Brown-Rho Scaling in the Strong Coupling Lattice QCD

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- **Introduction**
- **Chiral Condensate and Polyakov loop in SC-LQCD**
- **Meson masses in SCL-LQCD**
- **Summary**

Hadron Mass AO, N. Kawamoto, K. Miura, Mod. Phys. Lett. A 23 (2008), 2459.
1/d effects N. Kawamoto, K. Miura, AO, T. Ohnuma, PRD 75 (2007), 014502.
NLO ($1/g^2$) K. Miura, T. Z. Nakano and AO, PTP 122 (2009), 1045.
K. Miura, T. Z. Nakano, AO, N. Kawamoto, PRD 80 (2009), 074034.
NNLO ($1/g^4$) T. Z. Nakano, K. Miura, AO, PTP 123 (2010), 825.
NNLO + Polyakov loop T. Z. Nakano, K. Miura, AO, PRD 83 (2011), 016014.

Hadron Mass in Nuclear Matter

- **Medium meson mass modification**
  - **may be the signal of partial restoration of chiral sym.**

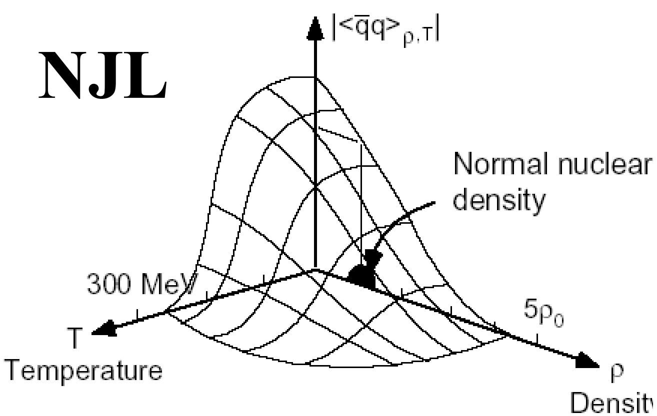
    Brown, Rho, PRL66('91)2720; Kunihiro,Hatsuda, PRep 247('94),221; Hatsuda, Lee, PRC46('92)R34.

    Brown-Rho scaling (*20-th year anniversary*)

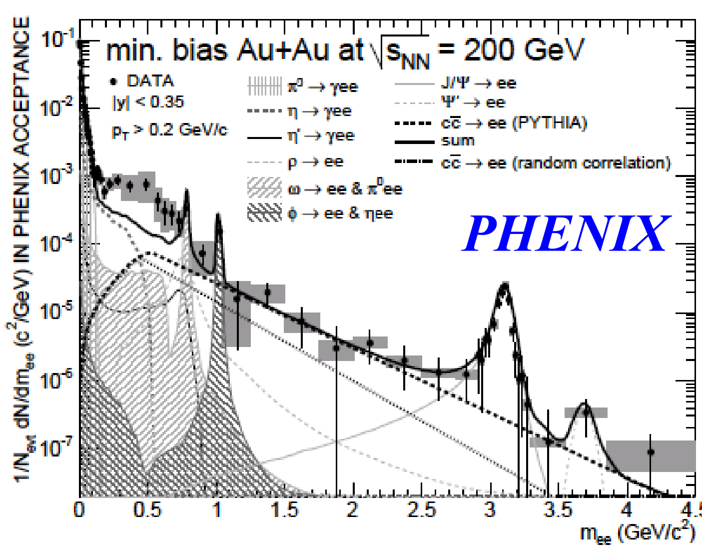
$$M_N^*/M_N = M_\sigma^*/M_\sigma = M_\rho^*/M_\rho = M_\omega^*/M_\omega = f_\pi^*/f_\pi$$

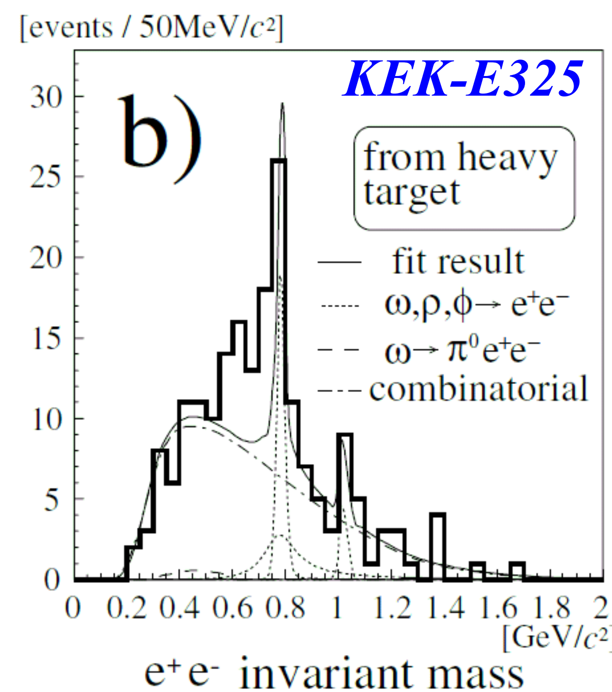
  - **and is suggested experimentally.**

    CERES Collab., PRL75('95),1272; PHENIX Collab., arXiv:0706.3034; KEK-E325 Collab.(Ozawa et al.), PRL86('01),5019.



W. Weise, Nucl. Phys. A 553 (1993) 59c

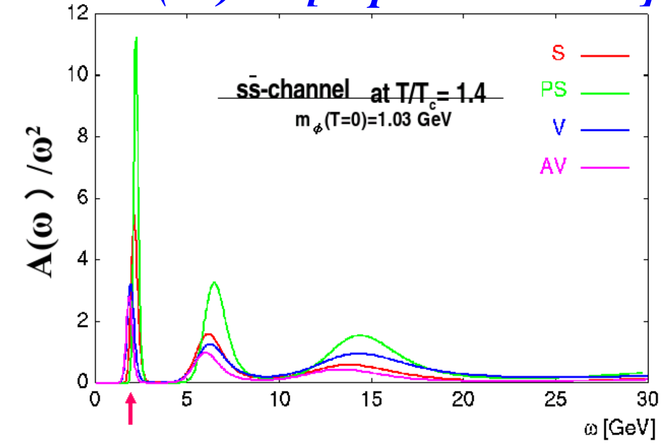




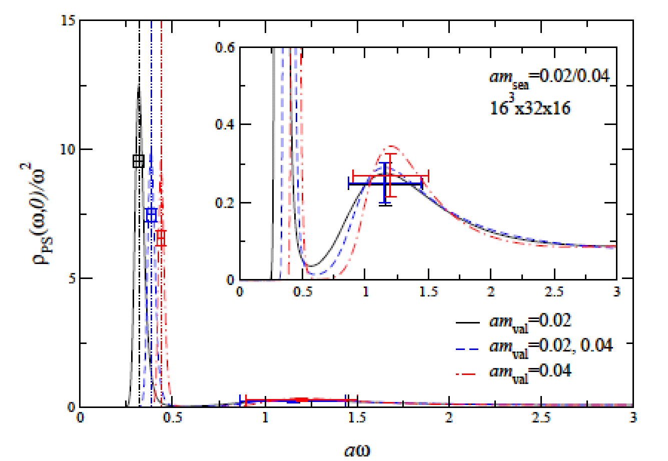
Hadron Mass in QCD

- Lattice QCD
  → Succcessful at μ=0, Sign prob. at finite μ
  M. Asakawa, T. Hatsuda, Y. Nakahara ('03);
  G. Aarts, Foley ('07, DW).
- QCD sum rule
  → Condensates have to be given.
  Hatsuda, Lee, PRC46('92)R34.; Gubler, Oka, Morita,
- Strong Coupling Lattice QCD (SC-LQCD)
  - Hadron masses in vacuum
    (Strong Coupling Limit ($1/g^2 \to 0$))
    Kluberg-Stern, Morel, Petersson, '83;
    Kawamoto, Shigemoto, '82.
  - To do: Finite (T, μ), $1/g^2$ corr., ...

Asakawa, Nakahara, Hatsuda,
NPA715(03)863[hep-lat/0208059].



G. Aarts, J. Foley (UKQCD),
JHEP 0702('07)062.
[DW QCD, PS (T=0)]



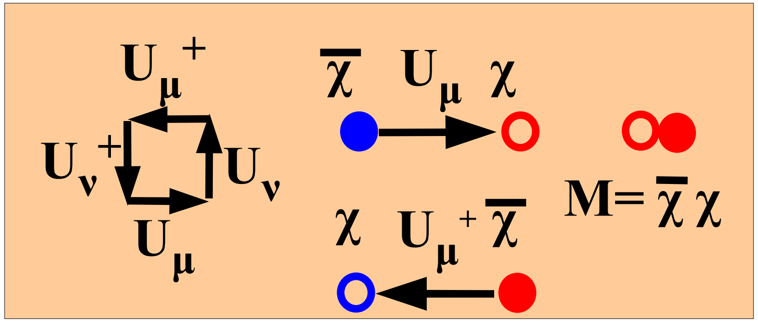
We discuss meson masses at finite (T, μ) in SCL-LQCD. *(AO, Miura, Kawamoto, 2008)*

Chiral condensate and Polyakov loop in Strong Coupling Lattice QCD

Strong Coupling Lattice QCD

- **Lattice QCD=ab initio, non-perturbative theory**

$$S_{\rm LQCD} = \frac{1}{2}\sum_{x,j}\left[\eta_{\nu,x}\bar{\chi}_x U_{\nu,x}\chi_{x+\hat{\nu}} - \eta^{-1}_{\nu,x}\bar{\chi}_{x+\hat{\nu}}U^{\dagger}_{\nu,x}\chi_x\right] - \frac{1}{g^2}\sum_{\Box}\mathrm{tr}\left[U_{\Box}+U^{\dagger}_{\Box}\right] + m_0\sum_x \bar{\chi}_x\chi_x$$

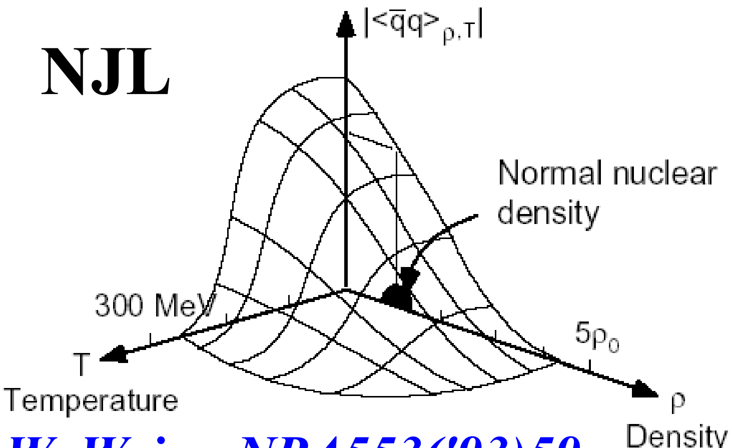


- **Strong Coupling Lattice QCD**
  - **$1/g^2 \ll 1$ → perturbative treatment of plaquetts**
    - **Effective action of color singlet objects (Mesons, Baryons, Loops)**
  - **Great successes in pure YM**
    - **Area law (Wilson), Strong and weak coupling (Creutz), Character expansion to higher orders (Munster), …**
  - **Chiral transition at finite T and μ:**
    → mainly discussed in the Strong Coupling Limit ($g \to \infty$)
    Kawamoto, Damgaard, Shigemoto; Bilic, Karsch, Redlich; Fukushima; Nishida, Fukushima, Hatsuda; …

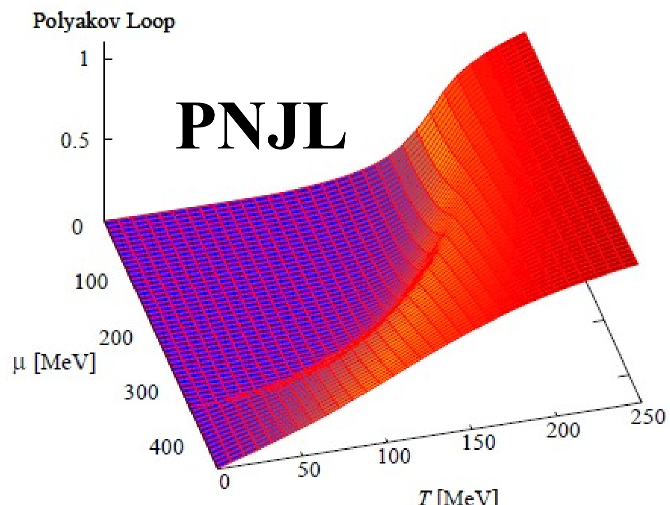
    → NLO, NNLO, and Polyakov loop effects in SC-LQCD

Chiral Condensate and Polyakov loop in SC-LQCD

NJL

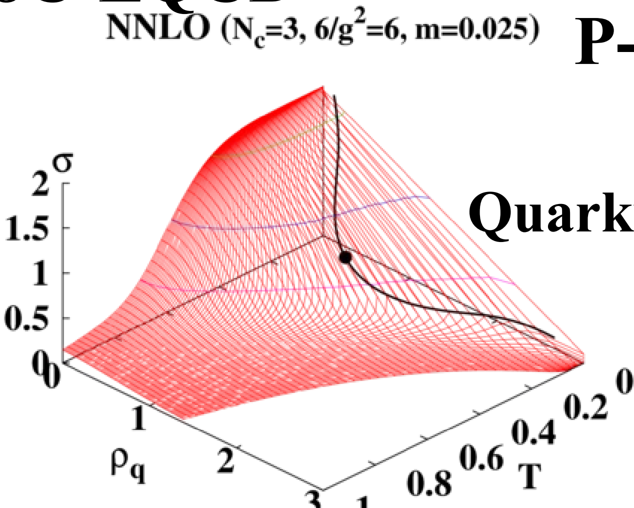


W. Weise, NPA553('93)59c



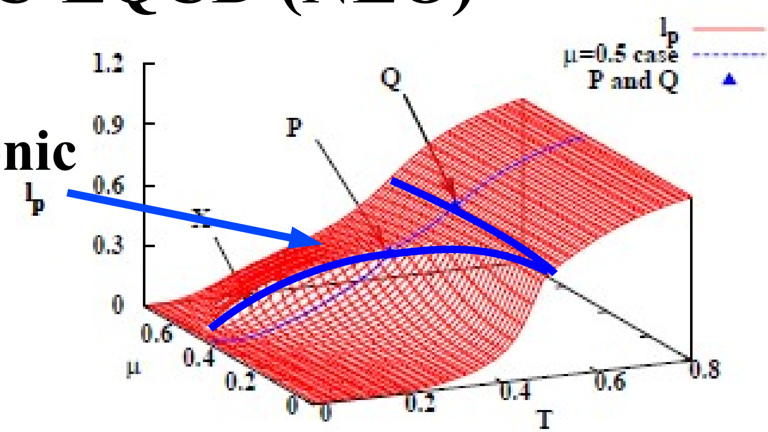
K. Fukushima, PRD77('08)114028

SC-LQCD



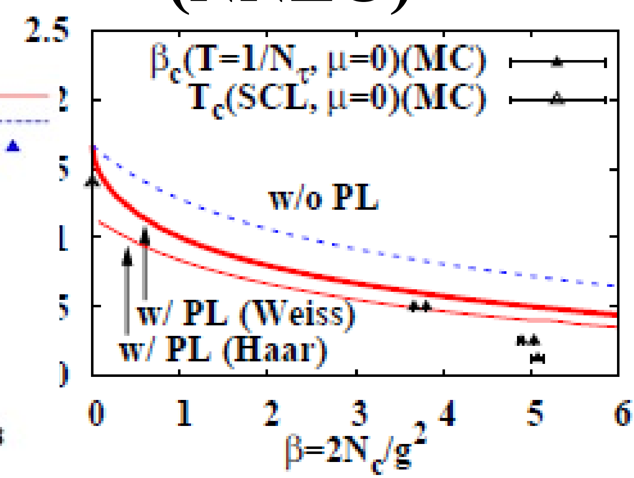
Nakano, Miura, Ohnishi, PTP123('10)825

P-SC-LQCD (NLO)



Miura,Nakano, AO, Kawamoto PoS Lat2010, 202; in prep.

P-SC-LQCD (NNLO)



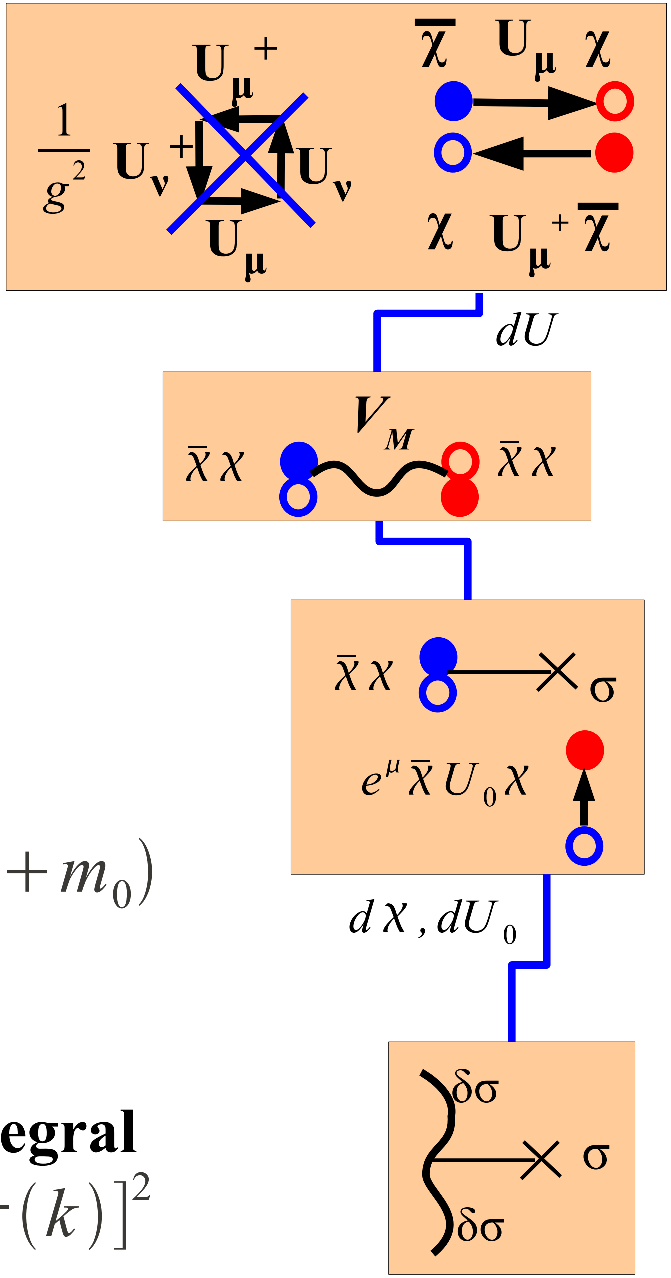
Nakano, Miura, AO, PRD83('11)016014

Qualitatively good in condensates. How about hadron masses ?

Meson masses in SCL-LQCD

Strong Coupling Limit of Lattice QCD

- **Finite T tratment** *Damgaard, Kawamoto, Shigemoto, 1984.*
  → Exact temporal link integral followed by spatial link integral, bosonization, and Fermion det.



- **QCD Lattice Action (staggered Fermion)**

$$S_{\mathrm{LQCD}} = \sum_{\nu} \bar{\chi} D_{\nu} \chi + m_0 \sum_{x} \bar{\chi}_x \chi_x + \cancel{S_G}$$

Strong Coupling Limit

$$\bar{\chi} D_{\nu} \chi = \frac{1}{2} \sum_{x} \left(\eta_{\nu,x} \bar{\chi}_x U_{\nu,x} \chi_{x+\hat{\nu}} - \eta_{\nu,x}^{-1} \bar{\chi}_{x+\hat{\nu}} U_{\nu,x}^{+} \chi_x \right)$$

- **Spatial link integral + Bosonization**

$$S_{\mathrm{eff}} = \underbrace{\frac{1}{2} \sigma V_M^{-1} \sigma}_{\text{Spatial hopping}} + \underbrace{\bar{\chi} (D_t + m_q) \chi}_{\text{Temporal hopping}} \qquad (m_q = \sigma + m_0)$$

- **Decomposition ($\sigma = \bar{\sigma} + \delta\sigma$)+ Fermion & U_0 integral**

$$S_{\mathrm{eff}}(\delta\sigma) = L^d N_\tau F_{\mathrm{eff}}(\bar{\sigma}) + \frac{1}{2} \sum_k G(k)^{-1} [\delta\sigma(k)]^2$$

Strong Coupling Limit of Lattice QCD

- **Effective Potential** *Fukushima ('04), Nishida ('04)*

$$F_{\rm eff}=\frac{N_c}{d}\bar{\sigma}^2+V_{\rm eff}(\bar{\sigma},T,\mu)$$

$$V_{\rm eff}=-T\log\left[\frac{\sinh((N_c+1)E_q/T)}{\sinh(E_q/T)}+2\cosh(N_c\mu/T)\right]\quad(E_q(m)=\mathrm{arcsinh}\,m)$$

- **Meson propagator**
  - **Meson self-energy comes from the quark determinant, whose derivative (minor det.) is obtained from recursion relation.** *Faldt, Petersson ('86)*

$$G^{-1}(\boldsymbol{k},\omega)=V_M^{-1}(\boldsymbol{k})+\mathrm{F.T.}\frac{\partial^2 V_{\rm eff}}{\partial m(\tau)\partial m(\tau')}$$

$$\exp(-V_{\rm eff}/T)=\int dU_0\begin{vmatrix} I_1 & e^{\mu} & 0 & & e^{-\mu}U^{+} \\ -e^{-\mu} & I_2 & e^{\mu} & & \\ 0 & -e^{-\mu} & I_3 & e^{\mu} & \\ \vdots & & & \ddots & \\ -e^{\mu}U & & & -e^{-\mu} & I_N \end{vmatrix}\quad(I_k=\sigma_k+m_0)$$

Prescriptions related to lattice staggered fermions

- **Mass = Pole energy of G at "zero" momentum**
  - **"Zero" momentum: $\underline{k}$ = -$\underline{k}$ (vector) → $\underline{k}$ = (0,0,0), (0,0,π), (0, π, 0)**

$$\kappa(\boldsymbol{k}) = \sum_{j=1}^{d} \cos k_j = -3, -1, 1, 3 \quad \text{for zero momentum} \, (\boldsymbol{k} = -\boldsymbol{k})$$

Four different types of meson appear !
(Bound state with doubler)

  - **"Zero" Euclidean energy: ω= -ω → ω = 0 or π**

→ Search for the pole with ($\underline{k}$, ω)= (δ_π, δ_π, δ_π, iM +δ_π) (δ_π =0 or π)

$$G^{-1}(\boldsymbol{k} = '\boldsymbol{0}', \omega = iM + \delta\pi) = \frac{2N_c}{\kappa} + \frac{4N_c}{d} \frac{\bar{\sigma}(\bar{\sigma} + m_0)}{\pm\cosh M + \cosh 2E_q} = 0$$

Hadron Mass in SCL-LQCD (Finite T)

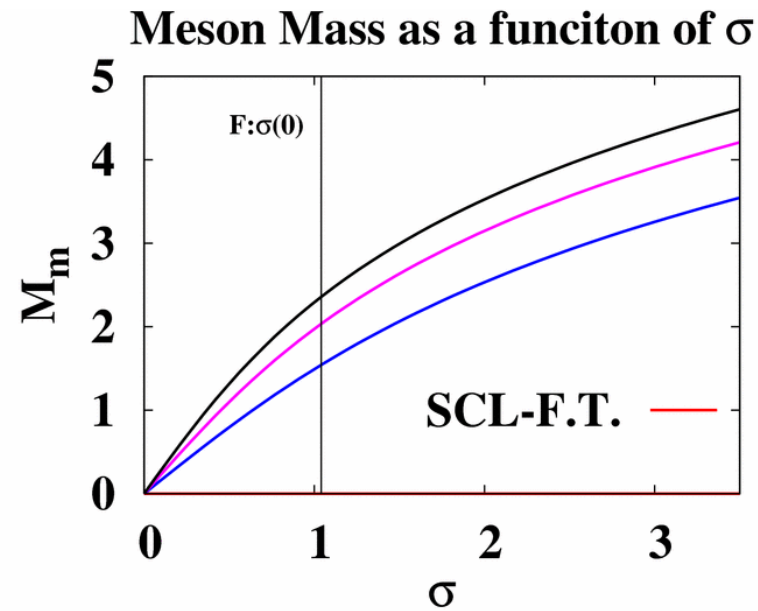
AO, N. Kawamoto, K. Miura, Mod. Phys. Lett. A 23 (2008)2459.

- **Meson Mass**

$$G^{-1}(\boldsymbol{k},\omega)=\frac{2N_c}{\kappa(\boldsymbol{k})}+\frac{4N_c\bar{\sigma}}{d}\frac{\bar{\sigma}+m_0}{\cos\omega+\cosh 2E_q}$$

$$\kappa(\boldsymbol{k})=\sum_{i=1}^{d}\cos k_i \quad \rightarrow \quad \kappa=-d,-d+2,\ldots d$$

$$\boxed{M=2\,\mathrm{arcsinh}\sqrt{(\bar{\sigma}+m_0)\left(\frac{d+\kappa}{d}\bar{\sigma}+m_0\right)}}$$



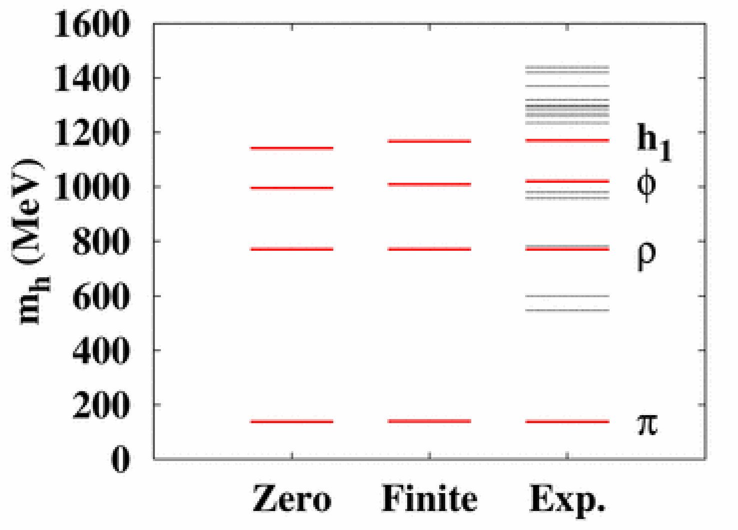
- **Equilibrium condition: $\partial V_{\mathrm{eff}}/\partial\sigma = -2N_c\sigma/d$**
  → Meson masses are determined by the chiral condensate, σ.
- **Chiral condensate is a function of (T, μ).**

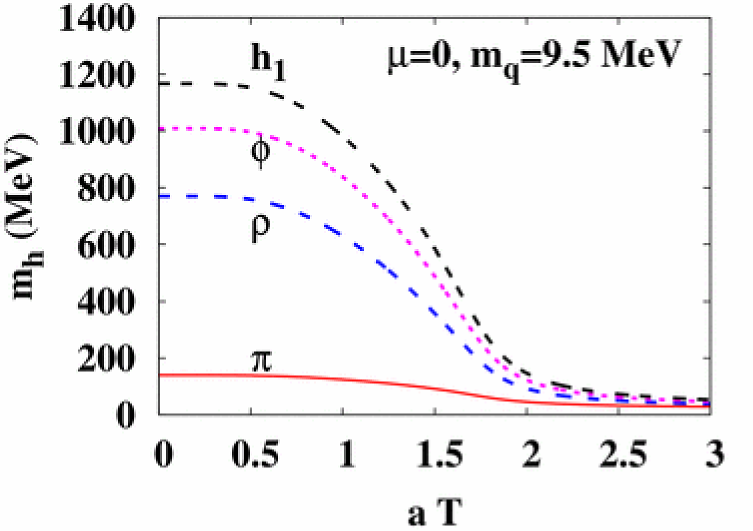
→ ***Approximate Brown-Rho scaling emerges in SCL-LQCD***

- **Many eservations: SCL-LQCD, LO in 1/d expansion, staggered fermion, mean field app. (no feed back of fluc.), ...**

Medium Modification of Meson Masses

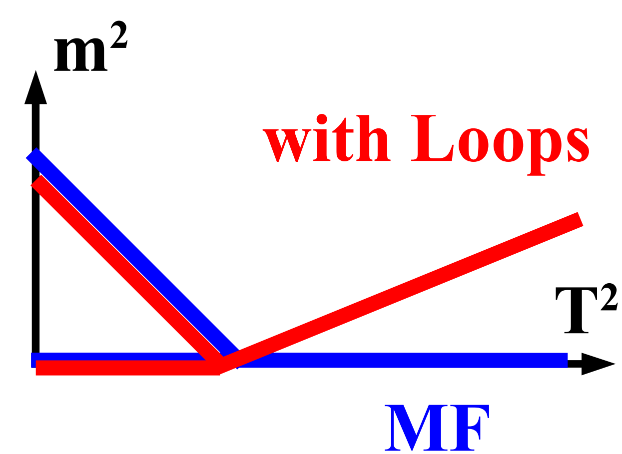
- **Scale fixing**
  - **Search for σ_{vac} to minimize free E.**
  - **Assign κ=-3, -1 as π and ρ**
  - **Determine m_0 and a^{-1} (lattice unit) to fit m_π / m_ρ (a=497 MeV)**
- **Medium modification**
  - **Search for σ(T, μ) → Meson mass**
  - **Vacuum mass ~ Zero T results**
    Kluberg-Stern, Morel, Petersson, 1982; Kawamoto, Shigemoto, 1982





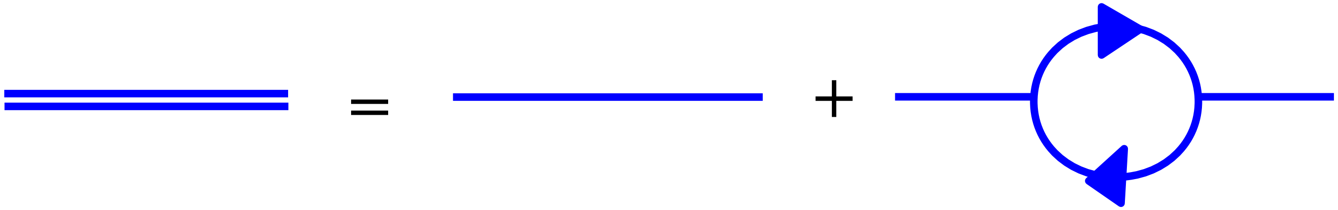
Summary

- **Chiral condensates and Polyakov loop at finite T and μ are investigated with SC-LQCD.**
  - **Partial restoration of χ sym. is expected at finite T and/or μ in SC-LQCD and P-SC-LQCD.**
  - **Qualitative behavior is similar to NJL and PNJL results.**
  - **Quantitative differences to be further discussed $\rightarrow$ T_c and μ_c, Density gap at finite μ,Critical point, ...**
- **Meson masses at finite T and μ are studied in SCL-LQCD.**
  - **Results with mean field approx. shows Brown-Rho scaling behavior.**
  - **Loop effects of mesons are expected to enhance meson masses after χ restoration**
    Hatsuda, Kunihiro / Kapusta text book
  - **Finite coupling effects and self-consistent treatment (SD type) would be interesting.**

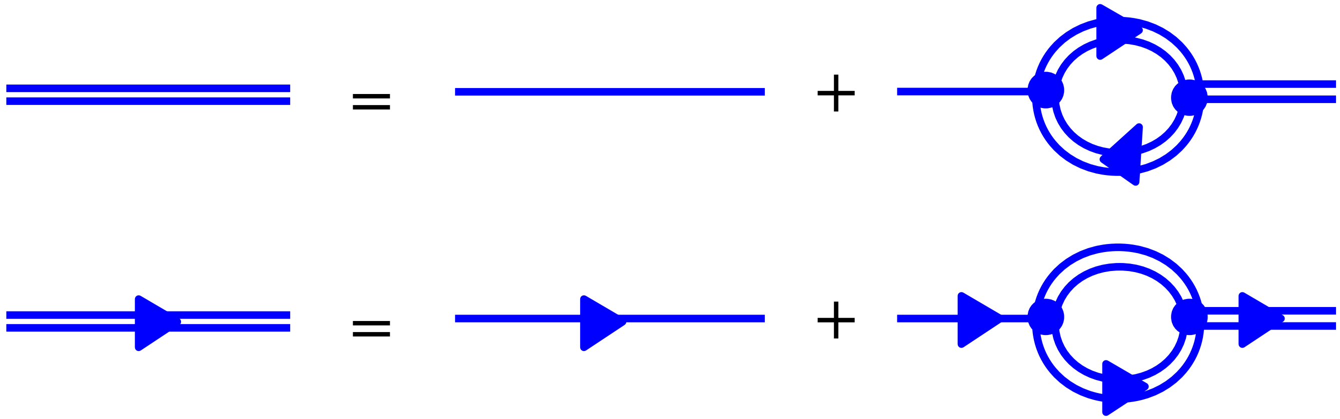


Homework: Can we do it ?

- **Present treatment**



- **Self-consistent treatment**



> ***Is it possible to carry out the self-consistent calculation of meson and quark propagator in SC-LQCD hopefully with NLO/NNLO/PL effects (in two weeks) ?***

Thank you !