
強結合格子 QCD における補助場 MC

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- Introduction
 - Grand Challenges in Quark-Hadron Sciences
- Auxiliary field MC in strong coupling lattice QCD
- Summary

Grand Challenges in Quark-Hadron Sciences

■ Nuclear matter on the lattice

- Lattice QCD で核物質の相図と状態方程式を明らかにする。
- Problem: 有限密度 (有限の μ) では Fermion det. が complex (符号問題)
- 強結合格子 QCD での補助場での積分 (可能性は?)

■ Real-time non-equilibrium dynamics of QCD for HIC

- QCD から出発して RHIC, LHC の重イオン衝突を解く
- 現状: Classical Yang-Mills (CYM)、および CYM と粒子運動との結合
→ 熱平衡化・エネルギー損失
- 夢?: QCD の real-time “quantum” non-eq. dynamics

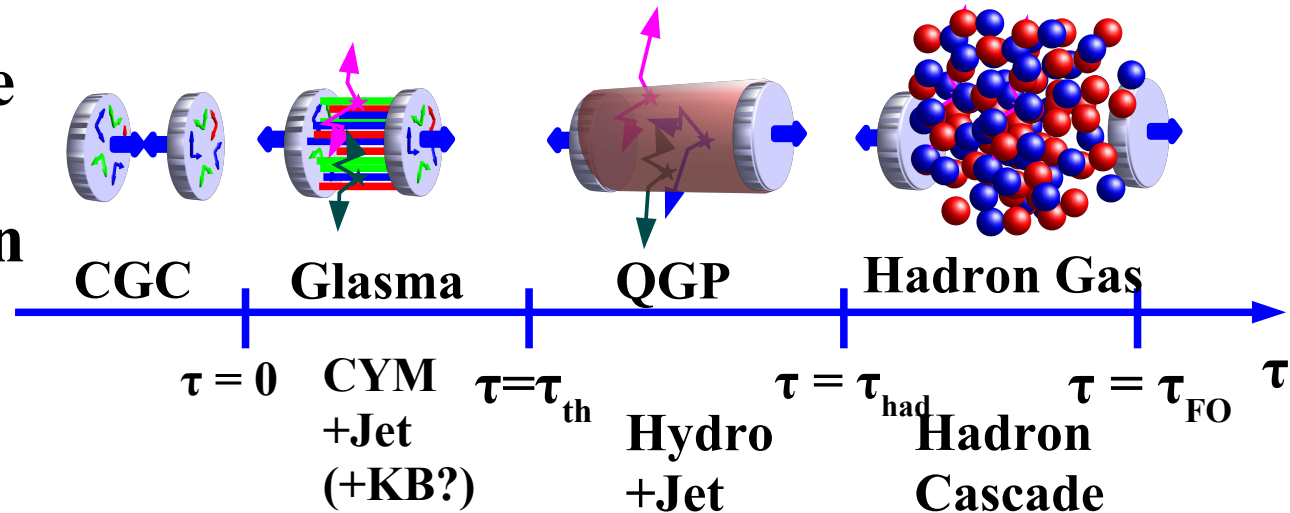
■ Dense matter EOS

- 実験・観測データを再現し、QCD 臨界点を含む有限温度状態方程式
- 現状: ハイペロン自由度を含むと 1.97 Msun の中性子星が支えられない、
カイラル有効モデルによる EOS では飽和性 or 臨界性が不十分
- 解決方法(?): RMF における3体力、baryon loop

Thermalization at RHIC & LHC

Success of hydrodynamics at RHIC & LHC

- Results are sensitive to initial conditions (Early thermalization + Fluctuation)



Entropy production in Classical Yang-Mills

Kunihiro, Muller, AO, Schafer, Takahashi, Yamamoto ('10)

- ランダムな初期条件 → 不安定な初期条件では？
- 問題：古典場では平衡状態が正確に記述できない

Can we solve pre-QGP dynamics in quantum field ?

- 現存する CYM+particle simulation では off-shell 効果なし
Dumitru, Nara, Strickland

- Candidate: CYM + Kadanoff-Baym

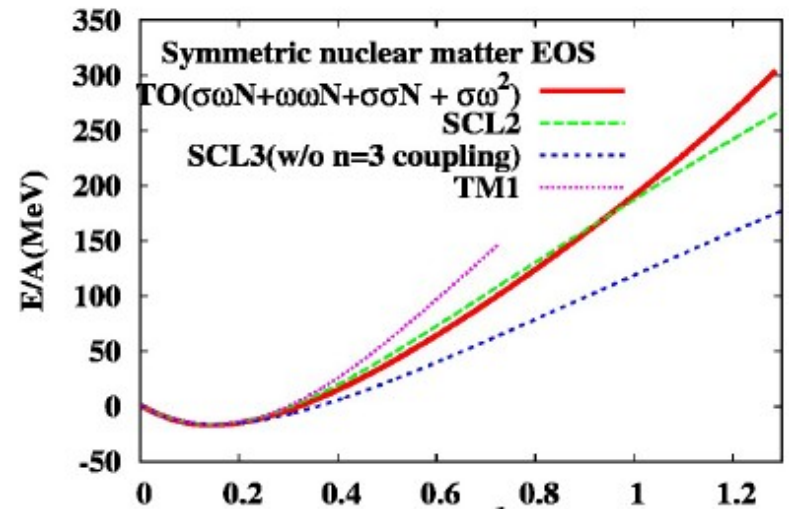
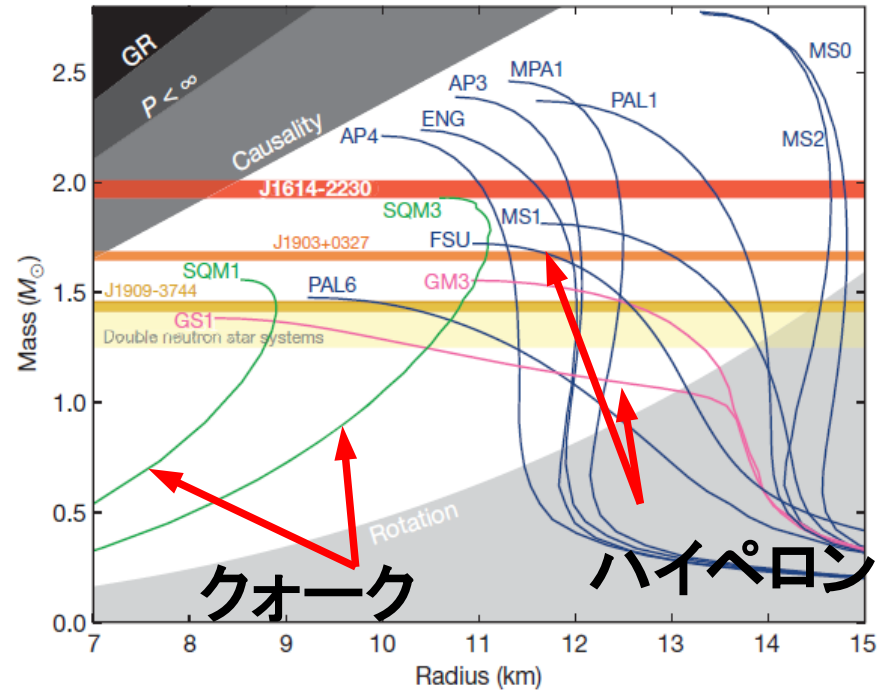
Dense matter EOS

■ 高密度物質 EOS の新しい段階

- 重い中性子星 (1.97 Msun) の発見
Demorest et al.('10)
→ バリオン・中間子結合に SU(3) を
仮定した RMF では支えられない。
- 超新星爆発・ブラックホール形成過程
でのクォーク物質生成の可能性
*Hatsuda ('87); Sagert et al.('09); AO, Ueda,
Nakano, Ruggieri, Sumiyoshi ('11)*
 - ◆ 現存するハドロン・クォーク物質 EOS
= ハドロン EOS と bag 模型 EOS を
Maxwell construction
(臨界点なし!)

**QCD 臨界点をもち、高密度で硬い
クォーク・ハドロン物質 EOS が必要**

Demorest et al., Nature 467 (2010) 1081.



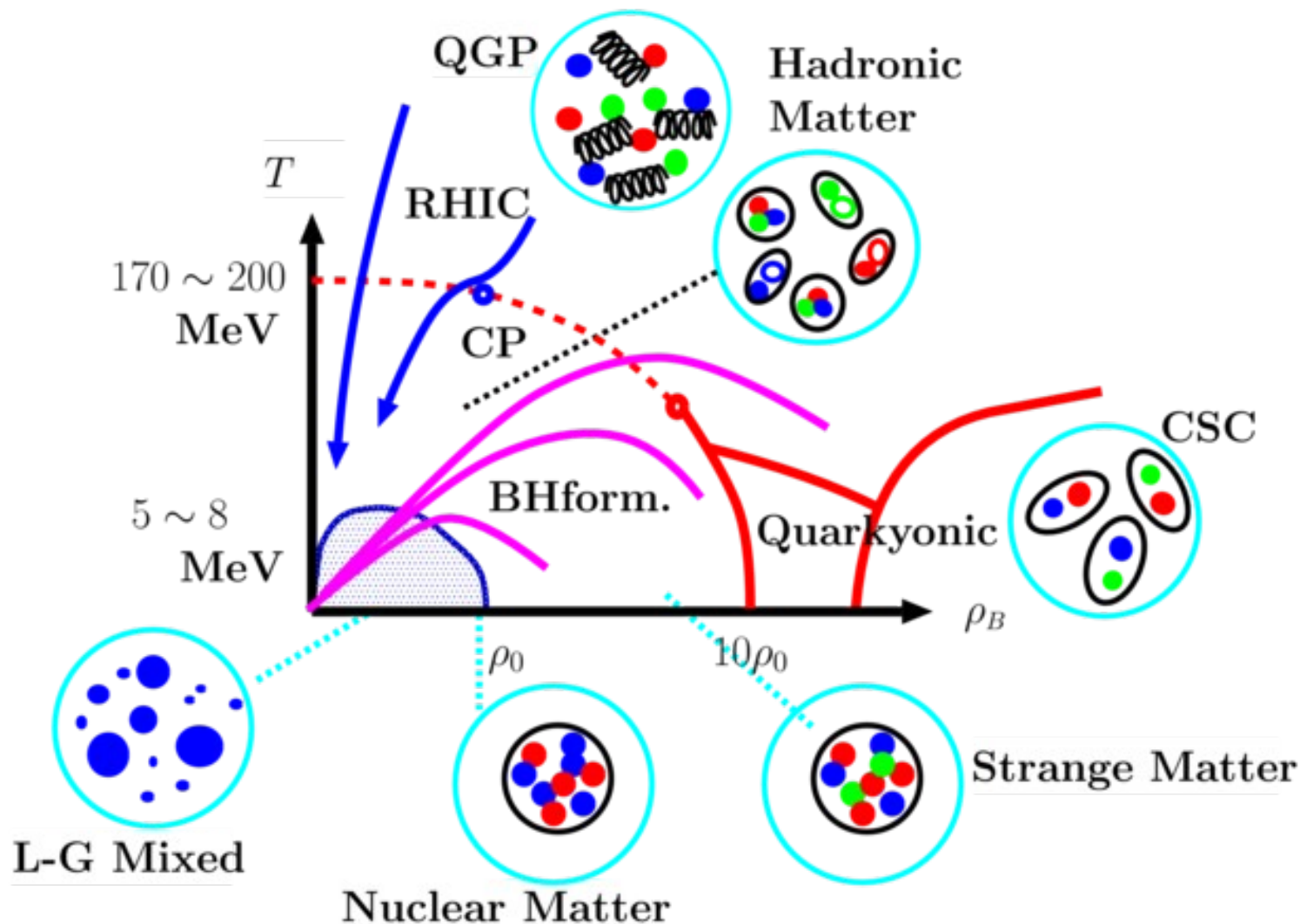
Tsubakihara, AO, in prep.

*Auxiliary field MC
in strong coupling lattice QCD*

QCD Phase diagram

■ Grand Challenge

"Nuclear matter phase diagram and EOS in lattice QCD"



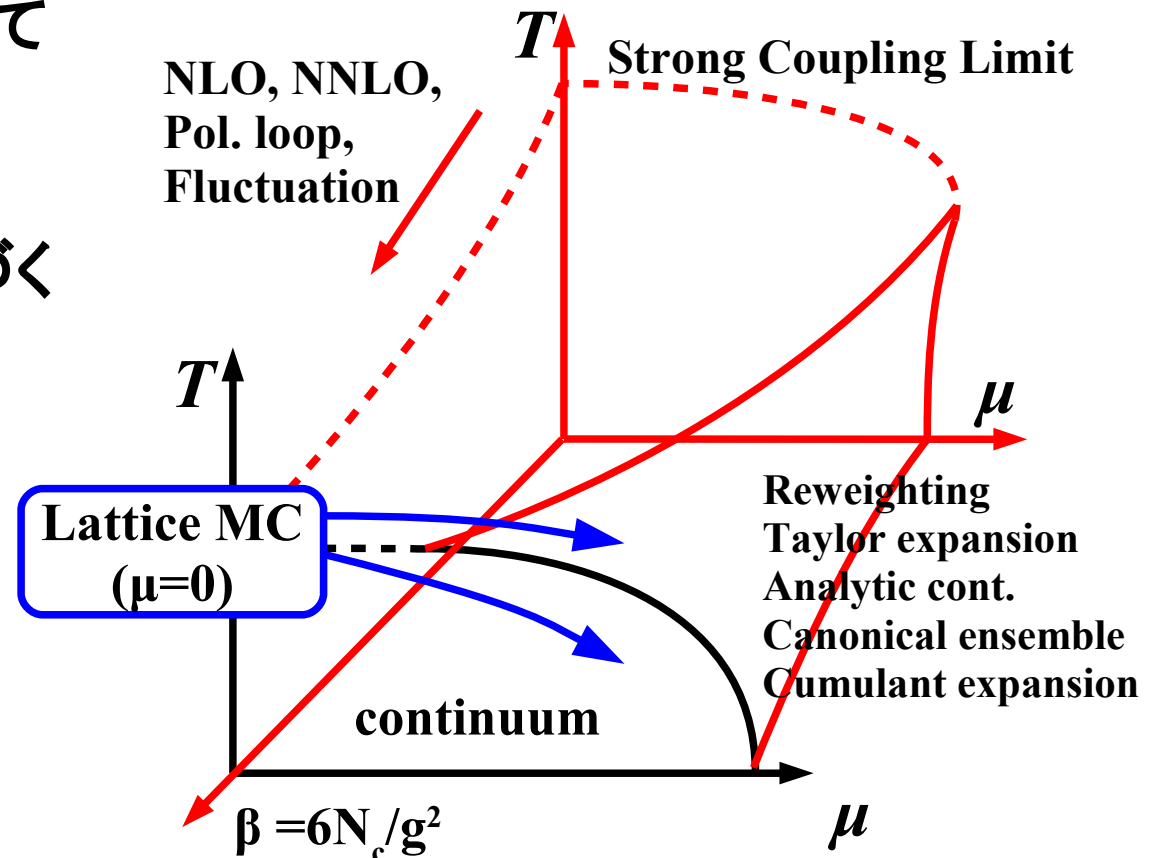
"Nuclear Matter on the Lattice" への道のり

■ 通常のアプローチ

- 有効模型: NJL(初田・国広、浅川・矢崎、...)、PNJL(福嶋、...)
- 格子 QCD : 符号問題 $\rightarrow \mu=0$ or $\mu=i\mu_1$ からの展開・接続

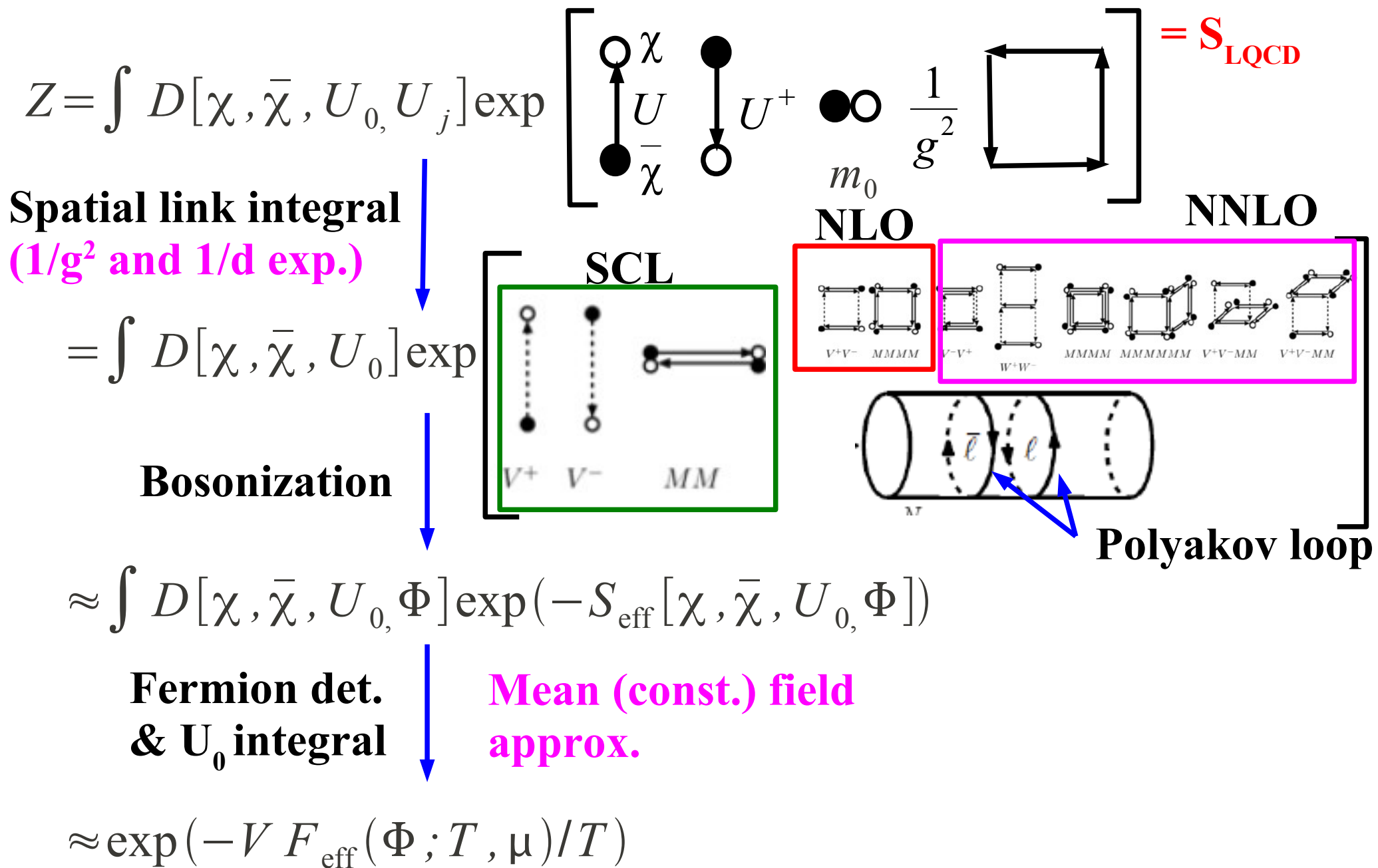
■ 強結合格子 QCD

- 強結合領域 ($1/g^2 \ll 1$) にて有限 μ 領域へ
- $1/g^2$ の展開を進めて $g \rightarrow 0$ (continuum) に近づく



「急がば回れ」か？
「困難は保存する」か？

SC-LQCD Procedures



強結合格子 QCD によるこれまでの取り組み

■ 強結合極限 (Strong coupling limit (SCL))

● QCD に基づく相図の予想

*Damgaard, Kawamoto, Shigemoto ('84);
Faldt, Petersson ('86); Fukushima ('04);
Nishida ('04)*

■ 強結合展開 (NLO, NNLO)

*Miura, Nakano, AO ('09); Miura, Nakano,
AO, Kawamoto ('09); Nakano, Miura, AO ('10)*

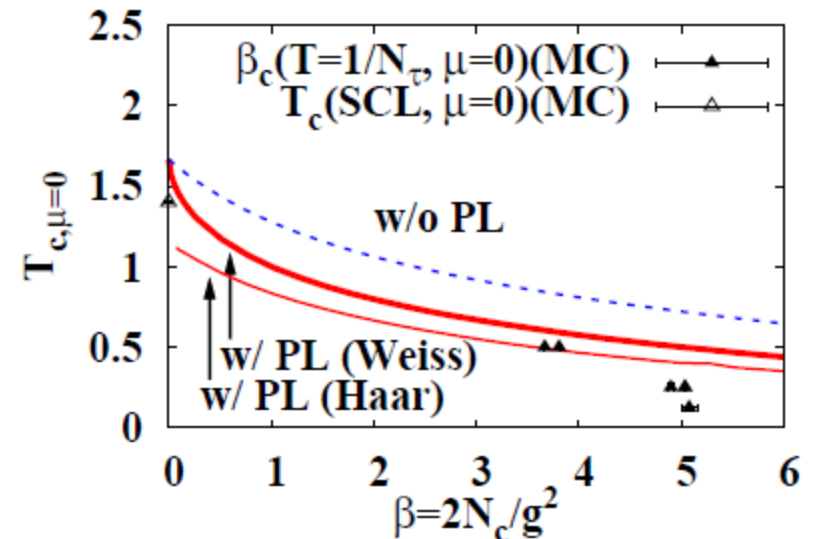
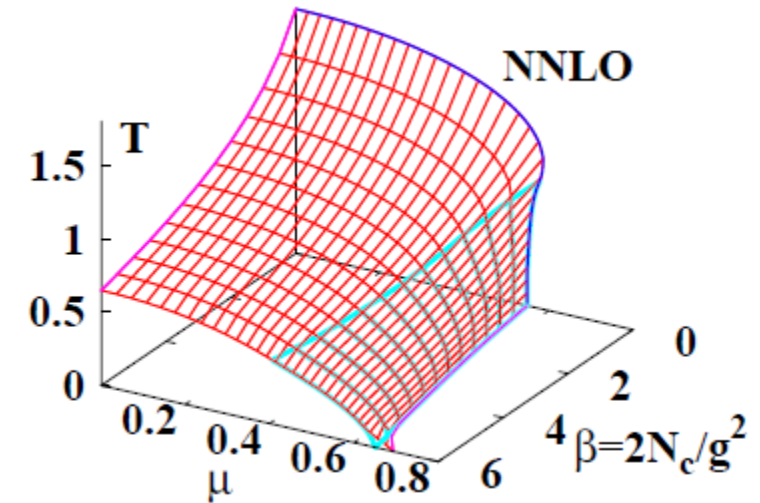
● SCL で大きすぎる T_c の低下

● CP の低 μ 側への移動 (NNLO)

■ ポリアコフループの導入

*Nakano, Miura, AO ('11);
Miura, Nakano, AO, Kawamoto ('11)*

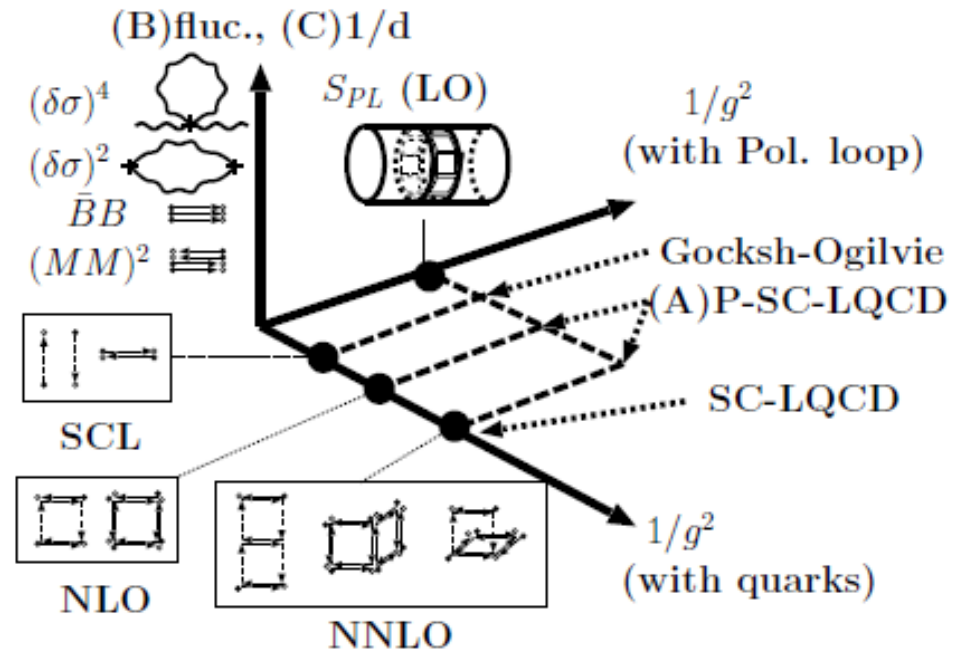
● $\mu=0$ での MC 計算結果 (T_c) を ほぼ再現



足りない部分は？

■ 強結合格子 QCD における近似

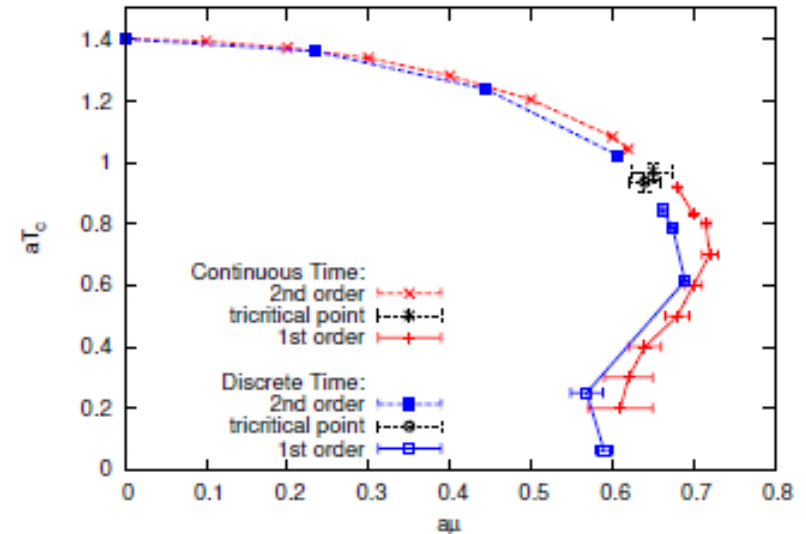
- 強結合 ($1/g^2$) 展開
- Large dimensional ($1/d$) 展開 ($d = \text{空間次元} = 3$)
- 平均場近似
導入した補助場が一定と制限
- Staggered Fermion



■ 最近の進展

P. de Forcrand, M. Fromm, PRL104('10)112005;
W. Unger, P. de Forcrand, arXiv:1107.1553.

- Monomer-Dimer-Polymer (MDP) simulation により、強結合極限において、揺らぎの効果を含んだ QCD 相図が求まる



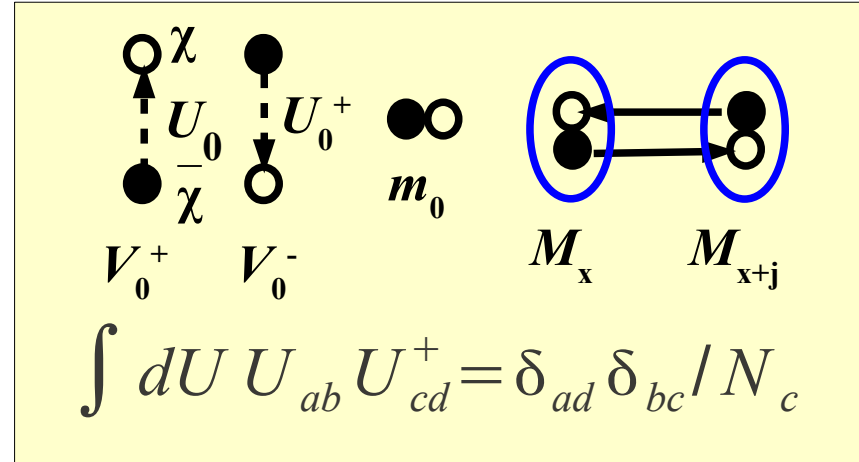
揺らぎと有限結合効果をとともに含む
 計算手法は？

Auxiliary Field MC (σ MC)

■ SCL effective action

Damgaard, Kawamoto, Shigemoto ('84)

- Ignore plaquette action ($1/g^0$)
- Integrate out spatial link variables of min. quark number diagrams (LO in $1/d$ expansion)



$$S_{\text{eff}} = \frac{1}{2} \sum_x [V^+(x) - V^-(x)] - \frac{1}{4N_c \gamma^2} \sum_{x,j} M_x M_{x+\hat{j}} + \frac{m_0}{\gamma} \sum_x M_x \quad (\gamma = a_s/a_\tau)$$

■ Effective action of Auxiliary Field

Faldt, Petersson ('86), Miura, Nakano, AO ('09)

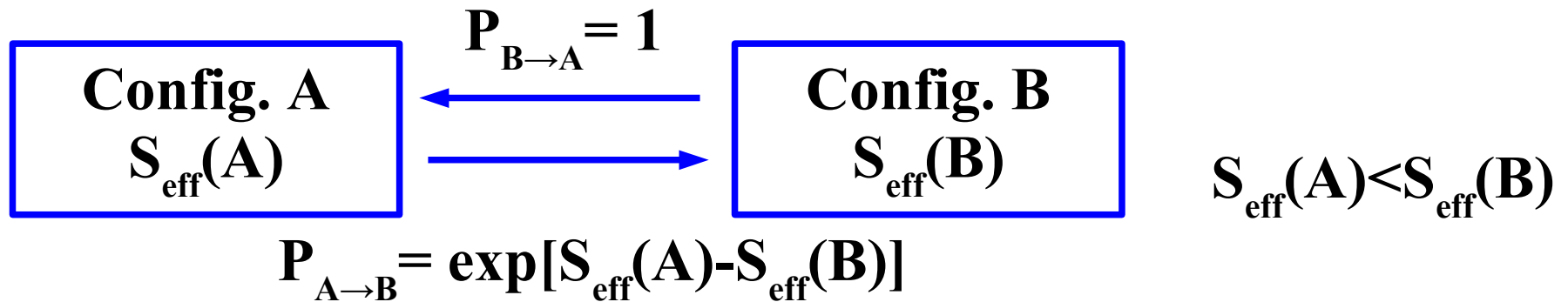
- Extended Hubbard-Stratonovich transf. + U_0 integral of q-det.

$$S_{\text{eff}} = \frac{\Omega}{4N_c \gamma^2} \sum_{k, f_M(\mathbf{k}) > 0} f_M(\mathbf{k}) [\sigma_k^* \sigma_k + \pi_k^* \pi_k] - \sum \log [X_N[\sigma(\mathbf{x})]^3 - 2X_N[\sigma(\mathbf{x})] + 2 \cosh(3N_\tau \mu)]$$

$$\Sigma(x) = \frac{\Gamma^x}{2N_c \gamma^2} [\sigma(x) + i\varepsilon(x)\pi(x)] + \frac{m}{\gamma}, \quad \sigma(x) = \sum_{k, f_M(\mathbf{k}) > 0} f_M(\mathbf{k}) e^{ikx} \sigma_k, \quad \pi(x) = \sum_{k, f_M(\mathbf{k}) > 0} f_M(\mathbf{k}) e^{ikx} \pi_k$$

Numerical Calculation

- 4^4 asymmetric lattice + Metropolis sampling of σ_k and π_k .
- Metropolis sampling
= One of the typical (popular) method of importance sampling

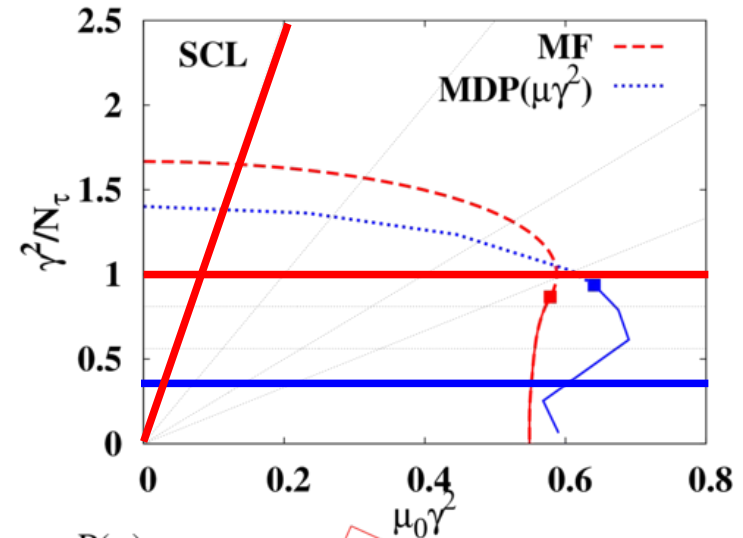


- Trial prob.: $P_{A \rightarrow B}^{\text{try}} = P_{B \rightarrow A}^{\text{try}}$ (detailed balance)
- Pickup prob.: According to S_{eff} .
- In equilibrium, $P(A) P_{A \rightarrow B} = P(B) P_{B \rightarrow A} \rightarrow P(A) \propto \exp[-S_{\text{eff}}(A)]$
- Typical sampling size: Thermalization= 5×10^4 , Sample= 2×10^6
- We have the sign problem, but the sign problem is less severe at larger μ .

σ distribution

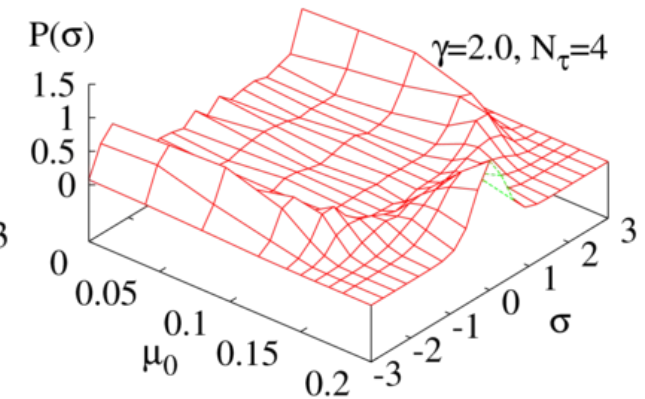
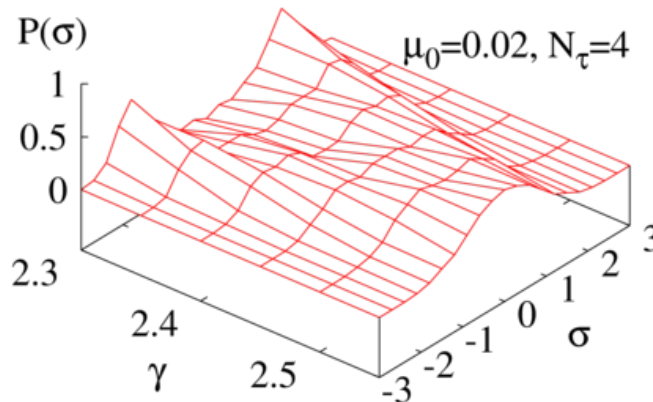
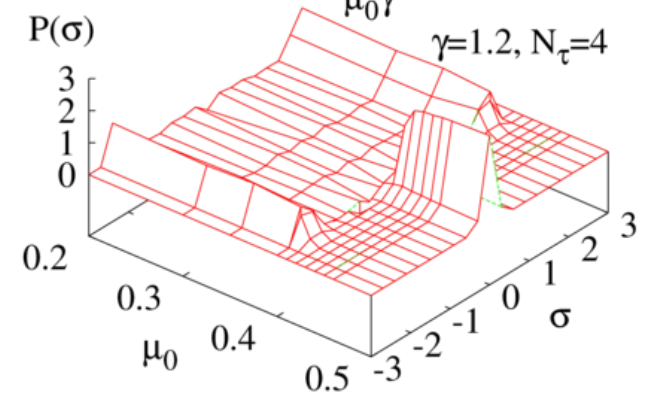
■ Low T simulation [$\gamma=1.2, N_\tau=4$ ($T=0.36$)]

- Two peaks ($\sigma \sim \sigma_{\text{vac}}$) \rightarrow One peak ($\sigma = 0$)
 \rightarrow First order phase transition
- Transition takes place at $\mu_0 \sim 0.4$
 $(\mu = \mu_0 \gamma^2 \sim 0.58)$



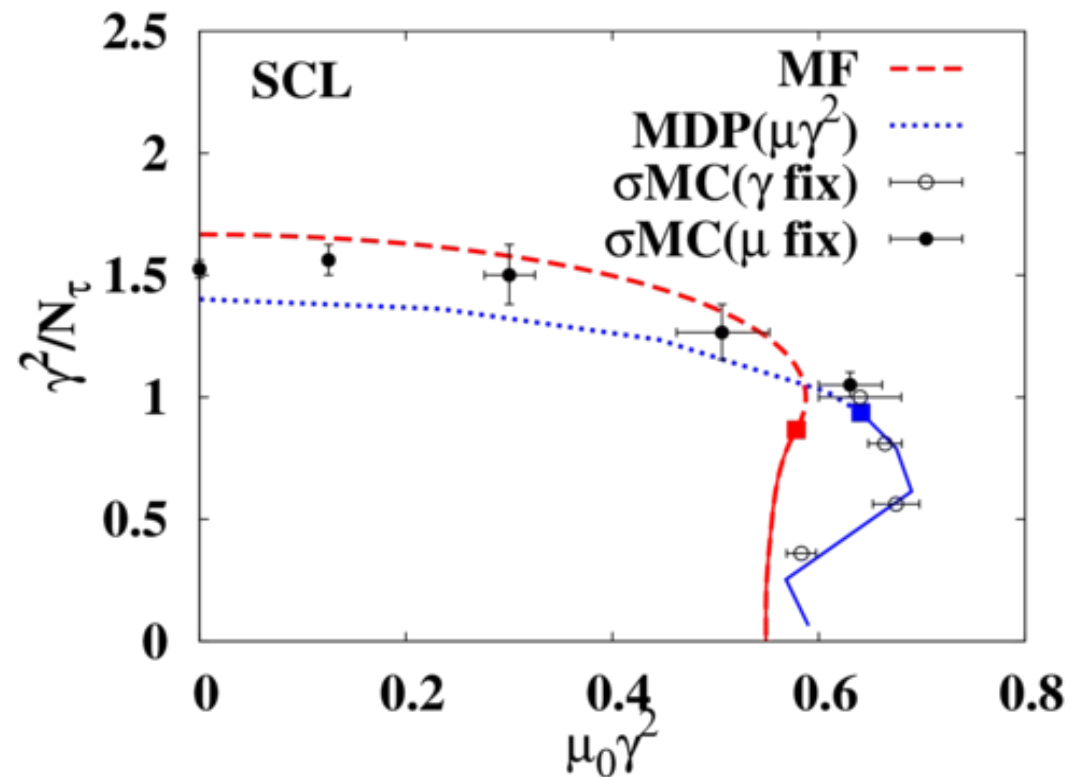
■ Medium T simulation [$\gamma=2, N_\tau=4$ ($T=1$)]

- Two peaks merges to be one
 \rightarrow Second order phase transition
- Transition takes place at $\mu_0 \sim 0.16$
 $(\mu = \mu_0 \gamma^2 \sim 0.64)$



Phase diagram

- T, μ are assumed to be given by $\gamma^2/N_\tau, \mu_0\gamma^2$
- Fluctuation of aux. field modifies the phase boundary.
 - Lower transition T , larger transition μ
- σ MC results are close to MDP results.
 - σ MC overestimate T_c in $\mu \sim 0$ region by $\sim 7\%$.
- When $\mu\gamma^2$ scaling is assumed, σ MC and MDP results reasonably match MF results.



Summary

- 強結合格子 QCD における補助場 MC は、揺らぎと有限結合効果をともに取り入れられる枠組みとして「育つ」可能性あり。
 - 有効作用が与えられれば、あとは厳密に解く！
 - 核子多体問題における Determinantal MC (Abe, Seki) と類似
 - 符号問題は存在するが、小さな lattice では計算可能。
また finite μ でむしろ弱い
 - 有限結合効果を取り入れても(おそらく)定式化・数値計算可能
- 問題
 - 大きな lattice では符号問題はより深刻。
 - baryon の伝播効果 ($1/\sqrt{d}$) を取り入れると大きな行列式の計算が必要
 - staggered Fermion からどの時点で切り替えるか？
(staggered Wilson, staggered overlap \rightarrow 中野)
 - もっとうまい方法はないだろうか？

Thank you

QCD phase transition in strong coupling limit

■ QCD phase transition at finite μ

- Lattice QCD Monte-Carlo simulation has the sign problem. There are many attempts to avoid the sign problem, but the results at large μ ($\mu/T > 1$ or $\mu > m_\pi$) are not yet reliable.

*(Reweighting, AC from Imaginary μ , Taylor expansion, cumulant expansion, ...)
Fodor, Katz ('02); de Forcrand, Philipsen('02); D'Elia, M. Lombardo ('03); Allton et al. ('04); Ejiri ('08); ...*

- Phase diagram is obtained by using the auxiliary field method of strong coupling lattice QCD, but those works rely on the mean field approximation.

Damgaard, Kawamoto, Shigemoto ('84); Damgaard, Hochberg, Kawamoto ('85); Bilic, Karsch, Redlich ('92); Fukushima ('03); Nishida ('03); Kawamoto, Miura, AO, Ohnuma ('07); Miura, Nakano, AO ('09); Miura, Nakano, AO, Kawamoto ('09); Nakano, Miura, AO ('10);

- Monomer-Dimer-Polymer (MDP) algorithm was proposed and has been demonstrated to work in the strong coupling limit (SCL), but its extension to finite coupling cases is not easy.

Karsch, Mutter ('89), de Forcrand, Fromm ('09), ...

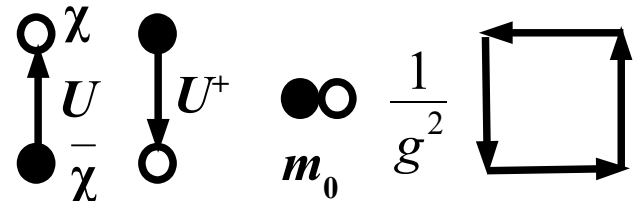
Problem in mean field approaches in SC-LQCD

- **Three types of approximations**
 - **Strong coupling expansion ($1/g^2$):** Small number of plaquettes
 - **Large dimensional approximation ($1/d$):** Small number of quarks
 - **Constant field assumption**
- **Phase diagram in mean field approaches in SCL may not match the phase diagram in MDP simulation result.**
→ **$1/d$ or constant field assumption ?**
 - **Higher orders in $1/d$ expansion:** Still difficult to handle (Bosonization breaks chiral and/or gauge symmetry.)
Azcoiti et al. ('03); Kawamoto, Miura, AO, Ohnuma ('07); AO, Nakano, Miura ('10).
 - **Fluctuations of aux. fields:** Not discussed seriously.

We try to extend the auxiliary field method of SC-LQCD to include fluctuations.

Lattice QCD action \rightarrow SCL quark & U_0 action

Lattice QCD action with (unrooted) staggered Fermion

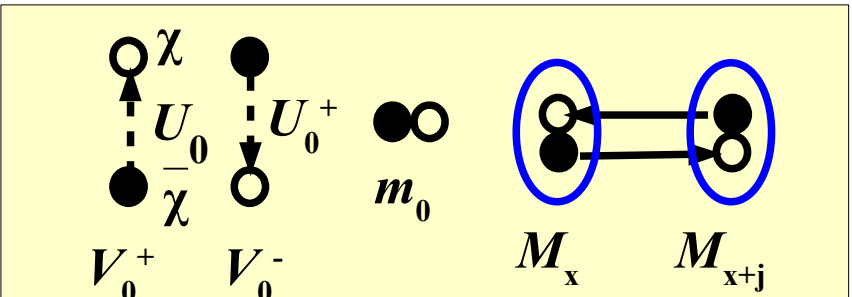
$$S_{LQCD} = S_F + S_G \quad S_G = -\frac{1}{g^2} \sum_{\text{plaq.}} \text{tr} [U_P + U_P^+] f_P$$


$$S_F = \frac{1}{2} \sum_x [V^+(x) - V^-(x)] + \frac{1}{2\gamma} \sum_{x,j} \eta_\mu(x) [\bar{\chi}_x U_{x,j} \chi_{x+\hat{j}} - \bar{\chi}_{x+\hat{j}} U_{x,j}^+ \chi_x] + \sum_x \frac{m_0}{\gamma} M_x$$

$$V^+(x) = e^\mu \bar{\chi}_x U_{x,0} \chi_{x+\hat{0}}, \quad V^-(x) = e^{-\mu} \bar{\chi}_{x+\hat{0}} U_{x,0}^+ \chi_x, \quad M_x = \bar{\chi}_x \chi_x, \quad a_\tau = a/\gamma, \quad f_P = 1 \text{ or } 1/\gamma$$

Strong coupling expansion (Strong coupling limit)

- Ignore plaquette action ($1/g^0$)
- Integrate out spatial link variables of min. quark number diagrams ($1/d$ expansion)



$$\int dU U_{ab} U_{cd}^+ = \delta_{ad} \delta_{bc} / N_c$$

$$S_{\text{eff}} = \frac{1}{2} \sum_x [V^+(x) - V^-(x)] - \frac{1}{4 N_c \gamma^2} \sum_{x,j} M_x M_{x+\hat{j}} + \frac{m_0}{\gamma} \sum_x M_x$$

Introduction of Auxiliary Fields

- MM term = Four Fermi (two-body) interacting term

→ Bosonization

Non-Local NJL
type ?

$$\begin{aligned}
 S^{(s)} &= -\frac{1}{4N_c\gamma^2} \sum_{x,j} M_x M_{x+\hat{j}} = -\frac{L^3}{4N_c\gamma^2} \sum_{\tau,\mathbf{k}} f_M(\mathbf{k}) \tilde{M}_{\mathbf{k}}(\tau) \tilde{M}_{-\mathbf{k}}(\tau) \\
 &= \frac{L^3}{4N_c\gamma^2} \sum_{\tau,\mathbf{k},f_M(\mathbf{k})>0} f_M(\mathbf{k}) \left[\varphi_{\mathbf{k}}(\tau)^2 + \phi_{\mathbf{k}}(\tau)^2 + \varphi_{\mathbf{k}}(\tilde{M}_{\mathbf{k}} + \tilde{M}_{-\mathbf{k}}) - i\phi_{\mathbf{k}}(\tilde{M}_{\mathbf{k}} - \tilde{M}_{-\mathbf{k}}) \right. \\
 &\quad \left. + \varphi_{\bar{\mathbf{k}}}(\tau)^2 + \phi_{\bar{\mathbf{k}}}(\tau)^2 + i\varphi_{\bar{\mathbf{k}}}(\tilde{M}_{\bar{\mathbf{k}}} + \tilde{M}_{-\bar{\mathbf{k}}}) + \phi_{\bar{\mathbf{k}}}(\tilde{M}_{\bar{\mathbf{k}}} - \tilde{M}_{-\bar{\mathbf{k}}}) \right] \\
 &= \frac{\Omega}{2N_c\gamma^2} \sum_{k,f_M(\mathbf{k})>0} f_M(\mathbf{k}) [\sigma_k^* \sigma_k + \pi_k^* \pi_k] + \frac{1}{2N_c\gamma^2} \sum_x M_x [\sigma(x) + i\varepsilon(x)\pi(x)]
 \end{aligned}$$

$$\sigma(x) = \sum_{k,f_M(\mathbf{k})>0} f_M(\mathbf{k}) e^{ikx} \sigma_k, \quad \pi(x) = \sum_{k,f_M(\mathbf{k})>0} f_M(\mathbf{k}) e^{ikx} \pi_k$$

$$\sigma_k = \varphi_k + i\phi_k, \quad \pi_k = \varphi_{\bar{\mathbf{k}}} + i\phi_{\bar{\mathbf{k}}}$$

$$V_{x,y} = \frac{1}{2} \sum_j \left(\delta_{x+\hat{j},y} + \delta_{x-\hat{j},y} \right), \quad f_M(\mathbf{k}) = \sum_j \cos k_j, \quad \bar{\mathbf{k}} = \mathbf{k} + (\pi, \pi, \pi)$$

Fermion Determinant

Faldt, Petersson, 1986

- Fermion action is separated to each spatial point and bi-linear
 → Determinant of $N_\tau \times N_c$ matrix

$$\exp(-V_{\text{eff}}/T) = \int dU_0 \begin{vmatrix} I_1 & e^\mu & 0 & & e^{-\mu} U^+ \\ -e^{-\mu} & I_2 & e^\mu & & \\ 0 & -e^{-\mu} & I_3 & e^\mu & \\ \vdots & & & \ddots & \\ -e^\mu U & & & -e^{-\mu} & I_N \end{vmatrix} \quad \begin{matrix} \uparrow \\ \downarrow \end{matrix} N_c \times N_\tau$$

$$= \int dU_0 \det \left[\underbrace{X_N[\sigma] \otimes \mathbf{1}_c}_{\text{magenta}} + \underbrace{e^{-\mu/T} U^+}_{\text{blue}} + \underbrace{(-1)^{N_\tau} e^{\mu/T} U}_{\text{red}} \right] \quad \begin{matrix} \updownarrow \\ \updownarrow \end{matrix} N_c$$

$$= X_N^3 - 2 X_N + 2 \cosh(3 N_\tau \mu)$$

$$I_\tau/2 = [\sigma(x) + i\varepsilon(x)\pi(x)]/2 N_c \gamma^2 + m_0/\gamma$$

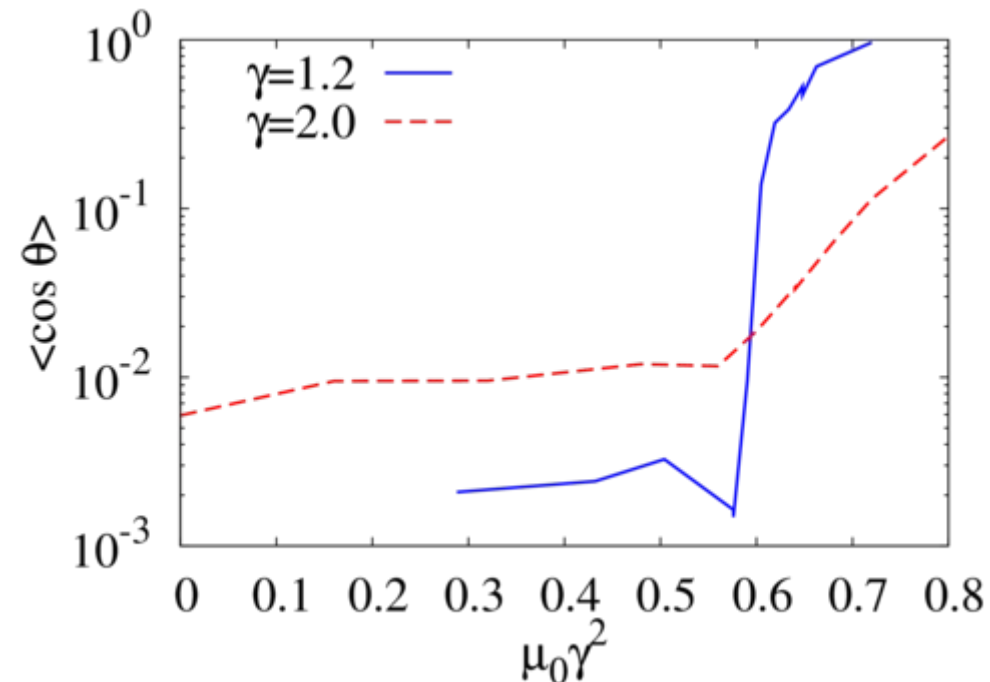
$$X_N = B_N + B_{N-2} (2; N-1)$$

$$B_N = I_N B_{N-1} + B_{N-2}$$

$$B_N = \begin{vmatrix} I_1 & e^\mu & 0 & & \\ -e^{-\mu} & I_2 & e^\mu & & \\ 0 & -e^{-\mu} & I_3 & e^\mu & \\ \vdots & & & \ddots & \\ -e^{-\mu} & & & & I_N \end{vmatrix}$$

Numerical Calculation

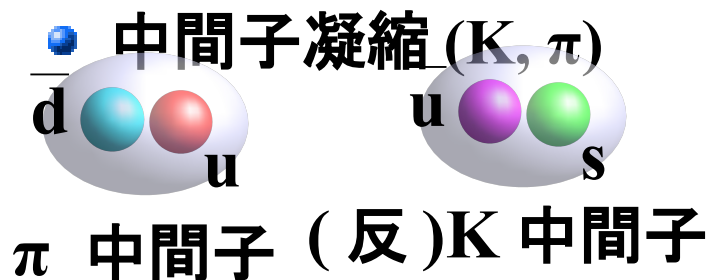
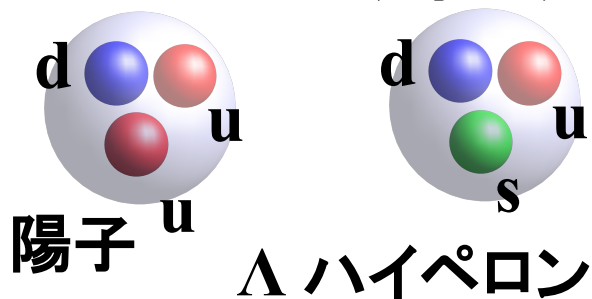
- Jump size is chosen to be new sampling prob. ~ 0.5
 - Always full Update of σ_k and π_k (This may not be very efficient.)
- Initial cond. = const. σ ($\sigma = -2.5, -2.0, \dots, 2.5$)
 - Chiral limit ($m=0$) simulation \rightarrow Symmetry in $\sigma \leftrightarrow -\sigma$
 - Deep Seff min. at $\sigma \sim \sigma_{\text{vac}}$ at low T
- Sign problem is not severe in 4^4 lattice.
 - $\langle \cos \theta \rangle \sim$ a few $\times 10^{-3}$ or more.
- Computer: My PC (Core i7)



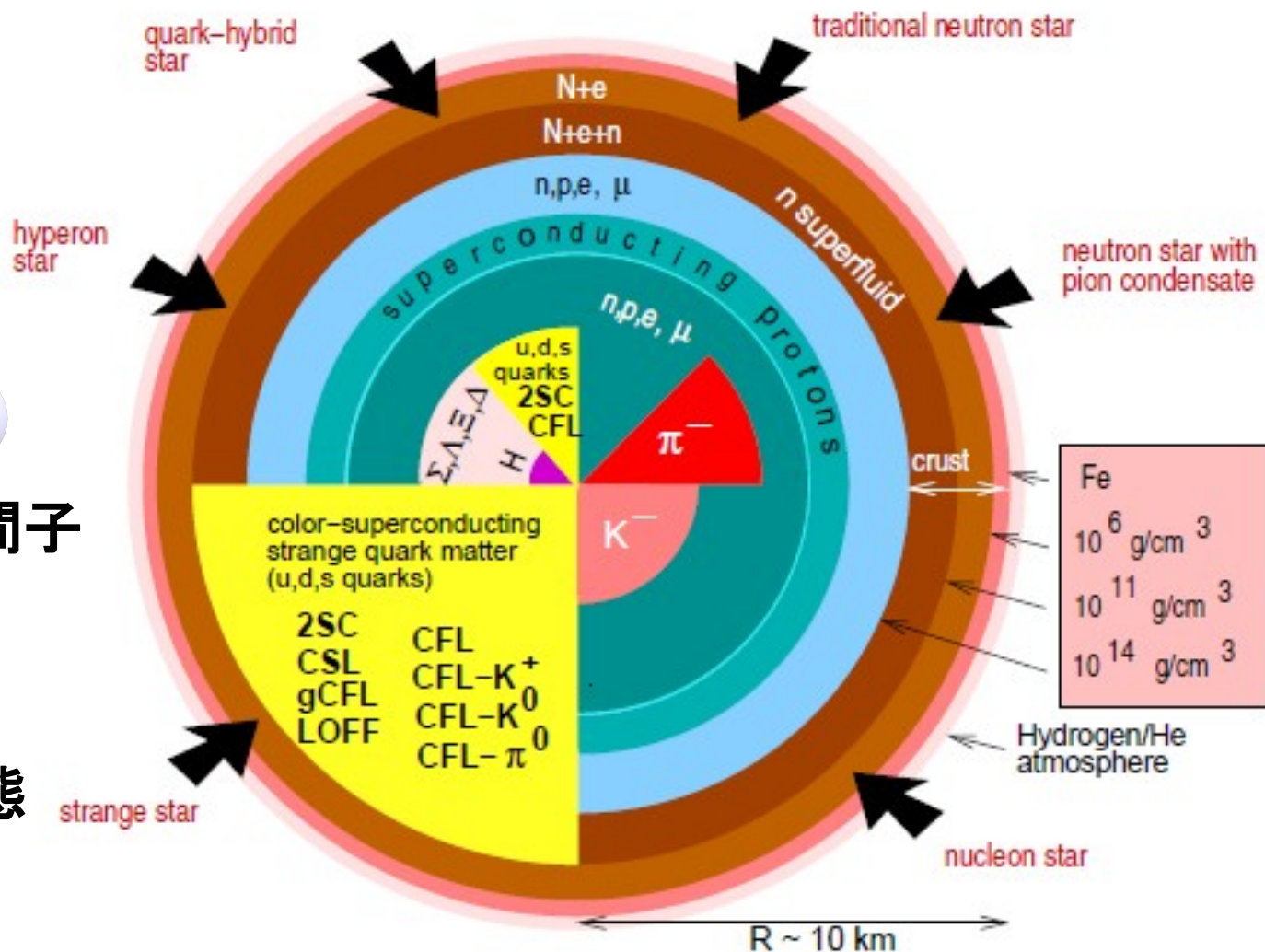
中性子星コアの状態

■ コア領域では様々な可能性

- ストレンジクォークを含むバリオン (ハイペロン) を含む物質



- クォーク物質
- クォーク対凝縮状態 (カラー超伝導)



F. Weber, Prog. Part. Nucl. Phys. 54 (2005) 193

重い中性子星ショック....

■ 2010年のビッグニュース

「 $1.97 \pm 0.04 M_{\odot}$ の質量をもつ中性子星が発見された」

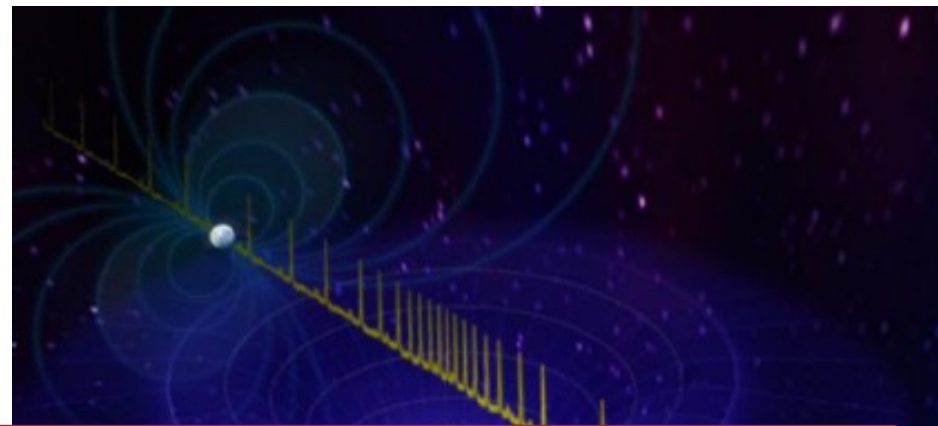
Demorest et al., Nature 467 (2010) 1081 (Oct.28, 2010).

● 一般相対論に基づく観測

「パルサー(中性子星)からくる光が伴星(白色矮星)の近くを通り、時間が遅れる(Shapiro delay)。」

論文での主張

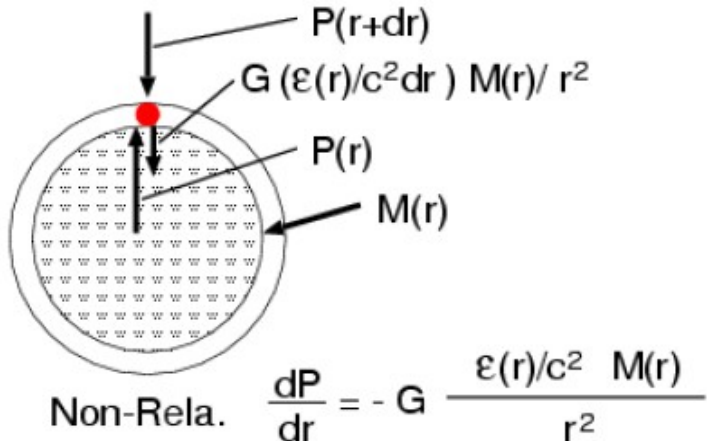
$(1.97 \pm 0.04) M_{\odot}$ の中性子星は、ハイペロン、中間子凝縮を含む状態方程式では支えられない。クォーク物質でも強い相互作用が必要である。



signature. We calculate the pulsar mass to be $(1.97 \pm 0.04)M_{\odot}$, which rules out almost all currently proposed²⁻⁵ hyperon or boson condensate equations of state (M_{\odot} , solar mass). Quark matter can support a star this massive only if the quarks are strongly interacting and are therefore not 'free' quarks¹².

状態方程式と中性子星の最大質量

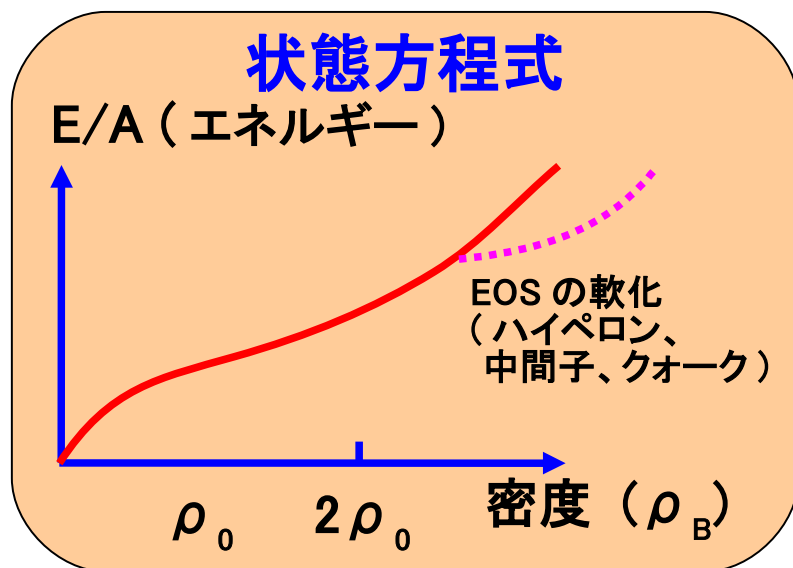
- TOV(Tolman-Oppenheimer-Volkoff) 方程式
= 一般相対論での圧力と重力の釣り合いを記述



$$\frac{dP}{dr} = -G \frac{(\epsilon/c^2 + P/c^2)(M + 4\pi r^3 P/c^2)}{r^2(1 - 2GM/rc^2)}$$

$$\frac{dM}{dr} = 4\pi r^2 \epsilon/c^2, \quad P = P(\epsilon) \quad (\text{EOS})$$

- 状態方程式 (EOS) を与えると質量・半径の関係式は一意的



TOV 方程式

