核物質の状態方程式とコンパクト天体現象

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- Introduction
- Neutron Stars, Supernovae, and Black Hole formation
- **Equation of State for Dense Baryonic Matter**
- Relativistic Mean Field based EOS
- Recent development in dense matter EOS and QCD phase diagram
- \bullet Three-body force in RMF and 1.97 M_{\odot} neutron star
- Critical point sweep during black hole formation
- Summary





核物質の状態方程式とコンパクト天体現象

Abstract:核物質の状態方程式は、原子核のバルクな性質を説明する基本的 な概念であるとともに、コンパクト天体現象を記述する上で欠かせないもの である。例えば、高密度での斥力がなければ精度よく測定されている中性子 星の質量(1.44Msun)を支えることは不可能である。コアの重力崩壊から始ま る爆発天体現象においても状態方程式は重要である。近年の多次元シミュ レーションでは超新星爆発を起こすことに成功するという大きな成果が報告 されているが、こうした計算で用いられている状態方程式は一般にソフトな ものであり、最近発見された重い中性子星(1.97Msun)とのconsistencyが問わ れる。

このセミナーでは、これまでに超新星爆発シミュレーションで用いられて いる核物質の状態方程式と高密度物質におけるエキゾチックな自由度(主とし てハイペロン)について概観したのち、ブラックホール形成時のQCD臨界点探 索、相対論的平均場理論(RMF)における3体力等、講演者らの最近の研究につ いて報告する。

石塚さんからのメール: 鈴木先生からのリクエストでセミナーの内容は天体核(原子核?)&宇宙物理 の境界領域のバックグラウンド的なことをざっくり話しつつ(若い人向け ない人向け 原子核物理と宇宙物理

Nuclear physicists thinks that

"Astrophysics is a laboratory of nuclear physics".

Astrophysicists thinks that

"Nuclear physics gives us inputs of Astrophysics".

(~ Kajino's closing at OMEG.)

Both of them are true, and hopefully we can keep win-win relation.



Macrophysics Environment (Τ, ρ, Element abundance,)







Ohnishi @ TUS seminar, Dec.9, 2011, Tokyo Univ. of Science, Noda, Japan

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┛中性子星

- •Cold -dense (~ 5 ρ_0) matter (static, v-less)
- •Many new forms of matter have been proposed !

■超新星

- •Warm (T ~ 20 MeV), dense (~ 1.6 ρ_0), dynamical, non-eq. v
- Important site of nucleosynthesis
- ■ブラックホール形成過程
- •Hot (T ~ 90 MeV), dense (~ 5 ρ 0), dynamical, non-eq. v
- •QCD critical point may be reached

■BH-RH_NS_RH_NS_NS 融合____数值相対論 Nuclear matter at various densities and temperatures is realized in nature !





■コア領域では様々な可能性

●ストレンジクォークを含むバリオン(ハイペロン)を含む物質



重い中性子星ショック...

■2010年のビッグニュース

「1.97 ± 0.04 M_o の質量をもつ中性子星が発見された」 Demorest et al., Nature 467 (2010) 1081 (Oct.28, 2010).

■一般相対論に基づく観測 「パルサー(中性子星)からくる光が伴星(白色矮星)の近くを通り、 時間が遅れる(Shapiro delay)。」

論文での主張 (1.97 ± 0.04) M_☉の中性子星は、 ハイペロン、中間子凝縮を含む 状態方程式では支えられない。 クォーク物質でも強い相互作用が



signature. We calculate the pulsar mass to be $(1.97 \pm 0.04)M_{\odot}$, which rules out almost all currently proposed²⁻⁵ hyperon or boson condensate equations of state (M_{\odot} , solar mass). Quark matter can support a star this massive only if the quarks are strongly interacting and are therefore not 'free' quarks¹².



Lorentz Festival 2011 (6/21, Ohnishi)



$1.97 \pm 0.04 M_{\odot}$ Neutron Star

Demorest et al., Nature 467 (2010) 1081 (Oct.28, 2010).



Neutron Star Radius

■中性子星の質量・半径同時測定

■TOV方程式を使うと M(質量)-R(半径)関係式とEOS は1対1対応

M, Rが同時に決まると、EOS に非常に強い制限
 [観測された(M, R) の"点"を通る必要がある!]

●X線バースト観測 → 半径(+質量)の情報



Ozel, Baym & Guver, PRD82('10)101301 [arXiv: 1002.3153]

Steiner, Lattimer, Brown, ApJ 722 (2010) 33 [arXiv:1005.0811]



Neutron Star Cooling

■表面温度測定と冷却曲線

 Cas A の正確な温度測定と冷却率の測定 Heinke, Ho, ApJ 719('10) L167 [arXiv:1007.4719] Page et al., PRL 106 ('11) 081101 [arXiv:1011.6142]

oneutron pair Obreaking & formation

■核物理への宿題:5ρ₀程度までのギャップを 正確に測定・計算できるか?



YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

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Takatsuka

5678

0.5

Numerical Simulation of Supernova Explosion

- v radiation hydrodynamics
- Baryons, Electons, Photons (Hydro)
- •+ neutrinos (Boltzmann)
- ●1-dim. (Spherical Sym.)
 → Exact v transport leads
 to failed supernova explosion failure.
 (Sumiyoshi et al., 2005)
- a2-dim. Hydrodynamics → merginal (Janka et al., 2002)



R. Buras, M. Rampp, H.-Th. Janka, K. Kifonidis, PRL90(03)241101



K. Sumiyoshi, S. Yamada, H. Suzuki, H. Shen, S. Chiba, H. Toki, ApJ629(05)922



Numerical Simulation of Supernova Explosion

Recent developments (approximate v transport)

- •Light progenitor (8-10 Msun, 1D)
- \rightarrow Successful explosion with simultaneous calc. of nucleosynthesis
- Heavy progenitor (15 Msun, 2D)
- → Standing accretion shock instability (SASI) causes late expl.





Marek, Janka, 2008

Kitaura, Janka, Hillebrandt, 2006



Numerical Simulation of Supernova Explosion

 More recent example Soft EOS + 2D + rotation
 leads stronger explosion
 (~ 10⁵⁰ erg ~ 10 % of observed E.)
 Y.Suwa, K.Kotake, T.Takiwaki, S.C.
 Whitehouse, M. Liebendoerfer, K.Sato,
 Publ.Astron.Soc.Jap. 62 (2010) L49.







Dynamical Black Hole Formation



УИСИЧАН ИНТИТИТЕ FOR ТИКОНАТИКА РИТИКА

Black Hole Formation (Failed Supernova)

""Hot" and "Dense" matter in BH formation process !

•T_{max} ~ 90 MeV (Nucleon), 70 MeV (w/ Hyperon) • $\rho_{Bmax} \sim 4 \rho_0$

v spectrum is sensitive to dense matter EOS (stiffness, hyperon, quark, ...)



Sumiyoshi, Ishizuka, AO, Yamada, Suzuki, 2009



Black Hole Formation (Failed Supernova)





AO, Ueda, Nakano, Ruggieri, Sumiyoshi, PLB 704 ('11),284 [arXiv:1102.3753]

Nakazato, Furusawa, Sumiyoshi, AO, Yamada, Suzuki, ApJ, to appear K.Nakazato, K.Sumiyoshi, S. Yamada, PRD77 (2008) 103006



Short Summary of Compact Astrophysical Phenomena

Neutron Stars

 $\mu_{\rm B}$ (center) ~ 1650 MeV (TM1, M_{max}=2.17 M_o)

 \rightarrow Much larger than Λ mass and hyperons are expected to admix.

•Challenge: How to find the mechanism to suppress hyperons OR stiffen to support 1.97 M_{\odot} with hyperons (very stiff quark matter ?)

Supernovae

•Some 2D v radiation hydrodynamics results show explosions. (Explosion energy is too small / soft (K ~ 180 MeV) EOS is used.)

•No consensus on the explosion mechanism.

Black hole formation

•Hot and dense nuclear matter is formed.

•v sp In all of these phenomena, nuclear matter EOS from low to high density is necessary !







コンパクト天体現象に用いる核物質状態方程式

- ■超新星爆発計算=v輸送を取り入れた流体模型
- ●時間スケール~数100 msec = v 以外は熱・化学平衡
- ・状態方程式:核子、電子、光子、原子核、ハイペロン、π,K,クォーク、…
- •輸送方程式(Boltzmann): v-A 断面積、e-捕獲率
- ■状態方程式 → 有限温度効果、広い密度・Yp範囲、公開
- 第一原理計算 (LQCD, GFMC, Variational, DBHF, G-matrix)
 → 飽和性の説明には現象論的3体力などが必要
- •Lattimer-Swesty (LS) EOS (J. M. Lattimer, F.D. Swesty, NPA535('91)331)
- →一様物質 → スキルム力(密度依存ゼロレンジカ)での平均場
- ◆非一様効果 → 圧縮性液滴
- Relativistic EOS (Shen EOS)
 - (H. Shen, H. Toki, K. Oyamatsu, K. Sumiyoshi, NPA637(1008)435)
- →一様物質 → Relativistic Mean Field (RMF, TM1)

非正義果 - Ohnishi @ Thomais - Ferensi 近似 + 排除体積効果を取り入れたa

コンパクト天体現象に用いる核物質状態方程式

■超新星爆発計算 = v 輸送を取り入れた流体模型 ●時間スケール~数100 msec = v 以外は熱・化学平衡 ・状態方程式:核子、電子、光子、原子核、ハイペロン、π,K,クォーク、… •輸送方程式(Boltzmann): v-A 断面積、e-捕獲率 ■状態方程式 → 有限温度効果、広い密度・Yp範囲、公開 ◎第一原理計算 (LQCD, GFMC, Variational, DBHF, G-matrix) $L_{ii}^2, \sigma_i \cdot \sigma_j L_{ii}^2$ and $(\mathbf{L} \cdot \mathbf{S})_{ij}^2$. The UIX model of V_{ijk} contains two static terms; the two-pion exchange Fujita-Miyazawa in-<u>▶</u>—₩ teraction, $V_{iik}^{2\pi}$, and a phenomenological, intermediate range ◆非repulsion V_{iik}^R . The strength of the $V_{iik}^{2\pi}$ interaction was determined by reproducing the binding energy of the triton via Rel Green's-function Monte Carlo (GFMC) calculations [20], while that of V_{iik}^{R} was adjusted to reproduce the saturation density of SNM **APR** paper A HOMMANT CENT LEPA TATTER PARTY AND CAR AND CON

Key quantities in Nuclear Matter EOS



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by Murakami

Notations for symmetry energy

$$\begin{split} E(\rho,\delta) &= E(\rho,0) + E_{sym}(\rho)\delta^{2} + o(\delta^{4}) \\ E(\rho,0) &= E(\rho_{0},0) + \frac{K_{0}}{2}\varepsilon^{2} + o(\varepsilon^{3}) \\ E_{sym}(\rho) &= E_{sym}(\rho_{0}) + L\varepsilon + \frac{K_{sym}}{2}\varepsilon^{2} + o(\varepsilon^{3}) \\ K_{0} &= 9\rho_{0}^{2}\frac{\partial^{2}E(\rho,0)}{\partial\rho^{2}}\Big|_{\rho=\rho_{0}} \\ \delta &= (\rho_{n} - \rho_{p})/\rho \\ \varepsilon &= (\rho - \rho_{0})/3\rho_{0} \\ S_{0} &= E_{sym}(\rho_{0}) \\ L &= 3\rho_{0}\frac{\partial E_{sym}(\rho)}{\partial\rho}\Big|_{\rho=\rho_{0}} = (3/\rho_{0})P_{0} \\ K_{sym} &= 9\rho_{0}^{2}\frac{\partial^{2}E_{sym}(\rho)}{\partial\rho^{2}}\Big|_{\rho=\rho_{0}} \\ K_{\tau} &\approx K_{sym} - 6L \end{split}$$



DBHF and **Dirac** Phenomenology

Dirac Bruckner-Hartree-Fock *R. Brockmann, R. Machleidt, PRC42('90),1965*

•Non Rel. Brueckner calculations do not reproduce saturation point (Coester Line).

■Relativity gives additional repulsion, → Saturation point !

Dirac phenomenology *E.D. Cooper, S. Hama, B.C. Clark, R.L. Mercer, PRC47('93),297*

 Scalar + Vector pA potenti (-400 MeV + 350 MeV)
 → Cross Section, &Spin Observables







σω Model

Serot, Walecka, Adv.Nucl.Phys.16 (1986),1

aConsider only σ and ω mesons

Lagrangian

$$L = \bar{\psi} (i\gamma^{\mu} \partial_{\mu} - M + g_{s}\sigma - g_{\nu}\gamma^{\mu}\omega_{\mu})\psi + \frac{1}{2} \partial_{\mu}\sigma \partial^{\mu}\sigma - \frac{1}{2}m_{s}^{2}\sigma^{2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_{\nu}^{2}\omega_{\mu}\omega^{\mu} (F_{\mu\nu} = \partial_{\mu}\omega_{\nu} - \partial_{\nu}\omega_{\mu})$$

σ, ω

Equation of Motion

$$\frac{\partial}{\partial x^{\mu}} \left[\frac{\partial L}{\partial (\partial_{\mu} \varphi_i)} \right] - \frac{\partial L}{\partial \varphi_i} = 0$$

Euler-Lagrange Equation

 $\sigma: \left[\partial_{\mu} \partial^{\mu} + m_{s}^{2}\right] \sigma = g_{s} \psi \psi$

$$\omega: \partial_{\mu} F^{\mu\nu} + m_{\nu}^{2} \omega^{\nu} = g_{\nu} \psi \gamma^{\nu} \psi \rightarrow \left[\partial_{\mu} \partial^{\mu} + m_{\nu}^{2} \right] \omega^{\nu} = g_{\nu} \psi \gamma^{\nu} \psi$$
$$\psi: \left[\gamma^{\mu} \bigotimes i \partial_{\mu} - g_{\nu} V_{\mu} \bigotimes - (M - g_{s} \sigma) \right] \psi = 0$$



EOM of ω (for beginners)

Euler-Lagrange Eq.

 $\partial_{\mu}F^{\mu\nu} + m_{\nu}^{2}\omega^{\nu} = g_{\nu}\psi\gamma^{\nu}\psi$

Divergence of LHS and RHS

$$\partial_{\nu}\partial_{\mu}F^{\mu\nu} + m_{\nu}^{2}(\partial_{\nu}\omega^{\nu}) = m_{\nu}^{2}(\partial_{\nu}\omega^{\nu}) = g_{\nu}\left(\partial_{\nu}\bar{\psi}\gamma^{\nu}\psi\right) = 0$$

LHS: derivatives are sym. and $F_{\mu\nu}$ is anti-sym. RHS: Baryon Current = Conserved Current

Put it in the Euler-Lagrange Eq.

$$\partial_{\mu}F^{\mu\nu} = \partial_{\mu}(\partial^{\mu}\omega^{\nu} - \partial^{\nu}\omega^{\mu}) = \partial_{\mu}\partial^{\mu}\omega^{\nu} - \partial^{\nu}(\partial_{\mu}\omega^{\mu}) = \partial_{\mu}\partial^{\mu}\omega^{\nu}$$







Nuclear Matter in σω Model

Serot, Walecka, Adv.Nucl.Phys.16 (1986),1

Uniform Nuclear Matter

$$E/V = \gamma_N \int^{P_F} \frac{d^3 p}{(2\pi)^2} E^* + \frac{1}{2} m_s^2 \sigma^2 - \frac{1}{2} m_v^2 \omega^2 + g_v \rho_B \omega$$

$$\sigma = \frac{g_s}{m_s^2} \rho_s = \gamma_N \frac{g_s}{m_s^2} \int^{P_F} \frac{d^3 p}{(2\pi)^2} \frac{M^*}{E^*} \qquad \left(M^* = M + U_s = M - g_s \sigma, E^* = \sqrt{p^2 + M^{*2}} \right)$$
$$\omega = \frac{g_v}{m^2} \rho_B = \gamma_N \frac{g_v}{m^2} \int^{P_F} \frac{d^3 p}{(2\pi)^3} \qquad 20$$

$$\gamma_{N} = \text{Nucleon degeneracy}$$

$$(=4 \text{ in sym. nuclear matter})$$

$$Problem: EOS \text{ is too stiff} \\ K \sim (500-600) \text{ MeV }! \\ \rightarrow How \text{ can we solve }?$$



$\sigma \omega$ model --- pros and cons

Pros (merit)

- **•**Foundation is clear: based on the success of Dirac phen. and DBHF.
- •Simple description of scalar and vector potential in σ and ω mesons. •Saturation is well described in two parameters.
- Natural explanation of large LS potential in nuclei.
- Cons (shortcomings)
- **•**Relation with the bare NN interaction is not clear.
- Specially, pion effects are not included.
- •Symmetry energy is too small.
- Incompressibility is too large (K ~ 600-700 MeV) (c.f. Empirical value K ~ (200-300) MeV)
- Chiral symmetry is not respected.



High Quality RMF models

Variety of the RMF models

 \rightarrow MB couplings, meson masses, meson self-energies

• σN , ωN , ρN couplings are well determined \rightarrow almost no model deps. in Sym. N.M. at low ρ

ω⁴ term is introduced to simulate
DBHF results of vector pot. *TM: Y. Sugahara, H. Toki, NPA579('94)557; R. Brockmann, H. Toki, PRL68('92)3408.*

• σ^3 and σ^4 terms are introduced to soften EOS at ρ_0 . *J. Boguta, A.R.Bodmer NPA292('77)413,*

J. Boguta, A.R.Boamer NPA292('//)413, NL1:P.-G.Reinhardt, M.Rufa, J.Maruhn, W.Greiner, J.Friedrich, ZPA323('86)13. NL3: G.A.Lalazissis, J.Konig, P.Ring, PRC55('97)540.

■→ Large differences are found at high ρ <u>K. Tsubakihara, H. M</u>

ρ_B (fm⁻³) K. Tsubakihara, H. Maekawa, H. Matsumiya, AO,PRC81('10)065206.

0

0.1

60

40

20

0

-20

E/A (MeV)



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0.3

0.4

0.5

0.6

How to determine higher order terms ?

Nucleon-meson coupling can be well determined from data !

Higher order terms are not well determined, and give EOS uncertainties at high density !

We need some guiding principle to obtain hadronic Lagrangian including higher order (higher mass dimensional) coupling. c.f. Naive dim. analysis,

Chiral effecitve field, Quark Meson Coupling



(Miyatsu, Saite), E.I. RMF parameters. In SCL, g_3 and g_4 are from the expansion of f_{SCL} .

	$g_{\sigma N}$	$g_{\omega N}$	$g_{\rho N}$	$g_3(MeV)$	g_4	c_ω	$m_{\sigma}(\text{MeV})$	$m_{\omega}(\text{MeV})$	$m_{\rho}(\text{MeV})$
NL1[26]	10.138	13.285	4.976	2401.9	-36.265	0	492.25	795.359	763
NL3[27]	10.217	12.868	4.474	2058.35	-28.885	0	508.194	782.501	763
TM1[28]	10.0289	12.6139	4.6322	1426.466	0.6183	71.3075	511.198	783	770
SCL[29]	10.08	13.02	4.40	1255.88	13.504	200	502.63	783	770





■構成粒子

- •超流動核子、π、K、ハイペロン、クォーク、クォーク対、…
- ■非圧縮率(K):決まっていない
- ●GMR (原子核の圧縮振動) → K = 210 ± 30 MeV (非相対論的平均場)
- ●重イオン反応→ 平均場の運動量依存性がK依存性を隠す
 (Sahu, Cassing, Mosel, AO, 2000; Danielewicz, et al., 2002; Isse et al., 2005)
- ■対称エネルギーの密度依存性:分りつつある。
- ●不安定核半径の精密測定で推定可能 (Oyamatsu, Iida, 2007)
- 。実験からの制限 (Murakami, 2011)
- ■核物質中のハドロン・ポテンシャル:進んでいる

•ハイペロン $U_{\Lambda}(\rho_0) = -30$ MeV, $U_{\Sigma}(\rho_0) = +(15-90)$ MeV, $U_{\Xi}(\rho_0) \sim -15$ MeV

■pionic atom → $U_{\pi}(2\rho_0, Y_p \sim 0.2) > +50$ MeV (AO, Jido, Sekihara, Tshubakihara)



Our consensus is S₀=31-34 MeV and L=50-110 MeV

Now preparing a summary article on outcomes of NuSYM11.

Murakami

核物質中でのハイペロン・ポテンシャル

■1粒子ポテンシャル $U_Y(r) \simeq g_{\sigma Y} \sigma + g_{\omega Y} \omega + g_{\rho Y} R$

 •核物質中では σ, ω, R (=ρ中間子期待値)は、与えられている
 → ハイペロン・中間子結合定数によりハイペロン・ポテンシャルが決まる。


S Potential in Nuclear Matter



Relativistic EOS of Supernova Matter with Hyperons

Extention of the Relativistic (Shen) EOS to SU_f(3) with updated Hyperon Potentials in Nuclear Matter (Ishizuka, AO, Tsubakihara, Sumiyoshi, Yamada, J. Phys. G35(08),085201)

Relativistic (Shen) EOS (Shen, Toki, Oyamatsu, Sumiyoshi, PTP 100('98), 1013) Rel. Mean Field (RMF) + Local Density Approx.(Nuclear Formation)

•SU_f(3) Extention of RMF (Schaffner, Mishustin, PRC53 (1996), 1416) $\mathscr{L} = \mathscr{L}_{Free}(B, \sigma, \omega_{\mu}, \vec{R}_{\mu}, \zeta, \phi_{\mu}) - U_{\sigma}(\sigma) + \frac{1}{4}c_{\omega}(\omega^{\mu}\omega_{\mu})^{2}$ $-\sum_{B} \bar{\psi}_{B} \left(g_{\sigma B}\sigma + g_{\omega B}\phi + g_{\rho B}\vec{R} \cdot \vec{t}_{B} + g_{\zeta B}\zeta - g_{\phi B}\gamma^{\mu}\phi_{\mu} \right) \psi_{B}$

Coupling ~ Quark Number Counting

• $g_{\sigma Y}$ is tuned to fit Hyperon Potential in Nuclear Matter $U_A = -30 \text{ MeV}, U_{\Sigma} = +30 \text{ MeV}, U_{\Xi} = -15 \text{ MeV}$ • Nuclear Formation is included using Shen EOS table



Hyperon Composition in Dense Matter

- **Hyperon start to emerge at (2-3)** ρ_0 in Neutron Star Matter !
- **Hyperon composition in NS is sensitive to Hyperon potential.**
- • $U_{\Lambda} \sim -30$ MeV: Well-known
- •U_{Σ}, U_{Ξ} ~ -30 MeV (Old conjecture) $\rightarrow \Sigma$ - appears prior to Λ
- • $U_{\Sigma} > 0$ (repulsive) \rightarrow No Σ in NS Σ atom (phen. fit), QF prod. S. Balberg, A. Gal, NPA625('97)435; H. Noumi et al., PRL89('02)072301; T. Harada, Y. Hirabayashi, NPA759('05)143; M. Kohno et al. PRC74('06)064613.

•U₂ ~ -(12-15) MeV (K⁻,K⁺) reaction, twin hypernuclei *P. Khaustov et al. (E885),PRC61('00)054603; S. Aoki et al., PLB355('95)45.*



J. Schaffner-Bielich, NPA804('08)309.





Neutron Star



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Finite Temperature and Supernova





Where do we see Hyperons?

Hyperon Fraction is sensitive to Ye, T, and \rho_{\rm B}.

- •Yv ~ 0 (Neutron Star) $\rightarrow \rho_B > 2 \rho_0$
- •Ye ~ 0.4 (Supernova, early stage) \rightarrow T > 40 MeV or ρ_B > 3 ρ_0

Hyperons would be important in Late Stage(Nstar cooling), BH formation, and Heavy-Ion Collisions





Black Hole Formation (Failed Supernova)

aCan we obtain information on Σ repulsion strength from astrophysics ?

•v duration time during black hole formation is more sensitive to the Σ potential depth in nuclear matter !

Nakazato, Furusawa, Sumiyoshi, AO, Yamada, Suzuki, ApJ, to appear





RMF is a phenomenological MODEL !

- Baryon one-loop approximation (Hartree approximation) makes RMF a phenomenological model.
- \rightarrow We need DATA and AB INITIO results.
- •Saturation point (ρ_0 and E/A(ρ_0)) from mass formula
- Ruclear binding energies
- \mathcal{O}_{v} and U_{s} from DBHF results
- $P(\rho_B)$ from heavy-ion data
- Λ separation energy from single Λ hypernuclear data
- KA bond energy from double A hypernuclear data
- •Σ atomic shift
- Σ and Ξ potential depth from quasi-free production data
- •Pyre neutron matter EOS from ab initio calculations (not used here)

•Neutron Star Max. Mass ~ 1.4 *The Judgement Day, Oct. 28, 2010.*







$1.97 \pm 0.04 M_{\odot}$ Neutron Star

Demorest et al., Nature 467 (2010) 1081 (Oct.28, 2010).



Which type of EOSs are rejected ?





Glendenning & Moszkowski (1991)

RMF with hyperons

on, p, Y, σ , ω , ρ / σ^3 , σ^4

• Give $x_{\sigma} = g_{\sigma Y}/g_{\sigma N}$ and fix $x_{\omega} = g_{\omega Y}/g_{\omega N}$ to fit Λ separation energy.

• $x_{\sigma} = 0.6 \rightarrow m^*/m = 0.7, x_{\omega} = 0.653$ (similar to quark number counting result, x=2/3)

__ \ **\/** _ 1 6 M 2.5 Proper number density (fm⁻³) 2 . n+p0.1 n+p+F M_{max} / M_{sun} M / M_{sun} (0.7, 300)(0.78, 300)0.01 n+p+H, g_H=0 0.5 (0.78, 240)0.00 0 0 2 3 4 5 6 7 8 9 10 11 12 15.5 14.5 15 16 14 r (km) 0.3 0.4 0.5 0.6 0.7 0.8 0.9 $\log (\epsilon_c g/cm^3)$ \mathbf{x}_{σ} N.K.Glendenning, S.A.Moszkowski, PRL67('91)2414

TABLE I. Values of the hyperon-to-nucleon scalar and vector coupling that are compatible with the binding of -28 MeV for Λ hyperons in nuclear matter for two values of the nucleon (Dirac) effective mass at saturation density.

	X	
xσ	$m^{+}/m = 0.7$	m*/m=0.78
0.2	0.131	0.091
0.3	0.261	0.233
0.4	0.392	0.375
0.5	0.522	0.517
0.6	0.653	0.568
0.7	0.783	0.800
0.8	0.913	0.942
0.9	1.04	1.08
1	1.17	1.23

У ТР У РС У ИКАЧА И КЛІТИТЕ ГОР ТИБОЛЕГІСИ. РИТЭКЗ

Appendix 48

How can we solve it ?

No Hyperons, No Kaons

 \rightarrow How can it be consistent with YN interaction ?

Stiff nuclear matter EOS + transition to quark matter at small $\rho_B \rightarrow$ How can it be consistent with HIC data at AGS-SPS energies ?

Three-body force for baryons, quarks, ...







SU(3)_f "violating" coupling

Naïve RMF assumption = BM coupling follows SU(3)_f.

Short range BB interaction comes from quark Pauli blocking + one-gluon exch. *Oka, Yazaki; Faessler et al.; Fujiwara et al.; HAL QCD collab.*

Short-range BB repulsion is sensitive to (S,T) in the s-channel. When we include those interactions in the 8 bosonized form, BM coupling violates SU(3)_f.

$$V = \sum_{\alpha,\beta} (\bar{\psi}\bar{\psi})_{\alpha} \Gamma_{\alpha\beta}(\psi\psi)_{\beta} \rightarrow -\frac{1}{2} \sum_{\alpha} m_{\alpha}^{2} \omega_{\alpha}^{2} + \sum_{\alpha} g_{\alpha} \omega_{\alpha}(\psi\Gamma\psi)_{\alpha}$$

E.g., Σ atomic shift $\rightarrow g_{\sigma\Sigma} \sim g_{\sigma\Sigma} (SU(3)) \times (0.2-0.3)$



Tsubakihara et al., (2010)

8+1

8



RMF with 3BF

Tsubakihara, AO, in prep.







Appendix 52

RMF with $3BF + SU(3)_{f}$ "violation"

Two types of modification

Tsubakihara, AO, in prep.

•3-baryon repulsion \rightarrow EOS becomes stiff gradually at high density. (Fitting meson mass (E325) and Uv in RBHF)

•R=
$$g_{\omega\Lambda}$$
 / $g_{\omega N}$ ~ 0.8 (~ 2/3 (SU(3))

 $\blacksquare \rightarrow M_{max} \sim 2.02~M_{\odot}$ with hyperons (~ 1.4 M_{\odot} w/o 3BF, violation)





Ohnishi @ DCEN2011, *Sep.20-Oct.28*, *2011*, *YITP*, *Kyoto*, *Japan* 53

Critical Point Sweep during Black Hole Formation



From Supernova Matter EOS to Phase Diagram

Supernova matter EOS

•Lattimer-Swesty EOS (Skyrme-type int. + Droplet) J.M.Lattimer, F.D.Swesty, NPA535('91)331.

•Shen EOS (Relativistic Mean Field + Thomas Fermi) *H.Shen et al.*, *NPA637('98)435;PTP100('98)1013*.

Ishizuka EOS (Shen EOS + Hyperons)
C. Ishizuka, AO, K.Tsubakihara, K.Sumiyoshi, S.Yamada, JPG 35 ('08)085201.

Does quark matter exist in compact stars ?

•Suggested in Supernovae: Warm(~20 MeV), mildely dense (~1.8 ρ₀) *T. Hatsuda, MPLA2('87)805; I. Sagert et al., PRL102 ('09) 081101; Nishimura talk.*

•Probable in Neutron Stars: Cold (T~0), Dense (ρ_B~5 ρ₀) E.g. N. Glendenning, "Compact Stars"; F. Weber, Prog.Part.Nucl.Phys.54('05)193

•How about Black hole formation ?

M. Liebendorfer et al., ApJS 150('04)263; K. Sumiyoshi et al., PRL97('06) 091101; K.Sumiyoshi, C.Ishizuka, AO, S.Yamada, H.Suzuki, ApJL690('09),L43;

K.Nakazato et al., ApJ, to appear [arXiv:1111.2900] (Nakazato, Poster)



Purpose and Methods

We compare (T, μ_B) during BH formation and QCD phase transition boundary by using

•v radiation Hydrodynamics (1D) for BH formation *Sumiyoshi et al.*, *PRL97('06)091101*;

•Shen EOS (npeµ) Shen et al., NPA637('98)435;PTP100('98)1013

•Grav. collapse of 40 M_{sun} star with WW95 initial condition. S.E.Woosley, T.A.Weaver, ApJS 101 ('95) 181.

•Chiral Effective Models for phase boundary and Critical Point

NJL (Nambu, Jona-Lasinio), PNJL (Polyakov loop extended NJL), PNJL with 8 quark int., PQM (Pol. loop ext. quark meson) models Nambu, Jona-Lasinio('61); Hatsuda, Kunihiro('94), Fukushima('04); Ratti, Thaler, Weise('06); Roessner et al.('07); Kashiwa, Kouno, Matsuzaki, Yahiro('08), Schaefer, Pawlowski, Wambach ('07), Skokov et al. ('10).

•Vector coupling: unknown \rightarrow compare results with $G_v/G_s=0, 0.2$

Flavor SU(2) models are considered.

The second Reference of Science, Noda, Japan 56



QCD phase diagram in Asymmetric Matter

- Characteristic features of Compact Star Matter
- Hot and/or Dense
- •Unbalanced n and p yields (Isospin Asymmetric Matter)

Isospin chemical potential $\delta \mu = (\mu_n - \mu_p)/2 = (\mu_d - \mu_u)/2 > 0$

T_{CP} (critical point T) decreases at finite $\delta\mu$

Decrease of effective number of flavors



How is quark matter formed during BH formation ?

■Highest $\mu_B \sim 1300 \text{ MeV} > \mu_c$ (1000-1100 MeV in eff. models) → Quark matter is formed before BH formation

■Highest T ~ 90 MeV > T_{CP} (at $\delta\mu$ ~50 MeV) Core evolves below CP, Off-center goes above CP → *CP sweep*

Convenient to consider 3D phase diagram (T, \mu_{\rm B}, \delta\mu)



How is quark matter formed during BH formation ?

Model dependence to form quark matter \rightarrow Three ways





Swept Region of Phase Diagram during BH formation

- CP location in Symmetric Matter
- •Lattice QCD μ_{CP}=(400-900) MeV
- •Effecitve models µ_{CP}=(700-1050) MeV
- **CP in Asymmetric Matter** (E.g. δμ=50 MeV)
- •T_{CP} decreases at finite $\delta\mu$. \rightarrow Accessible (T, μ_B) region during BH formation



M.A.Stephanov, Prog.Theor.Phys.Suppl.153 ('04)139; FK02:Z. Fodor, S.D.Katz, JHEP 0203 (2002) 014 LTE:S. Ejiri et al., Prog.Theor.Phys.Suppl. 153 (2004) 118; Can: S. Ejiri, PRD78 (2008) 074507 Stat.:A. Andronic et al., NPA 772('06)167



What happens at CP sweep ?

aLarge density fluctuation is expected around CP.

Three layers (hadron, mixed, quark) merges to be one at a time.



Summary

- **Compact Astrophysical Phenomena**
- Possibilities of varous state of matter, New observations, New calculations
- Dense baryonic matter Equation of State from Relativistic Mean Field
- Successfully applied to Compact Astrophysical Phenomena.
- •Needs further studies: Higher order terms, pion contribution, Esym, Hyperons,
- Standard RMF with hyperons cannot support 1.97 M_{\odot} neutron star.
- •Various data / DBHF results can be fitted in RMF.
- •Vector Coupling ~ $SU(3)_f$, linear BM coupling (BMB)
- •RMF with 3BF + SU(3)_f "violation" may help to support the heavy NS. \rightarrow Can we support 1.97 M_o neutron star with hyperons ? Open problem.
- **Quark matter formation and critical point sweep may take place in neutron stars and black hole formation.**



Thank you for your attention !



Ohnishi @ TUS seminar, Dec.9, 2011, Tokyo Univ. of Science, Noda, Japan

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■なぜ面白い?

- ●高温・高密度物質が「平衡状態」で作られている → 相転移現象がみえる?
- ■物質の構成要素は? → 原子核・ハドロン・クォークの3階層の関連

■問題点

- ◎高密度側
- ◆非圧縮率、対称エネルギーの密度依存性→ Unknown
- →核物質中のハドロン・ポテンシャル
- ◆クォーク物質への相転移の性質は?

●低密度側

◆物質の構成粒子は? Fermi gas? Nuclei? Pasta?

↓「原子核熱的分布」と「一様物質」をスムースにつないだEOS はない。

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Theories/Models for Nuclear Matter EOS

- ■Ab initio Approaches to Nuclear Matter → LQCD, Variatonal, GFMC, BHF(G-Matrix), DBHF, ...
- ■LQCD-MC: Not (yet) applicable to cold dense matter, A ≤ 4 SC-LQCD: Nuclear matter does not bound
- **•**Variatioal, BHF: Need phen. 3-body repulsion to reproduce saturation point.
- **•GFMC:** Limited to be $A \le 12$.
- **•DBHF:** Good, but E/A is not enough. Not yet extensively investigated.
- ■→ Not easy to handle, Not yet satisfactory for phen. purposes
- Mean Field Models (~ Nuclear Density Fuctional approach)
- Skyrme Hartree-Fock(-Bogoliubov)
- •Nuclear Mass is very well explained (HFB, Total B.E. $\Delta E \sim 0.6$ MeV)
- Causality is violated at very high densities.
- Relativistic Mean Field

Relativistic, Meson-Baryon coupling, Meson self-energies Y TP Ohnishi @ TUS seminar, Dec. 9, 2011, Tokyo Univ. of Science, Noda, Japan

EOS in Dirac-Brueckner-Hartree-Fock

R. Brockmann, R. Machleidt, PRC42('90),1965

Non Relativistic Brueckner Calculation

→ Nuclear Saturation Point cannot be reproduced (Coester Line)

Relativistic Approach (DBHF)

→ Relativity gives additional repulsion, leading to successful description of the saturation point.





Bruckner-Hartree-Fock

Self-consistent treatment of

Effective interaction (G-matrix) in the Bruckner Theory

o— PAR: Paris

- ● — A: Bonn A - ■ — B: Bonn B - ● — C: Bonn C

• O — CD: CD-Bonn

A- R93: Reid93

40

30

– □— V14: Argonne V14 – ☆— V18: Argonne V18

- and Single particle energy from G-matrix
- •Need 3-body force to reproduce saturation point.
- $\rightarrow FY type 2 \pi exchange$ + phen. or Z-diagram



Z.H.Li, U. Lombardo, H.-J. Schulze, W. Zuo, L. W. Chen, H. R. Ma, PRC74('06)047304.



Bruckner-Hartree-Fock theory with Hyperons

Microscopic G-matrix calculation with realistic NN, YN potential and microscopic (or phen.) 3N force (or 3B force).

•Interaction dep. (V18, N93, ...) is large \rightarrow Need finite nuclear info. *E.Hiyama, T.Motoba, Y.Yamamoto, M.Kamimura / M.Tamura et al.*

•NS collapses with hyperons w/o 3BF.



H.J.Schulze, A.Polls, A.Ramos, I.Vidana, PRC73('06),058801.

S. Nishizaki, T. Takatsuka, Y. Yamamoto, PTP108('02)703.



QCD Phase Diagram





Neutron Star Composition





Hyperons in Dense Matter

Hyperons are HOT now !

•What makes NS matter core ? Nucleons ? Quarks ? Hyperons ?

•How can we suppress hyperon appearance in NS ? or How can hyperonic matter be so stiff ? or Which inter-quark interaction supports 1.97 M_{\odot} NS ?

We stick to hyperonic matter (rather than quark matter), and discuss possible mechanism to stiffen the EOS at high d


$1.97 \pm 0.04 M_{\odot}$ Neutron Star

■重い中性子星(2倍の太陽質量)の観測

Demorest et al., Nature 467 (2010) 1081 (Oct.28, 2010).

PSR J1614-2230 (NS-WD binary), 1.97 ± 0.04 Msun

●一般相対性理論に基づく時間の遅れ(Shapiro delay)による質量決定 (模型によらない)

- ●幸運な公転面の向き
- ●美しい観測結果







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K.Nakazato et al., ApJ, to appear [arXiv:1111.2900] (Nakazato, Poster)



Chiral Symmetry

Fundamental symmetry of massless QCD, and its spontaneous breaking generates hadron masses.

Nambu, Jona-Lasinio ('61)

Non-linear representation (chiral pert.) leads to density dependent coupling from pion loops.

Kaiser, Fritsch, Weise ('02), Finelli, Kaiser, Vretener Weise ('04)



 ϵ (MeV/fm³) σ (MeV) σ (MeV)





Dynamical Black Hole Formation





RMF is a phenomenological MODEL !

- Baryon one-loop approximation (Hartree approximation) makes RMF a phenomenological model.
- \rightarrow We need DATA and AB INITIO results.
- •Saturation point (ρ_0 and E/A(ρ_0)) from mass formula
- Nuclear binding energies
- **•**U_v and U_s from DBHF results
- •P(ρ_B) from heavy-ion data
- Λ separation energy from single Λ hypernuclear data
- • $\Lambda\Lambda$ bond energy from double Λ hypernuclear data
- • Σ atomic shift
- Σ and Ξ potential depth from quasi-free production data
- •Pure neutron matter EOS from ab initio calculations (not used here)





RMF models

Variety of the RMF models

 \rightarrow MB couplings, meson masses, meson self-energies

• σN , ωN , ρN couplings are well determined \rightarrow almost no model deps. in Sym. N.M. at low ρ

 ω⁴ term is introduced to simulate DBHF results of vector pot. *TM1&2: Y. Sugahara, H. Toki, NPA579('94)557; R. Brockmann, H. Toki, PRL68('92)3408.*

 $\circ \sigma^3$ and σ^4 terms are introduced to soften EOS at ρ_0 .

J. Boguta, A.R.Bodmer NPA292('77)413, NL1:P.-G.Reinhardt, M.Rufa, J.Maruhn, W.Greiner, J.Friedrich, ZPA323('86)13. NL3: G.A.Lalazissis, J.Konig, P.Ring, PRC55('97)540.

• \rightarrow Large differences are found at high ρ



K. Tsubakihara, H. Maekawa, H. Matsumiya, AO, PRC81('10)065206.



Vector potential in RMF

aVector potential from ω dominates at high density !

$$U_{v}(\rho_{B}) = g_{\omega}\omega \sim \frac{g_{\omega}^{2}}{m_{\omega}^{2}}\rho_{B}$$

Dirac-Bruckner-Hartree-Fock shows suppessed vector potential at high ρ_B.
 R. Brockmann, R. Machleidt, PRC42('90)1965.

•Collective flow in heavy-ion collisions suggests pressure at high ρ_B. *P. Danielewicz, R. Lacey, W. G. Lynch, Science298('02)1592.*

•Self-interaction of $\omega \sim c_{\omega}(\omega_{\mu}\omega^{\mu})^2$

 \rightarrow DBHF results & Heavy-ion data



K. Tsubakihara, H. Maekawa, H. Matsumiya, AO, PRC81('10)065206. ρ_B (fm)

VIEWARIALIMIS

RMF with Hyperons (Single A hypernuclei)

aRMF for A hypernuclei

■x ~ 1/3: *R. Brockmann, W. Weise, PLB69('77)167; J. Boguta and S. Bohrmann, PLB102('81)93. x* ~ 2/3: *N. K. Glendenning, PRC23('81)2757, PLB114('82)392;*

Tensor: Y. Sugahara, H. Toki, PTP92('94)803; H. Shen, F. Yang, H. Toki, PTP115('06)325; J. Mares, B. K. Jennings, PRC49('94)2472.

ρ-dep. coupling: H. Lenske, Lect. Notes Phys. 641('04)147; C. M. Keil, F. Hofmann, H. Lenske, *PRC* 61('00)064309.

SU(3) or SU(6) (ς, φ): J. Schaffner, C. B. Dover, A. Gal, C. Greiner, H. Stoecker, PRL71('93)1328; Schaffner et al., Ann.Phys.235('94)35; J. Schaffner, I. N. Mishustin, PRC 53('96)1416. Chiral SU(3) RMF: K. Tsubakihara, H. Maekawa, H. Matsumiva, AO, PRC81('10)065206.



K. Tsubakihara, H. Maekawa, H. Matsumiya, AO, PRC81('10)065206.

RMF with Hyperons (Double A hypernuclei)

aNagara event $\Delta B_{\Lambda\Lambda} \sim 1.0$ MeV (weakly attractive)

•TM & NL-SH based RMF

H. Shen, F. Yang, H. Toki, PTP115('06)325.

Model 1: $x_{\sigma} = 0.621$, $x_{\omega} = 2/3$ (no ς , φ) Model 2: $R_{\varsigma} = g_{\varsigma\Lambda} / g_{\sigma N} = 0.56 - 0.57$, $R_{\varphi} = g_{\varphi\Lambda} / g_{\omega N} = -\sqrt{2/3}$

•Chiral SU(3) RMF K. Tsubakihara, H. Maekawa, H. Matsumiya, AO, PRC81('10)065206. SU(3)f for vector coupling $x_{\omega}=0.64, R_{\phi}=0.504$ Det. (KMT) int. mixes σ and ς M. Kobayashi, T. Maskawa,

PTP44('70)1422; G. 't Hooft, PRD14('76)3432.

$$\rightarrow$$
 $x_{\sigma}=0.335, R_{\varsigma}=0.509$





Hyperon Composition in Dense Matter

Hyperon start to emerge at (2-3) ρ_0 in Neutron Star Matter !

Hyperon composition in NS is sensitive to Hyperon potential.

 $\bullet U_{\Lambda} \sim -30$ MeV: Well-known

•U₂ ~ -(12-15) MeV (K⁻,K⁺) reaction, twin hypernuclei *P. Khaustov et al. (E885),PRC61('00)054603; S. Aoki et al., PLB355('95)45.*

• U_{Σ} ~ -30 MeV (Old conjecture) $\rightarrow \Sigma$ - appears prior to Λ

• $U_{\Sigma} > 0$ (repulsive) \rightarrow No Σ in NS Σ atom (phen. fit), QF prod. S. Balberg, A. Gal, NPA625('97)435; H. Noumi et al., PRL89('02)072301; T. Harada, Y. Hirabayashi, NPA759('05)143; M. Kohno et al. PRC74('06)064613.



J. Schaffner-Bielich, NPA804('08)309.



Hyperon Composition in Dense Matter

Comparison of Hyperon Composition

•U_Σ=-30 MeV, U_Ξ = -28 MeV → SU(3) sym. matter at $ρ_B \sim 10 ρ_0$ Schaffner, Mishustin ('94)

•U_{Σ}=+30 MeV, U_{Ξ} = -15 MeV $\rightarrow \Sigma$ baryons are strongly suppressed. *C.Ishizuka, AO, K.Tsubakihara, K.Sumiyoshi, S.Yamada, JPG35('08)085201.*



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 Σ atom data

1000

500

100

50

A tomic Shift(eV)

4-3

5->4

Exd.

10 -> 9

a Σ^{-} atom data suggested repulsion in the interior of nuclei !

C.J.Batty, E.Friedman, A.Gal, PLB335('94)273 Batty's DD potential is very repulsive inside nuclei.

 \rightarrow No Σ baryon in dense matter.





S atom in RMF

 V_{opt} of Σ in Si

40

30

20

10

-10

-20

V_{opt}(MeV)

Re. part(SR) Im. part(SR)

Re. part(WR) Im. part(WR)

RMF fit of Si and Pb Σ^{-} atom $\alpha_{\omega} = g_{\omega\Sigma} / g_{\omega N} \sim 2/3(M), 0.69 (T)$ $\alpha_{\rho} = g_{\rho\Sigma} / g_{\rho N} \sim 2/3(M), 0.434(T)$ *J.Mares, E.Friedman, A.Gal, B.K.Jennings, NPA594('95)311; Tsubakihara et al.('10)*

•Much smaller $g_{\rho\Sigma}$ than naïve SU(3) ($g_{\rho\Sigma}$ / $g_{\rho N}$ =2), which has been applied in some of previous works.





Σ atom and Neutron Star

- Σ may not feel *very* repulsive potential in neutron star....
- ${\scriptstyle \bullet} \rho^{\gamma} \text{-type fit} \rightarrow very \ repulsive}$
- **RMF** fit \rightarrow small isovector potential



T. Harada, Y. Hirabayashi, NPA767('06)206





Neutron Star Mass

- **Large fraction of hyperons** softenes EOS at $\rho_{\rm B} > (0.3-0.4)$ fm⁻³
- \odot NS star max. mass red. ~ 1 M_{sun}.
- •RMF generally predicts stiff EOS at high density.
- (Scalar attraction saturation,
 - or Z-graph in NR view.)
- •Some of RMF with Y do not support 1.44 M_{sun}.
- **a**Additional Repulsion at high ρ?
- •Vector mass mod.
- \rightarrow stronger repulsion at high ρ. *M. Naruki et al.*, *PRL96('06)092301*.

•Another term such as NNωσ.

C. Ishizuka, AO, K. Tsubakihara, K. Sumiyoshi, S. Yamada, JPG35('08)085201. K. Tsubakihara, H. Maekawa, H. Matsumiya, AO, PRC81('10)065206.

