Hadron interaction from heavy-ion collisions

Akira Ohnishi

based on K. Morita, T. Furumoto, AO,
Phys. Rev. C91 (2015), 024916 [arXiv:1408.6682]

How can we measure the radius of a star ?

- **Two photon intensity correlation**
  Hanbury Brown & Twiss, Nature 10 (1956), 1047.
  - **Simultaneous two photon observation probability is enhanced from independent emission cases**
    → angular diameter of Sirius=0.0063”

A TEST OF A NEW TYPE OF STELLAR INTERFEROMETER ON SIRIUS

By R. HANBURY BROWN

Jodrell Bank Experimental Station, University of Manchester

AND

Dr. R. Q. TWISS

Services Electronics Research Laboratory, Baldock

NATURE November 10, 1956 Vol. 178

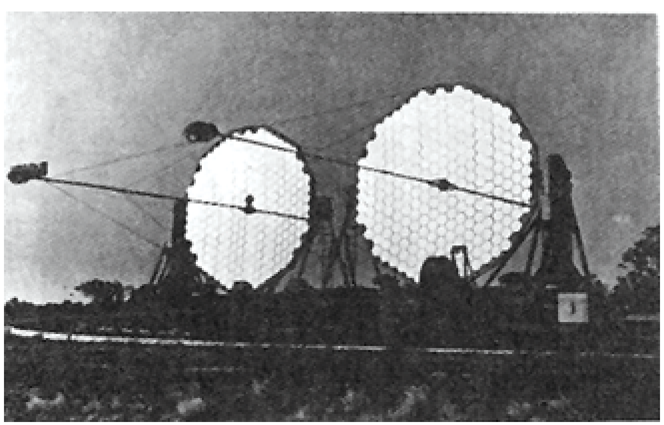
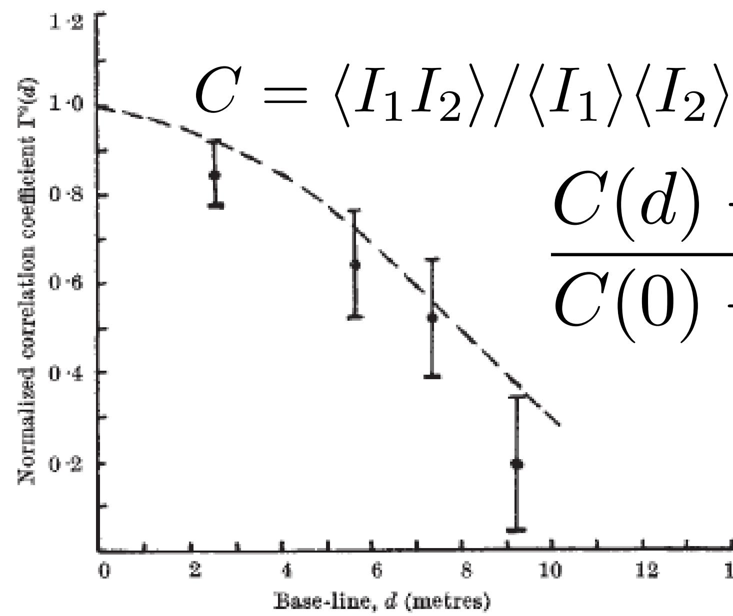


Figure 2. Picture of the two telescopes used in the HBT experiments. The figure was extracted from Ref.[1].

HBP telescope (from Goldhaber, ('91))



$$C = \langle I_1 I_2 \rangle / \langle I_1 \rangle \langle I_2 \rangle$$

$$\frac{C(d)-1}{C(0)-1}$$

Fig. 2. Comparison between the values of the normalized correlation coefficient $\Gamma^2(d)$ observed from Sirius and the theoretical values for a star of angular diameter 0·0063". The errors shown are the probable errors of the observations

HBT ('56)

Two particle intensity correlation

- **Wave function symmetrization from quantum statistics**

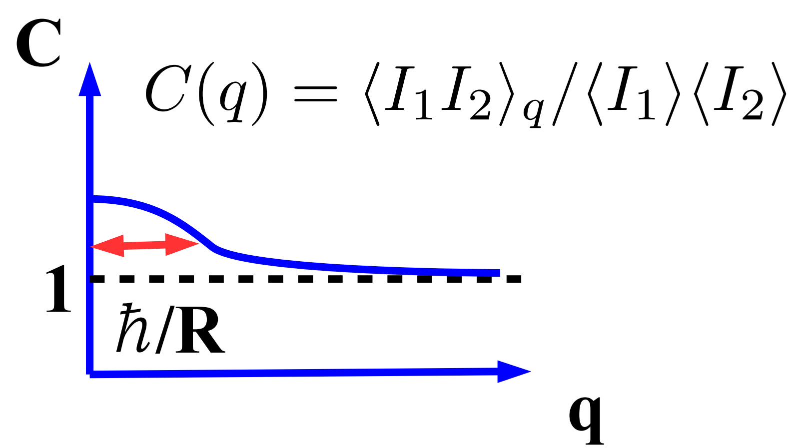
$$C(\mathbf{q}) = \int d^3r\, S(\mathbf{q},\mathbf{r}) \left| \frac{1}{\sqrt{2}} (e^{i\mathbf{q}\cdot\mathbf{r}} + e^{-i\mathbf{q}\cdot\mathbf{r}}) \right|^2 \simeq 1 + \exp(-4q^2R^2)$$

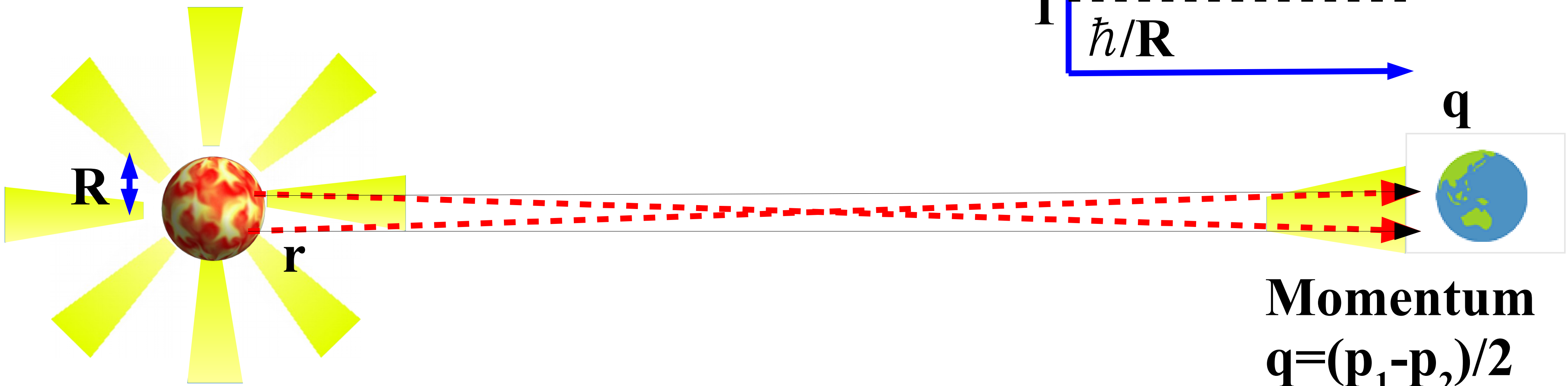
Source fn.
(r=relative
coordinate)

(symmetrized w.f.)^2

Static spherical
source case

→ **Small relative momenta are favored**
due to symmetrization
of the relative wave function.



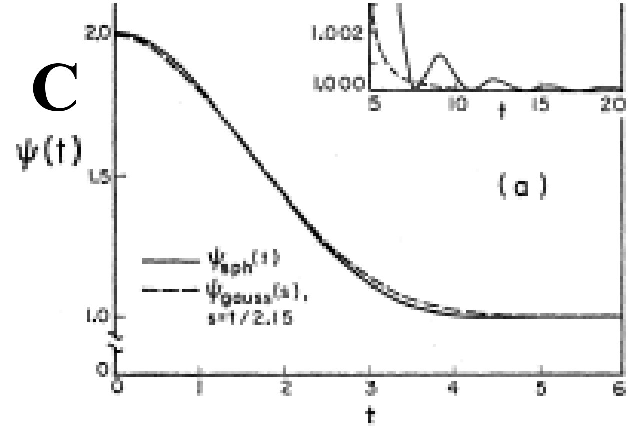


Momentum
q=(p$_1$-p$_2$)/2

How can we measure source size in nuclear reactions ?

- **Two pion interferometry**
  G. Goldhaber, S. Goldhaber, W. Lee,
  A. Pais, Phys. Rev. 120 (1960), 300
  - **Two pion emission probability is enhanced at small relative momenta**
    → Pion source size ~ 0.75 ℏ / μc

C

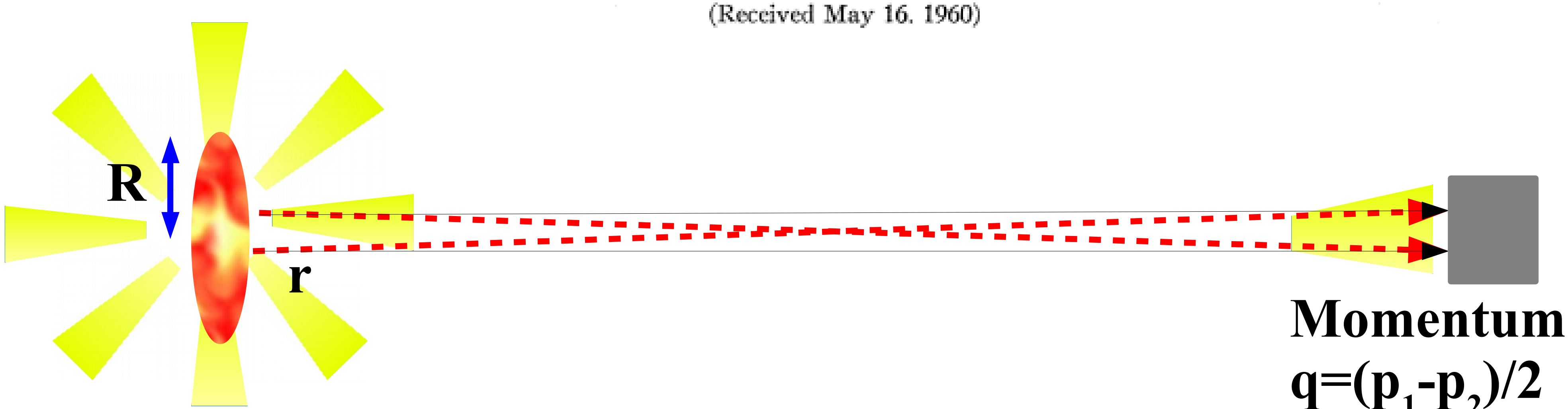


q (relative momentum)

PHYSICAL REVIEW VOLUME 120, NUMBER 1 OCTOBER 1, 1960

Influence of Bose-Einstein Statistics on the Antiproton-Proton Annihilation Process*

GERSON GOLDHABER, SULAMITH GOLDHABER, WONYONG LEE, AND ABRAHAM PAIS†
Lawrence Radiation Laboratory and Department of Physics, University of California, Berkeley, California
(Received May 16. 1960)



How does interaction modifies correlation ?

- **Interaction modifies the relative wave function, and modifies correlation.**
  e.g. W. Bauer, C.-K. Gelbke, S. Pratt, Annu. Rev. Nucl. Part. Sci. 42 (1992)77

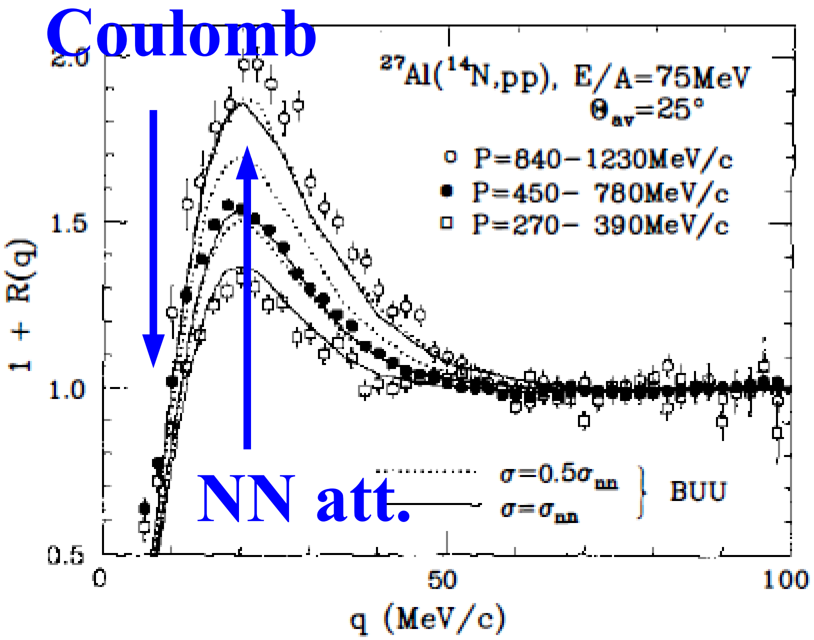
$$C(\mathbf{q}) = \int d^3r\, S(\mathbf{q},\mathbf{r}) \left|\underline{\psi^{(-)}_{12}(\mathbf{r};\mathbf{q})}\right|^2$$

int. → relative w.f.

$$\simeq 1 - \frac{1}{2}\exp(-4q^2R^2) + \frac{1}{2}\int d^3r\, \underline{S_{12}(\mathbf{r})} \left[\underline{|\chi_0(r)|^2} - \underline{|j_0(qr)|^2}\right]$$

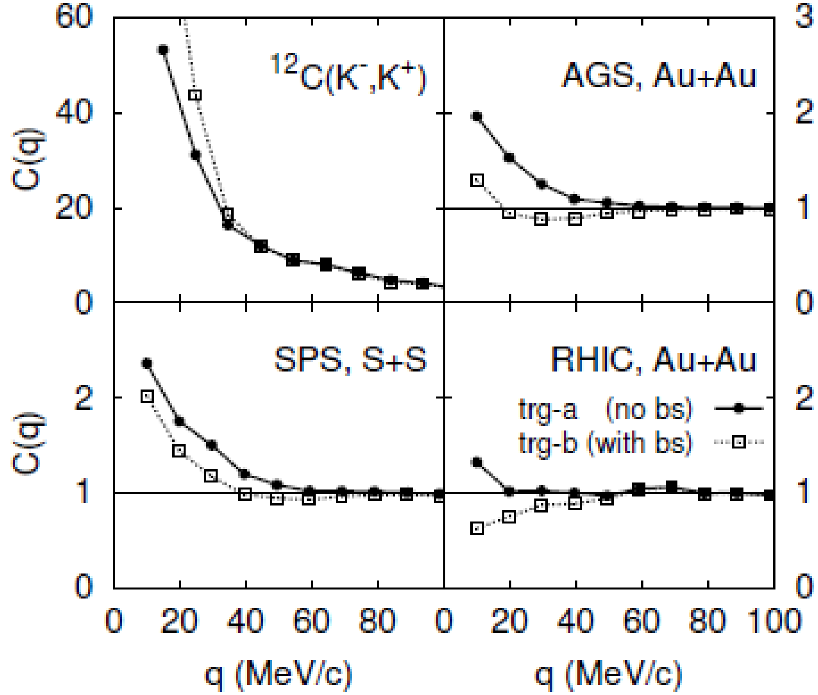
Fermion **Source** **w.f.** **free**

- **pp correlation from heavy-ion collisions is well understood by the (vacuum) NN interaction and the source function from a transport model.**

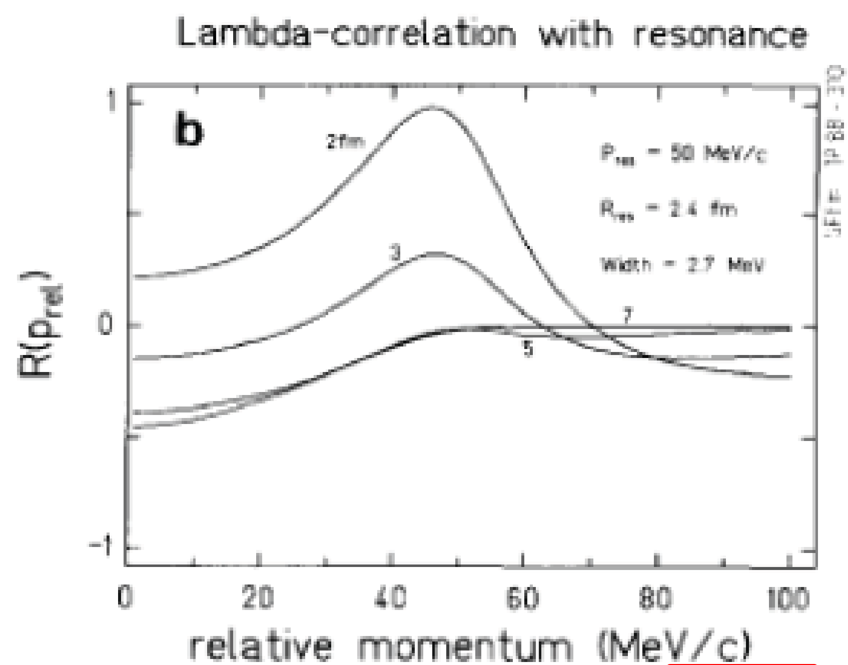


Idea of Reversal: Can we determine hadron interactions ?

- **If we know correlation and source, it should be possible to get knowledge of hadron intearctions !**
- **How about ΛΛ interaction ?**
  - **Λ particle is too short-lived to perform scattering experiments.**
    Mass=1115.6 MeV, τ= 2.6 x 10^{-10} s, cτ = 7.89 cm
    quark content = uds (p=uud, n=udd)



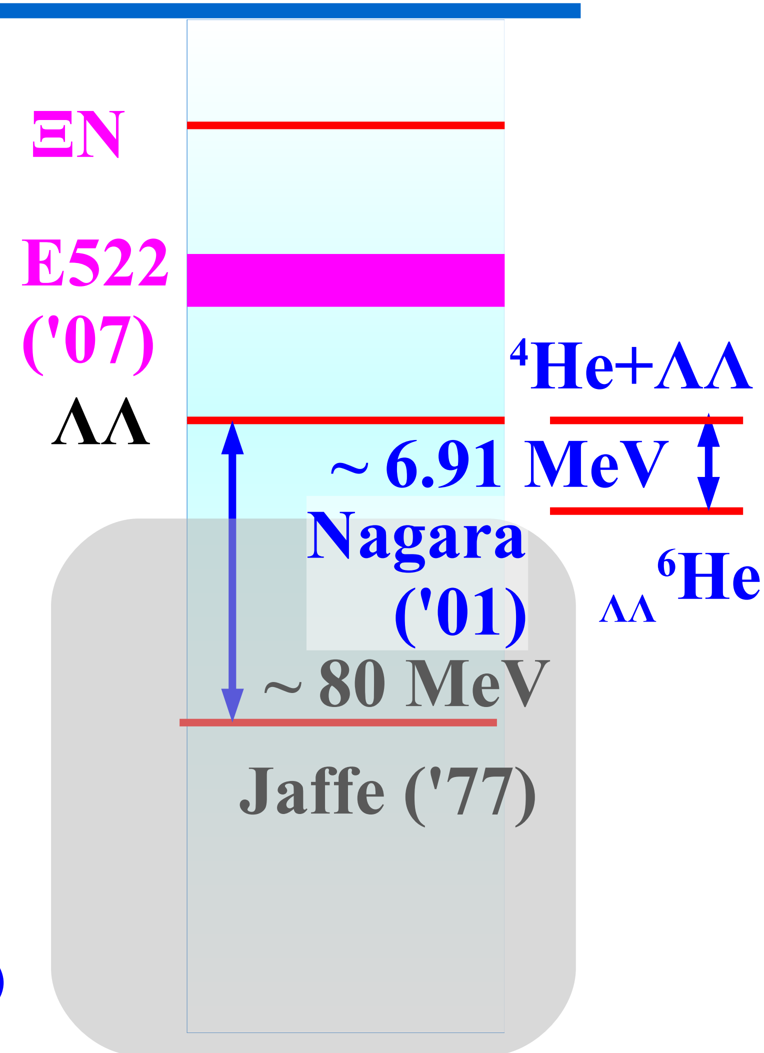
AO, Y. Hirata, Y. Nara, S. Shinmura, Y. Akaishi Nucl. Phys. A 670 (2000), 297c



C. Greiner, B. Muller, PLB219 (1989) 199.

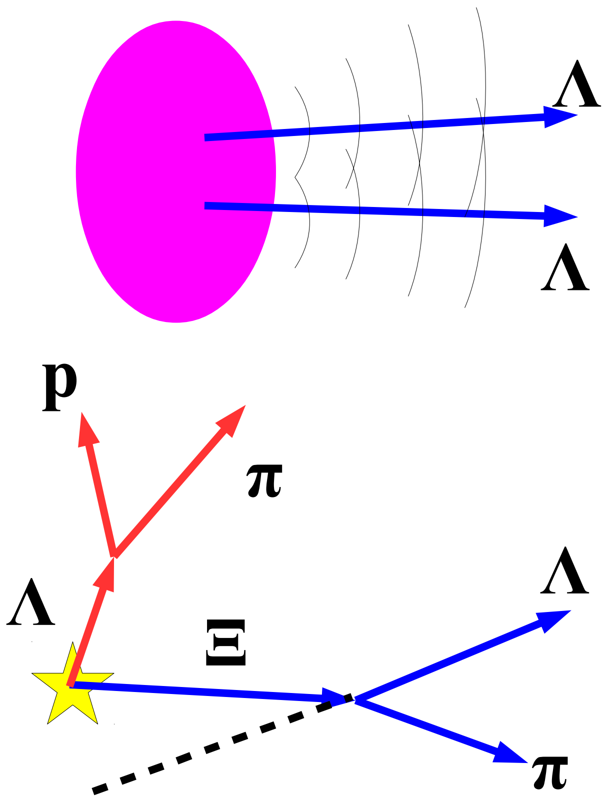
Relevance of ΛΛ interaction to physics

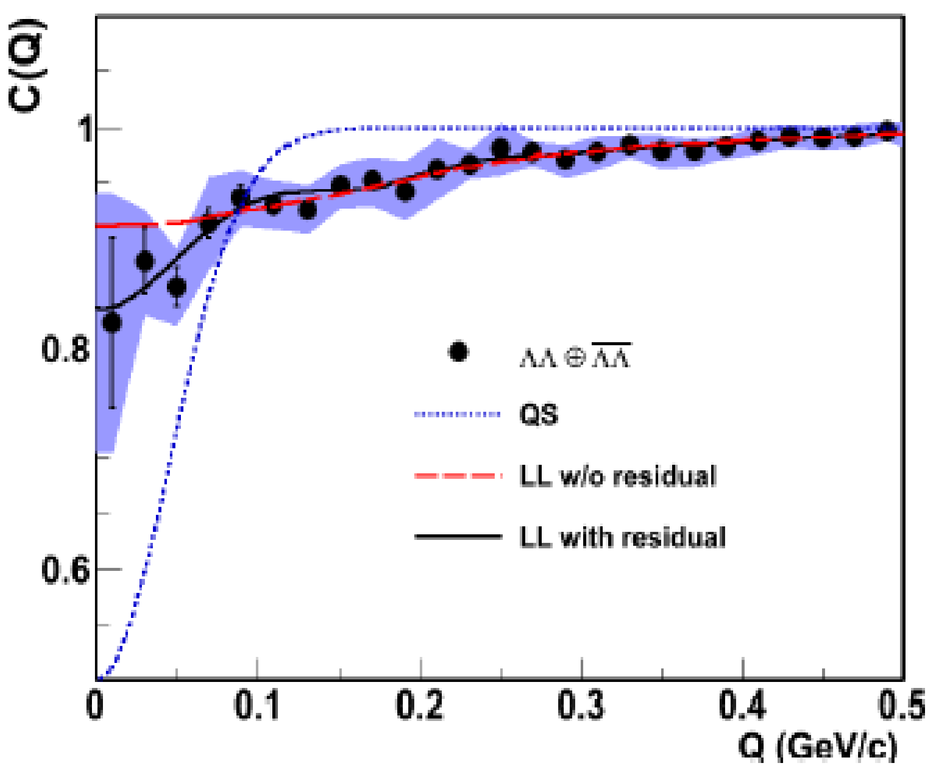
- H-particle: 6-quark state (uuddss)
  - Prediction: *R.L.Jaffe, PRL38(1977)195*
  - Ruled-out by double Λ hypernucleus
    Takahashi et al.,PRL87('01) 212502
  - Resonance or Bound “H” ?
    Yoon et al.(KEK-E522) ('07)
  - Lattice QCD
    HAL QCD & NPLQCD ('11)
- Neutron Star Matter EOS
  - Hyperon Puzzle
    Demorest et al. ('10), Antoniadis et al. ('13)
  - Cooling Puzzle (ΛΛ superfluidity)
    T. Takatsuka, R. Tamagaki, PTP 112('04)37
- QGP signal, BB interaction model, ….



Measurement at RHIC

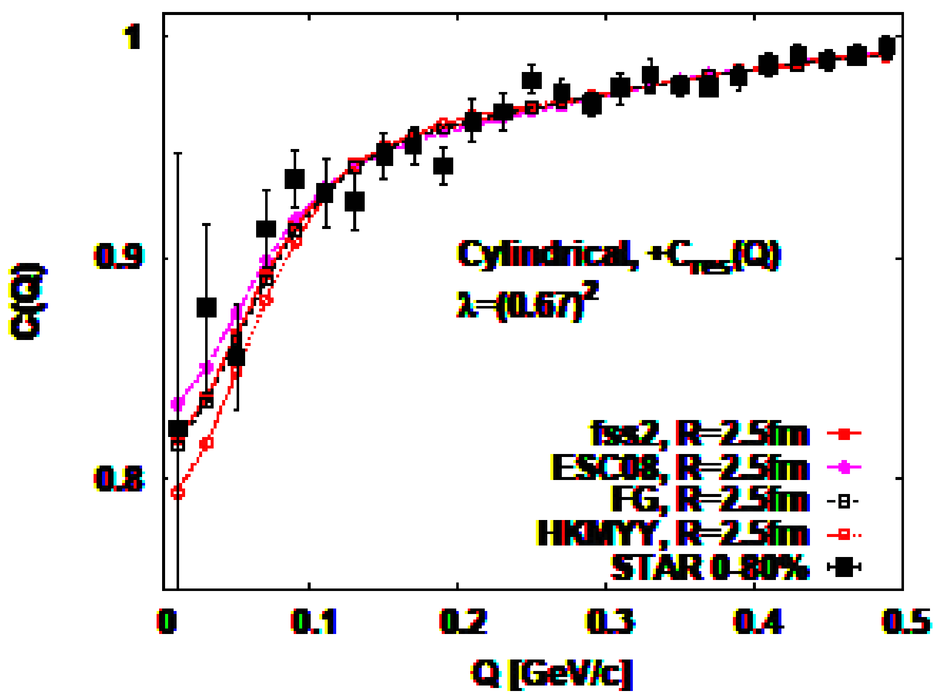
- **STAR collaboration at RHIC measured ΛΛ correlation !**
  Adamczyk et al. (STAR Collaboration), PRL 114 ('15) 022301.
  - **RHIC (Relativistic Heavy-Ion Collider @ Brookhaven Nat. Lab.) Au+Au (100 GeV+100 GeV per nucleon pair)**
  - **Track analysis enhances S/N ratio.**
- **Enhanced correlation from free Fermion correlation**

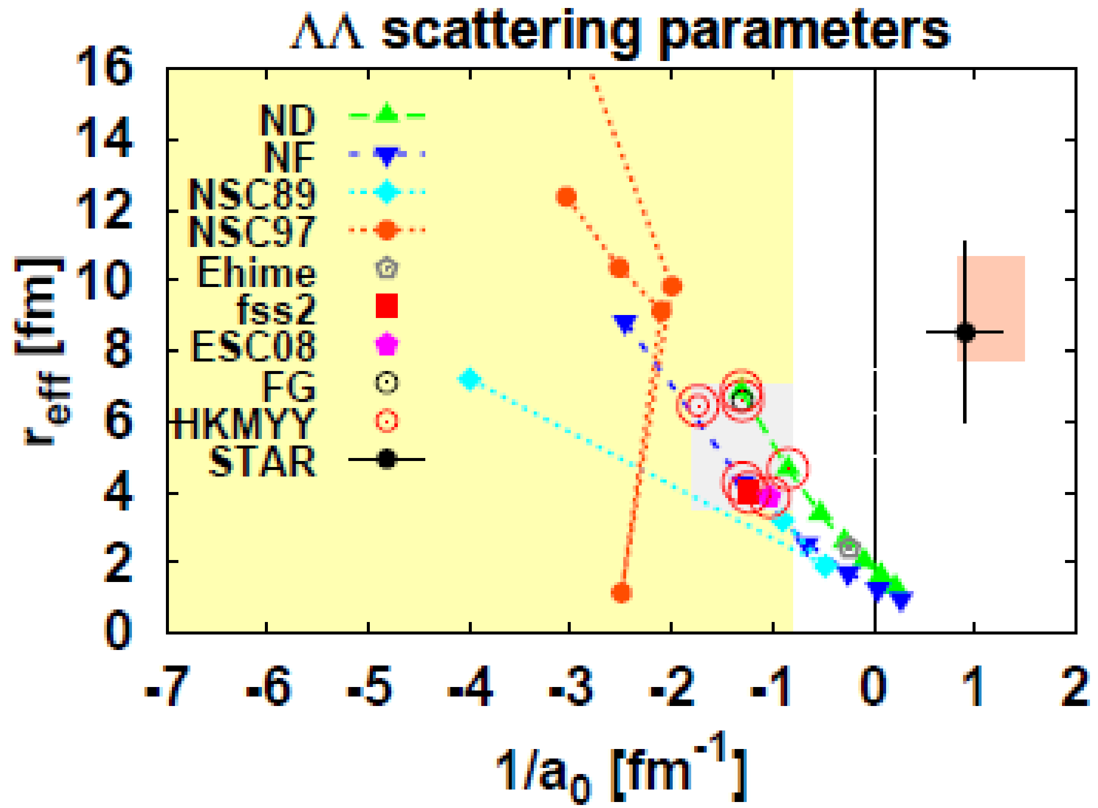




$\Lambda\Lambda$ interaction from $\Lambda\Lambda$ correlation at RHIC

- HBT + $\Lambda\Lambda$ int. + Expansion + Feed down + "Residual" source
  - $1/a_0 < -0.8\ \mathrm{fm}^{-1}$
  - Source size dependence is small, $\chi^2/\mathrm{DOF} \sim 1$
- Question: STAR analysis $a_0 = 1.10 \pm 0.37$ fm > 0





K.Morita, T.Furumoto, AO, PRC91('15)024916 [arXiv:1408.6682]
Data: Adamczyk et al. (STAR Collaboration), PRL 114 ('15) 022301.

Summary

- **From two-particle intensity correlation in heavy-ion collisions, it is possible to constrain interaction.**
  - **Standard HBT: Correlation (+interaction) → Source size**
  - **Another use of HBT: Correlation + Source info. → Interaction**
- **STAR data suggest weakly attractive Λ Λ interaction, which is consistent with double Λ hypernuclear data.**
  - **We asked an experimentalist (H.Huang) to measure ΛΛ in 2010 (YIPQS molecule), and STAR really analyzed it !**
- **This method can be applied to other hadron interactions.**
  - **Ω^- p correlation (HALQCD predict a bound state) Morita, AO, Hatsuda, in prep.**
  - **K^- p correlation (related to Λ(1405)) Miyahara, Hyodo, Morita, AO, in prog.**
  - **Ξ^- p, Λp, ….**

Thank you !

ΛΛ interaction models

- **Boson exchange potential**

  Nijmegen potentials (ND, NF, NSC89, NSC97, ESC08)

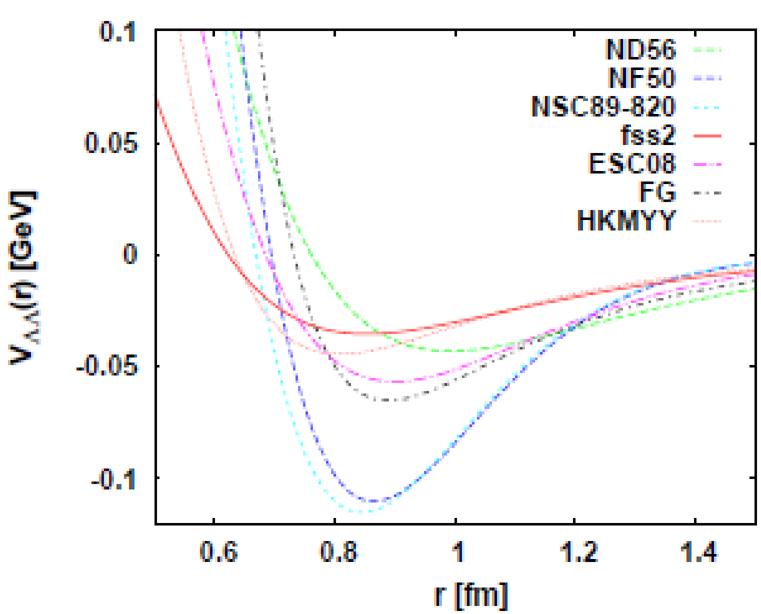
  Nagels+('77, '79), Maessen+('89), Rijken+('99,'10)

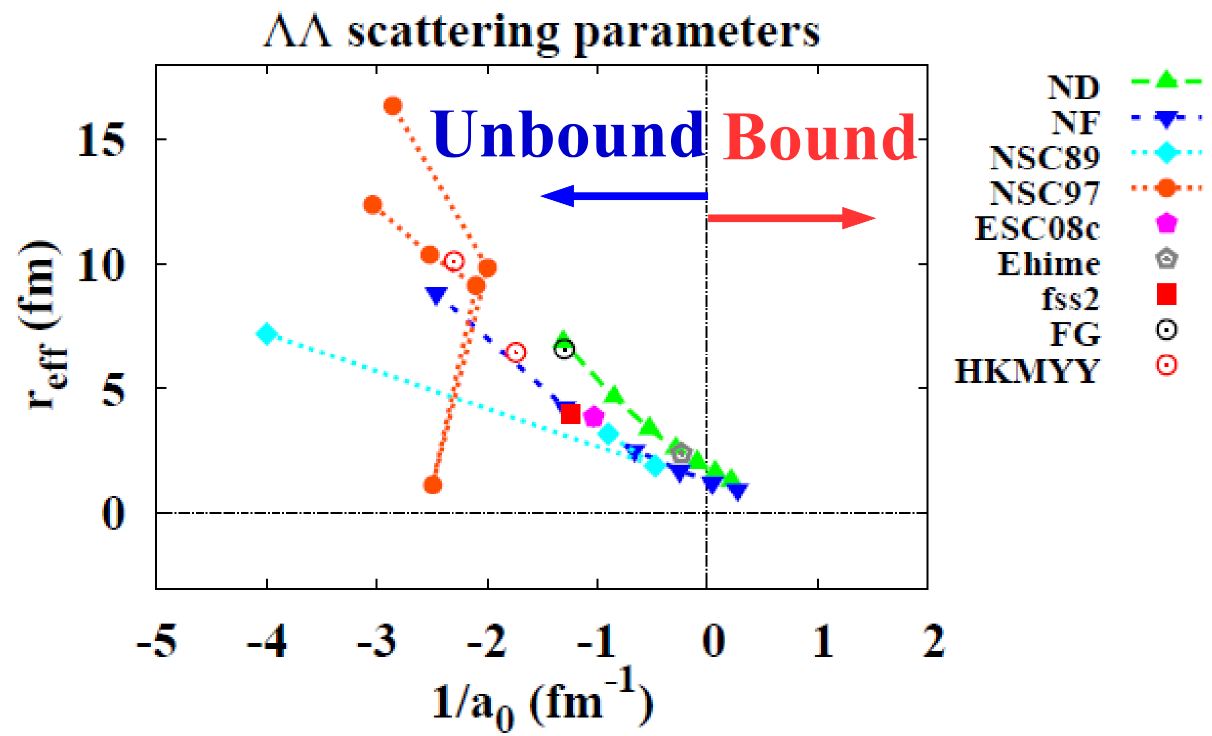
  Ehime *Ueda et al. ('98)*

- **Quark model interaction:** **fss2** **Fujiwara et al.('07)**

- **Tuned potential to Nagara**

  Filikhin, Gal ('02) (FG), Hiyama et al. ('02, '10)(HKMYY)





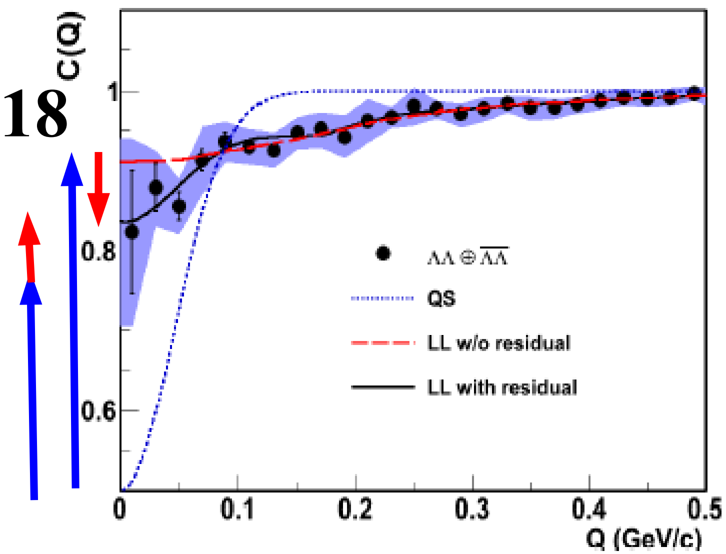
Measurement at RHIC

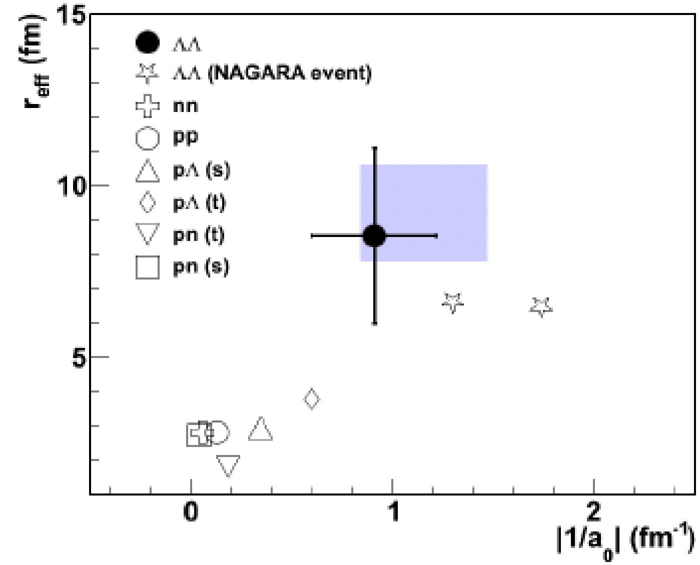
- **Lednicky-Lyuboshits (LL '81) model**
  R. Lednicky, V. L. Lyuboshits, Sov.J.Nucl.Phys.35('82)770; Yad.Fiz.35 ('81) 1316.
  - **Analytic model including a_0 and r_{eff}.**
- **Intercept (pair purity) parameter is treated as a free parameter.**
- **STAR Collab. analysis implies positive scatt. length.**

$$a_0 = 1.10 \text{ fm} \ , \quad r_{\rm eff} = 8.52 \text{ fm} \ , \quad \lambda = 0.18$$

$\lambda = 0.18$

$\lambda = (0.67)^2$



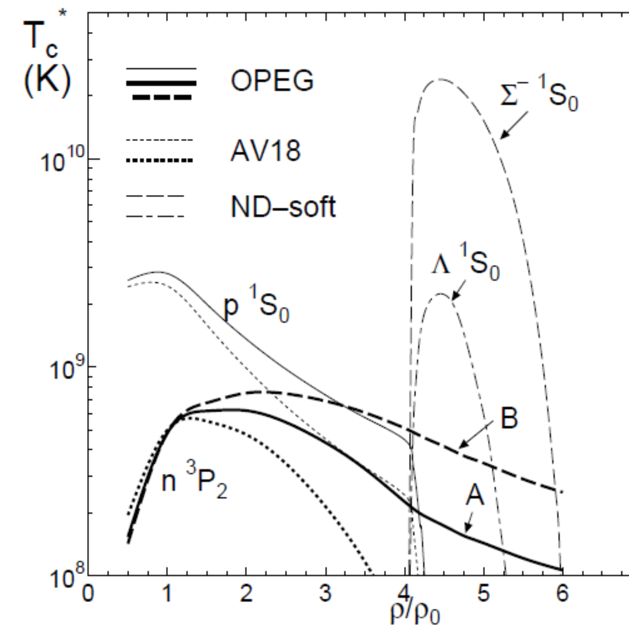


Adamczyk et al. (STAR Collaboration), PRL 114 ('15) 022301.

Which is correct ? Positive a_0 or Negative a_0 ?

ΛΛ interaction and Neutron Star

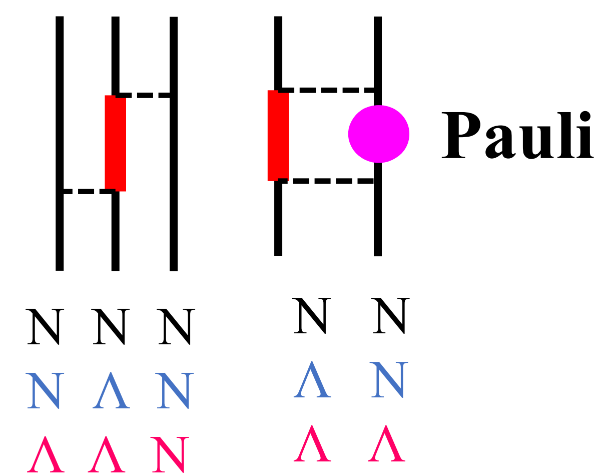
- **Hyperon superfluidity puzzle**
  - "Exotic" components are necessary to explain fast cooling of some of NSs.
  - "Exotic" superfluidity is necessary to forbid too fast cooling.
  - Nagara-fit ΛΛ interaction is too weak for Λ superfluidity.



T. Takatsuka, R. Tamagaki, PTP 112('04)37

- **Massive Neutron Star Puzzle**
  - Hyperons should appear in dense matter.
  - EOSs w/ Y are too soft to support 2 $M_\odot$ NS
  - Three-body repulsion involving Y may help to stiffen EOS at high density.

    NS session (Gandolfi, Bombaci, Yamamoto, Takatsuka, Togashi, Lonardoni, Muto, Miyatsu, Nakamoto)Kohno ('13) / Myint, Shinmura, Akaishi ('03) / *Machleidt*.



Impact of ΛΛ interaction

- **ΛΛ interaction is relevant to the existence of H dibaryon.**
  - **One of the long-standing problem in hadron physics.**
- **ΛΛ interaction is important to understand BB interaction models.**
  - **No pion exchange → middle and core range int. are visible.**
  - **No quark pauli blocking in the flavor singlet (H) channel.**
- **ΛΛ and ΛΛN interaction is important to neutron stars.**
  - **Hypern superfluidity is preferred to explain NS cooling.**
  - **3-body force including Y is necessary to support 2 $M_\odot$ NS. $V_{\Lambda\Lambda}$ in vacuum & medium → $V_{\Lambda\Lambda N}$**

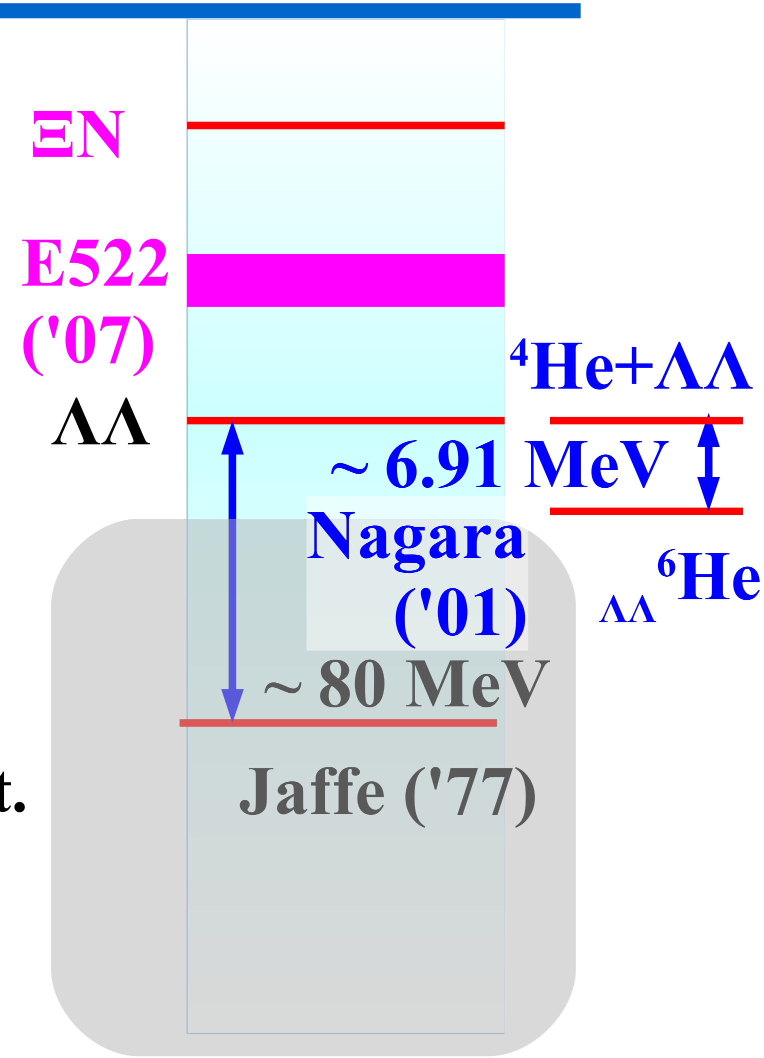
Other observables than double hypernuclei ?
→ ΛΛ intensity correlation from heavy-ion collisions
Recent data by STAR Collab.
Talk by N. Shah (Adamczyk et al., PRL 114 ('15) 022301.)

Contents

- Introduction
- Λ Λ correlation and interaction from heavy-ion collisions
  K. Morita, T. Furumoto, AO, PRC
  - Comparison of correlation function obtained with various ΛΛ model interactions with STAR data.
  - Expansion effects, Feeddown effects, and Residual source effects
  - Favored ΛΛ interaction
- ΛΛ scattering length: Negative or Positive ?
  - STAR analysis: $a_0 > 0$, MFO analysis: $a_0 < 0$
  - Re-analysis based on Lednicky-Lyuboshits '81 model
- Summary

Relevance of ΛΛ interaction to physics

- **Deeply bound H ?**
  - **Strong Attraction from Color Mag. Int.**
    → 80 MeV below ΛΛ
    R.L.Jaffe, PRL38(1977)195
  - **No observation for a long time.**
  - **Repulsive Instanton Ind. Int.**
    S.Takeuchi, M.Oka, PRL66 ('91)1271
  - **Nagara event $_{\Lambda\Lambda}{}^{6}$He**
    No deeply bound "H", Weakly Att. ΛΛ int.
    Takahashi et al.,PRL87('01) 212502
  - **Resonance or Bound "H" ?**
    - **2 σ "bump" at $E_{\Lambda\Lambda}$ ~ 15 MeV**
      Yoon et al.(KEK-E522) ('07)
    - **Bound H at large *ud* quark masses**
      HAL QCD & NPLQCD ('11), Haidenbauer, Meissner ('11)

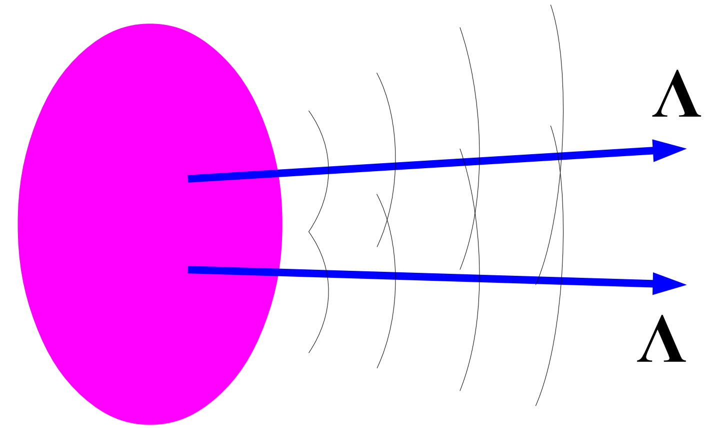


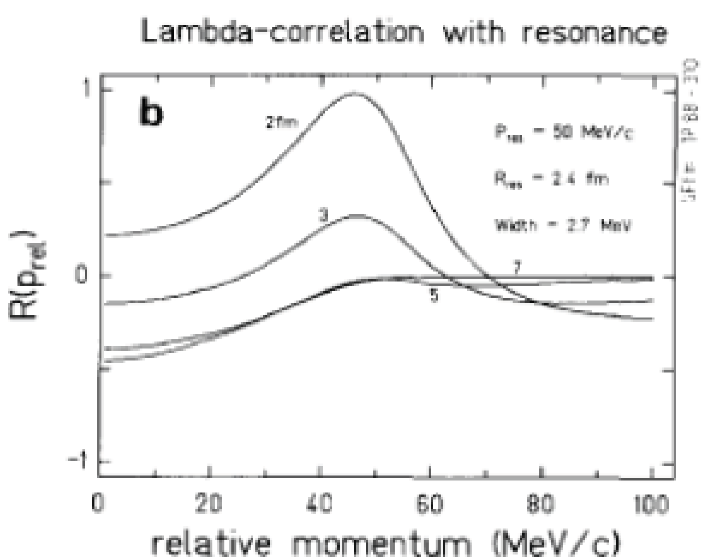
ΛΛ correlation and interaction from heavy-ion collisions

K. Morita, T. Furumoto, AO,
Phys. Rev. C91 (2015), 024916 [arXiv:1408.6682]

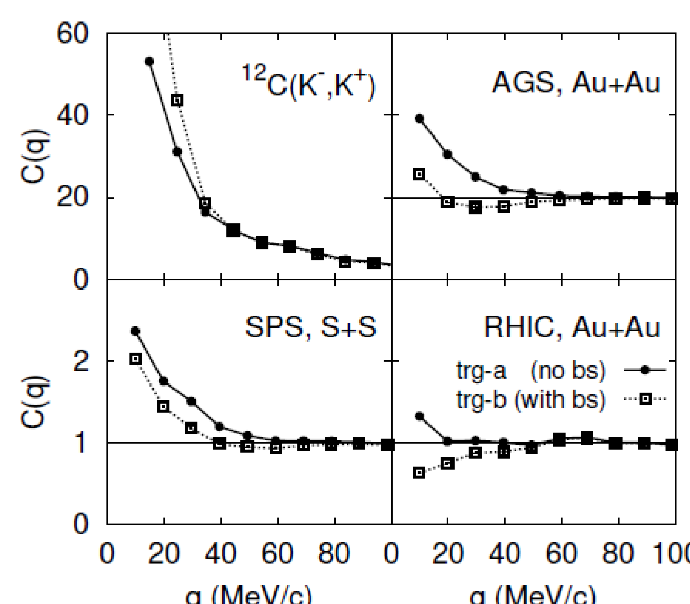
ΛΛ correlation in HIC

- **Merit of HIC to measure ΛΛ correlation**
  - **Source is "Simple and Clean" !**
    T, μ, flow, size, … are well-analyzed.
  - **Nearly Stat. prod.**
    → Many exotics will be produced.
    Schaffner-Bielich, Mattiello, Sorge ('00), Cho et al.(ExHIC Collab.) ('11)
  - **Discovery of "H" and/or Constraint on ΛΛ int.**
    Bound state exhaust the low q strength → suppressed C(q).





C. Greiner, B. Muller, PLB219('89)199.



AO, Hirata, Nara, Shinmura, Akaishi, NPA670('00)297c

ΛΛ correlation in HIC and ΛΛ interaction

- **Two particle correlation from chaotic source** *Bauer, Gelbke, Pratt ('92)*

$$C_{\Lambda\Lambda}(q)=\frac{\int dx_1 dx_2 S(x_1,p+q) S(x_2,p-q) |\psi^{(-)}(x_{12},q)|^2}{\int dx_1 dx_2 S(x_1,p+q) S(x_2,p-q)}$$

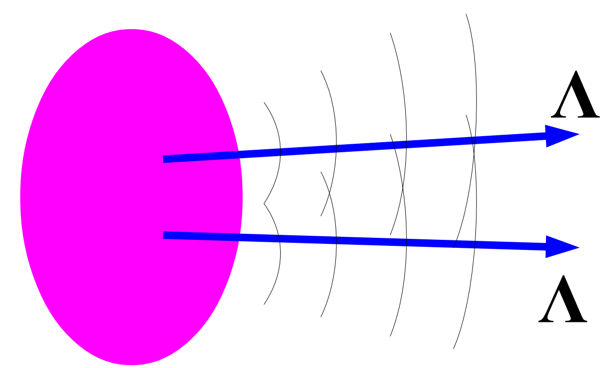
- **Static spherical source, s-wave only** **s-wave w.f. enh.**

$$C_{\Lambda\Lambda}(q)\simeq 1 \underbrace{-\frac{1}{2}\exp(-4q^2R^2)}_{\text{HBT}} + \frac{1}{2}\int dr\, S_{12}(r)\left(|\chi_0(r)|^2-|j_0(qr)|^2\right)$$

HBT

$q=(p_1-p_2)/2$, χ_0: s-wave wf, $S_{12}(x)=r^2 \exp(-r^2/4R^2)/2R^3\sqrt{\pi}$

- **HBT term: Suppression due to anti-symmetryzation for Fermions**
  0 x 3/4 (triplet) + 2 x 1/4 (singlet) = 1/2
  Hanbury Brown, Twiss ('56),
  Goldhaber, Goldhaber, Lee, Pais ('60)
- **Enhancement of wf by ΛΛ attraction**



Correlation function and scattering length

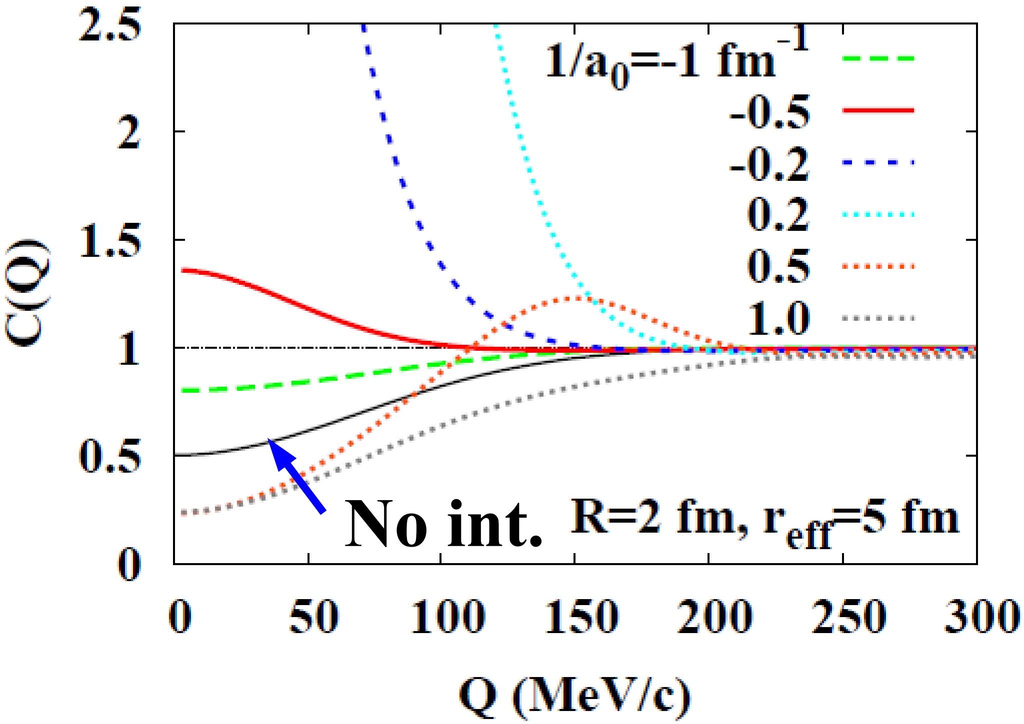
- Low E scatt. is mainly determined by the scattering length (a_0)

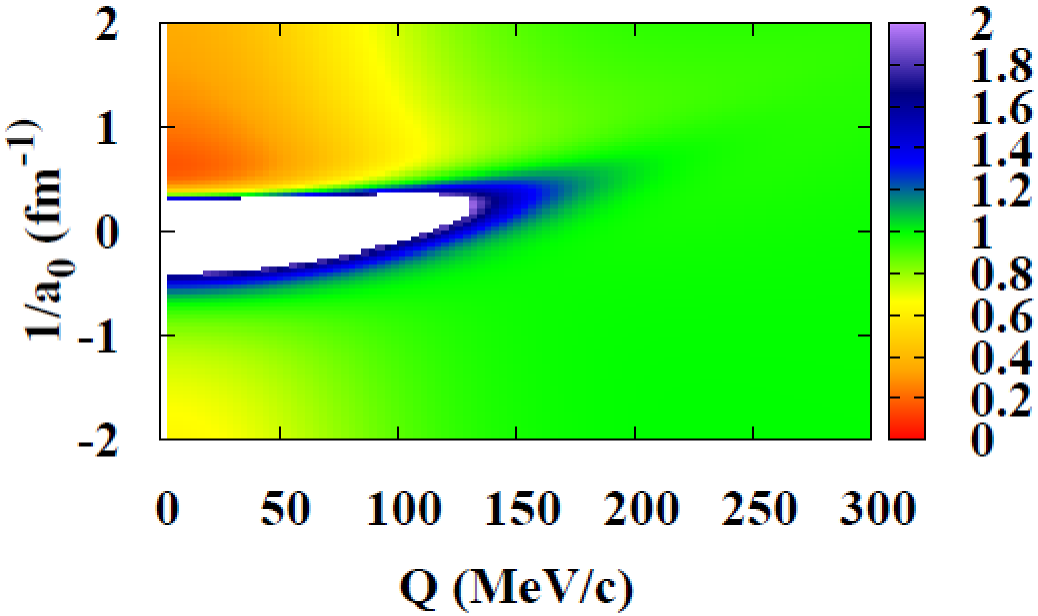
  $q \cot \delta = -1/a_0 + r_{eff}\, q^2/2 + O(q^4)$ Nuclear phys. convention

  - Negative a_0 : enhanced correlation
  - Positive a_0 : suppressed correlation

    w/ bound state → scatt. wf has a node

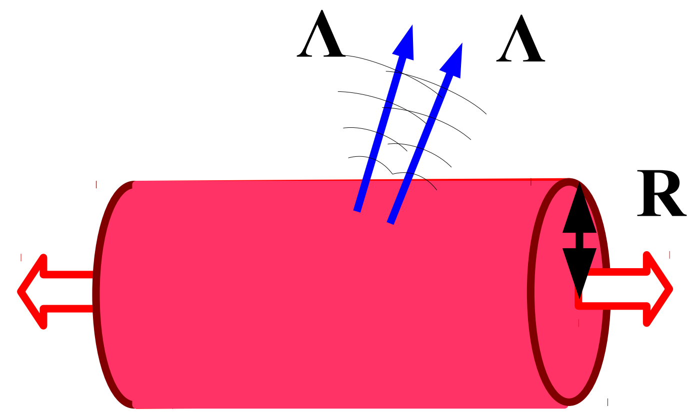
    w/o bound state → repulsion suppress wf



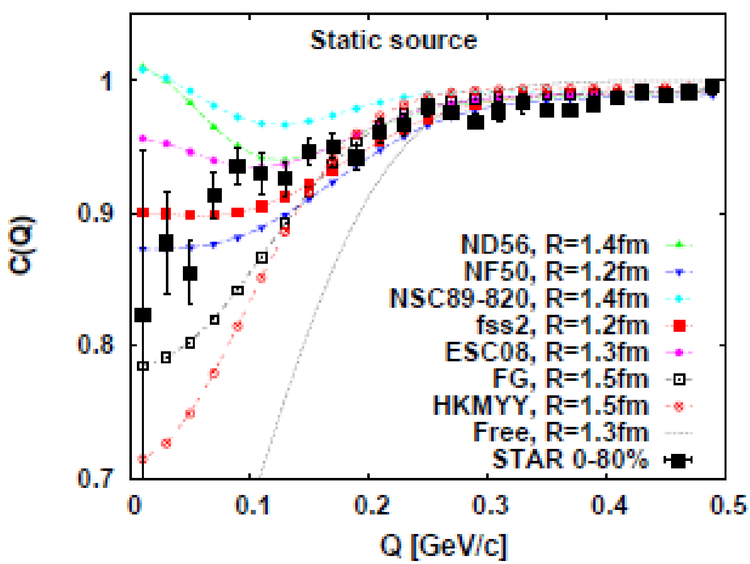


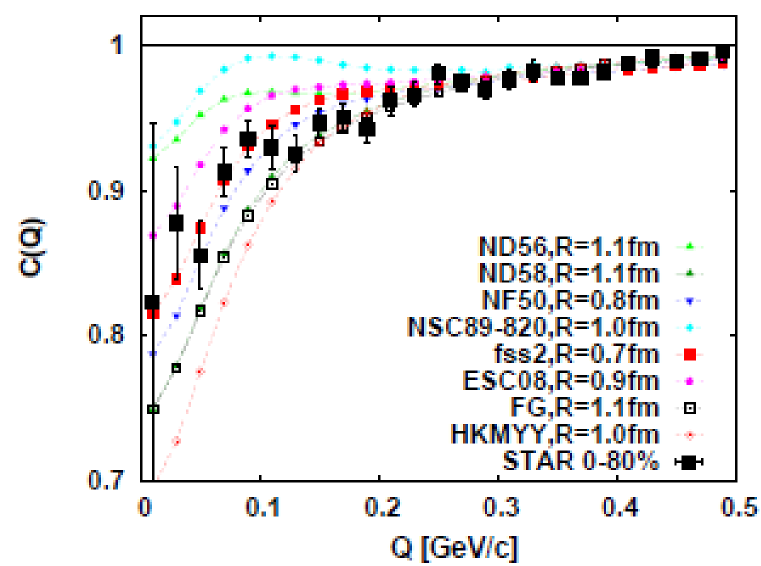
Source models

- Static & Spherical source
  - Source size R = (1-1.5) fm, χ^2/DOF ~ 2
- Cylindrical source
  - Bjorken expansion in long. direction
  - Transverse flow from pT spectrum
  - Source size R = (0.7-1.1) fm, χ2/DOF ~ 1.5



$$S_{\rm cyl}(x,k) = \frac{m_T \cosh(y - Y_L)}{(2\pi)^3\sqrt{2\pi(\Delta\tau)^2}} n_f(u\cdot k, T)\ \exp\left[-\frac{(\tau-\tau_0)}{2(\Delta\tau)^2} - \frac{x^2+y^2}{2R^2}\right]$$





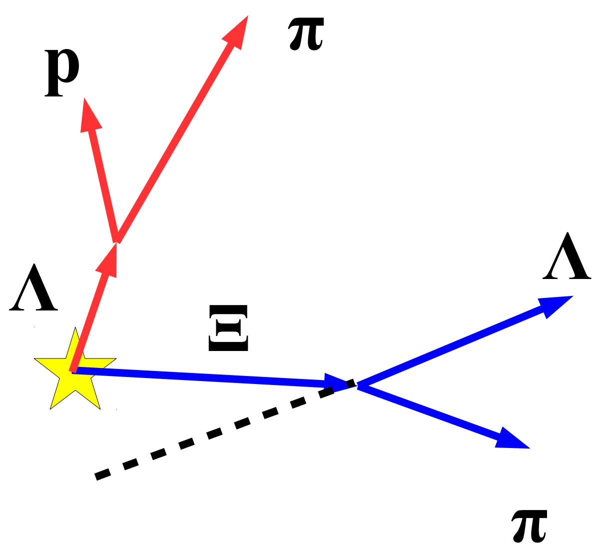
Additional Source

- **Feed down effects**

$$C_{\mathrm{corr}}(Q) = 1 + \lambda(C_{\mathrm{bare}}(Q) - 1)$$

λ = Purity of ΛΛ pair

  - **Short-lived Y* → mod. of source fn.**
  - **Ξ → Λπ can be excluded (cτ=8.71 cm)**
  - **$\Sigma^0 \to \Lambda\gamma$ is difficult to reject**
  - **Data based purity $\lambda=(0.67)^2$**
    Σ^0/Λ=0.278 (p+Be, 28.5 GeV/c) ***Sullivan et al. ('87)***
    Ξ/Λ = 15 % (RHIC)



- **"Residual" source**
  - **High-momentum tail → R_{res} ~ 0.5 fm (STAR collab.)**

$\Lambda\Lambda$ scattering length
Positive or Negative ?

- **Lednicky-Lyuboshits (LL '81) model**
  R. Lednicky, V. L. Lyuboshits, Sov.J.Nucl.Phys.35('82)770; Yad.Fiz.35 ('81) 1316.
  - **Analytic model including FSI effects**

$$C(Q) = N\left[1 + \lambda\left(-\exp(-R^2Q^2)/2 + C_{\text{int}}(Q) - 1\right) + a_{\text{res}}\exp(-R_{\text{res}}^2 Q^2)\right]$$

$$C_{\text{int}}(Q) = 1 + \frac{|f(k)|^2}{4R^2}\left(1 - \frac{r_{\text{eff}}}{2\sqrt{\pi}R}\right) + \frac{\text{Re}f(k)}{\sqrt{\pi}R}F_1(QR) - \frac{\text{Im}f(k)}{2R}F_2(QR)$$

$$f(k) = \left(-1/a_0 + r_{\text{eff}}k^2/2 - ik\right)^{-1} \quad (k = Q/2)$$ **scattering amp.**

$$F_1(z) = \int 0^z e^{x^2 - z^2}/z\ , \quad F_2(z) = (1 - e^{-z^2})/z$$

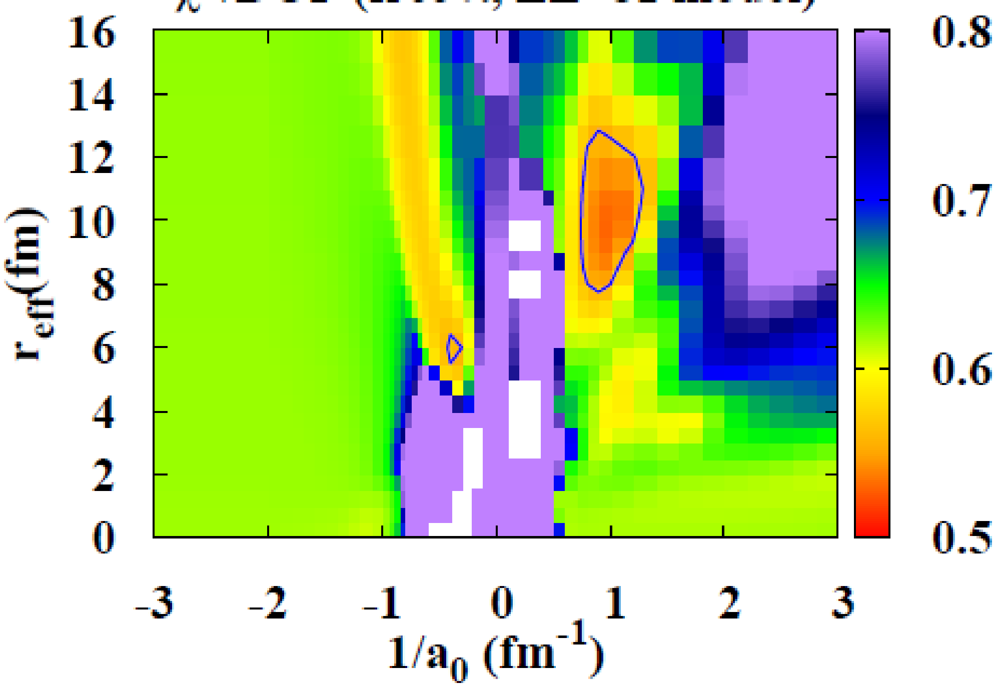
$$k\cot\delta = -\frac{1}{a_0} + \frac{1}{2}r_{\text{eff}}k^2 + \mathcal{O}(k^4)$$ **phase shift**

Let us examine LL '81 model results with fixed λ !

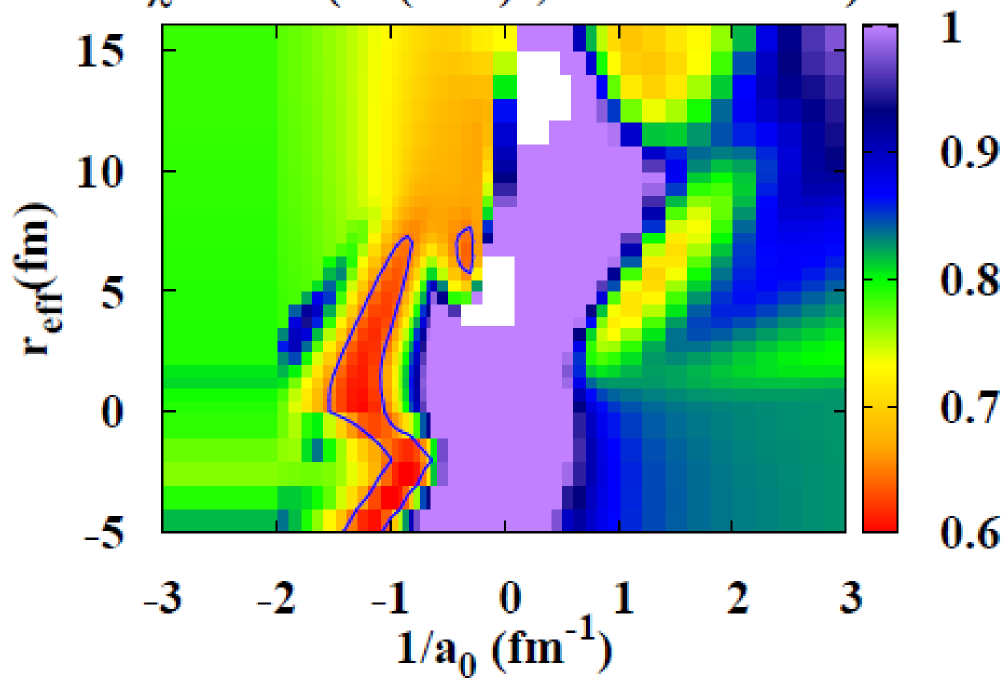
a_0 ($\Lambda\Lambda$): Positive or Negative ?

- **Key parameter = intercept parameter λ**
  - λ = free parameter ~ 0.18 → positive a_0 at χ^2 min. (χ^2/DOF ~ 0.56)
  - Measured Σ^0 / Λ ratio
    $\lambda = (\Lambda / (\Lambda + \Sigma^0))^2 \sim (0.67)^2$ → negative a_0 at χ^2 min.
    (χ^2/DOF ~ 0.65)

χ^2/DOF (free λ, LL '81 model)



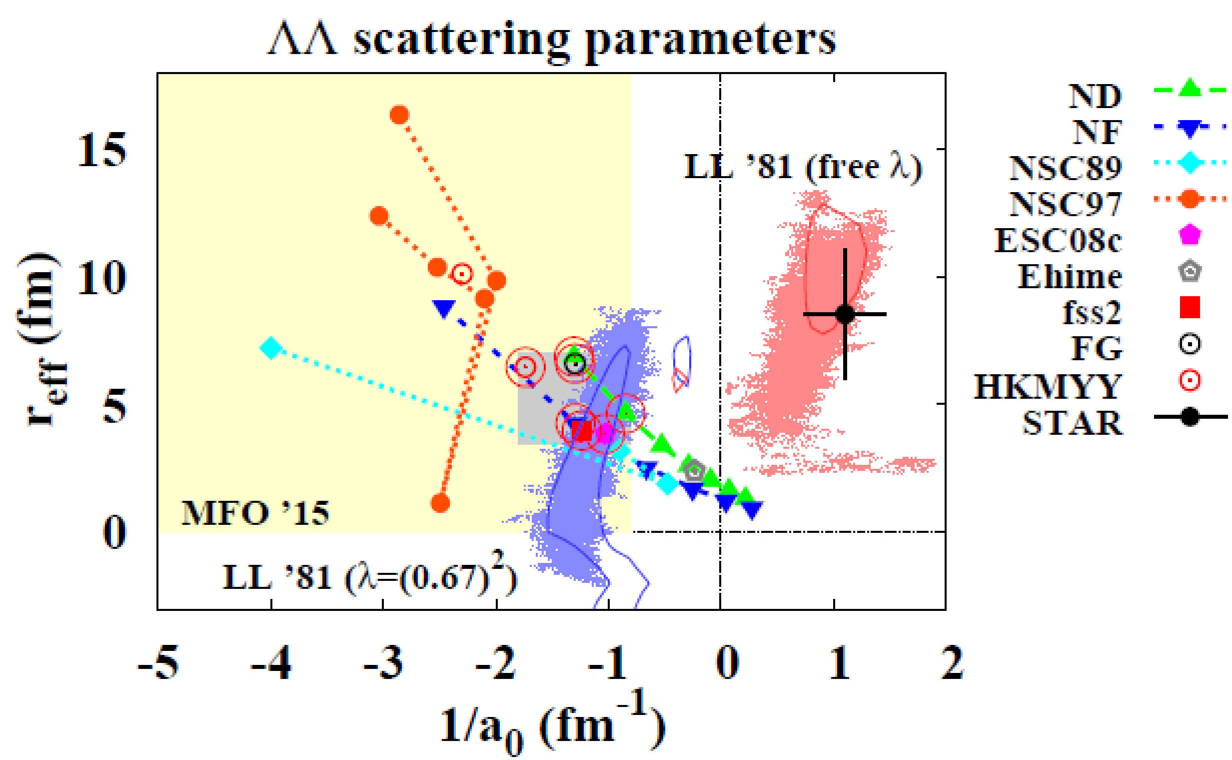
χ^2/DOF ($\lambda=(0.67)^2$, LL '81 model)



a_0 (ΛΛ): Positive or Negative ?

- χ^2 analysis
  - Gradient method (gnuplot) & Markov Chain MC (MCMC) in LL '81 model
  - Free λ result ~ STAR ('15) analysis : $a_0 > 0$
  - Fixed λ result ~ Our analysis : $a_0 < 0$

Difference btw STAR collab. analysis and our analysis comes from λ

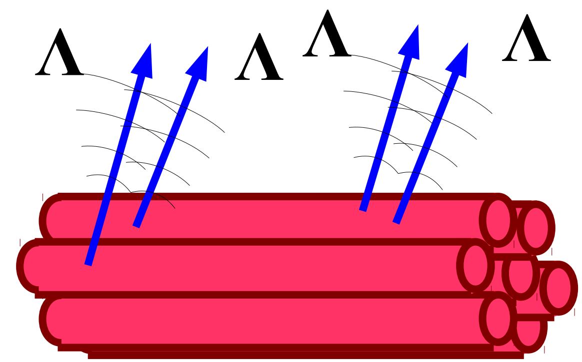


What is the origin of the long tail ?

- **Do we have a physical origin ?**
- **Two source model + LL '81 model**

$$S_{12}(\mathbf{x}) = \frac{w}{(2R_1\sqrt{\pi})^3}\exp\left(-\frac{x^2}{4R_1^2}\right) + \frac{1-w}{(2R_2\sqrt{\pi})^3}\exp\left(-\frac{x^2}{4R_2^2}\right)$$

  - **Fix Λ Λ interaction, and obtain R_1 and R_2.**
    R_1 = (0.65-1.30) fm
    R_2 = (0.33-0.54) fm
    for fss2, ESC08, FG, HKMYY interactions.
    χ^2/DOF = (0.6-0.65)



- **Λ might be produced from small tubes.**
  - **FSI with hot medium (~ π gas) is small for Λ**

Summary

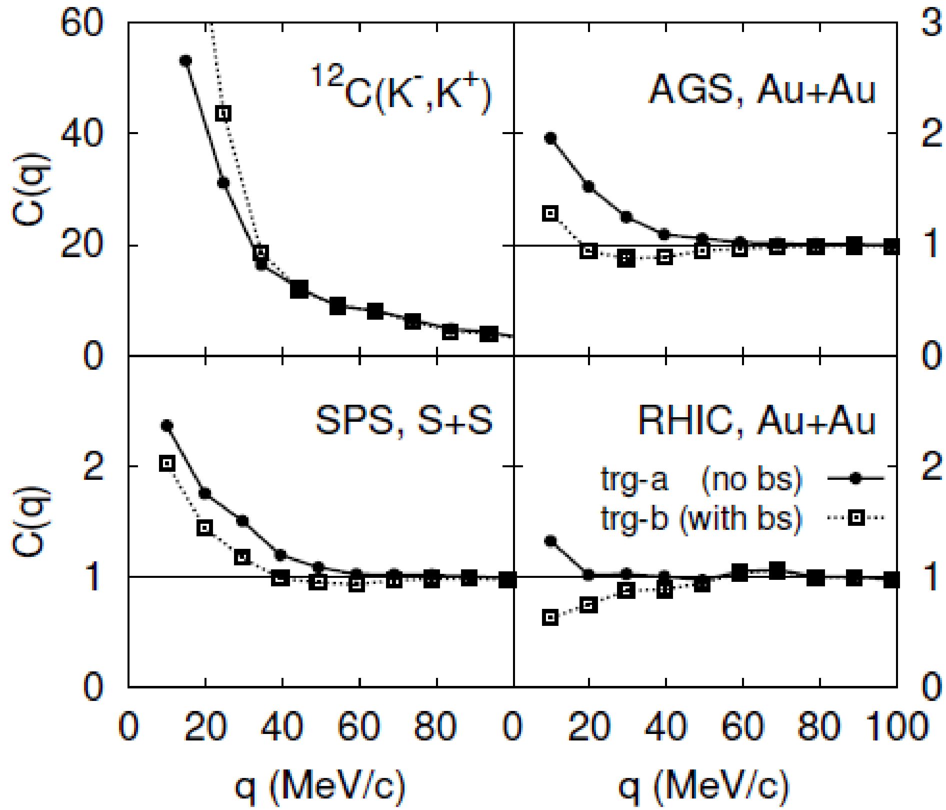
- ΛΛ intensity correlation in high-energy heavy-ion collisions has sensitivity to ΛΛ interaction.
- Our analysis of the STAR data implies that the favored ΛΛ interaction has negative scattering length ($1/a_0 < -0.8\ \mathrm{fm}^{-1}$, $\delta > 0$), which is consistent with the Nagara event analysis.
  - Anti-symm. + ΛΛ interaction + Expansion + Feeddown + Residual
- Difference between the STAR collab. analysis and ours lies in the assumption on the pair purity (chaoticity, intercept) parameter.
  - $\Sigma^0/\Lambda \sim 0.67$: p+Be, consistent with stat. model (T~170 MeV)
- Further studies are necessary to pin-down ΛΛ interaction.
  - Higher precision data are expected, Σ^0 detection is desired, Comparison with $A(K^-, K^+)\Lambda\Lambda$ reaction *C.J.Yoon et al.('07)/J-PARC*
- We can access other hh interactions using correlations.
  - ΩN interaction from ΩN correlation *K. Morita's talk*

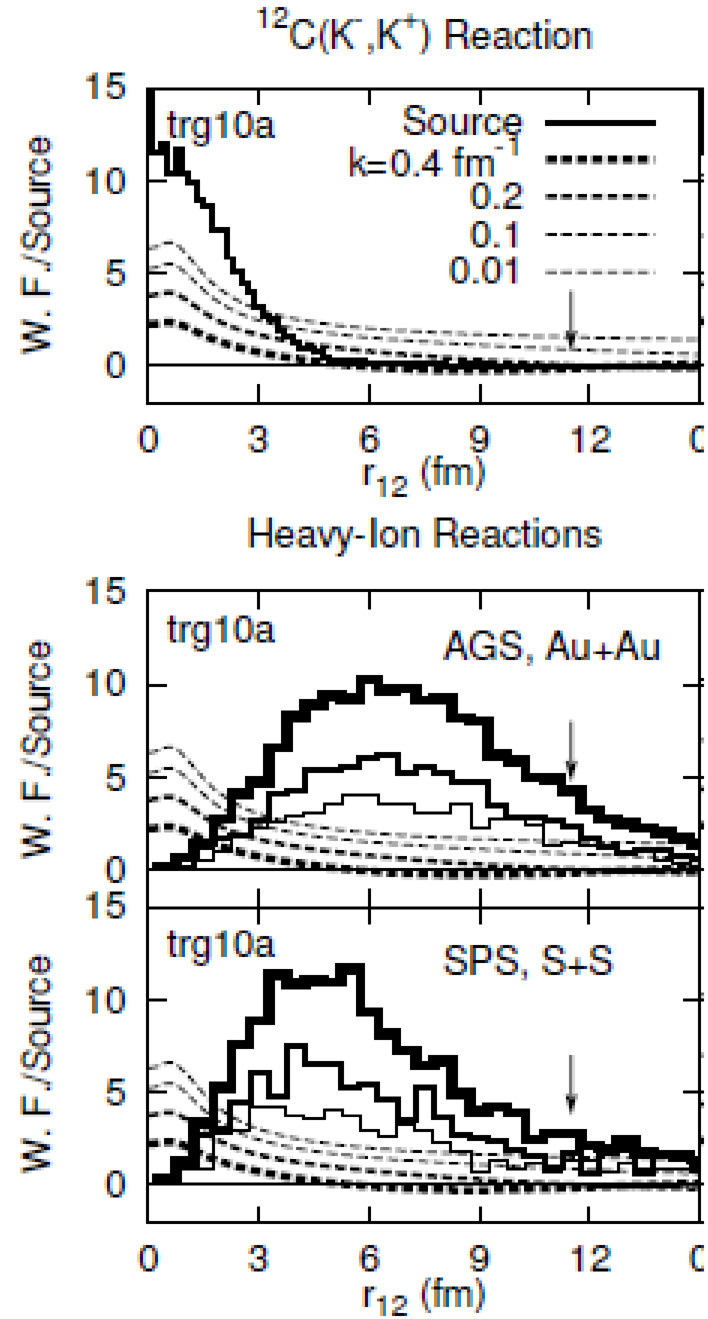
Nuclear Physics, Compact Stars, and Compact-star Mergers 2016 (NPCSM 2016)

- **Dates :**
  Oct.17 (Mon.)-Nov.18 (Fri.), 2016 (2016 Fall, 5 weeks).
- **Organizers:**
  Ohnishi (YITP, co-chair), M. Shibata(YITP, co-chair), Y. Suwa(YITP, LOC), K. Morita(YITP, LOC), H.Tamura (Tohoku U.), T.Harada (Osaka EC), H.Nakada(Chiba U.), K. Iida (Kochi U.), Sekiguchi (Toho U.), K. Sumiyoshi (Numazu), Ioka(KEK), Kotake(Fukuoka U.), Sotani(NAOJ), Wanajo (RIKEN)

Previous Work (before RHIC & Nagara)

- **Hadronic transport (JAM)**
  + Two Range Gaussian $V_{\Lambda\Lambda}$
  - **w/ bound state → w.f. node suppresses C(q**

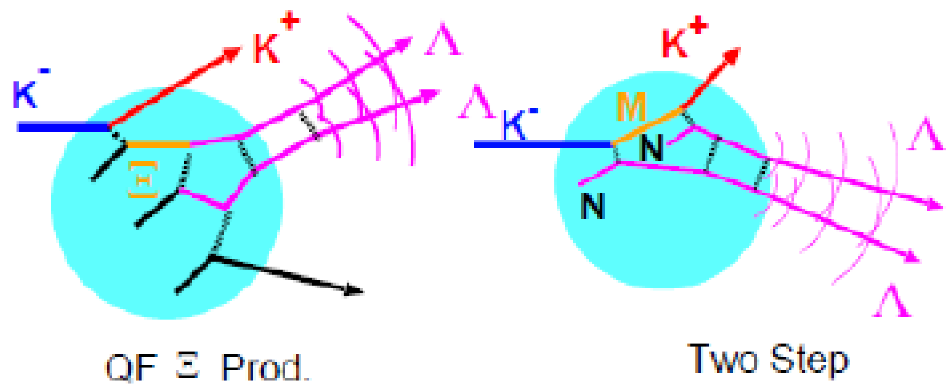


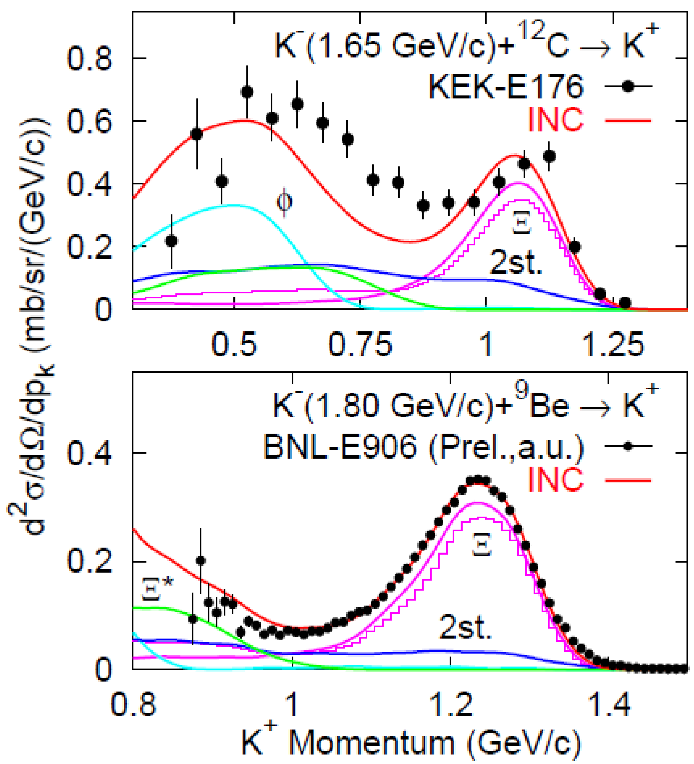


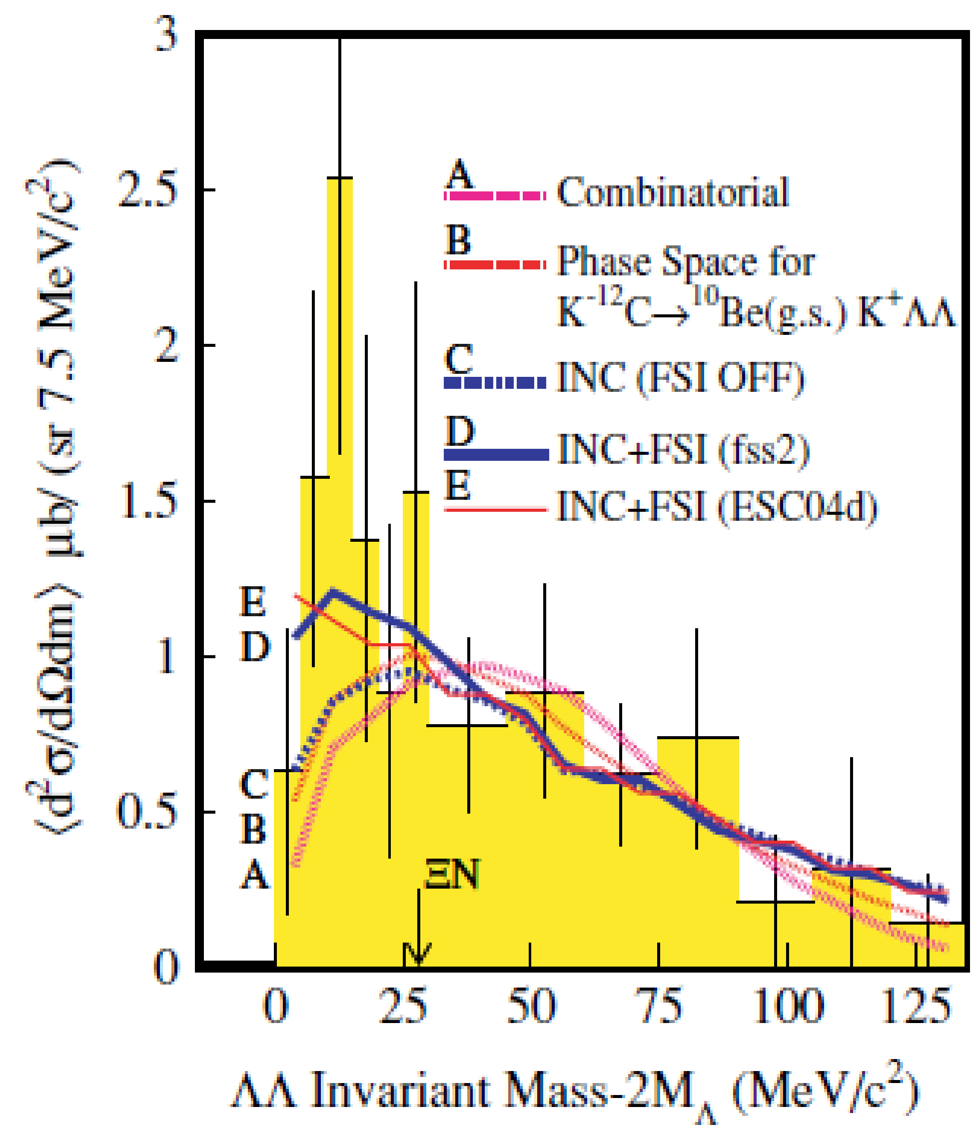
AO, Hirata, Nara, Shinmura, Akaishi, NPA670('00)297c [arXiv:nucl-th/9903021]; SNP2000 proc. p175.
JAM: Nara,Otuka, AO, Niita, Chiba, PRC61 ('00), 024901.

ΛΛ correlation from $(K^-, K^+\Lambda\Lambda)$ reaction

- **Enhancement at ~ 2 M(Λ)+ 10 MeV,**







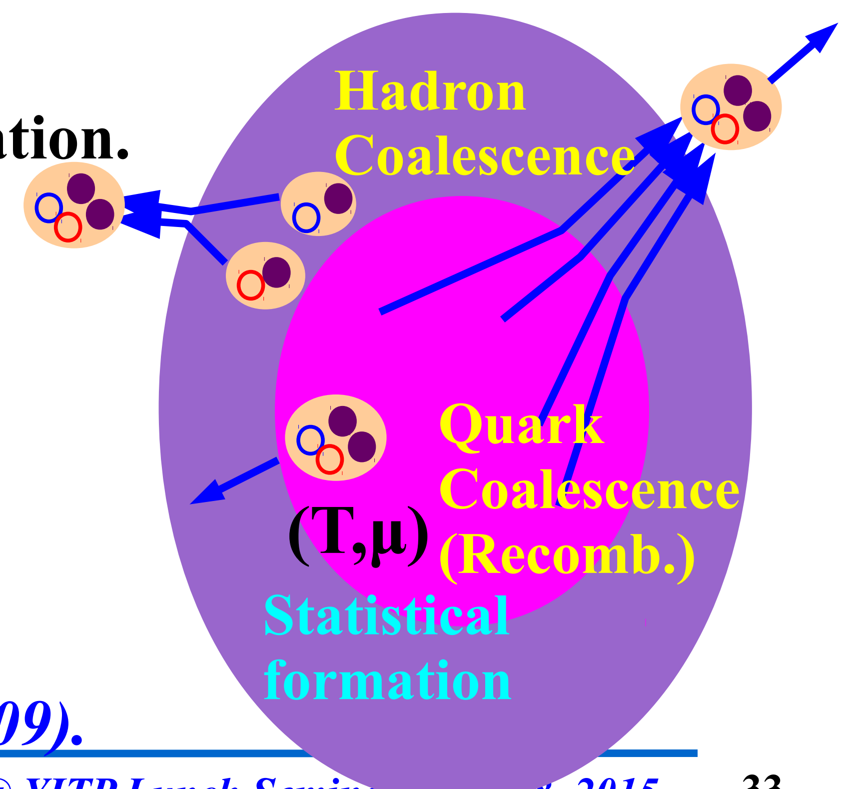
C.J.Yoon, …, (KEK-E522),AO, PRC75 (2007) 022201(R)

Exotics from Heavy Ion Collisions

- **High-Energy Heavy-Ion Collisions**
  - **Too complex → Statistical → Simple and Clean !**
  - **High *T* & Large volume → Abundant hadrons**
  - **RHIC & LHC → Nearly 4π detector / Vertex detector**

→ Let's regard RHC & LHC as Exotic Hadron Factories

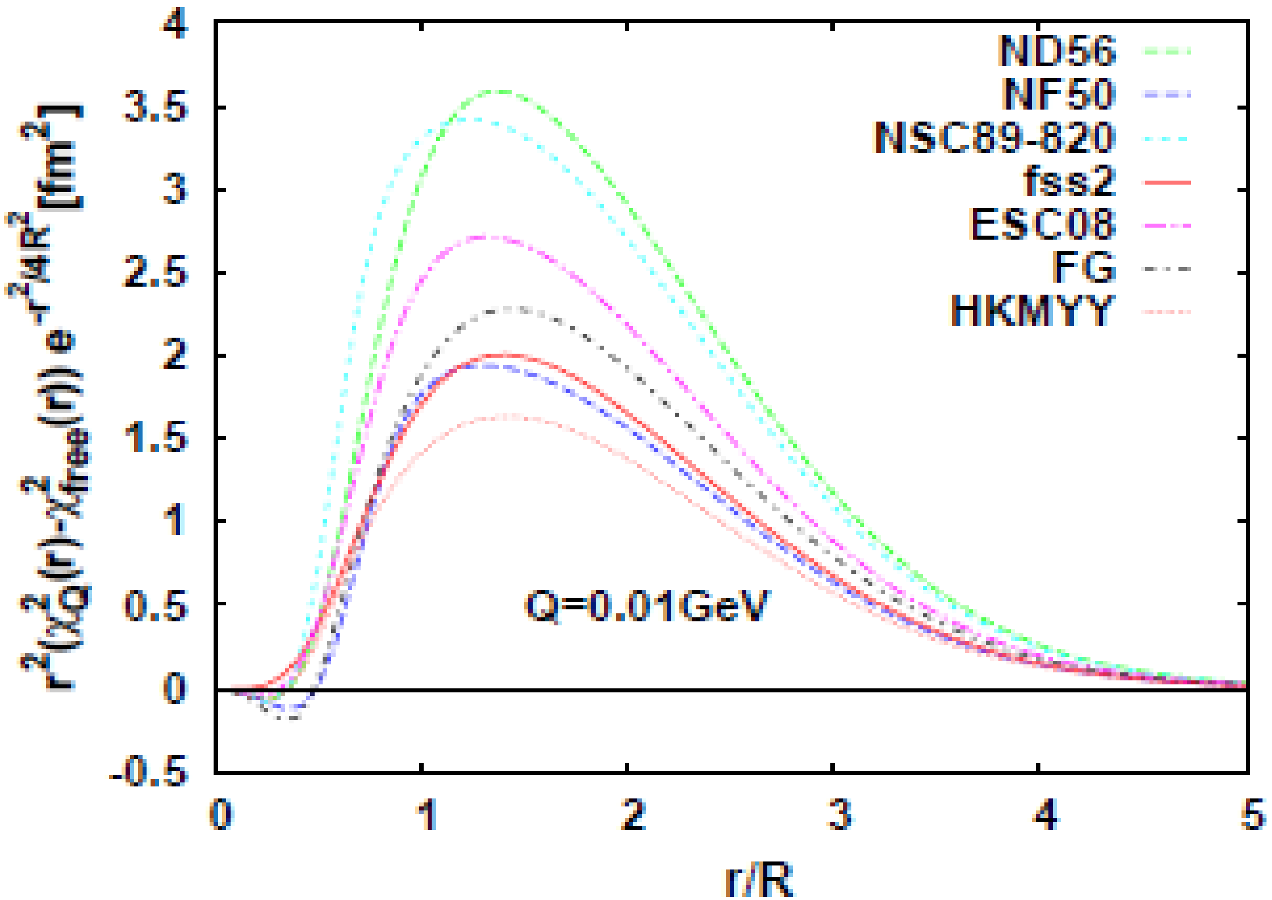
- **What can we learn ? Existence, Mass, Width, Size, Interaction, ..**
  - **Formation mechanism of hadrons**
    = Statistical , Coalescence, Fragmentation.
    → Yields are sensitive to hadron size in Coal.
    Chen, Greco, Ko, Lee, Liu, ('04)
  - **Correlation func. ~ ∫ Source x $|\mathrm{w.f.}|^2$**
    → Once we know source, Corr. Func. is sensitive to w.f. and pairwise interaction.
    Bauer, Gelbke, Pratt ('92); Lednicky ('09).



ΛΛ wave function

- **Correlation function**

$$C_{\Lambda\Lambda}(q) \simeq 1 - \frac{1}{2}\exp(-4q^2R^2) + \frac{1}{2}\int dr\, S_{12}(r)\left(|\chi_0(r)|^2 - |j_0(qr)|^2\right)$$



ΛΛ interaction models

- **Boson exchange potential**

  Nijmegen potentials (ND, NF, NSC89, NSC97, ESC08)
  Nagels+('77, '79), Maessen+('89), Rijken+('99,'10)
  Ehime Ueda et al. ('98)

- **Quark model interaction: fss2 Fujiwara et al.('07)**

- **Tuned potential to Nagara**

  Filikhin, Gal ('02) a0=-0.77 fm, reff=6.59 fm
  Hiyama et al. ('02)(HKMYY) a0=-0.575 fm, reff=6.45 fm
  Hiyama et al. ('10)(HKMYY) a0=-0.44 fm, reff=10.1 fm

- **Tuned potential to A(K-,K+)ΛΛ**

  Gasparayan et al. ('12)(GHH) a0=-1.2 fm
  A.M.Gasparyan, J. Haidenbauer, C. Hanhart. PRC85('12)015204.

Nagara event

- $_{\Lambda\Lambda}{}^{6}$He hypernuclei

  Takahashi et al., PRL87('01)212502

  (KEK-E373 experiment)

Lambpha

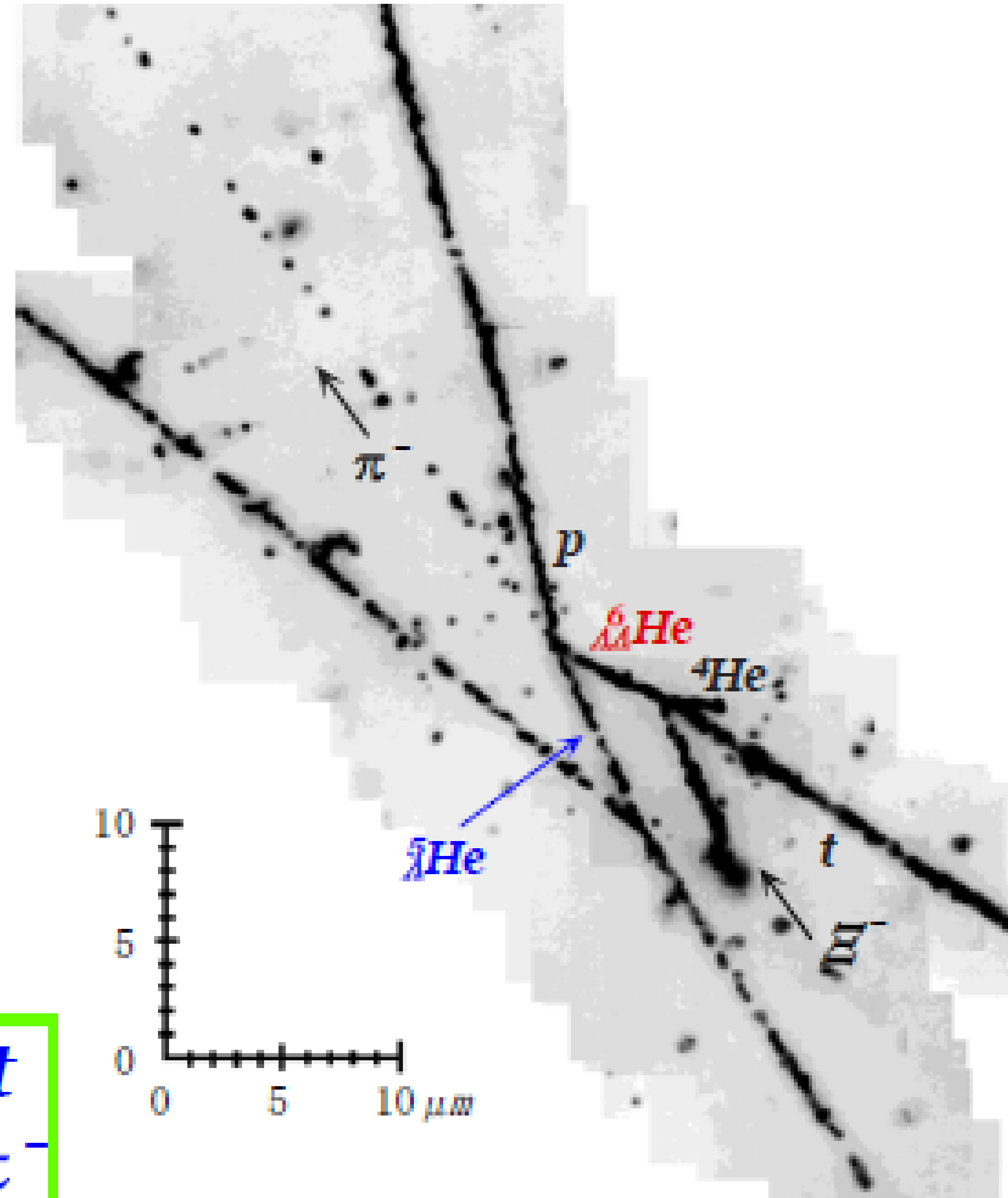
$m({}^{6}_{\Lambda\Lambda}\mathrm{He}) = 5951.82 \pm 0.54\,\mathrm{MeV}$

$B_{\Lambda\Lambda} = 7.25 \pm 0.19^{+0.18}_{-0.11}\,\mathrm{MeV}$

$\Delta B_{\Lambda\Lambda} = 1.01 \pm 0.20^{+0.18}_{-0.11}\,\mathrm{MeV}$

(assumed $B_{\Xi^-} = 0.13$ MeV)

→ $B_{\Lambda\Lambda}$ = 6.91 MeV (PDG modified(updated) Ξ^- mass)

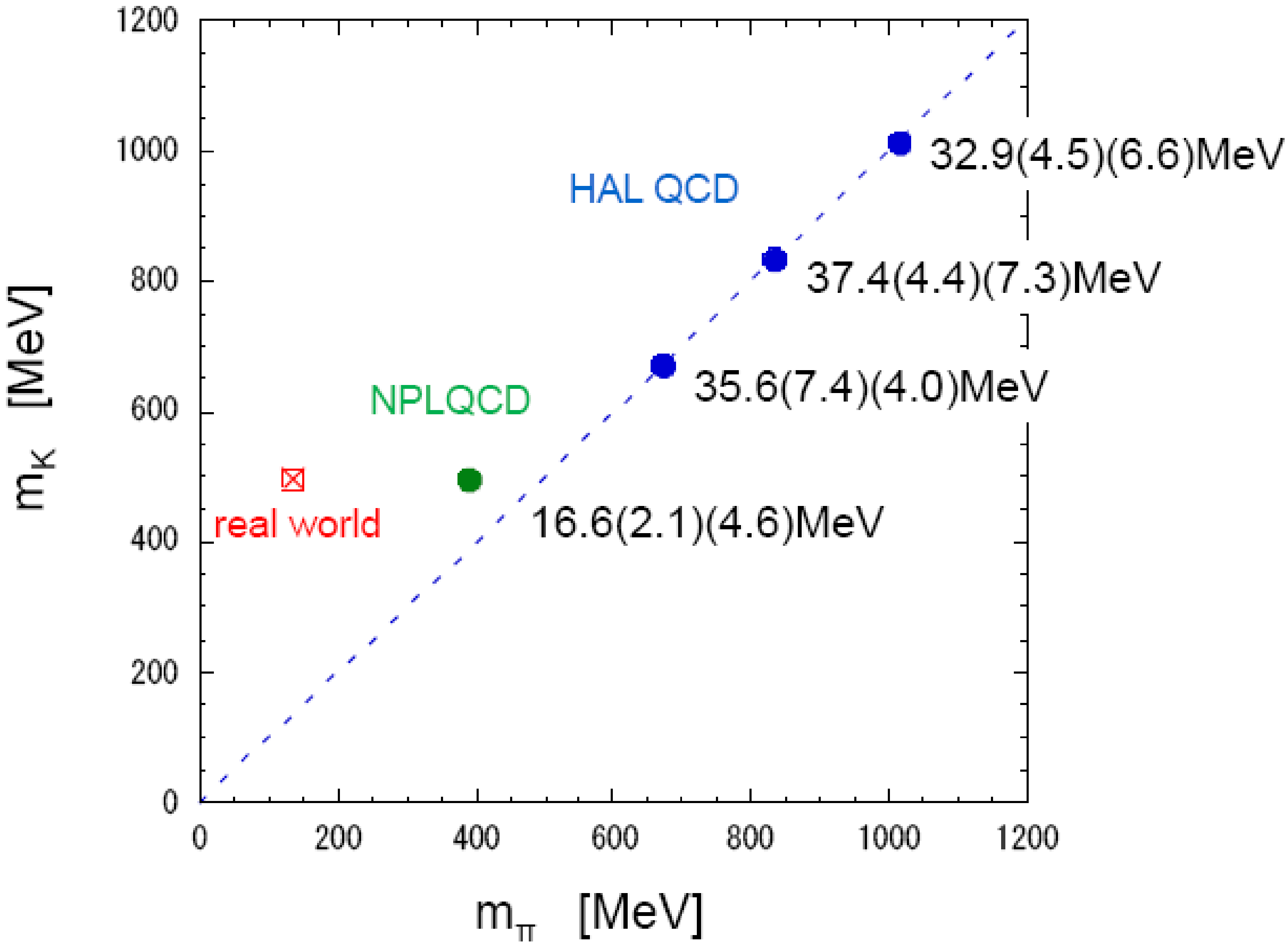


$$\Xi^- + {}^{12}\mathrm{C} \longrightarrow {}^{6}_{\Lambda\Lambda}\mathrm{He} + {}^{4}\mathrm{He} + t$$

$${}^{6}_{\Lambda\Lambda}\mathrm{He} \longrightarrow {}^{5}_{\Lambda}\mathrm{He} + p + \pi^-$$

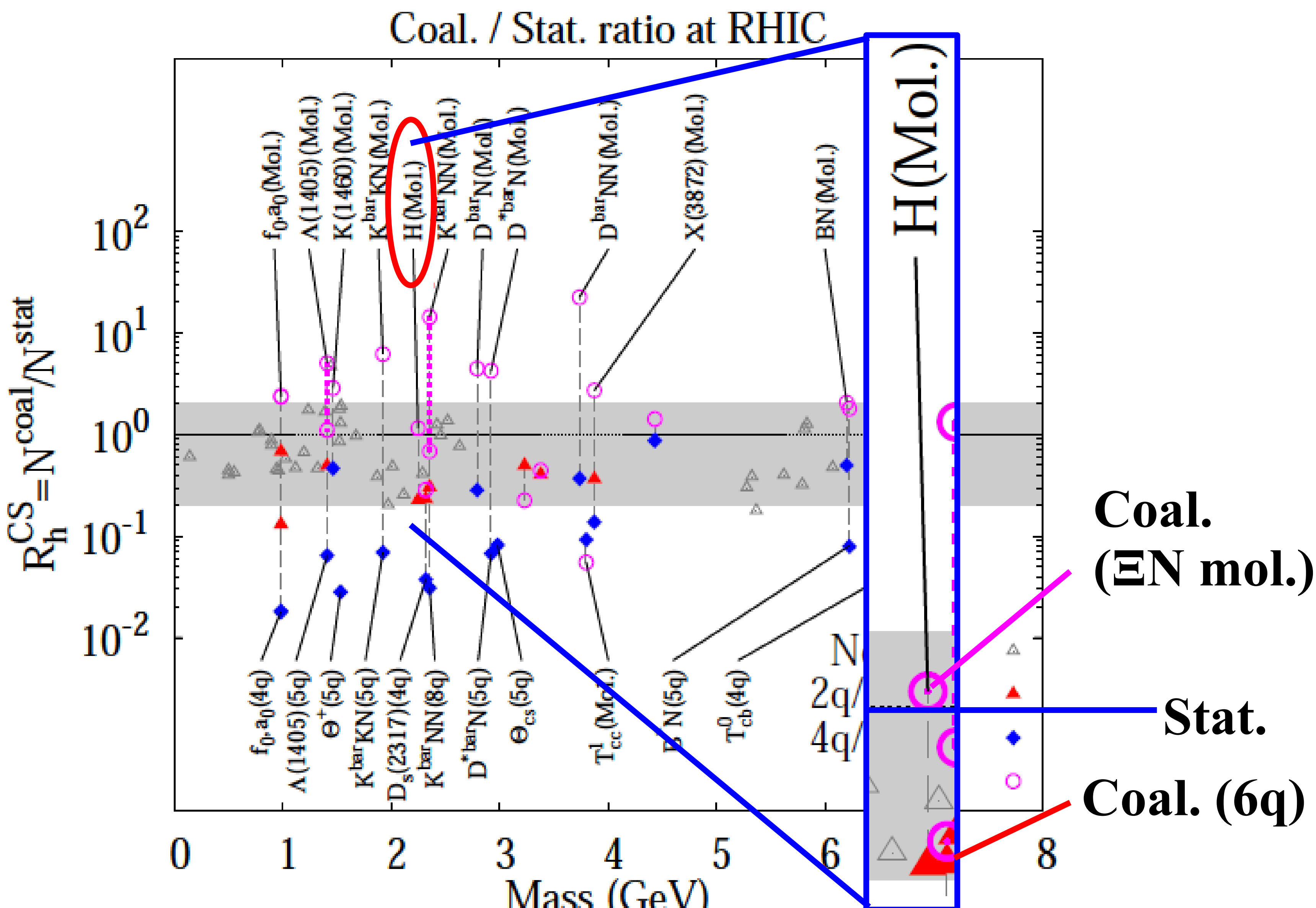
Lattice QCD predicts bound "H"

- **"H" bounds with heavy π (M_π > 400 MeV)**



NPLQCD Collab., PRL 106 (2011) 162001; HAL QCD Collab.,PRL 106 (2011) 162002

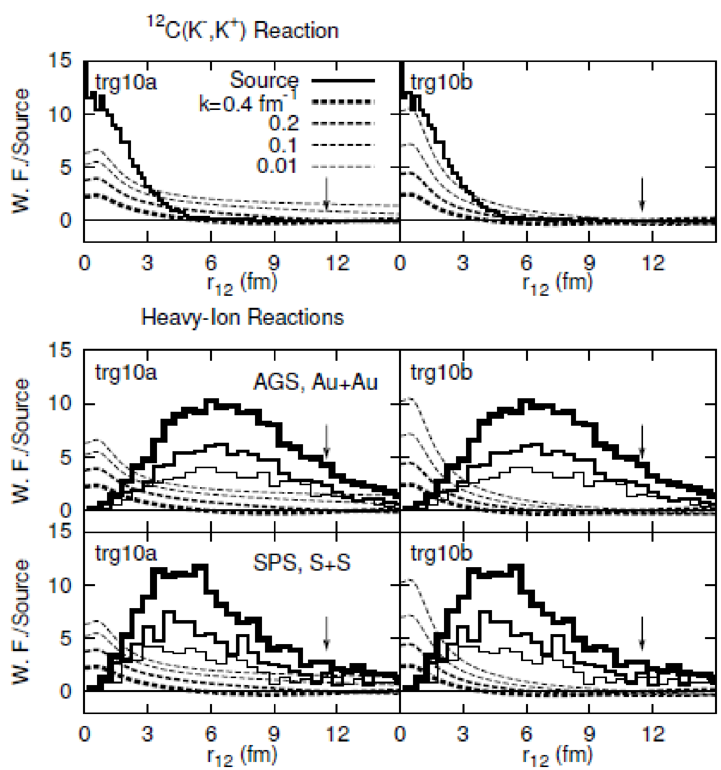
Exotics from Heavy-Ion Collisions

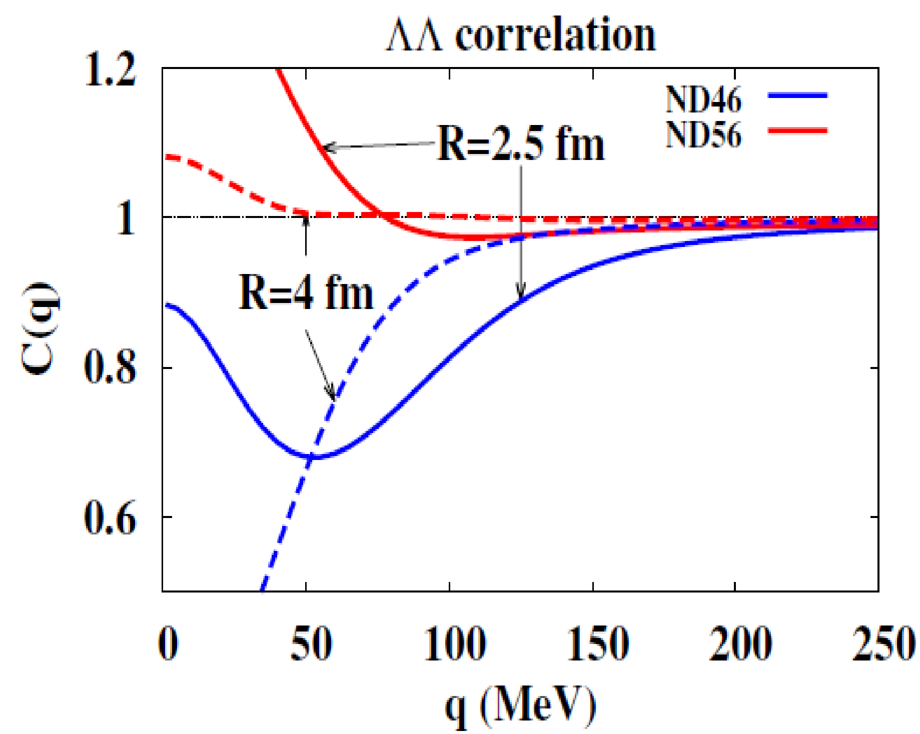


Cho,Furumoto,Hyodo,Jido, Ko, Lee,Nielsen,AO,Sekihara,Yasui,Yazaki (ExHIC Collab.), PRL('11)212001; arXiv:t:1107.1302

Fate of the prediction

- **Conjecture in 2000**
  Suppressed ΛΛ correlation may suggest the existence of bound H
  - **Bound H → Node in scattering ΛΛ wf → suppressed correlation**
    AO, Hirata, Nara, Shinmura, Akaishi, NPA670('00)297c [arXiv:nucl-th/9903021]; SNP2000 proc. p175.
  - **When the source (homogeneity) size is small, we find a dip with/without bound state.**

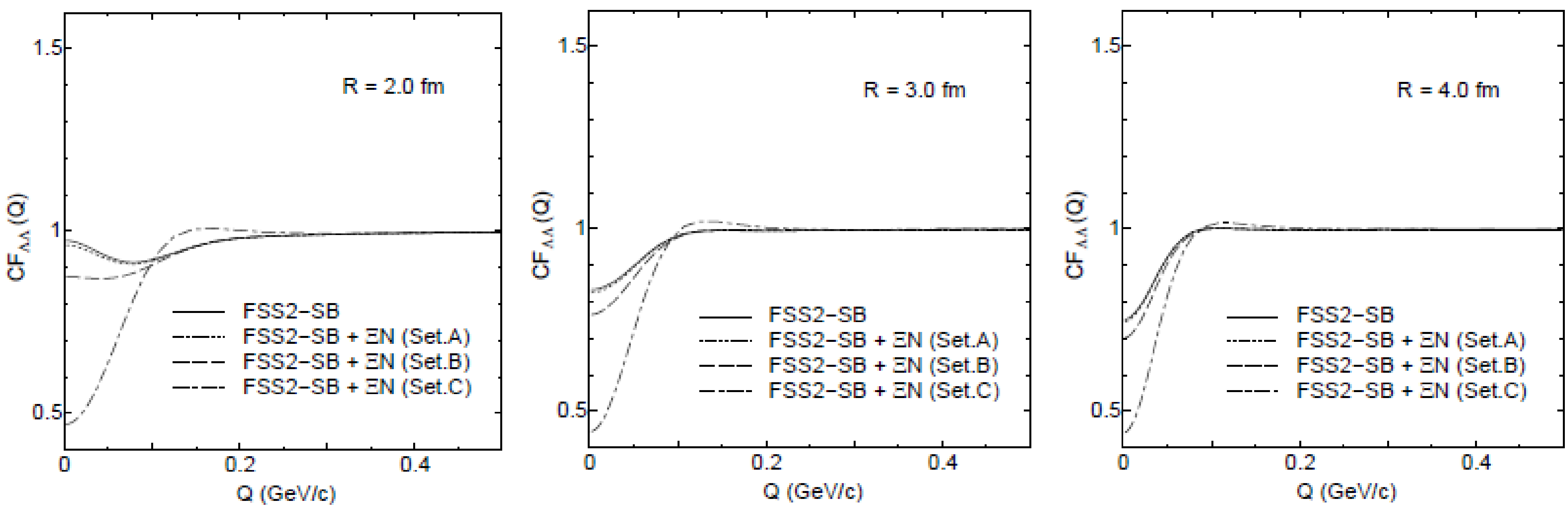




Source size dependence

- **Larger size → Smaller Q region**
- **No dip structure for larger size.**
  (Anti-symmetrization effects > Interaction effects)
  → Sensitive only to the scattering length.

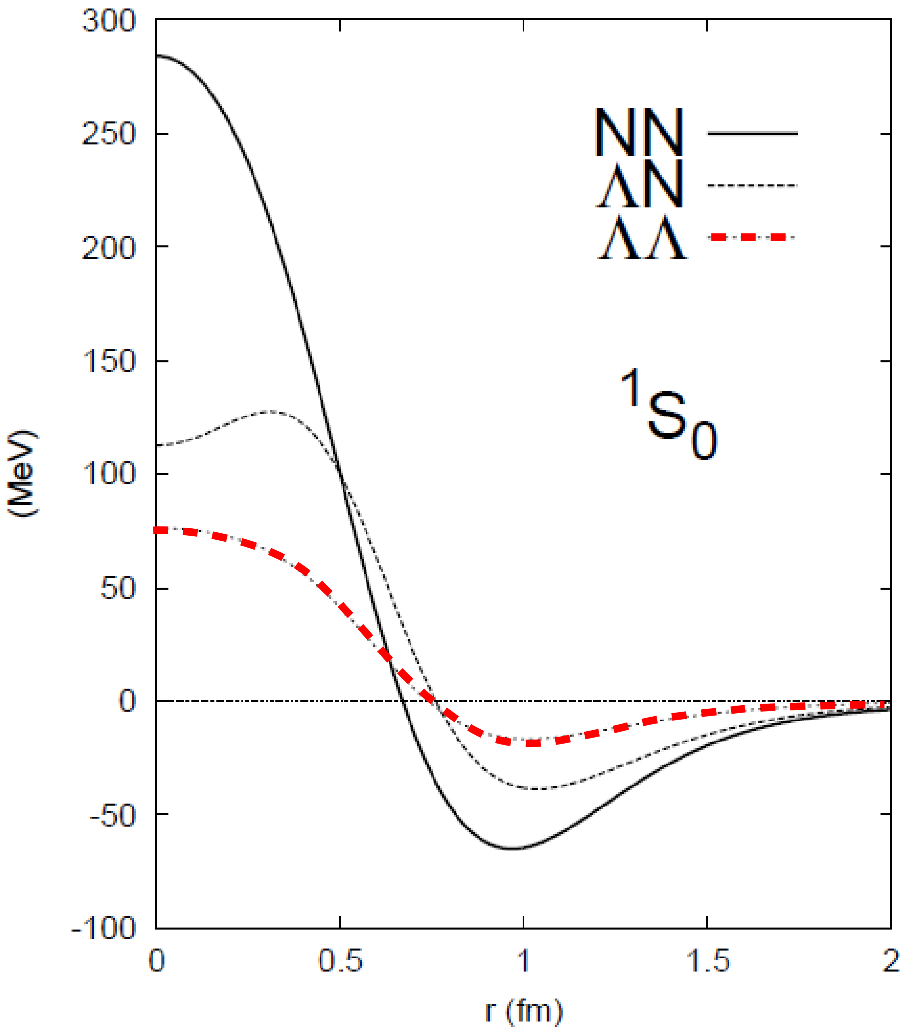
$$C(Q \to 0) \simeq \frac{1}{2} - \frac{2}{\sqrt{\pi}} \frac{a_0}{R} + \left(\frac{a_0}{R} \right)^2 \quad \textit{(if "Interaction Range" << R)}$$



AO, Furumoto, in prep.

ΛΛ potential

fss2 Phase shift equivalent potential

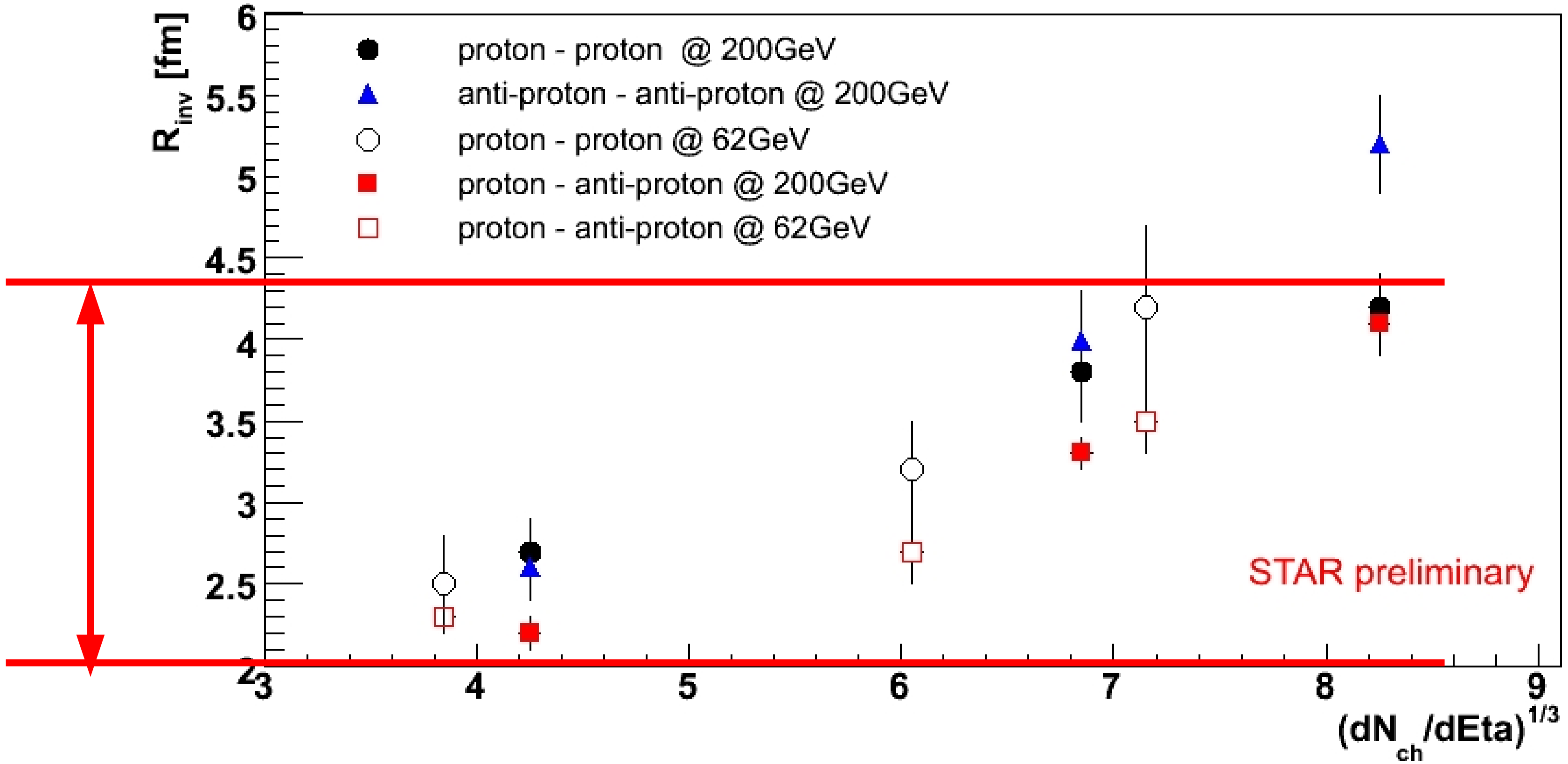


Y. Fujiwara, Y. Suzuki, C. Nakamoto, Prog.Part.Nucl.Phys. 58 (2007) 439-520

- **fss2**
  - **a_0 = - 0.82 fm, r_{eff}=4.1 fm**
- **Nagara fit**
  - **Filikhin, Gal ('02)**
    a0=-0.77 fm, reff=6.59 fm
  - **Hiyama et al. ('02)(HKMYY)**
    a0=-0.575 fm, reff=6.45 fm
  - **Hiyama et al. ('10)(HKMYY)**
    a0=-0.44 fm, reff=10.1 fm

Toward ΛΛ correlation at RHIC: Source Size

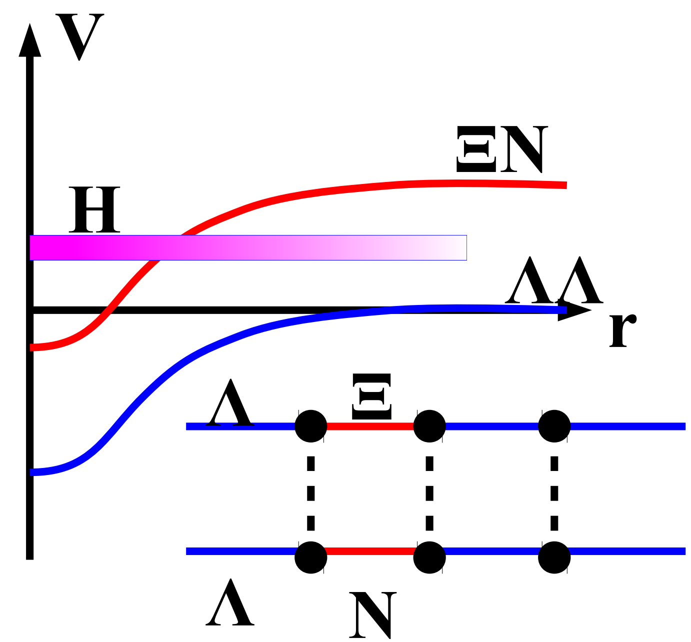
- Source size : R = (2-4.5) fm
  - Smaller than last collision point dist. results in hadron cascade (JAM)
    → Interaction in the early stage at RHIC
  - Smaller than π, K homogeneity length→ Further smaller for Λ ?



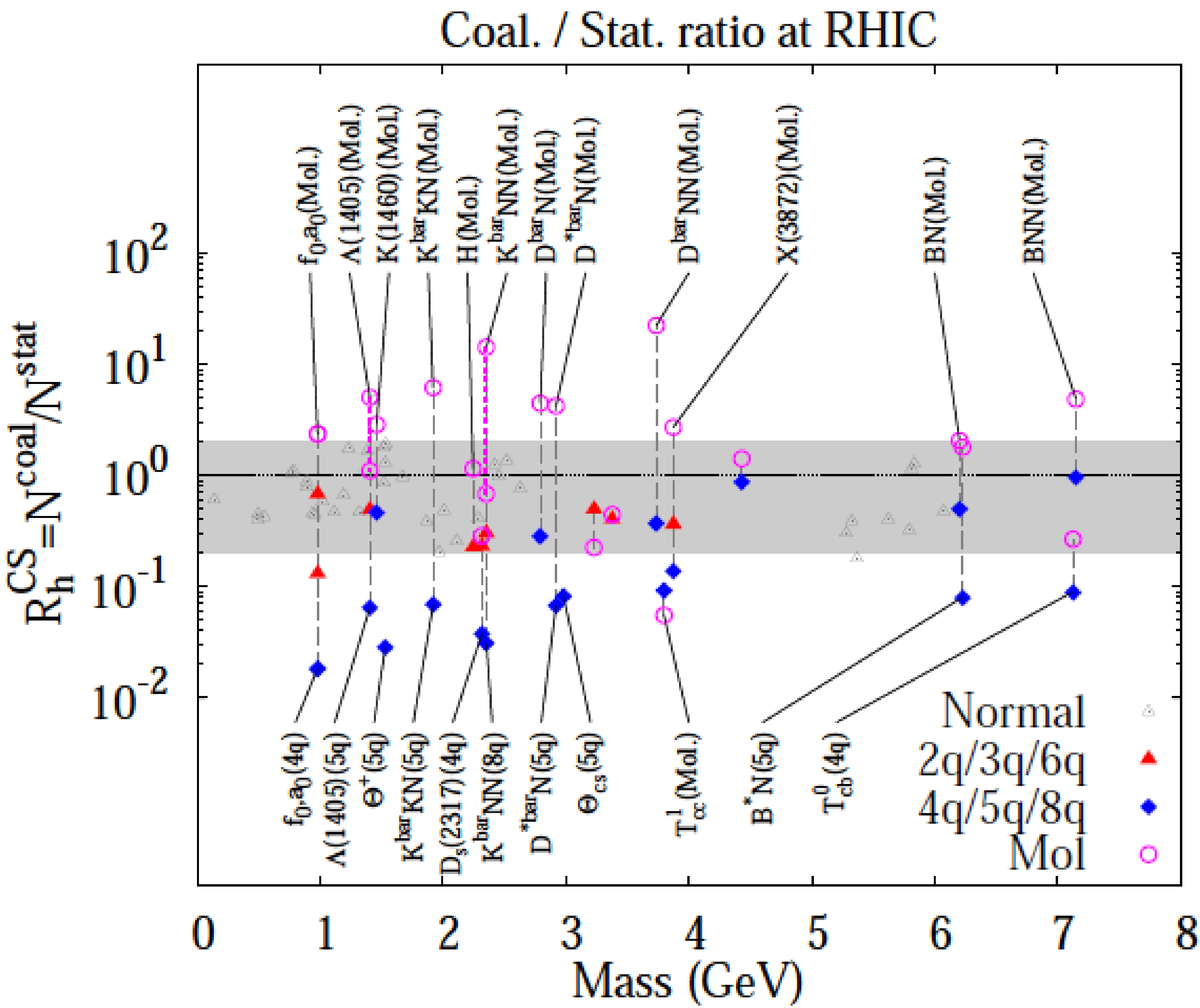
A. Kisiel (H. P. Zbroszczyk) (STAR)

Toward ΛΛ correlation at RHIC: ΛΛ interaction

- **ΛΛ interaction**
  After Nagara, "plausible" ΛΛ interaction becomes weaker.
  Bond energy $\Delta B_{\Lambda\Lambda}$=0.7 MeV (old guess=(3-6) MeV)
  - **fss2 (quark model interaction): No bound state**
    Y. Fujiwara, M. Kohno, C. Nakamoto, Y. Suzuki, PRC64('01)054001
    Bond energy $\Delta B_{\Lambda\Lambda}$= (1.2-1.9) MeV (depending on ΛN int.)
  - **Nijmegen model D (boson exch., Rc=0.46 fm): with bound state**
    M.M. Nagels, T.A. Rijken, J.J. de Swart, PRD15('77)2547
    B.E.(H) ~ 1.6 MeV
- **Resonance "H" btw ΛΛ- ΞN threshold**
  → Couple channel calc. is required
  - **One range gaussian coupling potential is assumed.**
  - **ΞN potential (diagonal) effects on C(q) is almost negligible.**



Coalescence / Statistical Ratio

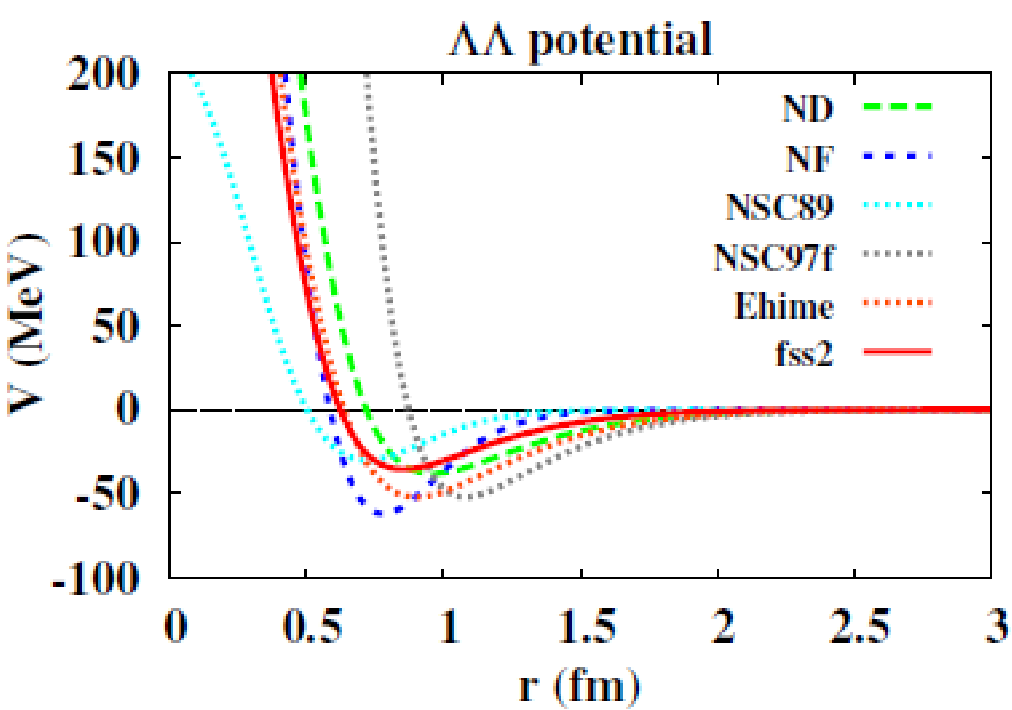


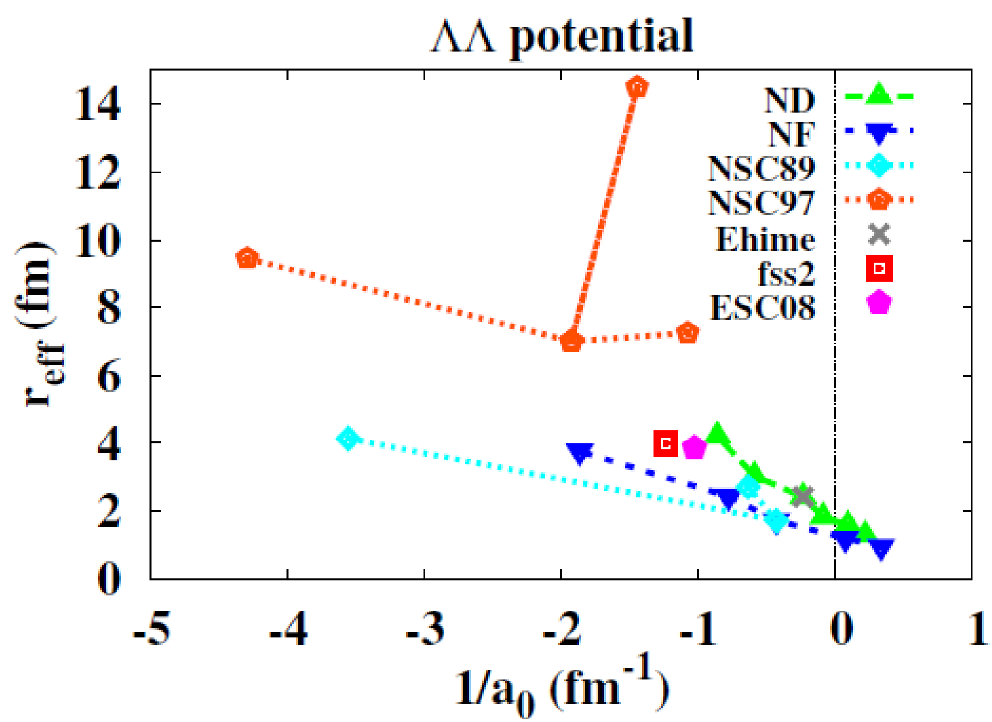
S. Cho et al. (ExHIC Collab.), PRL106('11)212001; PRC 84 ('11) 064910

Exotic Interactions from Heavy Ion Collisions --- H particle and ΛΛ interaction ---

ΛΛ interaction

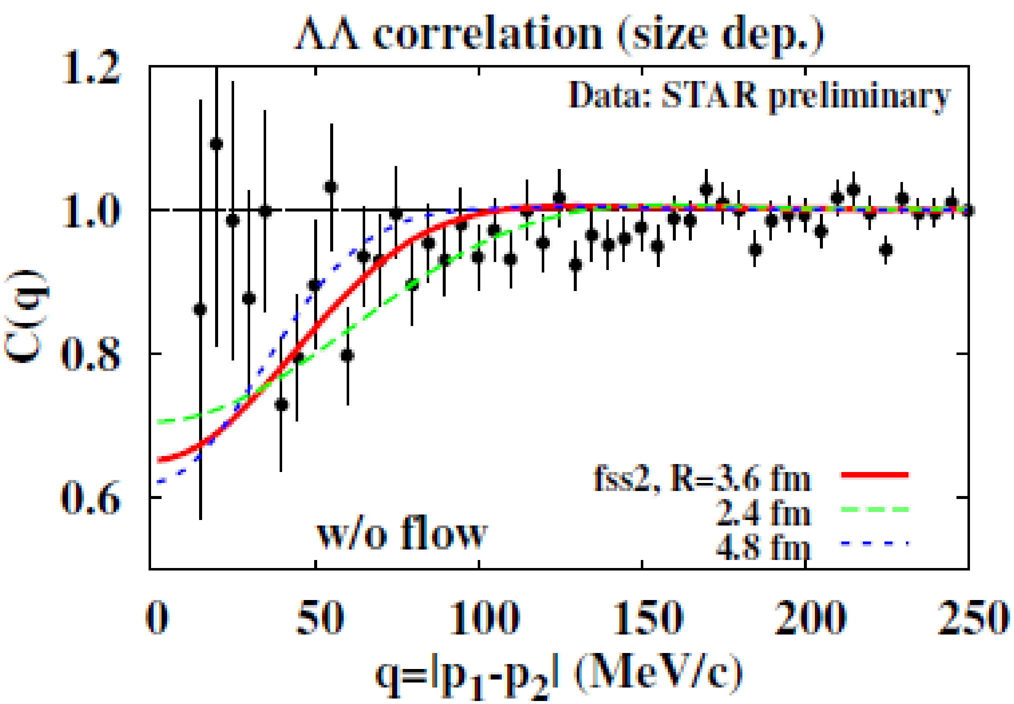
- **Type of ΛΛ interactoin**
  - **Meson exchange models: Nijmegen model D, F, Soft Core (89, 97), ESC08**
    Nagels, Rijken, de Swart ('77, '79), Maessen, Rijken, de Swart ('89), Rijken, Stoks, Yamamoto ('99); Rijken, Nagels, Yamamoto ('10).
  - **Quark cluster model interaction: fss2**
    Fujiwara, Fujita, Kohno, Nakamoto, Suzuki ('00)
  - **Phenomenological model: Ehime** T. Ueda et al. ('99).

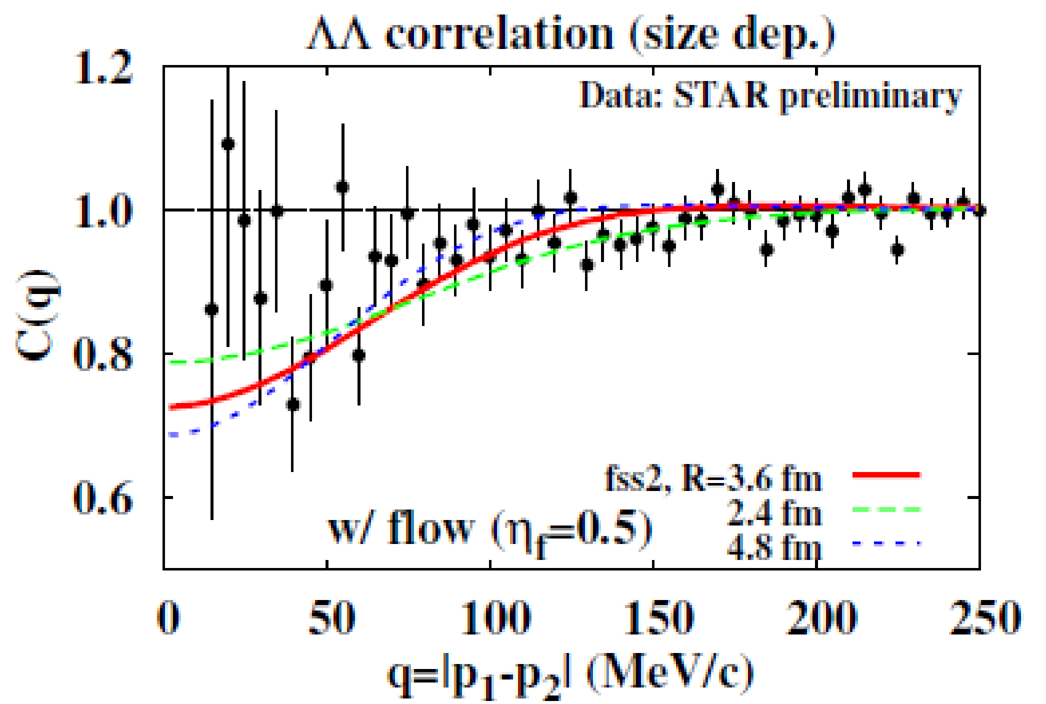




Flow Effects

- Too small source size ~ 1.7 fm with Σ^0 feed down effects ?
- Flow effects make the "apparent" size smaller.
  - Relative momentum is enhanced by the flow.
    → Actual size ~ 3 fm (consistent with proton source size)





Transverse rapidity, $Y_T = \eta_f \, r_T / R$

Morita

Summary

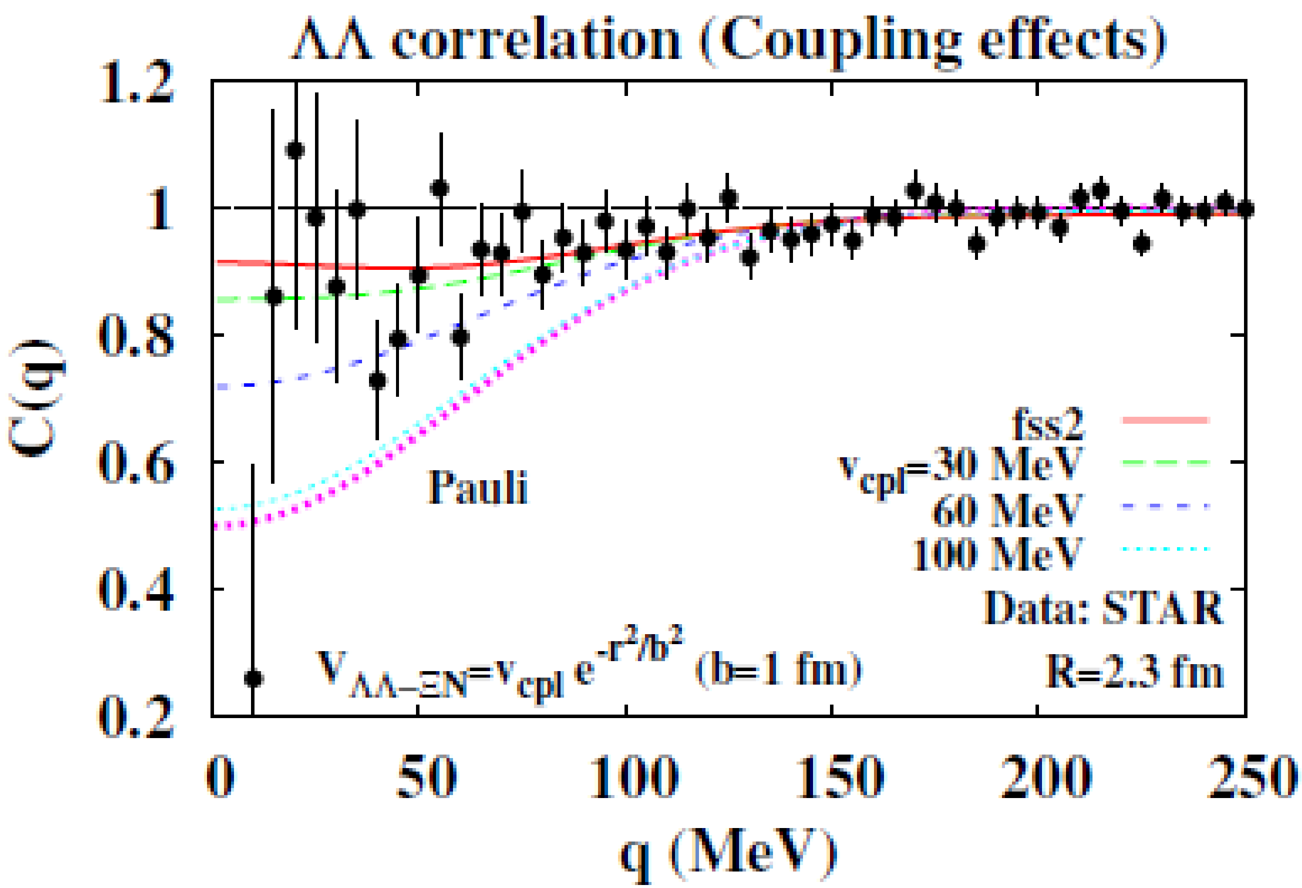
- **Various exotic hadrons will be formed in heavy ion collisions, and it is the right time to search them.**
  - **Extended hadron molecule production is enhanced in HICs.**
  - **Compact multiquark states may be suppressed in HICs.**
  - **Systematic study of exotics from e^+e^- reaction may be promising.**
- **ΛΛ correlation observed at RHIC is useful to distinguish proposed ΛΛ potentials.**

  N.Shah et al. (STAR Collab.), Acta Phys. Pol. Suppl. 5 ('12) 593.

  - **Preferred ΛΛ interactions have $1/a_0 < -0.8\ \mathrm{fm}^{-1}$, $r_{eff} > 3$ fm.**
    Weakly attractive. Consistent with Nagara event (a_0=-(0.7-1.3) fm)
    E. Hiyama et al. PRC66('02)024007; A.M.Gasparyan et al. PRC85('12)015204.
  - **Source size is R ~ 3 fm, if we take account the flow effects. Consistent with proton source size (2.5-4) fm *(STAR preliminary)*.**
  - **Existence of resonance "H" requires higher statistics.**
  - **Other YY (and hh) correlations would be measurable in HIC.**

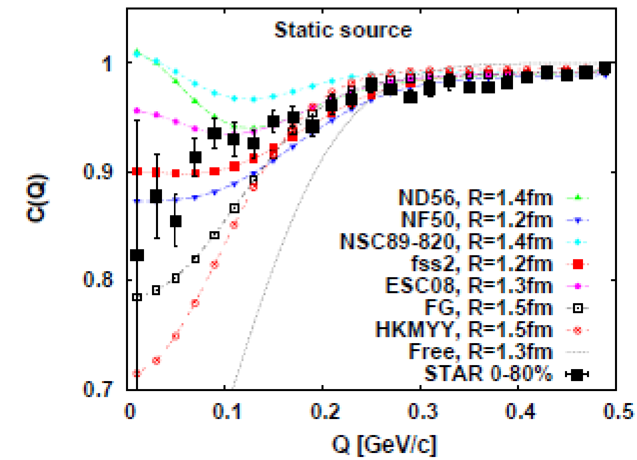
Coupling Effects

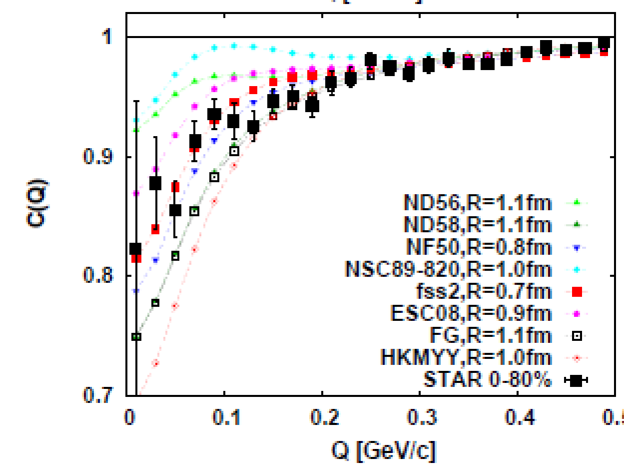
- Coupled channels effects with ΞN channel is considered.
  - Coupling with ΞN channel suppresses C(q) at low q. (~ Imag. pot.)
  - Unreasonably large coupling would meaningfully modify C(q).

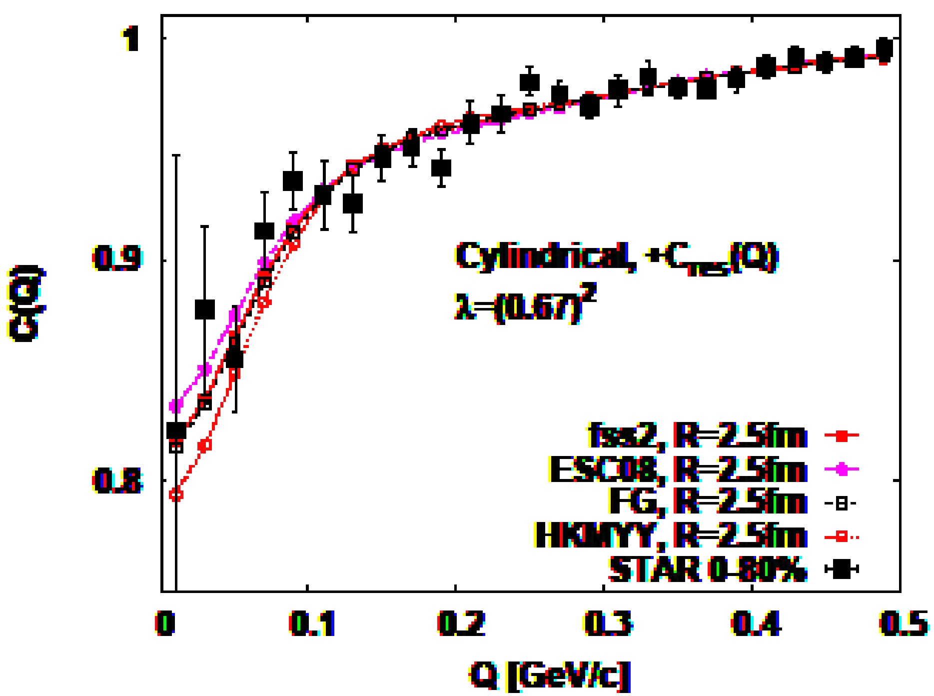


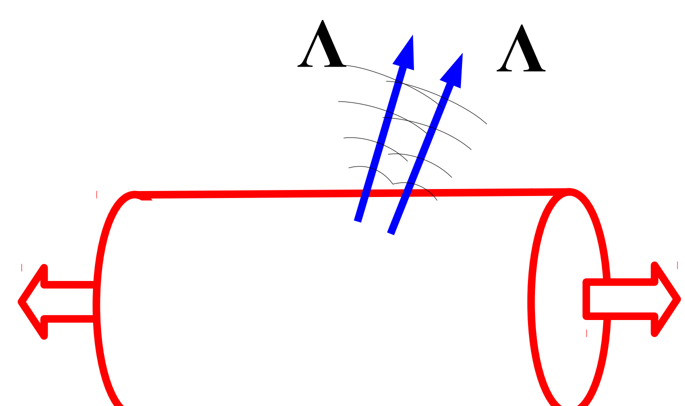
Source models

- Static & Spherical source
- Cylindrical model
  - Bjorken expansion in long. direction
  - Transverse flow from pT spectrum
- Cylindrical + Feed down (Σ0) + Residual



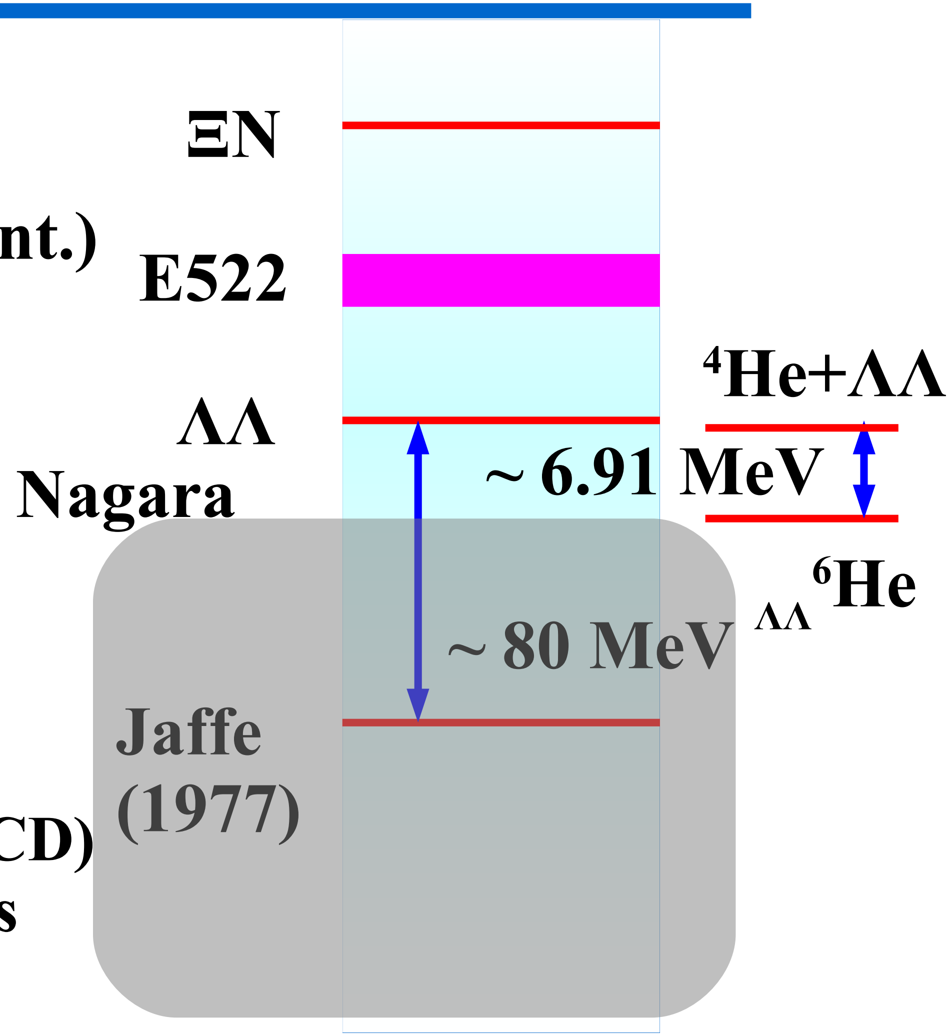






Where is the S=-2 dibaryon (uuddss) "H" ?

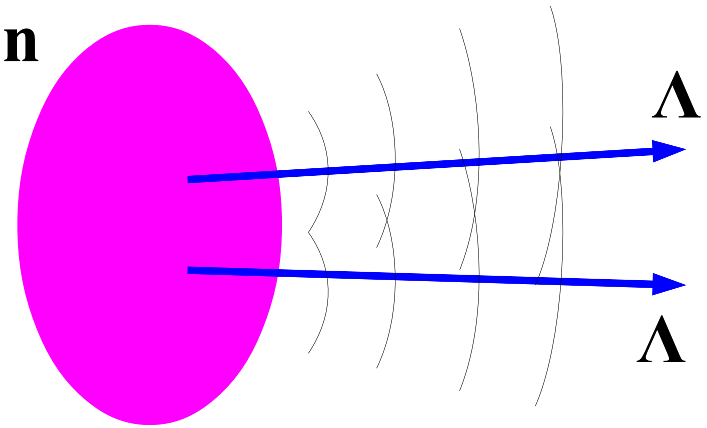
- Jaffe's prediction (1977)
  → 80 MeV below ΛΛ
  (strong attraction from color mag. int.)
- Double hypernuclei ${}_{\Lambda\Lambda}{}^{6}$He (Nagara)
  → No deeply bound "H"
- Resonance or Bound "H" ?
  - KEK-E522 (Yoon et al., ('07))
    → "bump" at $E_{\Lambda\Lambda}$ ~ 15 MeV
  - Lattice QCD (HAL QCD & NPLQCD)
    → bound H at large ud quark mass



- How about HIC ?
  - RHIC & LHC = Hadron Factory including Exotics
  - "H" would be formed as frequently as stat. model predicts.
    Cho,Furumoto,Hyodo,Jido, Ko, Lee,Nielsen,AO,Sekihara,Yasui,Yazaki (ExHIC Collab.), PRL('11)212001; arXiv:t:1107.1302

ΛΛ correlation in HIC

- **Merit of HIC to measure ΛΛ correlation**
  - **Source is "simple and clean" !**
    T, μ, flow, size, … are well-analyzed.
  - **Source size is big and probes w.f. tail.**
  - **Discovery of "H" and/or Constraint on ΛΛ int.**



- **Gaussian Source + s-wave int.**

c.f. P. Danielewicz; Bauer, Gelbke, Pratt, Annu. Rev. Nucl. Part. Sci. 42('92)77.

$$C_{\Lambda\Lambda}(q) = \frac{\int dx_1 dx_2 S(x_1, p+q) S(x_2, p-q) |\psi^{(-)}(x_{12}, q)|^2}{\int dx_1 dx_2 S(x_1, p+q) S(x_2, p-q)}$$

$$\simeq 1 - \frac{1}{2}\exp(-q^2 R^2) + \frac{1}{2}\int dr\, S_{12}(r) \left(|\chi_0(r)|^2 - |j_0(qr)|^2 \right)$$

(χ_0 : s-wave wave func., $S_{12}(x) = (R\sqrt{\pi})^{-3} \exp(-r^2/R^2)$)

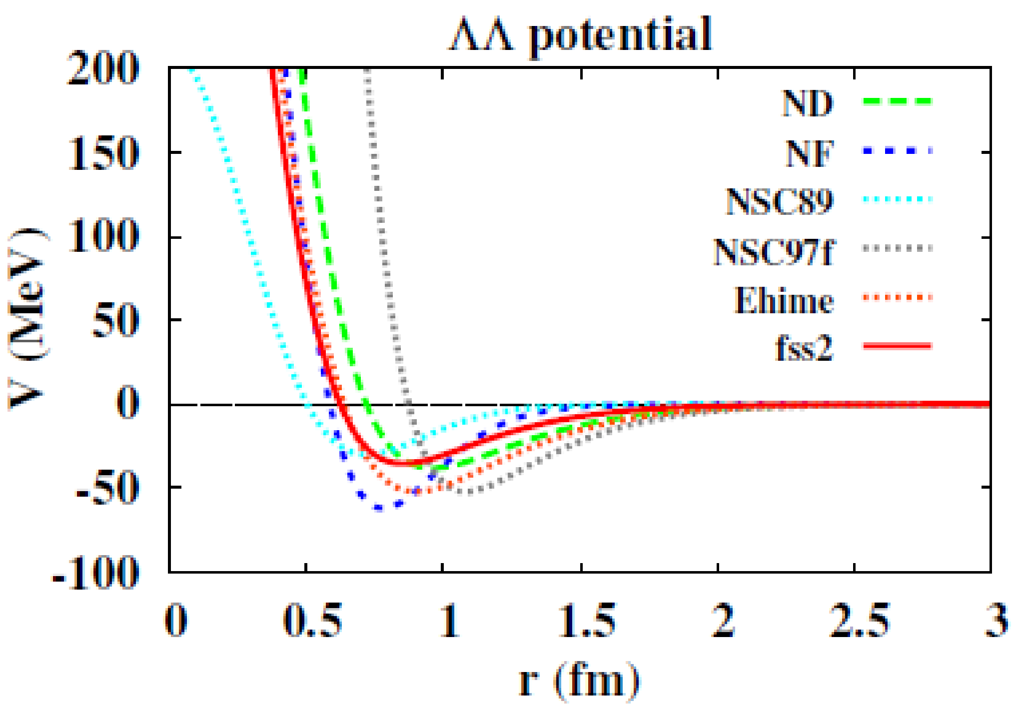
ΛΛ corr. in (K^-,K^+) and AA reactions

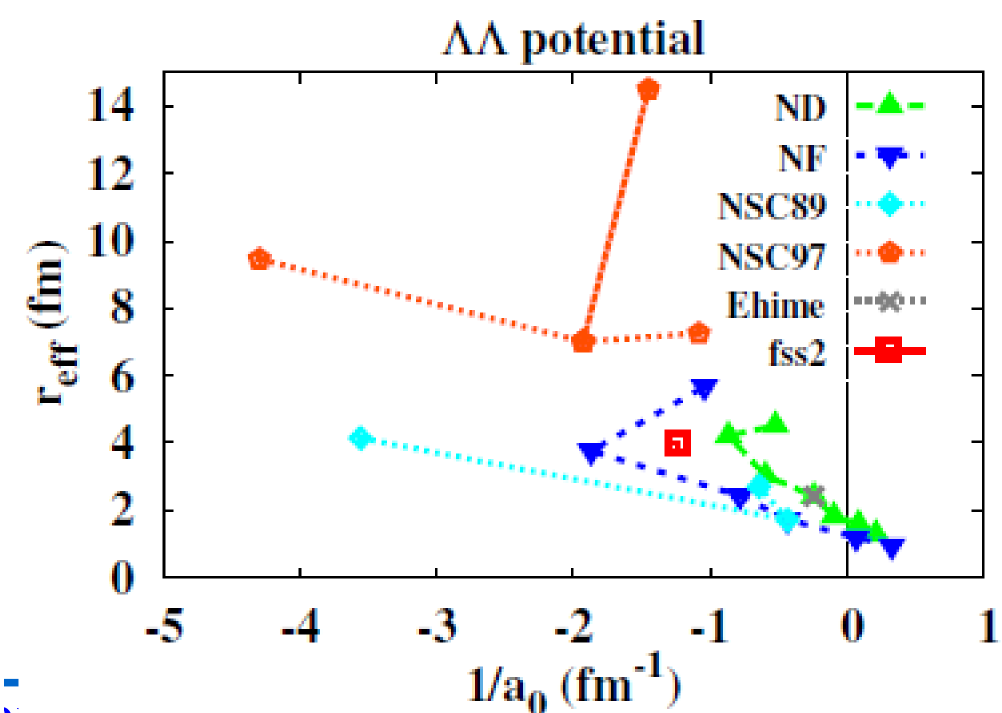
Question:
Can we constrain ΛΛ interaction
from correlation data at RHIC ?
Is the constraint consistent
with ΛΛ corr. in (K^-, K^+) reaction ?
Does H exist as a bound state or a resonance ?

- **Source: Gaussian (HIC), Cascade model results (K-,K+)**
- **Interaction: ND, NF, NSC89, NSC97, Ehime, fss2**
  (two or three range gaussian fit)
- **Other channel: ΛΛ-ΞN couple channels, Σ^0 decay feed effects**

ΛΛ interaction

- **Type of ΛΛ interactoin**
  - **Meson exchange models: Nijmegen model D, F, Soft Core (89, 97)**
    Nagels, Rijken, de Swart ('77, '79), Maessen, Rijken, de Swart ('89), Rijken, Stoks, Yamamoto ('99)
  - **Quark cluster model interaction: fss2**
    Fujiwara, Fujita, Kohno, Nakamoto, Suzuki ('00)
  - **Phenomenological model: Ehime**
- **Two (or three) range gaussian fit results are used in the analysis.**

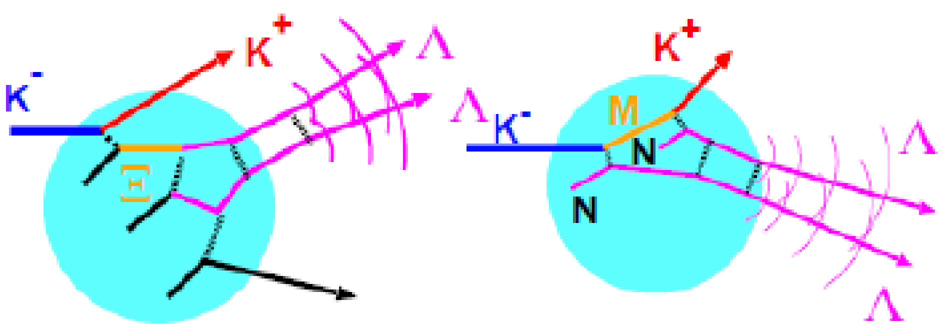


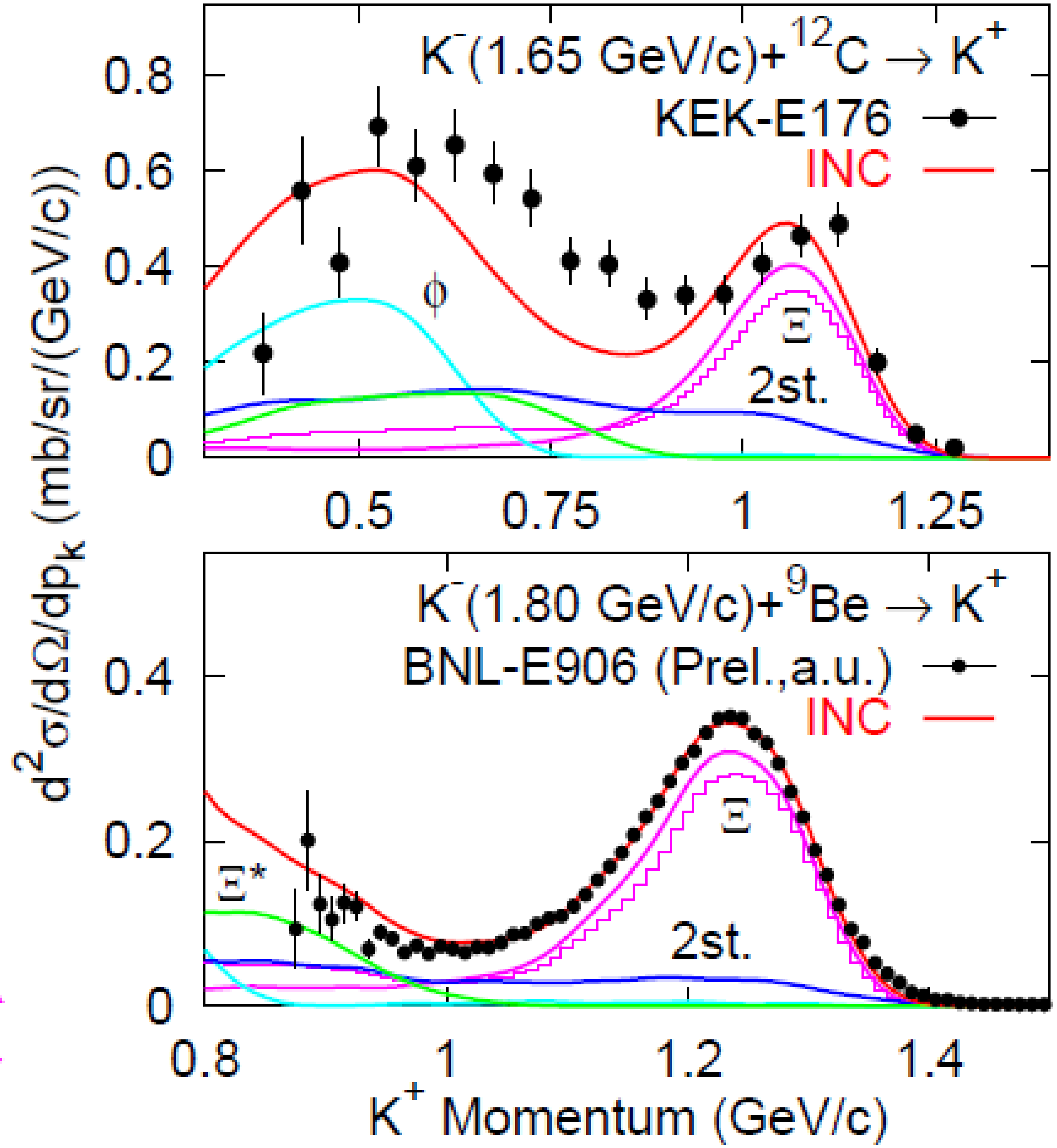


ΛΛ Correlation in (K^-,K^+) Reaction

ΛΛ Correlation in (K^-,K^+) Reaction (1)

- K^+ production mechanism
  - QF Ξ production
  - Heavy meson production and Decay
    Gobbi, Dover, Gal, PRC50 (1994) 1594.
  - Two step procecces
    Nara, AO, Harada, Engel, NPA614 (1997) 433

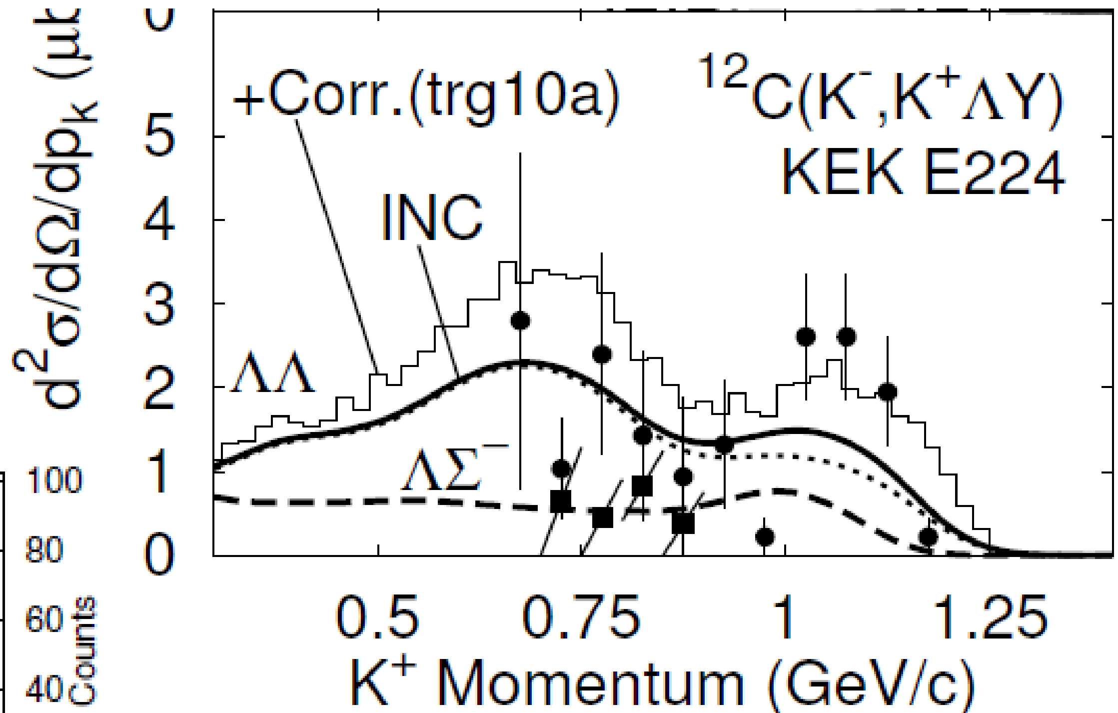




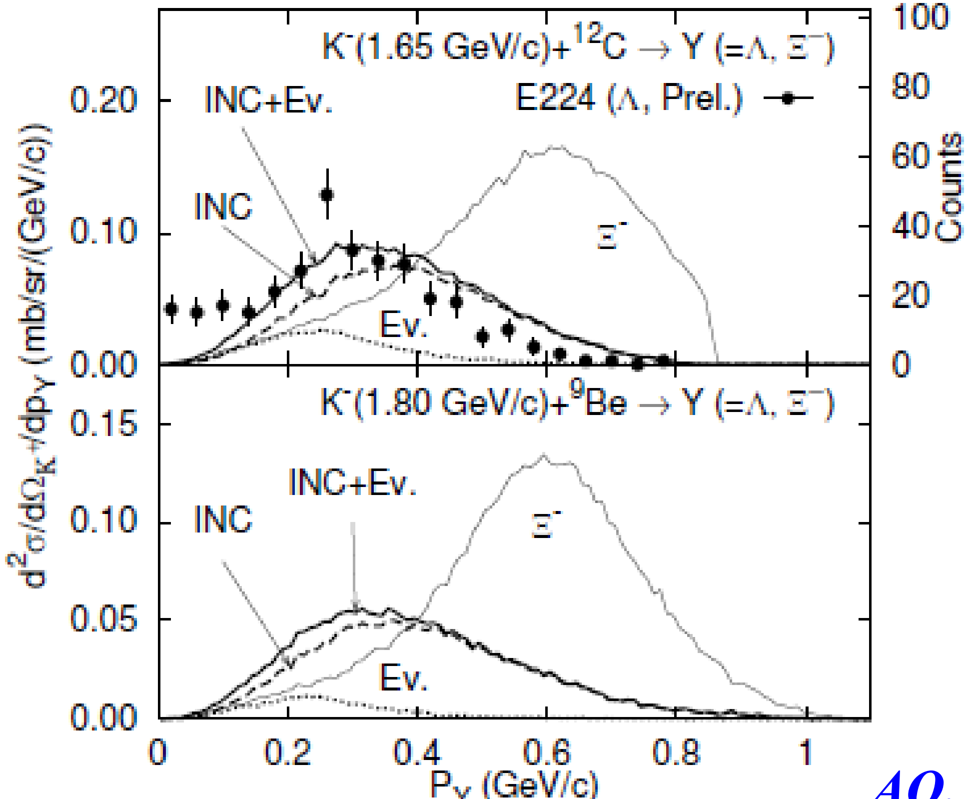
AO, Hirata, Nara, Shinmura, Akaishi, Few-Body Syst. Suppl. 12 (2000), 367

ΛΛ Correlation in (K^-,K^+) Reaction (2)

- Λ production mechanism
  - Cascade procecces
  - Evaporation from hyper compound nuclei

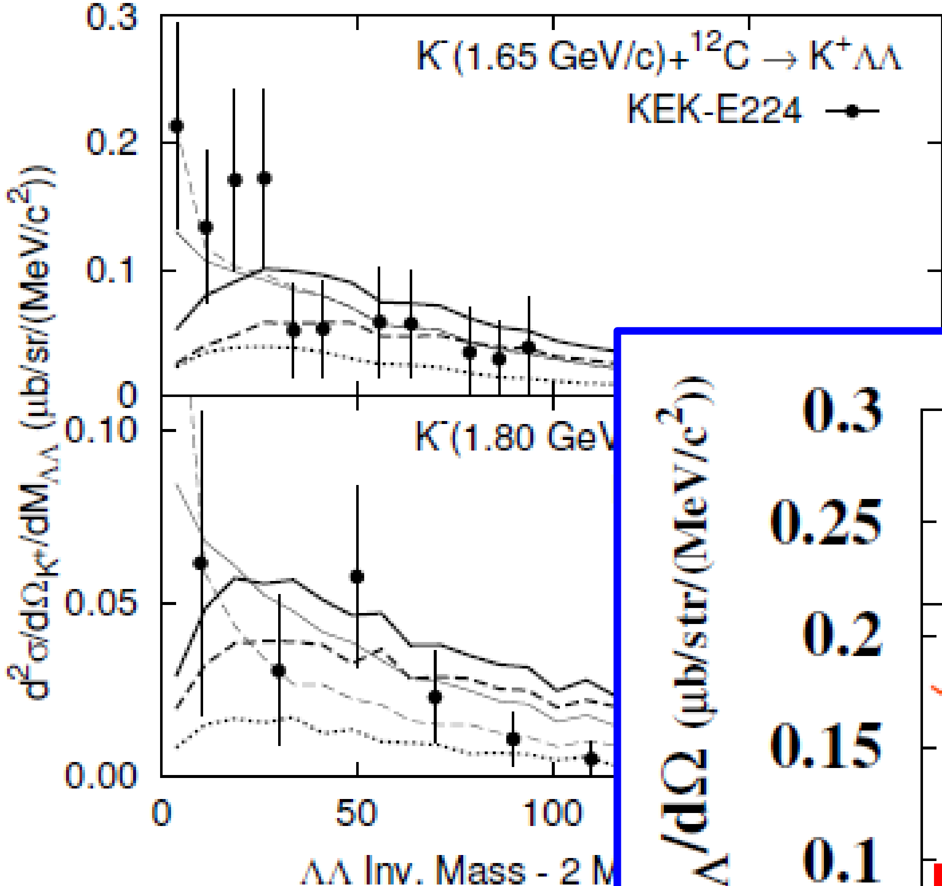


AO, Hirata, Nara, Shinmura, Akaishi, NPA670(2000), 297c

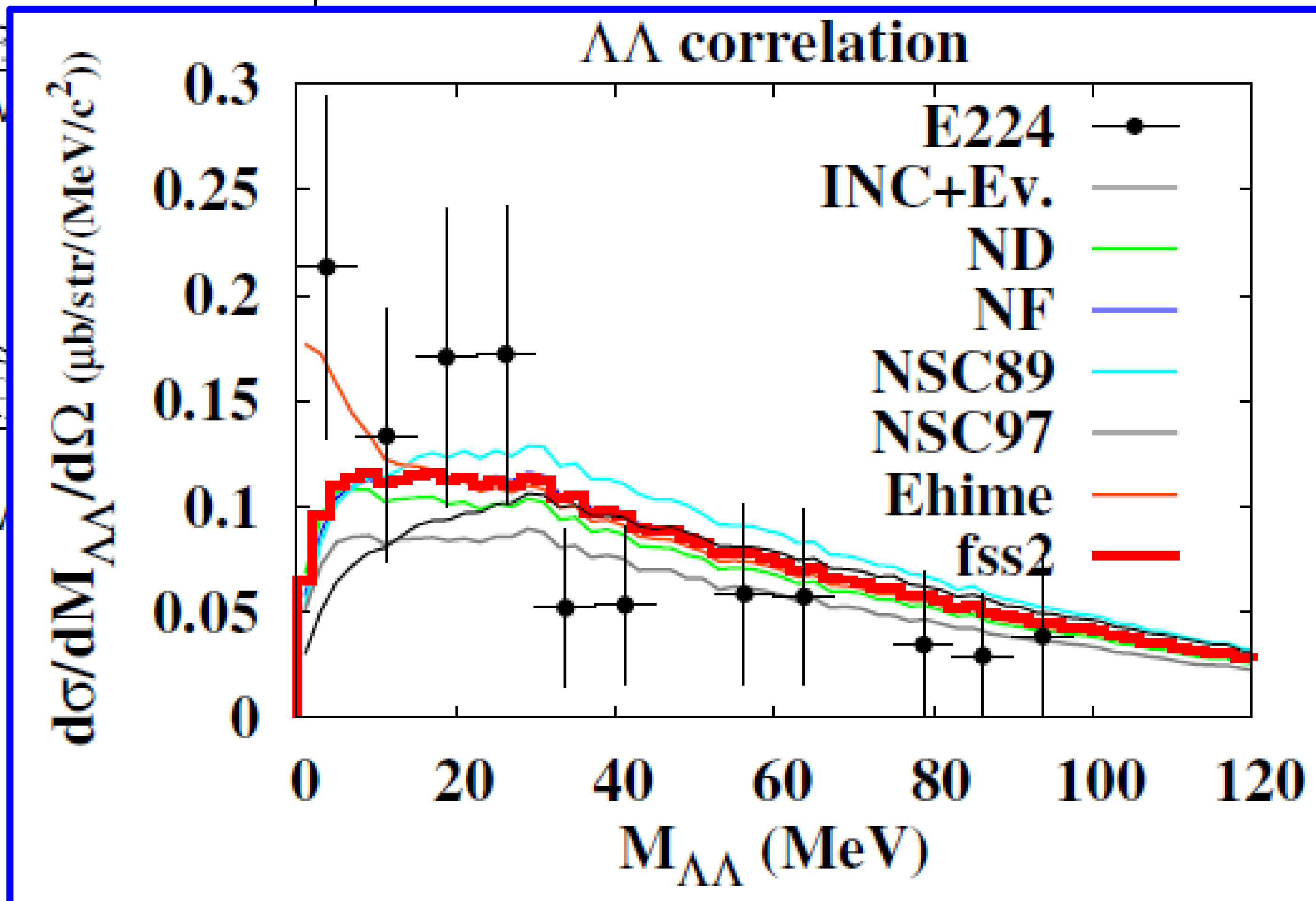


AO, Hirata, Nara, Shinmura, Akaishi, NPA691(2001), 242c

ΛΛ Invariant Mass Spectrum



ΛΛ int. constrained in HIC are consistent with ΛΛ inv. mass spectrum in ($K^-, K^+ΛΛ$)



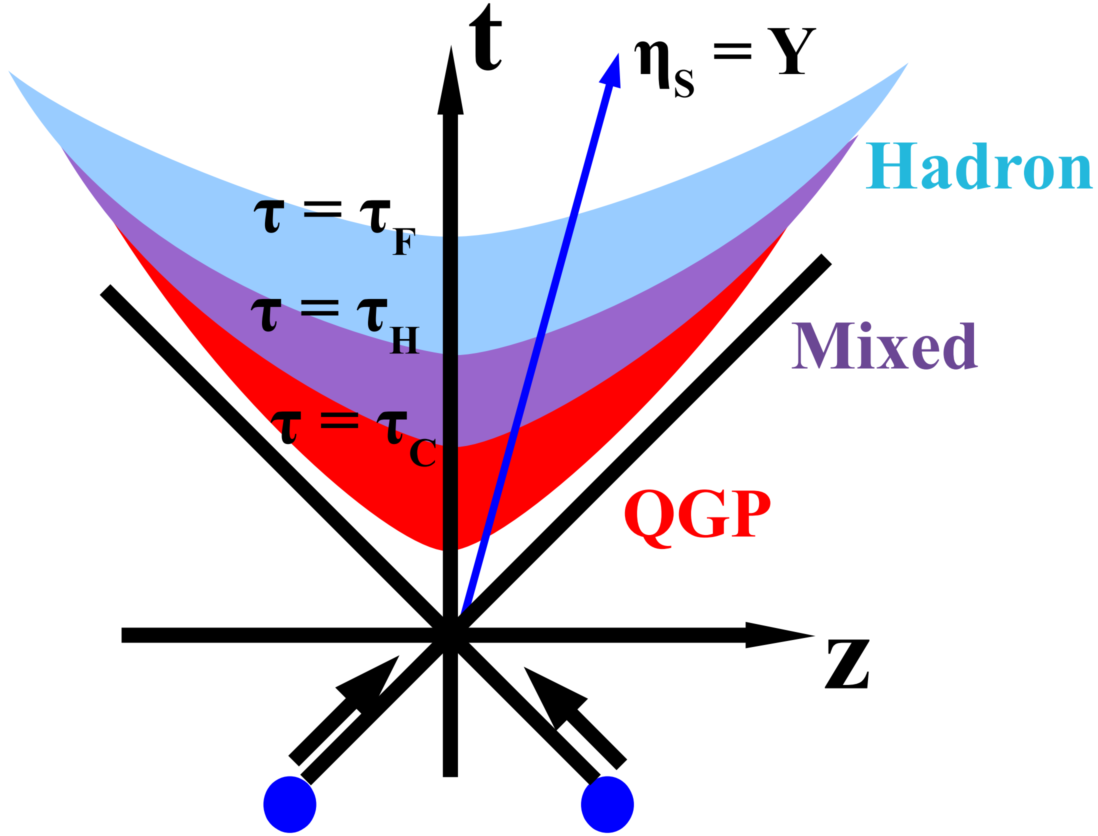
AO, Hirata, Nara, Shinmura, Akaishi, NPA684(2001), 595c

Exotic Hadron Yields in Heavy Ion Collisions

Schematic picture of HIC

- HIC picture based on the first order phase transition
  - $\tau = \tau_C$, $T=T_C$, $V=V_C$ → QGP start to hadronize (quark coal.)
  - $\tau = \tau_H$, $T=T_H=T_C$, $V=V_H$ → Hadronization is over (stat. model)
  - $\tau = \tau_F$, $T=T_F$, $V=V_F$ → Hadronic Freeze-out (hadron coal.)

	RHIC	LHC
$N_u = N_d$	245	662
$N_s = N_{\bar{s}}$	150	405
$N_c = N_{\bar{c}}$	3	20
$N_b = N_{\bar{b}}$	0.02	0.8
V_C	1000 fm^3	2700 fm^3
$T_C = T_H$	175 MeV	175 MeV
V_H	1908 fm^3	5152 fm^3
μ_B	20 MeV	20 MeV
μ_s	10 MeV	10 MeV
V_F	11322 fm^3	30569 fm^3
T_F	125 MeV	125 MeV



L.W.Chen, V.Greco, C.M.Ko, S.H.Lee, W.Liu, PLB 601('04)34.

Statistical Model

- **Statistical model**

$$N_h^{\rm stat} = V_H \frac{g_h}{2\pi^2} \int_0^\infty \frac{p^2 dp}{\gamma_h^{-1} e^{E_h/T_H} \pm 1}$$

(N_h = dN_h/dy (y=rapidity), V_H=Chem. freeze-out vol.)

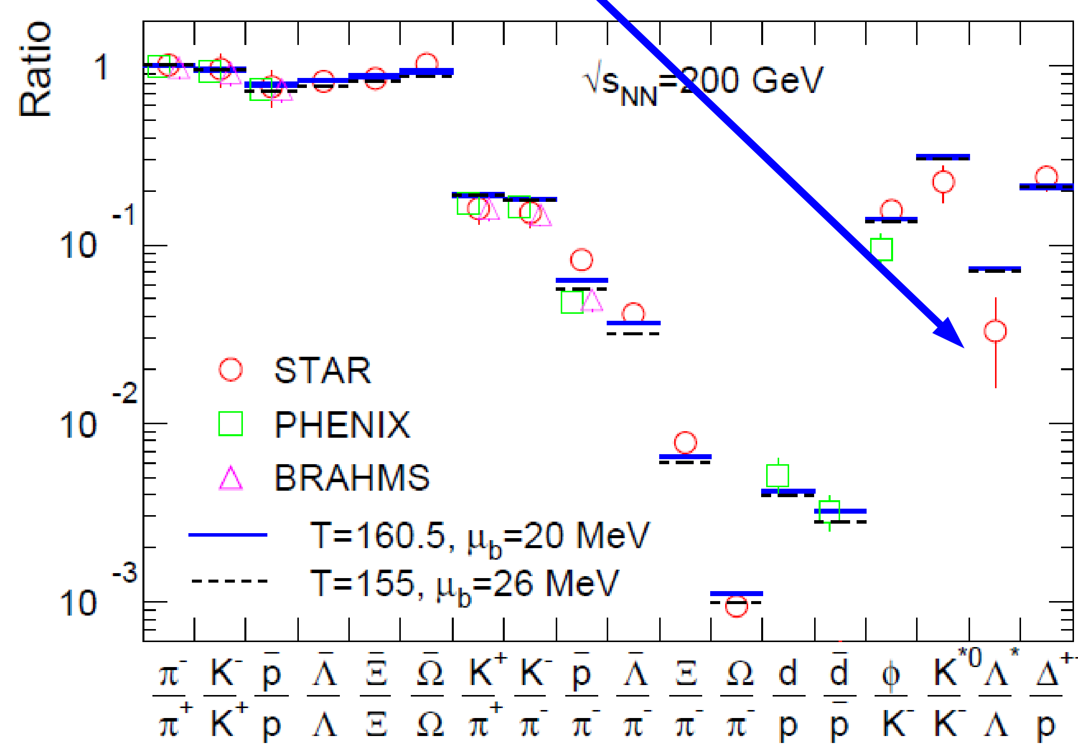
→ Successful to predict the hadron yield ratio at RHIC

- **Fugacity factor γ**
  - **u,d,s: chem. equil.**
  - **c,b: enhanced by initial hard processes**

Fugacities of c and b quarks are set to reproduce expected c and b quark numbers.

$$\gamma_h = \gamma_c^{n_c+n_{\bar{c}}} \gamma_b^{n_b+n_{\bar{b}}} e^{(\mu_B B + \mu_s S)/T_H}$$

L effects: Kanada-En'yo, Muller ('06)



A. Andronic, P. Braun-Munzinger, J. Stachel, NPA772('06)167.

Coalescence model

- **Yield = Overlap of const. dist. & Hadron intrinsic Wigner func. (Sudden approximation)**
  Sato, Yazaki (1984), Hwa, Yang (2003), Greco, Ko, Levai (2003), Fries, Muller, Nonaka, Bass (2003), Chen, Ko, Lee (2003)

$$N_h^{\mathrm{coal}} = g_h \int \underbrace{\left[\prod_{i=1}^{n} \frac{1}{g_i} \frac{p_i \cdot d\sigma_i}{(2\pi)^3} \frac{\mathrm{d}^3 \mathrm{p}_i}{E_i} f(x_i, p_i)\right]}_{\text{Dist. of constituents}} \times \underbrace{f^W(x_1, \cdots, x_n : p_1, \cdots, p_n)}_{\text{Intrinsic Wigner func.}}$$

- **Yield in HIC**
  - **Quark & hadron dist. = Transverse Boltzmann + Bjorken**
    Chen, Ko, Liu, Nielsen (2007)
  - **Hadron intr. Wigner func. = s-wave and p-wave HO w.f.**
    Kanada-En'yo, Muller (2006)

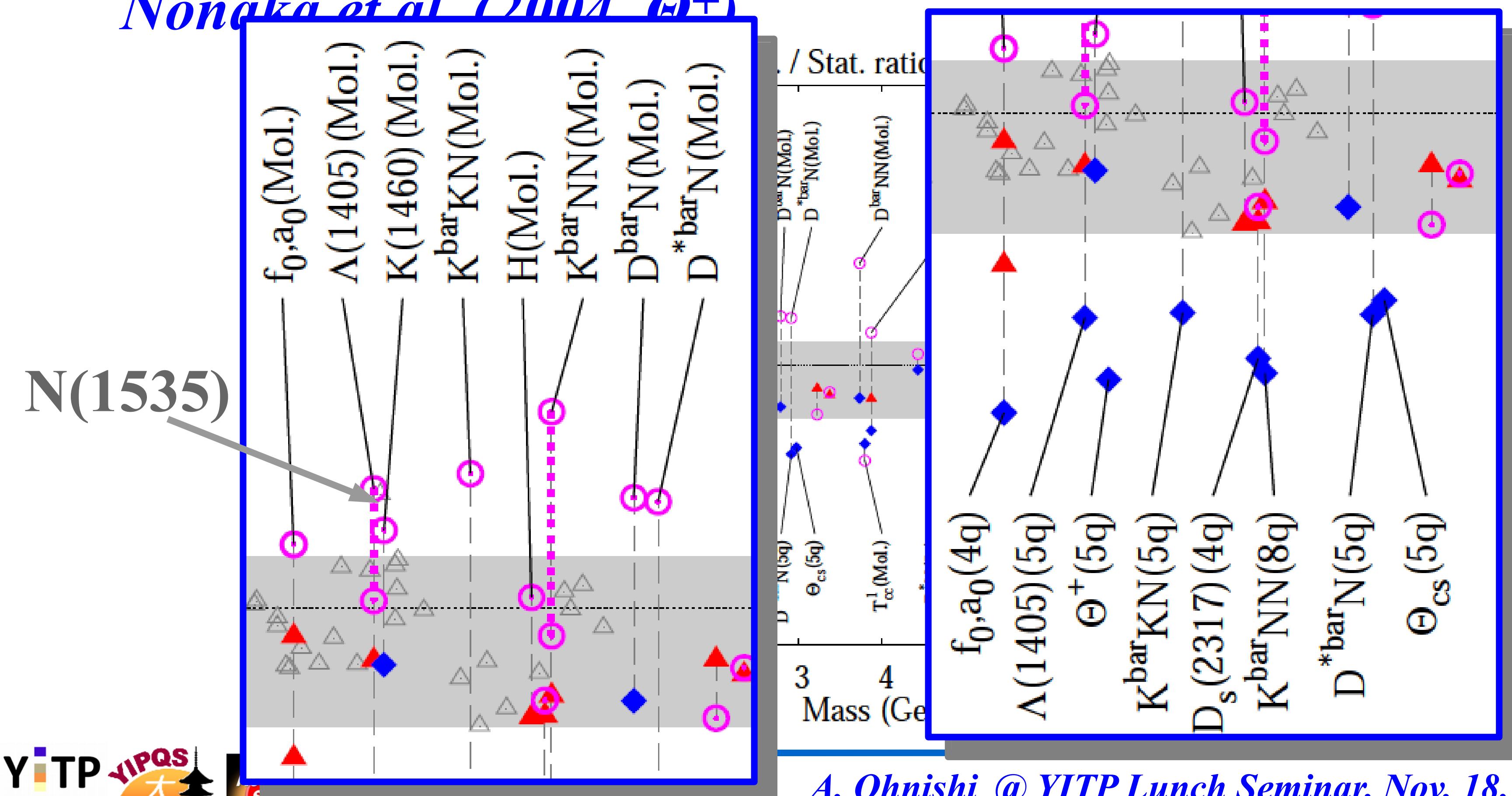
$$N_h^{\mathrm{coal}} \simeq g_h \prod_{j=1}^{n} \frac{N_j}{g_j} \prod_{i=1}^{n-1} \frac{(4\pi\sigma_i^2)^{3/2}}{V(1+2\mu_i T\sigma_i^2)} \left[\frac{4\mu_i T\sigma_i^2}{3(1+2\mu_i T\sigma_i^2)}\right]^{l_i}$$

σ = Gaussian width, μ=reduced mass, N = constituent yield

  - **Available structure information → σ (or ħω)**

Coalescence / Statistical Ratio

- **Normal hadrons: Rcs = (0.2-2)** *Normal band*
- **Extended hadronic molecule: Large yield is expected, Rcs > 2.**
  $\Lambda(1405)=\bar{K}N$, $\bar{K}NN$, $\bar{K}KN$, $\bar{D}NN$, … ($\hbar\omega$=(6-50) MeV)
- **Compact Multiquark states will be suppressed in HICs, Rcs < 0.2**
  $f_0/a_0(qq\bar{q}\bar{q})$, $\Theta^+(uudd\bar{s})$, H(uuddss), $\Theta_{cs}(uuds\bar{c})$, …
  Nonaka et al. (2004, Θ^+)



Enhancement of Hadronic Molecules: Why ?

- **Simple estimate: 2-body, Gaussian w.f. + Thermal dist. of constiuents**

$$N_h \propto \int \frac{d^D x\, d^D p}{(2\pi\hbar)^D} \underline{f_W(x,p)}\, \underline{f_{\mathrm{th}}(x,p)} = \left[\left(\frac{4}{\hbar^2}\right) \left((\Delta p)^2 + \mu T\right) \left((\Delta x)^2 + 2R^2\right) \right]^{-D/2}$$

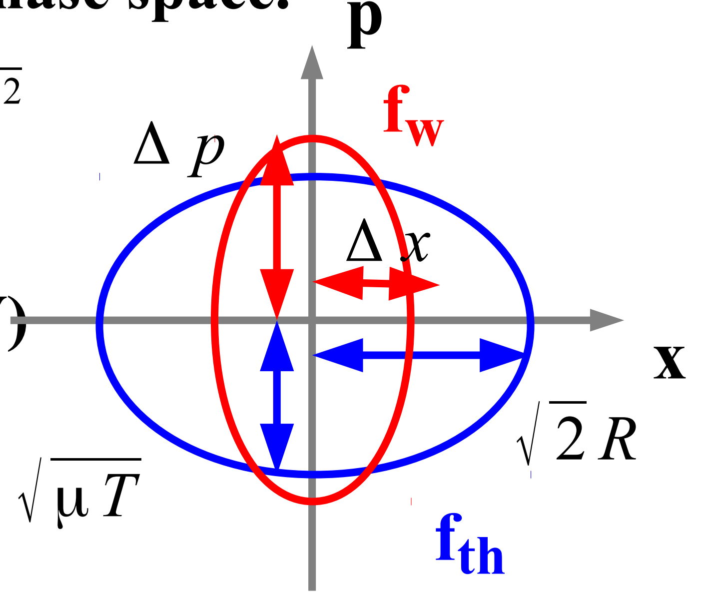
Intrinsic **Constituents (thermal)**

$$f_W(x,p) = \left(\frac{\hbar}{\Delta x \Delta p}\right)^D \exp\left(-\frac{x^2}{2(\Delta x)^2} - \frac{p^2}{2(\Delta p)^2}\right), \quad f_{\mathrm{th}}(x,p) = \left(\frac{\hbar^2}{2\mu T R^2}\right)^{D/2} \exp\left(-\frac{x^2}{4R^2} - \frac{p^2}{2\mu T}\right)$$

- **N_h is large when f_w shape is similar to f_{th} in phase space.**

$$\Delta x : \Delta p = \sqrt{2} R : \sqrt{\mu T} \rightarrow \hbar\omega = \sqrt{\hbar^2 T / 2\mu R^2}$$

- **Example: T=170 MeV, μ=500 MeV (red. mass), R = 5 fm (source size) → optimal hω ~ 16 MeV (<< 300-500 MeV)**



Large source size & Moderate T prefer extended hadrons

Why are Multi-quark Configs. Suppressed ?

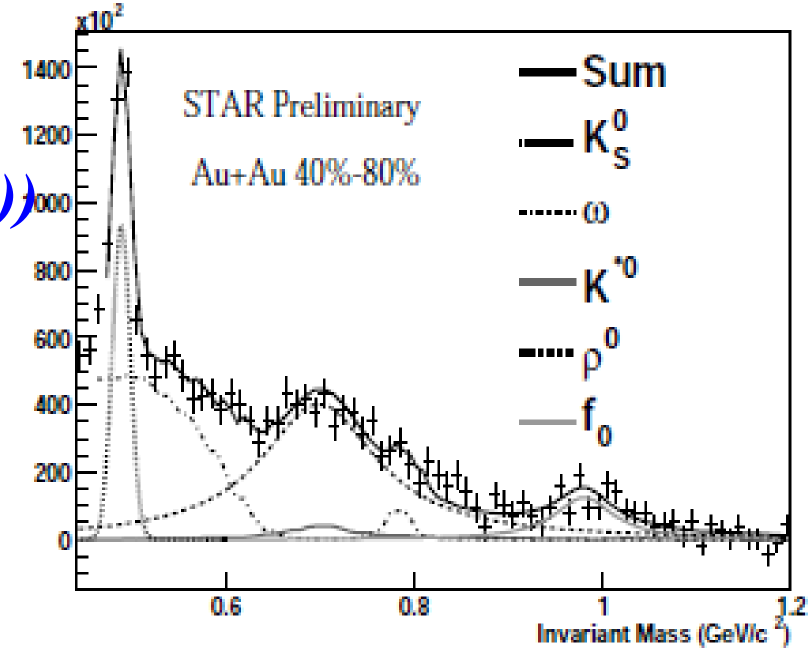
- **Hadron yield is sensitive to the structure in coal.**
  - **Additional q penalty factor**

s-wave $$\frac{1}{g_i}\frac{N_i}{V}\frac{(4\pi\sigma_i^2)^{3/2}}{(1+2\mu_i T\sigma_i^2)}$$ **~ 0.36**

p-wave $$\frac{1}{g_i}\frac{N_i}{V}\frac{2}{3}\frac{(4\pi\sigma_i^2)^{3/2}2\mu_i T\sigma_i^2}{(1+2\mu_i T\sigma_i^2)^2}$$ **~ 0.09**

Nonaka et al. (2004, Θ^+)
Kanada-En'yo, B. Muller (2006, $\Lambda(1520)$)
Large V disfavors multi-quarks !

- **STAR data (2003): N(f_0(980)) ~ 8.4**
  [f_0(980)/ ρ^0 ~ 0.2, stat. N(ρ^0) ~ 42]
  Stat: 5.6, 2q:0.76-3.8, 4q:0.1, Mol: 13
  → Tetra-quark picture
  underestimate the measured yield of f_0 .



P. Fachini [STAR Collaboration], Nucl. Phys. A 715,462 (2003).