Auxiliary field Monte-Carlo for finite density lattice QCD at strong coupling – a small step toward the sign problem –

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in collaboration with
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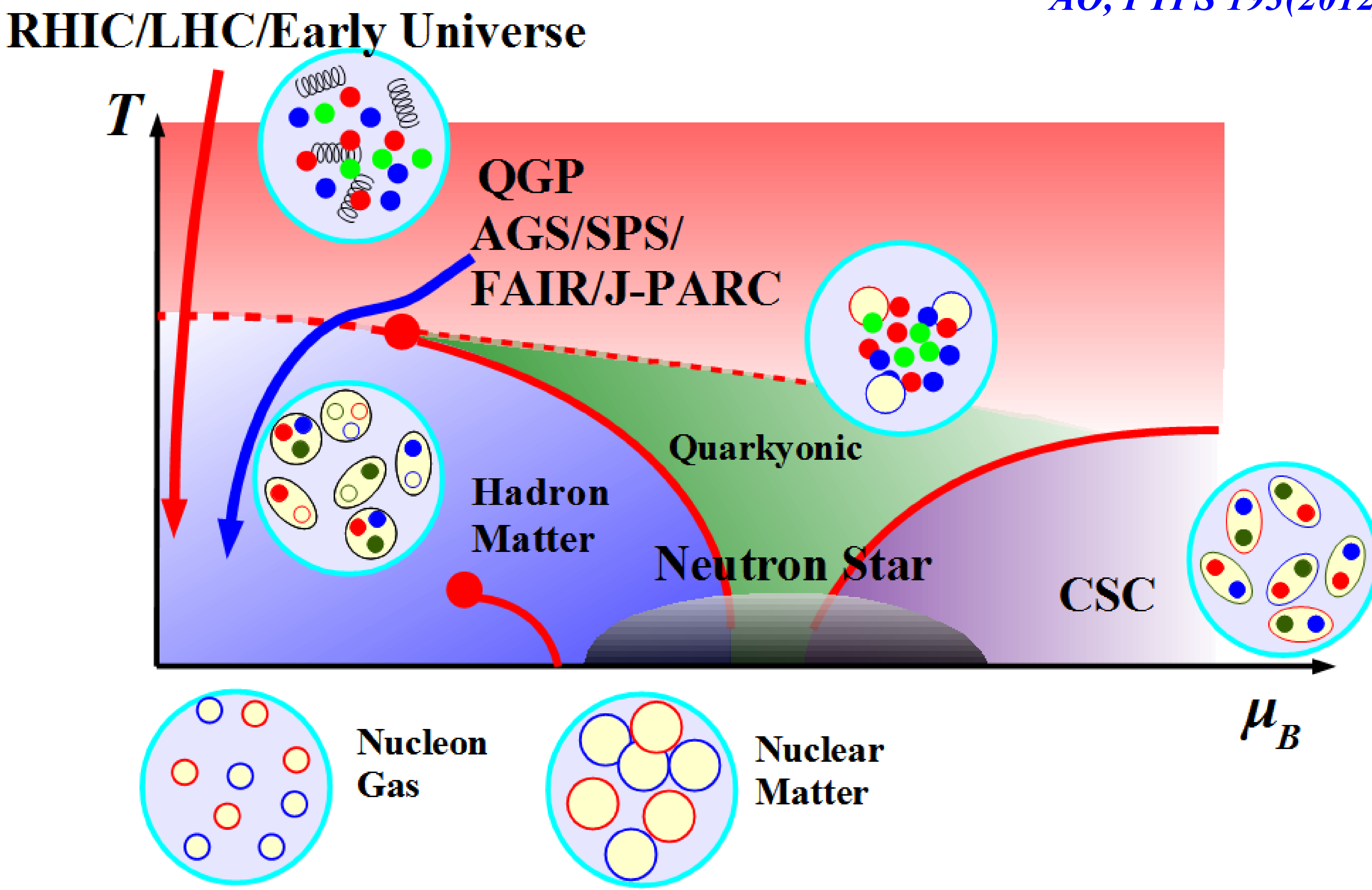
AO, T. Ichihara, T.Z. Nakano, PoS LATTICE 2012 (2012), 088 [arXiv:1211.2282]
T. Ichihara, Master thesis
AO, T. Ichihara, T.Z. Nakano, in preparation

QCD phase diagram

AO, PTPS 193(2012), 1.



QCD phase diagram

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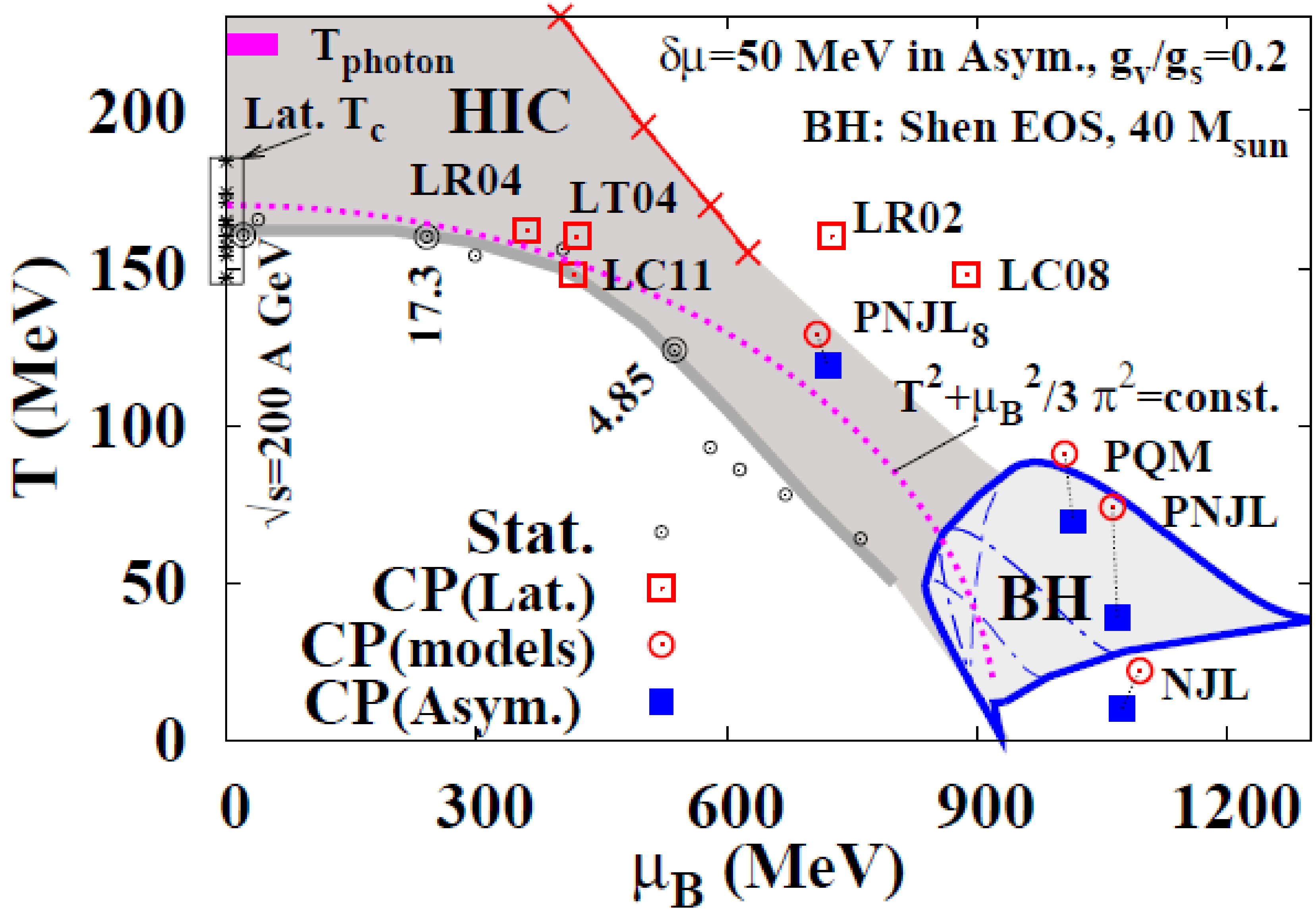


Dream ! Ab initio calc. of phase diagram and nuclear matter EOS

Cold Dense Matter: Unreachable in Lattice QCD ?

- **Sign prob. in lattice QCD Monte-Carlo simulation**

$$Z = \int D[\psi, \bar{\psi}, U] \ \exp(-\bar{\psi} D \psi - S_G)$$
$$= \int D[U] \ \det D \ \exp(-S_G) = \int D[U] \ |\det D| \ \exp(i\theta) \ \exp(-S_G)$$

  - **Fermion matrix $[\gamma_5 D(\mu) \gamma_5]^+ = D(-\mu^*)$**
    $\rightarrow$ [det $D(\mu)$]* = det $D(-\mu^*)$ $\rightarrow$ Complex weight in MC

- **Phase quenched simulation (+ reweighting) does not work in cold dense matter**

  - **Phase quenched configs.**
    ~ Finite isospin μ ($\mu_u = - \mu_d$, No flavor mixing as in large Nc)
    $\rightarrow$ π condense at $\mu_u - \mu_d > m_\pi$ at low T
    We cannot obtain baryonic matter from π cond. configurations !

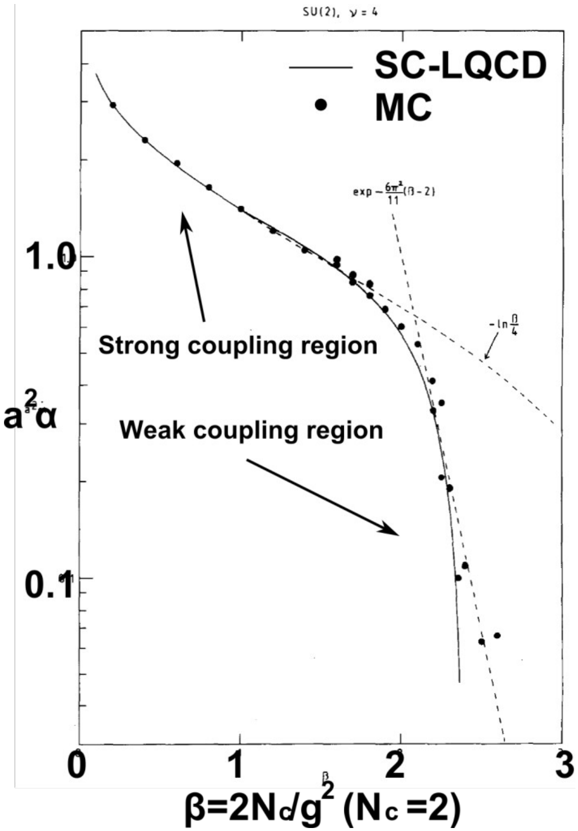
  - **No go theorem**
    Spritorff ('06), Han, Stephanov ('08), Hanada, Yamamoto ('11), Hidaka, Yamamoto ('11)

Approaches to Finite Density QCD

- "Standard" Lattice QCD MC approach
  - Direct / Expansion around μ=0 / Analytic continuation …
- Step-by-Step approach: Lattice QCD → NN interaction → Nuclei
  - Interaction of 3-body, 4-body, … *HAL QCD (Aoki, Hatsuda, Ishii, ...)*
- Complex Langevin *Aarts et al. ('10)*
  - It may converge to a wrong results.
- Strong coupling lattice QCD
  - Pure Yang-Mills → Roughening transition in "surface" observables
  - With quarks → LO (SCL), NLO ($1/g^2$), NNLO ($1/g^4$) in MF

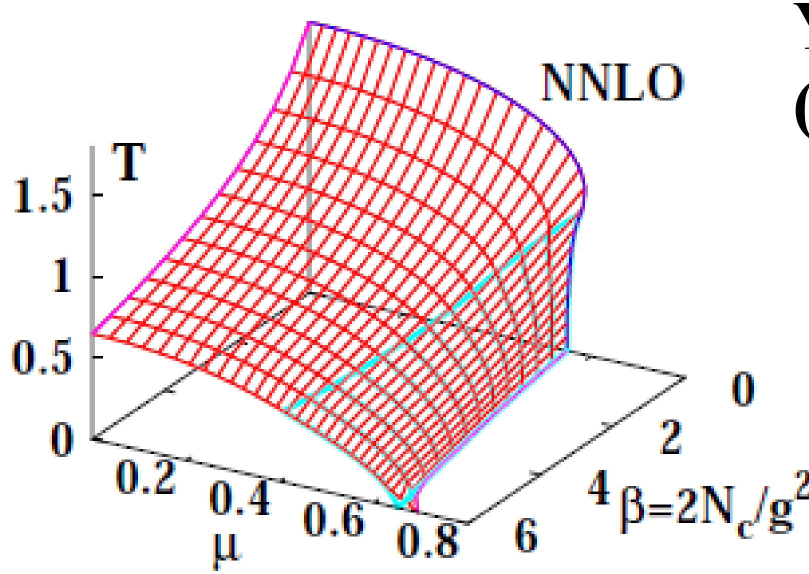
Strong Coupling Lattice QCD

Pure YM



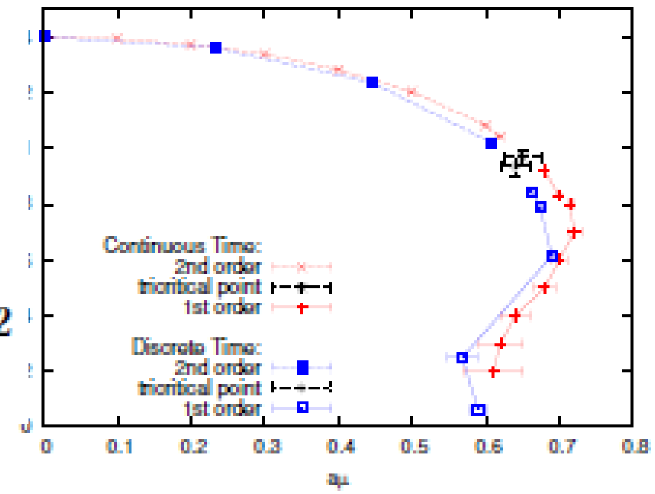
Wilson ('74), Creutz ('80), Munster ('80, '81), Lottini, Philipsen, Langelage's ('11)

YM+Quarks (MF)



Kawamoto ('80), Kawamoto, Smit ('81), Damagaard, Hochberg, Kawamoto ('85), Bilic, Karsch, Redlich ('92), Fukushima ('03); Nishida ('03), Kawamoto, Miura, AO, Ohnuma ('07). Miura, Nakano, AO, Kawamoto ('09) Nakano, Miura, AO ('10)

YM+Q+Fluc. (MDP) (SCL($1/g^2=0$))



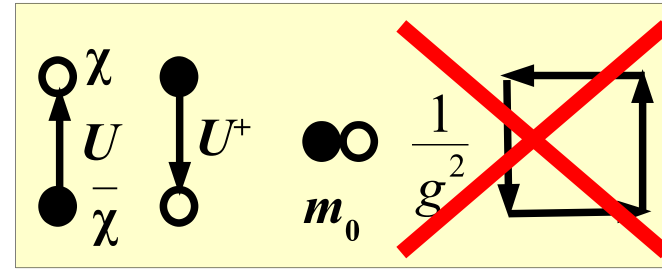
Monomer-Dimer-Polymer (MDP) simulation Mutter, Karsch ('89), de Forcrand, Fromm ('10), de Forcrand, Unger ('11)

Challenge: YM+Q+Fluc.+Finite Coupling Effects

de Forcrand, Fromm, Langelage, Miura, Philipsen, Unger ('11), AO, Ichihara, Nakano ('12)

Strong Coupling Effective Action

- **Lattice QCD action at strong coupling**



anisotropy

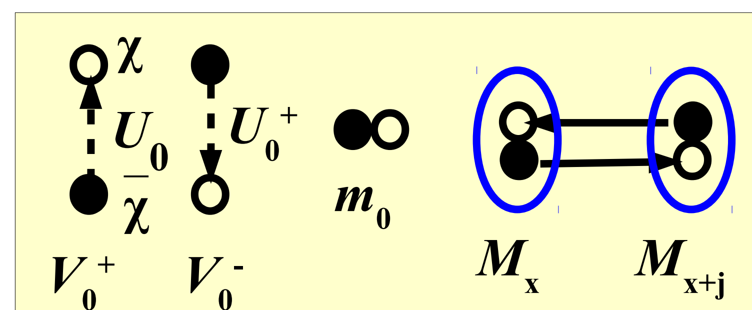
$$S_{\rm LQCD} = \frac{\gamma}{2}\sum_x \left[V^+(x) - V^-(x)\right] + m_0 \sum_x M_x + \frac{1}{2}\sum_{x,j}\eta_j(x)\left[\bar{\chi}_x U_{j,x}\chi_{x+\hat{j}} - \bar{\chi}_{x+\hat{j}} U^+_{j,x}\chi_x\right] + S_G$$

$$V^+(x) = e^{\mu/\gamma^2}\bar{\chi}_x U_{x,0}\chi_{x+\hat{0}},\quad V^-(x) = e^{-\mu/\gamma^2}\bar{\chi}_{x+\hat{0}} U^+_{x,0}\chi_x,\quad M_x = \bar{\chi}_x\chi_x$$

- **Strong Coupling Limit (LO in $1/g^2$ and $1/d$)**

$$S_{\rm eff} = \frac{\gamma}{2}\sum_x \left[V^+(x) - V^-(x)\right] - \frac{1}{4N_c}\sum_{x,j} M_x M_{x+\hat{j}} + m_0 \sum_x M_x$$

Damgaard, Kawamoto, Shigemoto ('84)



- **Integrate spatial links first.**
- **Leading order in $1/g^2$ and $1/d$**
- **Temporal Link + Nearest Neighbor Int.**

(d=spatial dim.)

Introduction of Auxiliary Fields

- **Bosonization of Nearest Neighbor Four-Fermi Interaction**

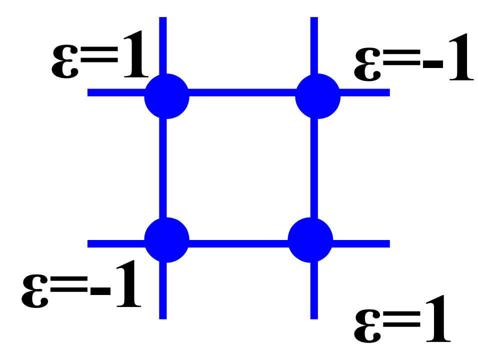
$$-\alpha \sum_{j,x} M_x M_{x+\hat{j}} = -\alpha L^3 \sum_{\boldsymbol{k},\tau} f(\boldsymbol{k}) M_{-\boldsymbol{k},\tau} M_{\boldsymbol{k},\tau} \quad \left(f_M(\boldsymbol{k}) = \sum_j \cos k_j \right)$$

$$= \alpha L^3 \sum_{\boldsymbol{k},\tau, f_M(\boldsymbol{k})>0} f_M(\boldsymbol{k}) \left[\sigma_k^* \sigma_k + \pi_k^* \pi_k \right] + \alpha \sum_{x,\pm j} \bar{\chi}_x \left(\sigma + i \varepsilon \pi \right)_{x \pm \hat{j}} \chi_x$$

- **Effective Action of Auxiliary Fields**

$$S_{\text{eff}}(\sigma, \pi) = \frac{L^3}{4 N_c} \sum_{k, f_M(\boldsymbol{k})>0} f_M(\boldsymbol{k}) \left[\sigma_k^* \sigma_k + \pi_k^* \pi_k \right] - \sum_{\boldsymbol{x}} \log \left[\det D_{\boldsymbol{x}} \right]$$

- **Negative Eigenvalues in Four-Fermi Int.**
  → Complex "mass" σ+iεπ (sign prob.)
  in Extended Hubbard-Stratonovich transf.
  $\varepsilon = (-1)^{x0+x1+x2+x3} = \exp(i\,\pi(x_0+x_1+x_2+x_3))$
  → Phase cancellation of nearest neighbor spatial site det for π field having low k



Miura, Nakano, AO ('09); AO, Ichihara, Nakano ('12)

Auxiliary Field Monte-Carlo

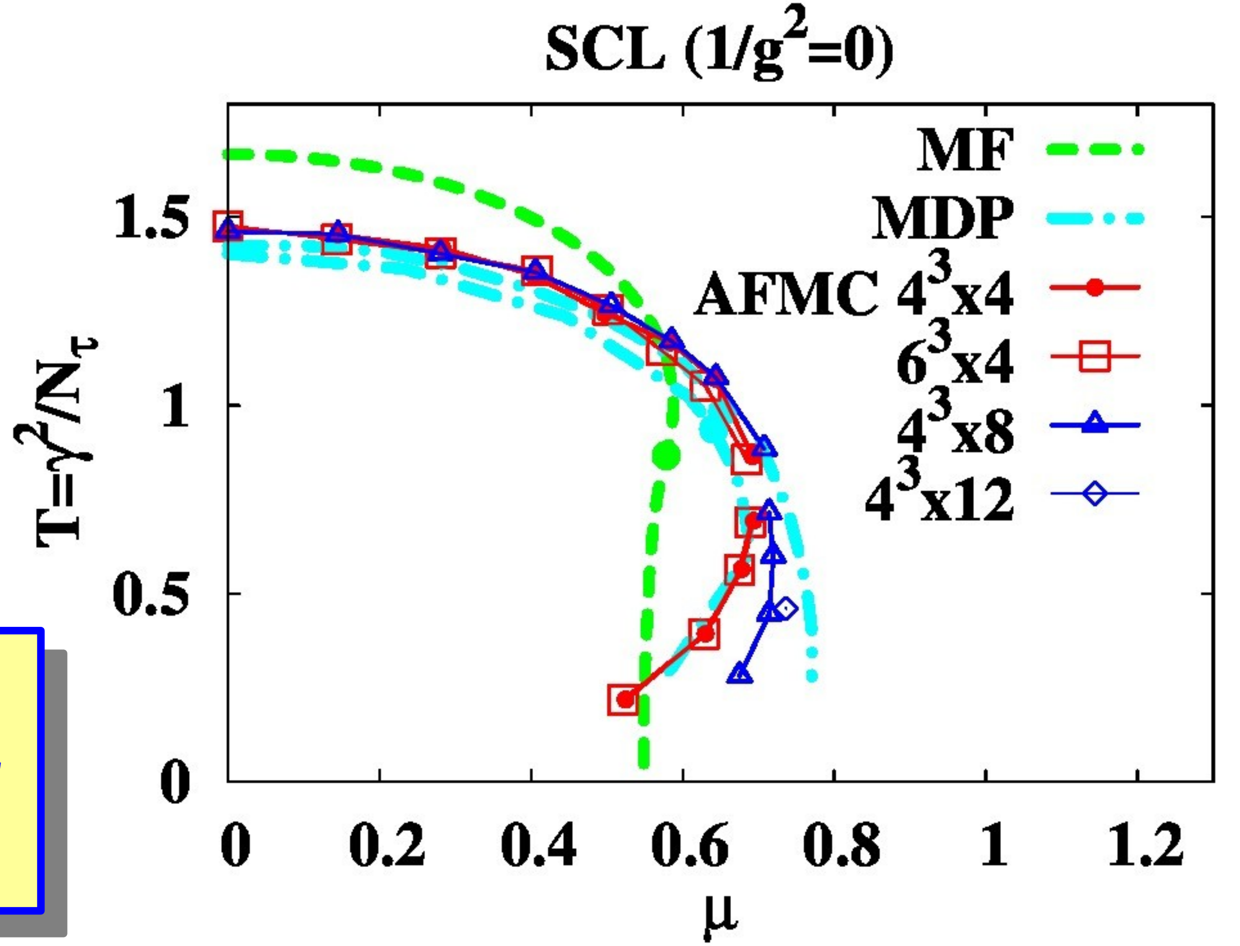
- **Mean Field and AFMC**

$$Z=\int D[\bar{\chi},\chi,U]\ \exp\left(-S_{\mathrm{LQCD}}\right)$$
$$\sim\int D[\bar{\chi},\chi,U_0]\ \exp\left(-S_{\mathrm{SCL}}[\bar{\chi},\chi,U_0]\right)$$
$$\left(\text{LO in } 1/g^2 \text{ and } 1/d \text{ expansions}\right)$$
$$=\int D[\sigma,\pi]\ \exp\left(-S_{\mathrm{eff}}[\sigma,\pi]\right)\quad\left(\text{AFMC}\right)$$
$$\sim\exp\left(-S_{\mathrm{eff}}[\sigma_{\mathrm{eq}},\pi_{\mathrm{eq}}=0]\right)\quad\left(\text{MF}\right)$$

- **Auxiliary Field Monte-Carlo**
  - **A generic method in Fermion many-body problem.**
  - **Problem**
    Large computation cost + sign problem from repulsive int.

Phase Diagram

- **AFMC predicts smaller T_c (μ=0), and extended Nambu-Goldstone phase at finite μ compared with mean field results.**
- **AFMC results are almost consistent with MDP results.**
  de Focrand, Fromm ('10), de Forcrand, Unger ('11)
  - **N_τ=4 results ~ MDP (N_τ=4)**
  - **N_τ= ∞ Extrapolation ~ Continuous Time MDP**



AFMC can be an alternative to discuss finite density LQCD !

Average Phase Factor

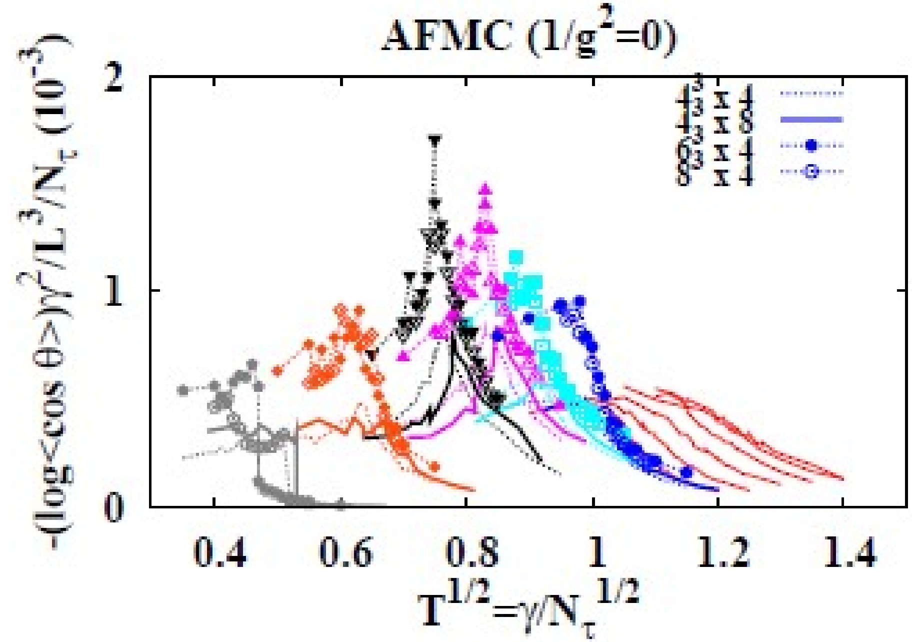
- **Sign problem is a problem at large volume, where**

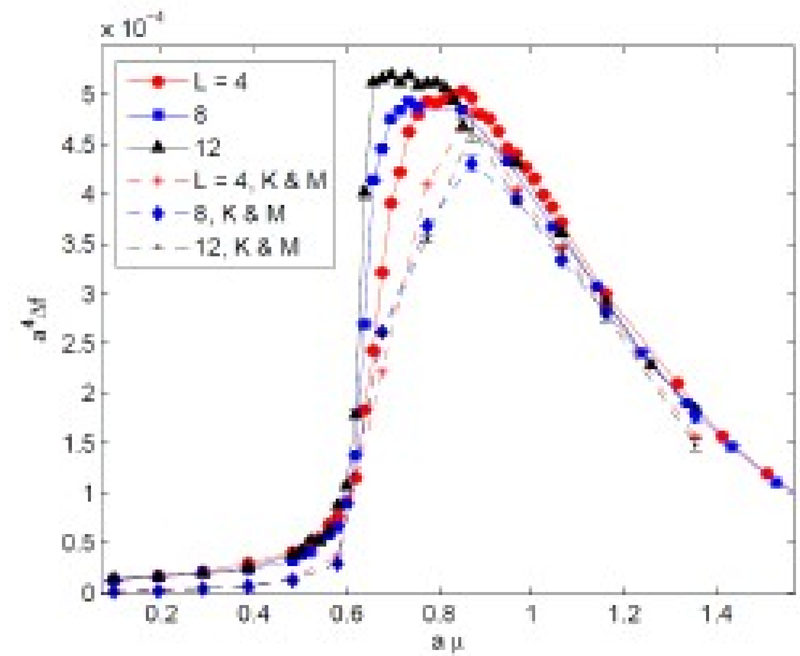
$$\langle e^{i\theta}\rangle = \frac{\int D[\Phi]\ \exp(-S[\Phi])}{\int D[\Phi]\ \exp(-\mathrm{Re}\,S[\Phi])} \equiv \exp(-L^3 N_\tau \Delta f)$$

becomes very small.

- **Sign problem is more serious in AFMC than in MDP**

$$\Delta f(\mathrm{MDP}) \sim 5\times 10^{-4}$$
$$\Delta f(\mathrm{AFMC}) \sim 1\times 10^{-3}$$





Summary

- Sign problem is a serious problem in lattice QCD.
  - It is not only a computer power problem, but an authentic problem in phase quenched simulations.
- Serious esitmate of the phase boundary in strong-coupling lattice QCD started recently.
  - Mean field estimate of the phase boundary with $1/g^2$, $1/g^4$ effects.
  - Monomer-Dimer-Polymer (MDP) simulation is applied to phase diagram in the strong coupling limit
    → Partition func. = sum of various loop configurations.
- Auxiliary Field Monte-Carlo (AFMC) for strong-coupling lattice QCD is proposed.
  - MDP phase diagram is confirmed.
  - AFMC is extensible to include finite coupling effects in a straightforward manner.
  - Conjecture: AFMC+ RG may solve the sign problem.

Thank you for your attention !

Long and Winding Road

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