Constraint on higher order symmetry energy parameters and its relevance to neutron star properties

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in collaboraton with
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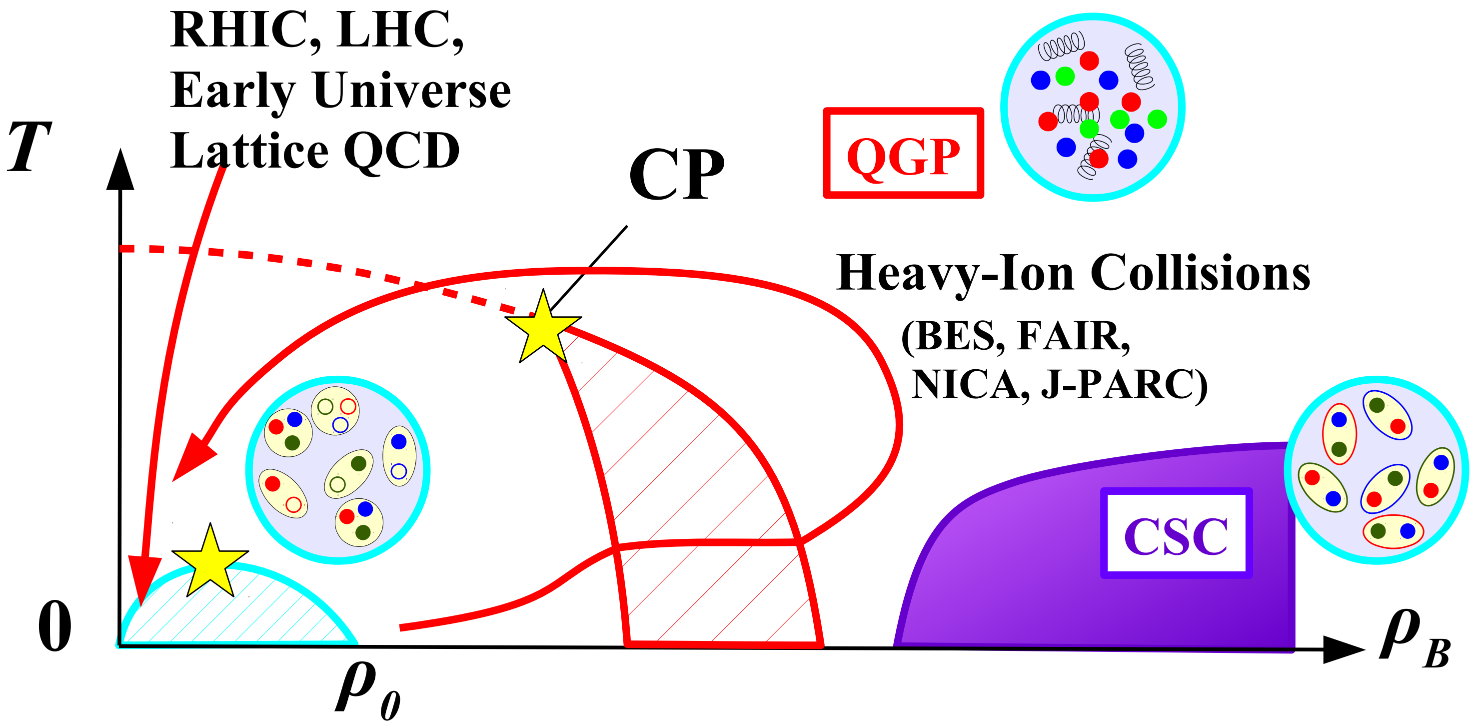
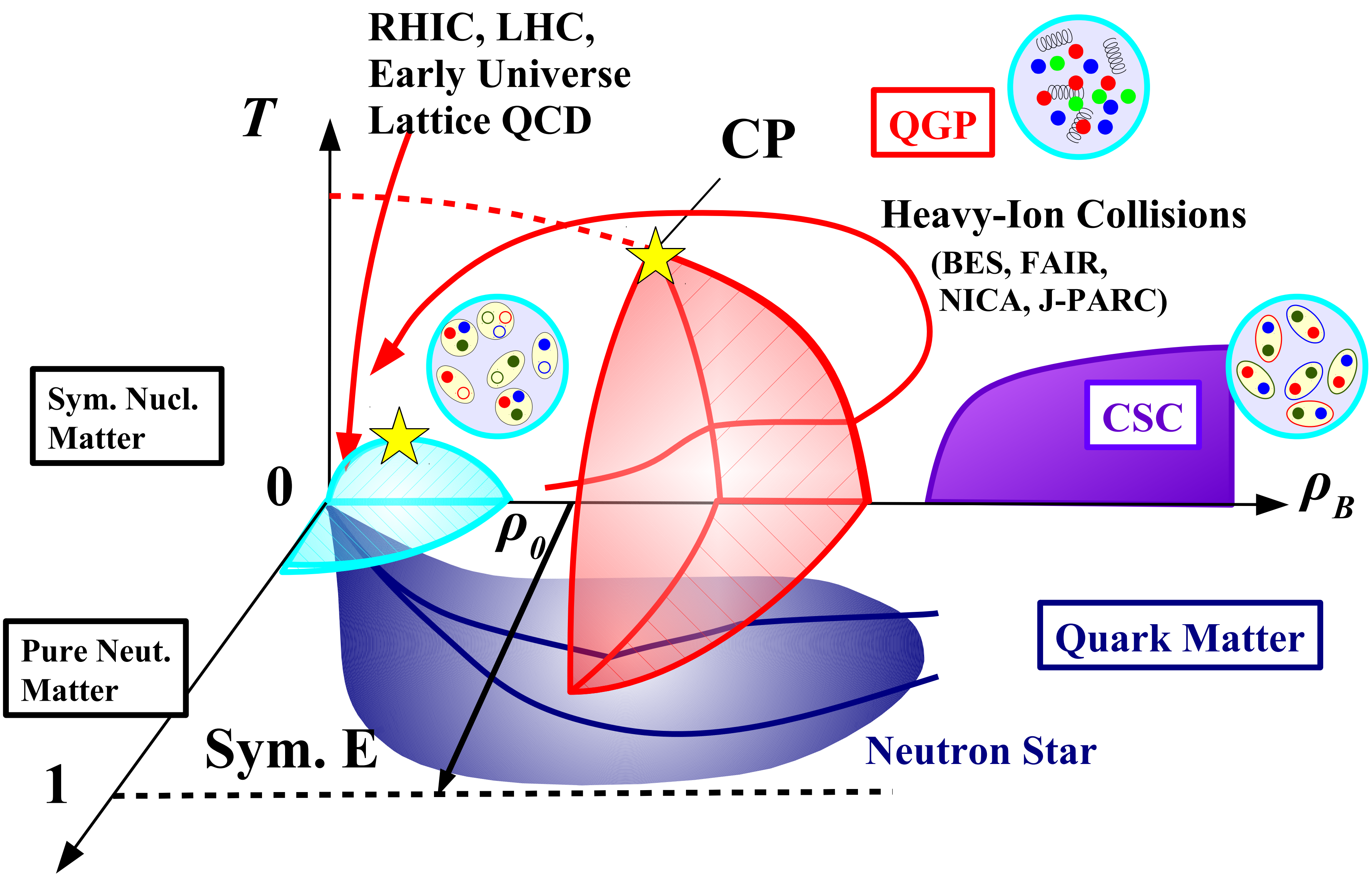
Int. workshop on "Hadron structure and interaction in dense matter"
Nov. 11-12, 2018, Tokai, Japan

- *I. Tews, J. M. Lattimer, AO, E.E.Kolomeitsev, ApJ 848('17) 105 [arXiv:1611.07133]*
- *AO, Kolomeitsev, Lattimer, Tews, X.Wu, in prog.*

QCD Phase Diagram



QCD Phase Diagram



$\delta=(N-Z)/A$ (or $Y_Q(\text{hadron})=Q_h/B \sim (1-\delta)/2$)

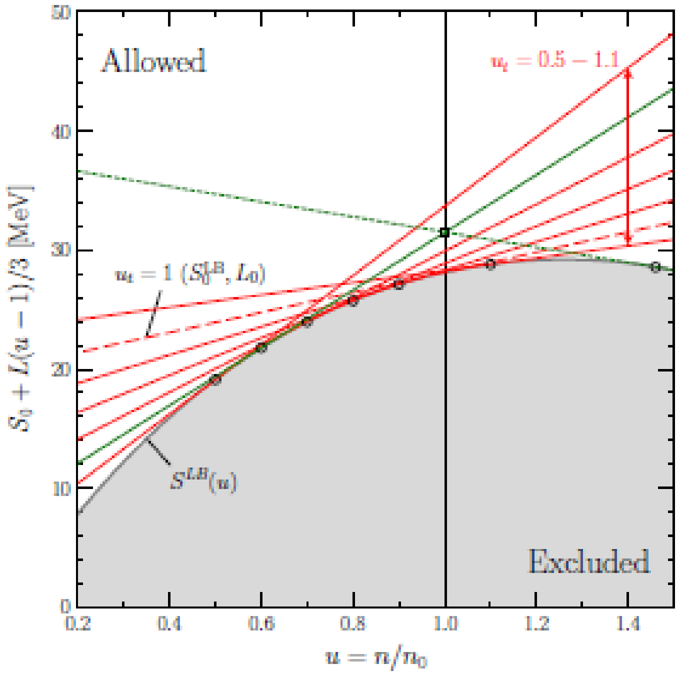
Symmetry Energy Parameters & Neutron Star Radius

- **Nuclear Matter Symmetry Energy parameters (S_0, L) are closely related to Neutron Star Properties,**
  e.g. $R_{1.4} = R_{NS}(M = 1.4 M_\odot)$
- **How can we constrain (S_0, L) ?**
  → Nuclear Exp't. & Theory, Astro. Obs., Unitary gas
- **Conjecture: UG gives the lower bound of neutron matter energy.**
  Tews, Lattimer, AO, Kolomeitsev (TLOK), ApJ ('17)

$$
\begin{aligned}
S(n) =& E_{\mathrm{PNM}} - E_{\mathrm{SNM}} \geq E_{\mathrm{UG}} - \underset{\uparrow}{E_{\mathrm{SNM}}} \\
E_{\mathrm{UG}} =& \xi E_{\mathrm{FG}} \ (\xi \simeq 0.38)
\end{aligned}
$$

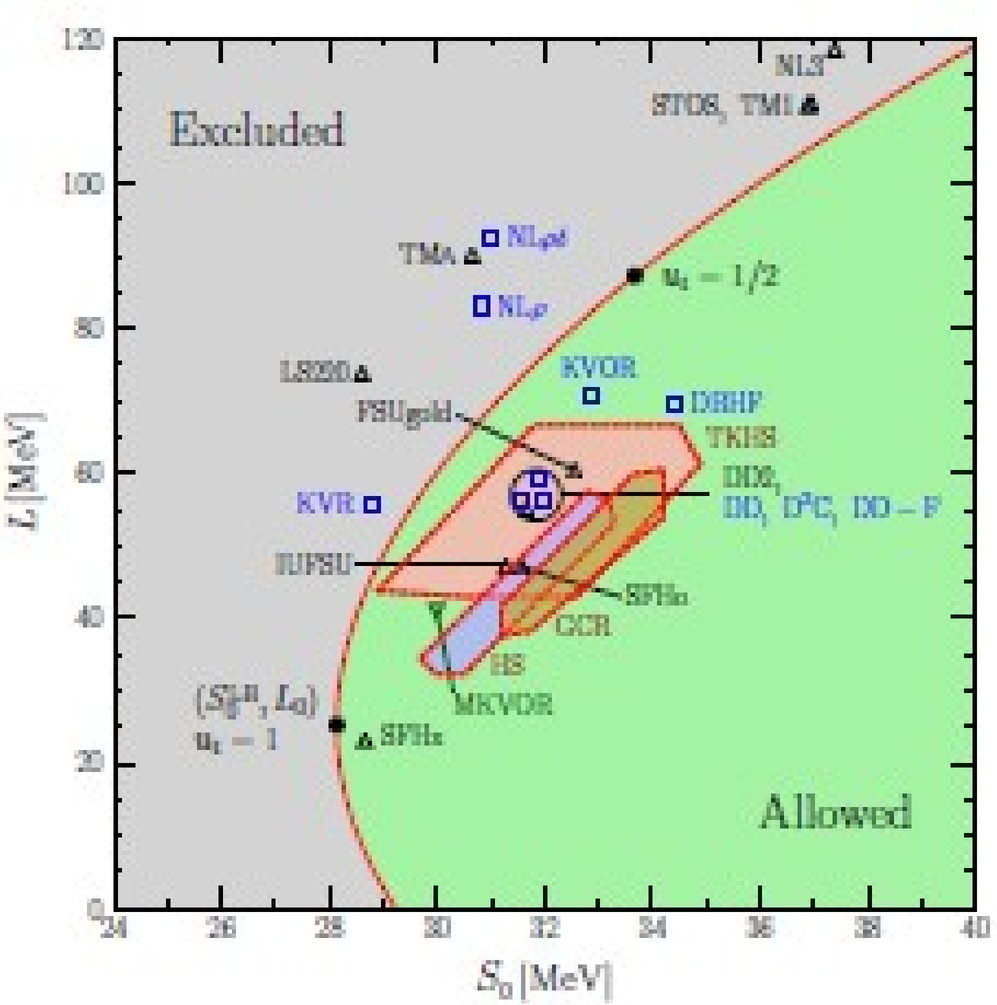
Sym. Nucl. Matter EOS is relatively well known.

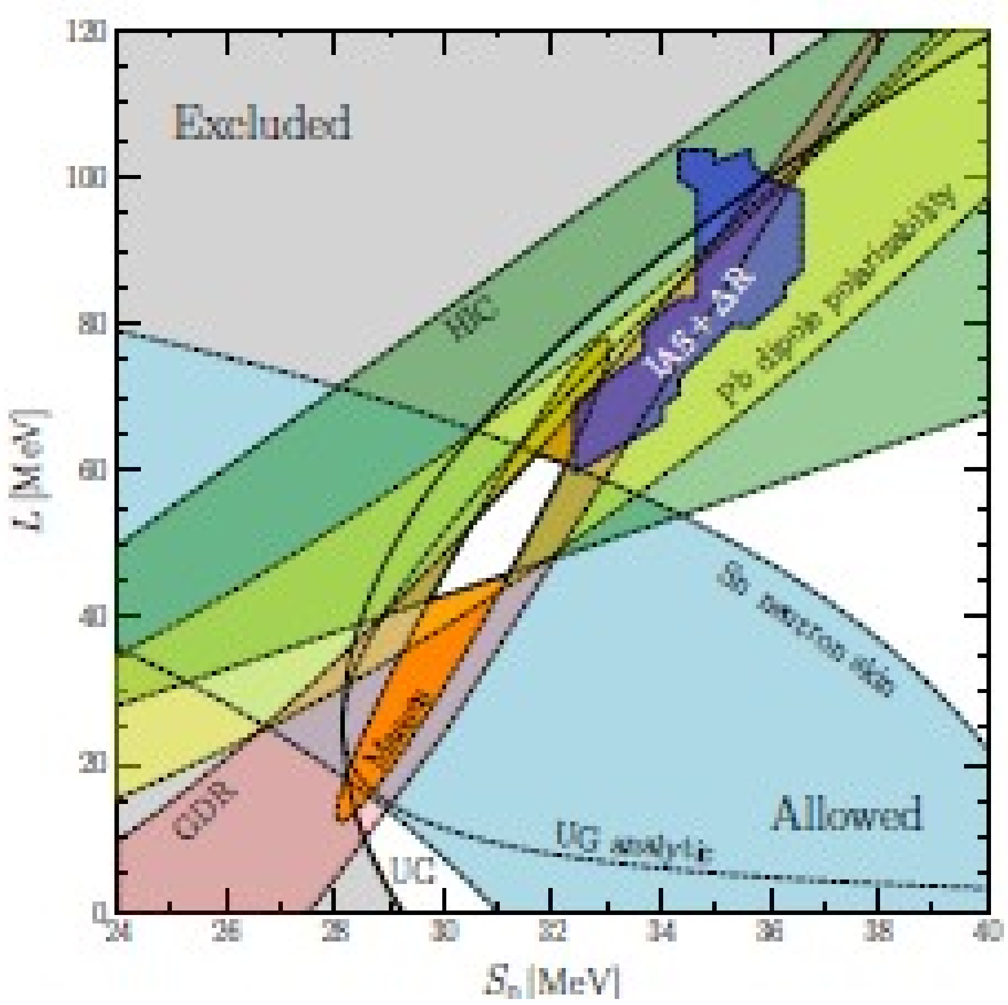
→ For a given L, lower bound of S_0 exists



Constraint on (S_0, L) from Lower Bound of PNM Energy

- **Unitary gas + 2 $M_\odot$ constraints rule out 5 EOSs out of 10 numerically tabulated and frequently used in astrophys. calc.**





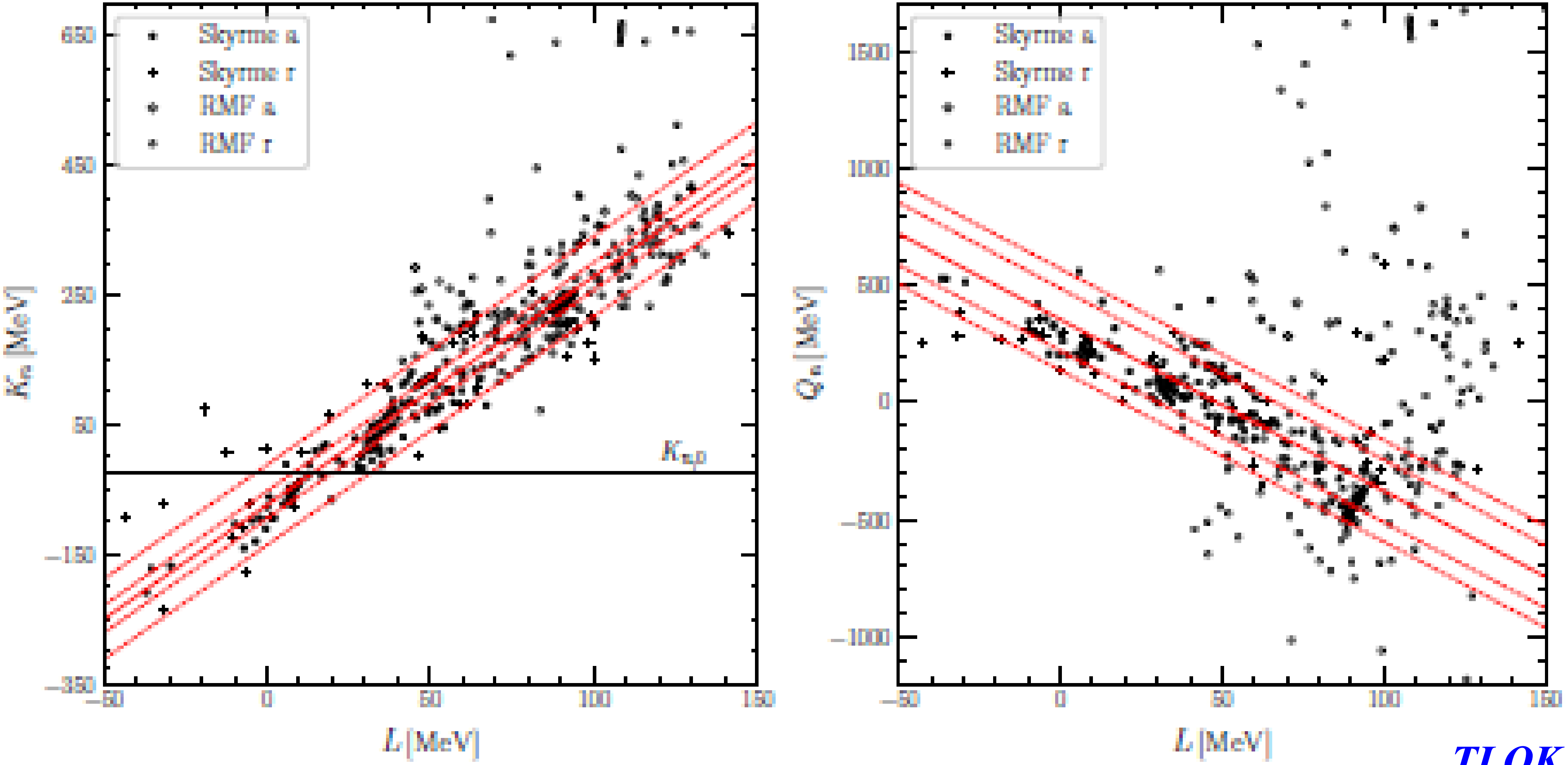
TLOK

Further Constraints on Higher-Order Sym. E. parameters

- **K_n and Q_n are correlated with L in "Good" theoretical models.**

$$K_n = 3.534L - (74.02 \pm 21.17)\mathrm{MeV}$$

$$Q_n = -7.313L + (354.03 \pm 133.16)\mathrm{MeV}$$



TLOK

Purpose & Contents

- **Quesion:**
  What are the effects of these higher-order sym. E. parameters on MR curve of NS ?
- **This work:**
  TLOK + 2 $M_\odot$ constraints + k_F expansion → $R_{1.4}$
- **Contents**
  - **Introduction**
  - **Symmetry Energy Parameters, Nuclear Matter EOS, and Neutron Star Radius**
  - **Implications to quark-hadron physics in cold dense matter**
    - **Neutron chemical potential, QCD phase transition**
  - **Summary**

Symmetry Energy Parameters, Nuclear Matter EOS, and Neutron Star Radius

Fermi momentum (k_F) expansion

TLOK

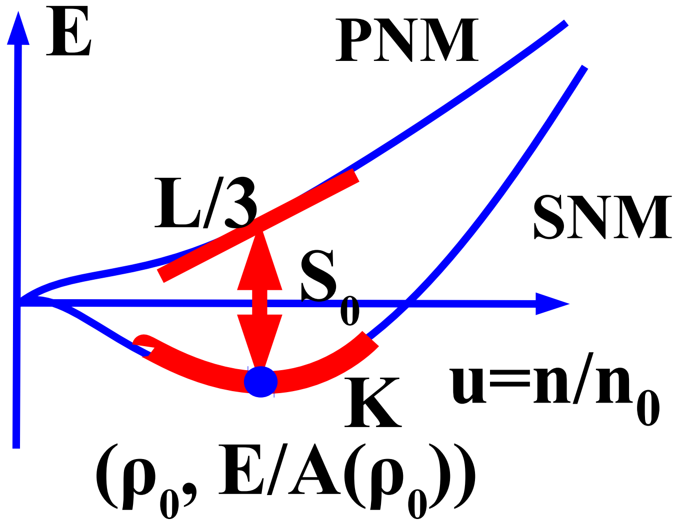
- **Saturation & Symmetry Energy Parameters**

$$E_{\rm NM}(u,\alpha) = E_{\rm SNM}(u) + \alpha^2\, S(u)$$

$$E_{\rm SNM}(u) \simeq E_0 + \frac{K_0}{18}(u-1)^2 + \frac{Q_0}{162}(u-1)^3$$

$$S(u) \simeq S_0 + \frac{L}{3}(u-1) + \frac{K_s}{18}(u-1)^2 + \frac{Q_s}{162}(u-1)^3$$

$$(u = n/n_0, \alpha = (n_n - n_p)/n)$$



Energy does not approach zero at n → 0.

- **Fermi momentum expansion (~ Skyrme type EDF)**
  - **Generated many-body force is given by** $k_F \propto u^{1/3}$

m*

$$E_{\rm SNM}(u) \simeq \boxed{T_0\, u^{2/3}} + \underline{a_0 u} + \underline{b_0 u^{4/3}} + \underline{c_0 u^{5/3}} + \underline{d_0 u^2}$$

$$S(u) \simeq \boxed{T_s u^{2/3}} + \underline{a_s u} + \underline{b_s u^{4/3}} + \underline{c_s u^{5/3}} + \underline{d_s u^2}$$

Kin. E. **Two-body** **Density-dep. pot.**

Expansion Coefficients

TLOK

- **Coefficients (a,b,c,d) are represented by Saturation and Symmetry Energy Parameters**

$$\begin{array}{llll}
a_0 = -\,4T_0 & +20E_0 & +\,K_0 & -Q_0/6 \\
b_0 = \;\;6T_0 & -45E_0 & -5K_0/2 & +Q_0/2 \\
c_0 = -\,4T_0 & +36E_0 & +2K_0 & -Q_0/2 \\
d_0 = \;\;\;T_0 & -10E_0 & -\,K_0/2 & +Q_0/6
\end{array}$$

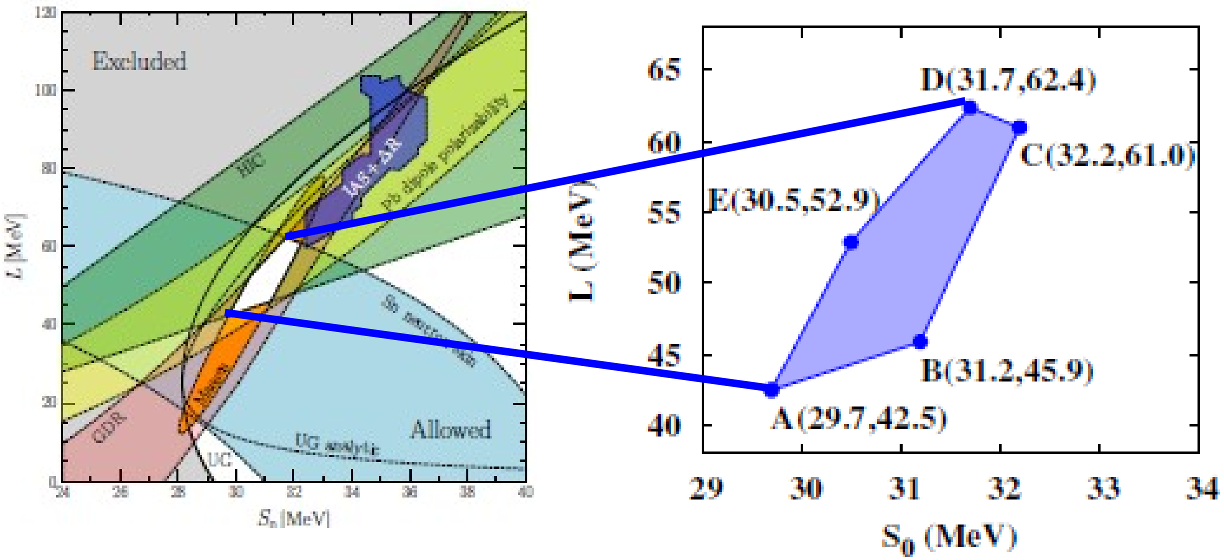
$$\begin{array}{llll}
a_s = -\,4T_s & +20S_0 - 19L/3 & +\,K_s & -Q_s/6 \\
b_s = \;\;6T_s & -45S_0 + 15L & -5K_s/2 & +Q_s/2 \\
c_s = -\,4T_s & +36S_0 - 12L & +2K_s & -Q_s/2 \\
d_s = \;\;\;T_s & -10S_0 + 10L/3 & -\,K_s/2 & +Q_s/6
\end{array}$$

$$\left(T_0 = \frac{3}{5}\frac{\hbar^2 k_F(n_0)^2}{2m}, \quad T_s = T_0(2^{1/3} - 1) \right)$$

Tedious but straightforward calc.

TLOK+$2M_\odot$ constraints

- TLOK constraints
  - (S_0, L) is in Pentagon.
  - (K_n, Q_n) are from TLOK constraint.
  - K_0=(190-270) MeV
  - (n_0,E_0) is fixed
    n_0=0.164 fm-3, E_0=-15.9 MeV (small uncertainties)
  - Q_0 is taken to kill d_0 parameter
    (Coef. of u^2. Sym. N. M. is not very stiff at high-density)

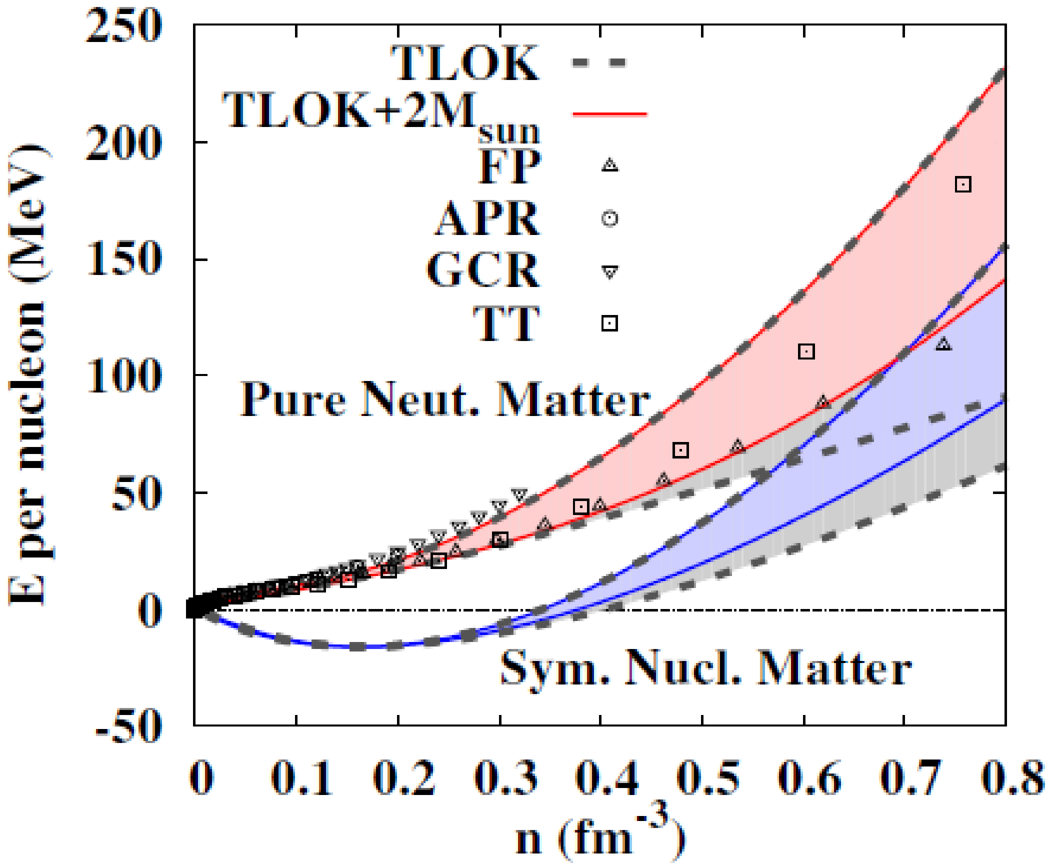


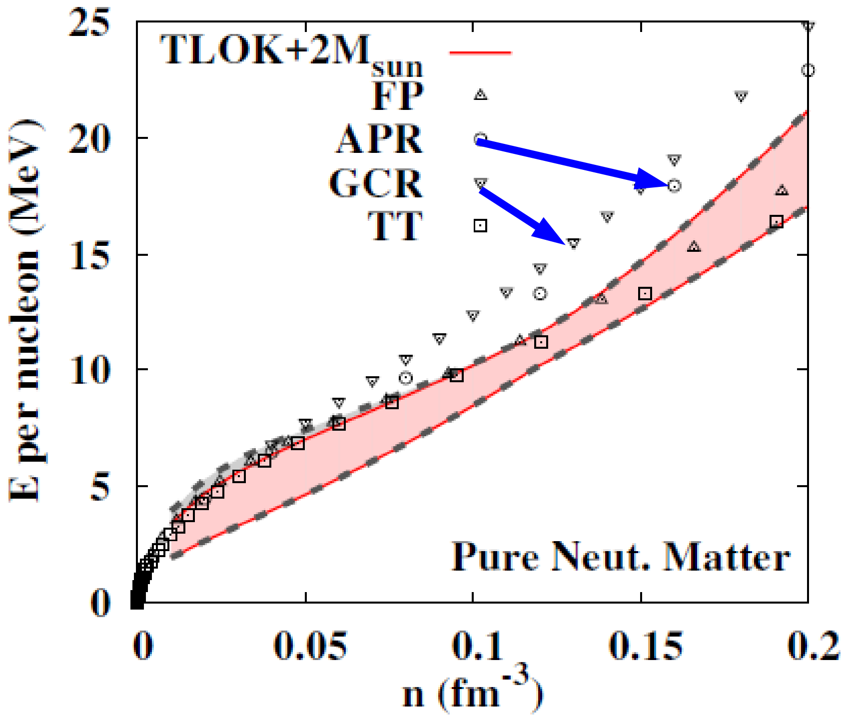
- $2 M_\odot$ constraint
  - EOS should support $2 M_\odot$ neutron stars.

AO, Kolomeitsev, Lattimer, Tews, Wu (OKLTW), in prog.

TLOK+2M$_\odot$ constraints on EOS

- **2M$_\odot$ constraint narrows the range of EOS.**
- **Consistent with FP and TT(Togashi-Takano) EOSs.**
- **APR and GCR(Gandolfi-Carlson-Reddy) EOSs seems to have larger S$_0$ values.**



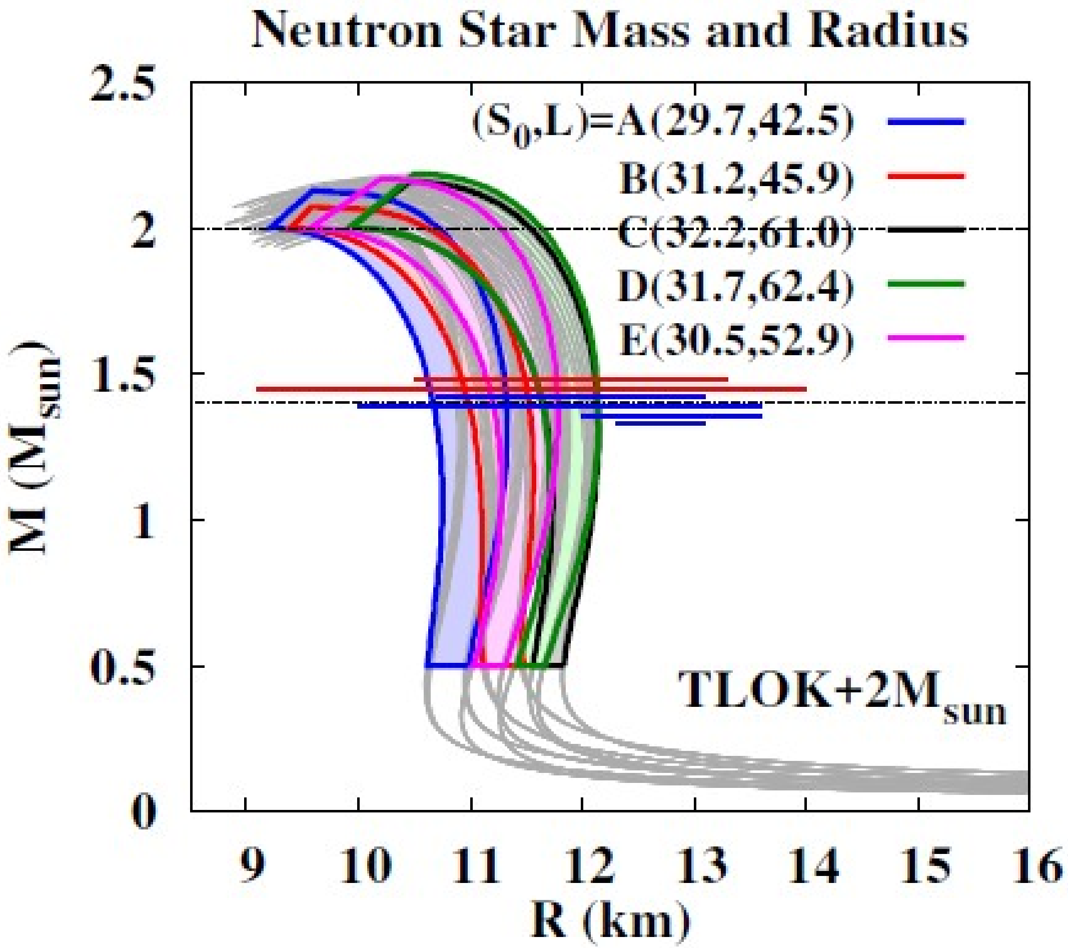


OKLTW, in prog.

Neutron Star MR curve

OKLTW, in prog.

- **TLOK + 2 $M_\odot$ constraints → $R_{1.4}$=(10.6-12.2) km**
  - **E and P are linear fn. of Sat. & Sym. E. parameters**
    → Min./Max. appears at the corners of pentagon (ABCDE).
  - **For a given (S_0, L),**
    unc. of $R_{1.4}$ ~ 0.5 km
    = unc. from higher-order parameters
  - **Unc. from (S_0, L) ~ 1.1 km**
    → We still need to fix (S_0, L) more precisely.

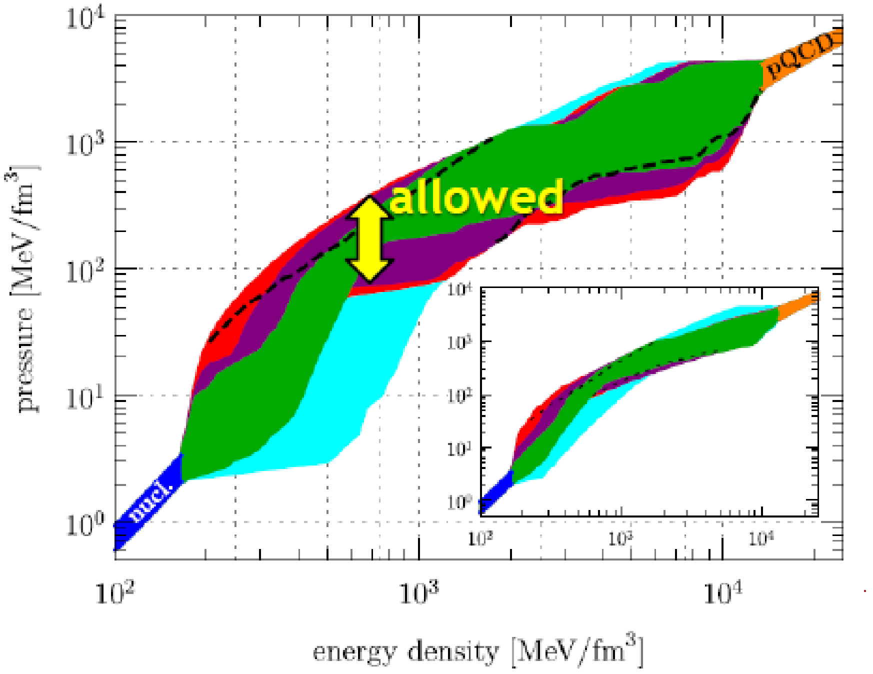


Impact of GW from binary neutron star merger

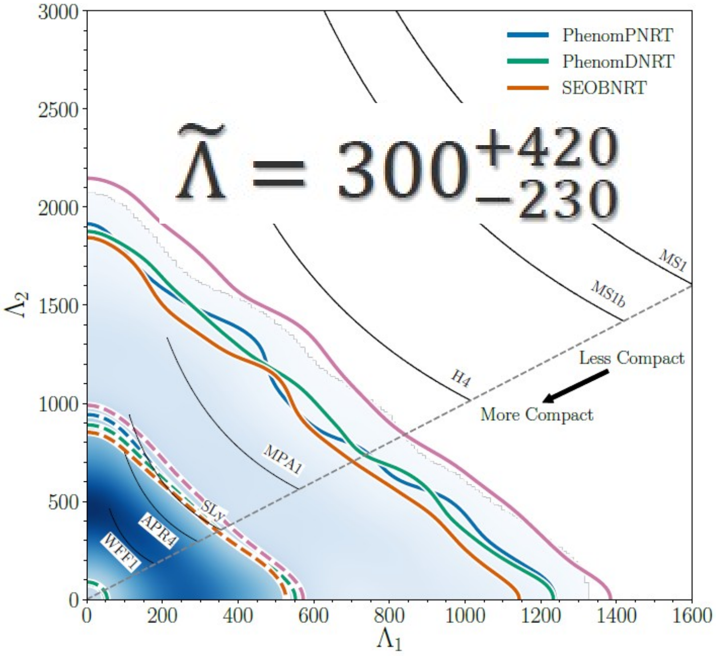
- **GW170817 from NS-NS → Multi messenger astrophysics (Kyutoku's talk)**
- **Neutron Star Radius**
  - **Inspiral region → Tidal deformability (Λ) → NS radius (e.g. R1.4)**
- **Neutron Star Maximum Mass**
  - **No GW signal from Hyper Massive NS → Mmax**
    $\mathrm{Mmax}(T=0,\omega=0) < \mathrm{Mmax}(T=0,\omega) < M < \mathrm{Mmax}(T,\omega)$
- **Nucleosynthesis site of r-process nuclei**
  - **kilonova/macronova from decay energy of the synthesized elements**
  - **r-process nucleosynthesis seems to occur in BNSM !**
- **Central Engine of (Short) Gamma-Ray Bursts**
- **GW as standard siren (Hubble constant)**

Courtesy of Y. Sekiguchi @ YKIS2018b

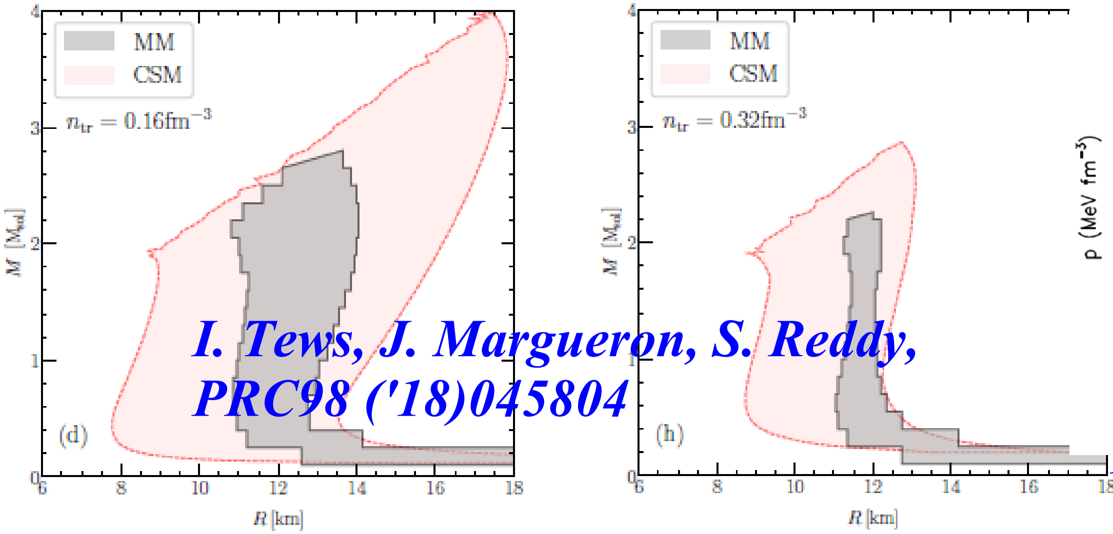
Various Constraints



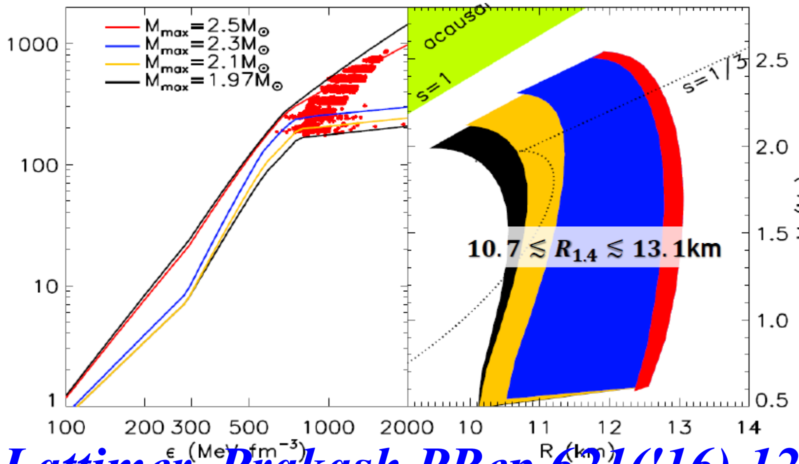
Annala+, PRL120('18)172703



Abbott+,1805.11579



I. Tews, J. Margueron, S. Reddy, PRC98 ('18)045804

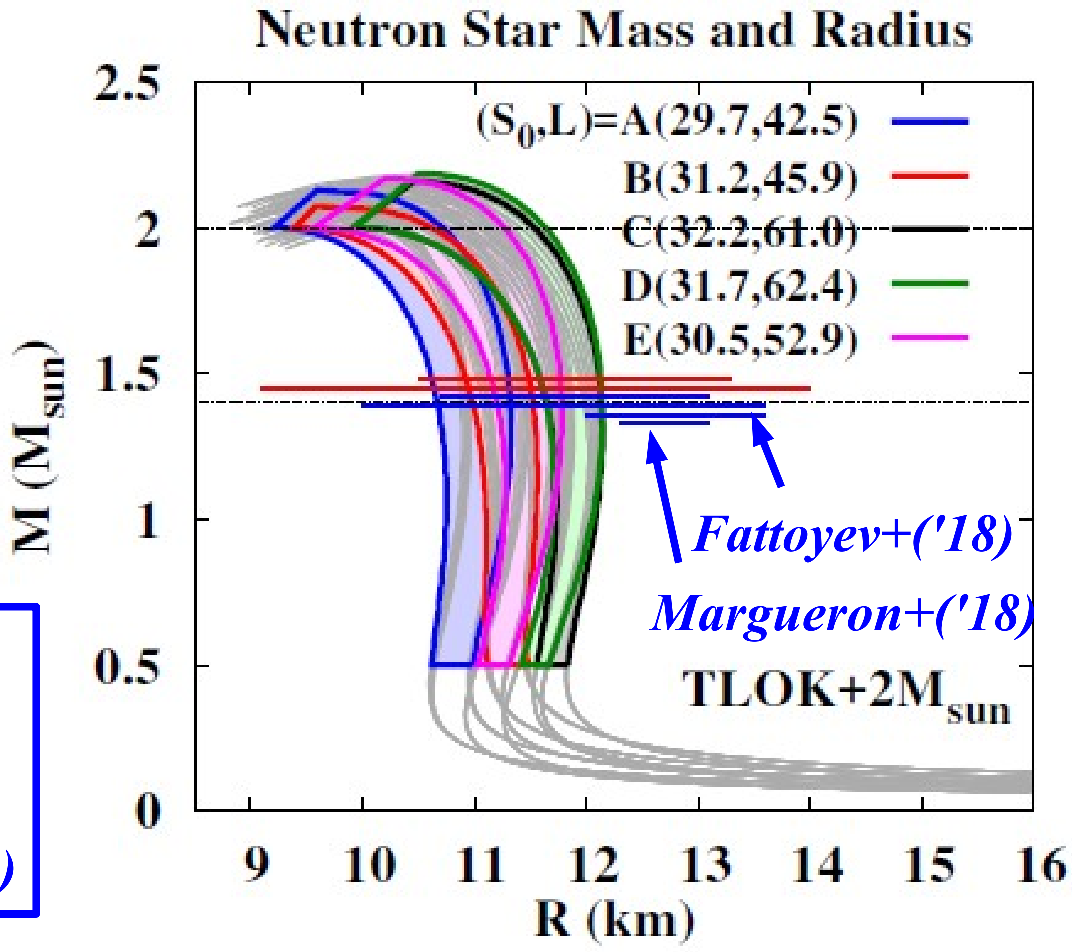


Lattimer, Prakash PRep.621('16),127

Neutron Star MR curve

- **Our constraint is consistent with many of previous ones.**
  - $R_{1.4}$=(10.6-12.2) km *Present work (TLOK + 2 $M_\odot$)* *OKLTW, in prog.*
  - LIGO-Virgo (Tidal deformability Λ from BNSM)
    (10.5-13.3) km *Abbott+('18b)*
    (9.1-14.0) km *De+('18) (Λ)*
  - Theoretical Estimates
    (10.7-13.1) km
    Lattimer, Prakash('16)
    (10.0-13.6) km
    Annala+('18) (χEFT+pQCD)
    (10-13.6) km
    Tews+('18)(χEFT+ c_s)

(12.0-13.6) km
Fattoyev+('18) (PREX)
12.7 ± 0.4 km
Margueron+('18) (n expansion)



Fattoyev+('18)

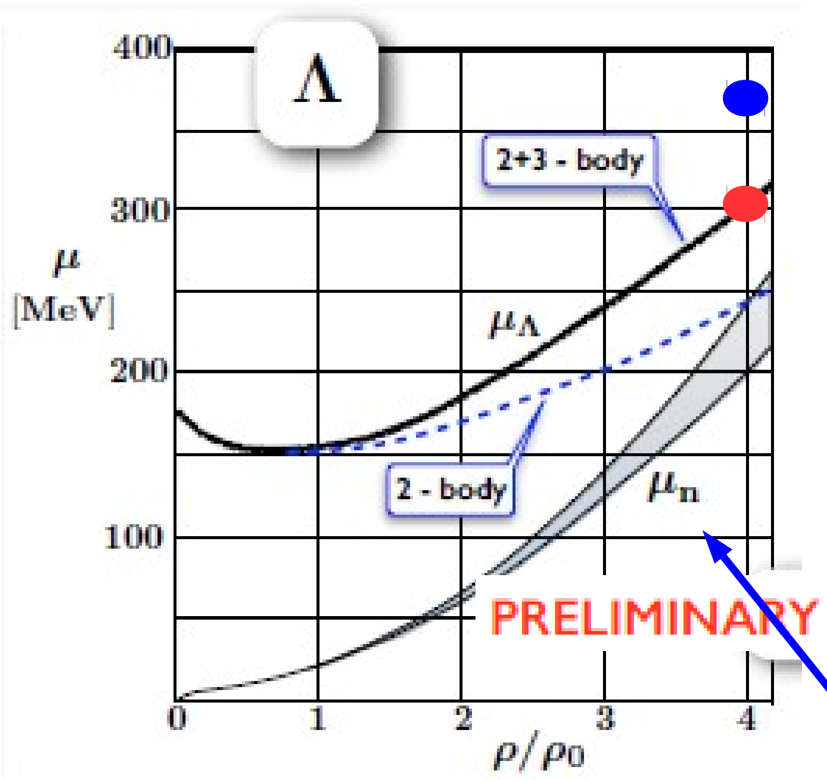
Margueron+('18)

TLOK+2M_{sun}

Implications to quark-hadron physics in cold dense matter (1) Neutron Chemical Potential and Hyperon Puzzle

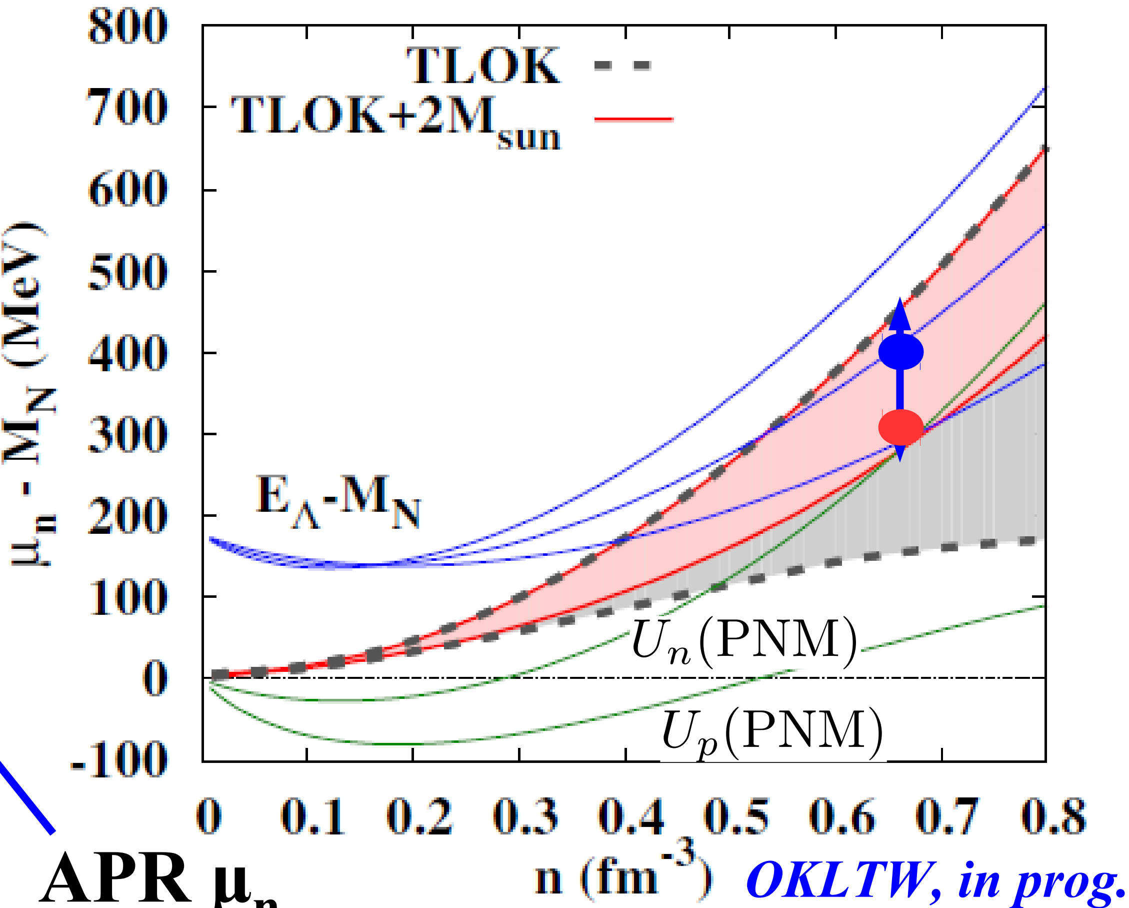
Neutron Chemical Potential in NS

- Λ appears in neutron stars if E_Λ (p=0) = $M_\Lambda + U_\Lambda < \mu_n$
- W. Weise's conjecture: U_Λ in χEFT (2+3 body) is stiff enough.
- But μ_n is larger with TLOK+2$M_\odot$ constraints



W. Weise, NFQCD2018 (2018.06);
Gerstung, Kaiser, Weise, in prog.

μ_n in NS matter



APR μ_n

OKLTW, in prog.

Neutron Chemical Potential in NS

- **Neutron Chemical Potential**

$$\mu_n + M_N = \frac{\partial(nE)}{\partial n_n} = E + u\frac{\partial E}{\partial u} + 2\alpha(1-\alpha)S(u)$$

- **Single particle potential**

$$U_\Lambda(u) = \frac{\partial(nV)}{\partial n_\Lambda}$$

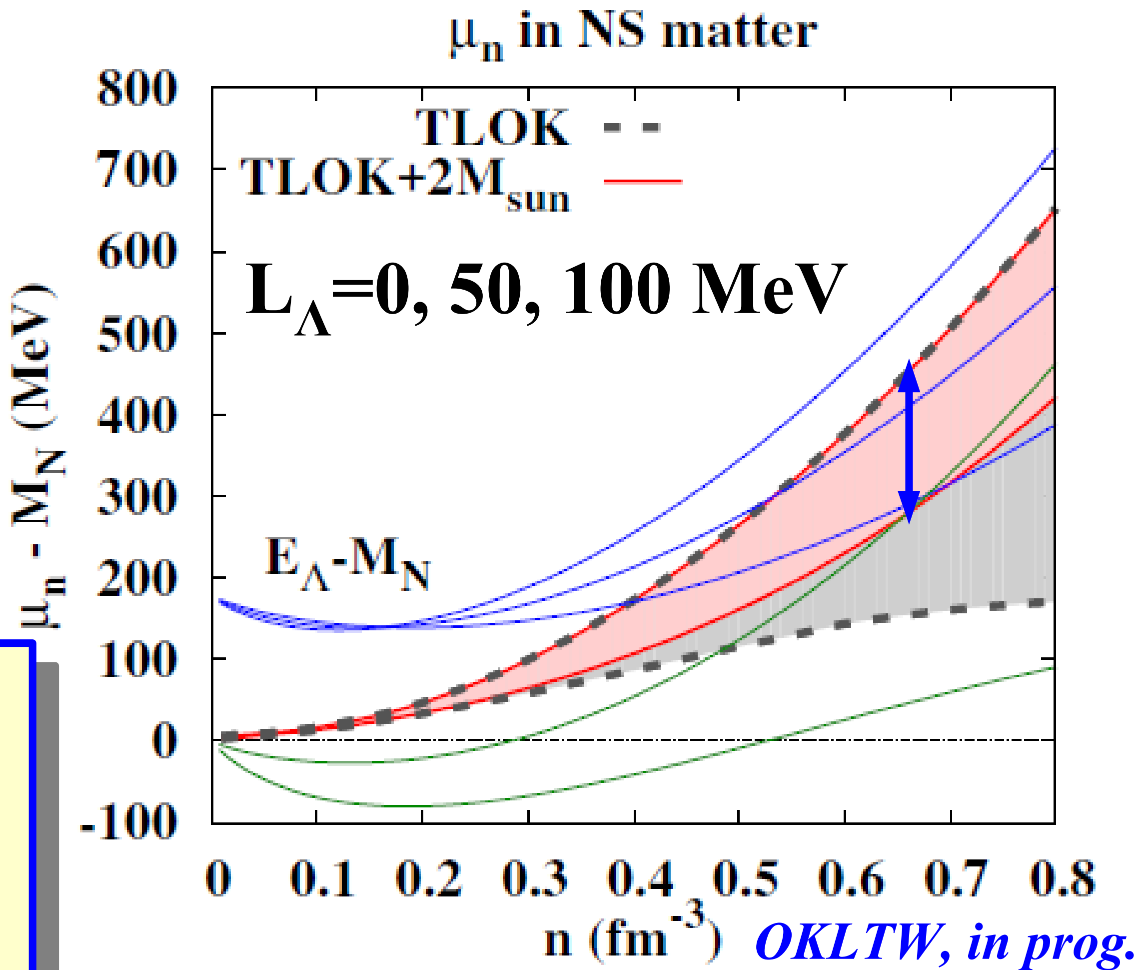
$$\simeq U_{0\Lambda} + \frac{L_\Lambda}{3}(u-1)$$

$$U_{0\Lambda} \simeq -30 \text{ MeV}$$

$$L_\Lambda = ???$$

(L_Λ<0 in most of RMF before 2010)

Sym. E. and L_Λ determine the onset density of Λ. (Already mentioned in Millener,Dover,Gal paper)



OKLTW, in prog.

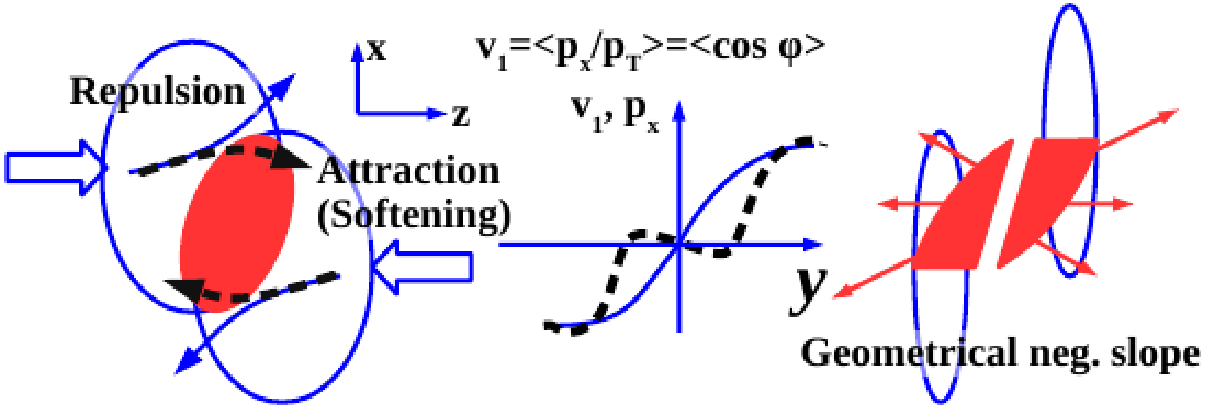
Implications to quark-hadron physics in cold dense matter (2)
QCD phase transition density and order in cold dense matter

QCD phase transition in cold dense matter

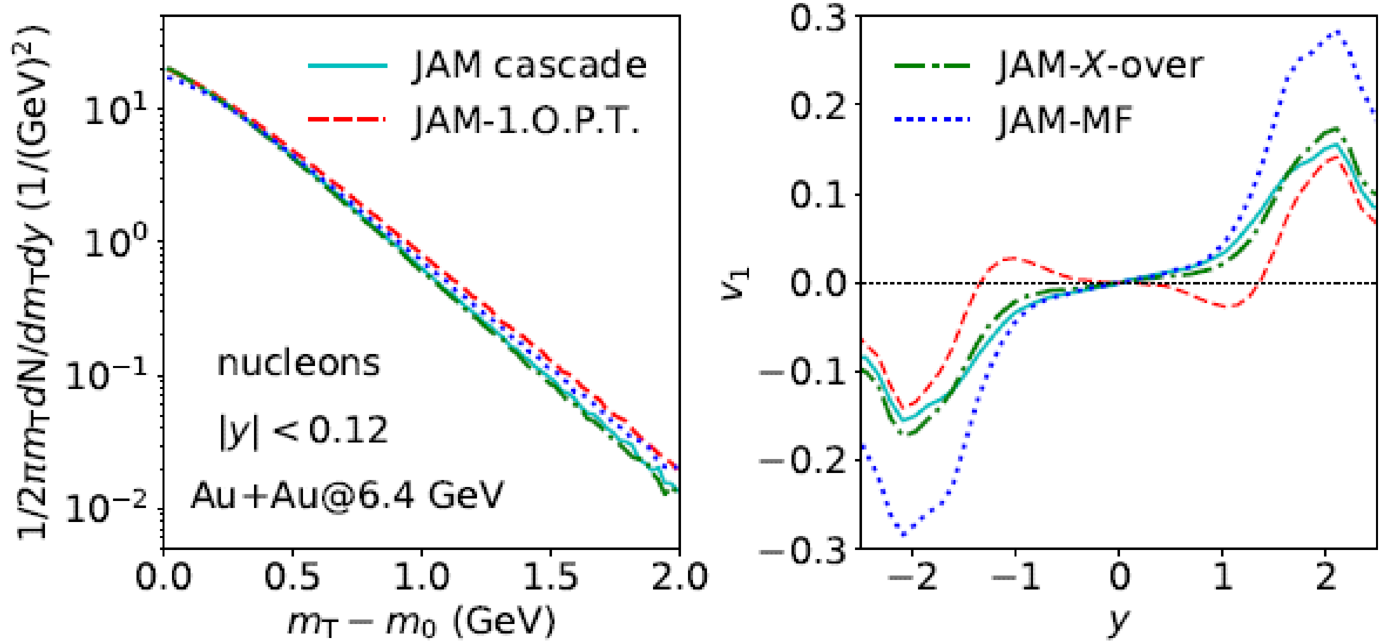
- **Transition to quark matter in cold-dense matter**
  1st order or crossover ?
- **Crossover: Masuda, Hatsuda, Takatsuka, Kojo, Baym, ...**
- **1st order p.t.**
  - **Many effective models predict, e.g. Asakawa-Yazaki CP**
  - **Recent phenomenological support: Negative Directed Flow in HIC**
    Y.Nara, H.Niemi, AO, H.Stoecker, PRC94('16)034906.
    Y. Nara, H. Niemi, AO, J. Steinheimer, X.-F. Luo, H. Stoecker, EPJA 54 ('18)18
  - **The phase transition density may be above NS central density**
    X.Wu, AO, H.Shen, PRC to appear (arXiv:1806.03760)

Negative Directed Flow

- **Directed Flow** $v_1 = \langle \cos\phi \rangle = \langle p_x/p_T \rangle$, Slope $= dv_1/dy$



- **Negative Directed Flow slope at $\sqrt{s_{NN}}$= 11.5 GeV (STAR ('14))**
→ Strong softening of EOS is necessary at n > (5-10) n_0



Y.Nara, H.Niemi, AO, H.Stoecker, PRC94('16)034906.

Y. Nara, H. Niemi, AO, J. Steinheimer, X.-F. Luo, H. Stoecker EPJA 54 ('18)18

Isospin & Hypercharge Sym. E in quark matter

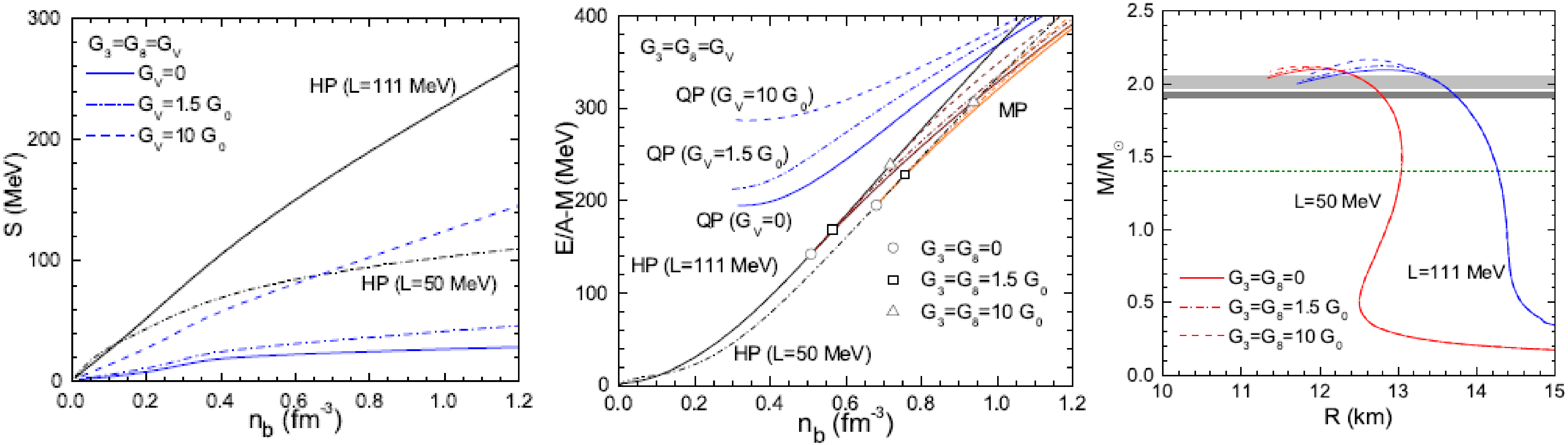
- **Two types of vector int. in NJL**

  X.Wu, AO, H.Shen, PRC to appear (arXiv:1806.03760)

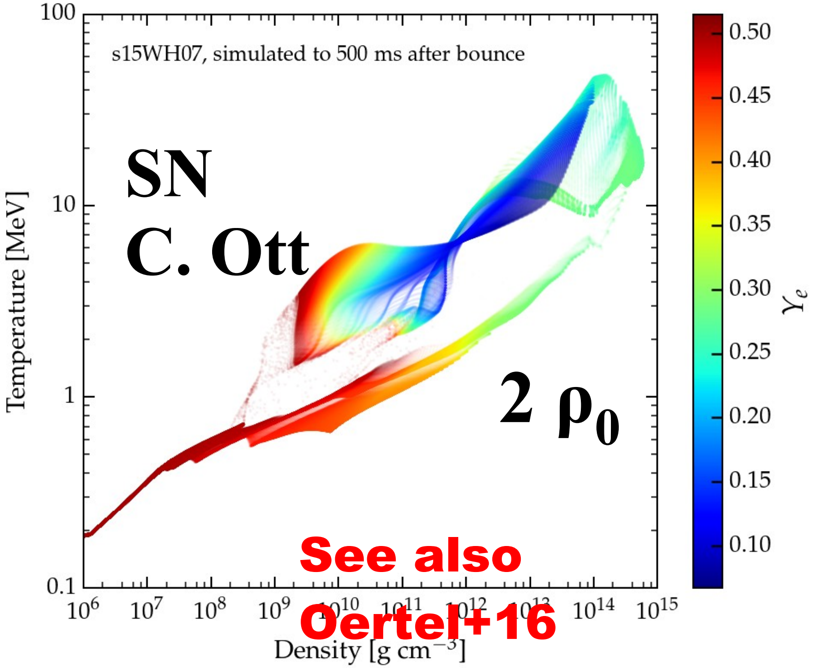
$$\mathcal{L}_v = -G_0(\bar{q}\gamma_\mu q)^2 - G_v \sum_i \left[(\bar{q}\gamma_\mu\lambda_i q)^2 + (\bar{q}i\gamma_5\gamma_\mu\lambda_i q)^2\right]$$

- **Isospin & Hypercharge Sym. E**

$$E = \alpha^2 S(n) + \alpha_Y^2 S_Y(n)\ ,\ \alpha = -2\langle T_z\rangle/B\ ,\ \alpha_Y = \langle B+S\rangle/B$$



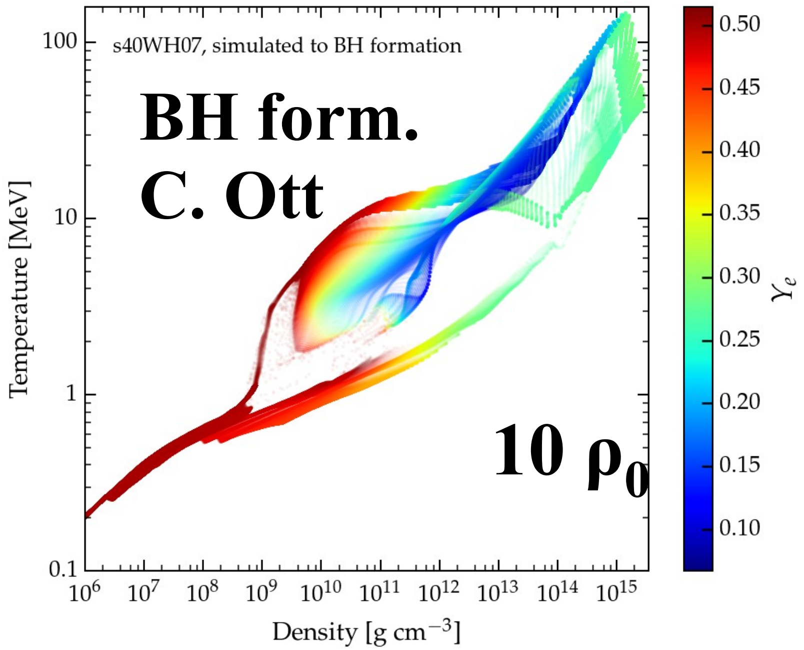
(ρ, T, Y_e) during SN, BH formation, BNSM



SN
C. Ott

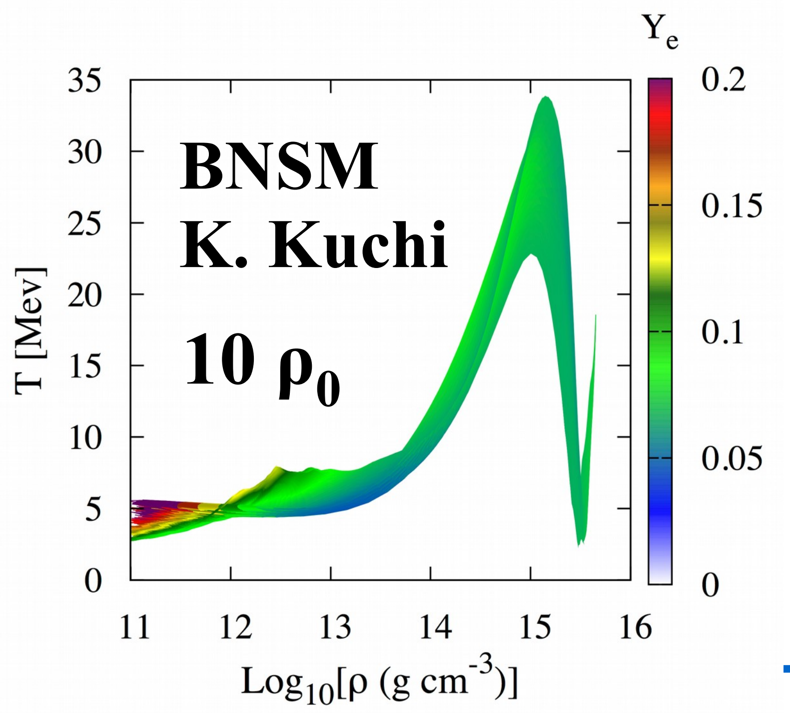
$2\ \rho_0$

See also Oertel+16



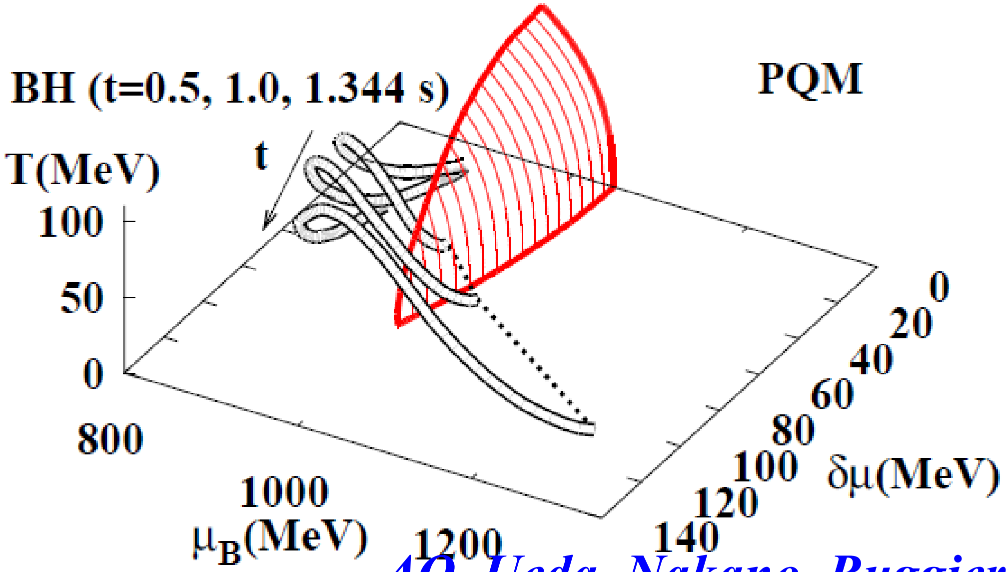
BH form.
C. Ott

$10\ \rho_0$



BNSM
K. Kuchi

$10\ \rho_0$



AO, Ueda, Nakano, Ruggieri, Sumiyoshi, PLB704('11),284

Reservations and Prospects

Reservations

- Only massless electrons are considered and Crust EOS is ignored.
  - With μ, chemical potential may be reduced a little.
- Non-relativistic kinetic energy is used.
  - With rel. K.E., E per nucleon is modified by 0.03 MeV @ 10 n_0 as long as Sat. and Sym. E parameters are fixed.
- Function form is limited to k_F expansion with $u^{k/3}$ (k=2-6).
  - $R_{1.4}$ range becomes narrower with k=2-5.
  - Density expansion gives EOSs very sensitive to parametrs.
- Smooth E(u) (= No phase transition) is assumed.
  - We expect QCD phase transition at (5-10) n0
    from recent BES data of directed flow *Nara, Niemi, AO, Stoecker ('16)*
  - Transition to quark matter may not soften EOS drastically.
- Causality is violated at high densities, n > (4-6) n_0.

To Do (or Prospect)

- **Baryons other than nucleons Λ, Δ, Ξ, Σ, …**
- **Connecting to Hadron Resonance Gas (HRG) EOS**
  - **HRG EOS**
    mass and kinetic E of hadrons with M<2 GeV + simple potential E

    $$\varepsilon_{\mathrm{HRG}} = \mathcal{T} + cn^2$$

    or Lattice EOS in HIC(No saturation, No constraint from NS).
  - **We need to guess the potential energy density more seriously for consistent understanding of HIC, Nuclear, and NS physics.**

    $$\varepsilon = \mathcal{T} + \enclose{circle}{\mathcal{V}} \longleftarrow \textbf{Nuclear and NS physics}$$
- **Connecting to Quark(-Gluon) matter EOS**
  - **Embed model-H singularities** *E.g. Nonaka, Asakawa ('04)*
  - **“Interpolation” of nuclear and quark matter EOS**

Summary

- **Tews-Lattimer-AO-Kolomeitsev ('17) constraints (S0, L, K_n, Q_n) and 2 $M_\odot$ constraint with the aid of Fermi momentum (k_F) expansion lead to the costraint on 1.4 $M_\odot$ neutron star radius of (10.6-12.2) km.**
  - **Consistent with many of other constraint.**
- **Onset density of hyperons may be sensitive to the symmetry energy in addition to potential parameters, (U_{0B}, L_B).**
  - **We need to know the slope of potential in addition to the depth.**
- **Global EOS (HIC and Nuclear/NS matter) needs to be given in a way where HIC physicists and NS physicists admit.**
  E.g. "Hadron Resonance Gas (HRG)+Potential from NS"

Thank you for your attention .

Further Constraint on Q_n

- **2 $M_\odot$ requirement constrains Qn further.**

$$Q_n > -9.3L + 480\,\mathrm{MeV}$$

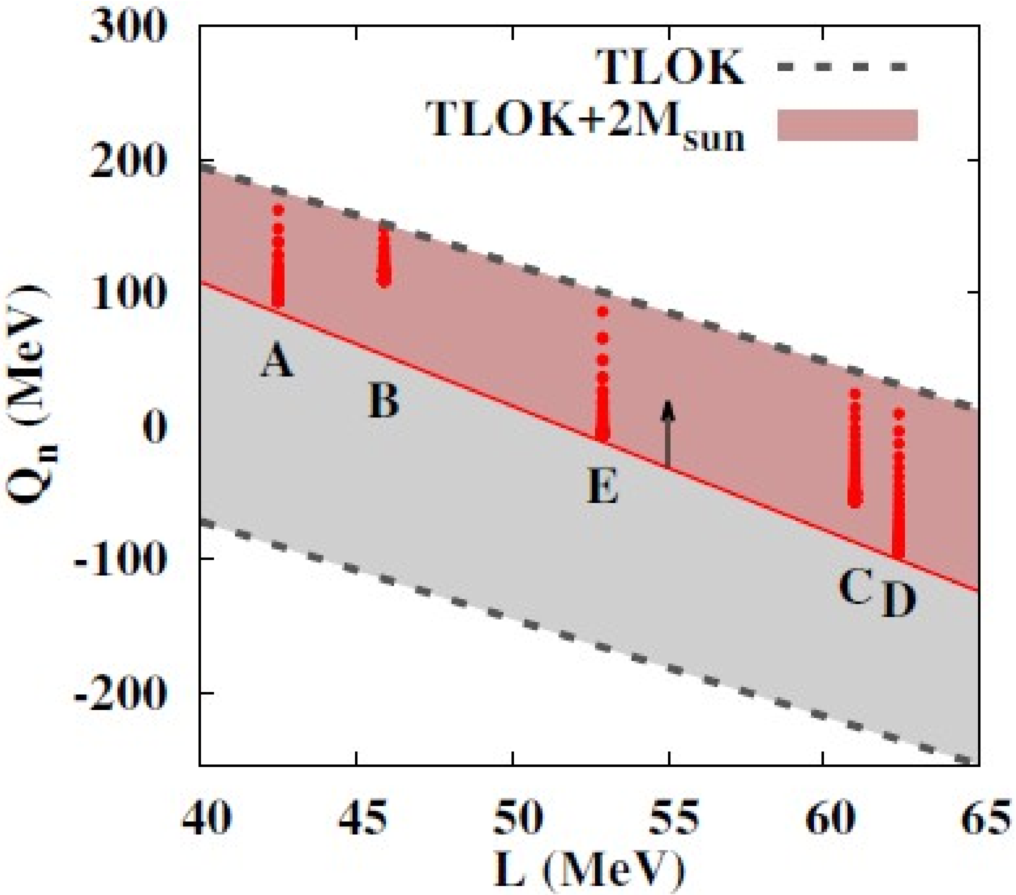
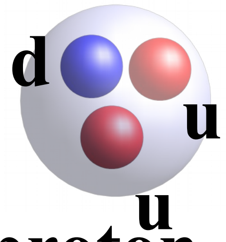


FIG. 4. Constraint on Q_n

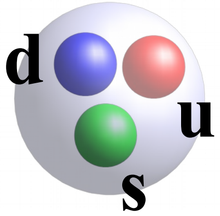
AO, Kolomeitsev, Lattimer, Tews, Wu (OKLTW), in prog.

Neutron star – Is it made of neutrons ?

- **Possibilities of various constituents in neutron star core**
  - **Strange Hadrons**

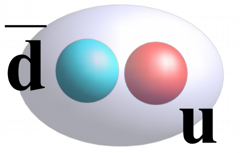


proton

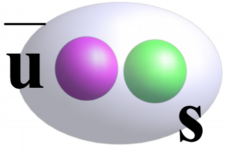


Λ hyperon

  - **Meson condensate (K, π)**

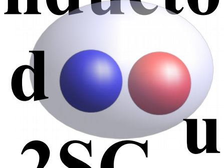


π



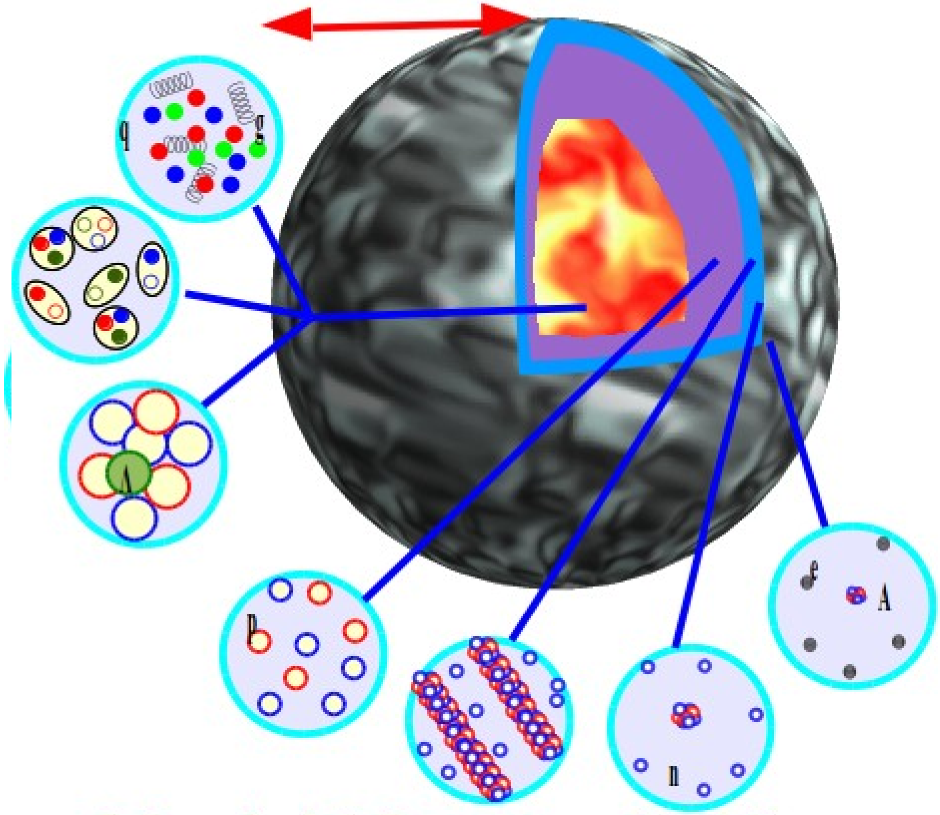
anti kaon

  - **Quark matter**
  - **Quark pair condensate (Color superconductor)**



2SC

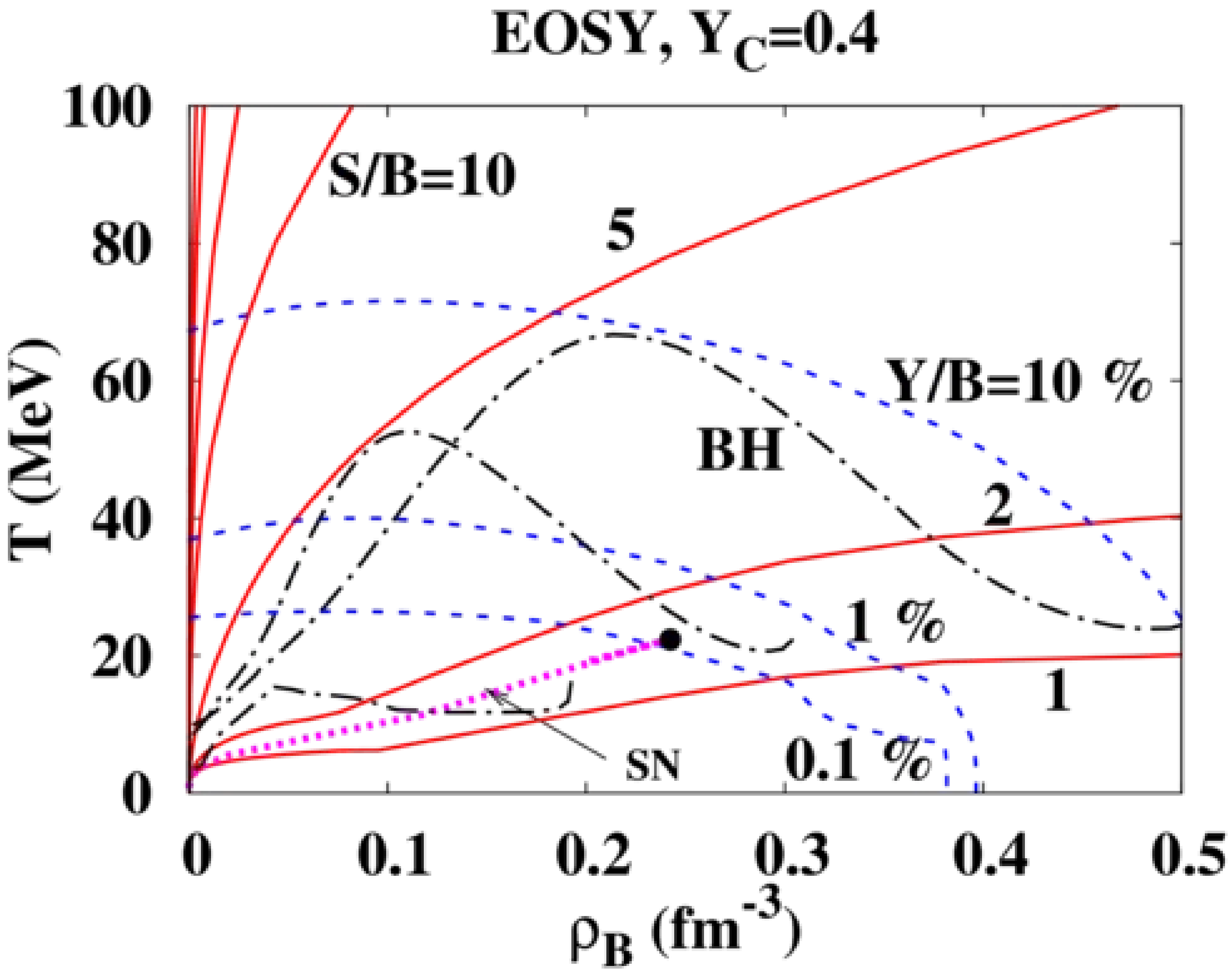
R ~ 10 km



$M \sim 1.4\ M_\odot$, $\rho_c \sim (3\text{-}10)\ \rho_0$

> ***NS core = Densest stable matter existing in our universe.***

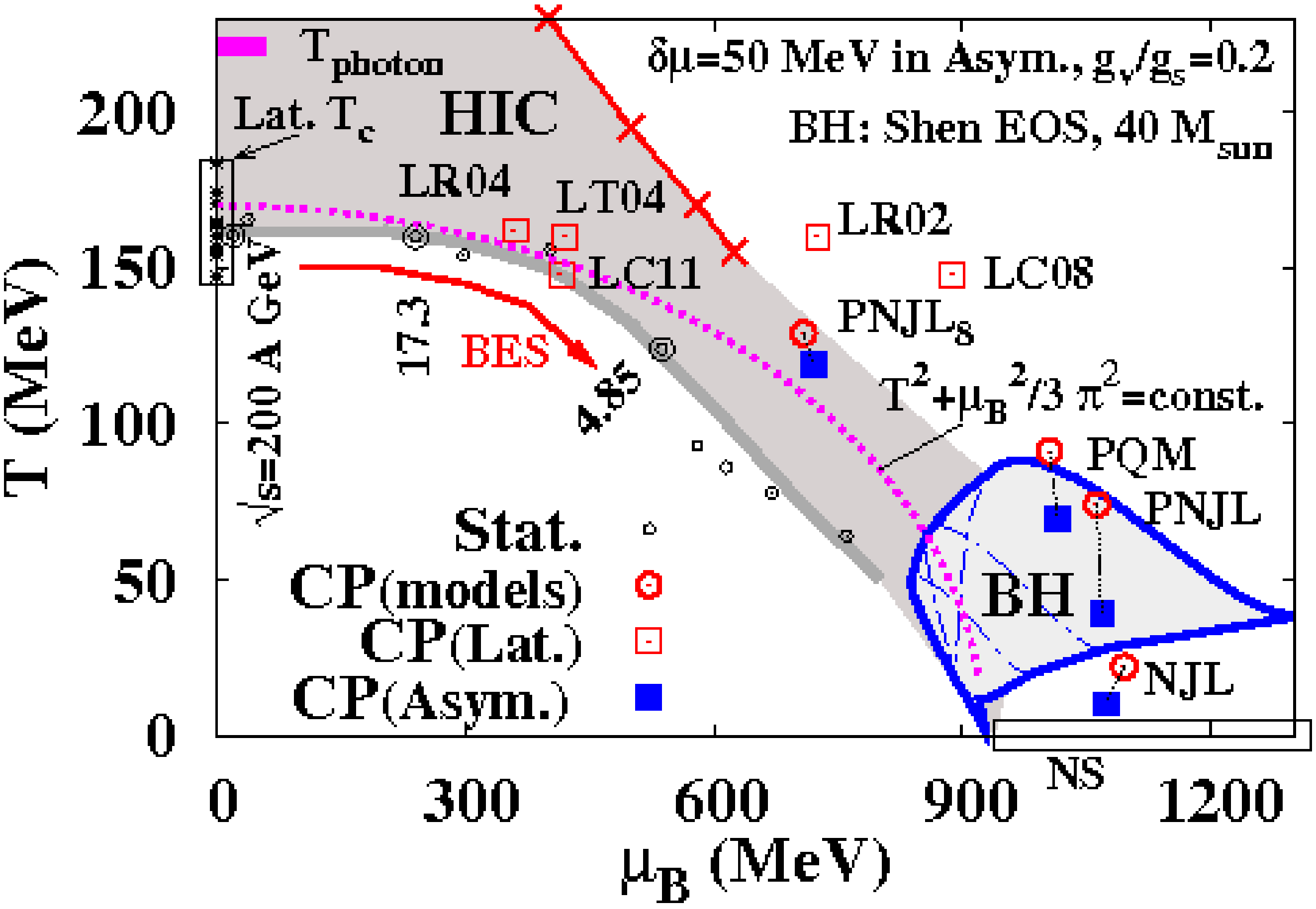
(ρ, T) during SN & BH formation



Shen EOS
+ hyperons

Ishizuka,AO, Tsubakihara, Sumiyoshi, Yamada, JPG 35('08) 085201;
AO et al., NPA 835('10) 374.

QCD phase diagram (Exp. & Theor. Studies)



QCD phase transition is not only an academic problem, but also a subject which would be measured in HIC or Compact Stars

AO, PTPS 193('12)1

Unitary Gas Constraint

Tews, Lattimer, AO, Kolomeitsev (TLOK), ApJ ('17)

- **Conjecture:**
  Unitary gas gives the lower bound of neutron matter energy.

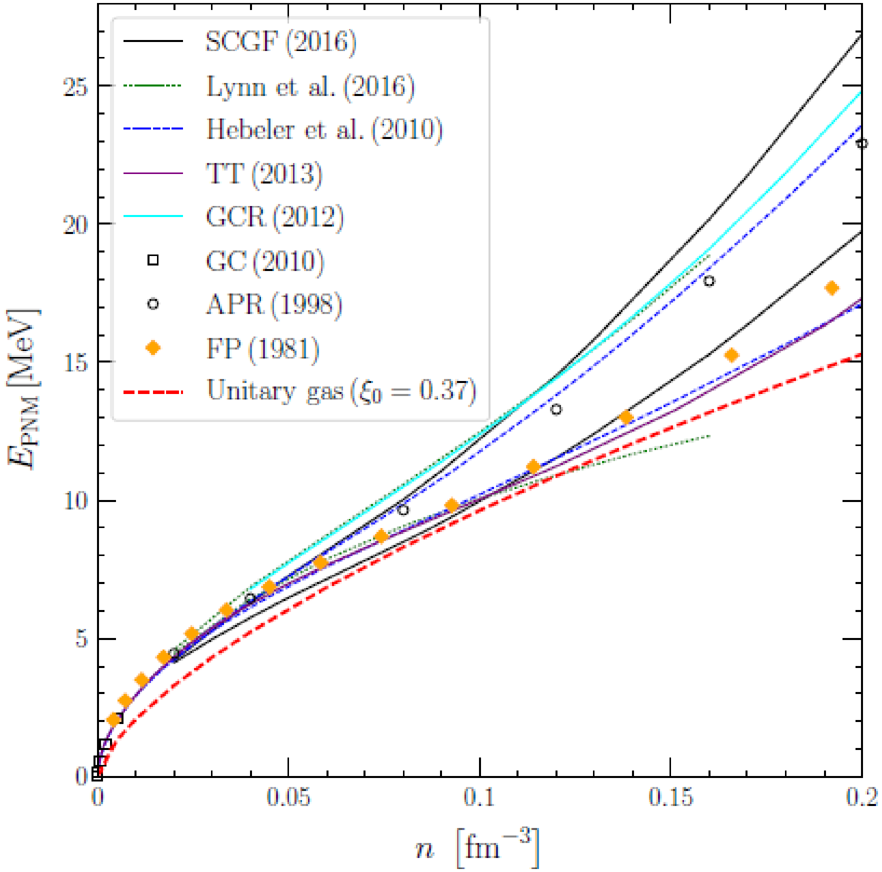
$$S(n) = E_\mathrm{PNM} - E_\mathrm{SNM} \geq E_\mathrm{UG} - E_\mathrm{SNM}$$

$$E_\mathrm{UG} = \xi E_\mathrm{FG} \ (\xi \simeq 0.38)$$

Sym. Nucl. Matter EOS is relatively well known.

- **$a_0 = \infty$ in unitary gas**
  → lower bound energy of $a_0 < 0$ systems (w/o two-body b.s.) ?

- **Supported by (most of) ab initio calc.**



Potential Energy Density

- **Potential Energy Density in the Fermi momentum expansion**

$$\mathcal{V} = nV = \sum_{i,j\in B} n_i n_j v_{ij}(n)$$

Density-dependent NN interactions vij (i, j=p or n) are known.

- **Single particle potential**

$$U_i = \frac{\partial \mathcal{V}}{n_i} = \sum_j n_j v_{ij}(n) + \sum_{jk} n_j n_k \frac{\partial v_{jk}(n)}{\partial n_i}$$

$$= U_{0i} + \frac{L_i}{3}(u-1) + \mathcal{O}((u-1)^2)$$

$$\simeq au + bu^{4/3}$$

rearrangement term

Again, a and b are given as a linear function of U_{0i} and L_i.