Unitary gas constraint on nuclear symmetry energy

Akira Ohnishi (YITP)

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- **Introduction: What is unitary gas ?**
- **Relevance to nuclear symmetry energy**
- **Conclusion**

Based on
Symmetry Parameter Constraints from a Lower Bound on the Neutron-Matter Energy,
I. Tews, J. M. Lattimer, A. Ohnishi, E. E. Kolomeitsev,
arXiv:1611.07133v2 [nucl-th]

What is Unitary Gas ?

- **Q: What are the ground state properties of the many-body system composed of spin ½ fermions interacting via a zero-range, infinite scattering length contact interaction. (Bertsch ('99), Seattle)**

$$H = \sum_i \frac{p_i^2}{2m} + \sum_{i<j} v(r_{ij}), \quad k \cot \underline{\delta(k)} = -\frac{1}{a_0} + \frac{1}{2} r_{\text{eff}} k^2,$$

phase shift

$$\text{Unitary Gas:} \quad a_0 k_F \to \infty, \quad r_{\text{eff}} k_F \to 0.$$

  - **a_0=scattering length, r_{eff}=effective range, k_F = Fermi momentum**
  - **Negative a0 means there is no bound state (BCS regime).**

- **A: Proportional to Fermi gas energy with a positive coef.**

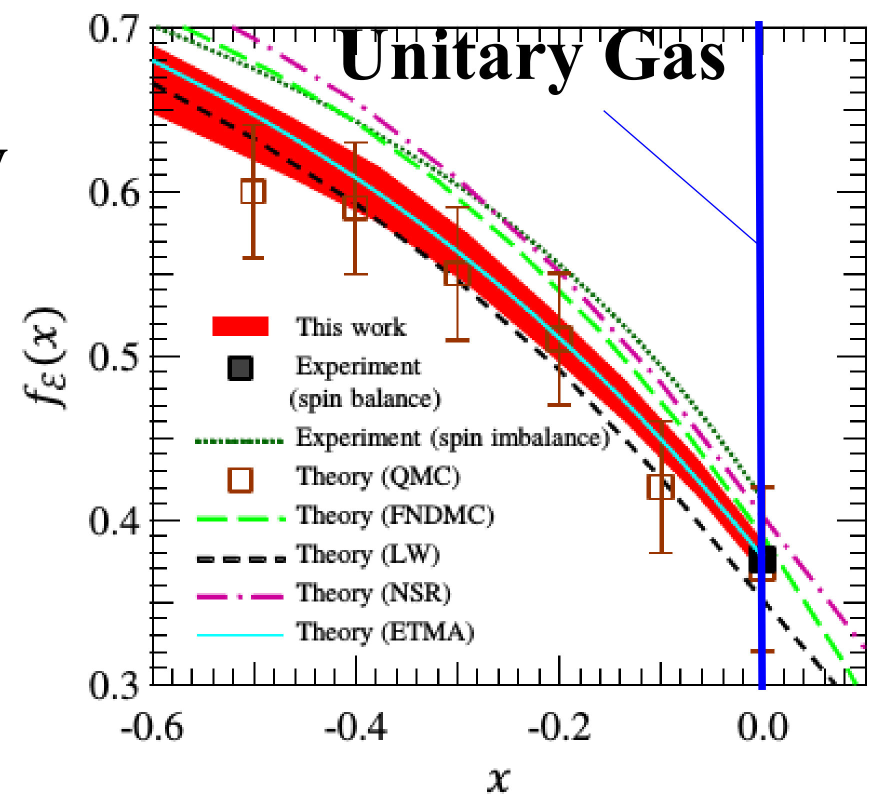
$$E_{\text{UG}} = \xi E_{\text{FG}} = \frac{3}{5} \frac{\hbar^2 k_F^2}{2m} \times \xi \quad (\xi \simeq 0.37, \text{Bertsch parameter})$$

  - **There is no typical scale length scale than k_F in unitary gas !**

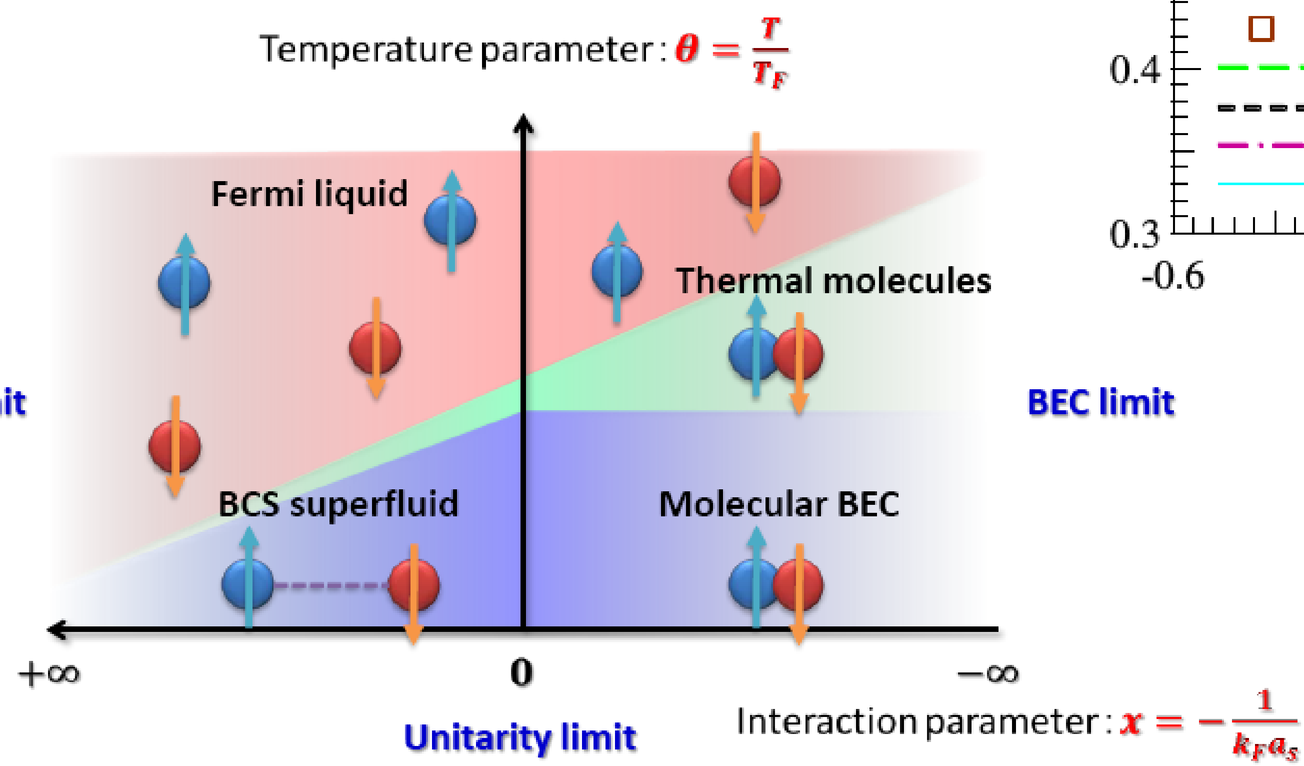
What is Unitary Gas ?

H. Horikoshi et al., arXiv:1612.04026

Cold atom energy / Fermi gas energy



$x = -1/a_0\, k_F$



Next Questions (T=0, spin-half fermions)

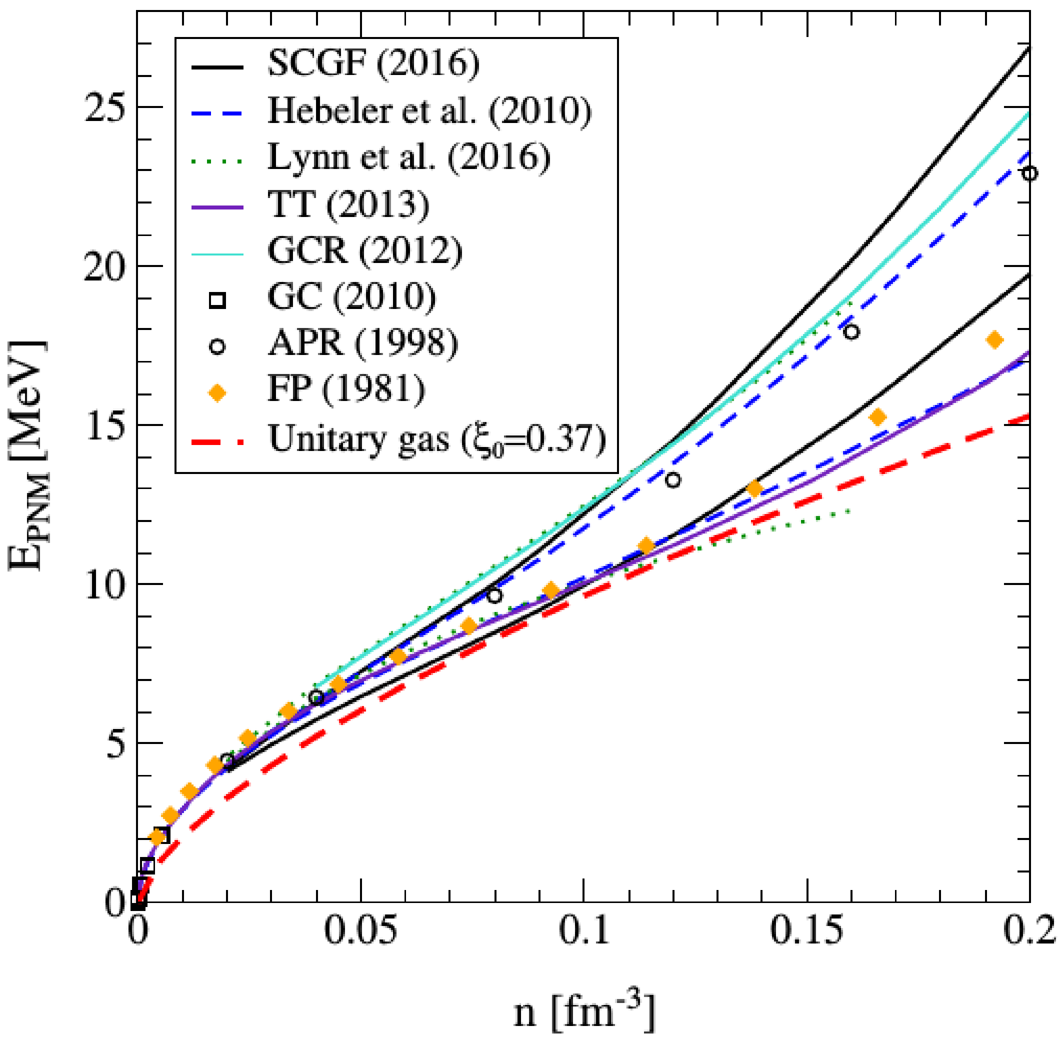
- **Zero range s-wave int.**
  E_{UG} = lower bound of E(a_0 < 0, r_{eff}=0, s-wave two-body) ?
  - **True (measured & theoretically confirmed)**

- **Finite range s-wave int.**
  E_{UG} = lower bound of E(a_0 < 0, s-wave two-body) ?
  - **Yes, at least for $r_{eff}\, k_F < 5$**
    Quantum MC, Gandolfi et al. ('15), Schwenk, Pethick ('05)
  - **Objection ! Attractive Hartree term ($\propto$ n) appears for finite r_{eff}.**
    P. van Wyk, Y. Ohashi et al. (in prep.)

- **Nuclear Interaction with p-wave, d-wave, 3-body force, ….**
  E_{UG} = lower bound of E (neutron matter) ?
  - **Large nn scattering length (a_0 = –18.9 fm) → Close to UG**
  - **Ab initio calc. support this conjecture at n < 1.5 n_0.**
    (n_0 = saturation density)

This Lunch Seminar

Neutron Matter EOS at low densities



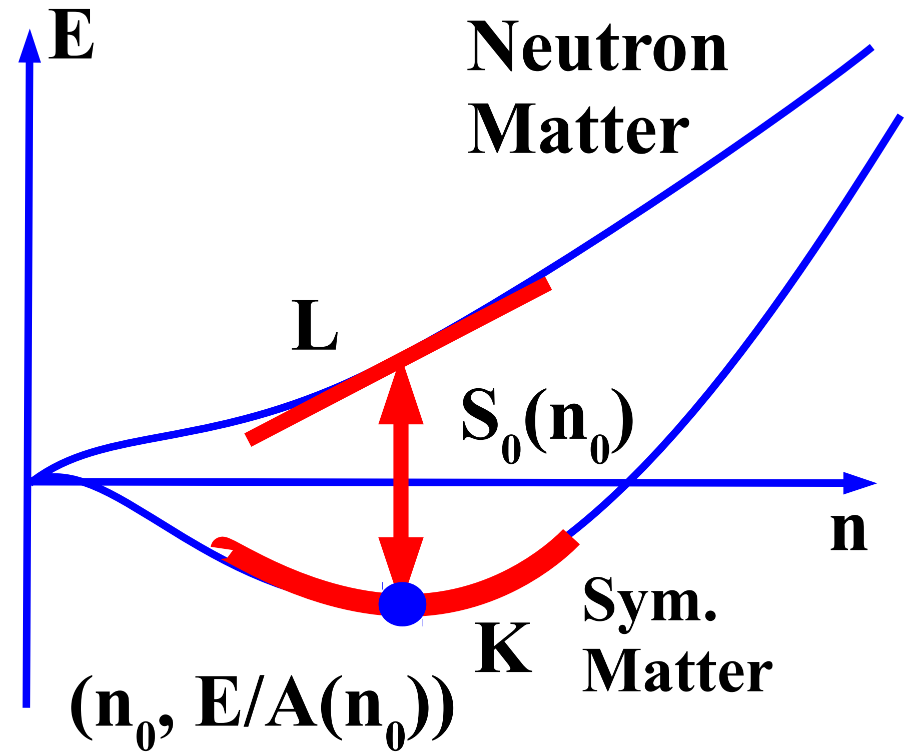
MC, variational, resummation, ….

Most of ab initio calculations with realistic nuclear Hamiltonian suggest E(neutron matter) > E(unitary gas)

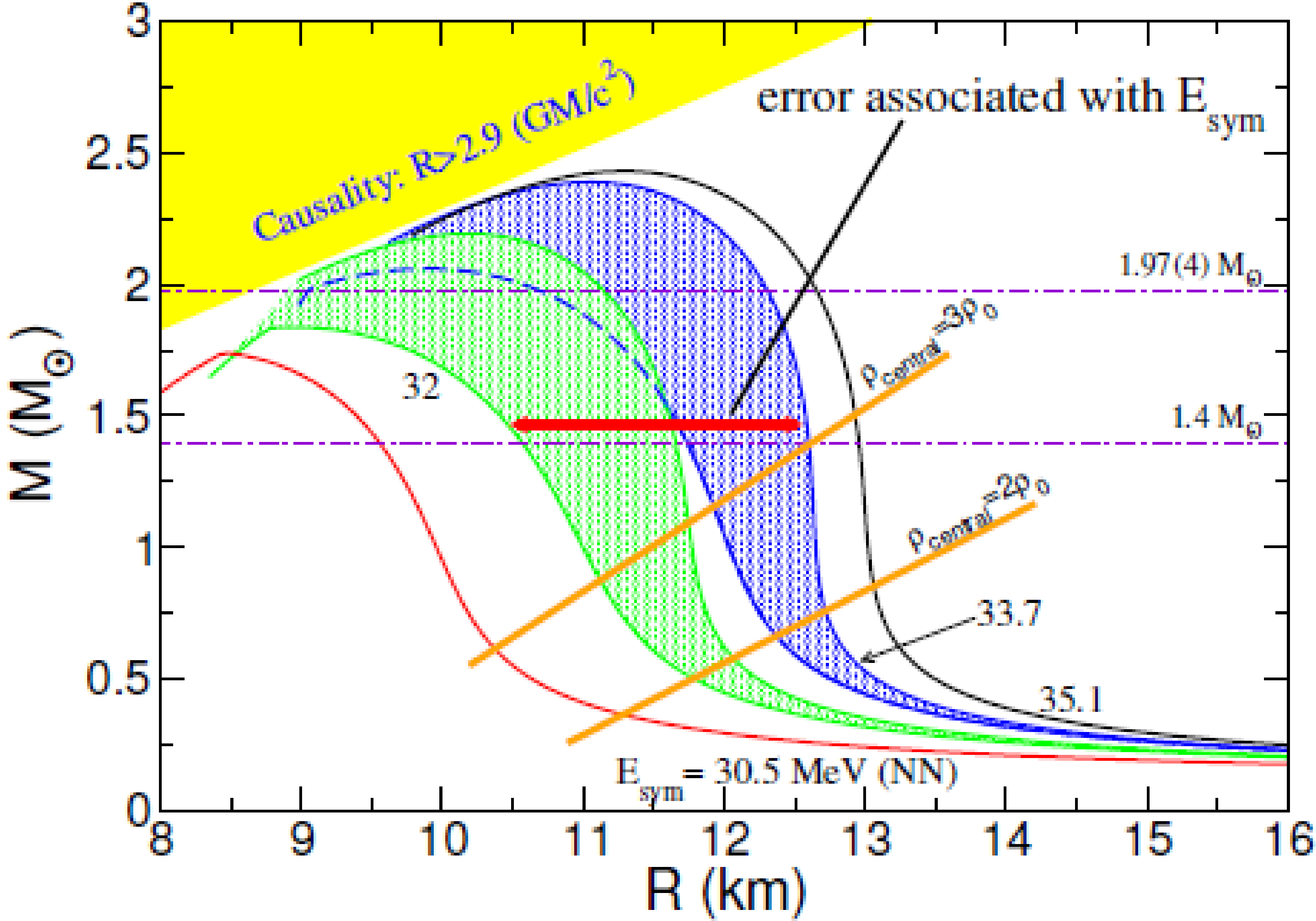
Tews, Lattimer, AO, Kolomeitsev, arXiv:1611.07133

Unitary Gas Constraint on Nuclear Symmetry Energy

- **Sym. E. = E(neutron matter) – E(sym. nucl. matter(N=Z))**
  (E=Energy / particle)
- **Sym. E. can be measured by laboratory experiments,**
  and determines the neutron star radius.



Symmetry Energy affects MR Relation of NS



Gandolfi, Carlson, Reddy, PRC 032801, 85 (2012).

Unitary Gas Constraint on Nuclear Symmetry Energy

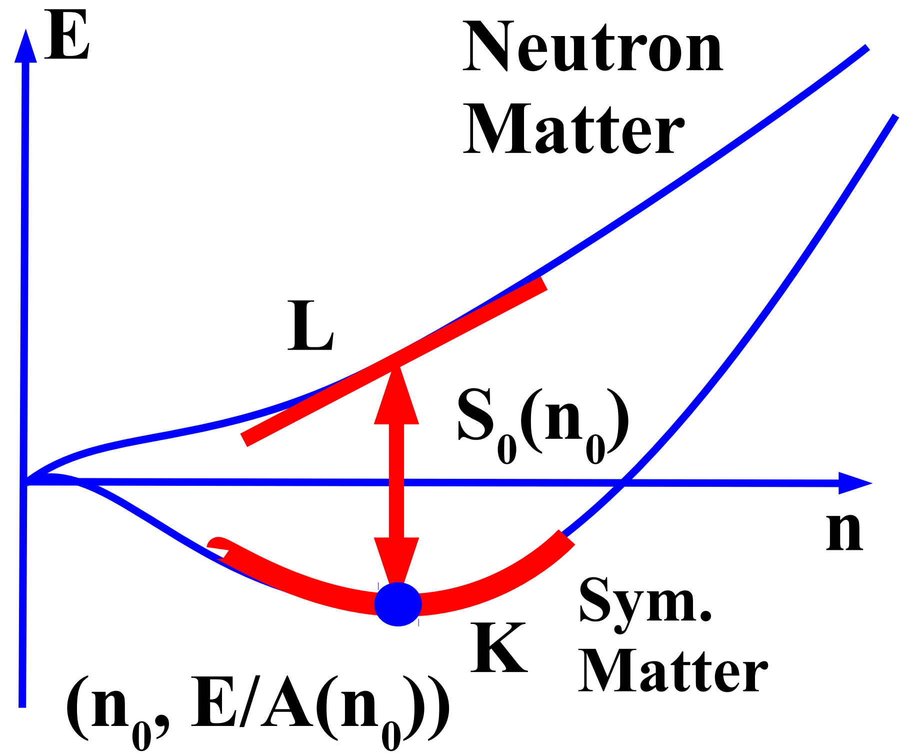
- **Sym. E. = E(neutron matter) – E(sym. nucl. matter(N=Z))**
  (E=Energy / particle)
- **Sym. E. can be measured by laboratory experiments, and determines the neutron star radius.**
- **Unitary Gas Conjecture**
  + Sym. nucl. matter EOS
  → Lower Bound of Sym. E.

$$E_{\mathrm{PNM}}(u) \geq E_{\mathrm{UG}}(u) = E^0_{\mathrm{UG}}\, u^{2/3}$$

$$\to S(u) = E_{\mathrm{PNM}}(u) - E_{\mathrm{SNM}}(u)$$

$$\geq \underline{E^0_{\mathrm{UG}} u^{2/3}} - \underline{E_{\mathrm{SNM}}(u)}$$

UG $k_F^2 \propto n^{2/3}$ **Sym. NM**



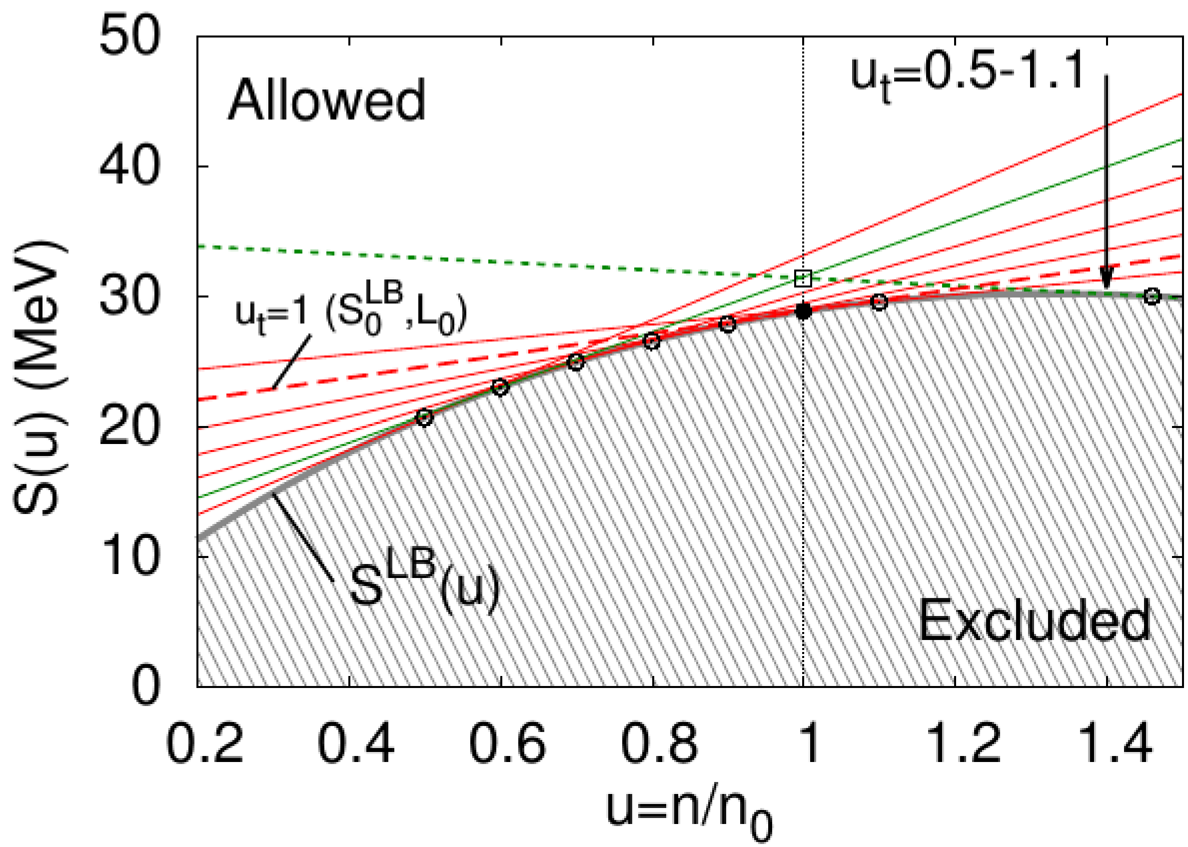
$u=n/n_0$ (n_0=sat. density)

Tews, Lattimer, AO, Kolomeitsev, arXiv:1611.07133

Symmetry Energy Parameters (S_0 , L)

- **Approximate density dep. of Sym. E.**

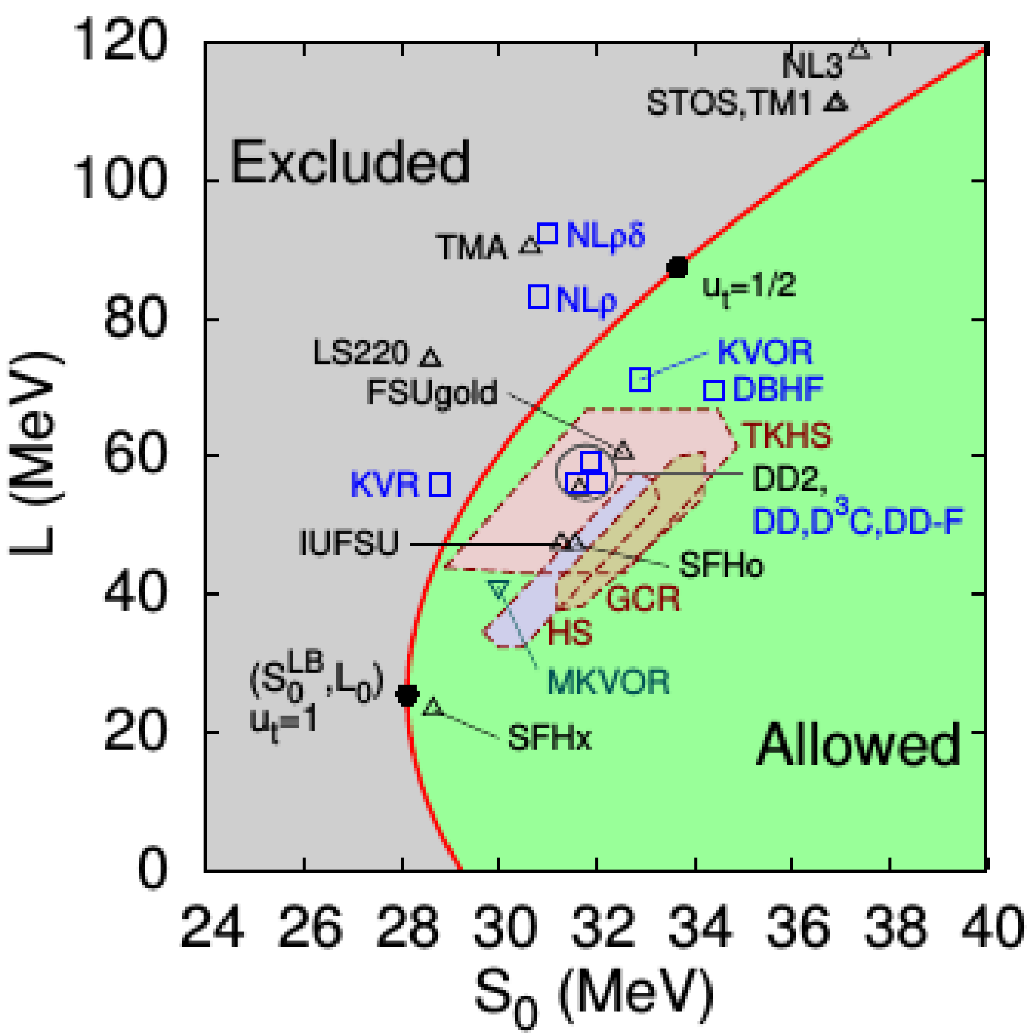
$$S(u) = S_0 + \frac{L}{3}(u-1) \geq E^0_{\rm UG} u^{2/3} - (E_0 + \frac{K}{18}(u-1)^2)$$

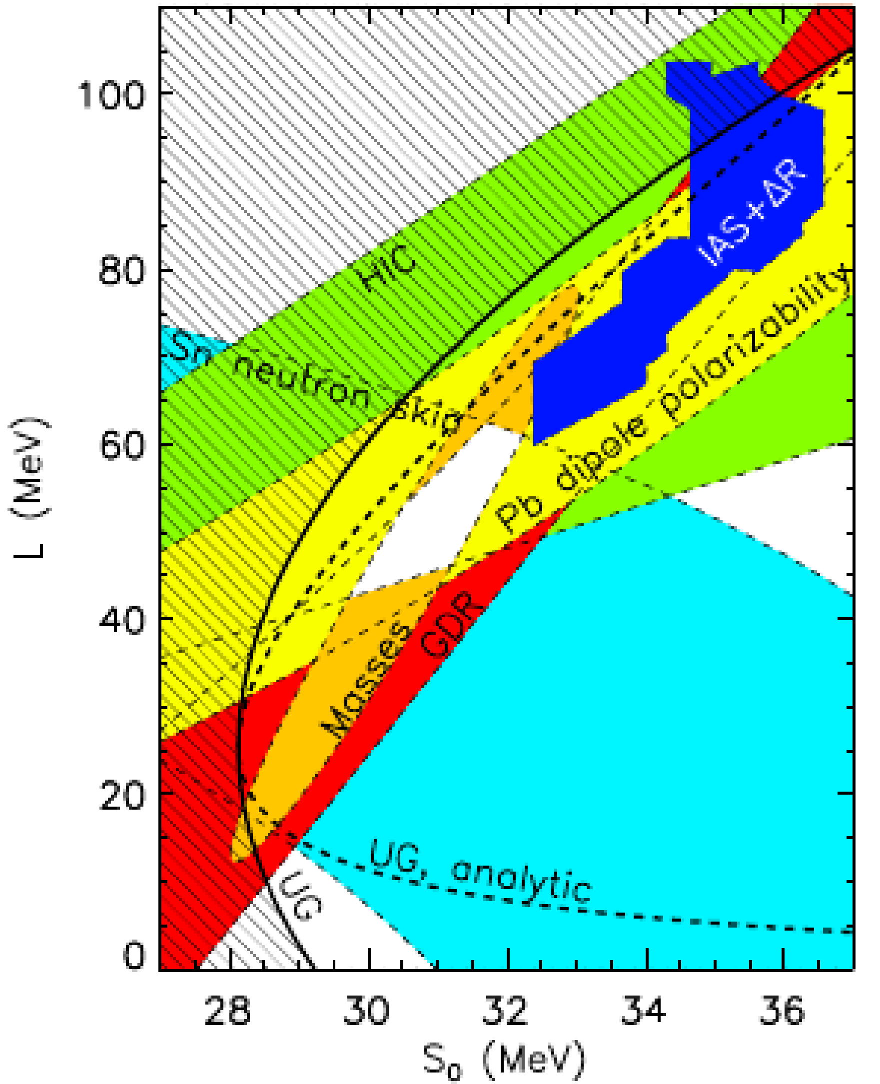


For a given slope at n_0 (L/3), there is a lower bound in S_0.

Tews, Lattimer, AO, Kolomeitsev, arXiv:1611.07133

Unitary Gas Constraint on S_0 and L





Triangles: EOS in Fischer, Hempel, Sagert, Suwa, Schaffner-Bielich ('14) → 3 (SFHo, SFHx, DD2) out of 10 remains.

UG constraint is consistent with experiments.

Tews, Lattimer, AO, Kolomeitsev, arXiv:1611.07133

Summary

- **Cold atoms around the unitary limit are quantum simulator of neutron matter, in which the scattering length is very large, a_0= – 18.9 fm.**
- **The neutron matter energy with realistic nuclear interaction seems to be greater than that of unitary gas.**
  - **Question: $E(1/a_0 k_F = 0, r_{eff}=0) < E$ (two-body, s-wave, $a_0<0$) ??**
- **If this conjecture is true, symmetry energy is non-trivially constrained, provided that we know the symmetric nuclear matter EOS, $S > E_{UG} - E_{SNM}$.**
  Symmetry energy parameters are also constrained.
- **Only a few tabulated EOSs among active astrophysics use have survived $2M_\odot$ constraint and the unitary gas constraint.**
  (We need more EOSs.)

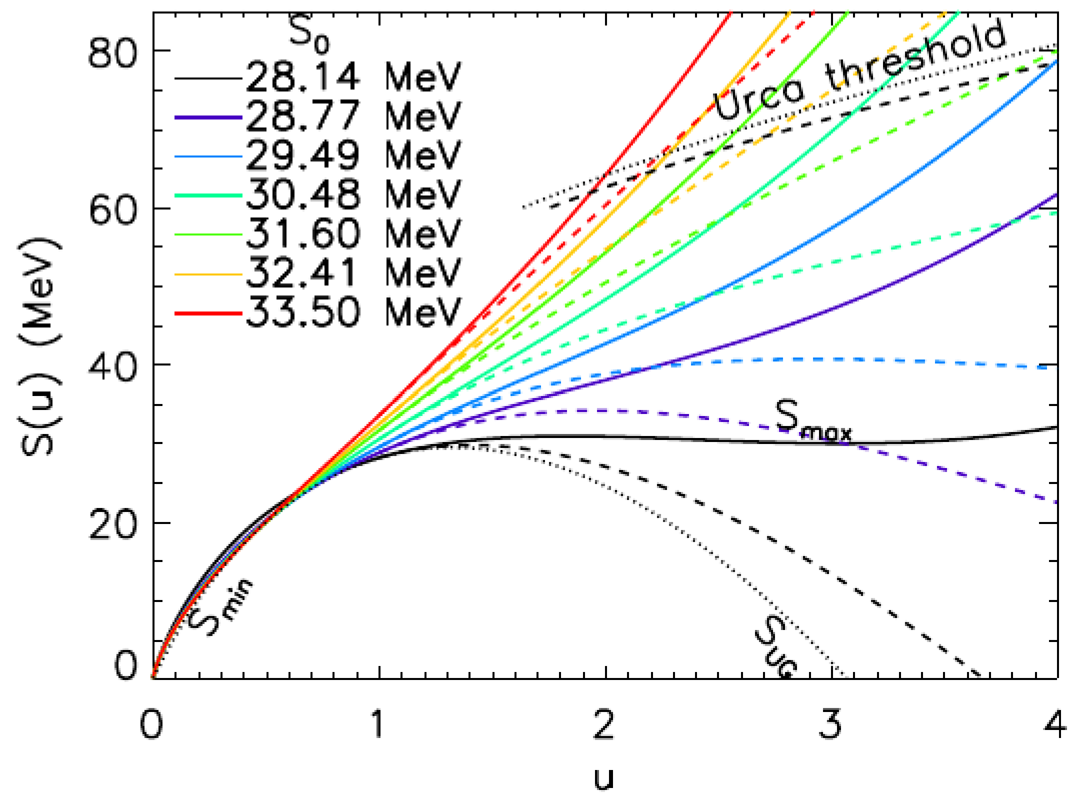
Do I have time ?

- **Further constraint from Urca process**
  - **Direct Urca process ($n \to p+\bar{\nu}+e$, $p \to n+\nu+e^+$) cools NS rapidly.**
  - **Only a small fraction of NSs cools rapidly.**
    $\to$ Direct Urca is allowed only at high density.
  - **Direct Urca is allowed when**

$$Y_p = \frac{Z}{N+Z} > \frac{1}{9}$$

$$\to \frac{S(u)}{u^{1/3}} \geq \frac{9}{28}\hbar c \left(\frac{\pi^2 n_0}{3}\right)^{1/3}$$

  - **Unitary Gas**
    + No Direct Urca at $n/n_0 < 4$
    $\to S_0 < 31.5$ MeV



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