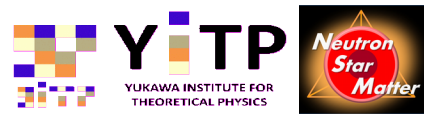

Three Baryon Interaction in the Quark Cluster Model

– 3B Interaction from Determinant Interaction of Quarks as an example –

Akira Ohnishi, Kouji Kashiwa, Kenji Morita
YITP, Kyoto U.

QH seminar, Nov. 11, 2016

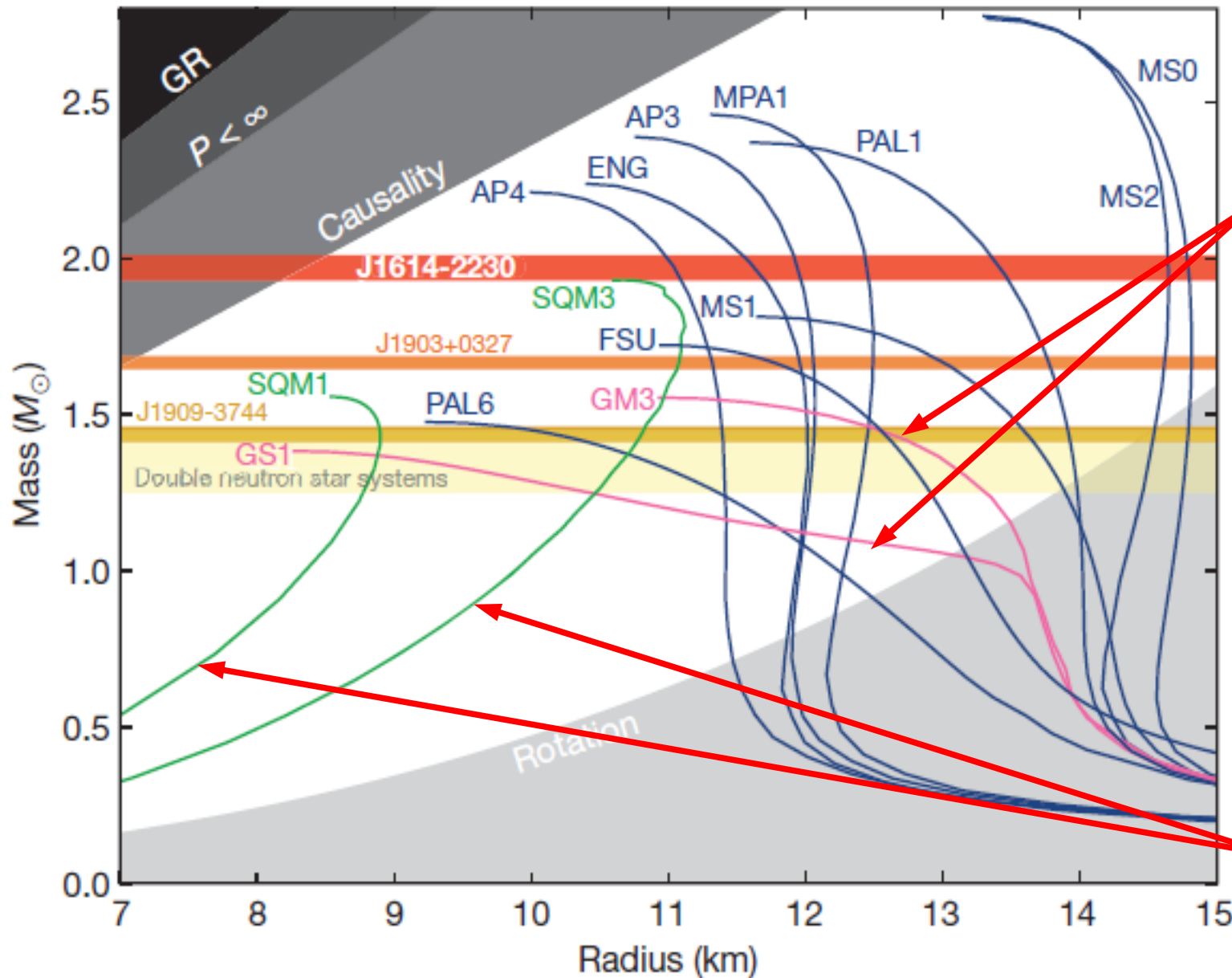
AO, K. Kashiwa, K. Morita, arXiv:1610.06306



- **Introduction**
 - **Hyperon Puzzle**
 - **Kobayashi-Maskawa-'t Hooft interaction**
- **Three-baryon force from KMT interaction from quark cluster model**
 - **Quark cluster model**
 - **Norm of 3B states**
 - **3B potential from KMT interaction**
- **Summary**

Hyperon Puzzle (or Hyperon Crisis)

Demorest et al., *Nature* 467 (2010) 1081 (Oct.28, 2010).



EOS with hyperons or Kaons

Quark matter EOS

Proposed Solutions

- **Hyperonic EOS cannot support massive NS ($M \sim 2 M_{\odot}$).**

Demorest et al. (2010), Antoniadis et al. (2013)

- **Proposed Solutions**

- **Hyperonic Three-Body Force (or density dep. coupling)**

Bednarek et al. ('12), Jiang et al. ('12); Long et al. ('12); Yamamoto et al. ('14); Lonardonì et al. ('15); Tsubakihara et al. ('13), T. Miyatsu et al. ('13), ...

- **Crossover Transition to Quark Matter**

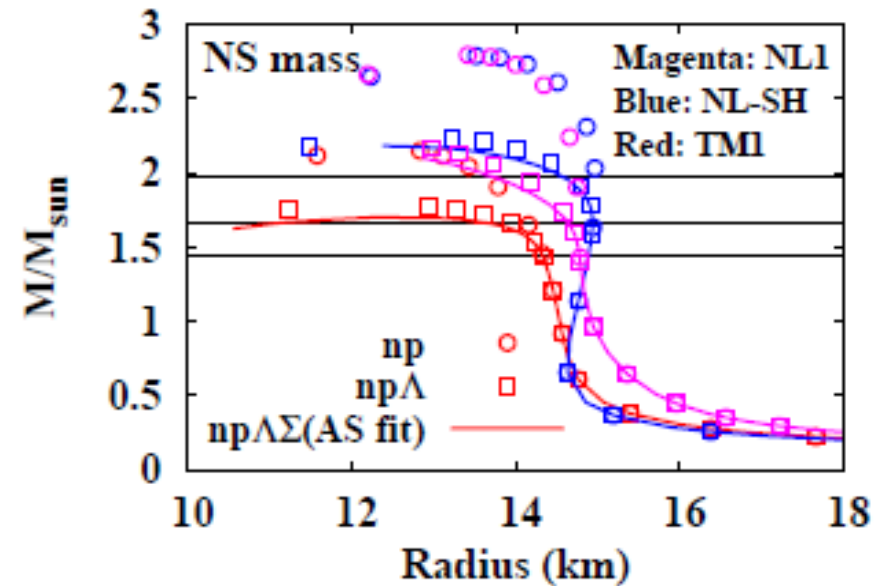
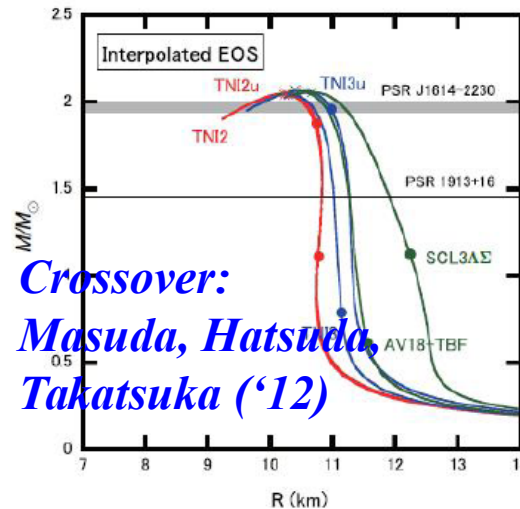
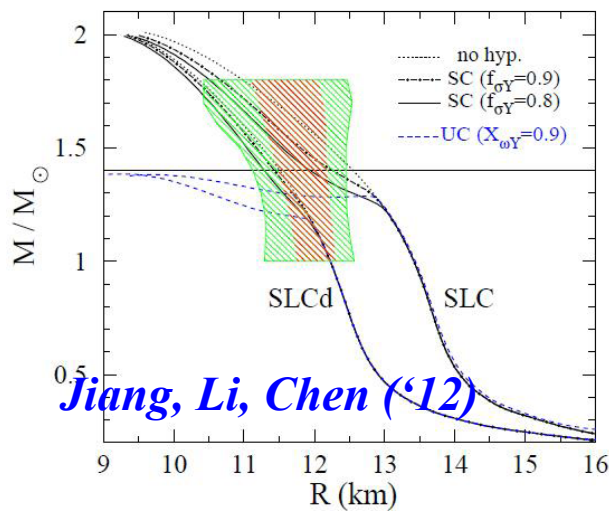
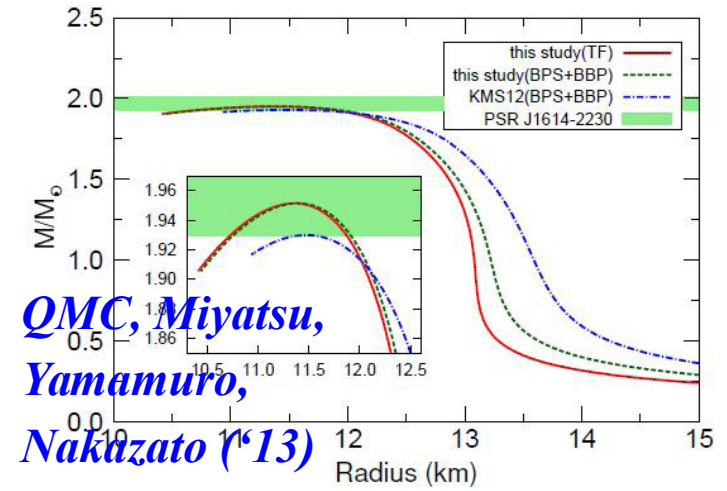
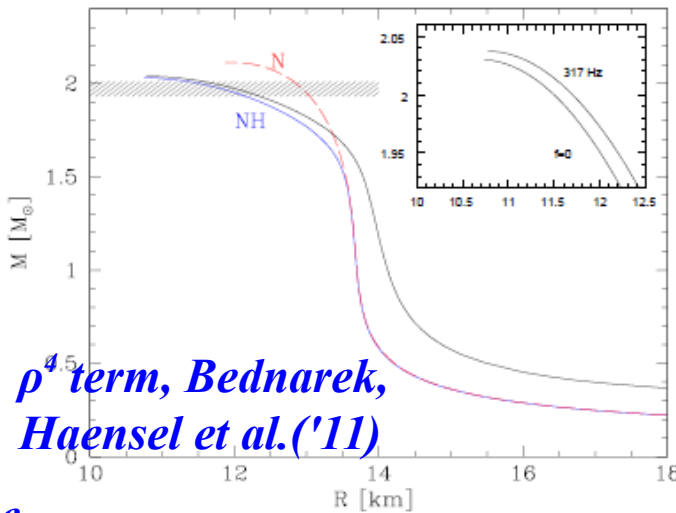
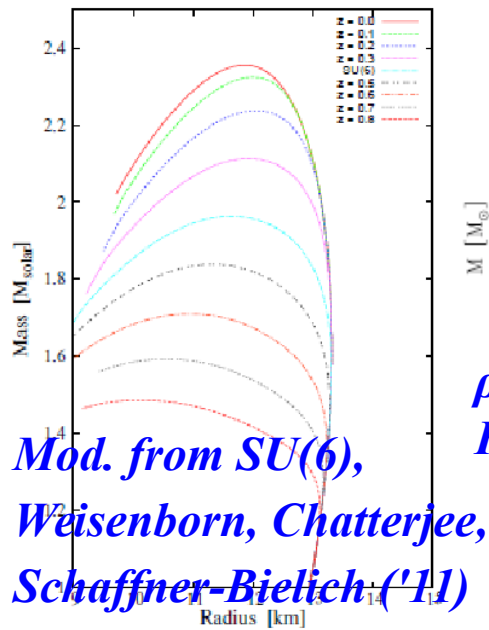
Bonanno et al.('12); Masuda et al. ('13); Bejger et al.('16), ...

- **Modified Gravity**

Astashenok et al. ('14)

- **Three-nucleon interaction is known to be necessary.
How can we determine YNN (+YYN, YYY) potential ?**

Massive Neutron Stars with Hyperons



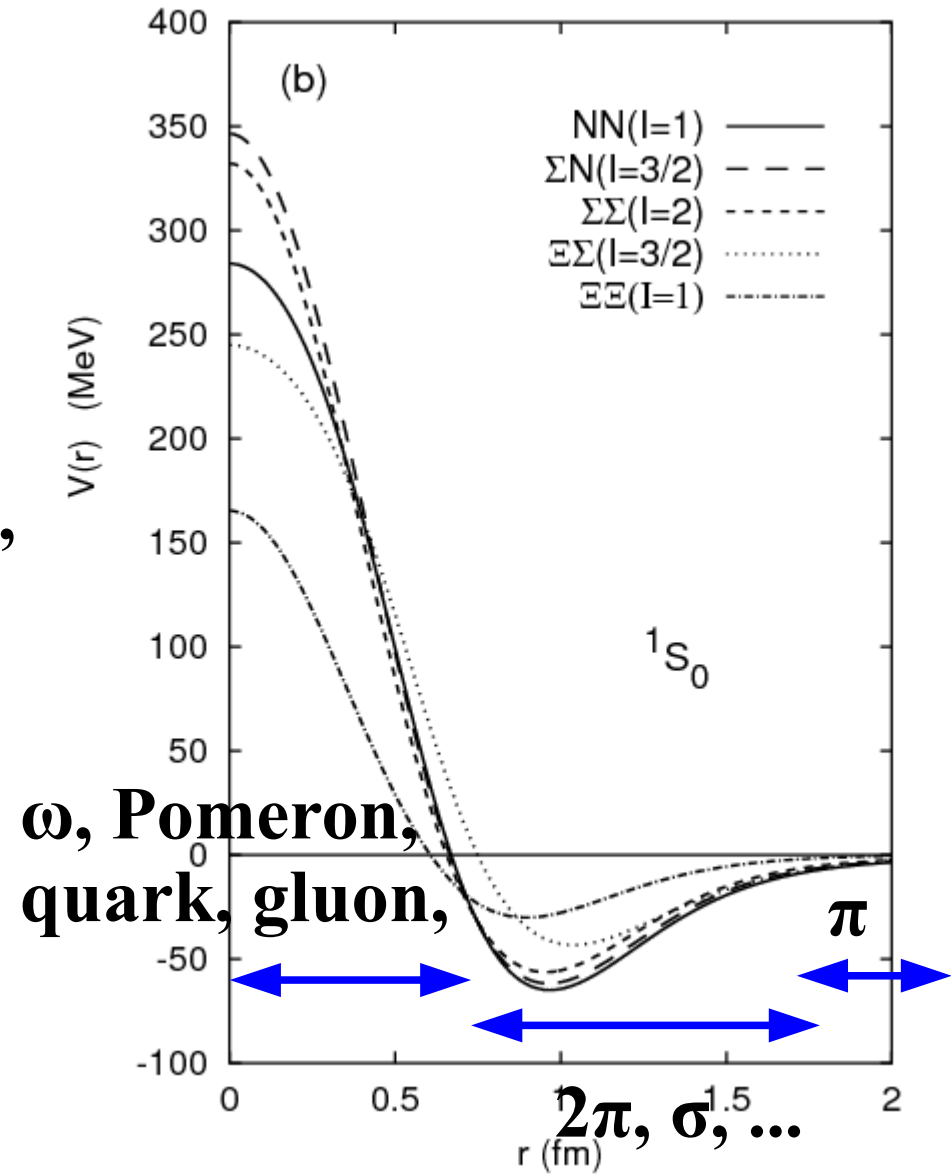
Tsubakihara, Harada, AO, arXiv:1402.0979

Baryon-Baryon Force

- Long-range ($r > 2$ fm): π exch.
- Intermediate ($r \sim 1$ fm): multi π exch., boson exch., ...
- Short range ($r < 0.6$ fm): vector boson exch., Pomeron exch., quark exclusion + one gluon exch., ...

V.G. Neudachin, Yu.F. Smirnov, R. Tamagaki, PTP 58 ('77) 1072; M. Oka, K. Yazaki, PTP ('81)572.

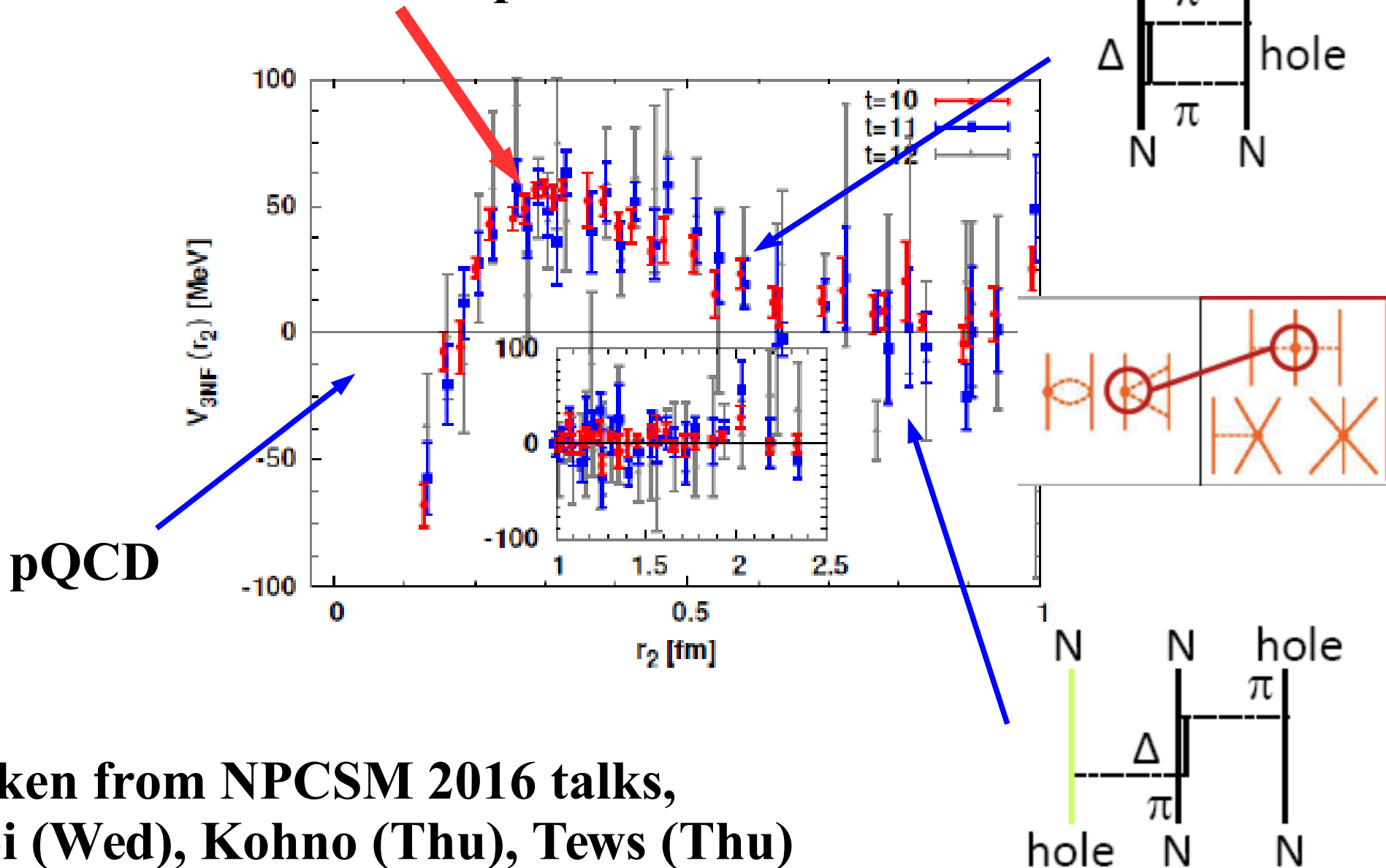
Quark model description of 3B repulsion should be a promising approach !



Fujiwara, Suzuki, Nakamoto ('07)

Three-Baryon force

What makes 3B repulsion at $r \sim 0.5$ fm ?



Taken from NPCSM 2016 talks,
Doi (Wed), Kohno (Thu), Tews (Thu)

Kobayashi-Maskawa-'t Hooft (KMT) interaction

■ KMT interaction

Kobayashi, Maskawa ('70), 't Hooft ('76)

$$\mathcal{L} = g_D (\det \Phi + \text{h.c.}) , \quad \Phi_{ij} = \bar{q}_j (1 - \gamma_5) q_i$$

- Determinant interaction in flavor for three quarks ($SU(3)_f$)

- Responsible for $U(1)_A$ anomaly

η - η' mass diff.

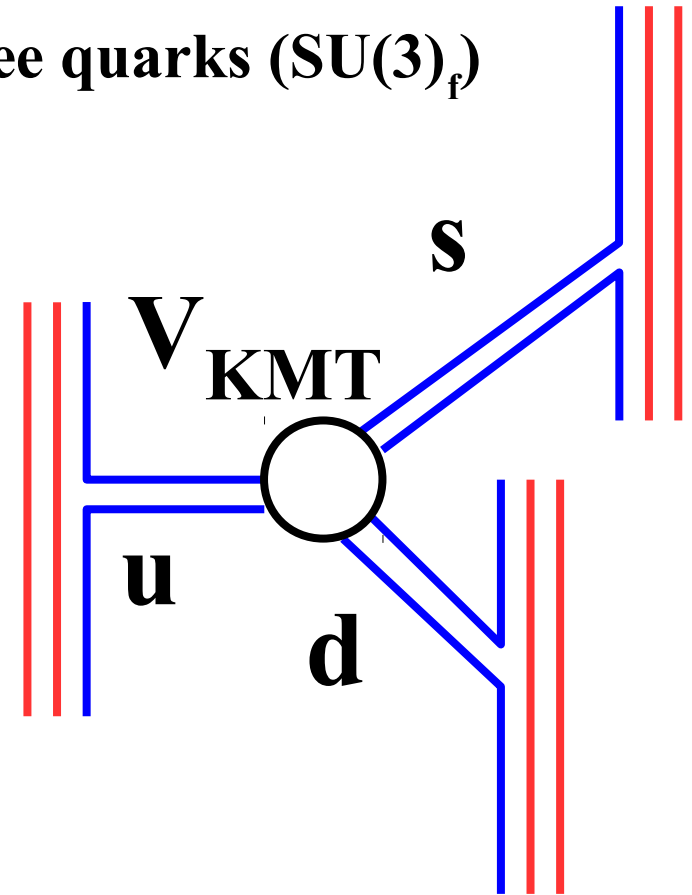
→ $g_D = -9.29$ *Hatsuda, Kunihiro ('94)*

– 12.36 *Rehberg, Klevanski, Hufner ('96)*

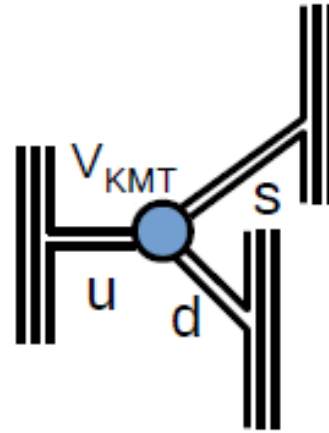
- KMT interaction should generate 2B and 3B interaction when hyperons are involved.

- Repulsive in $\Lambda\Lambda$ system
→ Pushes up H particle energy.

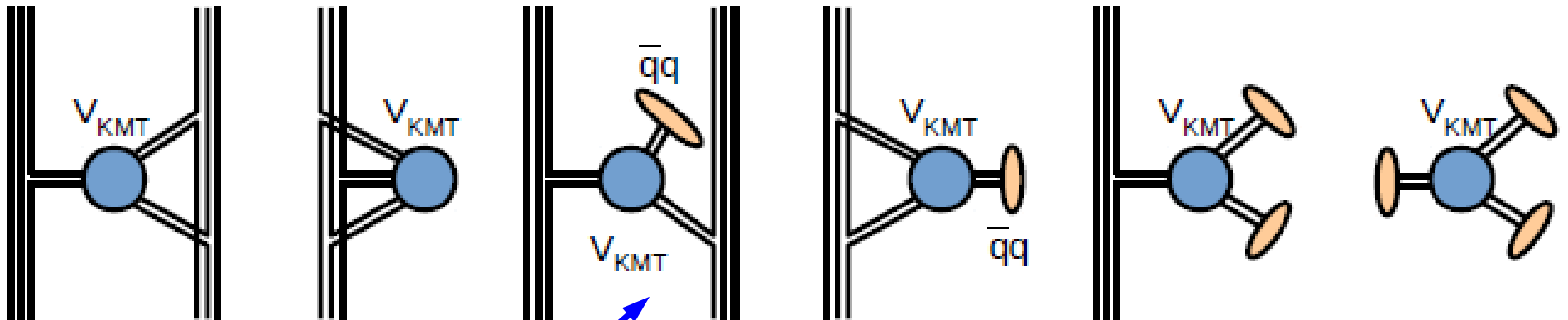
Takeuchi, Oka ('91)



Kobayashi-Maskawa-'t Hooft (KMT) interaction



3B force
(AO, Kashiwa, Morita)



Repulsion in $\Lambda\Lambda$ int.
Takeuchi, Oka ('91)

quark mass, vac. E.
(Hatsuda, Kunihiro)

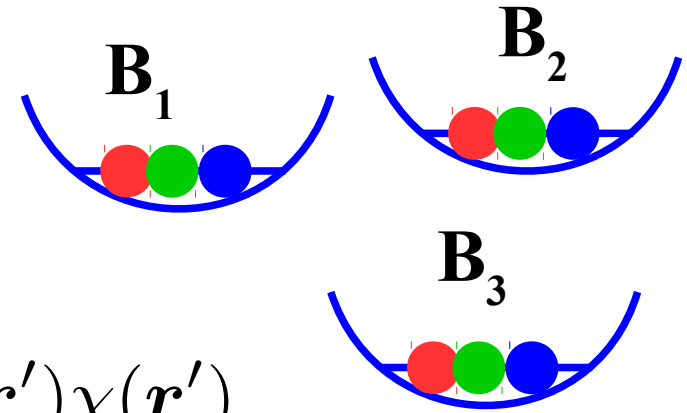
Does anomaly support massive NS ?

Quark Cluster model

- **Totally anti-symmetrized wave function of baryons**

$$|\Psi\rangle = \mathcal{A}|\chi_{12}B_1B_2\rangle$$

$$|\Psi\rangle = \mathcal{A}|\chi_{123}B_1B_2B_3\rangle$$



- **Resonating Group Method**

$$\int d\mathbf{r}' H(\mathbf{r}, \mathbf{r}')\chi(\mathbf{r}') = E \int d\mathbf{r}' N(\mathbf{r}, \mathbf{r}')\chi(\mathbf{r}')$$

$$\rightarrow -\frac{\hbar^2}{2\mu}\nabla^2\chi^{(N)} + (V\chi^{(N)}) = E\chi^{(N)} \quad (\chi^{(N)} = \mathcal{N}^{1/2}\chi)$$

$$H(\mathbf{r}, \mathbf{r}') = \langle \mathbf{r} B_1 B_2 \dots | H | \mathcal{A}(\mathbf{r}' B_1 B_2 \dots) \rangle$$

$$N(\mathbf{r}, \mathbf{r}') = \langle \mathbf{r} B_1 B_2 \dots | \mathcal{A}(\mathbf{r}' B_1 B_2 \dots) \rangle$$

- **When (wave length of χ) \gg (baryon size),**

$$V(\mathbf{r}) \simeq \Delta K + \langle V\mathcal{A} \rangle / \langle \mathcal{A} \rangle$$

Norm Kernel

- Antisymmetrizer makes the calculation complicated !

$$A = [1 - 9(P_{36} + P_{39} + P_{69}) + 27(P_{369} + P_{396}) + 54(P_{25} P_{39} + P_{35} P_{69} + P_{38} P_{69})] A_B - 216P_{25} P_{38} P_{69} ,$$

$$A_B = \sum_{\mathcal{P}} (-1)^{\pi(\mathcal{P})} \mathcal{P}$$

Toki, Suzuki, Hecht ('82)

- Recent work

Nakamoto, Suzuki ('16)

→ Norm kernel
of 3 octet B



(a1) D
 $C_p=1$



(a2) BC
 $C_p=1$



(a3) BX
 $C_p=-1$



(b1) 1QX
 $C_p=-1/3$



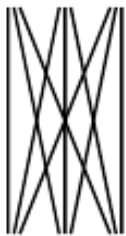
(b2) 1QX+BC
 $C_p=-1/3$



(b3) 1QX+BX
 $C_p=1/3$



(b4) 1QX+BXs
 $C_p=1/3$



(e) QC+QA
 $C_p=0$



(c1) 2QX
 $C_p=1/9$



(c2) 2QX+BC
 $C_p=1/9$



(c3) 2QX+BX
 $C_p=-1/9$



(d1) QC
 $C_p=1/9$



(d2) QC+BC
 $C_p=1/9$



(d3) QC+BX
 $C_p=-1/9$

Norm Kernel

- Single baryon w.f. $|\psi_A\rangle = \mathcal{A}/\sqrt{3!} \times \varepsilon_{abc}/\sqrt{3!} \times [|\text{Flavor}\rangle \otimes |\text{Spin}\rangle \otimes |\text{Spatial w.f.}\rangle]^{abc}$.

- Two barvon w.f.

$$|\psi_A(n_\uparrow, n_\downarrow)\rangle = \frac{1}{\sqrt{6!}} |\mathcal{A}[\psi(n_\uparrow)\psi(n_\downarrow)]\rangle$$

Norm

$$\begin{aligned} \mathcal{N}_A &= \langle \psi_A(n_\uparrow, n_\downarrow) | \psi_A(n_\uparrow, n_\downarrow) \rangle \\ &= \langle \psi(n_\uparrow)\psi(n_\downarrow) | \mathcal{A}[\psi(n_\uparrow)\psi(n_\downarrow)] \rangle \\ &= \frac{1}{(3!)^2} \sum_{i,j,k,l} c_i^*(n_\uparrow) c_j^*(n_\downarrow) c_k(n_\uparrow) c_l(n_\downarrow) \varepsilon_{abc} \varepsilon_{def} \varepsilon_{a'b'c'} \varepsilon_{d'e'f'} \\ &\quad \times \langle \phi_i^{abc}(n_\uparrow) \phi_j^{def}(n_\downarrow) | \mathcal{A}[\phi_k^{a'b'c'}(n_\uparrow) \phi_l^{d'e'f'}(n_\downarrow)] \rangle \\ &= \sum_{i,j,k,l} c_i^*(n_\uparrow) c_j^*(n_\downarrow) c_k(n_\uparrow) c_l(n_\downarrow) \sum_P C_P(\phi_i\phi_j, \phi_k\phi_l) F_P(\phi_i\phi_j, \phi_k\phi_l) \end{aligned}$$

B	$ \text{Flavor}\rangle$	$ \text{Spin}\rangle$
n_\uparrow	$ ddu\rangle/\sqrt{2}$	$ \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow\rangle/\sqrt{6}$
p_\uparrow	$ ud\bar{u}\rangle/\sqrt{2}$	$ \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow\rangle/\sqrt{6}$
Λ_\uparrow	$ uds\rangle$	$ \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow\rangle/\sqrt{2}$
Σ_\uparrow^-	$ dds\rangle/\sqrt{2}$	$ \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow\rangle/\sqrt{6}$
Σ_\uparrow^0	$ uds\rangle$	$ \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow\rangle/\sqrt{6}$
Σ_\uparrow^+	$ uus\rangle/\sqrt{2}$	$ \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow\rangle/\sqrt{6}$
Ξ_\uparrow^-	$ ssd\rangle/\sqrt{2}$	$ \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow\rangle/\sqrt{6}$
Ξ_\uparrow^0	$ ssu\rangle/\sqrt{2}$	$ \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow\rangle/\sqrt{6}$

Anti-symmetrization

$$\mathcal{A}[1^a 1^b 1^c 2^d 2^e 2^f] = 1^a 1^b 1^c 2^d 2^e 2^f - 1^a 1^b 2^d 1^c 2^e 2^f + 1^a 2^e 2^d 1^c 1^b 2^f + \dots$$

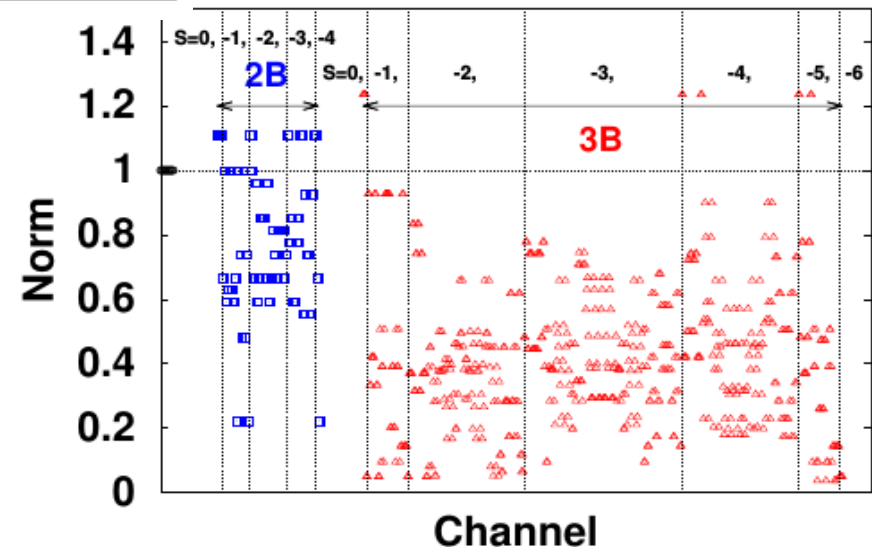
$$C_P = -\frac{1}{(3!)^2} \varepsilon_{abd} \varepsilon_{cef} \varepsilon_{abc} \varepsilon_{def} = -\frac{1}{36} 2\delta_{dc} 2\delta_{cd} = -\frac{1}{3}$$

$$F_P(\phi_i\phi_j, \phi_k\phi_l) = \langle \phi_i(n_\uparrow)\phi_j(n_\downarrow) | P[\phi_k(n_\uparrow)\phi_l(n_\downarrow)] \rangle_{\text{fss}} = 0 \text{ or } 1 \quad \leftarrow \text{Numerical}$$

Norm Kernel

Baryon(s)	\mathcal{N}_A	\mathcal{T}_A	\mathcal{T}	$\mathcal{T}_{nB}(n = 2, 3)$
$(NN)_{(S,T)=(0,1),(1,0)}$	10/9	0	0	0
$N_\uparrow\Lambda_\uparrow, N_\downarrow\Lambda_\downarrow$	1	20/3	20/3	20/3
$N_\uparrow\Lambda_\downarrow, N_\downarrow\Lambda_\uparrow$	1	10/3	10/3	10/3
$(\Lambda\Lambda)_{S=0}$	1	18/3	18/3	18/3
$(NNN)_{(S,T)=(1/2,1/2)}$	100/81	0	0	0
$n_\uparrow n_\downarrow \Lambda, p_\uparrow p_\downarrow \Lambda$	25/27	350/27	14	12/3
$n_\uparrow p_\uparrow \Lambda_\uparrow, n_\downarrow p_\downarrow \Lambda_\downarrow$	25/27	750/27	30	50/3
$n_\uparrow p_\uparrow \Lambda_\downarrow, n_\downarrow p_\downarrow \Lambda_\uparrow$	25/27	250/27	10	10/3
$n_\uparrow p_\downarrow \Lambda, n_\downarrow p_\uparrow \Lambda$	25/27	425/27	17	21/3
$N\Lambda_\uparrow\Lambda_\downarrow$	45/54	1035/54	23	21/3

Not very small



AO, Kashiwa, Morita ('16)

KMT matrix element

- Reduction of KMT interaction to 3 quark pot.

$$\begin{aligned} V_{\text{KMT}} &\simeq -2g_{\text{D}} \int d^3x \varepsilon_{ijk} u^\dagger(\mathbf{x}) q_i(\mathbf{x}) d^\dagger(\mathbf{x}) q_j(\mathbf{x}) s^\dagger(\mathbf{x}) q_k(\mathbf{x}) \\ &= -2g_{\text{D}} \varepsilon_{ijk} \sum_{\{\alpha, \beta, \gamma\}} \hat{T}_\alpha^{u,i} \hat{T}_\beta^{d,j} \hat{T}_\gamma^{s,k} \delta(\mathbf{x}_\alpha - \mathbf{x}_\beta) \delta(\mathbf{x}_\beta - \mathbf{x}_\gamma) \end{aligned}$$

- Flavor exchanging operator

$$\hat{T}^{\text{KMT}} = \sum_{\{\alpha, \beta, \gamma\}} \varepsilon_{ijk} \hat{T}_\alpha^{u,i} \hat{T}_\beta^{d,j} \hat{T}_\gamma^{s,k}$$

$$\mathcal{T}_A \equiv \langle \psi_A | \hat{T}^{\text{KMT}} | \psi_A \rangle \quad \mathcal{T} = \mathcal{T}_A / \mathcal{N}_A$$

- Subtract the two-body part

$$\mathcal{T}_{3B}(n_\uparrow n_\downarrow \Lambda_\uparrow) = \mathcal{T}(n_\uparrow n_\downarrow \Lambda_\uparrow) - \mathcal{T}(n_\uparrow \Lambda_\uparrow) - \mathcal{T}(n_\downarrow \Lambda_\uparrow)$$

KMT matrix element

$\langle \phi | V_{\text{KMT}} | \phi' \rangle = \sum_{\{\alpha, \beta, \gamma\}} \langle q_\alpha q_\beta q_\gamma | V_{\text{KMT}} | q'_\alpha q'_\beta q'_\gamma \rangle \prod_{i \neq \{\alpha, \beta, \gamma\}} \langle q_i | q'_i \rangle$
irrelevant quarks

↑ ↑ **product w.f.**

$= -2g_D \langle \sigma | \sigma' \rangle \sum_{\{\alpha, \beta, \gamma\}} F_{\alpha\beta\gamma}^{\text{KMT}}(f, f') R_{\alpha\beta\gamma}^{\text{KMT}}(\varphi, \varphi'),$

$\langle \sigma | \sigma' \rangle = \prod_{\alpha} \langle \sigma_{\alpha} | \sigma'_{\alpha} \rangle,$

flavor matching (numerical)

$F_{\alpha\beta\gamma}^{\text{KMT}}(f, f') = \langle f | \varepsilon_{ijk} \hat{T}_{\alpha}^{u,i} \hat{T}_{\beta}^{d,j} \hat{T}_{\gamma}^{s,k} | f' \rangle$

$= \delta_{u, f_{\alpha}} \delta_{d, f_{\beta}} \delta_{s, f_{\gamma}} \sum_{ijk} \varepsilon_{ijk} \delta_{i, f'_{\alpha}} \delta_{j, f'_{\beta}} \delta_{k, f'_{\gamma}} \prod_{\mu \neq \{\alpha, \beta, \gamma\}} \delta_{f_{\mu}, f'_{\mu}},$

$R_{\alpha\beta\gamma}^{\text{KMT}}(\varphi, \varphi') = \langle \varphi | \delta(\mathbf{x}_{\alpha} - \mathbf{x}_{\beta}) \delta(\mathbf{x}_{\beta} - \mathbf{x}_{\gamma}) | \varphi' \rangle$

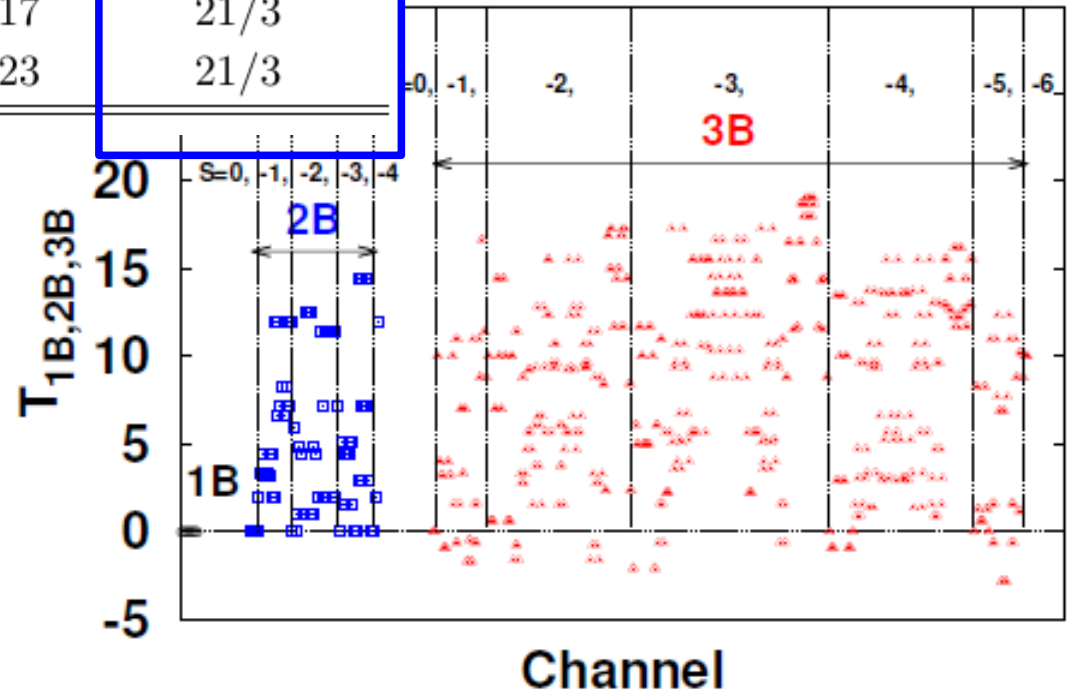
$= \int d^3x \varphi_{\alpha}^*(\mathbf{x}) \varphi_{\beta}^*(\mathbf{x}) \varphi_{\gamma}^*(\mathbf{x}) \varphi'_{\alpha}(\mathbf{x}) \varphi'_{\beta}(\mathbf{x}) \varphi'_{\gamma}(\mathbf{x}) \prod_{\mu \neq \alpha, \beta, \gamma} \langle \varphi_{\mu} | \varphi'_{\mu} \rangle.$

KMT matrix element

Baryon(s)	\mathcal{N}_A	\mathcal{T}_A	\mathcal{T}	$\mathcal{T}_{nB}(n = 2, 3)$
$(NN)_{(S,T)=(0,1),(1,0)}$	10/9	0	0	0
$N_\uparrow\Lambda_\uparrow, N_\downarrow\Lambda_\downarrow$	1	20/3	20/3	20/3
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$(\Lambda\Lambda)_{S=0}$	1	18/3	18/3	18/3
$(NNN)_{(S,T)=(1/2,1/2)}$	100/81	0	0	0
$n_\uparrow n_\downarrow \Lambda, p_\uparrow p_\downarrow \Lambda$	25/27	350/27	14	12/3
$n_\uparrow p_\uparrow \Lambda_\uparrow, n_\downarrow p_\downarrow \Lambda_\downarrow$	25/27	750/27	30	50/3
$n_\uparrow p_\uparrow \Lambda_\downarrow, n_\downarrow p_\downarrow \Lambda_\uparrow$	25/27	250/27	10	10/3
$n_\uparrow p_\downarrow \Lambda, n_\downarrow p_\uparrow \Lambda$	25/27	425/27	17	21/3
$N\Lambda_\uparrow\Lambda_\downarrow$	45/54	1035/54	23	21/3

**Big for np Λ
(S=3/2)**

**KMT matrix elements
strongly depend
on the channel**



3B potential from KMT interaction

■ 3q int. → 3B potential

$$V_{3B}^{KMT} = -2g_D T_{3B} \int d^3x \varphi_{R_a}^*(\mathbf{x}) \varphi_{R_b}^*(\mathbf{x}) \varphi_{R_c}^*(\mathbf{x}) \varphi_{R_d}(\mathbf{x}) \varphi_{R_e}(\mathbf{x}) \varphi_{R_f}(\mathbf{x})$$

$$V_{3B}^{KMT}(R_1, R_2, R_3) \simeq V_0 T_{3B} \exp \left[-\frac{2\nu}{3} (R_{12}^2 + R_{23}^2 + R_{31}^2) \right]$$

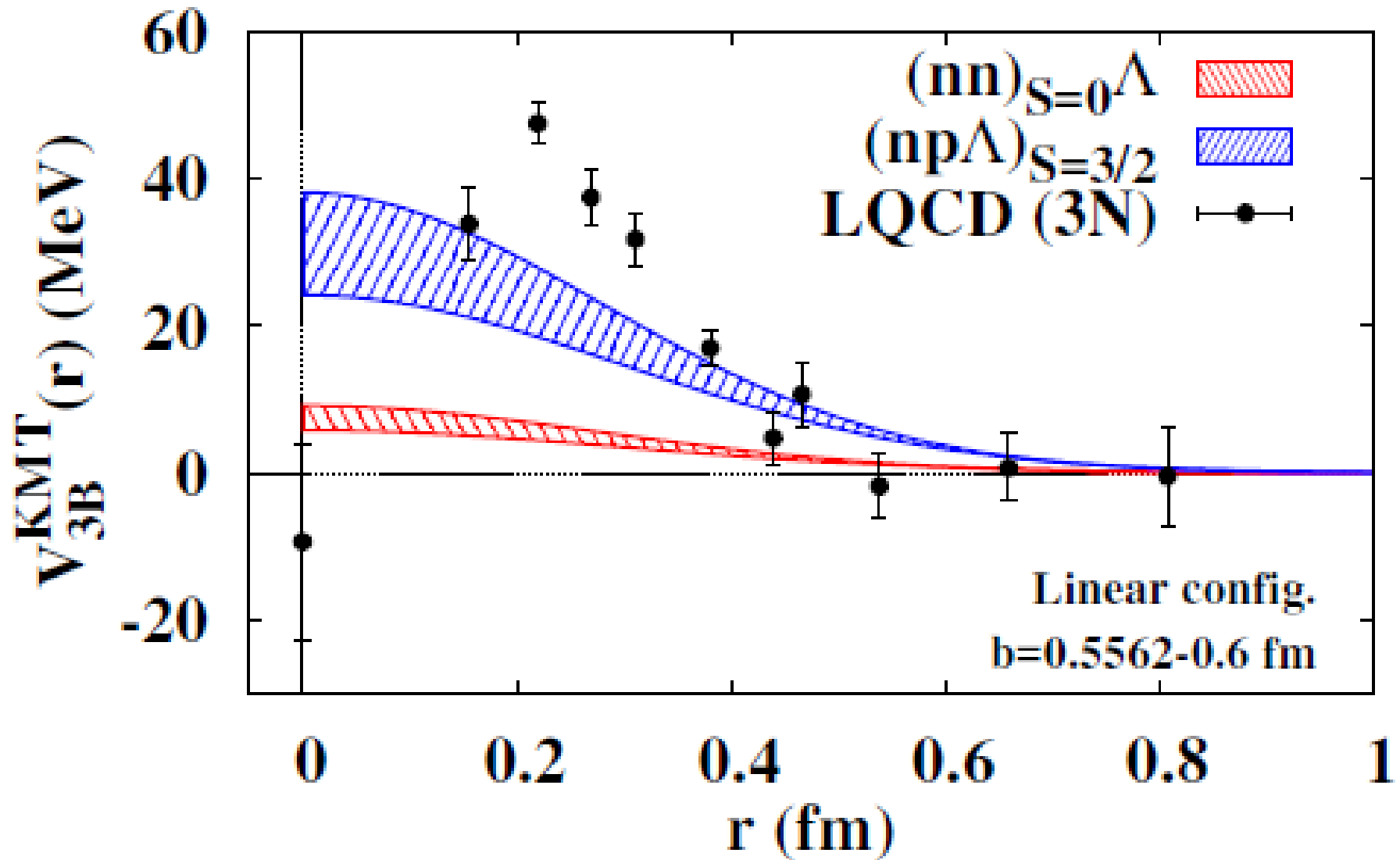
$$V_0 \equiv \frac{-2g_D}{(\sqrt{3}\pi b^2)^3} = \frac{-2g_D \Lambda^5}{(\sqrt{3}\pi b^2 \Lambda^2)^3} \Lambda = \begin{cases} 1.45 \text{ MeV} & (b = 0.6 \text{ fm}) , \\ 2.29 \text{ MeV} & (b = 0.5562 \text{ fm}) . \end{cases}$$

Parameters are taken from

*Hatsuda, Kunihiro ('94), Rehberg, Klevanski, Hufner ('96),
Fujiwara, Suzuki, Nakamoto ('07), Oka, Yazaki ('81)*

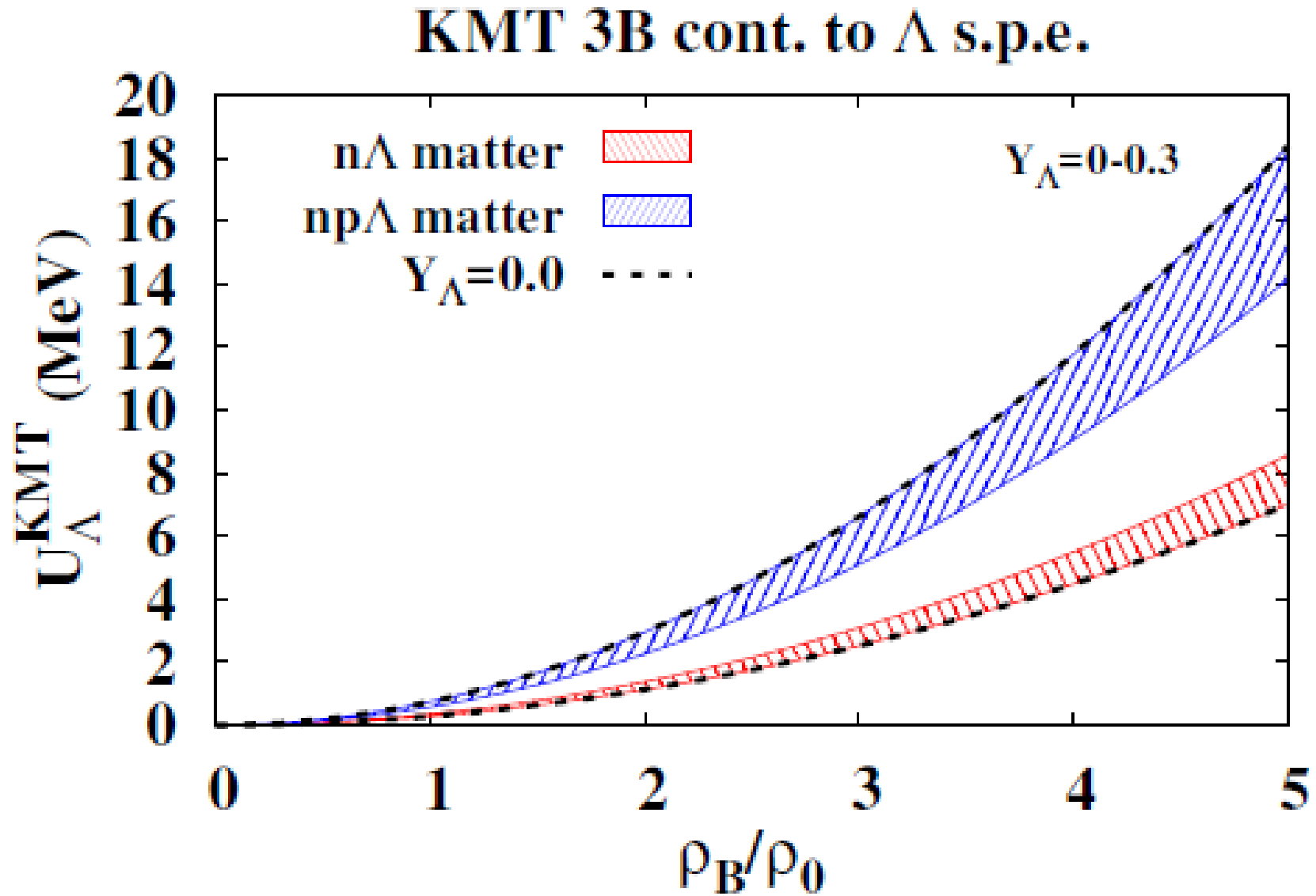
3B potential from KMT interaction

KMT 3B Potential



Lattice data: Doi et al. (HAL QCD) ('07)

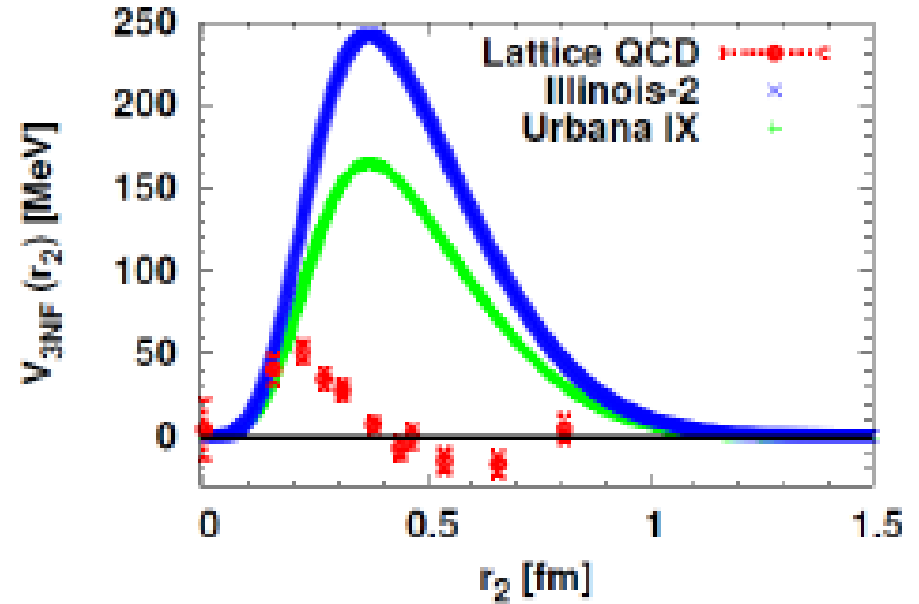
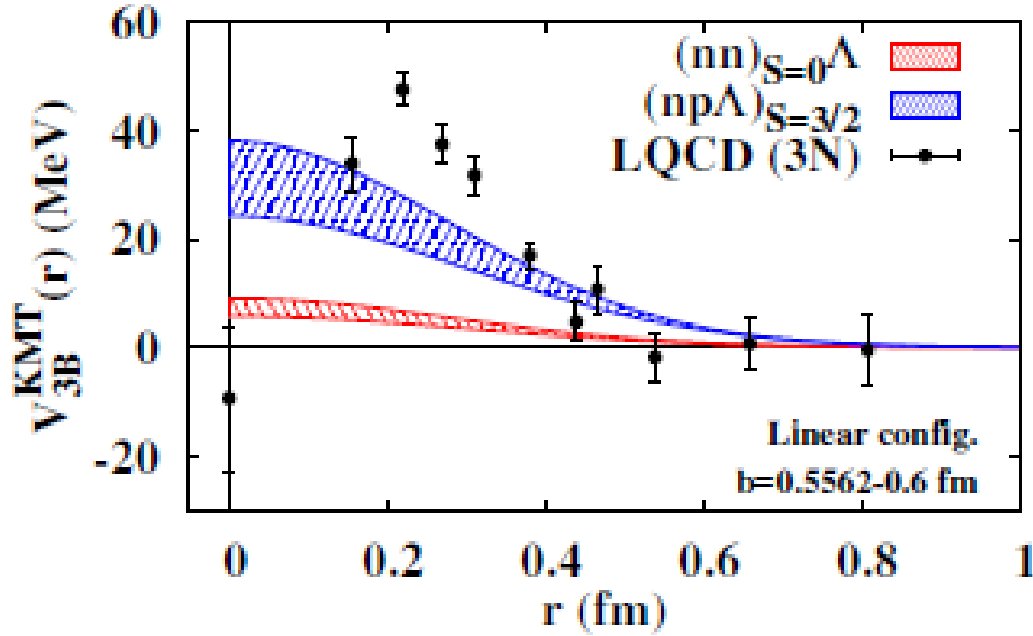
KMT-3B Contribution to Λ potential



Density is assumed to be uniform. No correlation effects.

3B potential from KMT: Repulsive enough ?

KMT 3B Potential



Summary

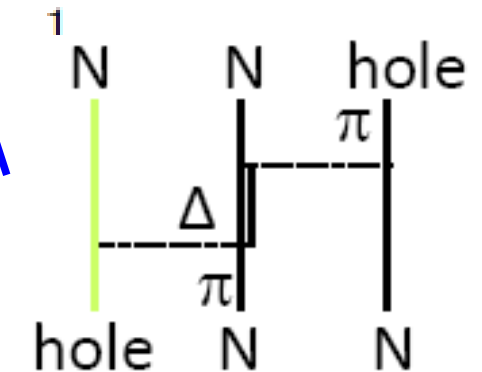
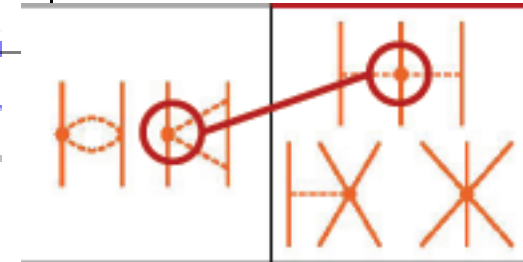
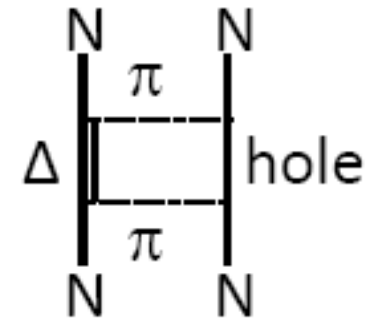
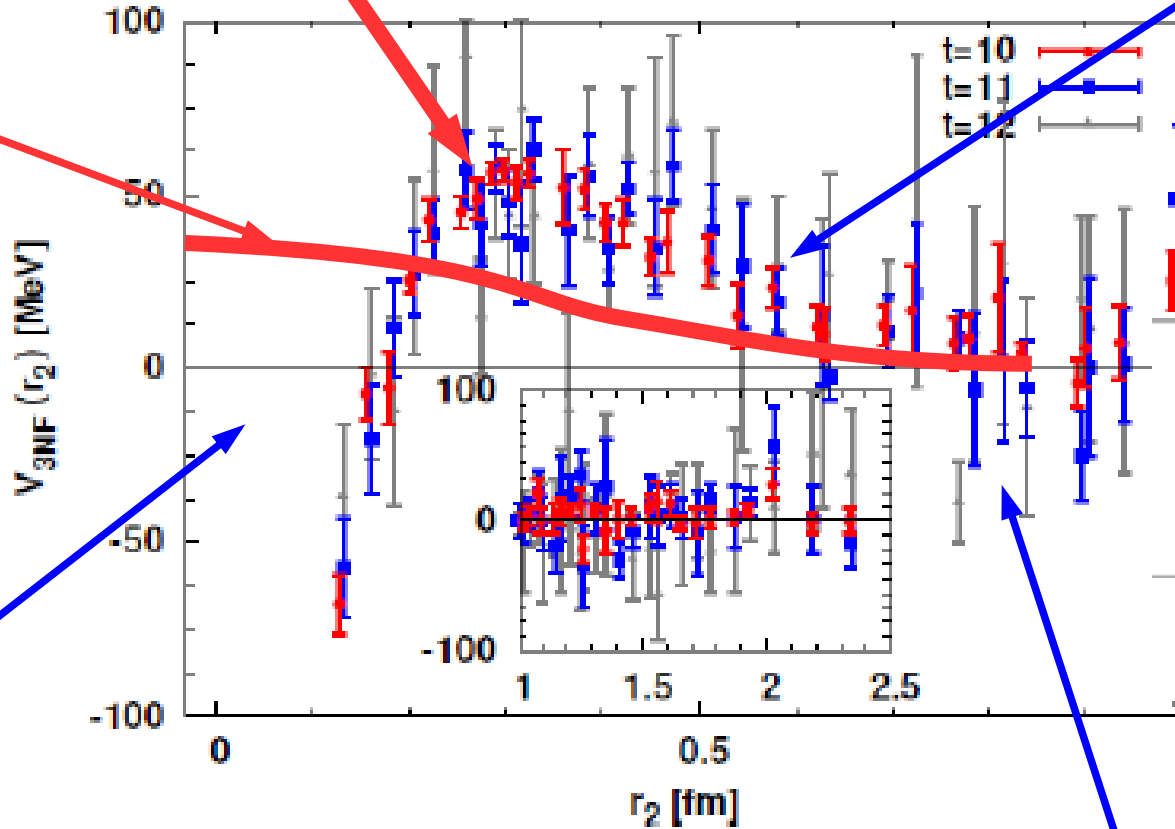
- **Quark model three-baryon (3B) potential may be a promising method to evaluate the 3B potential at short distances.**
- **Kobayashi-Maskawa-'t Hooft (KMT) interaction generates 3q potential among u,d,s quarks, and generates 3B potential only when hyperons are involved.**
- **Expectation value of the KMT interaction is evaluated in the cases where 3B are located at the same spatial point. Matrix elements strongly depend on the baryon trio.**
- **3B potential from KMT interaction is obtained.**
 - **It is comparable in strength to the lattice 3N potential.**
 - **More repulsive in $np\Lambda$ than in $nn\Lambda$ (Negative contribution to symmetry energy.)**
- **3B pot. from KMT is not strong enough to solve the hyperon puzzle, but contributes to hyperon suppression.**

Three-Baryon force

What makes 3B repulsion at $r \sim 0.5$ fm ?

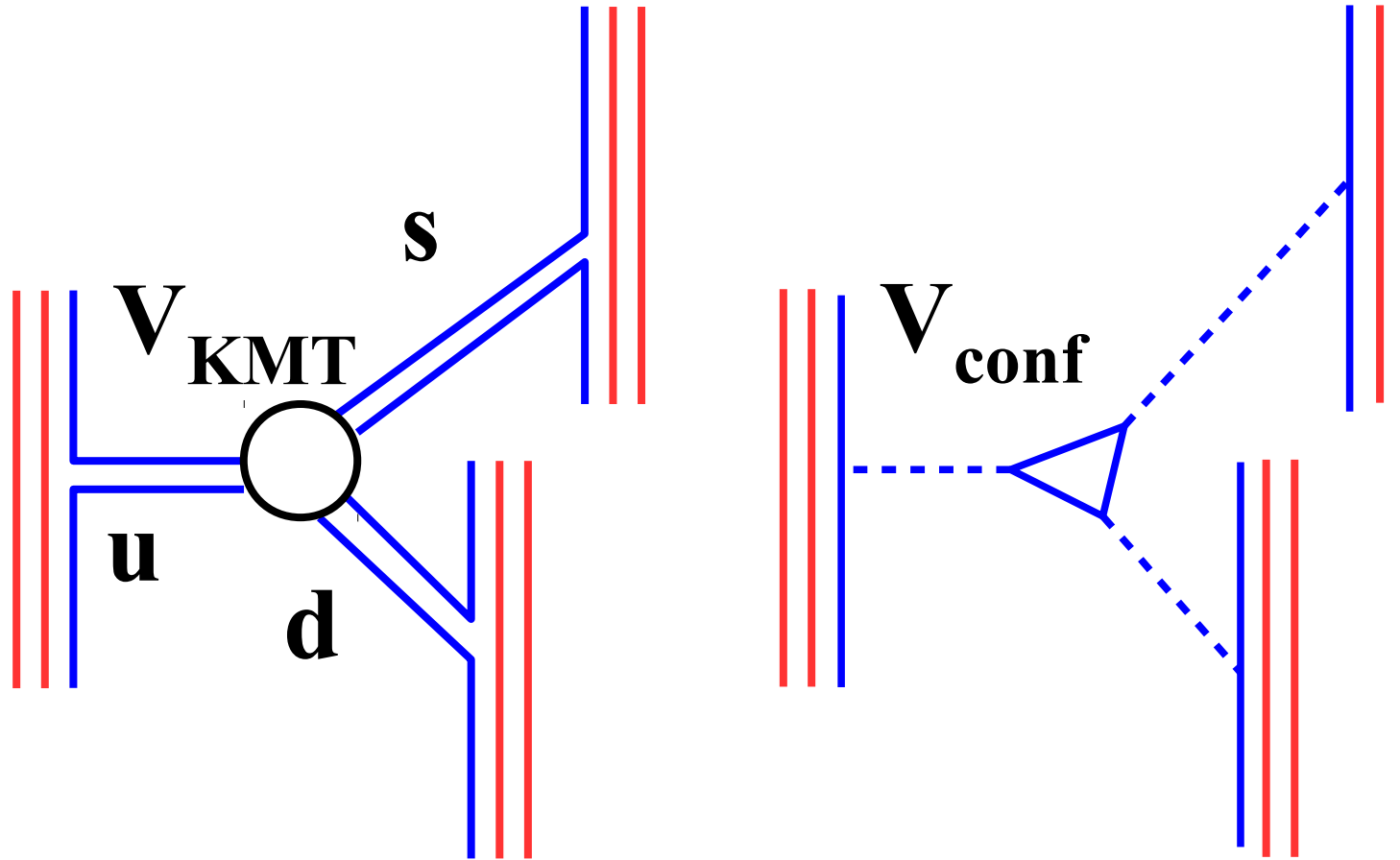
KMT
(Δ NN)

pQCD



Taken from NPCSM 2016 talks,
Doi (Wed), Kohno (Thu), Tews (Thu)

Confinement Potential \rightarrow 3B Potential ?



$$V_{\text{conf}} = \sum_{\{\alpha, \beta, \gamma\}} \varepsilon_{abc} \varepsilon_{a'b'c'} f(\mathbf{x}_\alpha, \mathbf{x}_\beta, \mathbf{x}_\gamma)$$

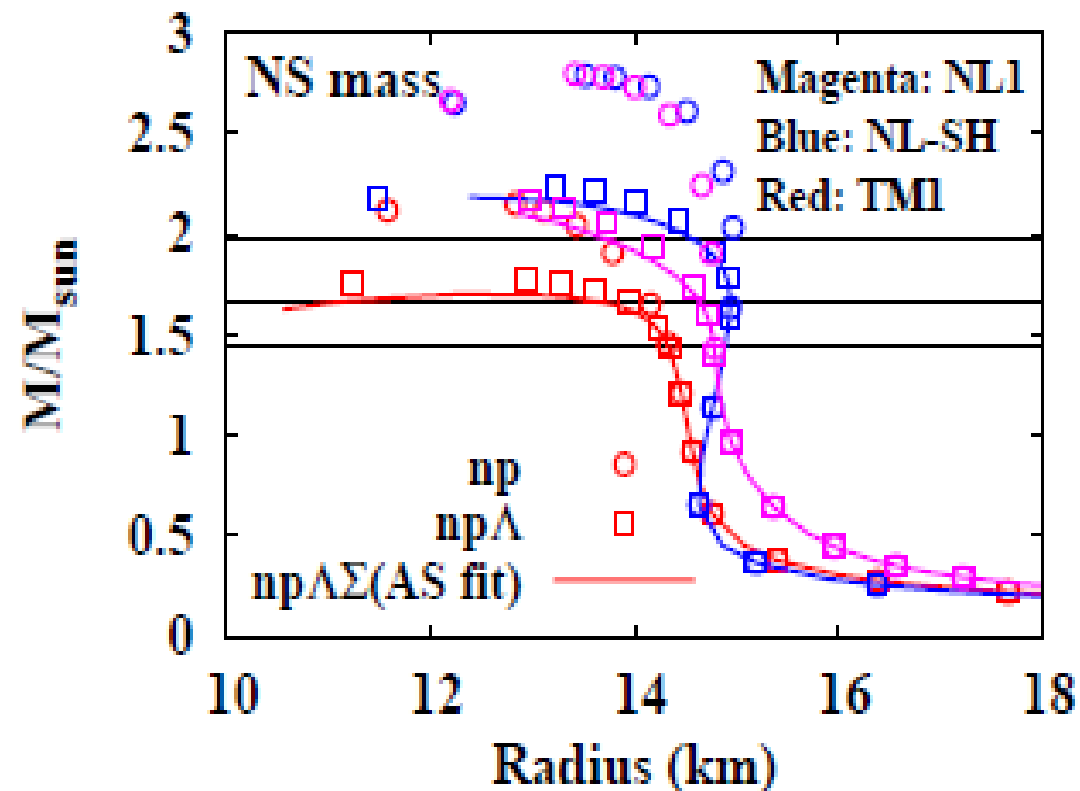
Takahashi, Suganuma, Nemoto, Matsufuru ('02)

Thank you for your attention !

Massive Neutron Stars with Hyperons

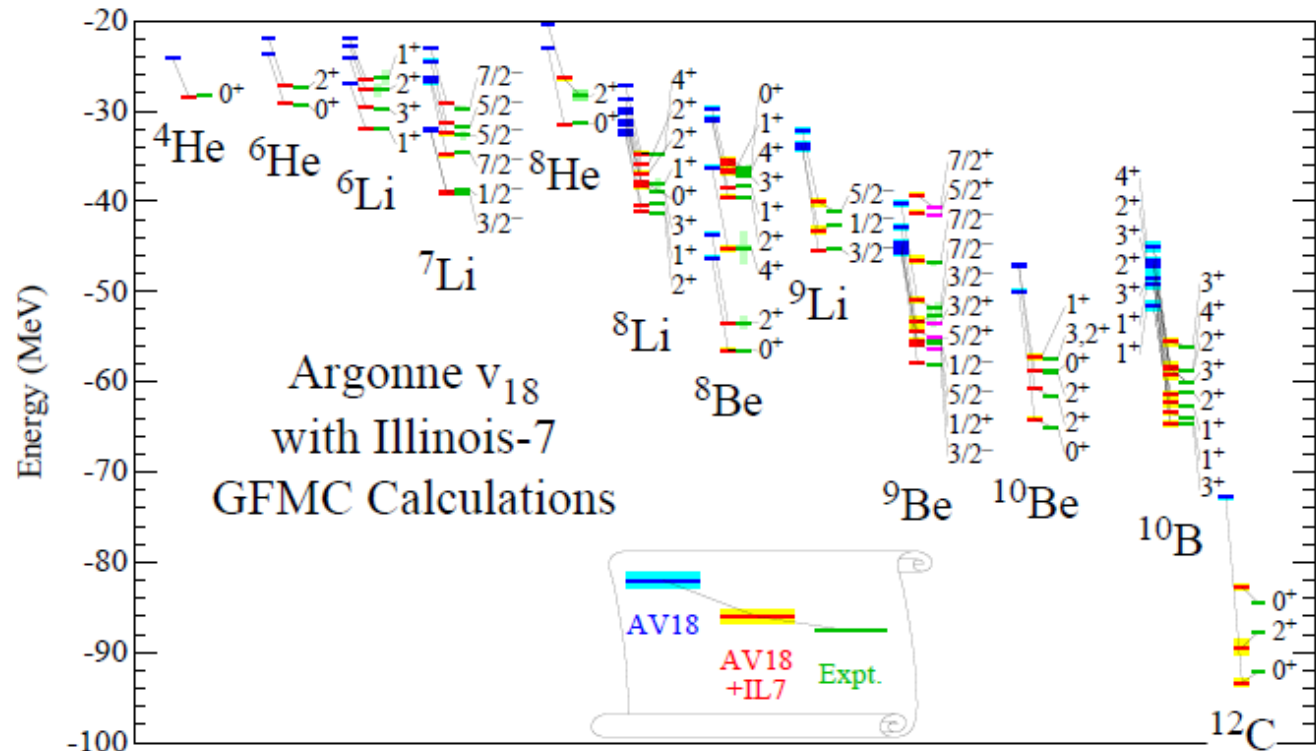
Tsubakihara, Harada, AO, arXiv:1402.0979

- Ruled-out EOS with hyperons = GM3
Glendenning & Moszkowski (1991)
- We did NOTHING special and find $2 M_{\odot}$ NS can be supported.
 - “Typical” RMF for nucl. matter
NL1, NL-SH, TM1
Reinhardt et al. ('86); Sharma, Nagarajan, Ring ('93); Sugahara, Toki ('94).
 - ss mesons are introduced
 - Hypernuclear data
 Λ , $\Lambda\Lambda$ hypernuclei
 Σ atomic shifts
SU(3) relation to isoscalar
-vector couplings



What is necessary to solve the massive NS puzzle ?

- There are many “model” solutions.
- Ab initio calculation including three-baryon force (3BF)
 - Bare 2NF+Phen. 3NF(UIX, IL2-7) + many-body theory (verified in light nuclei).
 - Chiral EFT (2NF+3NF) + many-body theory
 - Dirac-Bruckner-HF (no 3NF)



J. Carlson et al. ('14)

3BF including Hyperons

- 3BF incl. YNN, YYN and YYY should exist and contribute to EOS.

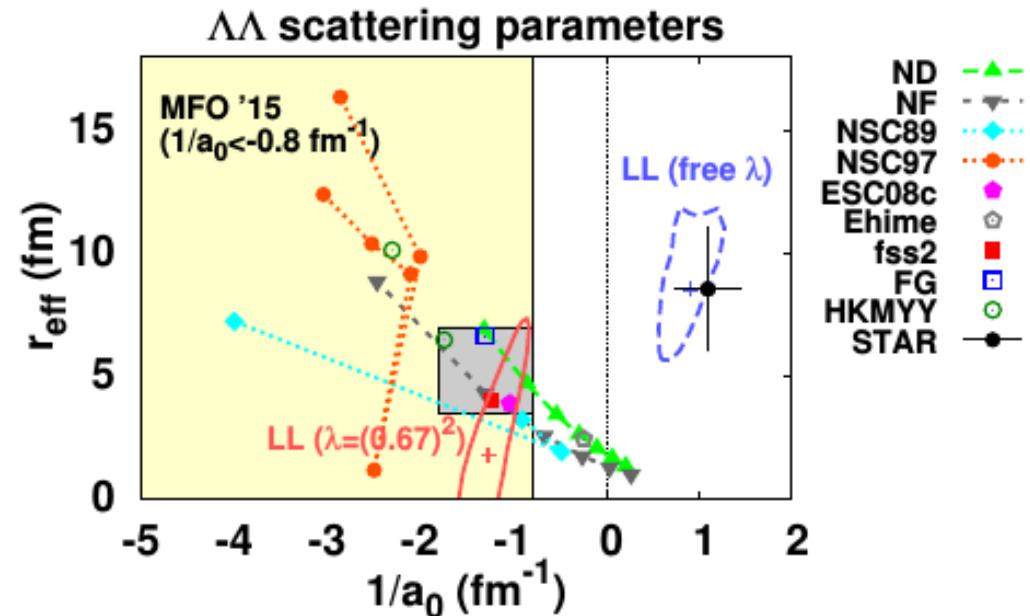
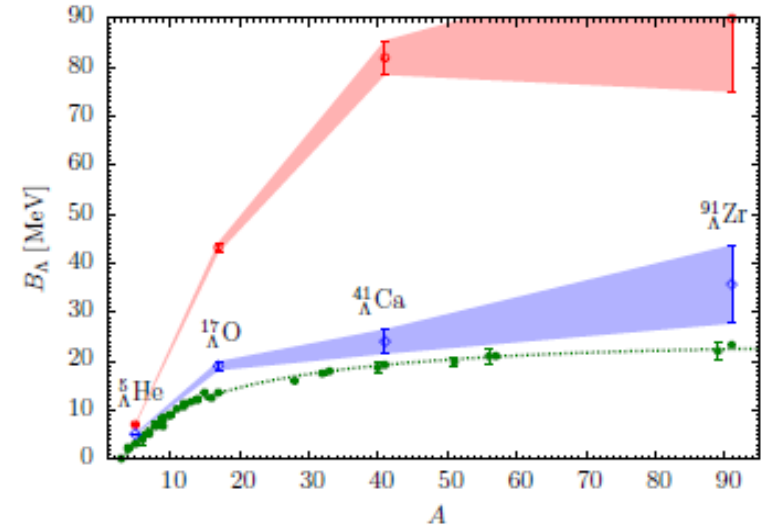
Nishizaki, Takatsuka, Yamamoto ('02)

- Chiral EFT, Multi-Pomeron exch., Quark Pauli, Lattice 3BF, SJ, ..
Kohno('10); Heidenbauer+('13); Yamamoto+('14); Nakamoto, Suzuki; Doi+(HALQCD,'12); Tamagaki('08); ...

- Quant. MC study *Lonardon et al.('14)*

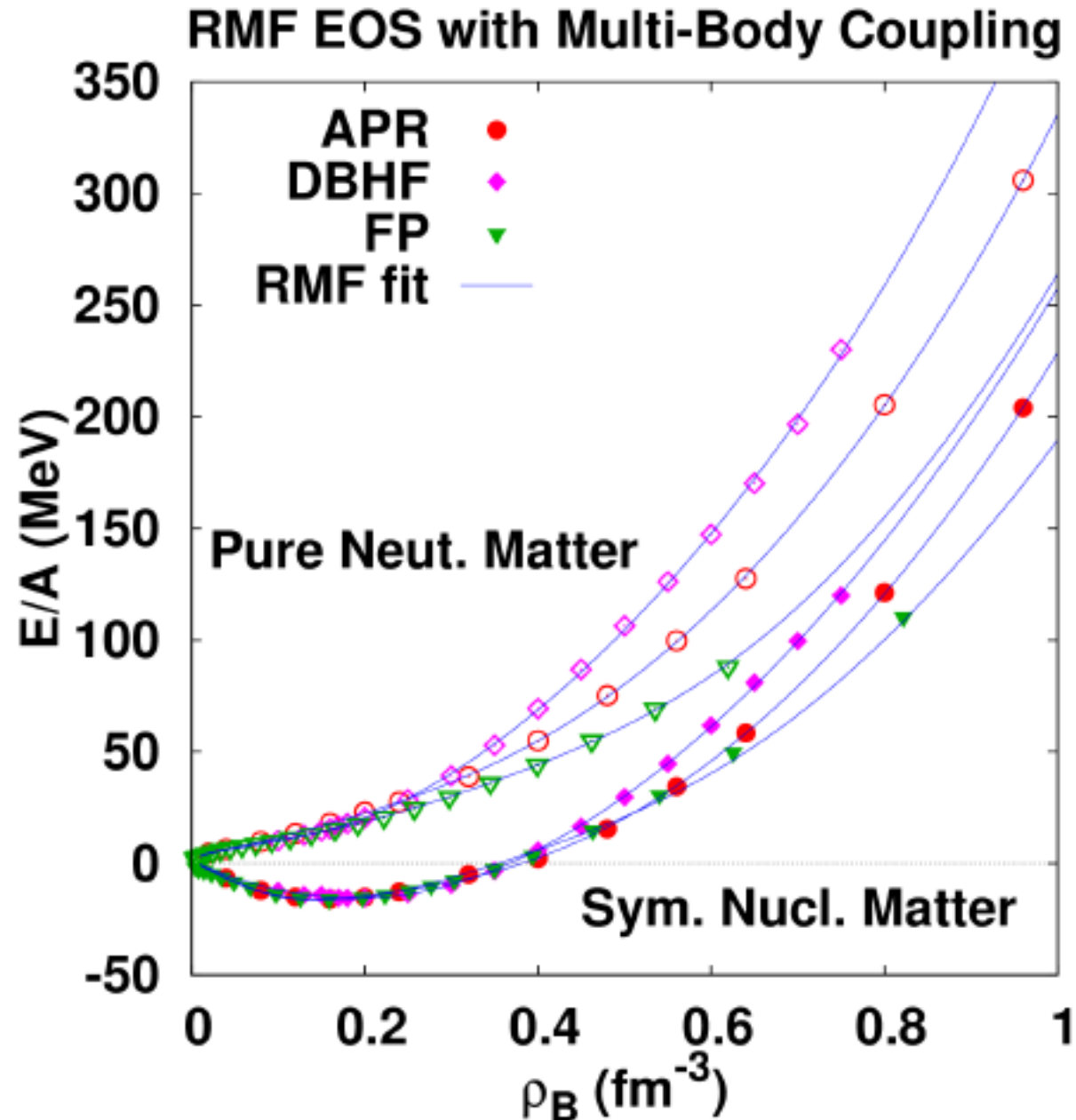
- Quark Meson Coupling
Miyatsu et al.; Thomas (HHIQCD)

- $\Lambda\Lambda$ *K. Morita, T. Furumoto, AO, PRC91('15)024916*

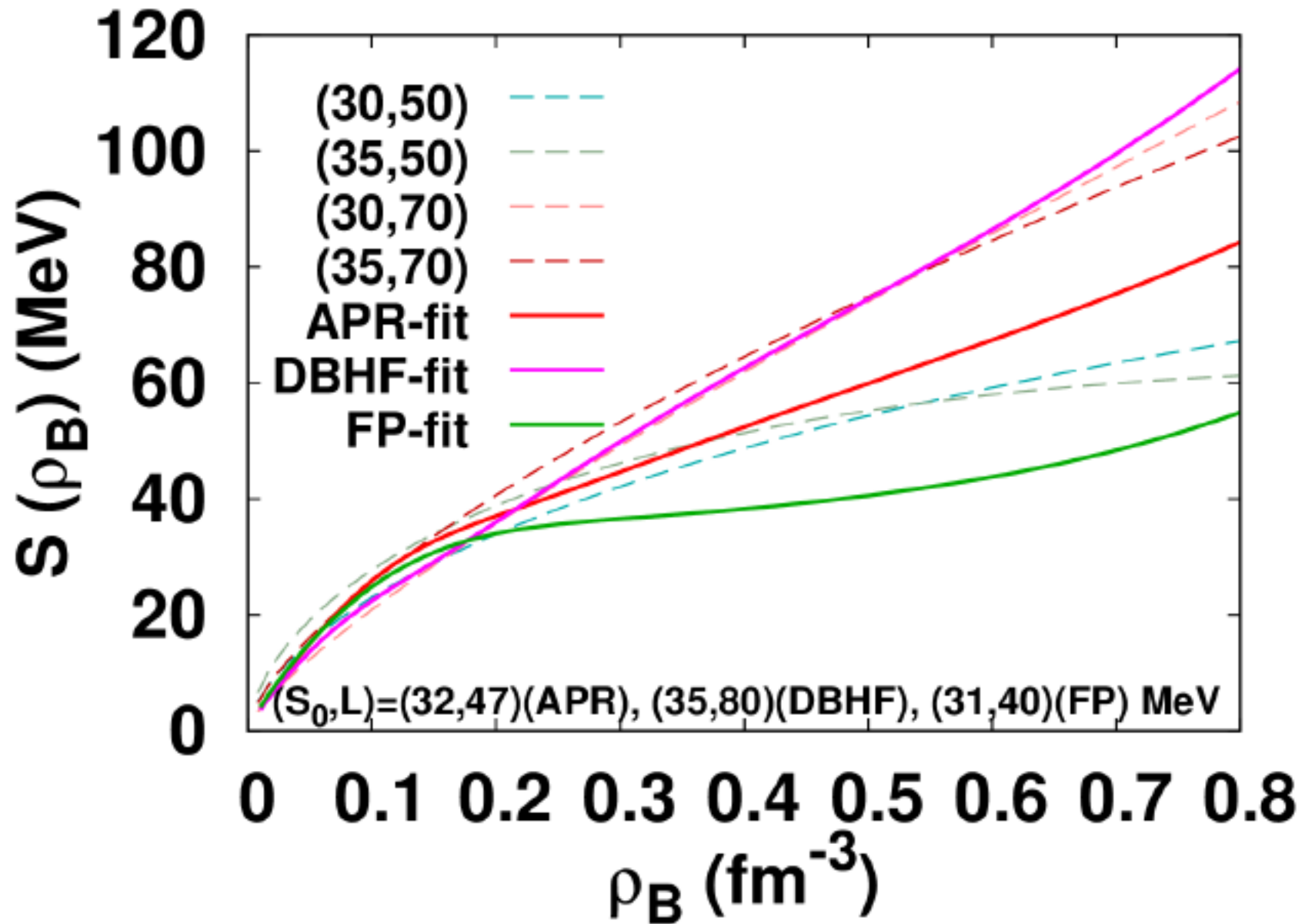


Caveat: Missing data

Fitting "Ab initio" EOS via RMF

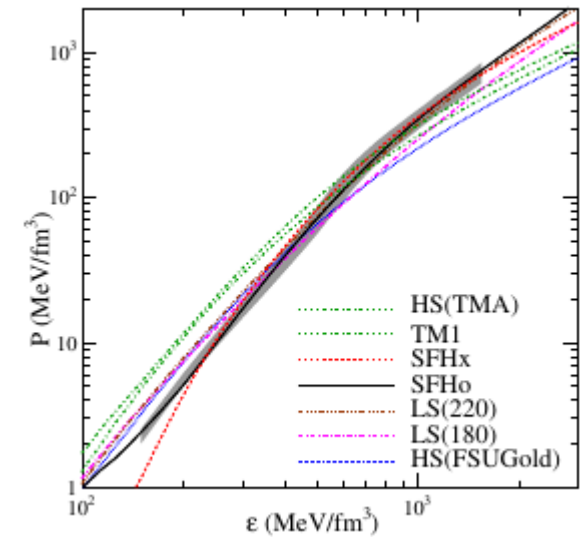
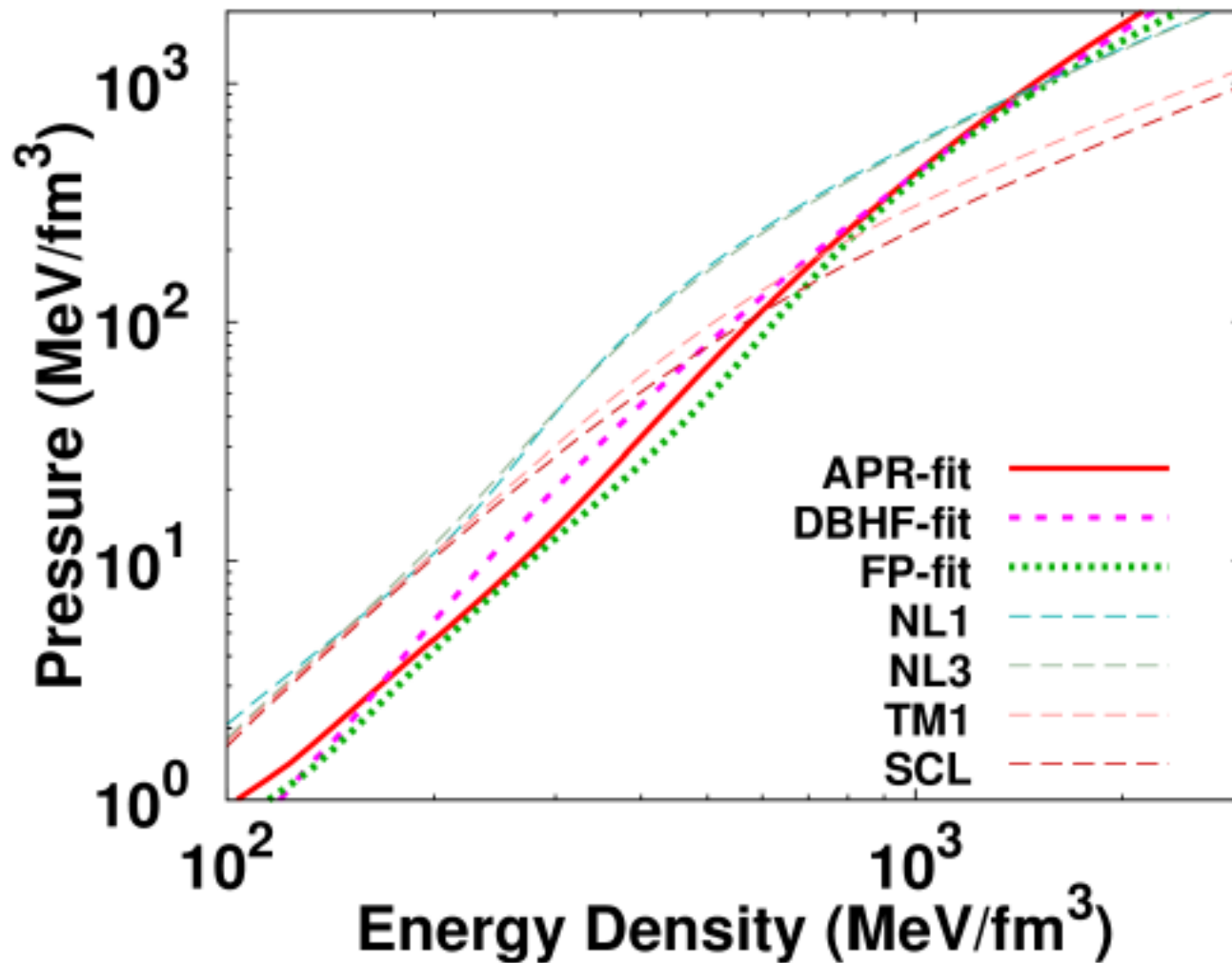


Symmetry Energy



Neutron Star Matter EOS

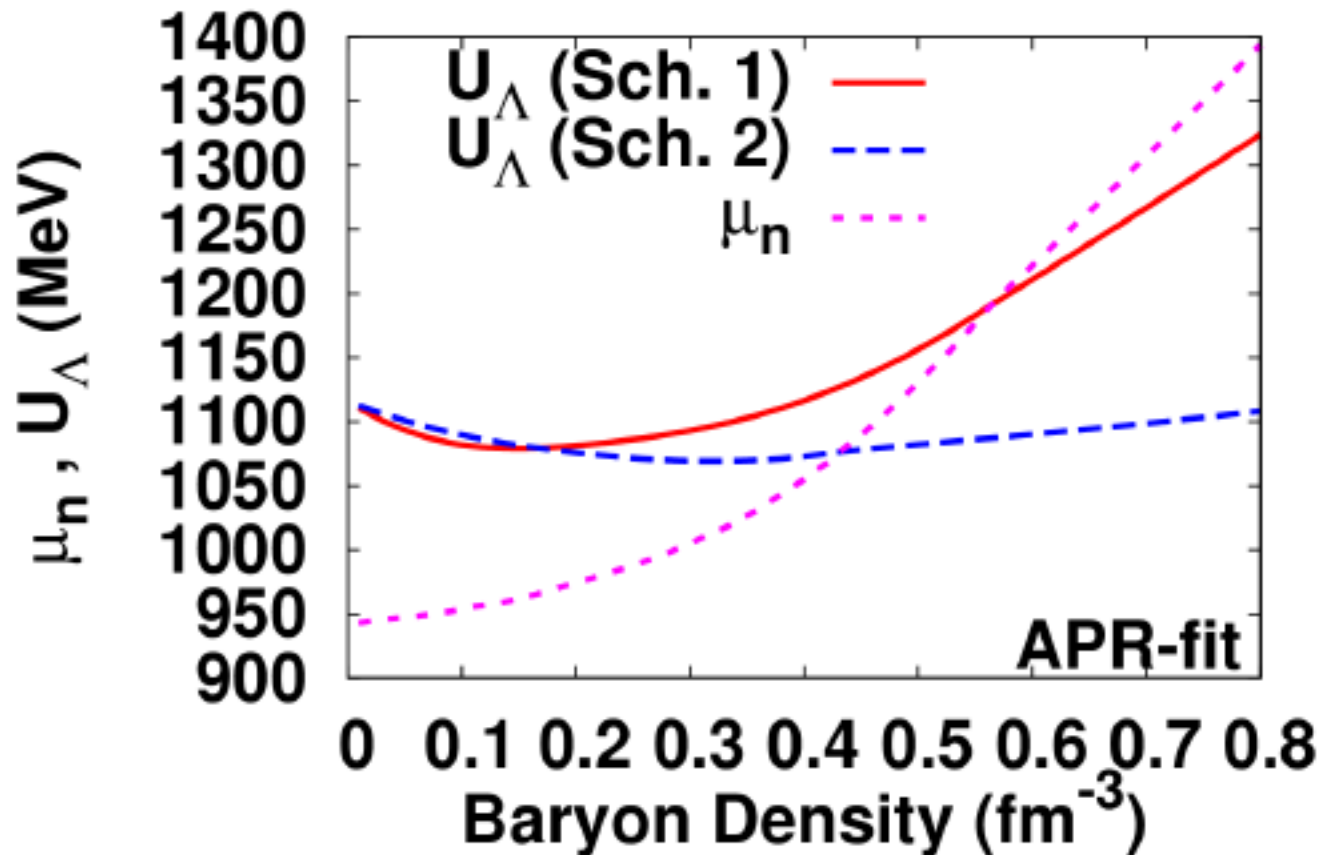
Neutron Star Matter EOS



*A. W. Steiner, M. Hempel,
T. Fischer,
ApJ 774 (2013) 17
(TMA+NSE w/ excl. vol.)*

NS matter in “ab initio”-fit + Λ

- Λ potential in nuclear matter at $\rho_0 \sim -30$ MeV
 - Scheme 1: $U_\Lambda(\rho) = \alpha U_N(\rho)$
 - Scheme 2: $U_\Lambda(\rho) = 2/3 U_{N^{n=2}}(\rho) + \beta U_{N^{n>2}}(\rho)$



M-R curve of Neutron Stars

