

Hadron-Hadron Correlation and Interaction from Heavy-Ion Collisions

Akira Ohnishi (YITP, Kyoto U.)

K. Morita, T. Furumoto, AO, PRC91('15), 024916 ($\Lambda\Lambda$).

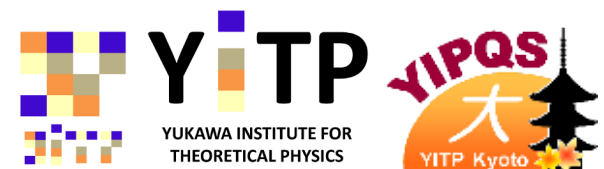
K. Morita, T. Furumoto, AO, EPJ Web Conf. 97 (2015), 00020 ($\Lambda\Lambda$).

AO, K. Morita, K. Miyahara, T. Hyodo, NPA954 ('16)294 ($\Lambda\Lambda$, $K-p$).

K. Morita, AO, F. Etminan, T. Hatsuda, PRC94('16), 031901(R) (ΩN).

S. Cho et al.(ExHIC Collab.), Prog.Part.Nucl.Phys.95('17)279 ($\Lambda\Lambda$, $K-p$).

T. Hatsuda, K. Morita, AO, K. Sasaki, NPA967('17), 856 ($\Xi-p$).



Pre-History

(K^-, K^+) reaction

- (K^-, K^+) reaction = doorway to produce $S=-2$ systems

- $K^- p \rightarrow K^+ \Xi^- \rightarrow \Xi$ nuclei, stopped Ξ to $\Lambda\Lambda$ nuclei, ...

- What is the origin of large yield at smaller K^+ momentum ?

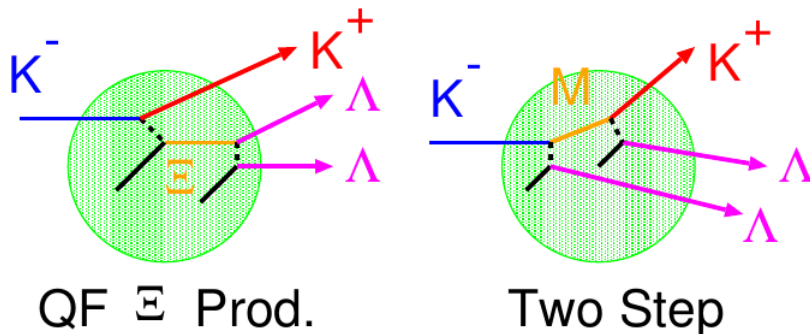
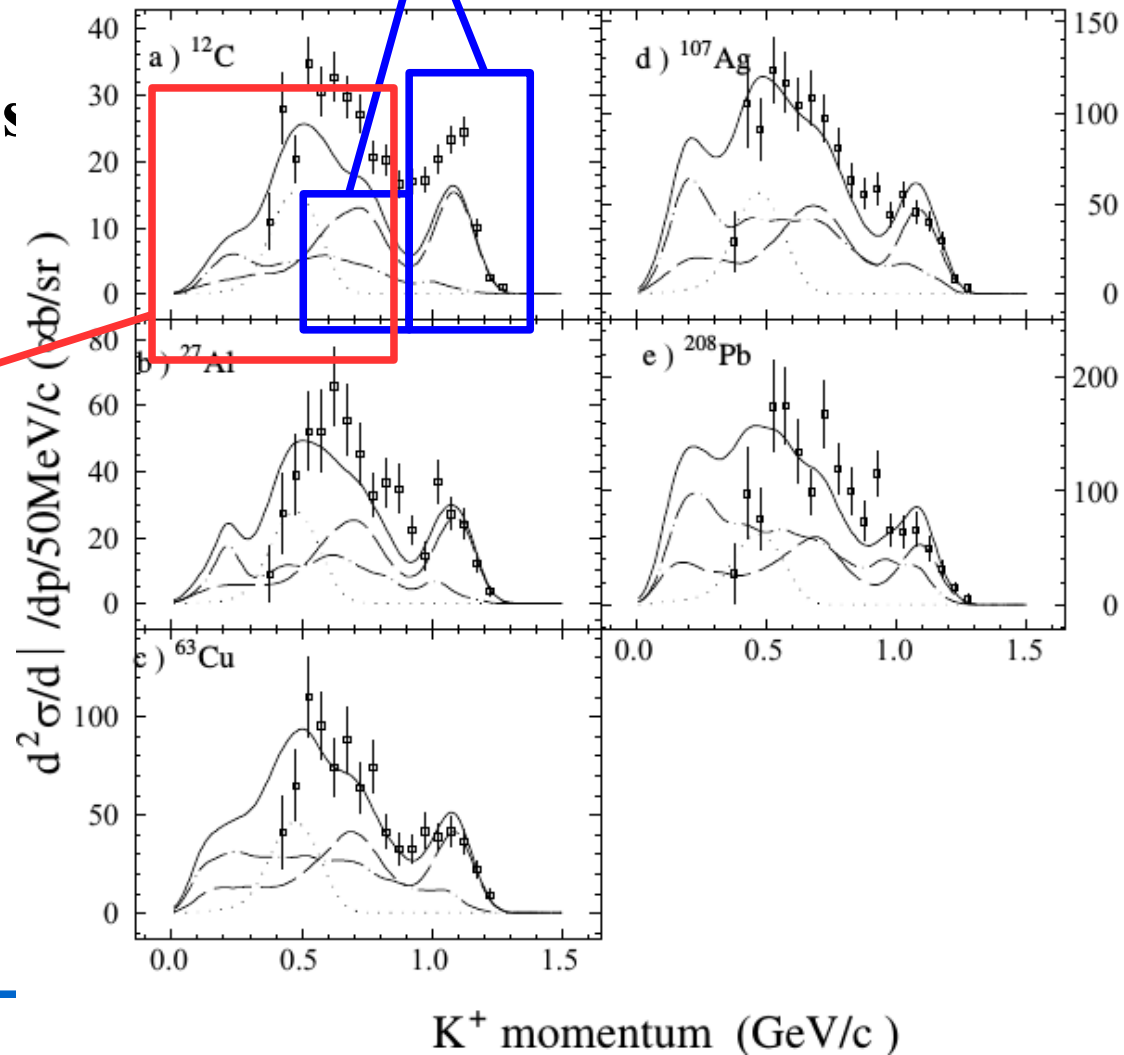
T. Iijima et al., NPA546('92) 588.

→ Various two step processes

Y. Nara, AO, T. Harada, A. Engel, NPA614('97)433

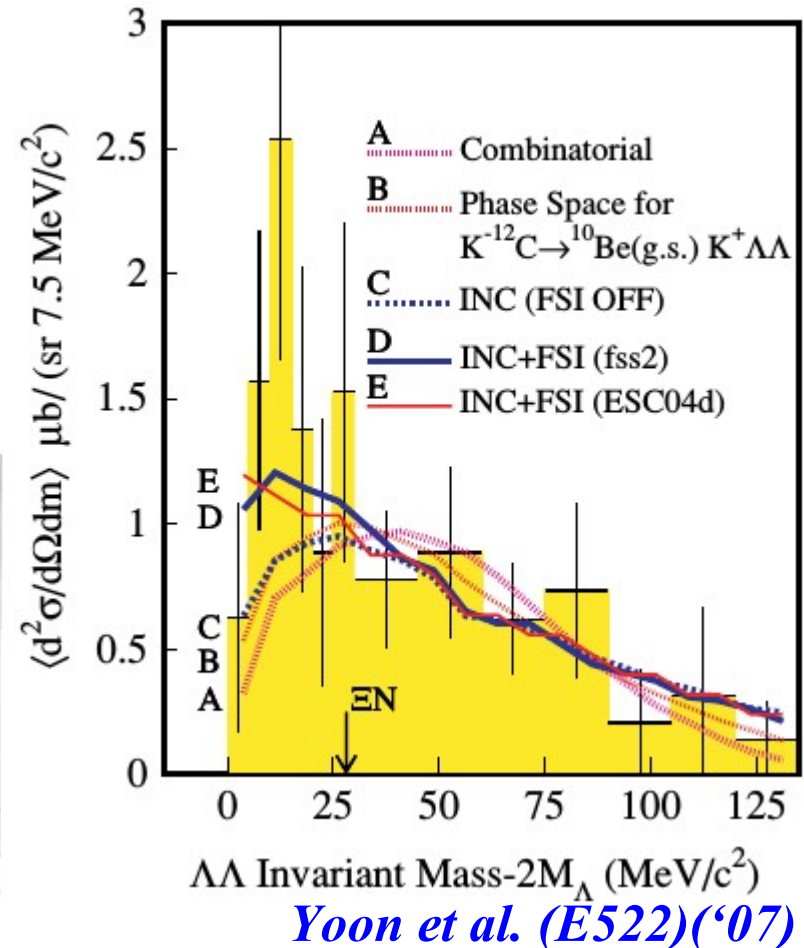
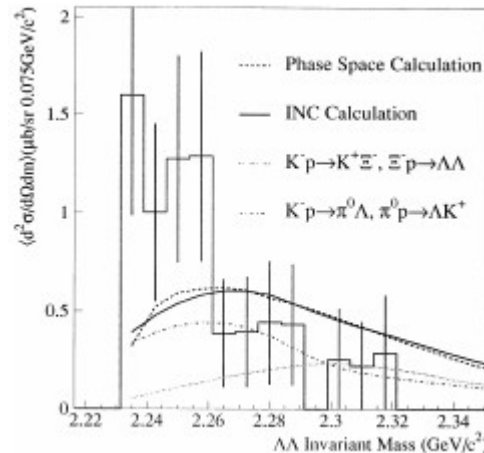
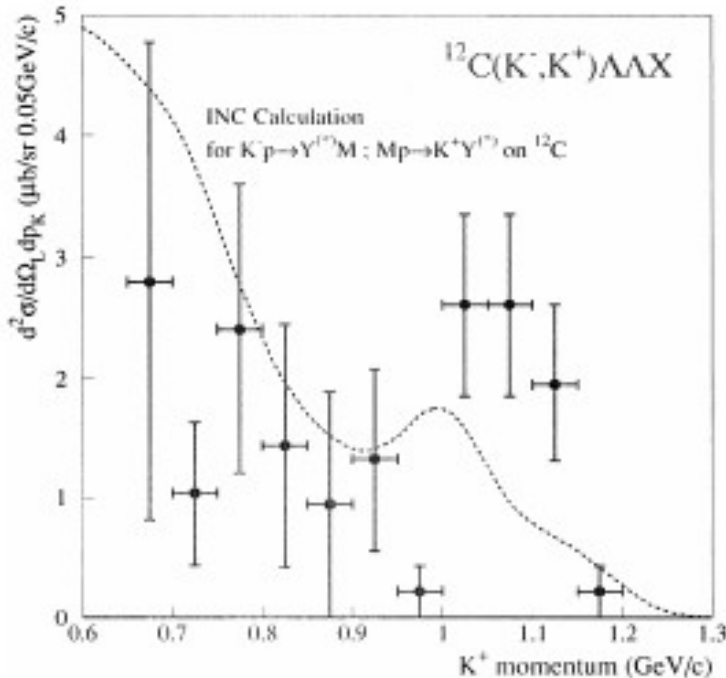
Quasi Free $\Xi(\Xi^*)$

Multi-step



Two Λ production in (K^- , K^+) reactions

- Experimentalists really measured two Λ emission !
J.K.Ahn et al. (KEK-PS E224 Collab.), PLB444('98)267.
C. J. Yoon et al. (KEK-PS E522 Collab. +AO), PRC75('07)022201
- Invariant mass spectrum of $\Lambda\Lambda$ is enhanced from our cascade calculation.
 → FSI enhancement ? or H particle ?



Ahn et al. (E224)('98)

Yoon et al. (E522)('07)

From correlation to interaction

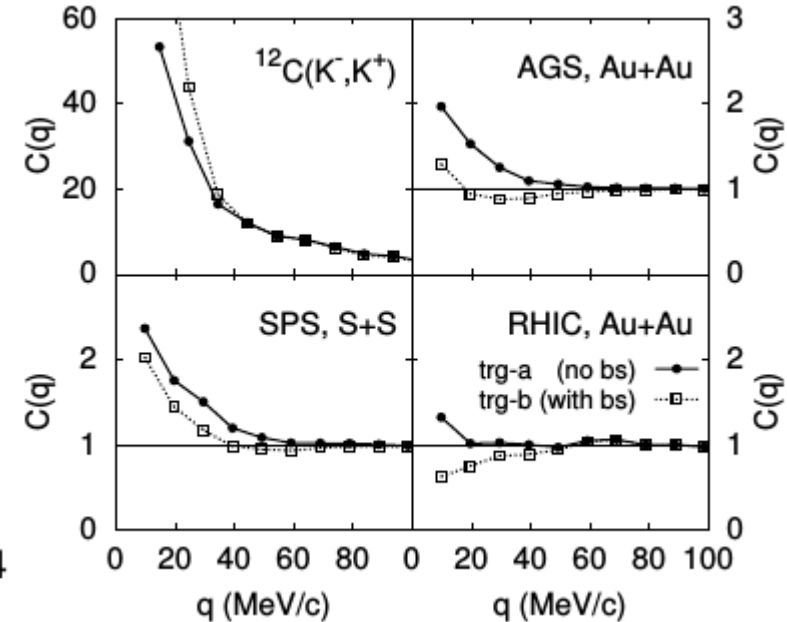
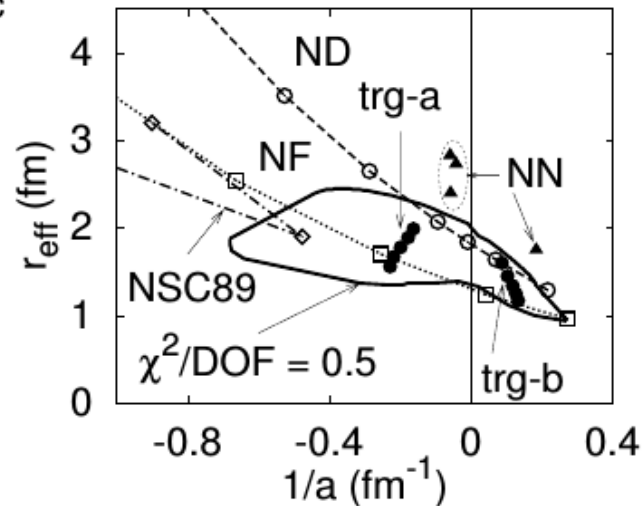
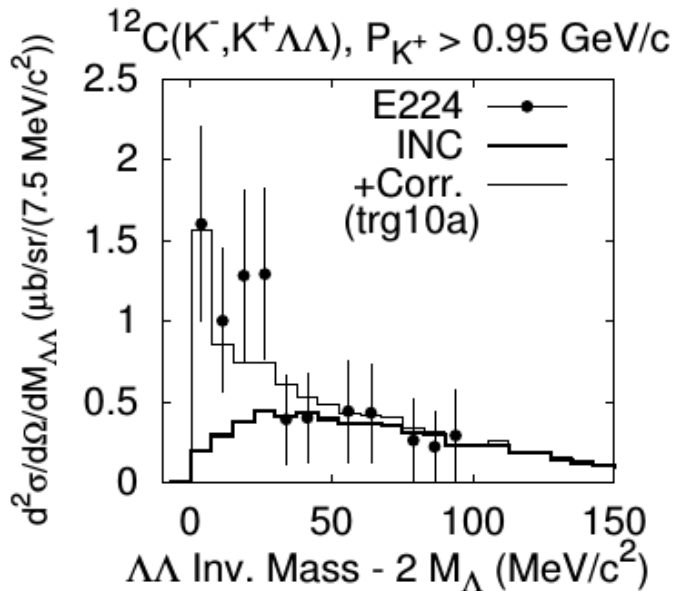
- Enhancement of $\Lambda\Lambda$ may show $\Lambda\Lambda$ interaction effects !

AO, Y. Hirata, Y. Nara, S. Shinmura, Y. Akaishi,
NPA670('00)297c; NPA684('01)595; NPA691('01)242c

- Enh. is roughly explained by $\Lambda\Lambda$ final state int.

- It should be clearer to measure in heavy-ion collisions.
 Enh. w/o bound state, Suppression w/ bound state.

→ I asked Prof. Huan Z. Hunag (STAR) in **ExHIC 2010 meeting**
 and they measured it !



AO, Hirata, Nara, Shinmura, Akaishi ('00)

Contents

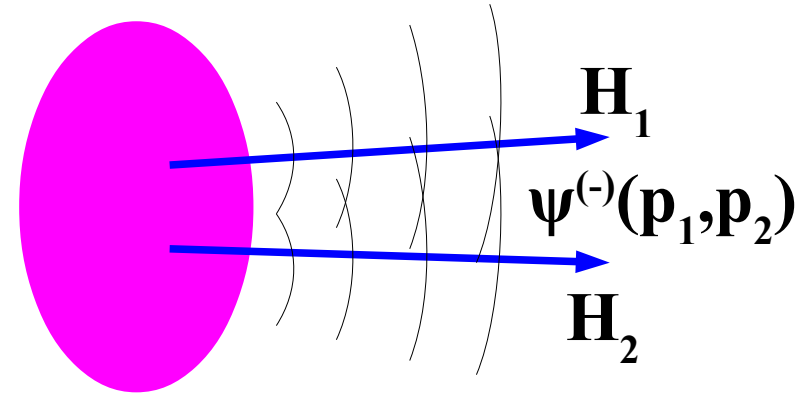
- **Pre-History**
- **Two Particle Correlation and Correlation Function**
 - **Koonin-Pratt formula & Lednicky-Lyuboshits model**
- **Hadron-Hadron Correlation and Hadron-Hadron Interaction**
 - **$\Lambda\Lambda$ correlation and $\Lambda\Lambda$ interaction**
 - **Ω^- p correlation and lattice ΩN interaction**
 - **K^- p correlation from chiral SU(3) dynamics**
 - **Ξ^- p correlation and lattice ΞN interaction**
- **Summary**

Two Particle correlation and Correlation Function

Hadron-Hadron Correlation in HIC

■ Hadron-Hadron Correlation Func. (Koonin-Pratt (KP) formula)

S. E. Koonin, PLB 70 ('77) 43; S. Pratt, T. Csorgo and J. Zimanyi, PRC42 ('90) 2646; W. Bauer, C.-K. Gelbke, S. Pratt, Annu. Rev. Nucl. Part. Sci. 42 ('92)77; R. Lednicky, V. L. Lyuboshits, Sov. J. Nucl. Phys. 35 ('82) 770.



$$C(\mathbf{q}) = \frac{E_1 E_2 dN_{12}/d\mathbf{p}_1 d\mathbf{p}_2}{(E_1 dN_1/d\mathbf{p}_1)(E_2 dN_2/d\mathbf{p}_2)} \simeq \int d\mathbf{r} \underbrace{S_{12}(\mathbf{r})}_{\text{Source fn.}} \underbrace{\left| \psi_{12}^{(-)}(\mathbf{r}, \mathbf{q}) \right|^2}_{\text{int.} \rightarrow \text{relative w.f.}}$$

\mathbf{q} : rel. mom. (referred to also as \mathbf{k}^*)

■ Static sph. Gaussian source, Int. for s-wave, Identical fermions

$$C(q) \simeq 1 - \frac{1}{2} \exp(-4q^2 R^2) + \frac{1}{2} \int d^3 r \underbrace{S_{12}(r)}_{\text{Source}} \left[\underbrace{|\psi_0(r)|^2}_{\text{s-wave w.f.}} - \underbrace{|j_0(qr)|^2}_{\text{free}} \right]$$

↖ **Fermion (Quant. Stat.)**

Lednický-Lyuboshits (LL) model

■ Lednický-Lyuboshits analytic model

- **Asymp. w.f. + Eff. range corr. + $\psi^{(-)} = [\psi^{(+)}]^*$**

$$\psi_0(r) \rightarrow \psi_{\text{asy}}(r) = \frac{e^{-i\delta}}{qr} \sin(qr + \delta) = \mathcal{S}^{-1} \left[\frac{\sin qr}{qr} + f(q) \frac{e^{iqr}}{r} \right]$$

$$\begin{aligned} \Delta C_{\text{LL}}(q) &= \int dr S_{12}(r) (|\psi_{\text{asy}}(r)|^2 - |j_0(qr)|^2) \\ &= \frac{|f(q)|^2}{2R^2} F_3\left(\frac{r_{\text{eff}}}{R}\right) + \frac{2\text{Re}f(q)}{\sqrt{\pi}R} F_1(x) - \frac{\text{Im}f(q)}{R} F_2(x) \end{aligned}$$

($x = 2qR$, $R = \text{Gaussian size}$, F_1, F_2, F_3 : Known functions)

■ Phase shifts

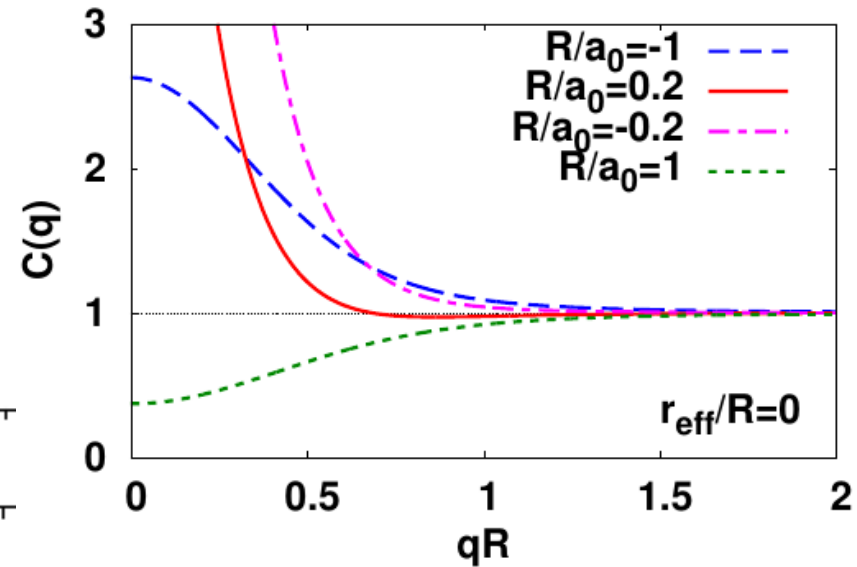
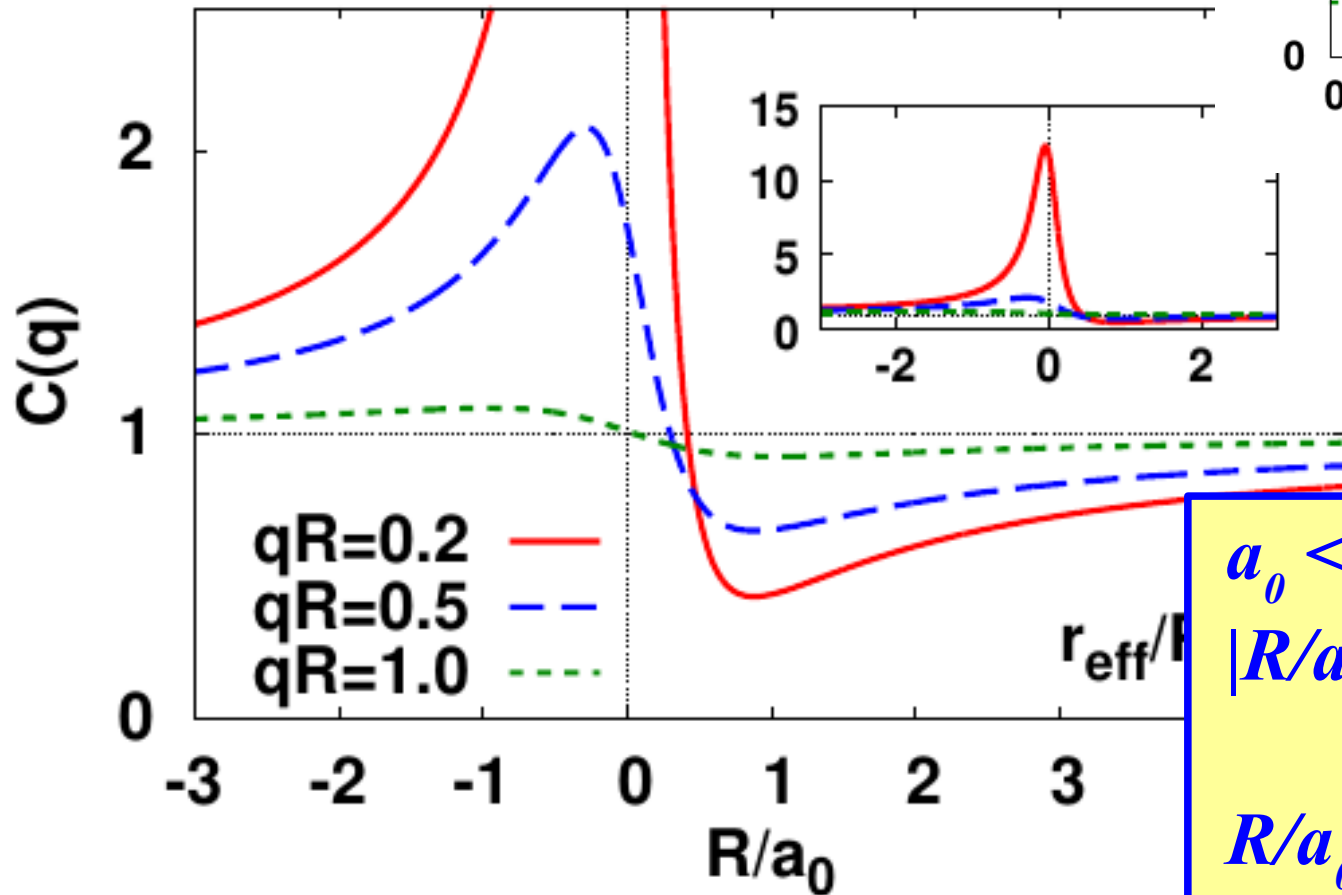
$$q \cot \delta = -\frac{1}{a_0} + \frac{1}{2} r_{\text{eff}} q^2 + \mathcal{O}(q^4) \rightarrow \delta \simeq -a_0 q + \mathcal{O}(q^3)$$

$$\sin(qr + \delta) \simeq \sin(q(r - a_0) + \dots) \quad \text{Node at } \mathbf{r} \sim \mathbf{a}_0$$

for small q

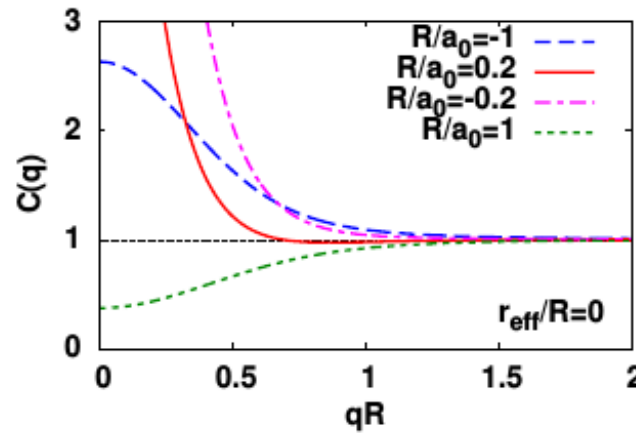
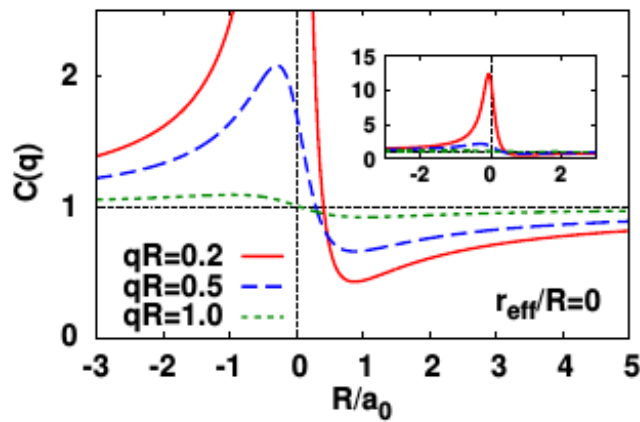
Interaction Dependence of Correlation Function

$$q \cot \delta = -1/a_0 + r_{\text{eff}} q^2/2 + O(q^4)$$



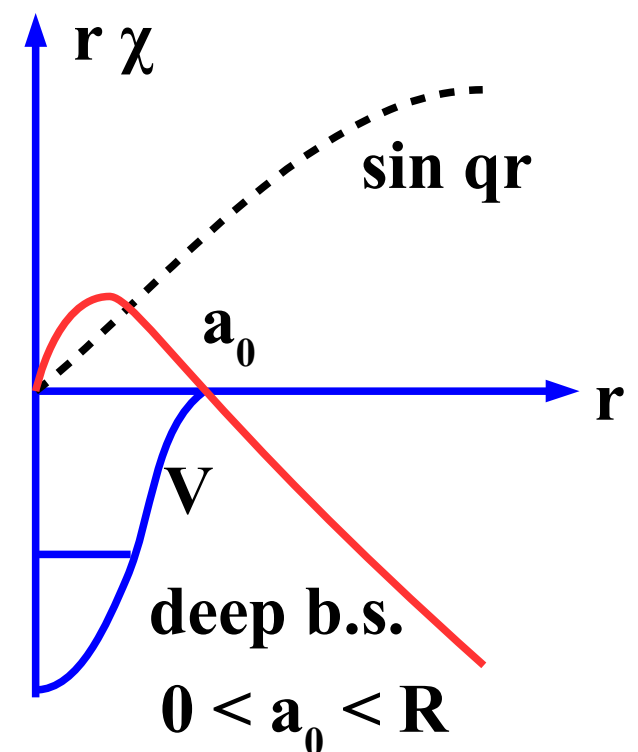
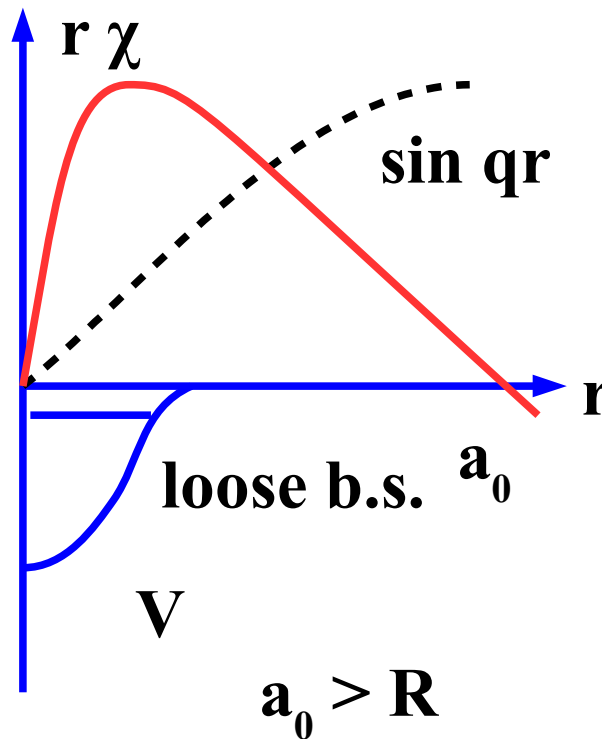
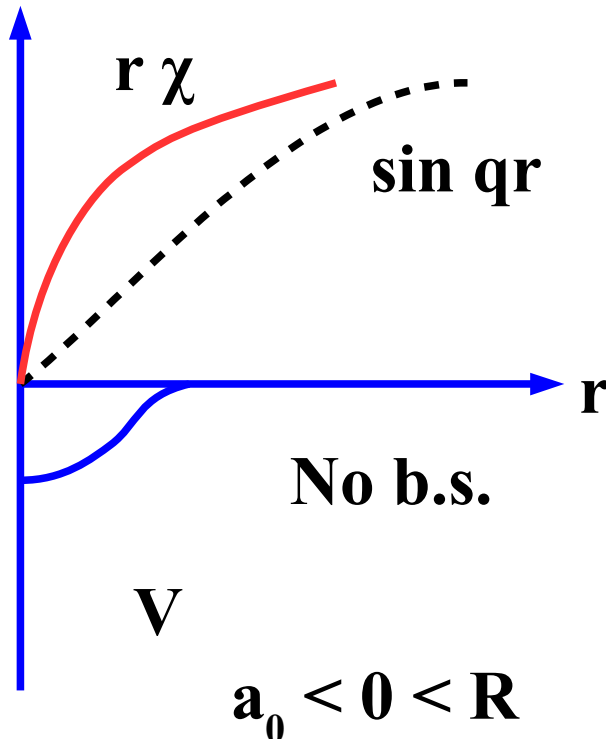
$a_0 < 0 \rightarrow$ *Enhancement*
 $|R/a_0| \ll 1 \rightarrow$ *Strong enh.*
at small q
 $R/a_0 > 1 \rightarrow$ *Suppression*

Interaction Dependence of Correlation Function



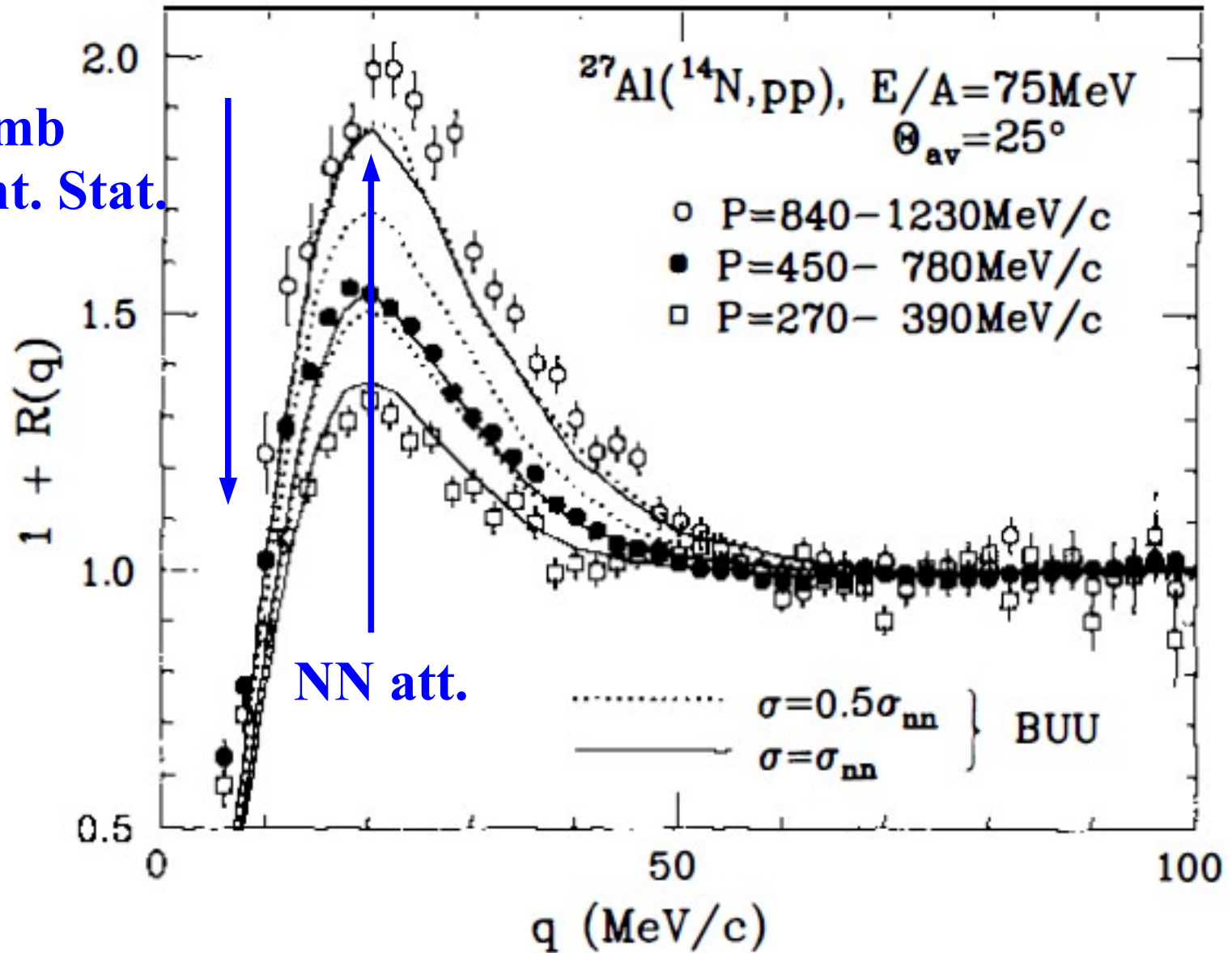
$$q \cot \delta = -1/a_0 + r_{\text{eff}} q^2/2 + O(q^4)$$

w.f. node suppresses CF



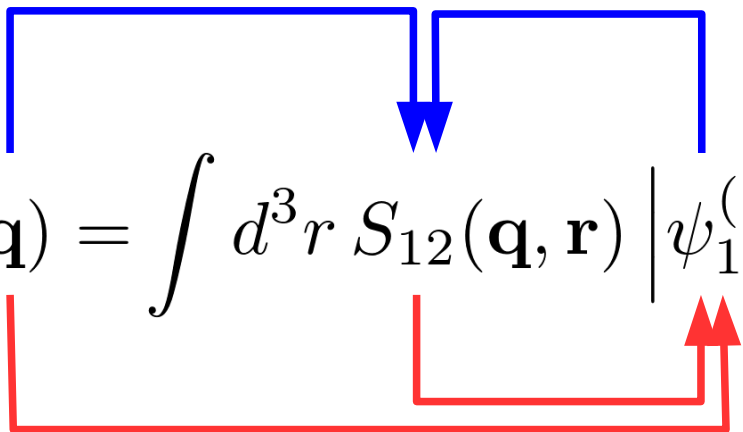
How does interaction modifies correlation ?

Coulomb
+Quant. Stat.



Bauer et al. ('92)

Standard Usage
Known Int. + Corr. Fn. → Source Size


$$C(\mathbf{q}) = \int d^3r S_{12}(\mathbf{q}, \mathbf{r}) \left| \psi_{12}^{(-)}(\mathbf{r}; \mathbf{q}) \right|^2$$

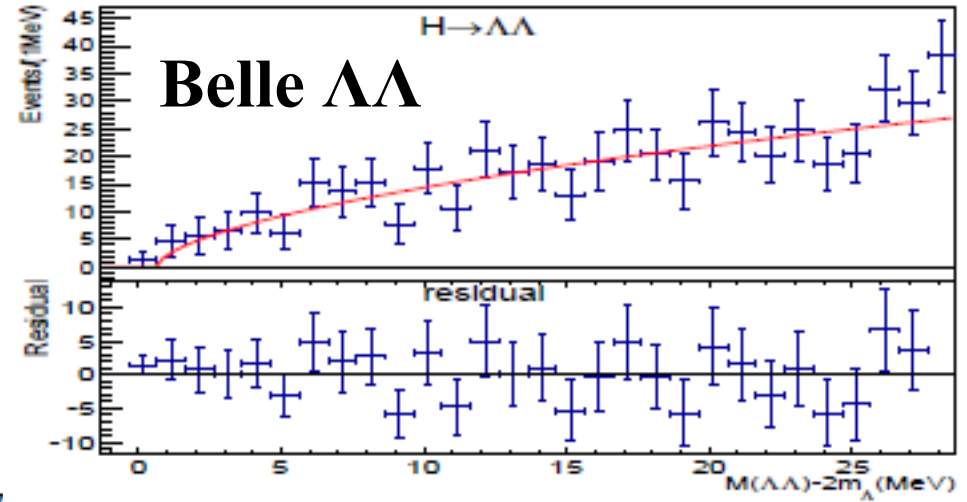
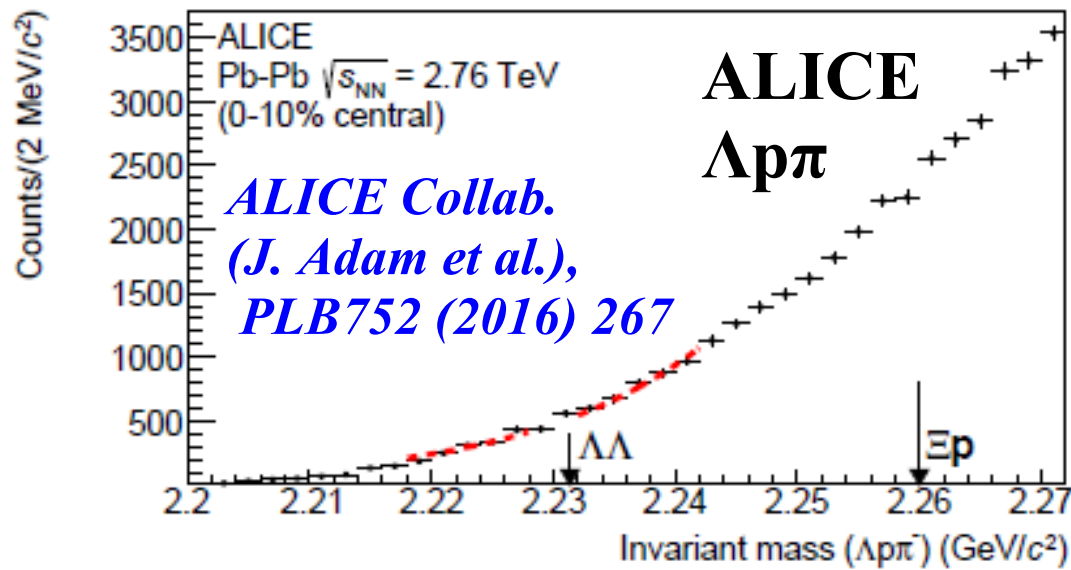
Idea of Reversal:
Can we determine hh interaction
from hh correlation ?

Let us try ! Examples: $\Lambda\Lambda$, $\Omega^- p$, and $K^- p$

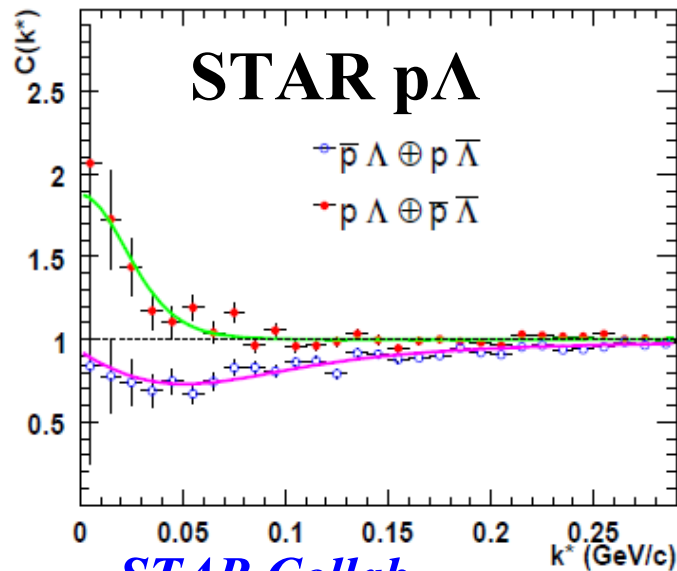
Previous works ($\Lambda\Lambda$): C. Greiner, B. Muller, PLB219('89)199.;

AO, Y. Hirata, Y. Nara, S. Shinmura, Y. Akaishi, NPA 670 ('00), 297c

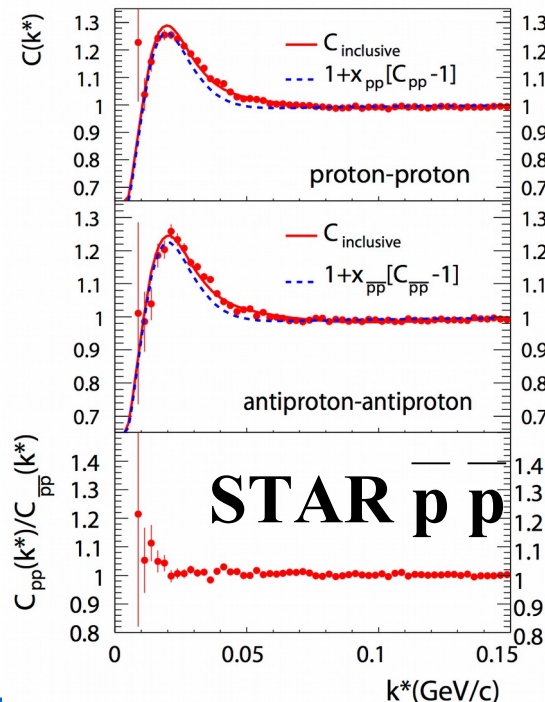
$\Lambda\Lambda$ invariant mass / BB correlation function (as of 2016)



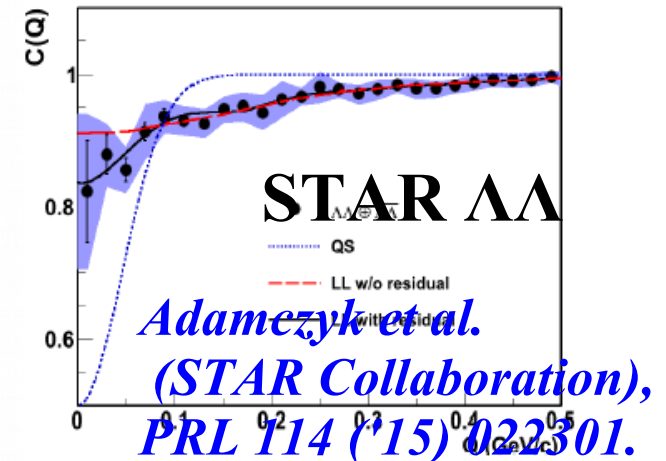
*Belle Collaboration (Kim, B.H. et al.),
PRL110('13)222002.*



STAR Collab.
(J. Adams et al.),
PRC74('06)064906.



STAR Collab.
(J. Adams et al.)
Nature 527('15)345



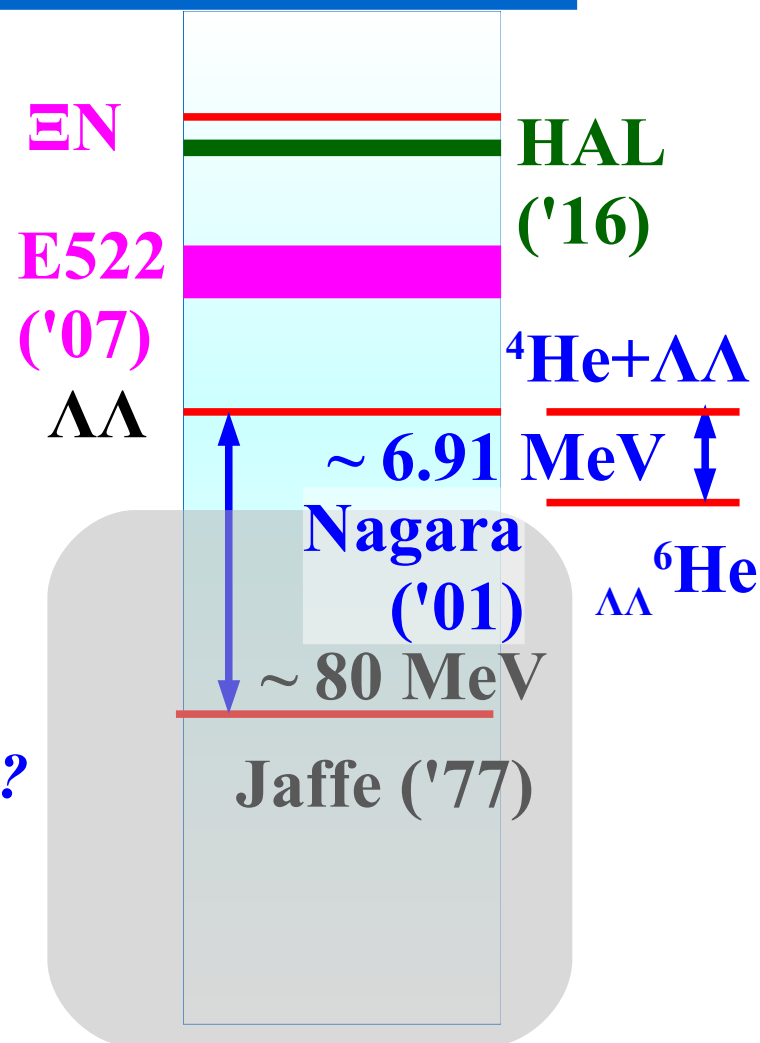
Adamezyk et al.
(STAR Collaboration),
PRL 114 ('15) 022301.

$\Lambda\Lambda$ correlation and $\Lambda\Lambda$ interaction

Relevance of $\Lambda\Lambda$ interaction to physics

■ H-particle: 6-quark state (uuddss)

- Prediction: *R.L.Jaffe, PRL38(1977)195*
- Ruled-out by double Λ hypernucleus
Takahashi et al., PRL87('01) 212502
- Resonance or Bound “H” ?
Yoon et al.(KEK-E522)+AO ('07)
- Lattice QCD
HAL QCD & NPLQCD ('11)
HAL QCD ('16): H as a loosely bound ΞN ?



■ Neutron Star Matter EOS

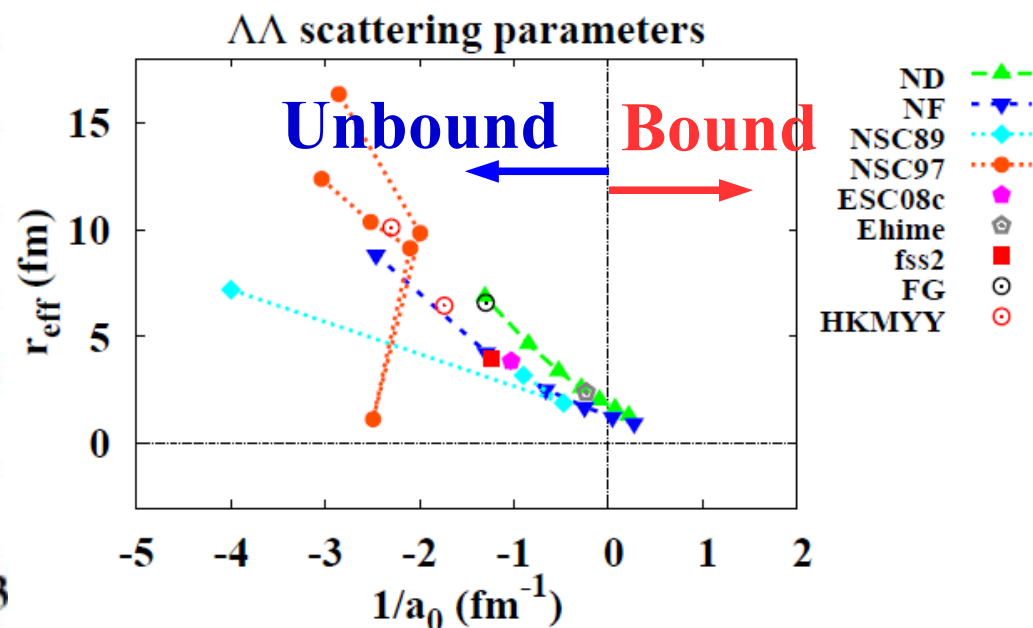
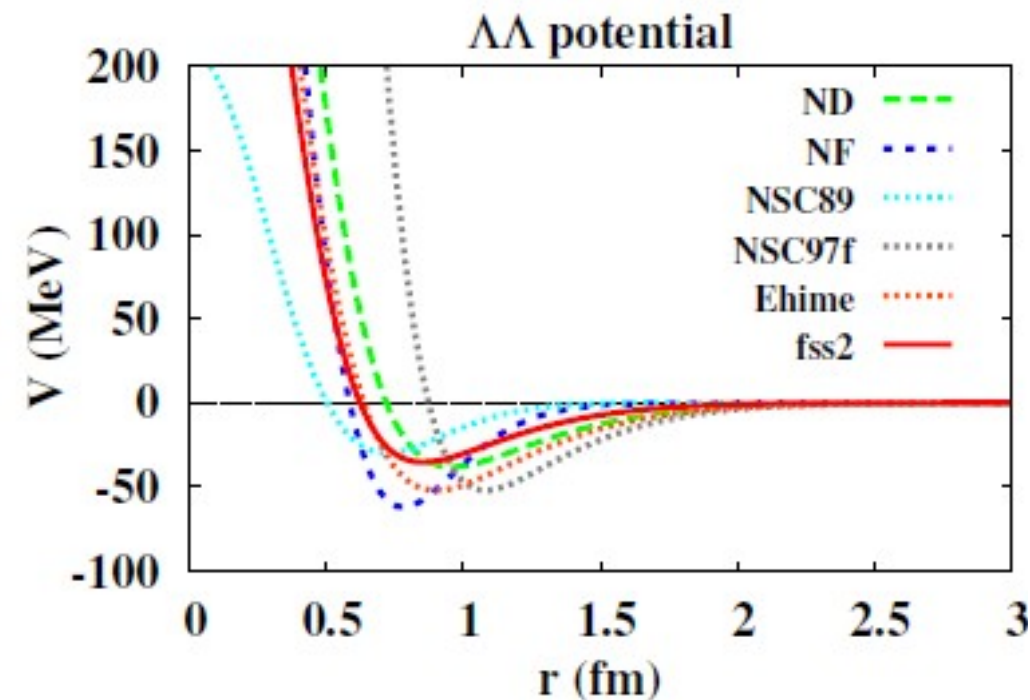
- Hyperon Puzzle
Demorest et al. ('10), Antoniadis et al. ('13)
- Cooling Puzzle ($\Lambda\Lambda$ superfluidity)
T. Takatsuka, R. Tamagaki, PTP 112('04)37

$\Lambda\Lambda$ interaction

Proposed $\Lambda\Lambda$ interactions

- **Meson Ex. models:** Nijmegen model D, F, Soft Core (89, 97), ESC08
Nagels, Rijken, de Swart ('77, '79), Maessen, Rijken, de Swart ('89), Rijken, Stoks, Yamamoto ('99); Rijken, Nagels, Yamamoto ('10).
- **Quark cluster model interaction:** fss2
Fujiwara, Fujita, Kohno, Nakamoto, Suzuki ('00)
- **Phenomenological model:** Ehime **T. Ueda et al. ('99).**

Two (or three) range gaussian fit results are used in the analysis.



$\Lambda\Lambda$ correlation in HIC

■ Merit of HIC to measure $\Lambda\Lambda$ correlation

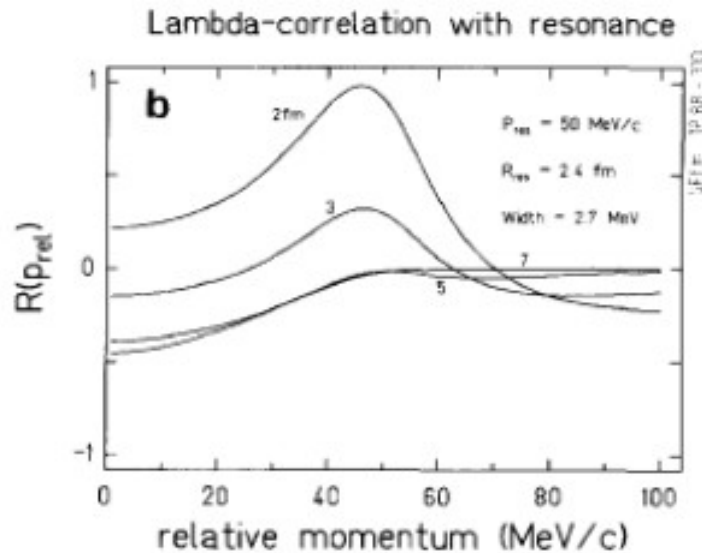
- Source is “Simple and Clean” !
T, μ , flow, size, ... are well-analyzed.

- Nearly Stat. prod.
→ Many exotics will be produced.

Schaffner-Bielich, Mattiello, Sorge ('00), Cho et al.(ExHIC Collab.) ('11)

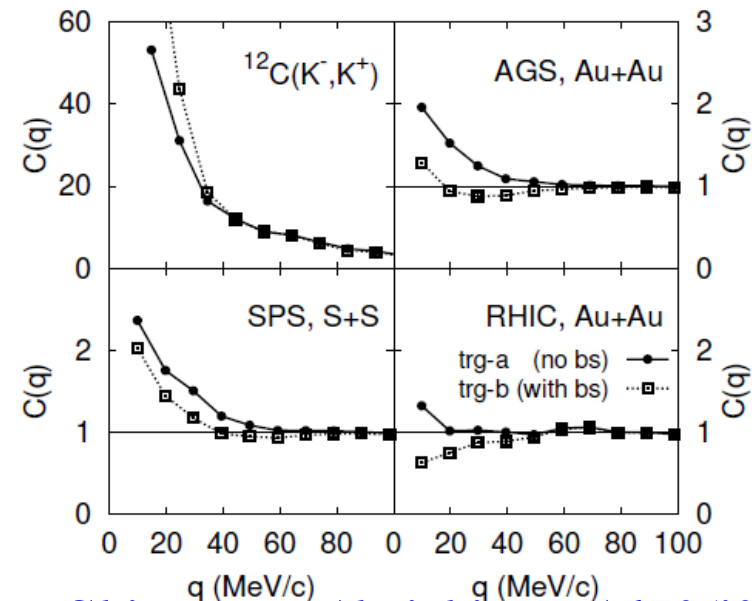
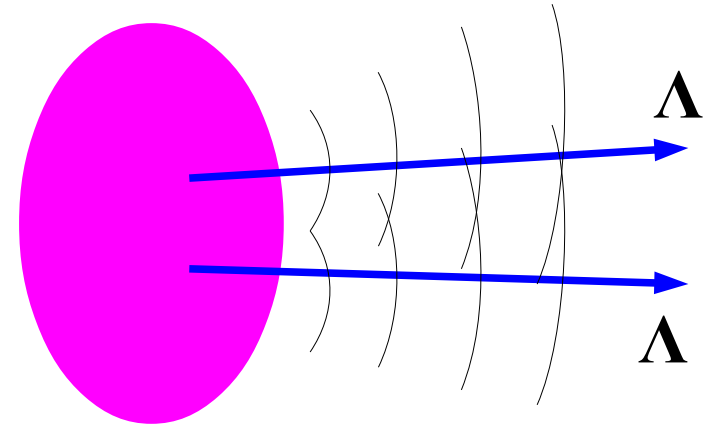
- Discovery of “H” and/or Constraint on $\Lambda\Lambda$ int.

Bound state exhaust the low q strength → suppressed $C(q)$.



C. Greiner, B. Muller, PLB219('89)199.

AO, Hirata, Nara, Shinmura, Akaishi, NPA670('00)297c



$\Lambda\Lambda$ correlation at RHIC

- STAR collaboration at RHIC measured $\Lambda\Lambda$ correlation !

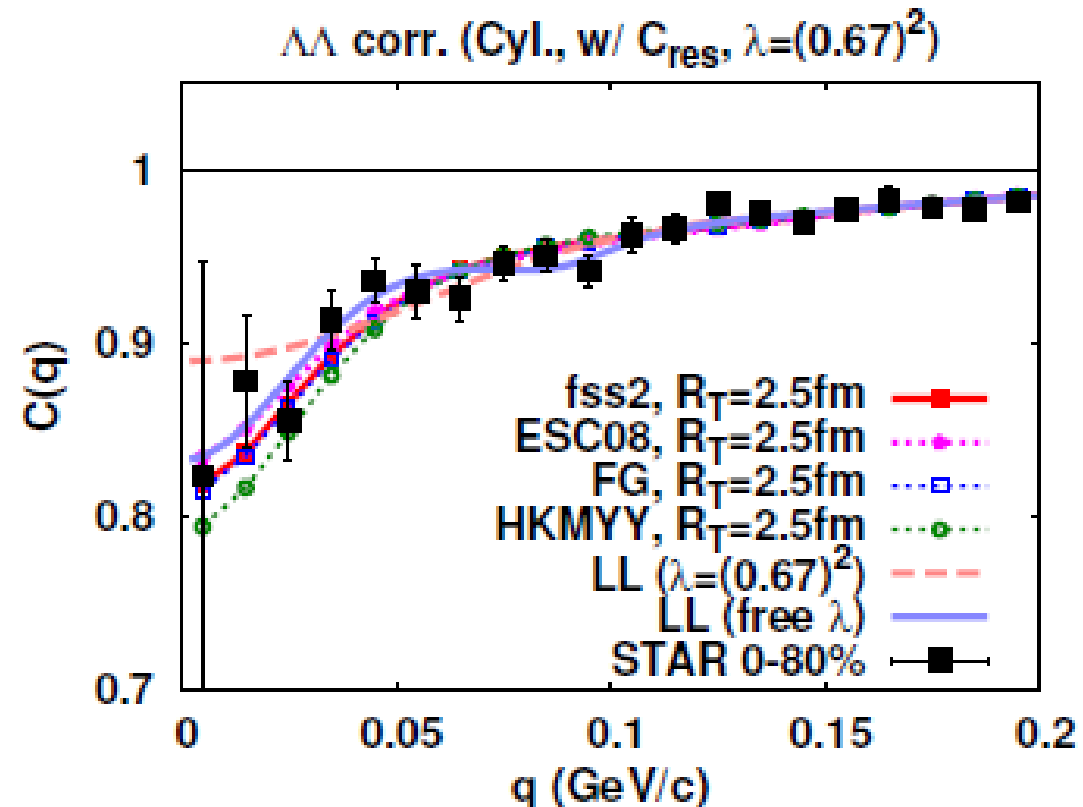
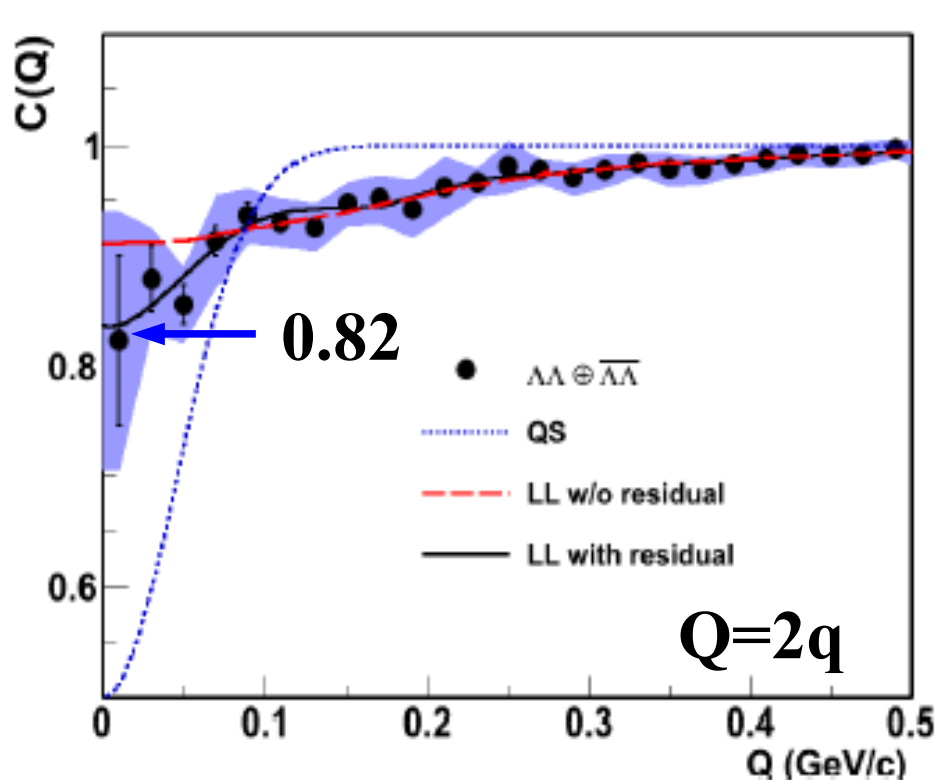
Adamczyk et al. (STAR Collaboration), PRL 114 ('15) 022301.

- RHIC, Au+Au ($\sqrt{s_{NN}}=200$ GeV), Weak decay vertex analysis.

- Theoretical Analysis well explains the data

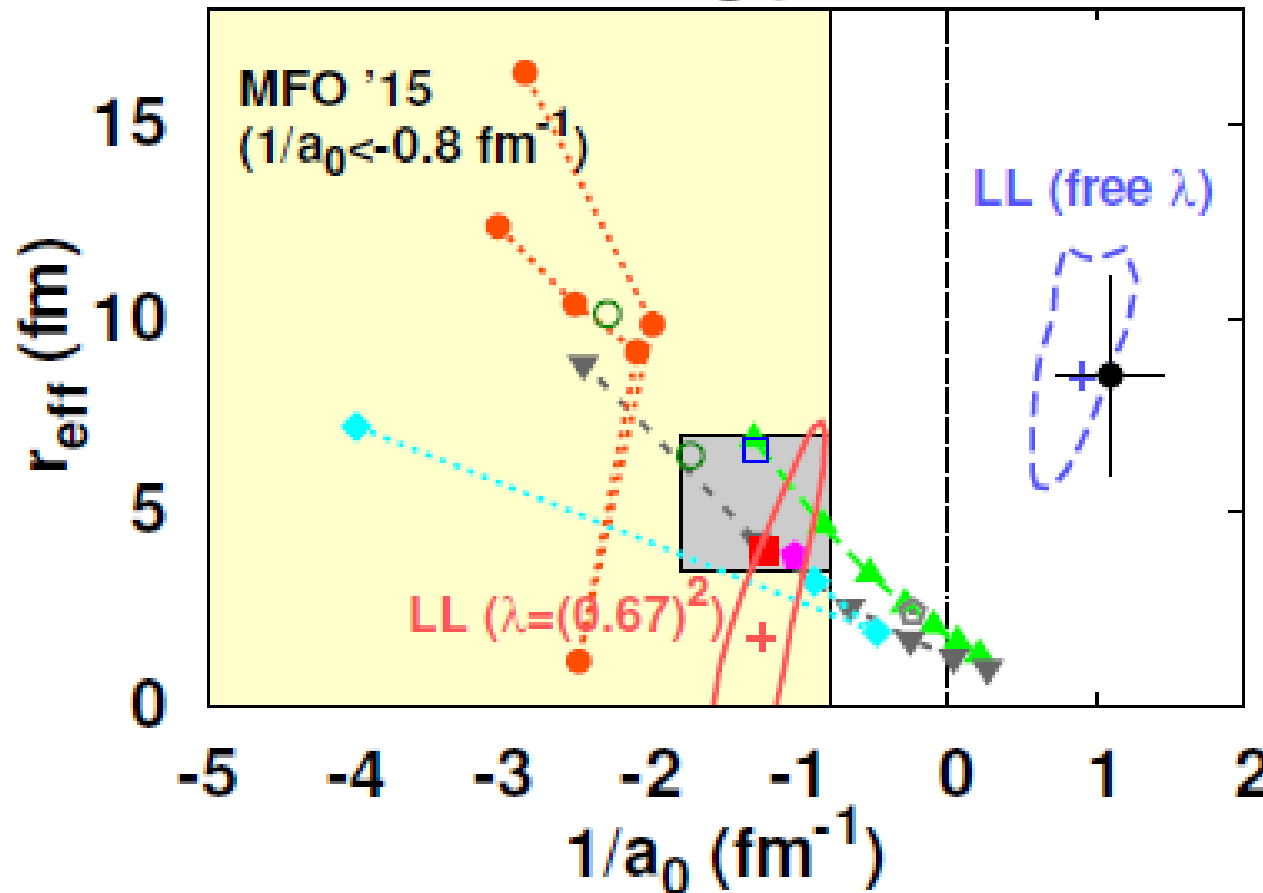
K.Morita et al., T.Furumoto, AO, PRC91('15)024916;

AO, K.Morita, K.Miyahara, T.Hyodo, NPA954 ('16), 294.



$\Lambda\Lambda$ interaction from $\Lambda\Lambda$ correlation

$\Lambda\Lambda$ scattering parameters



- Nijmegen potentials (ND, NF, NSC89, NSC97, ESC08) *Nagels+('77, '79), Maessen+('89), Rijken+('99, '10)*
- Ehime *Ueda et al. ('98)*
- Quark model interaction: fss2 *Fujiwara et al.('07)*
- Potential fitted to Nagara *Filikhin, Gal ('02) (FG), Hiyama et al. ('02, '10)(HKMYY)*

$$q \cot \delta = -1/a_0 + r_{\text{eff}} q^2/2 + O(q^4)$$

Positive a_0 (STAR) \longleftrightarrow Negative a_0 (MFO'15)

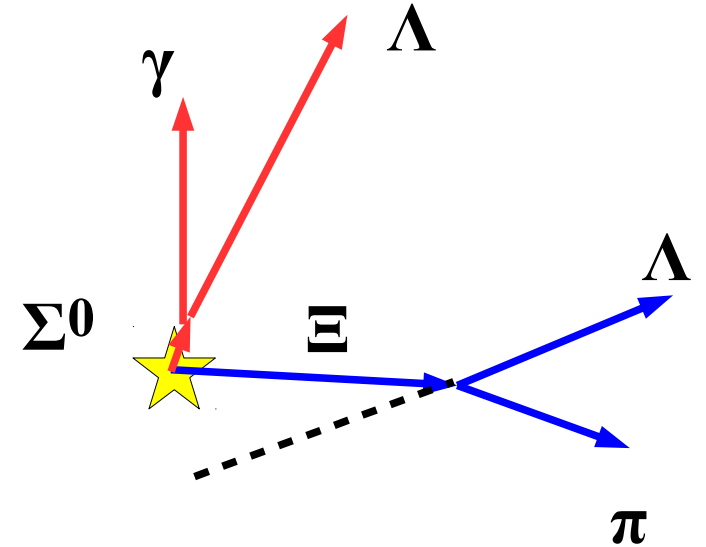
Additional Source

■ Feed down effects

$$C_{\text{corr}}(Q) = 1 + \lambda(C_{\text{bare}}(Q) - 1)$$

λ = Purity of $\Lambda\Lambda$ pair

- Short-lived Y^* \rightarrow mod. of source fn.
- $\Xi \rightarrow \Lambda\pi$ can be excluded ($c\tau=8.71$ cm)
- $\Sigma^0 \rightarrow \Lambda\gamma$ is difficult to reject
- Data based purity $\lambda=(0.67)^2$
 $\Sigma^0/\Lambda=0.278$ (p+Be, 28.5 GeV/c) *Sullivan et al. ('87)*
 $\Xi/\Lambda = 15\%$ (RHIC)



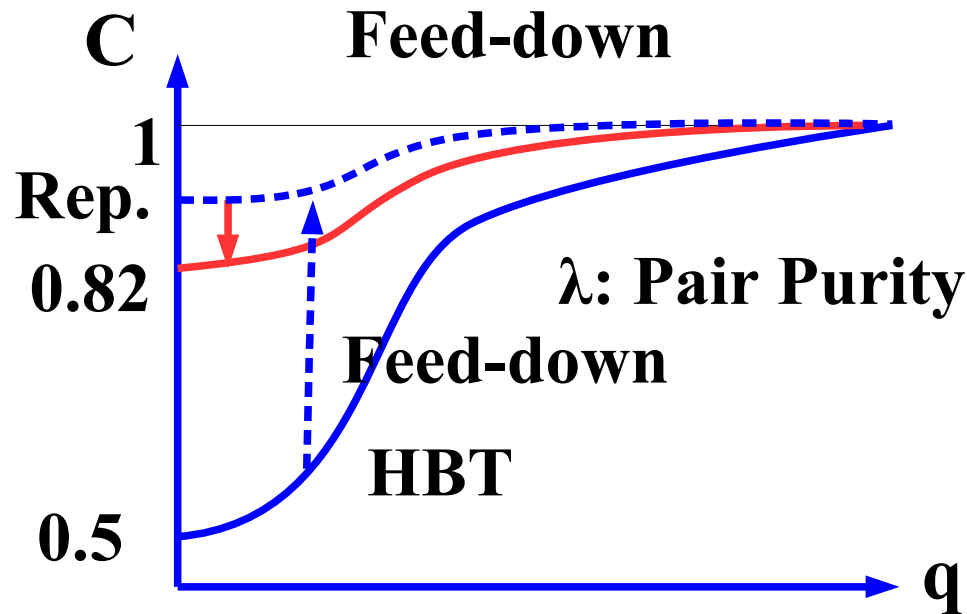
■ “Residual” source

- High-momentum tail $\rightarrow R_{\text{res}} \sim 0.5$ fm (STAR collab.)

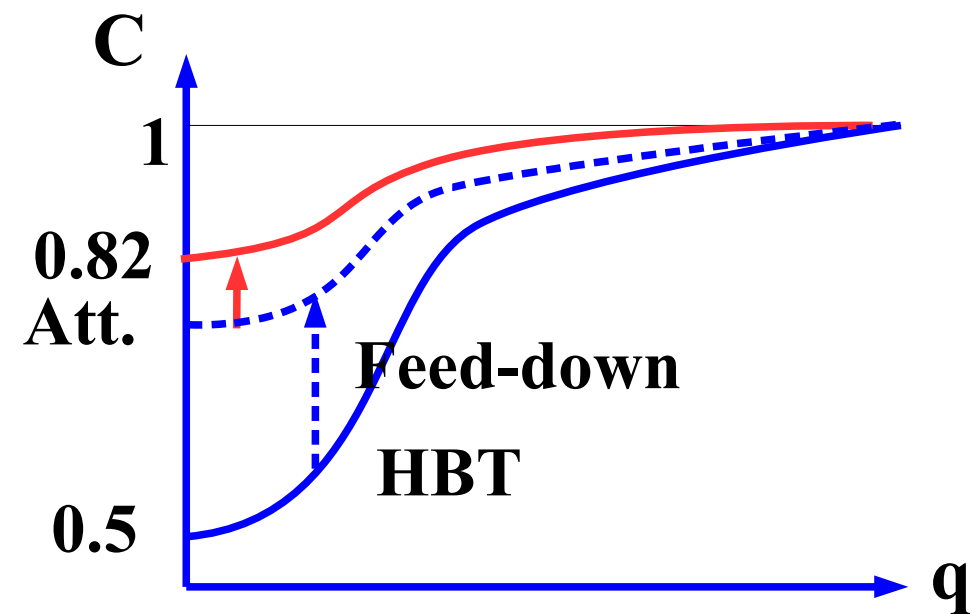
Feed-Down Effects & Residual Source

- Correlation Fn. w/ Feed-down & Residual source effects.

$$C_{\text{corr}}(q) = 1 + \lambda(C_{\text{bare}}(q) - 1) + a_{\text{res}} \exp(-4r_{\text{res}}^2 q^2)$$



STAR:
 $\lambda \sim 0.18$ (free para.)



Morita et al. (MFO15):
 $\lambda \sim 0.45$

$\Sigma^0/\Lambda=0.278$ (p+Be, 28.5 GeV/c)
Sullivan et al. ('87)
 $\Xi/\Lambda = 15\%$ (RHIC)

AO, Morita, Mihayara, Hyodo ('16)

*Pair purity (λ) should be determined experimentally !
 Puzzle: Residual source*

New Data from LHC-ALICE

V. M. Sarti (ALICE Collab.), MESONS 2018

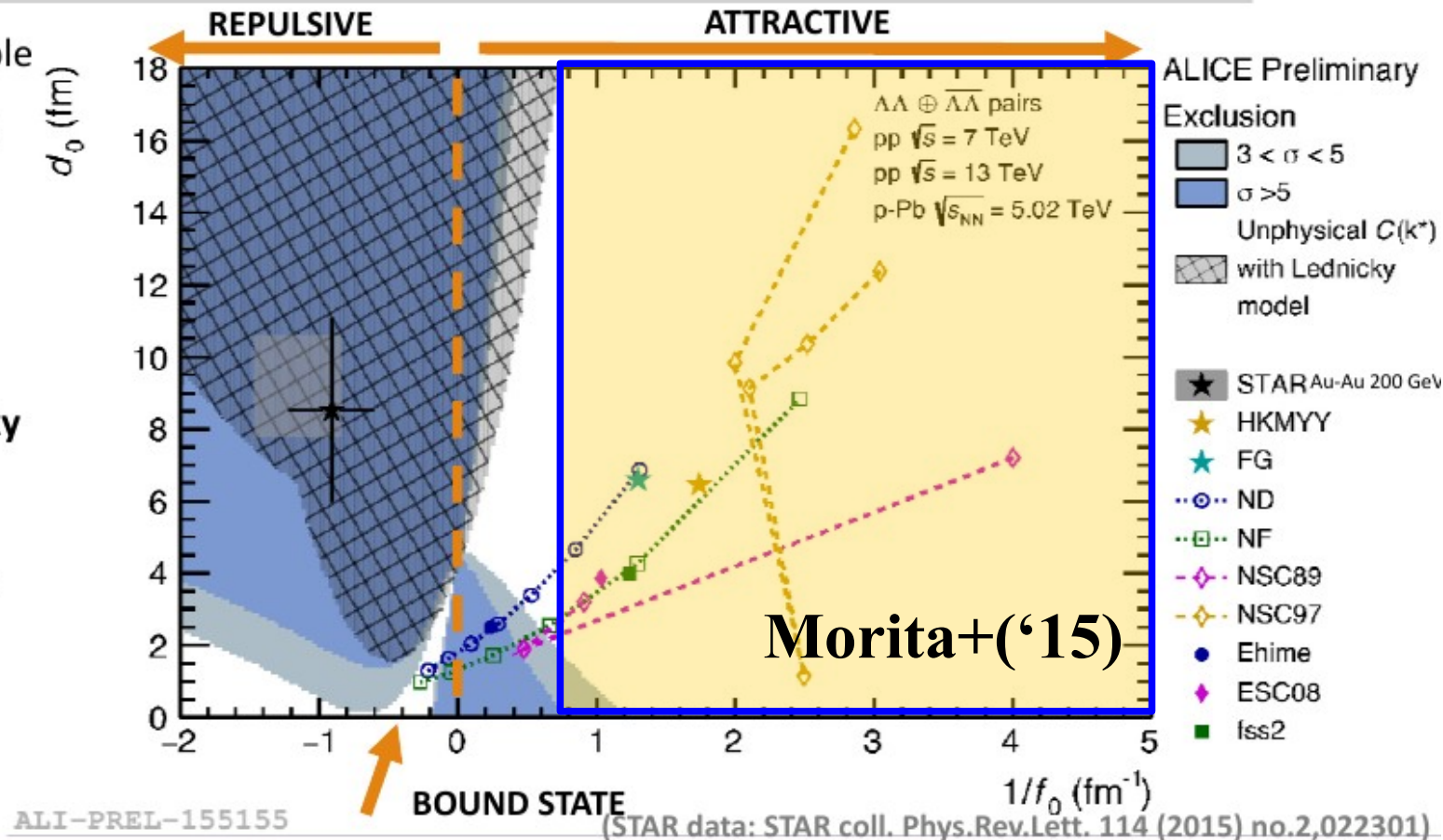


ALICE

Λ - Λ Correlations: Combined Exclusion Plot



- Combination of all available datasets: **pp 7 TeV, pp 13 TeV, p-Pb 5.02 TeV**
- Test of the **agreement between data and the prediction by the Lednicky model by $n\sigma$**
- Small source size, large d_0 and negative f_0 limit the prediction power of Lednicky



$\Omega^- p$ correlation

Ω^- p interaction

■ Ω^- : quark content=sss, $J^\pi=3/2^+$, $M=1672$ MeV

■ Ω^- p bound state as a $S=-3$ dibaryon ?

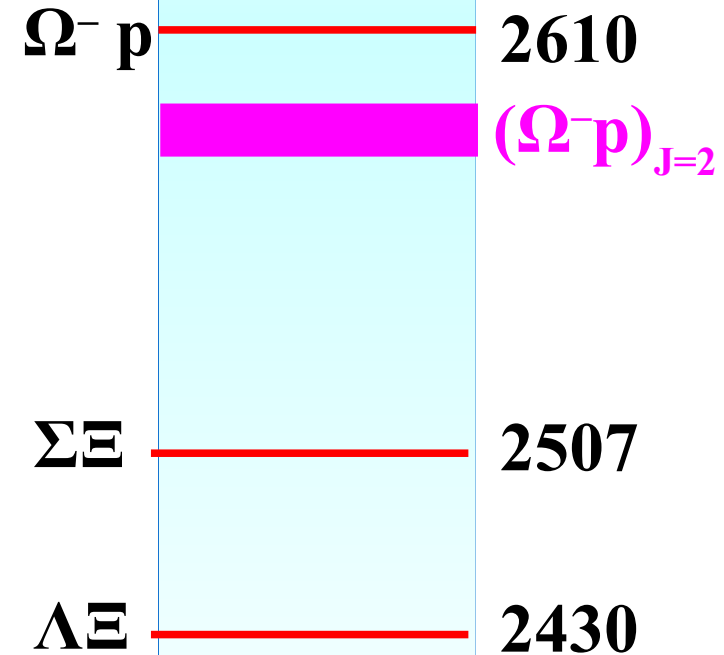
F.Etminan et al. (HAL QCD Collab.),

NPA928('14)89.

- No quark Pauli blocking in ΩN , $H=uuddss$, and $d^*=\Delta\Delta$ channels.
Oka ('88), Gal ('16)

- $J=2$ state (5S_2) couples to Octet-Octet baryon pair only with $L \geq 2$
→ Small width is expected.
Etminan et al. (HAL QCD)('14)

- Correlation is measurable at RHIC !
Neha Shah (STAR), private commun.



*Let us try to discover the first(?) dibaryon (after deuteron) !
(First dibaryon with $S<0$!)*

$\Omega^- p$ potential from lattice QCD

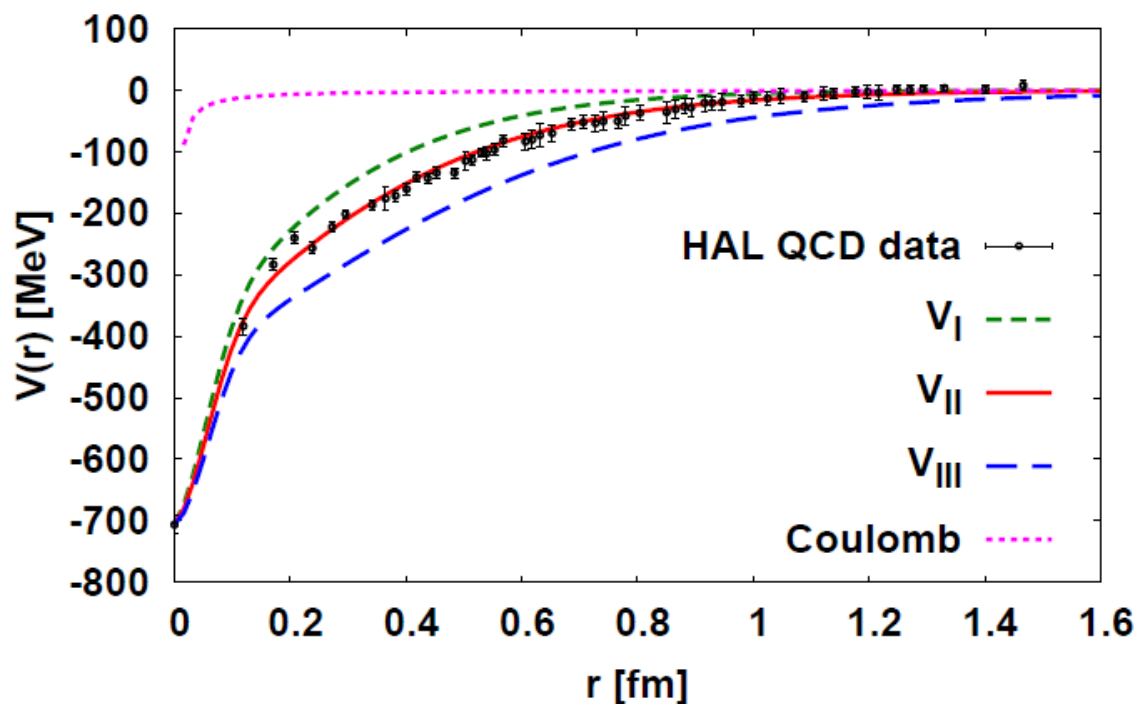
- Lattice QCD predicts $\Omega^- p$ bound state at large quark mass, $m_\pi=875$ MeV (B.E. ~ 19 MeV) in 5S_2 channel.

F. Etminan et al. (HAL QCD Collab.), NPA928('14)89.

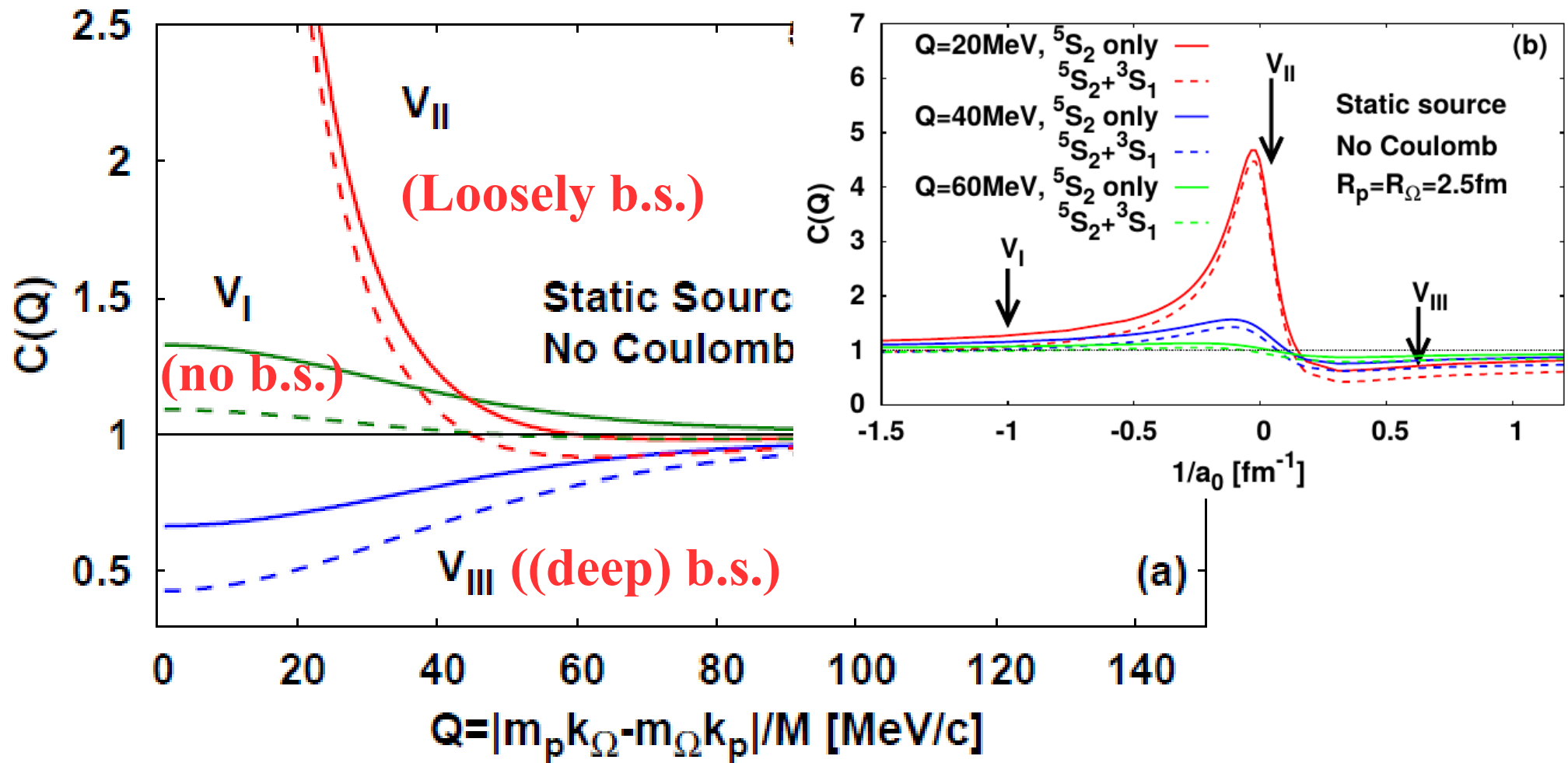
- Extrapolation to physical quark mass

- VI \rightarrow Weaker potential (no b.s.)
- VII \rightarrow Same potential (shallow b.s.)
- VIII \rightarrow Stronger potential (deep b.s.)

| Spin-2 $N\Omega$ Potentials | | V_I | V_{II} | V_{III} |
|-----------------------------|-----------------------|-------|----------|-----------|
| | E_B [MeV] | – | 0.05 | 24.8 |
| without Coulomb | a_0 [fm] | –1.0 | 23.1 | 1.60 |
| | r_{eff} [fm] | 1.15 | 0.95 | 0.65 |
| | E_B [MeV] | – | 6.3 | 26.9 |
| with Coulomb | a_0 [fm] | –1.12 | 5.79 | 1.29 |
| | r_{eff} [fm] | 1.16 | 0.96 | 0.65 |



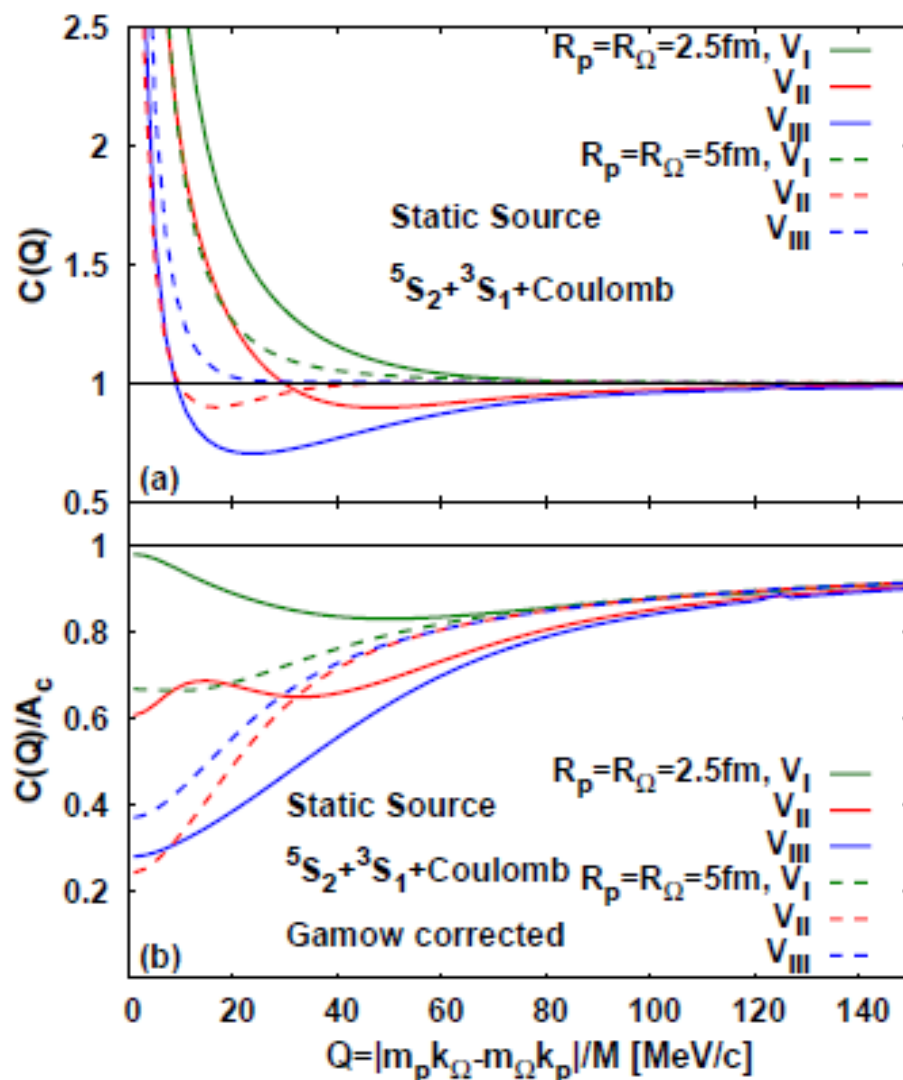
Ω - p correlation



(w/o Coulomb, Strong absorption at $r < 2$ fm in 3S_1 (decay to 8-8 in S-wave))

K. Morita, AO, F. Etminan, T. Hatsuda, PRC94('16)031901(R) [arXiv:1605.06765 [hep-ph]]

Ω - p correlation w/ Coulomb

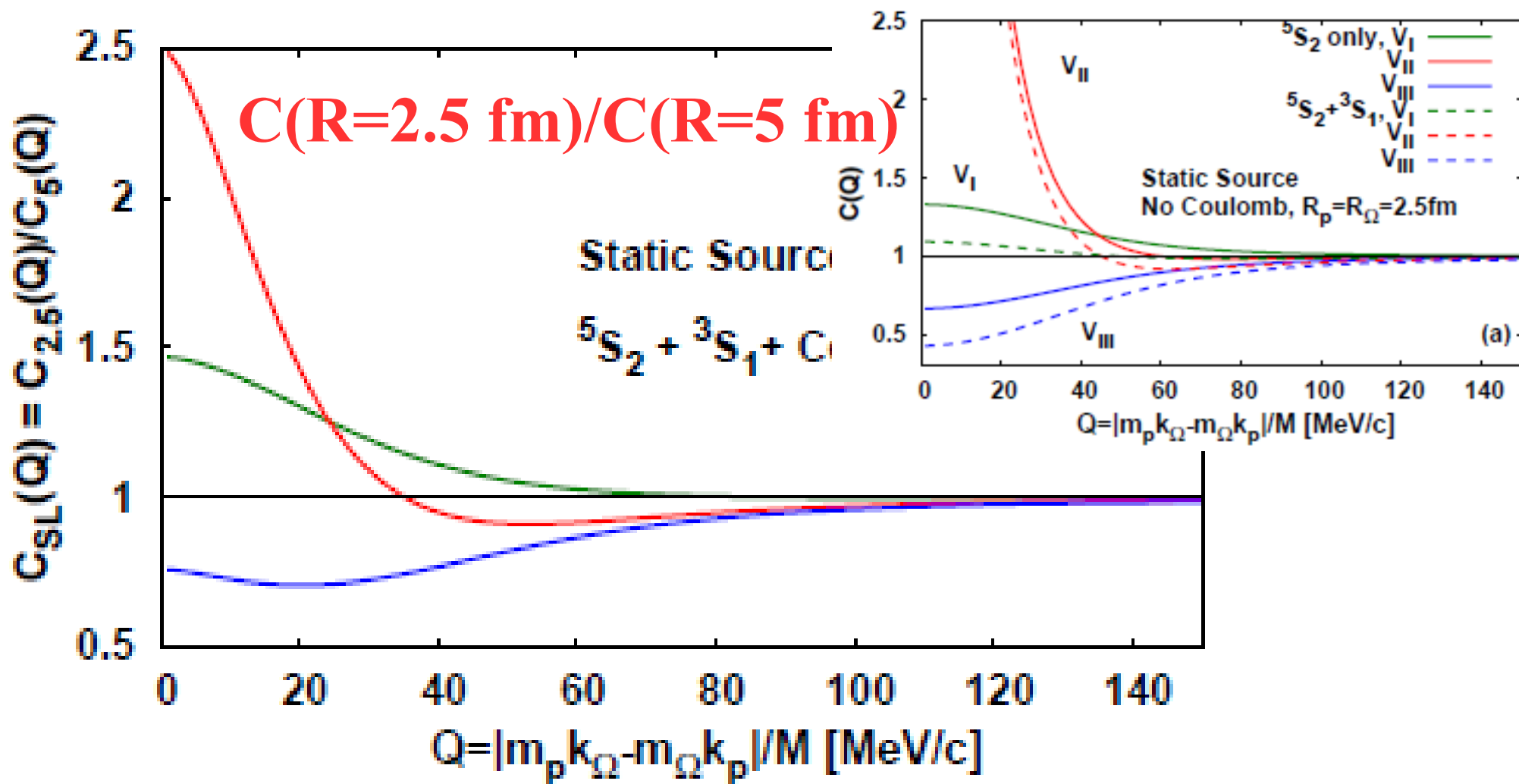


With Coulomb

Coulomb + Gamow corr.

Coulomb potential washes out the features of V_I , V_{II} , V_{III} , and Gamow correction is not enough.

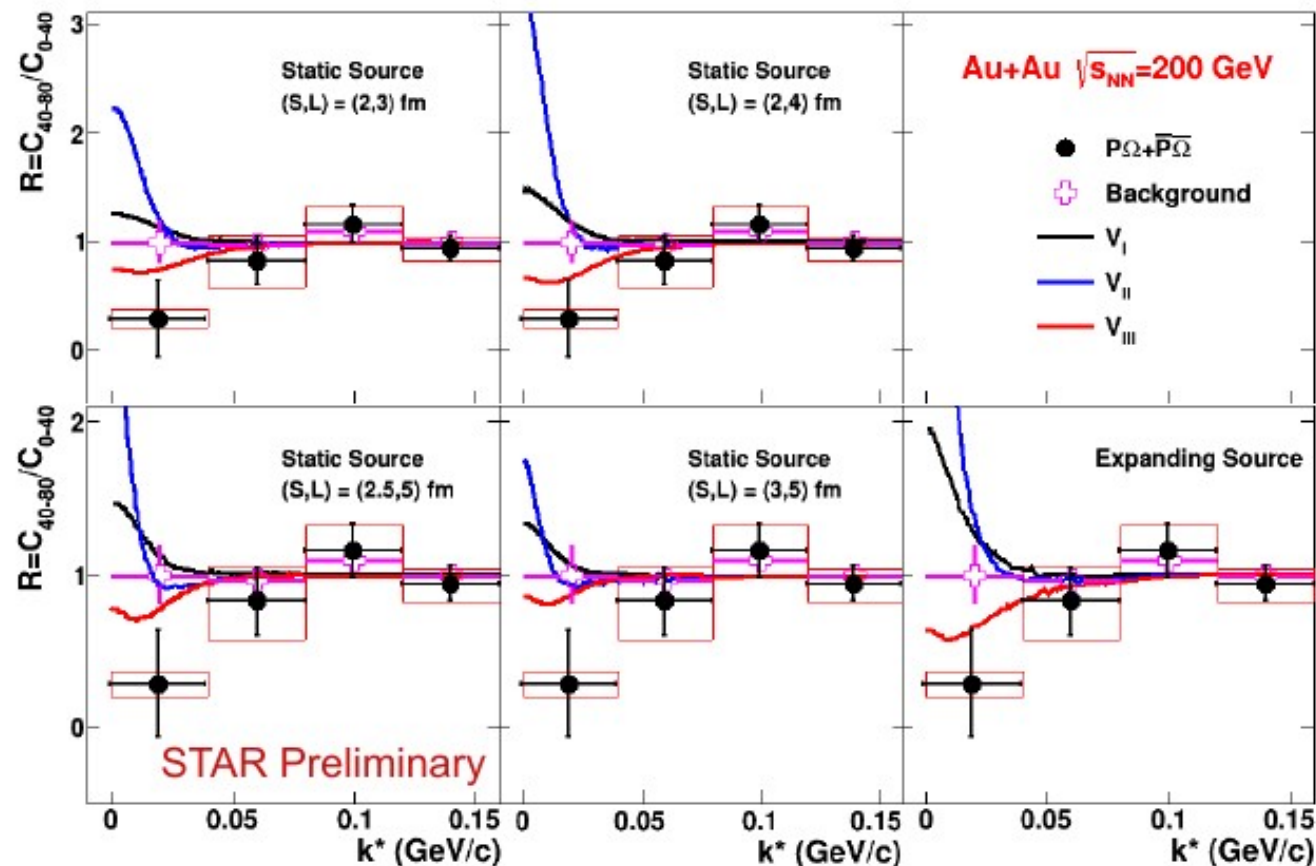
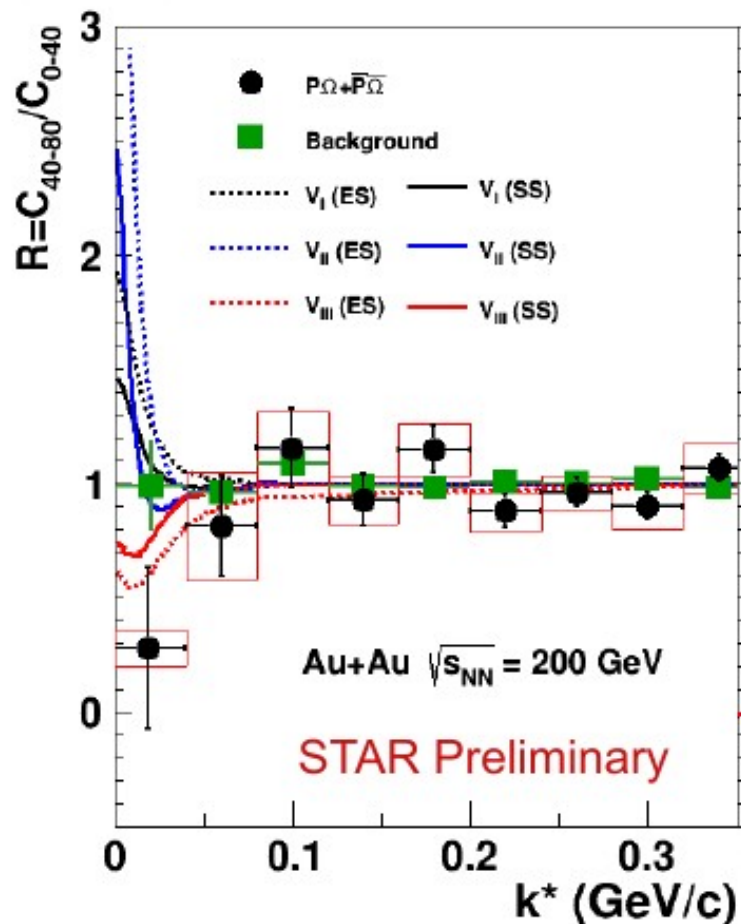
Ω - p correlation: Small / Large Ratio



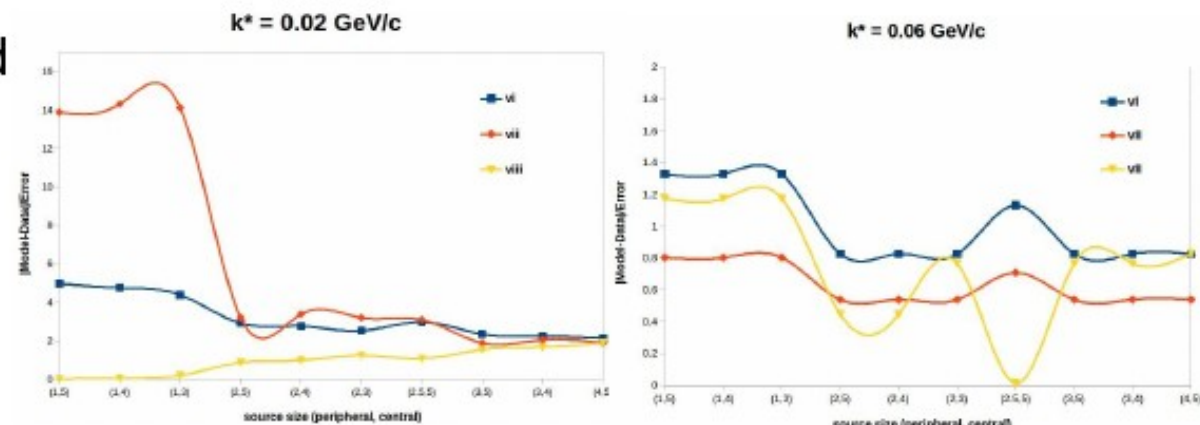
By taking small ($R=2.5 \text{ fm}$) / large ($R=5 \text{ fm}$) ratio, we approximately see the corr. fn. w/o Coulomb !



Source Size Analysis on $p\Omega$ Correlation Function



The source parameters are derived from other PID and centrality dependence measurements at RHIC



K^-p correlation

$K^- p$ interaction

- $\Lambda(1405)$ $\bar{K}N$ quasi-bound state

*Dalitz, Tuan ('60); Koch ('94); Kaiser, Siegel, Weise ('95);
AO, Nara, Koch ('97)*

- Positive scattering length in K^- atoms

*M.Iwasaki et al. PRL78('97)3067;
M.Bazzi et al. [SIDDHARTA Collab.],
PLB704('11)113.*

- Kaonic nuclei ?

*Nogami ('63); Akaishi, Yamazaki ('02);
Shevchenko, Gal, Mares ('07); Ikeda, Sato ('07);
Dote, Hyodo, Weise ('09)*

→ Needs precise info. on $\bar{K}N$ int.

- Scattering amplitude and Potential fitting scattering and SIDDARTA data in chiral approach

*Ikeda, Hyodo, Weise ('11,'12),
Miyahara, Hyodo ('16)*

$K^- p$

1435

$\Lambda(1405)$

$\Sigma(1385)$

$\pi\Sigma$

1325

How about $K^- p$ correlation ?

$K^- p$ scattering and $K^- p$ correlation

- $K^- p$ scattering: Plane wave + Outgoing spherical wave

$$\Psi_{K^- p}^{(+)}(\mathbf{r}) \rightarrow \exp(i\mathbf{q} \cdot \mathbf{r})\chi(K^- p) + \frac{e^{iqr}}{r} \left[f_{K^- p \rightarrow K^- p} \chi(K^- p) + f_{K^- p \rightarrow \bar{K}^0 n} \chi(\bar{K}^0 n) \right]$$

- $K^- p$ correlation: Plane wave + Incoming spherical wave

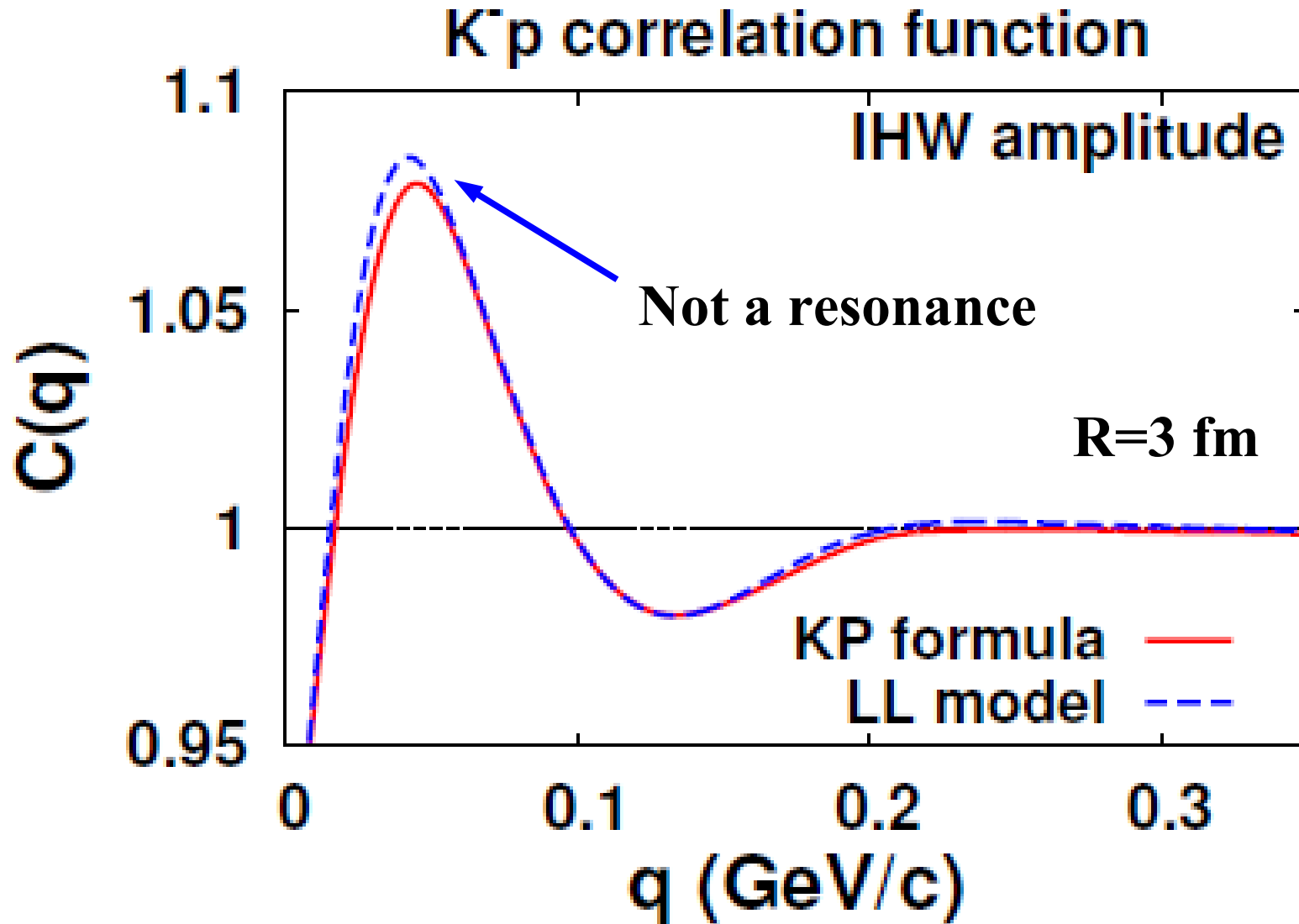
$$\Psi_{K^- p}^{(-)}(\mathbf{r}) \rightarrow \exp(i\mathbf{q} \cdot \mathbf{r})\chi(K^- p) + \frac{e^{-iqr}}{r} \left[\tilde{f}_{K^- p \rightarrow K^- p} \chi(K^- p) + \tilde{f}_{K^- p \rightarrow \bar{K}^0 n} \chi(\bar{K}^0 n) \right]$$

$$\tilde{f}_{K^- p \rightarrow K^- p} = \frac{\tilde{\mathcal{S}} - 1}{2iq} \neq f_{K^- p \rightarrow K^- p}, \quad \tilde{\mathcal{S}} = \left[\frac{\mathcal{S}_{I=0}^{-1} + \mathcal{S}_{I=1}^{-1}}{2} \right]^{-1}$$

$\Psi^{(-)} \neq (\Psi^{(+)})^* \rightarrow K^- p$ correlation probes

different combination of $I=0, 1$ from $K^- p$ scattering

K^-p correlation



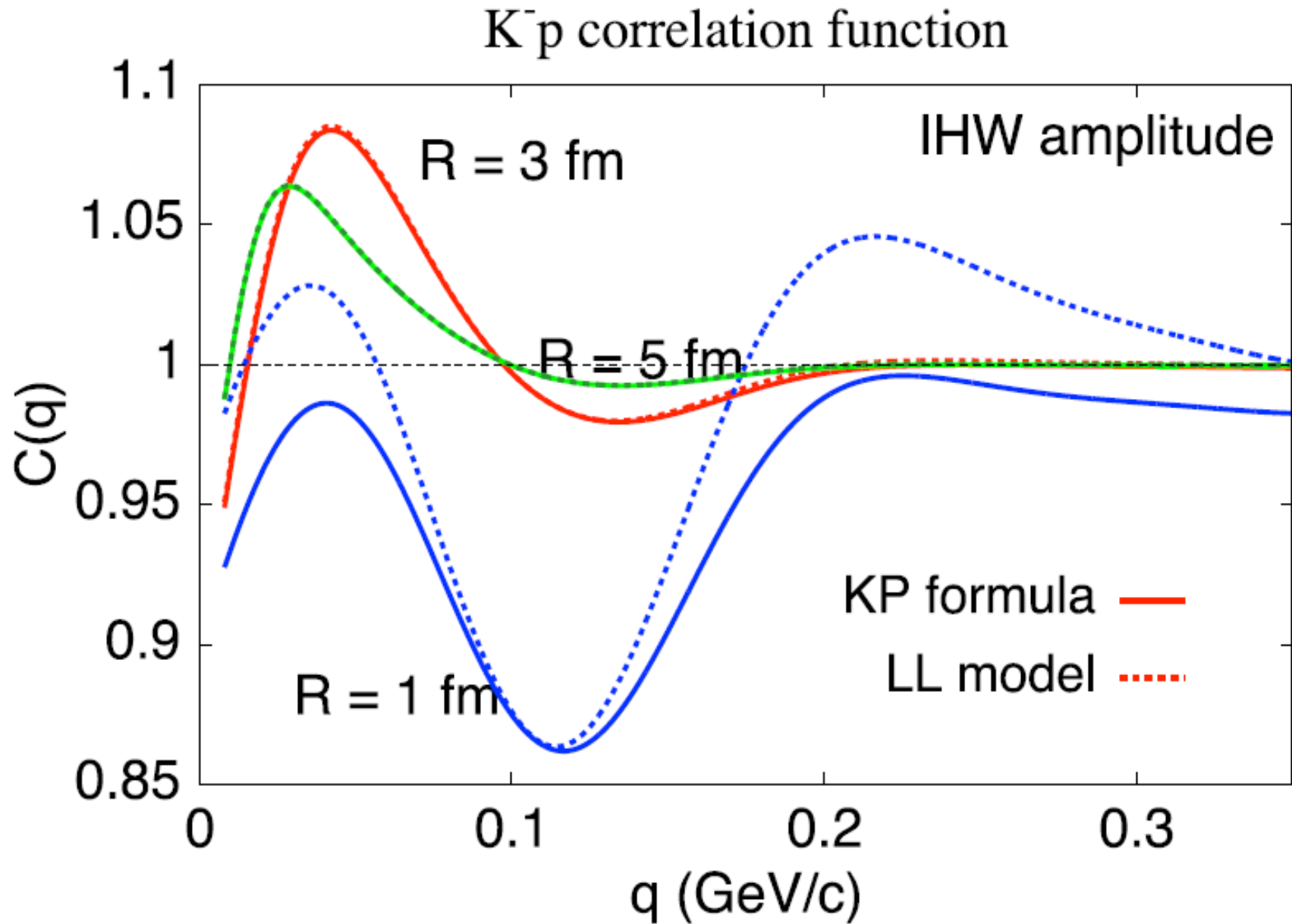
Calc: AO, K. Morita, K. Miyahara, T. Hyodo, NPA954 ('16)294 ($\Lambda\Lambda$, K^-p).

Potential: K. Miyahara, T. Hyodo, PRC93 ('16) 015201.

Amplitude: Y. Ikeda, T. Hyodo, W. Weise, PLB 706 ('11) 63; NPA 881 ('12) 98.

Fitting SIDDHARTA data: M. Bazzi et al. [SIDDHARTA Collab.], PLB 704 ('11) 113.

K^-p correlation



$\psi^{(-)}$: Another source of fake peak ?

K. Miyahara

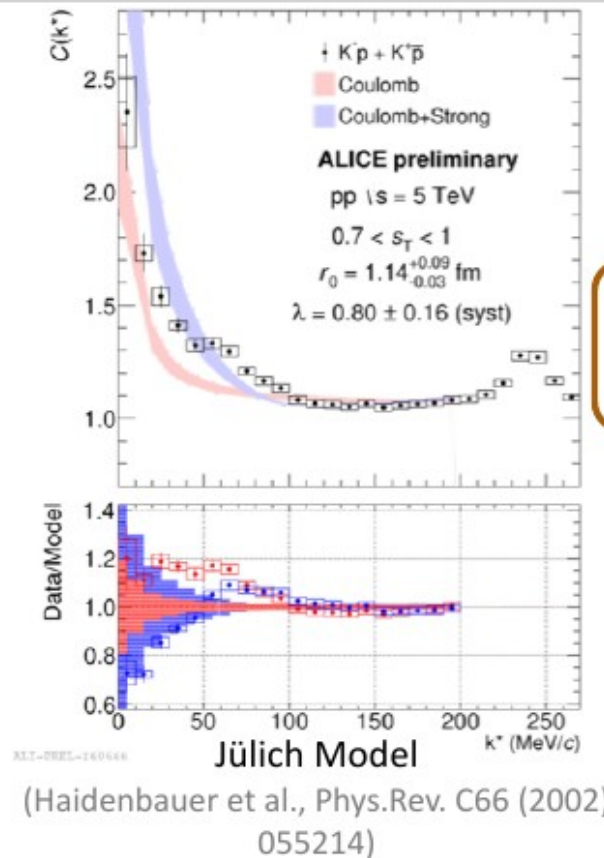
New Data from LHC-ALICE

V. M. Sarti (ALICE Collab.), MESONS 2018

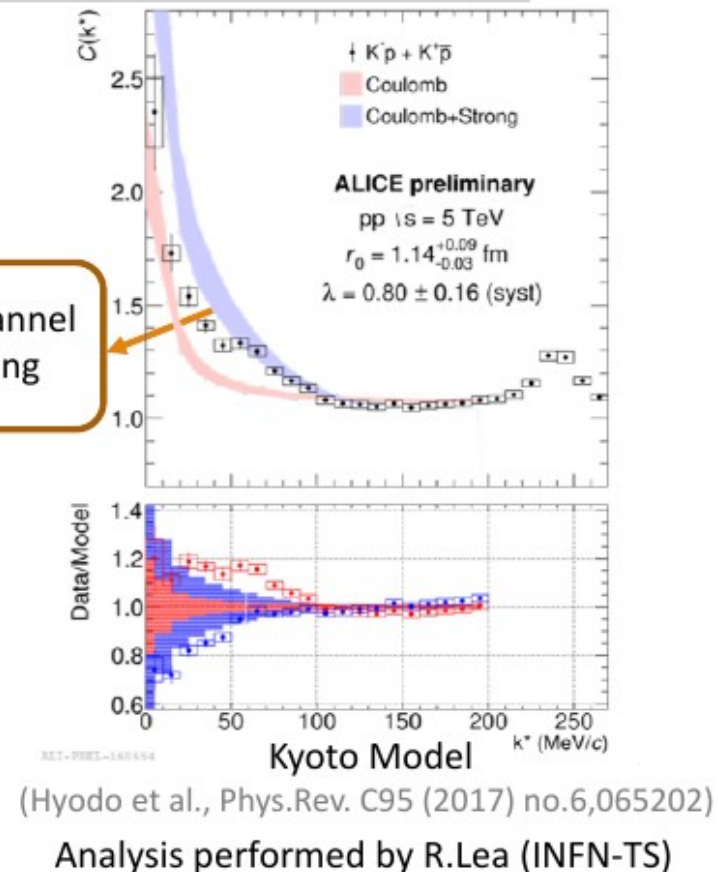
ALICE K^- -p Correlation-pp 5 TeV, 7 TeV, 13 TeV



- Analysis on datasets:
 - pp 5 TeV, 7 TeV, 13 TeV
- Short range $\bar{K}N$ interaction:
 - $\Lambda(1405)$, kaonic atoms and kaonic clusters
- Kaonic atoms and scattering data

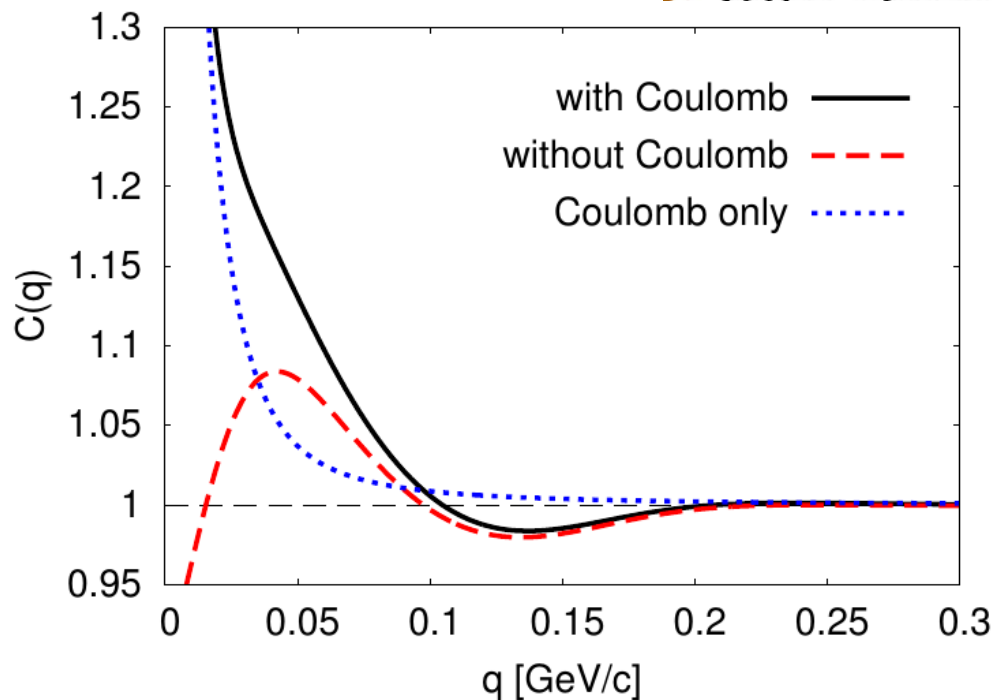


\bar{K}^0n channel opening

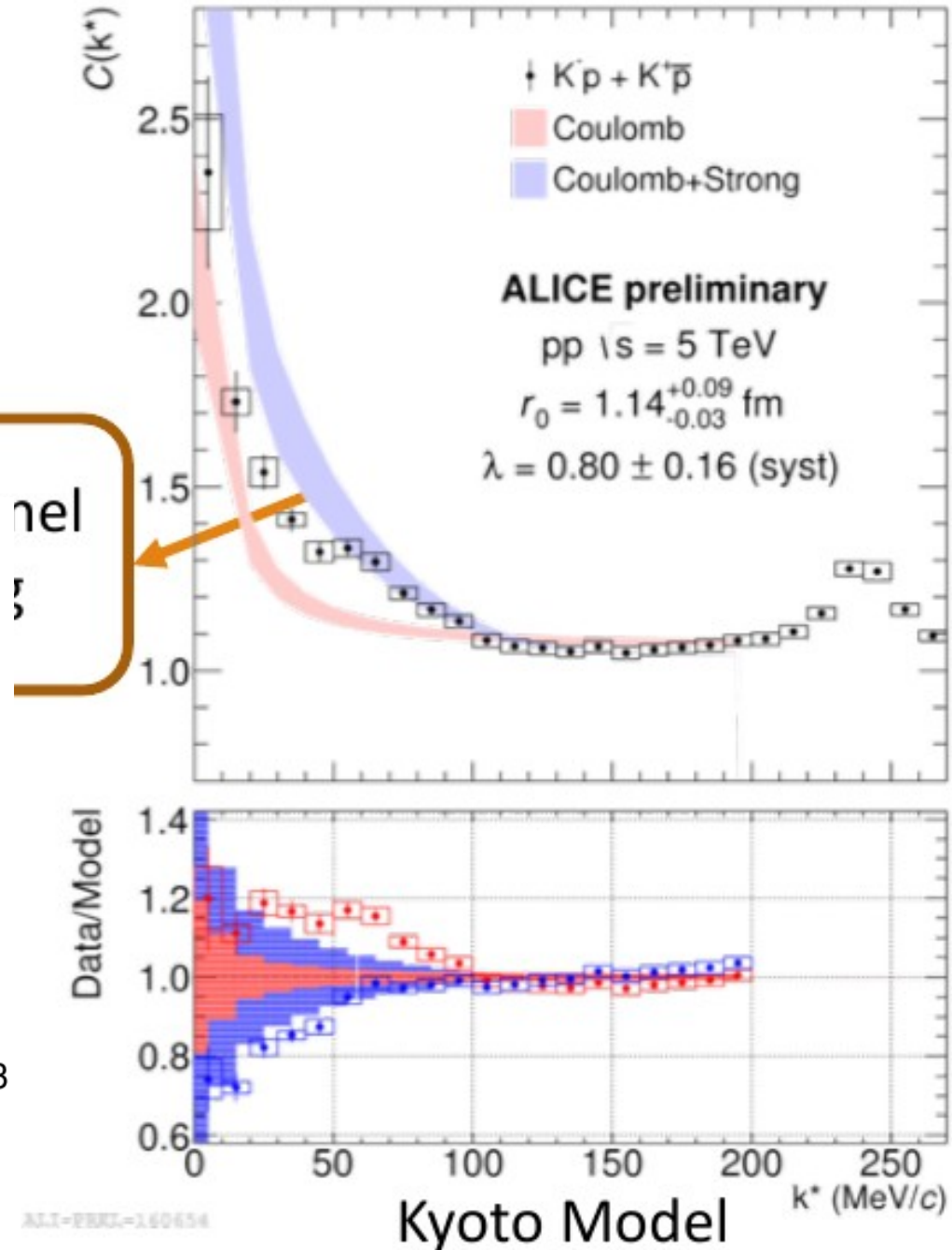


New Data from LHC-ALICE

- Data shows shoulder at $q \sim 50$ MeV
 - Threshold effect ?
 - Interference of $I=0$ and $I=1$ amplitudes ? (Cho et al. ('17))



Cho et al. (ExHIC Collab.) ('17)



ALICE-PHOS-140654

Kyoto Model

Coupled Channels Effects Revisited

■ J. Haidenbauer (arXiv:1808.05049)

- Coupled channels effects strongly modifies the correlation function of $K^- p$ and $\Xi^- p$.
- Threshold difference needs to be taken care of.
- Source of other channels need to be added.
- No Coulomb, No Flow, Gaussian source with a fixed size, ...

- Two channel closure approximation is used in our previous works, and source function of other channels were missed (forgotten to add).

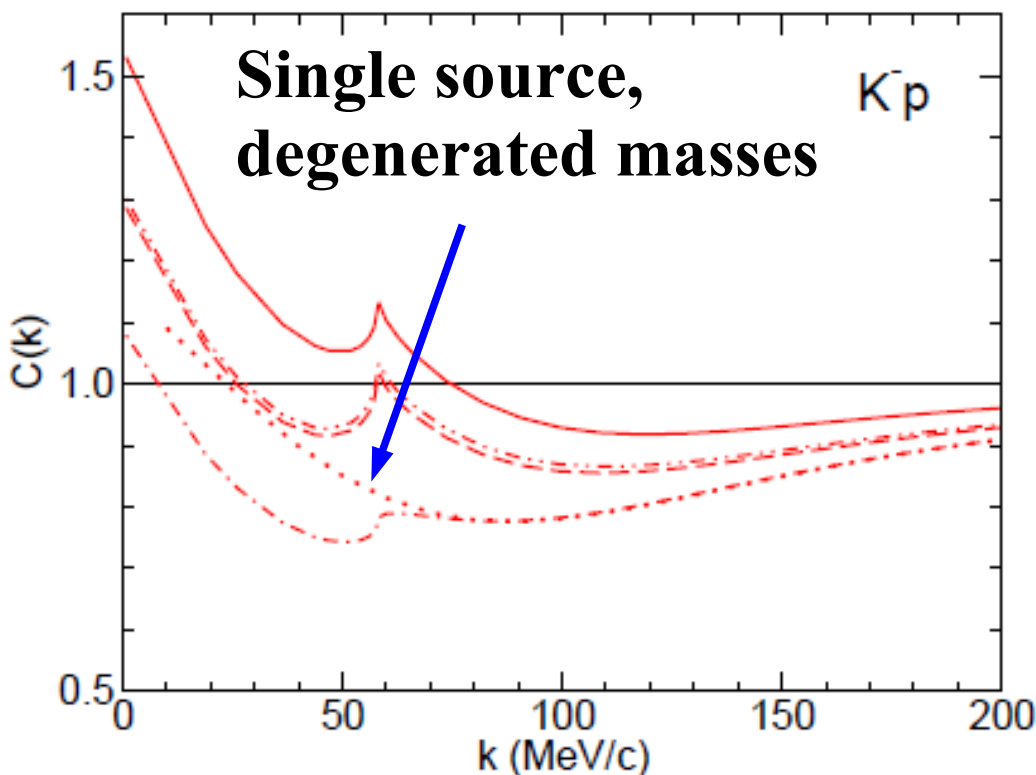
Cho et al. ('17), AO et al. ('16)

$$\tilde{S} = (S_{2 \times 2})^{-1} \neq (S^\dagger)_{2 \times 2}$$

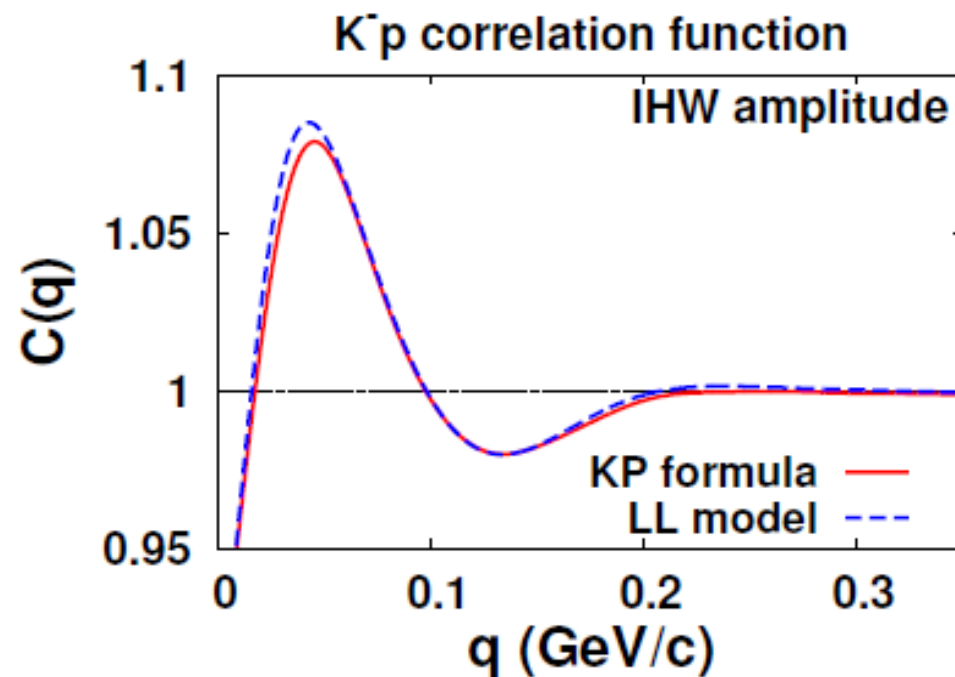
→ We need to revisit $K^- p$ correlation ...

Coupled Channels Effects

- Results by J. Haidenbauer (arXiv:1808.05049) seem to be different from our results (AO et al. ('16))
→ Coupled channels effect is strong ...



Haidenbauer, arXiv:1808.05049



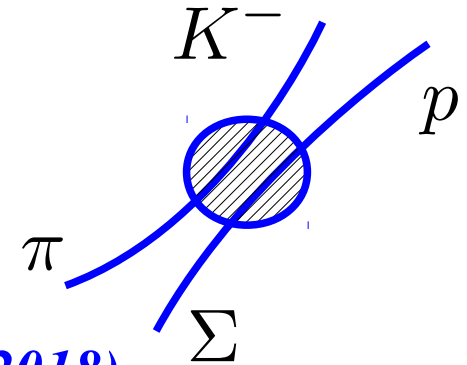
AO, Morita, Mihayara, Hyodo ('16)

Correlation Function with Coupled Channels

- Contribution of the source in other channels

$$C_\alpha(\mathbf{q}) = \sum_\beta \int d\mathbf{r} S_\beta(\mathbf{r}) \left| \psi_{\beta\alpha}^{(-)}(\mathbf{r}, \mathbf{q}) \right|^2$$

Source fn.



- Asymptotic wave function

K. Miyahara, T. Hyodo, W. Weise, Phys. Rev. C 98, 025201 (2018), arXiv:1804.08269.

$$|\Psi_\alpha^{(\pm)}\rangle = \sum_\beta \psi_{\beta\alpha}^{(\pm)} |\beta\rangle$$

No outgoing w.f. for $\beta \neq \alpha$

$$\psi_{\beta\alpha}^{(+)}(r) \rightarrow \frac{-1}{2ik_\alpha} \left[\delta_{\beta\alpha} \frac{e^{-ik_\beta r}}{r} - \sqrt{\frac{v_\alpha}{v_\beta}} S_{\beta\alpha} \frac{e^{ik_\beta r}}{r} \right] \quad (v_\alpha = k_\alpha/\mu_\alpha)$$

- Asymptotically outgoing wave function

$$\psi_{\beta\alpha}^{(-)}(r) = \frac{1}{k_\alpha} \sum_\gamma \psi_{\beta\gamma}^{(+)}(r) S_{\gamma\alpha}^\dagger k_\gamma \sqrt{\frac{v_\alpha}{v_\gamma}} \rightarrow \frac{1}{2ik_\alpha} \left[\delta_{\beta\alpha} \frac{e^{ik_\beta r}}{r} - \sqrt{\frac{v_\alpha}{v_\beta}} S_{\beta\alpha}^\dagger \frac{e^{-ik_\beta r}}{r} \right]$$

- Difference of \tilde{S} and S^\dagger may lead to different results....

E^-p correlation

Relevance of ΞN interaction to physics

- H-particle: 6-quark state (uuddss) may be realized as a loosely bound state of ΞN ($I=0$)

K. Sasaki et al. (HAL QCD, '16, '17)

- Repulsive ΞN interaction ($I=1$) may help to support $2 M_{\odot}$ Neutron Star

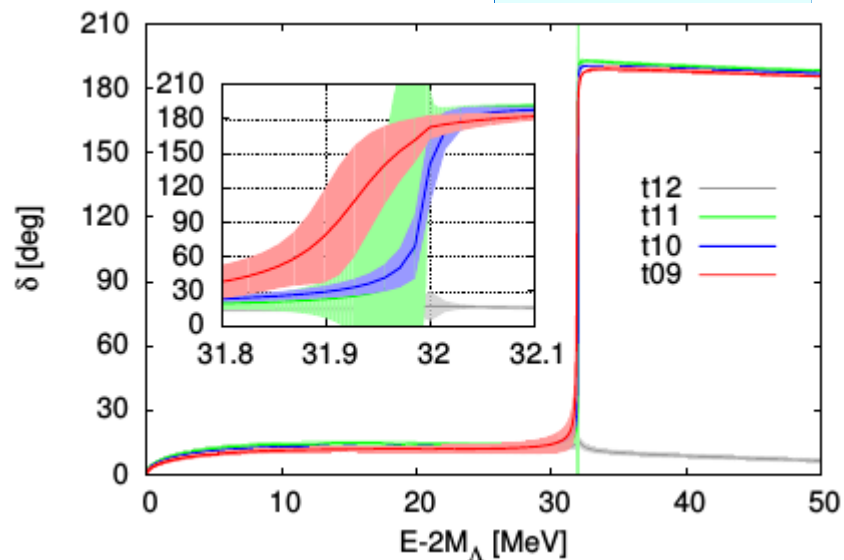
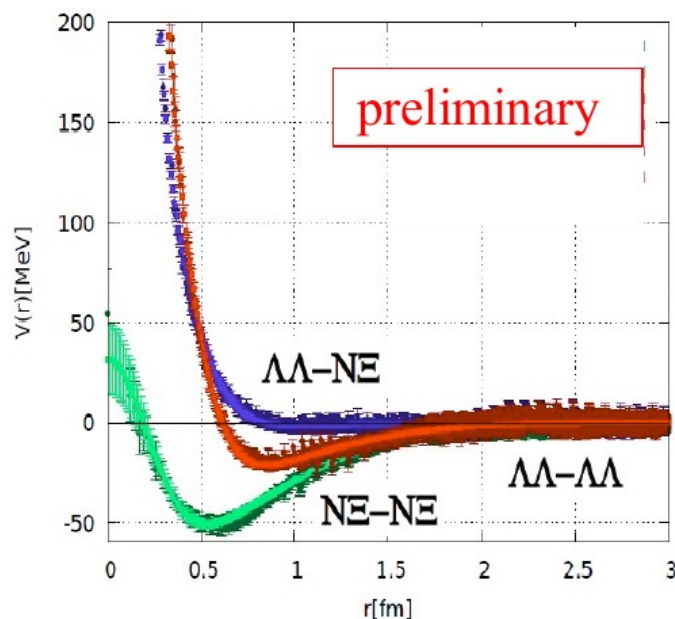
Weissborn et al., NPA881 ('12) 62.

ΞN

E522 ('07)

$\Lambda\Lambda$

HAL ('16)



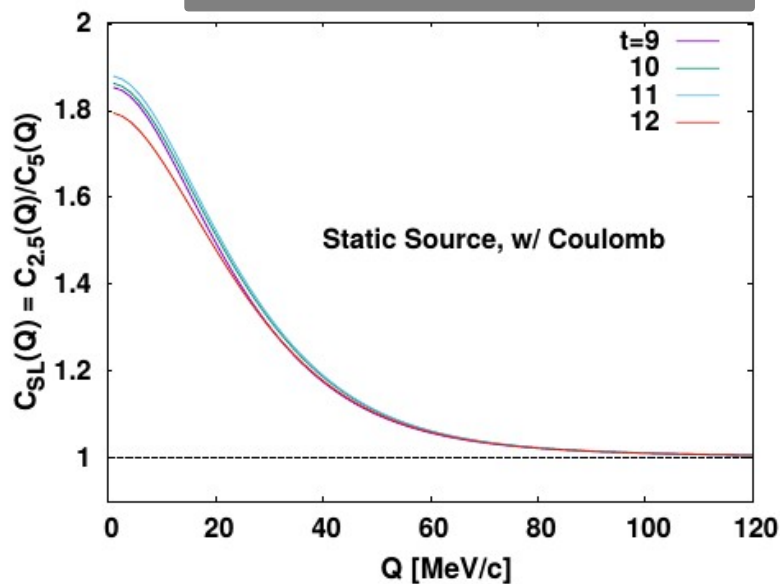
K. Sasaki et al. (HAL QCD Collab.), EPJ Web Conf. 175 ('18) 05010.

$\Xi^- p$ correlation

- Prediction of the correlation function by using ΞN potential (HAL QCD Collab.) + Coulomb potential

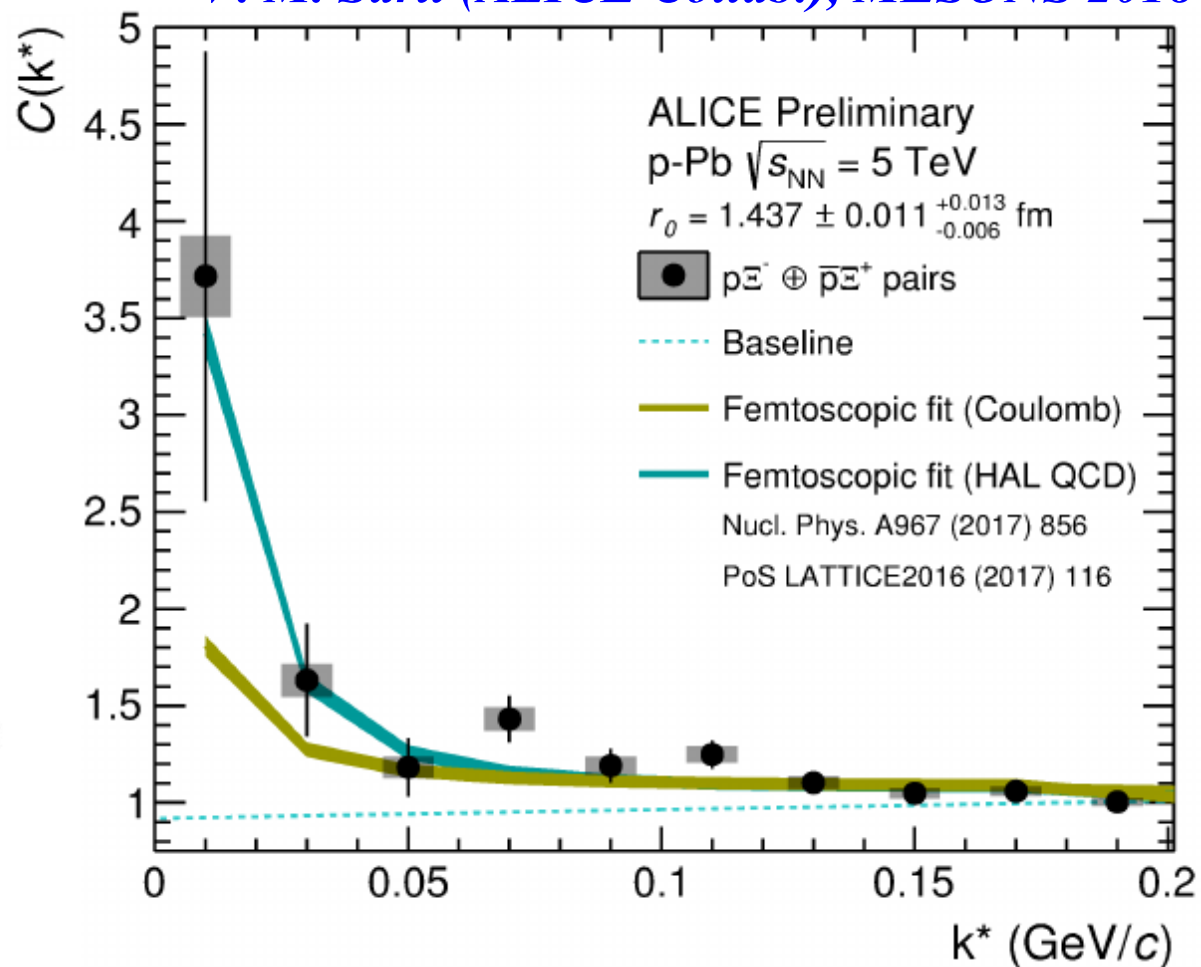
$$|\psi|_{\text{spin av.}}^2 = \frac{1}{2} \sum_{I=0,1} \left[\frac{1}{4} |\psi_I^{J=0}|^2 + \frac{3}{4} |\psi_I^{J=1}|^2 \right]$$

HAL prediction is examined!



T. Hatsuda, K. Morita, AO, K. Sasaki, NPA967('17), 856.

V. M. Sarti (ALICE Collab.), MESONS 2018



Summary

- High energy collisions (incl. heavy-ion collisions) are hadron factories, and can be utilized to perform hadron physics as well.
- Hadron-Hadron correlation contains information on interactions.
 - Correlations in various pairs have been measured: $\pi\pi$, KK , pp , nn , $\bar{p}\bar{p}$, $\Lambda\Lambda$, Λp , $K^- p$, $\Omega^- p$, $\Xi^- p$, ...
 - When the pair purity is large enough, corr. fn. has sensitivity to hh interaction.
- Some of hh correlations have been discussed.
 - $\Lambda\Lambda$ correlation data constrain (a_0, r_{eff}) region of $\Lambda\Lambda$ interaction.
 - $\Omega^- p$ correlation suggests the existence of a $S=-3$ dibaryon.
 - $K^- p$ correlation shows “shoulder” structure.
 - $\Xi^- p$ correlation has examined the prediction by HAL QCD Collab.
- Many other type of pairs are waiting for us.

Challenge, Lessons, and Reflections

■ Challenge

- “Static Gaussian source + single channel problem” can be now done by experimentalists.
D. L. Mihaylov et al. EPJC78('18), 394.
- Coupled channels problem needs to be investigated further, including the effects of threshold difference.
- Source fn. needs to be studied more.
Combination with transport models
Gaussian source for pp and e^+e^- collisions ?

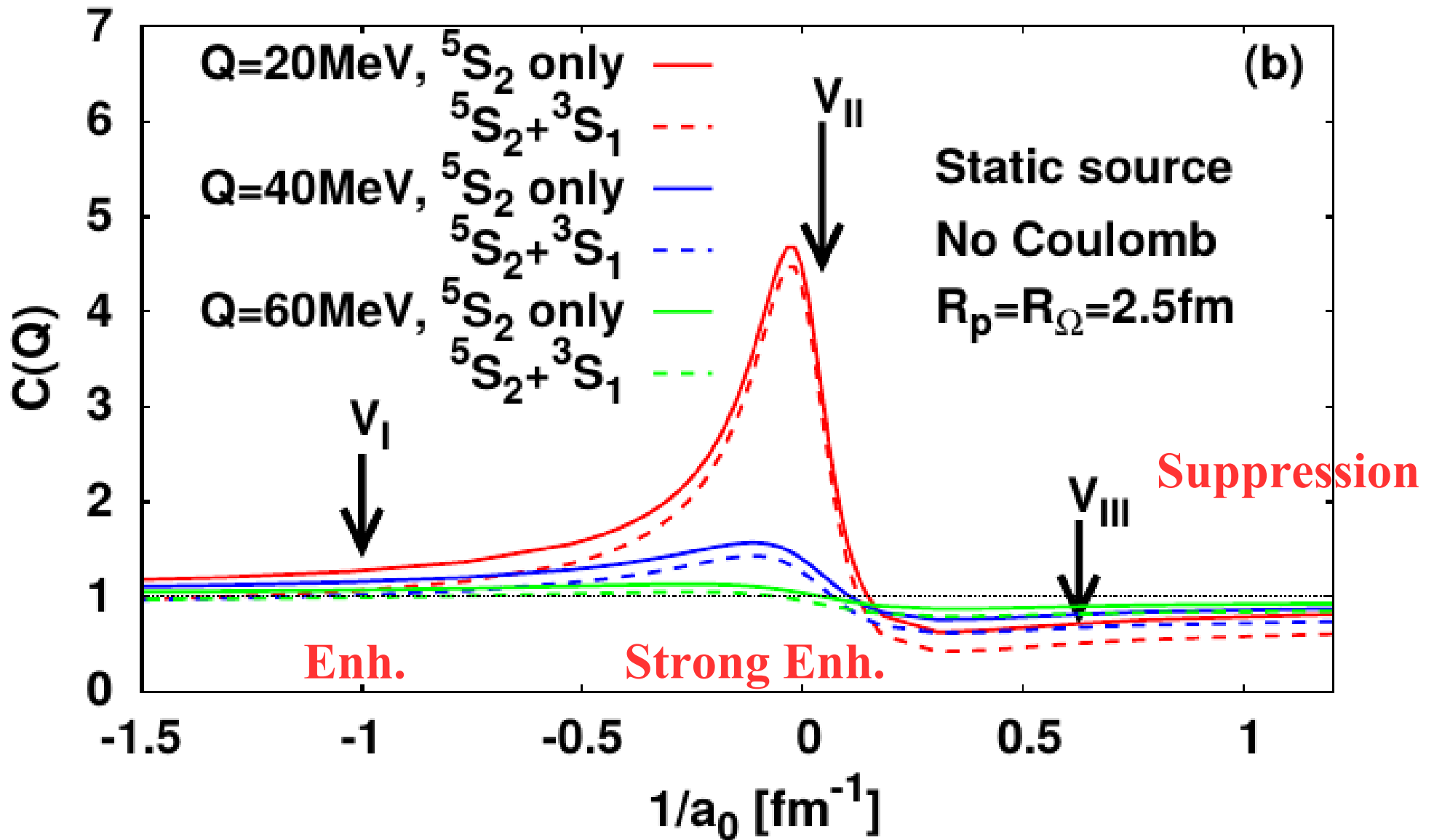
■ Lessons

- Let's draw some graph on observables which Experimentalists have not shown ! It can be a prediction !
- It can take 20 years before your prediction is examined.

- Reflections: We need to write “original” papers rather than proceedings.

Thank you !

Interaction Dependence of Correlation Function

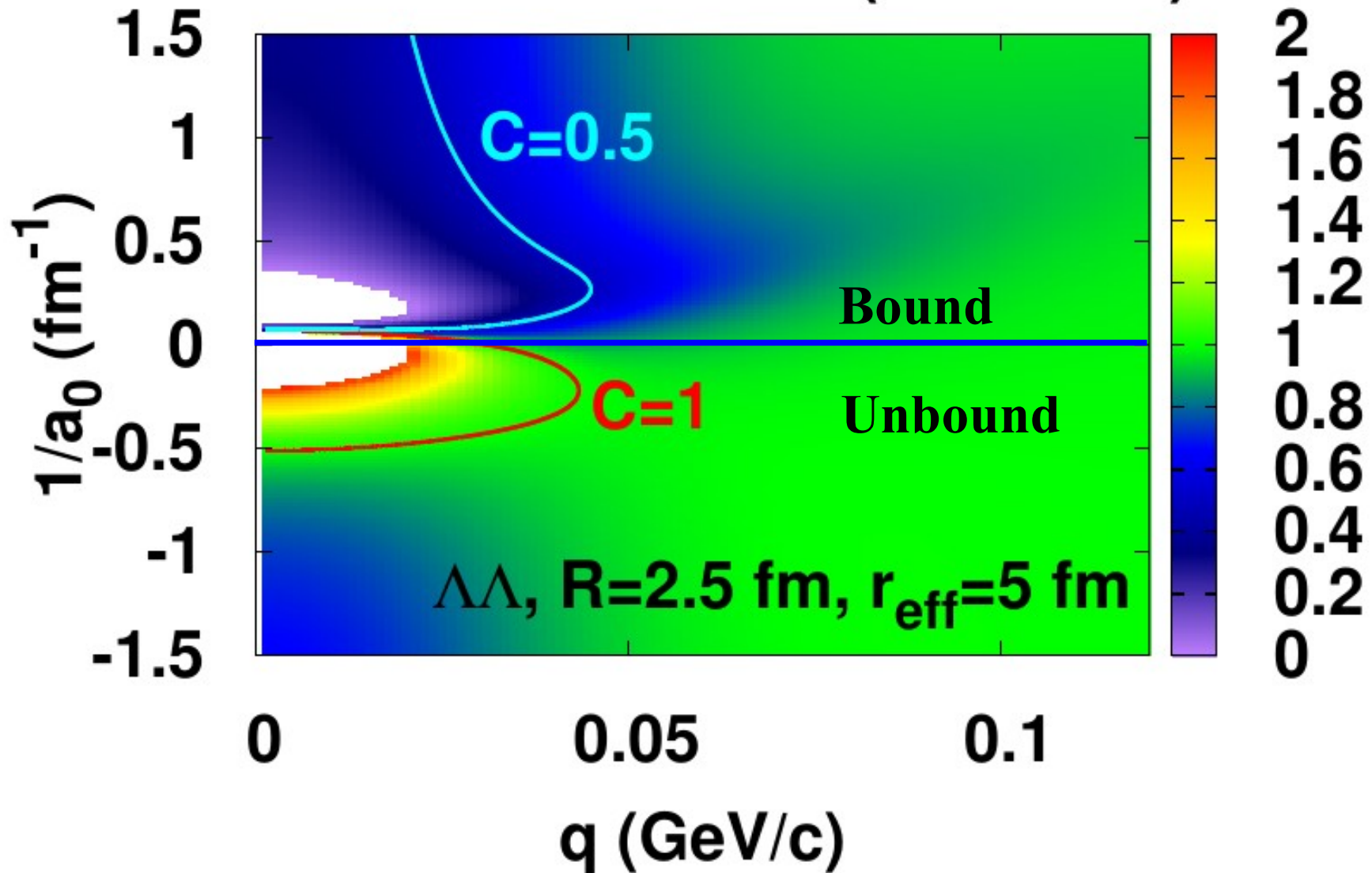


$$q \cot \delta = -1/a_0 + r_{\text{eff}} q^2/2 + O(q^4)$$

K. Morita, AO, F. Etminan, T. Hatsuda, PRC94('16)031901(R) [arXiv:1605.06765 [hep-ph]]

Interaction Dependence of Correlation Function

Correlation function (LL model)



$$q \cot \delta = -1/a_0 + r_{\text{eff}} q^2/2 + O(q^4)$$

AO, K. Morita, K. Miyahara, T. Hyodo, NPA954 ('16), 294.

Interaction dependence of CF

■ Scattering wave function w/ incoming plain wave

$$\varphi^{(+)}(\mathbf{r}) = e^{i\mathbf{q}\cdot\mathbf{r}} + \psi^{(+)}(r) - j_0(qr) \rightarrow e^{i\mathbf{q}\cdot\mathbf{r}} + f(q) \frac{e^{iqr}}{r}$$

$$\psi^{(+)}(r) \rightarrow \frac{e^{i\delta}}{qr} \sin(qr + \delta) = \frac{1}{2iqr} (S e^{iqr} - e^{-iqr}) \quad (S = e^{2i\delta})$$

■ Scatt. w. f. with outgoing plain wave boundary condition

$$\varphi^{(-)}(\mathbf{r}) = e^{i\mathbf{q}\cdot\mathbf{r}} + \psi^{(-)}(r) - j_0(qr) \rightarrow e^{i\mathbf{q}\cdot\mathbf{r}} + \tilde{f}(q) \frac{e^{-iqr}}{r}$$

$$\psi^{(-)}(r) \rightarrow \frac{e^{-i\delta}}{qr} \sin(qr + \delta) = \frac{1}{2iqr} \left(e^{iqr} - \frac{1}{S} e^{-iqr} \right)$$

$$\tilde{f} = - \frac{S^{-1} - 1}{2iq} (= f^* \quad \text{if } S^{-1} = S)$$

Lednický-Lyuboshits (LL) model

■ Lednický-Lyuboshits analytic model

- **Asymp. w.f. + Eff. range corr. + $\psi^{(-)} = [\psi^{(+)}]^*$**

$$\psi_{\text{asy}}(r) = \frac{e^{-i\delta}}{qr} \sin(qr + \delta) = \mathcal{S}^{-1} \left[\frac{\sin qr}{qr} + f(q) \frac{e^{iqr}}{r} \right]$$

$$\begin{aligned} \Delta C_{\text{LL}}(q) &= \int dr S_{12}(r) (|\psi_{\text{asy}}(r)|^2 - |j_0(qr)|^2) \\ &= \frac{|f(q)|^2}{2R^2} F_3 \left(\frac{r_{\text{eff}}}{R} \right) + \frac{2\text{Re}f(q)}{\sqrt{\pi}R} F_1(x) - \frac{\text{Im}f(q)}{R} F_2(x) \end{aligned}$$

($x = 2qR$, $R = \text{Gaussian size}$, F_1, F_2, F_3 : Known function)

How can we measure the radius of a star ?

Two photon intensity correlation

Hanbury Brown & Twiss, Nature 10 (1956), 1047.

- Simultaneous two photon observation probability is enhanced from independent emission cases
→ **angular diameter of Sirius=0.0063"**

A TEST OF A NEW TYPE OF STELLAR INTERFEROMETER ON SIRIUS

By R. HANBURY BROWN

Jodrell Bank Experimental Station, University of Manchester

AND

DR. R. Q. TWISS

Services Electronics Research Laboratory, Baldock

NATURE November 10, 1956 VOL. 178

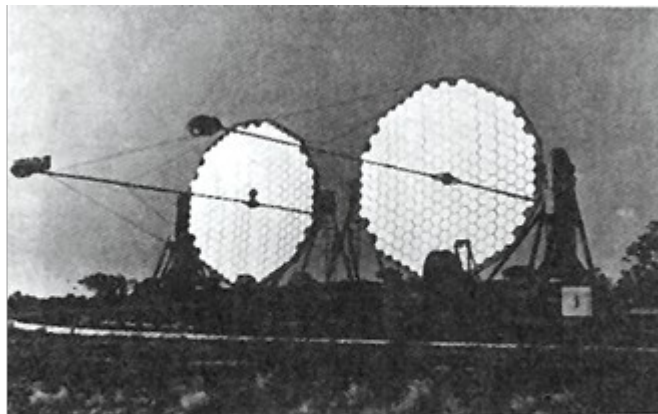


Figure 2. Picture of the two telescopes used in the HBT experiments. The figure was extracted from Ref.[1].

HBP telescope (from Goldhaber, ('91))

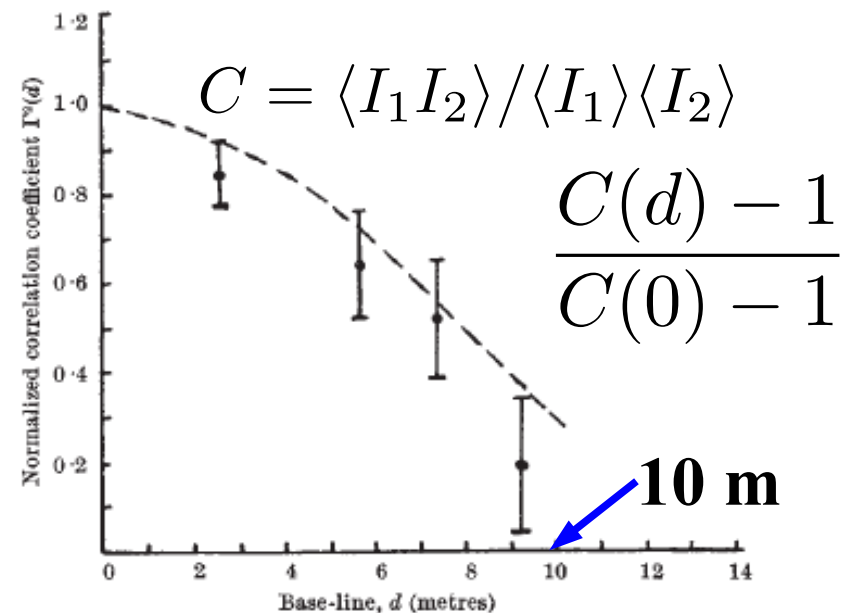


Fig. 2. Comparison between the values of the normalized correlation coefficient $\Gamma^2(d)$ observed from Sirius and the theoretical values for a star of angular diameter 0.0063". The errors shown are the probable errors of the observations

HBT ('56)

Two particle intensity correlation

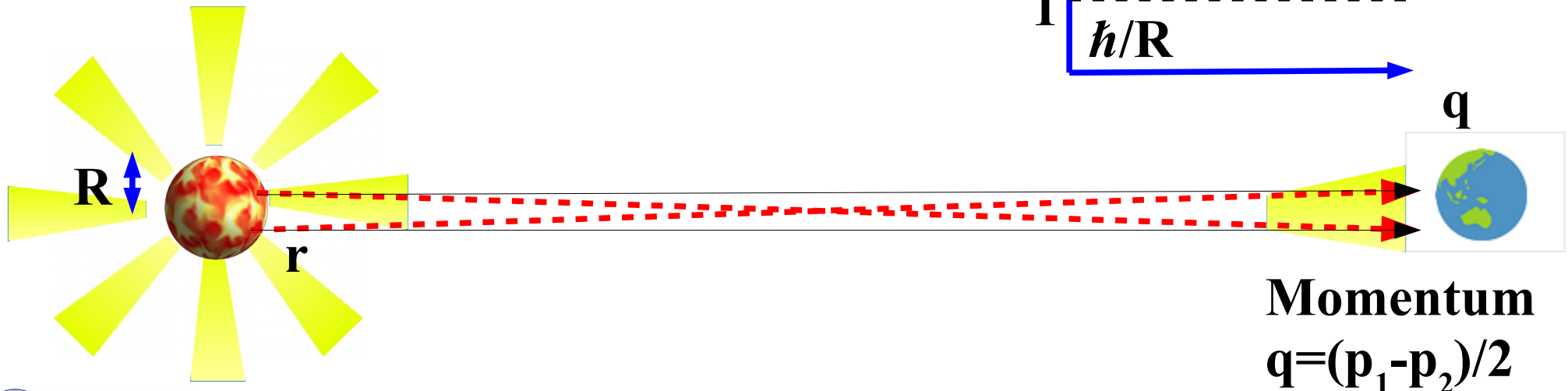
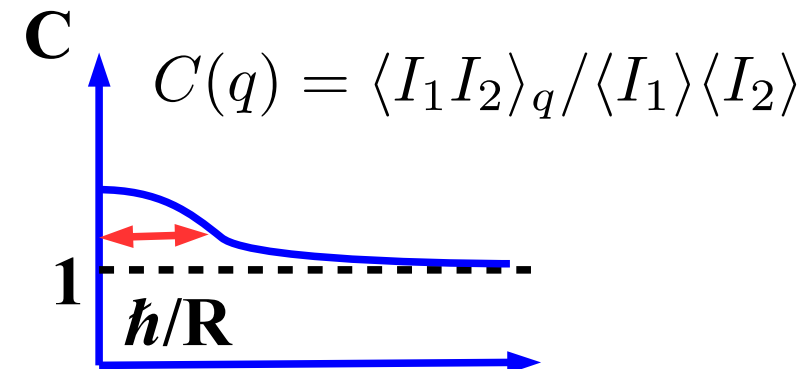
Wave function symmetrization from quantum statistics

$$C(\mathbf{q}) = \int d^3r S(\mathbf{q}, \mathbf{r}) \left| \frac{1}{\sqrt{2}} (e^{i\mathbf{q}\cdot\mathbf{r}} + e^{-i\mathbf{q}\cdot\mathbf{r}}) \right|^2 \simeq \underline{1 + \exp(-4q^2 R^2)}$$

Source fn.
(\mathbf{r} =relative coordinate)
(symmetrized w.f.)²

Static spherical source case

→ Small relative momenta are favored due to symmetrization of the relative wave function.



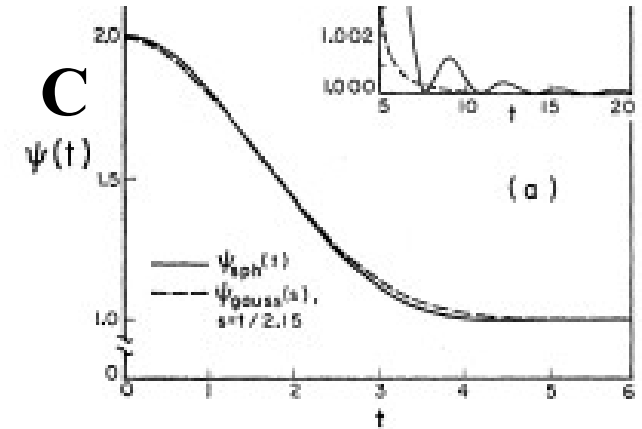
How can we measure source size in nuclear reactions ?

Two pion interferometry

G. Goldhaber, S. Goldhaber, W. Lee, A. Pais, Phys. Rev. 120 (1960), 300

- Two pion emission probability is enhanced at small relative momenta

→ Pion source size $\sim 0.75 \hbar / \mu c$



q (relative momentum)

PHYSICAL REVIEW

VOLUME 120, NUMBER 1

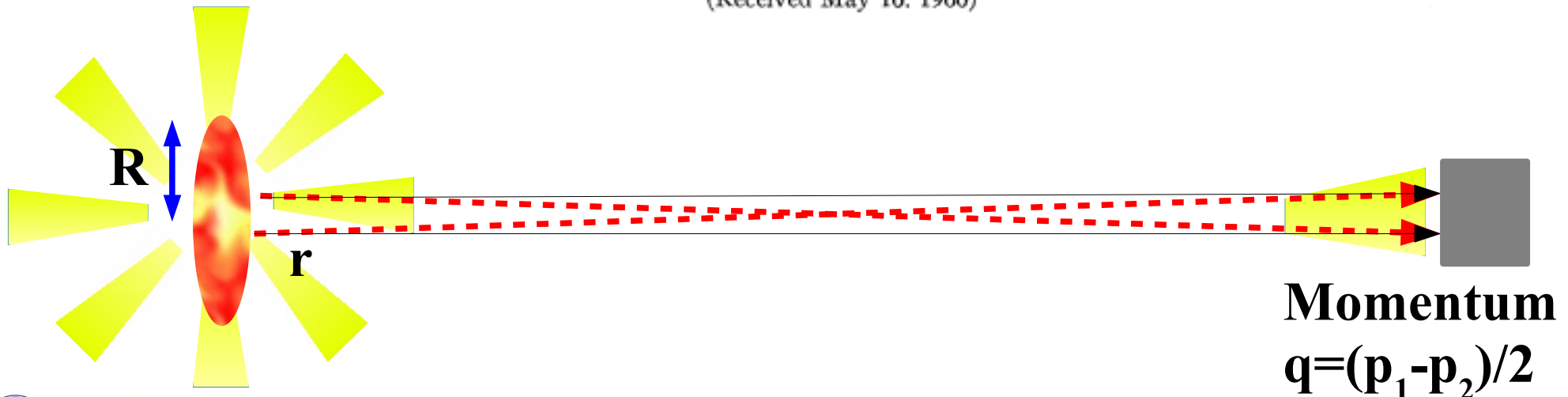
OCTOBER 1, 1960

Influence of Bose-Einstein Statistics on the Antiproton-Proton Annihilation Process*

GERSON GOLDHABER, SULAMITH GOLDHABER, WONYONG LEE, AND ABRAHAM PAIS†

Lawrence Radiation Laboratory and Department of Physics, University of California, Berkeley, California

(Received May 16, 1960)



How does interaction modifies correlation ?

- Interaction modifies the relative wave function, and modifies correlation.

S. E. Koonin, PLB 70 ('77) 43; S. Pratt, T. Csorgo and J. Zimanyi, PRC42 ('90) 2646; W. Bauer, C.-K. Gelbke, S. Pratt, Annu. Rev. Nucl. Part. Sci. 42 (1992)77.

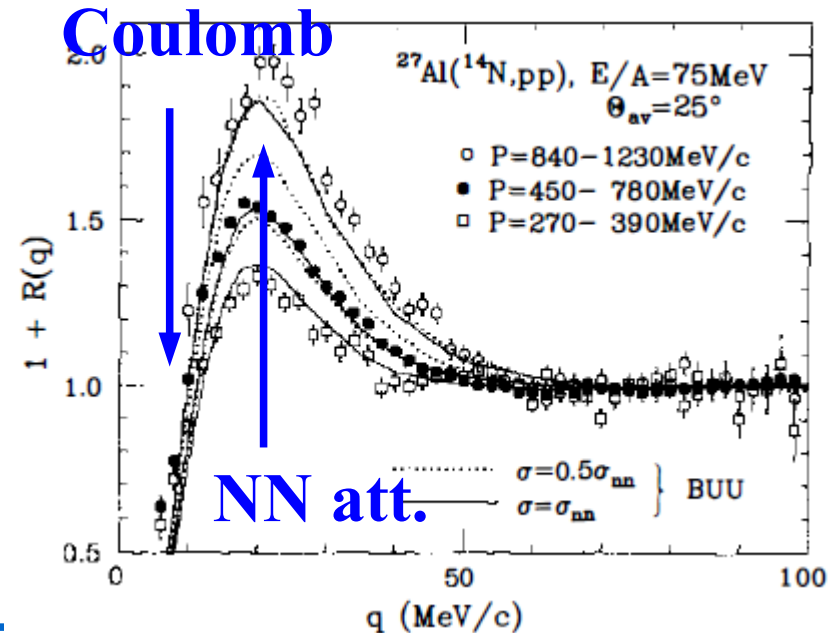
$$C(\mathbf{q}) = \int d^3r S(\mathbf{q}, \mathbf{r}) \left| \underline{\psi_{12}^{(-)}(\mathbf{r}; \mathbf{q})} \right|^2 \quad \text{int.} \rightarrow \text{relative w.f.}$$

$$\simeq 1 - \frac{1}{2} \exp(-4q^2 R^2) + \frac{1}{2} \int d^3r \underline{S_{12}(\mathbf{r})} \left[\underline{|\chi_0(r)|^2} - \underline{|j_0(qr)|^2} \right]$$

Fermion
Source
w.f.
free

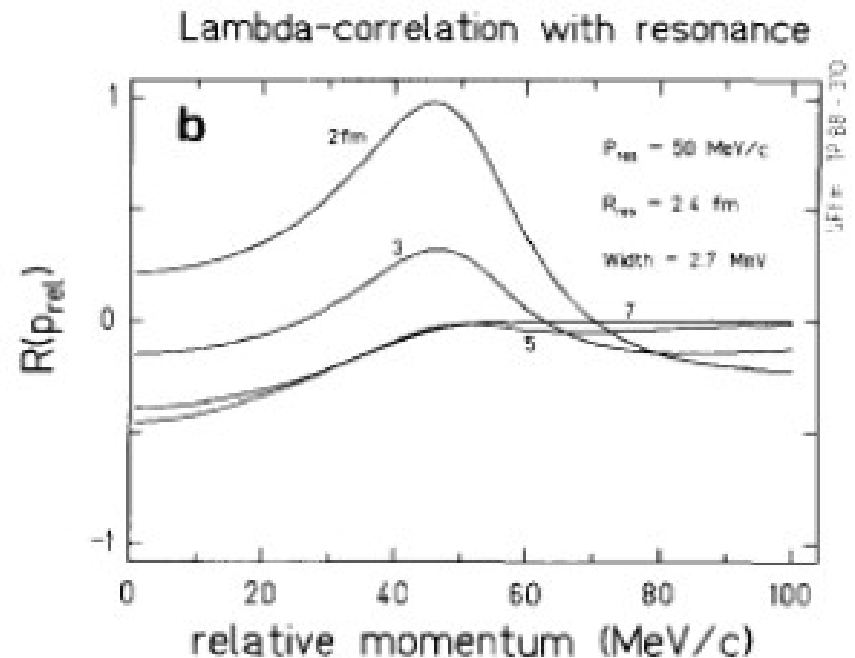
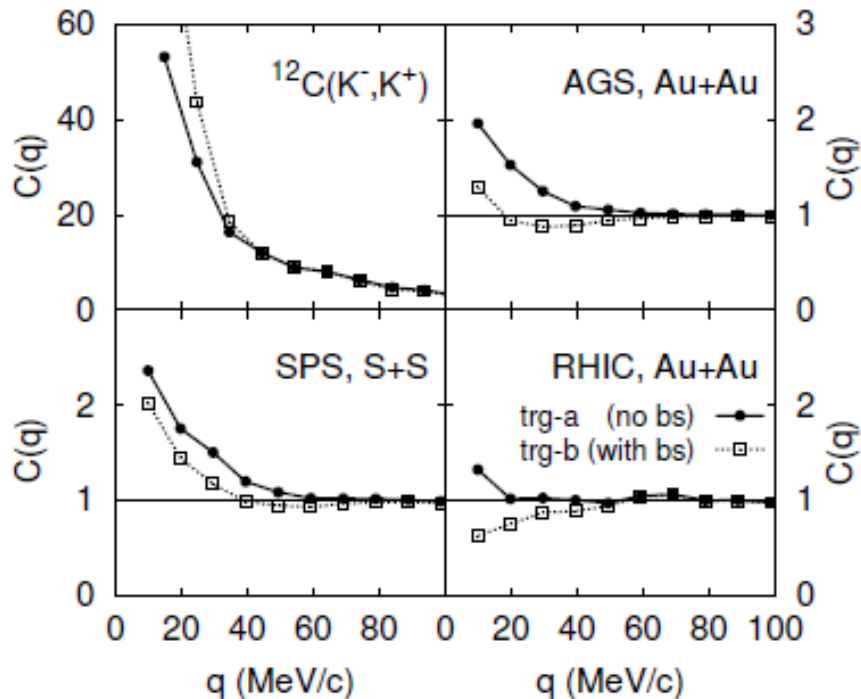
(Koonin-Pratt formula)

- pp correlation from heavy-ion collisions is well understood by the (vacuum) NN interaction and the source function from a transport model.



Correlation \rightarrow Interaction

- If we know correlation and source, it should be possible to get knowledge of hadron interactions !
- How about $\Lambda\Lambda$ interaction ?
 - Λ particle is too short-lived to perform scattering experiments.
 Mass=1115.6 MeV, $\tau=2.6 \times 10^{-10}$ s, $c\tau = 7.89$ cm
 quark content = uds (p=uud, n=udd)



AO, Y. Hirata, Y. Nara, S. Shinmura, Y. Akaishi, C. Greiner, B. Muller, *PLB*219 (1989) 199.
Nucl. Phys. A 670 (2000), 297c