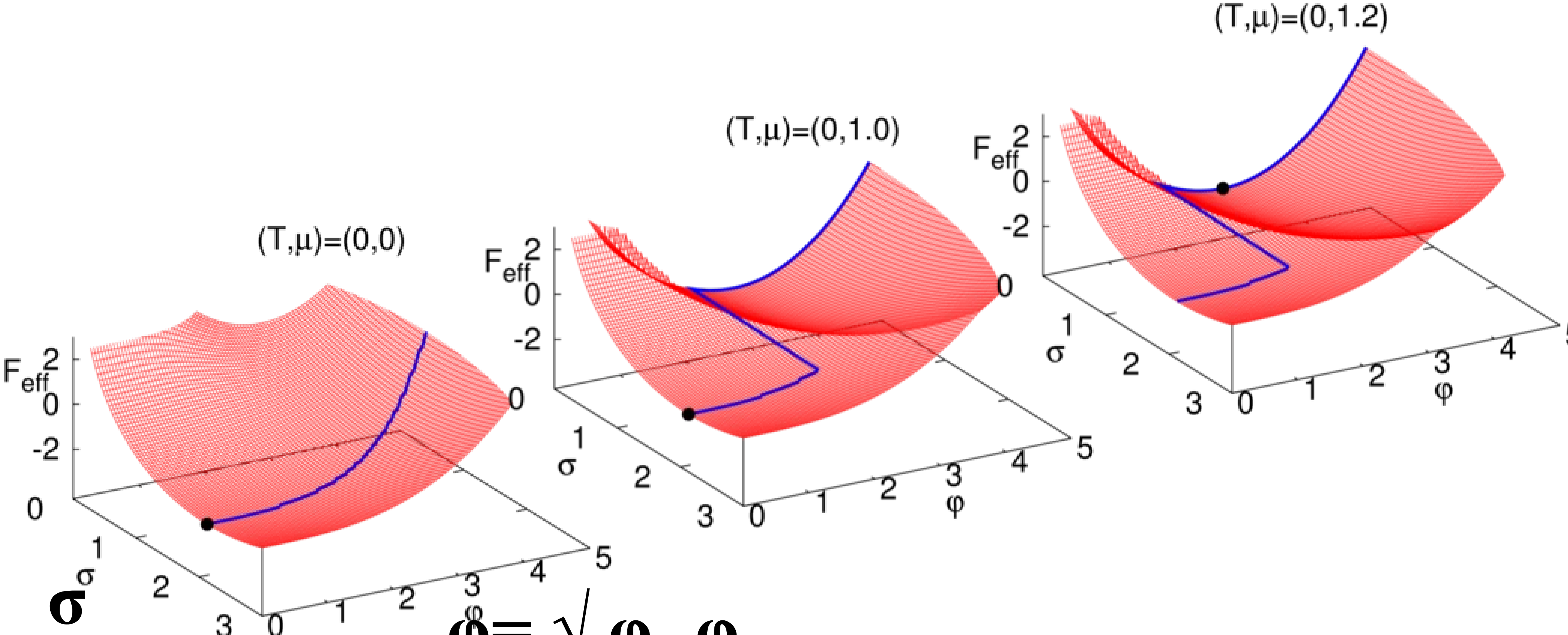
Potential Surface at μ=0

- **Another mean field φ connects two types of potential !**
  - **φ = 0 → Zero T treatment (log σ) type**
  - **φ ≠ 0 → Finite T treatment (arcsinh σ) type**
- **Smooth change from Low T to High T → 2nd order**



Potential Surface at finite μ (T=0)

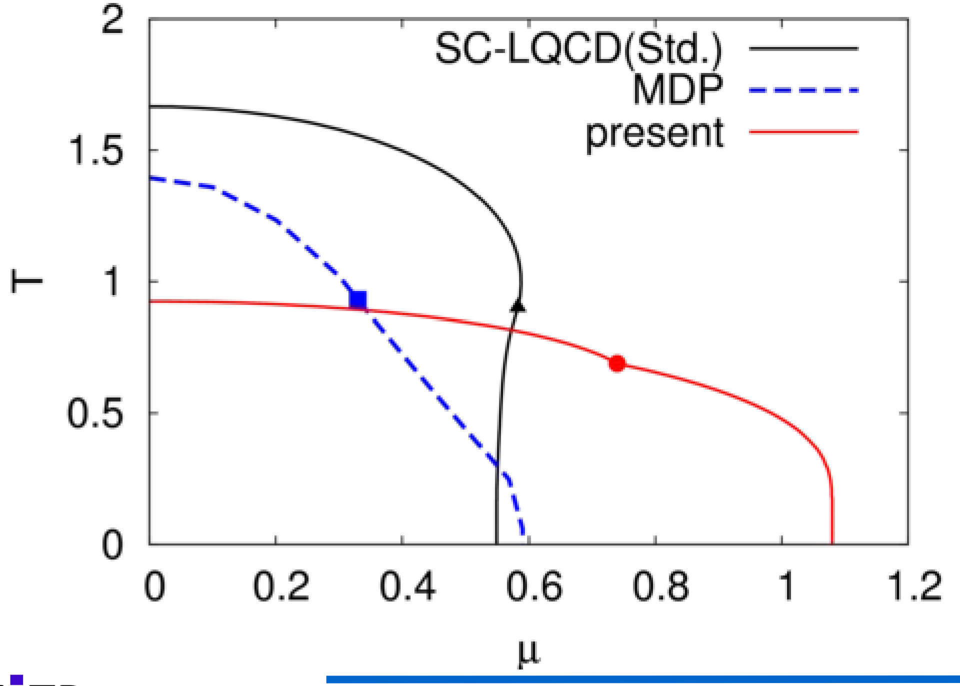
- **Two types of potentials are separated by a ridge at finite μ → first order transition**
- **High μ transition takes place as $(\varphi, \sigma)=(0, \text{finite}) \rightarrow (\text{finite}, 0)$**

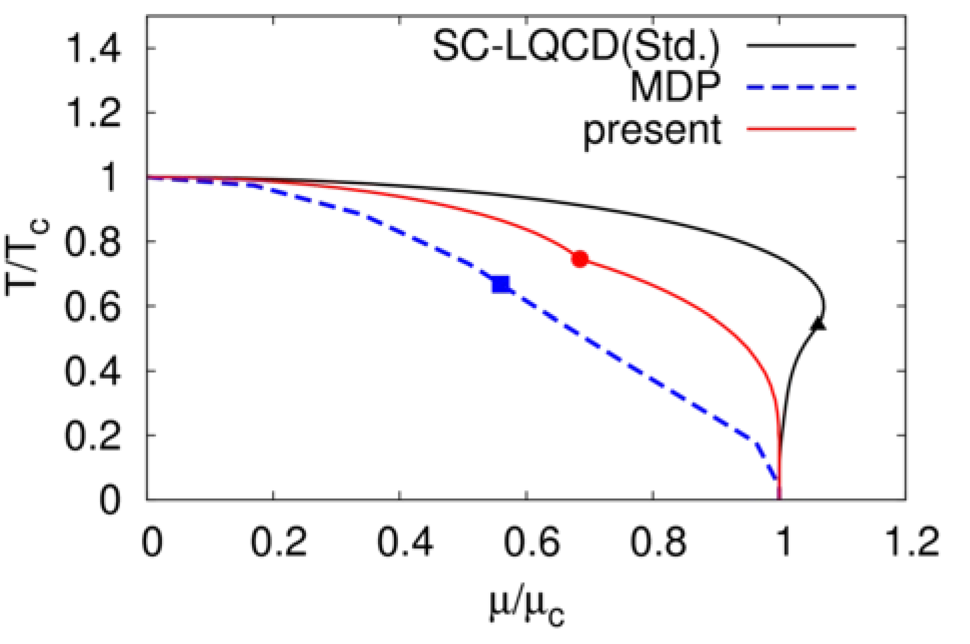


$$\phi = \sqrt{\varphi_+ \varphi_-}$$

Phase diagram

- **Comparison of the phase diagrams**
  MDP simulation *de Forcrand, Fromm ('10)*
  SCL-LQCD *Fukushima('04), Nishida ('04)*
  Present treatment
- **T_c: 1.4 (MDP), 1.67 (SCL-LQCD), 0.92 (=$(8/9)^{2/3}$)(Present)**
- **μ_c: 0.59 (MDP), 0.55 (SCL-LQCD), 1.08 (Present)**
  → Desired direction, but too much. Shape is improved.

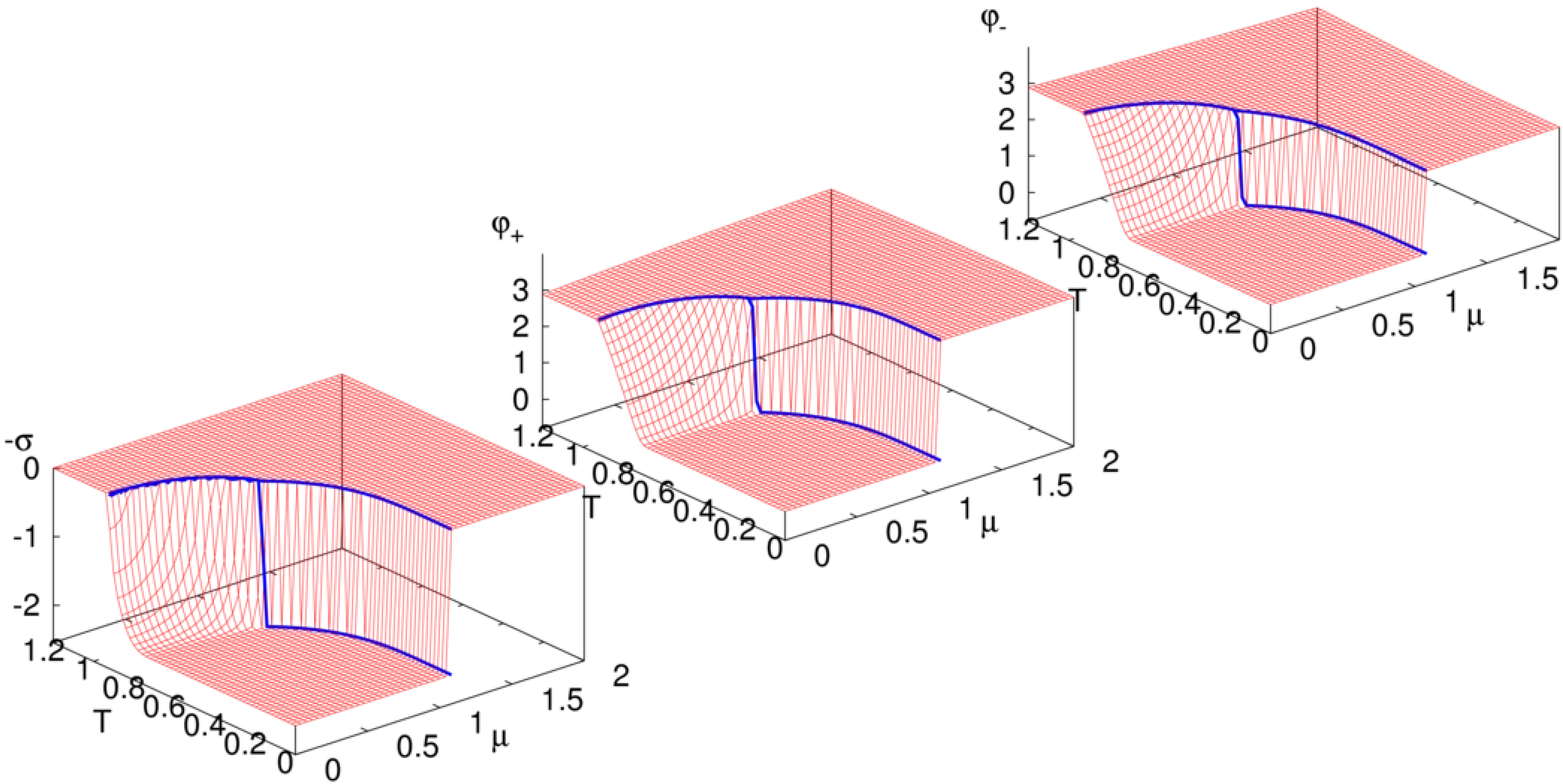




Summary

- **Strong coupling lattice QCD is a promising tool to understand QCD phase diagram, qualitatively and quantitatively.**
- **We have investigated the origin of the discrepancies between**
  Mean field treatments
  and Monomer-Dimer-Polymer (MDP) simulation
  in the strong coupling limit of lattice QCD.
  - **Within the Zero T & mean field treatment, mean field connecting different temporal sites would be necessary.**
  - **We have examined the consequences of a new type of mean field $\varphi_{\pm} \sim \langle \eta \, \bar{\chi}_x \, \chi_{x\pm 0} \rangle$ is introduced in the Zero T treatment. Phase diagram shape is improved, but the effects are too much.**
- **We should examine the effects of**
  Polyakov loop, quark momentum integral,
  and fluctuation of aux. Fields.
  → to be continued...

Backup

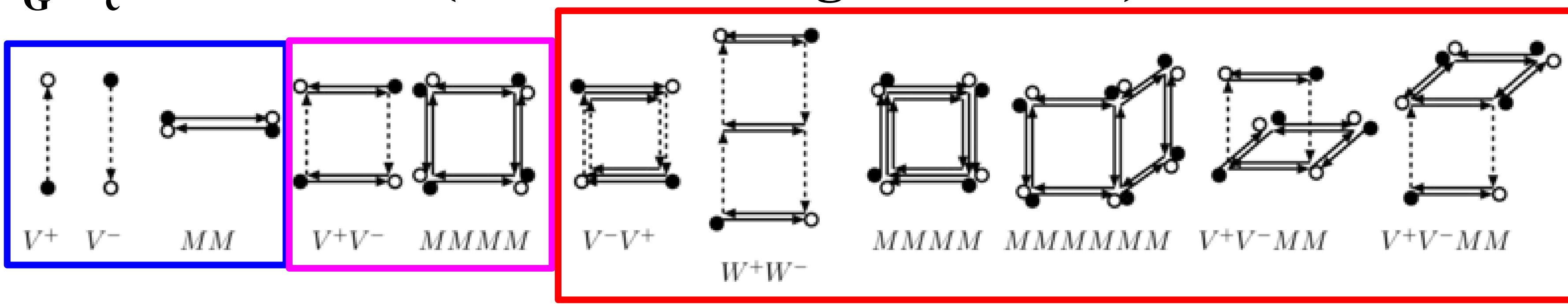

-σ
0
-1
-2
1.2
1
0.8
0.6
0.4
0.2
0
T
0
0.5
1
μ
1.5
2
φ+
3
2
1
0
1.2
1
0.8
0.6
0.4
0.2
0
T
0
0.5
1
μ
1.5
2
φ-
3
2
1
0
1.2
1
0.8
0.6
0.4
0.2
0
T
0
0.5
1
μ
1.5

SC-LQCD with $1/g^2$ corrections (1)

- **Effective Action with finite coupling corrections**
  Integral of $\exp(-S_G)$ over spatial links with $\exp(-S_F)$ weight → S_{eff}

$$S_{\rm eff}=S_{\rm SCL}-\log\langle\exp(-S_G)\rangle=S_{\rm SCL}-\sum_{n=1}\frac{(-1)^n}{n!}\langle S_G^n\rangle_c$$

$<S_G^n>_c$=Cumulant (connected diagram contr.) *c.f. R.Kubo('62)*



$$S_{\rm eff}=\frac{1}{2}\sum_x(V_x^+-V_x^-)-\frac{b_\sigma}{2d}\sum_{x,j>0}[MM]_{j,x}$$ *SCL (Kawamoto-Smit, '81)*

$$+\frac{1}{2}\frac{\beta_\tau}{2d}\sum_{x,j>0}[V^+V^-+V^-V^+]_{j,x}-\frac{1}{2}\frac{\beta_s}{d(d-1)}\sum_{x,j>0,k>0,k\neq j}[MMMM]_{jk,x}$$ *NLO (Faldt-Petersson, '86)*

$$-\frac{\beta_{\tau\tau}}{2d}\sum_{x,j>0}[W^+W^-+W^-W^+]_{j,x}-\frac{\beta_{ss}}{4d(d-1)(d-2)}\sum_{\substack{x,j>0,|k|>0,|l|>0\\|k|\neq j,|l|\neq j,|l|\neq|k|}}[MMMM]_{jk,x}[MM]_{j,x+\hat{l}}$$

$$+\frac{\beta_{\tau s}}{8d(d-1)}\sum_{x,j>0,|k|\neq j}[V^+V^-+V^-V^+]_{j,x}\left([MM]_{j,x+\hat{k}}+[MM]_{j,x+\hat{k}+\hat{0}}\right)$$ *NNLO (Nakano, Miura, AO, '09)*

SC-LQCD with $1/g^2$ corrections (2)

- **Extended Hubbard-Stratonovich transformation**

$$\exp(\alpha A B) = \int d\varphi\, d\phi \exp[-\alpha(\varphi^2 - (A+B)\varphi + \phi^2 - i(A-B)\phi)]$$

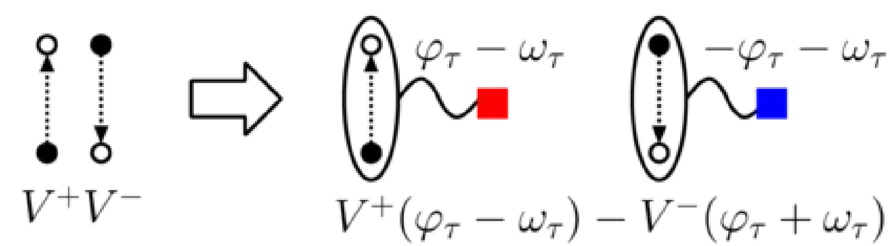
$$\approx \exp[-\alpha(\bar{\psi}\psi - A\psi - \bar{\psi} B)]_{\text{stationary}}$$

Miura, Nakano, AO (09), Miura, Nakano, AO, Kawamoto (09)

4, 8, 12 Fermion int. term → bi-linear form of quarks.

Ex.: $V^+ V^- \to \varphi_\tau^2 - \omega_\tau^2 + \varphi_\tau (V^+ - V^-) - \omega_\tau (V^+ + V^-)$

- **Effective Action after bosonization (and in gluonic dressed fermion)**



$$S_{eff} = S_{eff}^{(F)} + S_{eff}^{(X)}$$

$$S_{eff}^{(F)} = \frac{1}{2}\sum_x [Z_- V_x^+(\mu) - Z_+ V_x^-(\mu)] + m_q \sum_x M_x$$

$$= Z_\chi \left\{ \frac{1}{2}\sum_x [e^{-\delta\mu} V_x^+(\mu) - e^{+\delta\mu} V_x^-(\mu)] + \tilde{m}_q \sum_x M_x \right\} = Z_\chi \sum \bar{\chi} G^{-1} \chi$$

→ **w.f. renorm. factor (Z_χ)**, **quark mass (m_q)**, **chem. pot. shift ($\delta\mu$)**

Phase Diagram Evolution

- **Shape of the phase diagram is compressed in T direction with β**

  → ***Improvements in $R=\mu_c/T_c$!***

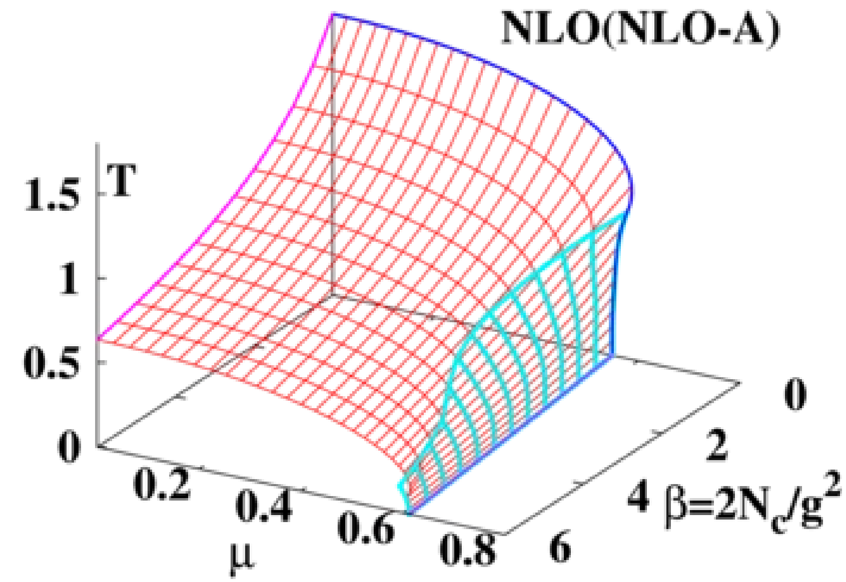
  - **MC (R > 1) → SCL (R = (0.3-0.45))**
    → NLO/NNLO (R ~ 1)
    → Real World (R~(2-4))

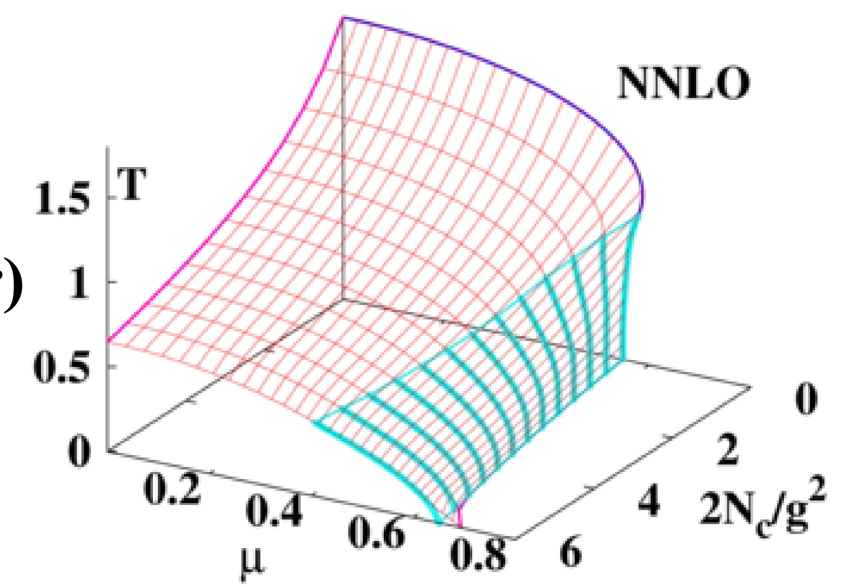
- **Critical Point**

  - **NLO: μ(CP) ~ Const.**
  - **NNLO: μ(CP) decreases with β**
    → ***Improvements !*** **(N_f=4 → 1st order)**

    Kronfeld ('07), Pisarski, Wilczek ('84)

  - **μ(CP)/T(CP) ~ 1 ↔ MC (μ/T > 1)**
    Ejiri, ('08), Aoki et al.(WHOT,'08), Allton et al., ('03,'05)





Miura, Nakano, AO, Kawamoto ('09)
Nakano, Miura, AO ('09)

Zero T treatment in SCL-LQCD

- **Link integral in the Strong Coupling Limit (no plaquette)**
  → Effective action of quarks (exact)

$$S_{\text{eff}} = \sum_{x,\nu} S_{\nu,x} + m_0 \sum_x M_x$$

$$S_{\nu,x} = -\frac{1}{4N_c}[MM]_{+\nu,x} + \frac{1}{2^{N_c}}\left(\eta_{\nu,x}[\bar{B}B]_{+\nu,x} - \eta_{\nu,x}^{-1}[\bar{B}B]_{-\nu,x}\right)$$
$$-\frac{1}{576}[MM]^2_{+\nu,x} - \frac{5}{576}[\bar{B}B]_{+\nu,x}[\bar{B}B]_{-\nu,x}$$

- **Approximations in SCL-LQCD**
  - **LO in 1/d expansion = min. quark number config.**
    → Baryonic action (6q), M^4 (8q) term, M^6 term (12q) are ignored. (d=3=spatial dim.)
  - **Mean field approximation**
    → fluctuations in aux. fields are ignored.

Monomer-Dimer-Polymer simulation

- Monomer-Dimer-Polymer simulation (MDP)

  Karsch, Mutter ('89), Rossi, Wolff ('84, U(3))

  → Integrate out link variables first in the strong coupling limit

  - Sign problem is weakened → Phase diagram in SCL

    de Forcrand, Fromm ('10)

  - $T_c(\mu=0)$ and $\mu_c(T=0)$ qualitatively agree with SCL-LQCD (MF) results.

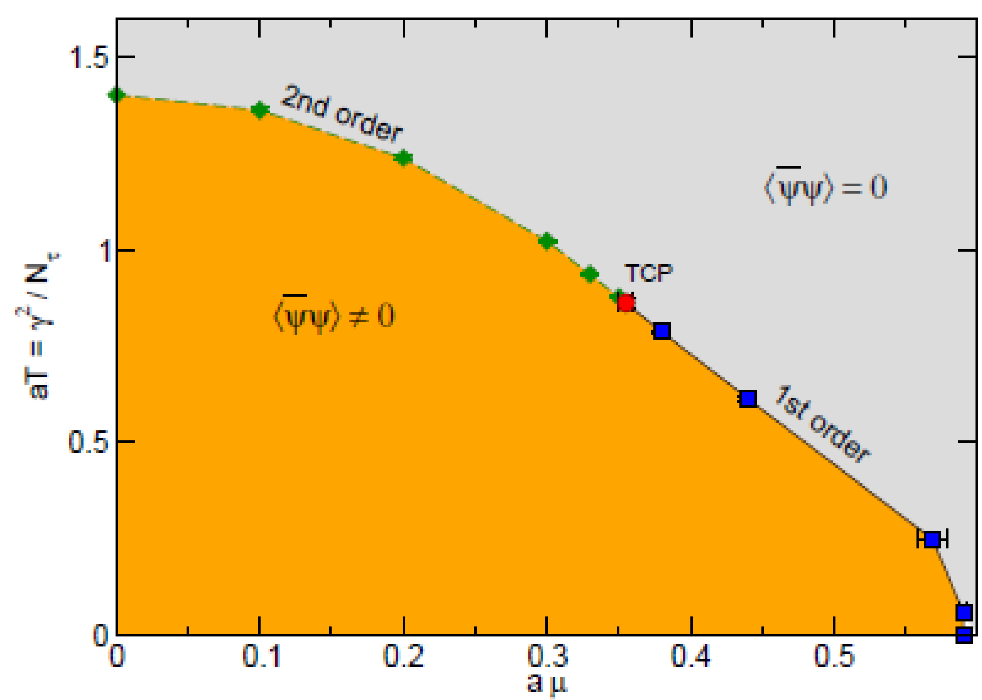
    aT_c=5/3 (MF), 1.41(3) (MDP)

    $a\mu_c$=0.549(MF), 0.593(MDP)

    $(aT_{TCP}, a\mu_{TCP})$

    =(0.867, 0.578)(MF),

    (0.86(2), 0.355(5))(MDP)



Strong Coupling Limit of Lattice QCD: Zero T (Std.)

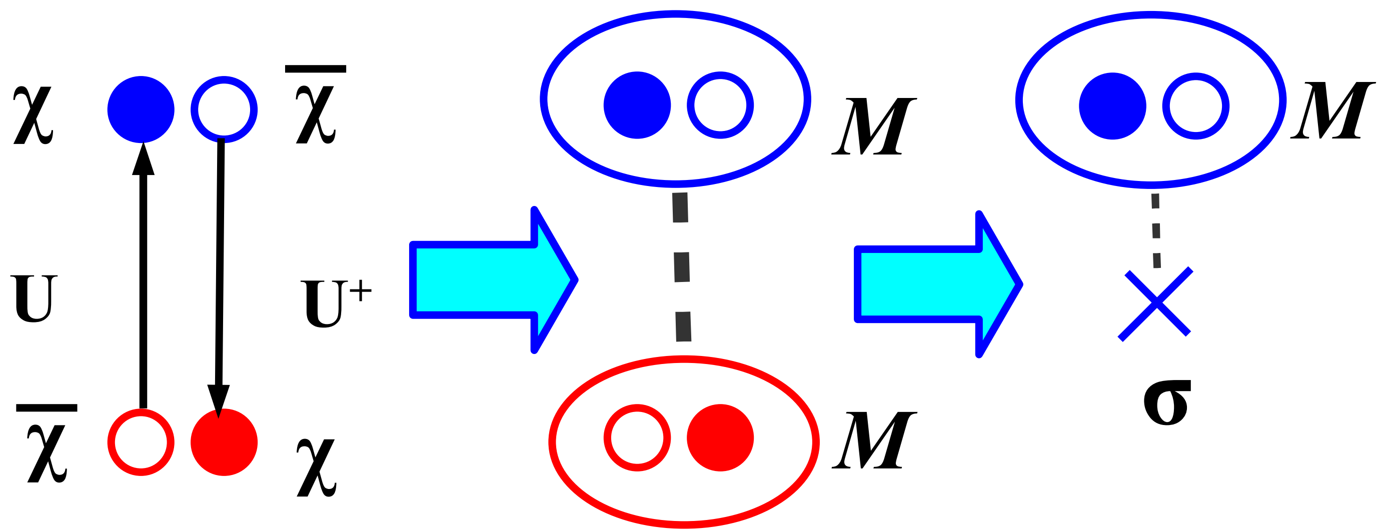
Kawamoto-Smit('81), Damgaad-Kawamoto-Shigemoto ('84)

- **Zero T treatment (U_μ integral + 1/d expansion)**

$$S_{\mathrm{eff}}=\frac{1}{4N_c}\sum_x\sum_{j=0}^{d}M_x M_{x+\hat{j}}+m_0\sum_x M_x+O(1/\sqrt{d})$$

$$\simeq N_\tau L^3\times\ \frac{1}{2}b_\sigma\sigma^2+\sum_x(b_\sigma\sigma+m_0)M_x$$

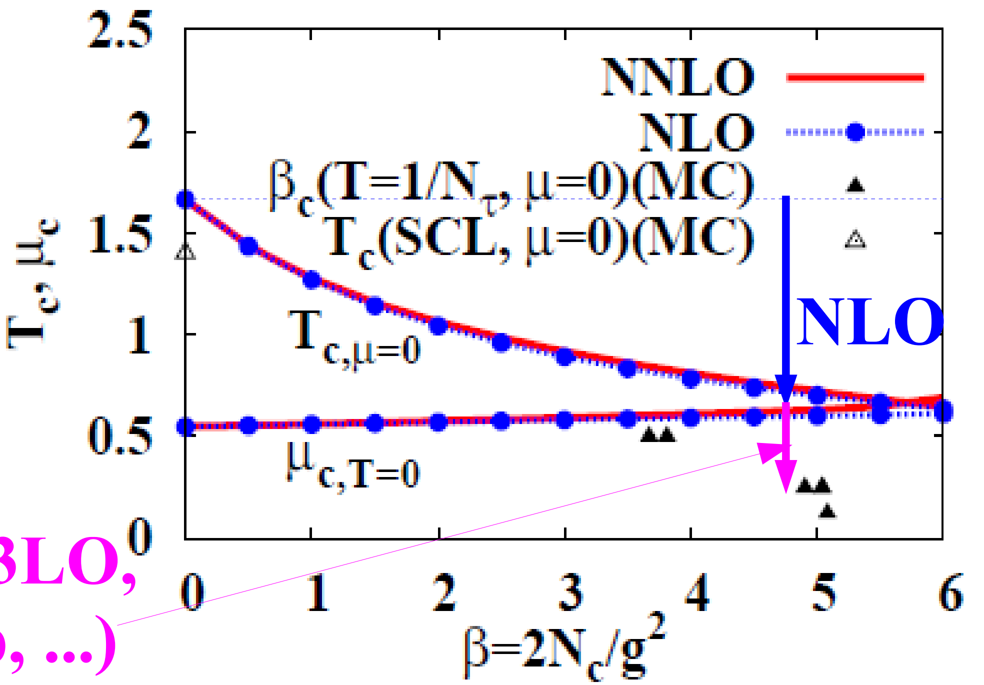
$$F_{\mathrm{eff}}=\frac{1}{2}b_\sigma\sigma^2-N_c\log(b_\sigma\sigma+m_0)$$



Quark-Gluon Dynamics → Hadronic Composites(+ U_0)

Critical Temperature and Chemical Potential

- **Critical Temperature (μ =0) → rapid decrease with $\beta=2N_c/g^2$**
  - **W.F. Renom. factor Z_χ → suppression of mass**
  - **T_c is still larger than MC results**
    de Forcrand ('06), Gottlieb et al. ('87), Gavai et al. ('90), de Forcrand, Fromm ('09)
- **Critical Chem. Pot. (T=0) → weak deps. on β**
  - **Suppression of mass ~ Suppression of $\tilde{\mu}$**
  - **Consistent with previous results**
    Bilic-Demeterfi-Petersson, '92
- **NNLO effects are small on $T_c(\mu=0)$ and $\mu_c(T=0)$.**



?(1/d, N3LO, Pol. loop, ...)

Nakano, Miura, AO ('09)

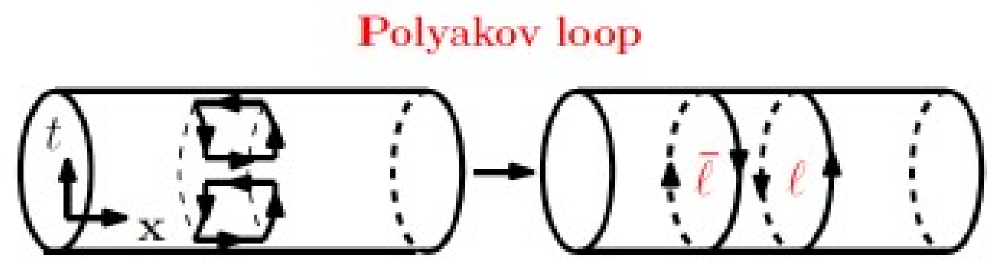
Polyakov Loop Effects

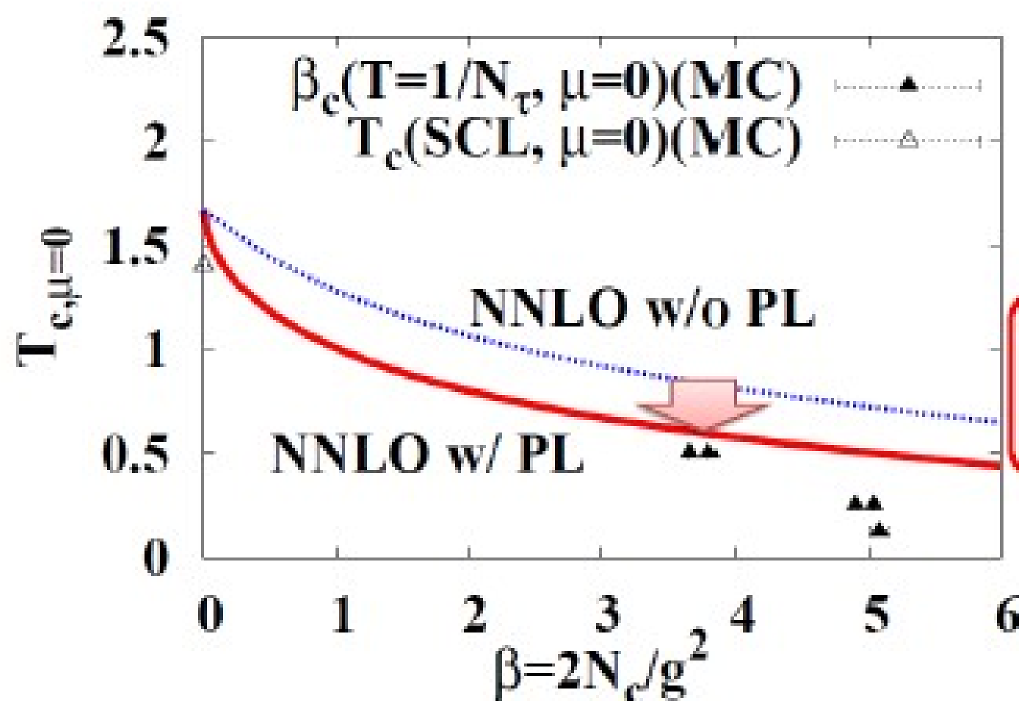
- T_c(NLO) ~ T_c(NNLO) > T_c(MC)
  → Slow convergence ?
  Deconfinement ?
  → Resummation is necessary !

- **NNLO SC-LQCD with Polyakov loop effects**
  Nakano, Miura, AO, in prep.
  c.f. PNJL (Fukushima / Ratti-Weise et al. / Kyushu group)

  - **Pros**
    Chiral & Deconf. transition
    Large effects on T_c

  - **Cons**
    Expansion is not systematic in $1/g^2$
    Does not improve at SCL
    Boyd, Karsch et al. / de Forcrand & Fromm ('09)

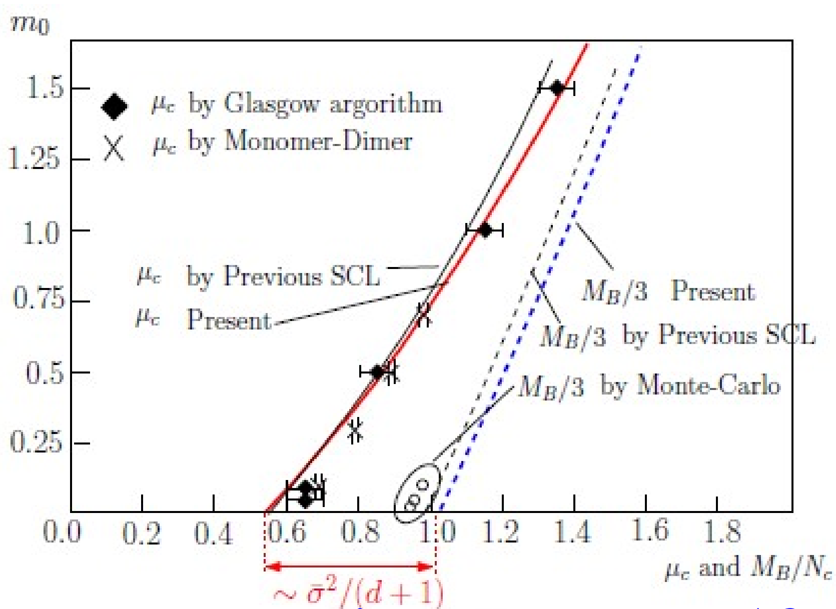




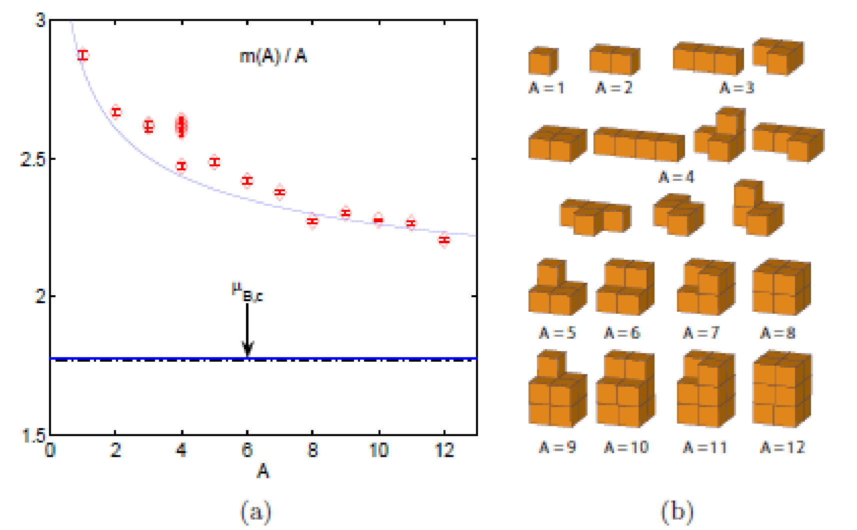
Cold Nuclear Matter in Lattice QCD

- **Baryon mass puzzle in SCL-LQCD: $N_c \mu_c < M_B$**
  → QCD phase transition takes place before baryons appear.
  Kluberg-Stern, Morel, Petersson ('83), Damgaard, Hochberg,Kawamoto ('85), Karsch, Mutter ('89), Barbour et al.('97), Bringoltz('07), Miura, Kawamoto, AO ('07)

- **Possible Solutions**
  - **Regard the matter at $\mu > \mu_c$ as nuclear matter** *de Forcrand, Fromm ('09)*
  - **Finite coupling effects: Decrease of quark mass**



Miura, Kawamoto, AO ('07)



de Forcrand, Fromm ('09)

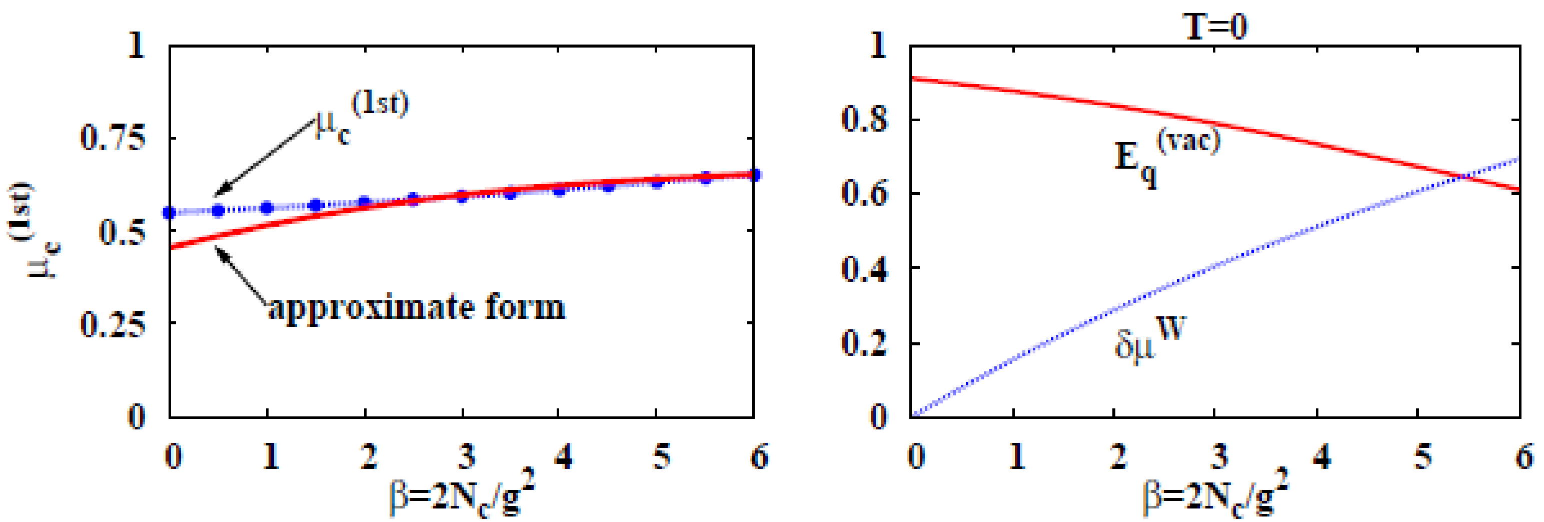
Constituent Quark Mass in NNLO SC-LQCD

- **Mechanism of "stable" $\mu_c(T=0)$ in NLO/NNLO SC-LQCD**
  = Effects of quark mass reduction & repulsive vector pot. cancel

Transition Condition at $T=0: E_q(\tilde{m}_q)=\tilde{\mu}\simeq\mu-\beta'_\tau\omega_\tau$

$\to\mu\simeq E_q(\tilde{m}_q)+\beta'_\tau\omega_\tau$

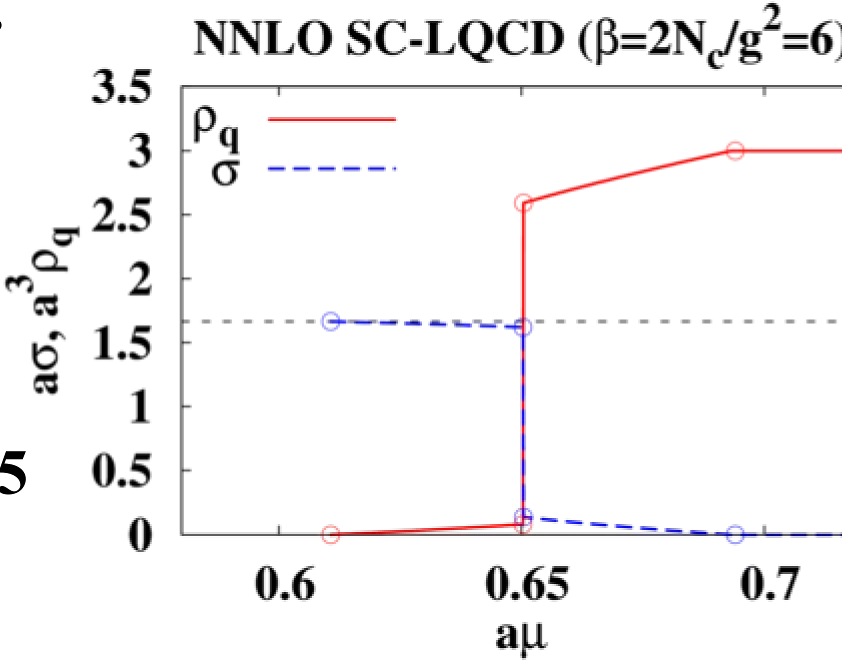
Pocket formula $\mu_{c,T=0}\simeq\frac{1}{2}\left[E_q(\sigma=\sigma_{\text{vac}},\omega_\tau=0)+\delta\mu(\sigma=0,\omega=N_c)\right]$

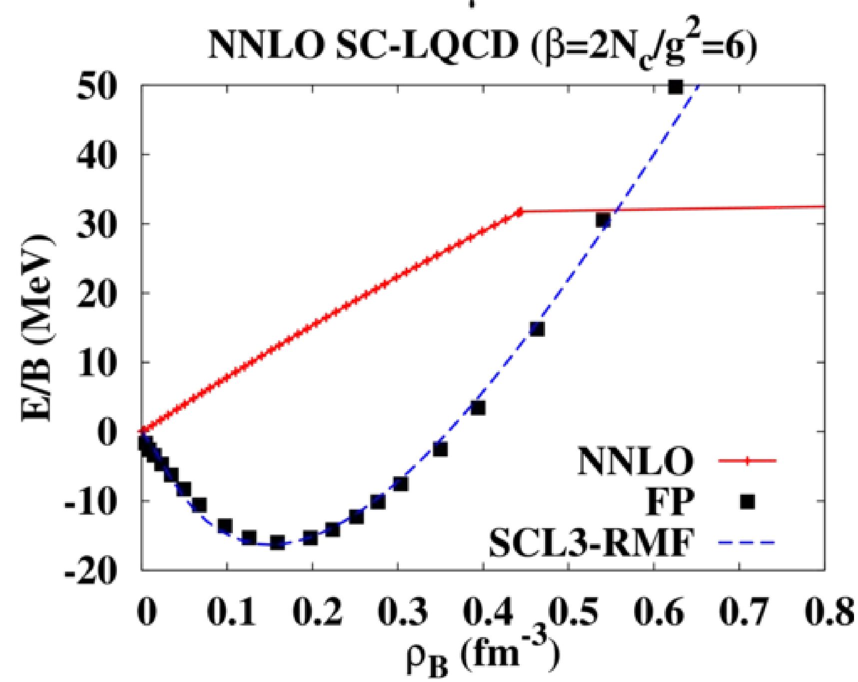


Quark mass ($\approx E_q$) is smaller than μ_c for $\beta > 5.5$.
→ "Baryon mass puzzle" may be solved !

Nuclear Matter on the Lattice at Strong Coupling

- **Do we observe finite density matter before 1st order phase transition ?**
  → Yes !
  - **$E_q(\mu=0, T=0, \beta=6)=0.61$**
    $\mu_c^{(1st)}(T=0, \beta=6)=0.65$
    → "Nuclear matter" in $0.61<\mu<0.65$
- **EOS of "Nuclear matter"**
  - **a^{-1} = 500 MeV**
    Bilic, Demeterfi, Petersson ('92)
    → Density in the order of ρ_0
  - **No saturation**
  - **1st order transition at ρ_B=0.4 fm^{-1}.**





Nuclear matter on the lattice.
Can we attack it soon ?

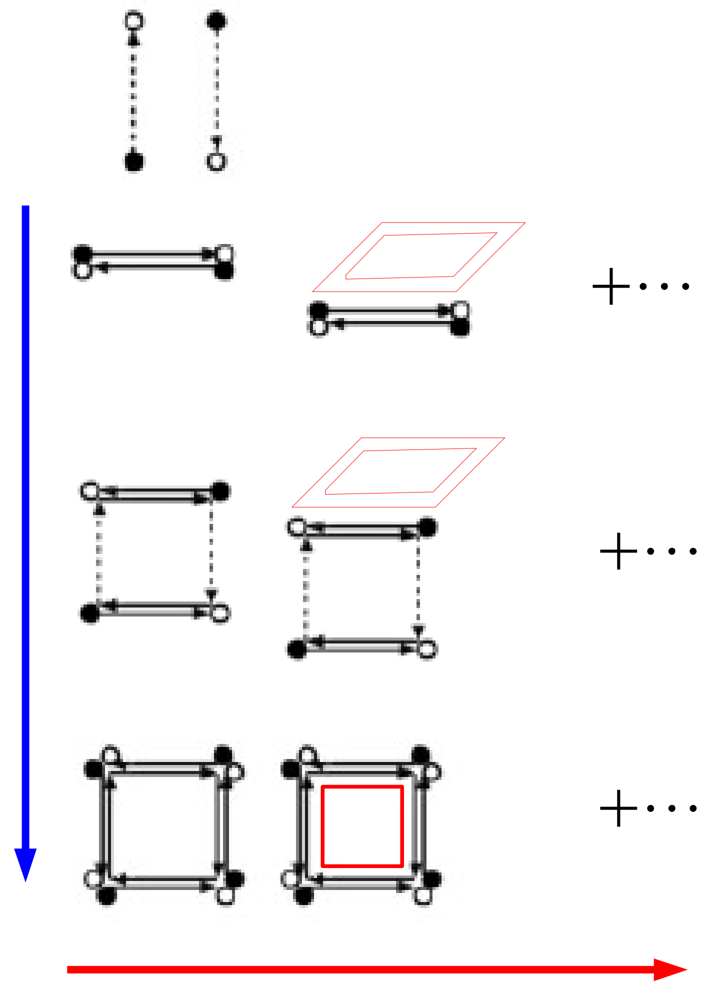
Possibilities ?

- **SC-LQCD action can be improved by the plaquette contribution**
  → Effective action of fermions and Polyakov loop
  with coef. evaluated in MC

$$
\begin{aligned}
S_{\rm eff} &= S_{\rm SCL} + \Delta S_{\rm NLO} + \Delta S_{\rm NNLO} + O(1/g^6) \\
&= S_{\rm YM} + \frac{1}{2}\sum_x (V^+ - V^-) \\
&\quad + a_{0s}(N_\tau, \beta) \sum M_x M_{x+\hat{j}} \\
&\quad + a_{1t}(N_\tau, \beta) \sum V^+_x V^-_{x+\hat{j}} \\
&\quad + a_{1s}(N_\tau, \beta) \sum MMMM \\
&\quad + \cdots
\end{aligned}
$$

E.g. $a_{1s} = \langle \mathrm{Plaq.} \rangle$

Fermionic Strong Coupling Expansion



Gluonic SCE

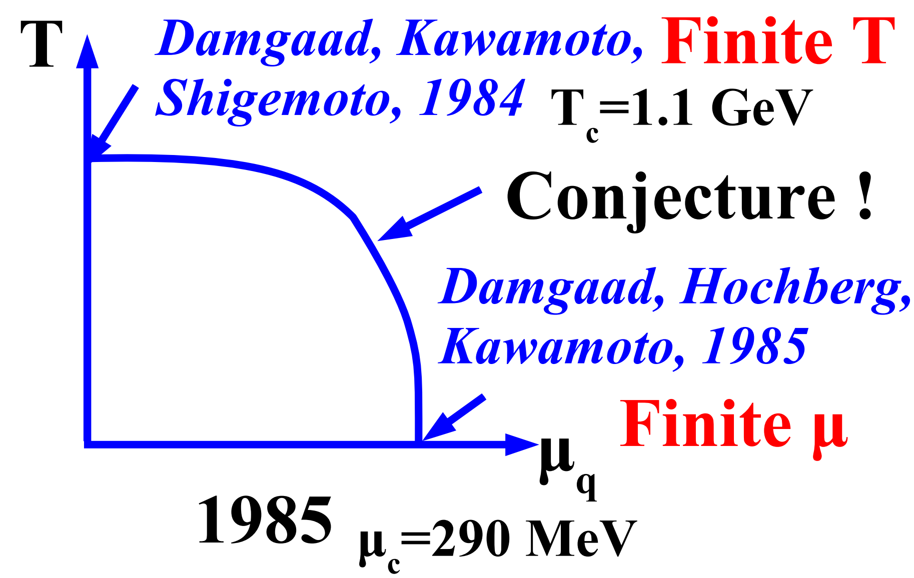
→ I asked Nakamura-san to join our grant-in-aid project !

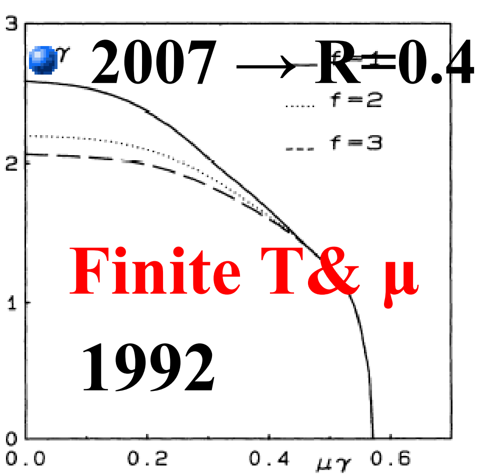
Evolution of Phase Diagram

- **Phase Diagram "Shape" becomes closer to that of Real World,**

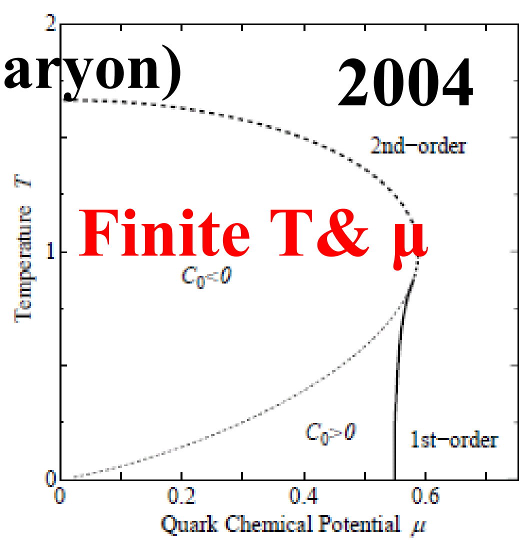
  $R=\mu_c/T_c \sim (2\text{-}4)$

  - **1985 → R=0.26 (Zero T / Finite T)**
  - **1992 → R=0.28 (Finite T & μ)**
  - **2004 → R= 0.33 (Finite T& μ)**
  - **2007 → R=0.45 (Baryon)**

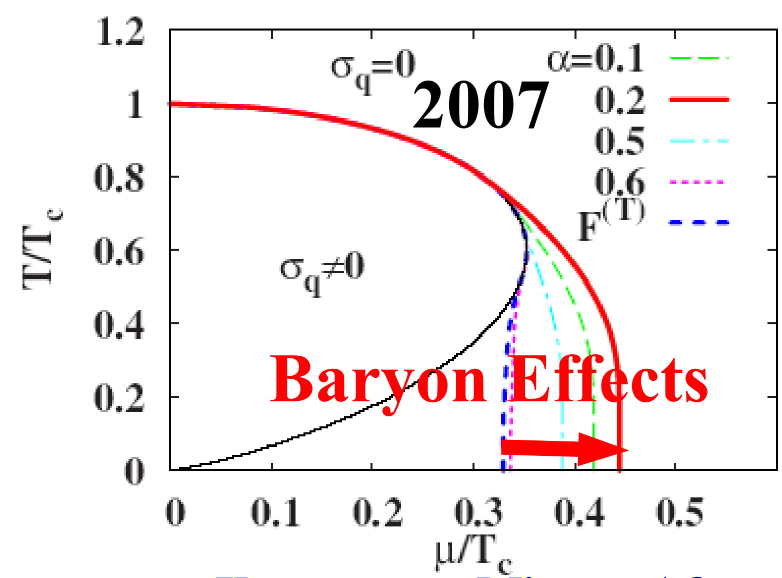




Bilic, Karsch, Redlich, 1992



Fukushima, 2004

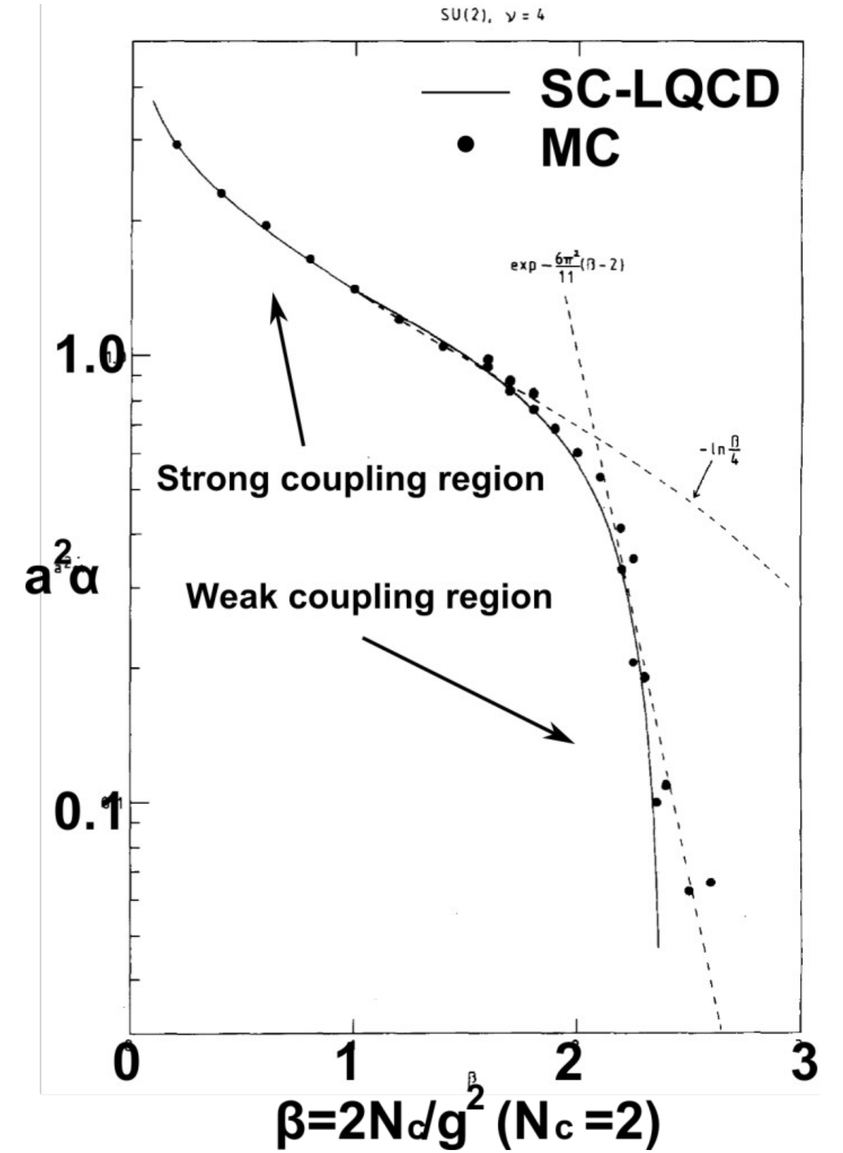


Kawamoto, Miura, AO, Ohnuma, 2007

Strong Coupling Lattice QCD

- Large bare coupling → $1/g^2$ expansion
- Success in pure YM
  → Lattice MC & $1/g^2$ Expansion
  Wilson, '74; Creutz '80; Munster '81

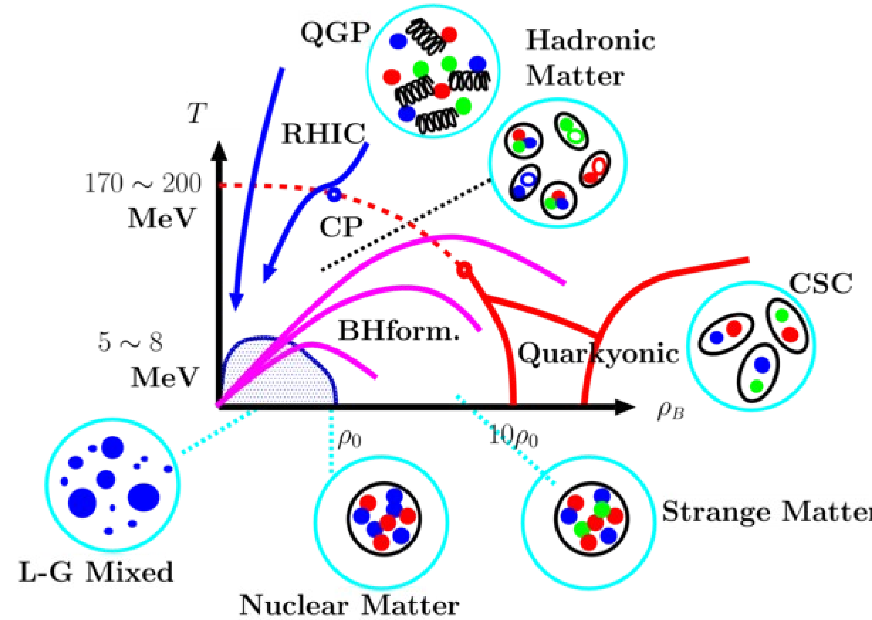
  → ***Scaling region would be accessible in SC-LQCD !***
- Chiral transition at finite T and μ in Strong Coupl. Limit (SCL) & Next-to-leading order (NLO)
  Kawamoto-Smit '81, Damgaard-Kawamoto-Shigemoto, '84(U(3)), Faldt-Petersson'86 (SU(3)), Fukushima'04(SU(3)), Nishida '04 (SU(3)), Bilic-Karsch-Petersson, '92(NLO), Kawamoto-Miura-AO-Ohnuma '07 (Baryons)

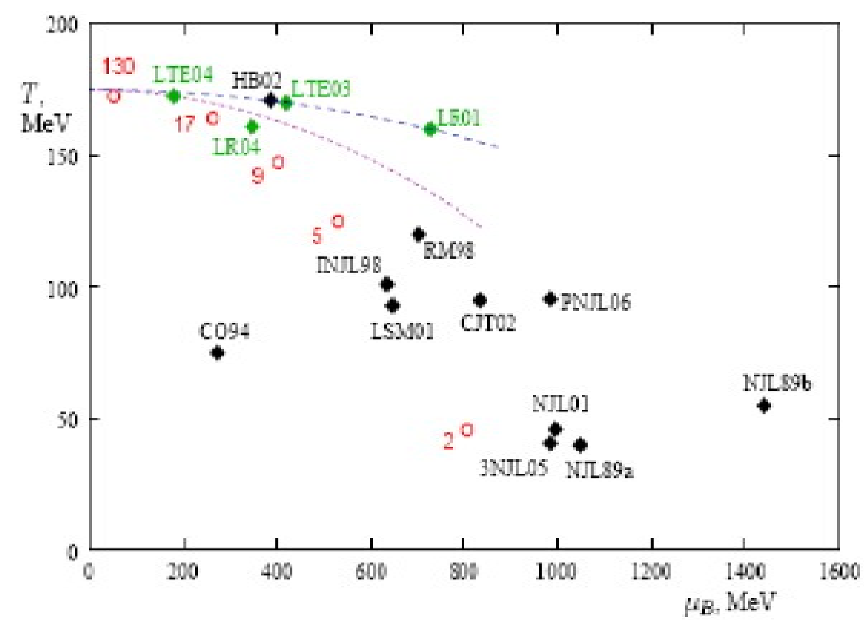


Munster, '81

QCD Phase diagram

- Phase transition at high T
  - Lattice MC & RHIC
- High μ transition has rich physics
  - Various phases, CEP, Astrophysical applications, ...
  - Models & Approximations are necessary !
    - Lattice MC works only for small μ (Tayler, AC, DOS, Canonical, …) or in the Strong Coupling Limit(SCL) (MDP) *Karsch, Mutter ('89), de Forcrand, Fromm ('09)*
    - Eff. Models: NJL, PNJL, PLSM, ...
    - Approximations:
      Large Nc, Strong Coupling, ...





NNLO Phase diagram

- **With increasing β, phase diagram is compressed in T direction.**
- **For finite β, 1st order boundary has a negative slope, $dT_c/d\mu<0$.** *c.f. Bilic, Demeterfi, Petersson ('92)*
- **Existence of the partially chiral restored phase in the higher μ direction of the hadron phase.**