
*Nuclear matter symmetry energy
and neutron star properties
--- Neutron star radius from gravitational wave
vs nuclear experiments --*

Akira Ohnishi

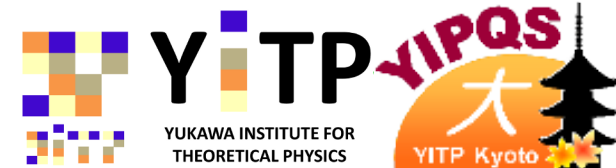
(Yukawa Inst. for Theor. Phys., Kyoto U.)

in collaboraton with

**E. E. Kolomeitsev (Matej Bel U.), James M. Lattimer (Stony Brook),
Ingo Tews (LANL), Xuhao Wu (Nankai U./YITP)**

*Gravitational wave physics and astronomy: Genesis,
2nd annual area symposium, Nov.26-28, 2018, Kyoto, Japan*

- I. Tews, J. M. Lattimer, AO, E.E.Kolomeitsev, ApJ 848('17) 105
[arXiv:1611.07133]*
- AO, Kolomeitsev, Lattimer, Tews, X.Wu, in prog.*



Contents

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Introduction

Impact of GW from binary neutron star merger

- **GW170817 from NS-NS → Multi messenger astrophysics**

B.P.Abbott+ [LIGO Sci. and Virgo Collab.], PRL119('17)161101

- **Neutron Star Maximum Mass**

- **No prompt collapse, No GW signal from Hyper Massive NS**
→ $M_{\max}(T=0, \omega=0) < M_{\max}(T=0, \omega) < M < M_{\max}(T, \omega)$

- **Neutron Star Radius**

- **Inspiral region → Tidal deformability (Λ) → NS radius (e.g. R1.4)**

- **Nucleosynthesis site of r-process nuclei**

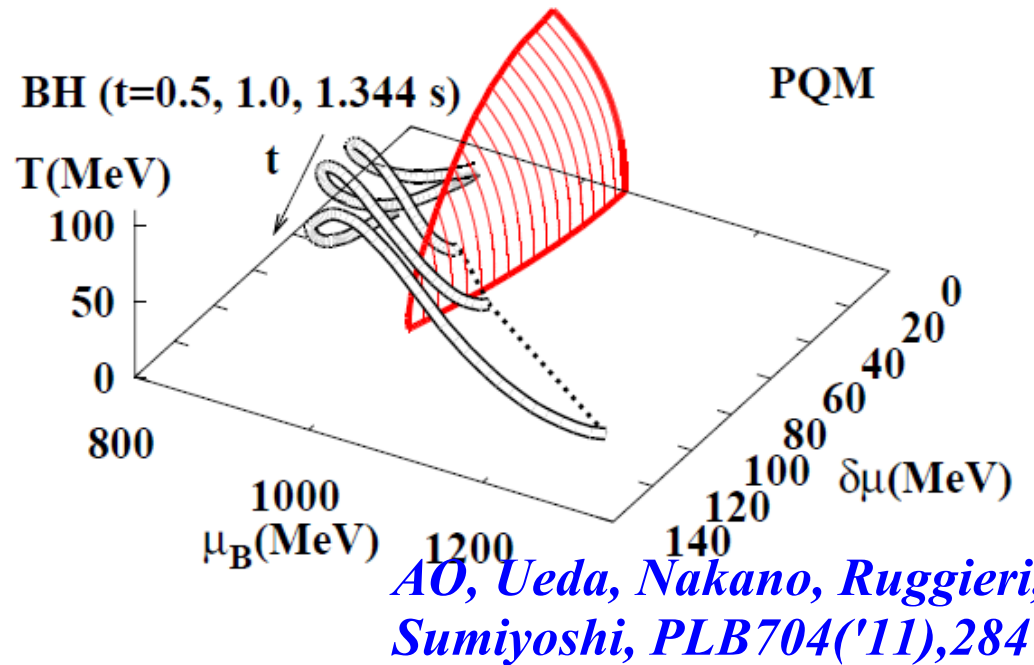
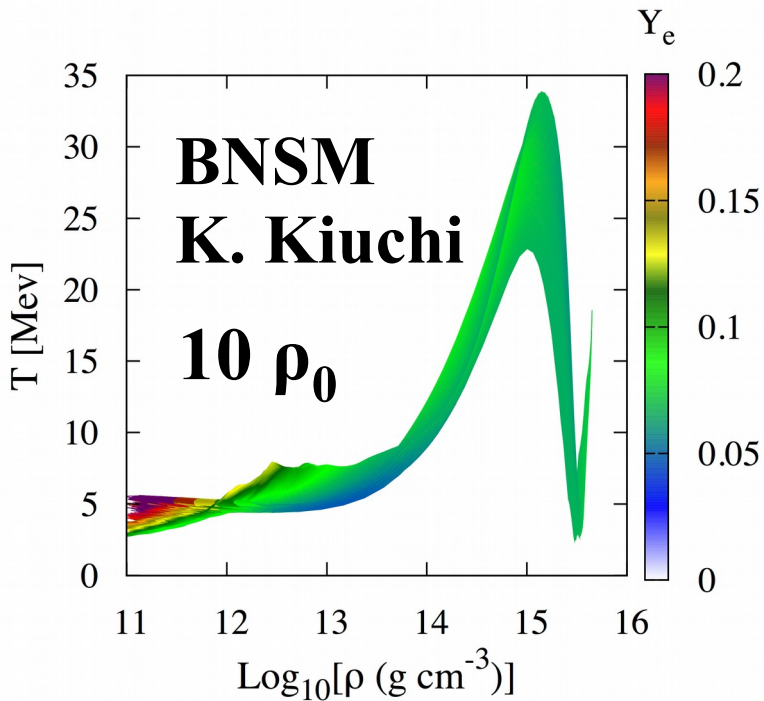
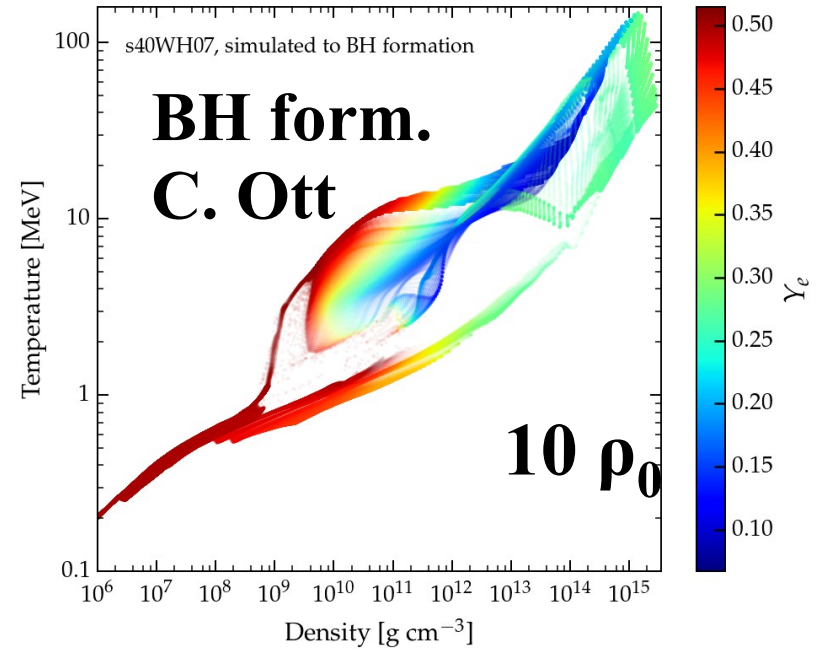
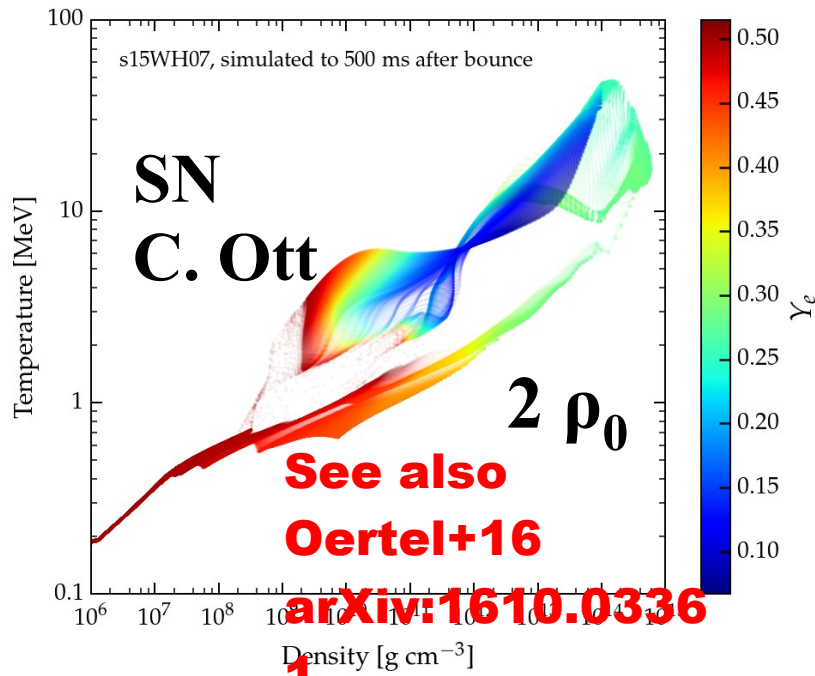
- **kilonova/macronova from decay energy of the synthesized elements**
- **r-process nucleosynthesis seems to occur in BNSM !**

- **Central Engine of (Short) Gamma-Ray Bursts**

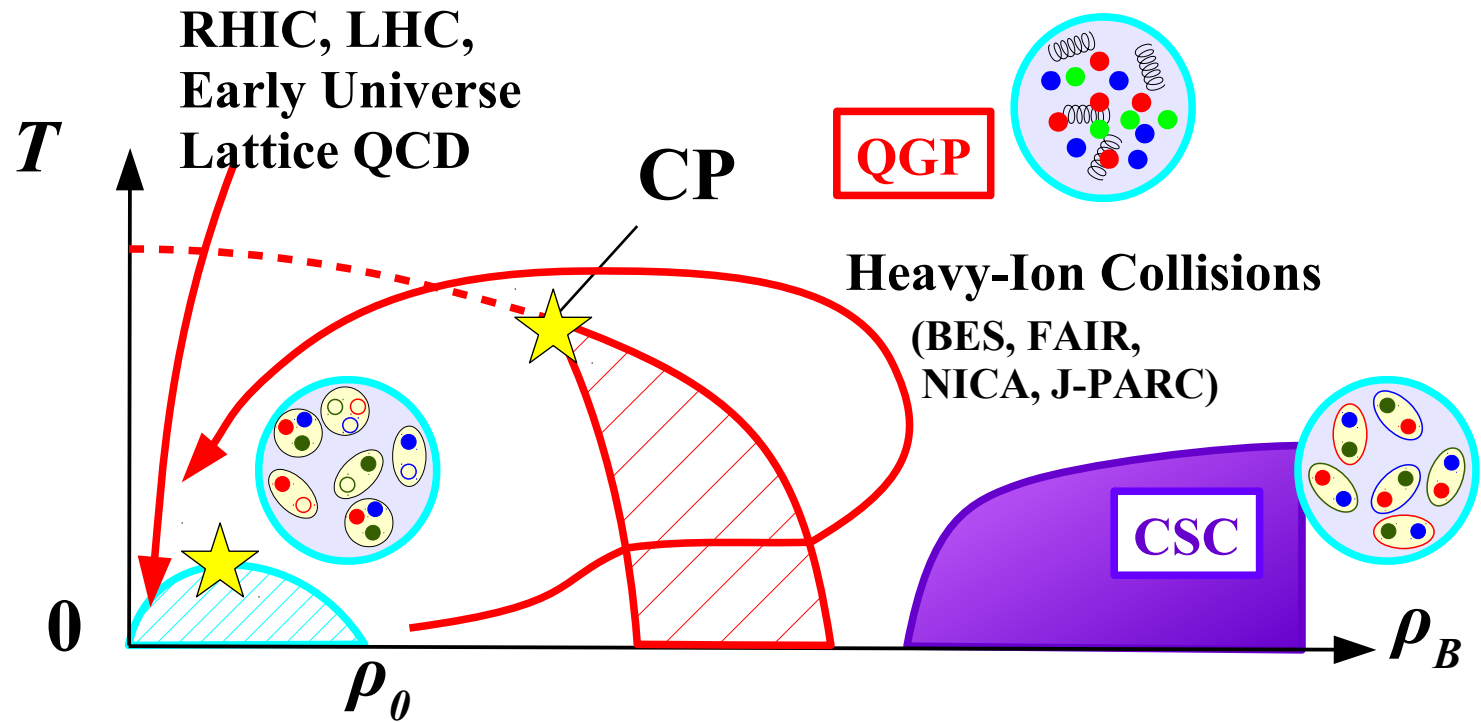
- **GW as standard siren (Hubble constant)**

Courtesy of Y. Sekiguchi @ YKIS2018b

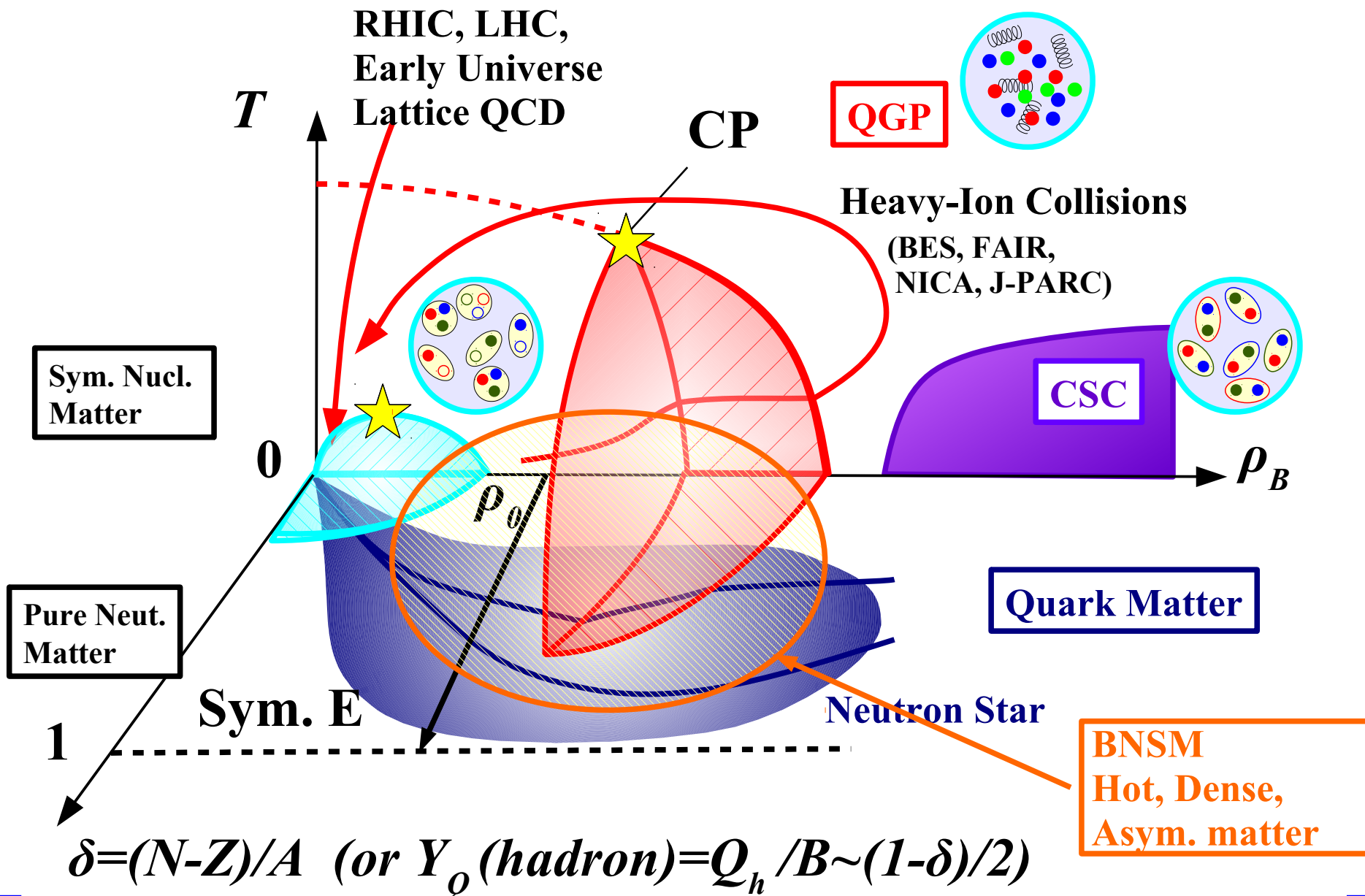
(ρ, T, Y_e) during SN, BH formation, BNSM



QCD Phase Diagram

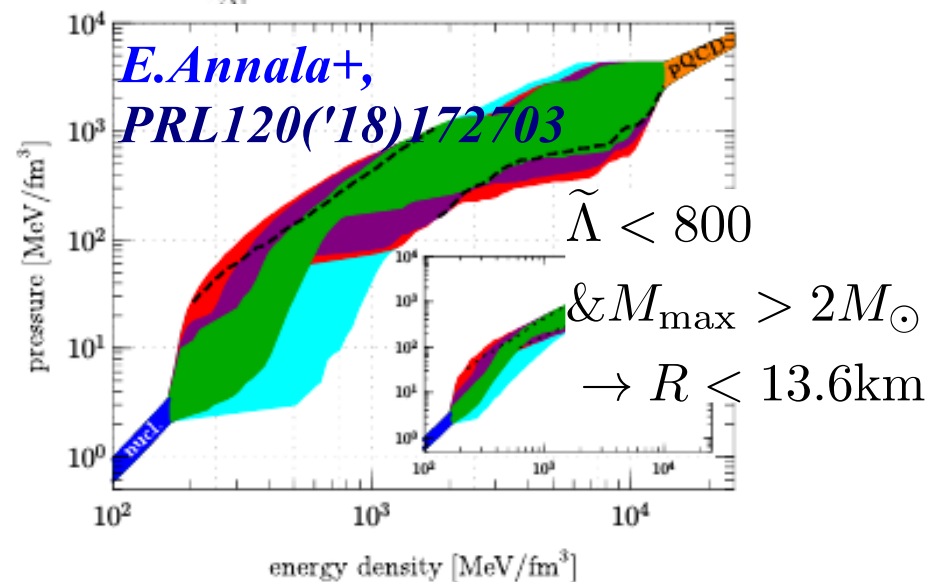
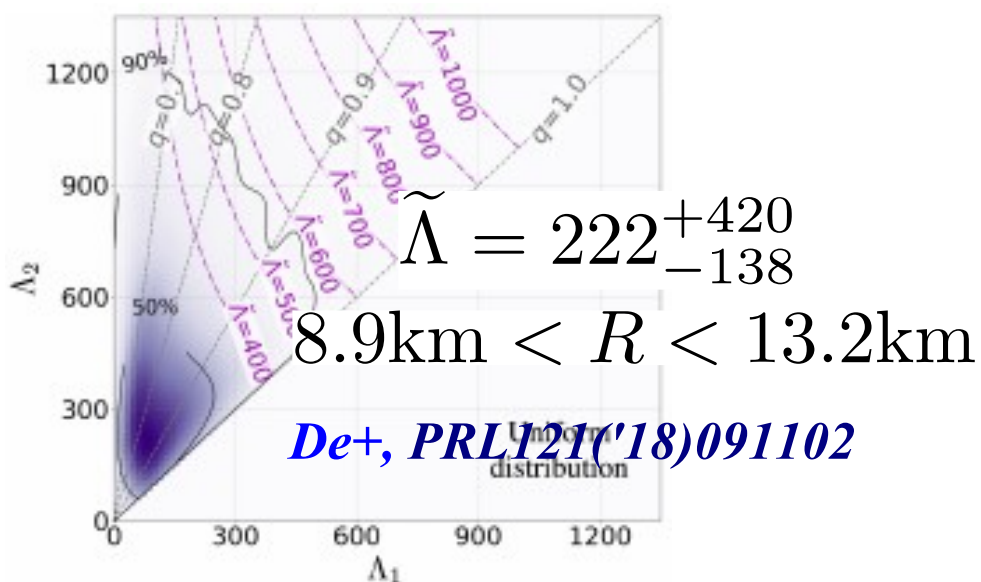
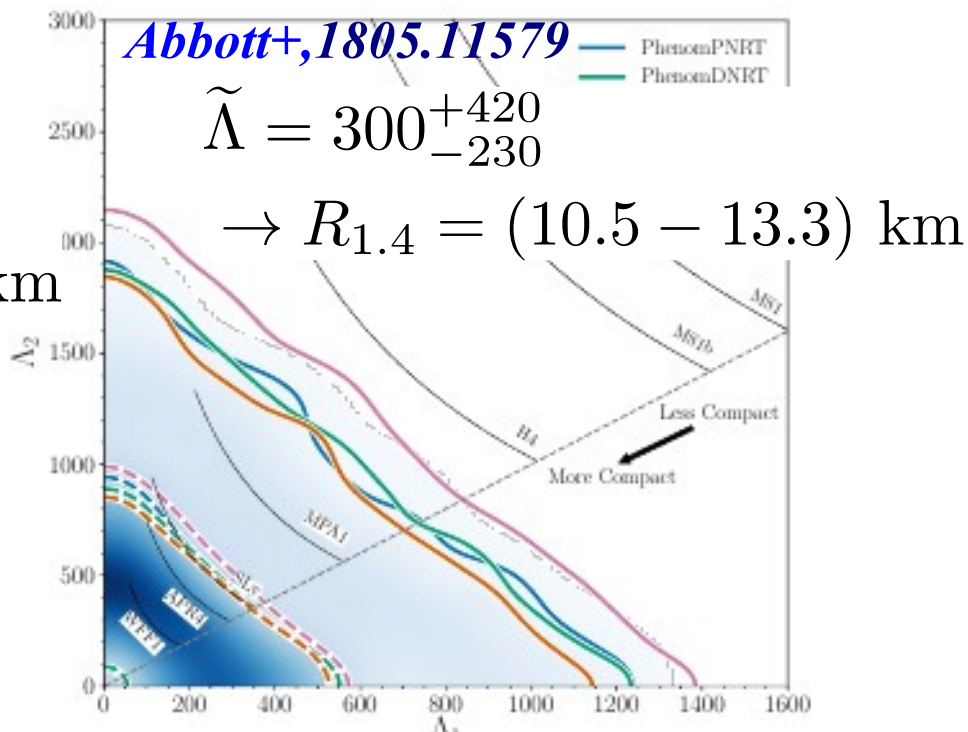
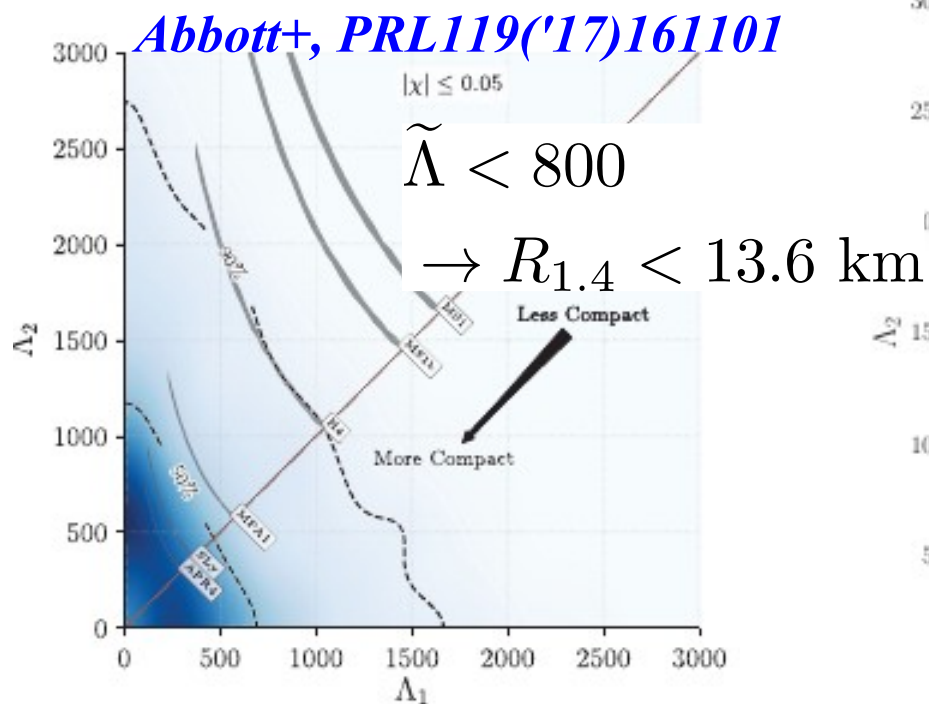


QCD Phase Diagram

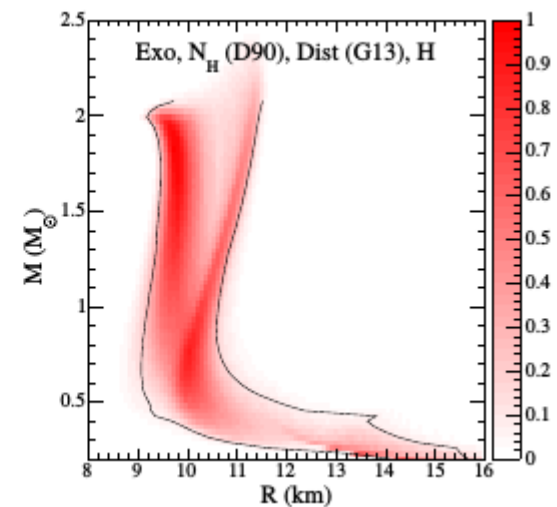
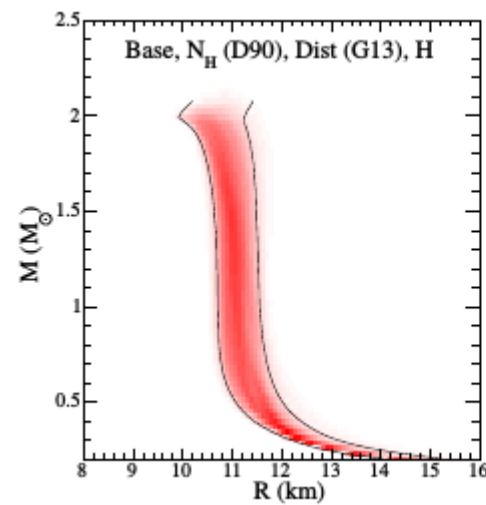
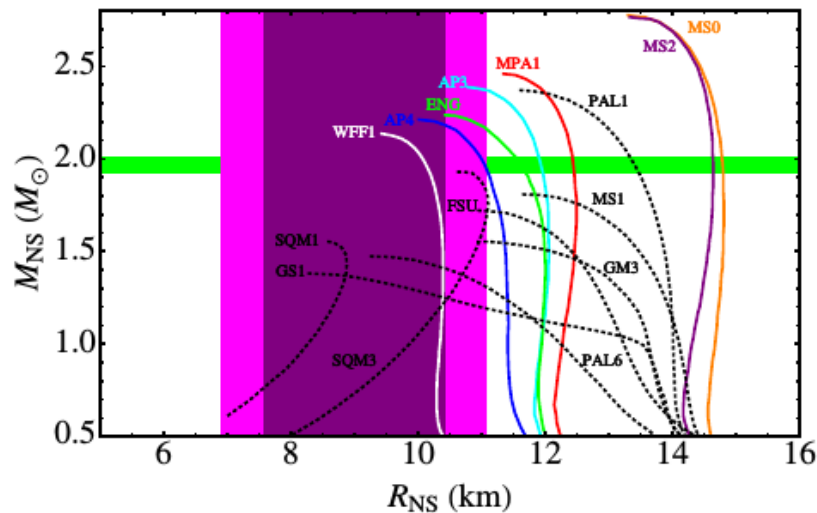
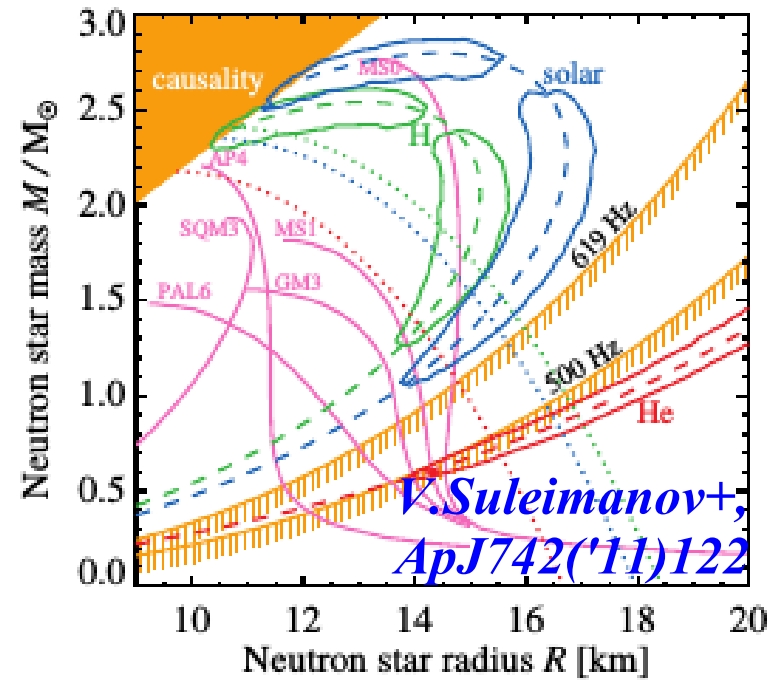
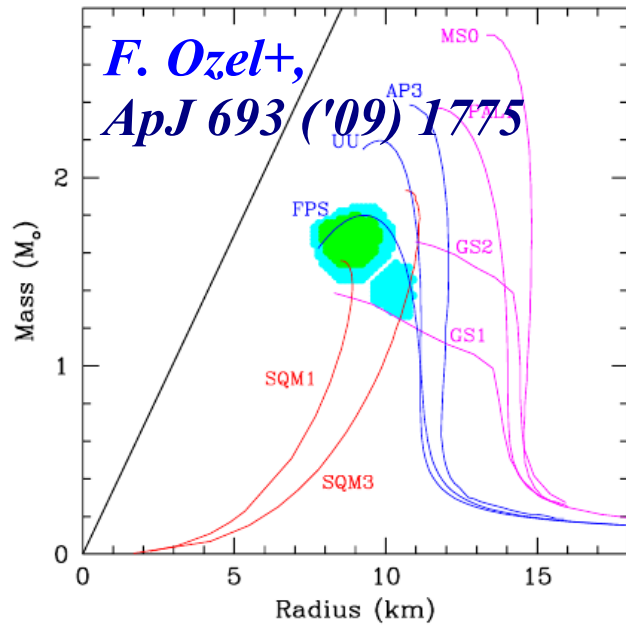


AO, arXiv:1712.01088

Constraints on EOS from GW170817



MR curve from X-ray burst



S. Guillot+, ApJ 772 ('13) 7

J.M. Lattimer, A.W. Steiner, ApJ 784 ('14) 123

Neutron Star Radius from Astronomical Observations

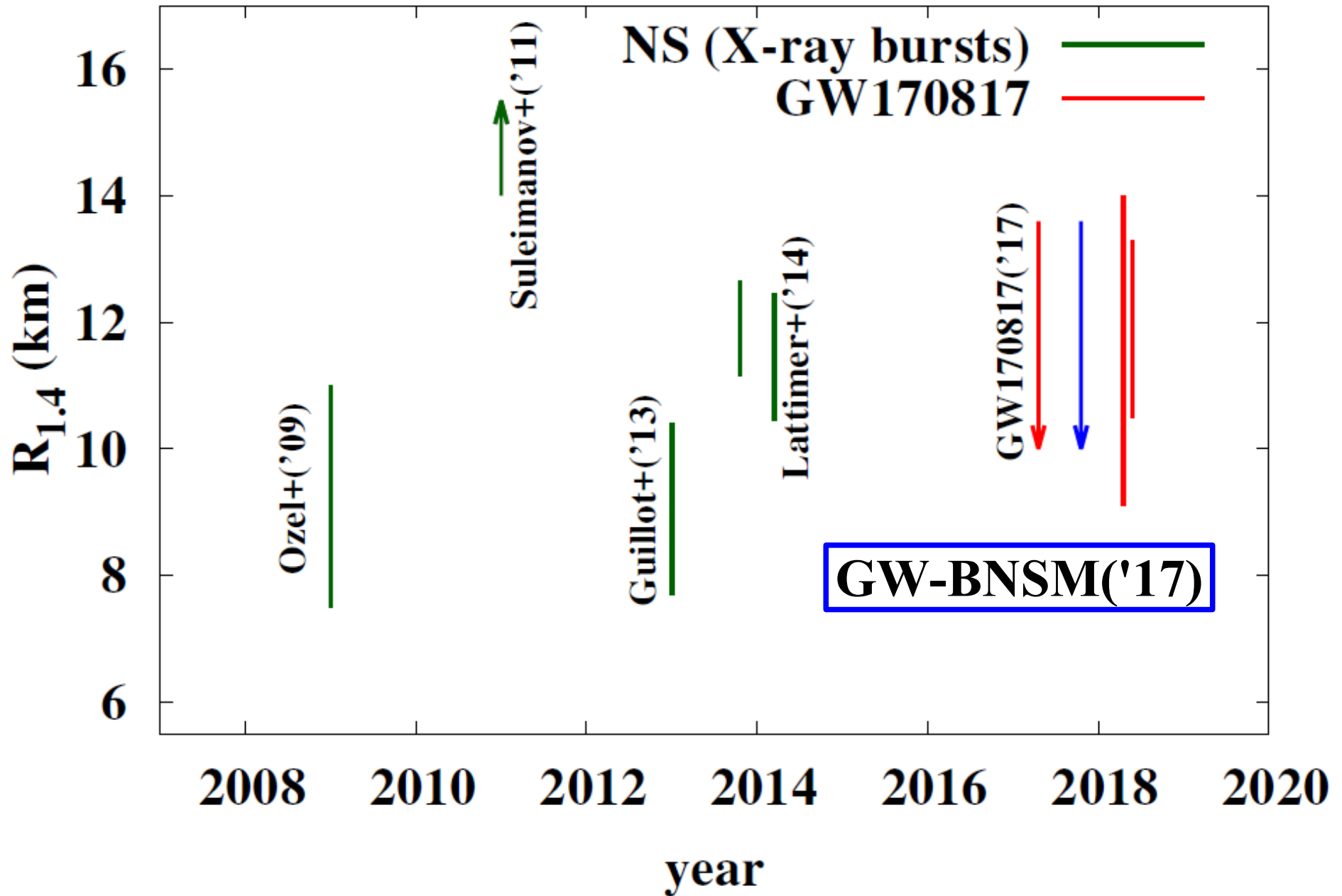
■ From X-ray bursts

- $R=9$ or 11 km (7.5-11 km) *F. Özel+, ApJ693('09)1775*
[touch down = Eddington limit?]
- $R > 14$ km *V.Suleimanov+, ApJ742('11)122*
[color correction factor ?]
- $R=9.1^{+1.3}_{-1.4}$ km *S.Guillot+, ApJ772('13)7*
[Common R ?; Denied later by the authors in ApJ 796('14)L3 (1409.4306)]
- $R=10.4-12.7$ km *J.M.Lattimer, A.W.Steiner, ApJ784('14)123*
($R=(11.15-12.66)$ km (normal EOS), $R=(10.45-12.45)$ km (Exo EOS))

■ Gravitational Waves

- $\bar{\Lambda} < 800$ *B.P.Abbott+, PRL119('17)161101* $\rightarrow R < 13.6$ km
- $\bar{\Lambda}=300^{+420}_{-230}$ $\rightarrow R=(10.5-13.3)$ km *B.P.Abbott+ (LIGO-Virgo), 1805.11579*
- $\bar{\Lambda}=222^{+420}_{-138}$ $\rightarrow R=(9.1-14.0)$ km *S.De+, PRL121('18)091102*
- $\bar{\Lambda} < 800$ & $M_{\max} > 2 M_{\odot}$ $\rightarrow R < 13.6$ km *E.Annala+, PRL120('18)172703*

Time dependence of Neutron Star Radius ($R_{1.4}$)

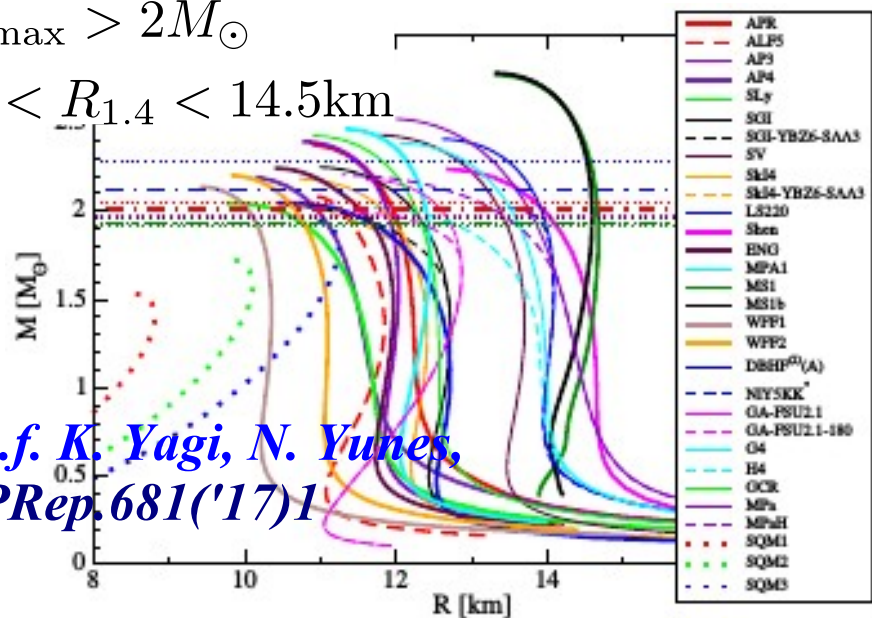


Constraints from Nuclear Physics (+ α)

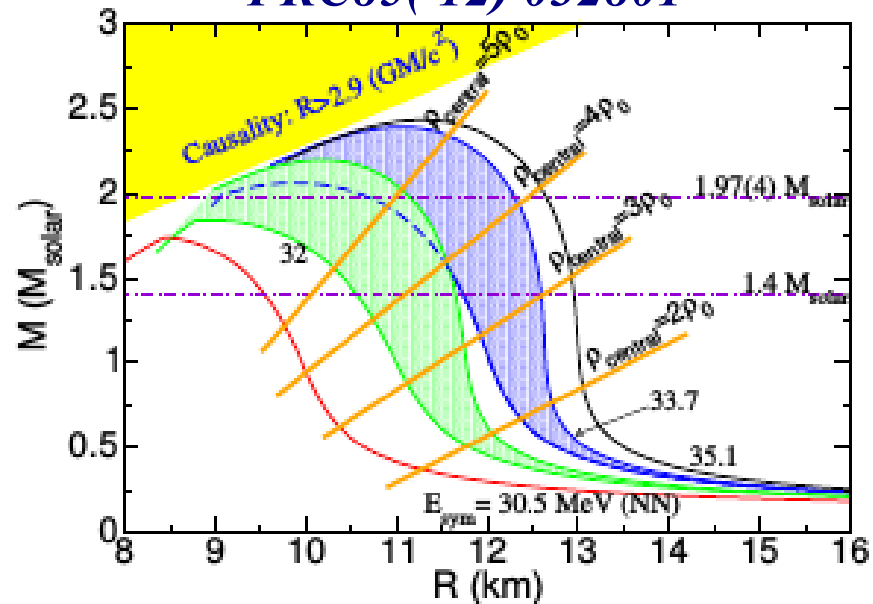
$$M_{\max} > 2M_{\odot}$$

$$10 < R_{1.4} < 14.5 \text{ km}$$

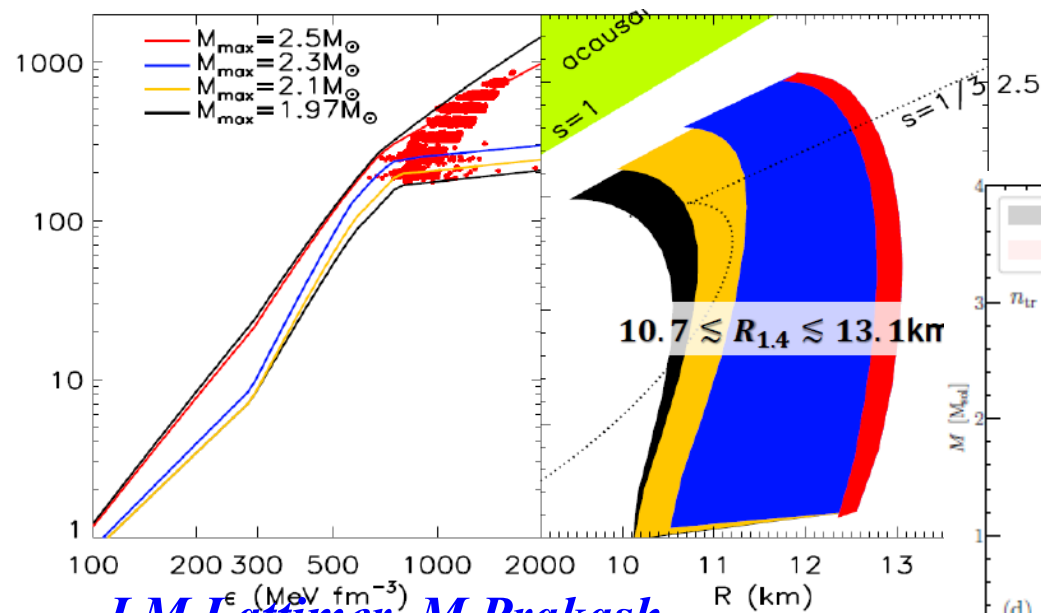
*c.f. K. Yagi, N. Yunes,
PRep.681('17)1*



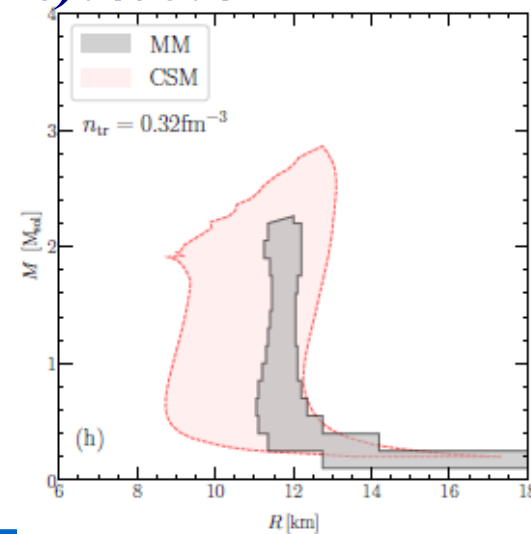
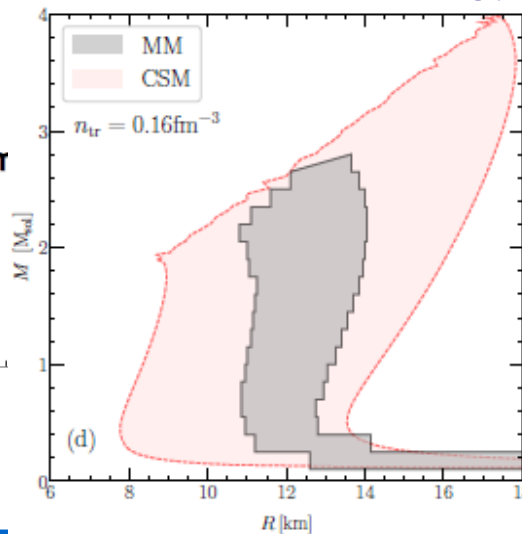
*S. Gandolfi, J. Carlson, S. Reddy,
PRC85('12) 032801*



*I. Tews, J. Margueron, S. Reddy,
PRC98 ('18)045804*



*J.M. Lattimer, M. Prakash,
PRep.621('16)127*



Neutron Star Radius from Nuclear Physics (+ α)

■ Impact of $2 M_{\odot}$ neutron star

P. Demorest+, Nature 467('10)1081 (1010.5788),

J. Antoniadis+, Science 340('13)6131(1304.6875)

- $R=(6-15)$ km (before 2010) \rightarrow $R=(10-15)$ km (with $2 M_{\odot}$)

■ Impact of symmetry energy parameters

- $S_0 \rightarrow R=(10.5-13)$ km *S. Gandolfi, J. Carlson, S. Reddy, PRC85('12)032801*

- $(S_0, L) \rightarrow R=(10.7-13.1)$ km *J.M. Lattimer, M. Prakash, PRep.621('16)127*

■ Chiral Effective Field Theory (Chiral EFT)

- χ EFT+pQCD+GW $\rightarrow R=(10.0-13.6)$ km

E. Annala, T. Gorda, A. Kurkela, A. Vuorinen, PRL120('18)172703

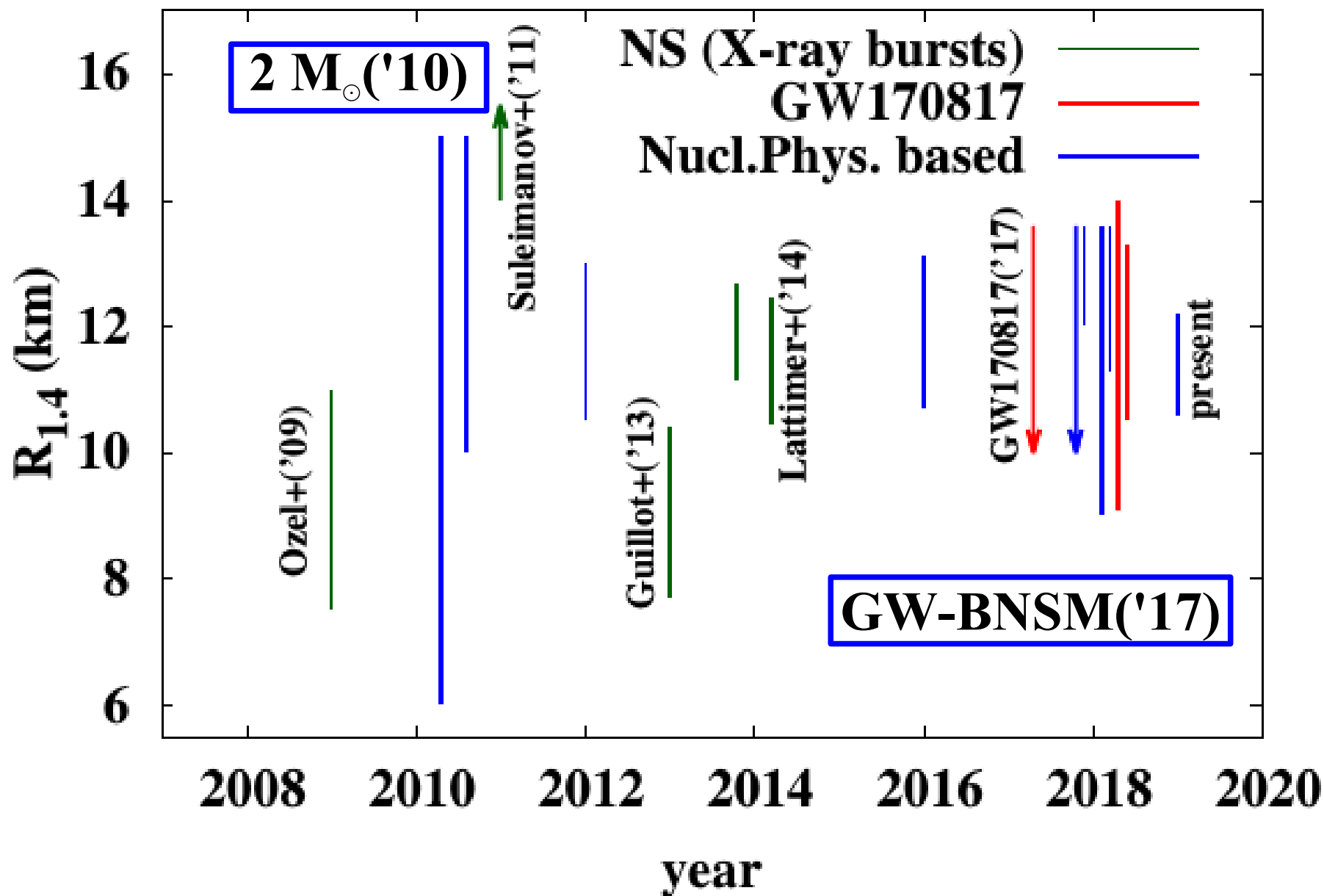
- χ EFT+c_s² $\rightarrow R=(9.0-13.6)$ km [min. model $R=(11.3-13.6)$ km]

I. Tews, J. Margueron, S. Reddy, PRC98 ('18)045804

- Neutron skin thickness from ν scatt. (PREX) $\rightarrow R=(12.0-13.6)$ km

F.J. Fattoyev, J. Piekarewicz, C.J. Horowitz, PRL120 ('18)172702

Time dependence of Neutron Star Radius ($R_{1.4}$)

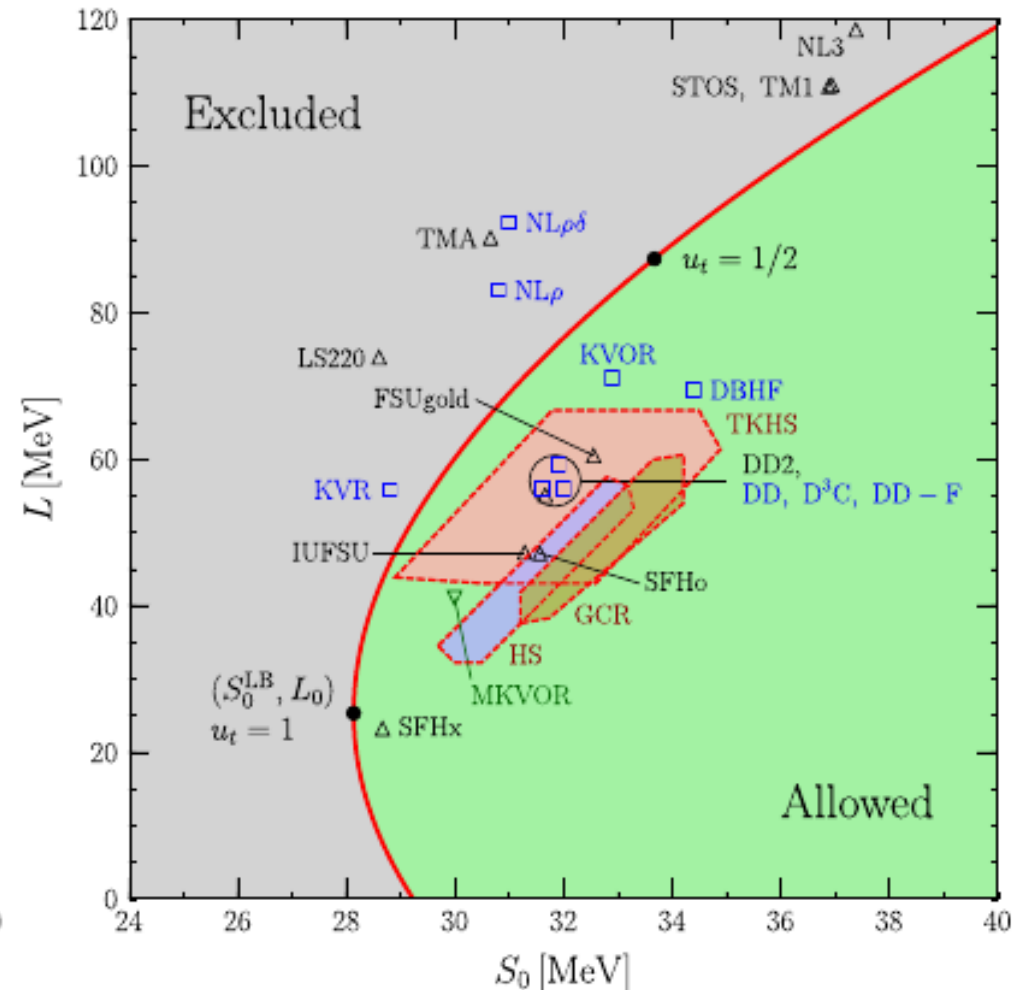
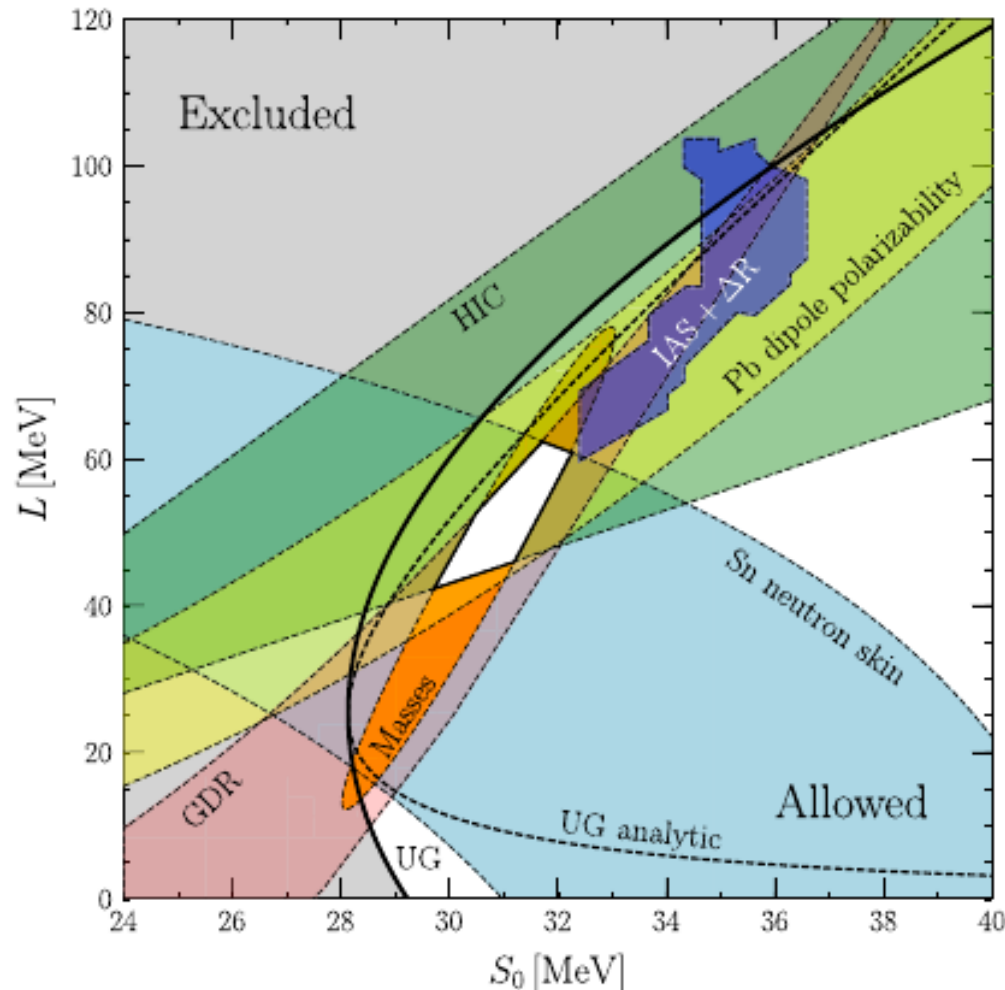


*Symmetry Energy Parameters (S_0 , L) affect
Neutron Star Radius.
How about higher-order parameters ?*

*Symmetry Energy Parameters,
Nuclear Matter EOS,
and Neutron Star Radius*

Constraint on (S_0, L) from Lower Bound of PNM Energy

- Unitary gas ($E_{\text{PNM}} > E_{\text{UG}}$) + $2 M_{\odot}$ constraints rule out 5 EOSs (incl. LS220, Shen) out of 10 numerically tabulated ones.



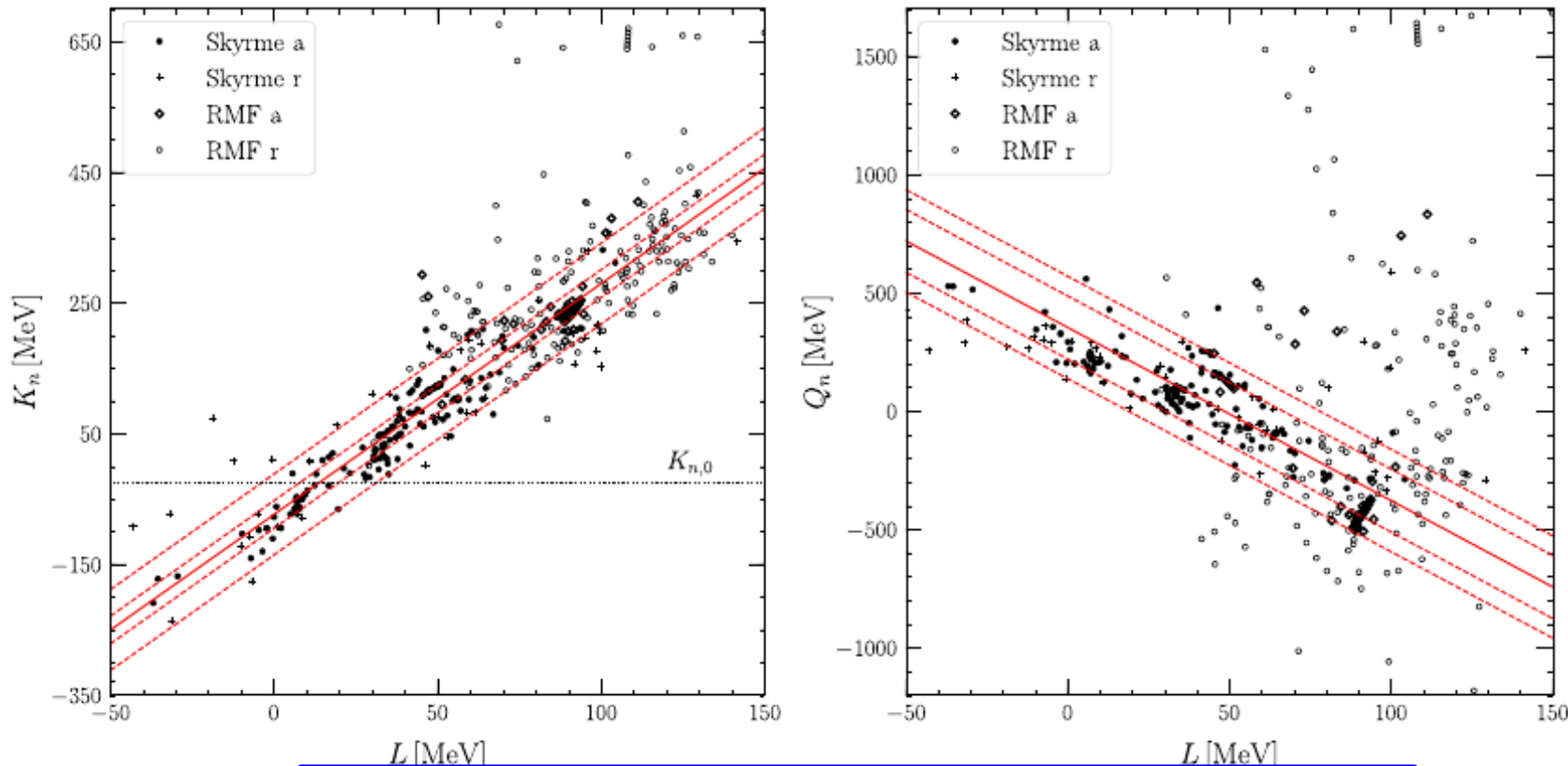
I. Tews, J.M.Lattimer, AO, E.E.Kolomeitsev (TLOK), ApJ 848 ('17)105

Further Constraints on Higher-Order Sym. E. parameters

- K_n and Q_n are correlated with L in “Good” theoretical models.

$$K_n = 3.534L - (74.02 \pm 21.17)\text{MeV}$$

$$Q_n = -7.313L + (354.03 \pm 133.16)\text{MeV}$$



Regard theoretical models as data !

I. Tews, J.M.Lattimer, AO, E.E.Kolomeitsev (TLOK), ApJ 848 ('17)105

Fermi momentum (k_F) expansion

■ Saturation & Symmetry Energy Parameters

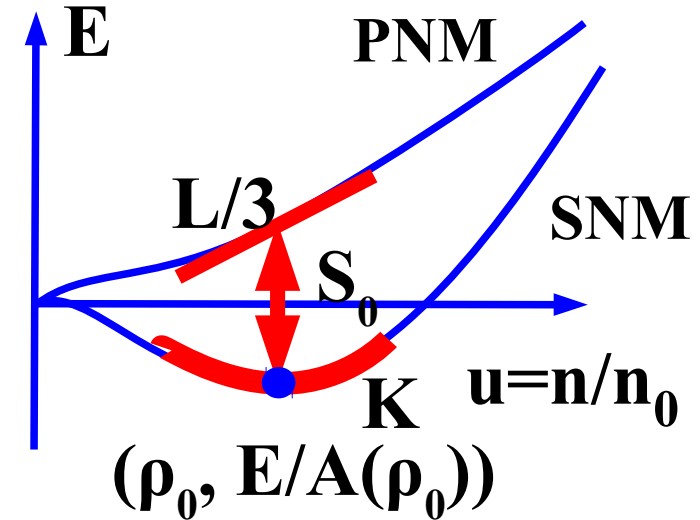
TLOK

$$E_{\text{NM}}(u, \alpha) = E_{\text{SNM}}(u) + \alpha^2 S(u)$$

$$E_{\text{SNM}}(u) \simeq E_0 + \frac{K_0}{18}(u-1)^2 + \frac{Q_0}{162}(u-1)^3$$

$$S(u) \simeq S_0 + \frac{L}{3}(u-1) + \frac{K_s}{18}(u-1)^2 + \frac{Q_s}{162}(u-1)^3$$

$$(u = n/n_0, \alpha = (n_n - n_p)/n)$$



Energy does not approach zero at $n \rightarrow 0$.

■ Fermi momentum expansion (~ Skyrme type EDF)

- Generated many-body force is given by $k_F \propto u^{1/3}$ m^*

$$E_{\text{SNM}}(u) \simeq \boxed{T_0 u^{2/3}} + \underline{a_0 u} + \underline{b_0 u^{4/3}} + \underline{c_0 u^{5/3}} + \underline{d_0 u^2}$$

$$S(u) \simeq \boxed{T_s u^{2/3}} + \underline{a_s u} + \underline{b_s u^{4/3}} + \underline{c_s u^{5/3}} + \underline{d_s u^2}$$

Kin. E. Two-body Density-dep. pot.

Expansion Coefficients

- Coefficients (a,b,c,d) are represented by Saturation and Symmetry Energy Parameters

TLOK

$$a_0 = -4T_0 \quad +20E_0 \quad + K_0 \quad -Q_0/6$$

$$b_0 = 6T_0 \quad -45E_0 \quad -5K_0/2 \quad +Q_0/2$$

$$c_0 = -4T_0 \quad +36E_0 \quad +2K_0 \quad -Q_0/2$$

$$d_0 = T_0 \quad -10E_0 \quad -K_0/2 \quad +Q_0/6$$

$$a_s = -4T_s \quad +20S_0 - 19L/3 \quad + K_s \quad -Q_s/6$$

$$b_s = 6T_s \quad -45S_0 + 15L \quad -5K_s/2 \quad +Q_s/2$$

$$c_s = -4T_s \quad +36S_0 - 12L \quad +2K_s \quad -Q_s/2$$

$$d_s = T_s \quad -10S_0 + 10L/3 \quad -K_s/2 \quad +Q_s/6$$

$$\left(T_0 = \frac{3 \hbar^2 k_F (n_0)^2}{5 \cdot 2m}, \quad T_s = T_0 (2^{1/3} - 1) \right)$$

Tedious but straightforward calc.

$TLOK+2M_{\odot}$ constraints

TLOK constraints

- (S_0, L) is in Pentagon.

- (K_n, Q_n) are from TLOK constraint.

- $K_0=(190-270)$ MeV

- (n_0, E_0) is fixed

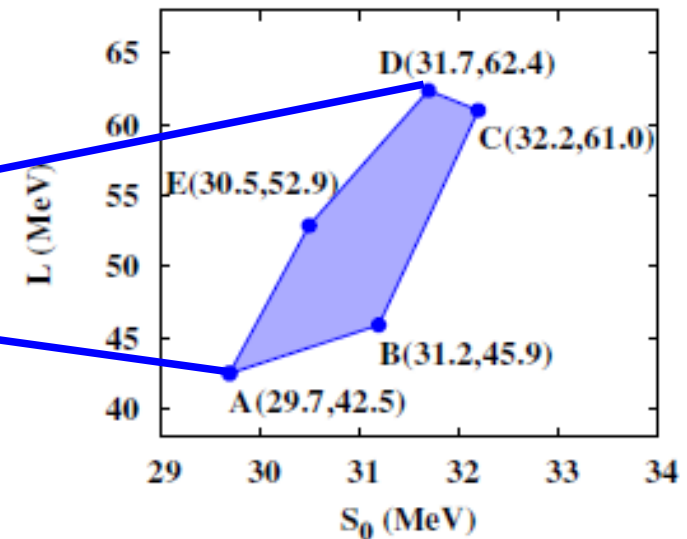
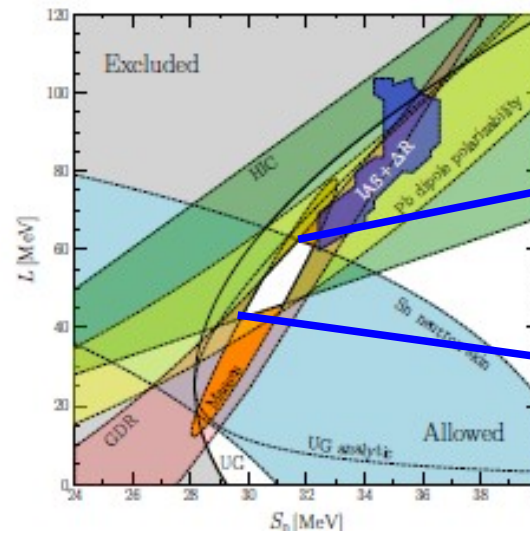
$n_0=0.164$ fm⁻³, $E_0=-15.9$ MeV (small uncertainties)

- Q_0 is taken to kill d_0 parameter

(Coef. of u^2 . Sym. N. M. is not very stiff at high-density)

2 M_{\odot} constraint

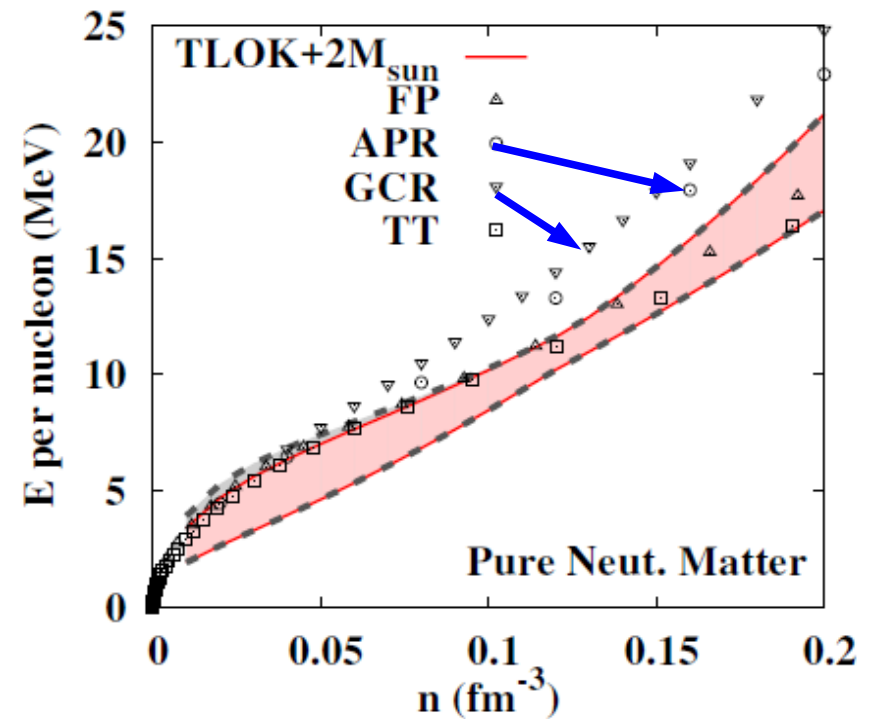
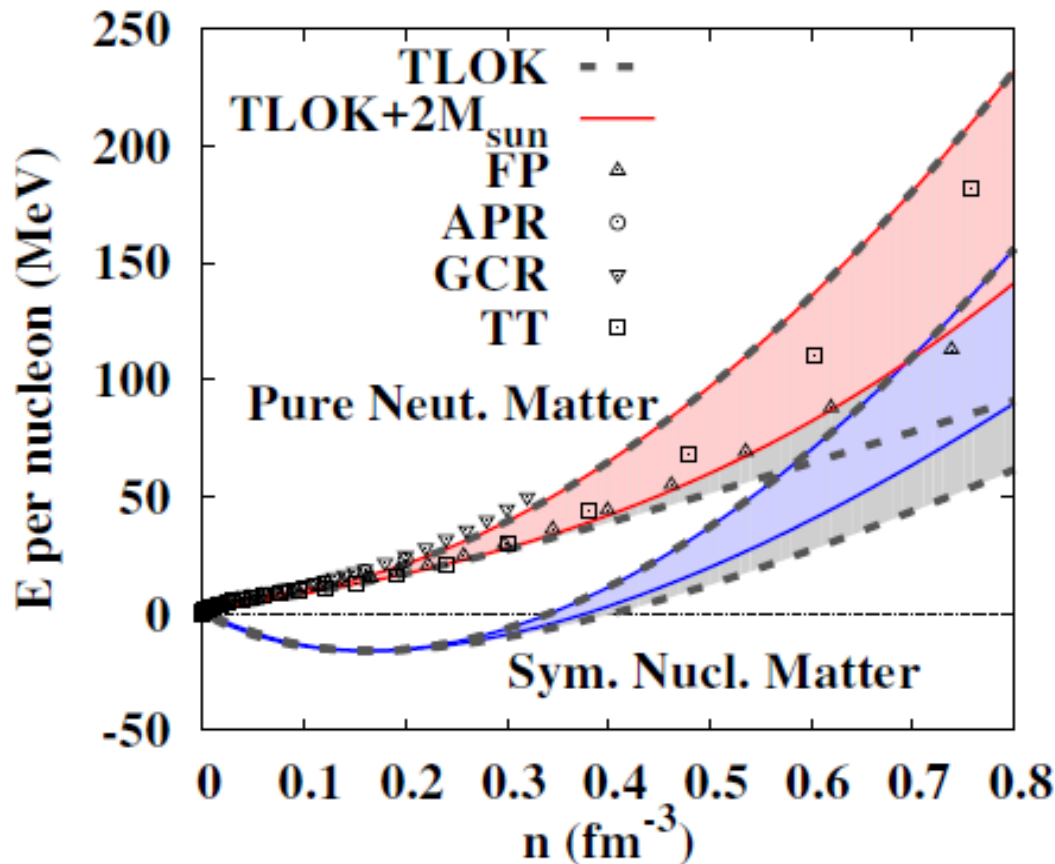
- EOS should support 2 M_{\odot} neutron stars.



AO, Kolomeitsev, Lattimer, Tews, Wu (OKLTW), in prog.

$TLOK+2M_{\odot}$ constraints on EOS

- $2M_{\odot}$ constraint narrows the range of EOS.
- Consistent with FP and TT(Togashi-Takano) EOSs.
- APR and GCR(Gandolfi-Carlson-Reddy) EOSs seems to have larger S_0 values.



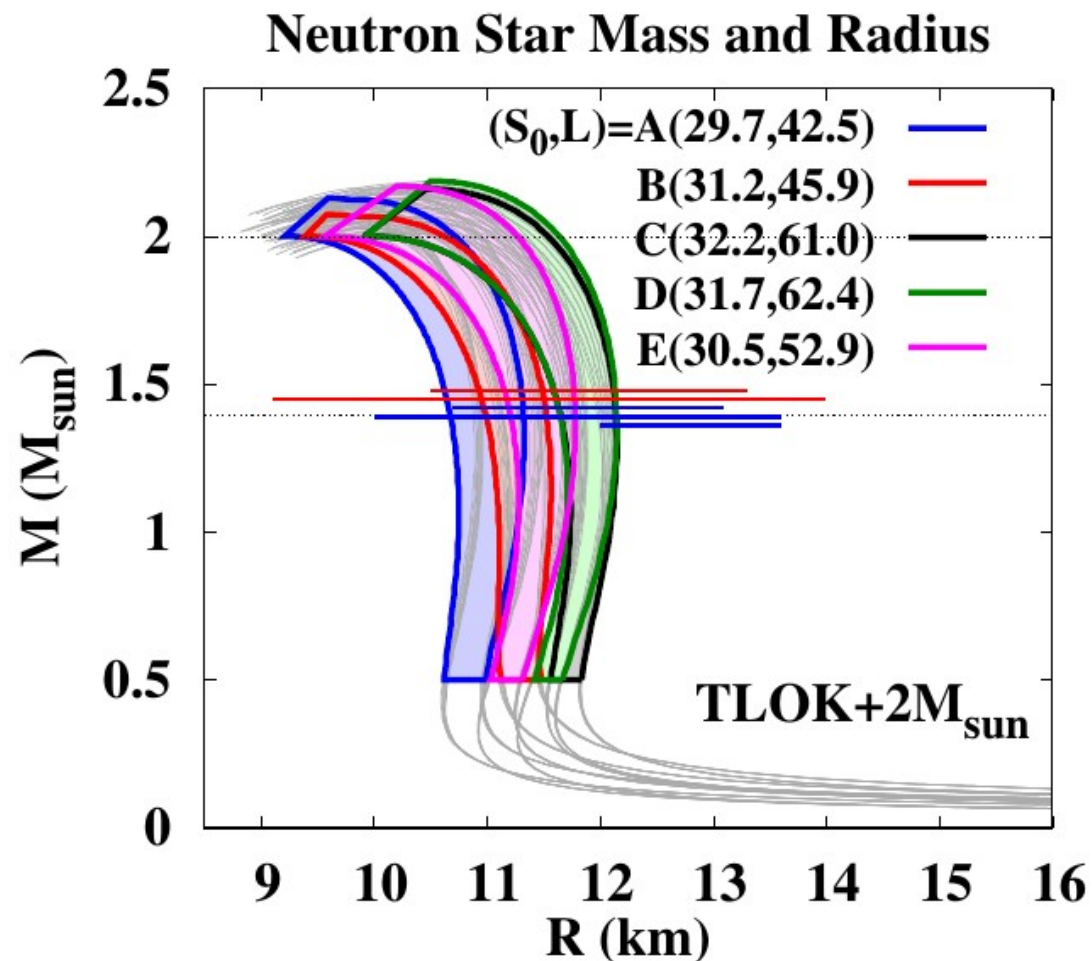
OKLTW, in prog.

Neutron Star MR curve

■ TLOK + $2 M_{\odot}$ constraints $\rightarrow R_{1.4} = (10.6-12.2)$ km

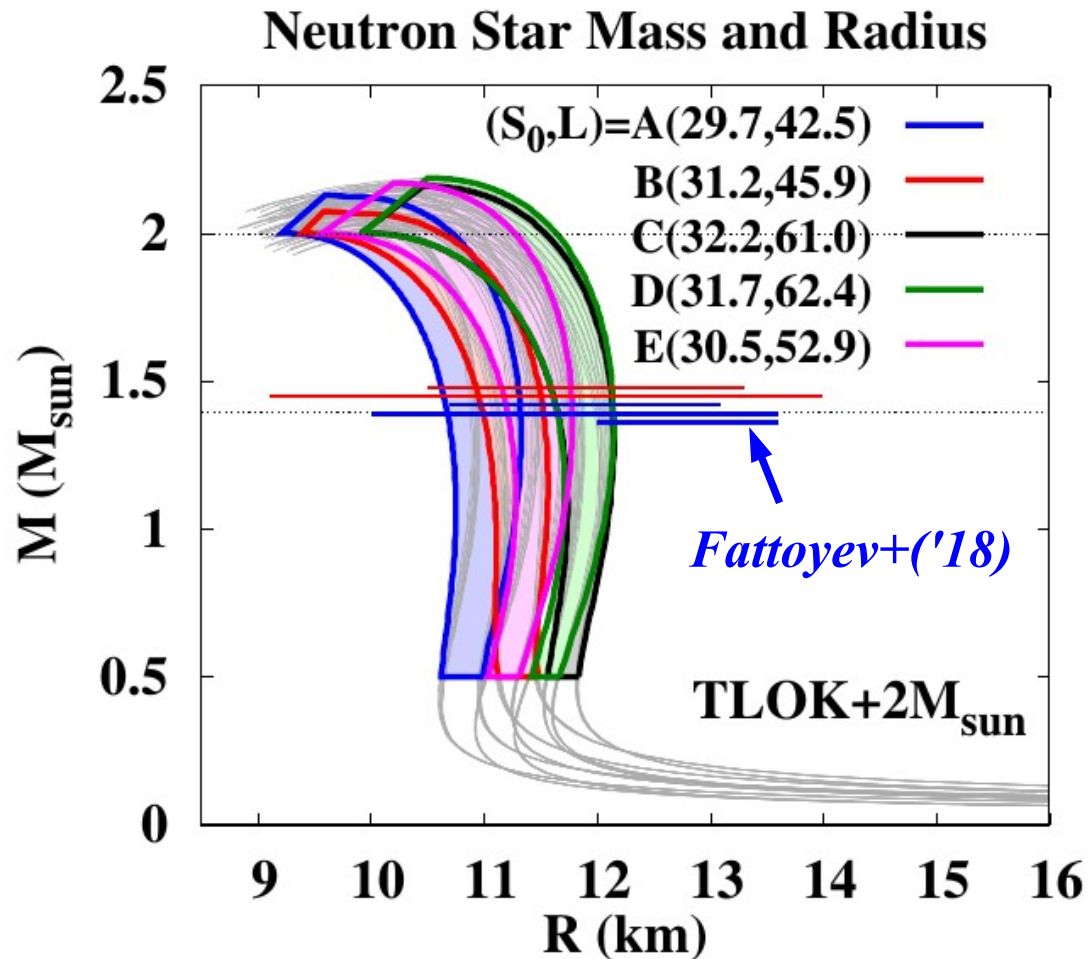
OKLTW, in prog.

- E and P are linear fn. of Sat. & Sym. E. parameters
 \rightarrow Min./Max. appears at the corners of pentagon (ABCDE).
- For a given (S_0, L) ,
unc. of $R_{1.4} \sim 0.5$ km
= unc. from higher-order parameters
- Unc. from $(S_0, L) \sim 1.1$ km
 \rightarrow We still need to fix (S_0, L) more precisely.

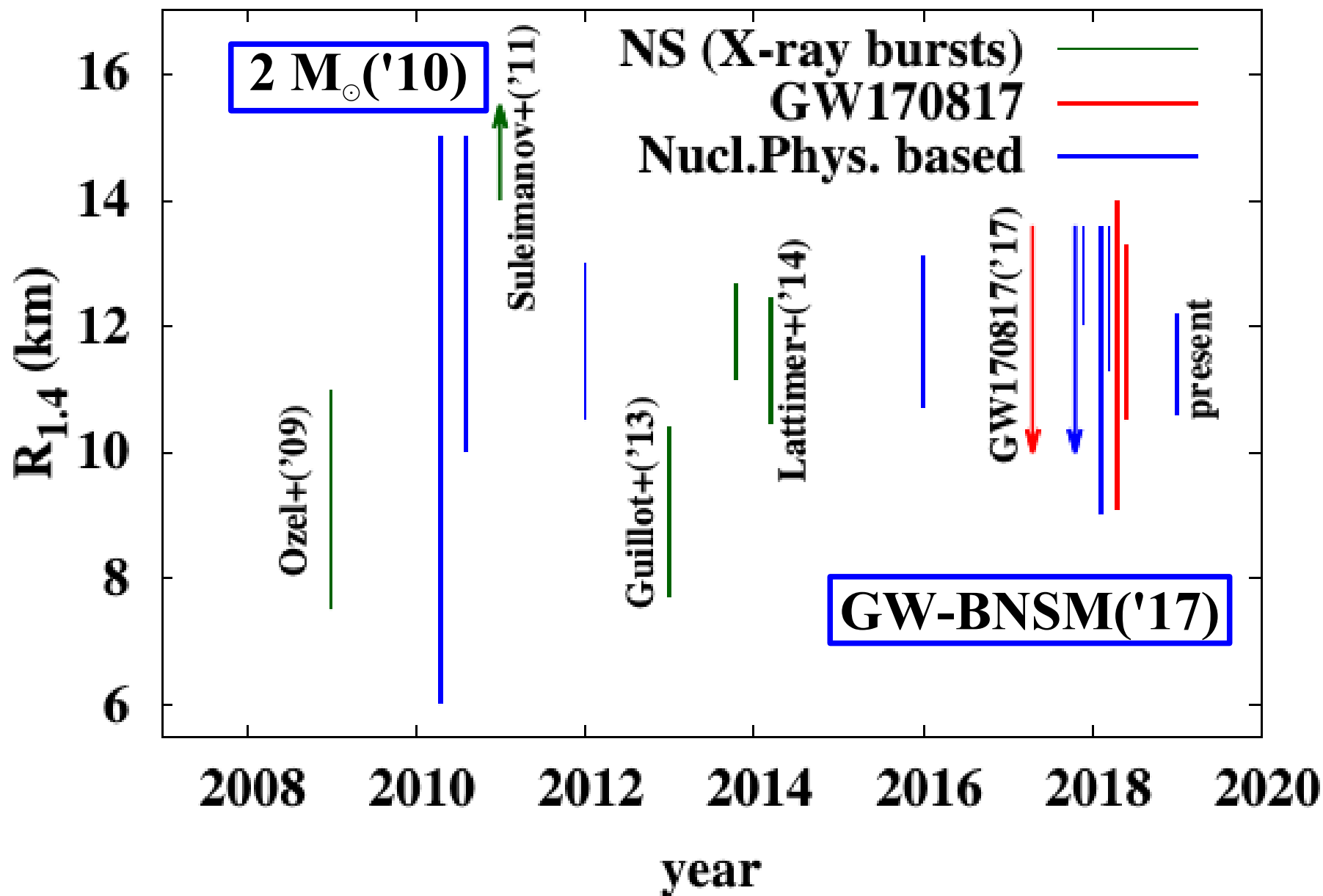


Neutron Star MR curve

- Our constraint is consistent with many of previous ones.
 - $R_{1.4}=(10.6-12.2)$ km *Present work (TLOK + 2 M_{\odot}) OKLTW, in prog.*
 - LIGO-Virgo (Tidal deformability Λ from BNSM)
 - (10.5-13.3) km *Abbott+('18b)*
 - (9.1-14.0) km *De+('18) (Λ)*
 - Theoretical Estimates
 - (10.7-13.1) km
Lattimer+, PRep.621('16)127
 - (10.0-13.6) km
Annala+, PRL120('18)172703
 - (9-13.6) km
Tews+, PRC98 ('18)045804
 - (12.0-13.6) km
F.J.Fattoyev+(PREX), PRL120 ('18)172702



Time dependence of Neutron Star Radius ($R_{1.4}$)



*Implications to quark-hadron physics
in cold dense matter*

Questions !

■ Hyperon puzzle

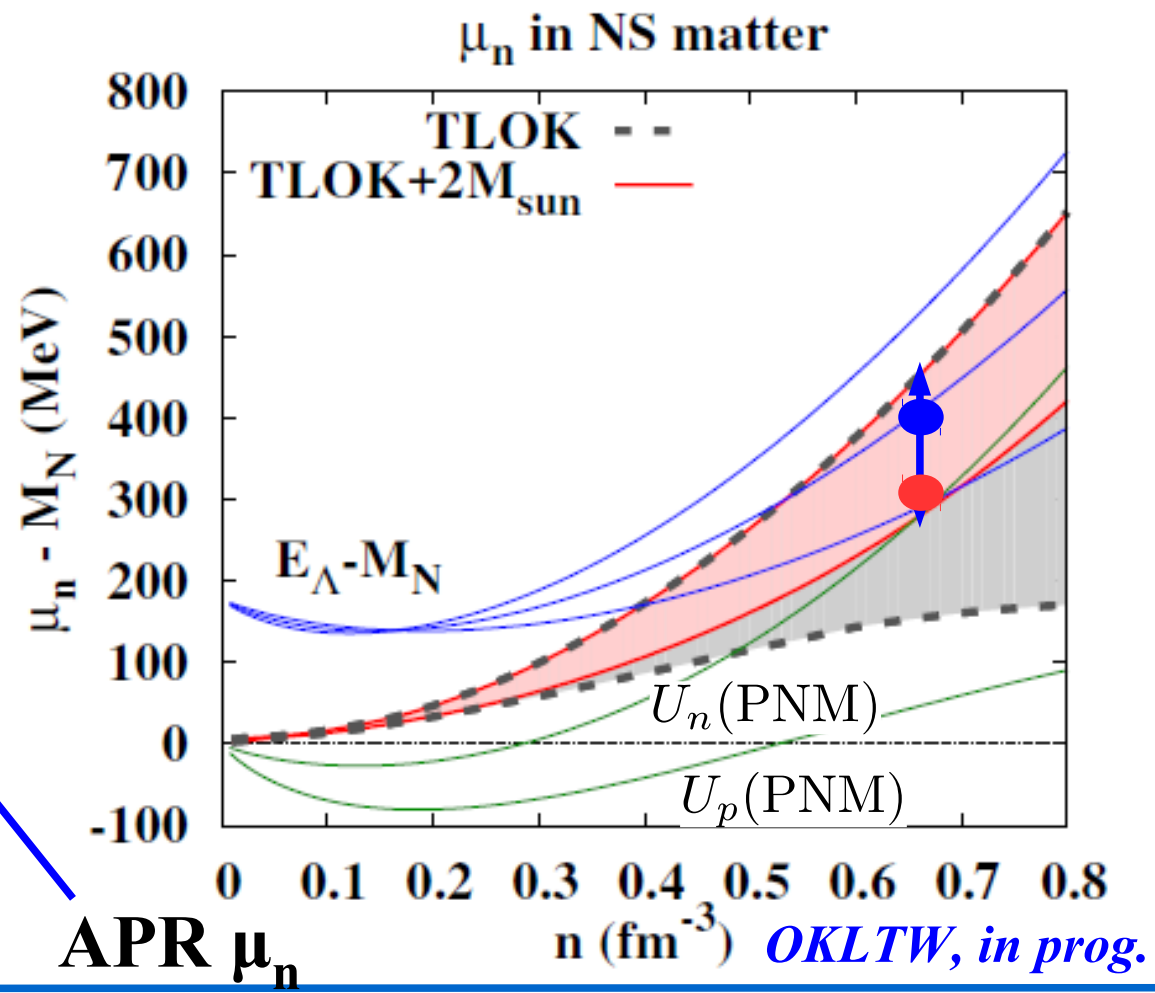
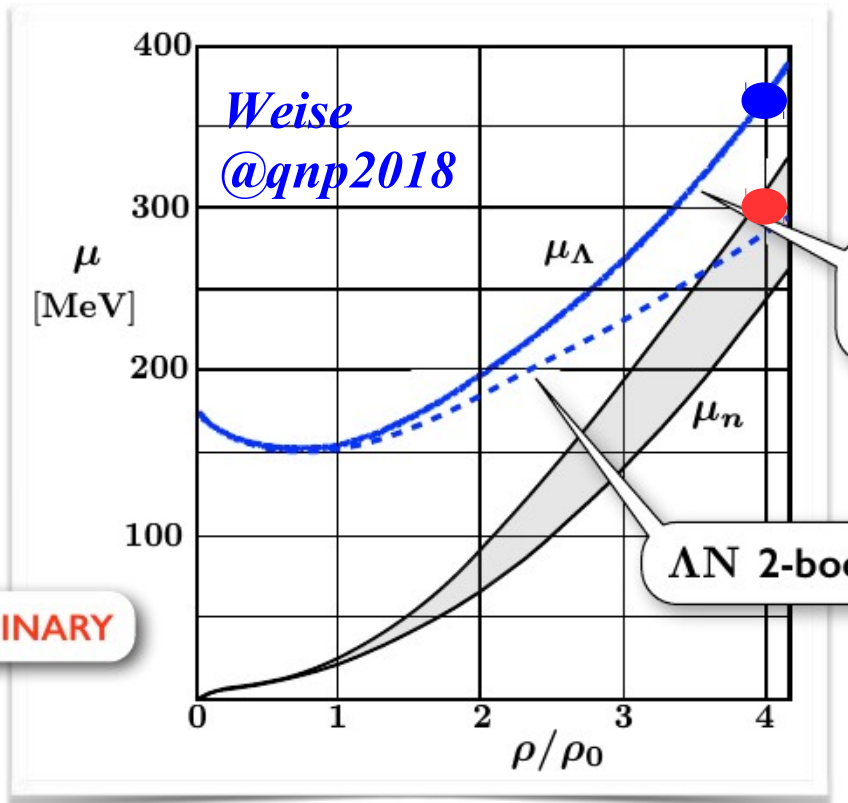
- At what density do hyperons appear ? $\rightarrow U_{\Lambda} = \mu_n$
- In STANDARD EOS with hyperons with $U_{\Lambda}(n_0) = -30$ MeV, Λ appears at $n = (2-3)n_0$
- Density dep. of U_{Λ} is essential.
- Neutron chemical potential strongly depends on sym. E.

■ QCD phase transition in cold dense matter

- Do we have the first order phase transition in cold dense matter ?
If yes, at which density ?
- Recent high-energy heavy-ion collision data suggest strong softening of EOS at $n = (5-10) n_0$.
- With hadronic matter EOS with $L = 50$ MeV and NJL model, mixed phase would appear at $n = (5-10) n_0$ in neutron stars.

Neutron Chemical Potential in NS

- Λ appears in neutron stars if $E_{\Lambda}(p=0) = M_{\Lambda} + U_{\Lambda} < \mu_n$
- U_{Λ} in χ EFT (2+3 body) is stiff.
- But μ_n is larger with TLOK+ $2M_{\odot}$ constraints



*W. Weise, NFQCD2018 (2018.06);
Gerstung, Kaiser, Weise, in prog.*

APR μ_n

OKLTW, in prog.



Neutron Chemical Potential in NS

Neutron Chemical Potential

$$\mu_n + M_N = \frac{\partial(nE)}{\partial n_n} = E + u \frac{\partial E}{\partial u} + 2\alpha(1 - \alpha)S(u)$$

Single particle potential

$$U_\Lambda(u) = \frac{\partial(nV)}{\partial n_\Lambda}$$

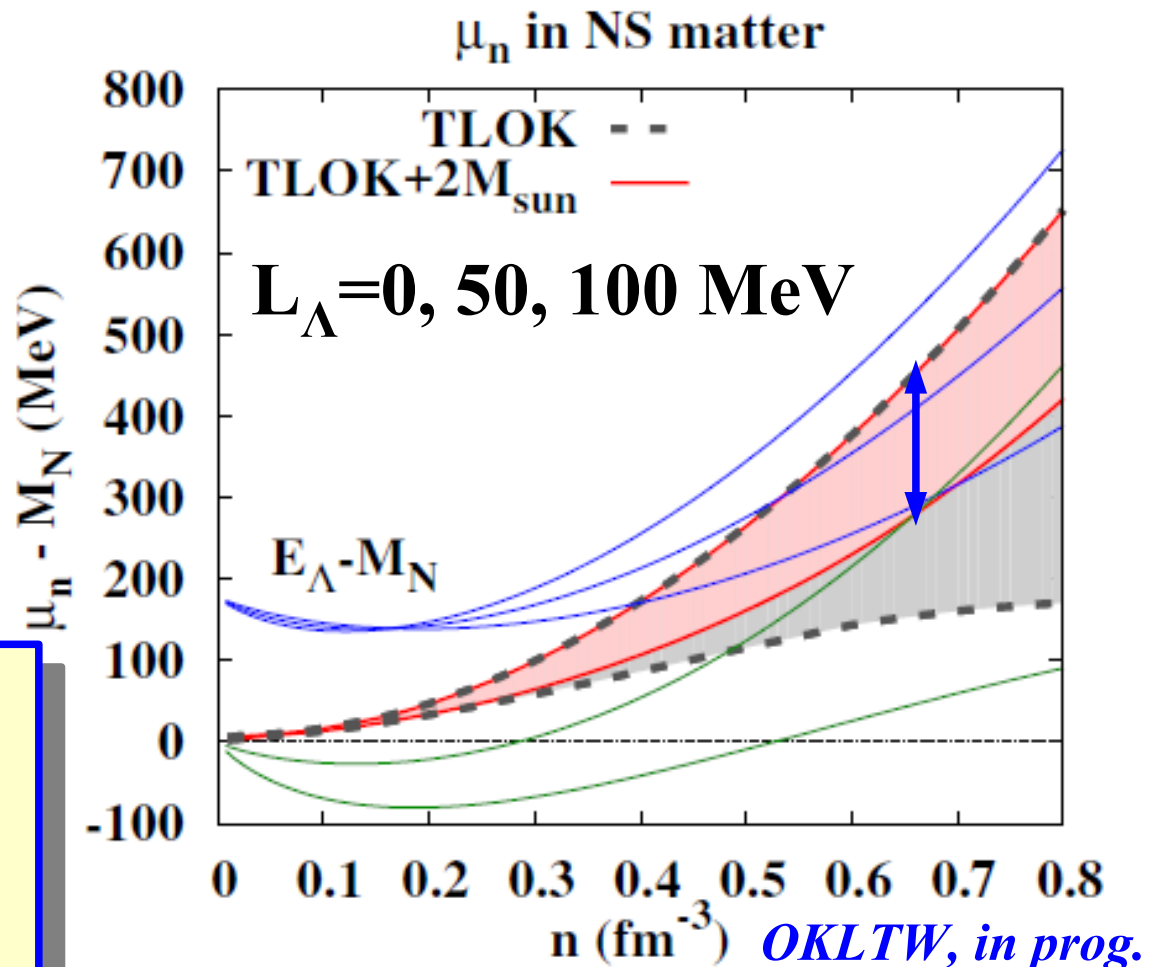
$$\simeq U_{0\Lambda} + \frac{L_\Lambda}{3}(u - 1)$$

$$U_{0\Lambda} \simeq -30 \text{ MeV}$$

$$L_\Lambda = ???$$

($L_\Lambda < 0$ in most of RMF before 2010)

Sym. E and L_Λ determine the onset density of Λ . (Already mentioned in Millener, Dover, Gal paper)

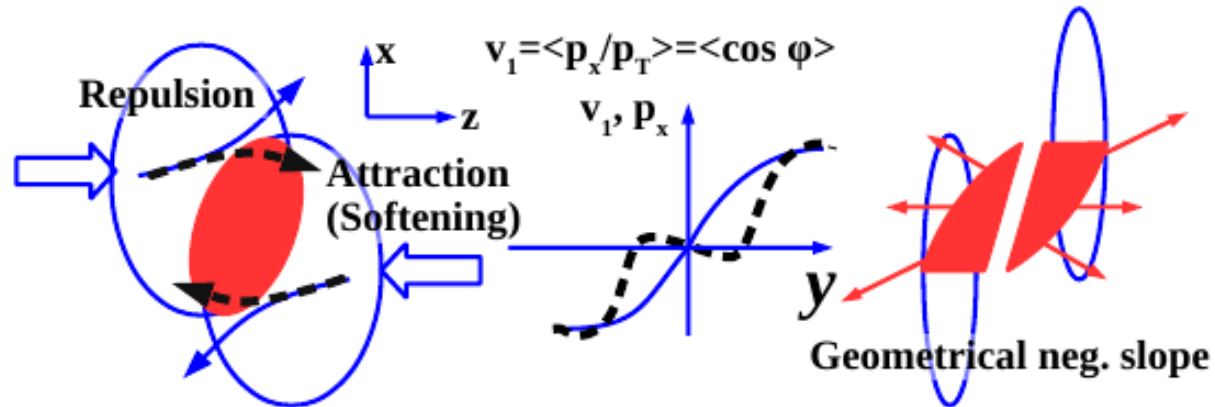


QCD phase transition in cold dense matter

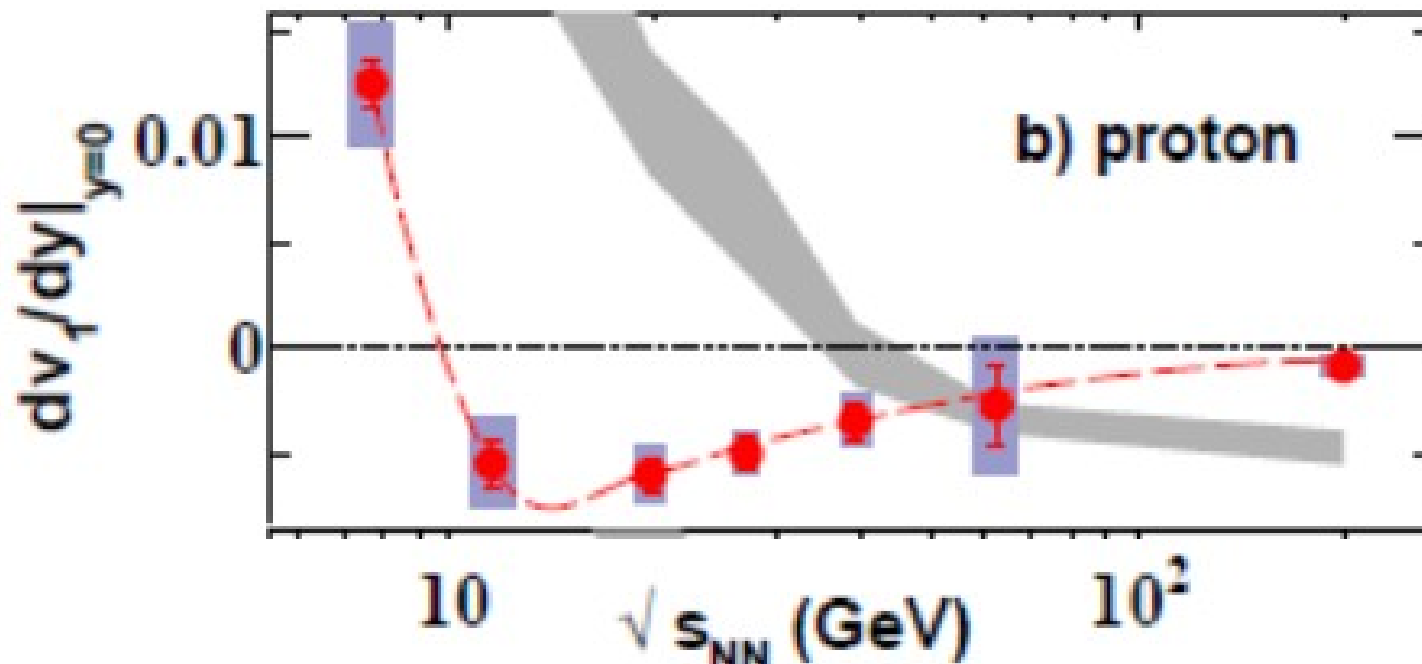
- **Transition to quark matter in cold-dense matter**
1st order or crossover ?
- **Crossover: Masuda, Hatsuda, Takatsuka, Kojo, Baym, ...**
- **1st order p.t.**
 - **Many effective models predict, e.g. Asakawa-Yazaki CP**
 - **Recent phenomenological support: Negative Directed Flow in HIC**
Y.Nara, H.Niemi, AO, H.Stoecker, PRC94('16)034906.
Y. Nara, H. Niemi, AO, J. Steinheimer, X.-F. Luo, H. Stoecker, EPJA 54 ('18)18
 - **The phase transition density may be above NS central density**
X.Wu, AO, H.Shen, PRC to appear (arXiv:1806.03760)

Negative Directed Flow

- Directed Flow $v_1 = \langle \cos \phi \rangle = \langle p_x / p_T \rangle$, Slope = dv_1 / dy



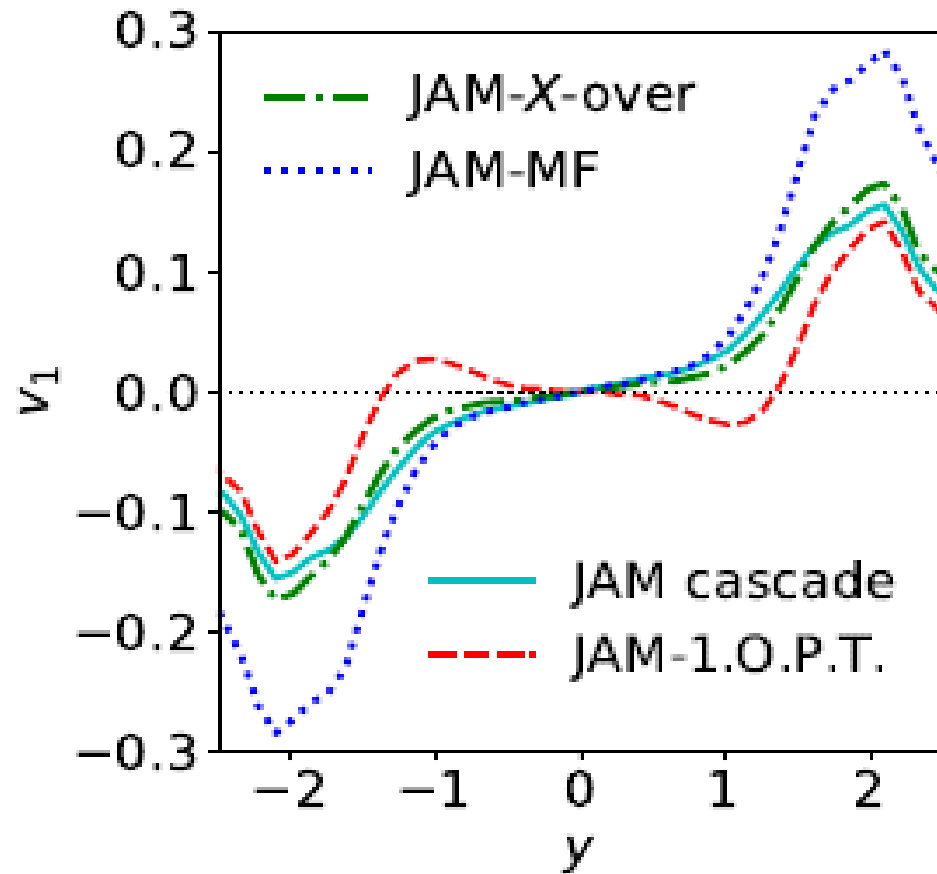
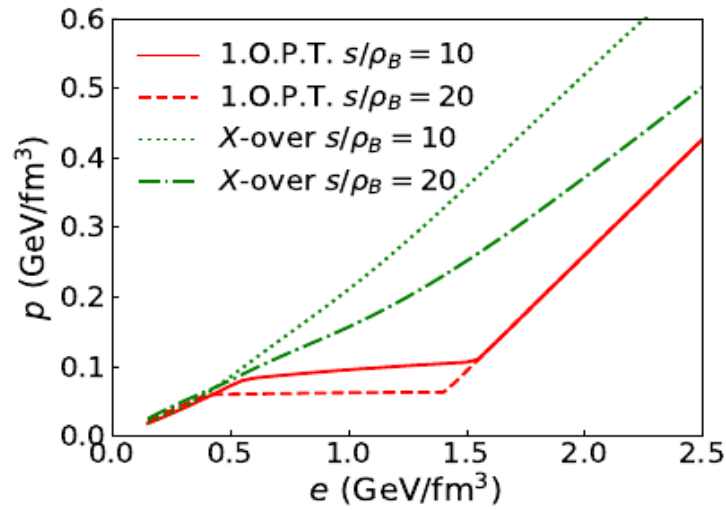
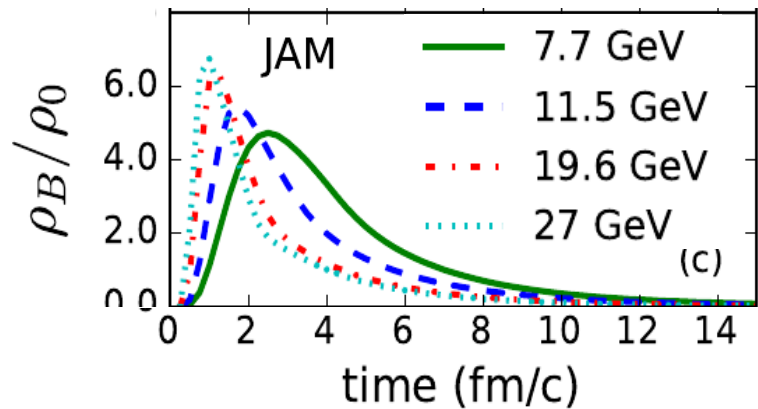
- Negative Flow in Heavy-Ion Collisions



STAR Collab. (L. Adamczyk et al.), *Phys.Rev.Lett.* 112 ('14), 162301

Negative Directed Flow

- Negative Directed Flow slope at $\sqrt{s_{NN}} = 11.5$ GeV (STAR ('14))
 → Strong softening of EOS is necessary at $n > (5-10) n_0$



*Y.Nara, H.Niemi, AO, H.Stoecker, PRC94('16)034906.
 Y. Nara, H. Niemi, AO, J. Steinheimer, X.-F. Luo,
 H. Stoecker, EPJA 54 ('18)18*

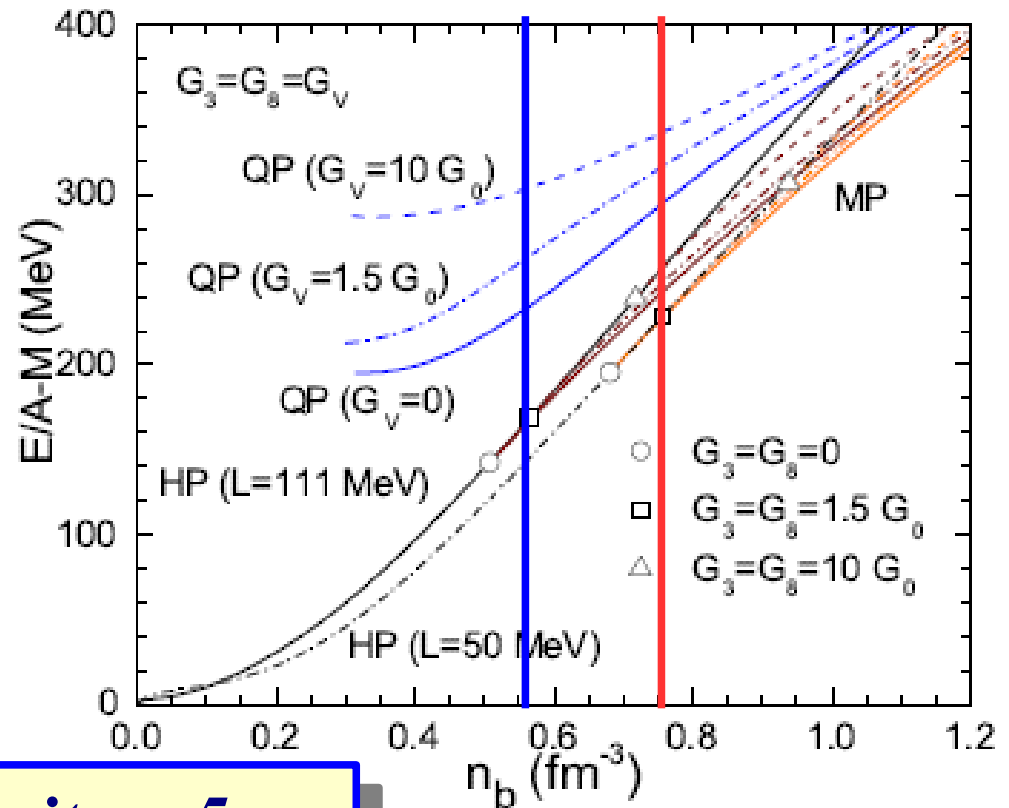
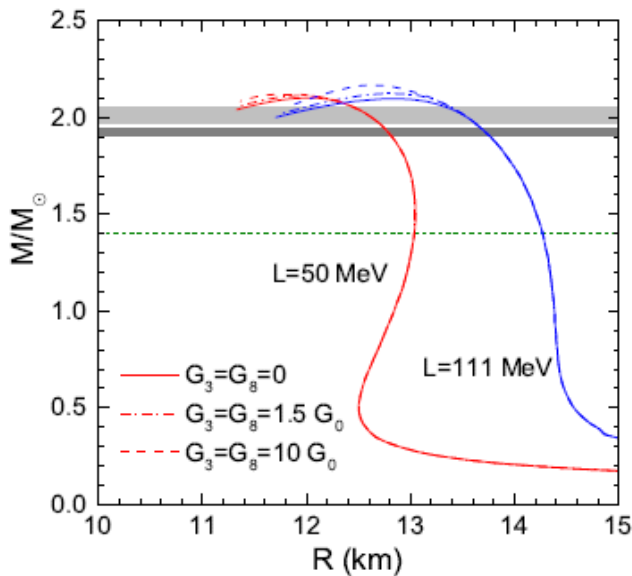
Isospin & Hypercharge Sym. E in quark matter

- Two types of vector int. in NJL \rightarrow Isospin & Hypercharge Sym. E

X.Wu, AO, H.Shen, PRC to appear (arXiv:1806.03760)

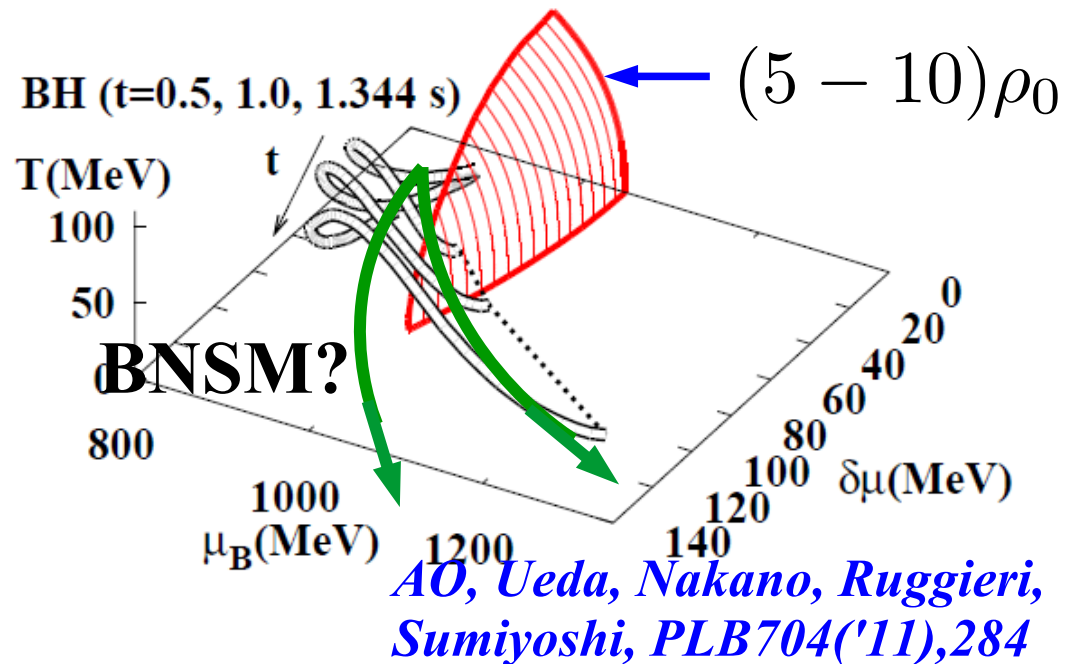
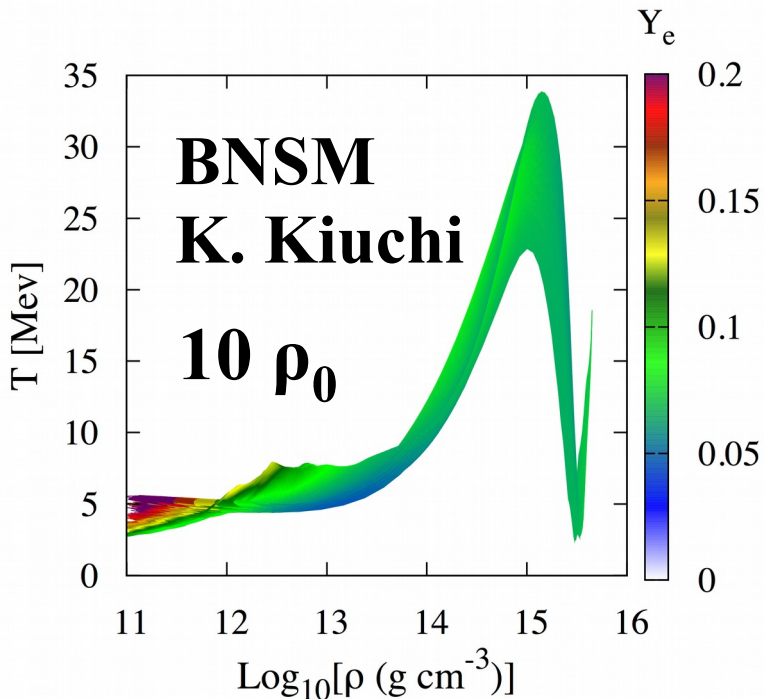
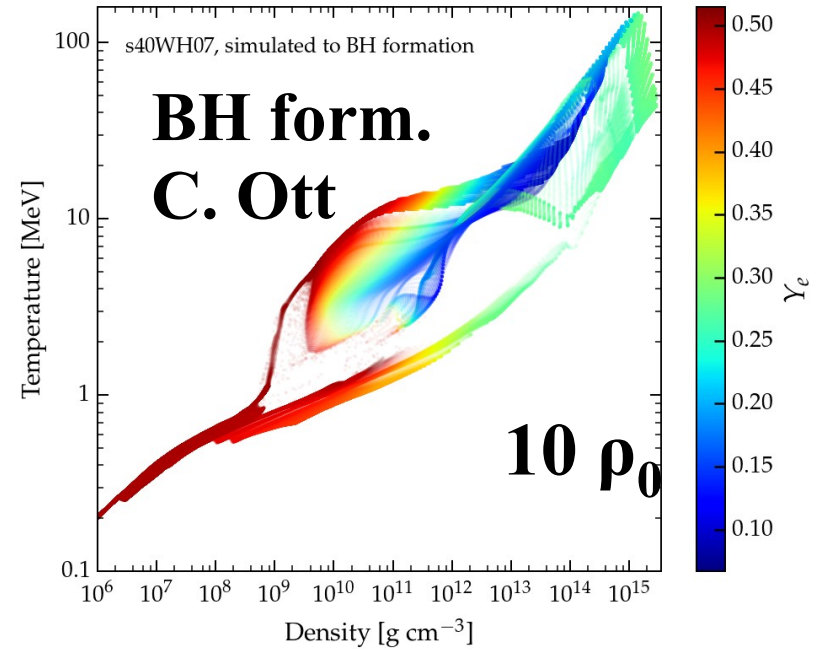
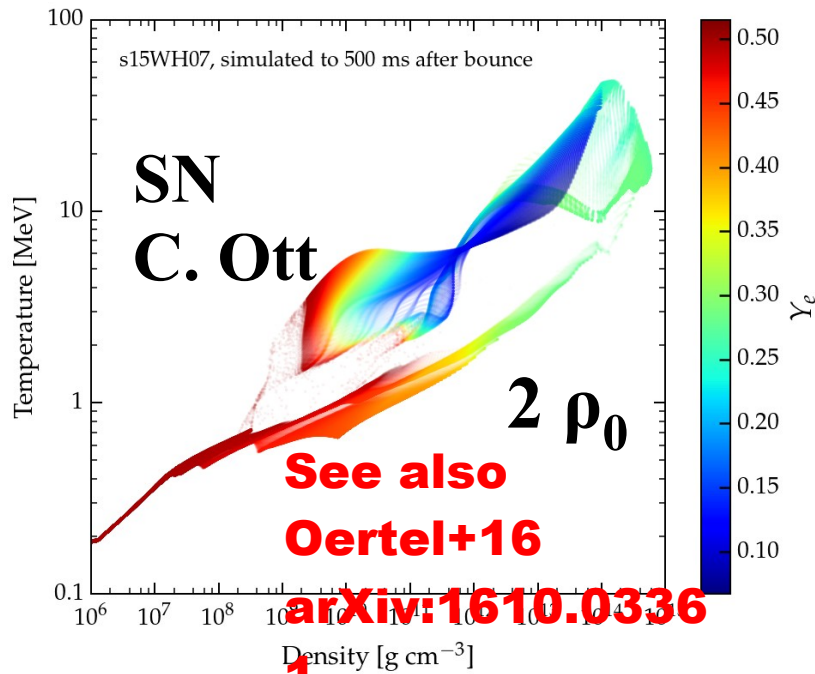
$$\mathcal{L}_v = -G_0(\bar{q}\gamma_\mu q)^2 - G_v \sum_i [(\bar{q}\gamma_\mu \lambda_i q)^2 + (\bar{q}i\gamma_5\gamma_\mu \lambda_i q)^2]$$

$$E = \alpha^2 S(n) + \alpha_Y^2 S_Y(n), \quad \alpha = -2\langle T_z \rangle / B, \quad \alpha_Y = \langle B + S \rangle / B$$



$L=50$ MeV \rightarrow transition density $\sim 5 n_0$

(ρ, T, Y_e) during SN, BH formation, BNSM



Summary

- **Constraint on symmetry energy parameters (S_0, L, K_n, Q_n) together with $2 M_\odot$ constraint gives the $1.4 M_\odot$ neutron star radius in the range of (10.6-12.2) km.**
 - Consistent with the constraint from GW.
 - Fermi momentum (k_F) expansion is invoked. Smooth extrapolation to $2 n_0$ seems to work.
 - Let's wait for the NICER data and next NS-NS merger event.
- **Onset density of hyperons may be sensitive to the symmetry energy in addition to potential parameters, (U_{0B}, L_B).**
 - We need to know the slope of potential in addition to the depth.
- **QCD phase transition with strong EOS softening is expected at $n=(5-10)n_0$ in almost sym. n.m. from heavy-ion data.**
 - GW data from HMNS would clarify 3D phase diagram structure.

Thank you for your attention !

Reservations and Prospects

Reservations

- Only massless electrons are considered and Crust EOS is ignored.
 - With μ , chemical potential may be reduced a little.
- Non-relativistic kinetic energy is used.
 - With rel. K.E., E per nucleon is modified by 0.03 MeV @ $10 n_0$ as long as Sat. and Sym. E parameters are fixed.
- Function form is limited to k_F expansion with $u^{k/3}$ (k=2-6).
 - $R_{1,4}$ range becomes narrower with k=2-5.
 - Density expansion gives EOSs very sensitive to parameters.
- Smooth $E(u)$ (= No phase transition) is assumed.
 - We expect QCD phase transition at (5-10) n_0 from recent BES data of directed flow *Nara, Niemi, AO, Stoecker ('16)*
 - Transition to quark matter may not soften EOS drastically.
- Causality is violated at high densities, $n > (4-6) n_0$.

To Do (or Prospect)

- Baryons other than nucleons Λ , Δ , Ξ , Σ , ...
 - Connecting to Hadron Resonance Gas (HRG) EOS
 - HRG EOS
- mass and kinetic E of hadrons with $M < 2$ GeV + simple potential E

$$\varepsilon_{\text{HRG}} = \mathcal{T} + cn^2$$

or Lattice EOS in HIC (No saturation, No constraint from NS).

- We need to guess the potential energy density more seriously for consistent understanding of HIC, Nuclear, and NS physics.

$$\varepsilon = \mathcal{T} + \mathcal{V} \leftarrow \text{Nuclear and NS physics}$$

- Connecting to Quark(-Gluon) matter EOS
 - Embed model-H singularities *E.g. Nonaka, Asakawa ('04)*
 - “Interpolation” of nuclear and quark matter EOS

Further Constraint on Q_n

- $2 M_{\odot}$ requirement constrains Q_n further.

$$Q_n > -9.3L + 480 \text{ MeV}$$

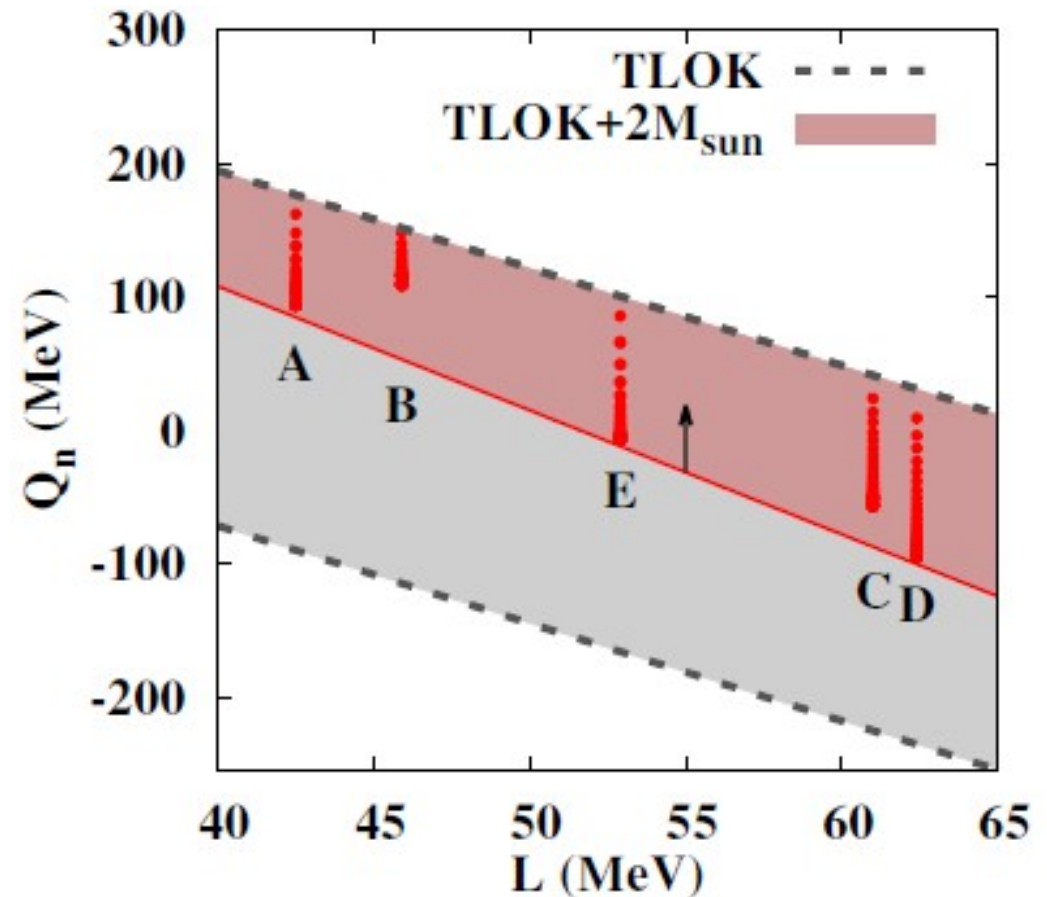


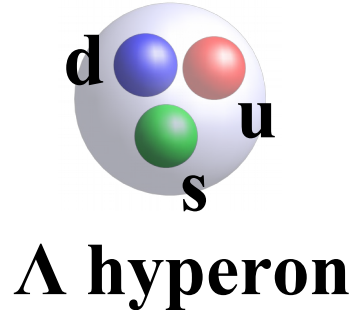
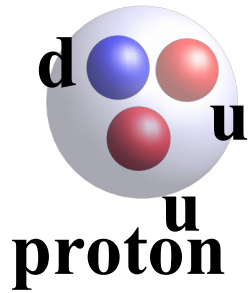
FIG. 4. Constraint on Q_n

AO, Kolomeitsev, Lattimer, Tews, Wu (OKLTW), in prog.

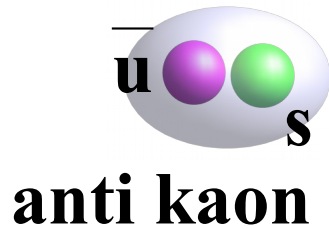
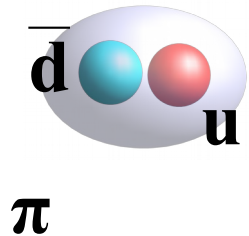
Neutron star – Is it made of neutrons ?

- Possibilities of various constituents in neutron star core

- Strange Hadrons

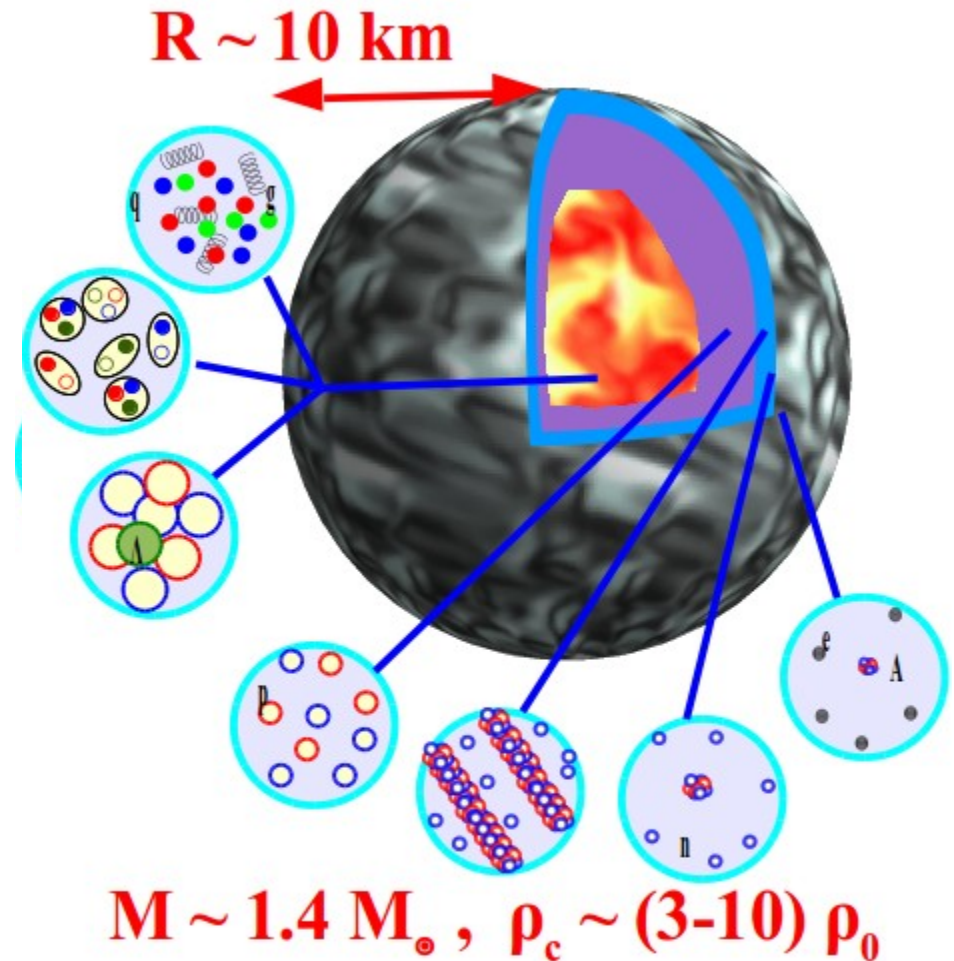


- Meson condensate (K , π)



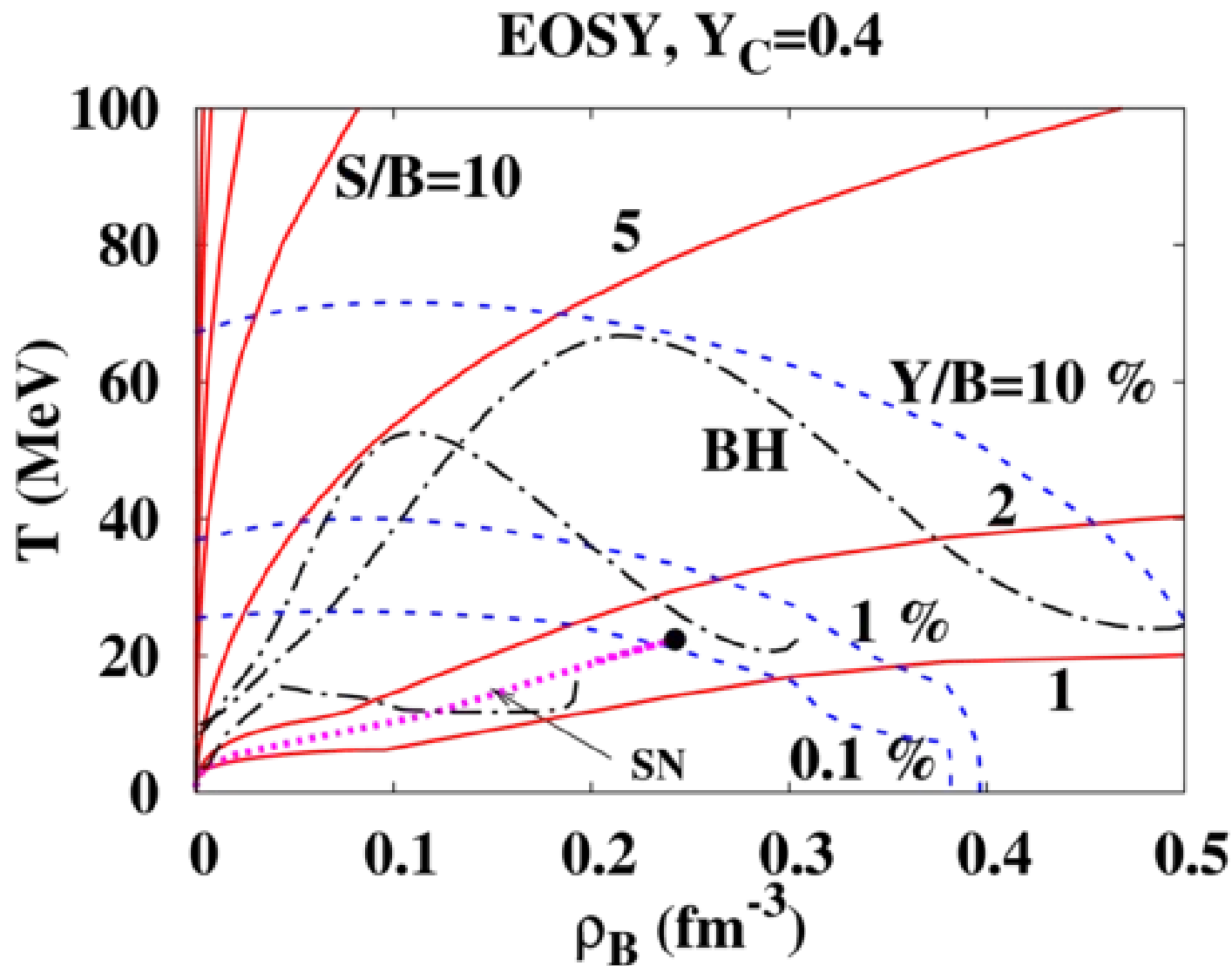
- Quark matter

- Quark pair condensate (Color superconductor)



NS core = Densest stable matter existing in our universe.

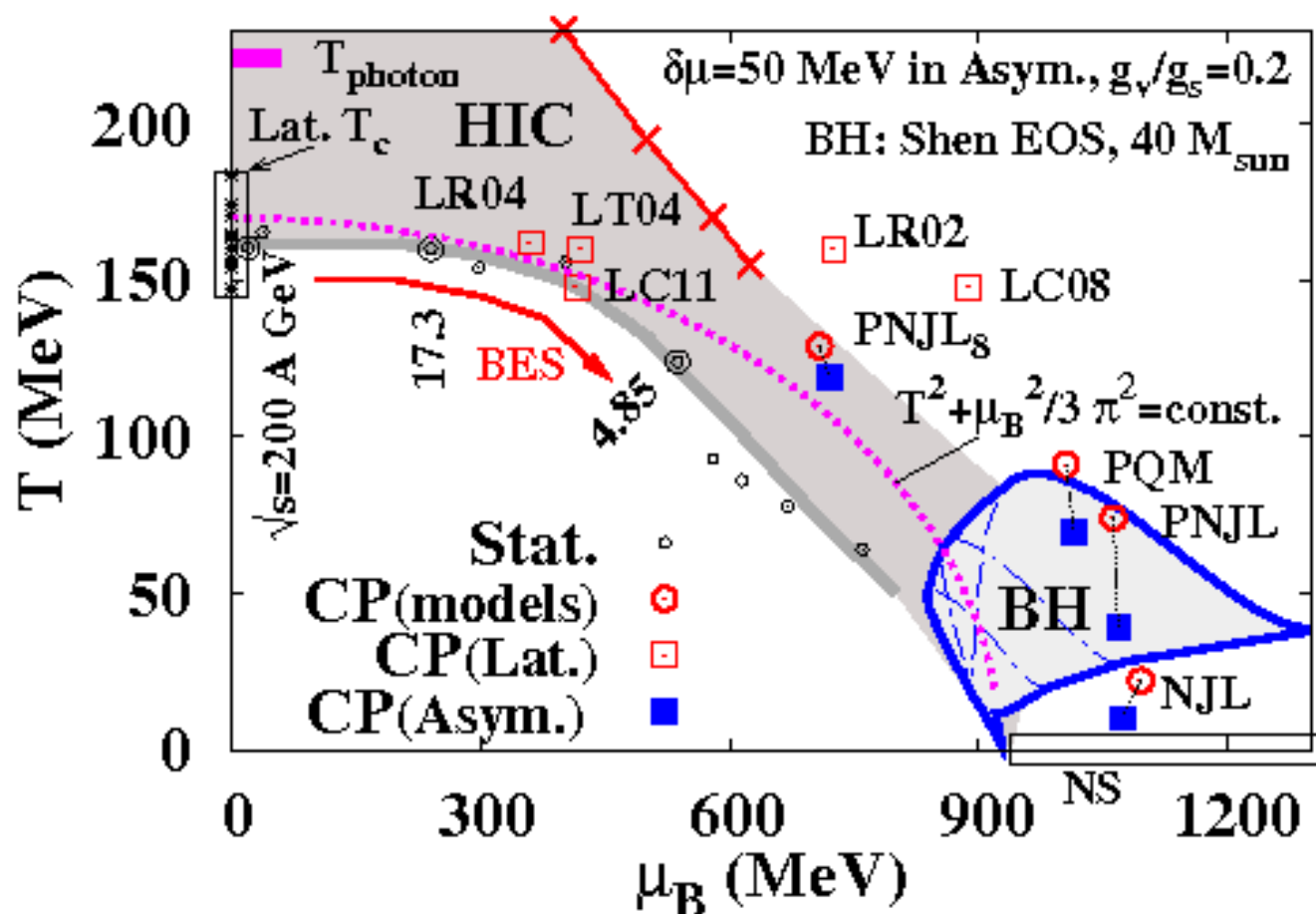
(ρ, T) during SN & BH formation



Shen EOS
+ hyperons

*Ishizuka, AO, Tsubakihara, Sumiyoshi, Yamada, JPG 35('08) 085201;
AO et al., NPA 835('10) 374.*

QCD phase diagram (Exp. & Theor. Studies)



QCD phase transition is not only an academic problem, but also a subject which would be measured in HIC or Compact Stars

Unitary Gas Constraint

Tews, Lattimer, AO, Kolomeitsev (TLOK), ApJ ('17)

■ Conjecture:

Unitary gas gives the lower bound of neutron matter energy.

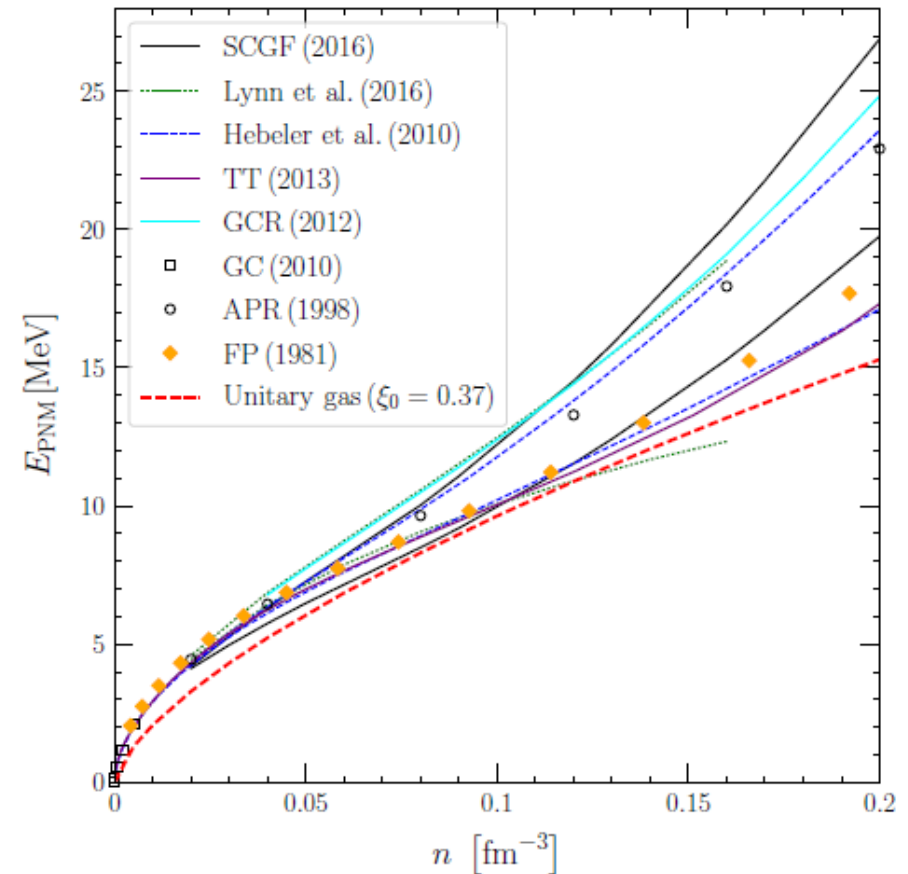
$$S(n) = E_{\text{PNM}} - E_{\text{SNM}} \geq E_{\text{UG}} - E_{\text{SNM}} \leftarrow \text{Sym. Nucl. Matter EOS is relatively well known.}$$

$$E_{\text{UG}} = \xi E_{\text{FG}} \quad (\xi \simeq 0.38)$$

■ $a_0 = \infty$ in unitary gas

→ lower bound energy
of $a_0 < 0$ systems
(w/o two-body b.s.) ?

■ Supported by (most of) ab initio calc.



Potential Energy Density

- Potential Energy Density in the Fermi momentum expansion

$$\mathcal{V} = nV = \sum_{i,j \in B} n_i n_j v_{ij}(n)$$

Density-dependent NN interactions v_{ij} ($i, j = p$ or n) are known.

- Single particle potential

$$U_i = \frac{\partial \mathcal{V}}{\partial n_i} = \sum_j n_j v_{ij}(n) + \sum_{jk} n_j n_k \frac{\partial v_{jk}(n)}{\partial n_i}$$

$$= U_{0i} + \frac{L_i}{3} (u - 1) + \mathcal{O}((u - 1)^2)$$

$$\simeq au + bu^{4/3}$$

rearrangement
term

Again, a and b are given as a linear function of U_{0i} and L_i .