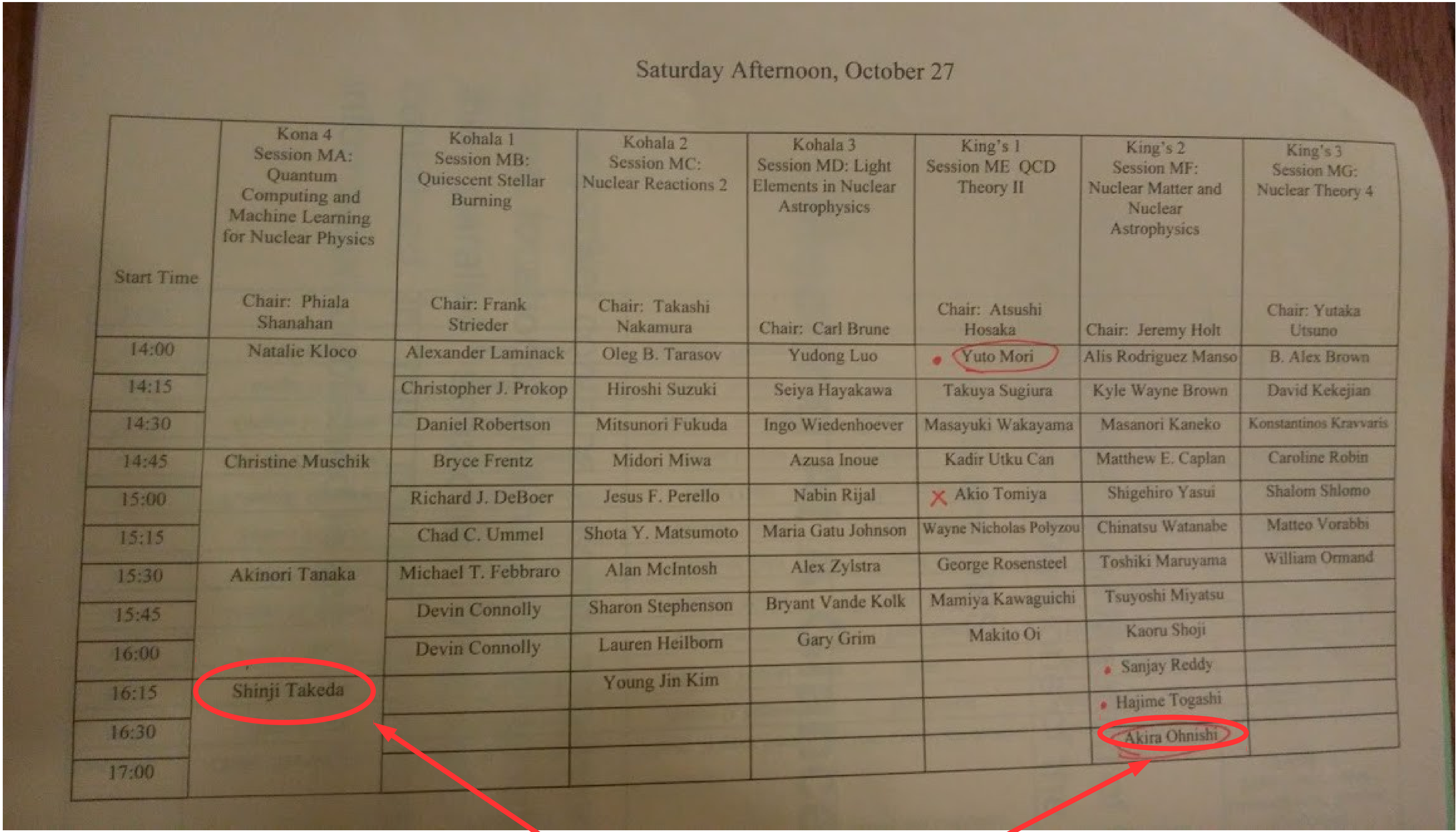
Higher-order symmetry energy parameters and neutron star properties

Akira Ohnishi
(Yukawa Inst. for Theor. Phys., Kyoto U.)
in collaboraton with
E. E. Kolomeitsev (Matej Bel U.), James M. Lattimer (Stony Brook), Ingo Tews (LANL), Xuhao Wu (Nankai U./YITP)

5th Joint Meeting of the NP Divisions
of APS and JPS
Oct. 23-27, 2018, Waikoloa, Hawaii, USA

- *I. Tews, J. M. Lattimer, AO, E.E.Kolomeitsev, ApJ 848('17) 105 [arXiv:1611.07133]*
- *AO, Kolomeitsev, Lattimer, Tews, X.Wu, in prog.*

First, I would like to thank the audience …



Saturday Afternoon, October 27

Start Time	Kona 4 Session MA: Quantum Computing and Machine Learning for Nuclear Physics Chair: Phiala Shanahan	Kohala 1 Session MB: Quiescent Stellar Burning Chair: Frank Strieder	Kohala 2 Session MC: Nuclear Reactions 2 Chair: Takashi Nakamura	Kohala 3 Session MD: Light Elements in Nuclear Astrophysics Chair: Carl Brune	King's 1 Session ME QCD Theory II Chair: Atsushi Hosaka	King's 2 Session MF: Nuclear Matter and Nuclear Astrophysics Chair: Jeremy Holt	King's 3 Session MG: Nuclear Theory 4 Chair: Yutaka Utsuno
14:00	Natalie Kloco	Alexander Laminack	Oleg B. Tarasov	Yudong Luo	• Yuto Mori	Alis Rodriguez Manso	B. Alex Brown
14:15		Christopher J. Prokop	Hiroshi Suzuki	Seiya Hayakawa	Takuya Sugiura	Kyle Wayne Brown	David Kekejian
14:30		Daniel Robertson	Mitsunori Fukuda	Ingo Wiedenhoever	Masayuki Wakayama	Masanori Kaneko	Konstantinos Kravvaris
14:45	Christine Muschik	Bryce Frentz	Midori Miwa	Azusa Inoue	Kadir Utku Can	Matthew E. Caplan	Caroline Robin
15:00		Richard J. DeBoer	Jesus F. Perello	Nabin Rijal	✗ Akio Tomiya	Shigehiro Yasui	Shalom Shlomo
15:15		Chad C. Ummel	Shota Y. Matsumoto	Maria Gatu Johnson	Wayne Nicholas Polyzou	Chinatsu Watanabe	Matteo Vorabbi
15:30	Akinori Tanaka	Michael T. Febbraro	Alan McIntosh	Alex Zylstra	George Rosensteel	Toshiki Maruyama	William Ormand
15:45		Devin Connolly	Sharon Stephenson	Bryant Vande Kolk	Mamiya Kawaguichi	Tsuyoshi Miyatsu	
16:00		Devin Connolly	Lauren Heilborn	Gary Grim	Makito Oi	Kaoru Shoji	
16:15	Shinji Takeda		Young Jin Kim			• Sanjay Reddy	
16:30						• Hajime Togashi	
17:00						Akira Ohnishi	

Last talks

Symmetry Energy Parameters & Neutron Star Radius

- **Nuclear Matter Symmetry Energy parameters (S_0, L) are closely related to Neutron Star Properties,**
  e.g. $R_{1.4} = R_{NS}(M = 1.4 M_\odot)$

  c.f. Workshop (Tue, 2WAA & 2WAB), this session, …

- **How can we constrain (S_0, L) ?**
  → Nuclear Exp't. & Theory, Astro. Obs., Unitary gas

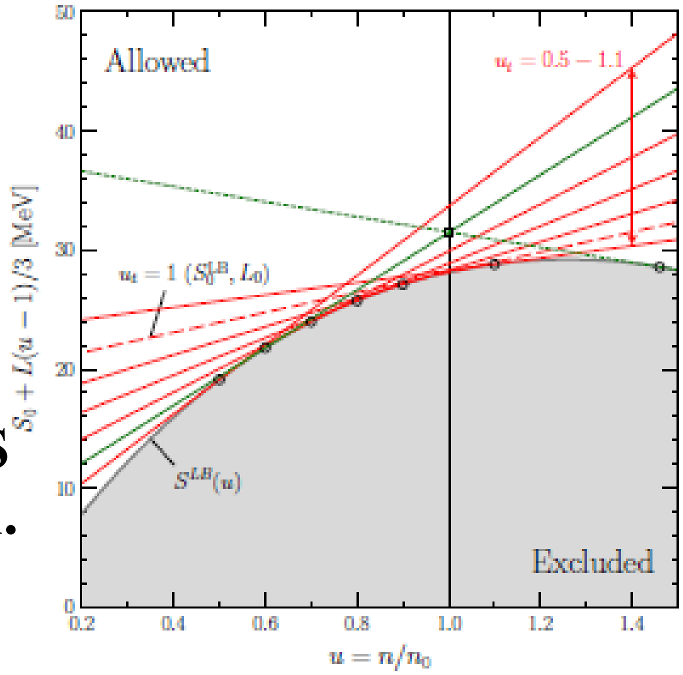
- **Conjecture: UG gives the lower bound of neutron matter energy.**
  Tews, Lattimer, AO, Kolomeitsev (TLOK), ApJ ('17)

$$S(n) = E_\mathrm{PNM} - E_\mathrm{SNM} \geq E_\mathrm{UG} - E_\mathrm{SNM}$$

$$E_\mathrm{UG} = \xi E_\mathrm{FG} \ (\xi \simeq 0.38)$$

↑

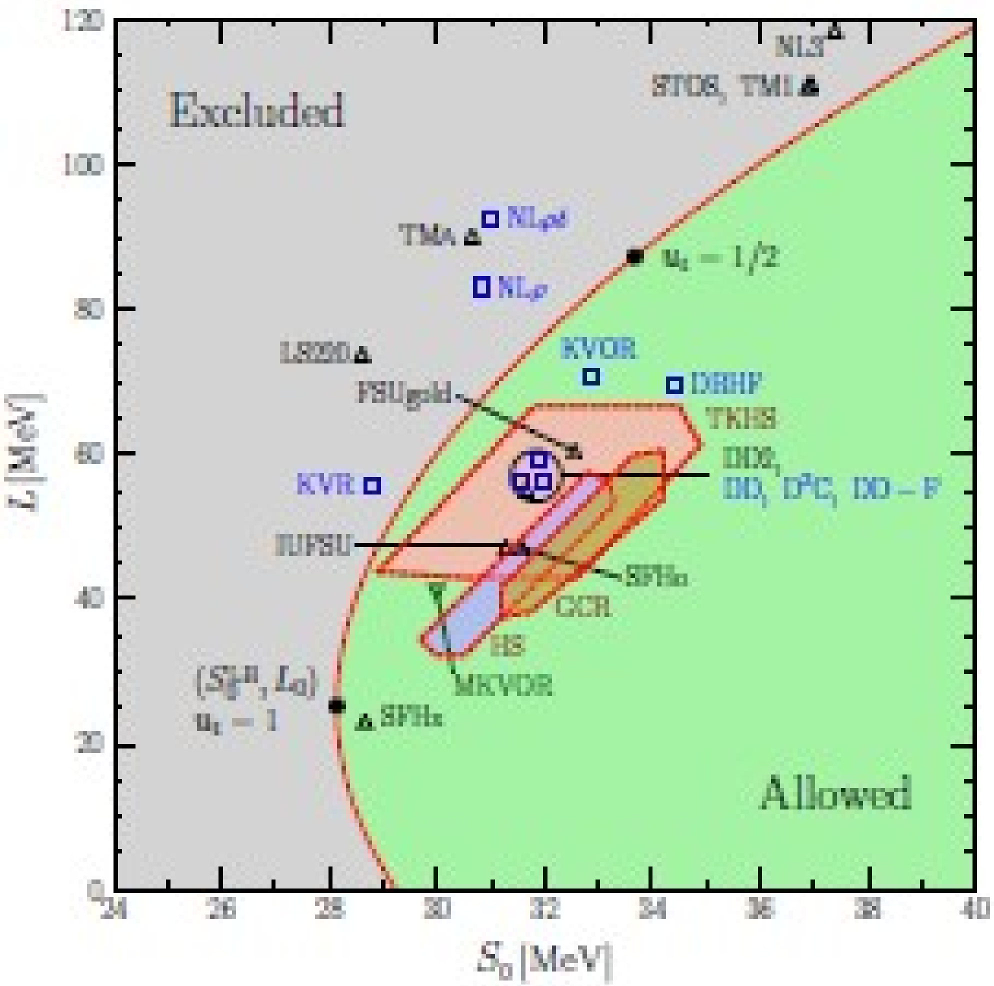
Sym. Nucl. Matter EOS is relatively well known.

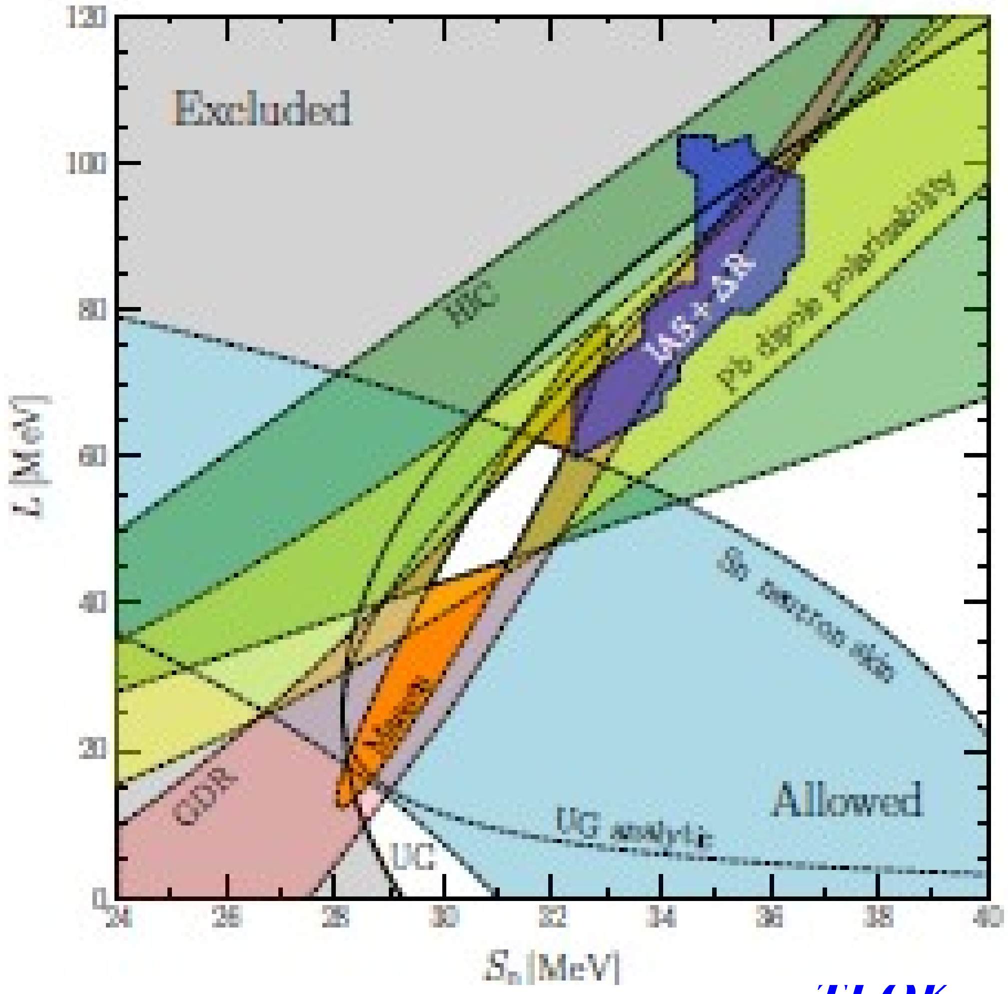


→ For a given L, lower bound of S_0 exists

Constraint on (S_0, L) from Lower Bound of PNM Energy

- **Unitary gas + 2 $M_\odot$ constraints rule out 5 EOSs out of 10 numerically tabulated and frequently used in astrophys. calc.**





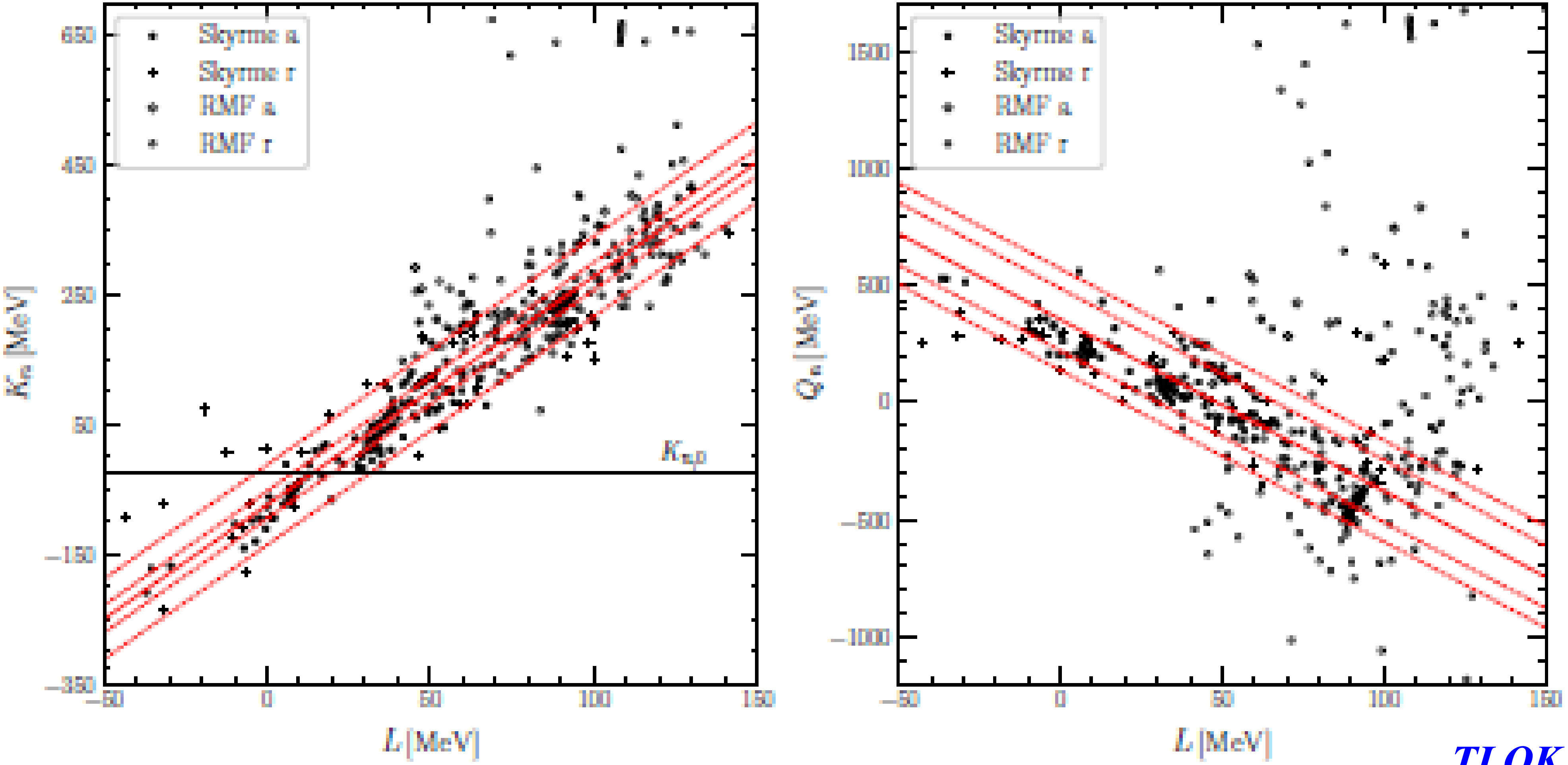
TLOK

Further Constraints on Higher-Order Sym. E. parameters

- **K_n and Q_n are correlated with L in "Good" theoretical models.**

$$K_n = 3.534L - (74.02 \pm 21.17)\mathrm{MeV}$$

$$Q_n = -7.313L + (354.03 \pm 133.16)\mathrm{MeV}$$



TLOK

Purpose & Contents

- **Quesion:**
  What are the effects of these higher-order sym. E. parameters on MR curve of NS ?
- **This work:**
  TLOK + 2 $M_\odot$ constraints + k_F expansion → $R_{1.4}$
- **Contents**
  - **Introduction**
  - **Fermi momentum (k_F) expansion of EOS**
  - **Neutron Star MR curve**
  - **(Limitations & Prospect)**
  - **Summary**

Fermi momentum (k_F) expansion

- **Saturation & Symmetry Energy Parameters**

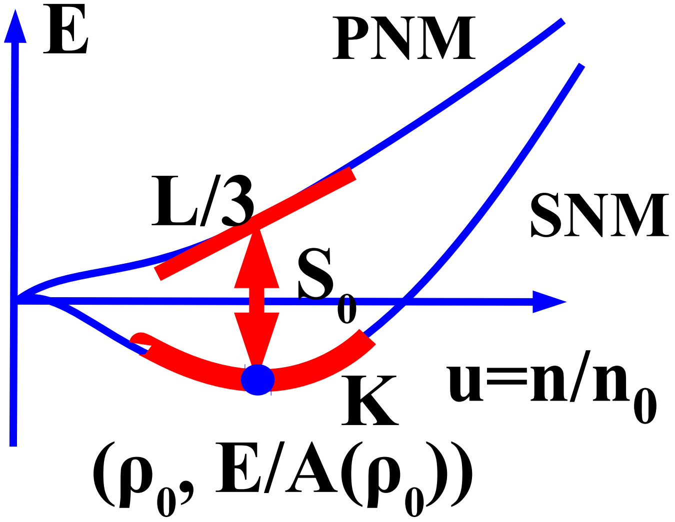
TLOK

$$E_{\rm NM}(u,\alpha) = E_{\rm SNM}(u) + \alpha^2\, S(u)$$

$$E_{\rm SNM}(u) \simeq E_0 + \frac{K_0}{18}(u-1)^2 + \frac{Q_0}{162}(u-1)^3$$

$$S(u) \simeq S_0 + \frac{L}{3}(u-1) + \frac{K_s}{18}(u-1)^2 + \frac{Q_s}{162}(u-1)^3$$

$$(u = n/n_0, \alpha = (n_n - n_p)/n)$$



Energy does not approach zero at n → 0.

- **Fermi momentum expansion (~ Skyrme type EDF)**
  - **Generated many-body force is given by** $k_F \propto u^{1/3}$

m*

$$\begin{aligned} E_{\rm SNM}(u) &\simeq \boxed{T_0\, u^{2/3}} + \underline{a_0 u} + \underline{b_0 u^{4/3}} + \underline{c_0 u^{5/3}} + \underline{d_0 u^2} \\ S(u) &\simeq \boxed{T_s u^{2/3}} + \underline{a_s u} + \underline{b_s u^{4/3}} + \underline{c_s u^{5/3}} + \underline{d_s u^2} \end{aligned}$$

Kin. E. **Two-body** **Density-dep. pot.**

Expansion Coefficients

- **Coefficients (a,b,c,d) are represented by Saturation and Symmetry Energy Parameters**

TLOK

$$
\begin{aligned}
a_0 &= -\,4T_0 & &+20E_0 & &+\,K_0 & &-Q_0/6\\
b_0 &= \quad 6T_0 & &-45E_0 & &-5K_0/2 & &+Q_0/2\\
c_0 &= -\,4T_0 & &+36E_0 & &+2K_0 & &-Q_0/2\\
d_0 &= \qquad T_0 & &-10E_0 & &-\,K_0/2 & &+Q_0/6
\end{aligned}
$$

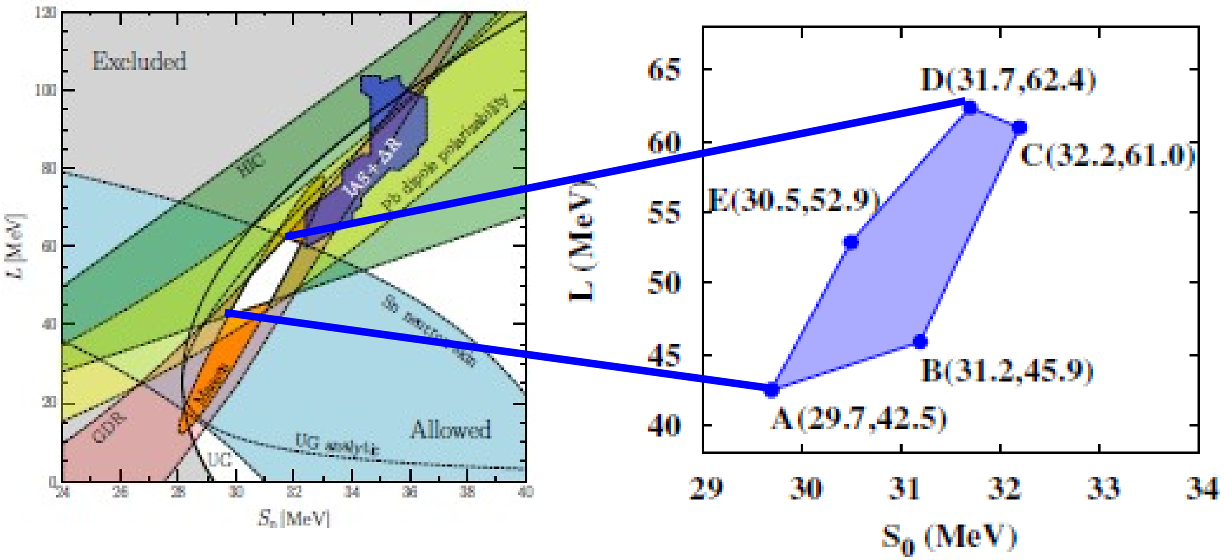
$$
\begin{aligned}
a_s &= -\,4T_s & &+20S_0-19L/3 & &+\,K_s & &-Q_s/6\\
b_s &= \quad 6T_s & &-45S_0+15L & &-5K_s/2 & &+Q_s/2\\
c_s &= -\,4T_s & &+36S_0-12L & &+2K_s & &-Q_s/2\\
d_s &= \qquad T_s & &-10S_0+10L/3 & &-\,K_s/2 & &+Q_s/6
\end{aligned}
$$

$$\left(T_0=\frac{3}{5}\frac{\hbar^2 k_F(n_0)^2}{2m},\quad T_s=T_0(2^{1/3}-1)\right)$$

Tedious but straightforward calc.

TLOK+2$M_\odot$ constraints

- **TLOK constraints**
  - **(S_0, L) is in Pentagon.**
  - **(K_n, Q_n) are from TLOK constraint.**
  - **K_0=(190-270) MeV**
  - **(n_0,E_0) is fixed**
    n_0=0.164 fm-3, E_0=-15.9 MeV (small uncertainties)
  - **Q_0 is taken to kill d_0 parameter**
    (Coef. of u^2. Sym. N. M. is not very stiff at high-density)

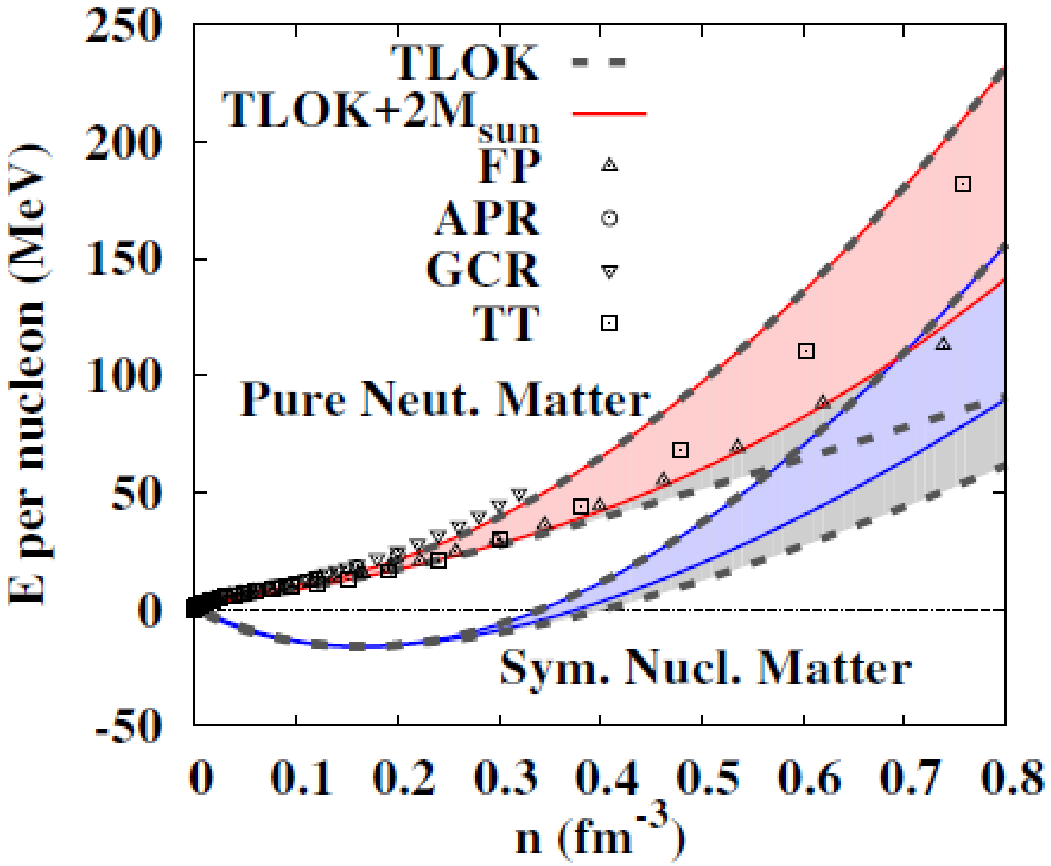


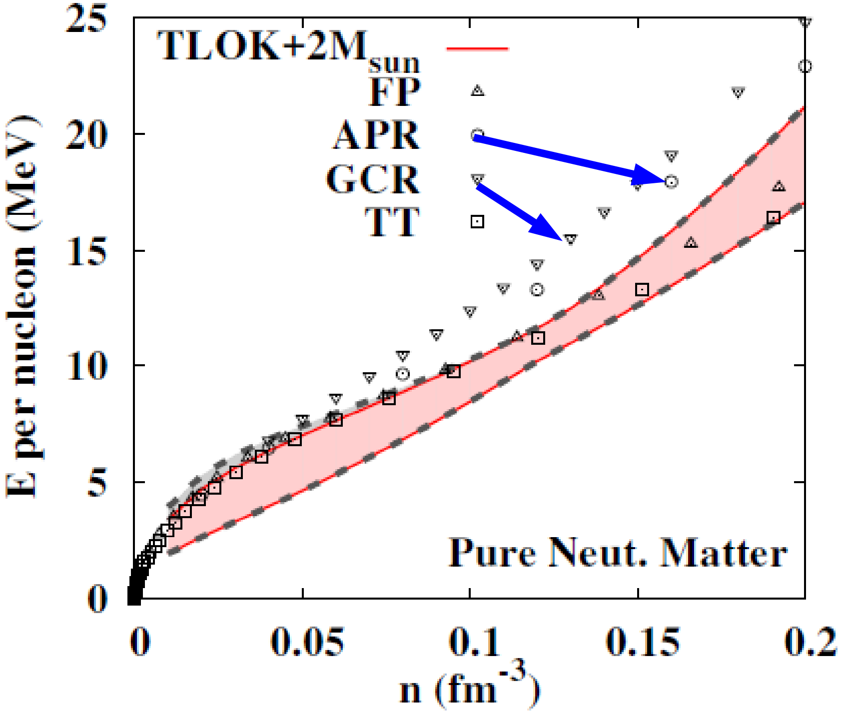
- **2 $M_\odot$ constraint**
  - **EOS should support 2 $M_\odot$ neutron stars.**

AO, Kolomeitsev, Lattimer, Tews, Wu (OKLTW), in prog.

TLOK+2M$_\odot$ constraints on EOS

- **2M$_\odot$ constraint narrows the range of EOS.**
- **Consistent with FP and TT(Togashi-Takano) EOSs.**
- **APR and GCR(Gandolfi-Carlson-Reddy) EOSs seems to have larger S_0 values.**





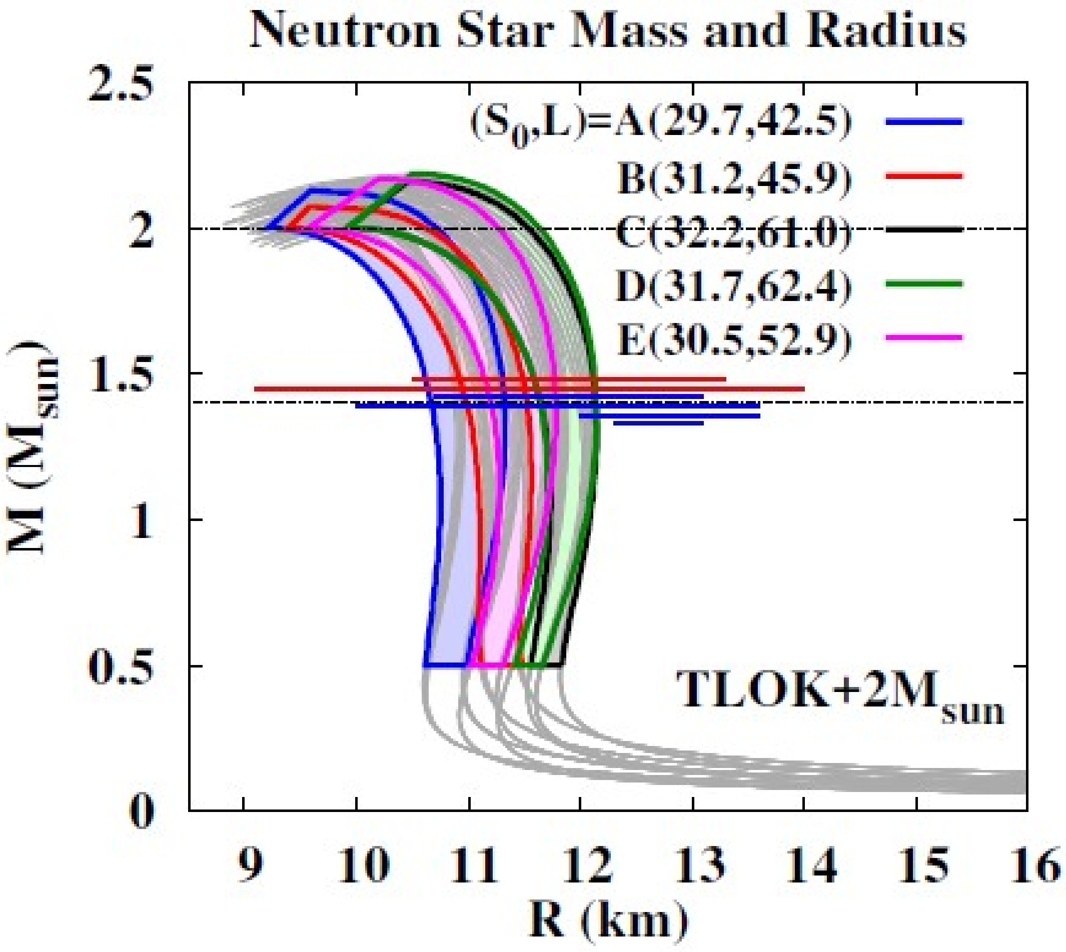
OKLTW, in prog.

Neutron Star MR curve

- TLOK + 2 $M_\odot$ constraints → $R_{1.4}$=(10.6-12.2) km

OKLTW, in prog.

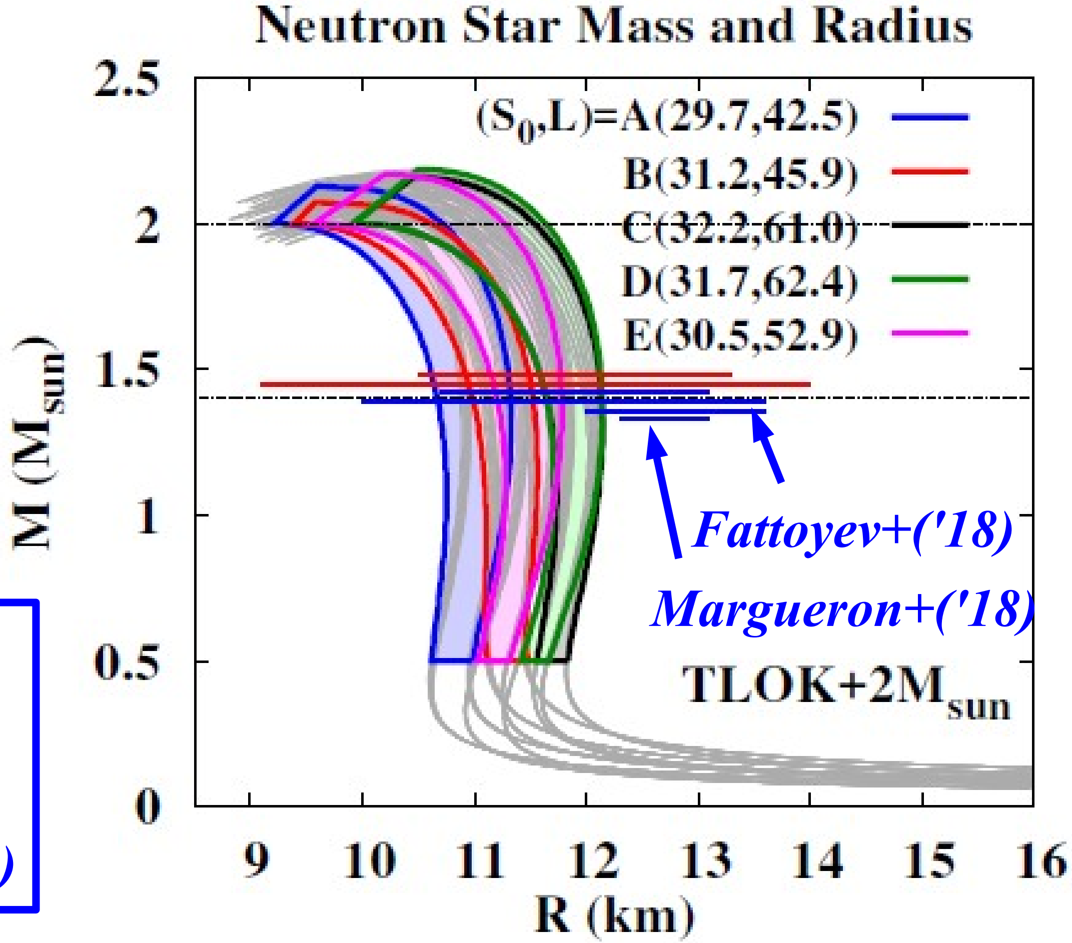
  - E and P are linear fn. of Sat. & Sym. E. parameters
→ Min./Max. appears at the corners of pentagon (ABCDE).
  - For a given (S_0, L),
unc. of $R_{1.4}$ ~ 0.5 km
= unc. from higher-order parameters
  - Unc. from (S_0, L) ~ 1.1 km
→ We still need to fix (S_0, L) more precisely.



Neutron Star MR curve

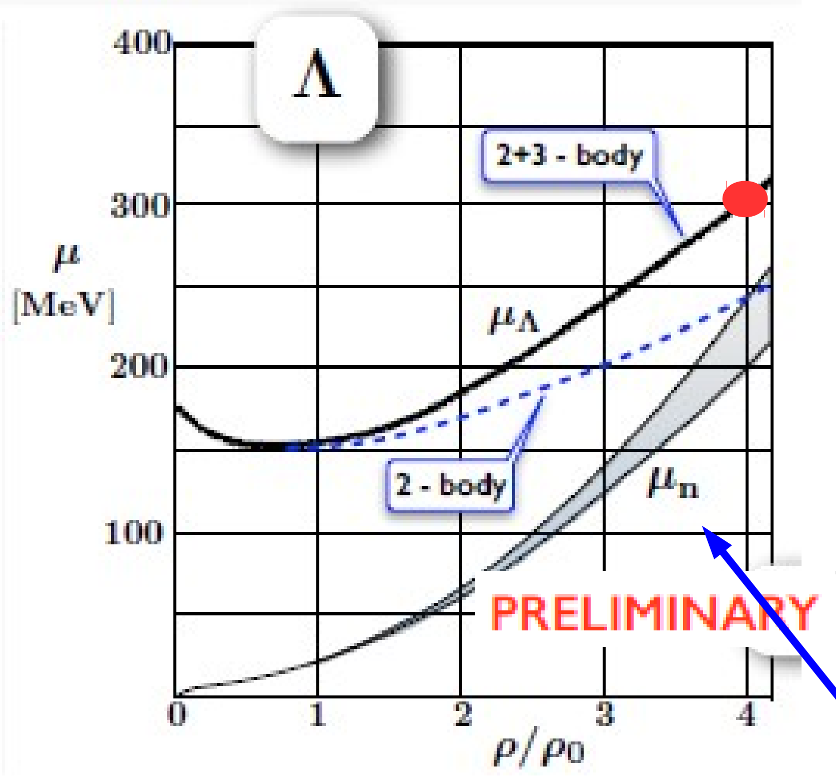
- **Our constraint is consistent with many of previous ones.**
  - $R_{1.4}$=(10.6-12.2) km *Present work (TLOK + 2 $M_{\odot}$)* *OKLTW, in prog.*
  - LIGO-Virgo (Tidal deformability Λ from BNSM)
    (10.5-13.3) km *Abbott+('18b)*
    (9.1-14.0) km *De+('18) (Λ)*
  - Theoretical Estimates
    (10.7-13.1) km
    Lattimer, Prakash('16)
    (10.0-13.6) km
    Annala+('18) (χEFT+pQCD)
    (10-13.6) km
    Tews+('18)(χEFT+ c_s)

    (12.0-13.6) km
    Fattoyev+('18) (PREX)
    12.7 ± 0.4 km
    Margueron+('18) (n expansion)



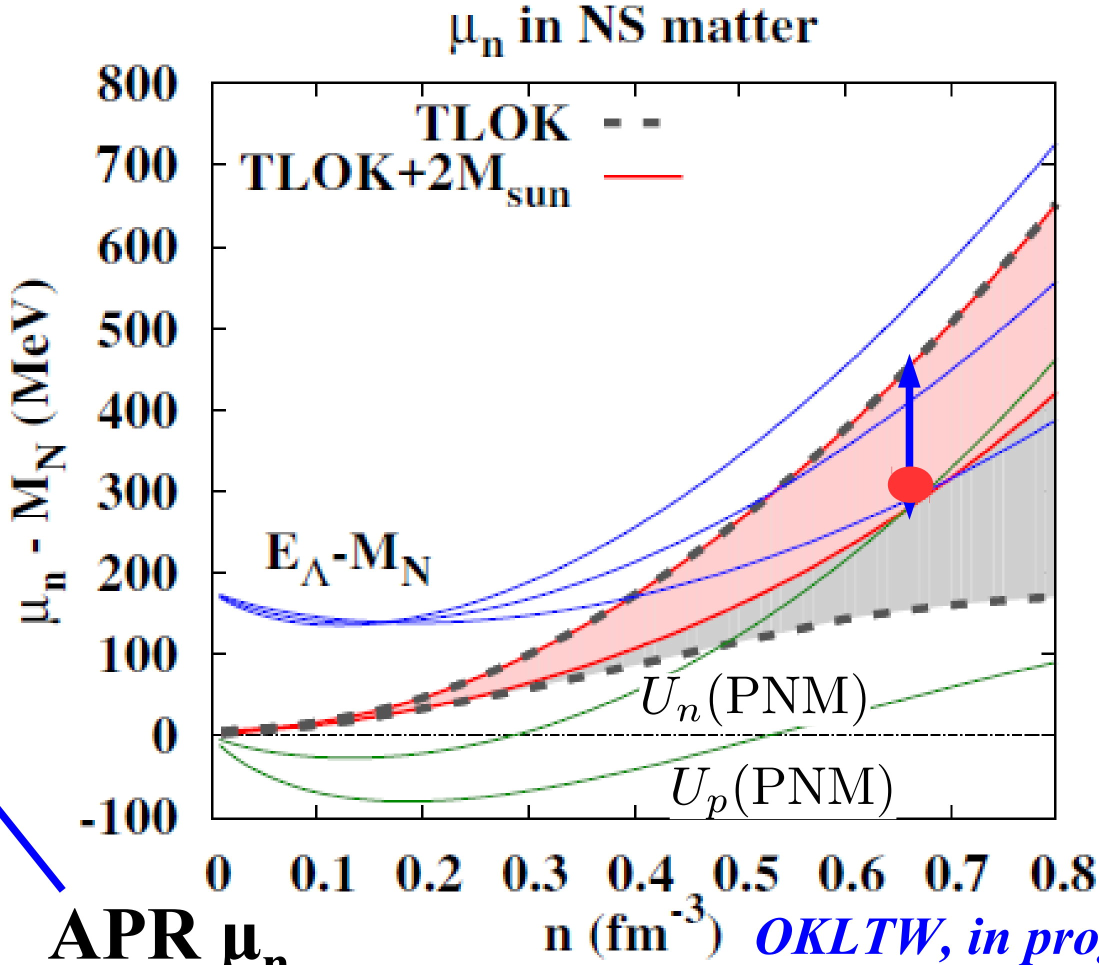
Neutron Chemical Potential in NS

- Λ appears in neutron stars if E_Λ (p=0) = $M_\Lambda + U_\Lambda < \mu_n$
- W. Weise's conjecture: U_Λ in χEFT (2+3 body) is stiff enough.
- But μ_n is larger with TLOK+$2M_\odot$ constraints



W. Weise, NFQCD2018 (2018.06);
Gerstung, Kaiser, Weise, in prog.

APR μ_n



OKLTW, in prog.

Neutron Chemical Potential in NS

- **Neutron Chemical Potential**

$$\mu_n + M_N = \frac{\partial(nE)}{\partial n_n} = E + u\frac{\partial E}{\partial u} + 2\alpha(1-\alpha)S(u)$$

- **Single particle potential**

$$U_\Lambda(u) = \frac{\partial(nV)}{\partial n_\Lambda}$$

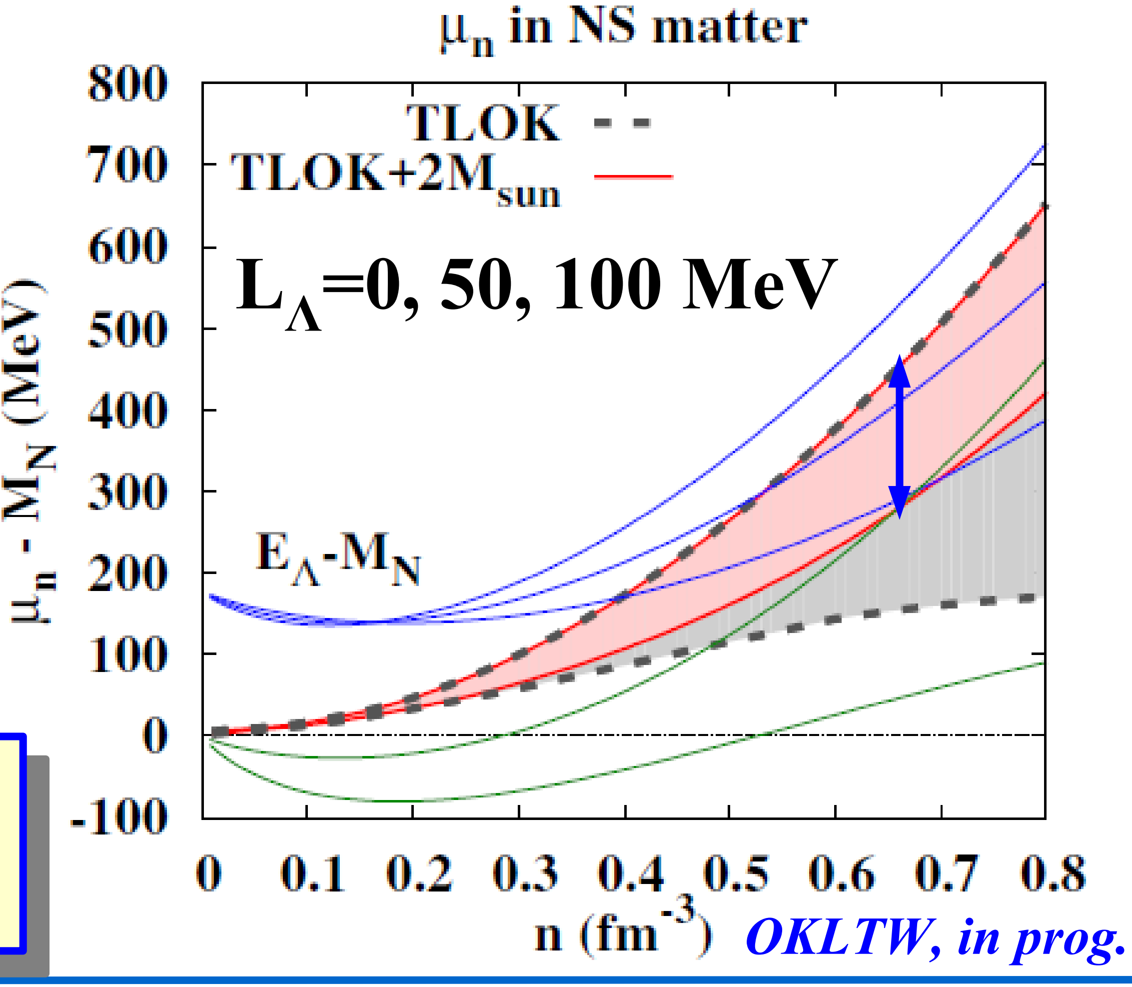
$$\simeq U_{0\Lambda} + \frac{L_\Lambda}{3}(u-1)$$

$$U_{0\Lambda} \simeq -30\ \text{MeV}$$

$$L_\Lambda = ???$$

(L_Λ<0 in most of RMF before 2010)

Sym. E. and L_Λ determine the emerging density of Λ.



OKLTW, in prog.

Reservations

- **Only massless electrons are considered and Crust EOS is ignored.**
  - **With μ, chemical potential may be reduced a little.**
- **Non-relativistic kinetic energy is used.**
  - **With rel. K.E., E per nucleon is modified by 0.03 MeV @ 10 n_0 as long as Sat. and Sym. E parameters are fixed.**
- **Function form is limited to k_F expansion with $u^{k/3}$ (k=2-6).**
  - **$R_{1.4}$ range becomes narrower with k=2-5.**
  - **Density expansion gives EOSs very sensitive to parametrs.**
- **Smooth E(u) (= No phase transition) is assumed.**
  - **We expect QCD phase transition at (5-10) n0**
from recent BES data of directed flow *Nara, Niemi, AO, Stoecker ('16)*
  - **Transition to quark matter may not soften EOS drastically.**
- **Causality is violated at high densities, n > (4-6) n_0.**

To Do (or Prospect)

- **Baryons other than nucleons Λ, Δ, Ξ, Σ, …**
- **Connecting to Hadron Resonance Gas (HRG) EOS**
  - **HRG EOS**
    mass and kinetic E of hadrons with M<2 GeV + simple potential E

    $$\varepsilon_{\mathrm{HRG}} = \mathcal{T} + cn^2$$

    or Lattice EOS in HIC(No saturation, No constraint from NS).
  - **We need to guess the potential energy density more seriously for consistent understanding of HIC, Nuclear, and NS physics.**

    $$\varepsilon = \mathcal{T} + \enclose{circle}{\mathcal{V}} \longleftarrow \textbf{Nuclear and NS physics}$$
- **Connecting to Quark(-Gluon) matter EOS**
  - **Embed model-H singularities** *E.g. Nonaka, Asakawa ('04)*
  - **“Interpolation” of nuclear and quark matter EOS**

Summary

- Tews-Lattimer-AO-Kolomeitsev ('17) constraints (S0, L, K_n, Q_n) and 2 $M_\odot$ constraint with the aid of Fermi momentum (k_F) expansion lead to the costraint on 1.4 $M_\odot$ neutron star radius of (10.6-12.2) km.
  - Consistent with many of other constraint.
- Appearance of hyperons and Deltas may be sensitive to the symmetry energy in addition to potential parameters, (U_{0B}, L_B).
  - We need to know the slope of potential in addition to the depth.
- Global EOS (HIC and Nuclear/NS matter) needs to be given in a way where HIC physicists and NS physicists admit.
  E.g. “Hadron Resonance Gas (HRG)+Potential from NS”

Thank you for your attention and for staying until the end of scientific program of the meeting.

Further Constraint on Qn

- **2 M$_\odot$ requirement constrains Qn further.**

$$Q_n > -9.3L + 480\,\mathrm{MeV}$$

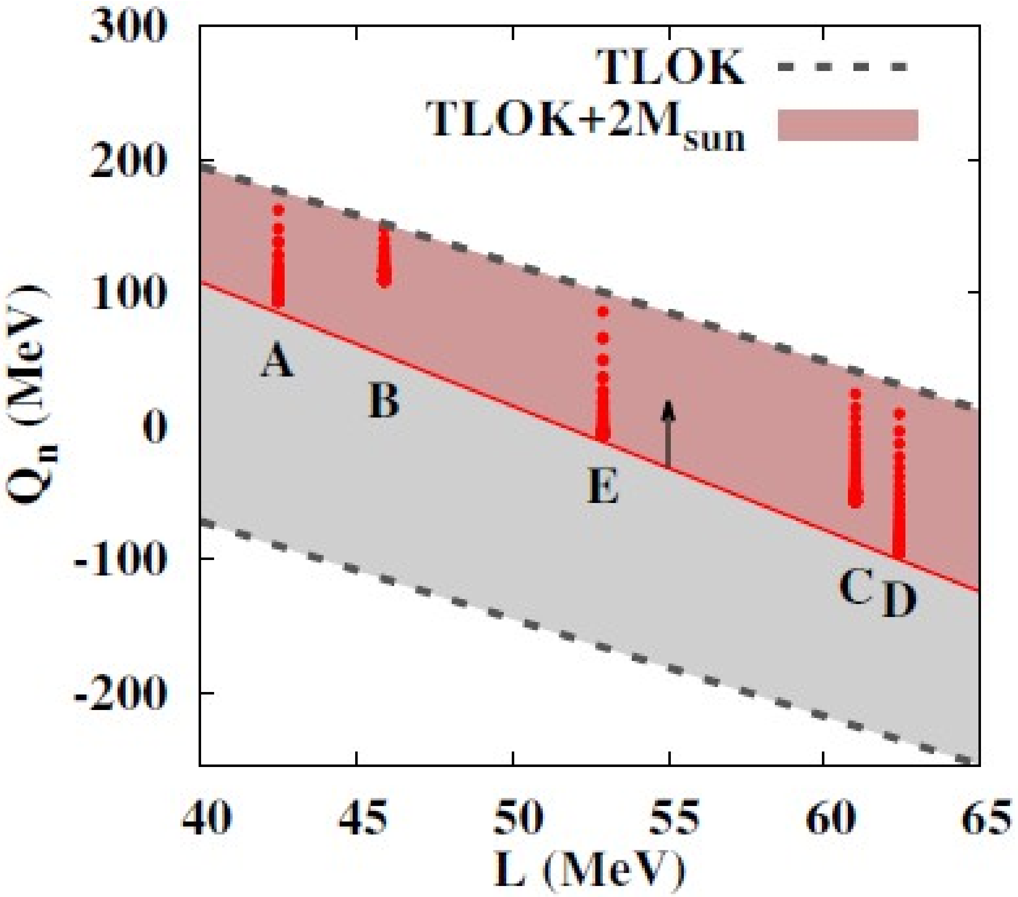


FIG. 4. Constraint on Q_n

AO, Kolomeitsev, Lattimer, Tews, Wu (OKLTW), in prog.

Unitary Gas Constraint

Tews, Lattimer, AO, Kolomeitsev (TLOK), ApJ ('17)

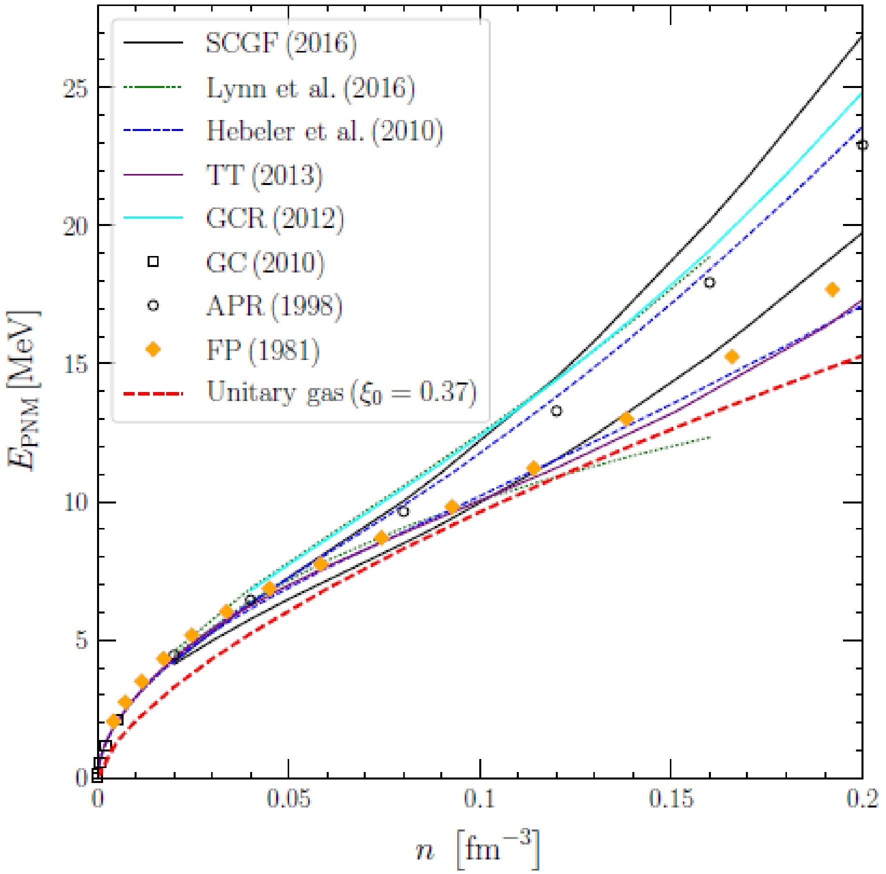
- **Conjecture:**
  Unitary gas gives the lower bound of neutron matter energy.

$$S(n) = E_{\rm PNM} - E_{\rm SNM} \geq E_{\rm UG} - E_{\rm SNM}$$

$$E_{\rm UG} = \xi E_{\rm FG} \;\; (\xi \simeq 0.38)$$

← **Sym. Nucl. Matter EOS is relatively well known.**

- **$a_0 = \infty$ in unitary gas**
  → lower bound energy of $a_0 < 0$ systems (w/o two-body b.s.) ?
- **Supported by (most of) ab initio calc.**



Potential Energy Density

- **Potential Energy Density in the Fermi momentum expansion**

$$\mathcal{V} = nV = \sum_{i,j\in B} n_i n_j v_{ij}(n)$$

Density-dependent NN interactions vij (i, j=p or n) are known.

- **Single particle potential**

$$U_i = \frac{\partial \mathcal{V}}{n_i} = \sum_{j} n_j v_{ij}(n) + \sum_{jk} n_j n_k \frac{\partial v_{jk}(n)}{\partial n_i}$$

$$= U_{0i} + \frac{L_i}{3}(u-1) + \mathcal{O}((u-1)^2)$$

$$\simeq au + bu^{4/3}$$

rearrangement term

Again, a and b are given as a linear function of U_{0i} and L_i.