強結合格子 QCD から物質の相図と状態方程式へ
––– 現状と展望 –––

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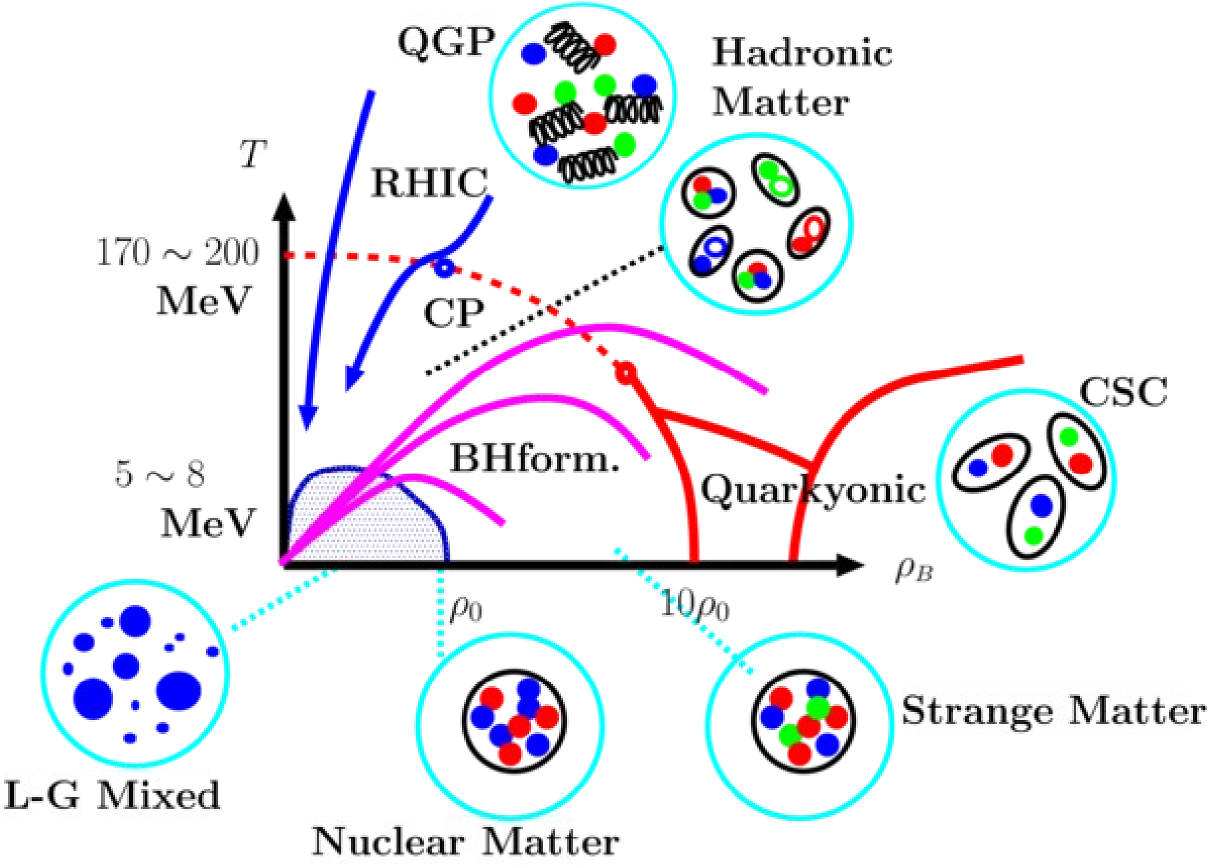
- **Introduction**
- **Finite Coupling Effects in Strong Coupling Lattice QCD**
- **Polyakov Loop Effects**
- **Conclusion**

Miura,Nakano,AO,Kawamoto, PRD80('09),074034.
Nakano, Miura, AO, PTP123('10)825.
Nakano, Miura, AO, arXiv:1009.1518
Nakano, Miura, AO; Miura, Nakano, AO, Kawamoto (LAT10)

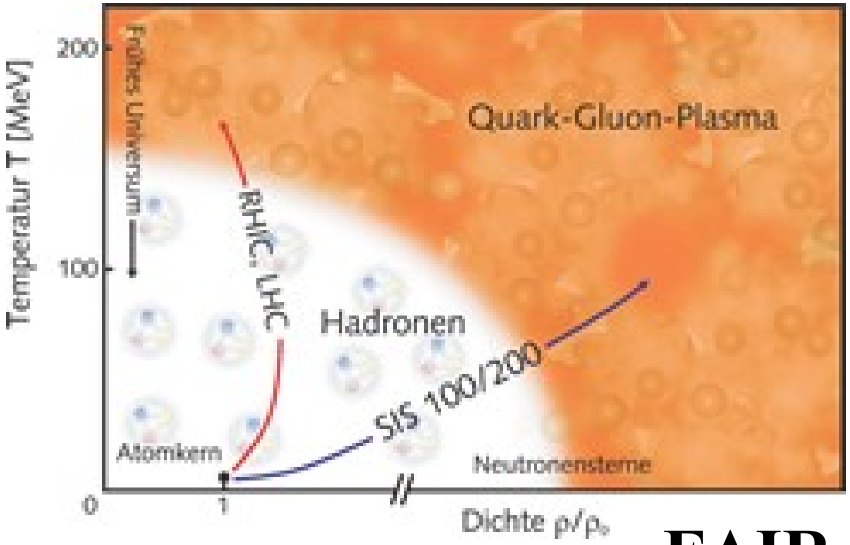
QCD Phase diagram

- **Phase transition at high T → Lattice MC, RHIC, LHC**
- **High μ transition has rich physics**
  → Various phases, CEP, Astrophysical applications, …
  but Lattice MC has sign problem at finite density.

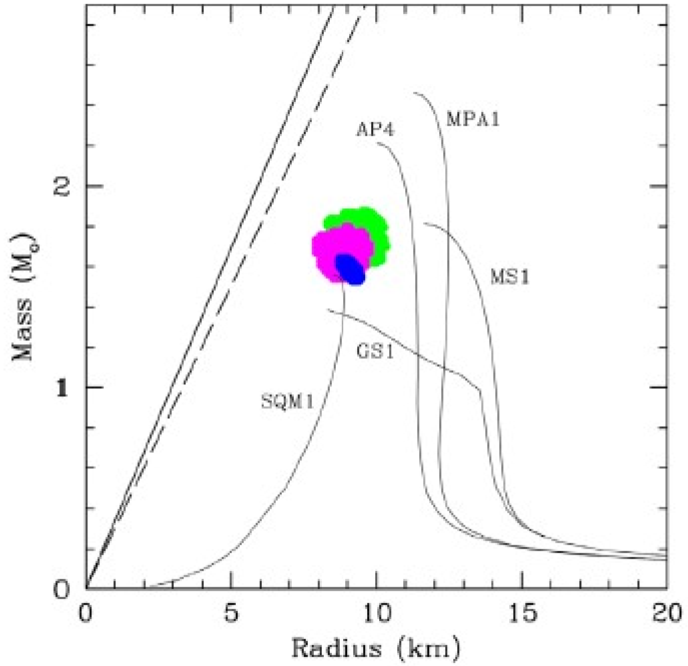


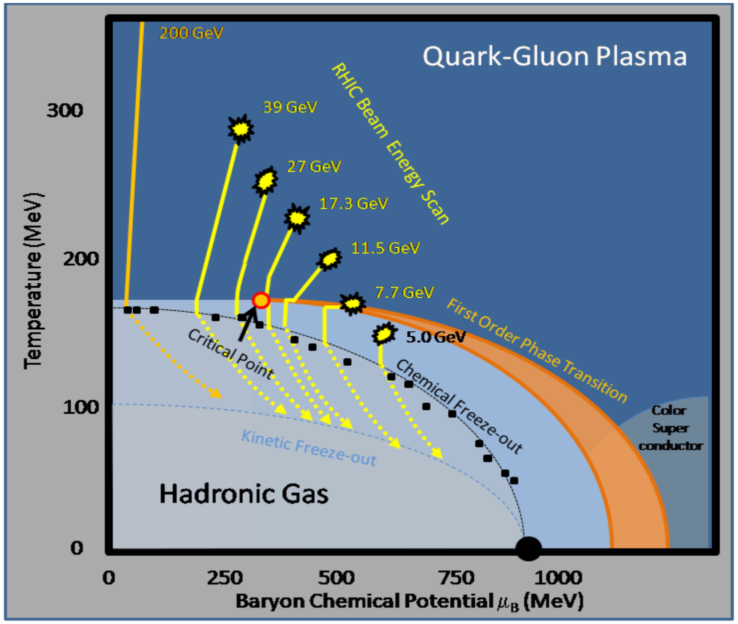
Experimental & Observational Approaches

- **FAIR / Low E prog. of RHIC aim at searching for baryon rich QGP and Critical End Point.**
- **Neutron Star observation of radius & mass (simultaneously) reveals EOS of dense matter.**



FAIR



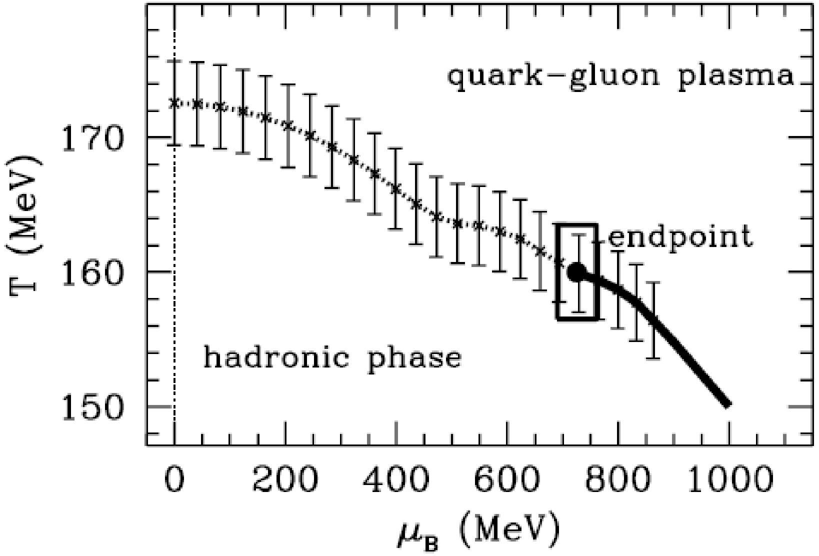


RHIC

Ozel, Baym & Guver, arXiv: 1002.3153 [astro-ph.HE]

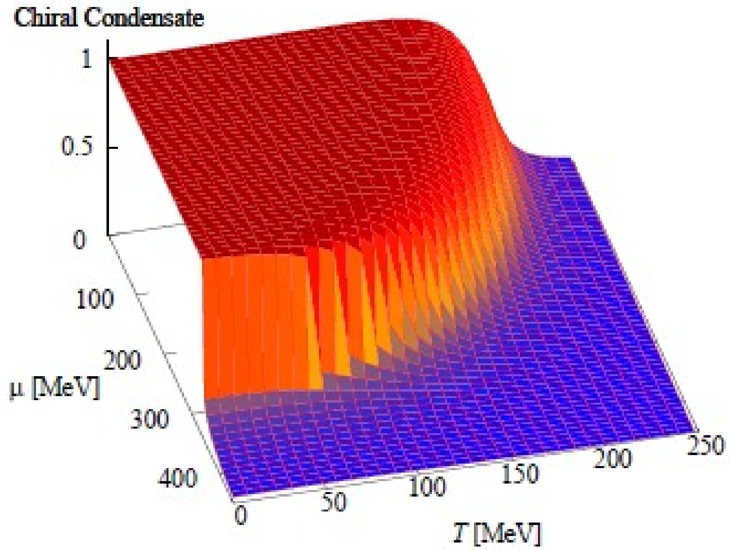
How can we attack QCD phase diagram ?

- **Lattice QCD Monte-Carlo simulation with some prescriptions**
  - **Taylor expansion, Analytic cont., Reweighting methodd, Canonical ensemble, …**
    → 符号問題を避ける様々な方法が提案されている。
    but 異なるグループで異なる結果



Z.Fodor, S.D.Katz, JHEP 0203, 014

- **Effective models of QCD**
  - **Nambu-Jona-Lasinio (NJL), Polyakov NJL (PNJL), Random matrix, ...**
    → MC-LQCD で計算可能な領域や実験データを再現し、信頼性を高めた有効模型
    but まだ大きな模型依存性



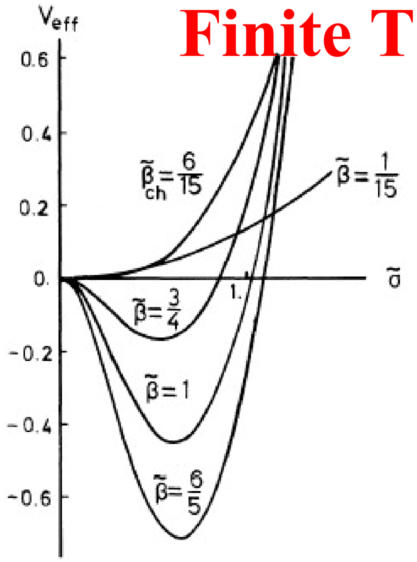
K.Fukushima, PRD77('08)114028.

Another Approach: Strong Coupling Lattice QCD

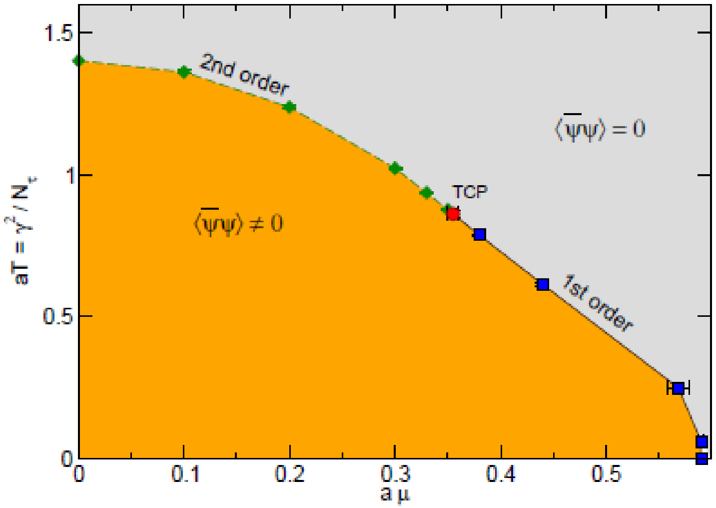
- 強結合領域ではグルーオンの伝播項は小さい
  → Plaquette (グルーオンループ) の少ない配位が支配的
  → 1/g2 の次数を決めて、グルーオンの積分を解析的に実行
- Pure Yang-Mills / 強結合極限での大きな成功
  - クォークの閉じ込め
  - カイラル対称性の破れと回復
  - MC 計算による相図

Wilson('74), Munster ('81)

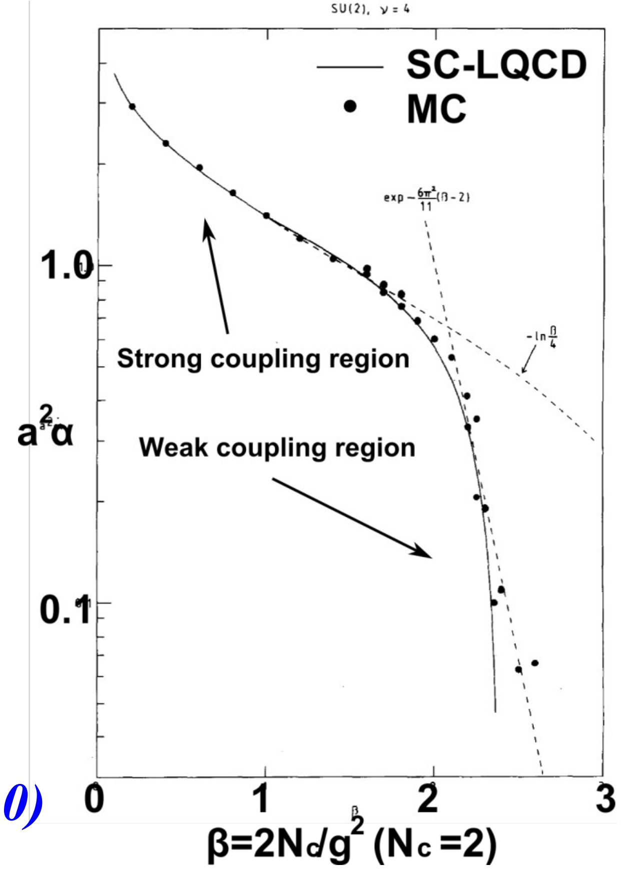
Finite T, U(3)



Damgaard,Kawamoto,Shigemoto ('84)



de Forcrand, Fromm ('10)



Towards the phase diagram in the real world

- **Fermion を含む場合、強結合極限 (Strong coupling limit; SCL) では大きな成功を収めたが ...**
  - **強結合極限 (g → ∞)**
  - **Staggered fermion (連続領域で $N_f \to 4$)**
  - **解析的な計算では Large d 近似 (1/d 展開の LO) ＋平均場近似**
  - **Polyakov loop (非閉じ込め相転移の秩序変数) を含まない**
    → 非閉じ込め相転移が記述できない
  - **バリオン質量問題 ($N_c\,\mu_c < M_N$; 核物質ができる前に相転移)**
- **有限結合効果・ポリアコフループ効果を含む強結合格子 QCD**

NLO ($1/g^2$): Miura,Nakano,AO,Kawamoto, PRD80('09),074034.
NNLO ($1/g^4$): Nakano, Miura, AO, PTP123('10)825.
Polyakov loop: Nakano, Miura, AO, arXiv:1009.1518
Miura, Nakano, AO, Kawamoto (LAT10)

→ 現実の QCD 相図の理解に向けた研究の進展

SC-LQCD: Setups & Disclaimer

- **Setups & Disclaimer**
  - **Effective action in SCL ($1/g^0$), NLO ($1/g^2$), NNLO ($1/g^4$) terms and Polyakov loop.**

    NLO: Faldt-Petersson ('86), Bilic-Karsch-Redlich ('92)
    Conversion radius > 6 in pure YM ? Osterwalder-Seiler ('78)
  - **One species of unrooted staggered fermion (N_f=4 @ cont.)**

    Moderate N_f deps. of phase boundary: BKR92, Nishida('04), D'Elia-Lombardo ('03)
  - **Leading order in 1/d expansion (d=3=space dim.)**
    → Min. # of quarks for a given plaquette configurations,
    no spatial B prop.
  - **Effective potential is obtained in mean field approximation**
  - **Different from "strong couling" in "large N_c"**

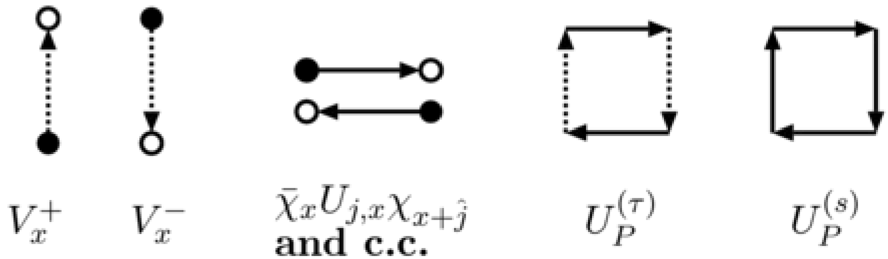
Still far from "Realistic", but SC-LQCD would tell us useful qualitative features of the phase diagram and EOS.

Effective Potential in NLO and NNLO Strong-Coupling Lattice QCD

Strong Coupling Lattice QCD

- **Lattice QCD action**

$$S_{LQCD}=\frac{1}{2}\sum_{x}[V_x^+-V_x^-]+m_0\sum_{x}M_x+\frac{1}{2}\sum_{x,j}\eta_{j,x}(\bar{\chi}_x U_{j,x}\chi_x-\bar{\chi}_{x+j}U_{j,x}^+\chi_x)$$
$$+\frac{1}{g^2}\sum_{P}(U_P+U_P^+)$$



Mesonic composites

$$M_x=\bar{\chi}_x\chi_x,\quad V_x^+=e^{\mu}\bar{\chi}_x U_{0,x}\chi_{x+\hat{0}},\quad V_x^-=e^{-\mu}\bar{\chi}_{x+\hat{0}}U_{0,x}^+\chi_x$$

- **Effective Action & Effective Potential (free energy density)**

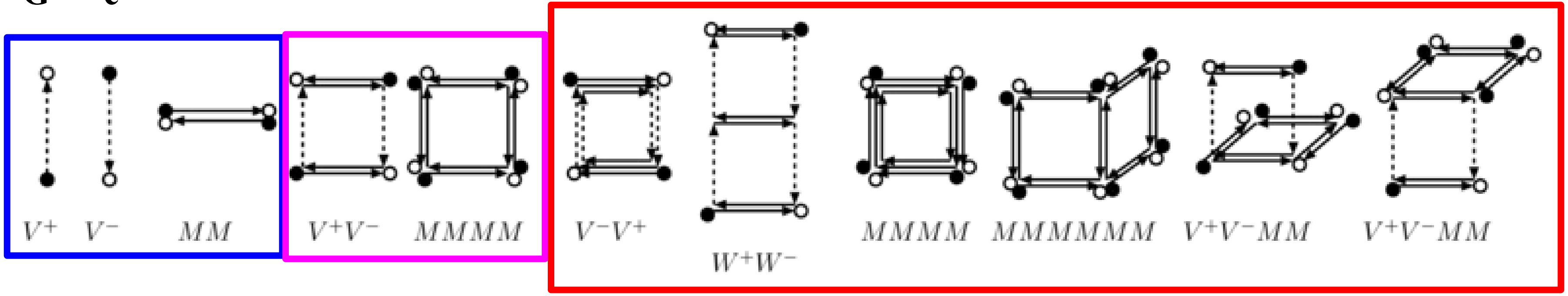
$$Z=\int D[\chi,\bar{\chi},U_{0,}U_j]\exp(-S_{\mathrm{LQCD}})$$
$$=\int D[\chi,\bar{\chi},U_0]\exp(-S_{\mathrm{SCL}})\langle\exp(-S_G)\rangle \qquad (U_j\,\text{integral})$$
$$\approx\int D[\chi,\bar{\chi},U_0]\exp(-S_{\mathrm{eff}}[\chi,\bar{\chi},U_{0,}\Phi_{\mathrm{stat.}}]) \qquad (\text{bosonization})$$
$$\approx\exp(-V\,F_{\mathrm{eff}}(\Phi;T,\mu)/T) \qquad (\text{fermion}+U_0\,\text{integral})$$

Finite Coupling Effects

- **Effective Action with finite coupling corrections**
  Integral of $\exp(-S_G)$ over spatial links with $\exp(-S_F)$ weight → S_{eff}

$$S_{\mathrm{eff}} = S_{\mathrm{SCL}} - \log\langle \exp(-S_G)\rangle = S_{\mathrm{SCL}} - \sum_{n=1} \frac{(-1)^n}{n!}\langle S_G^n\rangle_c$$

$\langle S_G^n\rangle_c$=Cumulant (connected diagram contr.) *c.f. R.Kubo('62)*



$$S_{\mathrm{eff}} = \frac{1}{2}\sum_x (V_x^+ - V_x^-) - \frac{b_\sigma}{2d}\sum_{x,j>0}[MM]_{j,x}$$

SCL (Kawamoto-Smit, '81)

$$+\frac{1}{2}\frac{\beta_\tau}{2d}\sum_{x,j>0}[V^+V^- + V^-V^+]_{j,x} - \frac{1}{2}\frac{\beta_s}{d(d-1)}\sum_{x,j>0,k>0,k\neq j}[MMMM]_{jk,x}$$

NLO (Faldt-Petersson, '86)

$$-\frac{\beta_{\tau\tau}}{2d}\sum_{x,j>0}[W^+W^- + W^-W^+]_{j,x} - \frac{\beta_{ss}}{4d(d-1)(d-2)}\sum_{\substack{x,j>0,|k|>0,|l|>0\\ |k|\neq j,|l|\neq j,|l|\neq|k|}}[MMMM]_{jk,x}[MM]_{j,x+\hat{l}}$$

$$+\frac{\beta_{\tau s}}{8d(d-1)}\sum_{x,j>0,|k|\neq j}[V^+V^- + V^-V^+]_{j,x}\left([MM]_{j,x+\hat{k}} + [MM]_{j,x+\hat{k}+\hat{0}}\right)$$

NNLO (Nakano, Miura, AO, '09)

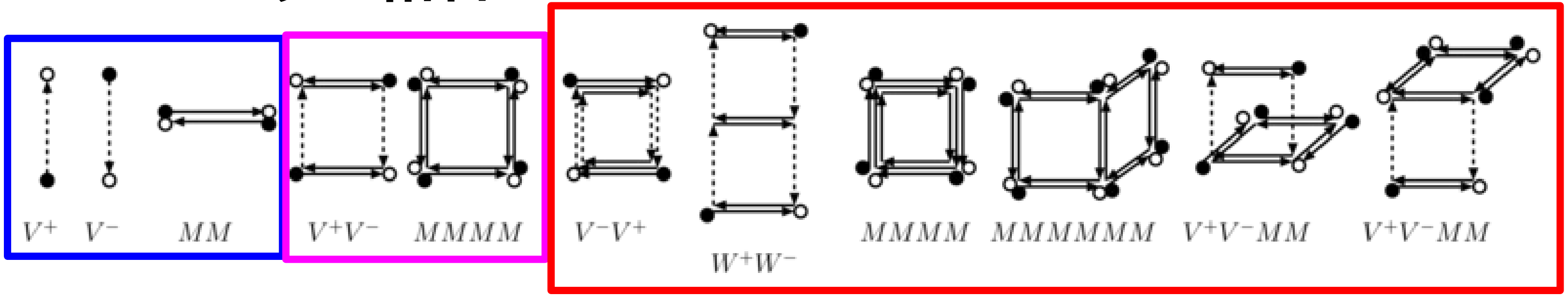
Finite Coupling Effects (cont.)

- **拡張された Hubbard-Stratonovich (EHS) 変換**
  - **異なる Composite の積の分解が可能**

Miura, Nakano, AO (09), Miura, Nakano, AO, Kawamoto (09)

$$
\begin{aligned}
&\exp(\alpha A B) \\
&= \int d\varphi\, d\phi \exp\left[-\alpha\left(\varphi^2 - (A+B)\varphi + \phi^2 - i(A-B)\phi\right)\right] \\
&\approx \exp\left[-\alpha\left(\bar{\psi}\psi - A\psi - \bar{\psi} B\right)\right]_{\text{stationary}}
\end{aligned}
$$

- **有限結合定数効果**
  - **波動関数繰り込み定数の変化 (時間方向の hopping が変化)**
  - **有効クォーク質量の変化 (4 Fermi + 8 Fermi + 12 Fermi)**
  - **有効化学ポテンシャルの変化 (ベクトル的な平均場)**
  - **ベクトル・スカラー結合**



Effective Potential in SC-LQCD with Finite Couplings

- **Effective Potential in NLO/NNLO SC-LQCD**

  Miura,Nakano,AO,Kawamoto,PRD80('09),074034;Nakano,Miura,AO,PTP123('10)825.

$$F_{\mathrm{eff}} = F_{\mathrm{eff}}^{(X)}(\sigma, \omega_\tau) + V_q(\tilde{m}_q ; \tilde{\mu}, T) - N_c \log Z_\chi$$

$$\sigma \approx \langle M \rangle \ (\text{chiral condensate}), \quad \omega_\tau \approx -\partial F_{\mathrm{eff}} / \partial \mu = \rho_q \ (\text{quark number density})$$

$$\tilde{m}_q = \frac{\tilde{b}_\sigma \sigma + m_0}{Z_\chi (1 + 4 \beta_{\tau\tau} \varphi_\tau)} \approx \frac{d}{2\mathrm{N}_c} \sigma \times \left(1 + \beta_{\sigma\sigma}^{(m)} \sigma^2 - \beta_{\sigma\omega}^{(m)} \sigma^2 \omega_\tau^2 + \ldots.\right)$$

$$\delta\mu = \mu - \tilde{\mu} = \log(Z_+ / Z_-) \approx \beta_\tau \omega_\tau \times \left(1 + \beta_{\omega\sigma}^{(\mu)} \sigma^2 + \ldots\right)$$

$$V_q(m, \mu, T) = -\frac{T}{L^d} \log\left\{ \int D[U_0] \, det(G^{-1}) \right\}$$

$$= -T \log\left[\frac{\sinh((N_c + 1) E_q(m)/T)}{\sinh(E_q(m)/T)} + 2\cosh(N_c \mu / T) \right]$$

$$E_q(m) = \mathrm{arcsinh}\, m \quad (\text{quark excitation energy})$$

NLO/NNLO SC-LQCD
≈ σω model of quarks non-linear couplings

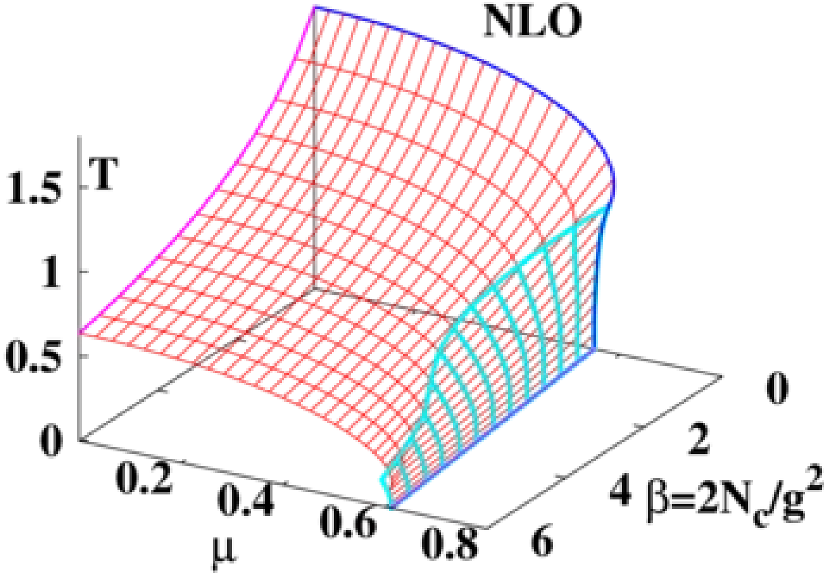
Phase Diagram Evolution

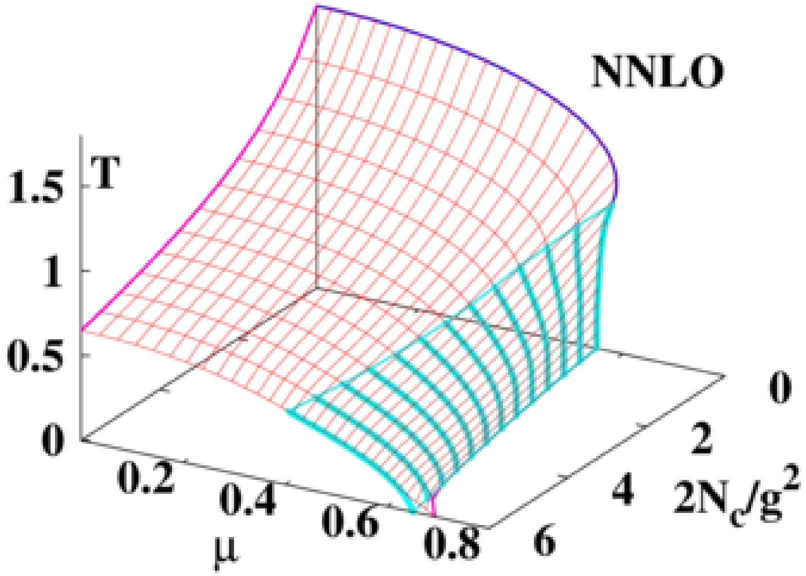
- **Shape of the phase diagram is compressed in T direction with β**

  → Improvements in $R=\mu_c/T_c$!

  - **MC (R > 1) → SCL (R = (0.3-0.45))**
    → NLO/NNLO (R ~ 1)
    → Real World (R~(2-4))

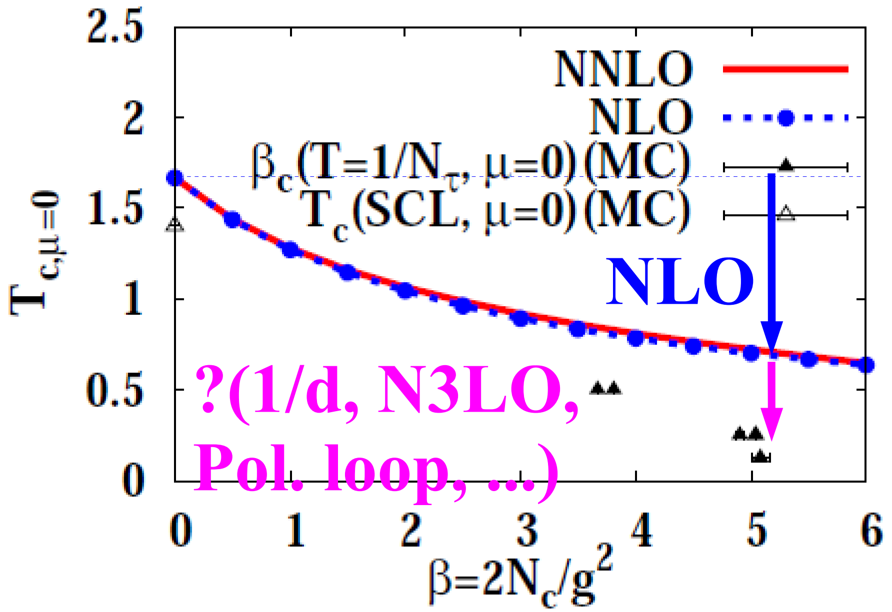
- **Critical Point**
  - **NLO: μ(CP) ~ Const.**
  - **NNLO: μ(CP) decreases with β**
    → Improvements !
    (Staggered → 1st order @ μ=0)
    D'Elia, Lombardo ('03), Pisarski, Wilczek ('84)
  - **μ(CP)/T(CP) ~ 1 ↔ MC (μ/T > 1)**
    Ejiri, ('08), Aoki et al.(WHOT,'08),
    Allton et al., ('03,'05)



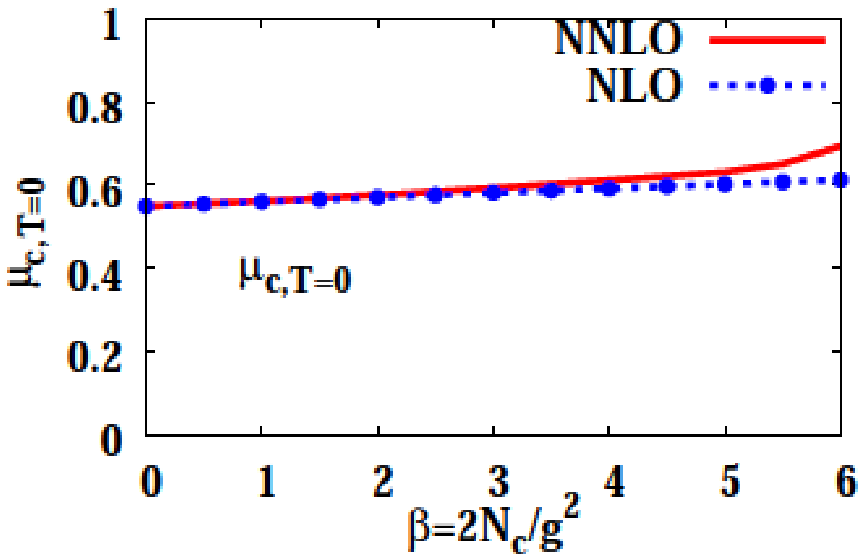


Critical Temperature and Chemical Potential

- **Critical Temperature (μ =0) → rapid decrease with $\beta=2N_c/g^2$**
  - **W.F. Renom. factor Z_χ**
    → suppression of mass
  - **T_c is still larger than MC results**
    de Forcrand ('06), Gottlieb et al. ('87),
    Gavai et al. ('90), de Forcrand, Fromm ('09)



- **Critical Chem. Pot. (T=0)**
  → weak deps. on β
  - **Suppression of mass**
    ~ Suppression of μ
  - **Consistent with previous results**
    Bilic-Demeterfi-Petersson, '92



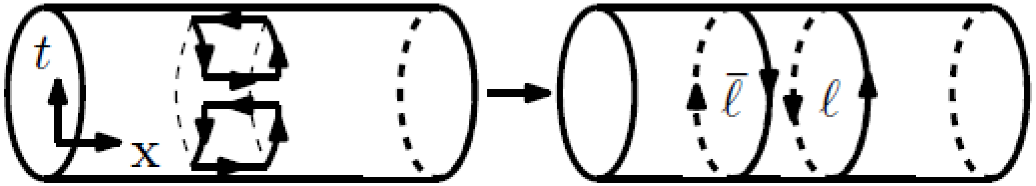
- **NNLO effects are small**
  on $T_c(\mu=0)$ and $\mu_c(T=0)$.

まだ何かが足りない.....

Polyakov Loop Effects in SC-LQCD

Polyakov loop effects in SC-LQCD

- **Polyakov Loop**



$$P = \frac{1}{N_c} \operatorname{tr} L\,, \qquad L = T\exp\left[-i\int_0^\beta dx_4\, A_4\right] = T\prod_{\tau=1}^{N_\tau} U_0(\tau, \boldsymbol{x})$$

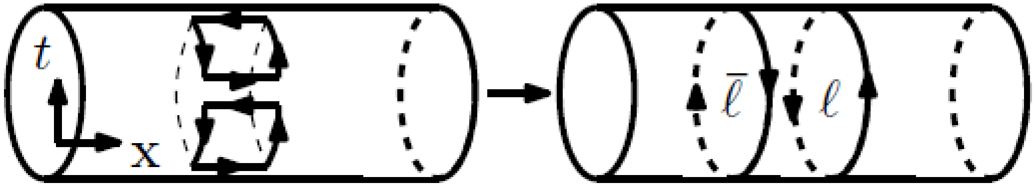
  - **Order parameter of the deconfinement transition in the heavy quark mass limit.**
    A.M. Polyakov, PLB72('78),477; L. Susskind, PRD20('79)2610; B. Svetitsky, Phys.Rept.132('86),1.
  - **Interplay between PL and χ cond. is known to be important in effective models**
    A. Gocksch, M. Ogilvie, PRD31(85)877; K. Fukushima, PLB591('04),277.

Polyakov loop will definitely affect QCD phase transition.
→ Let's evaluate its effects in SC-LQCD

Effective action with Polyakov loop

- **Polyakov Loop action in the leading order of 1/g²**
  - **After integrating out plaquette action over spatial links, we get**

$$\Delta S_p = -\left(\frac{1}{g^2 N_c}\right)^{N_\tau} N_c^2 \sum_{\boldsymbol{x}, j>0} \left[\bar{P}_{\boldsymbol{x}} P_{\boldsymbol{x}+\hat{j}} + \text{h.c.}\right] \quad (\text{LO in SC expansion})$$



  - **Polyakov loop coupling with fermion**

$$Z \sim \prod_{\boldsymbol{x}} \int dL(\boldsymbol{x}) e^{-\Delta S_p} \det_c \left[1 + L e^{-(E_q-\tilde{\mu})/T}\right]\left[1 + L^+ e^{-(E_q+\tilde{\mu})/T}\right]$$

$$= \prod_{\boldsymbol{x}} \int dP\, d\bar{P}\, H(P, \bar{P}) e^{-\Delta S_p} \left[1 + N_c P e^{-(E_q-\tilde{\mu})/T} + N_c \bar{P} e^{-2(E_q-\tilde{\mu})/T} + e^{-3(E_q-\tilde{\mu})/T}\right]$$

$$\times \left[1 + N_c \bar{P} e^{-(E_q+\tilde{\mu})/T} + N_c P e^{-2(E_q+\tilde{\mu})/T} + e^{-3(E_q+\tilde{\mu})/T}\right]$$

Finite Polyakov loop l enables one- and two-quark excitation

Effective potential with Polyakov loop

- **Haar measure method**
  - **Replace the Polyakov loop *P* with its representative value *l*, and Haar measure is included in the potential.**

$$\begin{aligned}\mathcal{F}_{\rm q} =& -N_c E - T\log\left[1+N_c\ell e^{-(E-\tilde{\mu})/T}+N_c\bar{\ell}e^{-2(E-\tilde{\mu})/T}+e^{-3(E-\tilde{\mu})/T}\right]\\ &-T\log\left[1+N_c\bar{\ell}e^{-(E+\tilde{\mu})/T}+N_c\ell e^{-2(E+\tilde{\mu})/T}+e^{-3(E+\tilde{\mu})/T}\right]-N_c\log Z_\chi\ ,\\ U_g =& -2T\beta_p\bar{\ell}\ell - T\log\left[1-6\ell\bar{\ell}+4\left(\ell^3+\bar{\ell}^3\right)-3\left(\ell\bar{\ell}\right)^2\right]\ ,\end{aligned}$$

E. M. Ilgenfritz, J. Kripfganz, ZPC29('85)79; A. Gocksch, M. Ogilvie, PRD31('85)877; K. Fukushima, PLB 553, 38 (2003); PRD 68('03)045004;K. Fukushima, PLB591('04)277.

- **Bosonization method**
  - **Introduce the auxiliary field *l* = <*P*>, and integrate out $\mathbf{U_0}$=L.**

$$\Delta S_p \approx \left(\frac{1}{g^2 N_c}\right)^{N_\tau} N_c^2 \sum_{\mathbf{x},j>0} 2\left(\bar{\ell}\ell-\bar{P}_{\mathbf{x}}\ell-\bar{\ell}P_{\mathbf{x}}\right) \simeq 2\beta_p L^d\bar{\ell}\ell - 2\beta_p\sum_{\mathbf{x}}\left(\bar{P}_{\mathbf{x}}\ell+\bar{\ell}P_{\mathbf{x}}\right)$$

→ Weise mean field approximation

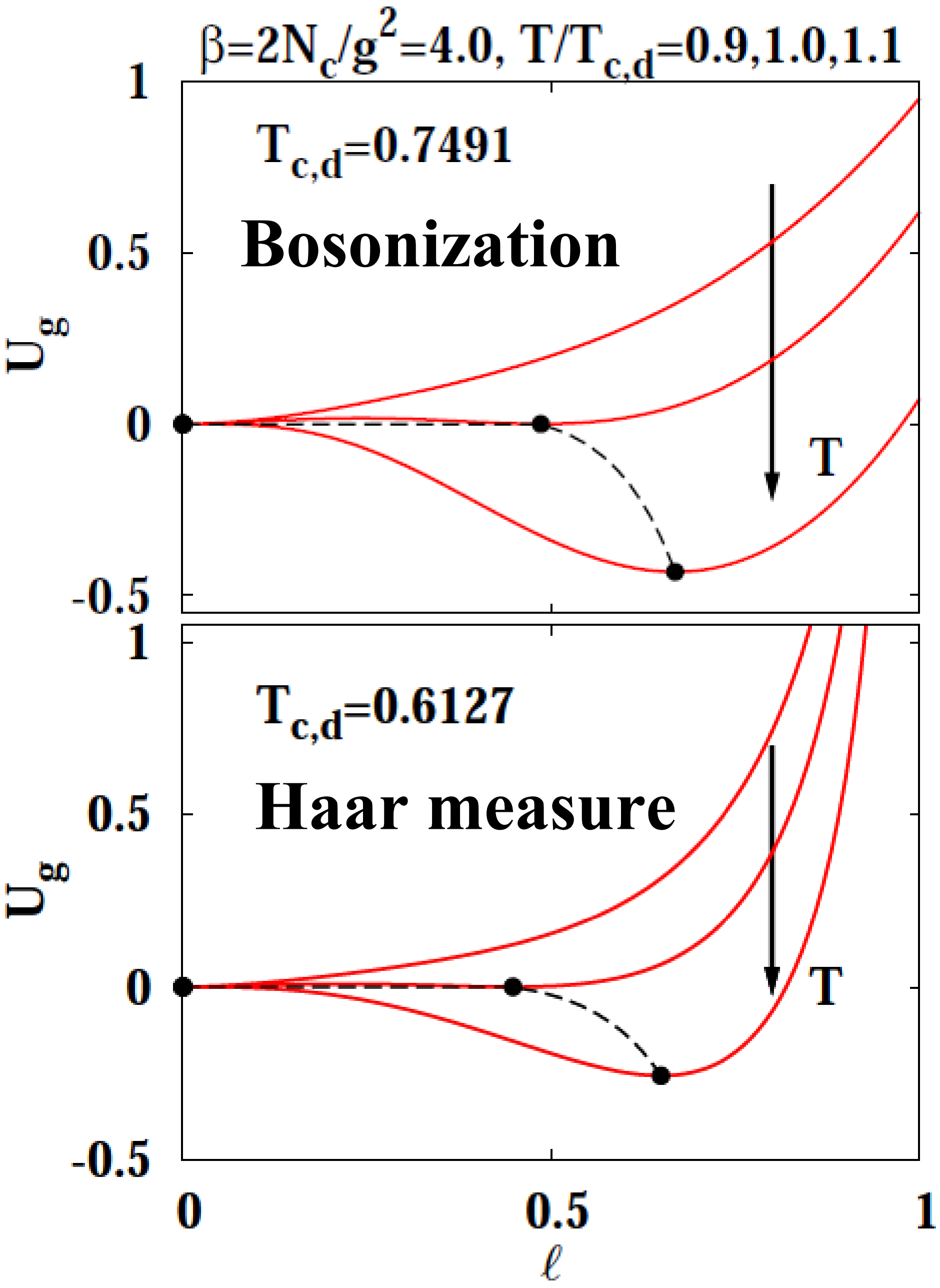
c.f. J. B. Kogut, M. Snow and M. Stone, NPB 200('82)211 (no quarks)

Polyakov loop potential

- Deconf. phase transition
  $l = 0$ at low T → $l \sim 1$ at high T
  is mainly governed by U_g
- Integral over U_0
  in Bosonization method
  - Fluctuation of PL
    → smooth potential
  - No singularity at $l = 1$
  - Correlation of l and l bar

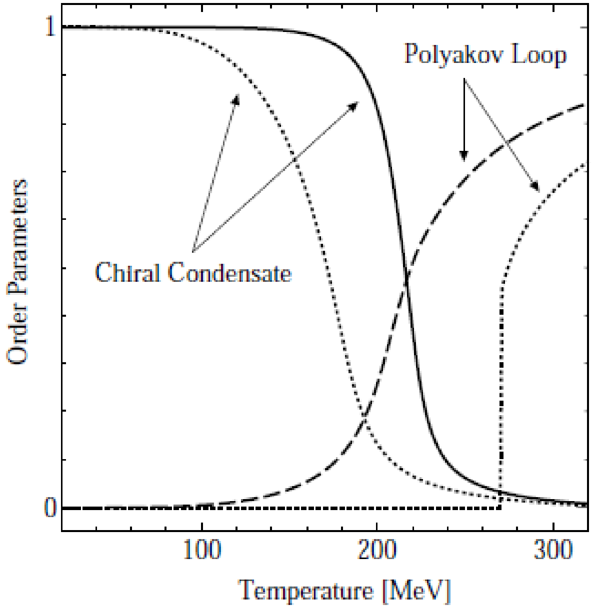
    $$\langle l_p \bar{l}_p \rangle = 1 \text{ even at } l = \bar{l} = 0$$

    → meson excitation is favored

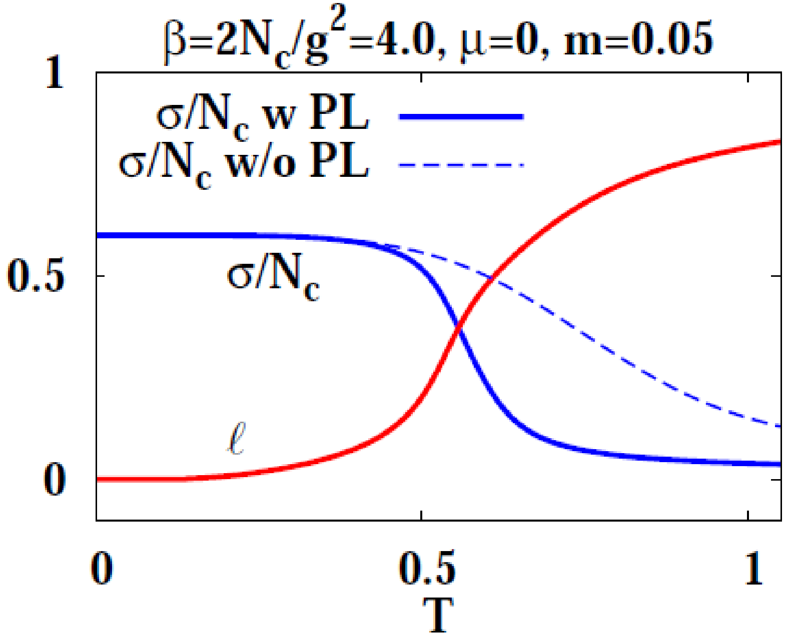


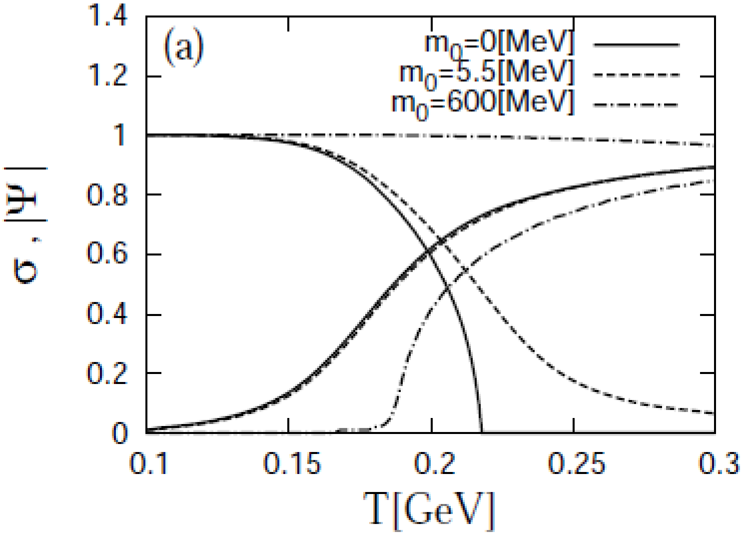
Chiral condensate and Polyakov loop

- **Chiral and Deconf. transition correlate !**
- **SC-LQCD w/o PL: quarks are confined.**
  → PL promote quarks to deconfine !
  (cf. Quarks are *not* confined in NJL
  → PL *confines* quarks in PNJL.)
- **Tc is suppressed with PL**



Fukushima ('04)

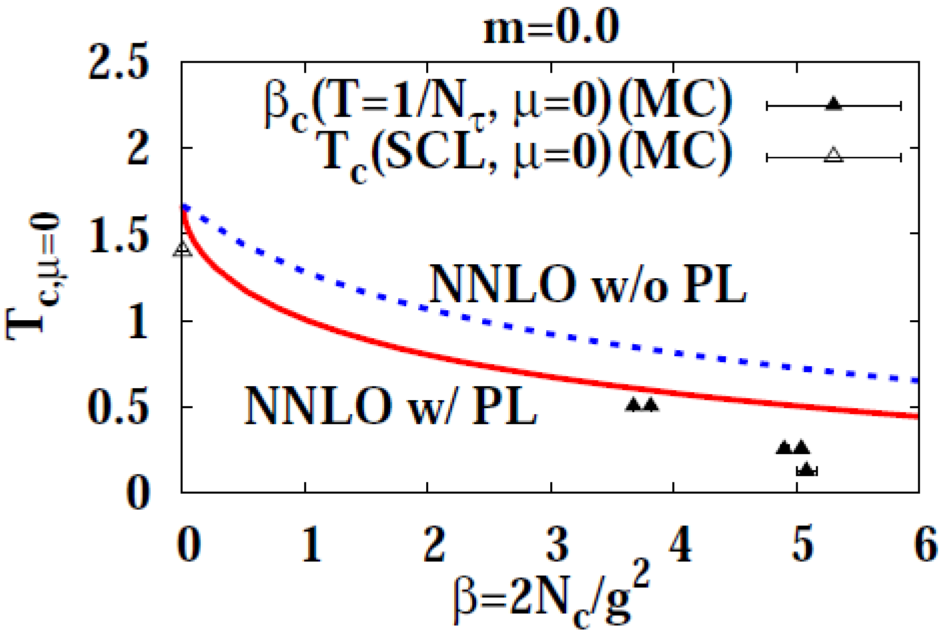




Nakano, Miura, AO, Lat10 & in prep. ***Sakai, Kashiwa, Kouno, Yahiro ('08)***

Critical Temperature at μ=0

- SC-LQCD w PL seems to be qualitatively promising.
  Is it *quantitatively* good ?
  - Improved from SC-LQCD w/o Polyakov loop.
  - Polyakov loop suppresses T_c. (cf. PNJL)
  - Quantitavely, not bad for $\beta < 4$ in T_c (β_c)
  - In the "scaling" region ($\beta>5$), we do not see further bending of T_c in SC-LQCD.

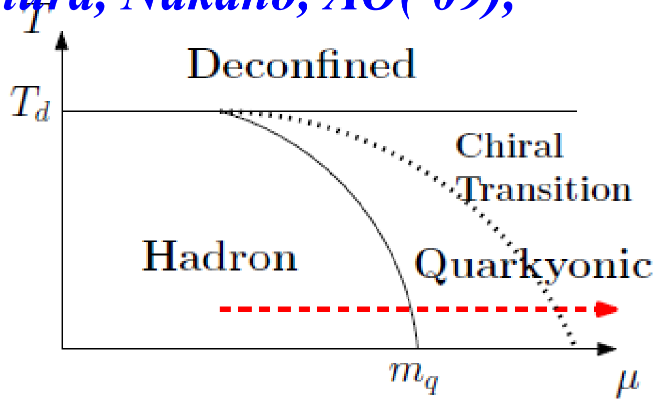


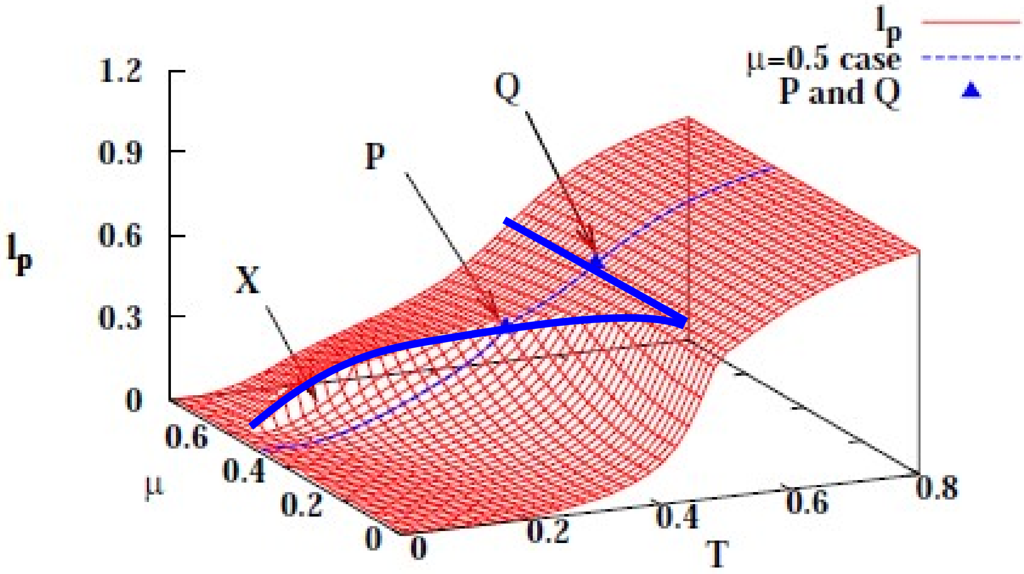
Nakano, Miura, AO, LAT10 & in prep.
MC Results:
Ph. de Forcrand, M. Fromm ('09),
Ph. de Forcrand, private comm.,
S.A.Gottlieb et al. ('87),
D'Elia, Lombardo ('03),
Z.Fodor, S. D. Katz ('02),
R.V.Gavaiet al. ('90)

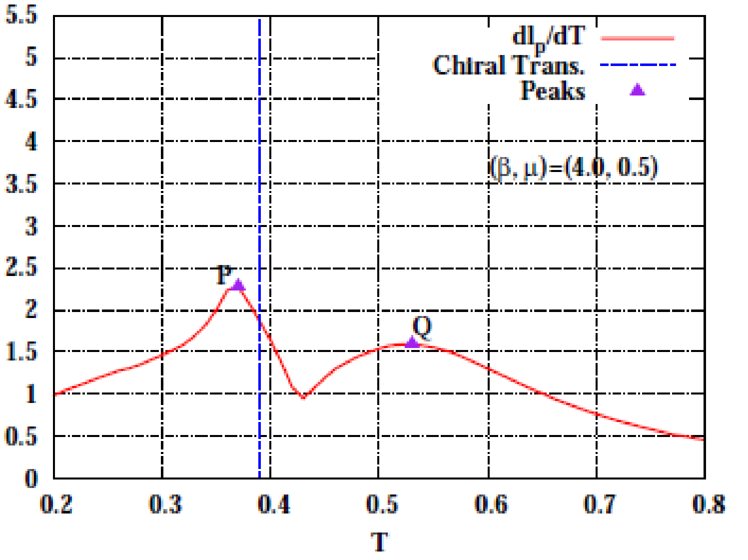
Quarkyonic matter

McLerran, Pisarski ('07), Hidaka, McLerran, Pisarski ('08), Kojo, Hidaka, McLerran, Pisarski ('10), Glozman et al('08), Fukushima ('08), Abuki, .., Ruggieri ('08), McLerran, Redlich, Sasaki ('09), Miura, Nakano, AO('09),

- **Quarkyonic matter**
  - **T_d is governed by gluons at large N_c , while high density matter is realized at $\mu \sim m_q$ → deviation of deconf. and chiral transitions**
  - **SC-LQCD with PL (Haar measure method) shows large region of "quarkyonic" matter**

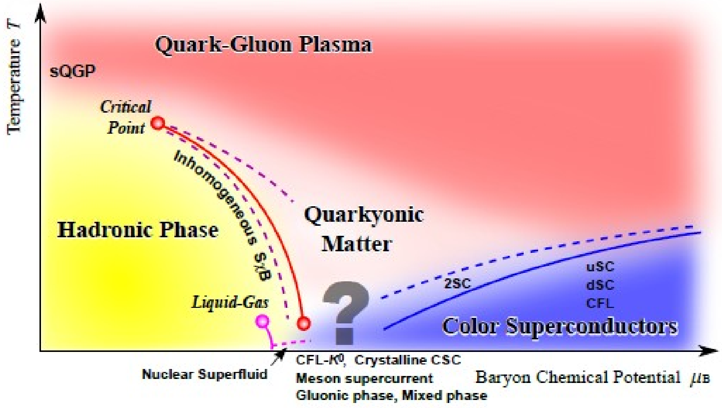






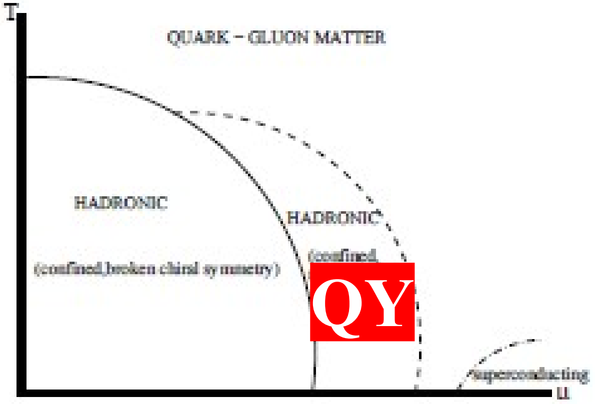
Miura, Nakano, AO, LAT10, in prep.

Comparison with Other Models

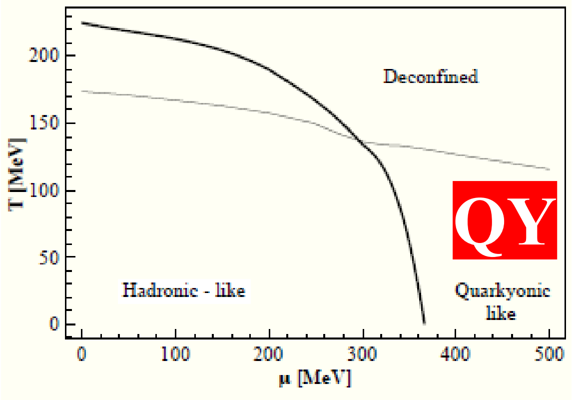


Fukushima, Hatsuda ('10)

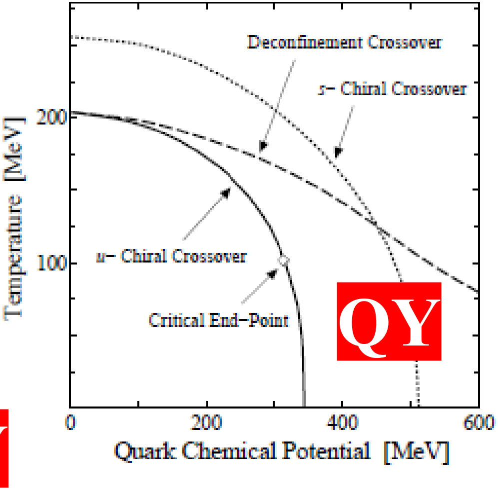
Grozman et al. (08)

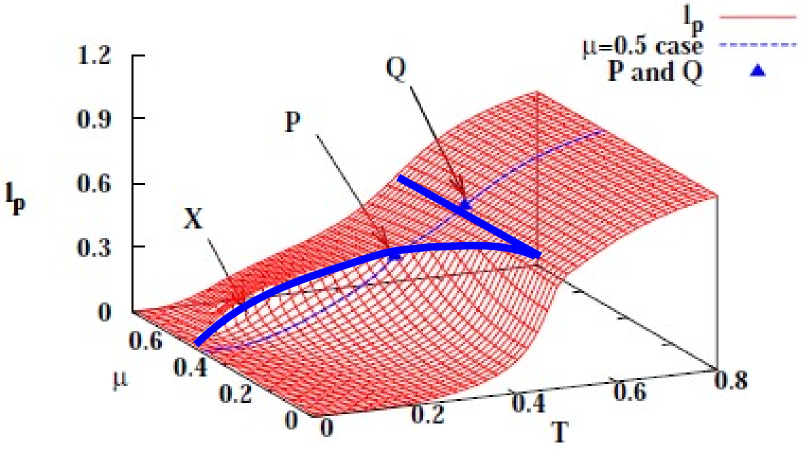


Abuki et al. (08)

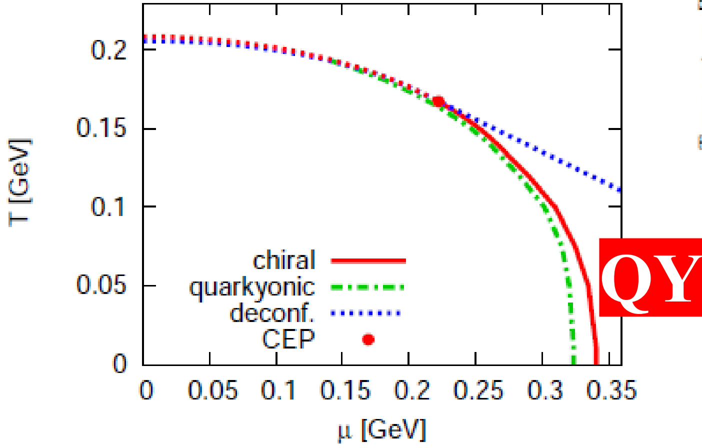


Fukushima (08)





Miura, Nakano, AO, LAT10, in prep.



McLerran, Pisarski, Sasaki ('09)

Summary

- Strong coupling lattice QCD (SC-LQCD) has been developed to describe the QCD phase diagram at finite T and μ.
  - Approximations: double expansion ($1/g^2$, $1/d$) and mean field.
  - Recent development: NLO and NNLO in $1/g^2$, Polyakov loop effects

    cf. Jolicoeur, Kluberg-Stern, Morel, Lev, Petersson ('84)
    → NNLO at T=0 treatment (no phase diagram study)
    Gocksch-Ogilvie model
    → SCL-LQCD + phen. string tension
- SC-LQCD may serve a qualitatively competitive framework to effective models such as PNJL in some aspects of the QCD phase diagram.
  - NNLO w/ PL (bosonization method) roughly (i.e. 10-20 % precision) explains T_c in MC simulations for $\beta < 4$.
  - NLO w/ PL (Haar measure method) predicts the existence of the quarkyonic matter.

Future directions

- Further studies are necessary to give nuclear matter saturation.
  - NNLO SC-LQCD solves the “Baryon Mass Puzzle”
    ($\mu_c > M_B/3$ @ SCL), but nuclear matter does not saturate.
  - Auxiliary field fluctuations and 1/d higher order terms are the plausible origin of saturation.
    → Nakano's next work ?
  - Combination with MC simulation may be an interesting direction to pursue.
- Problems....
  - With Polyakov loop, rigorous $1/g^2$ expansion is broken.
    How can we justify it ?
  - Transition at $\mu=0$ in the chiral limit is still 2nd order.
    How can we include the effects of anomaly ?
  - There is still an unsolved homework in the strong coupling limit.
    Any idea ?

Thank you for your attention !

Constituent Quark Mass in NNLO SC-LQCD

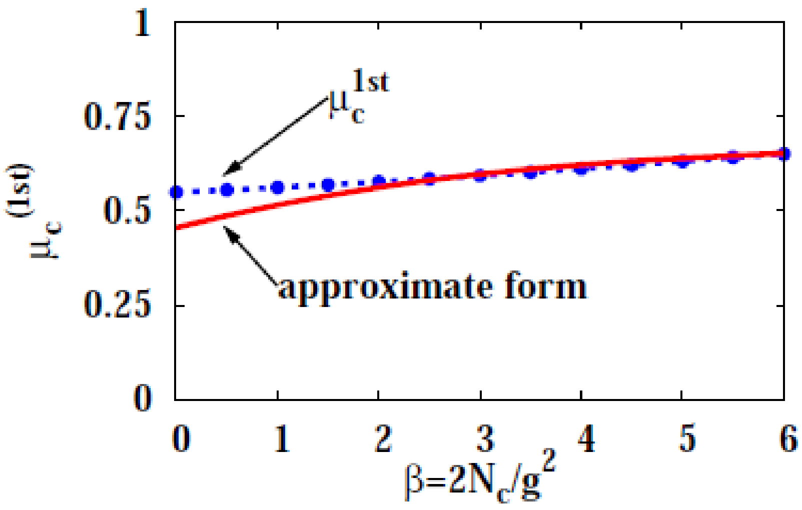
- **Mechanism of "stable" $\mu_c(T=0)$ in NLO/NNLO SC-LQCD**

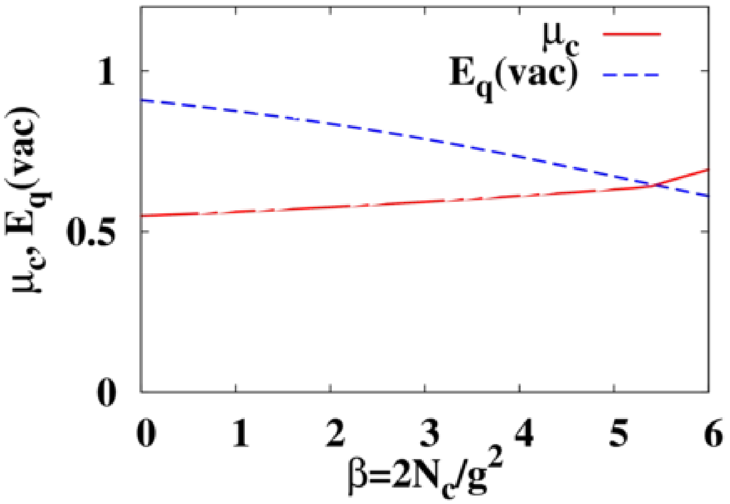
  = Effects of quark mass reduction & repulsive vector pot. cancel

Transition Condition at $T=0: E_q(\tilde{m}_q)=\tilde{\mu}\simeq\mu-\beta'_\tau\omega_\tau$

$\rightarrow\mu\simeq E_q(\tilde{m}_q)+\beta'_\tau\omega_\tau$

Pocket formula $$\mu_{c,T=0}\simeq\frac{1}{2}\left[E_q(\sigma=\sigma_{\text{vac}},\omega_\tau=0)+\delta\mu(\sigma=0,\omega=N_c)\right]$$





Quark mass ($\approx E_q$) is smaller than μ_c for β > 5.5.
→ "Baryon mass puzzle" may be solved !

Nuclear Matter on the Lattice at Strong Coupling

- **Do we observe finite density matter before 1st order phase transition ?**
  → Yes !
  - **$E_q(\mu=0, T=0, \beta=6)=0.61$**
    $\mu_c^{(1st)}(T=0, \beta=6)=0.65$
    → "Nuclear matter" in $0.61<\mu<0.65$
- **EOS of "Nuclear matter"**
  - **a^{-1} = 500 MeV**
    Bilic, Demeterfi, Petersson ('92)
    → Density in the order of ρ_0
  - **No saturation**
  - **1st order transition at ρ_B=0.4 fm^{-1}.**