Equilibration in classical Yang-Mills dynamics

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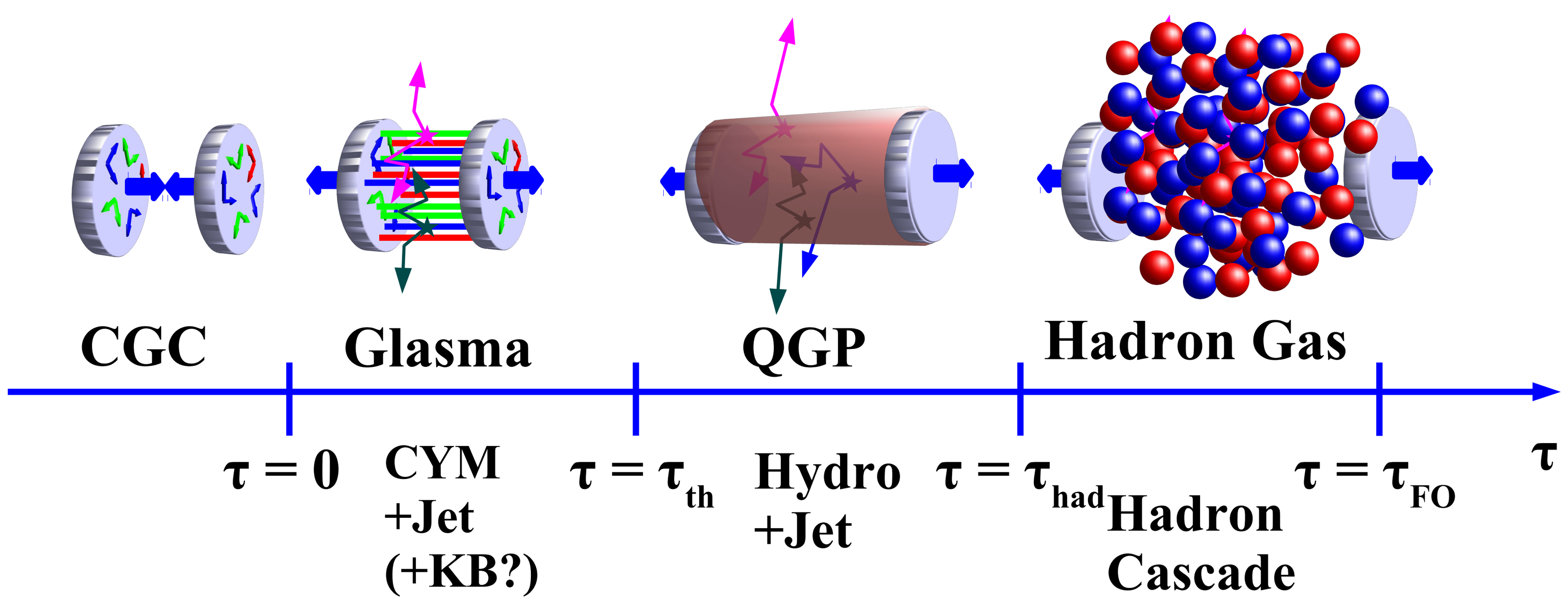
Introduction

- Time Evolution at RHIC & LHC



- $\tau < 0$ fm/c → Color Glass Condensate
- $0 < \tau < \tau_{eq}$ → Thermalization
- $\tau > \tau_{eq}$ → Success of (nearly ideal) Hydrodynamics ($\tau_{eq} < 1.5$ fm/c)

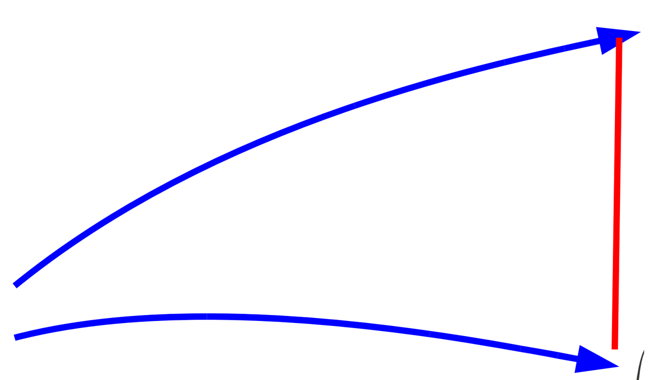
- Mechanism of Early Thermalization
  - decoherence: $\tau_{eq} \sim 1 / Q_s \sim 0.2$ fm/c (Fastest !) but entropy production is not enough for thermalization (R. J. Fries, B. Müller and A. Schäfer, PRC 79('09)034904)
  - Instability of Classical Yang-Mills (CYM) field
    - Weibel & Nielsen-Olesen Instability
    → Mechanism of strong field decay into particles toward thermalization is not known yet.
  - Chaotic behavior
    - Entropy production rate = Kolmogorov-Sinai entropy (S_{KS}) (T.Kunihiro, B. Müller, AO, A. Schäfer, PTP121('09)555)

$$S_{KS} = \sum_{\lambda_i > 0} \lambda_i, \quad |\delta X_i| \propto \exp(\lambda_i t)$$

This work:
Evaluate SKS of CYM → Equilibration time of CYM

Chaotic CYM Dynamics

- EOM and Deviation of two trajectories



$$\delta\dot{X}(t) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} H_{xx} & H_{xp} \\ H_{px} & H_{pp} \end{pmatrix} \delta X(t) = K(t)\,\delta X(t)$$

Equation of Motion $\quad \dot{X}(t) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} H_x \\ H_p \end{pmatrix}, \quad X = \begin{pmatrix} x \\ p \end{pmatrix}$

- Local Lyapunov exponent = Eigenvalue of K(t)
  → standard instability
- (Global) Lyapunov exponent → $|\delta X| \propto \exp(\lambda t)$
  → entropy production in a long time scale
- Intermediate Lyapunov exponent = Eigenvalue of U(t,t+ Δt)

$$U(t, t+\Delta t) = T\left[\exp\left(\int_t^{t+\Delta t} K(t')\,dt\right)\right]$$

→ U can be evaluated by using Trotter formula
Most relevant to entropy production

- Entropy production rate $\quad \frac{dS}{dt} = S_{KS} = \sum_{\lambda_i^{\rm ILE} > 0} \lambda_i^{\rm ILE}$
- Classical Yang-Mills field on the lattice

$$H = \frac{1}{2}\sum_{x,a,i} E_i^a(x)^2 + \frac{1}{4}\sum_{x,a,i,j} F_{ij}^a(x)^2$$

$$F_{ij}^a = \partial_i A_j^a(x) - \partial_j A_i^a(x) + \sum_{b,c} f^{abc} A_i^b(x) A_j^c(x)$$

- CYM is conformal !

$$s_{KS}^{(L)} = c_{KS}\left[\varepsilon^{(L)}\right]^{1/4} \to \tau_{eq} = \frac{\text{const}}{T\, c_{KS} (\varepsilon^{(L)})^{1/4}}$$

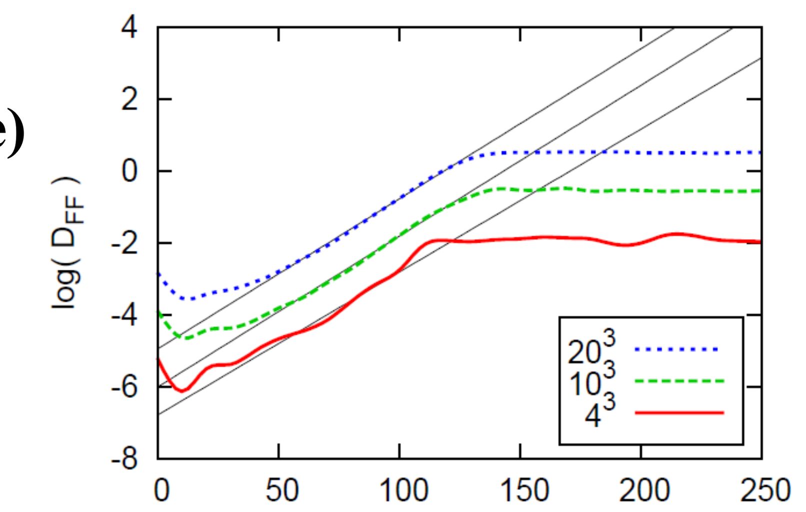
Equilibrium Entropy per site on the lattice

Conformal

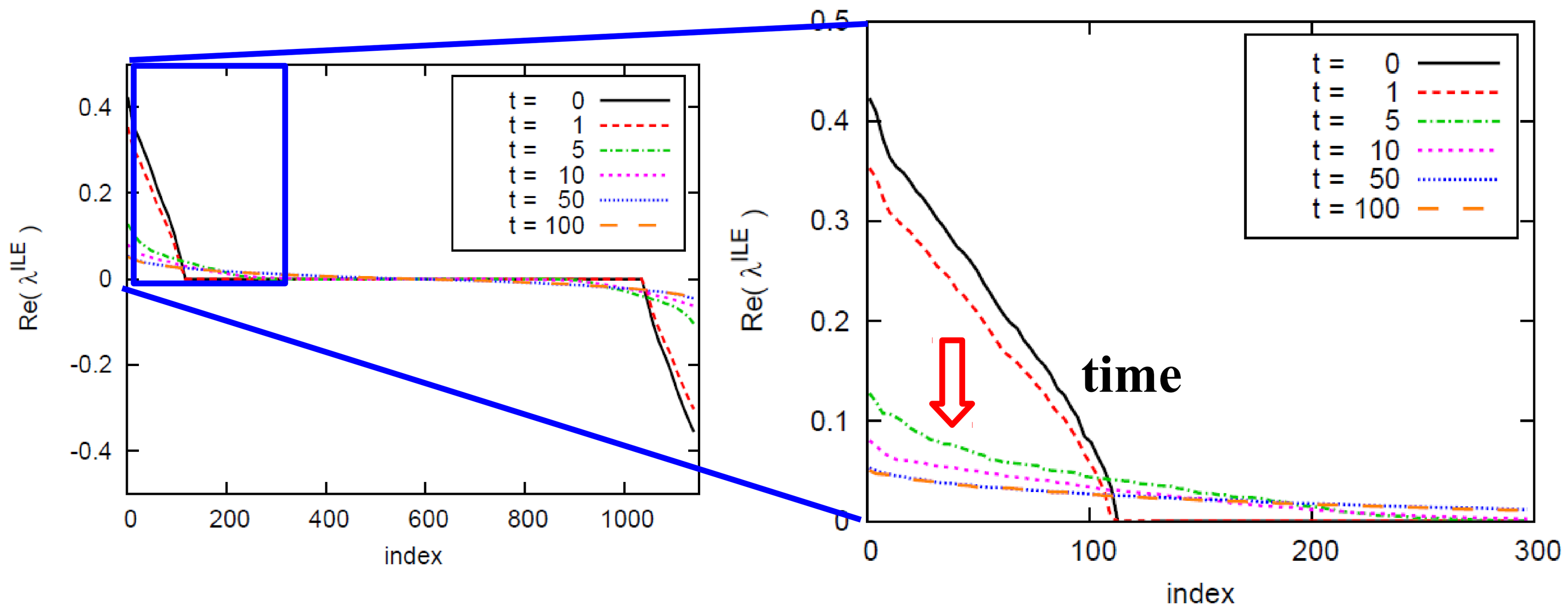
Lattice unit → Physical unit

Results

- Distance between two trajectories
  - Exponential growth → chaotic behavior
  - Growth rate ~ max. Lyapunov exponent = indep. of lattice size



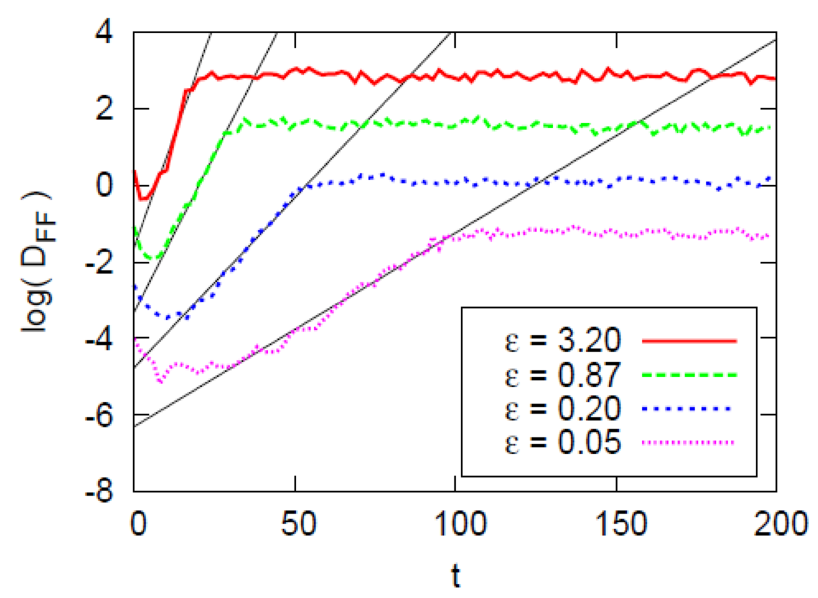
- Distribution of ILEs
  - $(\text{LLE})^2 \sim -$ (potential curvature)
  - Max. ILE rapidly decreases, while # of modes with positive ILE increases.
    → Exponential growth spread over various modes.

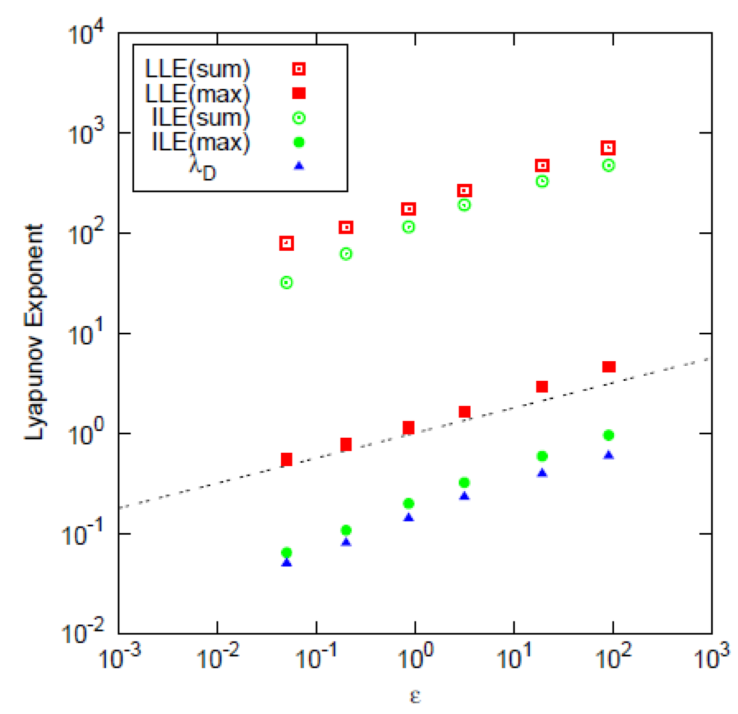


- Energy density dep.
  - Large ε (=E/V) → Rapid increase of distance

$$\frac{dS}{dt} = S_{KS} = \sum_{\lambda_i^{\rm ILE} > 0} \lambda_i^{\rm ILE} = c_{KS}\,\varepsilon^{1/4}, \quad c_{KS} \simeq 2 \times L^3 \quad (L = \text{spatial lattice size})$$

$$\tau_{eq} \simeq \frac{5}{T} + \tau_{delay} \simeq 3\,\text{fm}/c\,(T = 350\,\text{MeV})$$





Chaotic nature of CYM does not fully explain early thermalization, but its contribution is significant !

Summary & Discussion

- We have developed a method to evaluate the equilibration time of Classical Yang-Mills (CYM) system.
  - Entropy production rate
    = Kolmogorov-Sinai entropy (also in quantum system)
    = sum of positive *Intermediate* Lyapunov exponent
  - Equilibration time
    = "Equilibrium entropy" / "Entropy production rate"
- Spatial lattice simulation of CYM shows *conformal* nature of the entropy production rate, S_{KS}.
  - Mode-mode coupling is strong, and energy partition (~equilibration) proceeds with this coupling.
  - Sum of positive ILEs follow $S_{KS} \propto \varepsilon^{1/4}$ in CYM.
  - If equilibration in CYM is dominant, conformal nature suggests $\tau_{eq} \propto 1/T$.
- Equilibration time at RHIC is estimated

$$\tau_{eq} \simeq \frac{5}{T} + \tau_{delay} \simeq 3\,\text{fm}/c \quad \text{at} \quad T = 350\,\text{MeV}$$

  - τ_{eq} from CYM is not short enough, but non-negligible.
  - Initial cond.=random mag., No expansion, No quark, No quantum effects.