Strong-Coupling Lattice QCD with fluctuation and plaquette effects

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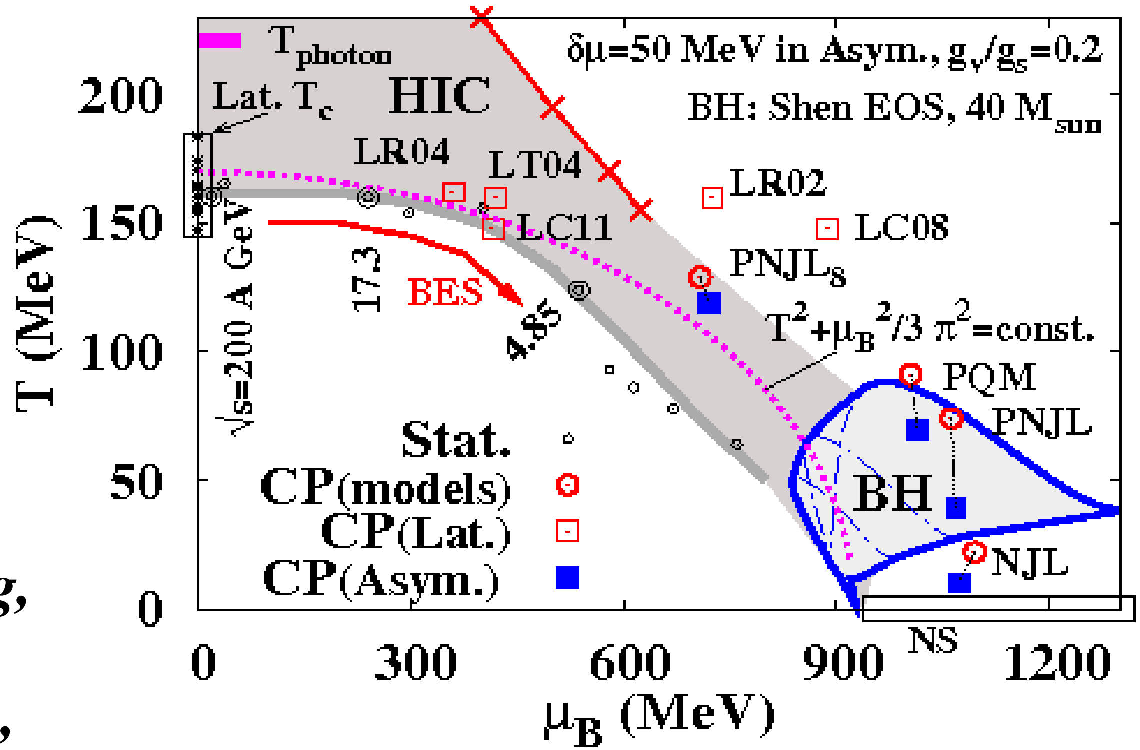
AO, T. Ichihara, Latitce 2015 & in prep.
T. Ichihara, AO, T.Z. Nakano, PTEP 2014, 123D02
T. Ichihara, K. Morita, AO, PTEP 2015, 113D01

QCD phase diagram and Sign Problem

- **Finite density QCD**
  Critical Point,
  Nuclear Matter EOS,
  Compact Stars, …
- **Obstacle**
  = Sign problem at finite μ.
- **Many methods proposed.**
  Taylor expansion, Re-weighting,
  Imag. μ+AC,
  Canonical, Fugacity expansion,
  Histogram method,
  Complex Langevin,
  Lefschetz thimble,
  Strong Coupling,
  …
  but it is still difficult
  to attack large μ/T region.

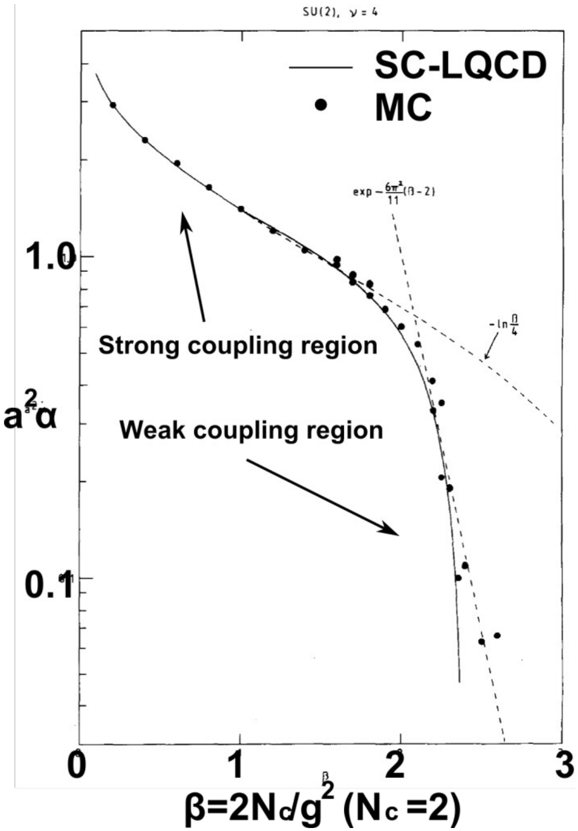
AO, PTPS193('12)1.



How can we explore
μ/T > 1 region in QCD ?

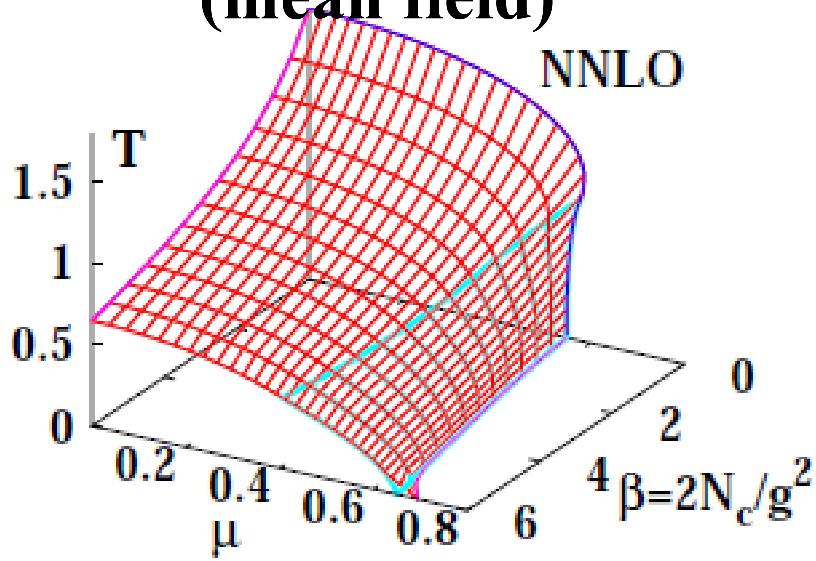
Strong Coupling Lattice QCD

Pure YM → Area Law



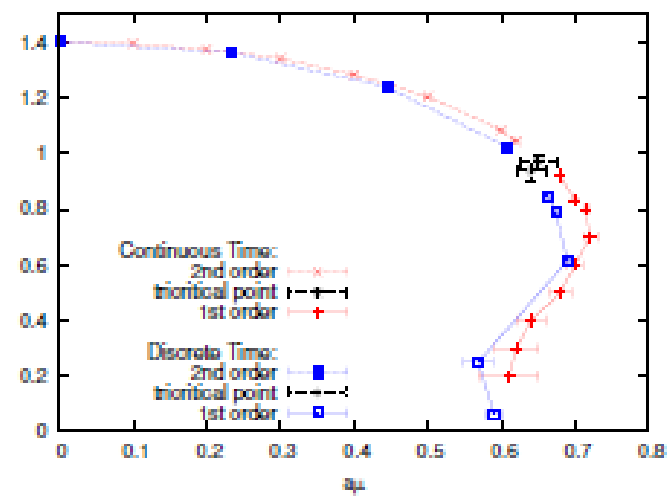
Wilson ('74), Creutz ('80), Munster ('80, '81), Lottini, Philipsen, Langelage's ('11)

Phase diagram (mean field)



Kawamoto ('80), Kawamoto, Smit ('81), Damagaard, Hochberg, Kawamoto ('85), Ilgenfritz, Kripfganz ('85), Bilic, Karsch, Redlich ('92), Fukushima ('03); Nishida ('03), Kawamoto, Miura, AO, Ohnuma ('07). Miura, Nakano, AO, Kawamoto ('09), Nakano, Miura, AO ('10)

Fluctuations

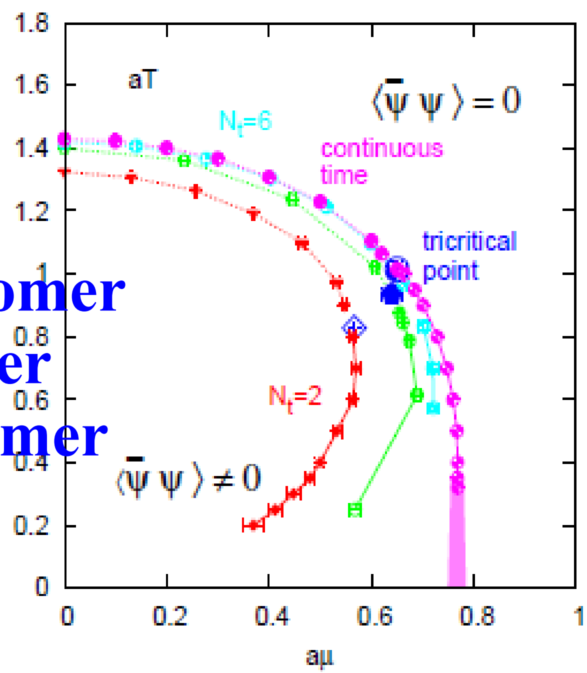


Mutter, Karsch ('89), de Forcrand, Fromm ('10), de Forcrand, Unger ('11), AO, Ichihara, Nakano ('12), Ichihara, Nakano, AO ('14), de Forcrand, Langelage, Philipsen, Unger ('14)

Phase diagram in the Strong Coupling Limit

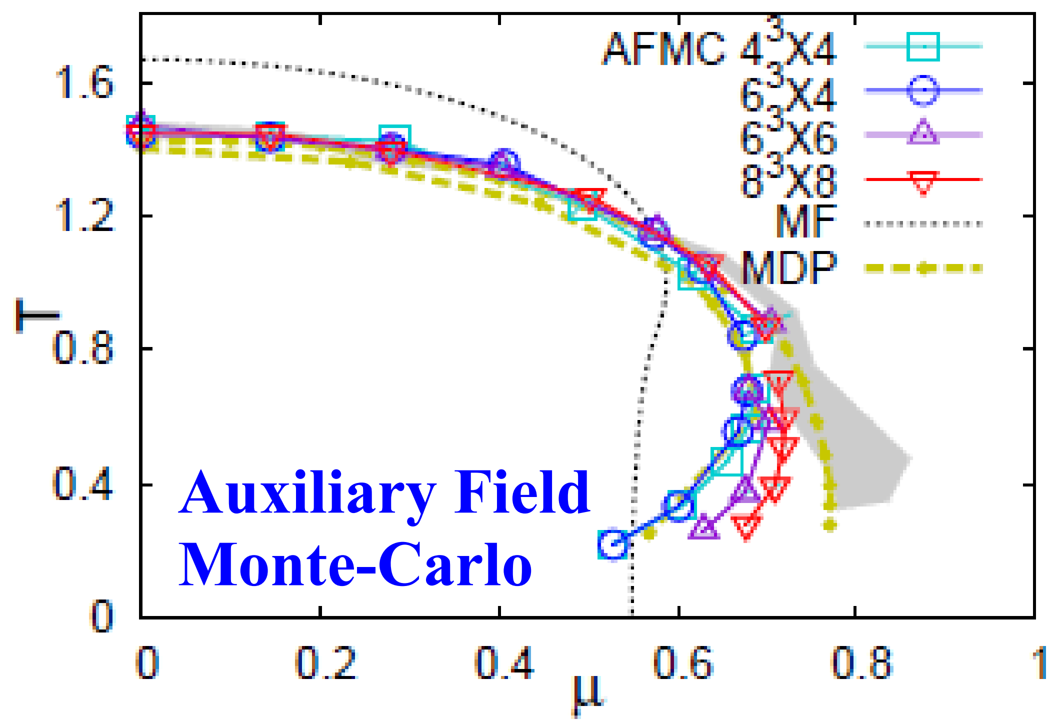
Wilson ('74), Kawamoto ('80), Kawamoto, Smit ('81), Aoki ('84), Damagaard, Hochberg, Kawamoto ('85), Bilic, Karsch, Redlich ('92), Fukushima ('03), Nishida ('03), Kawamoto, Miura, AO, Ohnuma ('07). Miura, Nakano, AO, Kawamoto ('09), Nakano, Miura, AO ('10), Mutter, Karsch ('89), de Forcrand, Fromm ('10), de Forcrand, Unger ('11), Misumi, Nakano, Kimura, AO('12), Misumi, Kimura, AO('12), AO, Ichihara, Nakano ('12), Ichihara, Nakano, AO ('14), Tomboulis ('13), ...

- **Integrate links first, and fermions later → Milder sign prob.**
- **Two indep. methods give consistent phase diagram in the *Strong Coupling Limit*.**



Monomer -Dimer -Polymer

de Forcrand, Fromm ('10), de Forcrand, Langelage, Philipsen, Unger ('14)



Auxiliary Field Monte-Carlo

T.Ichihara, AO, T.Z.Nakano, PTEP2014,123D02.

Cumulant Ratio: Phase transition signal ?

- **Cumulants** $\chi^{(n)} = \dfrac{\partial^n (P/T^4)}{\partial \hat{\mu}^n}, \quad \hat{\mu} = \mu_B / T$

  $\chi^{(4)}/\chi^{(2)} = \kappa\sigma^2 \quad (\kappa : \text{kurtosis})$

  - **$\kappa\sigma^2$ shows DOF at μ=0, and criticality at μ>0.**

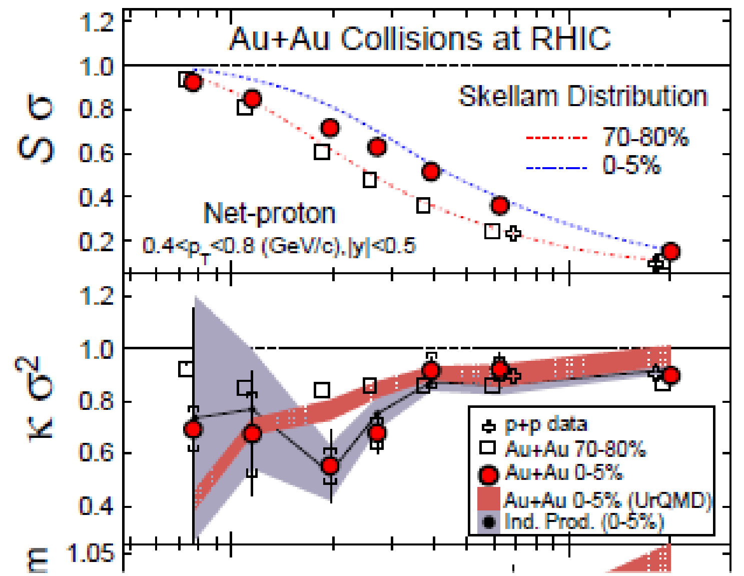
- **Lattice MC at μ=0**
  Bazarov et al.('14), Bellwied et al.('13), ….
  Gavai, Gupta ('05), Allton et al. ('05),

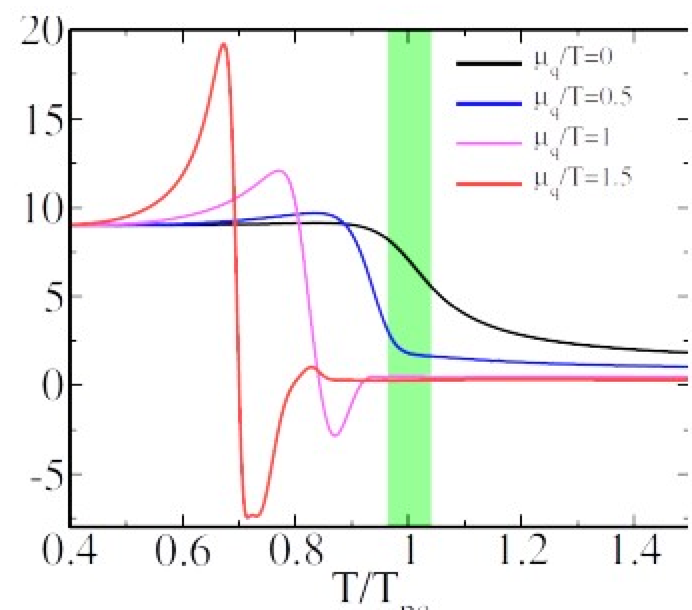
- **Lattice MC at μ>0 but large m_q**
  Jin, Kuramashi, Nakamura, Takeda, Ukawa ('15)

- **Scaling function analysis**
  Friman, Karsch, Redlich, Skokov ('11)

- **Lattice MC in χ-limit (SCL)**
  Ichihara, Morita, AO ('15)

STAR Collab., PRL 112('14)032302

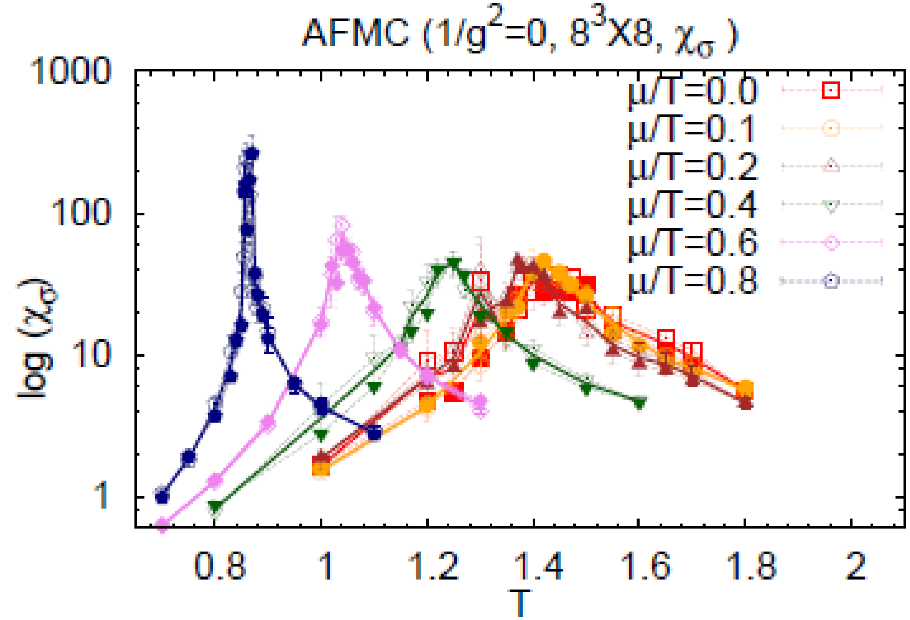




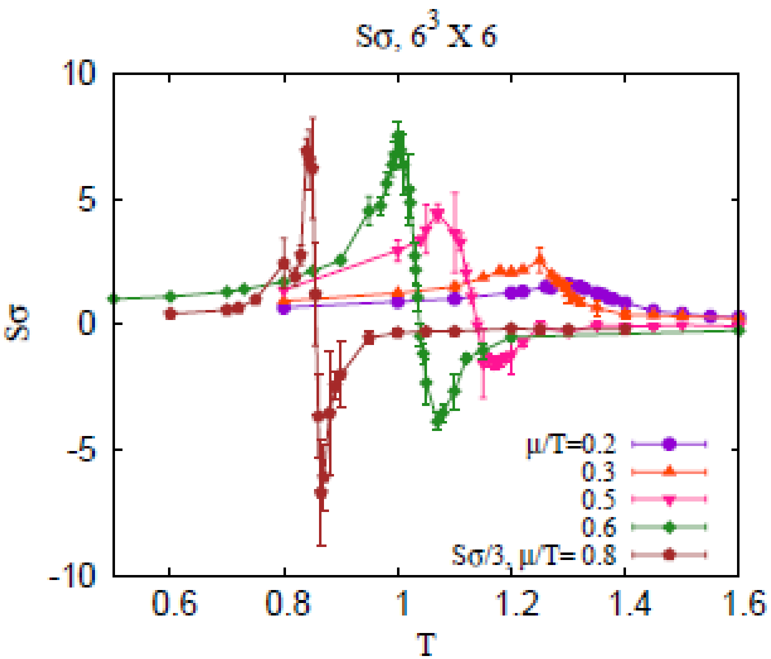
Friman et al. ('11)

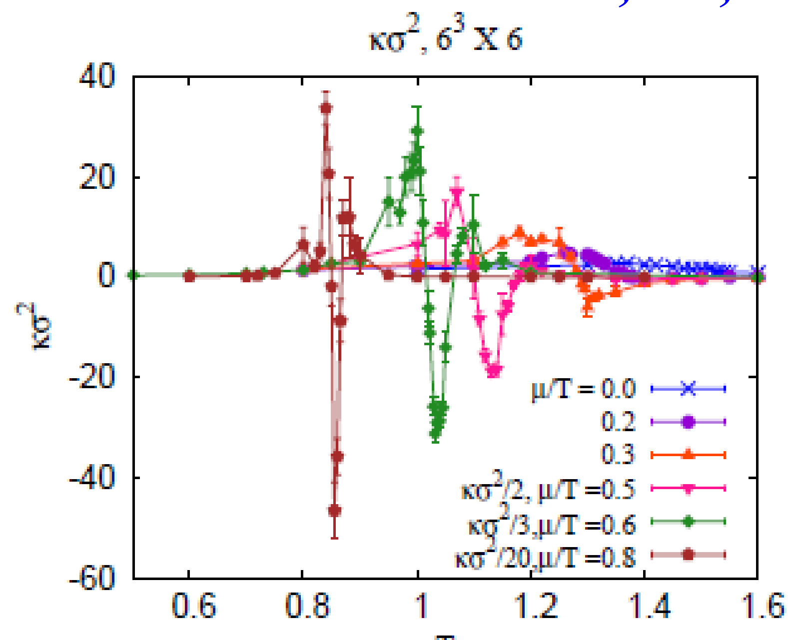
Susceptibilities, Skewness, Kurtosis, ...

- **Chiral susceptibility**
  → Divergent at V → ∞
- **Net baryon skewness**
  Sσ → +∞ from below
  - ∞ from above
- **Net baryon kurtosis**
  κσ² → +-+ structure



Ichihara, AO, Nakano ('14)





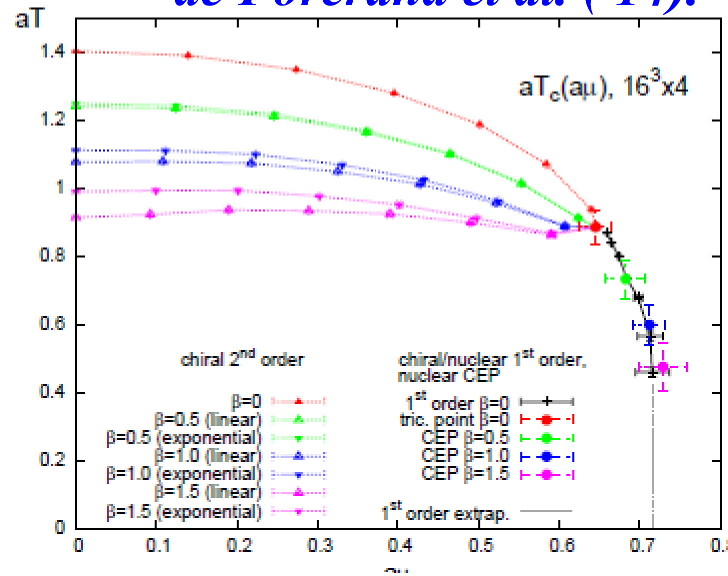
Ichihara, Morita, AO ('15)

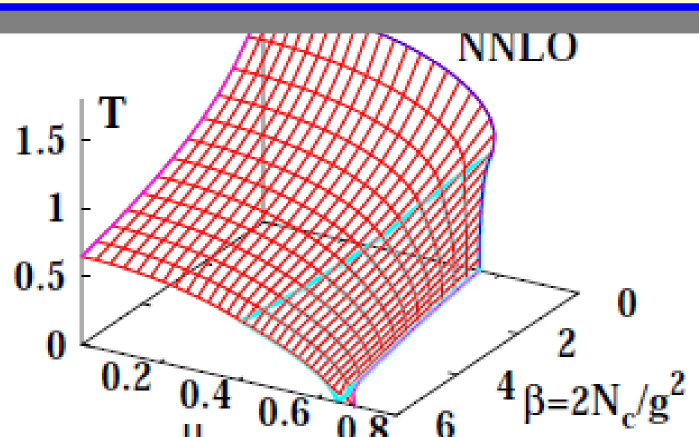
Finite $1/g^2$ corrections

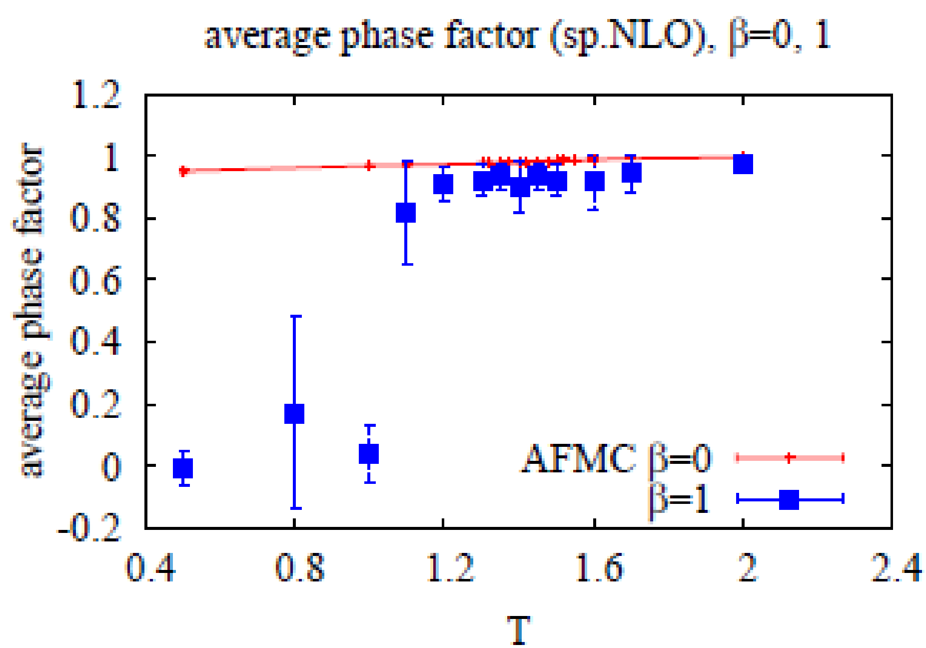
- **Mean field results: No sign prob.**
- **Monomer-Dimer-Polymer:**
  Phase diagram by reweighting
  de Forcrand, Langelage, Philipsen, Unger ('14)
- **Auxiliary Field Monte-Carlo:**
  Severe weight cancellation at finite $1/g^2$
  T.Ichihara, T.Z.Nakano, AO, Lattice 2014

Direct sampling method is not yet fully developed.

de Forcrand et al. ('14).







AO, Miura, Nakano, Kawamoto (Lattice 2009), Nakano, Miura, AO ('10, '11)

T.Ichihara, T.Z.Nakano, AO, Lattice 2014

Contents

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Auxiliary Field Action at Strong Coupling including $1/g^2$ Effects

Lattice QCD action

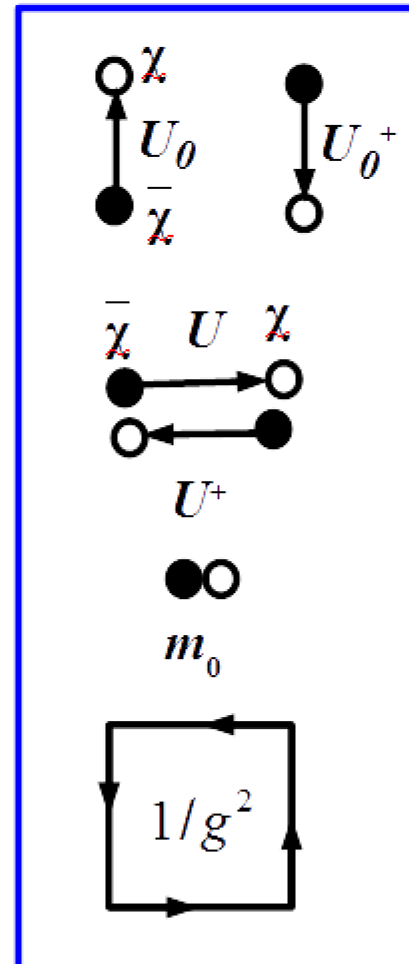
- **Lattice QCD action (unrooted staggered fermion)**

$$L=\frac{1}{2}\sum_x\left[V_x^+-V_x^-\right]+\frac{1}{2\gamma}\sum_{x,j}\eta_j(x)\left[\bar{\chi}_x U_j(x)\chi_{x+\hat{j}}-\bar{\chi}_{x+\hat{j}}U_j^+(x)\chi_x\right]$$

$$+\frac{m_0}{\gamma}\sum_x\bar{\chi}_x\chi_x+\frac{2N_c}{g^2}\left[\gamma S_\tau^{\text{plaq}}+\frac{1}{\gamma}S_s^{\text{plaq}}\right]$$

$$S_\alpha^{\text{plaq}}=\sum_{P_\alpha}\left[1-\frac{1}{N_c}\mathrm{Re\,Tr}\,U_{P_\alpha}\right]$$

$$V_x^+=\bar{\chi}_x U_0(x)e^{\mu/\gamma^2}\chi_{x+\hat{0}},\quad V_x^-=\bar{\chi}_{x+\hat{0}}U_0^+(x)e^{-\mu/\gamma^2}\chi_x$$



- **Staggered sign factor**
  $\eta_j(x)=(-1)^{**}(x_0+...+x_{j-1})$
- **$U(1)_L$ x $U(1)_R$ chiral sym.**
  $\chi_x\to\exp[i\,\theta\varepsilon(x)]\,\chi_x$, $\varepsilon(x)=(-1)^{**}(x_0+x_1+x_2+x_3)$
- **Anisotropy parameter γ ($T=\gamma^2/N_\tau$)** *E.g. Bilic et al. ('92)*

Strong Coupling Lattice QCD

- **Strong coupling limit**
  Damgaard, Kawamoto, Shigemoto ('84), Jolicoeur, Kluberg-Stern, Lev, Morel, Peterssson ('84).
  Spatial link integral→ Fermion action with four-Fermi int. (LO in 1/d expansion)

$$S_{\text{eff}}^{(\text{SCL})}=\frac{1}{2}\sum_x\left[V_x^+-V_x^-\right]+\frac{m_0}{\gamma}\sum_x M_x$$

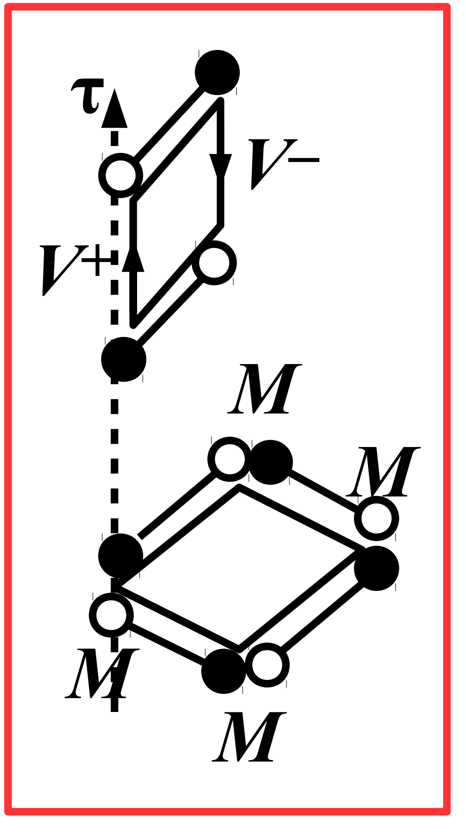
$$-\frac{1}{4N_c\gamma^2}\sum_{x,j}M_x M_{x+\hat{j}}\quad\left(M_x=\bar{\chi}_x\chi_x\right)$$

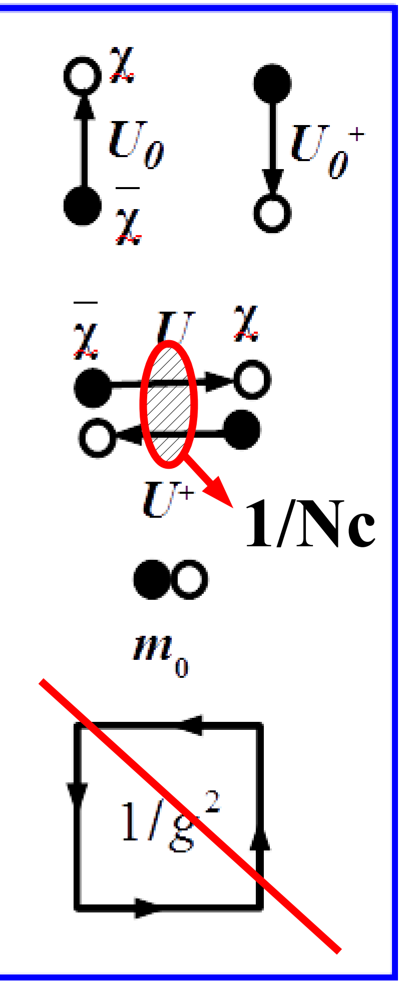
- **Eff. Action with 1/g² correction**
  Faldt, Petersson ('86), Miura, Nakano, AO, Kawamoto ('09)

$$S_{\text{eff}}^{(\text{NLO})}=S_{\text{eff}}^{(\text{SCL})}+\frac{\beta_\tau}{2}\sum_{x,j}\left[V_x^+V_{x+\hat{j}}^-+V_{x+\hat{j}}^+V_x^-\right]$$

$$-\frac{\beta_s}{\gamma^4}\sum_{x,k,j,k\neq j}M_x M_{x+\hat{j}}M_{x+\hat{k}}M_{x+\hat{k}+\hat{j}}$$

$$\beta_\tau=1/2\,N_c^2g^2\gamma,\quad \beta_s=1/16\,N_c^4g^2\gamma$$





Extended Hubbard-Stratonovich Transformation

- **Extended HS transf.** $\exp(\alpha AB) = \int d\varphi\, d\varphi^* \exp(-\alpha[\varphi^*\varphi + \varphi^* A + \varphi B])$
- **Bosonized Effective Action**

$$S_{\text{eff}}^{(\text{EHS})} = \frac{1}{2}\sum_x \left(Z_x^- V_x^+ - Z_x^+ V_x^-\right) + \frac{1}{\gamma}\sum_x m_x M_x + S_{\text{AF}}$$

$$m_x = m_0 + \frac{1}{4N_c}\sum_j (\sigma + i\varepsilon\pi)_{x\pm\hat{j}} + \beta_s \sum_j \left\{\varphi_x^{(j)*}(\Theta_x^{(j)})^{1/2} + \varphi_{x-\hat{j}}^{(j)*}(\Theta_{x-\hat{j}}^{(j)})^{1/2}\right\}$$

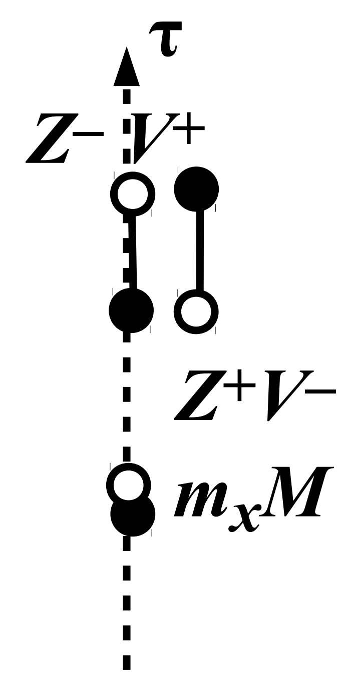
$$S_{\text{AF}} = \frac{L^d}{4N_c}\sum_{\mathbf{k},\tau,f(\mathbf{k})>0} f(\mathbf{k})\left[\sigma_{\mathbf{k},\tau}^*\sigma_{\mathbf{k},\tau} + \pi_{\mathbf{k},\tau}^*\pi_{\mathbf{k},\tau}\right]$$

$$+ \beta_s L^3 \sum_{\mathbf{k},\tau,f^{(j)}(\mathbf{k})>0} f^{(j)}(\mathbf{k})\left[\sigma_{\mathbf{k},\tau}^{(j)*}\sigma_{\mathbf{k},\tau}^{(j)} + \pi_{\mathbf{k},\tau}^{(j)*}\pi_{\mathbf{k},\tau}^{(j)}\right] + \beta_s \sum_{x,j}\varphi_x^{(j)*}\varphi_x^{(j)}$$

$$+ \beta_\tau L^3 \sum_{\mathbf{k},\tau,f(\mathbf{k})>0} f(\mathbf{k})\left[\Omega_{\mathbf{k},\tau}^*\Omega_{\mathbf{k},\tau} + \omega_{\mathbf{k},\tau}^*\omega_{\mathbf{k},\tau}\right] .$$

$$\Theta_x^{(j)} = \sum_{k,k\neq j}\left(\sigma^{(j)} + i\varepsilon\pi^{(j)}\right)_{x\pm\hat{k}}$$

$$Z_x^- = 1 + \beta_\tau \sum_j (\omega - \varepsilon\Omega)^*_{x\pm\hat{j}} , \quad Z_x^+ = 1 + \beta_\tau \sum_j (\omega + \varepsilon\Omega)_{x\pm\hat{j}}$$



SCL+1/g² corr. → x dep. mass + mod. of temporal coef.

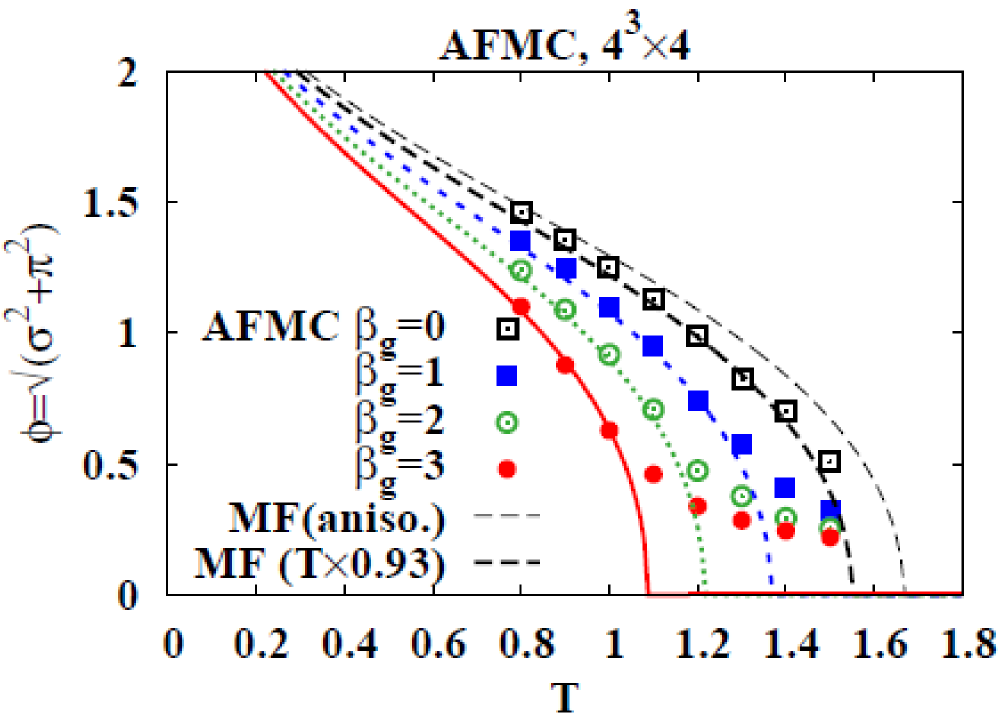
QCD phase diagram at strong coupling including Fluctuation and $1/g^2$ Effects

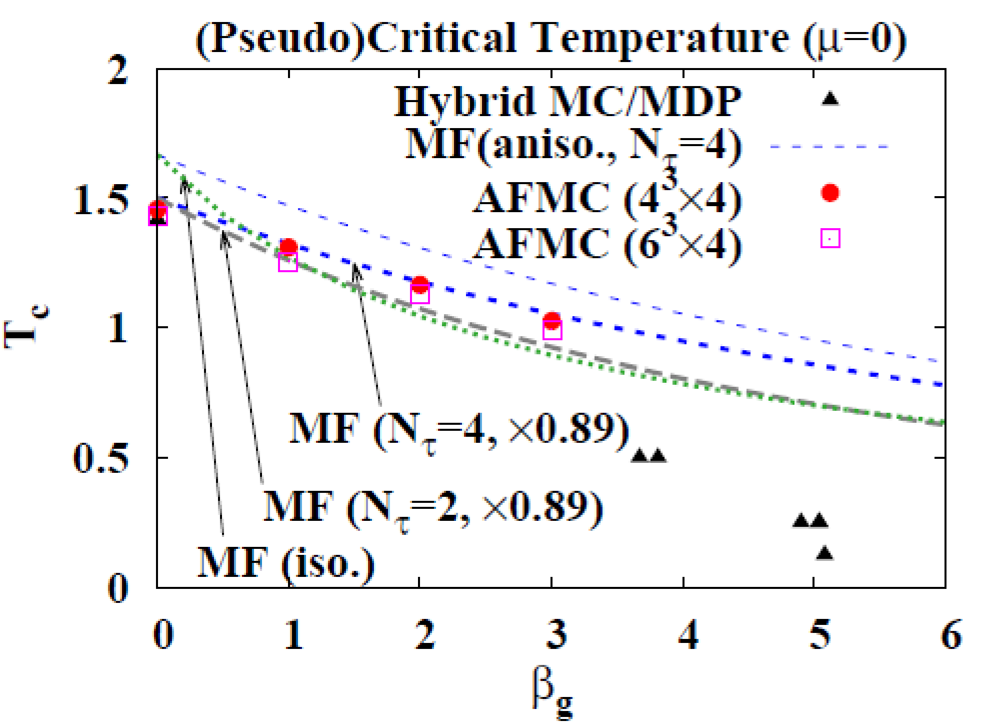
Lattice Setup

- **One species of unrooted starggered fermion**
  - $N_f = 4$ in the continuum limit
  - Chiral limit→ Chiral sym.: $U(1)_L \times U(1)_R$, chiral cond.=$(\sigma^2 + \pi^2)^{1/2}$
  - O(2) symmetry w/o anomaly → proxy for O(4) w/ anomaly (Second order phase transition at $\mu=0$)
- **Strong coupling expansion to $1/g^2$ order.**
  - LO (strong coupling lim.), NLO ($1/g^2$ corr.), $\beta_g=2N_c/g^2 = 0\sim3$.
  - Spatial link: LO in 1/d expansion (no spatial baryon hopping)
  - Temporal link : exact
- **Lattice size : 4^3 x 4, 6^3 x 4**
  - Anisotropic Lattice: $T = \gamma^2 / N_\tau$
- **Auxiliary Field Monte-Carlo**

Phase Transition at μ=0

- **Suppression of transition T from mean field results on anisotropic lattice with N_τ=4 by ~ 10 % for β_g=0~3.**
  - **Tc (aniso. MF, N_τ=4) > Tc (aniso. MF, N_τ=2), Tc (iso. MF)**
  - **Tc from max. -dσ/dT**



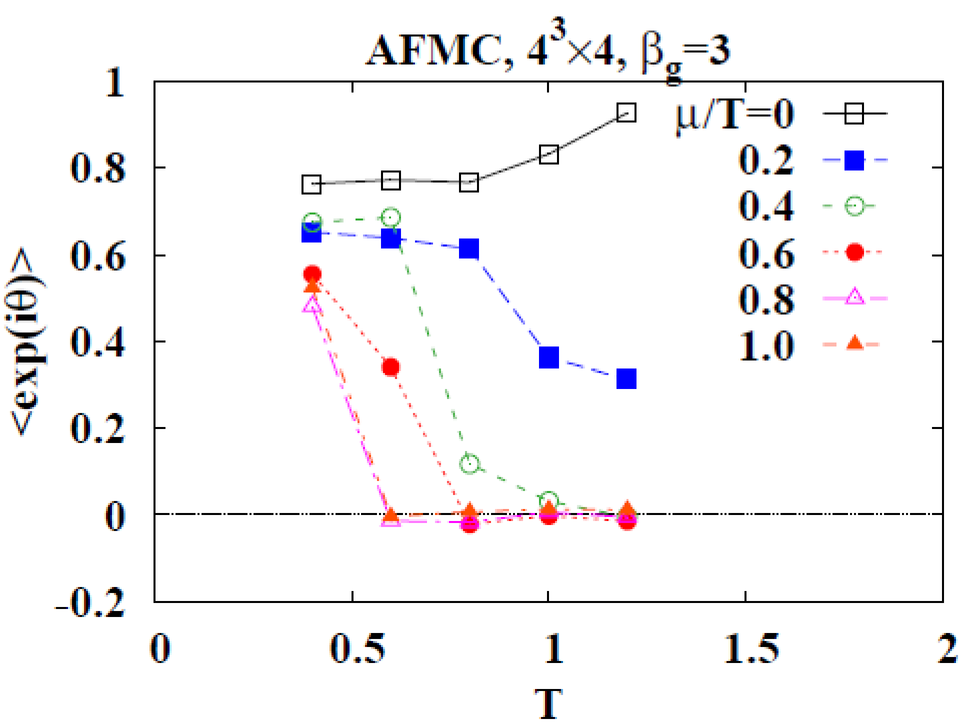


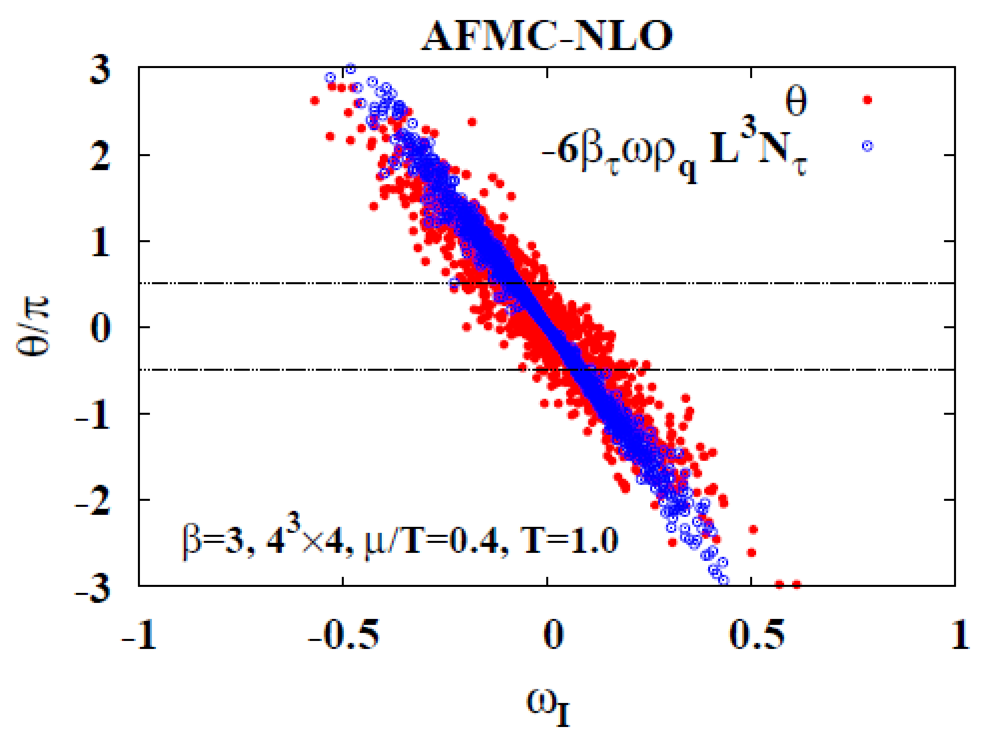
AO, T. Ichihara, Latitce 2015 & in prep.

Finite μ (β_g=3)

- **Sudden collapse of the average phase factor, < exp (i θ) >, where quark number density becomes finite.**
- **Strong correlation is found btw θ and one of the aux. field, ω_I = Im(ω (k=0)) → O(V^2) effect !**

$$\theta \sim -6\beta_\tau \rho_q \omega_I L^3 N_\tau$$





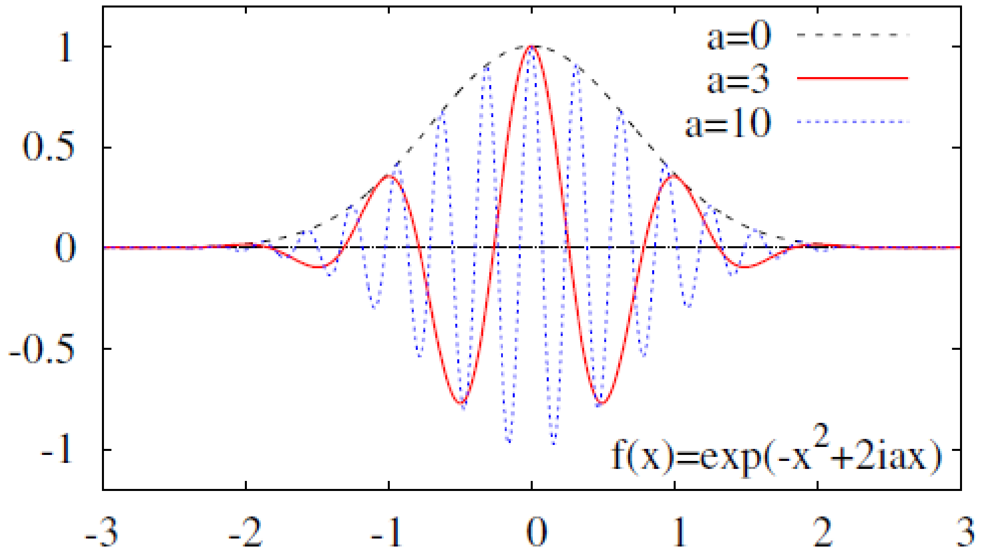
Why do we have a large complex phase ?

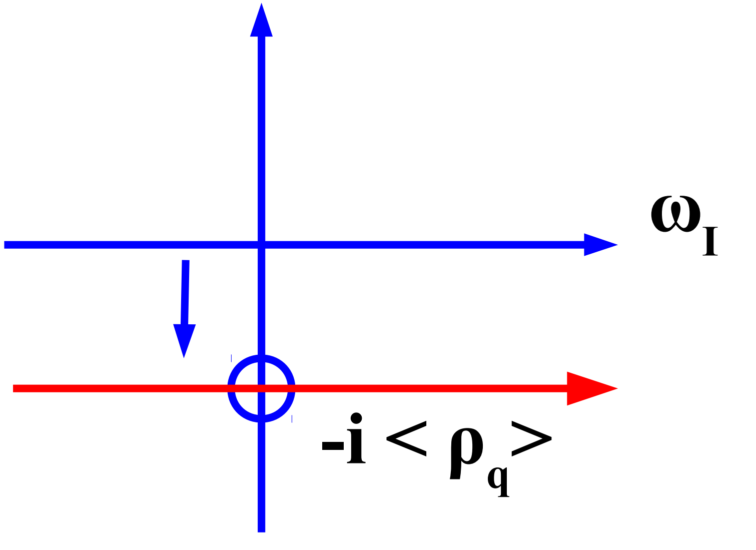
- **Effective action terms including $\omega_I(k=0)$**

$$S_{\omega_I} = \frac{1}{2}C\omega_I^2 + iC\omega_I\rho_q = \frac{1}{2}C(\omega_I + i\rho_q)^2 + \frac{1}{2}C\rho_q^2$$

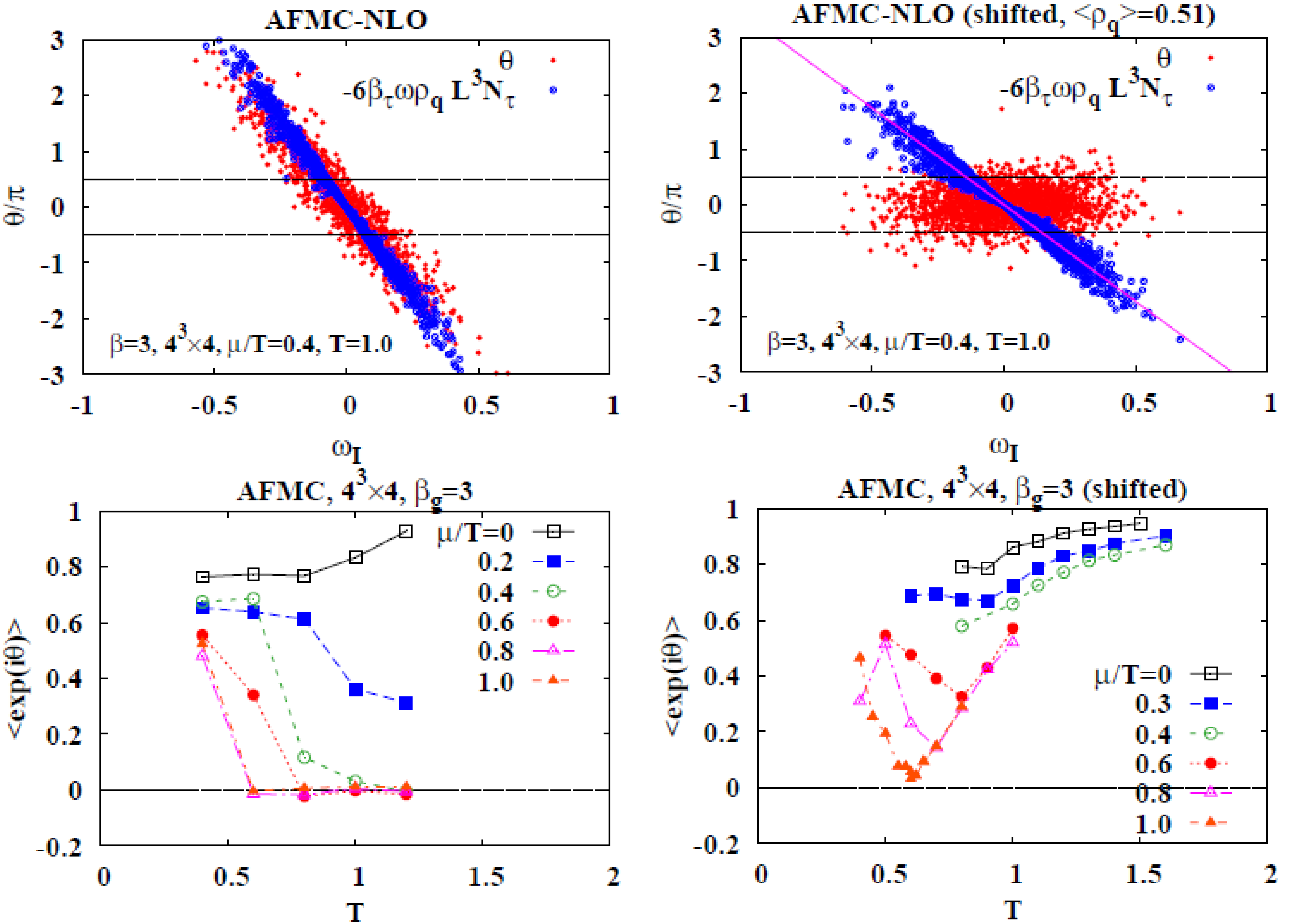
$$(C = 6\beta_\tau L^3 N_\tau)$$

  - **Textbook example of the sign problem,**
    and we know the answer !
    → Shift the integral path to imaginary direction.





Complex Shifted Integration Path



Discussion

- **Vector field (temporal component) in Relativistic Mean Field**

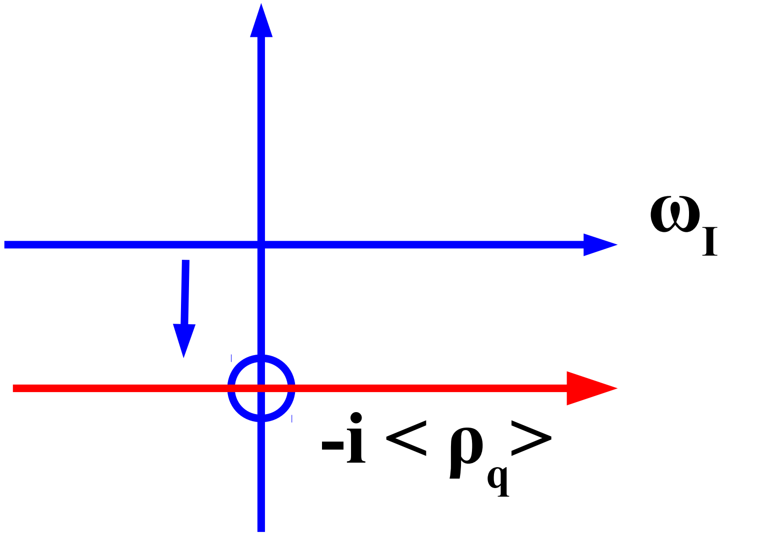
$$E/V = -\frac{1}{2} m_v^2 \omega^2 + g_v \rho_B \omega \rightarrow \frac{1}{2} m_v^2 \omega^2 + i g_v \rho_B \omega \quad (\text{Wick roation})$$

  - **Repulsive potential gives rise to the sign problem.**

- **Lefschetz thimble**

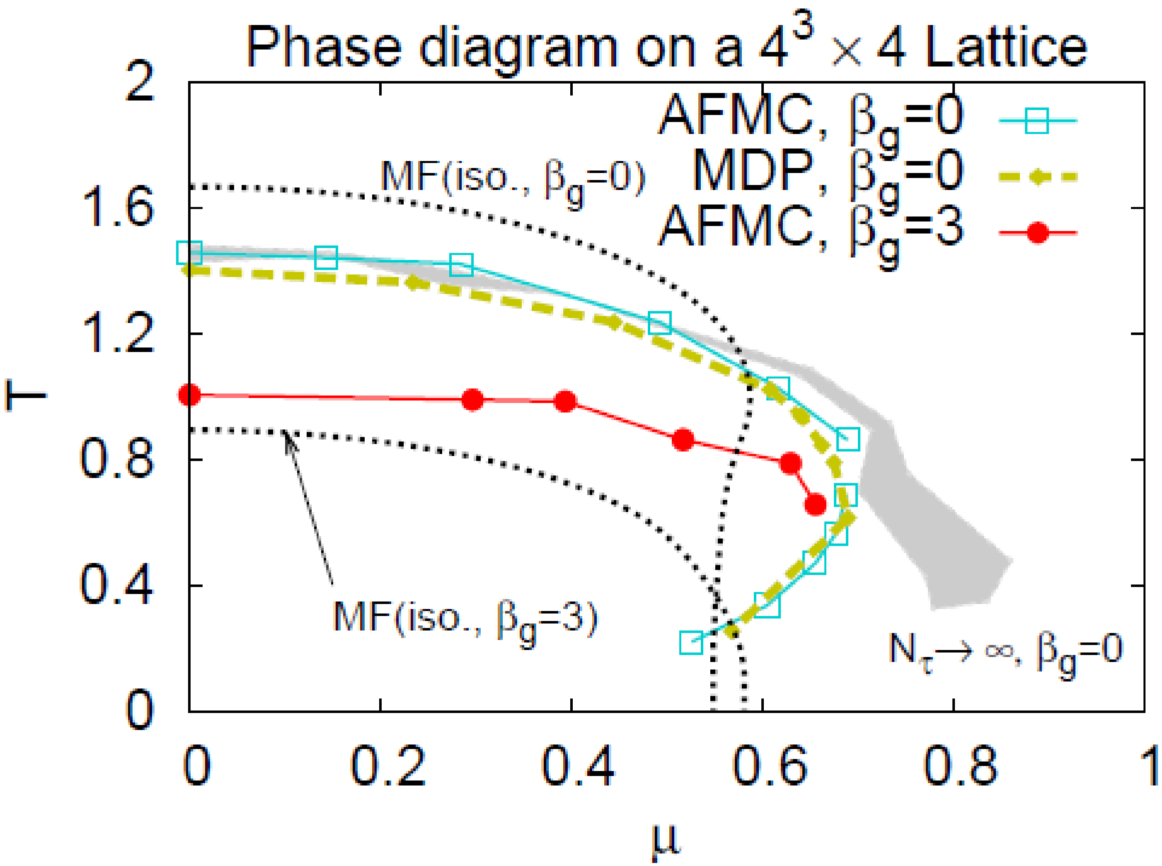
$$Z = \int D\Phi \exp(-S) = \int D\omega \exp(-S_\omega) \ , \quad S_\omega = -\log\left[\int D\Phi' \exp(-S)\right]$$

  - **Stationary point is at ω= -i ρ_q**
    → Shifted path ~ Thimble

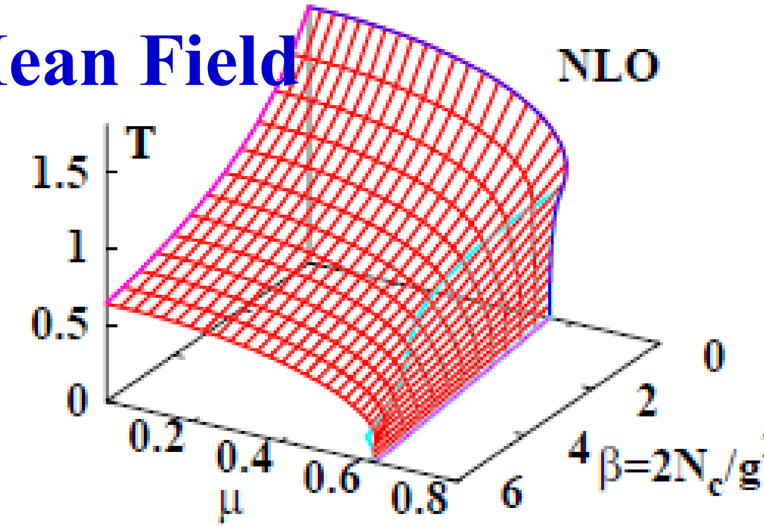


Phase diagram at finite β_g

- **Phase diagram on a $4^3 \times 4$ lattice at $\beta_g = 3$**
  - **Suppression of Tc(μ=0), Smaller curvature.**
  - **Similar to previous works**

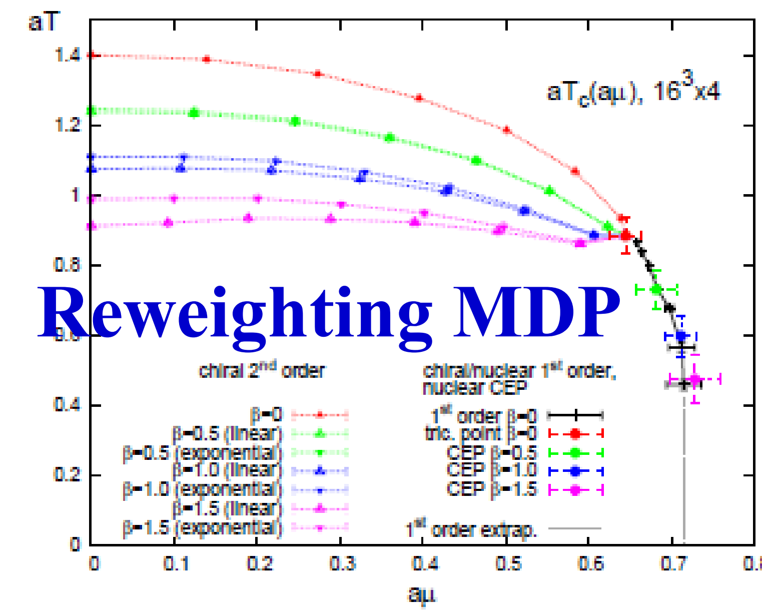


Mean Field



Miura, Nakano, AO, Kawamoto ('09), AO, Miura, Nakano, Kawamoto ('09)

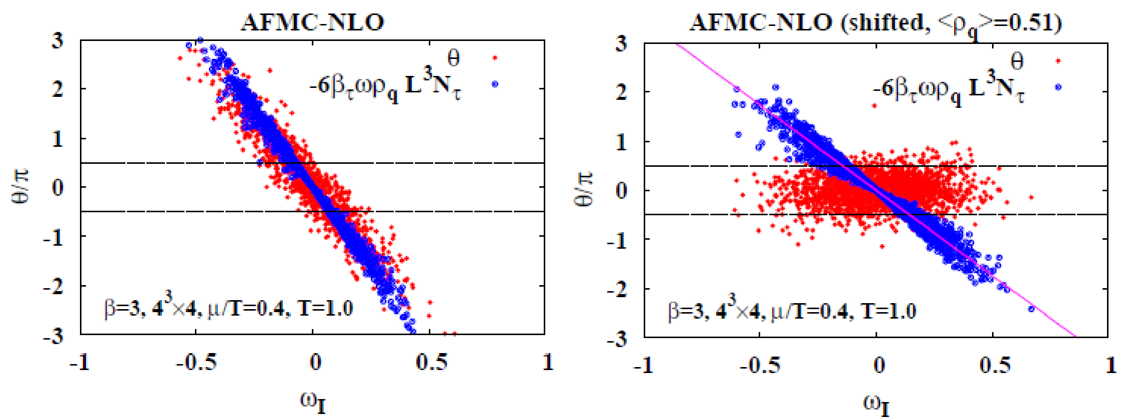
Reweighting MDP



de Forcrand, Langelage, Philipsen, Unger, PRL113('14)152002

Summary

- Strong-coupling lattice QCD including fluctuation and $1/g^2$ effects is developed, and applied to phase diagram.
  - Complex phase mainly comes from the temporal plaquette term, which can be removed by shifting the integration path.



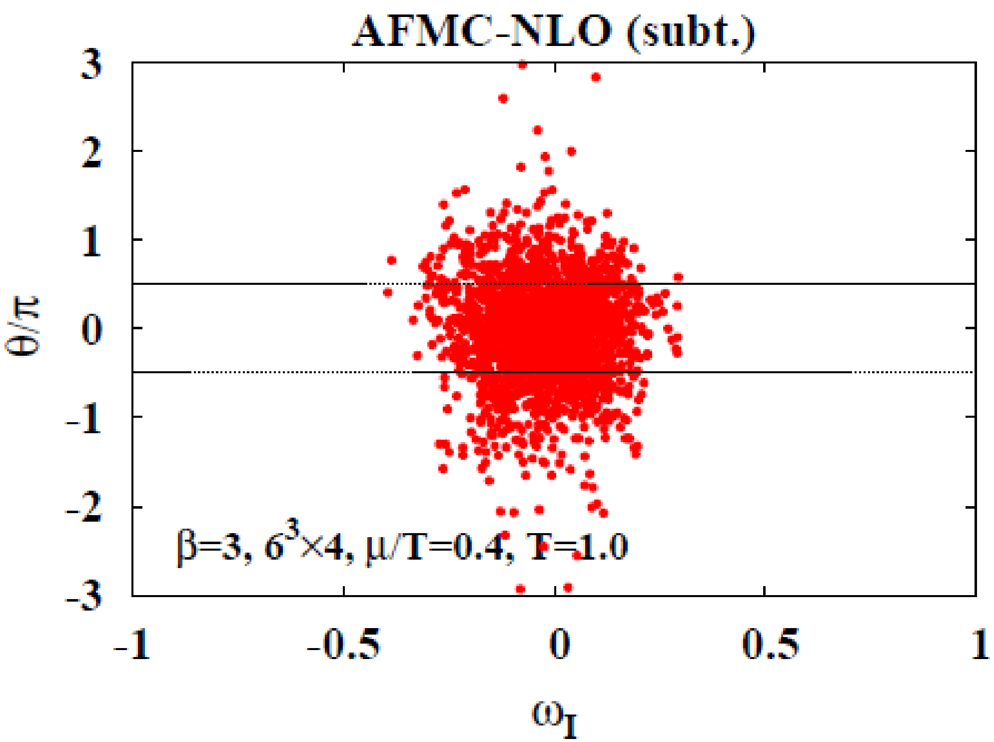
  - Preliminary phase diagram is obtained. As MF suggests, Tc decreases and the phase boundary becomes more flat.
  - Shifted aux. field corresponds to repulsive vector potential.
  - Caveat: Self-consistent subtraction is necessary for stability.
- Complex shift simulates integral on Lefschetz thimble. (It may be helpful in other systems ....)

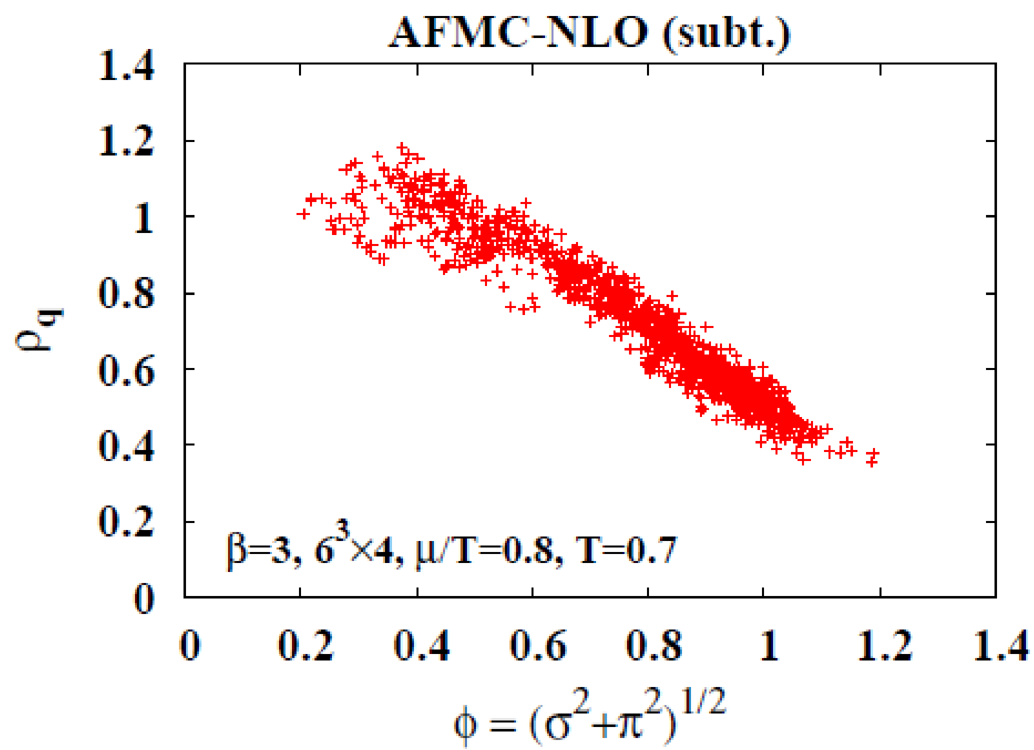
Thank you !

AO, T. Ichihara, Latitce 2015 & in prep.
T. Ichihara, AO, T.Z. Nakano, PTEP 2014, 123D02
T. Ichihara, K. Morita, AO, PTEP 2015, 113D01

Remaining Problem

- **Larger spread of θ distribution on lager lattice.**
  How to handle it ?
- **Phase coexisting region.**
  Two local minima have different ρ_q values,
  then how can we shift the path ?

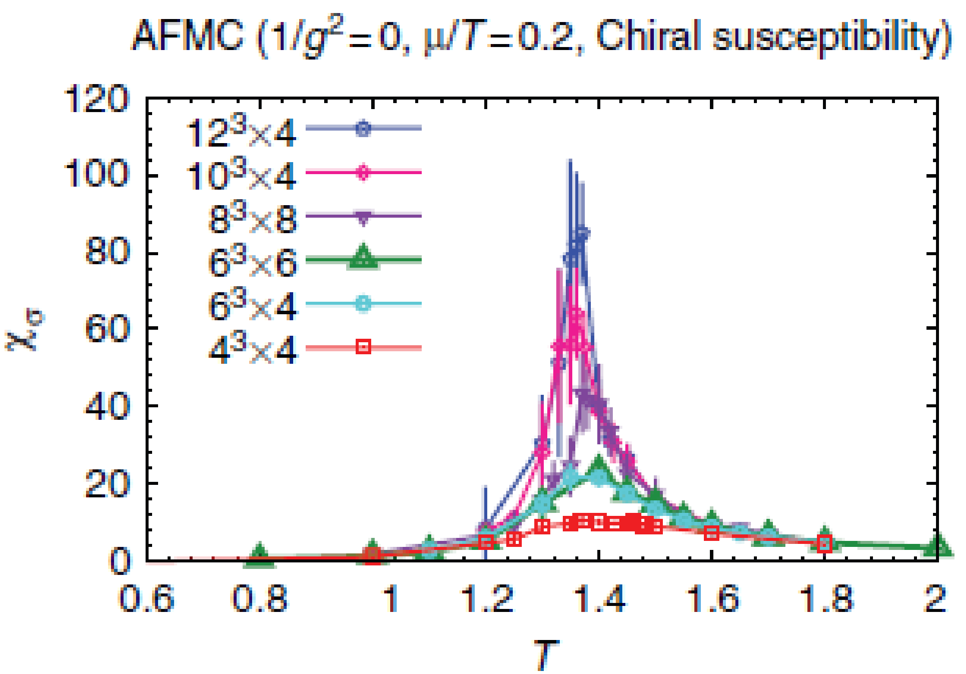


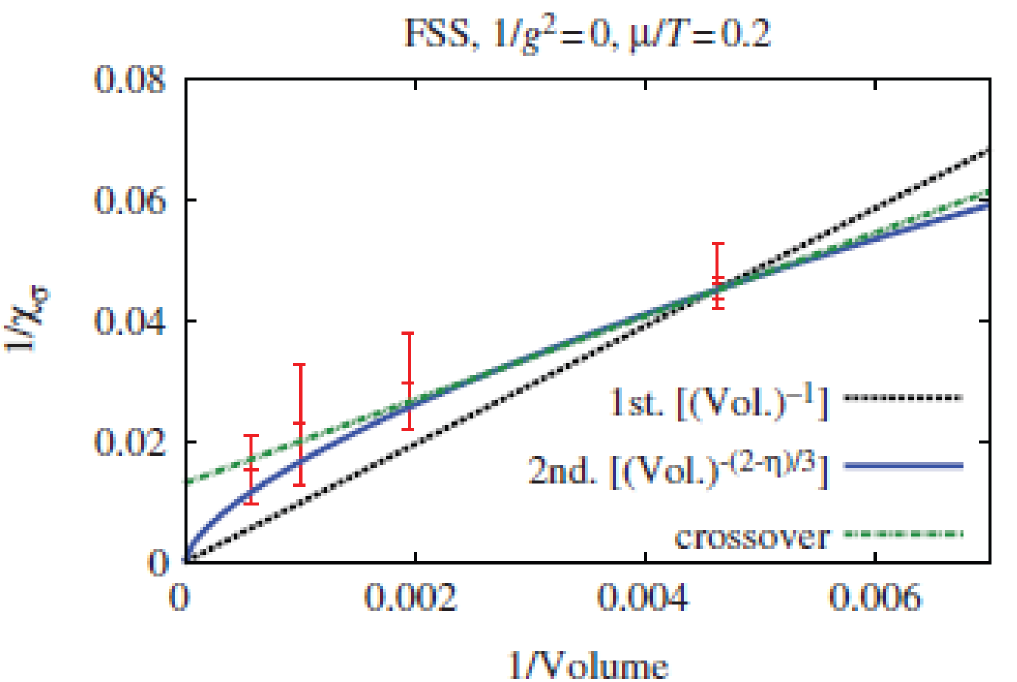


Finite Size Scaling of Chiral Susceptibility

- **Peak height of chiral susceptibility on lattice > $6^3 \mathrm{x} N_\tau$ is consistent with O(2) finite size scaling.**
  Y. Aoki, G. Endrodi, Z. Fodor, S. D. Katz, and K. K. Szabo, Nature 443('06), 675; M. Campostrini, M. Hasenbusch, A. Pelissetto, P. Rossi, and E. Vicari, PRB 63('01), 214503.

- **Pion mass = 0, but zero momentum pions are absorbed by σ because of chiral symmetry.**





T. Ichihara, AO, T.Z.Nakano, PTEP 2014, 123D02