核物質の状態方程式と相図
Nuclear Matter EOS and QCD phase diagram

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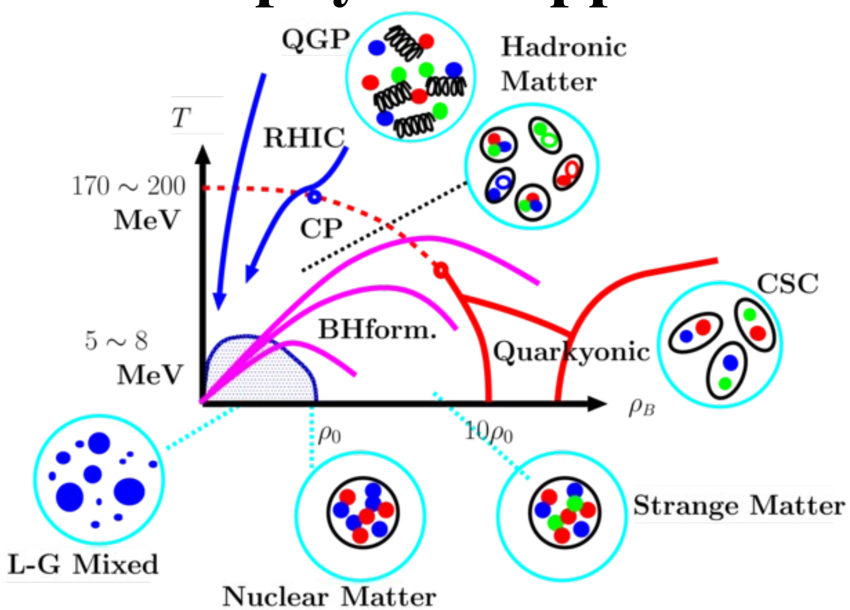
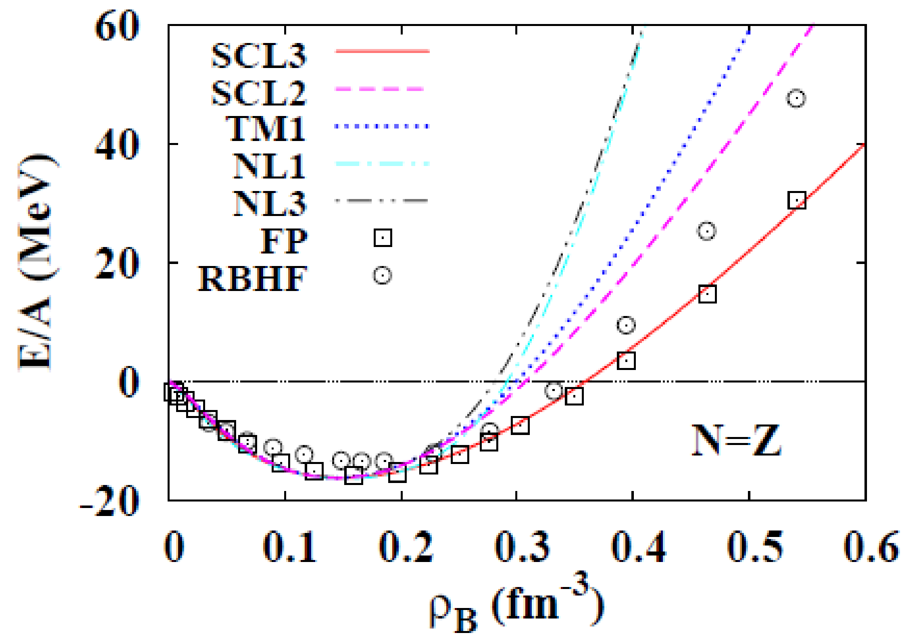
- **Introduction**
  --- Recent developments in nuclear matter EOS studies
- **Bottom-up approach**
  --- RMF study of Normal nuclei, hypernuclei, and supernova matter
- **Top-down approach**
  --- Strong coupling lattice QCD for phase diagram and EOS
- **Summary**

K. Tsubakihara, H. Maekawa, H. Matsumiya, AO, PRC81('10)065206.
K. Sumiyoshi, C. Ishizuka, AO, S. Yamada, H. Suzuki, ApJ Lett. 690 ('09)L43
C. Ishizuka, AO, K. Tsubakihara, K. Sumiyoshi, S. Yamada, JPG35('08)085201.
K. Miura, T.Z.Nakano,AO, N.Kawamoto, PRD80('09),074034.
T.Z. Nakano, K. Miura, AO, PTP123('10)825.

QCD Phase diagram and Nuclear Matter EOS

- **Phase diagram and EOS**
  = Two important aspects of Nuclear Matter
- **Dense nuclear matter has rich physics**
  → Many-body theory, Exotic compositions, CEP, Astrophysical applications, ...





Nuclear matter EOS
= Subjects in Nuclear, Quark-Hadron, Particle, Astro, and Condensed Matter Physics !

What is EOS ?

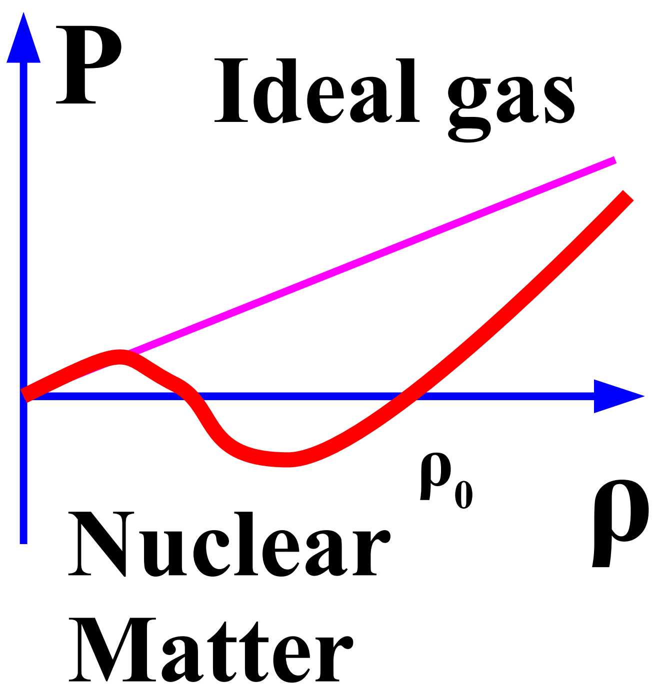
- **Equation of State (EOS) of Ideal Gas (理想気体の状態方程式)**

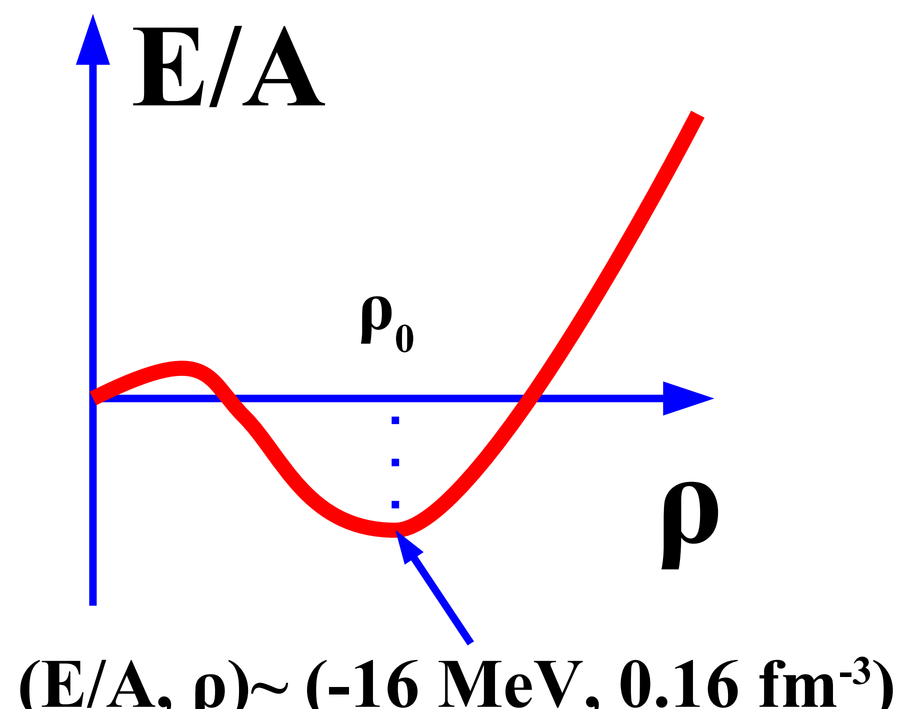
$$PV = NkT \rightarrow P = \rho T \quad (\rho = N/V, k = 1)$$

- **Self-binding system → Null pressure density (ρ_0) exists.**

$$P = P(\rho, T, \ldots), \quad E/A = -\epsilon_0 + \frac{K}{18\rho_0^2}(\rho - \rho_0)^2 + \cdots$$

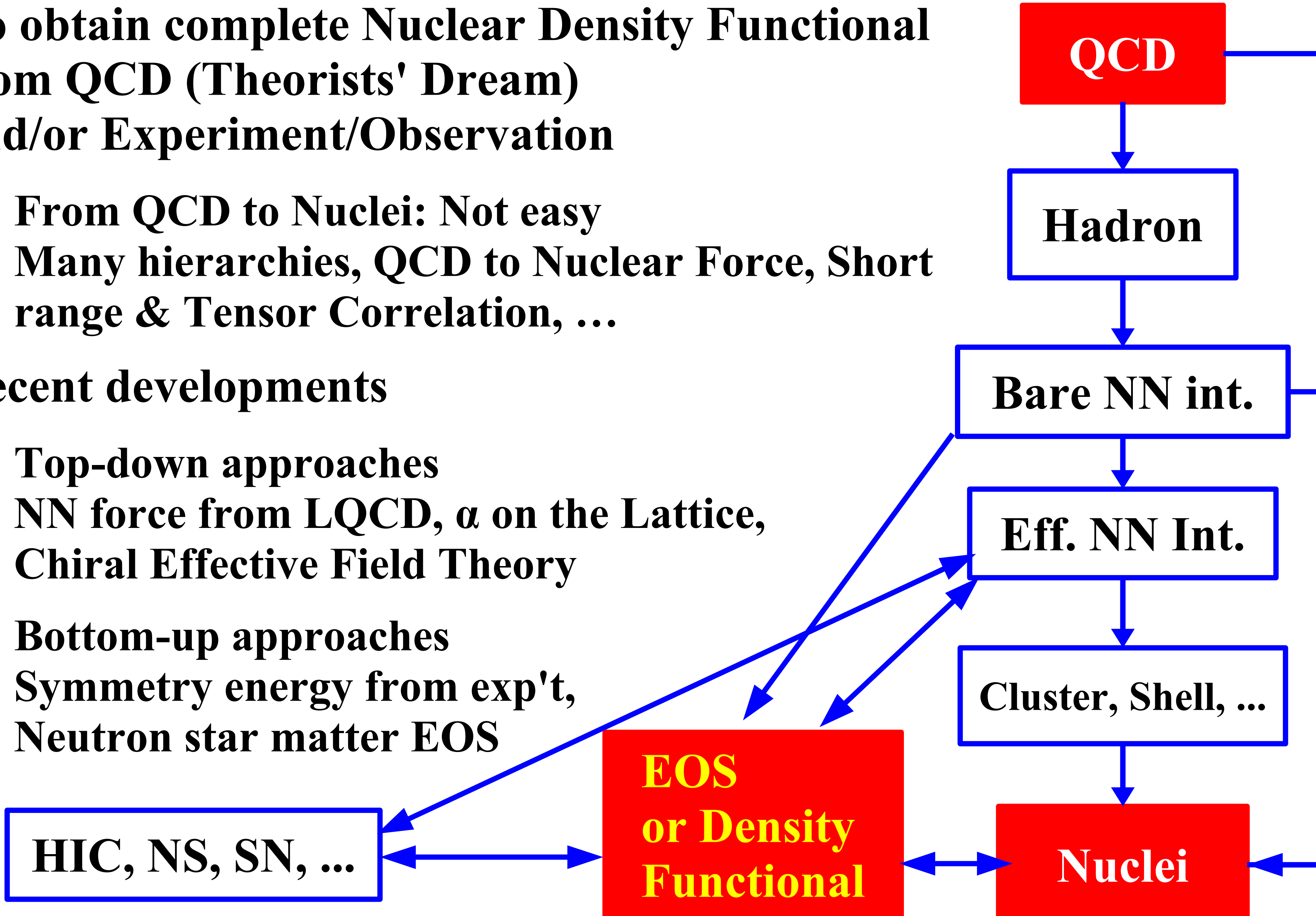
ε_0: Saturation E. (~ -16 MeV), ρ_0: Saturation density (~ 0.16 fm^{-3}),
K: incompressibility (~ 200-300 MeV)





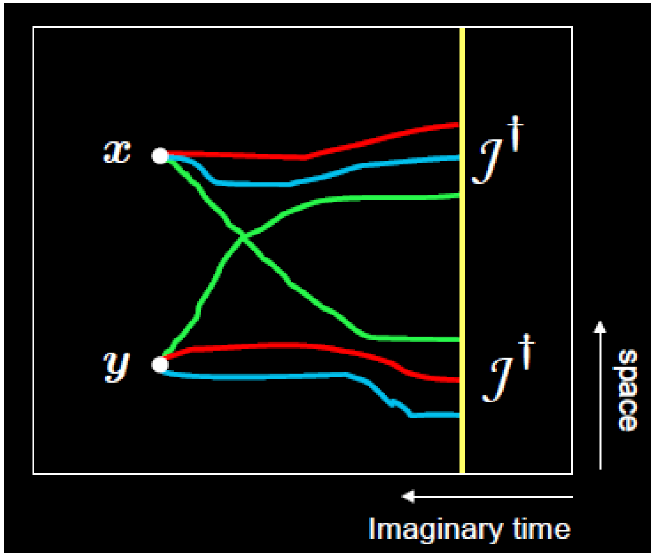
One of the "Ultimate" Goals in Nuclear Physics

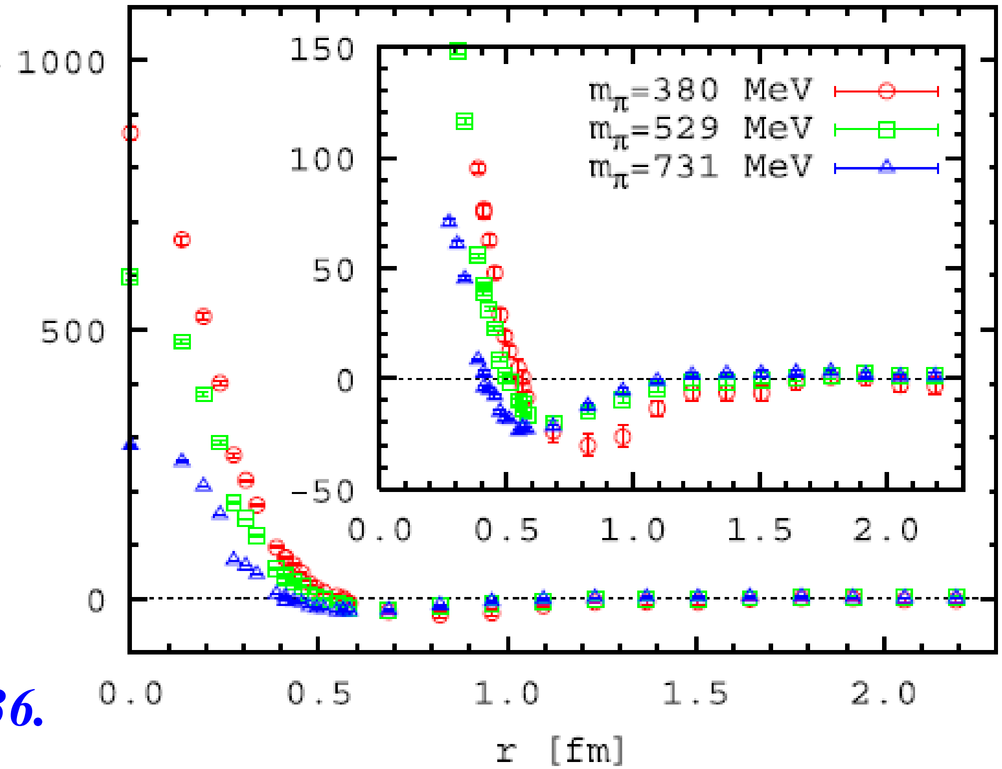
- **To obtain complete Nuclear Density Functional from QCD (Theorists' Dream) and/or Experiment/Observation**
  - **From QCD to Nuclei: Not easy**
    Many hierarchies, QCD to Nuclear Force, Short range & Tensor Correlation, …
- **Recent developments**
  - **Top-down approaches**
    NN force from LQCD, α on the Lattice, Chiral Effective Field Theory
  - **Bottom-up approaches**
    Symmetry energy from exp't, Neutron star matter EOS



Nuclear force on the Lattice

- **BS wave function → Lattice NN Pot.**
  - **Starting from wall source, and measure Bethe-Salpeter ampl.**
  - **By using Schrodinger-type Eq., NN potential is obtained.**
- **Lot of achievements !**
  - **One pion exchange potential tail.**
  - **Repulsive core from quark Pauli principle.**
  - **YN potential, MB potential, …**
- **Needs further studies for EOS**

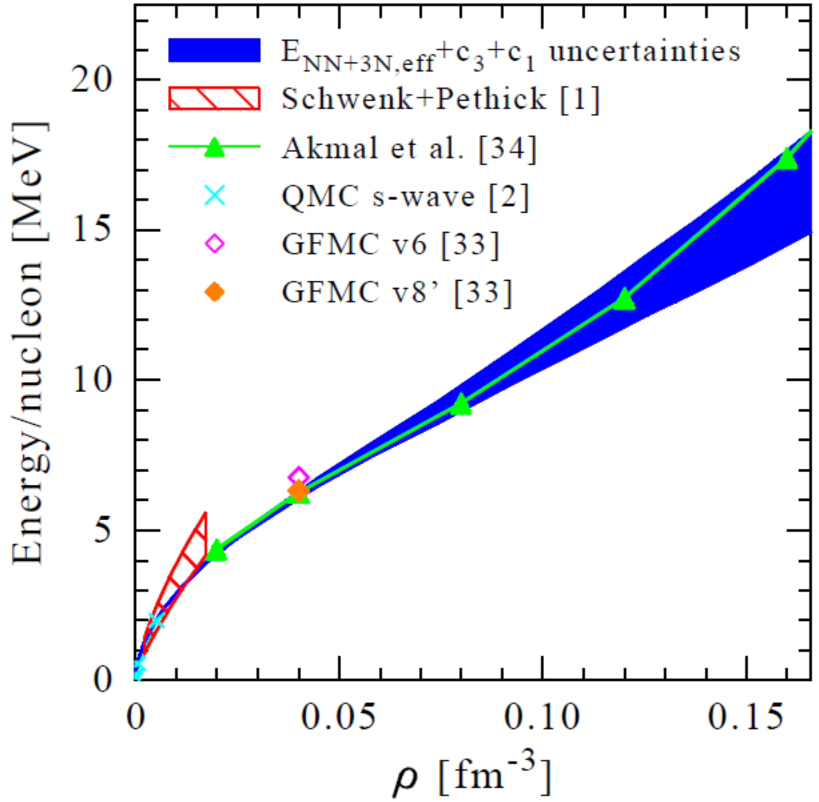




S. Aoki, T. Hatsuda, N. Ishii, PTP 123('10)89
Ishii, Aoki, Hatsuda, PRL 99 ('07) 022001
Nemura et al, arXiv:1005.5352 [hep-lat]
H. Nemura, Ishii, Aoki, Hatsuda, PLB673('09)136.

Ab Initio Calculations

- **Chiral EFT + RG evolution to low momenta**
  - **N3LO NN + NNLO 3N force**
    E. Epelbaum, H.-W. Hammer, U.-G. Meißner, RMP81('09)1773.
  - **3N force → ρ dep. NN force**
    S.K.Bogner, T.T.S.Kuo, A.Schwenk, PRep386('03)1.
- **Neutron matter results**
  - **Consistent with other "rigorous" results such as APR**
    A.Akmal, V.R.Pandharipande, D.G.Ravenhall, PRC58('98)1804.
    → Understanding of the origin of phen. 3-body repl. in APR.



K. Hebeler, A. Schwenk, arXiv:0911.0483

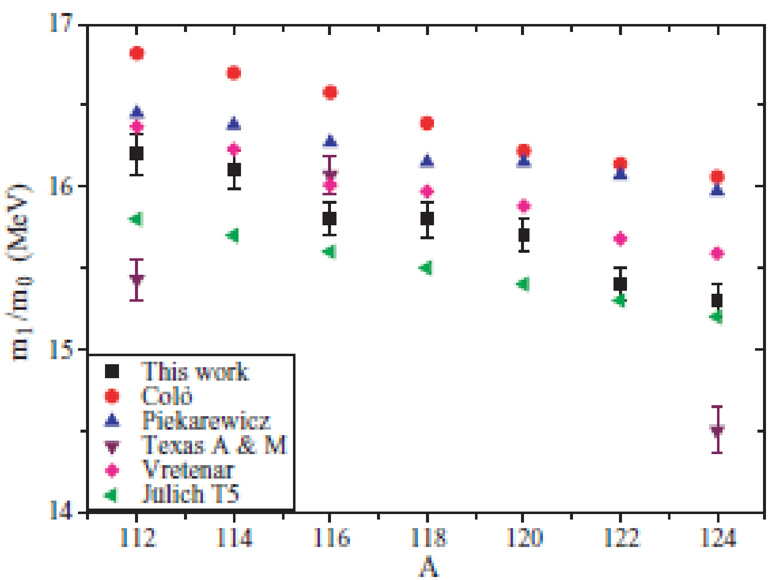
- **Related work:**
  - **QMC on the lattice**
    T. Abe, R. Seki, PRC79('09)054002.
  - **3NF from Exp. (Sekiguchi)**

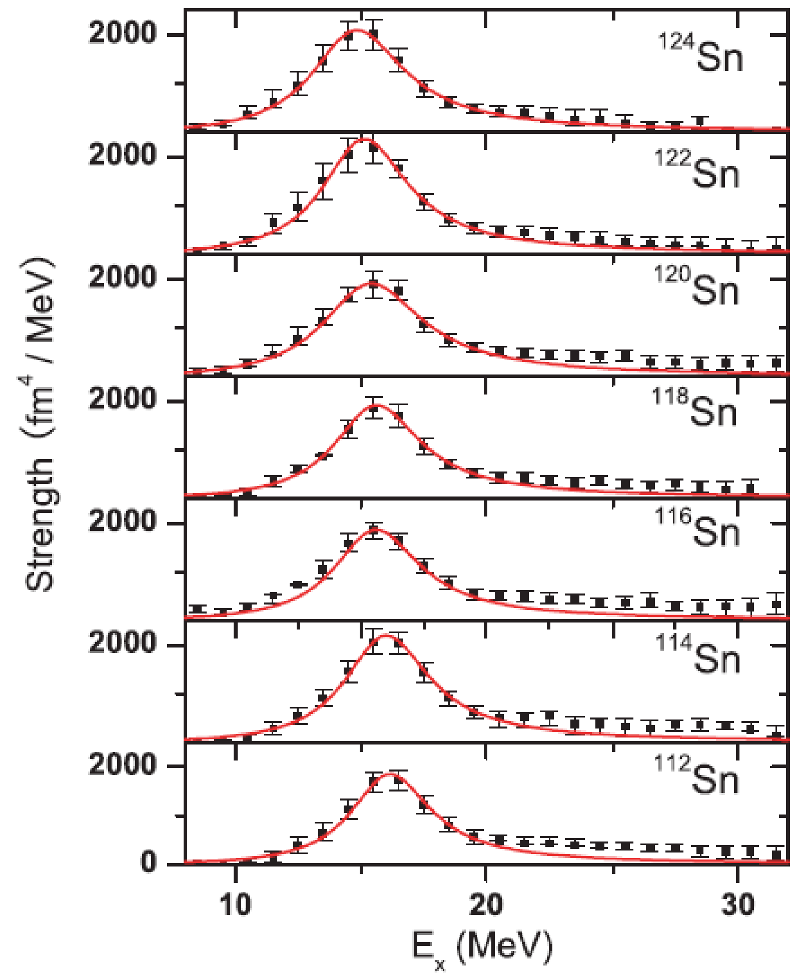
Symmetry Energy (1)

- **Recent data suggest that EOS becomes softer in asymmetric nuclear matter.**

$$K = K_{\rm sym} + K_{\rm asy}\,\delta^2, \quad K_{\rm asy} \sim -550\,{\rm MeV}$$
$$E_{\rm sym} \simeq 31.6\,(\rho/\rho_0)^{1.05}$$

- **Isoscalar Giant Monopole Resonance (ISGMR) of Sn isotopes**
  - **ISGMR in Isotope chain (^{112}Sn ~ ^{124}Sn) is systematically studied.**





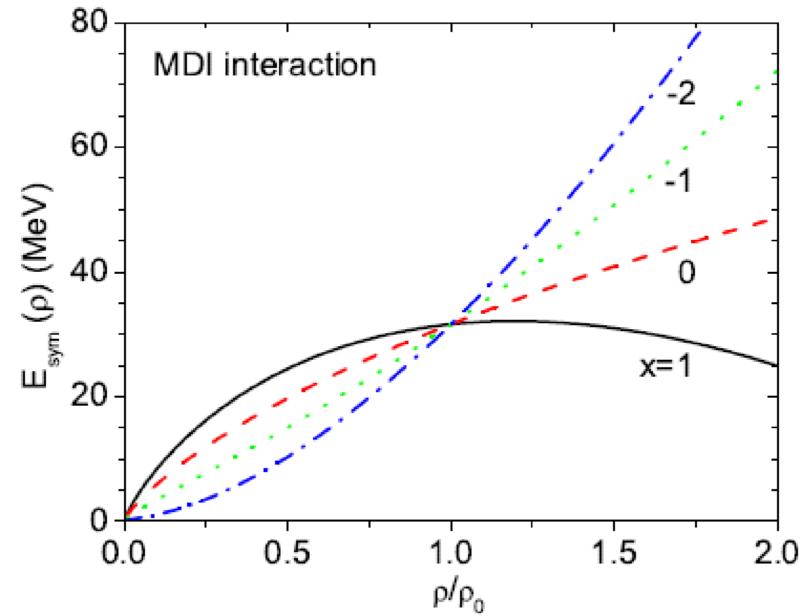
T. Li, U. Garg, et al., PRC81('10), 034309.

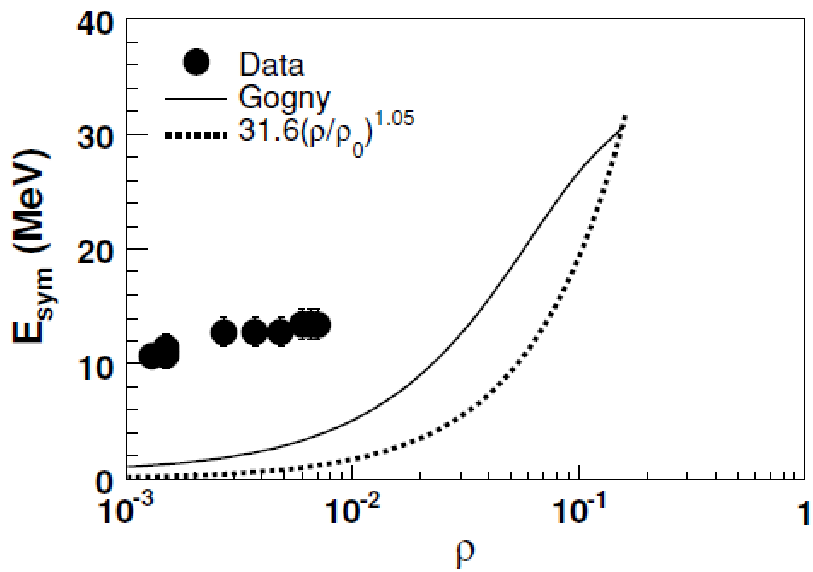
Symmetry Energy (2)

- **Symmetry energy in HIC**
  - **Isospin diffusion → K_{asy} ~ -550 MeV**

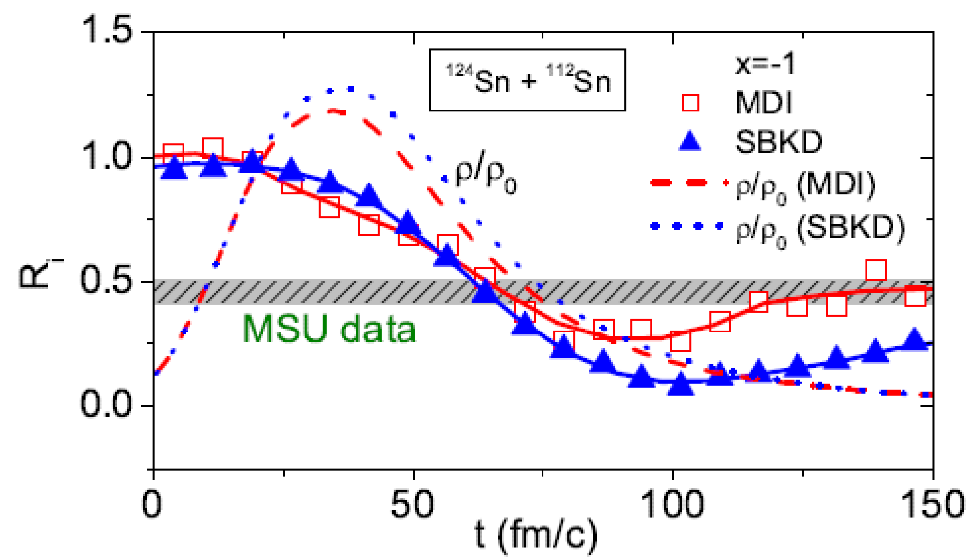
$$R_i = \frac{2X_{^{124}\mathrm{Sn}+^{112}\mathrm{Sn}} - X_{^{124}\mathrm{Sn}+^{124}\mathrm{Sn}} - X_{^{112}\mathrm{Sn}+^{112}\mathrm{Sn}}}{X_{^{124}\mathrm{Sn}+^{124}\mathrm{Sn}} - X_{^{112}\mathrm{Sn}+^{112}\mathrm{Sn}}}$$

  - **Light frag. dist.**
    → Larger Sym. E at low ρ





S. Kowalski, ..., A. Ono, PRC75('07)014601

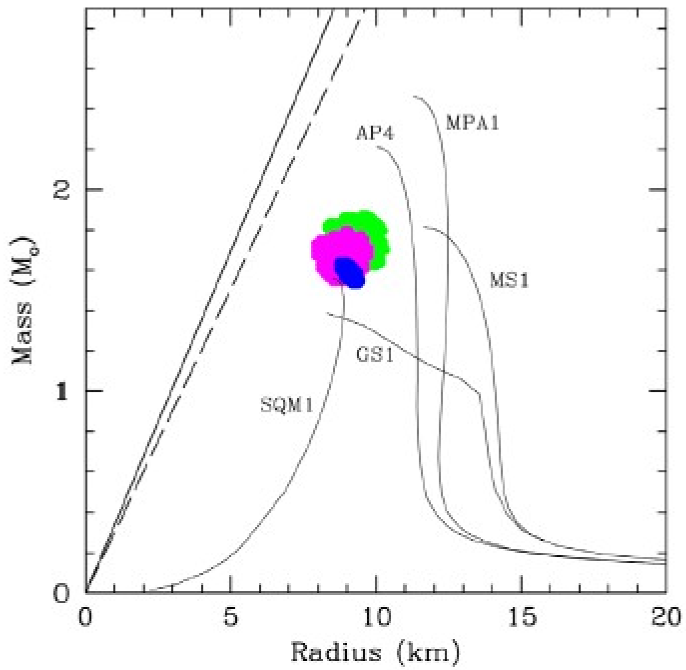


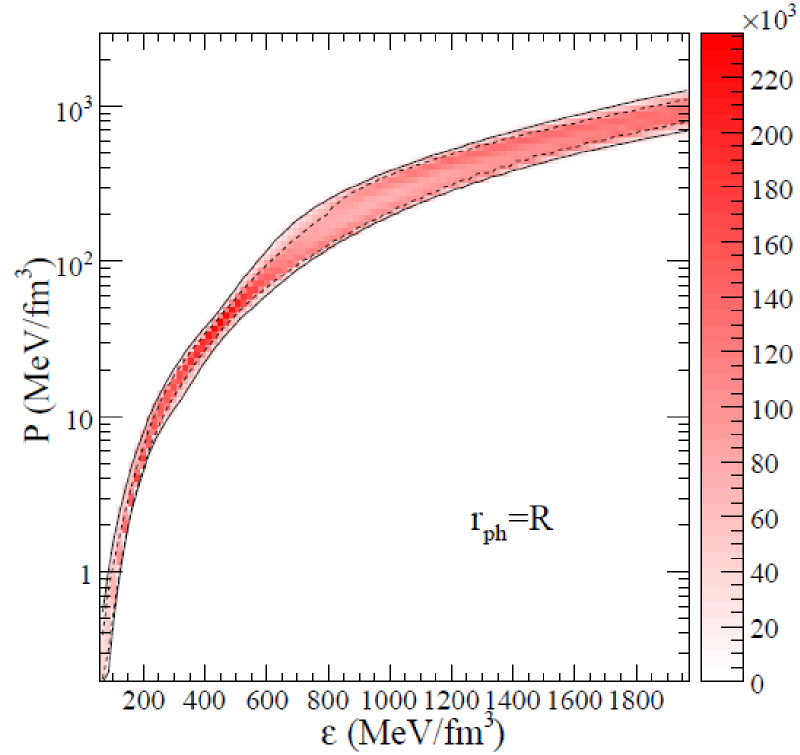
L.W.Chen, C.M.Ko, B.A.Li, PRL94('05),032701.

X-ray measurements of Neutron Stars

- **Neutron star mass (M)-radius (R) curve *uniquely*(*) determines NS matter EOS.**
  - **Radius measurement:**
    flux + temperature → apparent radius
    $$\frac{R_\infty}{D} = \frac{R}{D}\frac{1}{\sqrt{1-2GM/Rc^2}}$$
  - **Eddington flux would give another info.**
  - **Bayesian TOV inversion → EOS**

Thermonuclear Burst in X-ray Binaries
4U 1608-248
EXO 1745-248
4U 1820-30



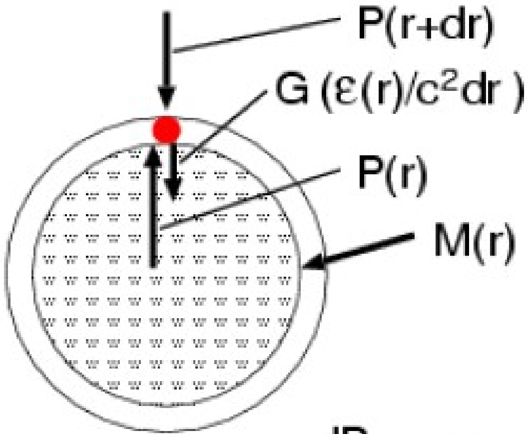


Ozel, Baym & Guver, arXiv: 1002.3153 [astro-ph.HE]

A. W. Steiner, J. M. Lattimer, Ed. Brown, arXiv:1005.0811

Tolman-Oppenheimer-Volkoff (TOV) equation

- **TOV Eq. = General Relativistic Balance of pressure and gravity**



Non-Rela. $\frac{dP}{dr} = -G \frac{\varepsilon(r)/c^2 \; M(r)}{r^2}$

$$\frac{dP}{dr} = -G \frac{(\varepsilon/c^2 + P/c^2)(M + 4\pi r^3 P/c^2)}{r^2(1 - 2GM/rc^2)}$$

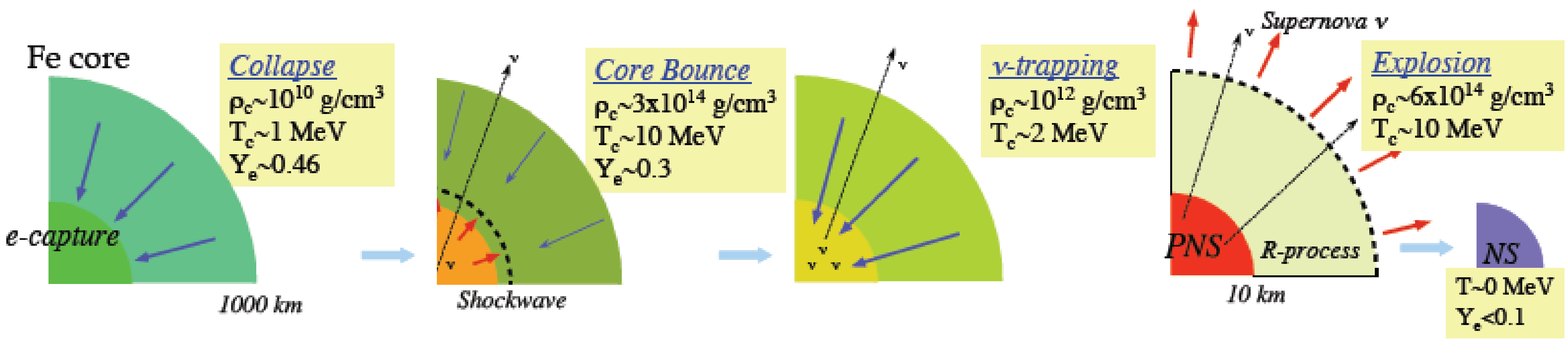
$$\frac{dM}{dr} = 4\pi r^2 \varepsilon/c^2 \,, \quad \frac{dP}{dr} = \frac{dP}{d\varepsilon}\frac{d\varepsilon}{dr}$$

$$P = P(\varepsilon) \,, \quad \frac{dP}{d\varepsilon} = \frac{dP}{d\varepsilon}(\varepsilon) \quad \text{(EOS)}$$

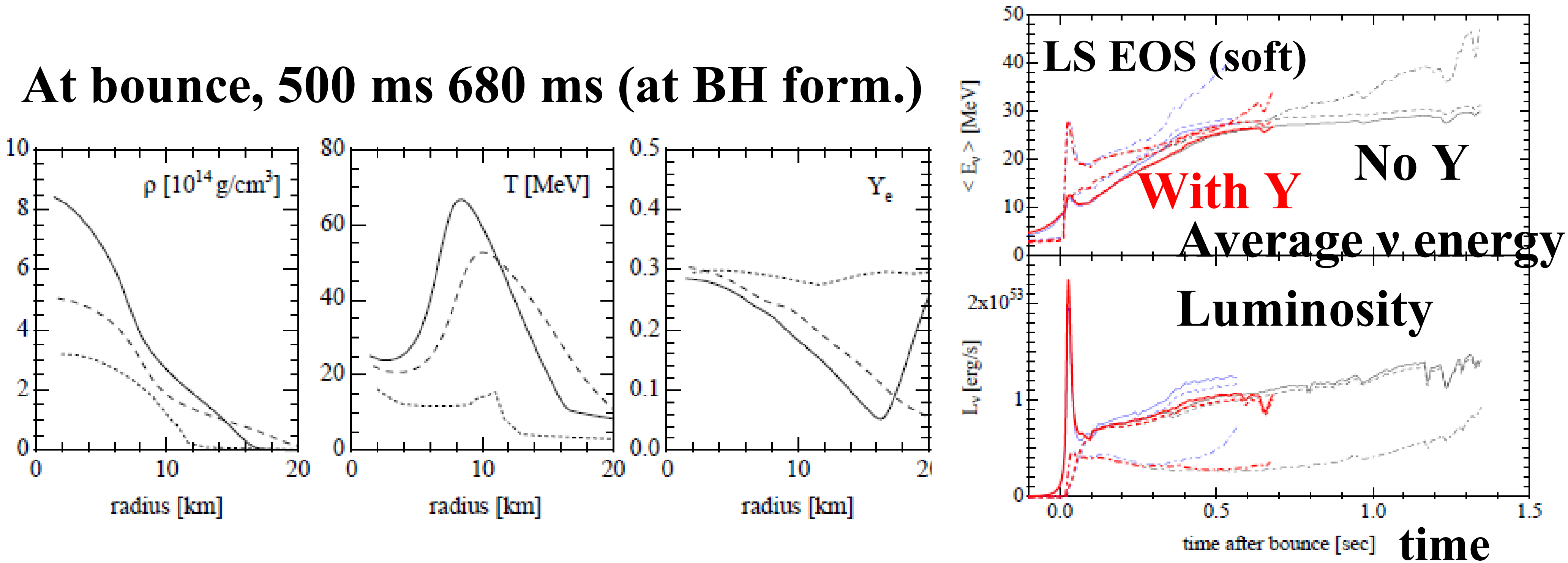
Neutron Star Mass = M(R) where P(R)=0

When you make a new EOS, please check the NS mass !

Black Hole Formation (Failed Supernova)



At bounce, 500 ms 680 ms (at BH form.)

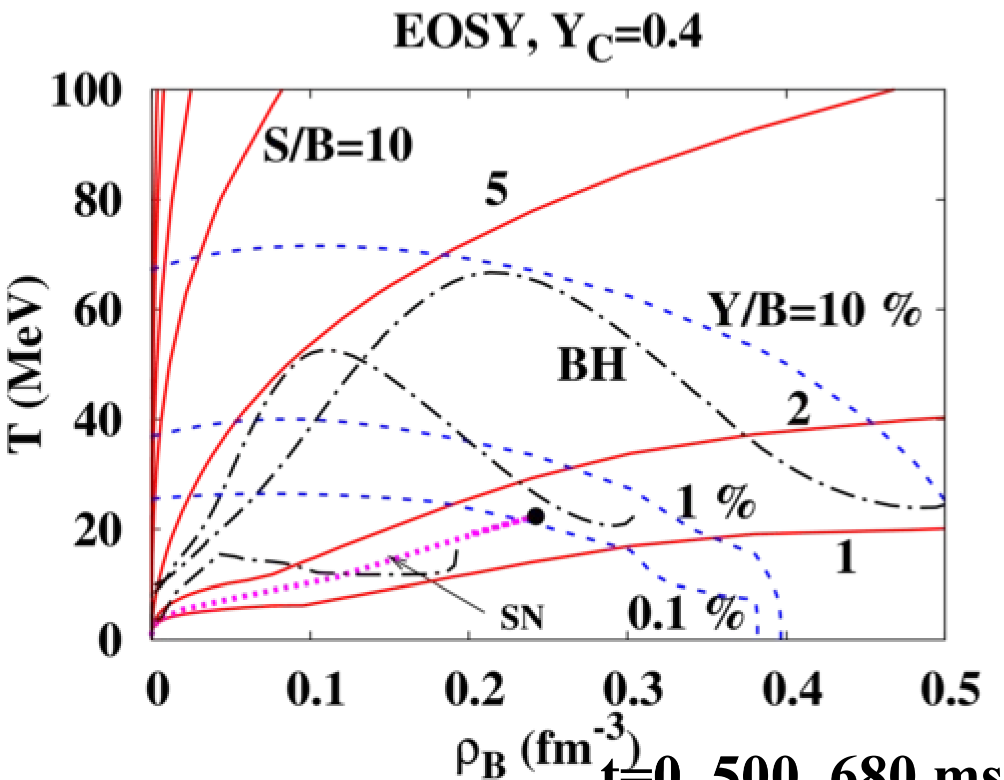


Sumiyoshi, Ishizuka, AO, Yamada, Suzuki, 2009

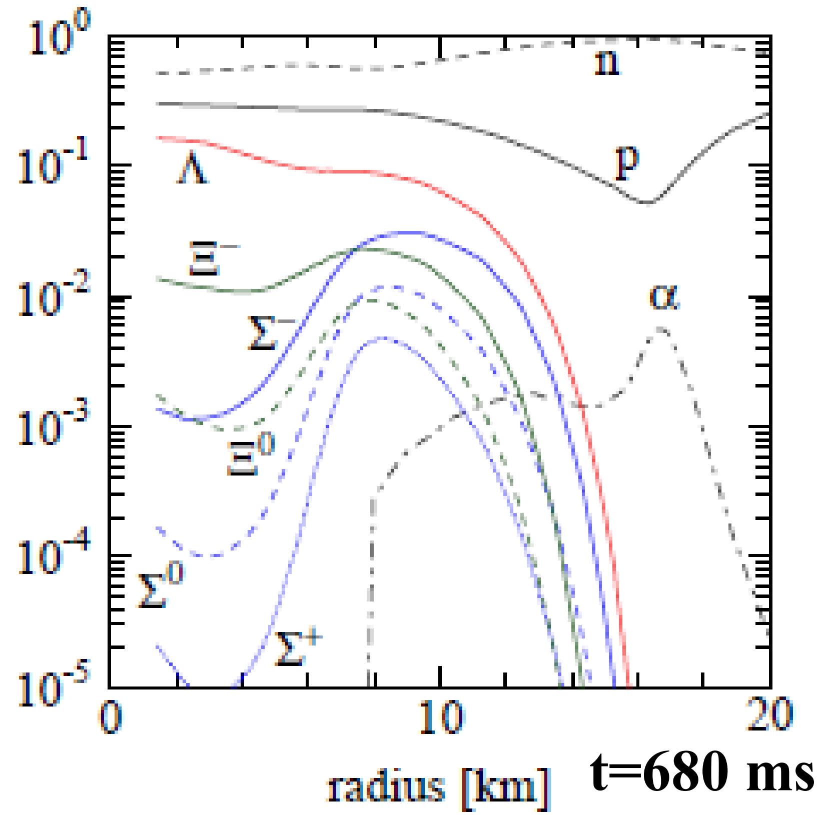
Black Hole Formation

- **Black Hole Formation: (ρ_B, T, Y_e) ~ (4 ρ_0, 70 MeV, 0.2)**
  → Hyperon fraction ~ 10 %

(K. Sumiyoshi, C. Ishizuka, AO, S. Yamada, H. Suzuki, ApJ690(09)L43)



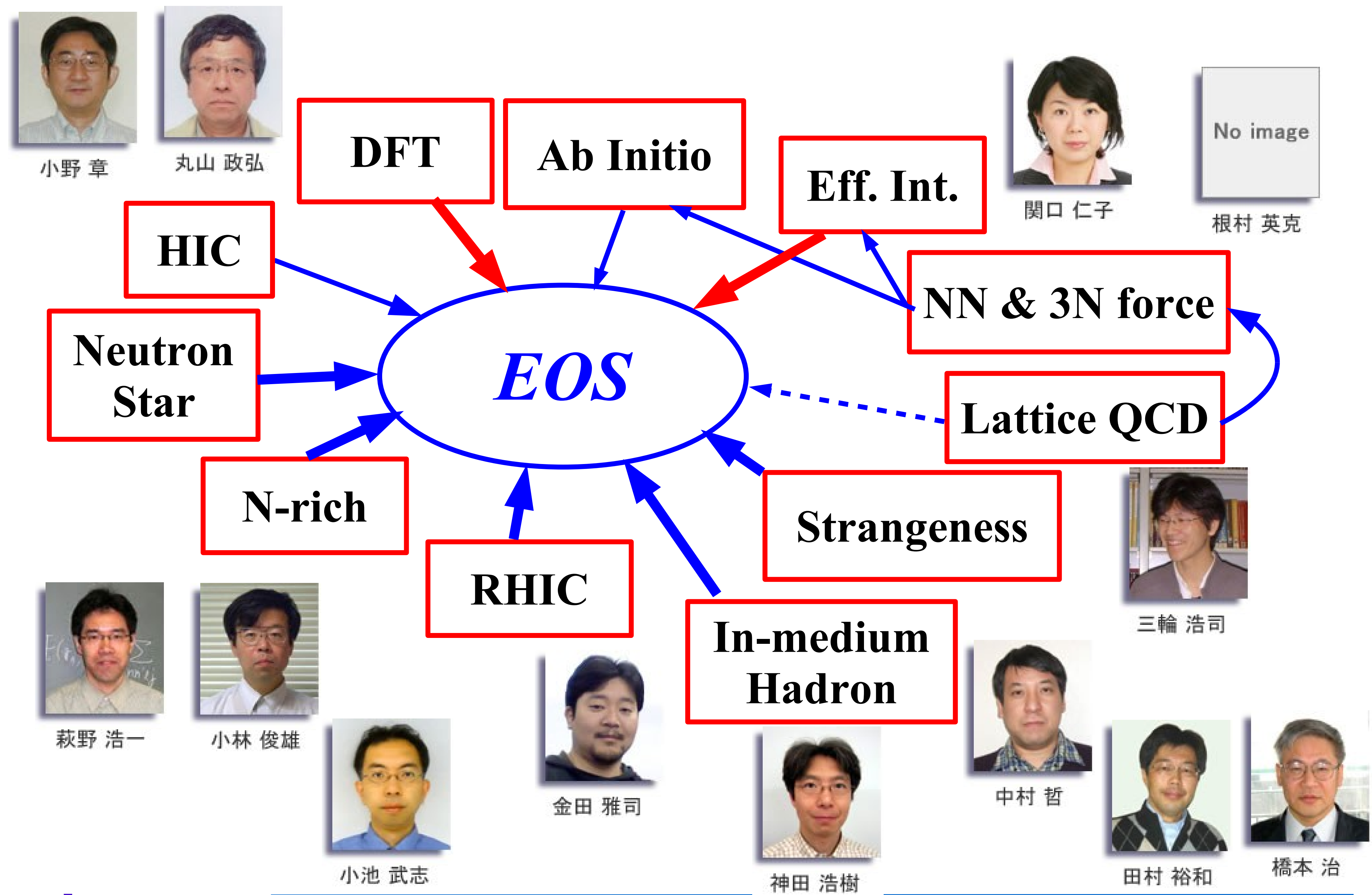
t=0, 500, 680 msec



t=680 msec

Hyperons are abundantly formed during BH formation !
→ EOS softening, Early collapse, Short ν duration

EOS and Related Physics



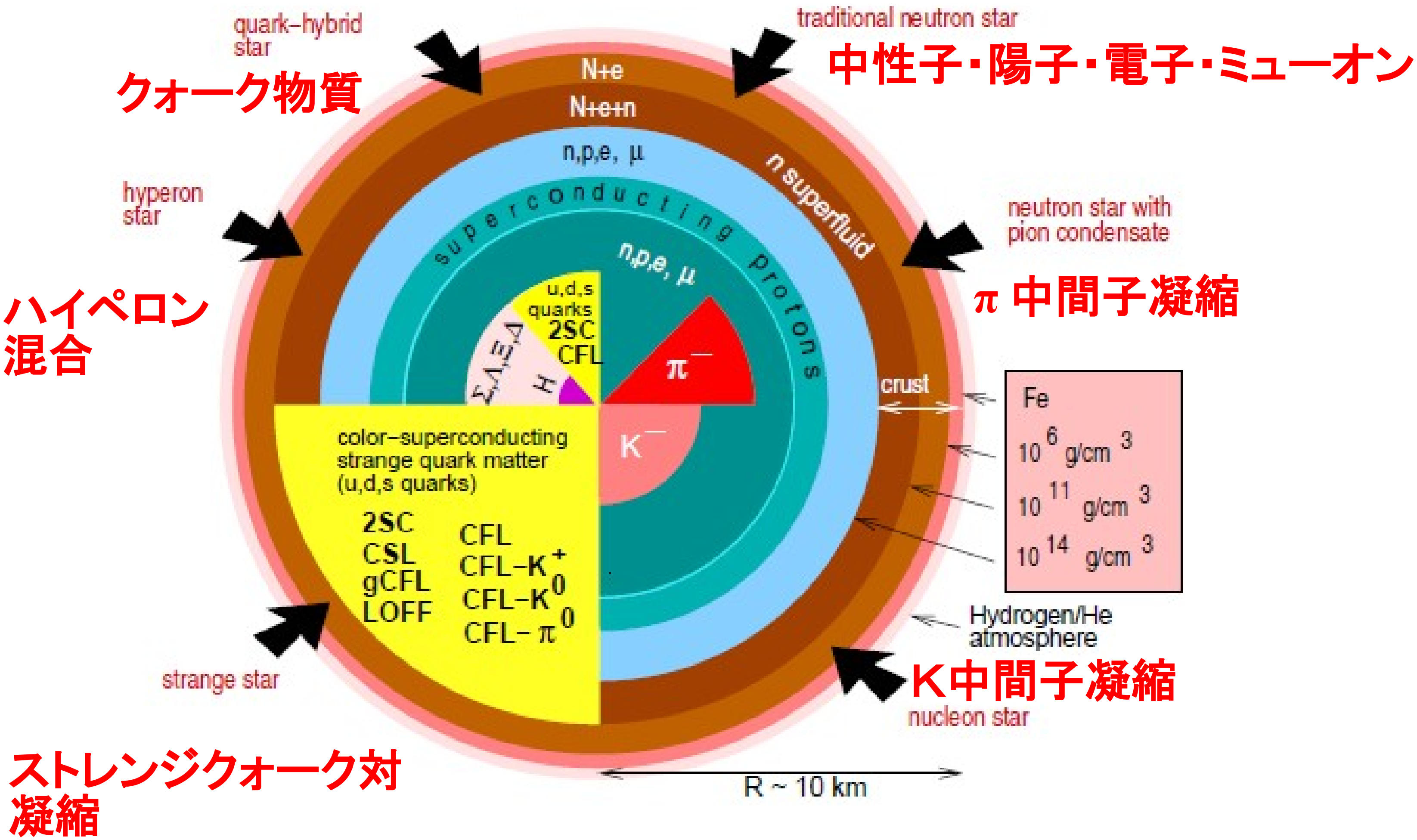
Recent experiments / observations provides rich information of EOS in 2D (ρ_B, Y_p) or 3D (ρ_B, Y_p, T).

How can we use it to obtain "THE" EOS ?

***Here "THE" means "The",
and "Tri-Hierarchical EOS"
(Quark-Hadron-Nuclear)***

Bottom-up Approach
Relativistic Mean Field study of Normal nuclei, hypernuclei, and dense matter

高密度星の中のストレンジネス

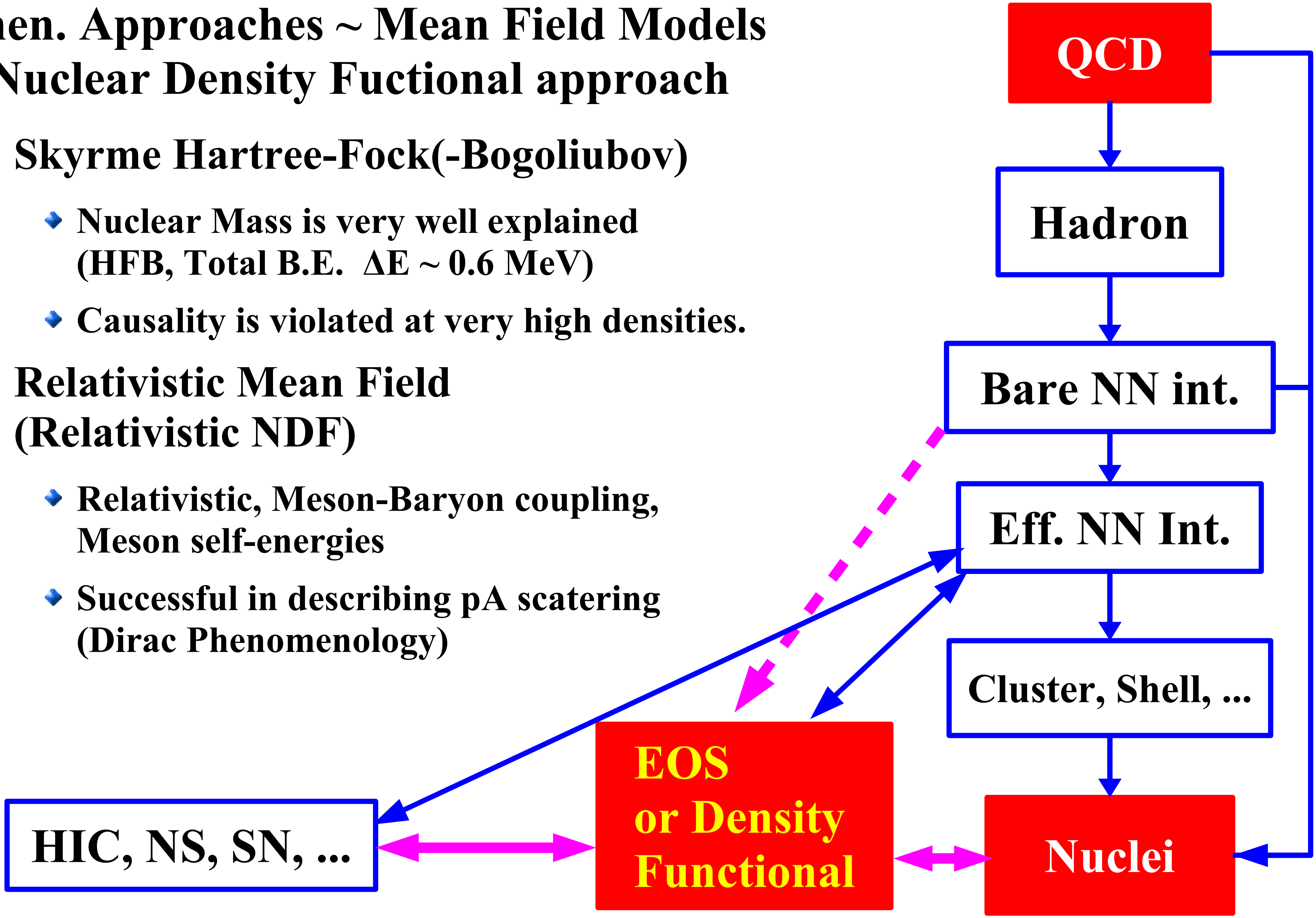


F. Weber, Prog. Part. Nucl. Phys. 54 (2005) 193

「中性子星」の内側 →ほぼ確実にストレンジネスを多く含む！

Phenomenological Approaches to EOS

- **Phen. Approaches ~ Mean Field Models ~ Nuclear Density Fuctional approach**
  - **Skyrme Hartree-Fock(-Bogoliubov)**
    - **Nuclear Mass is very well explained (HFB, Total B.E. $\Delta E \sim 0.6$ MeV)**
    - **Causality is violated at very high densities.**
  - **Relativistic Mean Field (Relativistic NDF)**
    - **Relativistic, Meson-Baryon coupling, Meson self-energies**
    - **Successful in describing pA scatering (Dirac Phenomenology)**



Relativistic Mean Field

- **Effective Lagrangian of Baryons and Mesons + Mean Field App.**

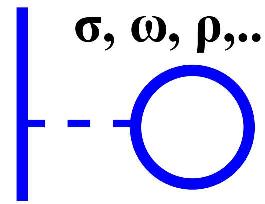
  B.D.Serot, J.D.Walecka, Adv.Nucl.Phys.16 ('86), 1

$$L = L_B^{\text{free}} + L_M^{\text{free}} + L_{BM} + L_M^{\text{Int}}$$

$$L_M^{\text{Int}} = -U_\sigma(\sigma) + \frac{1}{4} c_\omega (\omega_\mu \omega^\mu)^2 + \cdots, \quad U_\sigma = \frac{g_3}{3}\sigma^3 + \frac{g_4}{4}\sigma^4 + \cdots$$

$$L_{BM} = -\sum_{B,S} g_{BS} \bar{\psi}_B \varphi_S \psi_B - \sum_{B,V} g_{BV} \bar{\psi}_B \gamma^\mu V_\mu \psi_B$$

$$L_B^{\text{free}} = \bar{\psi}_B (i\gamma^\mu \partial_\mu - M_B)\psi_B \quad , \quad L_M^{\text{free}} = \sum_S \left[\frac{1}{2}\partial^\mu \varphi_S \partial_\mu \varphi_S - \frac{1}{2} m_S^2 \varphi_S^2\right] + \sum_V \left[-\frac{1}{4} V_{\mu\nu} V^{\mu\nu} + \frac{1}{2} V_\mu V^\mu\right]$$



- **Baryons and Mesons: B=N, Λ, Σ, Ξ, ...,   S= σ, ς, ...,   V= ω, ρ, φ, …**
- **Based on Dirac phenomenology & Dirac Bruckner-Hatree-Fock**

  E.D. Cooper, S. Hama, B.C. Clark, R.L. Mercer, PRC47('93),297
  R. Brockmann, R. Machleidt, PRC42('90),1965

- **Large scalar (att.) and vector (repl.) → Large spin-orbit pot.**
  Relativistic Kinematics → Effective 3-body repulsion
- **Non-linear terms of mesons → Bare 3-body and 4-body force**

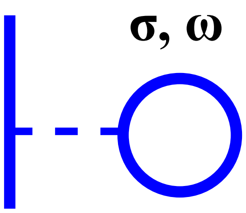
  Boguta, Bodmer ('77), NL1:Reinhardt, Rufa, Maruhn, Greiner, Friedrich ('86), NL3: Lalazissis, Konig, Ring ('97),TM1 and TM2: Sugahara, Toki ('94), Brockmann, Toki ('92)

σω Model --- first RMF model

Serot, Walecka, Adv.Nucl.Phys.16 (1986),1

- **Nucleon, σ (scalar-isoscalar) and ω (vector-isoscalar) mesons**

$$L=\bar\psi\left(i\gamma^\mu\partial_\mu-M+g_s\sigma-g_v\omega\right)\psi$$
$$+\frac{1}{2}\partial_\mu\sigma\,\partial^\mu\sigma-\frac{1}{2}m_s^2\sigma^2-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2}m_v^2\omega_\mu\omega^\mu$$
$$\left(F_{\mu\nu}=\partial_\mu\omega_\nu-\partial_\nu\omega_\mu\right)$$



- **Equation of Motion (Euler-Lagrange Equation)**

$$\frac{\partial}{\partial x^\mu}\left[\frac{\partial L}{\partial(\partial_\mu\phi_i)}\right]-\frac{\partial L}{\partial\phi_i}=0$$

$$\sigma:\left[\partial_\mu\partial^\mu+m_s^2\right]\sigma=g_s\bar\psi\psi$$

$$\omega:\partial_\mu F^{\mu\nu}+m_v^2\omega^\nu=g_v\bar\psi\gamma^\nu\psi\quad\rightarrow\quad\left[\partial_\mu\partial^\mu+m_v^2\right]\omega^\nu=g_v\bar\psi\gamma^\nu\psi$$

$$\psi:\left[\gamma^\mu\left(i\partial_\mu-g_v V_\mu\right)-\left(M-g_s\sigma\right)\right]\psi=0$$

→ **Nucleon EOM = Free Dirac Eq. with modified *E* and *M***

Nuclear Matter in σω Model

- **Uniform Nuclear Matter** *Serot, Walecka, Adv.Nucl.Phys.16 (1986),1*

$$E/V = \gamma_N \int^{P_F} \frac{d^3p}{(2\pi)^2} \sqrt{M^{*2}+p^2} + \frac{1}{2} m_s^2 \sigma^2 - \frac{1}{2} m_v^2 \omega^2 + g_v \rho_B \omega$$

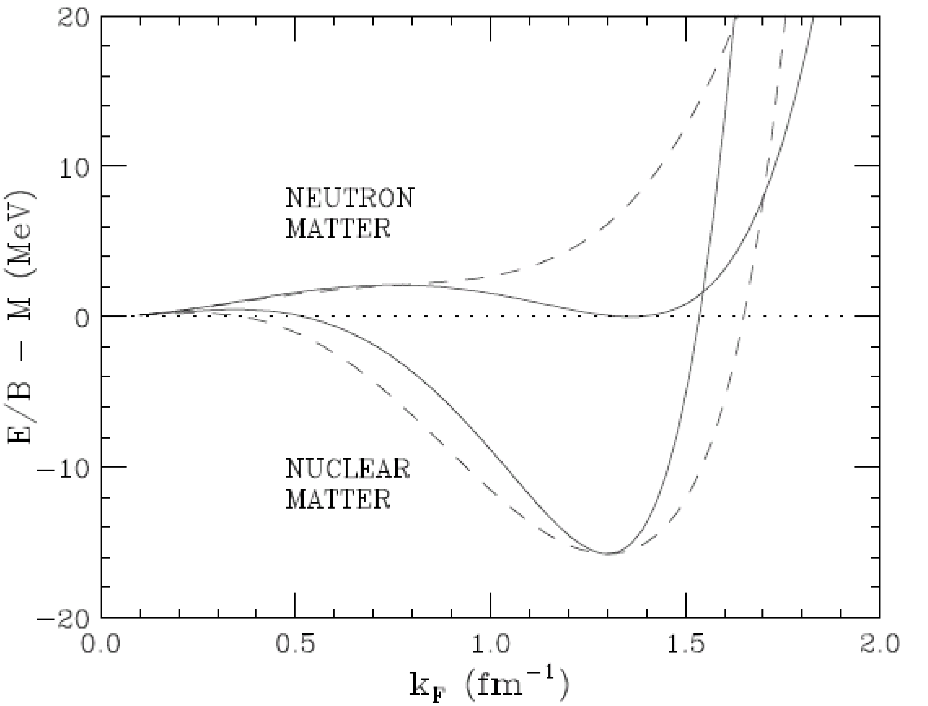
$$M^* = M + U_s = M - g_s \sigma, \quad \sigma = \frac{g_s}{m_s^2} \rho_s, \quad \omega = \frac{g_v}{m_v^2} \rho_B$$

γ_N = Nucleon degeneracy (=4 in sym. nuclear matter)
→ Too Stiff EOS (K ~ 600 MeV)

- **Too stiff EOS is improved with non-linear terms (σ^3, σ^4, ω^4)**

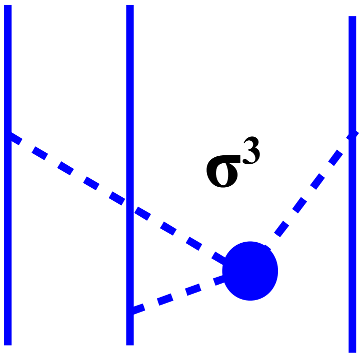
$$L = L_B^{\text{free}} + L_M^{\text{free}} + L_{BM} + L_M^{\text{Int}}$$

$$L_M^{\text{Int}} = -U_\sigma(\sigma) + \frac{1}{4} c_\omega (\omega_\mu \omega^\mu)^2 + \cdots$$



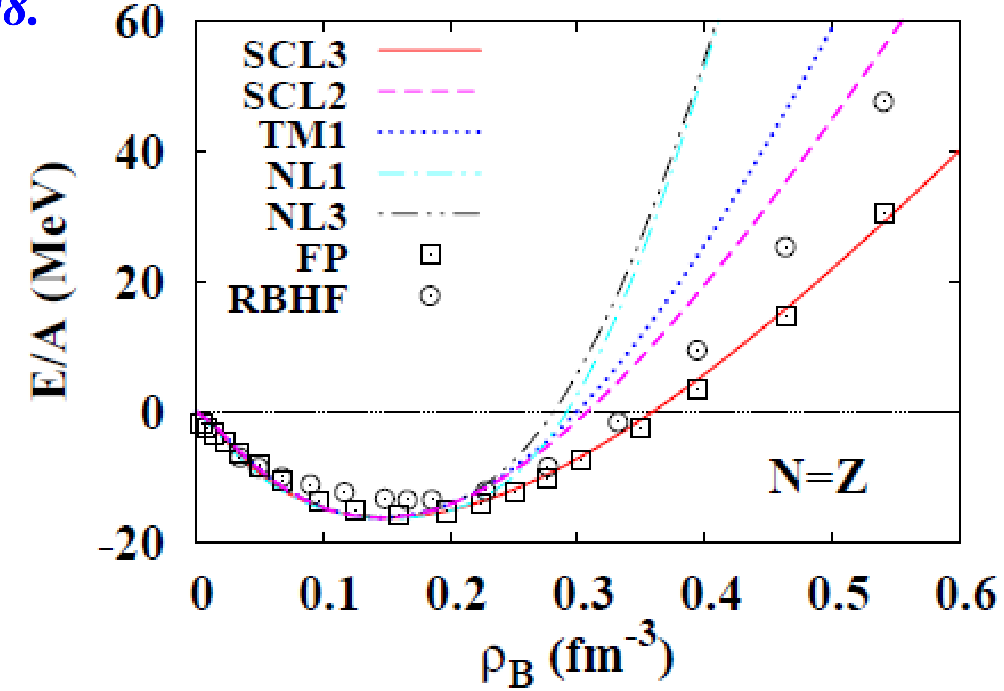
High Quality RMF models

- Variety of the RMF models
  → MB couplings, meson masses, meson self-energies



  - σN, ωN, ρN couplings are well determined
    → almost no model deps. in Sym. N.M. at low ρ
  - ω^4 term is introduced to simulate DBHF results of vector pot.
    TM: Y. Sugahara, H. Toki, NPA579('94)557; R. Brockmann, H. Toki, PRL68('92)3408.
  - σ^3 and σ^4 terms are introduced to soften EOS at ρ_0.
    J. Boguta, A.R.Bodmer NPA292('77)413, NL1:P.-G.Reinhardt, M.Rufa, J.Maruhn, W.Greiner, J.Friedrich, ZPA323('86)13. NL3: G.A.Lalazissis, J.Konig, P.Ring, PRC55('97)540.

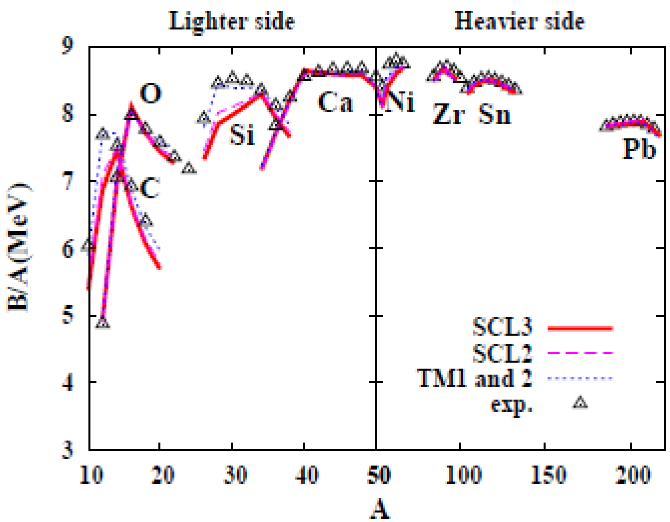
→ Large differences are found at high ρ

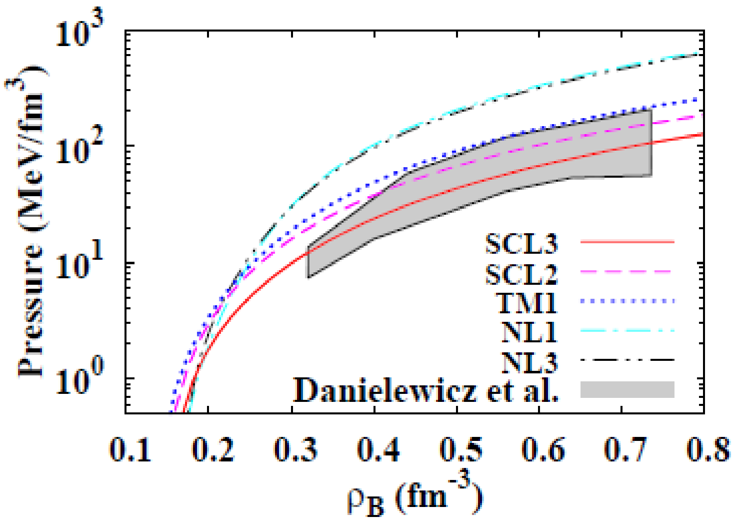


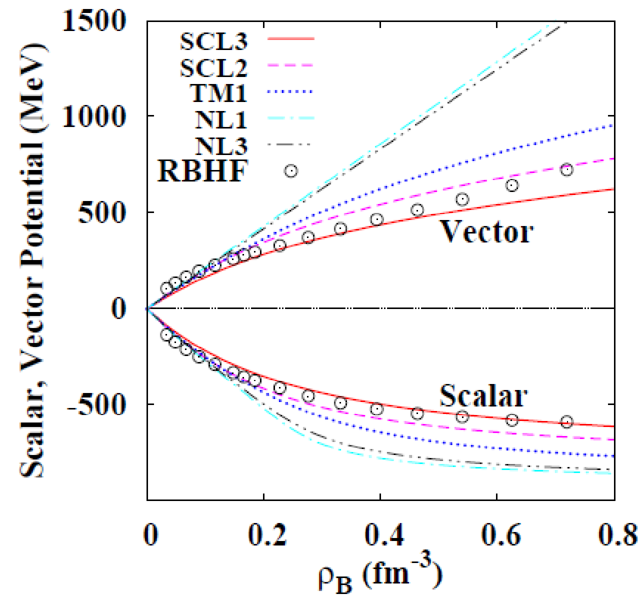
K. Tsubakihara, H. Maekawa, H. Matsumiya, AO,PRC81('10)065206.

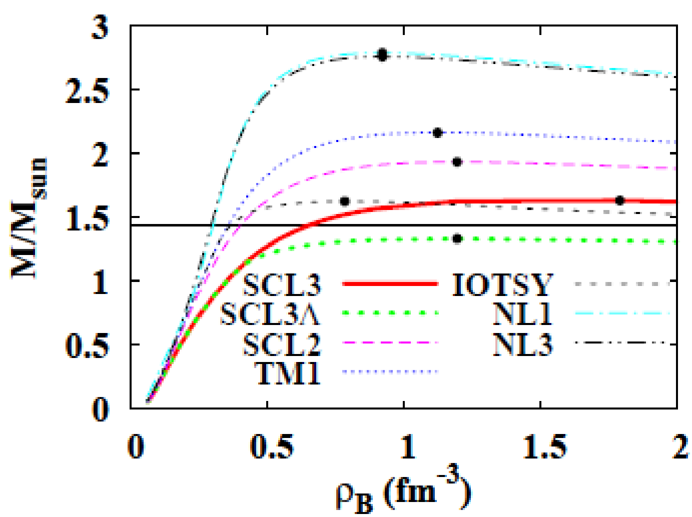
How to determine Non-Linear terms ? (1)

- **Method 1: Fit as many as known observables**
  - **EOS, Nuclear B.E., High density EOS from HIC, Vector potential in DBHF, Neutron Star, ...**





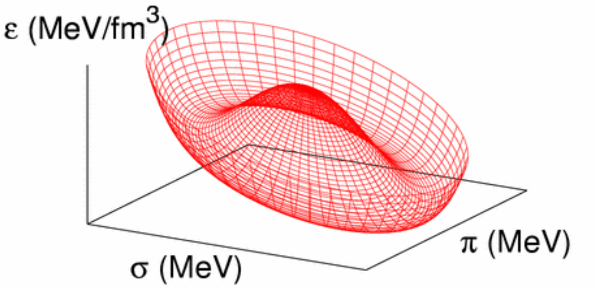


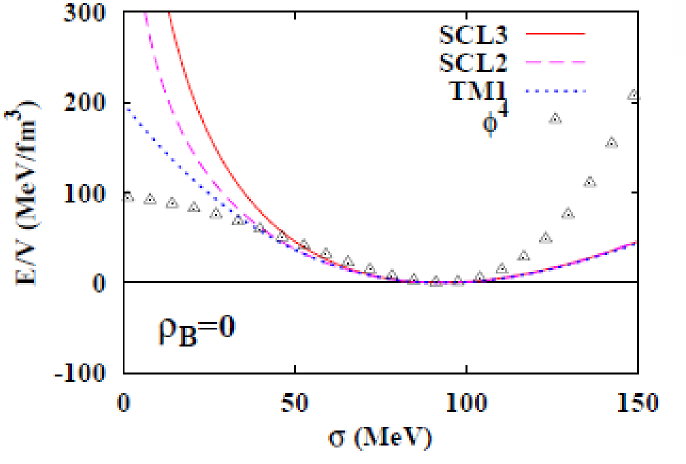


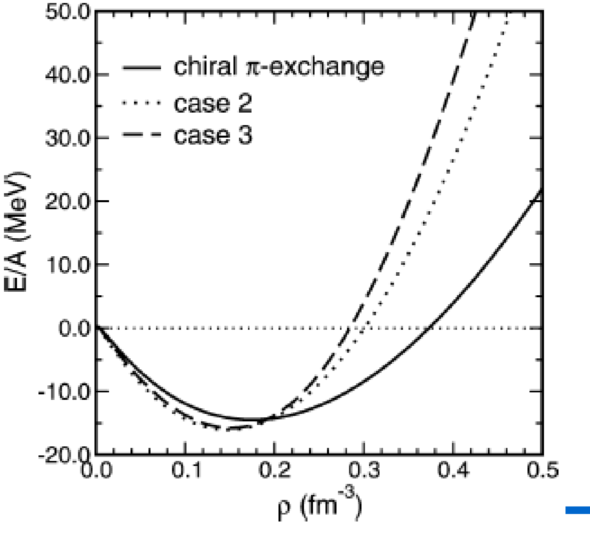
P. Danielewicz, R. Lacey,W. G. Lynch, Science298('02)1592.
R. Brockmann, R. Machleidt, PRC42('90)1965.
K. Tsubakihara, H. Maekawa, H. Matsumiya, AO,PRC81('10)065206.

How to determine Non-Linear terms ? (2)

- **Method (2): Fix parameters by using symmetry, such as the *Chiral Symmetry***
- **Chiral Symmetry**
  - **Fundamental symmetry of massless QCD, and its spontaneous breaking generates hadron masses.**
    Nambu, Jona-Lasinio ('61)
  - **Many of the linear σ models are unstable against finite density (chiral collapse).**
    → Log type chiral potential
    Sahu, Tsubakihara,AO('10), Tsubakihara, AO('07), Tsubakihara et al.('10)
  - **Non-linear representation (chiral pert.) leads to density dependent coupling from one- and two-pion exchanges.**
    Kaiser, Fritsch, Weise ('02),
    Finelli, Kaiser, Vretener, Weise ('04)

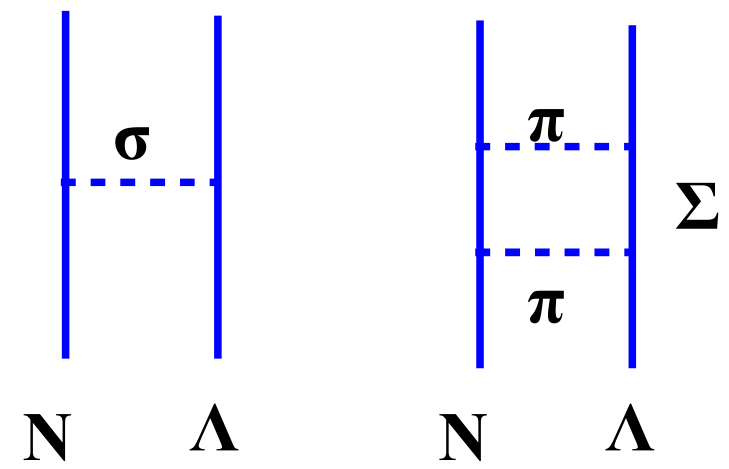




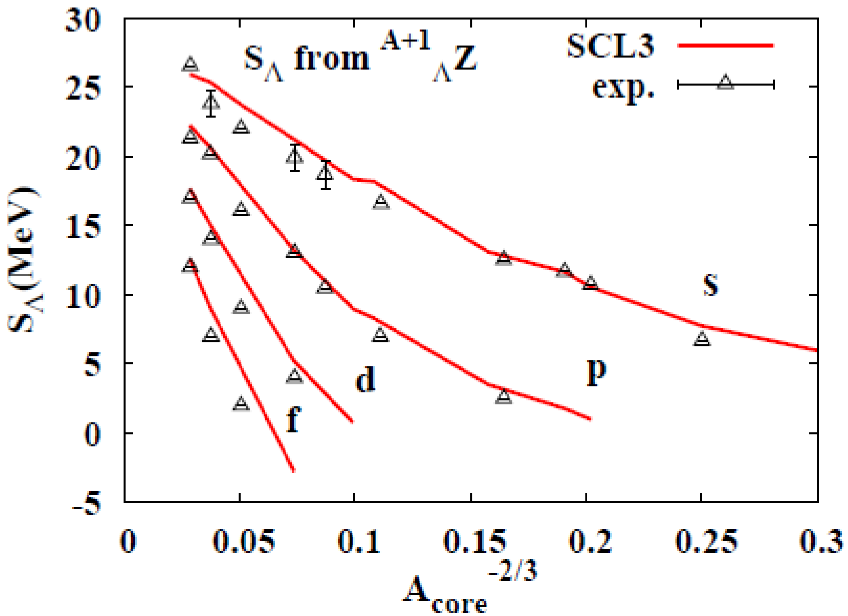


RMF with Hyperons --- Λ hypernuclei

- **Why Λ ?**
  - Λ is expected to appear in NS.
  - Coupling with π, σ, … are different
    → detailed study of Λ hypernnuclei will tell us what makes MF (OBEP or π)



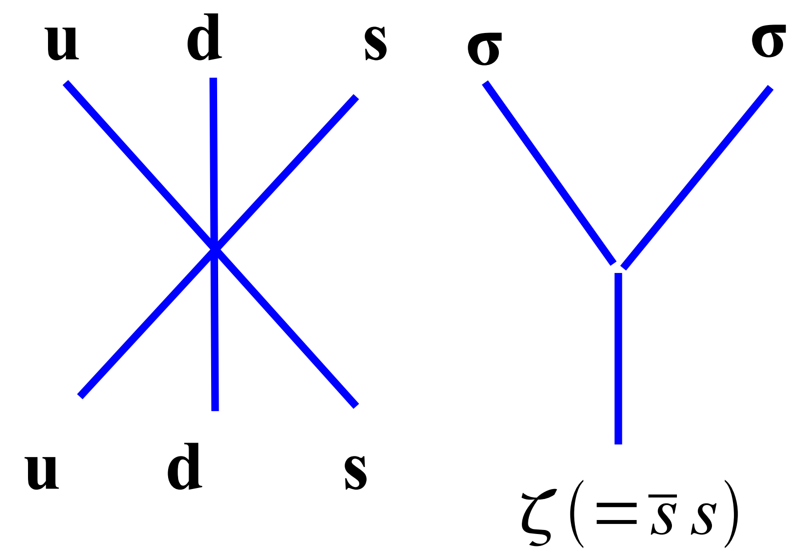
  - Coupling with mesons : $x_M = g_{M\Lambda}/g_{MN}$
    quark counting: $x_\sigma \sim 2/3$
    π exchanges: $x_\sigma \sim 1/3$
    → Which is true ?
- **Single Λ hypernuclei**
  - Λ Sep. E. → $U_\Lambda \sim$ -30 MeV $\sim 2/3\, U_N$
    → We can fit them by changing $g_{\sigma\Lambda}$, $g_{\omega\Lambda}$, $g_{\zeta\Lambda}$, …

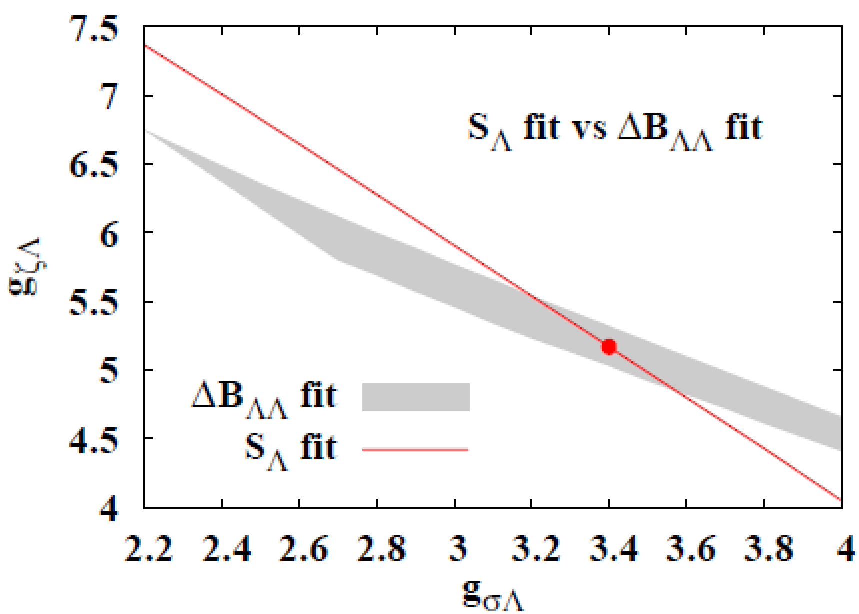


K. Tsubakihara, H. Maekawa, H. Matsumiya, AO, PRC81('10)065206.

RMF with Hyperons --- Double Λ hypernuclei

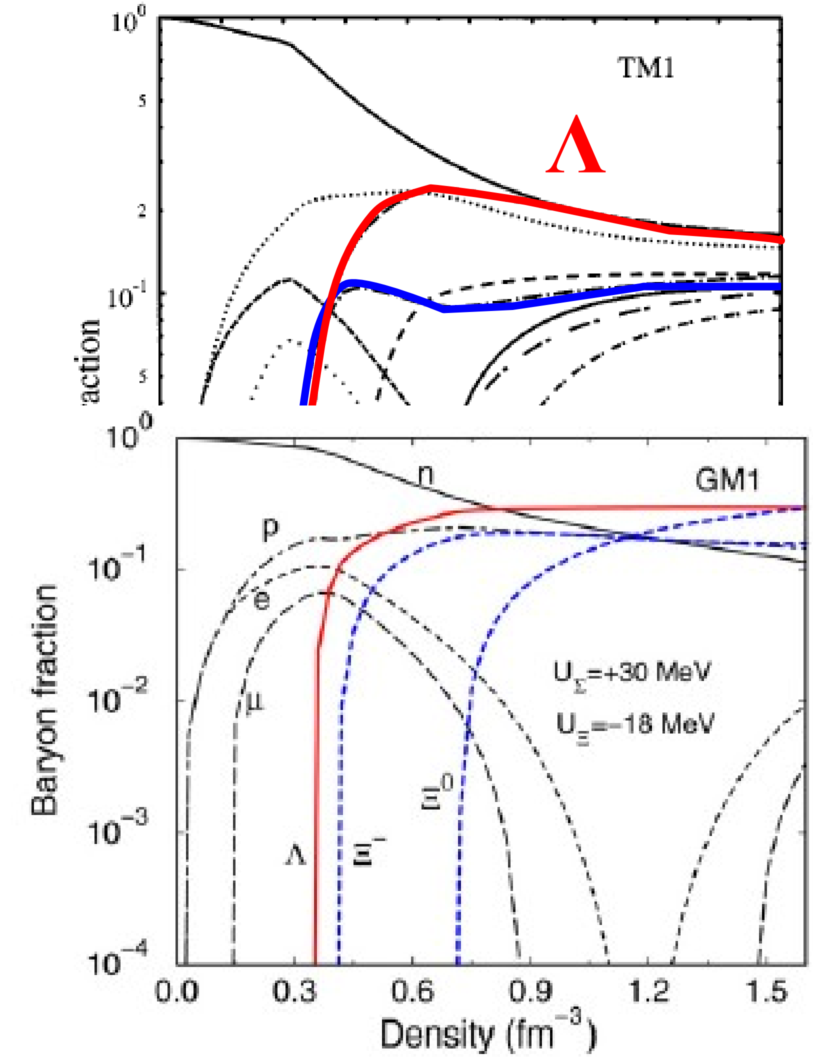
- Nagara event $\Delta B_{\Lambda\Lambda} \sim 1.0$ MeV (weakly attractive)
  - TM & NL-SH based RMF
    H. Shen, F. Yang, H. Toki, PTP115('06)325.
    Model 1: $x_\sigma = 0.621$, $x_\omega = 2/3$ (no ς, φ)
    Model 2: $R_\varsigma = g_{\varsigma\Lambda} / g_{\sigma N} = 0.56\text{-}0.57$,
    $R_\varphi = g_{\varphi\Lambda} / g_{\omega N} = -\sqrt{2}/3$
  - Chiral SU(3) RMF
    K. Tsubakihara, H. Maekawa, H. Matsumiya, AO, PRC81('10)065206.
    $SU(3)_f$ for vector coupling
    $x_\omega = 0.64$, $R_\varphi = 0.504$
    Det. (KMT) int. mixes σ and ς
    M. Kobayashi, T. Maskawa, PTP44('70)1422. G. 't Hooft, PRD14('76)3432.
    → $x_\sigma = 0.335$, $R_\varsigma = 0.509$





Hyperon Composition in Dense Matter

- **Hyperon start to emerge at (2-3)ρ_0 in Neutron Star Matter !**
- **Hyperon composition in NS is sensitive to Hyperon potential.**
  - **U_Λ ~ -30 MeV: Well-known**
  - **U_Ξ ~ -(12-15) MeV**
    (K^-,K^+) reaction, twin hypernuclei
    P. Khaustov et al. (E885),PRC61('00)054603;
    S. Aoki et al., PLB355('95)45.
  - **U_Σ~ -30 MeV (Old conjecture)**
    → Σ^- appears prior to Λ
  - **U_Σ > 0 (repulsive) → No Σ in NS**
    Σ atom (phen. fit), QF prod.
    H. Noumi et al., PRL89('02)072301;
    T. Harada, Y. Hirabayashi, NPA759('05)143;
    M. Kohno et al. PRC74('06)064613.

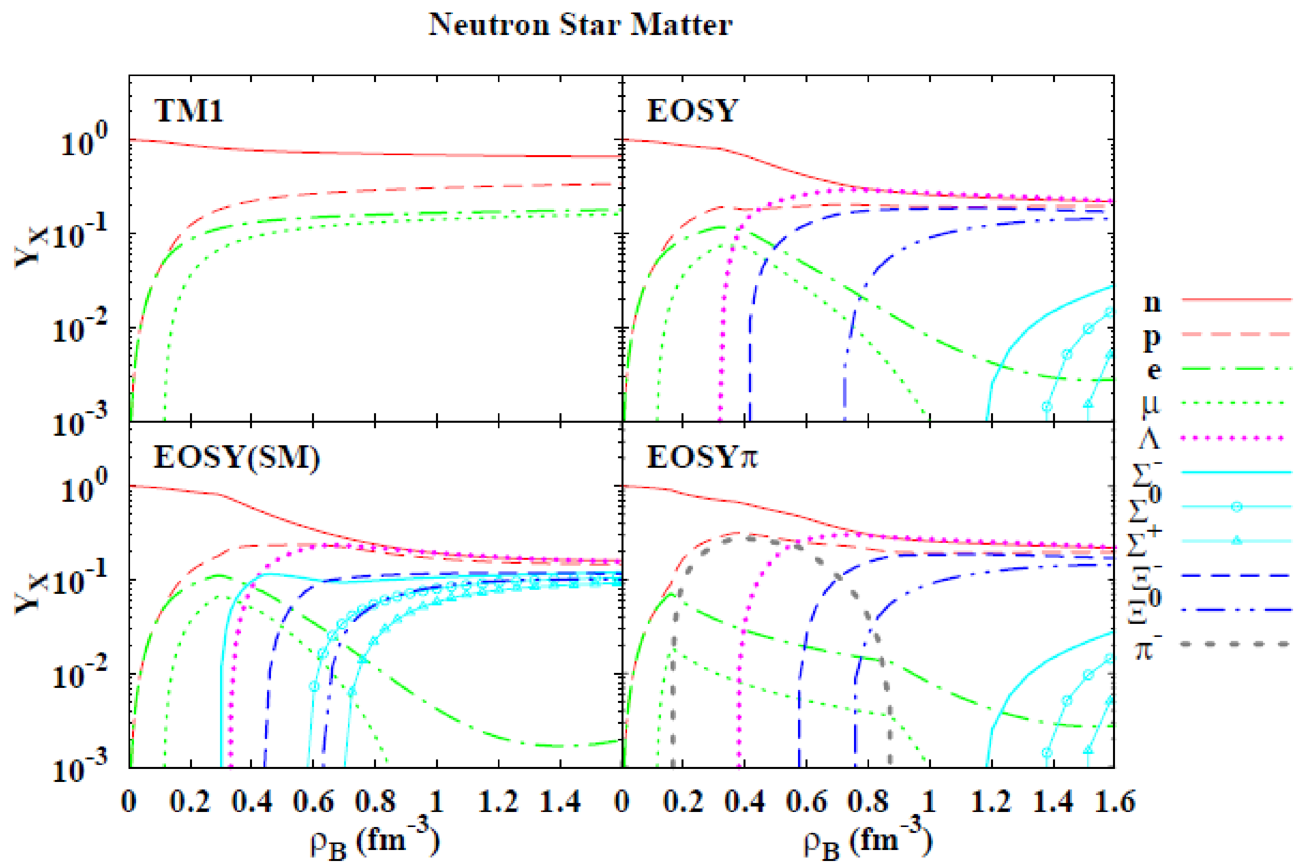


J. Schaffner-Bielich, NPA804('08)309.

Hyperon Composition in Dense Matter

- **Comparison of Hyperon Composition**
  - **U_Σ=-30 MeV, U_Ξ = -28 MeV → SU(3) sym. matter at $\rho_B \sim 10\ \rho_0$**
    Schaffner, Mishustin ('94)
  - **U_Σ=+30 MeV, U_Ξ = -15 MeV→ Σ baryons are strongly suppressed.**
    C.Ishizuka, AO, K.Tsubakihara, K.Sumiyoshi, S.Yamada, JPG35('08)085201.

→ Does Σ play no role in NS ?



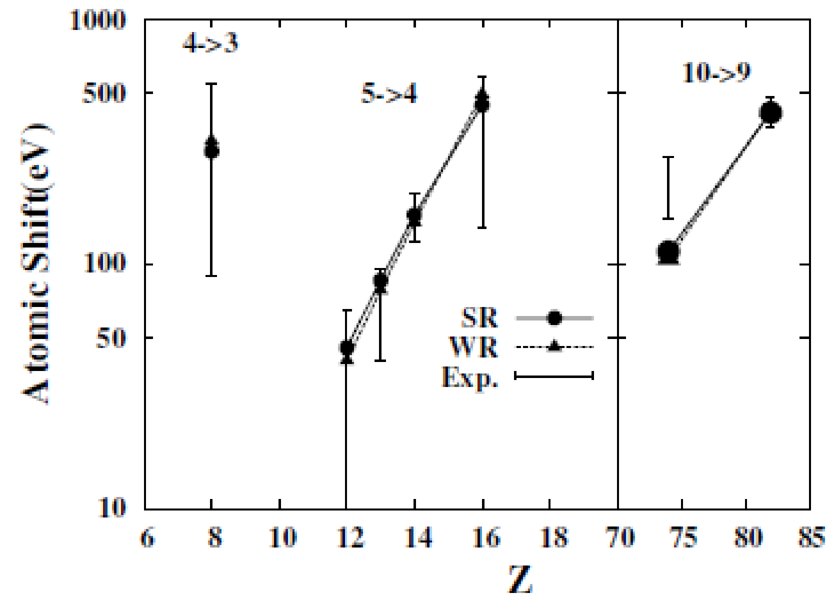
Σ^- atom data

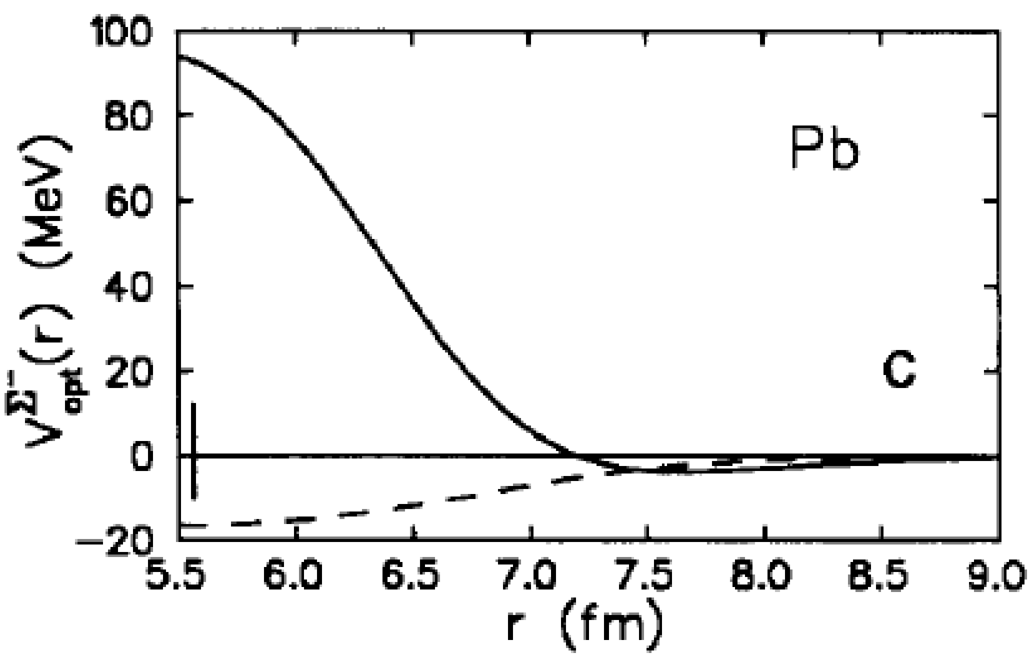
- **Σ^- atom data suggested repulsion in the interior of nuclei !**

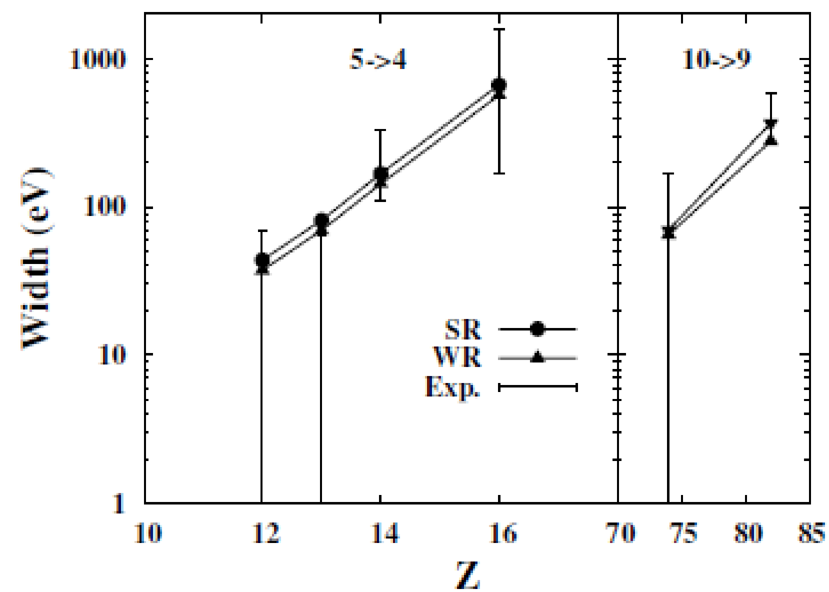
  C.J.Batty, E.Friedman, A.Gal, PLB335('94)273

  Batty's DD potential is very repulsive inside nuclei.

  → No Σ baryon in dense matter.







J.Mares, E.Friedman, A.Gal, B.K.Jennings, NPA594('95)311.

K.Tsubakihara, H.Maekawa, AO, EPJA33('07)295.

Σ^- atom in RMF

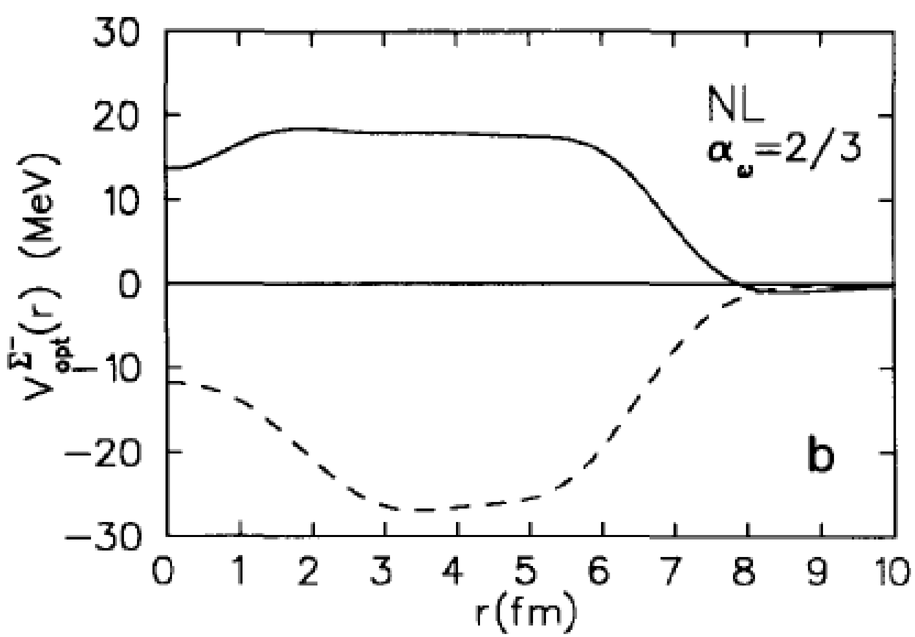
- **RMF fit of Si and Pb Σ^- atom**

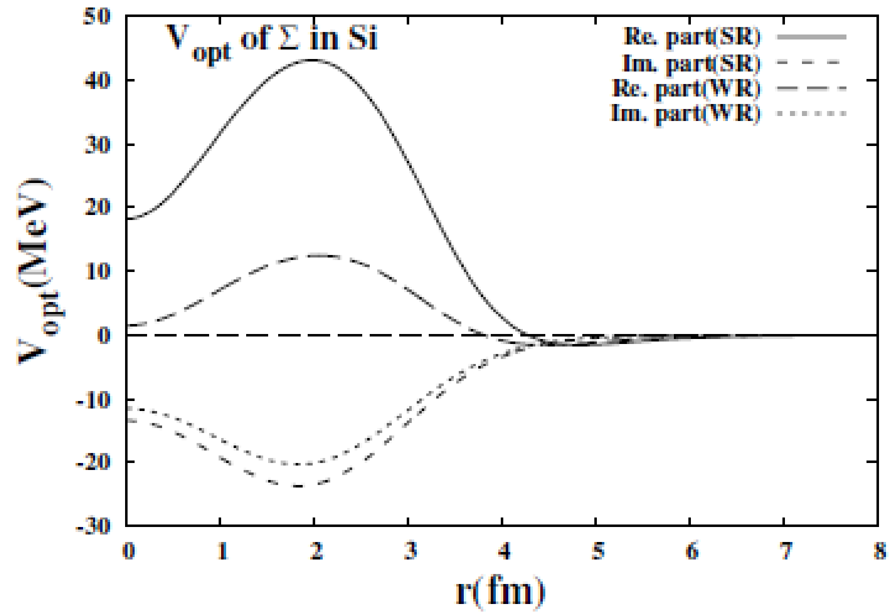
  $\alpha_\omega = g_{\omega\Sigma} / g_{\omega N} \sim 2/3(M), 0.69\ (T)$

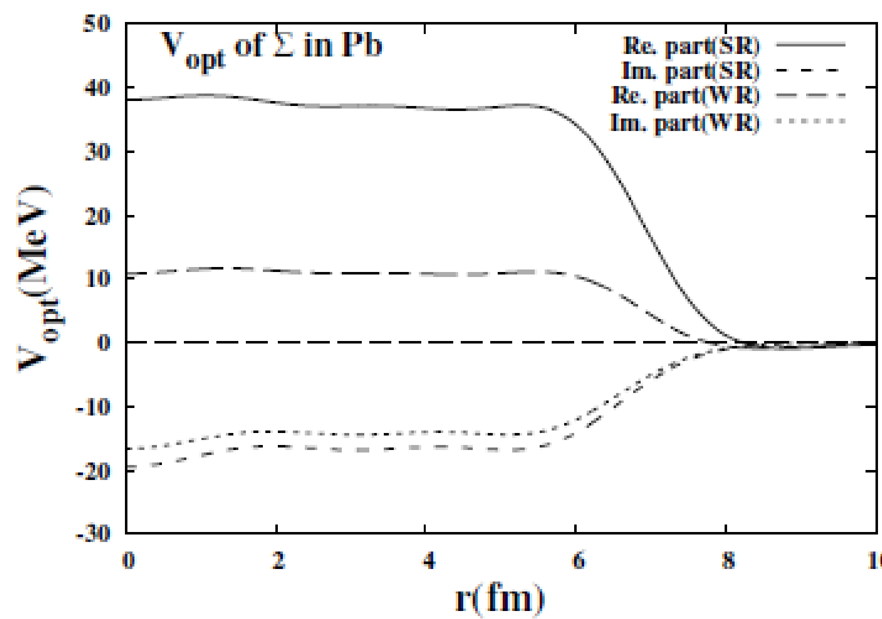
  $\alpha_\rho = g_{\rho\Sigma} / g_{\rho N} \sim 2/3(M), 0.434(T)$

  J.Mares, E.Friedman, A.Gal, B.K.Jennings, NPA594('95)311; Tsubakihara et al.('10)

  - **Much smaller $g_{\rho\Sigma}$ than naïve SU(3) ($g_{\rho\Sigma} / g_{\rho N}$=2), which has been applied in some of previous works.**



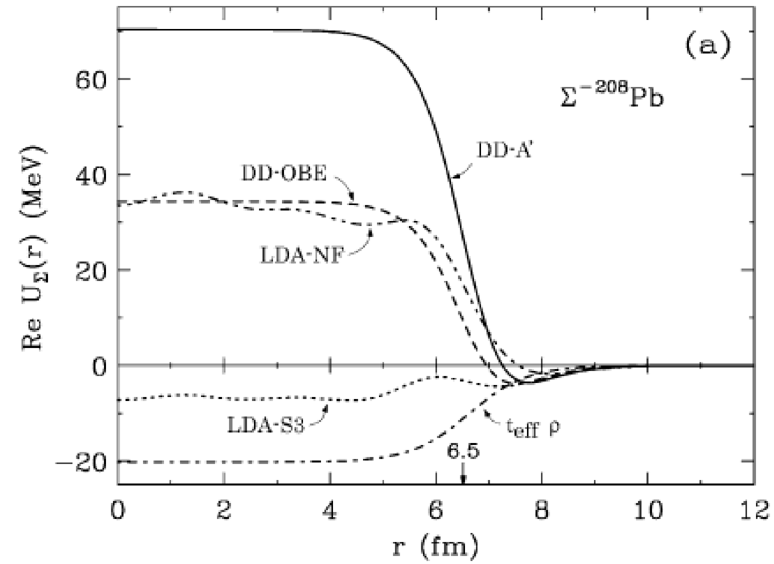




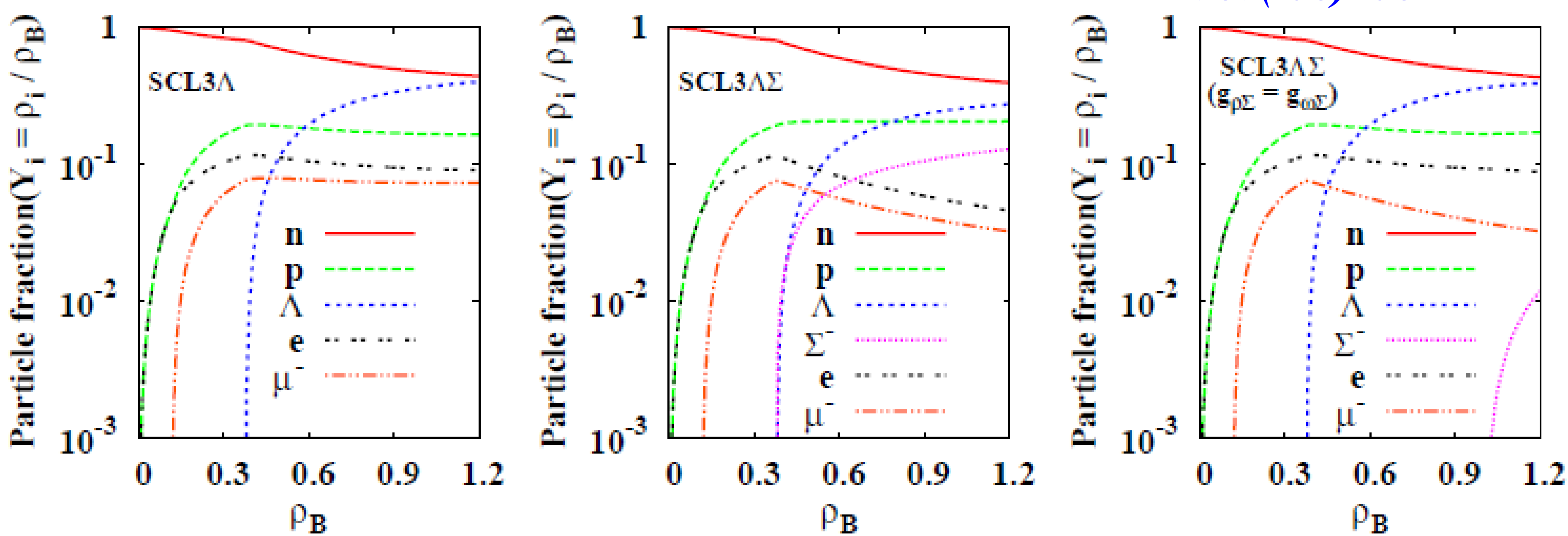
Σ atom and Neutron Star

- **Σ may not feel *very* repulsive potential in neutron star....**
  - **ρ^γ-type fit → very repulsive**
  - **RMF fit → small isovector potential**

→ QF prod. may support the latter.
Σ^- would appear in NS.



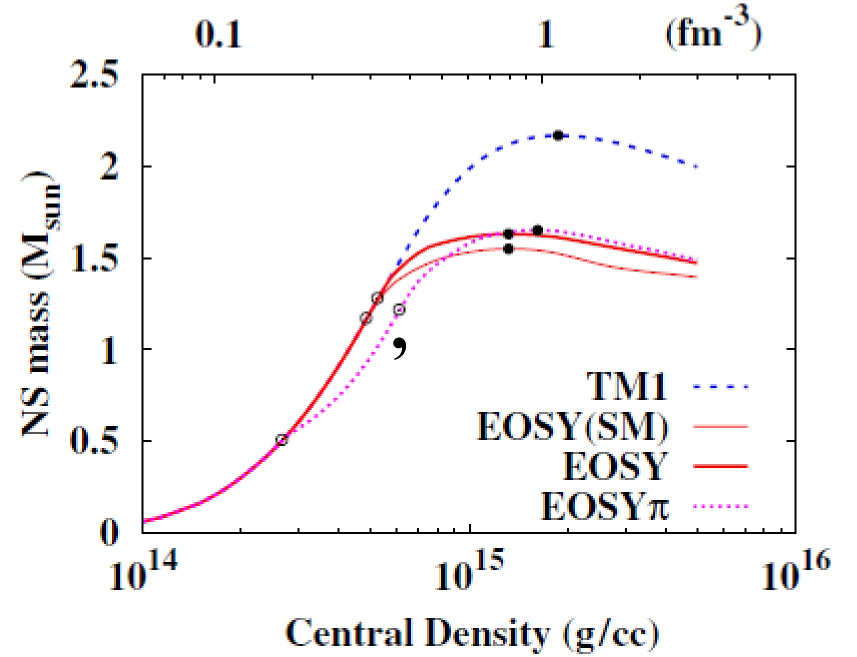
T. Harada, Y. Hirabayashi, NPA767('06)206

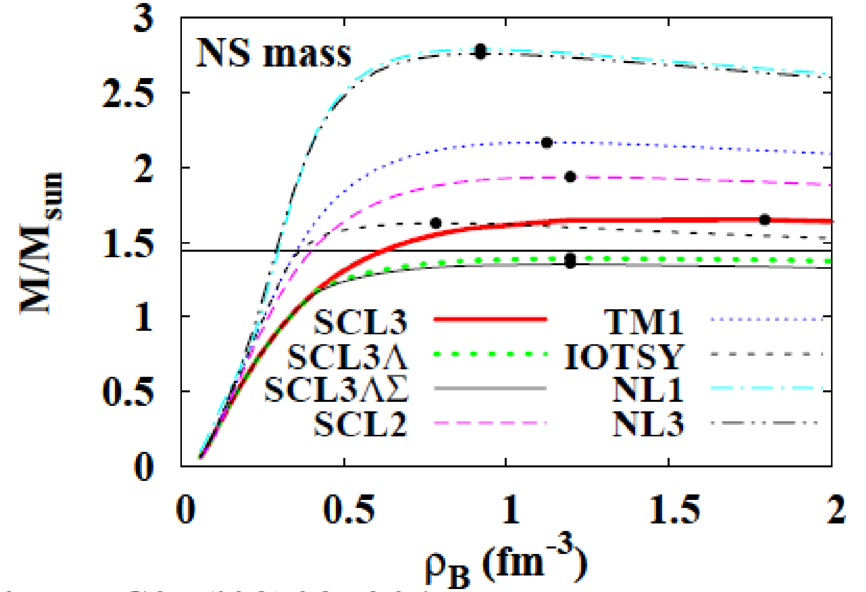


K. Tsubakihara, H. Maekawa, H. Matsumiya, AO, PRC81('10)065206.

Neutron Star Mass

- Large fraction of hyperons softenes EOS at $\rho_B > (0.3\text{-}0.4)\ \mathrm{fm}^{-3}$
  - NS star max. mass red. ~ 1 M_{sun}.
  - RMF generally predicts stiff EOS at high density.
    (Scalar attraction saturation,
    or Z-graph in NR view.)
  - Some of RMF with Y do not support 1.44 M_{sun}.
- Additional Repulsion at high ρ ?
  - Vector mass mod.
    → stronger repulsion at high ρ.
    M. Naruki et al., PRL96('06)092301.
  - Another term such as NNωσ.

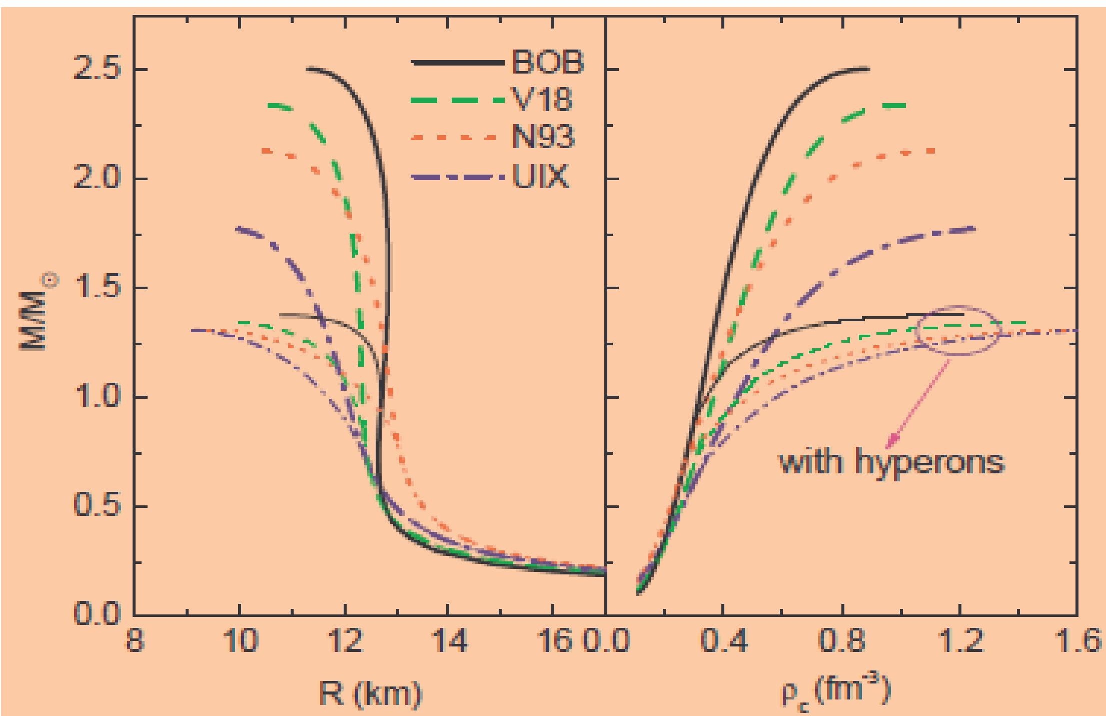




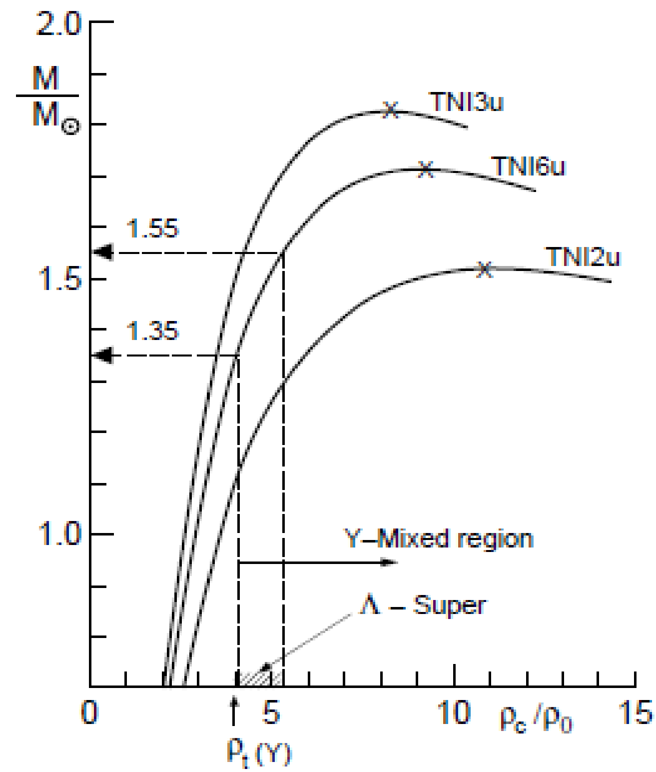
C. Ishizuka, AO, K. Tsubakihara, K. Sumiyoshi, S. Yamada, JPG35('08)085201.
K. Tsubakihara, H. Maekawa, H. Matsumiya, AO, PRC81('10)065206.

Bruckner-Hartree-Fock theory with Hyperons

- **Microscopic G-matrix calculation with realistic NN, YN potential and microscopic (or phen.) 3N force (or 3B force).**
  - **Interaction dep. (V18, N93, ...) is large → Need finite nuclear info.**
    E.Hiyama, T.Motoba, Y.Yamamoto, M.Kamimura / M.Tamura et al.
  - **NS collapses with hyperons w/o 3BF.**



H.J.Schulze, A.Polls, A.Ramos, I.Vidana, PRC73('06),058801.



S. Nishizaki, T. Takatsuka, Y. Yamamoto, PTP108('02)703.

So far, not bad.
Phen. approaches (such as RMF) explain
various aspects of nuclear matter EOS.
In addition, they may be suggesting the nature
of the mean field ($\pi + \sigma + \omega + \ldots$),
which may have impact on quark-hadron physics.

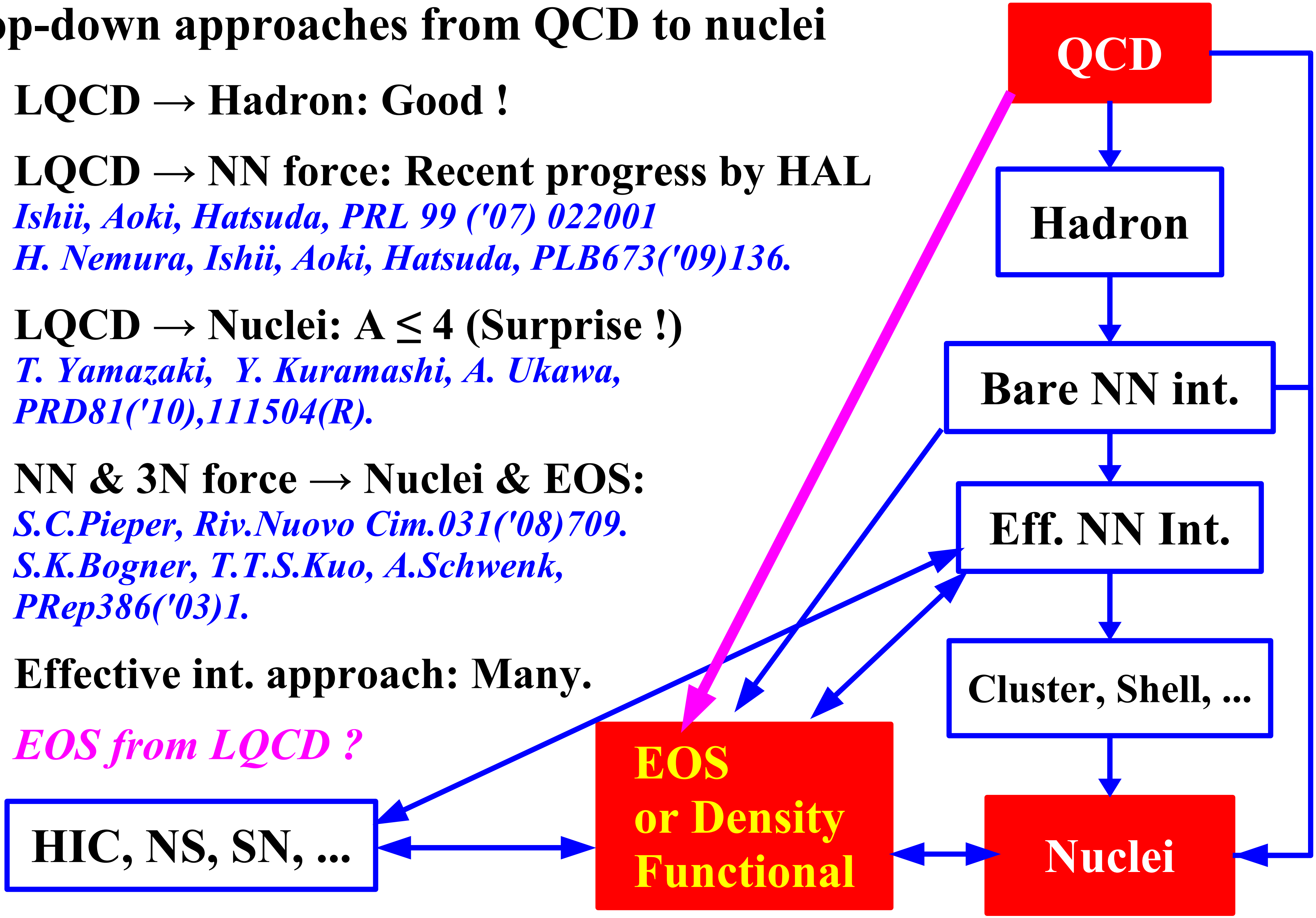
But phen. approaches require rich data are available
from experiment / observations.
Is there any way to describe nuclear matter
from QCD ?

Top-down Approach

Phase Diagram in Strong Coupling Lattice QCD with Polyakov loop and Finite CoupIng Effects

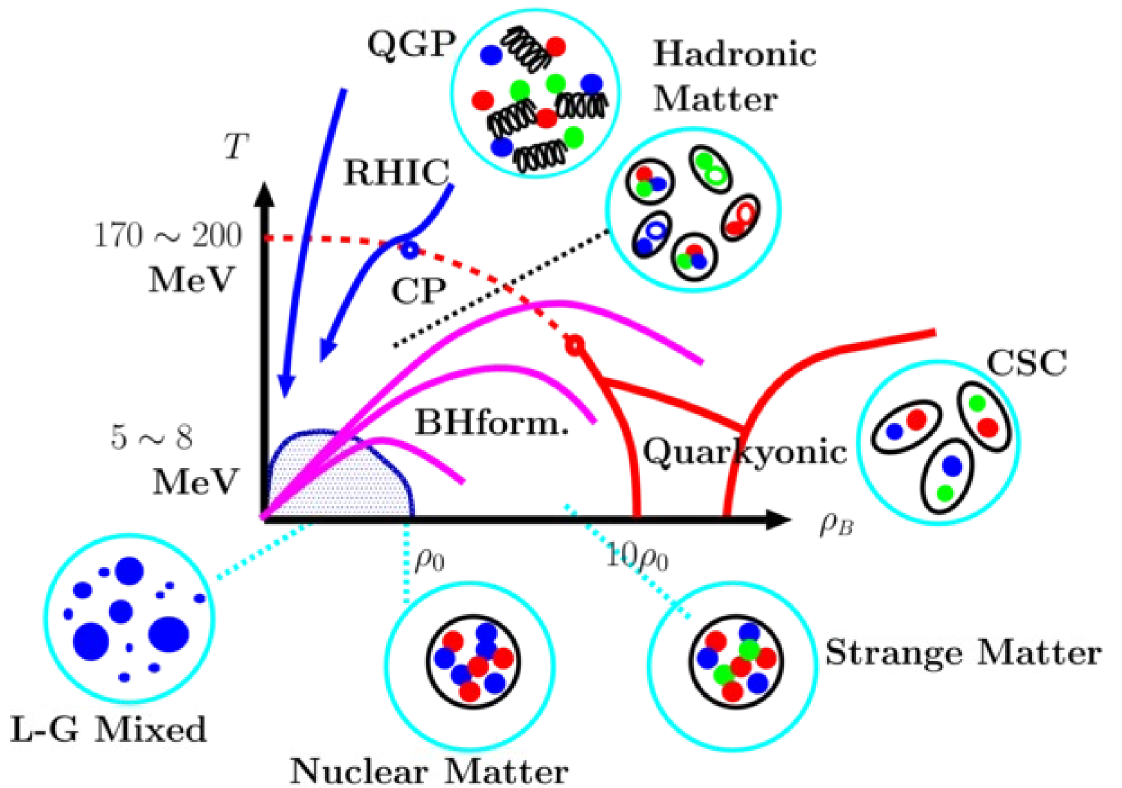
Top-down Approaches from QCD to Nuclei and/or EOS

- **Top-down approaches from QCD to nuclei**
  - **LQCD → Hadron: Good !**
  - **LQCD → NN force: Recent progress by HAL**
    Ishii, Aoki, Hatsuda, PRL 99 ('07) 022001
    H. Nemura, Ishii, Aoki, Hatsuda, PLB673('09)136.
  - **LQCD → Nuclei: A ≤ 4 (Surprise !)**
    T. Yamazaki, Y. Kuramashi, A. Ukawa, PRD81('10),111504(R).
  - **NN & 3N force → Nuclei & EOS:**
    S.C.Pieper, Riv.Nuovo Cim.031('08)709.
    S.K.Bogner, T.T.S.Kuo, A.Schwenk, PRep386('03)1.
  - **Effective int. approach: Many.**
  - ***EOS from LQCD ?***



QCD Phase diagram

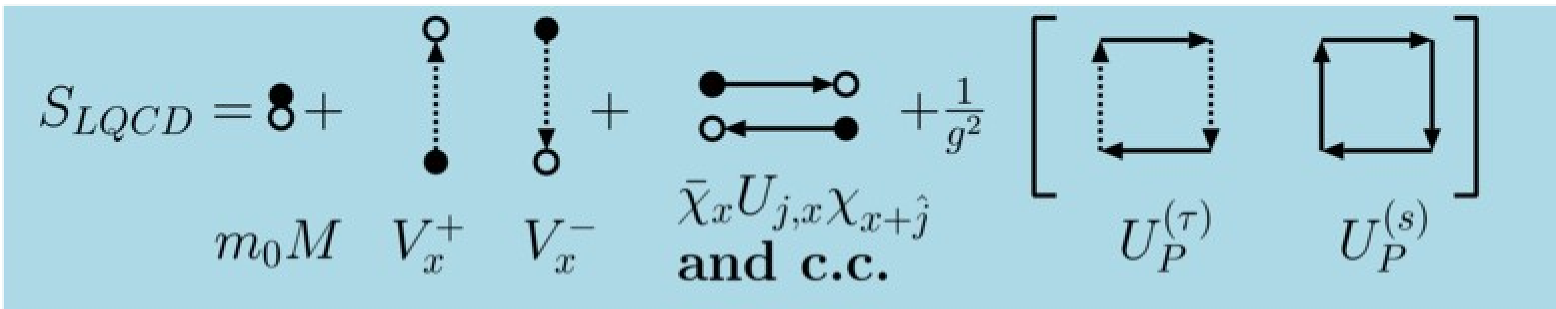
- **Phase transition at high T → Lattice MC, RHIC, LHC**
- **High μ transition has rich physics**
  → Various phases, CEP, Astrophysical applications, ...



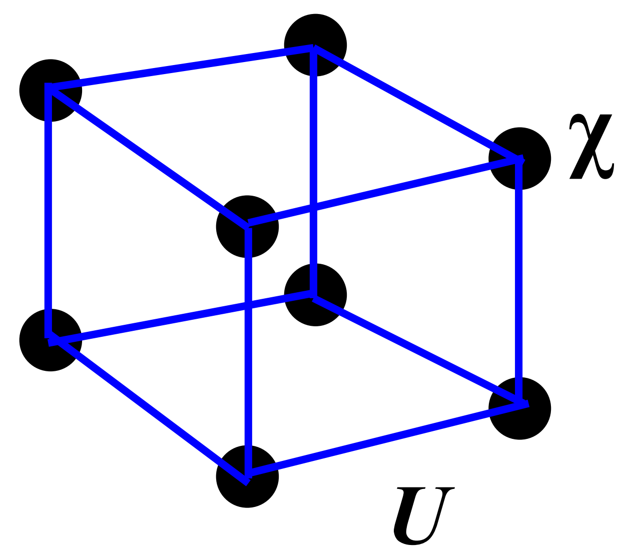
Sign problem in Lattice MC at finite density
→ We need approximations and/or eff. models

What is Strong Coupling Lattice QCD ?

- **Lattice QCD**
  - **Gluons on the link (link var.) + Fermions on the site**
  - **MC simulation → Non-pert. (SSB of χ sym., confinement, …)**

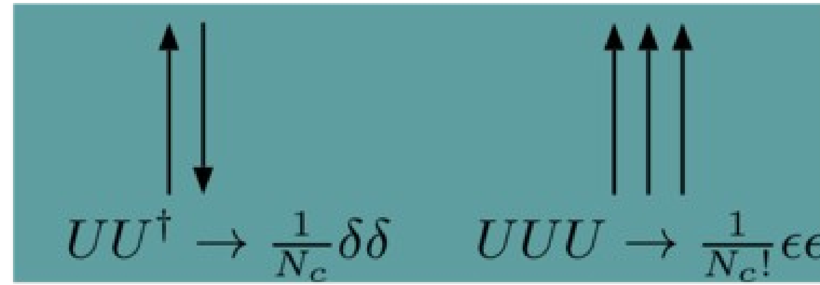


- **Strong Coupling Lattice QCD (SC-LQCD)**
  - **Strong coupling ($1/g^2$) expansion**
    ~ Plaquette ($\propto 1/g^2$) expansion
    c.f. pQCD (expansion in g)
  - **Integrate out (spatial) link analytically**
    → weaker sign problem
    c.f. MC (fermion intg first)



Basic tools in SC-LQCD

- **Group integral formula**



$$\int dU\, U_{ab} U^{+}_{cd} = \frac{1}{N_c}\delta_{ad}\delta_{bc}$$

$$\int dU\, U_{ab} U_{cd} U_{ef} = \frac{1}{6}\epsilon_{ace}\epsilon_{bdf}$$

$$\to \int dU \exp(-a\bar{\chi}(x)U\chi(y) + b\bar{\chi}(y)U^{+}\chi(x)) = 1 + ab\, M(x)M(y) + \cdots$$

- **1/d expansion → small # of quarks are favored**

$$\sum_{j=1}^{d} M_x M_{x+\hat{j}} = \frac{1}{d}\sum_{j=1}^{d}\frac{M_x}{\sqrt{d}}\frac{M_{x+\hat{j}}}{\sqrt{d}} \sim \text{const. at large } d \quad \to \quad \chi \propto d^{-1/4}$$

- **Bosonization**

$$\exp\left(\frac{1}{2}M^2\right) = \int d\sigma \exp\left(-\frac{1}{2}\sigma^2 - \sigma M\right)$$

- **Grassmann integral**

$$\int d\chi\, d\bar{\chi} \exp\left[\bar{\chi} A \chi\right] = det\ A = \exp\left[-(-\log det\ A)\right]$$

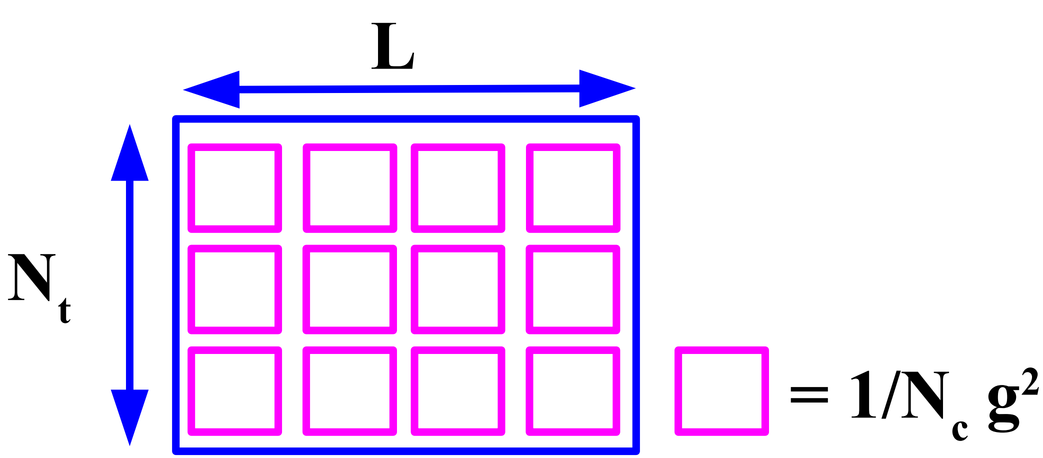
- **Temporal Link Integral, Matsubara product, Staggered Fermion,**

Achievements in SC-LQCD: Pure Gauge

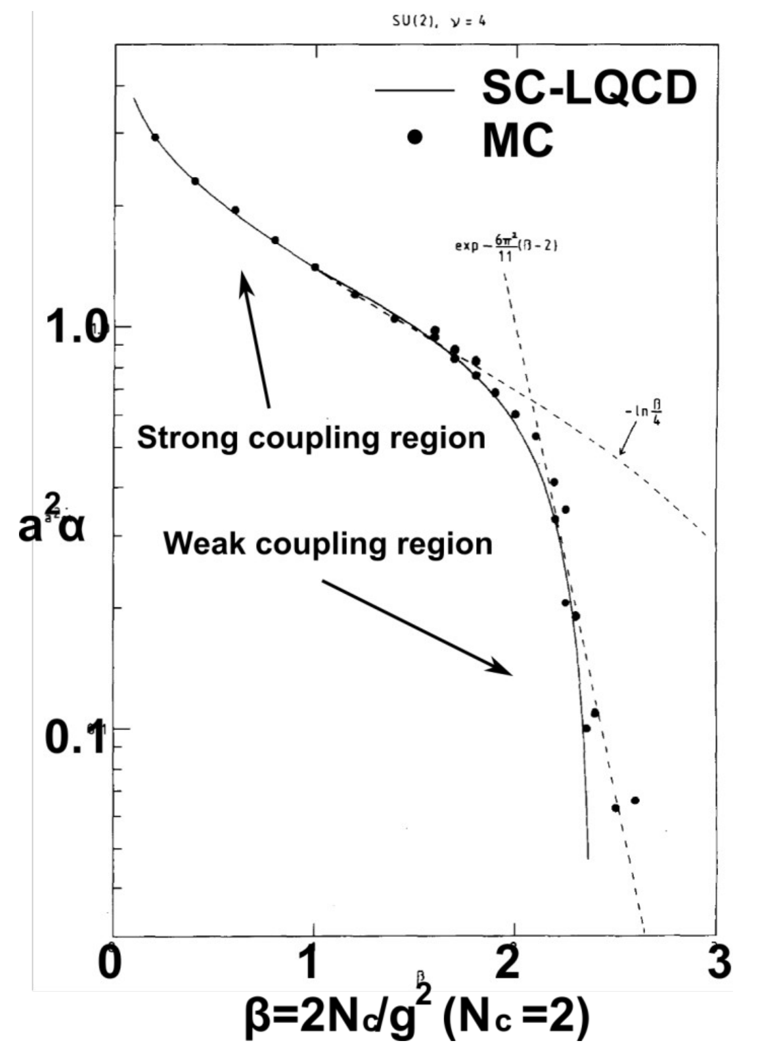
- **Quarks are confined in Strong Coupling QCD**
  - **Strong Coupling Limit (SCL)**
    → Fill Wilson Loop with Min. # of Plaquettes
    → Area Law (Wilson, 1974)

$$S_{\rm LQCD} = -\frac{1}{g^2}\sum_{\Box} \mathrm{tr}\left[U_{\Box} + U_{\Box}^{\dagger}\right]$$

  - **Smooth Transition from SCL to pQCD in MC (Creutz, 1980; Munster 1980)**



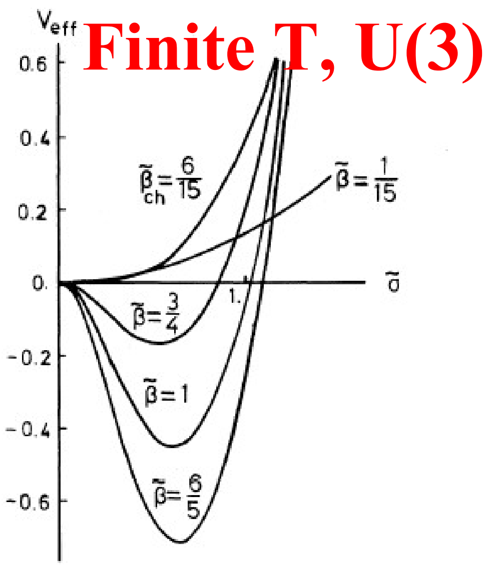
K. G. Wilson, PRD10(1974),2445
M. Creutz, PRD21(1980), 2308.
G. Munster, (1980, 1981)



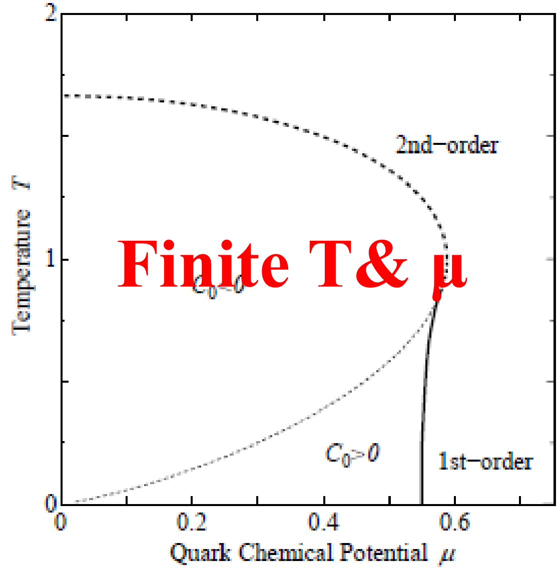
Munster, '80

Achievements in SC-LQCD: Chiral Phase Diagram

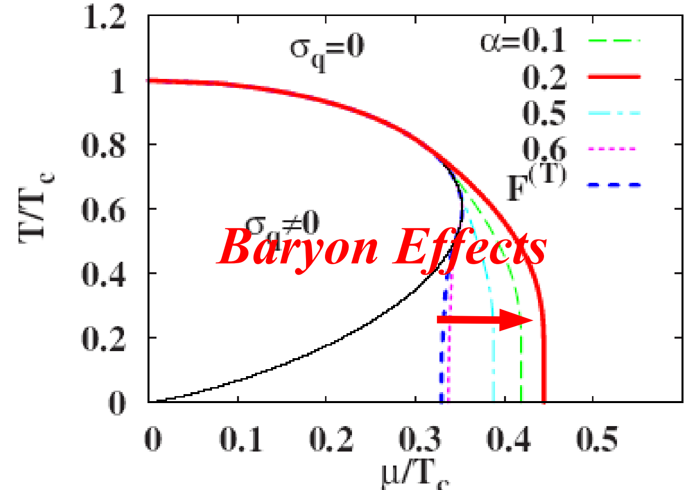
- **SCL-LQCD with Quarks → SSB of Chiral Symmetry**
  Kawamoto ('81), Kawamoto, Smit ('81), Kluberg-Stern, Morel, Napoly, Petersson ('81)
- **SCL-LQCD has been a powerful tool in "phase diagram" study !**
  - **Chiral restoration, Phase diagram, Baryon effects, Hadron masses, Finite coupling effects, ...**



Damgaard,Kawamoto, Shigemoto ('84)



Bilic, Karsch, Redlich ('92), Fukushima ('04), Nishida ('04)



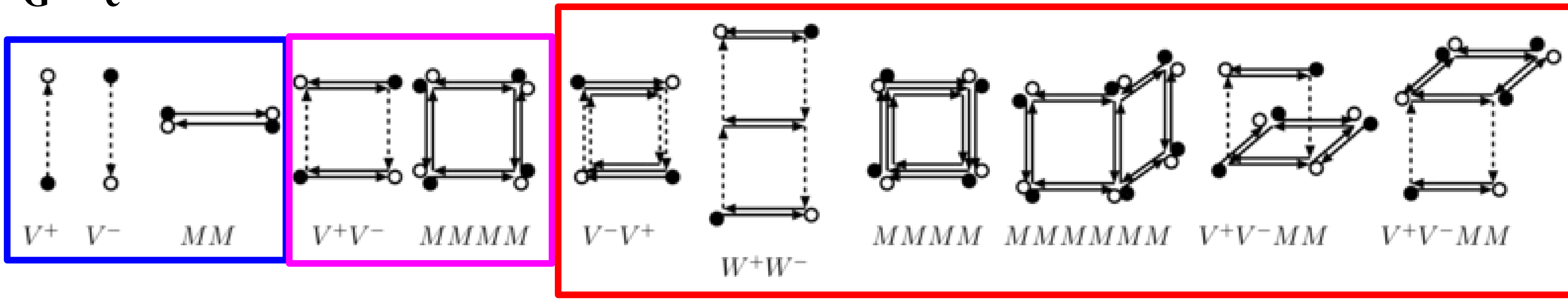
Kawamoto,Miura,AO,Ohnuma, PRD75 (07), 014502.

Effective Action in SC-LQCD

- **Effective Action with finite coupling corrections**
  Integral of exp(-S_G) over spatial links with exp(-S_F) weight → S_{eff}

$$S_{\text{eff}} = S_{\text{SCL}} - \log\langle \exp(-S_G)\rangle = S_{\text{SCL}} - \sum_{n=1} \frac{(-1)^n}{n!}\langle S_G^n\rangle_c$$

$\langle S_G^n\rangle_c$=Cumulant (connected diagram contr.) *c.f. R.Kubo('62)*



$$S_{\text{eff}} = \frac{1}{2}\sum_x (V_x^+ - V_x^-) - \frac{b_\sigma}{2d}\sum_{x,j>0}[MM]_{j,x}$$ *SCL (Kawamoto-Smit, '81)*

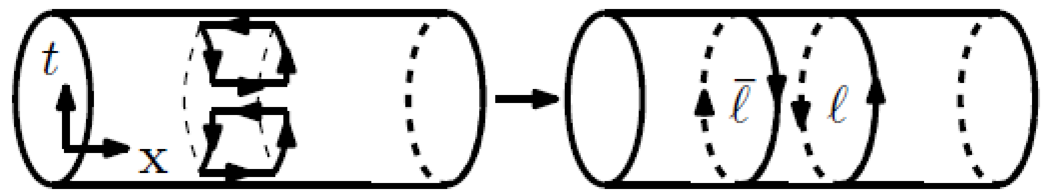
$$+\frac{1}{2}\frac{\beta_\tau}{2d}\sum_{x,j>0}[V^+V^- + V^-V^+]_{j,x} - \frac{1}{2}\frac{\beta_s}{d(d-1)}\sum_{x,j>0,k>0,k\neq j}[MMMM]_{jk,x}$$ *NLO (Faldt-Petersson, '86)*

$$-\frac{\beta_{\tau\tau}}{2d}\sum_{x,j>0}[W^+W^- + W^-W^+]_{j,x} - \frac{\beta_{ss}}{4d(d-1)(d-2)}\sum_{\substack{x,j>0,|k|>0,|l|>0\\|k|\neq j,|l|\neq j,|l|\neq|k|}}[MMMM]_{jk,x}[MM]_{j,x+\hat{l}}$$

$$+\frac{\beta_{\tau s}}{8d(d-1)}\sum_{x,j>0,|k|\neq j}[V^+V^- + V^-V^+]_{j,x}\left([MM]_{j,x+\hat{k}} + [MM]_{j,x+\hat{k}+\hat{0}}\right)$$ *NNLO (Nakano, Miura, AO, '09)*

Polyakov loop action

- **Polyakov Loop**



$$P = \frac{1}{N_c} Tr\, L\,, \quad L = T \exp\left[-i \int_0^\beta dx_4 A_4\right] = T \prod_{\tau=1}^{N_\tau} U_0(\tau, \boldsymbol{x})$$

$$\Delta S_p = -\left(\frac{1}{g^2 N_c}\right)^{N_\tau} N_c^2 \sum_{\boldsymbol{x}, j>0} \left[\bar{P}_{\boldsymbol{x}} P_{\boldsymbol{x}+\hat{j}} + \text{h.c.}\right] \quad (\text{LO in SC expansion})$$

- **Order parameter of econfinement in the heavy quark mass limit.**
  A.M. Polyakov, PLB72('78),477; L. Susskind, PRD20('79)2610; B. Svetitsky, Phys.Rept.132('86),1.
- **Polyakov loop coupling with fermion → interplay with χ cond.**

$$Z \sim \prod_{\boldsymbol{x}} \int dL(\boldsymbol{x})\, e^{-\Delta S_p} \det{}_c \left[1 + L e^{-(E_q - \tilde{\mu})/T}\right]\left[1 + L^+ e^{-(E_q + \tilde{\mu})/T}\right]$$

→ Finite Polyakov loop enables one- and two-quark excitation
A. Gocksch, M. Ogilvie, PRD31(85)877; K. Fukushima, PLB591('04),277.

Effective Potential

- **Effective action → Effective potential (Eff. E. density)**
  - **Bosonization (Extended Hubbard-Stratonovich transf.)**
    K. Miura, T.Z.Nakano,AO, N.Kawamoto, PRD80('09),074034.
  - **Integral over fermions and temporal links**
    Damgaad, Kawamoto, Shigemoto (84), Faldt-Petersson (86), Nishida (04)
  - **Effective Potential in NLO/NNLO SC-LQCD**
    Miura,Nakano,AO,Kawamoto,PRD80('09),074034;Nakano,Miura,AO,PTP123('10) 825.

$$
\begin{aligned}
Z &= \int D[\chi,\bar{\chi},U_0,U_j]\exp(-S_{\mathrm{LQCD}}) \\
&= \int D[\chi,\bar{\chi},U_0]\exp(-S_{\mathrm{SCL}})\langle\exp(-S_G)\rangle && (U_j\ \text{integral}) \\
&\approx \int D[\chi,\bar{\chi},U_0]\exp(-S_{\mathrm{eff}}[\chi,\bar{\chi},U_0,\Phi_{\mathrm{stat.}}]) && (\text{bosonization}) \\
&\approx \exp(-V\,F_{\mathrm{eff}}(\Phi;T,\mu)/T) && (\text{fermion}+U_0\ \text{integral})
\end{aligned}
$$

Effective potential in SC-LQCD

- **Integral over fermions and temporal links**

  Damgaad, Kawamoto, Shigemoto (84), Faldt-Petersson (86), Nishida (04)

$$V_q(m,\mu,T) = -\frac{T}{L^d}\log\left\{\int D[U_0]\, det(G^{-1})\right\}$$

$$= -T\log\left[\frac{\sinh((N_c+1)E_q(m)/T)}{\sinh(E_q(m)/T)} + 2\cosh(N_c\mu/T)\right]$$

$$E_q(m) = \text{arcsinh}\, m \quad (\text{quark excitation energy})$$

- **Effective Potential in NLO/NNLO SC-LQCD**

  Miura,Nakano,AO,Kawamoto,PRD80('09),074034;Nakano,Miura,AO,PTP123('10)825.

$$F_{\text{eff}} = F_{\text{eff}}^{(X)}(\sigma,\omega_\tau) + V_q(\tilde{m}_q;\tilde{\mu},T) - N_c \log Z_\chi$$

$$\sigma \approx \langle M\rangle \ (\text{chiral condensate}),\quad \omega_\tau \approx -\partial F_{\text{eff}}/\partial\mu = \rho_q \ (\text{quark number density})$$

$$\tilde{m}_q = \frac{\tilde{b}_\sigma\sigma + m_0}{Z_\chi(1+4\beta_{\tau\tau}\varphi_\tau)} \approx \frac{d}{2\mathrm{N}_c}\sigma\times\left(1+\beta_{\sigma\sigma}^{(m)}\sigma^2 - \beta_{\sigma\omega}^{(m)}\sigma^2\omega_\tau^2 + ...\right)$$

$$\delta\mu = \mu - \tilde{\mu} = \log(Z_+/Z_-) \approx \beta_\tau\omega_\tau\times\left(1+\beta_{\omega\sigma}^{(\mu)}\sigma^2 + ...\right)$$

> ***NLO/NNLO SC-LQCD***
> ***≈ σω model of quarks non-linear couplings***

Effective potential with Polyakov loop

- **Haar measure method**
  - **Replace the Polyakov loop *P* with its representative value *l*, and Haar measure is included in the potential.**

$$\mathcal{F}_{\mathrm{q}} = -N_c E - T\log\left[1 + N_c \ell e^{-(E-\tilde{\mu})/T} + N_c \bar{\ell} e^{-2(E-\tilde{\mu})/T} + e^{-3(E-\tilde{\mu})/T}\right] - T\log\left[1 + N_c \bar{\ell} e^{-(E+\tilde{\mu})/T} + N_c \ell e^{-2(E+\tilde{\mu})/T} + e^{-3(E+\tilde{\mu})/T}\right] - N_c \log Z_\chi \,,$$

$$U_g = -2T\beta_p \bar{\ell}\ell - T\log\left[1 - 6\ell\bar{\ell} + 4\left(\ell^3 + \bar{\ell}^3\right) - 3\left(\ell\bar{\ell}\right)^2\right] \,,$$

E. M. Ilgenfritz, J. Kripfganz, ZPC29('85)79; A. Gocksch, M. Ogilvie, PRD31('85)877; K. Fukushima, PLB 553, 38 (2003); PRD 68('03)045004;K. Fukushima, PLB591('04)277.

- **Bosonization method**
  - **Introduce the auxiliary field *l* = <*P*>, and integrate out U_0=L.**

$$\Delta S_p \approx \left(\frac{1}{g^2 N_c}\right)^{N_\tau} N_c^2 \sum_{\mathbf{x},j>0} 2\left(\bar{\ell}\ell - \bar{P}_{\mathbf{x}}\ell - \bar{\ell}P_{\mathbf{x}}\right) \simeq 2\beta_p L^d \bar{\ell}\ell - 2\beta_p \sum_{\mathbf{x}} \left(\bar{P}_{\mathbf{x}}\ell + \bar{\ell}P_{\mathbf{x}}\right)$$

→ Weise mean field approximation

c.f. J. B. Kogut, M. Snow and M. Stone, NPB 200('82)211 (no quarks)

Stationary Condition --- Multi-Order Parameter

- **Stationary Condition** $$\frac{\partial \mathcal{F}_{\rm eff}}{\partial \Phi} = 0$$

  Φ(4(NLO) / 10 (NNLO) aux. field$) \to (\sigma, \omega_\tau)$

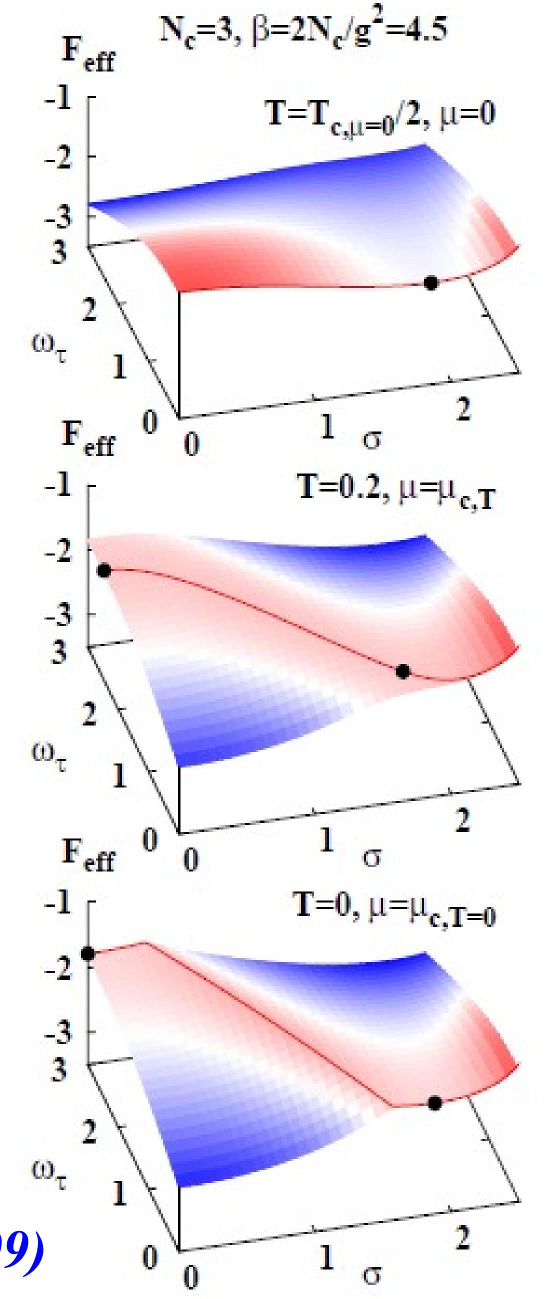
- **Multi-Order Parameter (σ, ω_τ)**

$$\sigma \approx -\frac{\partial F_{\rm eff}}{\partial m_0} = \text{Chiral Cond.}$$

$$\omega \approx -\frac{\partial F_{\rm eff}}{\partial \mu} = \text{Quark number density}$$

  - **Two indep. var. in V_q(m, μ)**
  - **Scalar (σ) and Vector (ω) potential for Quarks**

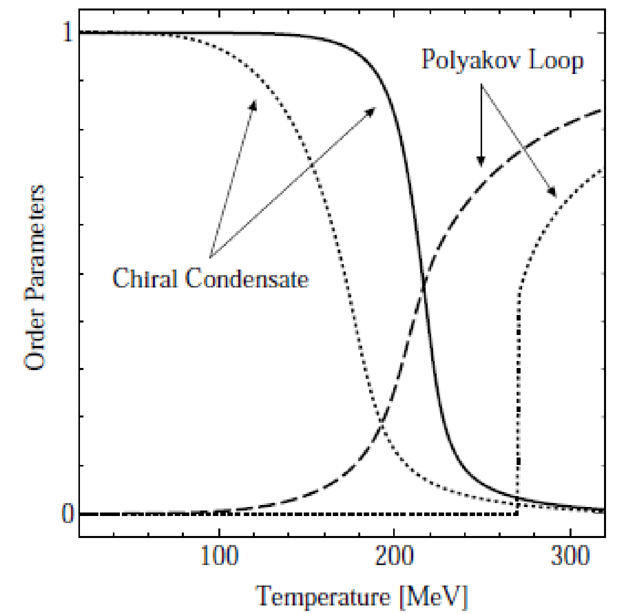
→ Saddle point in Feff(σ, ω_τ)



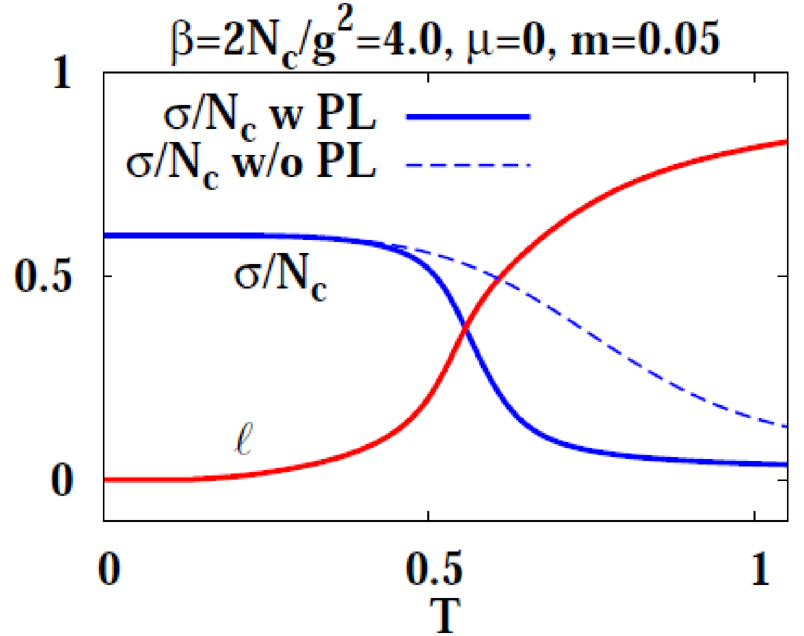
Miura, Nakano, AO, Kawamoto ('09)

Chiral condensate and Polyakov loop

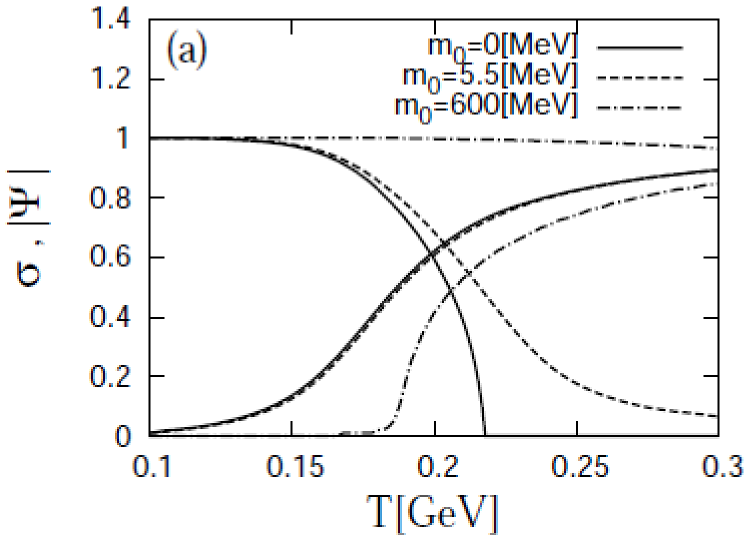
- Chiral and Deconf. transition correlate !
- SC-LQCD w/o PL: quarks are confined.
  → PL promote quarks to deconfine !
  (cf. Quarks are *not* confined in NJL
  → PL *confines* quarks in PNJL.)
- Tc is suppressed with PL



Fukushima ('04)



Nakano, Miura, AO, Lat10 & in prep.

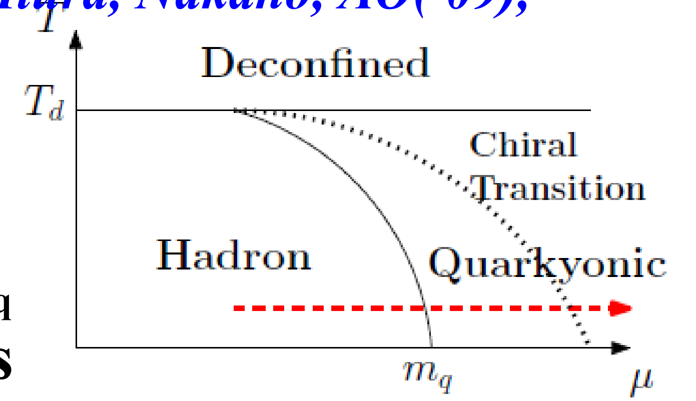


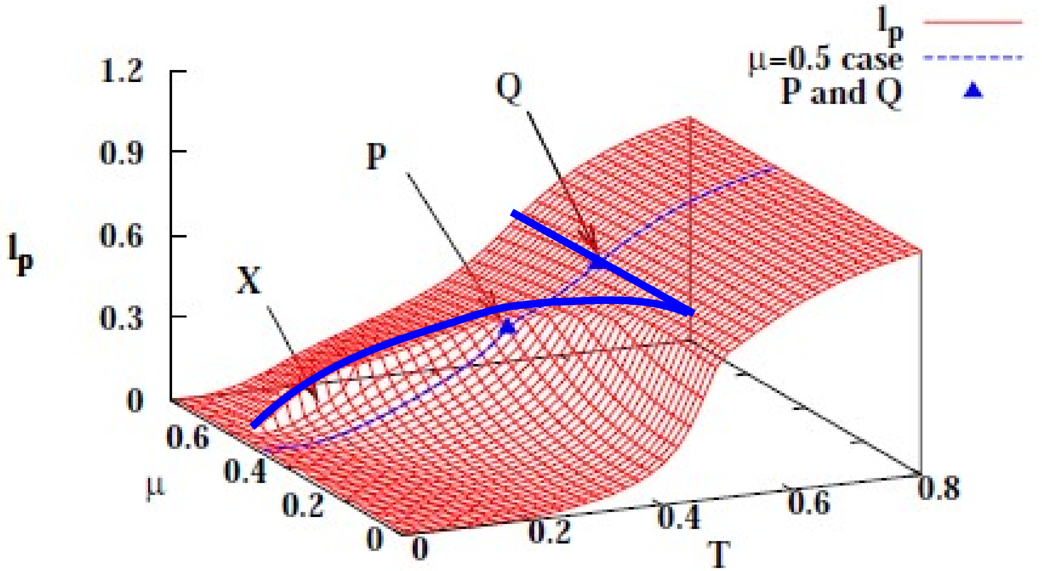
Sakai, Kashiwa, Kouno, Yahiro ('08)

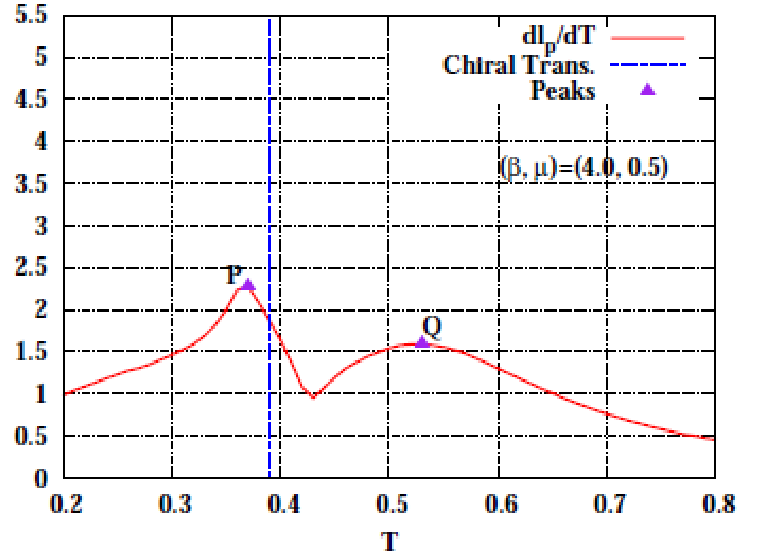
Quarkyonic matter

McLerran, Pisarski ('07), Hidaka, McLerran, Pisarski ('08), Kojo, Hidaka, McLerran, Pisarski ('10), Glozman et al('08), Fukushima ('08), Abuki, .., Ruggieri ('08), McLerran, Redlich, Sasaki ('09), Miura, Nakano, AO('09),

- **Quarkyonic matter**
  - **T_d is governed by gluons at large N_c , while high density matter is realized at $\mu \sim m_q$ → deviation of deconf. and chiral transitions**
  - **SC-LQCD with PL (Haar measure method) shows large region of "quarkyonic" matter**

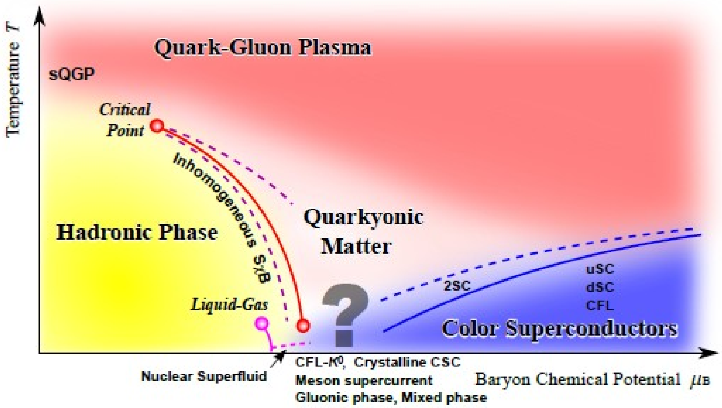






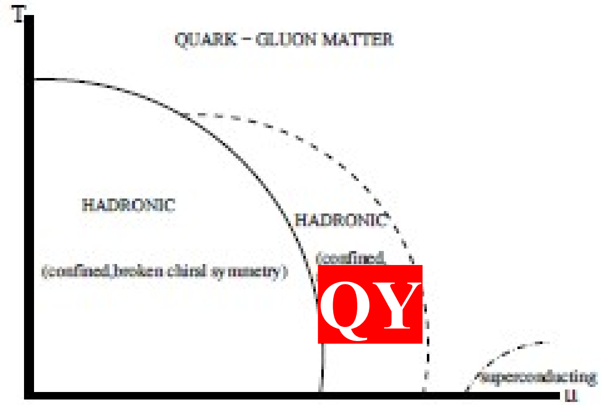
Miura, Nakano, AO, LAT10, in prep.

Comparison with Other Models

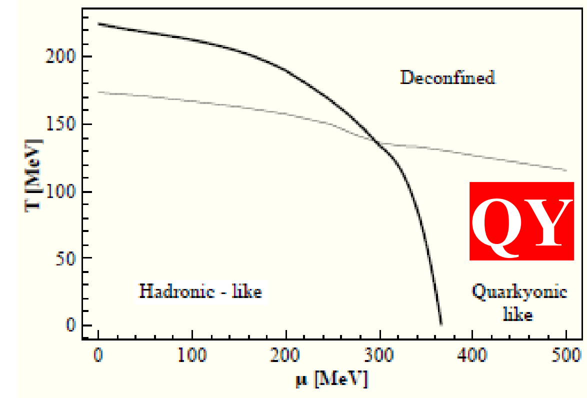


Fukushima, Hatsuda ('10)

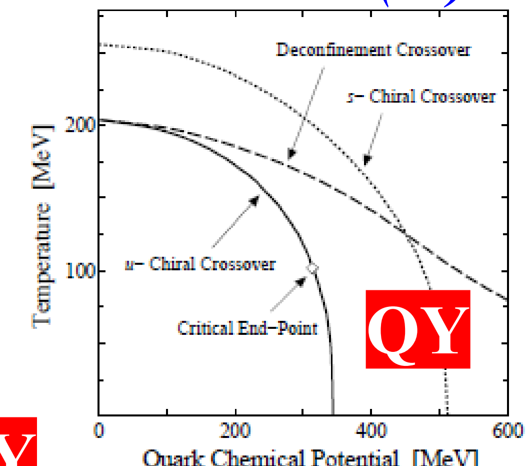
Grozman et al. (08)

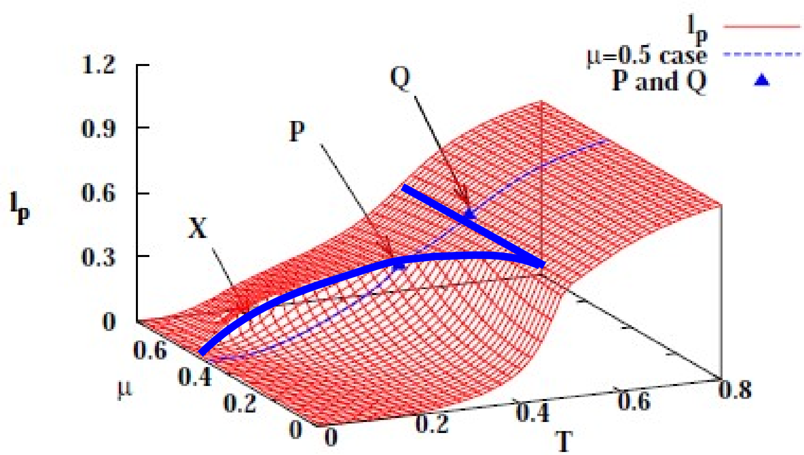


Abuki et al. (08)

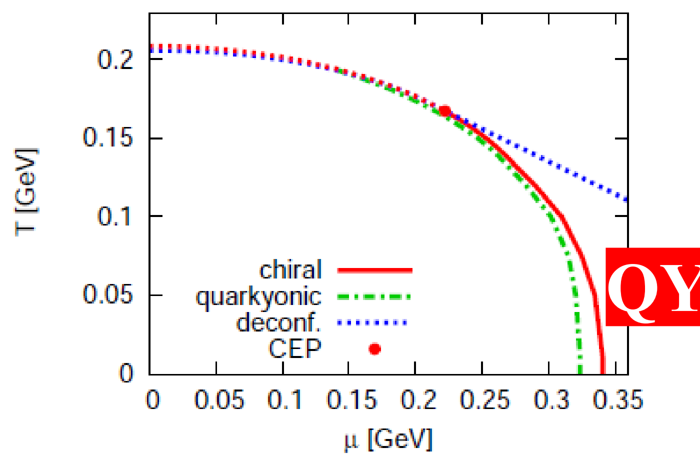


Fukushima (08)





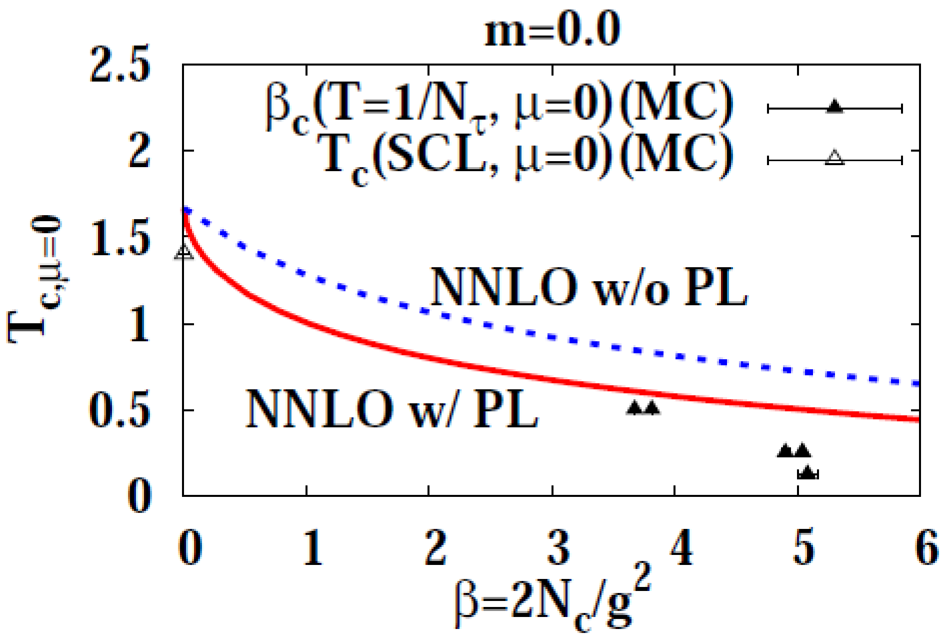
Miura, Nakano, AO, LAT10, in prep.



McLerran, Pisarski, Sasaki ('09)

Critical Temperature at μ=0

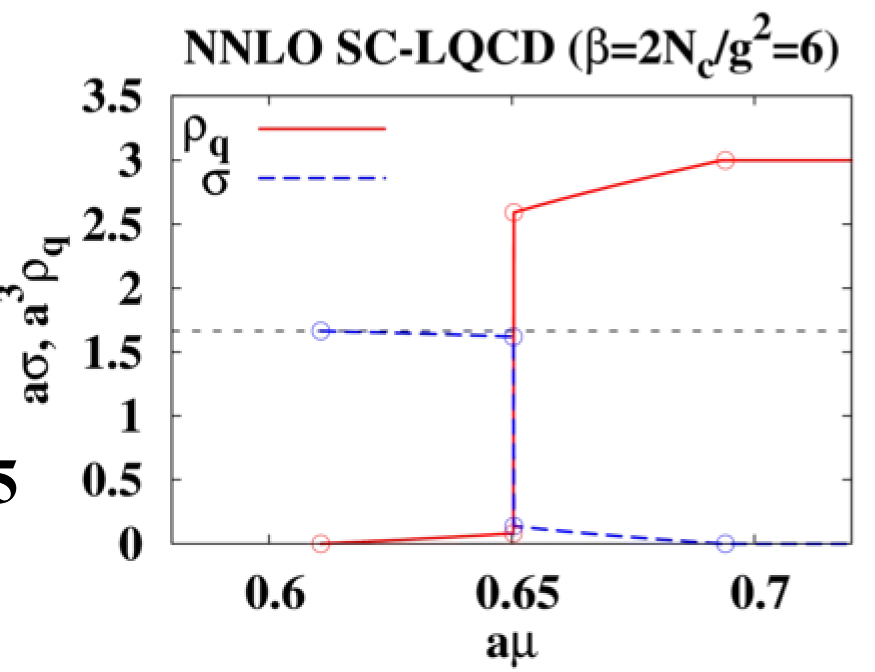
- **SC-LQCD w PL seems to be qualitatively promising.**
  Is it *quantitatively* good ?
  - **Improved from SC-LQCD w/o Polyakov loop.**
  - **Polyakov loop suppresses T_c. (cf. PNJL)**
  - **Quantitavely, not bad for β < 4 in T_c (β_c)**
  - **In the "scaling" region (β>5), we do not see further bending of T_c in SC-LQCD.**

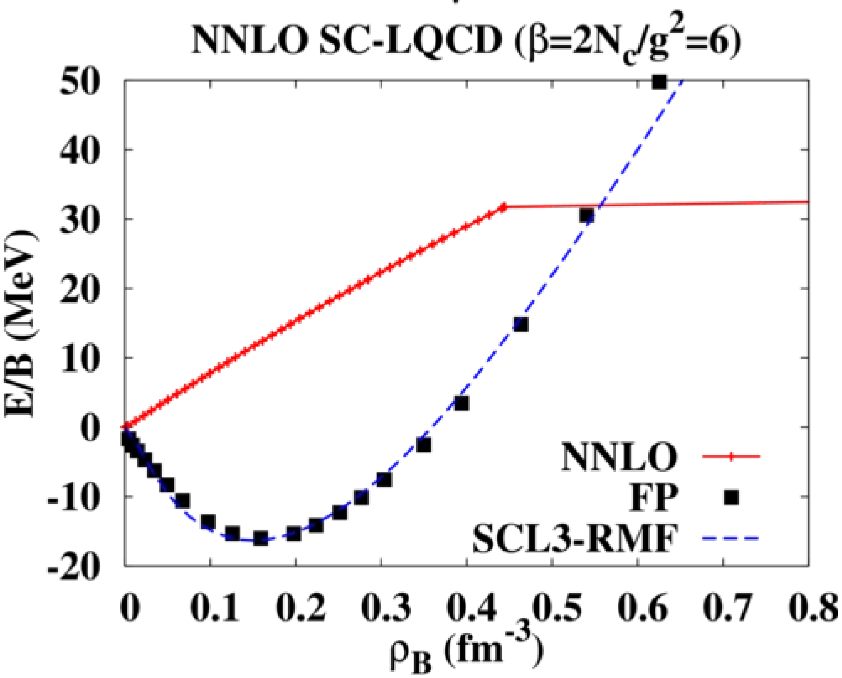


Nakano, Miura, AO, LAT10 & in prep.
MC Results:
Ph. de Forcrand, M. Fromm ('09),
Ph. de Forcrand, private comm.,
S.A.Gottlieb et al. ('87),
D'Elia, Lombardo ('03),
Z.Fodor, S. D. Katz ('02),
R.V.Gavaiet al. ('90)

Nuclear Matter on the Lattice at Strong Coupling

- **Do we observe finite density matter before 1st order phase transition ? → Yes !**
  - **$E_q(\mu=0, T=0, \beta=6)=0.61$**
    $\mu_c^{(1st)}(T=0, \beta=6)=0.65$
    → "Nuclear matter" in $0.61<\mu<0.65$
- **EOS of "Nuclear matter"**
  - **a^{-1} = 500 MeV**
    Bilic, Demeterfi, Petersson ('92)
    → Density in the order of ρ_0
  - **No saturation**
  - **1st order transition at ρ_B=0.4 fm^{-1}.**





Summary

- **Recent developments in nuclear matter EOS is reviewed.**
  - **asymmetric matter, ab initio approaches**
- **Bottom-up (phen.) approaches are still powerful to understand nuclear system on the earth and universe.**
  - **RMF as Covariant DF, is improved to explain DBHF results, Pressure from HIC, Hypernuclei, Atom data, …**
  - **"Effectiveness" of mesons would be evaluated via coupling const.**
  - **Key idea: non-linear terms ~ many-body force**
- **A top-down approach, Polyakov loop extended strong coupling lattice QCD (PSC-LQCD), seems to be promising.**
  - **MC results on Tc are roughly explained at $\beta<4$.**
  - **Existence of the quarkyonic phase is supported.**
  - **Qualitatively competitive to effective models such as PNJL in some aspects of the QCD phase diagram.**

Future works

- "Derivation" and/or "Justification" of RMF Lagrangian
  - Essential point: Many-body force (bare and effective)
    RMF: σ^3, σ^4, ω^4, …
    HF, BHF, …. ρ-dep. 2 body, ρ^3, ρ^α, …
  - Realization of chiral symmetry: linear or nonlinear repr.
    Chiral pert. based (e.g. Finelli, Kaiser, Vretenar, Weise)
    → large part of scalar/vector pot. comes from π,
    but bare scalar/vector field improves saturation.
  - Combination with Polyakov loop quark meson model seems to be an interesting direction to study.
- PSC-LQCD need further improvements to explain EOS.
  - NNLO SC-LQCD solves the "Baryon Mass Puzzle" ($\mu_c > M_B/3$ @ SCL), but nuclear matter does not saturate.
  - Combination with MC simulation may be an interesting direction.
  - Problems: Anomaly effects in starggered fermion, meson fluc.

Thank you for your attention !

I also thank my collaborators

K. Tsubakihara, H. Maekawa, H. Matsumiya, C. Ishizuka (Hokkaido U.), K. Sumiyoshi (Numazu CT), S. Yamada (Waseda U.), H. Suzuki (Tokyo Sci. U), H. Ueda (Kyoto U.), T. Sekihara, M. Ruggieri (YITP), N. Kawamoto (Hokkaido U.), T.Z. Nakano (Kyoto U.), K. Miura (INFN, Frascati)

This talk is mainly based on following papers.

K. Tsubakihara, H. Maekawa, H. Matsumiya, AO, PRC81('10)065206.
K. Sumiyoshi, C. Ishizuka, AO, S. Yamada, H. Suzuki, ApJ Lett. 690 ('09)L43
C. Ishizuka, AO, K. Tsubakihara, K. Sumiyoshi, S. Yamada, ,JPG35('08)085201.
K. Miura, T.Z.Nakano,AO, N.Kawamoto, PRD80('09),074034.
T.Z. Nakano, K. Miura, AO, PTP123('10)825.