複素作用から実作用へのマッピング法
Preweighting method in Monte-Carlo sampling with complex action

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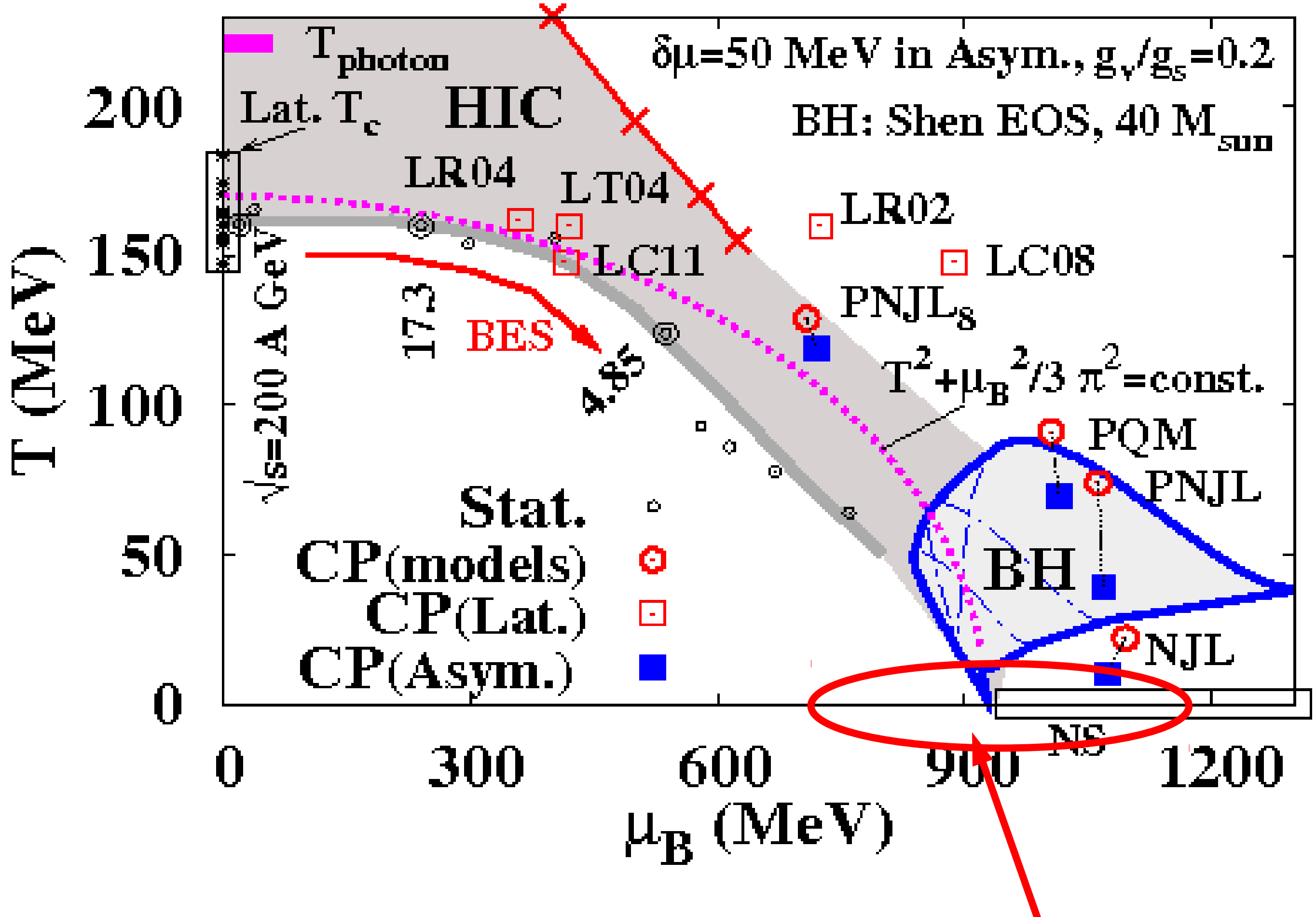
- **Introduction**
- **Preweighting**
- **Verification in AFMC-SCL Configs.**
- **Summary**

QCD phase diagram



AO, PTPS193('12)1.

Finite density LQCD and Sign Problem

- **Dirac operator at finite μ**

$$\gamma_5 D(\mu)\gamma_5 = [D(-\mu^*)]^+ \rightarrow [\det D(\mu)]^* = [\det D(-\mu^*)]$$

Fermion det. becomes real for μ=0 or μ=i μ_I ,
but for real finite μ, det D is complex.

- **Phase quenched simulation + Reweighting**

$$\langle O \rangle = \langle O e^{i\theta} \rangle_{pq} / \langle e^{i\theta} \rangle_{pq}$$

Error becomes large when the average phase factor is small.

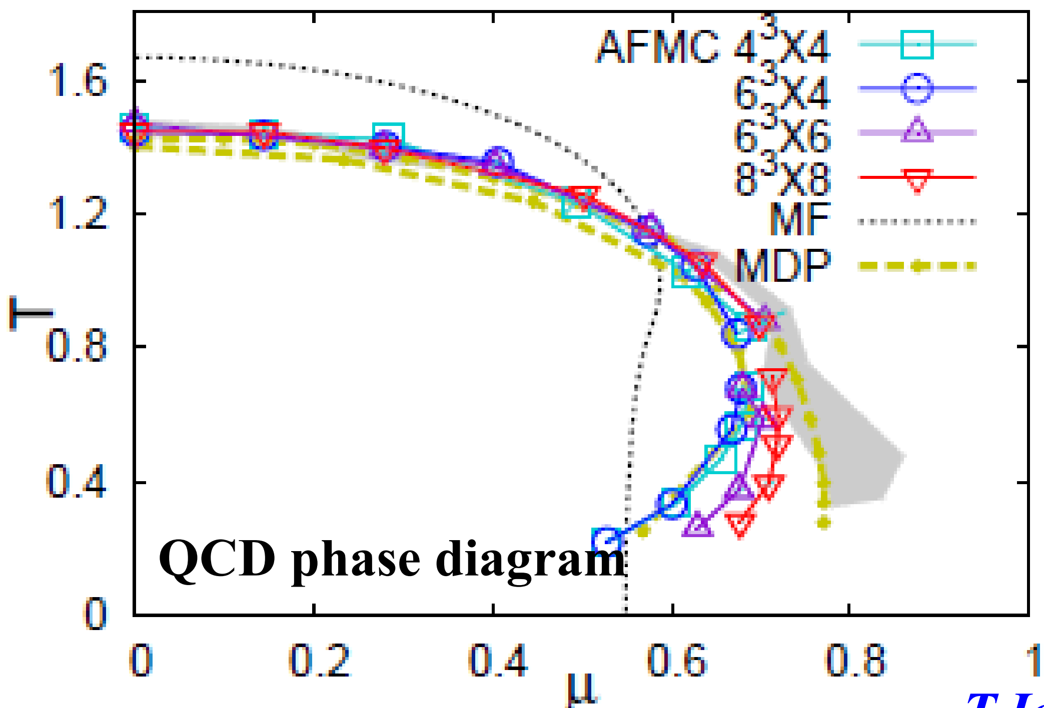
- **Many methods have been proposed.**
  Taylor expansion, Imag. μ +AC, Canonical, Re-weighting,
  Strong coupling LQCD, Fugacity expansion,
  Histogram method, Complex Langevin, Lefschetz thimble, ..

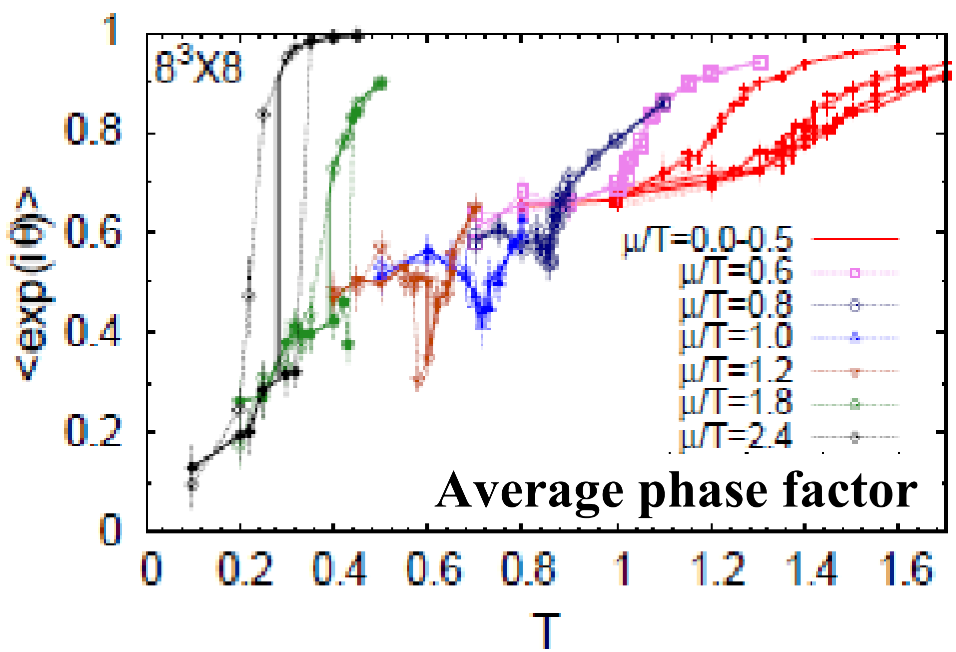
It is still difficult to go to low T region …..

Strong Coupling Lattice QCD

Wilson ('74), Kawamoto ('80), Kawamoto, Smit ('81),Aoki ('84), Damagaard, Hochberg, Kawamoto ('85), Bilic, Karsch, Redlich ('92), Fukushima ('03), Nishida ('03), Kawamoto, Miura, AO, Ohnuma ('07). Miura, Nakano, AO, Kawamoto ('09), Nakano, Miura, AO ('10), Mutter, Karsch ('89), de Forcrand, Fromm ('10), de Forcrand, Unger ('11), Misumi, Nakano, Kimura, AO('12), Misumi, Kimura, AO('12), AO, Ichihara, Nakano ('12), Ichihara, Nakano, AO ('14), ...

- **Integrate links first, and fermions later → Milder sign prob.**
  - **We can see the 1st order phase boundary in the *Strong Coupling Limit*.**

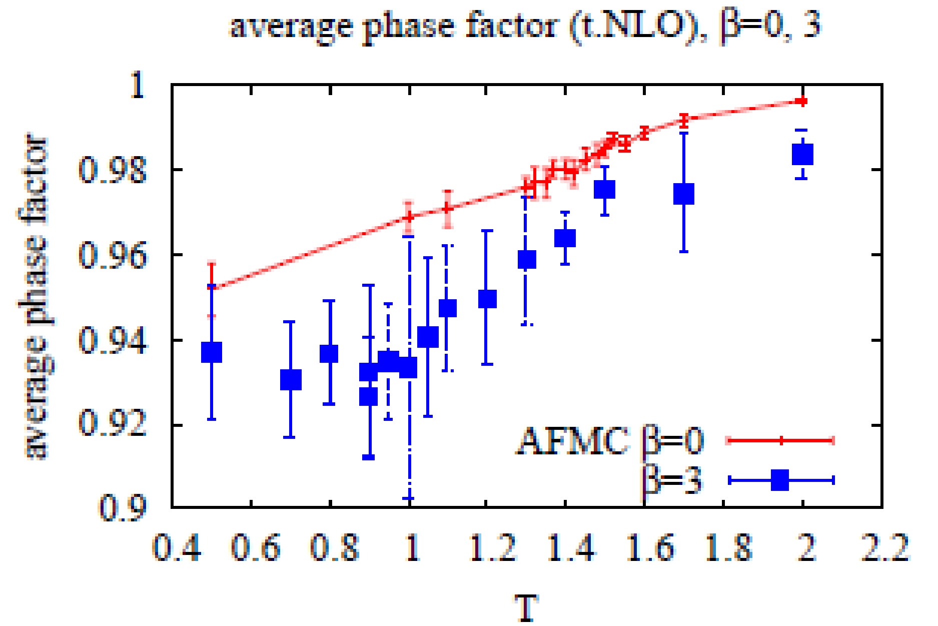


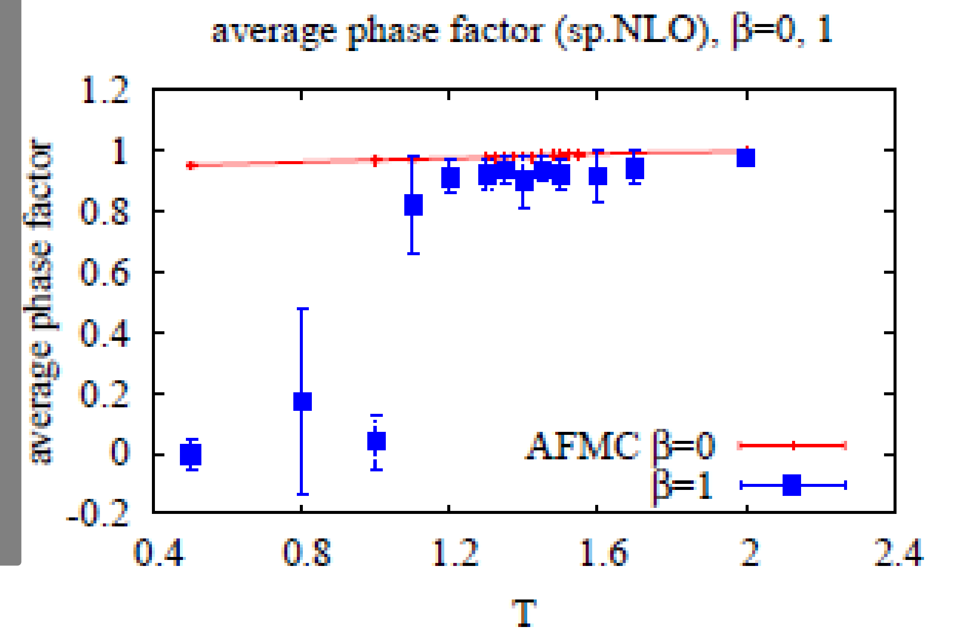


T.Ichihara, AO, T.Z.Nakano, PTEP2014,123D02.

Strong Coupling Lattice QCD (cont.)

- **Average phase factor $< e^{i\theta} >$ becomes much smaller when finite coupling correction is included.**





We try to develop a sampling method, preweighting, to reduce the sign problem.

Preweighting = Guess θ integral approximately, and suppress less-relevant conf. in advance.

T.Ichihara, T.Z.Nakano, AO, Lattice 2014

Preweighting

Preweighting Method

- **Assume θ dist. is Gaussian**

S. Ejiri, PRD77('08)014508

$$Z=\int D\Phi\, d\theta\, e^{-S_R[\Phi]+i\theta}=\int D\Phi\, d\theta\, e^{-S'_R[\Phi]}\boxed{\frac{e^{-\theta^2/2\Delta^2[\Phi]}}{\sqrt{2\pi\Delta[\Phi]}}e^{i\theta}}$$

θ-dist.

$$=\int D\Phi\, e^{-S'_R[\Phi]}\boxed{\exp\left(-\Delta^2[\Phi]/2\right)}$$

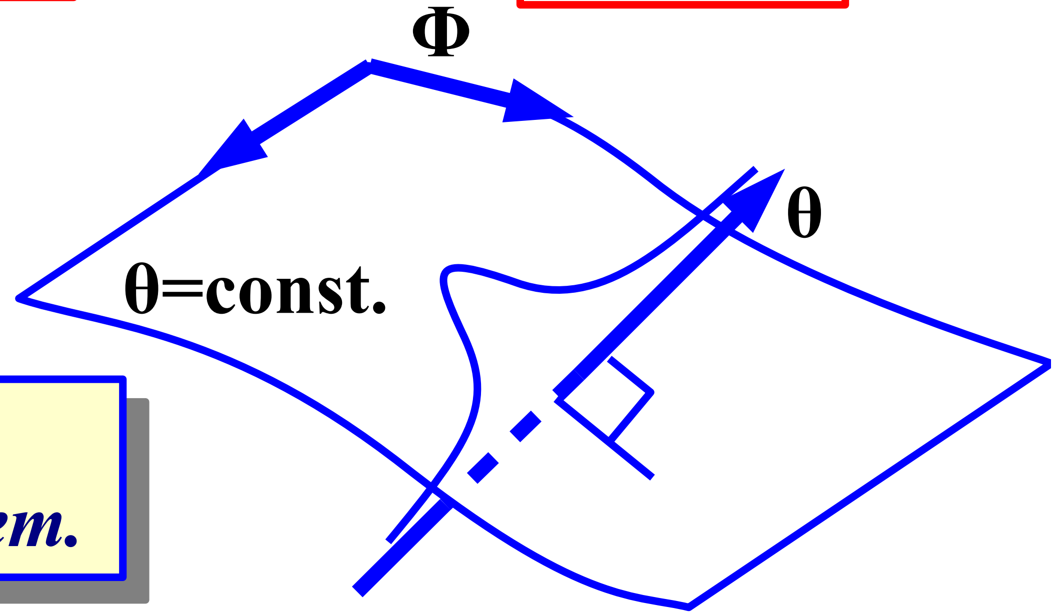
Φ dep. APF

- **Preweighting**

supp. large θ **Φ dep. supp. factor**

$$Z_{\mathrm{prew}}=\int D\Phi\, d\theta\, e^{-S_R[\Phi]}\boxed{\exp(-f(\theta))}=\int D\Phi\, e^{-S'_R[\Phi]}\boxed{F(\Delta[\Phi])}$$

$$F(\Delta)=\int d\theta\,\frac{e^{-\theta^2/2\Delta^2}}{\sqrt{2\pi\Delta}}e^{-f(\theta)}$$



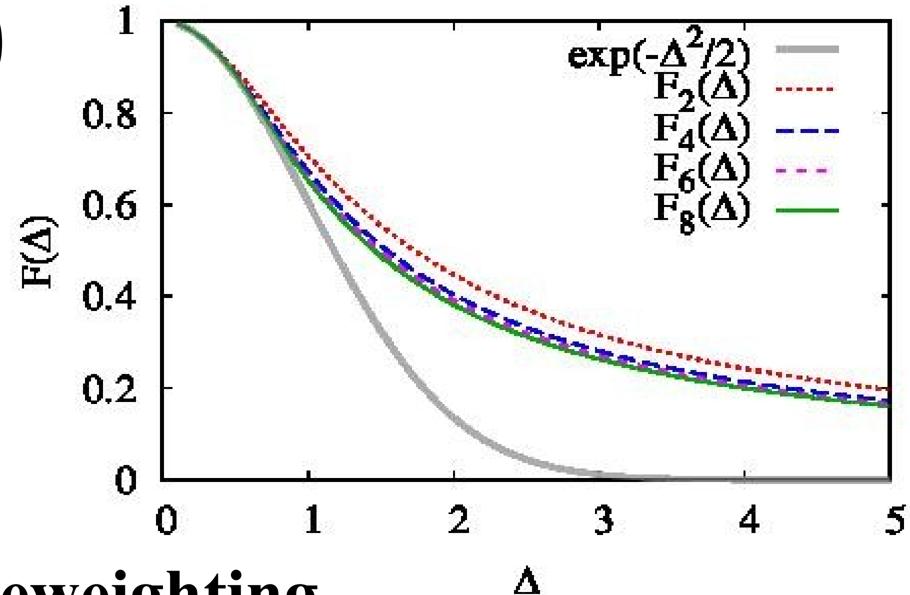
If F(Δ)=exp(-Δ²/2),
we can obtain Z w/o sign problem.

Preweighting Function

- **Can we find f(θ) which satisfies F(Δ)=exp(-Δ²/2) ?**
  → Yes, as perturbative series of Δ

$$f(\theta) = \frac{1}{2}\theta^2 + \frac{1}{12}\theta^4 + \frac{1}{45}\theta^6 + \frac{17}{1260}\theta^8 + \mathcal{O}(\theta^{10})$$

$$\rightarrow F(\Delta) = \exp(-\Delta^2/2) + O(\Delta^{10})$$



- **But there is no free lunch.**

$$F(\Delta) = \int d\theta \frac{e^{-\theta^2/2\Delta^2}}{\sqrt{2\pi}\Delta} e^{-f(\theta)}$$

$$\rightarrow \frac{1}{\sqrt{2\pi}\Delta} \int d\theta\, e^{-f(\theta)} \quad (\Delta \rightarrow \infty)$$

- **Practical method: Preweighting+Reweighting**
  - **MC with preweighting fn, e.g. f(θ)=θ²/2,**
  - **Make a histogram in Φ and obtain Δ[Φ]**
  - **Give Reweighting factor exp(-Δ²/2) / F(Δ)**

Let us examine in AFMC-SCL !

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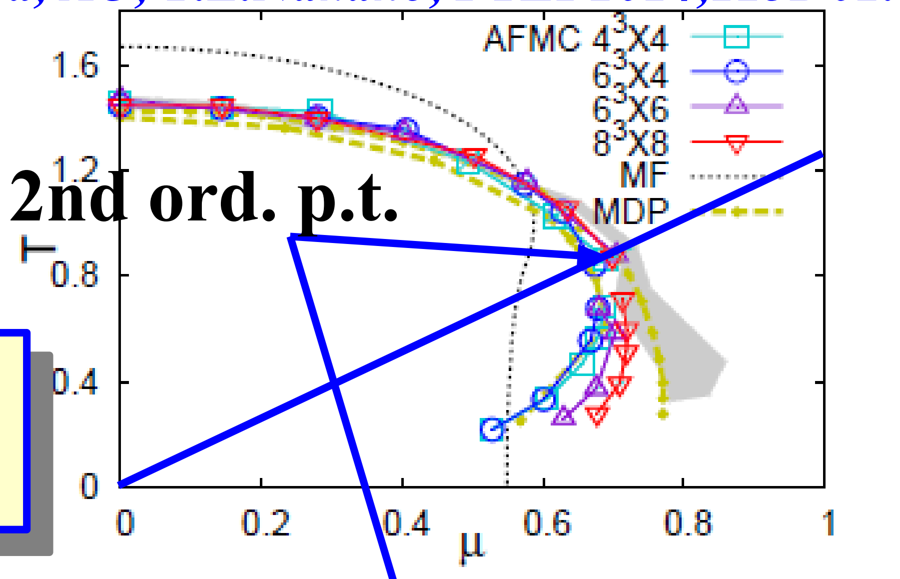
Verification in Auxiliary Field MC Confs. in the Strong Coupling Limit

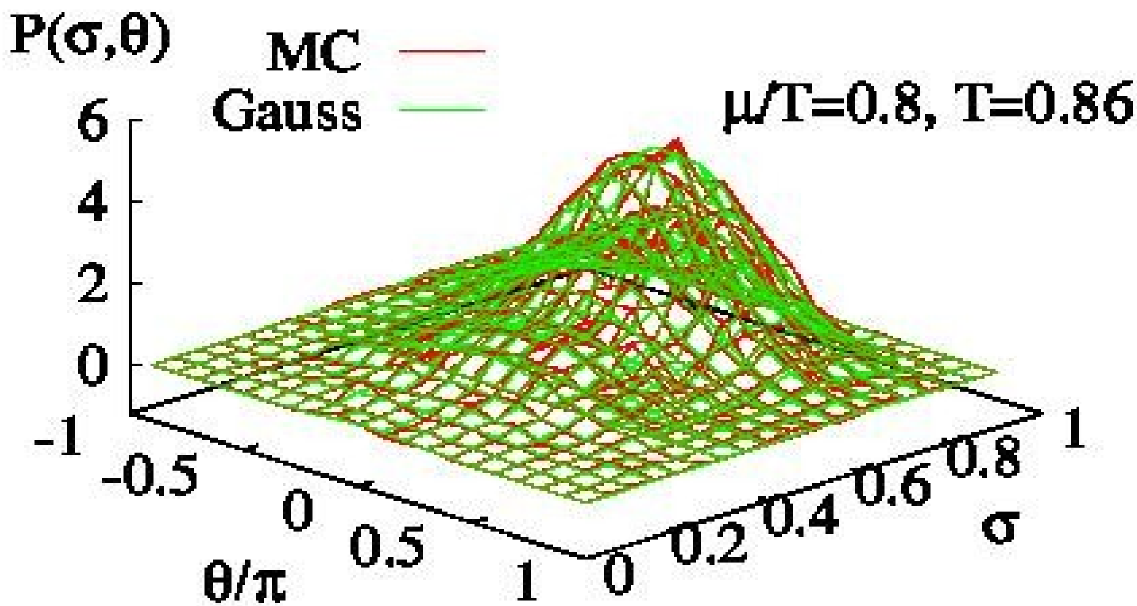
θ dist. in AFMC-SCL (μ/T=0.8)

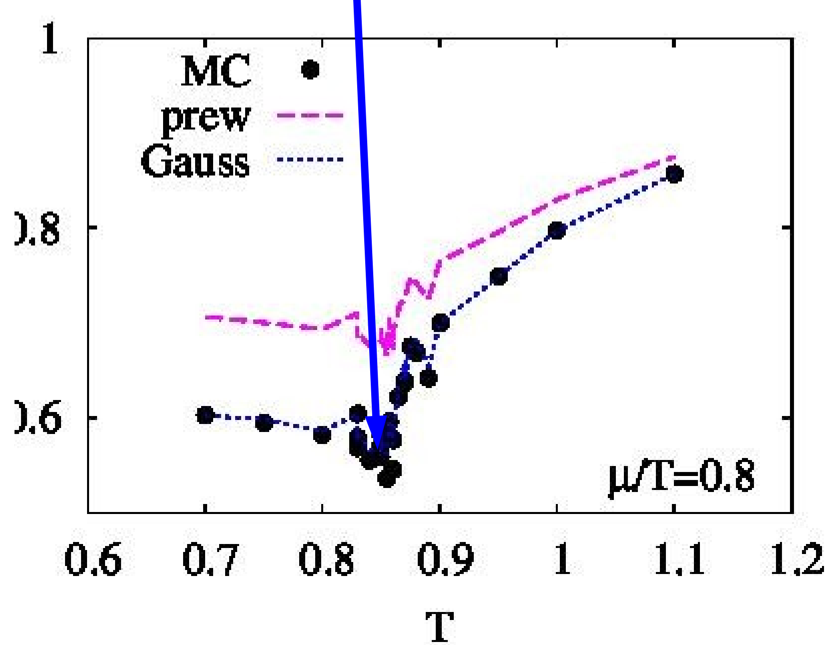
T.Ichihara, AO, T.Z.Nakano, PTEP2014,123D02.

- θ dist. ~ Gaussian
- average prew. factor ~ 0.7
- average phase factor ~ 0.5-0.6

Preweighting accounts for a large part of weight cancellation !



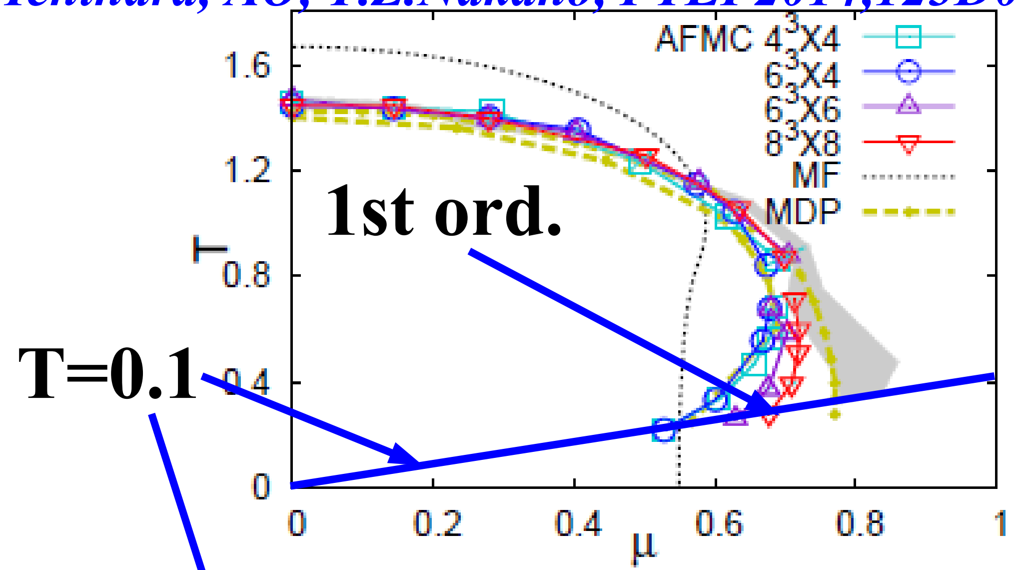


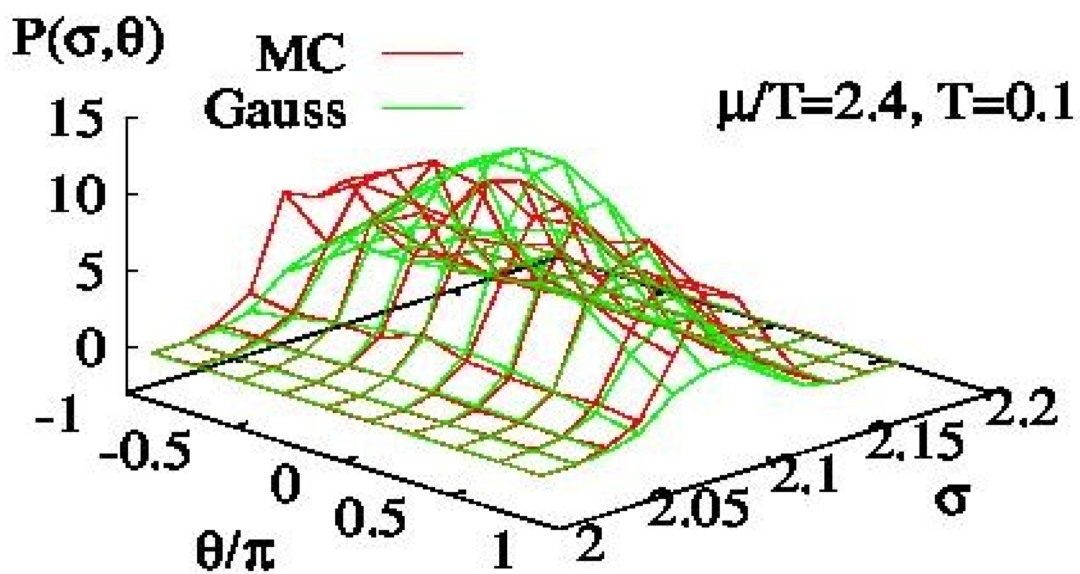


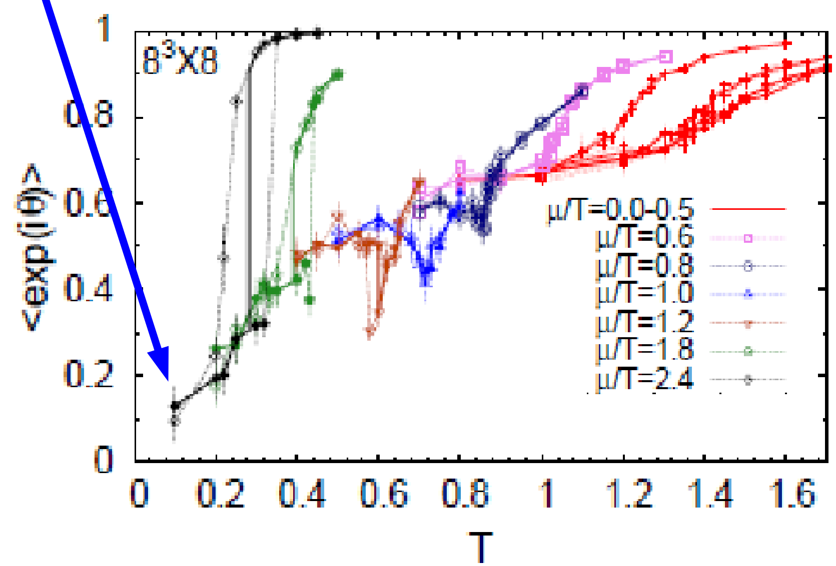
θ dist. in AFMC-SCL (μ/T=2.4)

T.Ichihara, AO, T.Z.Nakano, PTEP2014,123D02.

- **θ dist. ≠ Gaussian**
  - **θ distribution well extends around θ ~ π.**
    → We need to consider *mirror* contributions.



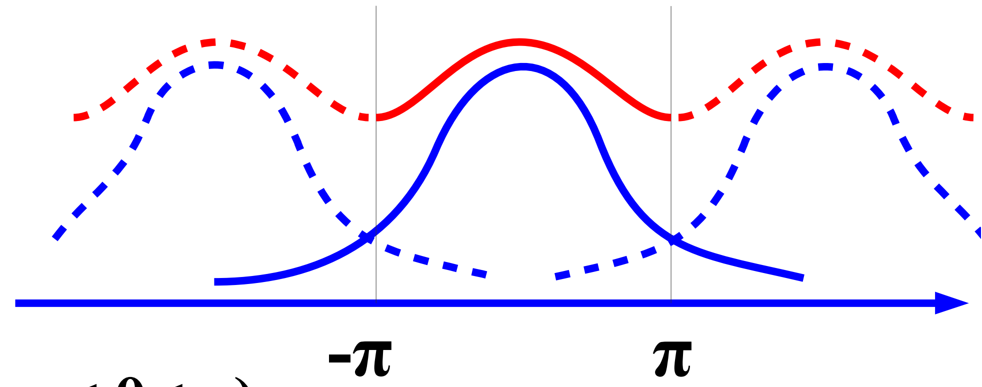




Mirror contributions

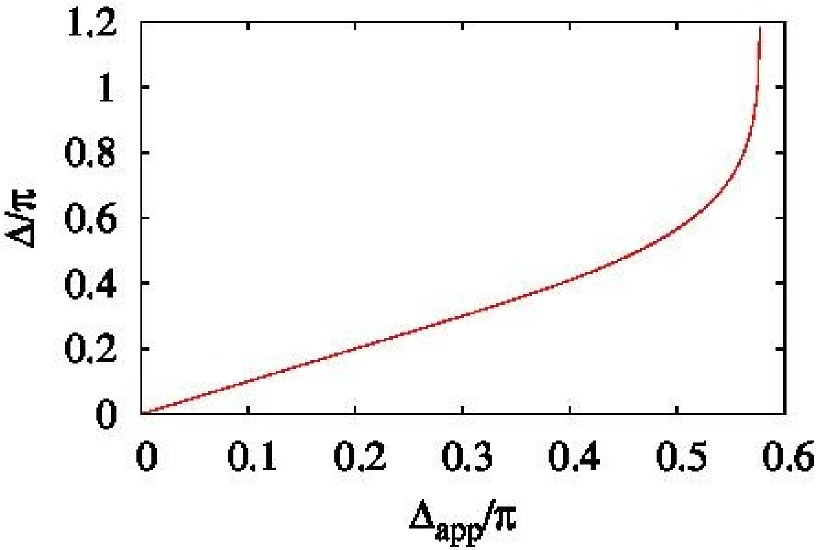
- **Mirrored Gaussian**

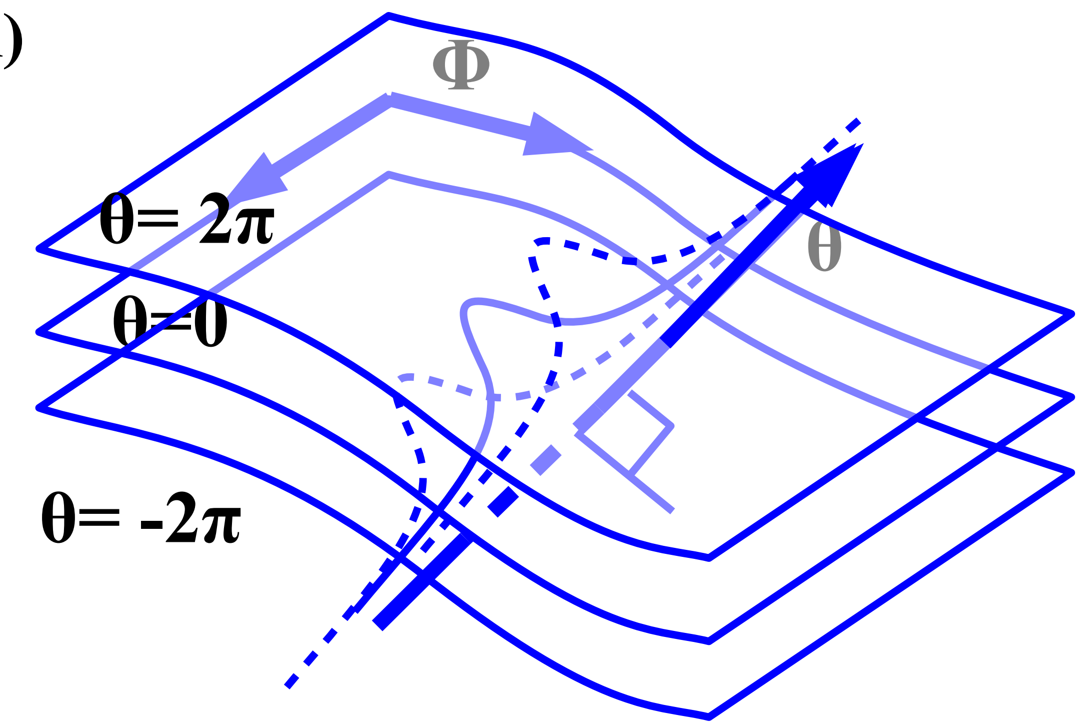
$$P(\theta) = \frac{1}{\sqrt{2\pi\Delta}} \sum_n \exp\left(-\frac{(\theta - 2\pi n)^2}{2\Delta^2}\right)$$



  - **Apparent std. dev. Δ (obtained in –π< θ< π)**
    → Actual Δ (defined in -∞< θ < ∞)

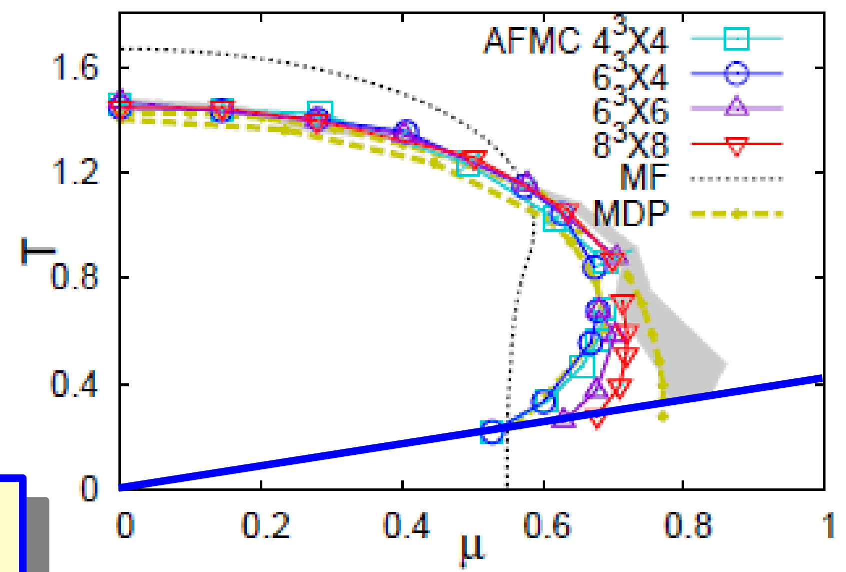
  - **Alternative**
    Obtain θ in -∞ < θ < ∞ (Ejiri)



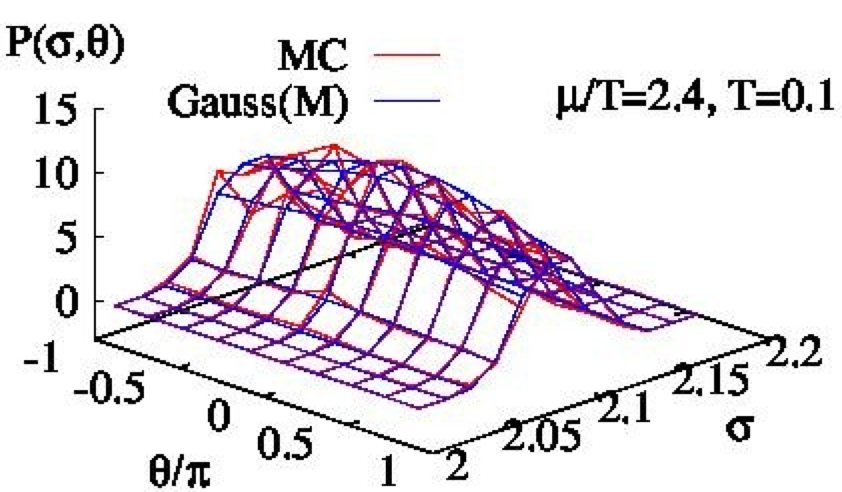


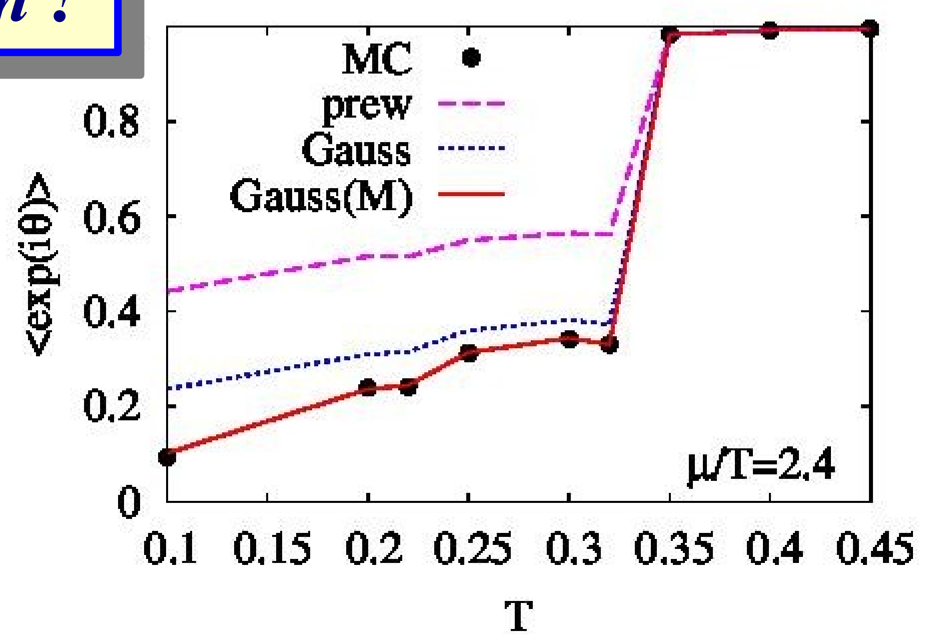
θ dist. in AFMC-SCL (μ/T=2.4, again)

- θ dist. ~ Mirrored Gaussian
  - APF is also well explained with Mirrored Gaussian.
  - average prew. factor ~ (0.4-0.6)
  - average phase factor ~ (0.1-0.3)



Preweighting accounts for around half of weight cancellation !



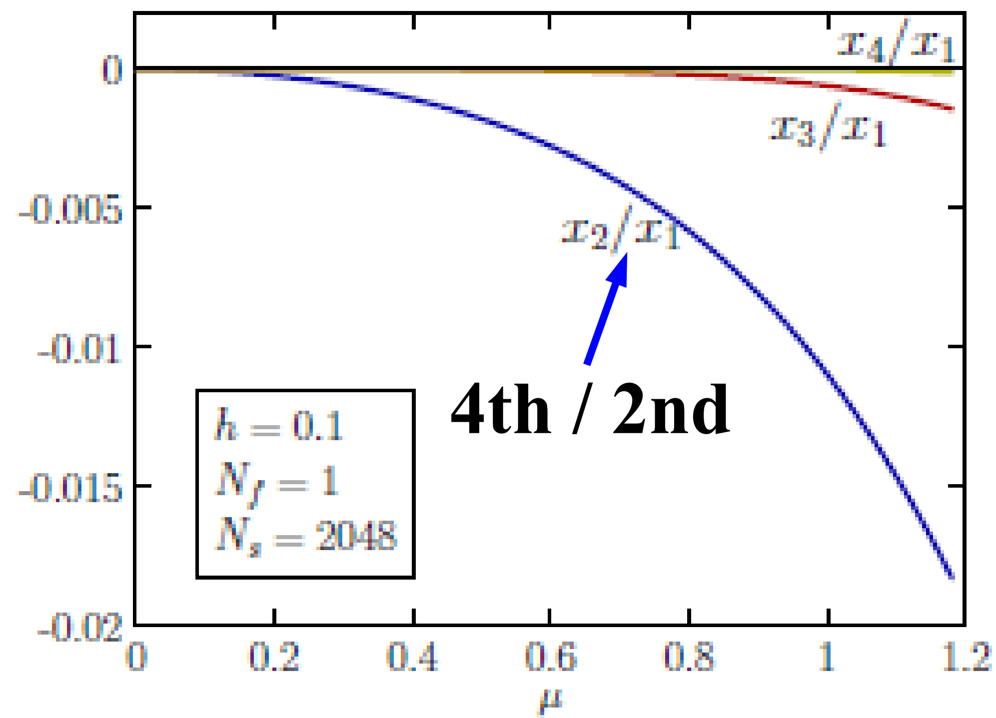


Summary

- 複素作用において前もって「重率」を与える手法 (preweighting) を提案した。
  - Phase quenched MC において $\exp(-\theta^2/2)$ の重率を余分にかけて sampling を行うことにより、位相が大きく揺らぐ配位を抑制。
  - Sample から θ の揺らぎを求め、重率が $\exp(-\Delta^2/2)$ となるよう reweighting 。(c.f. Historgram method, Ejiri)
- 強結合極限格子 QCD の補助場 MC(AFMC-SCL) 配位を用いて、preweighting + reweighting の有効性を確かめた。
  - preweighting により Average Phase Factor の減少の半分程度を説明する。
  - ガウス関数の回り込みまで考慮すると AFMC-SCL の結果は「θ がガウス分布」として計算した結果と誤差の範囲で一致。
  - Chiral condensate などの観測量についても同じ prew.+reweighting factor により数 % の精度で MC の結果を再現。

Discussion & Future works

- θ 分布の非ガウス性
  - SC & HP expansion: *Greensite, Myers, Splittorf, arXiv:1311.4568.*
  - Superposed Lorenzian in the π cond. phase
    M.P. Lombardo, K. Splittorff, J.J.M. Verbaarschot, PRD80 ('09) 054509
    (not relevant to SC-LQCD)
- Application to other systems
  - AFMC with $1/g^2$ terms
  - Link MC LQCD
    いかにして π cond. phase を消すか？
  - θ と相関の強い観測量
- $f(\theta)= \theta^2/2+ (\nabla\theta)^2/2$?
- $-\pi< \theta< \pi$ で陽に積分？



Greensite, Myers, Splittorf, arXiv:1311.4568.

Thank you !

Observations

- **Original MC (phase quenched)**

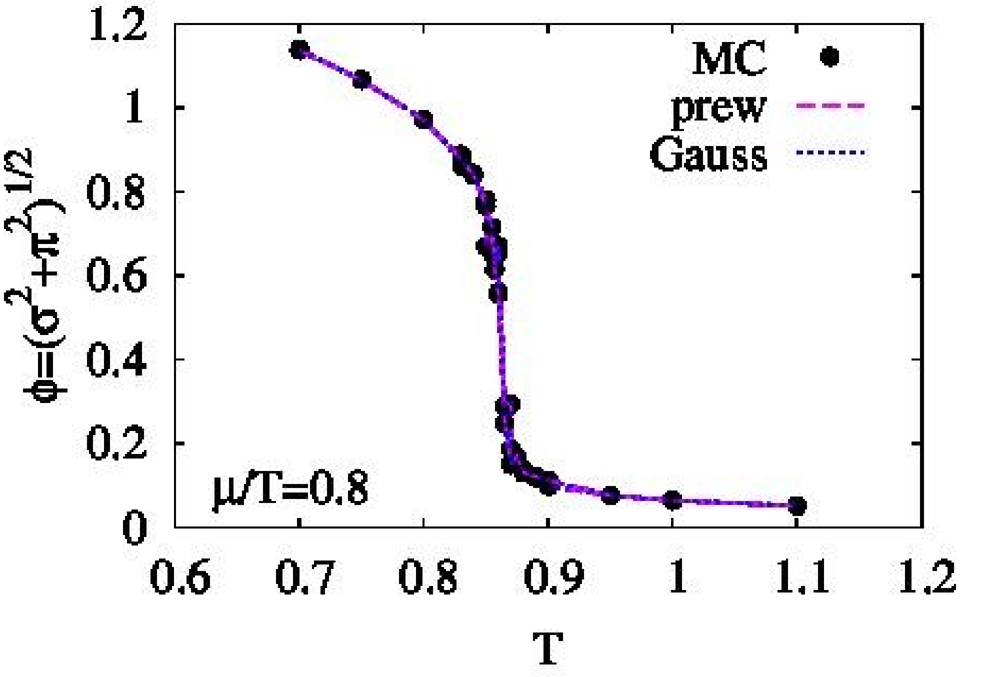
$$\langle O \rangle_{\text{orig}} = \frac{1}{\sum_i e^{i\theta_i}} \sum_i O_i e^{i\theta_i}$$

- **Preweighting (from p.q. samples)**

$$\langle O \rangle_{\text{prew}} = \frac{1}{\sum_i e^{-\theta_i^2/2}} \sum_i O_i e^{-\theta_i^2/2}$$

- **Gauss**

$$\langle O \rangle_{\text{Gauss}} = \frac{1}{\sum_i e^{-\Delta^2[\Phi_i]/2}} \sum_i O_i e^{-\Delta^2[\Phi_i]/2}$$



Apparent and Actual Std. Dev.

- **Apparent Δ**

$$\Delta_{\mathrm{app}}^2 = \int_{-\pi}^{\pi} P(\theta)\theta^2$$

- **"Actual" Δ**

$$P(\theta) = \sum_n P_G(\theta - 2n\pi)$$

$$\Delta^2 = \int_{-\infty}^{\infty} P_G(\theta)\theta^2$$