核物質の状態方程式とハイペロン

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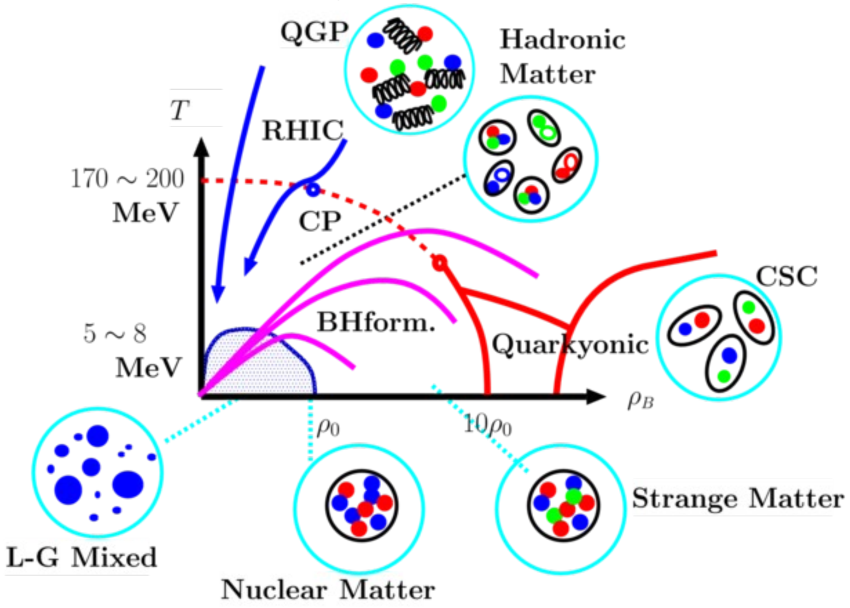
YUKAWA INSTITUTE FOR
THEORETICAL PHYSICS

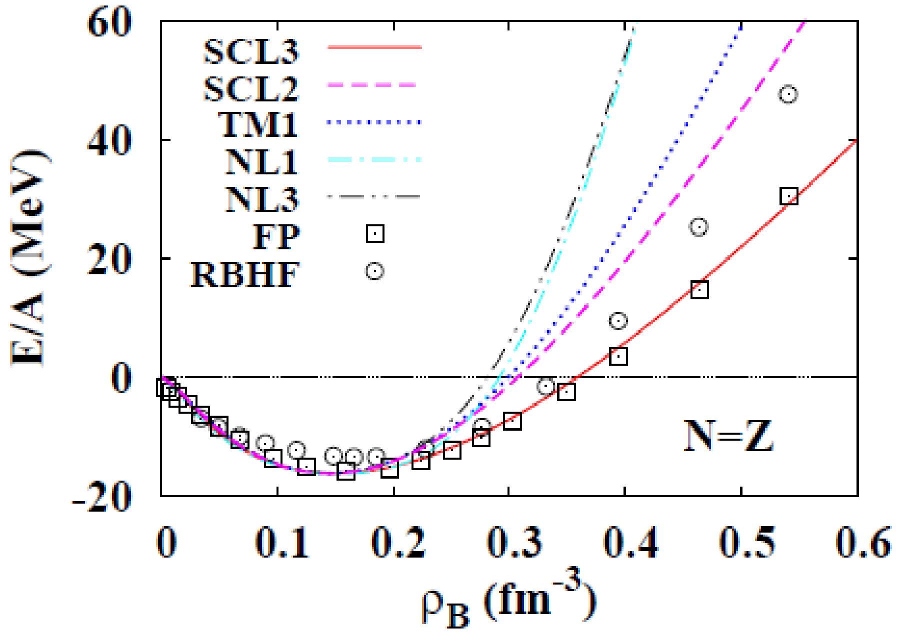
- **Introduction**
- **Recent developments in EOS studies**
- **Dense matter EOS with Hyperons**
- **Summary**

K. Tsubakihara, H. Maekawa, H. Matsumiya, AO, PRC81('10)065206.
C. Ishizuka, AO, K. Tsubakihara, K. Sumiyoshi, S. Yamada, JPG35('08)085201.
Miura,Nakano,AO,Kawamoto, PRD80('09),074034.
Nakano, Miura, AO, PTP123('10)825.

QCD Phase diagram and Nuclear Matter EOS

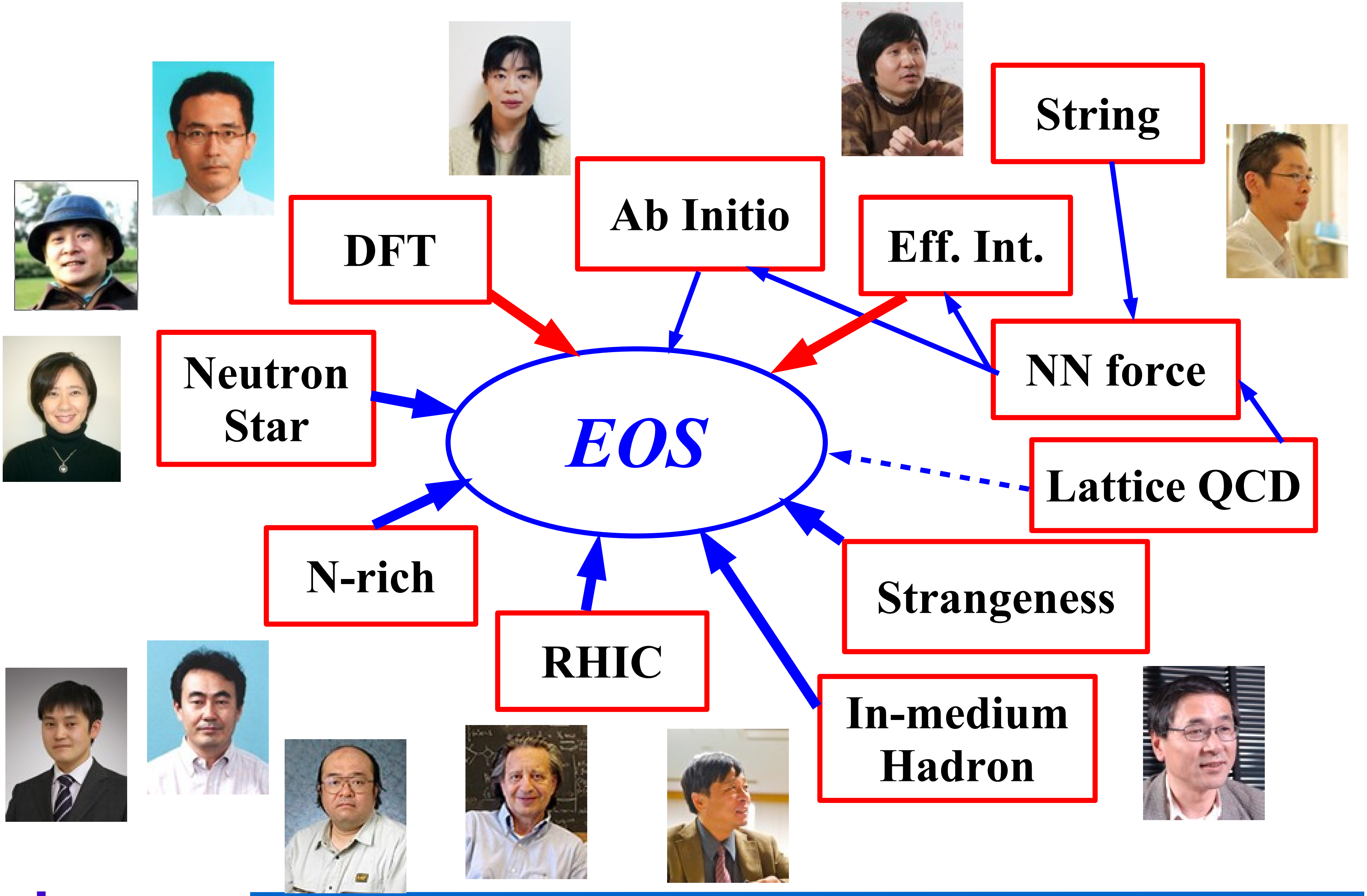
- **Phase diagram and EOS**
  = Two important aspects of Nuclear Matter
- **Dense nuclear matter has rich physics**
  → Many-body theory, Exotic compositions, CEP,
  Astrophysical applications, ...





Nuclear matter EOS
= Subjects in Nuclear, Quark-Hadron, Particle, Astro, and Condensed Matter Physics !

EOS and Related Physics

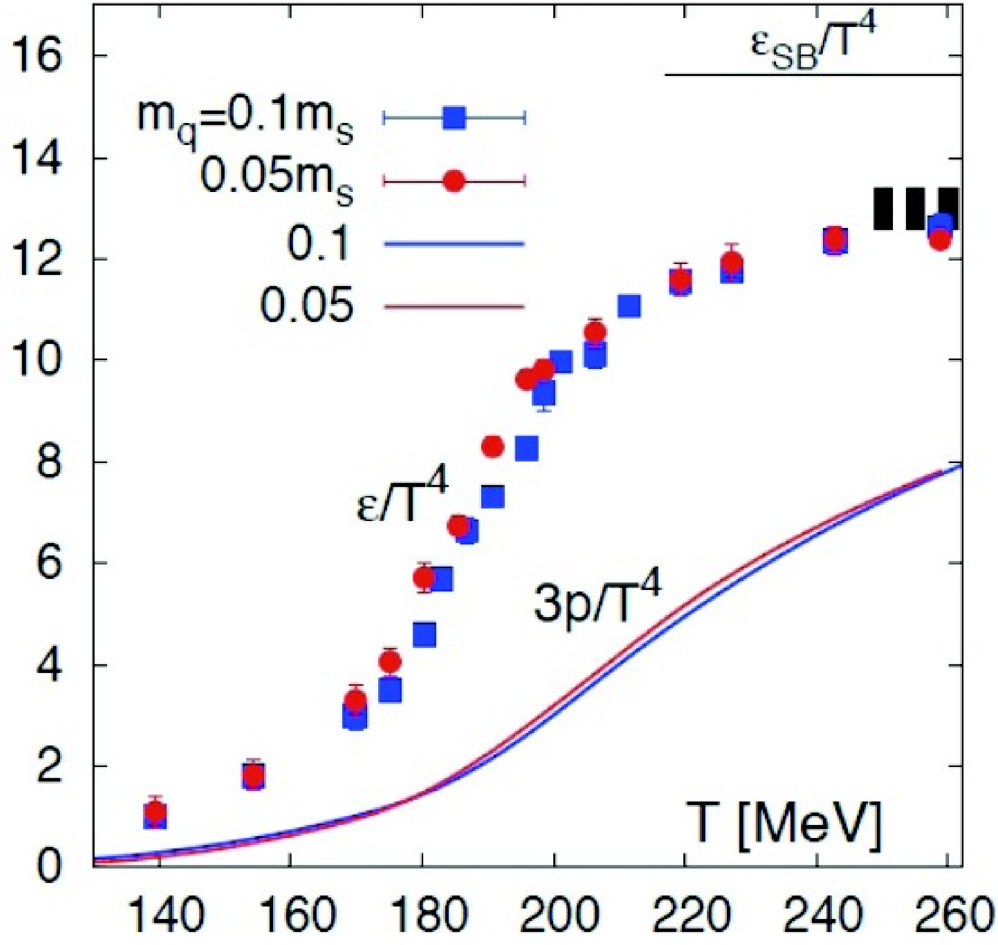


Recent Developments in EOS Studies

- **Ab initio Approaches**
  - **Lattice QCD approaches**
  - **Ab initio calculation from bare nuclear force**
- **Experimental / Observational developments**
  - **Collective flow in heavy-ion collisions**
  - **Symmetry energy**
  - **X ray measurements**
- **Applications**
  - **Neutron star matter EOS**
  - **Dense matter EOS and black hole formation**

Lattice QCD

- Monte-Carlo simulation of Lattice QCD
  = First principle & exact method at μ=0
  - "Physical point" (m_π = 154 MeV) results of EOS are now available.
  - T_c = (160-190) MeV
    Rapid ε, slow P growth at T_c
  - Slow conversion to SB limit.
- Finite μ → Sign problem !
  - Configuration weight (= Fermion det.) can be negative.
    → Important sampling
    is not possible.
  - New idea / Approximation / Eff. models are necessary.

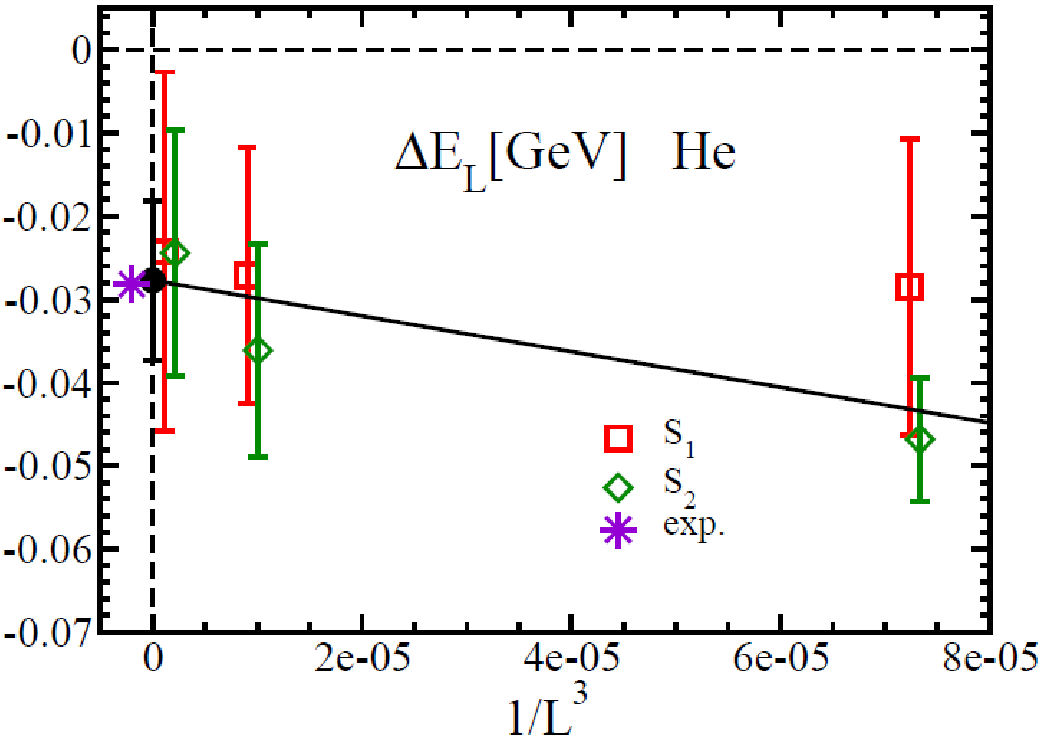


M. Chen et al. (HotQCD), PRD81('10),054504
Nf=2+1, P4, Nt=8

Helium Nucleus on the Lattice

- **Put sources on the lattice, and observe at late times.**
  - **No sign problem, but calculation cost is big.**
    E.g. Wick contraction = $(N_u\,!) \times (N_d\,!)$
  - **Calc. cost is reduced by using Symmetry + other techniques**
- **Simulation of He nuclei**
  - **Quench, heavy quarks**
    m_π =0.8 GeV, m_N=1.62 GeV
  - **It is not yet realistic, but B.E. = 27.7 MeV**

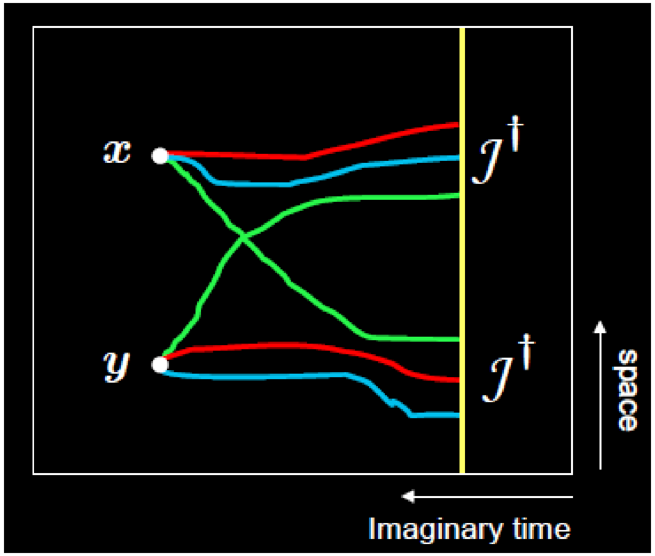
Accident ?
or Staring point of
Nuclei on the Lattice ?
→ Stay tuned !



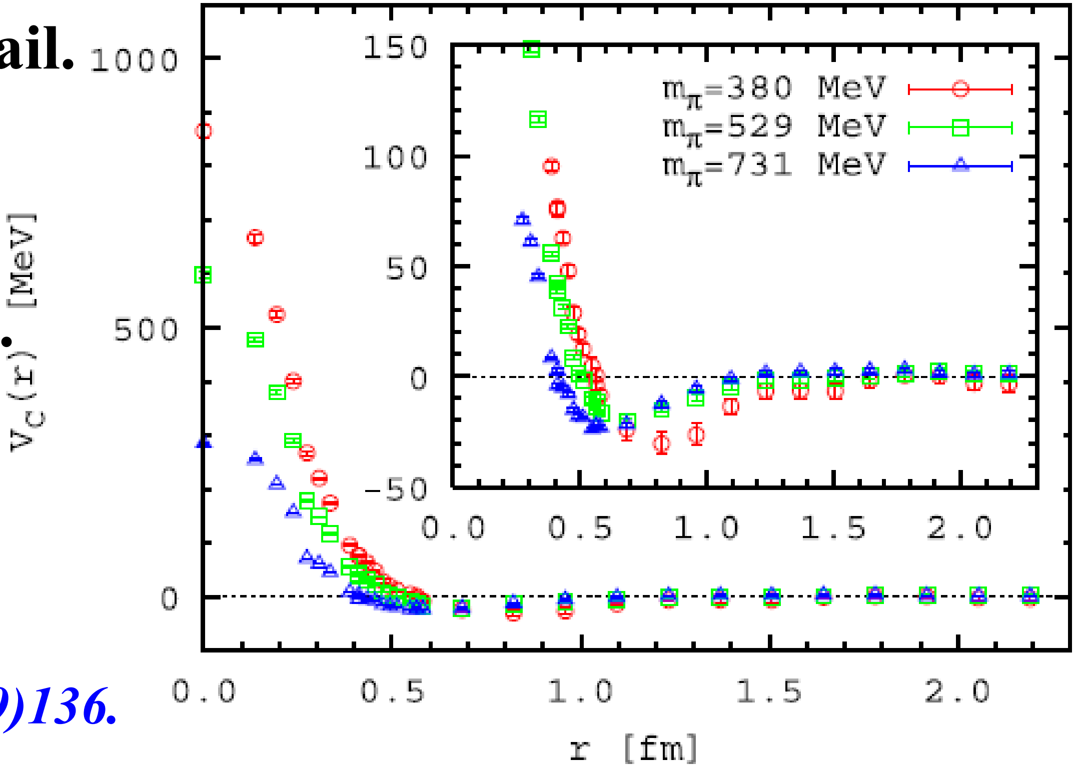
$$\Delta E_{\mathrm{He}} = 27.7(7.8)(5.5)\ \mathrm{MeV}$$

T. Yamazaki, Y. Kuramashi, A. Ukawa, PRD81('10),111504(R).

Nuclear force on the Lattice



- **BS wave function → Lattice NN Pot.**
  - **Starting from wall source, and measure Bethe-Salpeter ampl.**
  - **By using Schrodinger-type Eq., NN potential is obtained.**
- **Lot of achievements !**
  - **One pion exchange potential tail.**
  - **Repulsive core from quark Pauli principle.**
  - **YN potential, MB potential, …**
- **Needs further studies for EOS**



S. Aoki, T. Hatsuda, N. Ishii, PTP 123('10)89
Ishii, Aoki, Hatsuda, PRL 99 ('07) 022001
Nemura et al, arXiv:1005.5352 [hep-lat]
H. Nemura, Ishii, Aoki, Hatsuda, PLB673('09)136.

Nuclear Matter on the Lattice at Strong Coupling

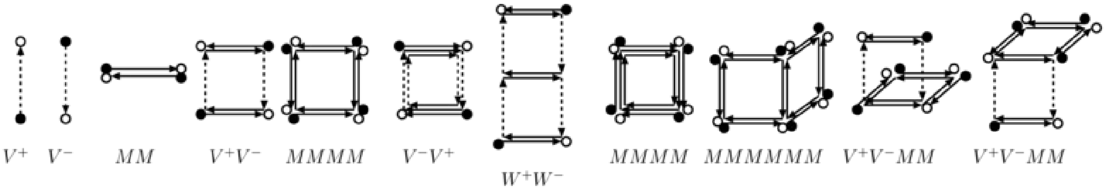
- **Sign problem is suppressed in Strong Coupling Lattice QCD**
  - **Integrate link variables first at a fixed order of $1/g^2$, and evaluate the partition func. in the mean field approx. or MC.**
  - **NNLO SC-LQCD → Nuclear matter can appear !**

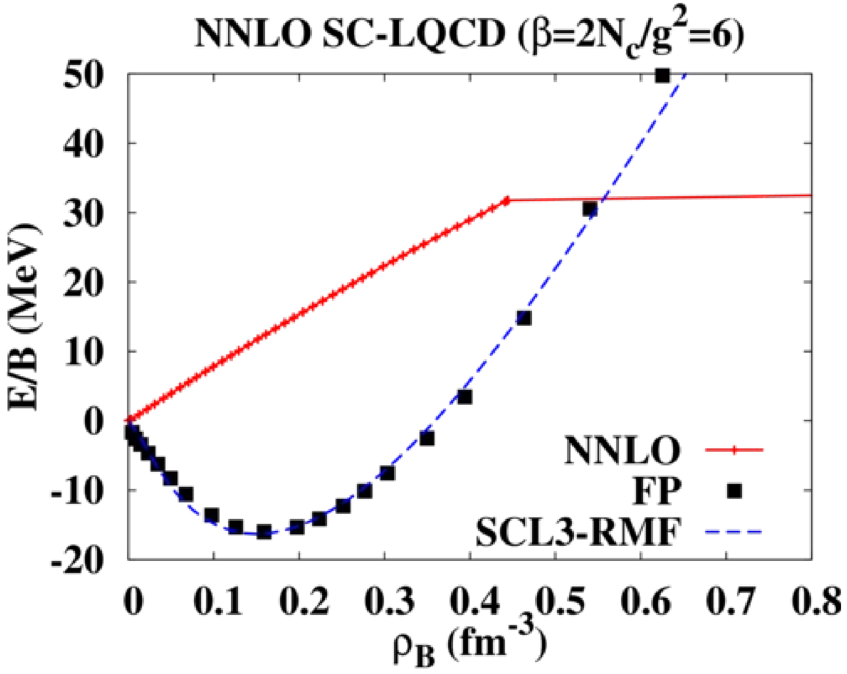
    T.Z. Nakano, K. Miura, AO, PTP123('10)825.

    With $a^{-1} = 500$ MeV, ρ_B and E/B is in the nuclear physics order

    Bilic, Demeterfi, Petersson ('92)
  - **1st order transition at ρ_B=0.4 fm^{-1}.**
  - **No saturation**
  - **Another approach: MDP simulation**

    P. de Forcrand, M. Fromm, PRL104('10)112005.





Ab Initio Calculations

- **Chiral EFT + RG evolution to low momenta**

  E. Epelbaum, H.-W. Hammer, U.-G. Meißner, RMP81('09)1773.
  S. K. Bogner, T.T.S.Kuo and A. Schwenk, PRep386('03)1.

  - **N3LO NN + NNLO 3N force**
  - **3N force → ρ dep. NN force**

- **Neutron matter results**

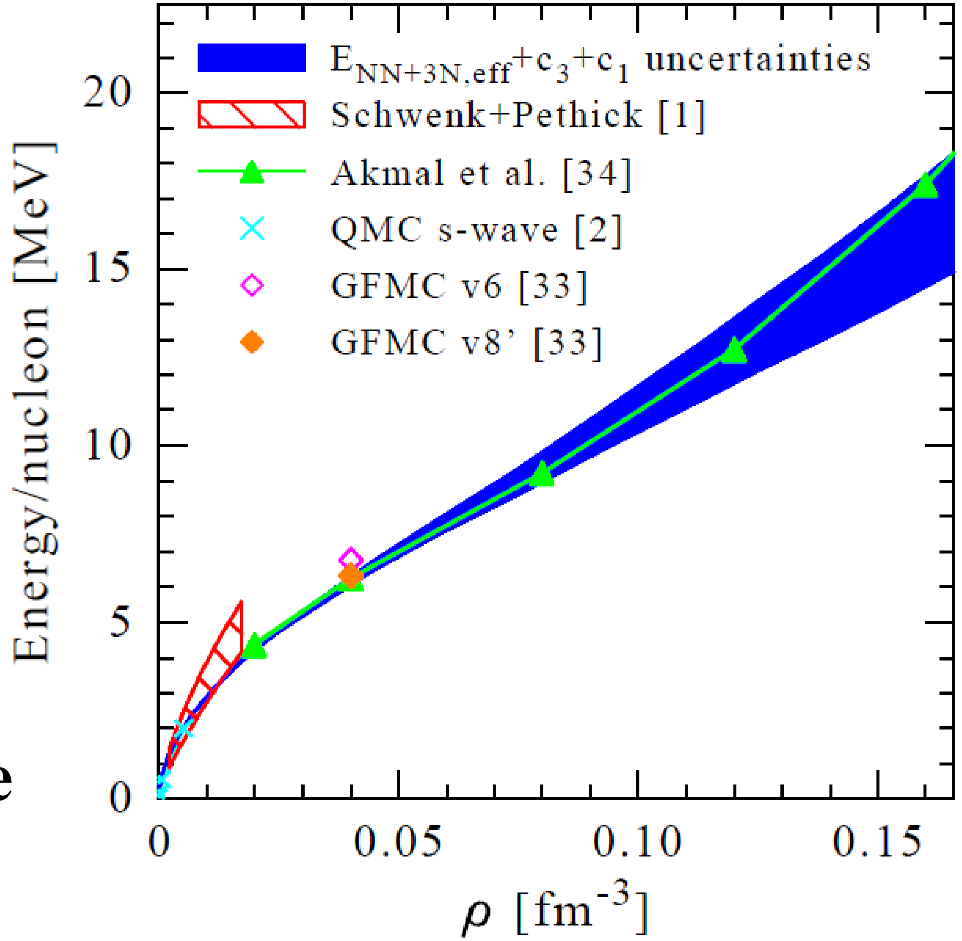
  - **Consistent with other "rigorous" results such as APR**

    A.Akmal, V.R.Pandharipande, D.G.Ravenhall, PRC58('98)1804.

    → Understanding of the origin of phen. 3-body repl. in APR.

- **Related work: QMC on the lattice**

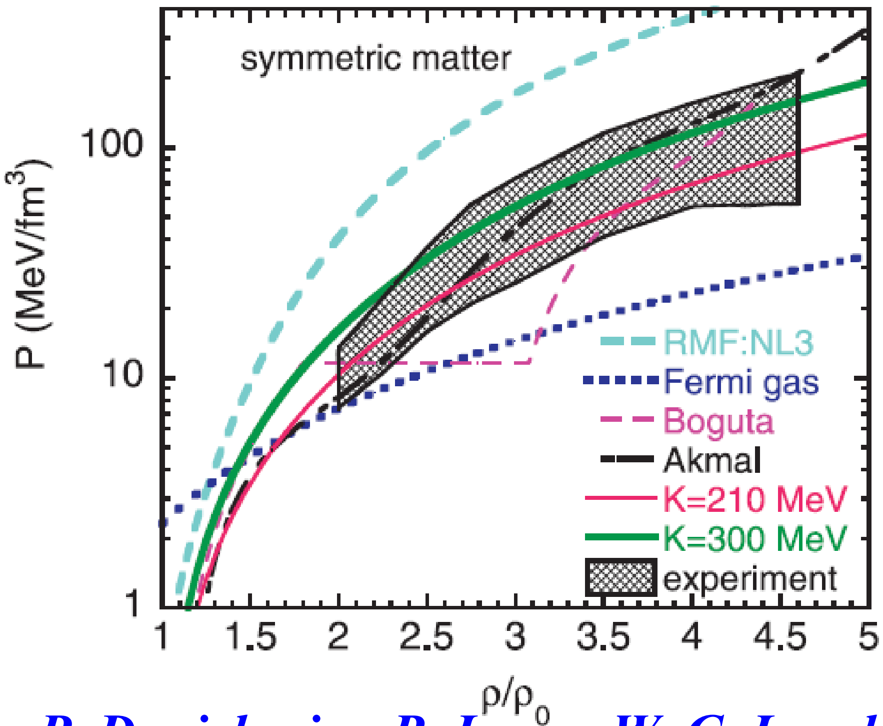
  T. Abe, R. Seki, PRC79('09)054002.



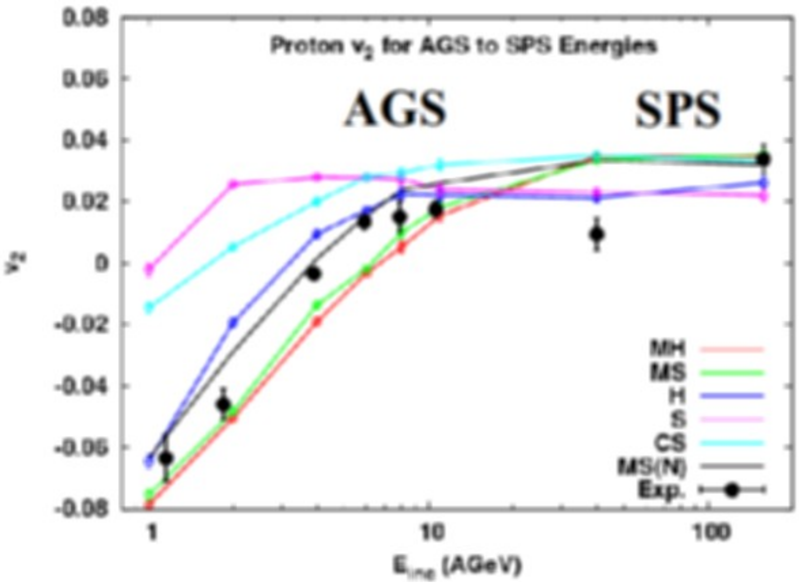
K. Hebeler, A. Schwenk, arXiv:0911.0483

Heavy-Ion Collisions and Nuclear Matter Stiffness

- **High-energy heavy-ion collisions**
  → Hot and dense nuclear matter → EOS of dense matter
  - **Systematic analyses of side- and elliptic-flow in 2-10 A GeV HIC give constraints on nuclear matter pressure at high density !**
  - **Ambiguities: mom. dep. int., pot. for Res. and mesons, ...**



P. Danielewicz, R. Lacey, W. G. Lynch, Science 298(2002), 1592.



M.Isse, AO, N.Otuka, P.K.Sahu, Y.Nara, PRC72('05)064908.

Symmetry Energy

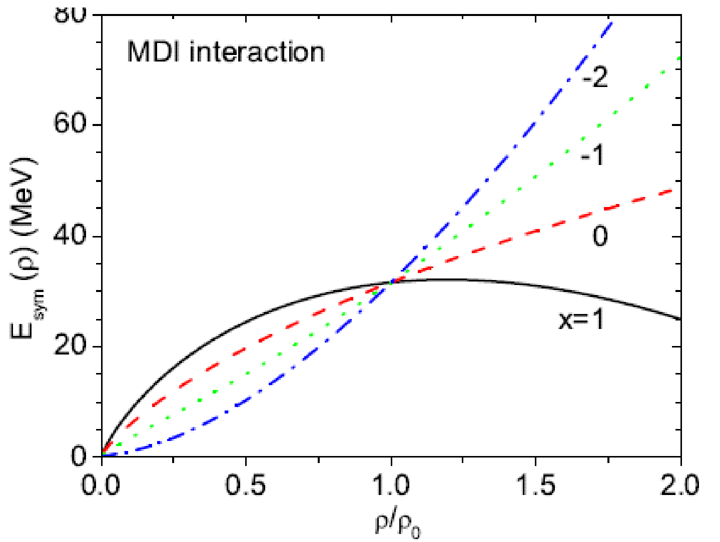
- **Two of recent data suggest that EOS becom asymmetric nuclear matter.**

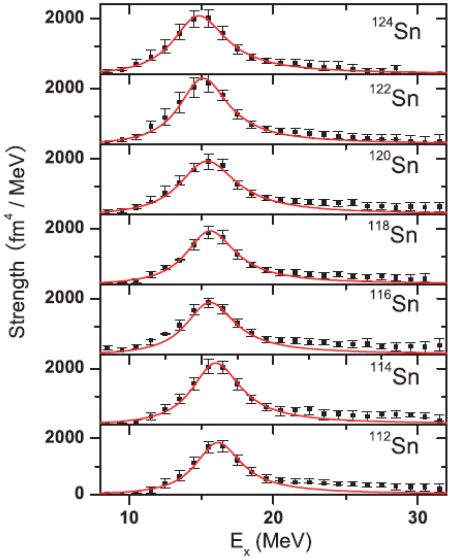
$$R_i = \frac{2X_{^{124}\mathrm{Sn}+^{112}\mathrm{Sn}} - X_{^{124}\mathrm{Sn}+^{124}\mathrm{Sn}} - X_{^{112}\mathrm{Sn}+}}{X_{^{124}\mathrm{Sn}+^{124}\mathrm{Sn}} - X_{^{112}\mathrm{Sn}+^{112}\mathrm{Sn}}}$$

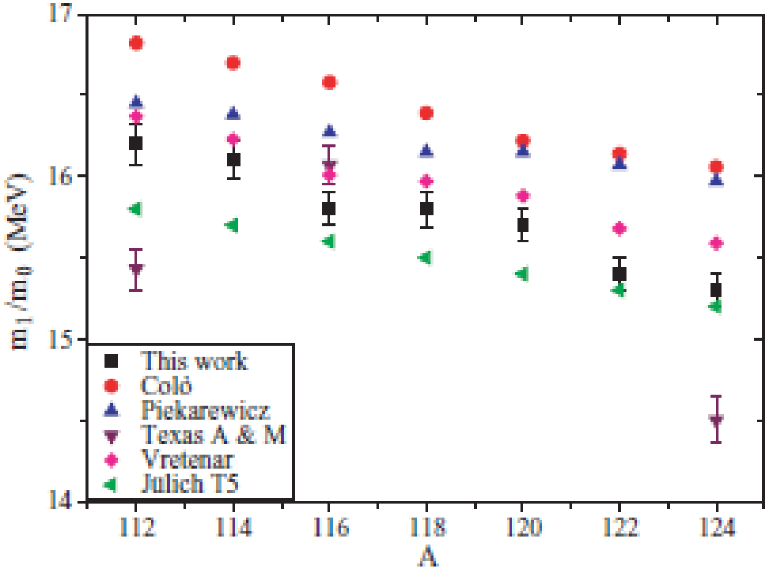
$$K = K_{\mathrm{sym}} + K_{\mathrm{asy}}\,\delta^2, \quad K_{\mathrm{asy}} \sim -550\,\mathrm{MeV}$$

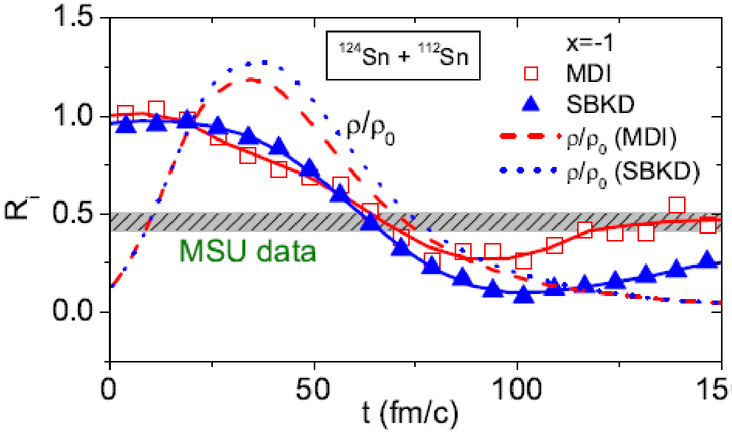
$$E_{\mathrm{sym}} \simeq 31.6\,(\rho/\rho_0)^{1.05}$$

  - **Isoscalar Giant Monopole Resonance (ISGMR) of Sn isotopes (^{112}Sn ~ ^{124}Sn)**
  - **Isospin diffusion in HIC**
  - **Related: n-rich nucl. rad (Kohama et al)**









L.W.Chen, C.M.Ko, B.A.Li, PRL94('05),032701.

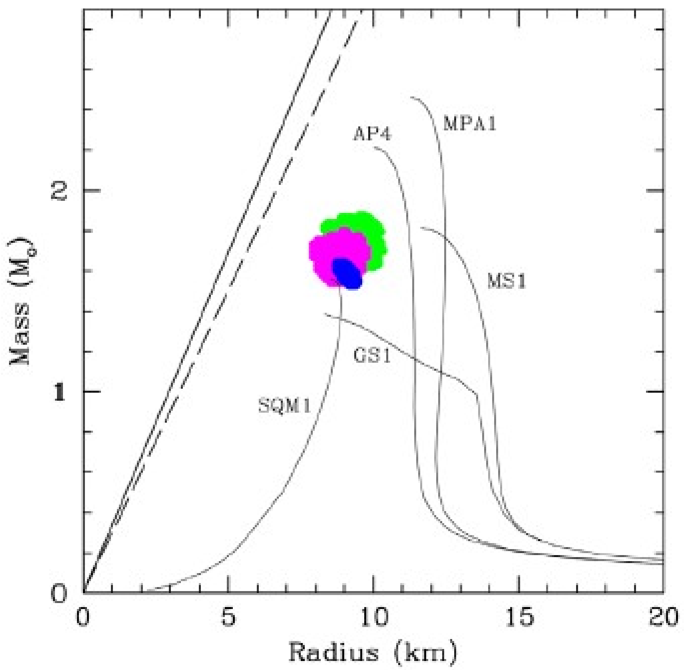
T. Li, U. Garg, et al., PRC81('10), 034309.

X-ray measurements of Neutron Stars

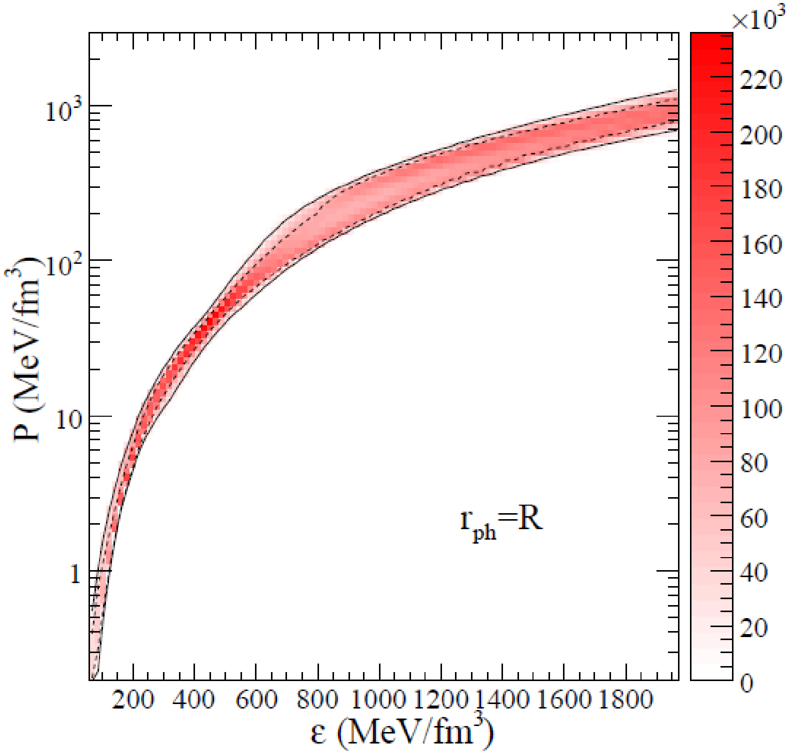
- **Neutron star mass (M)-radius (R) curve *uniquely*(*) determines NS matter EOS.**
  - **Radius measurement:**
    flux + temperature → apparent radius
  - **Eddington flux would give another info.**
  - **Bayesian TOV inversion → EOS**

$$\frac{R_\infty}{D}=\frac{R}{D}\frac{1}{\sqrt{1-2GM/Rc^2}}$$

Thermonuclear Burst
in X-ray Binaries
4U 1608-248
EXO 1745-248
4U 1820-30



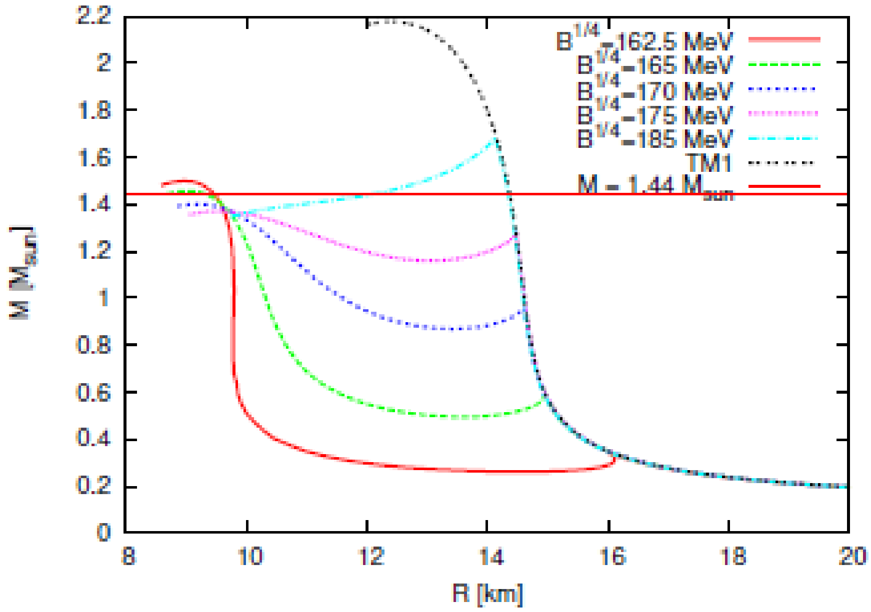
Ozel, Baym & Guver, arXiv: 1002.3153 [astro-ph.HE]



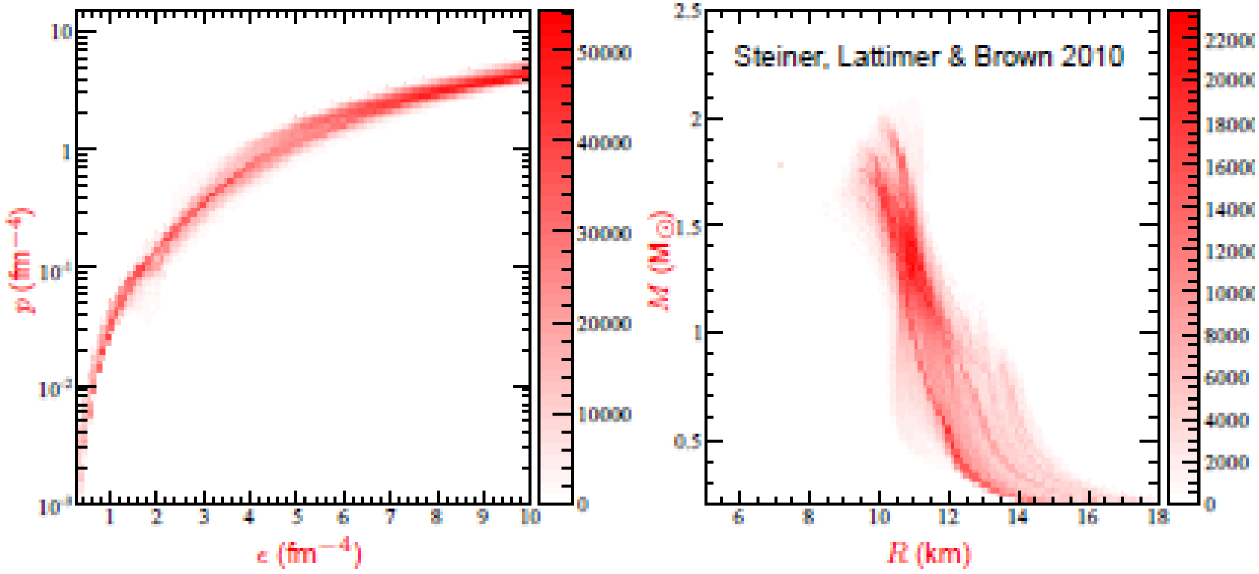
A. W. Steiner, J. M. Lattimer, Ed. Brown, arXiv:1005.0811

Some Results shown in NFQCD10

- **Bayesian TOV conversion (Lattimer, Brown, …)**
  - **Some NS observation of M&R**
  - **Determine “Plausile” NS EOS**
  - **牧島さんのコメント**
- **Quark-Hadron admixture (Sagert, Pagliara; Schaffner-Bielich)**
  - **Small B**
    → Larger NS Mass



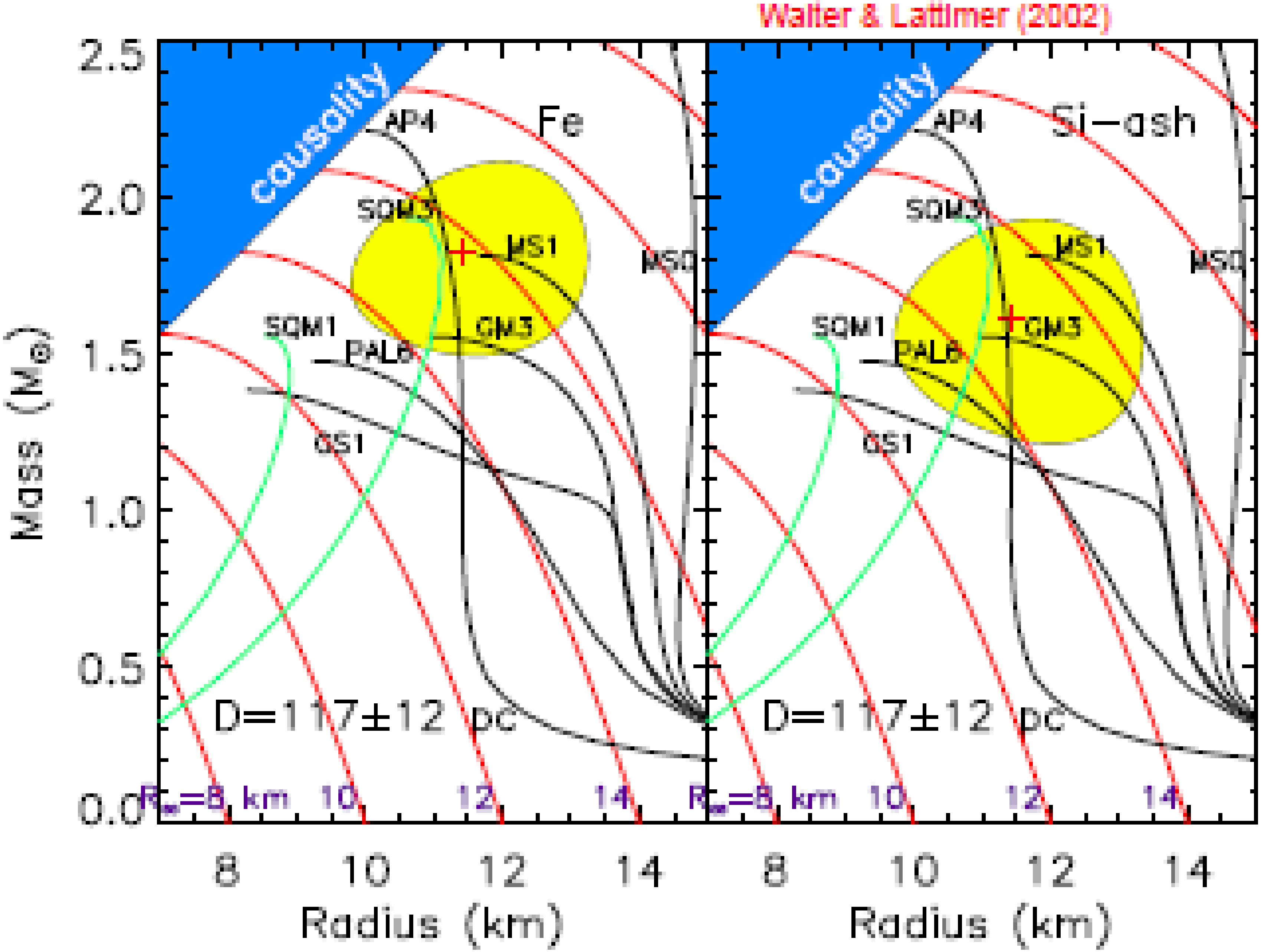
Sagert, Pagliara



Lattimer, Brown, et al.

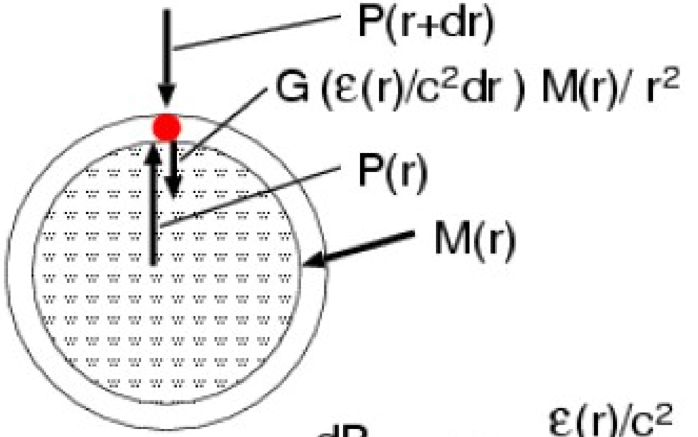
RX J1856-3754

By Lattimer



Tolman-Oppenheimer-Volkoff (TOV) equation

- **TOV Eq. = General Relativistic Balance of pressure and gravity**



Non-Rela. $\frac{dP}{dr} = -G \frac{\varepsilon(r)/c^2 \; M(r)}{r^2}$

$$\frac{dP}{dr} = -G\frac{(\varepsilon/c^2 + P/c^2)(M + 4\pi r^3 P/c^2)}{r^2(1-2GM/rc^2)}$$

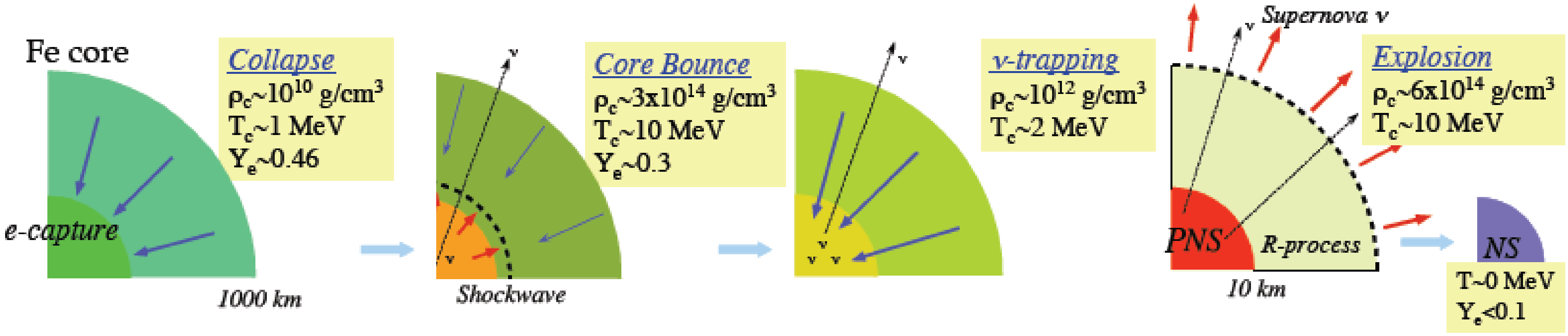
$$\frac{dM}{dr} = 4\pi r^2 \varepsilon/c^2 \; , \quad \frac{dP}{dr} = \frac{dP}{d\varepsilon}\frac{d\varepsilon}{dr}$$

$$P = P(\varepsilon) \; , \quad \frac{dP}{d\varepsilon} = \frac{dP}{d\varepsilon}(\varepsilon) \quad \text{(EOS)}$$

Neutron Star Mass = M(R) where P(R)=0

When you make a new EOS, please check the NS mass !

Black Hole Formation (Failed Supernova)



Collapse
$\rho_c \sim 10^{10}$ g/cm^3
$T_c \sim 1$ MeV
$Y_e \sim 0.46$

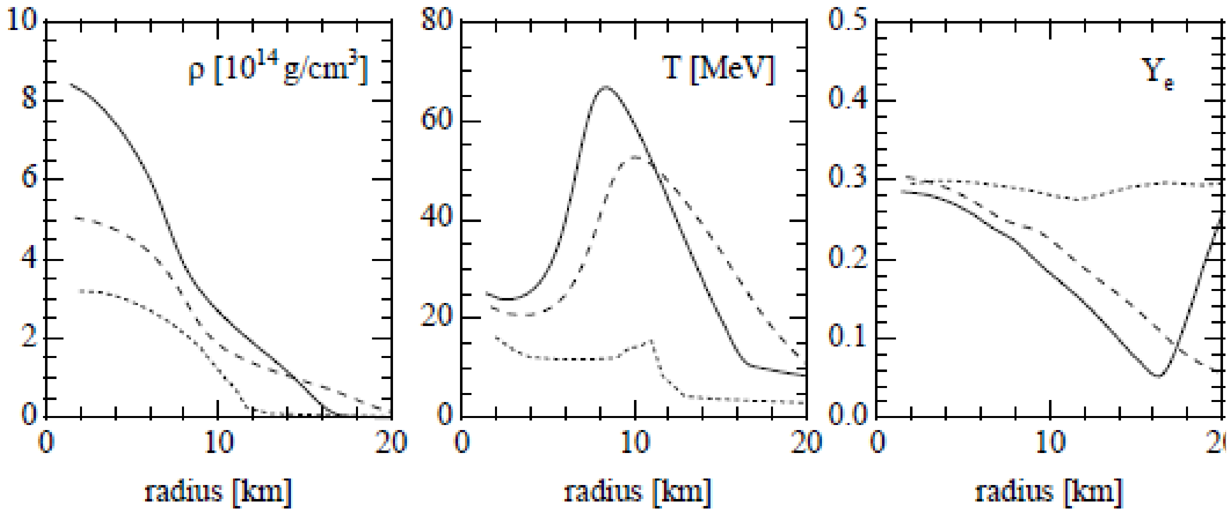
Core Bounce
$\rho_c \sim 3\times10^{14}$ g/cm^3
$T_c \sim 10$ MeV
$Y_e \sim 0.3$

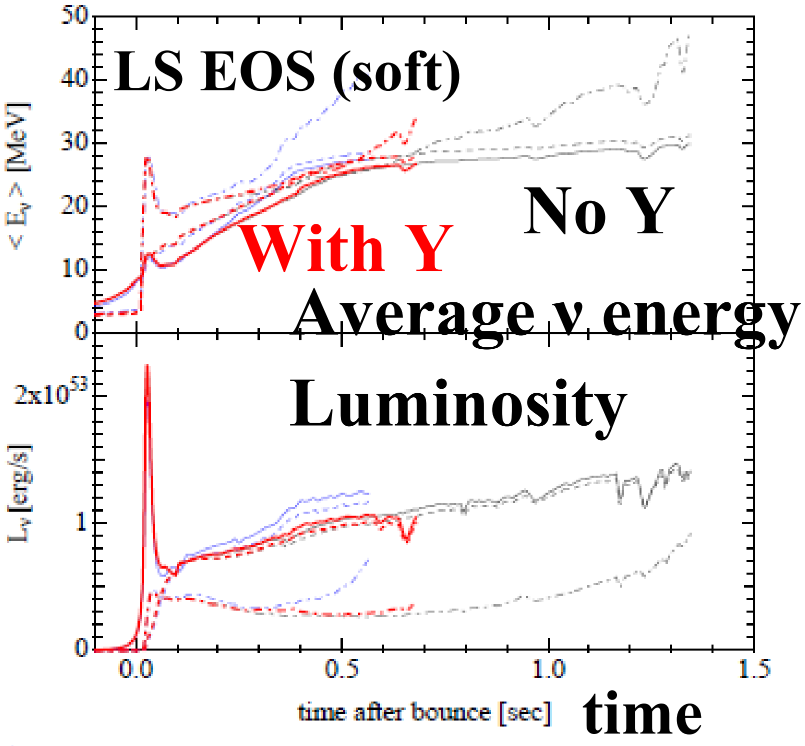
ν-trapping
$\rho_c \sim 10^{12}$ g/cm^3
$T_c \sim 2$ MeV

Explosion
$\rho_c \sim 6\times10^{14}$ g/cm^3
$T_c \sim 10$ MeV

$T \sim 0$ MeV
$Y_e < 0.1$

At bounce, 500 ms 680 ms (at BH form.)



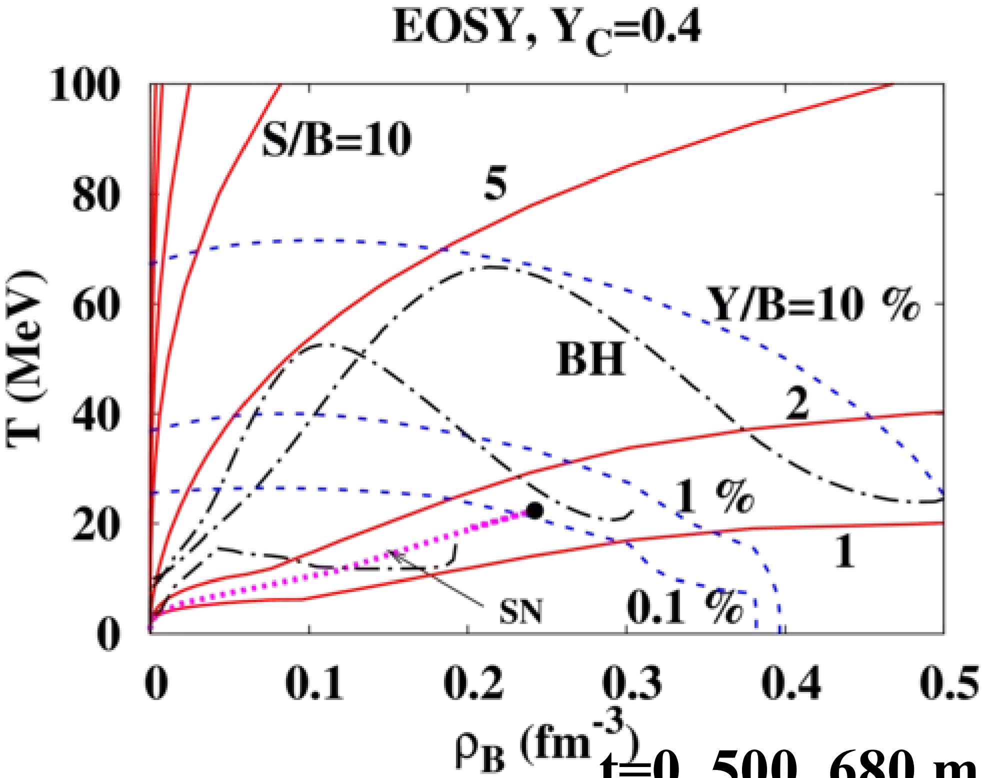


Sumiyoshi, Ishizuka, AO, Yamada, Suzuki, 2009

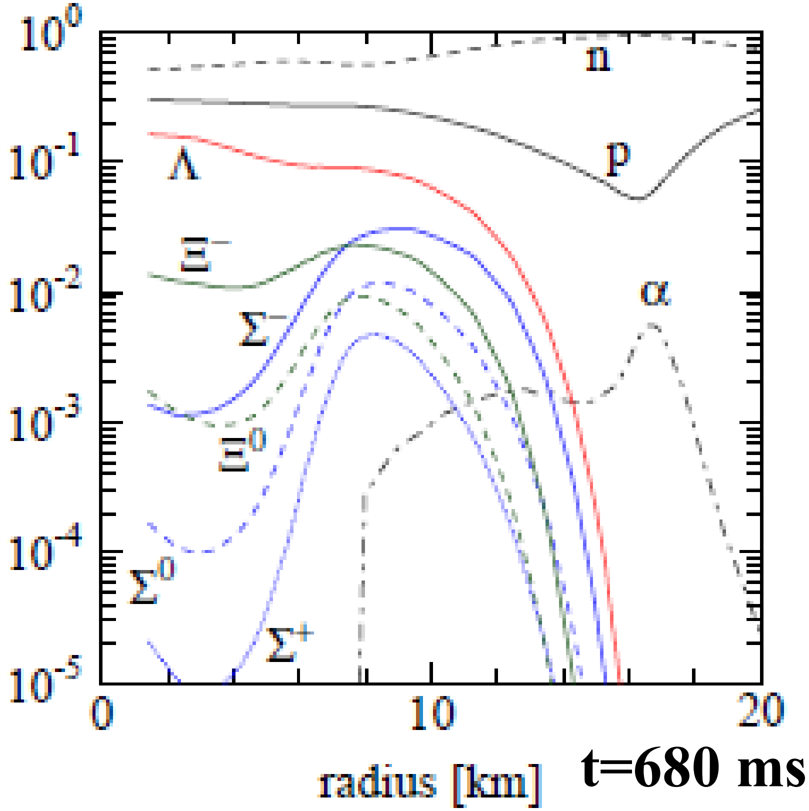
Black Hole Formation

- **Black Hole Formation: (ρ_B, T, Y_e) ~ (4 ρ_0, 70 MeV, 0.2)**
  → Hyperon fraction ~ 10 %

(K. Sumiyoshi, C. Ishizuka, AO, S. Yamada, H. Suzuki, ApJ690(09)L43)



t=0, 500, 680 msec



t=680 msec

Hyperons are abundantly formed during BH formation !
→ EOS softening, Early collapse, Short ν duration

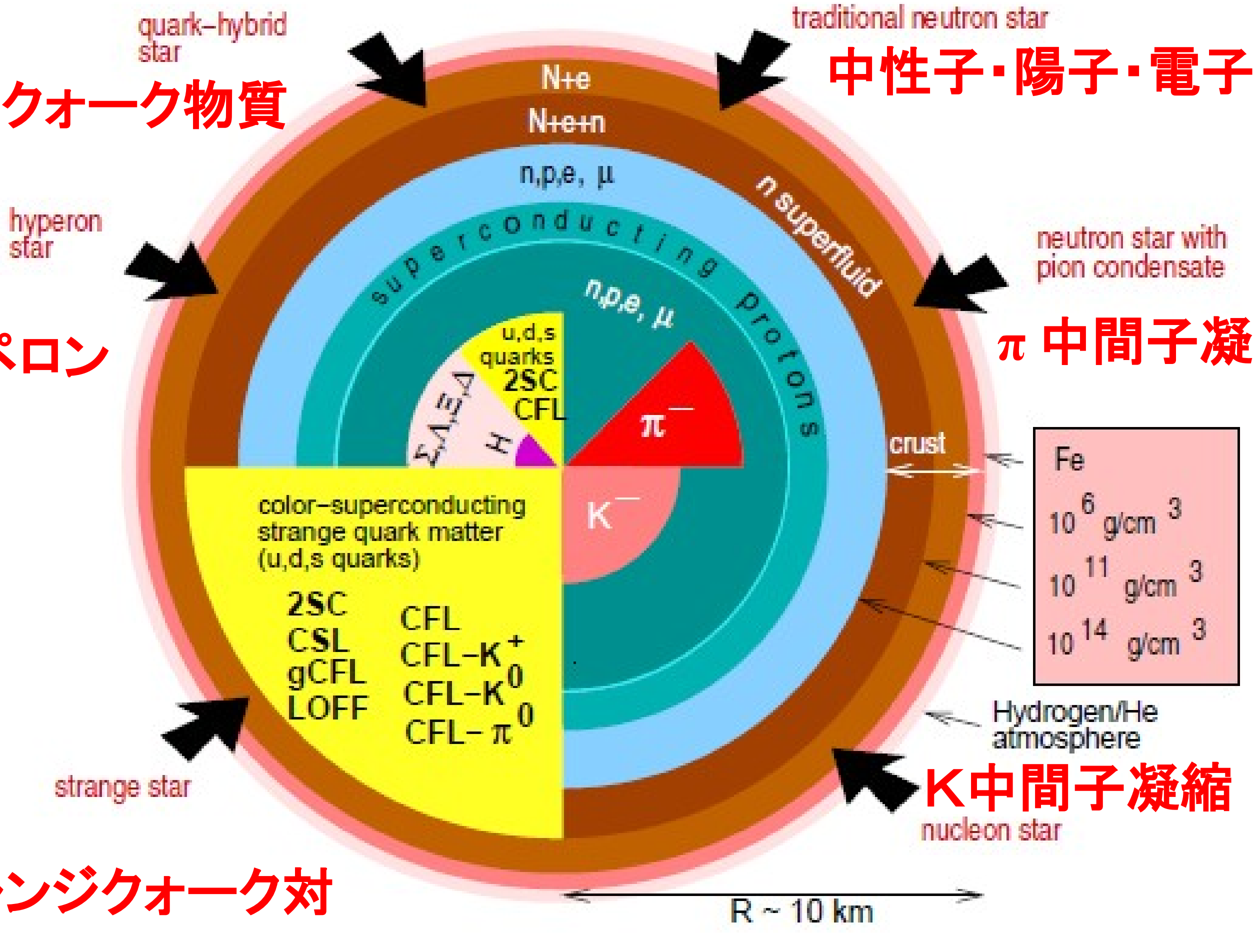
Recent experiments / observations provides rich information of EOS in 2D (ρ_B, Y_p) or 3D (ρ_B, Y_p, T).

How can we use it to obtain "THE" EOS ?

Here "THE" means "The",
and "Tri-Hierarchical EOS"
(Quark-Hadron-Nuclear)

Dense Nuclear Matter EOS with Hyperon Admixture

高密度星の中のストレンジネス



クォーク物質

中性子・陽子・電子・ミューオン

ハイペロン
混合

π 中間子凝縮

K中間子凝縮

ストレンジクォーク対
凝縮

F. Weber, Prog. Part. Nucl. Phys. 54 (2005) 193

「中性子星」の内側 →ほぼ確実にストレンジネスを多く含む！

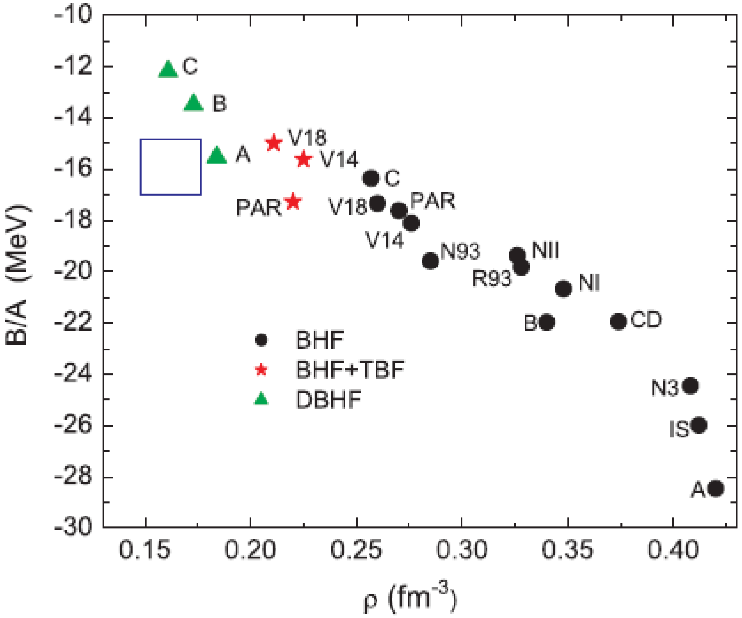
Theories/Models for Nuclear Matter EOS

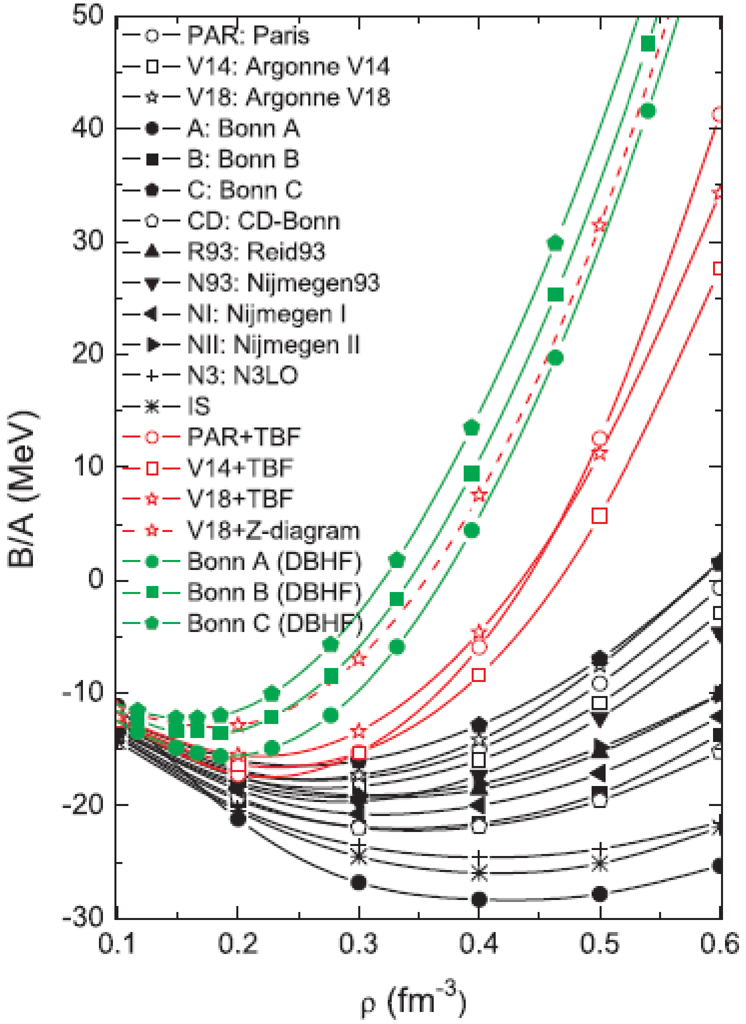
- **Ab initio Approaches to Nuclear Matter**
  → LQCD, Variatonal, GFMC, BHF(G-Matrix), DBHF, ...
  - **LQCD-MC: Not (yet) applicable to cold dense matter, $A \leq 4$**
    SC-LQCD: Nuclear matter does not bound
  - **Variatioal, BHF: Need phen. 3-body repulsion to reproduce saturation point.**
  - **GFMC: Limited to be $A \leq 12$.**
  - **DBHF: Good, but E/A is not enough. Not yet extensively investigated.**

  → Not easy to handle, Not yet satisfactory for phen. purposes
- **Mean Field Models (~ Nuclear Density Fuctional approach)**
  - **Skyrme Hartree-Fock(-Bogoliubov)**
    - **Nuclear Mass is very well explained (HFB, Total B.E. $\Delta E \sim 0.6$ MeV)**
    - **Causality is violated at very high densities.**
  - **Relativistic Mean Field**
    - **Relativistic, Meson-Baryon coupling, Meson self-energies**
    - **Successful in describing pA scatering (Dirac Phenomenology)**

Bruckner-Hartree-Fock

- **Self-consistent treatment of**
  Effective interaction (G-matrix) in the Bruckner Theory
  and Single particle energy from G-matrix
  - **Need 3-body force to reproduce saturation point.**
    → FY type 2 π exchange
    + phen. or Z-diagram

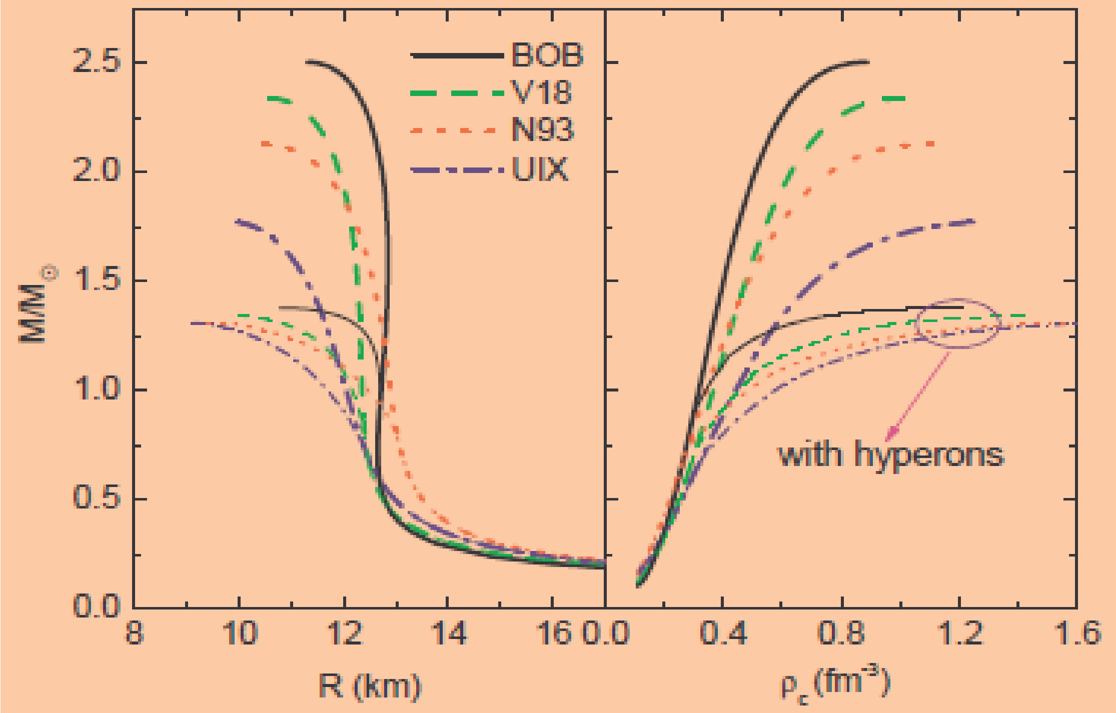




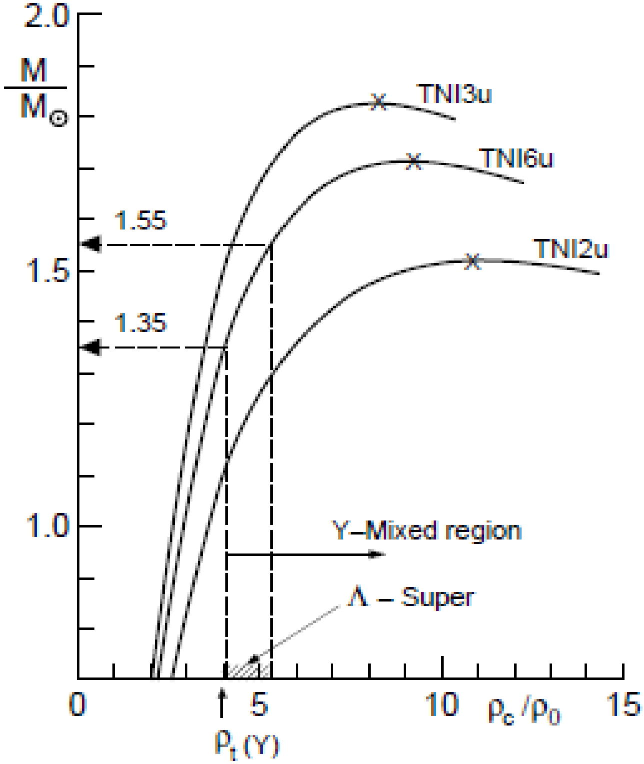
Z.H.Li, U. Lombardo, H.-J. Schulze, W. Zuo, L. W. Chen, H. R. Ma, PRC74('06)047304.

Bruckner-Hartree-Fock theory with Hyperons

- **Microscopic G-matrix calculation with realistic NN, YN potential and microscopic (or phen.) 3N force (or 3B force).**
  - **Interaction dep. (V18, N93, ...) is large → Need finite nuclear info.**
    E.Hiyama, T.Motoba, Y.Yamamoto, M.Kamimura / M.Tamura et al.
  - **NS collapses with hyperons w/o 3BF.**



H.J.Schulze, A.Polls, A.Ramos, I.Vidana, PRC73('06),058801.



S. Nishizaki, T. Takatsuka, Y. Yamamoto, PTP108('02)703.

Relativistic Mean Field

- **Effective Lagrangian of Baryons and Mesons + Mean Field App.**

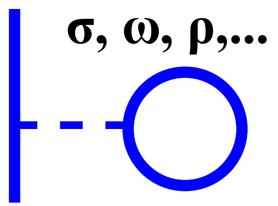
  B.D.Serot, J.D.Walecka, Adv.Nucl.Phys.16 ('86), 1

$$L = L_B^{\mathrm{free}} + L_M^{\mathrm{free}} + L_{BM} + L_M^{\mathrm{Int}}$$

$$L_M^{\mathrm{Int}} = -U_\sigma(\sigma) + \frac{1}{4} c_\omega (\omega_\mu \omega^\mu)^2 + \cdots$$

$$L_{BM} = -\sum_{B,S} g_{BS} \bar{\psi}_B \varphi_S \psi_B - \sum_{B,V} g_{BV} \bar{\psi}_B \gamma^\mu V_\mu \psi_B$$

$$L_B^{\mathrm{free}} = \bar{\psi}_B (i\gamma^\mu \partial_\mu - M_B) \psi_B \quad , \quad L_M^{\mathrm{free}} = \sum_S \left[\frac{1}{2} \partial^\mu \varphi_S \partial_\mu \varphi_S - \frac{1}{2} m_S^2 \varphi_S^2\right] + \sum_V \left[-\frac{1}{4} V_{\mu\nu} V^{\mu\nu} + \frac{1}{2} V_\mu V^\mu\right]$$



- **Baryons and Mesons: B=N, Λ, Σ, Ξ, ...,   S= σ, ς, ...,   V= ω, ρ, φ, …**
- **Based on Dirac phenomenology & Dirac Bruckner-Hatree-Fock**

  E.D. Cooper, S. Hama, B.C. Clark, R.L. Mercer, PRC47('93),297
  R. Brockmann, R. Machleidt, PRC42('90),1965

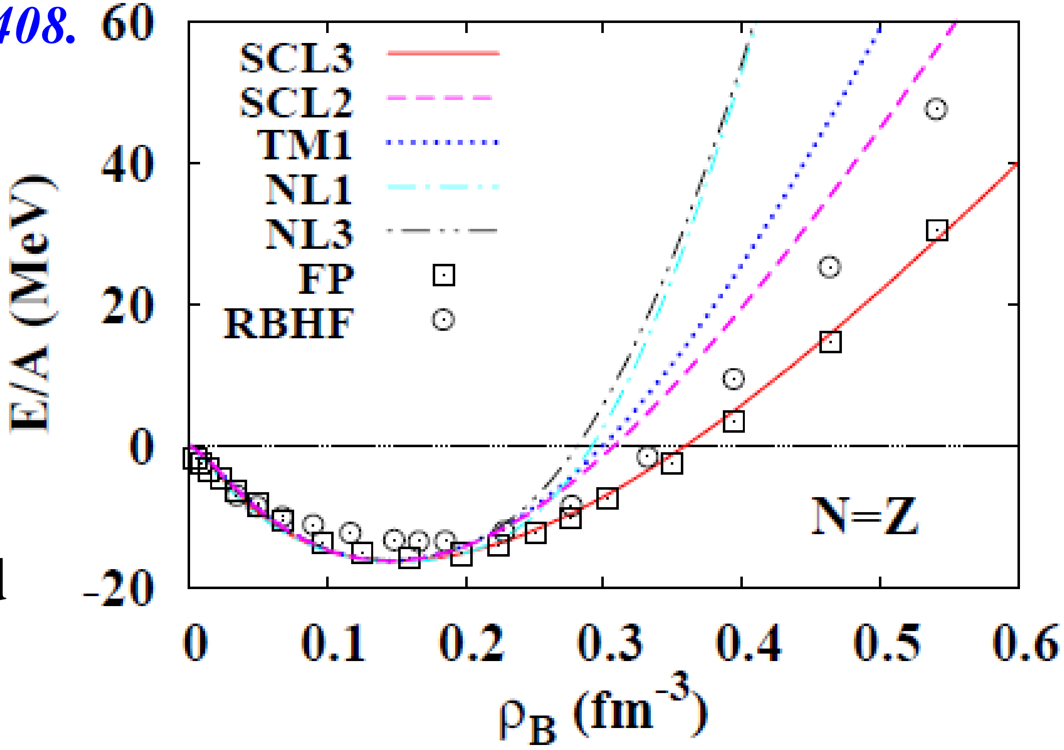
- **Large scalar (att.) and vector (repl.) → Large spin-orbit pot.**
  Relativistic Kinematics → Effective 3-body repulsion
- **Non-linear terms of mesons → Bare 3-body and 4-body force**

  Boguta, Bodmer ('77), NL1:Reinhardt, Rufa, Maruhn, Greiner, Friedrich ('86), NL3: Lalazissis, Konig, Ring ('97),TM1 and TM2: Sugahara, Toki ('94), Brockmann, Toki ('92)

RMF models

- Variety of the RMF models
  → MB couplings, meson masses, meson self-energies
  - σN, ωN, ρN couplings are well determined
    → almost no model deps. in Sym. N.M. at low ρ
  - ω^4 term is introduced to simulate DBHF results of vector pot.
    TM1&2: Y. Sugahara, H. Toki, NPA579('94)557;
    R. Brockmann, H. Toki, PRL68('92)3408.
  - σ^3 and σ^4 terms are introduced to soften EOS at ρ_0.
    J. Boguta, A.R.Bodmer NPA292('77)413,
    NL1:P.-G.Reinhardt, M.Rufa, J.Maruhn, W.Greiner, J.Friedrich, ZPA323('86)13.
    NL3: G.A.Lalazissis, J.Konig, P.Ring, PRC55('97)540.

    → Large differences are found at high ρ



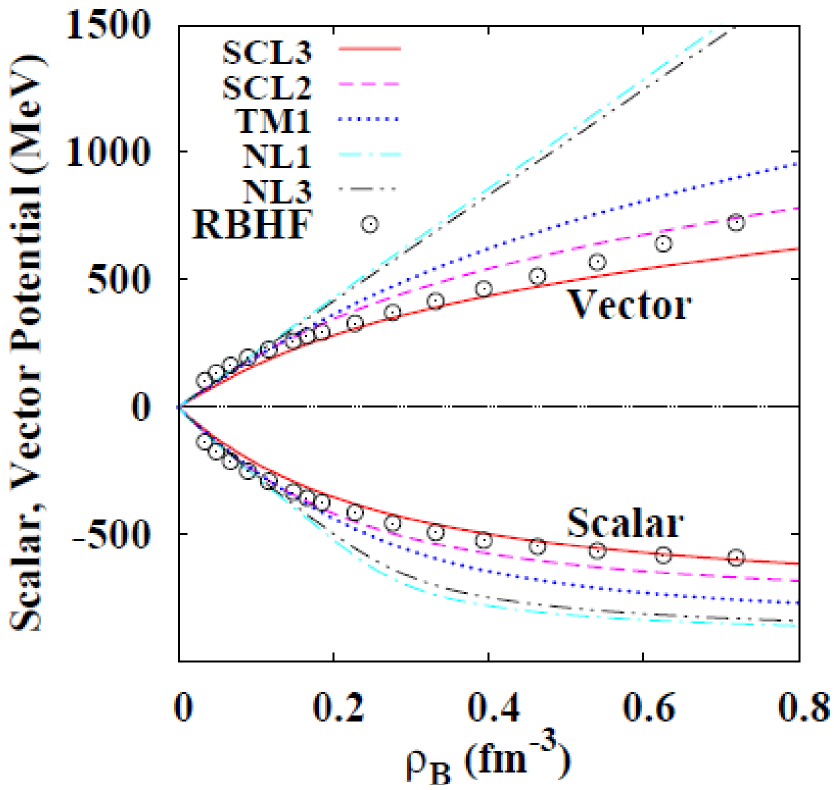
K. Tsubakihara, H. Maekawa, H. Matsumiya, AO,PRC81('10)065206.

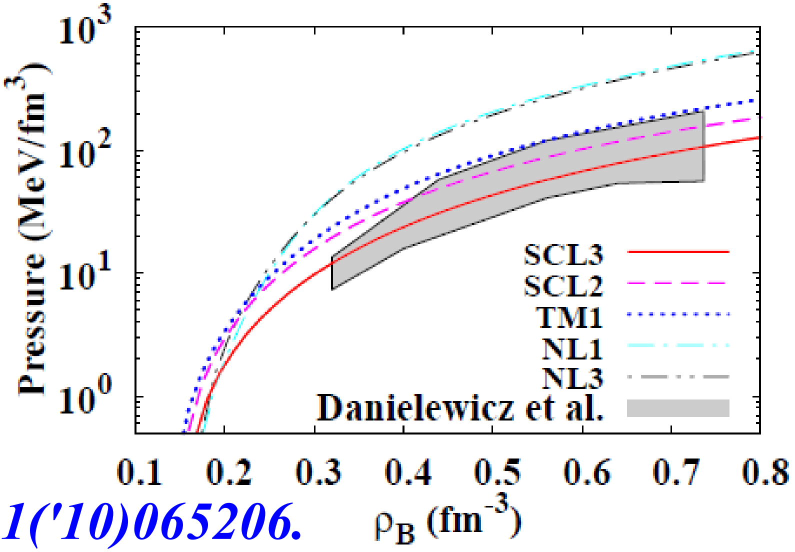
Vector potential in RMF

- **Vector potential from ω dominates at high density !**

$$U_v(\rho_B) = g_\omega \omega \sim \frac{g_\omega^2}{m_\omega^2}\rho_B$$

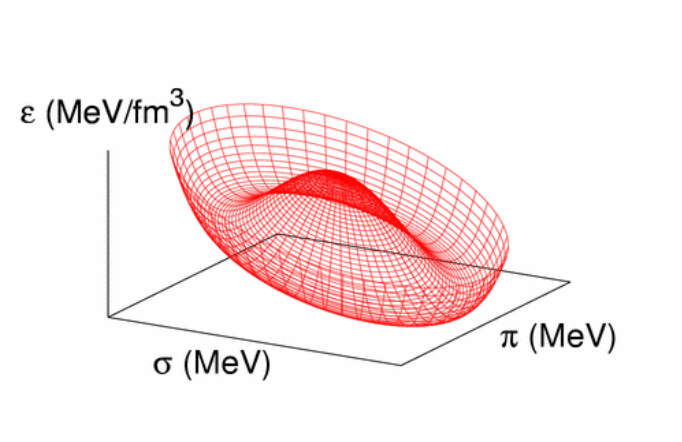
  - **Dirac-Bruckner-Hartree-Fock shows suppressed vector potential at high ρ_B.**
    R. Brockmann, R. Machleidt, PRC42('90)1965.
  - **Collective flow in heavy-ion collisions suggests pressure at high ρ_B.**
    P. Danielewicz, R. Lacey, W. G. Lynch, Science298('02)1592.
  - **Self-interaction of ω ~ $c_\omega(\omega_\mu\omega^\mu)^2$**
    → DBHF results & Heavy-ion data





K. Tsubakihara, H. Maekawa, H. Matsumiya, AO,PRC81('10)065206.

Chiral Symmetry



- **Fundamental symmetry of massless QCD, and its spontaneous breaking generates hadron masses.**

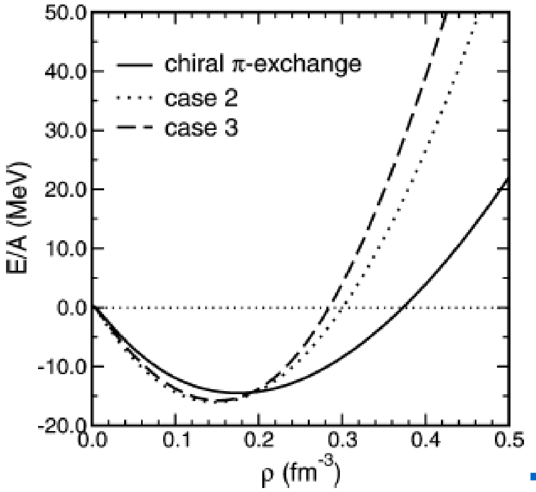
  Nambu, Jona-Lasinio ('61)

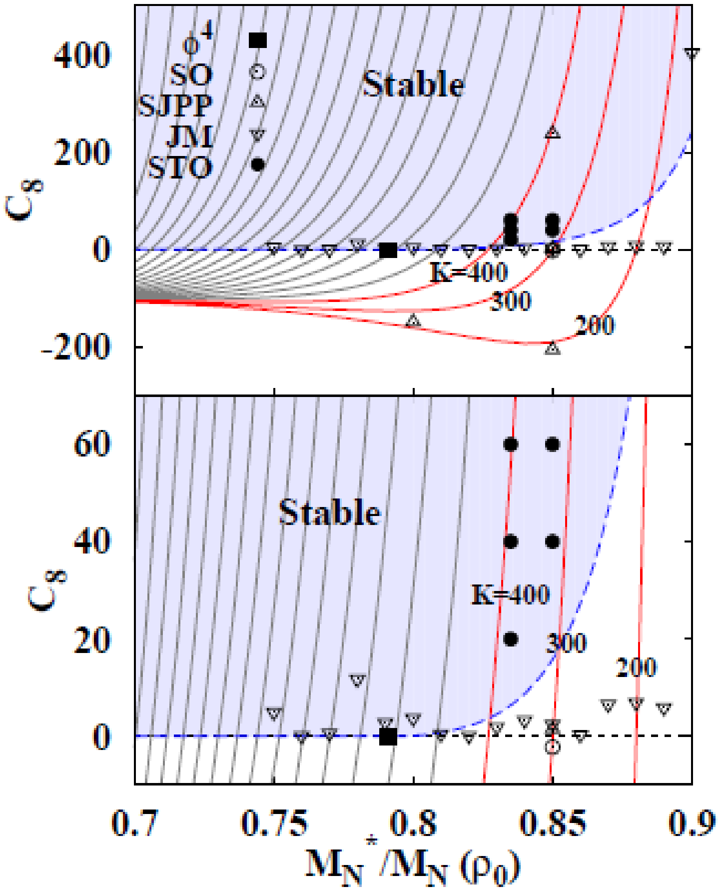
- **Many of the linear σ models are unstable against finite density (chiral collapse).**
  → Log type chiral potential

  Sahu, Tsubakihara, AO('10), Tsubakihara, AO('07

- **Non-linear representation (chiral pert.) leads to density dependent coupling from pion loops.**

  Kaiser, Fritsch, Weise ('02),
  Finelli, Kaiser, Vretener
  Weise ('04)





RMF with Hyperons (Single Λ hypernuclei)

- **RMF for Λ hypernuclei**

x ~ 1/3: R. Brockmann, W. Weise, PLB69('77)167; J. Boguta and S. Bohrmann, PLB102('81)93.
x ~ 2/3: N. K. Glendenning, PRC23('81)2757, PLB114('82)392;
Tensor: Y. Sugahara, H. Toki, PTP92('94)803; H. Shen, F. Yang, H. Toki, PTP115('06)325;
J. Mares, B. K. Jennings, PRC49('94)2472.
ρ-dep. coupling: H. Lenske, Lect. Notes Phys. 641('04)147; C. M. Keil, F. Hofmann, H. Lenske, PRC 61('00)064309.
SU(3) or SU(6) (ς, φ): J. Schaffner, C. B. Dover, A. Gal, C. Greiner, H. Stoecker,PRL71('93)1328; Schaffner et al., Ann.Phys.235('94)35; J. Schaffner, I. N. Mishustin, PRC 53('96)1416.
Chiral SU(3) RMF: K. Tsubakihara, H. Maekawa, H. Matsumiya, AO, PRC81('10)065206.

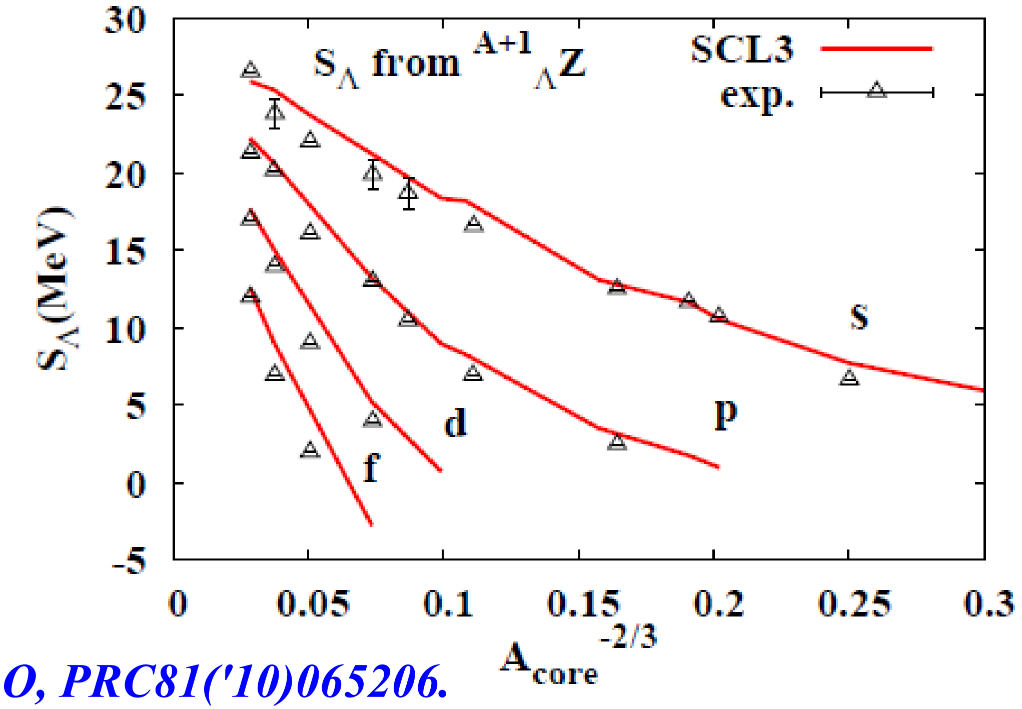
- **Sep. E. of Λ is well fitted by U_Λ ~ -30 MeV ~ 2/3 U_N**
- **Coupling with mesons**

  $$x_M = g_{M\Lambda}/g_{MN}$$

  quark counting: $x_\sigma \sim 2/3$

  π exchanges: $x_\sigma \sim 1/3$

  → Which is true ?



K. Tsubakihara, H. Maekawa, H. Matsumiya, AO, PRC81('10)065206.

RMF with Hyperons (Double Λ hypernuclei)

- **Nagara event $\Delta B_{\Lambda\Lambda} \sim 1.0$ MeV (weakly attractive)**
  - **TM & NL-SH based RMF**

    H. Shen, F. Yang, H. Toki, PTP115('06)325.

    Model 1: $x_\sigma = 0.621$, $x_\omega = 2/3$ (no ς, φ)

    Model 2: $R_\varsigma = g_{\varsigma\Lambda} / g_{\sigma N} = 0.56\text{-}0.57$, $R_\varphi = g_{\varphi\Lambda} / g_{\omega N} = -\sqrt{2}/3$
  - **Chiral SU(3) RMF**

    K. Tsubakihara, H. Maekawa, H. Matsumiya, AO, PRC81('10)065206.

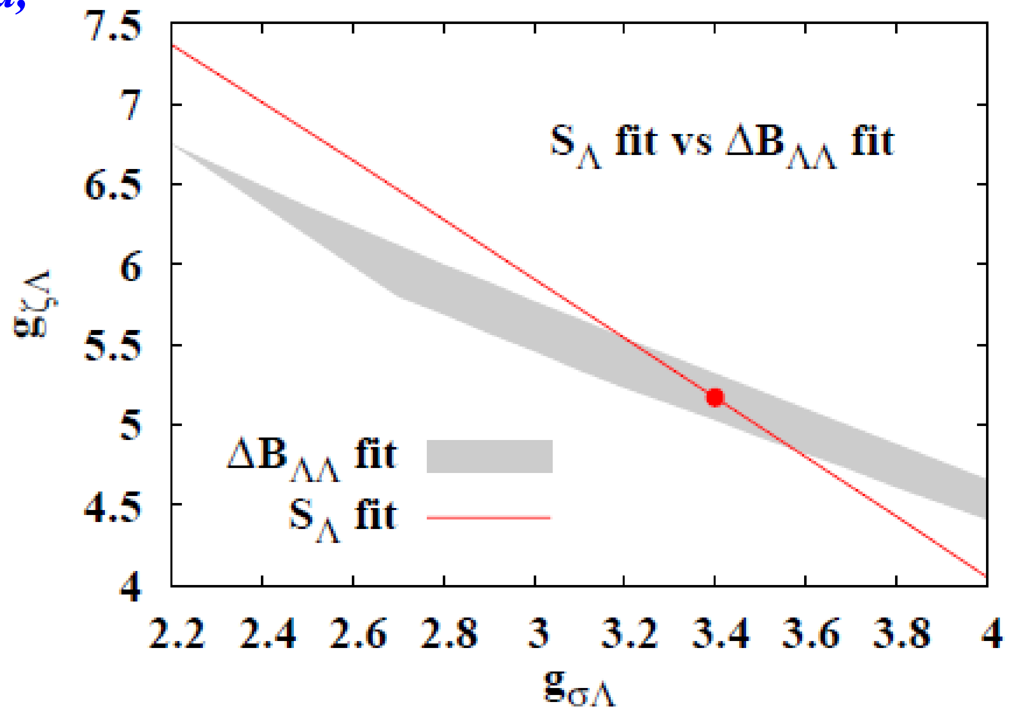
    SU(3)f for vector coupling

    $x_\omega = 0.64$, $R_\varphi = 0.504$

    Det. (KMT) int. mixes σ and ς

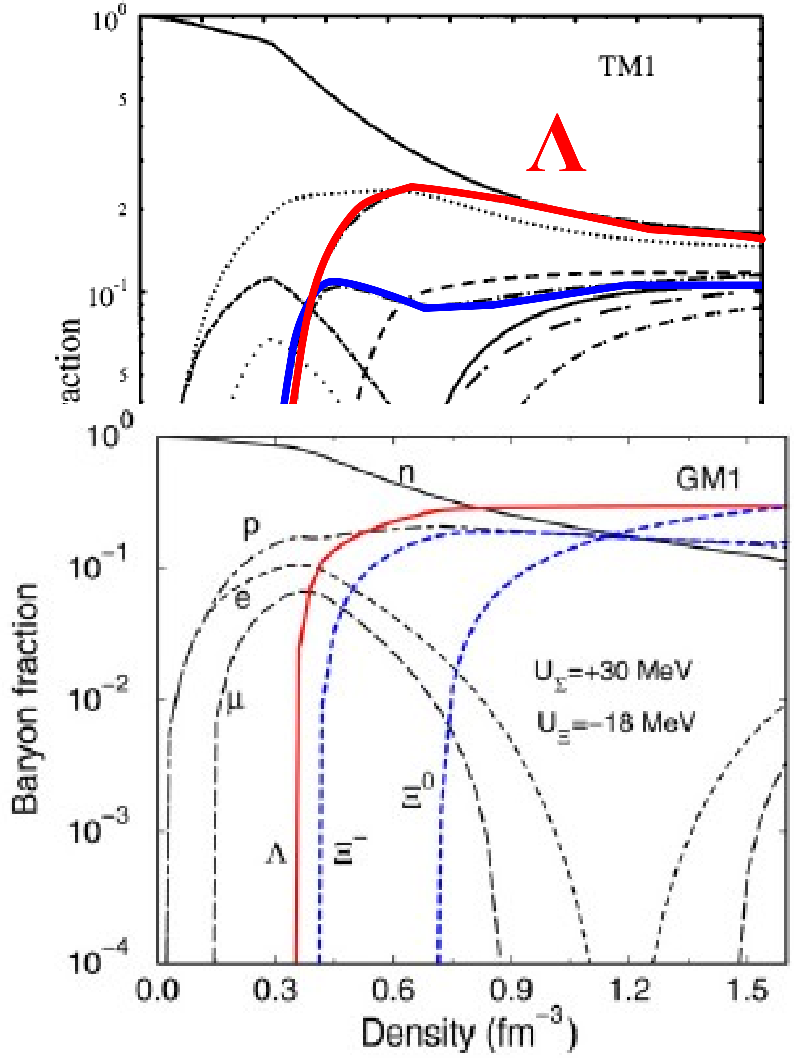
    M. Kobayashi, T. Maskawa, PTP44('70)1422;
    G. 't Hooft, PRD14('76)3432.

    → $x_\sigma = 0.335$, $R_\varsigma = 0.509$



Hyperon Composition in Dense Matter

- Hyperon start to emerge at (2-3)ρ_0 in Neutron Star Matter !
- Hyperon composition in NS is sensitive to Hyperon potential.
  - U_Λ ~ -30 MeV: Well-known
  - U_Ξ ~ -(12-15) MeV
    (K^-,K^+) reaction, twin hypernuclei
    P. Khaustov et al. (E885),PRC61('00)054603;
    S. Aoki et al., PLB355('95)45.
  - U_Σ~ -30 MeV (Old conjecture)
    → Σ- appears prior to Λ
  - U_Σ > 0 (repulsive) → No Σ in NS
    Σ atom (phen. fit), QF prod.
    H. Noumi et al., PRL89('02)072301;
    T. Harada, Y. Hirabayashi, NPA759('05)143;
    M. Kohno et al. PRC74('06)064613.



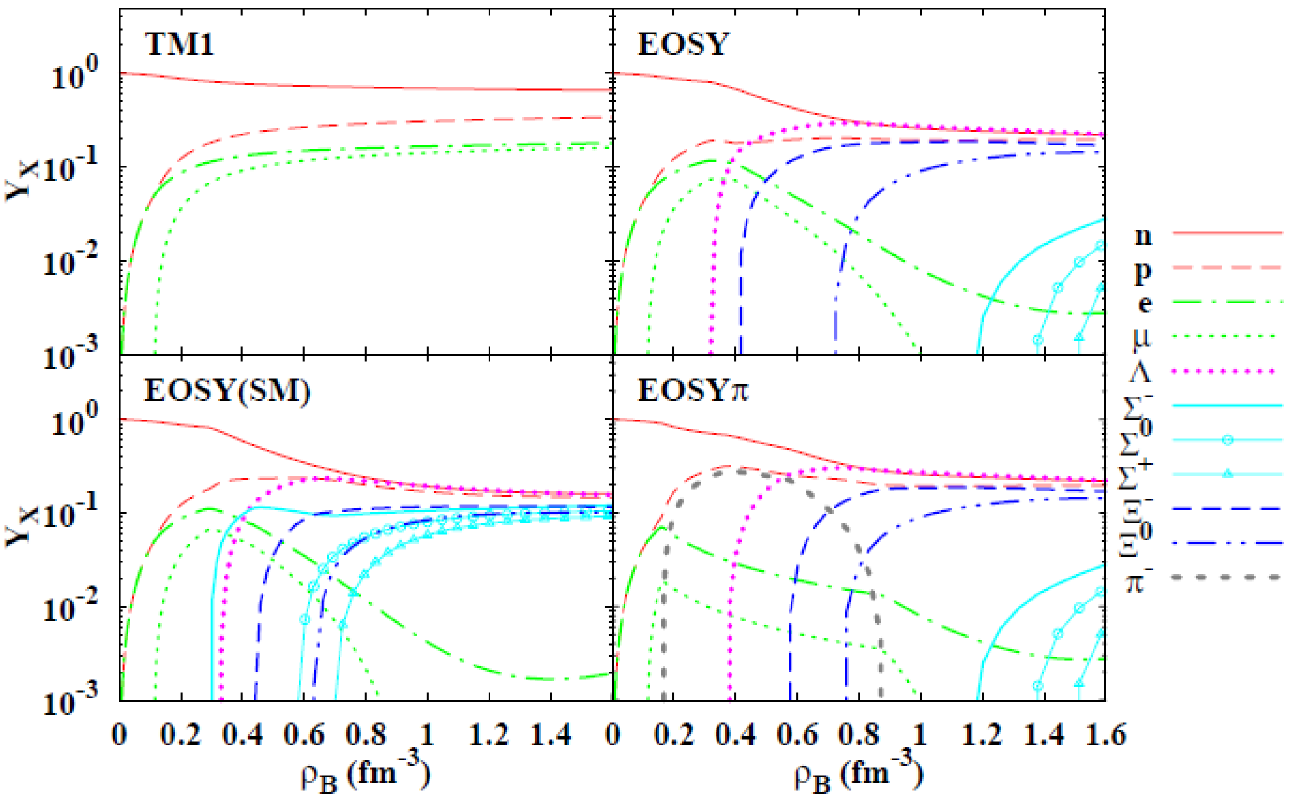
J. Schaffner-Bielich, NPA804('08)309.

Hyperon Composition in Dense Matter

- **Comparison of Hyperon Composition**
  - **U_Σ=-30 MeV, U_Ξ = -28 MeV → SU(3) sym. matter at $\rho_B \sim 10\ \rho_0$**
    Schaffner, Mishustin ('94)
  - **U_Σ=+30 MeV, U_Ξ = -15 MeV→ Σ baryons are strongly suppressed.**
    C.Ishizuka, AO, K.Tsubakihara, K.Sumiyoshi, S.Yamada, JPG35('08)085201.

→ Does Σ play no role in NS ?

Neutron Star Matter

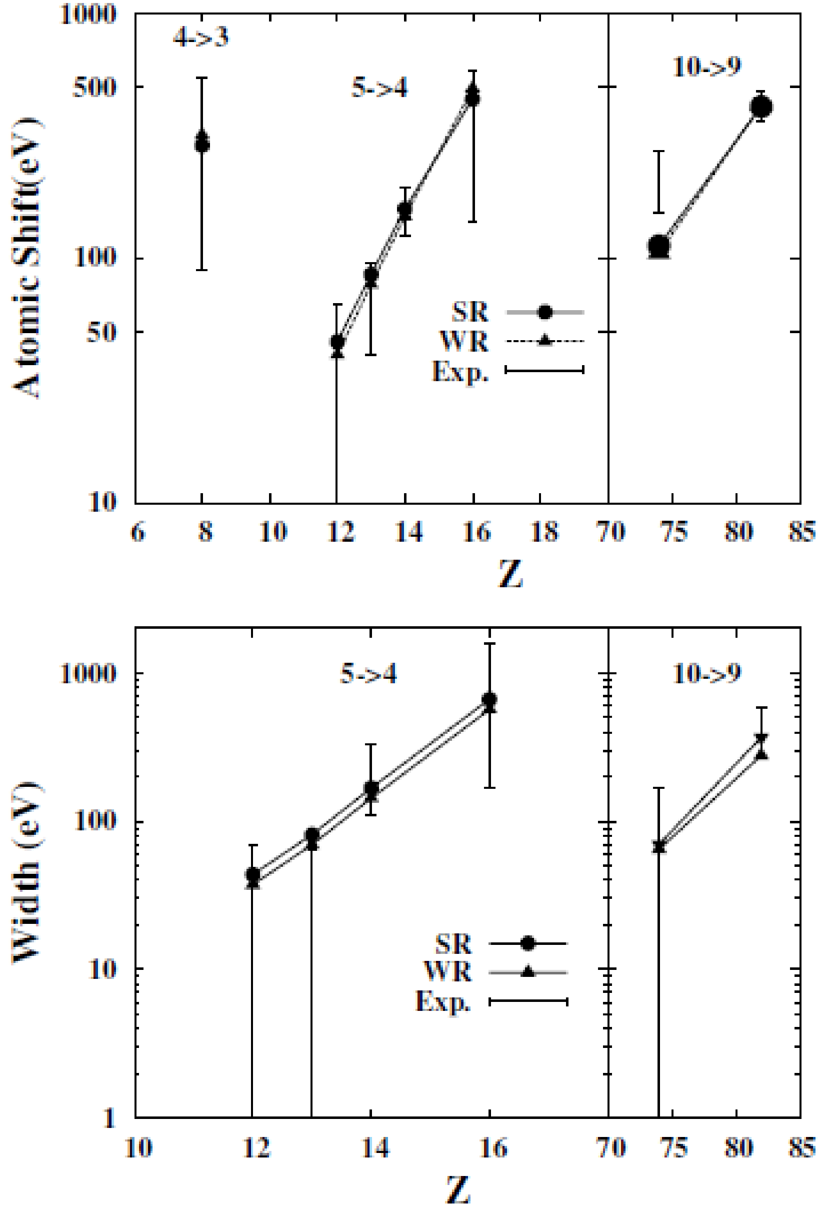


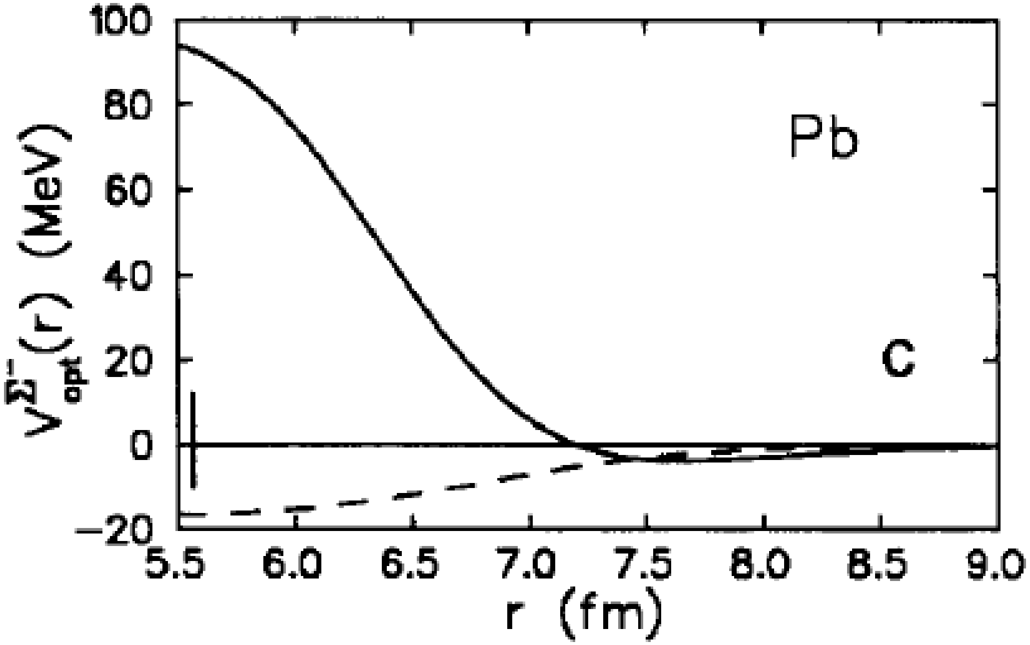
Σ^- atom data

- **Σ^- atom data suggested repulsion in the interior of nuclei !**

  C.J.Batty, E.Friedman, A.Gal, PLB335('94)273

  Batty's DD potential is very repulsive inside nuclei.
  → No Σ baryon in dense matter.





J.Mares, E.Friedman, A.Gal, B.K.Jennings, NPA594('95)311.

K.Tsubakihara, H.Maekawa, AO, EPJA33('07)295.

Σ^- atom in RMF

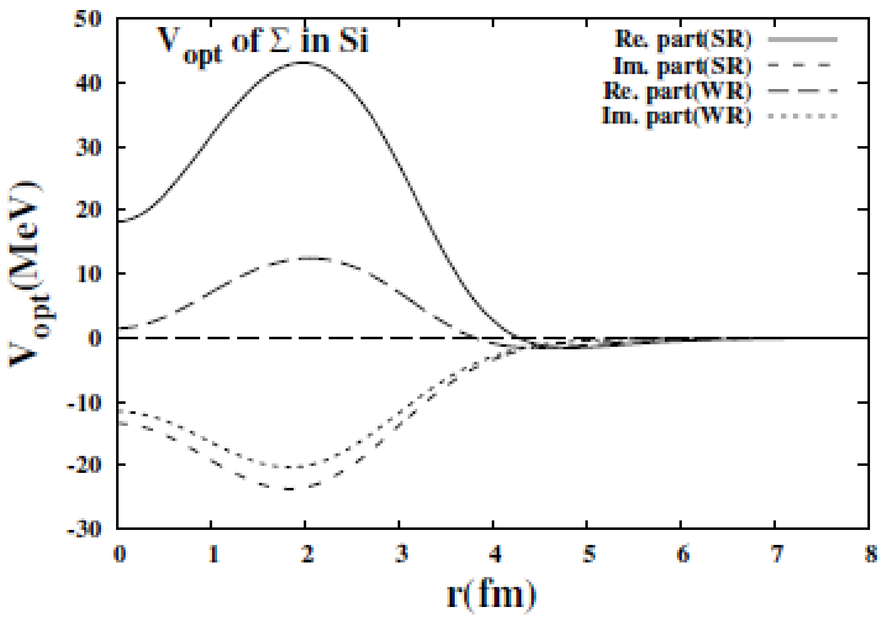
- **RMF fit of Si and Pb Σ^- atom**

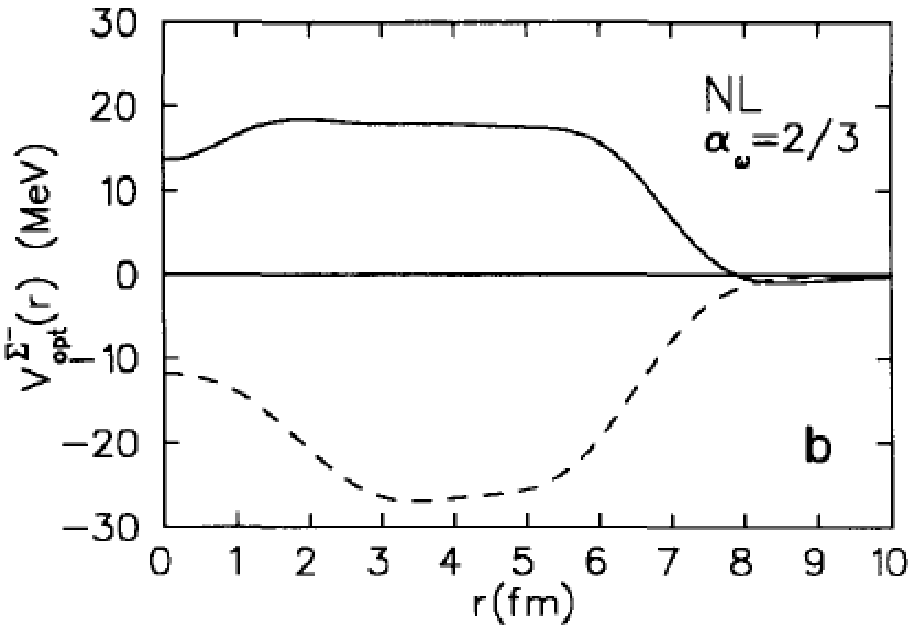
  $\alpha_\omega = g_{\omega\Sigma} / g_{\omega N} \sim 2/3(\mathrm{M}), 0.69\ (\mathrm{T})$

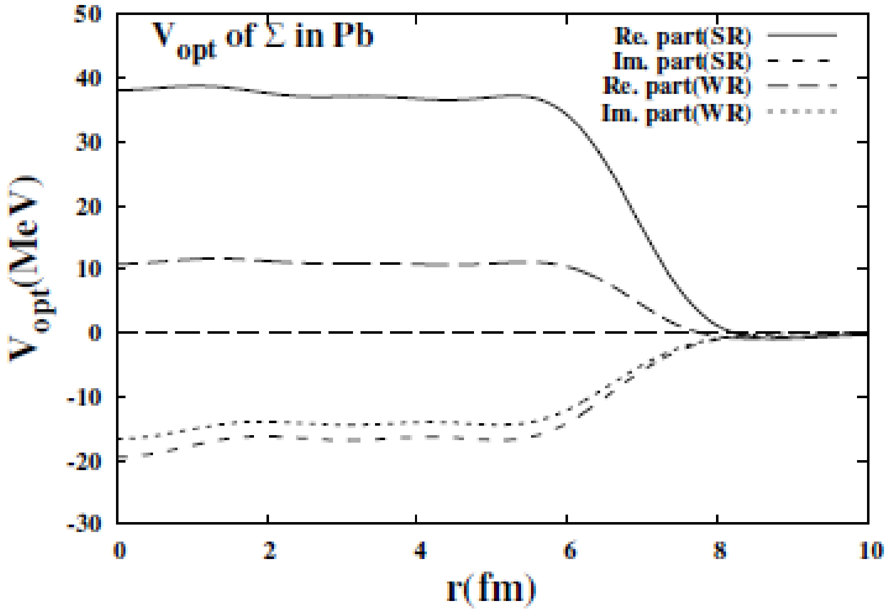
  $\alpha_\rho = g_{\rho\Sigma} / g_{\rho N} \sim 2/3(\mathrm{M}), 0.434(\mathrm{T})$

  J.Mares, E.Friedman, A.Gal, B.K.Jennings, NPA594('95)311; Tsubakihara et al.('10)

  - **Much smaller $g_{\rho\Sigma}$ than naïve SU(3) ($g_{\rho\Sigma} / g_{\rho N} = 2$), which has been applied in some of previous works.**



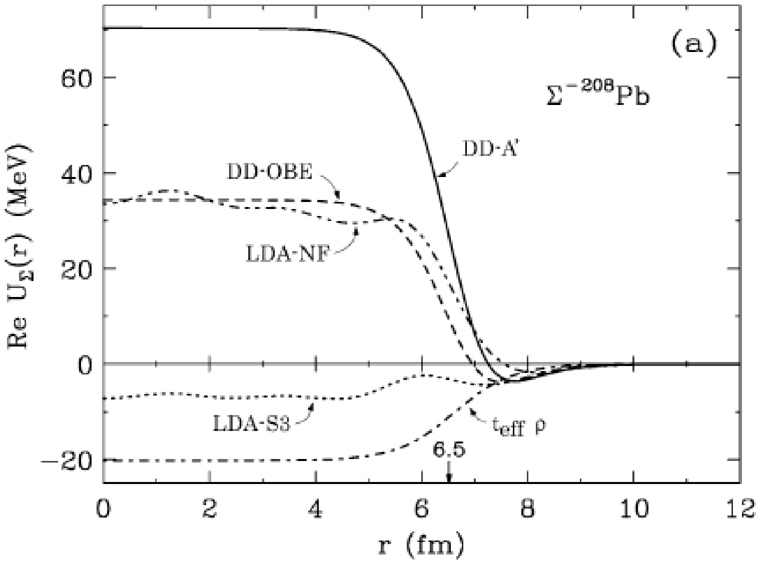




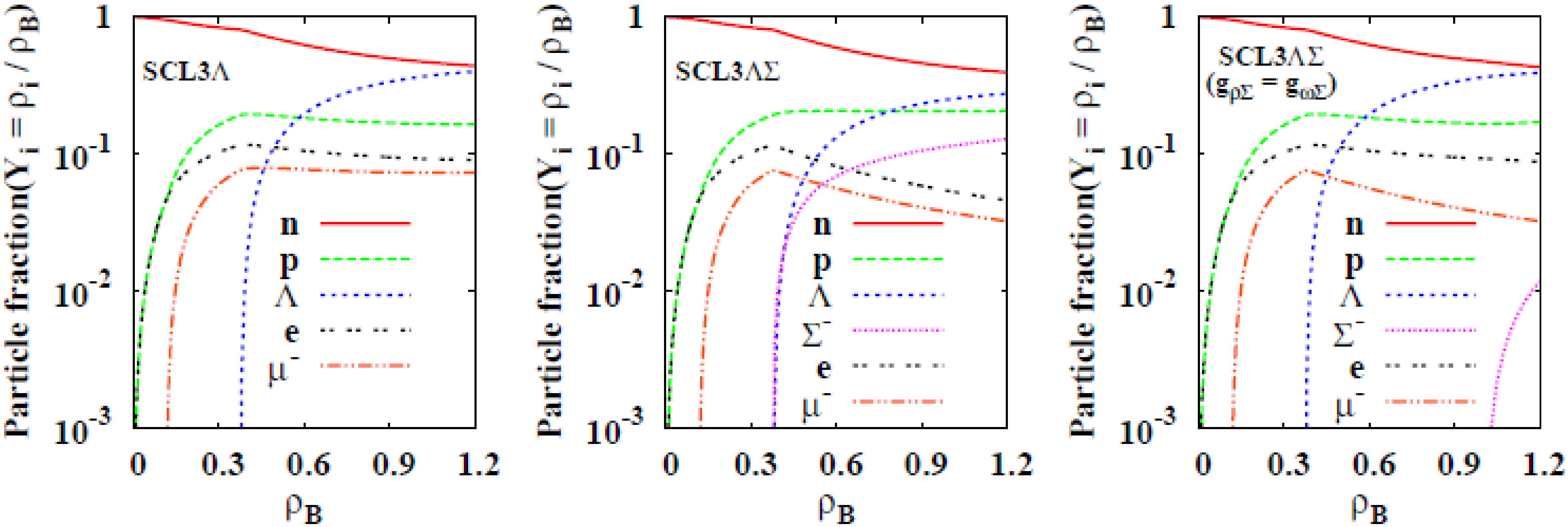
Σ atom and Neutron Star

- **Σ may not feel *very* repulsive potential in neutron star....**
  - **ρ^{γ}-type fit → very repulsive**
  - **RMF fit → small isovector potential**

**→ QF prod. may support the latter.
Σ^- would appear in NS.**



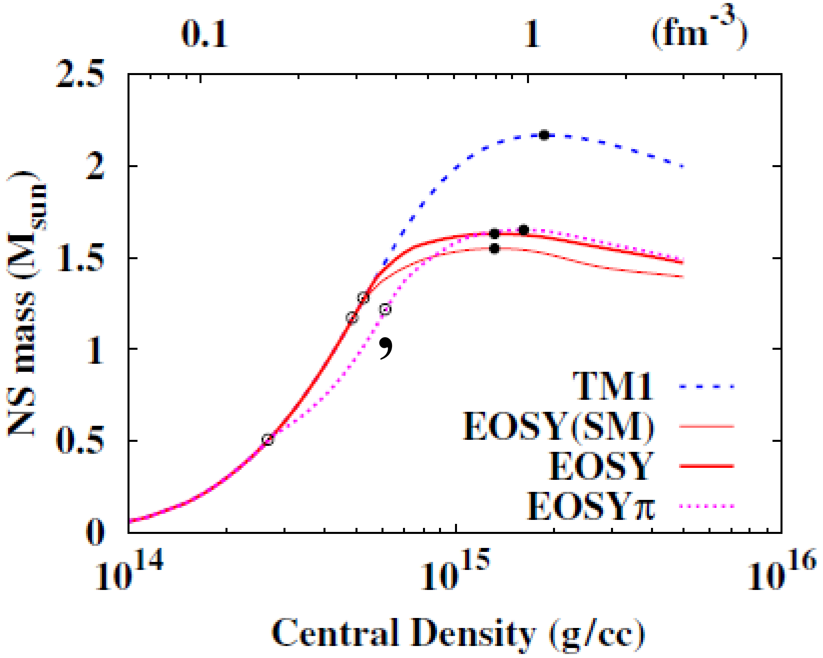
T. Harada, Y. Hirabayashi, NPA767('06)206

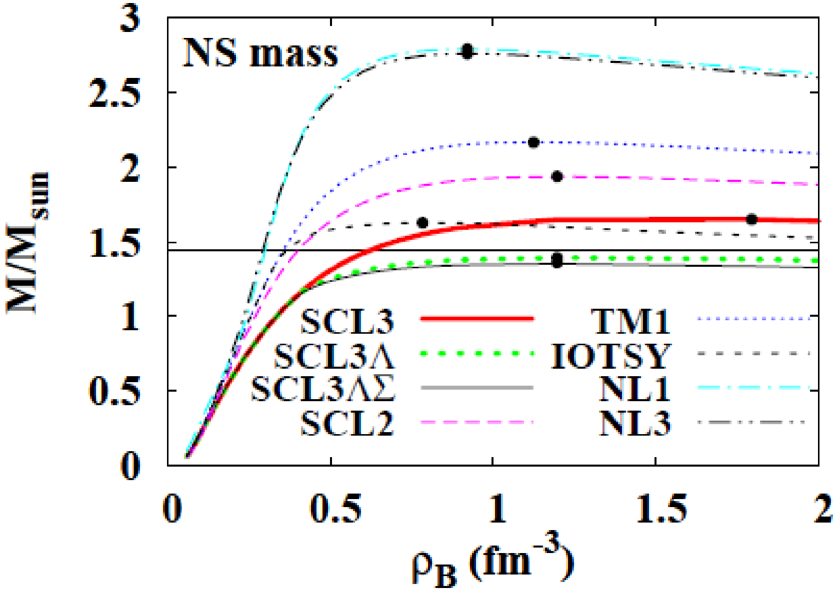


K. Tsubakihara, H. Maekawa, H. Matsumiya, AO, PRC81('10)065206.

Neutron Star Mass

- **Large fraction of hyperons softenes EOS at $\rho_B > (0.3\text{-}0.4)\ \mathrm{fm}^{-3}$**
  - **NS star max. mass red. ~ 1 M_{sun}.**
  - **RMF generally predicts stiff EOS at high density.**
    (Scalar attraction saturation,
    or Z-graph in NR view.)
  - **Some of RMF with Y do not support 1.44 M_{sun}.**
- **Additional Repulsion at high ρ ?**
  - **Vector mass mod.**
    → stronger repulsion at high ρ.
    M. Naruki et al., PRL96('06)092301.
  - **Another term such as NNωσ.**





C. Ishizuka, AO, K. Tsubakihara, K. Sumiyoshi, S. Yamada, JPG35('08)085201.
K. Tsubakihara, H. Maekawa, H. Matsumiya, AO, PRC81('10)065206.

Summary

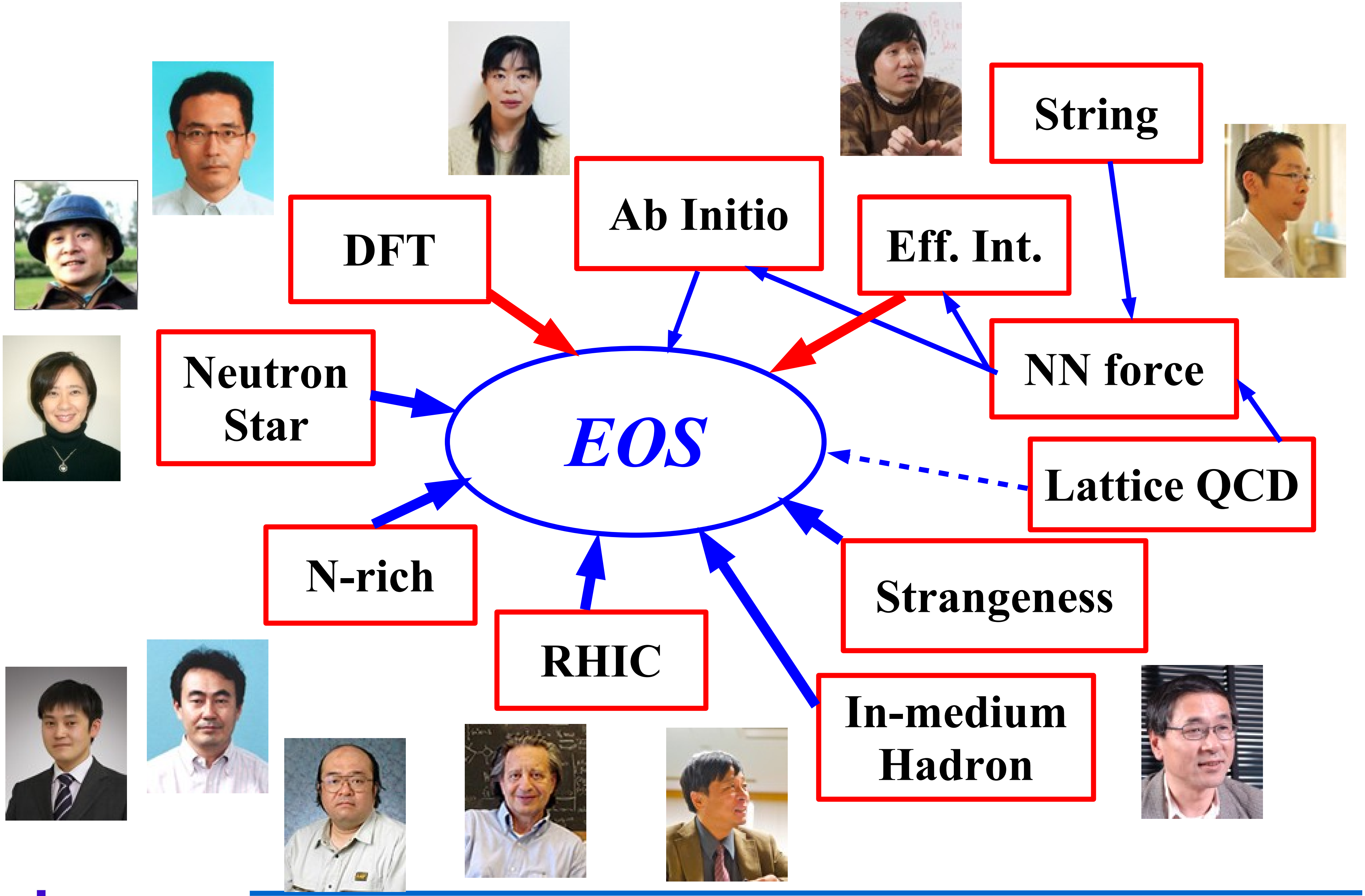
- **It is a long-standing problem to obtain Nuclear Matter EOS from 1st principle theory and/or experiments. Some constraints are obtained from recent experiments and observations.**
  - **Collective flow and Asym. relaxation in HIC, ISGMR of Sn isotopes → Pressure at high density, ρ_B deps. of Esym.**
  - **(M, R) observation of neutron star → EOS of neutron star matter**
- **First principle approaches become promising to understand EOS.**
  - **Lattice QCD (MC, SC), Ab initio approaches + chiral int., …**
- **Phenomenological approaches are also improved based on the developments in experiments and fundamental approaches.**
  - **DBHF results, Pressure from HIC, Hypernuclei, Atom data, …**

and they now start to suggest the nature of the mean field, and ...

Future works

- Theorists should further develop 1st principle / ab initio frameworks.
  - Lattice QCD, Lattice gas, diagoanization, diagonalization after reduction (eff. int.), AdS/CFT, ...
- Phenomenologists should should search for a model which satisfies the observational requirements as far as possible.
  - B.E. and rmsr of normal nuclei, hypernuclei, and exotic atoms,
    Sym. NM EOS, HIC data, Low density neutron matter,
    NS mass and radius, …
  - Low density matter is also important for SN and NS.
  - Essential point: Many-body force (bare and effective)
    RMF: σ^3, σ^4, ω^4, …
    HF, BHF, ... ρ-dep. 2 body, ρ^3, ρ^α, …
- Experimentalists should provide *key* data.
  - E.g. Isotope dep. of collective flow.

EOS and Related Physics

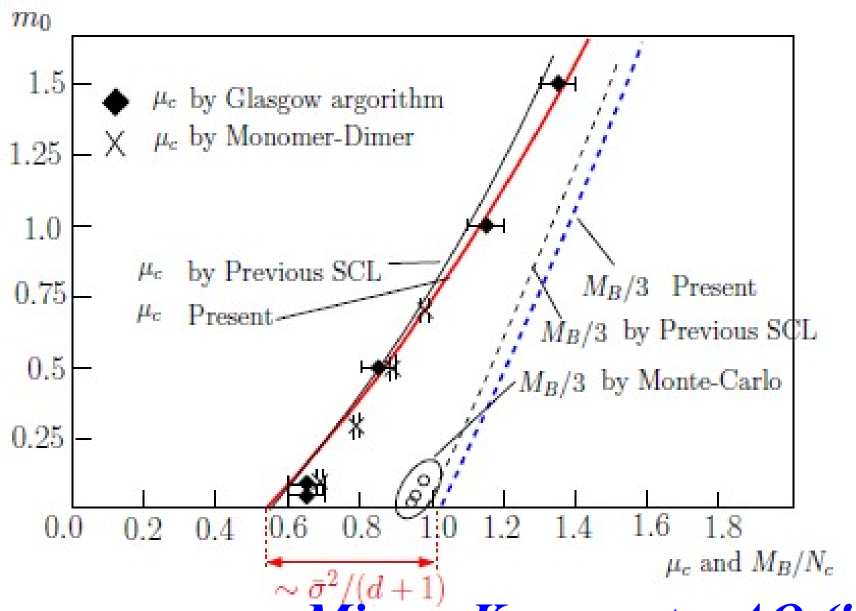


Thank you for your attention !

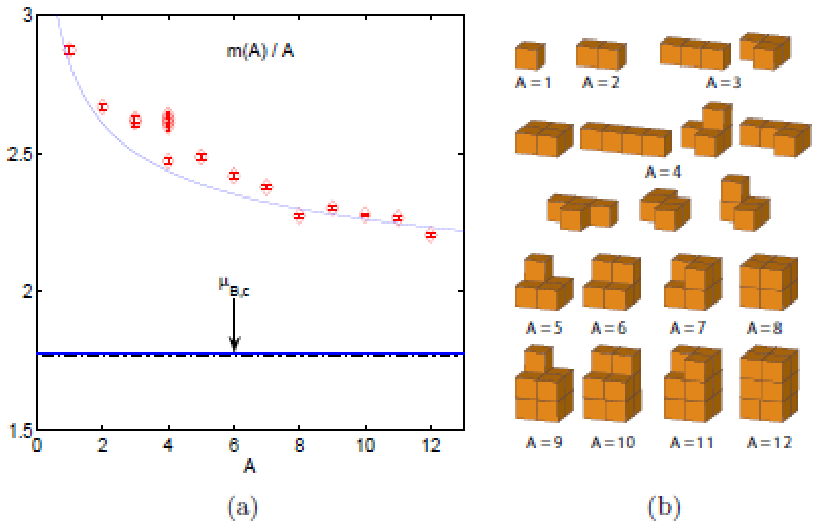
Cold Nuclear Matter in Lattice QCD

- **Baryon mass puzzle in SCL-LQCD: $N_c\mu_c < M_B$**
  → QCD phase transition takes place before baryons appear.
  Kluberg-Stern, Morel, Petersson ('83), Damgaard, Hochberg,Kawamoto ('85), Karsch, Mutter ('89), Barbour et al.('97), Bringoltz('07), Miura, Kawamoto, AO ('07)

- **Possible Solutions**
  - **Regard the matter at $\mu > \mu_c$ as nuclear matter** *de Forcrand, Fromm ('09)*
  - **Finite coupling effects: Decrease of quark mass**



Miura, Kawamoto, AO ('07)



(a) (b)

de Forcrand, Fromm ('09)

Constituent Quark Mass in NNLO SC-LQCD

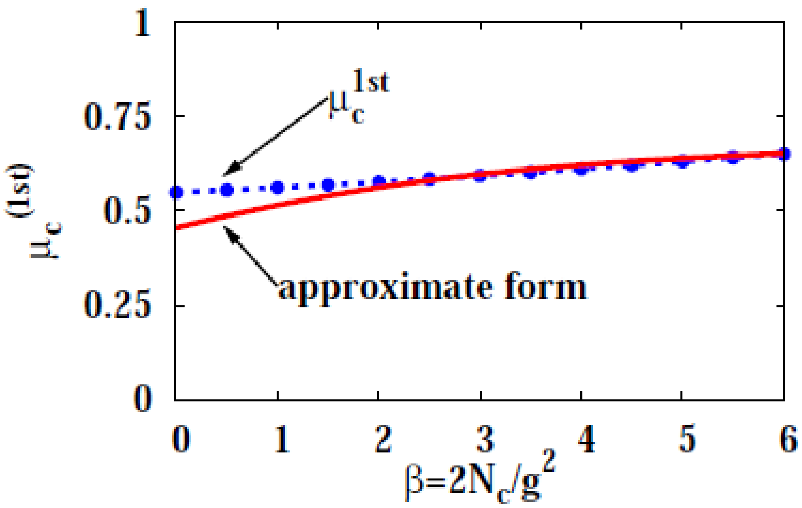
- **Mechanism of “stable” $\mu_c(T=0)$ in NLO/NNLO SC-LQCD**

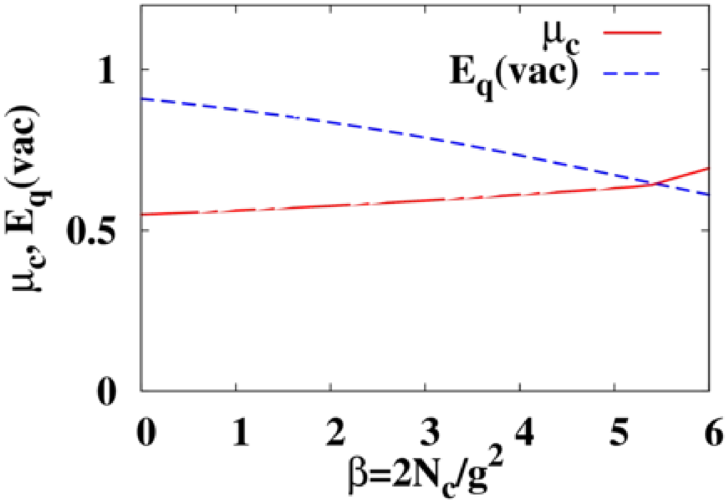
  = Effects of quark mass reduction & repulsive vector pot. cancel

  Transition Condition at $T=0: E_q(\tilde{m}_q)=\tilde{\mu}\simeq\mu-\beta'_\tau\omega_\tau$

  $\to\mu\simeq E_q(\tilde{m}_q)+\beta'_\tau\omega_\tau$

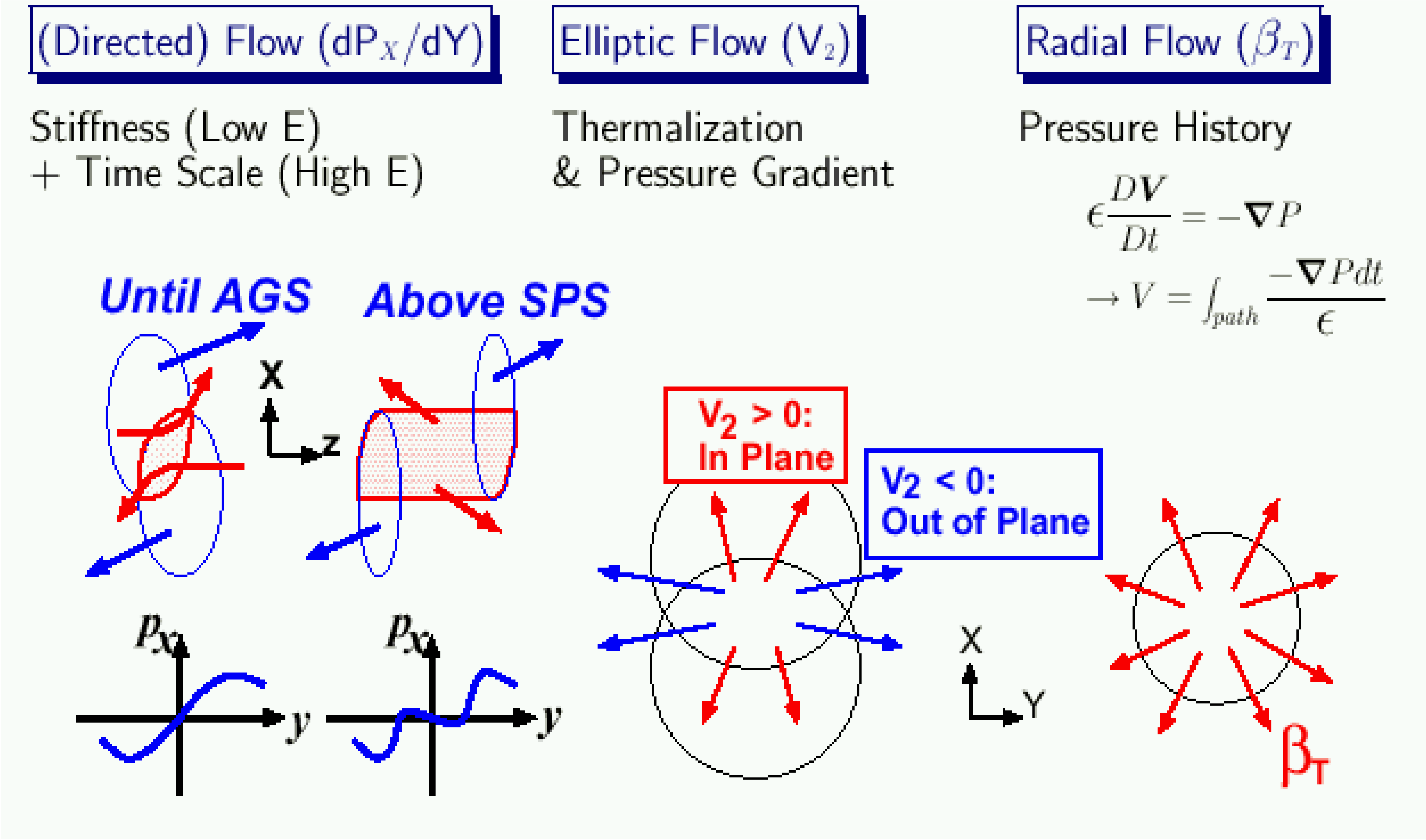
Pocket formula $\mu_{c,T=0}\simeq\frac{1}{2}\left[E_q(\sigma=\sigma_{\rm vac},\omega_\tau=0)+\delta\mu(\sigma=0,\omega=N_c)\right]$





Quark mass ($\approx E_q$) is smaller than μ_c for $\beta > 5.5$.
→ “Baryon mass puzzle” may be solved !

What is Collective Flow ?



(Directed) Flow (dP_X/dY)

Stiffness (Low E)
+ Time Scale (High E)

Elliptic Flow (V_2)

Thermalization
& Pressure Gradient

Radial Flow (β_T)

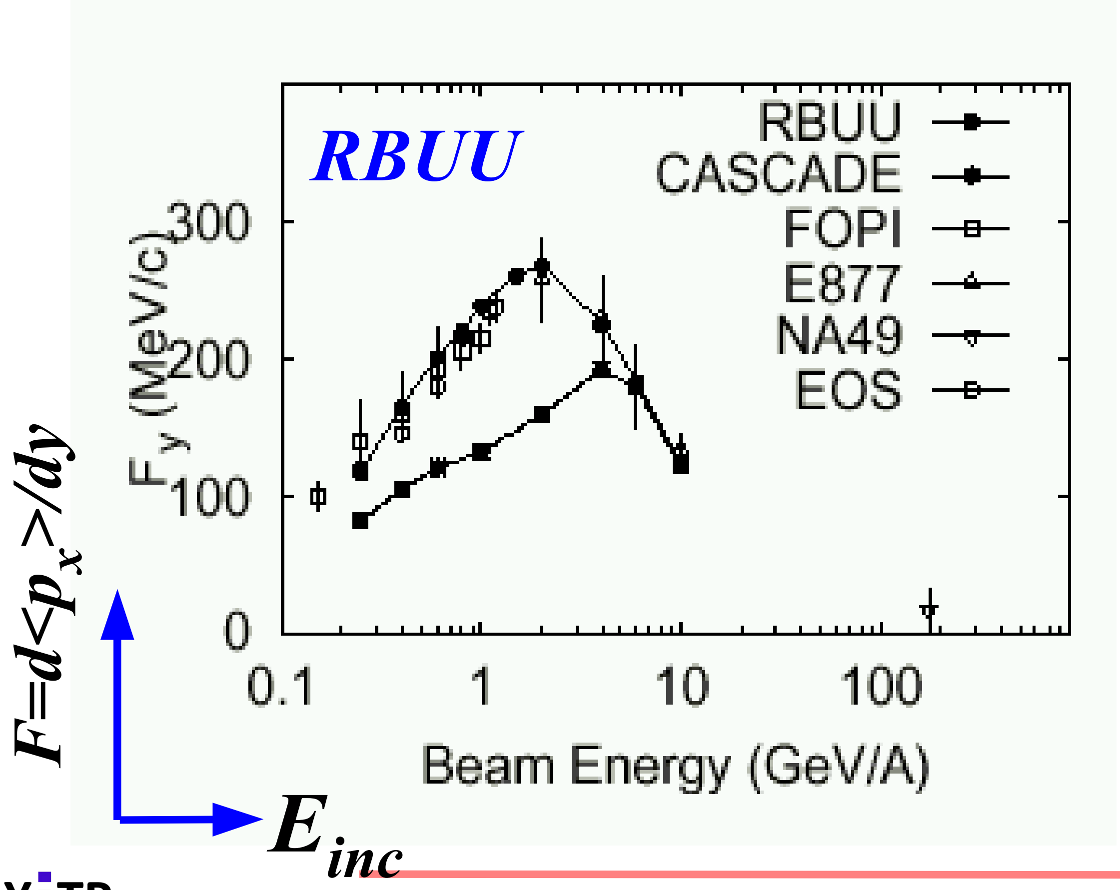
Pressure History

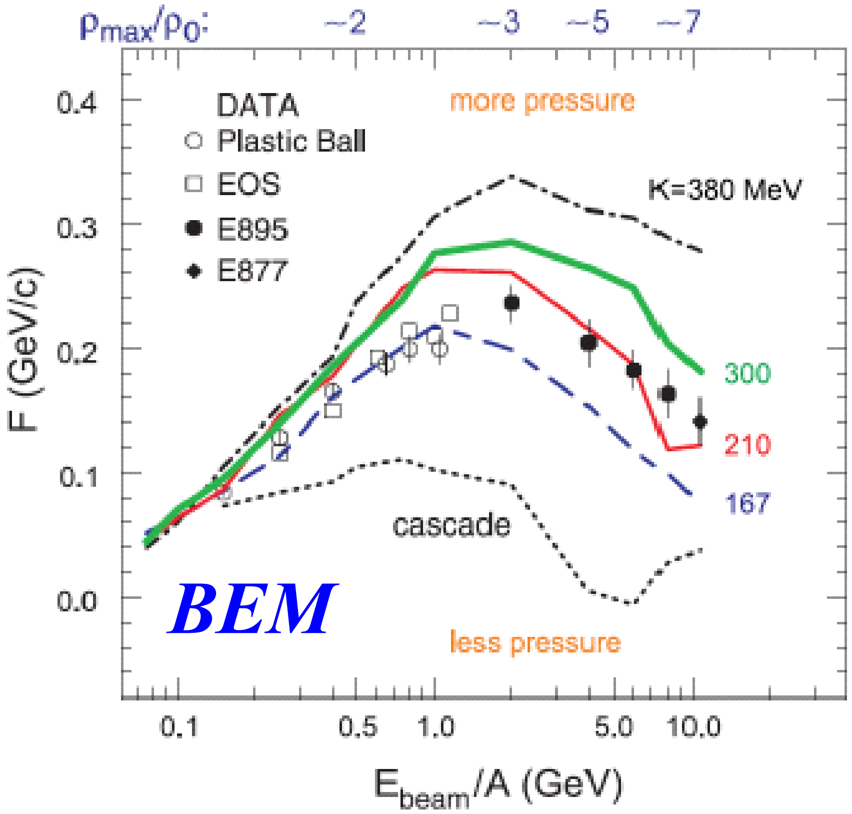
$$\epsilon \frac{D\mathbf{V}}{Dt} = -\nabla P$$

$$\rightarrow V = \int_{path} \frac{-\nabla P dt}{\epsilon}$$

Side Flow at AGS Energies

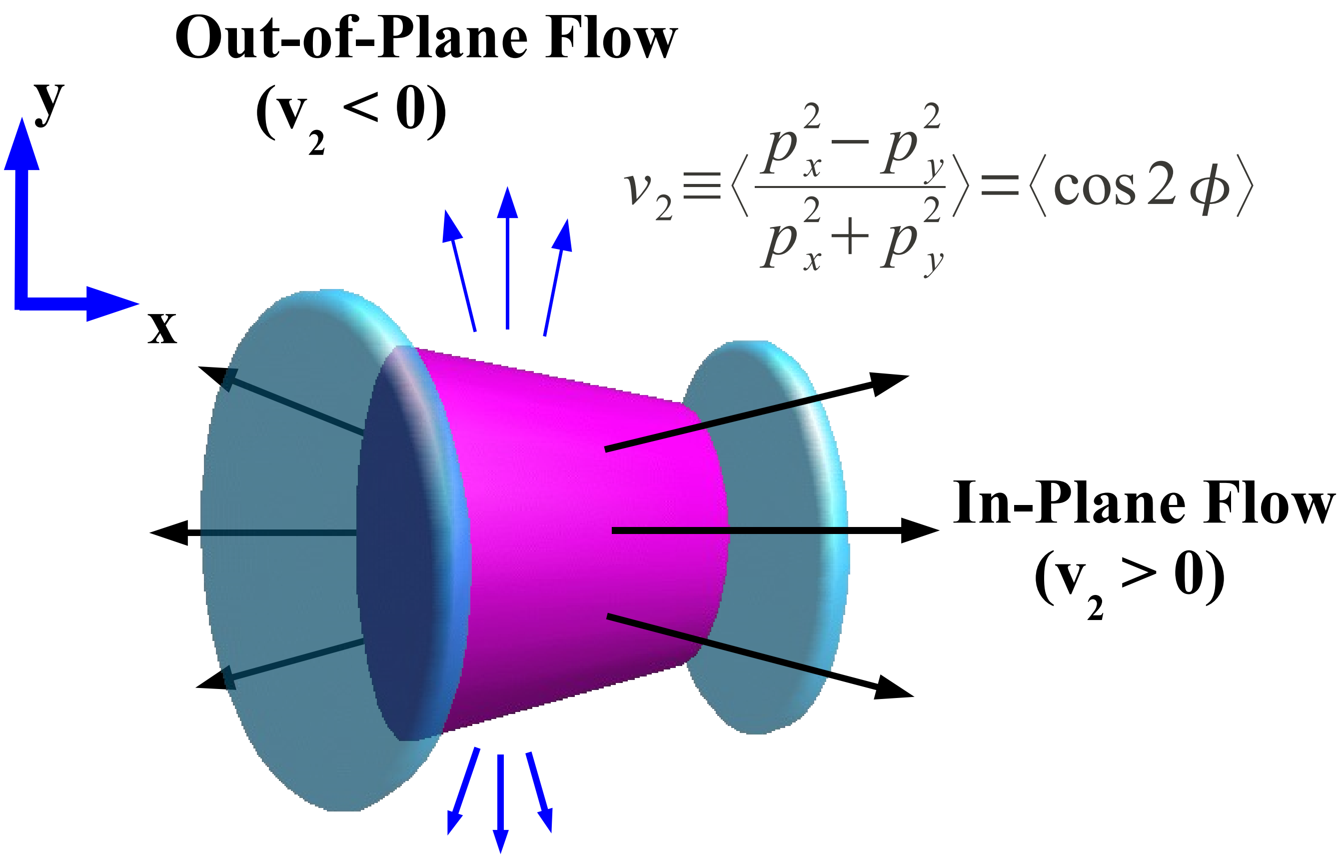
- **Relativistic BUU (RBUU) model: *K ~ 300 MeV***
  (Sahu, Cassing, Mosel, AO, Nucl. Phys. A672 (2000), 376.)
- **Boltzmann Equation Model (BEM): *K=167~210 MeV***
  (P. Danielewicz, R. Lacey, W.G. Lynch, Science 298(2002), 1592.)





Elliptic Flow

- **What is Elliptic Flow ? → Anisotropy in P space**
- **Hydrodynamical Picture**
  - **Sensitive to the Pressure Anisotropy in the Early Stage**
  - **Early Thermalization is Required for Large V2**



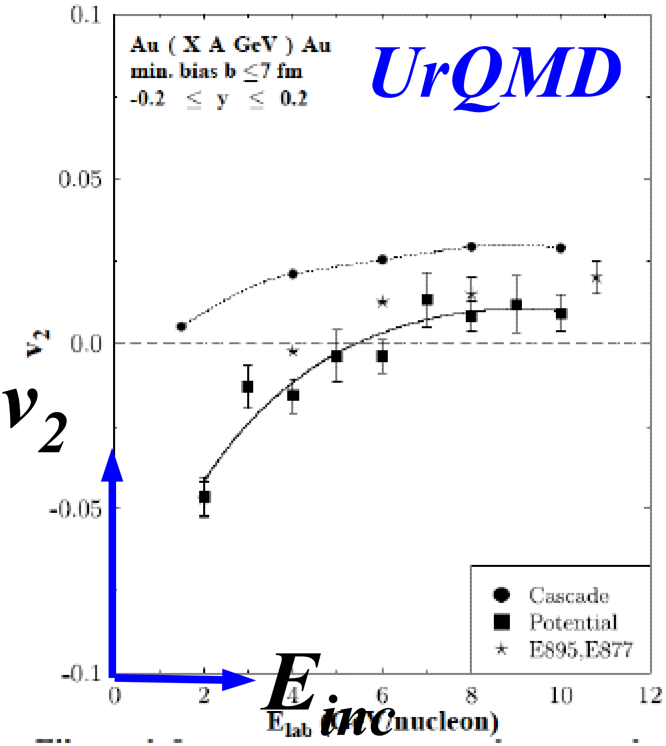
Out-of-Plane Flow
($v_2 < 0$)

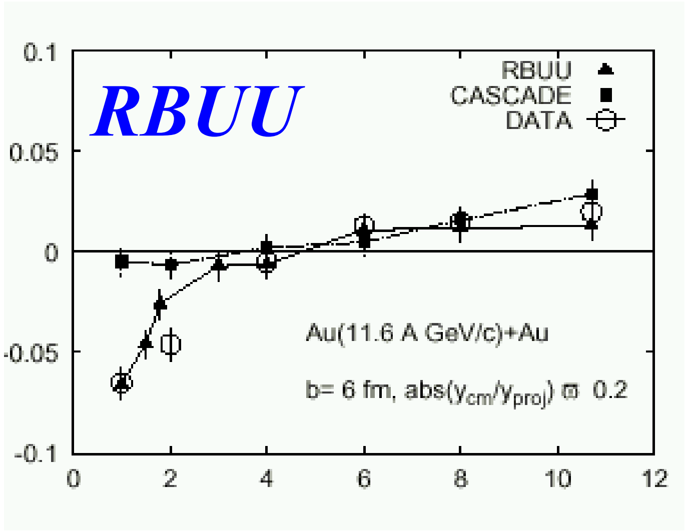
$$v_2 \equiv \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle = \langle \cos 2\phi \rangle$$

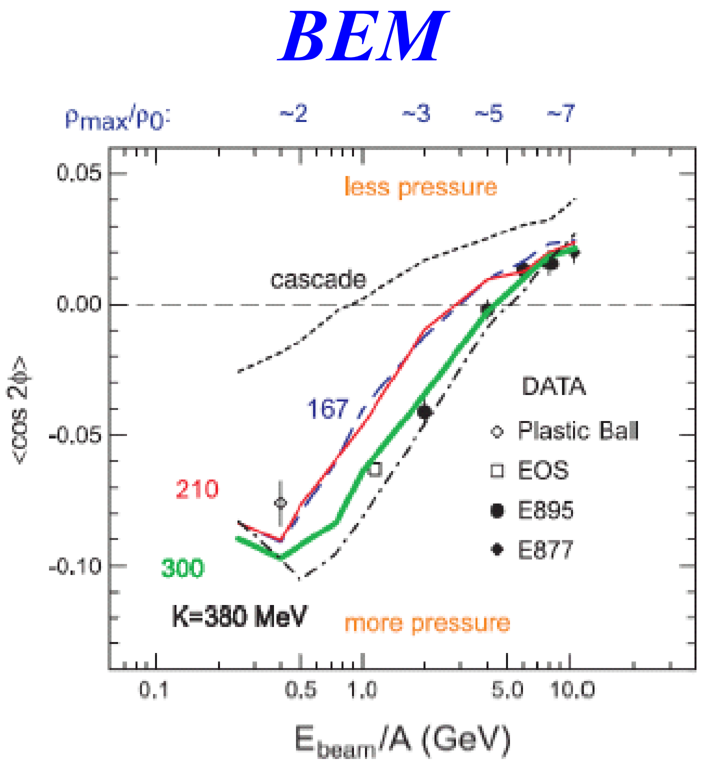
In-Plane Flow
($v_2 > 0$)

Elliptic Flow at AGS

- **Strong Squeezing Effects at low E (2-4 A GeV)**
  - **UrQMD: Hard EOS (S.Soff et al., nucl-th/9903061)**
  - **RBUU (Sahu-Cassing-Mosel-AO, 2000): K ~ 300 MeV**
  - **BEM(Danielewicz2002): K = 167 → 300 MeV**

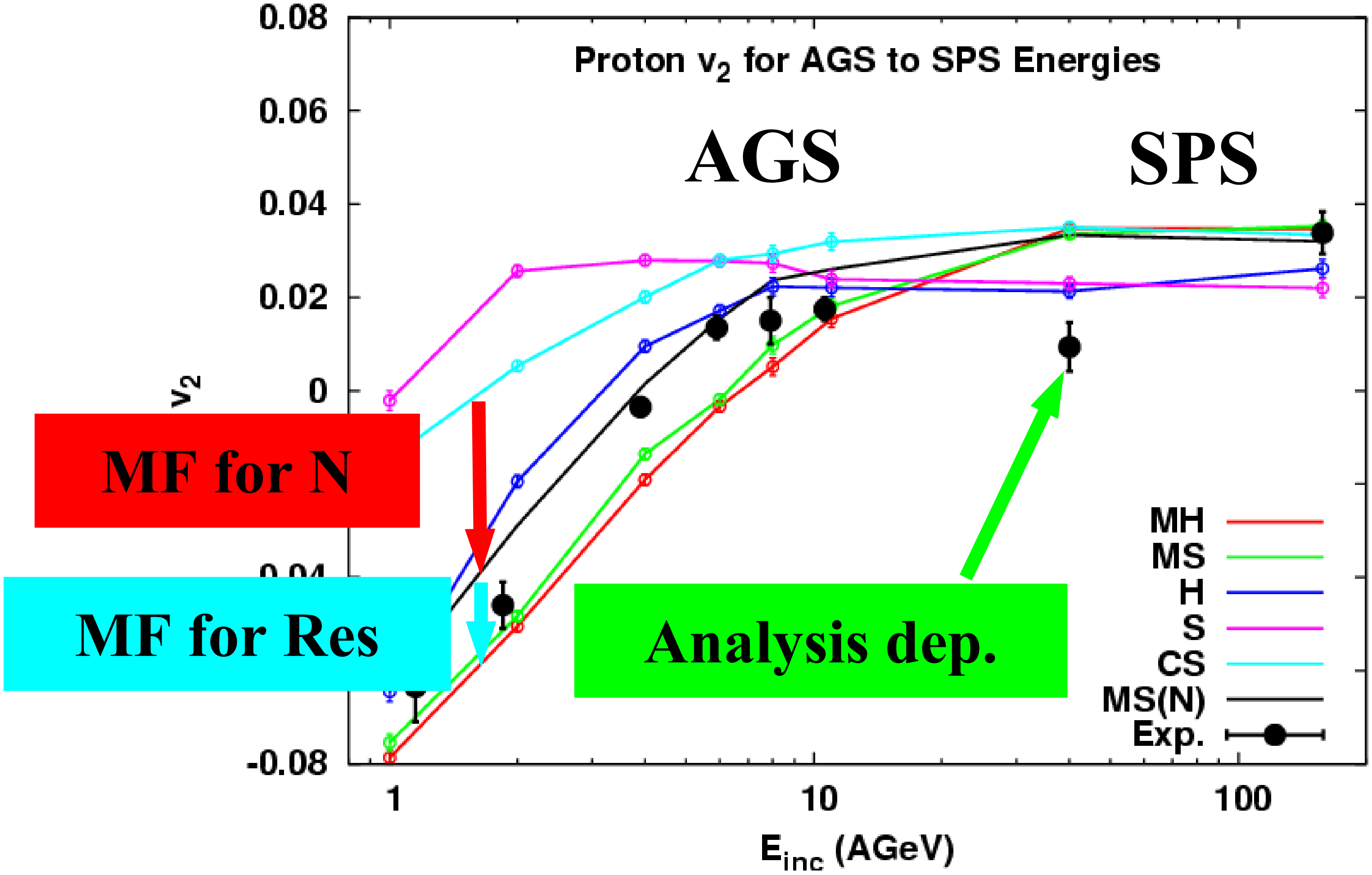






Elliptic Flow from AGS to SPS

- **JAM-MF with p dep. MF explains proton v2 at 1-158 A GeV**
  - **v2 is not very sensitive to K (incompressibility)**
  - **Data lies between MS(B) and MS(N)**



MF for N

MF for Res

Analysis dep.