Baryon-Baryon correlation from ~~heavy-ion~~ collisions and its implication to baryon-baryon interaction

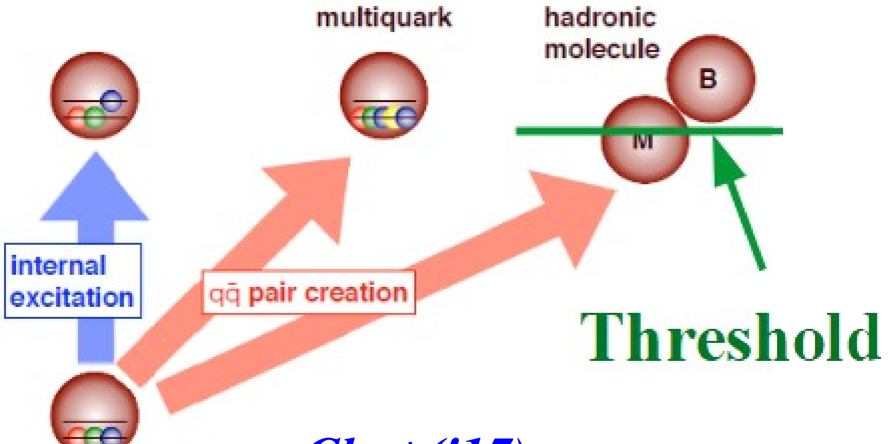
Akira Ohnishi (YITP, Kyoto U.)

FemTUM19, Sep. 12-13, 2019, Munich, Germany

K. Morita, T. Furumoto, AO, PRC91('15)024916 (ΛΛ)
AO, K. Morita, K. Miyahara, T. Hyodo, NPA954 ('16)294 (ΛΛ, K−p)
K. Morita, AO, F. Etminan, T. Hatsuda, PRC94('16), 031901(R) (ΩN)
S. Cho et al.(ExHIC Collab.), Prog.Part.Nucl.Phys.95('17)279 (ΛΛ, K−p)
T. Hatsuda, K. Morita, AO, K. Sasaki, NPA967('17), 856 (Ξ−p)
K. Morita, S. Gongyo, T. Hatsuda, T. Hyodo, Y. Kamiya, AO, arXiv:1908.05414 (NΩ, ΩΩ)

Hadron Physics at High-Energy Colliders ?

- **High-Energy Nuclear Collisions ($\sqrt{s_{NN}}$=40 GeV – 14 TeV) as a Hadron Factory**
  - **Higgs, BSM, SUSY, QGP, …**
  - **dN/dy ~ 1000 (RHIC, Au+Au) → 10^3-10^5 hadrons in one event**
  - **Various hadrons, nuclei (A<= 4) and anti-nuclei are formed.**
  - **Yield ~ Stat. Model calc.**
    (Formation processes are too complicated to be out of statistical.)
- **Trend in Hadron physics**
  - **Quark-gluon structure of hadrons (Multi-quark or Hadronic molecule)**
  - **Hadrons with heavy-quarks**
  - **Hadron-Hadron interaction**
    ΛN, ΣN, ΛΛ, ΞN, $\bar{K}$N, ….
  - **Hadrons in nuclear matter / EOS of nuclear matter**



Cho+('17)

Outline

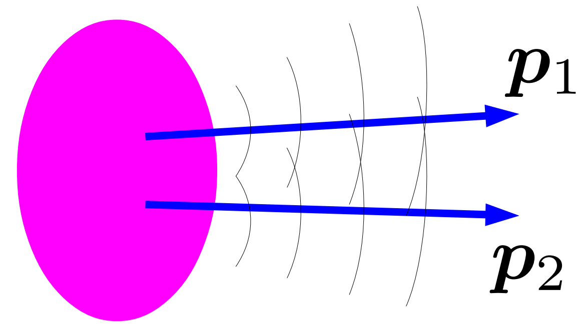
- Introduction: Hadron physics at High-Energy Colliders ?
- Correlation function and interaction
  - Two way to use the correlation function
  - Scattering length dependence of the correlation function
- Baryon-Baryon Correlation Function
  - Where is dibaryon ?
  - pΩ potential from lattice QCD
  - pΩ from STAR and ALICE
  - ΛΛ, pΞ and ΩΩ correlations
- Summary

Correlation Function and Interaction

Correlation Function

- **Emitting source function**

$$N_i(\boldsymbol{p}) = \int d^4x S_i(x, \boldsymbol{p})$$



- **Two-particle momentum dist.**
  - **Assumption: Two particles are produced independently, and the correlation is generated by the final state int.**
    Koonin('77), Pratt+('86), Lednicky+('82)

$$N_{12}(\boldsymbol{p}_1, \boldsymbol{p}_2) \simeq \int d^4x d^4y S_1(x, \boldsymbol{p}_1) S_2(y, \boldsymbol{p}_2) \underline{|\Psi_{\boldsymbol{p}_1, \boldsymbol{p}_2}(x, y)|^2}$$

two-body w.f.

$$\simeq \int d^4x d^4y S_1(x, \boldsymbol{p}_1) S_2(y, \boldsymbol{p}_2) \underline{|\varphi_{\boldsymbol{q}}(\boldsymbol{r})|^2}$$

relative w.f.

- **Correlation function**

$$C(\boldsymbol{p}_1, \boldsymbol{p}_2) = \frac{N_{12}(\boldsymbol{p}_1, \boldsymbol{p}_2)}{N_1(\boldsymbol{p}_1) N_2(\boldsymbol{p}_2)} \simeq \int d\boldsymbol{r} S_{12}(\boldsymbol{r}) |\varphi_{\boldsymbol{q}}(\boldsymbol{r})|^2$$

Correlation Function

- **Example: Free identical bosons (spin 0, non-relativistic), Gaussian source (static, simultaneous, spherical)**

$$S(\boldsymbol{x},\boldsymbol{p}) \propto \exp\left[-\frac{\boldsymbol{x}^2}{2R^2}-\frac{\boldsymbol{p}^2}{2MT}\right]$$

$$S(\boldsymbol{x},\boldsymbol{p}_1)S(\boldsymbol{y},\boldsymbol{p}_2) \propto \exp\left[-\frac{\boldsymbol{R}_{\rm cm}^2}{R^2}-\frac{\boldsymbol{r}^2}{4R^2}-\frac{\boldsymbol{P}^2}{4MT}-\frac{\boldsymbol{q}^2}{2\mu T}\right]$$

$$\Psi_{\boldsymbol{p}_1,\boldsymbol{p}_2}(\boldsymbol{x},\boldsymbol{y}) \propto \frac{1}{\sqrt{2}}\left[e^{i\boldsymbol{p}_1\cdot\boldsymbol{x}+i\boldsymbol{p}_2\cdot\boldsymbol{y}}+e^{i\boldsymbol{p}_1\cdot\boldsymbol{y}+i\boldsymbol{p}_2\cdot\boldsymbol{x}}\right]$$

$$= e^{i\boldsymbol{P}\cdot\boldsymbol{R}_{\rm cm}} \times \sqrt{2}\cos\boldsymbol{q}\cdot\boldsymbol{r}$$

- **Correlation function**

$$C(\boldsymbol{q}) = (4\pi R^2)^{-3/2}\int d\boldsymbol{r}\exp\left[-\frac{\boldsymbol{r}^2}{4R^2}\right]2\cos^2\boldsymbol{q}\cdot\boldsymbol{r}$$

$$= 1+\exp(-4q^2R^2)$$

Correlation Function → Source Size

How can we measure the radius of a star ?

- **Two photon intensity correlation**
  Hanbury Brown & Twiss, Nature 10 (1956), 1047.
  - **Simultaneous two photon observation probability is enhanced from independent emission cases**
    → angular diameter of Sirius=6.3 msec

Recent data (Wikipedia) 5.936±0.016 msec

A TEST OF A NEW TYPE OF STELLAR INTERFEROMETER ON SIRIUS

By R. HANBURY BROWN

Jodrell Bank Experimental Station, University of Manchester

AND

DR. R. Q. TWISS

Services Electronics Research Laboratory, Baldock

NATURE November 10, 1956 VOL. 178

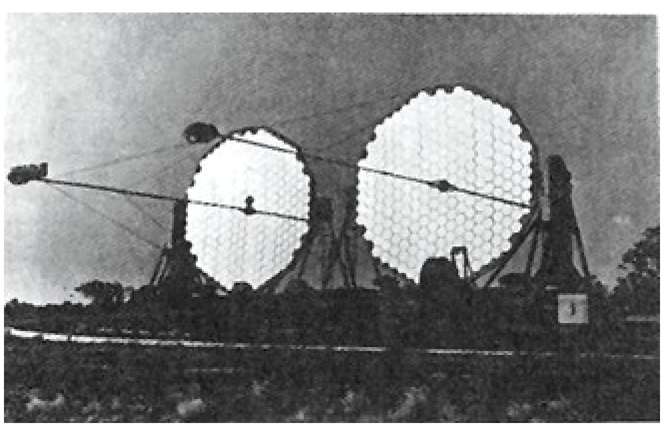


Figure 2. Picture of the two telescopes used in the HBT experiments. The figure was extracted from Ref.[1].

HBT telescope (from Goldhaber, ('91))

$$C = \langle I_1 I_2 \rangle / \langle I_1 \rangle \langle I_2 \rangle$$

$$\frac{C(d)-1}{C(0)-1}$$

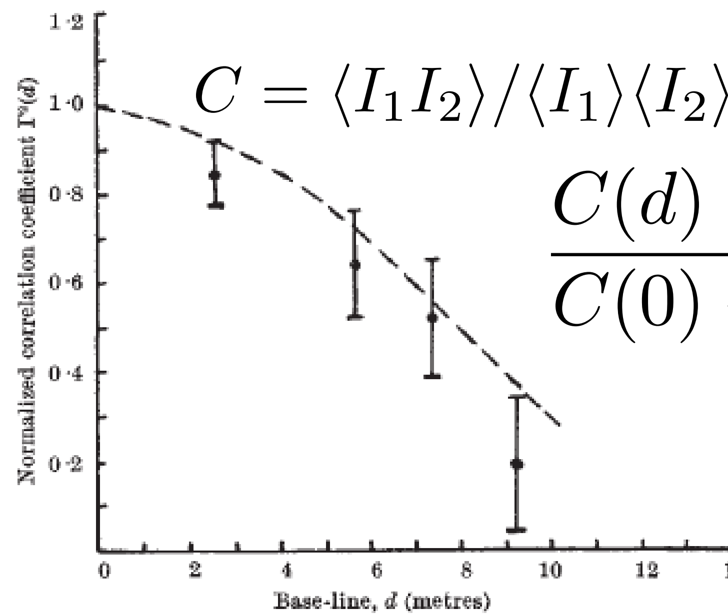


Fig. 2. Comparison between the values of the normalized correlation coefficient $\Gamma^2(d)$ observed from Sirius and the theoretical values for a star of angular diameter 0·0063″. The errors shown are the probable errors of the observations

HBT ('56)

Two particle intensity correlation

- **Wave function symmetrization from quantum statistics**

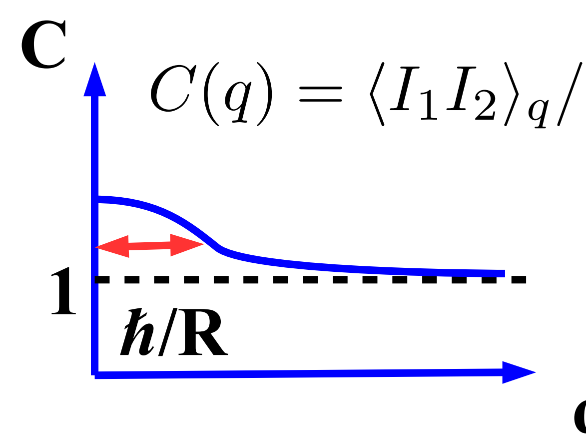
$$C(\mathbf{q}) = \int d^3r\, \underline{S(\mathbf{q},\mathbf{r})} \underline{\left| \frac{1}{\sqrt{2}} (e^{i\mathbf{q}\cdot\mathbf{r}} + e^{-i\mathbf{q}\cdot\mathbf{r}}) \right|^2} \simeq \underline{1 + \exp(-4q^2R^2)}$$

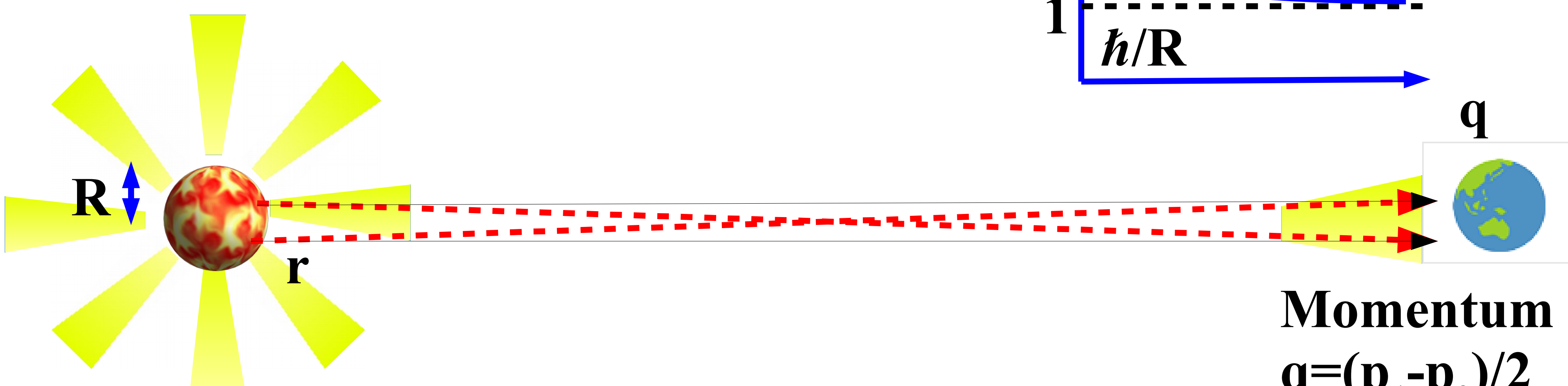
Source fn.
(r=relative coordinate)

(symmetrized w.f.)^2

Static spherical source case

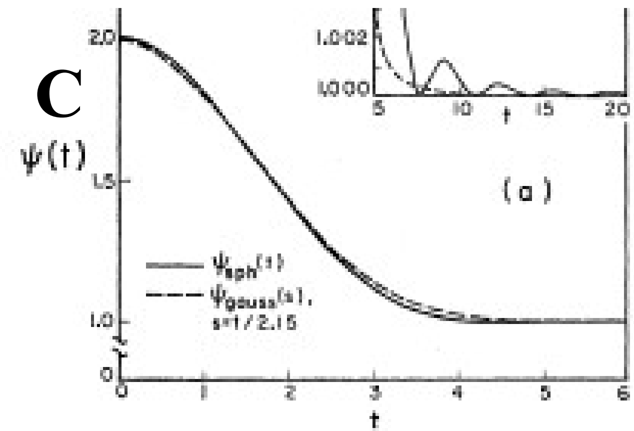
→ **Small relative momenta are favored due to symmetrization of the relative wave function.**





How can we measure source size in nuclear reactions ?

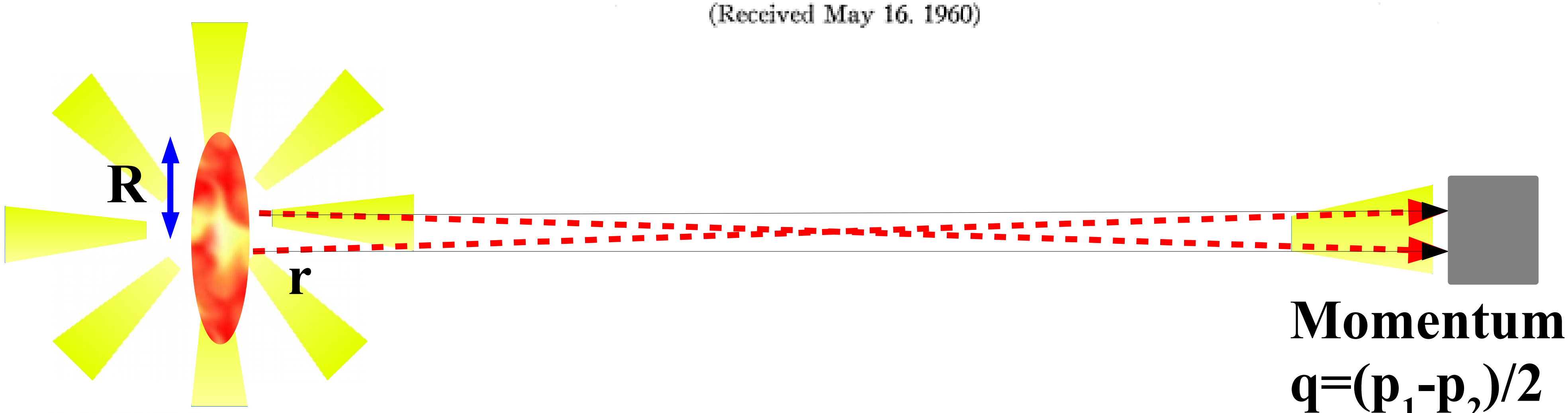
- **Two pion interferometry**
  G. Goldhaber, S. Goldhaber, W. Lee, A. Pais, Phys. Rev. 120 (1960), 300
  - **Two pion emission probability is enhanced at small relative momenta**
    → Pion source size ~ 0.75 ℏ / μc



PHYSICAL REVIEW VOLUME 120, NUMBER 1 OCTOBER 1, 1960

Influence of Bose-Einstein Statistics on the Antiproton-Proton Annihilation Process*

Gerson Goldhaber, Sulamith Goldhaber, Wonyong Lee, and Abraham Pais†
Lawrence Radiation Laboratory and Department of Physics, University of California, Berkeley, California
(Received May 16. 1960)



Another usage of correlation function

- **HBT, GGLP: Corr. Fn. + w.f. → Source Size**
  Another way: Corr. Fn. + Source Size
  → wave function → hadron-hadron interaction

- **Effect of hadron-hadron interaction on the wave function**
  - **Assumption: Only s-wave (L=0) is modified.**
  - **Non-identical particle pair, Gauss source.**

$$\varphi_{\boldsymbol{q}}(\boldsymbol{r}) = e^{i\boldsymbol{q}\cdot\boldsymbol{r}} - j_0(qr) + \chi_q(r)$$

$$\rightarrow C(\boldsymbol{q}) = \int d\boldsymbol{r} S(r) |\varphi_{\boldsymbol{q}}(\boldsymbol{r})|^2$$

$$= 1 + \int d\boldsymbol{r} S(r) \left\{ |\chi_q(r)|^2 - |j_0(qr)|^2 \right\}$$

K. Morita, T. Furumoto, AO, PRC91('15)024916

Corr. Fn. shows how much squared w. f. is enhanced
→ Large CF is expected with attraction

Wave function around threshold (S-wave, attraction)

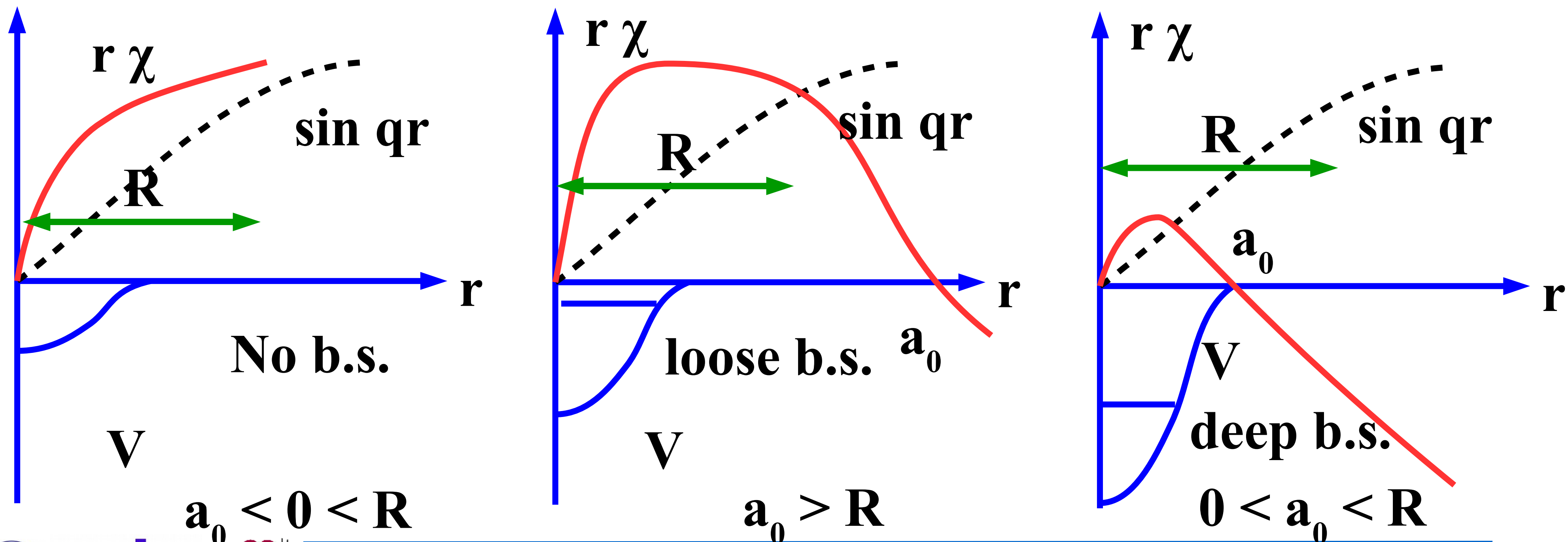
- **Low energy w.f. and phase shift**

$$u(r) = qr\chi_q(r) \to \sin(qr + \delta(q)) \sim \sin(q(r - a_0))$$

$$q \cot\delta = -\frac{1}{a_0} + \frac{1}{2} r_{\rm eff} q^2 + \mathcal{O}(q^4) \ (\delta \sim -a_0 q)$$

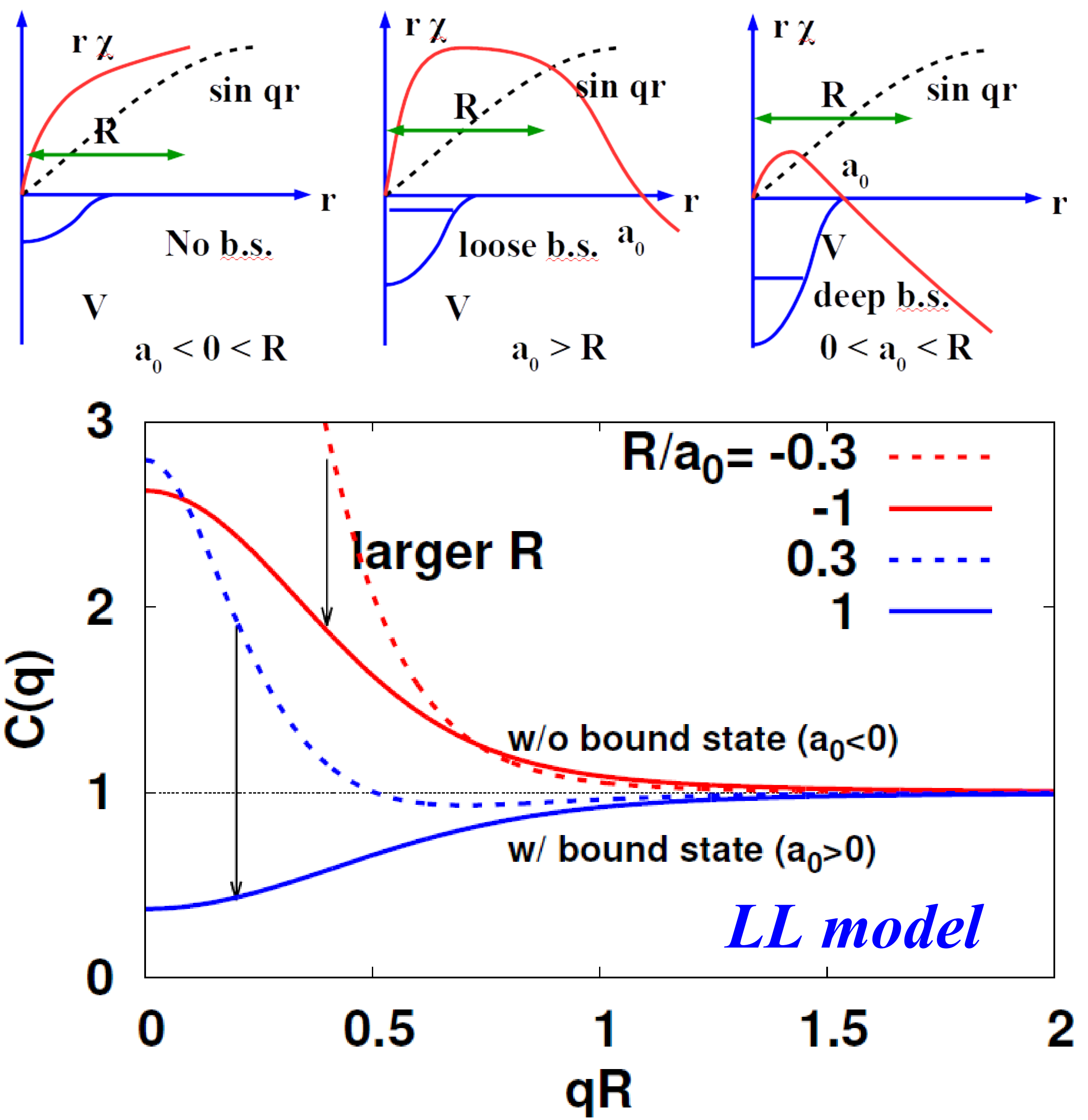
a_0 =scatt. length
r_{eff}=eff. range

  - **Wave function grows rapidly at small r with attraction.**
  - **With a bound state (a_0>0), a node appears around r=a_0**

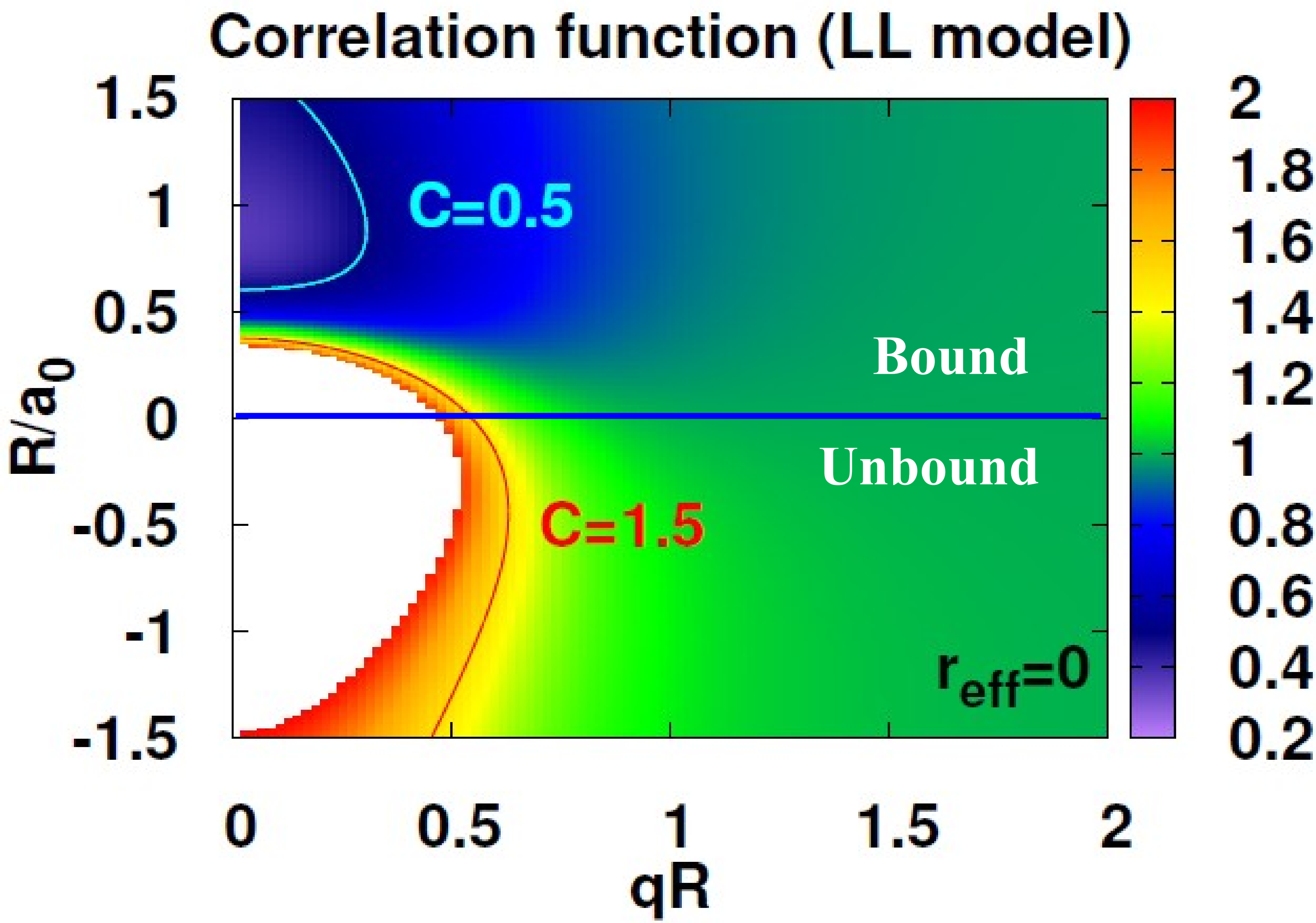


From correlation function to hadron-hadron interaction

- **Large $|a_0|$ ($|a_0| > R$)**
  → Large C(q)
  (unitary regime)
- **w/o bound state**
  ($a_0 < 0$, $|a_0| \sim R$)
  → C(q) > 1
- **With bound state**
  ($a_0 > 0$, $|a_0| \sim R$)
  → Region with
  C(q) < 1 appears



Correlation Function in LL model



LL model: R. Lednicky, V. L. Lyuboshits ('82)

Baryon-Baryon Correlation Function (mainly on pΩ correlation)

Where is dibaryon ?

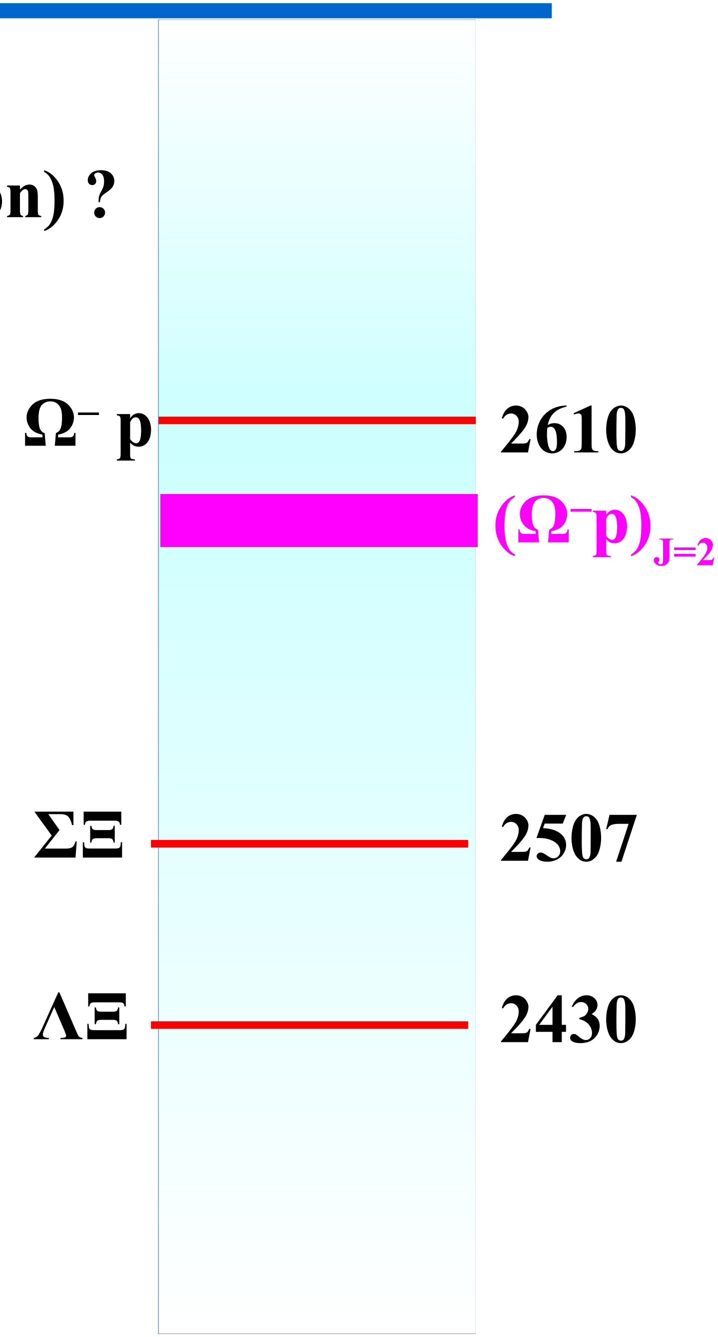
- **Deuteron = First dibaryon (pn bound state)**
- **H-particle: 6-quark state (uuddss)**
  - **Predicted (*Jaffe ('77)*), Ruled-out (ΛΛ nucl., *Takahashi+('01)*), Suggested as a resonance in exp. (*Yoon+ ('07)*) or as a bound state of ΞN (*HAL QCD ('16)*)**
- **Dibaryon would appear in channels, where *Oka ('88), Gal ('16)***
  - **The Pauli blocking of quarks does not operate,**
  - **and the Color-magnetic interaction is attractive**

Examples: H(=ΛΛ-NΞ-ΣΣ), NΩ, NΣ*, d*(=ΔΔ).

> ***Let us examine the existence of dibaryon states by using the correlation function !***

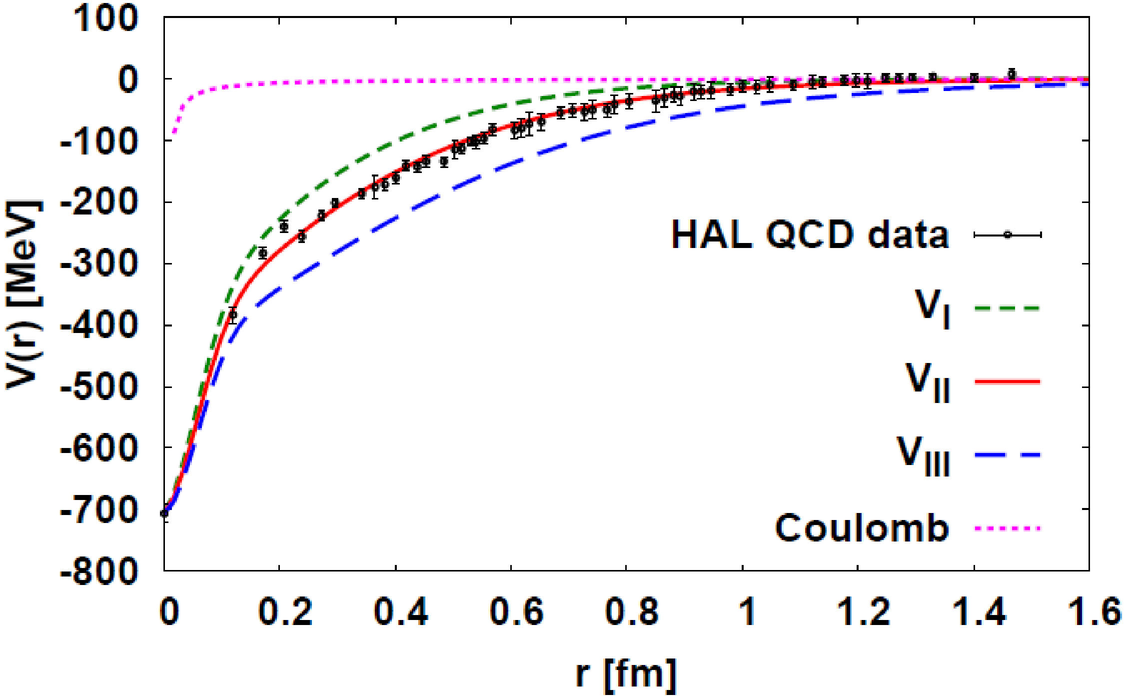
Ω N dibaryon

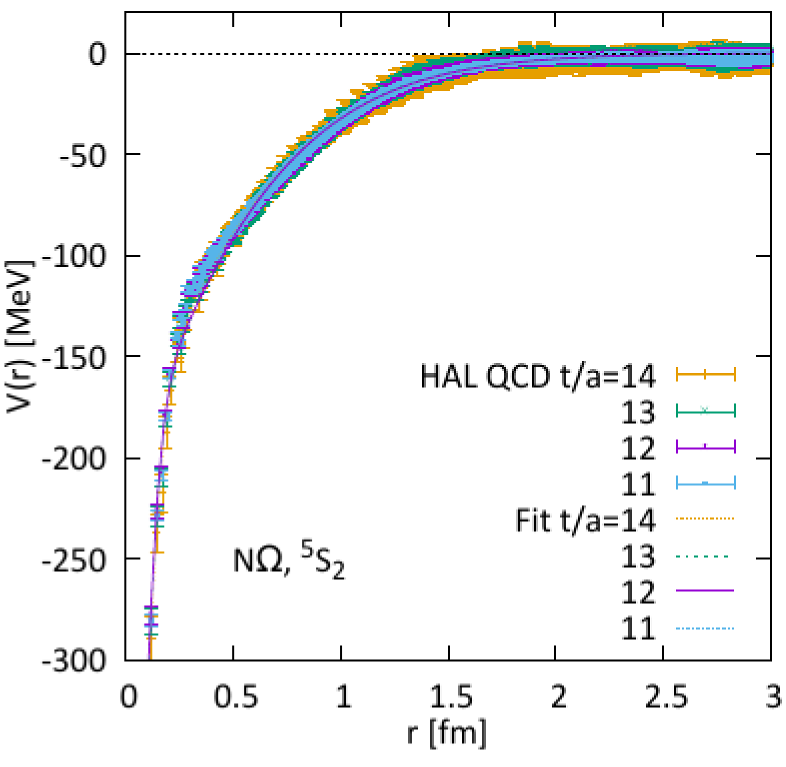
- **Ω : sss, Jπ=3/2+, M=1672 MeV**
- **Is there an ΩN bound state (S= –3 dibaryon) ?**
  - **Predicted as a dibaryon candidate**
    Goldman+ ('87), Oka ('88), Gal ('16)
  - **Lattice QCD predicts a bound state with narrow width for J=2 ($^5S_2$)**
    Etminan+ (HAL QCD)('14),
    Iritani+ (HAL QCD) ('19)
  - **Correlation function is measurable !**
    Adam+ (STAR)('19), ALICE, in prep.



ΩN potential from lattice QCD

- ΩN potential by HAL QCD Collab. (J=2)
  - m_π=875 MeV, B.E.~ 19 MeV
    F.Etminan et al. (HAL QCD Collab.), NPA928('14)89.
  - m_π=146 MeV, B.E.~ 2.2 MeV
    T. Iritani et al. (HAL QCD Collab.), PLB 792('19)284.





Calculation Details

- **NΩ potential from HAL QCD Collab.**
  Etminan+(HAL QCD) ('14), Iritani+ (HAL QCD)('19)
  - **J=1 potential is uncertain → Three models**
    Strong abs. at $r < r_0$ ($r_0 \sim 2$ fm) (*Morita+('16)*) (Standard)
    Complete absorption $\chi(J=1) = 0$ (Minimum)
    Same w.f. as that with J=2, $\chi(J=1) = \chi(J=2)$ (Reference)
  - **Statistical Error can be evaluated by using Jackknife potentials.**
- **Coulomb potential enhances CF even without strong int.**
  → Small-Large ratio of CF (*Morita+('16)*)
  - **Large source → Coulomb force dominate**
    Small source → Visible strong interaction effects
- **Source function: Blast wave, Gaussian source**

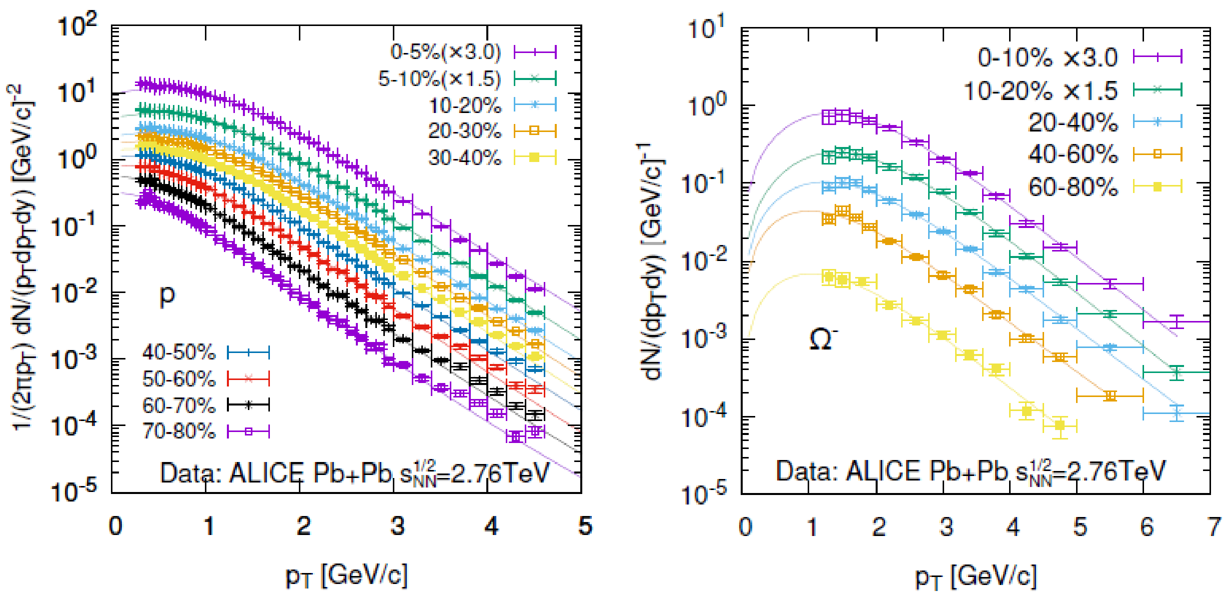
Emission Source Function

- **Gaussian Source ∝ exp($-r^2/4R^2$), R=(0.8-4) fm [Simple and convenient]**
- **Expanding source model [Reasonably realistic]**

Flow velocity $u_\mu(x)$

$$d^4x S_i(x,\boldsymbol{p}) = \tau_0 d\eta_s d^2 r_T \frac{d}{(2\pi)^3} \underline{n_f(u\cdot p, T)} \exp\left(-\frac{r_T^2}{2R_T^2}\right)$$

Fermi dist.



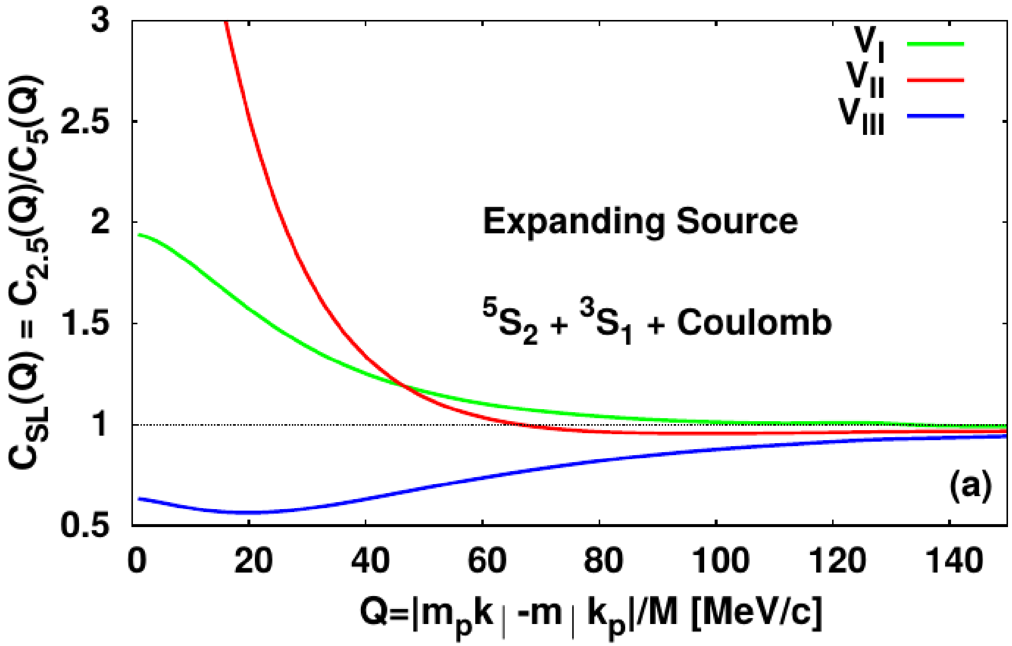
Morita+('19)

- **Transport model result [should be realistic] → Future work**

Comparison with STAR data

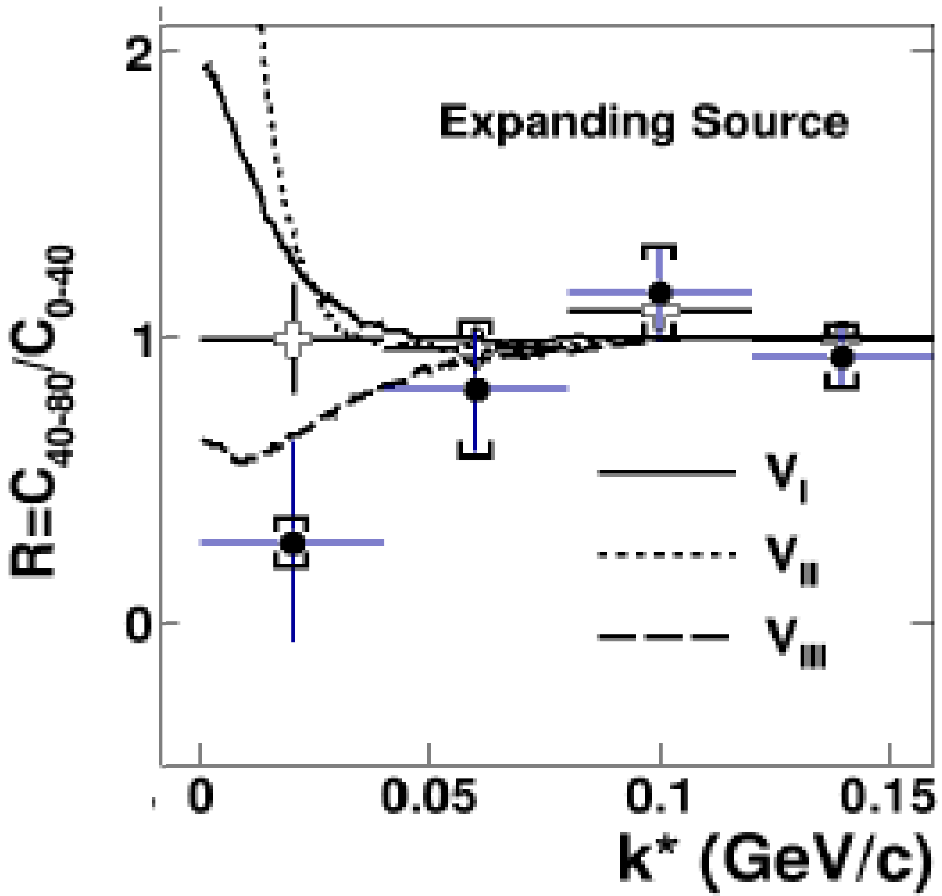
- **Dip structure in Small-Large ratio of CF**

C(R=2.5 fm)/C(R=5 fm)



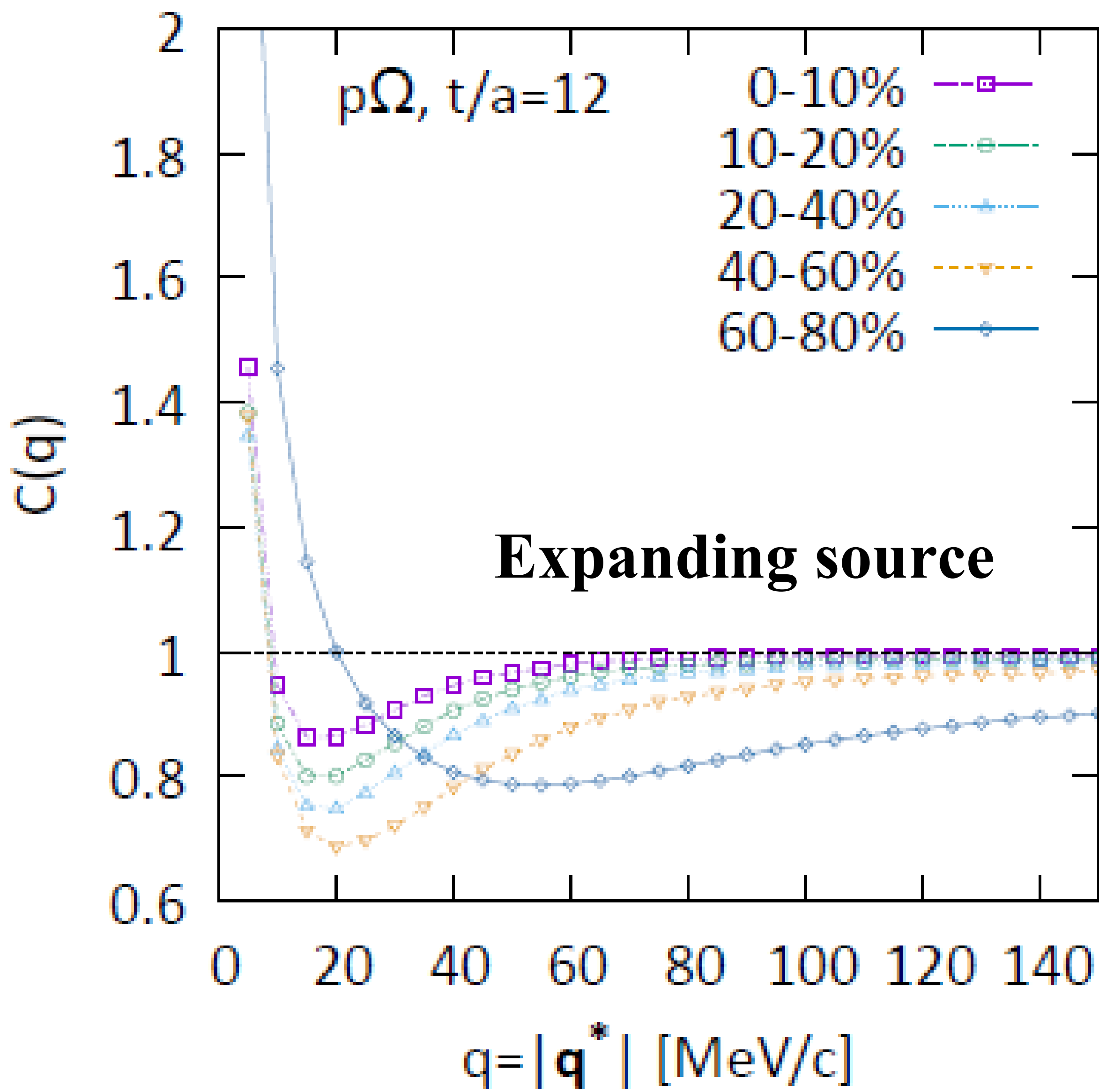
Morita, AO, Etminan, Hatsuda ('16)

STAR (1808.02511, PLB790 ('19) 490)



We may have a dibaryon state in ΩN channel

Source Size Dependence of Correlation Function



R ~ 8 fm

R ~ 4 fm

R ~ 2.5 fm

a_0 (pΩ)~ 3.4 fm

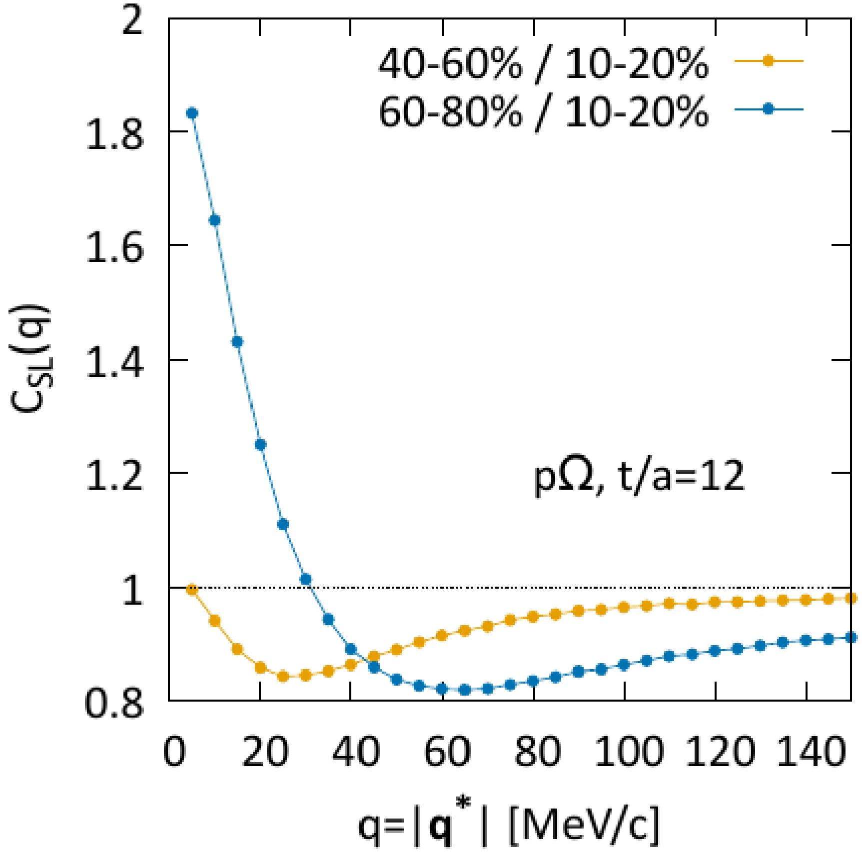
Centrality	τ_0 [fm/c]	R_T^Ω [fm]	R_T^p	α^Ω	β^Ω	α^p	β^p
0 – 10%	10.0	8.0	6.8	0.584	0.628	0.759	0.421
10 – 20%	9.085	6.75	6.23	0.618	0.579	0.750	0.425
20 – 40%	7.5	5.88	5.2	0.546	0.692	0.707	0.466
40 – 60%	5.5	4.38	3.92	0.444	0.858	0.604	0.6
60 – 80%	3.62	2.12	2.66	0.456	0.812	0.456	0.82

K. Morita, S. Gongyo, T. Hatsuda, T. Hyodo, Y. Kamiya, AO ('19)

Comparison with STAR data

- **Results with potential at nearly physical quark mass (= between V_{II} and V_{III})**
  → Dip is seen but is not deep enough to explain STAR data.

Morita, Gongyo, Hatsuda, Hyodo, Kamiya, AO, arXiv:1908.05414



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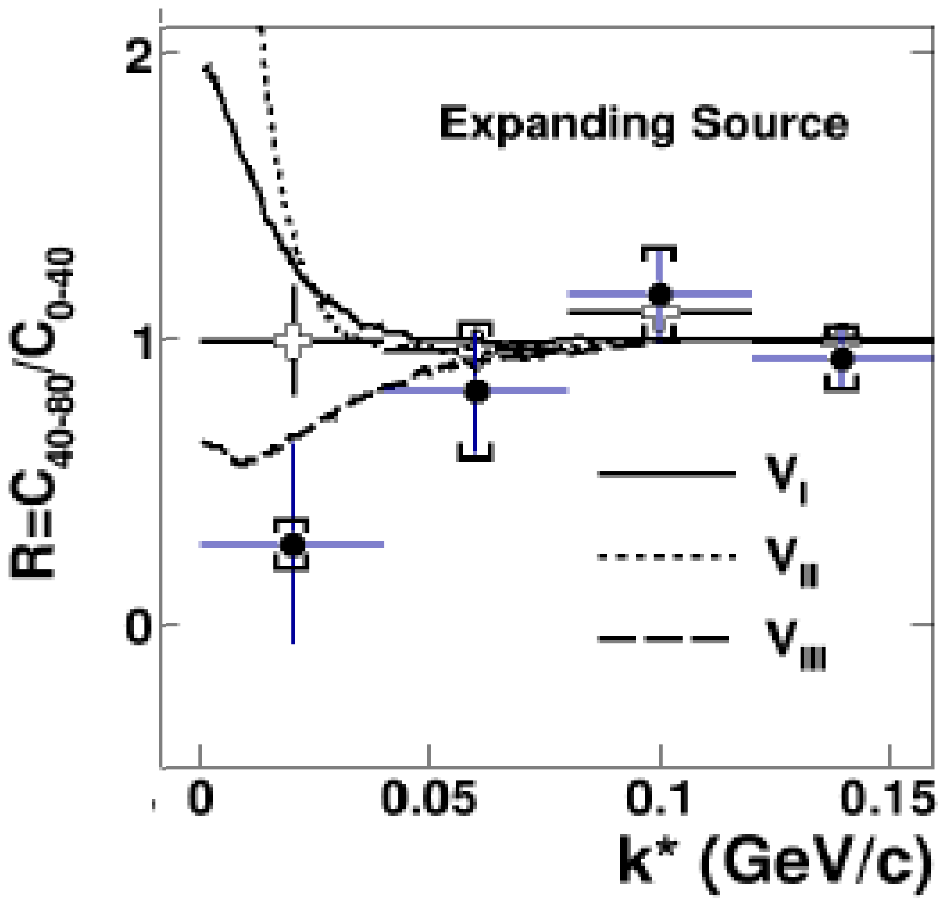
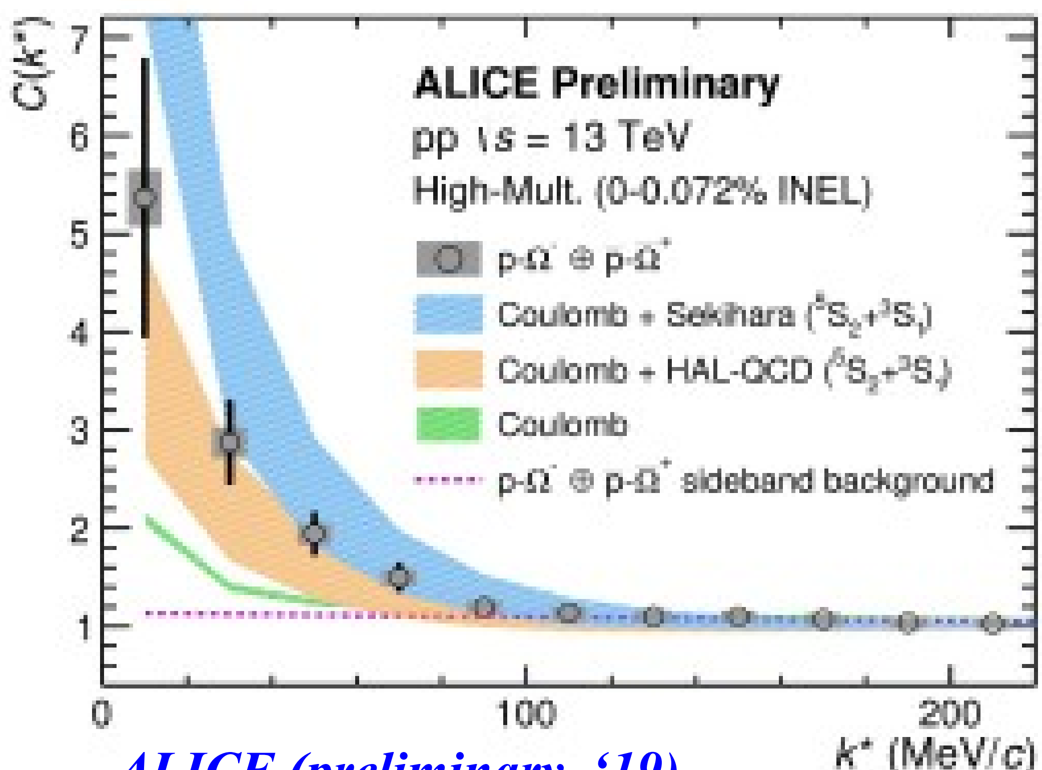
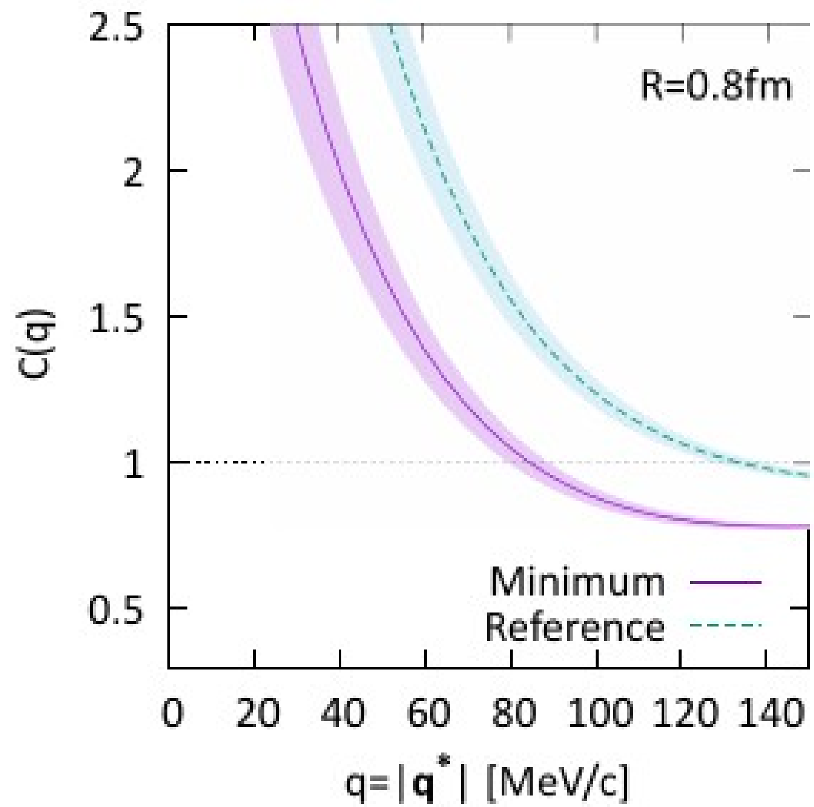


ALICE Preliminary

- **pp 13 TeV high-multiplicity events in ALICE**
  → Strong enhancement of CF at small q
  O. Vázquez Doce et al. (ALICE), Hadrons 2019

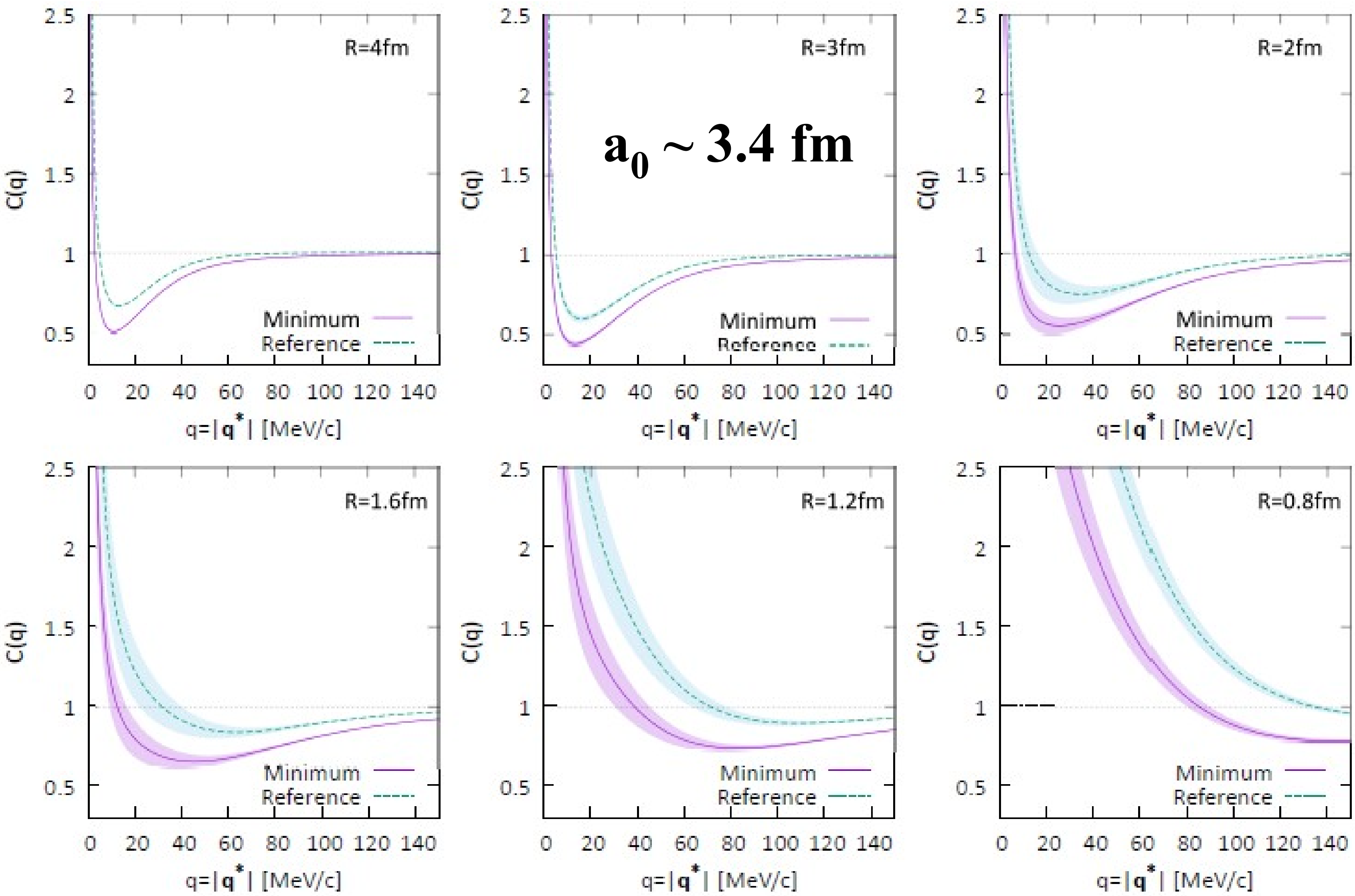


ALICE (preliminary, '19)



Morita+, arXiv:1908.05414

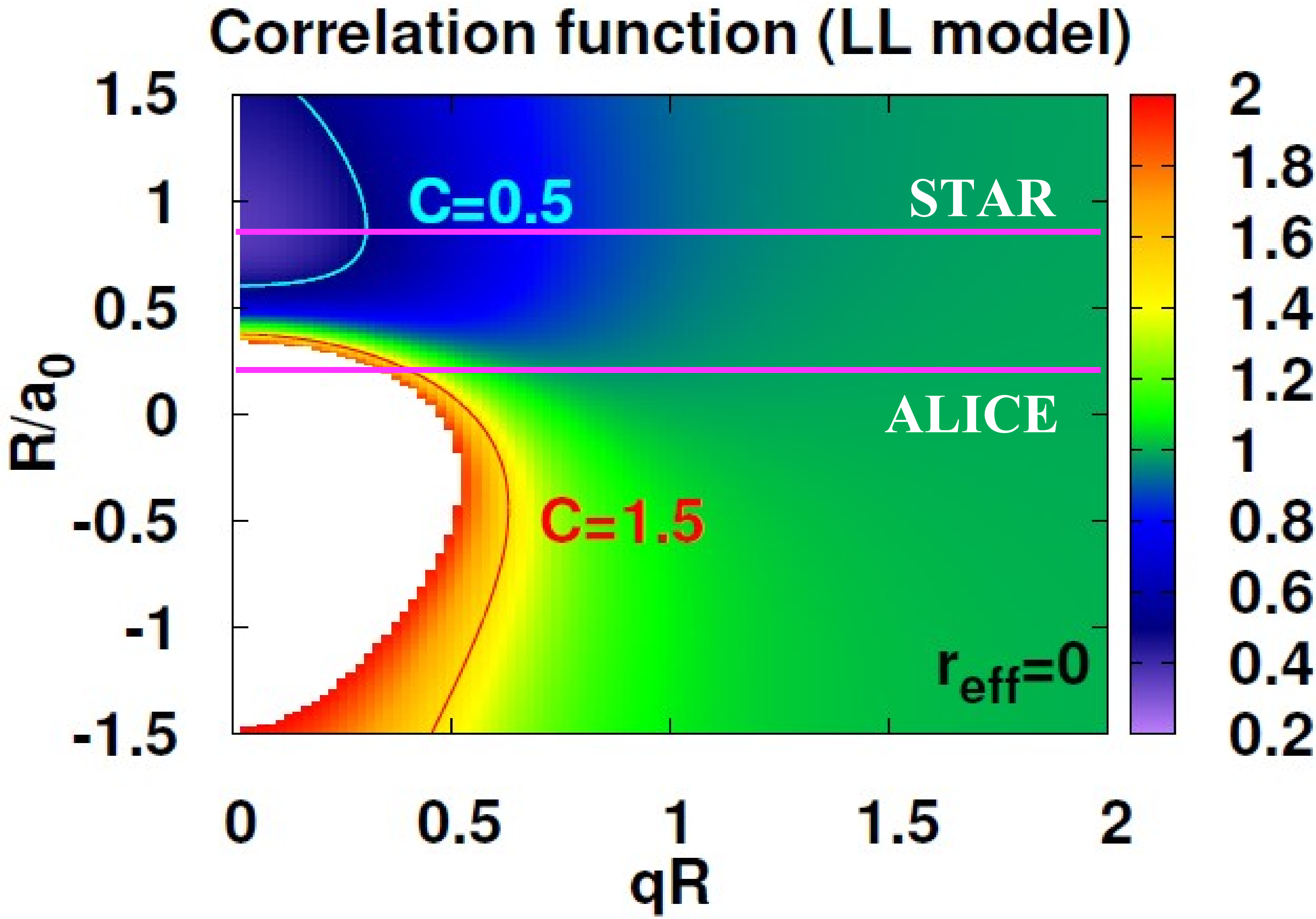
Source Size Dependence of Correlation Function



Gaussian Source

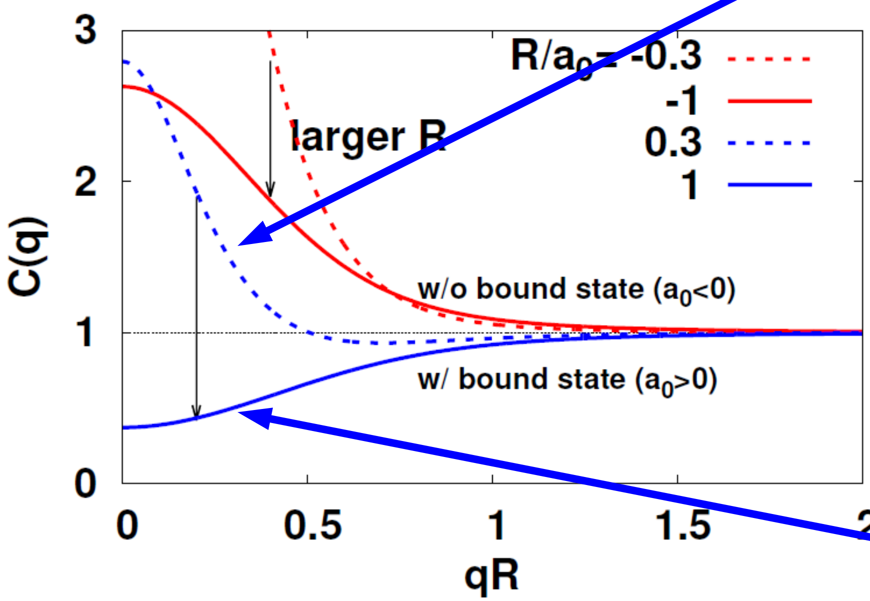
K. Morita, S. Gongyo, T. Hatsuda, T. Hyodo, Y. Kamiya, AO ('19)

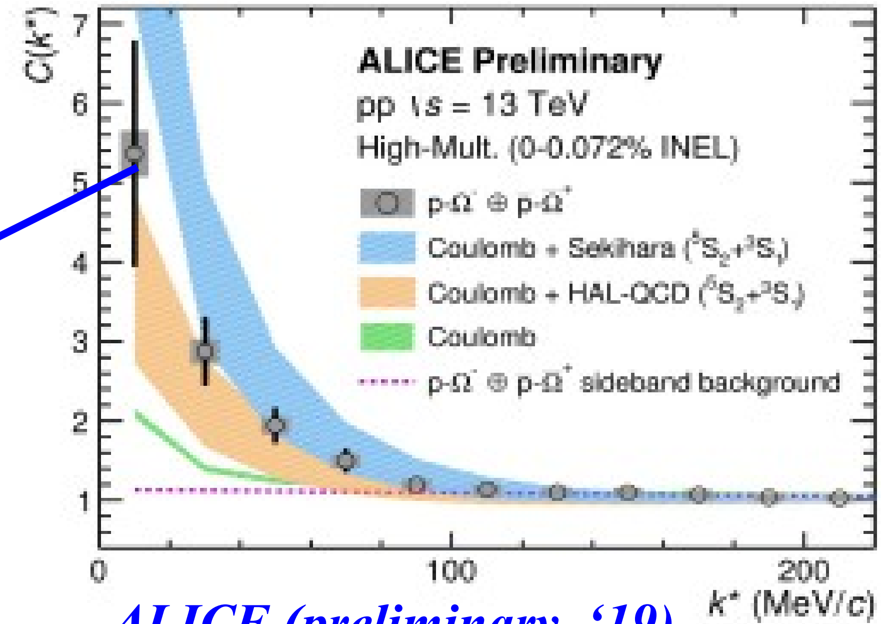
Correlation Function in LL model



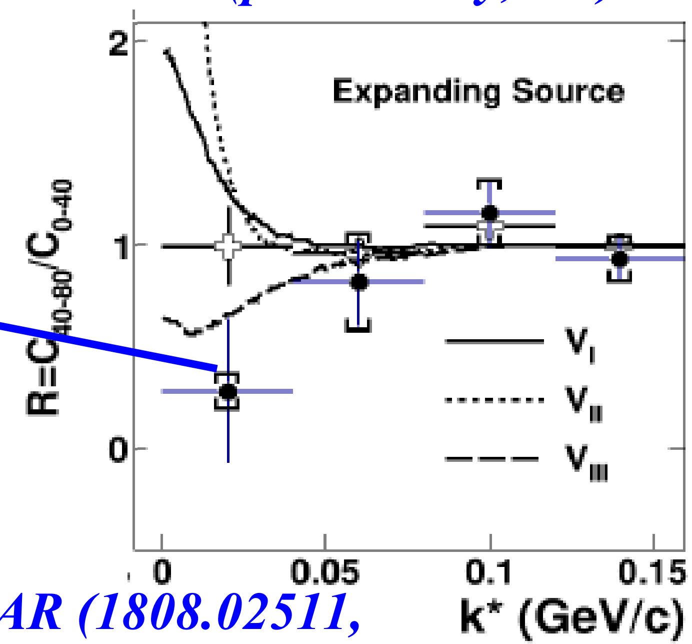
a_0 (pΩ)~ 3.4 fm, R(ALICE)~0.7 fm, R(STAR)~3 fm

STAR + ALICE = NΩ Dibaryon





ALICE (preliminary, '19)

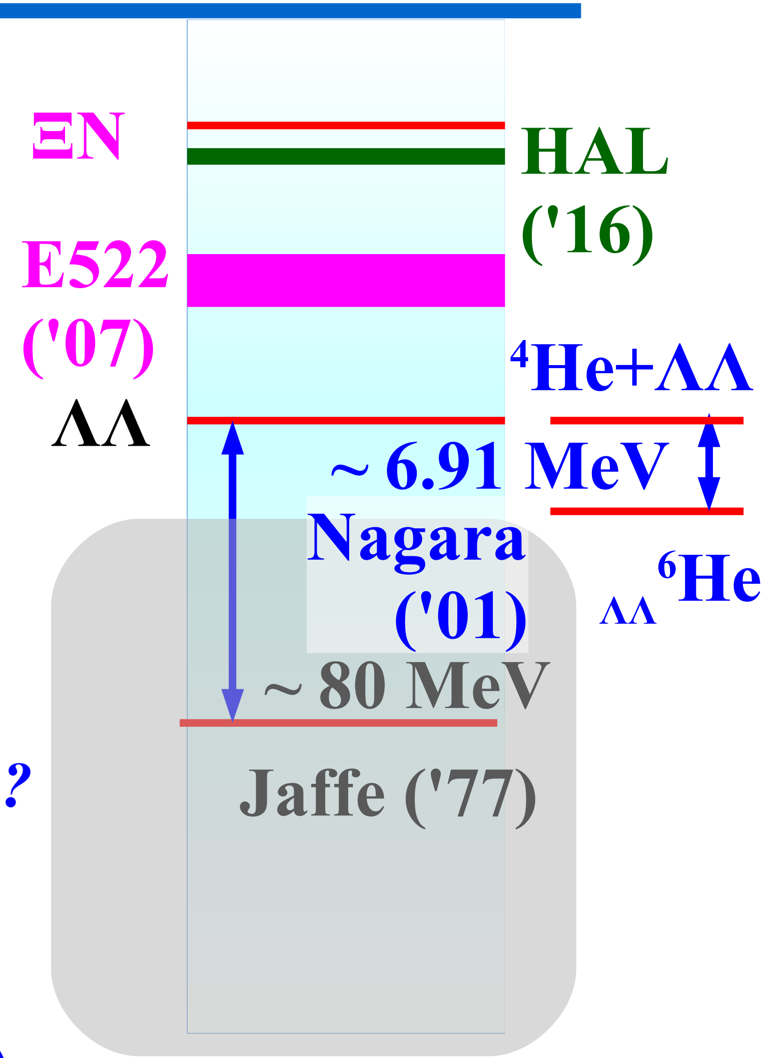


STAR (1808.02511, PLB790 ('19) 490)

Do I have time ?

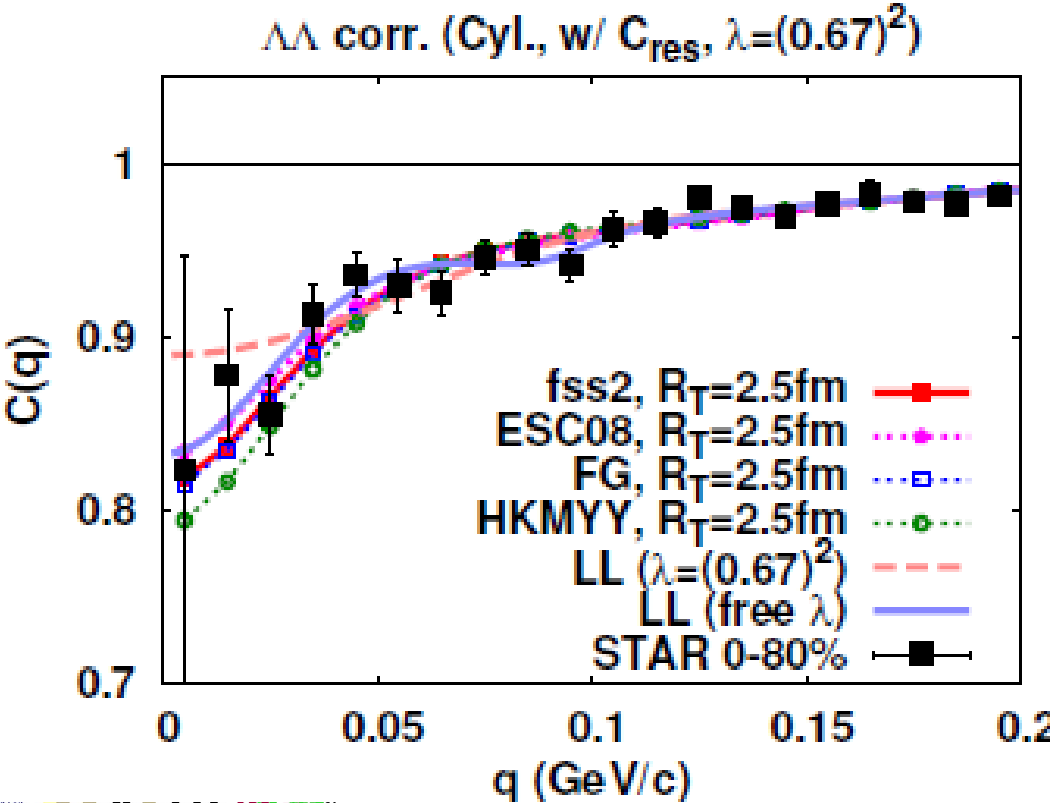
Relevance of ΛΛ interaction to physics

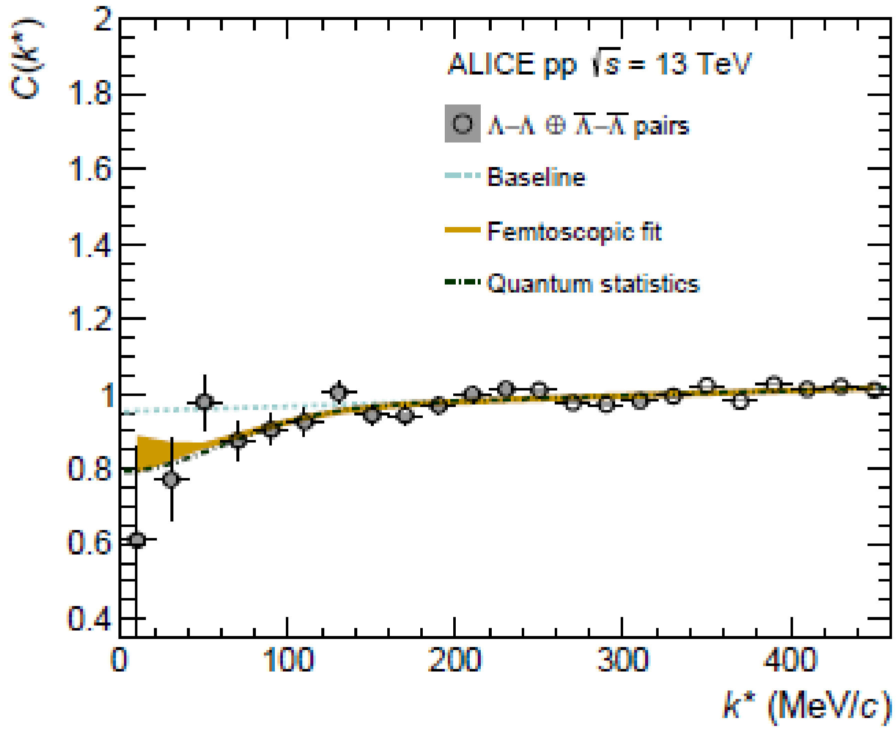
- **H-particle: 6-quark state (uuddss)**
  - **Prediction:** *R.L.Jaffe, PRL38(1977)195*
  - **Ruled-out by double Λ hypernucleus**
    Takahashi et al.,PRL87('01) 212502
  - **Resonance or Bound "H" ?**
    Yoon et al.(KEK-E522)+AO ('07)
  - **Lattice QCD**
    HAL QCD & NPLQCD ('11)
    HAL QCD ('16): H as a loosely bound ΞN ?
- **Neutron Star Matter EOS**
  - **Hyperon Puzzle**
    Demorest et al. ('10), Antoniadis et al. ('13)
  - **Cooling Puzzle (ΛΛ superfluidity)**
    T. Takatsuka, R. Tamagaki, PTP 112('04)37



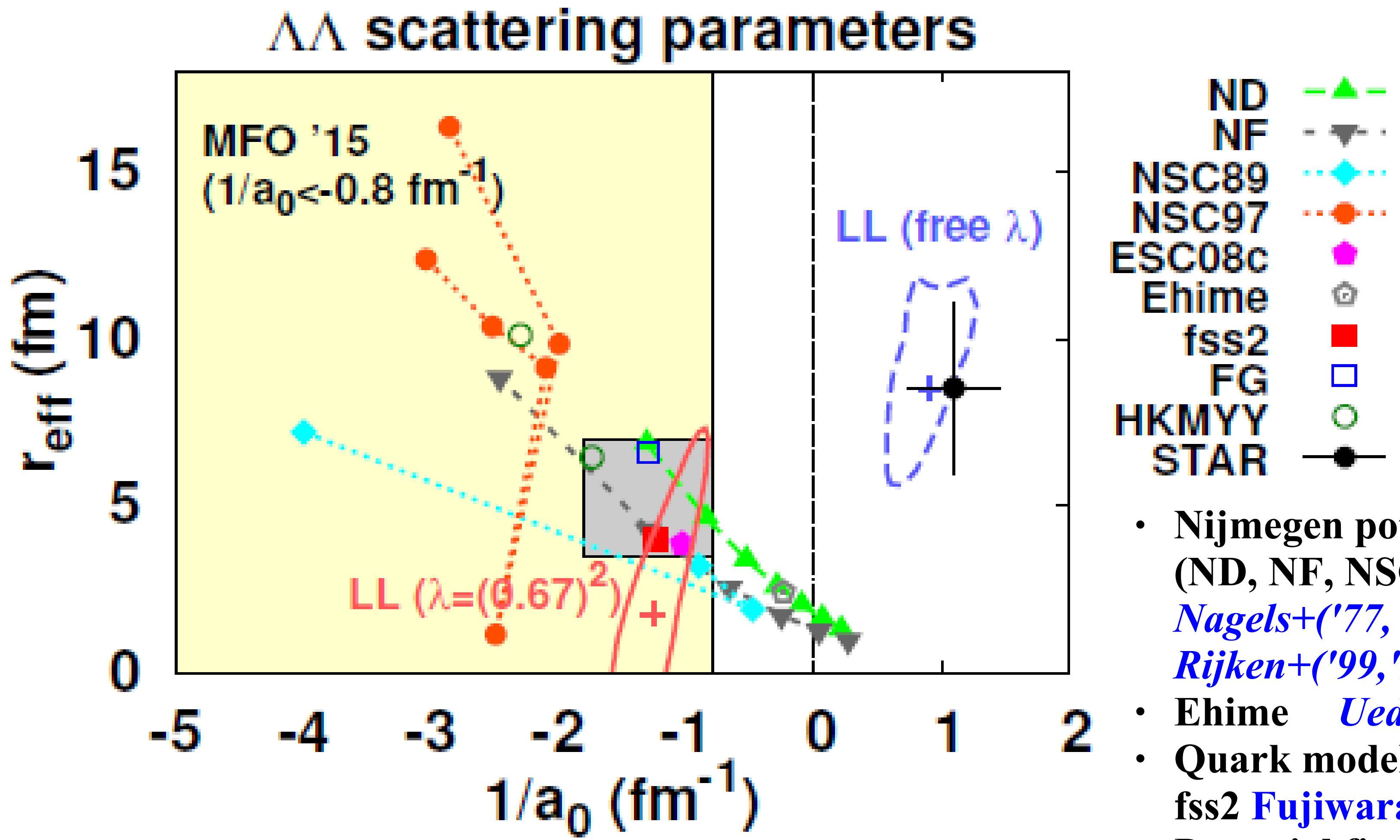
ΛΛ correlation at RHIC

- **STAR collaboration at RHIC measured ΛΛ correlation !**
  Adamczyk et al. (STAR Collaboration), PRL 114 ('15) 022301.
- **Theoretical Analysis well explains the data**
  K.Morita et al., T.Furumoto, AO, PRC91('15)024916;
  AO, K.Morita, K.Miyahara, T.Hyodo, NPA954 ('16), 294.
- **New Data from ALICE**
  S. Achara+(ALICE), PRC99('19), 024001; arXiv:1905.07209





$\Lambda\Lambda$ interaction from $\Lambda\Lambda$ correlation

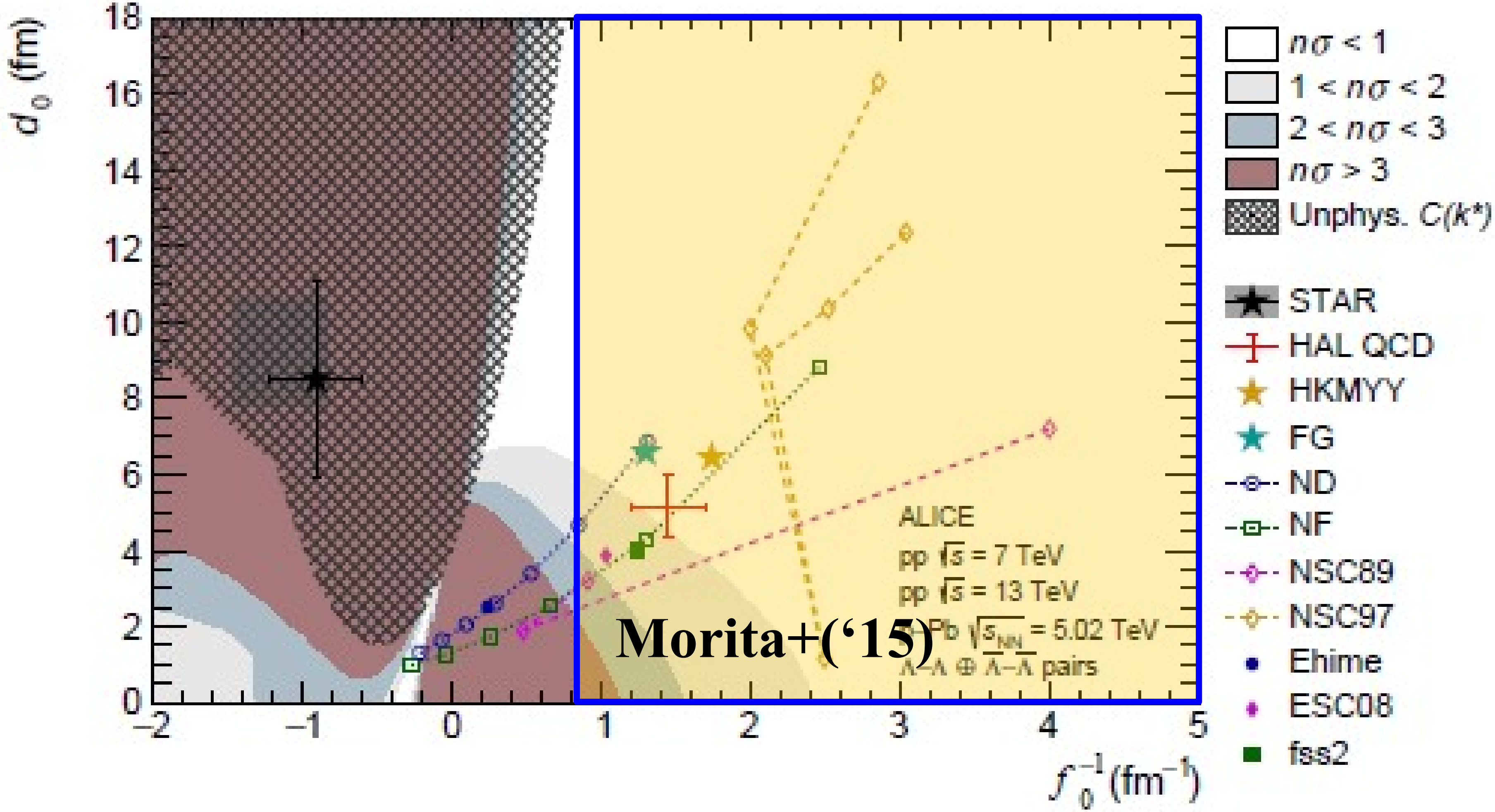


$$q \cot \delta = -1/a_0 + r_{eff}\, q^2/2 + O(q^4)$$

- **Nijmegen potentials (ND, NF, NSC89, NSC97, ESC08)** *Nagels+('77, '79), Maessen+('89), Rijken+('99,'10)*
- **Ehime** *Ueda et al. ('98)*
- **Quark model interaction:** fss2 Fujiwara et al.('07)
- **Potential fitted to Nagara** *Filikhin, Gal ('02) (FG), Hiyama et al. ('02, '10)(HKMYY)*

New Data from LHC-ALICE

ALICE (arXiv:1905.07209)

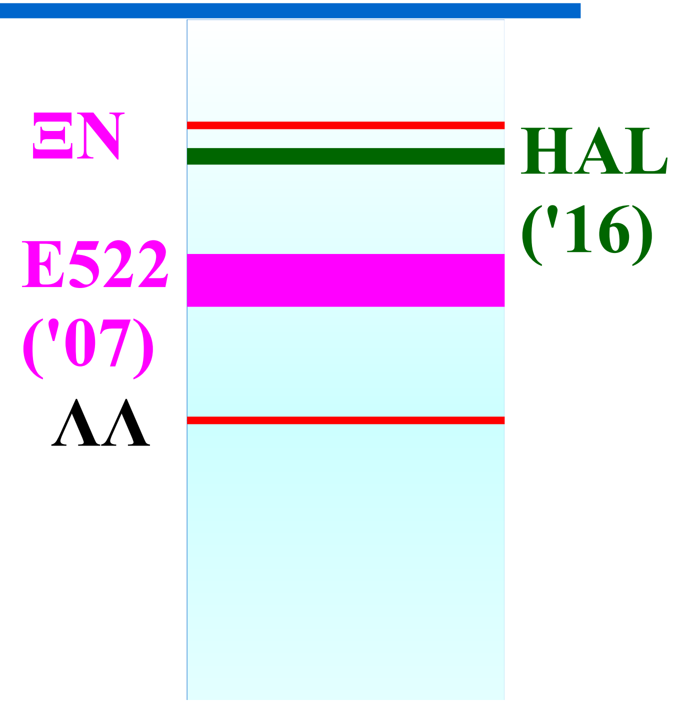


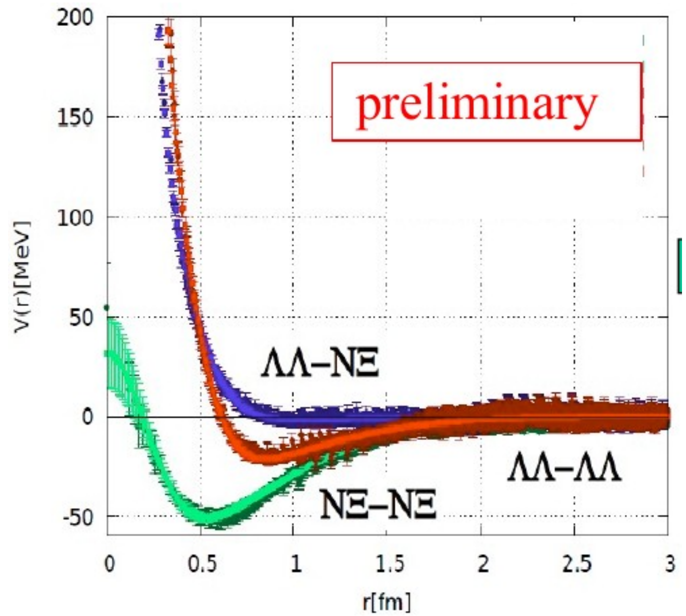
Weakly attractive $V_{\Lambda\Lambda}$.
Large reff → Becomes repulisve at low relatively density.

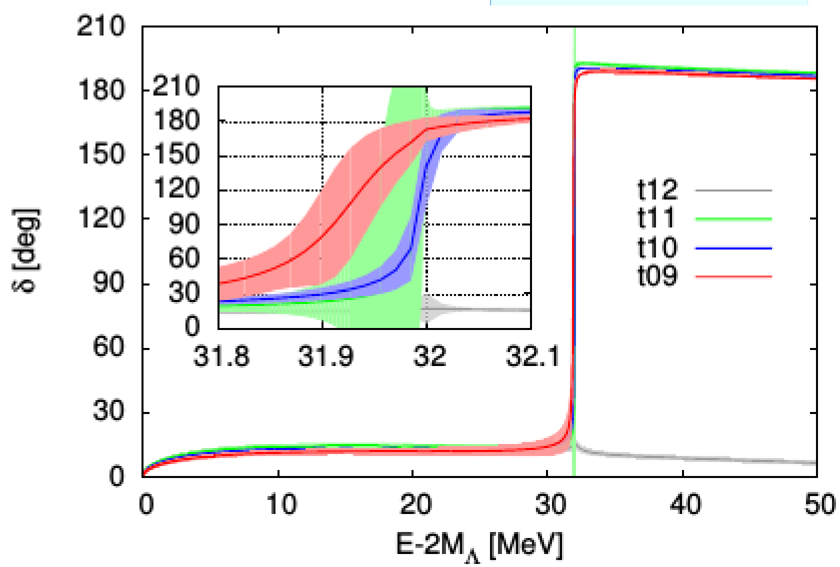
Relevance of ΞN interaction to physics

- **H-particle: 6-quark state (uuddss) may be realized as a loosely bound state of ΞN (I=0)**
  K. Sasaki et al. (HAL QCD, '16,'17)

- **Repulsive ΞN interaction (I=1) may help to support 2 $M_\odot$ Neutron Star**
  Weissborn et al., NPA881 ('12) 62.







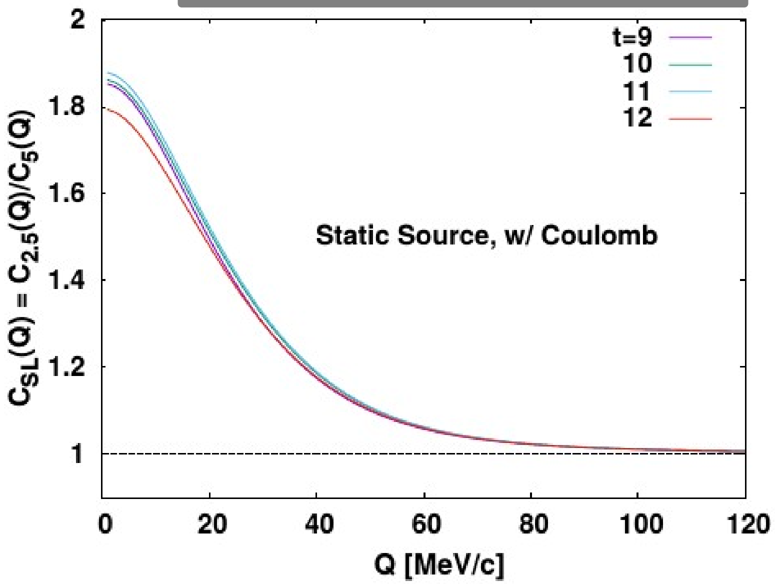
K. Sasaki et al. (HAL QCD Collab.), EPJ Web Conf. 175 ('18) 05010.

Ξ^- p correlation

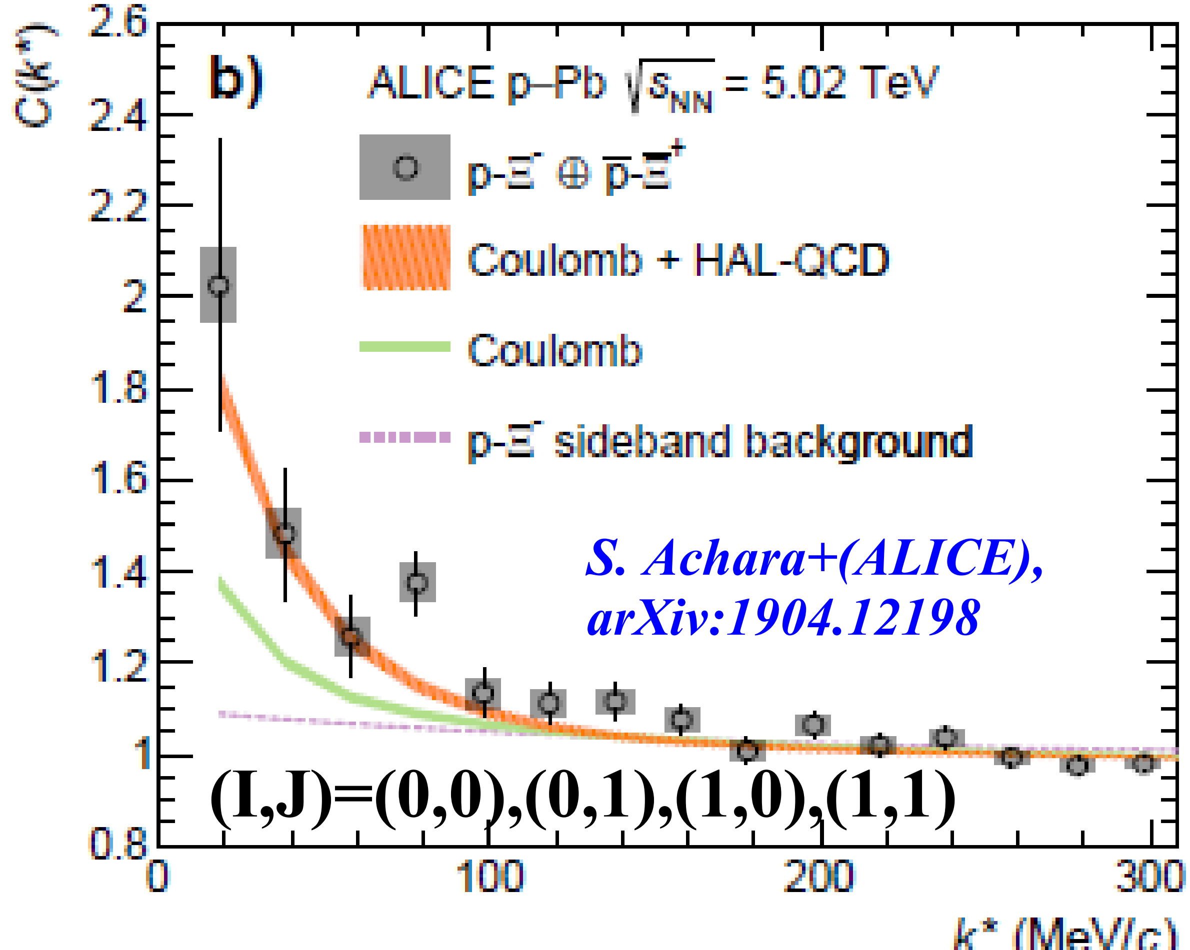
- **Prediction of the correlation function by using ΞN potential (HAL QCD Collab.) + Coulomb potential**

$$|\psi|^2_{\text{spin av.}} = \frac{1}{2}\sum_{I=0,1}\left[\frac{1}{4}|\psi_I^{J=0}|^2 + \frac{3}{4}|\psi_I^{J=1}|^2\right]$$

HAL prediction is examined !



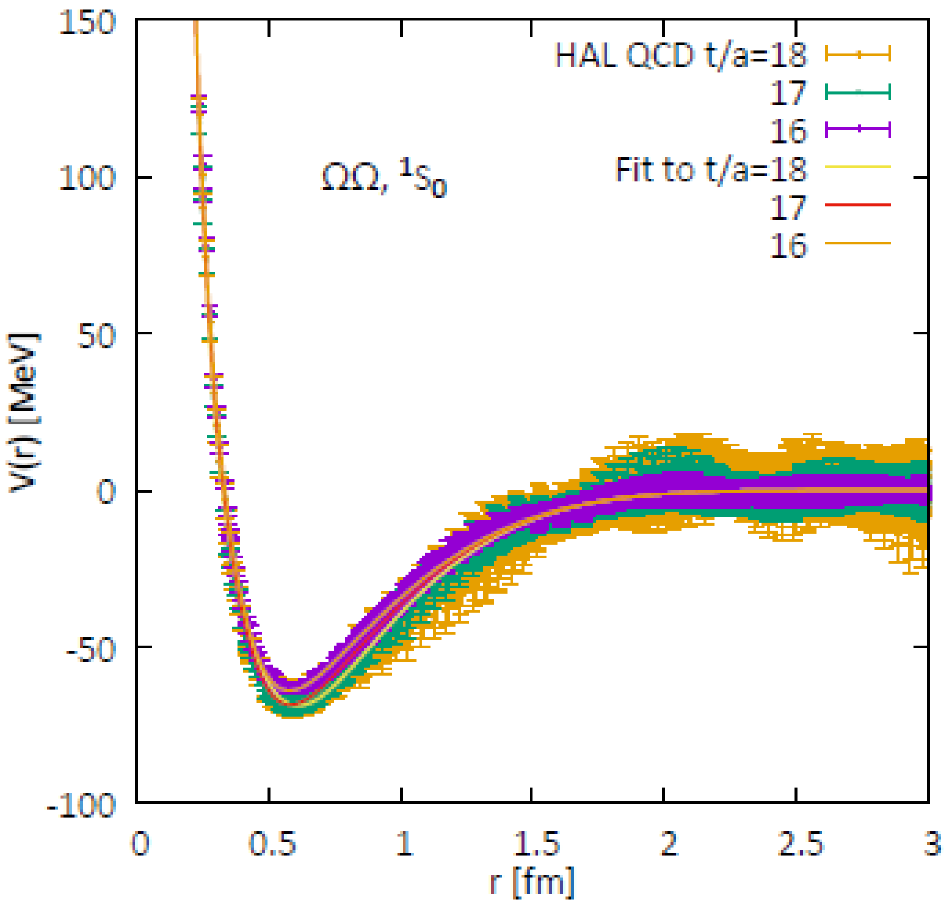
T. Hatsuda, K. Morita, AO, K. Sasaki, NPA967('17), 856.

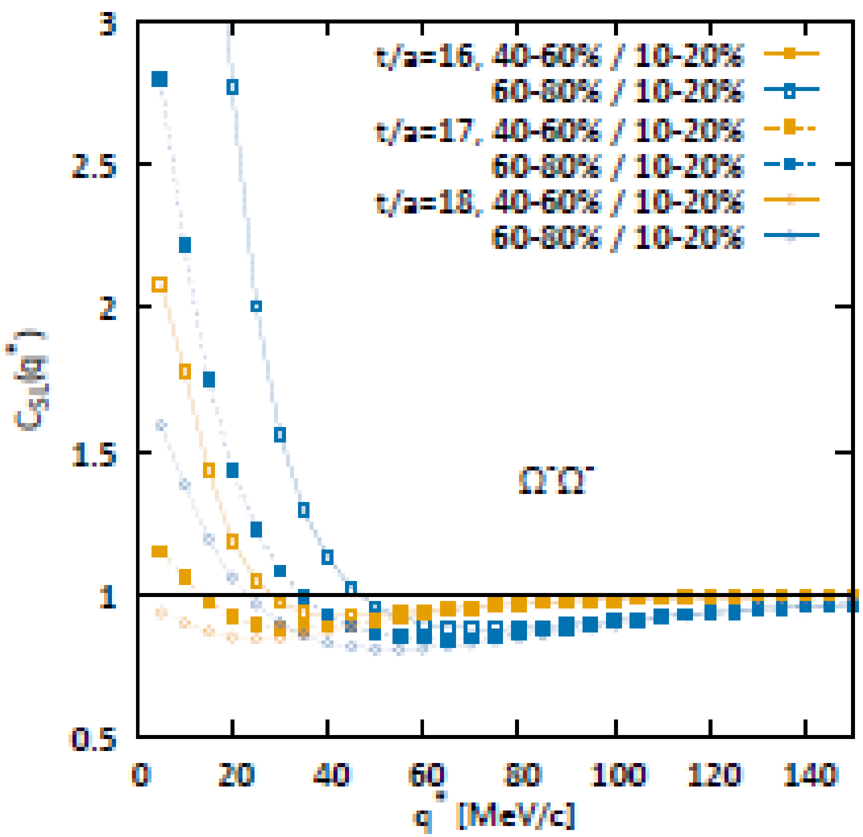


S. Achara+(ALICE), arXiv:1904.12198

ΩΩ correlation

ΩΩ potential: S. Gongyo et al. (HAL QCD Collab), Phys. Rev. Lett. 120, 212001 (2017), 1709.00654.





K. Morita, S. Gongyo, T. Hatsuda, T. Hyodo, T. Iritani, AO, K. Sasaki, in prep.

Summary

- **High-energy nuclear collisions produce many and various hadrons in one event, and can be utilized as the hadron factory.**
- **Two-hadron correlation functions have been used to measure the source size. Once the properties of the source is known, we can utilize CF to get knowledge of hadron-hadron interaction.**
- **Large CF at small q implies large $|a_0|/R$, and the source size dependence of CF may shows the sign of a_0.**
- **It seems (to me) that there are at least two dibayron states.**
  - **ALICE and STAR data strongly suggest the existence of S= –3 dibaryon as a bound state of NΩ.**
  - **ALICE data of Ξ^-p implies large $|a_0|/R$, and the existence of some kind of pole (b.s., res., virtual) around the threshold.**
  - **pp, pA and AA collision data will be helpful.**

To do (or Can do)

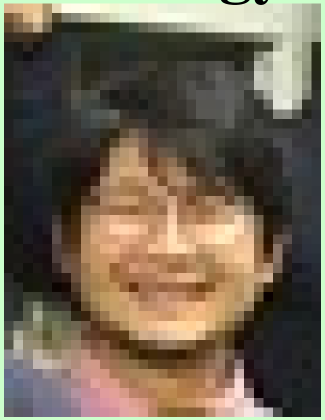
- **Coupled channel effects → Talk by Haidenbauer, Kamiya**
- **Ξ^- p correlation with updated HAL QCD potential (K. Sasaki et al.) → Sasaki's talk**
- **Λp correlation with various potentials (χEFT, Nijmegen, fss2, lattice) → Talk by Heidenbauer, Rijken**
- **K^- p correlation with amplitude from chiral SU(3) dynamics (e.g. Ikeda, Hyodo, Weise) → Kamiya's talk**
- **J=1 Ω^- p potential → Hyodo's talk**
- ***Let us use deuteron !***
  K^- d corr. (I=0 ampl.), Λd corr. ($^3_\Lambda$H B.E., 3BF), Ξ^- d, …
- ***Can we go to heavy-quarks ?***
  $c\tau(D)$ = 0.3 mm → $\gamma c\tau(D) = c\tau(D) \cosh y \sim 15$ cm (y=7)
  We may have enough D mesons at y=7 in fixed target LHC. (N. Yamanaka)

Thank you for attention !

Coauthors of arXiv:1908.05414

K. Morita

S. Gongyo

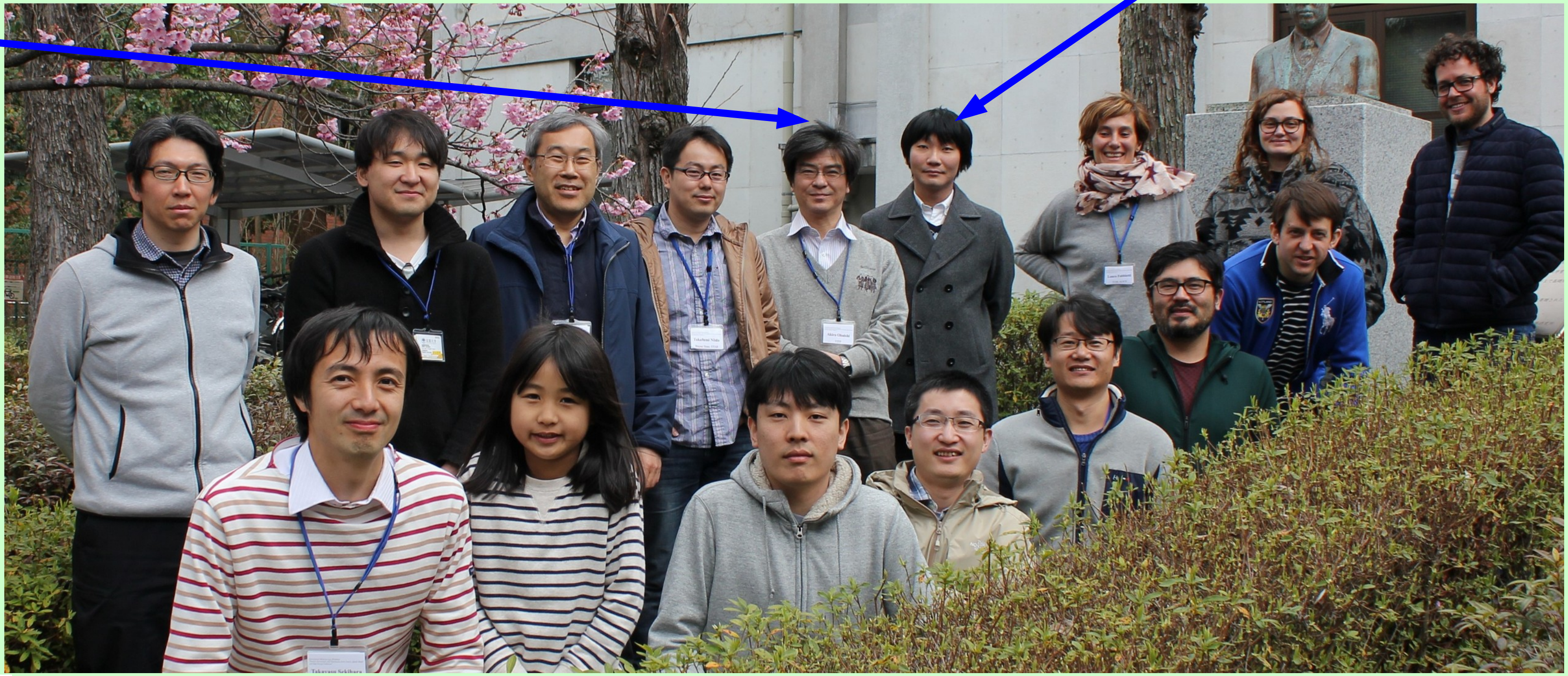


T. Hatsuda

T. Hyodo

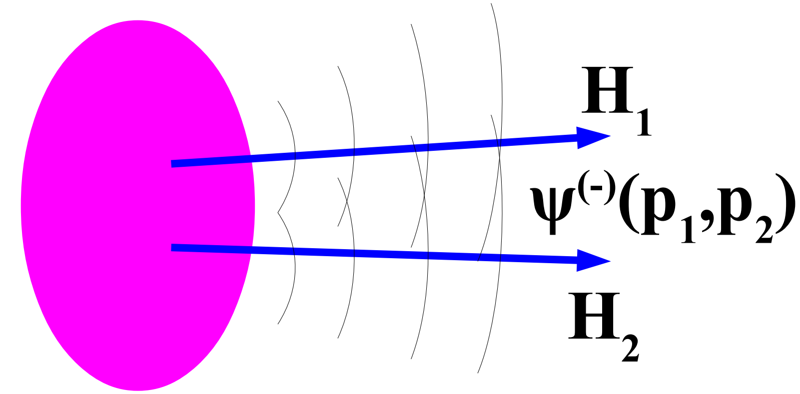
Y. Kamiya

AO



Hadron-Hadron Correlation in HIC

- **Hadron-Hadron Correlation Func. (Koonin-Pratt (KP) formula)**
  S. E. Koonin, PLB 70 ('77) 43; S. Pratt, T. Csorgo and J. Zimanyi, PRC42 ('90) 2646; W. Bauer, C.-K. Gelbke, S. Pratt, Annu. Rev. Nucl. Part. Sci. 42 ('92)77; R. Lednicky, V. L. Lyuboshits, Sov. J. Nucl. Phys. 35 ('82) 770.



$$C(\boldsymbol{q}) = \frac{E_1 E_2 dN_{12}/d\boldsymbol{p}_1 d\boldsymbol{p}_2}{(E_1 dN_1/d\boldsymbol{p}_1)(E_2 dN_2/d\boldsymbol{p}_2)} \simeq \int d\boldsymbol{r}\, \underline{S_{12}(\boldsymbol{r})}\, \underline{\left|\psi_{12}^{(-)}(\boldsymbol{r},\boldsymbol{q})\right|^2}$$

Source fn. **int. → relative w.f.**

q: rel. mom. (referred to also as k*)

- **Static sph. Gaussian source, Int. for s-wave, Identical fermions**

$$C(q) \simeq 1 - \frac{1}{2}\exp(-4q^2R^2) + \frac{1}{2}\int d^3r\, \underline{S_{12}(r)} \left[\underline{|\psi_0(r)|^2} - \underline{|j_0(qr)|^2}\right]$$

Fermion (Quant. Stat.) **Source** **s-wave w.f.** **free**

Lednicky-Lyuboshits (LL) model

- **Lednicky-Lyuboshits analytic model**
  - **Asymp. w.f. + Eff. range corr. + $\psi^{(-)} = [\psi^{(+)}]^*$**

$$\psi_0(r) \rightarrow \psi_{\rm asy}(r) = \frac{e^{-i\delta}}{qr}\sin(qr+\delta) = \mathcal{S}^{-1}\left[\frac{\sin qr}{qr} + f(q)\frac{e^{iqr}}{r}\right]$$

$$\Delta C_{\rm LL}(q) = \int d\boldsymbol{r} S_{12}(r)\left(|\psi_{\rm asy}(r)|^2 - |j_0(qr)|^2\right)$$

$$= \frac{|f(q)|^2}{2R^2} F_3\left(\frac{r_{\rm eff}}{R}\right) + \frac{2{\rm Re}f(q)}{\sqrt{\pi}R}F_1(x) - \frac{{\rm Im}f(q)}{R}F_2(x)$$

$$(x = 2qR, R = \text{Gaussian size}, \ F_1, F_2, F_3 : \text{Known functions})$$

- **Phase shifts**

$$q\cot\delta = -\frac{1}{a_0} + \frac{1}{2}r_{\rm eff}q^2 + \mathcal{O}(q^4) \rightarrow \delta \simeq -a_0 q + {\rm O}(q^3)$$

$$\sin(qr+\delta) \simeq \sin(q(r-a_0) + \cdots)$$

Node at r ~ a_0 for small q

Binding Energy, Scattering Length, Effecitve Range (pΩ)

- **m_π=875 MeV, B.E.~ 19 MeV**
  F.Etminan et al. (HAL QCD Collab.), NPA928('14)89.
- **m_π=146 MeV, B.E.~ 2.2 MeV**
  T. Iritani et al. (HAL QCD Collab.), PLB 792('19)284.

Spin-2 $N\Omega$ Potentials		$V_{\rm I}$	$V_{\rm II}$	$V_{\rm III}$
without Coulomb	$E_{\rm B}$ [MeV]	–	0.05	24.8
	a_0 [fm]	−1.0	23.1	1.60
	$r_{\rm eff}$ [fm]	1.15	0.95	0.65 → **0.63**
with Coulomb	$E_{\rm B}$ [MeV]	–	6.3	26.9
	a_0 [fm]	−1.12	5.79	1.29
	$r_{\rm eff}$ [fm]	1.16	0.96	0.65

t/a	a_0 [fm]	$r_{\rm eff}$ [fm]	E_B [MeV]
11	3.45	1.33	2.15
12	3.38	1.31	2.27
13	3.49	1.31	2.08
14	3.40	1.33	2.24

(m_π = 146 MeV, m_N = 955 MeV and m_Ω = 1712 MeV). By using the same parameter set for $t/a = 12$ in Table 1 with m_π = 146 MeV kept fixed but with physical baryon masses (m_p = 938 MeV and m_{Ω^-} = 1672 MeV), we find less binding than Eq. (10) as expected: $B_{p\Omega^-} \simeq 2.18(32)$ MeV and $\sqrt{\langle r^2 \rangle_{p\Omega^-}} \simeq 3.45(22)$ fm. On the other hand, if we additionally em-

Correlation Function with Coulomb potential

- **Correlation Function w/o Coulomb potential (spherical source)**

$$C_i(\boldsymbol{q}) = 1 - \int d^3r S^i(\boldsymbol{r})|j_0(q_i r)|^2 + \sum_j \omega_j \int d^3r S^j(r)|\psi_{ij}(r)|^2$$

- **Correlation Function with Coulomb potential (spherical source)**

$$C_i(\boldsymbol{q}) = \int d^3r S^i(r)\underline{|\psi^C(\boldsymbol{r},\boldsymbol{q})|}^2 - \int d^3r S^i(r)\underline{|\psi_0^C(q_i r)|^2}$$

Coulomb wf **s-wave Coulomb wf**

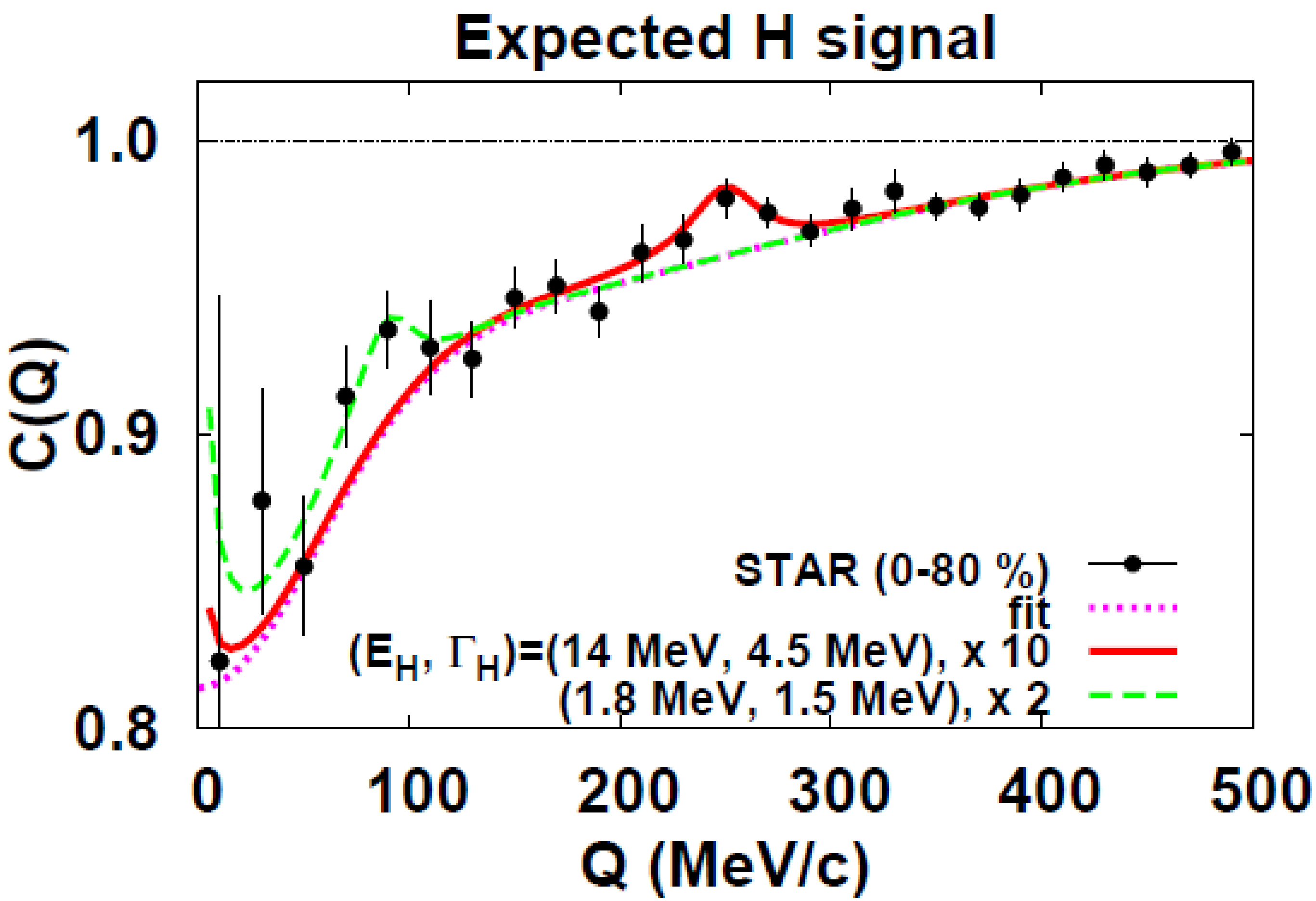
$$+ \sum_j \omega_j \int d^3r S^j(r)\underline{|\psi_{ij}(r)|}^2$$

s-wave wf, j → i

First two terms are large !

Detecting Dibaryon State from Invariant Mass Spectrum ?

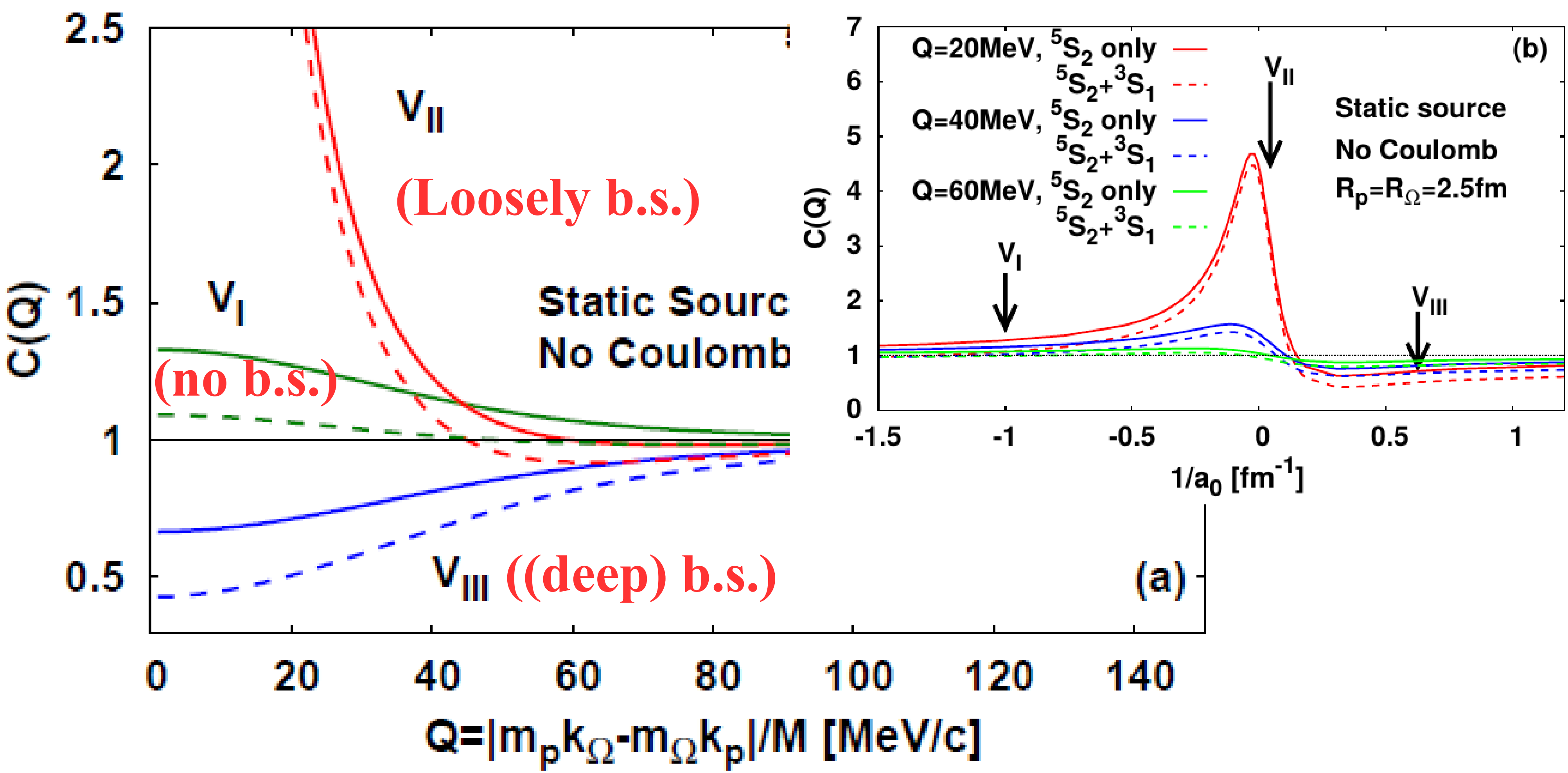
- **Strong signal of dibaryon(s) in correlation function**
  → How about invariant mass spectrum
  → Needs much more statistics



Morita+ ('15)

J. Haidenbauer will make an objection ...

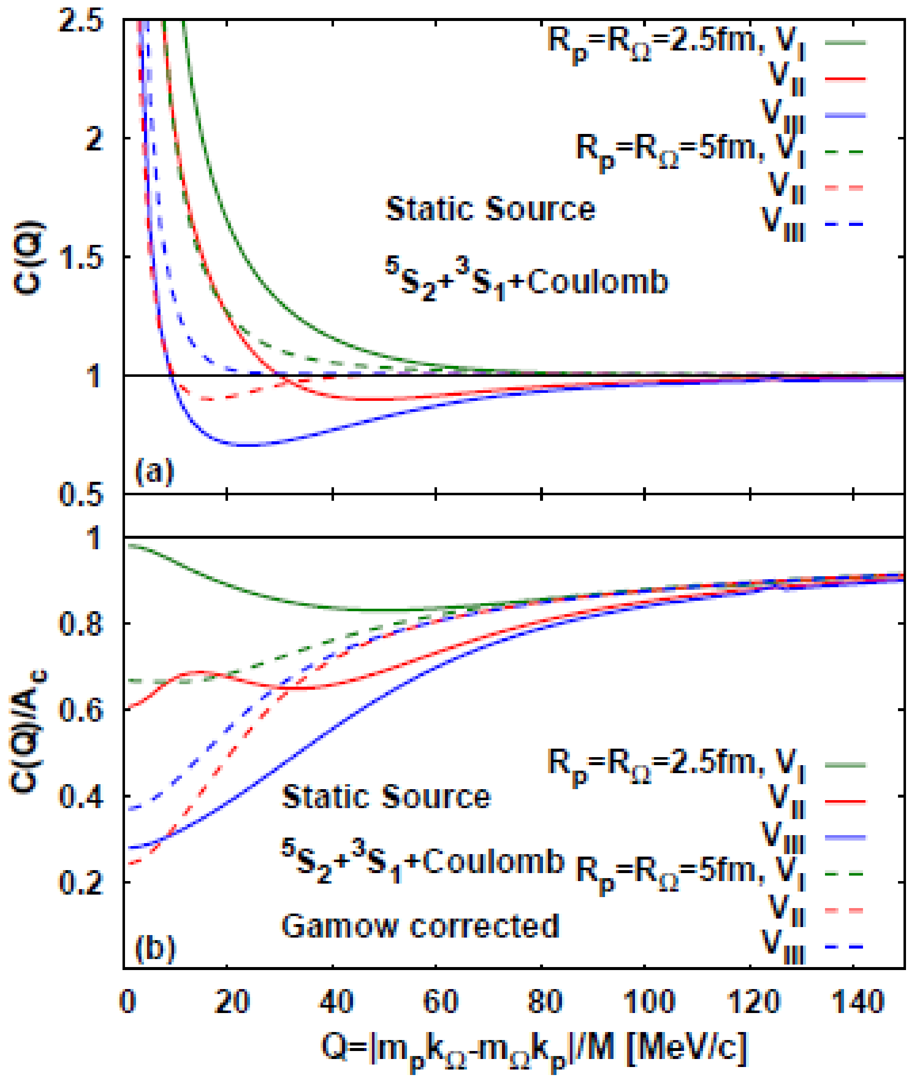
Ω⁻ p correlation



(w/o Coulomb, Strong absorption at r< 2 fm in $^3S_1$ (decay to 8-8 in S-wave))

K. Morita, AO, F. Etminan, T. Hatsuda, PRC94('16)031901(R) [arXiv:1605.06765 [hep-ph]]

$\Omega^{-}p$ correlation w/ Coulomb

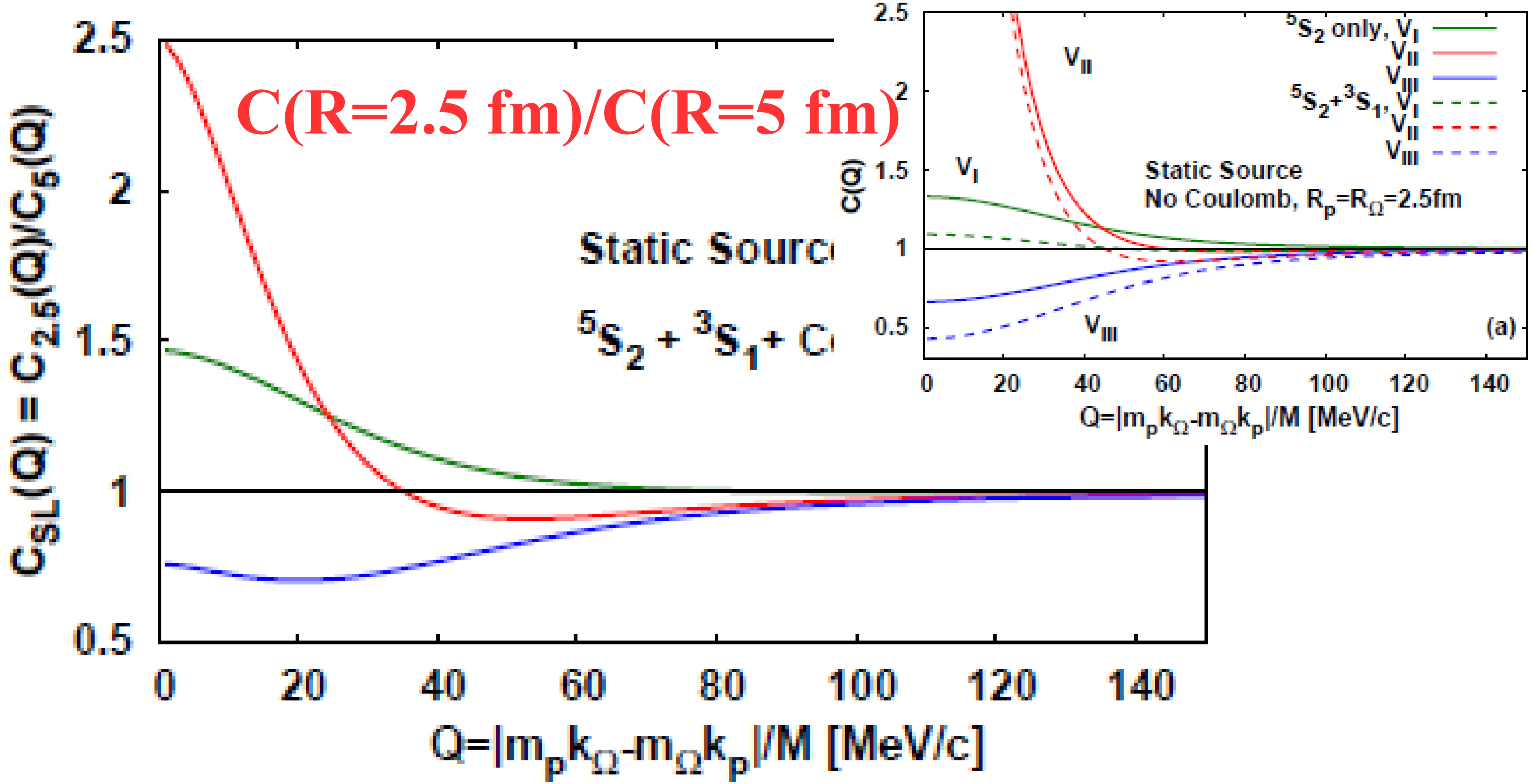


With Coulomb

Coulomb + Gamow corr.

Coulomb potential washes out the features of V_I, V_{II}, V_{III}, and Gamow correction is not enough.

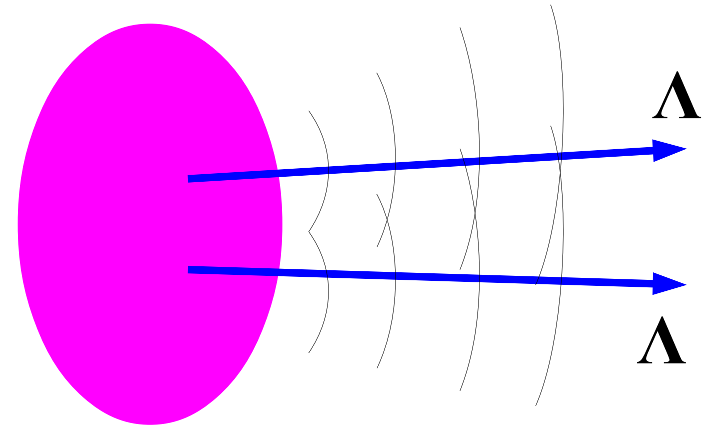
$\Omega^- p$ correlation: Small / Large Ratio

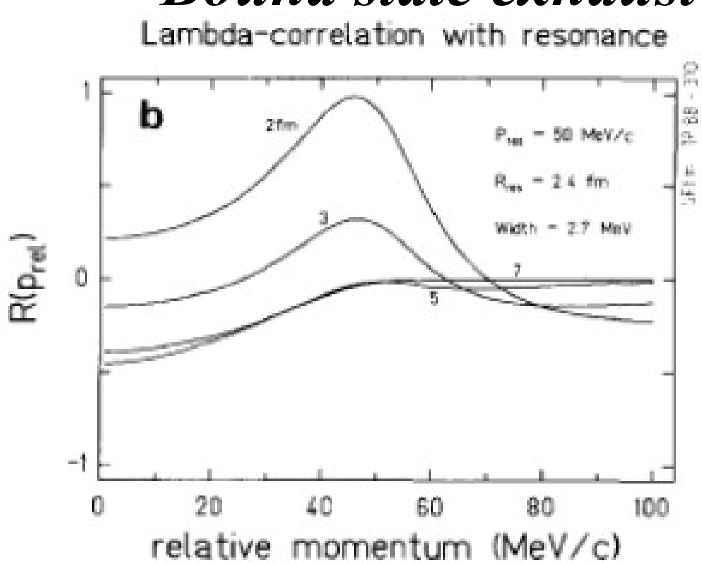


By taking small (R=2.5 fm) / large (R=5 fm) ratio, we approximately see the corr. fn. w/o Coulomb !

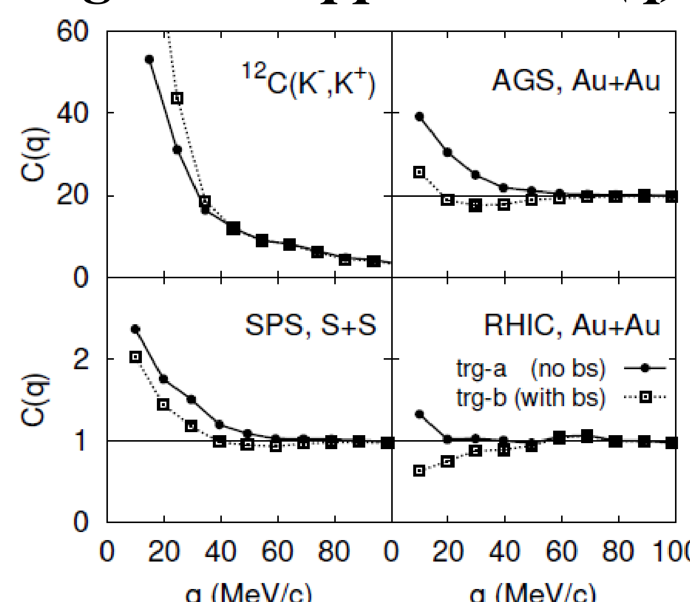
ΛΛ correlation in HIC

- **Merit of HIC to measure ΛΛ correlation**
  - **Source is "Simple and Clean" !**
    T, μ, flow, size, … are well-analyzed.
  - **Nearly Stat. prod.**
    → Many exotics will be produced.
    Schaffner-Bielich, Mattiello, Sorge ('00), Cho et al.(ExHIC Collab.) ('11)
  - **Discovery of "H" and/or Constraint on ΛΛ int.**
    Bound state exhaust the low q strength → suppressed C(q).



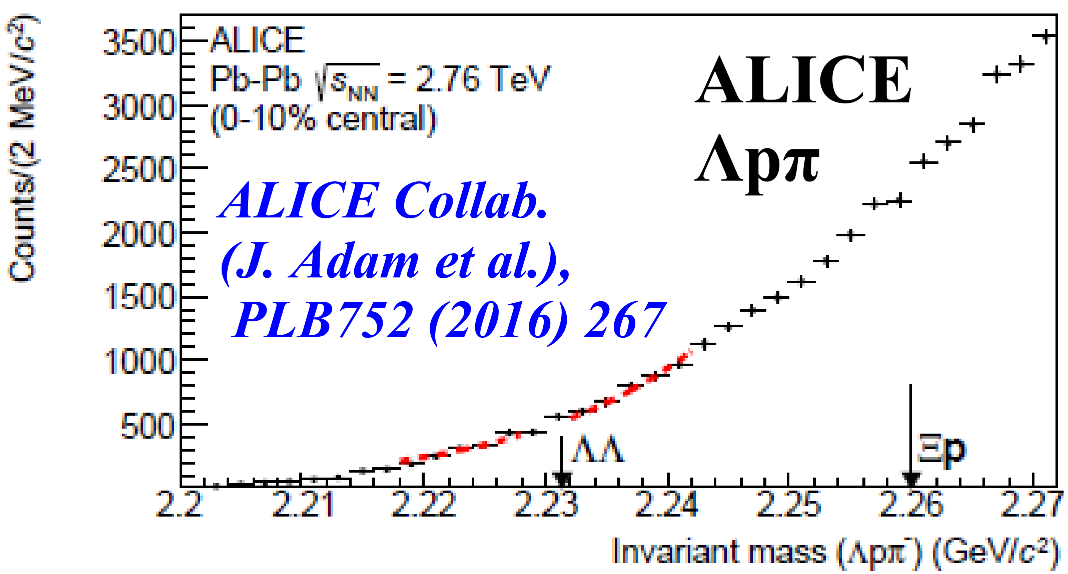


C. Greiner, B. Muller, PLB219('89)199.

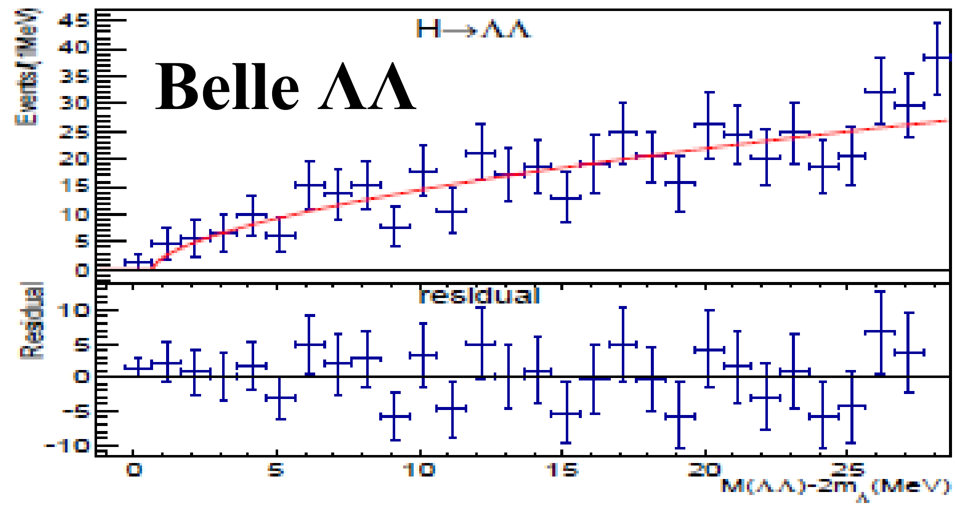


AO, Hirata, Nara, Shinmura, Akaishi, NPA670('00)297c

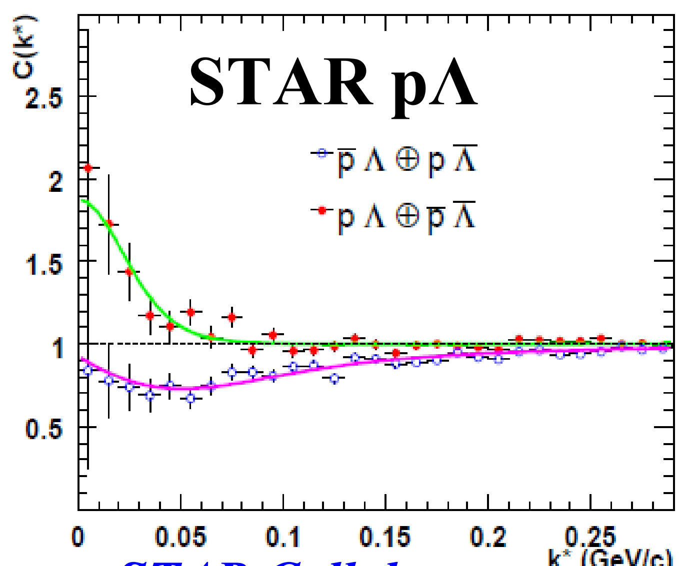
ΛΛ invariant mass / BB correlation function (as of 2016)



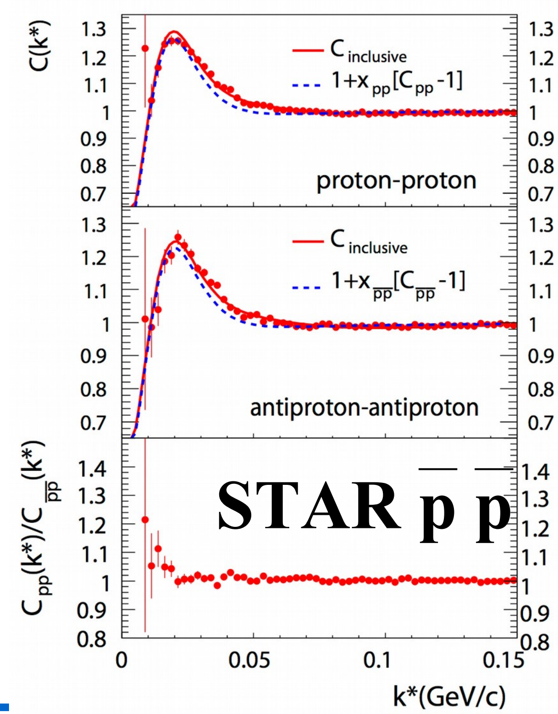
ALICE Collab. (J. Adam et al.), PLB752 (2016) 267



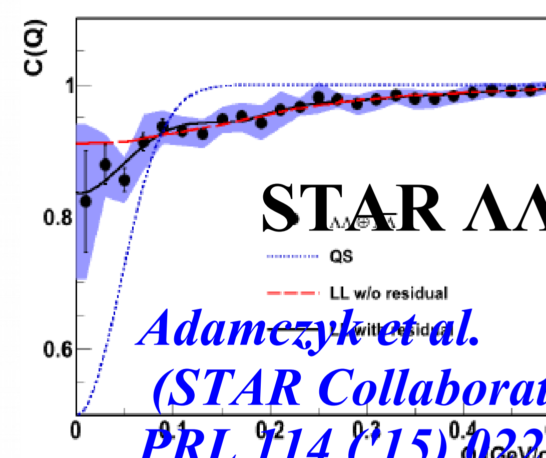
Belle Collaboration (Kim, B.H. et al.), PRL110('13)222002.



STAR Collab. (J. Adams et al.), PRC74('06)064906.



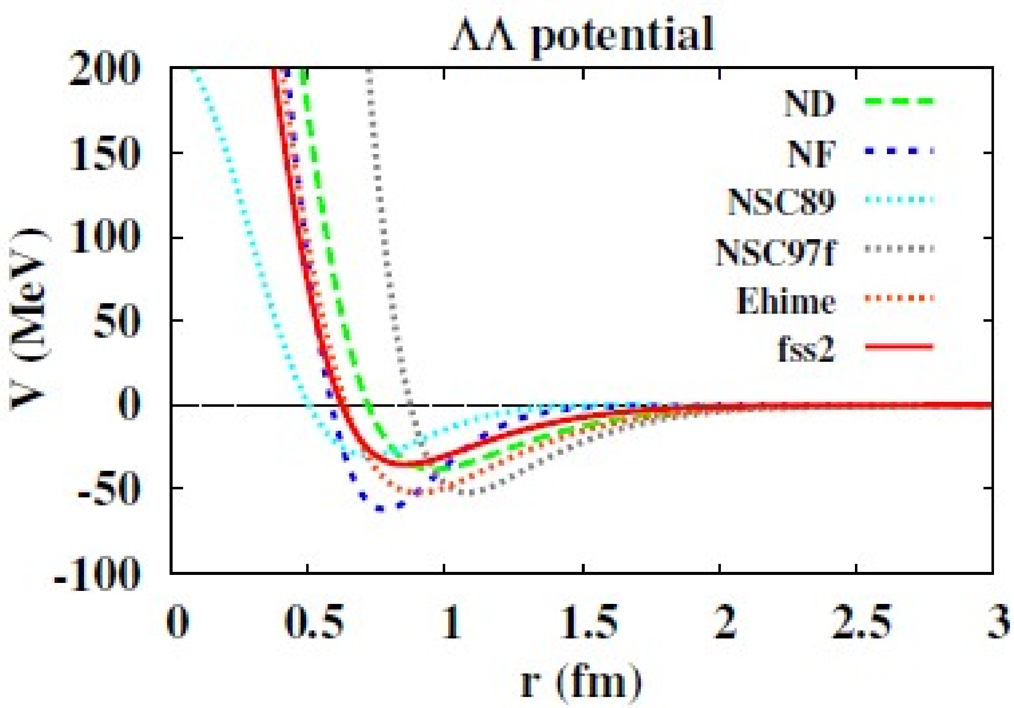
STAR Collab. (J. Adams et al.) Nature 527('15)345

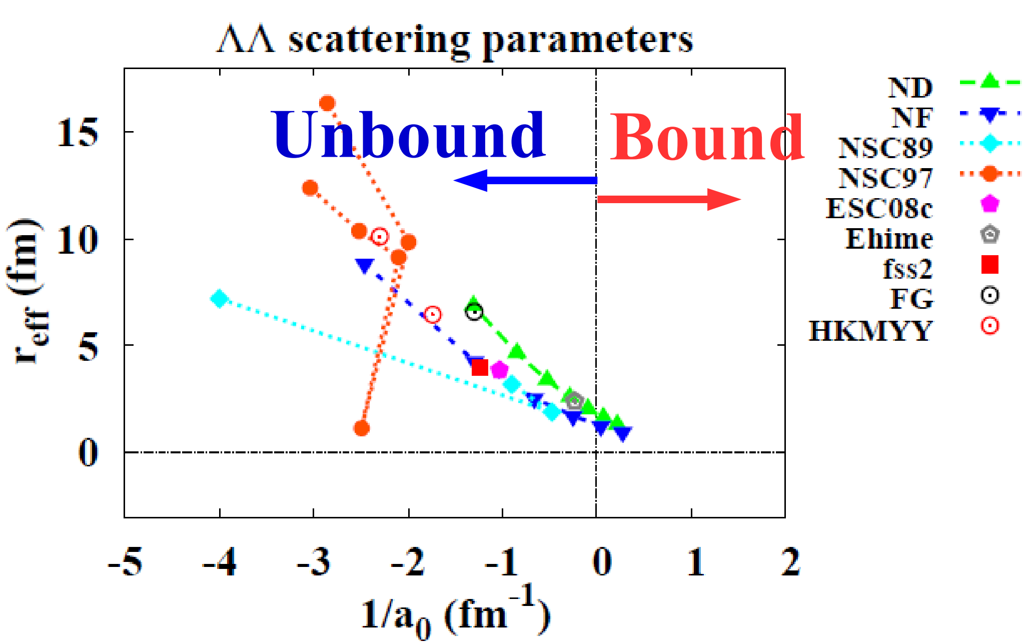


Adamczyk et al. (STAR Collaboration), PRL 114 ('15) 022301.

ΛΛ interaction

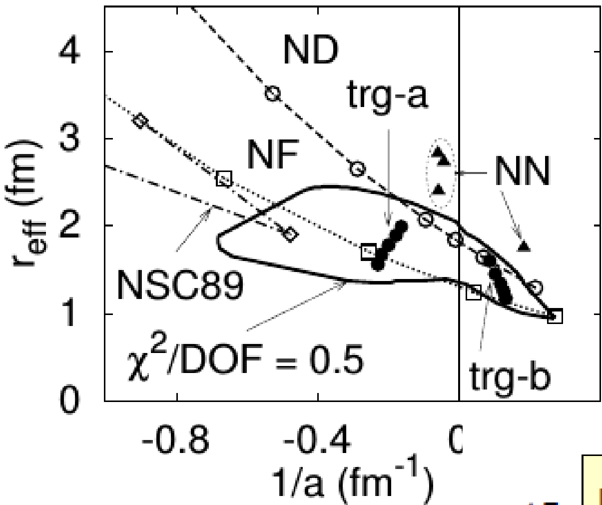
- **Propsed ΛΛ interactions**
  - **Meson Ex. models: Nijmegen model D, F, Soft Core (89, 97), ESC08**
    Nagels, Rijken, de Swart ('77, '79), Maessen, Rijken, de Swart ('89), Rijken, Stoks, Yamamoto ('99); Rijken, Nagels, Yamamoto ('10).
  - **Quark cluster model interaction: fss2**
    Fujiwara, Fujita, Kohno, Nakamoto, Suzuki ('00)
  - **Phenomenological model: Ehime** T. Ueda et al. ('99).
- **Two (or three) range gaussian fit results are used in the analysis.**



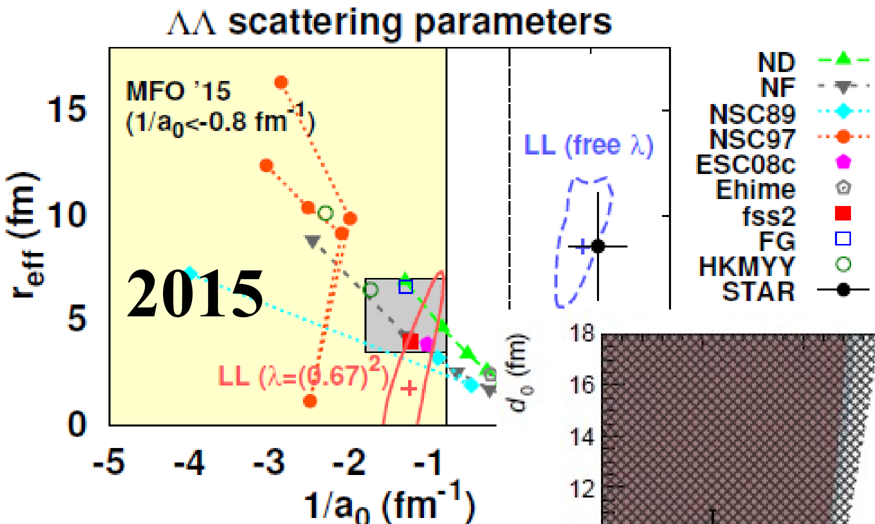


Time dependence of ΛΛ interaction

$^{12}\mathrm{C}(K^-, K^+\Lambda\Lambda)$

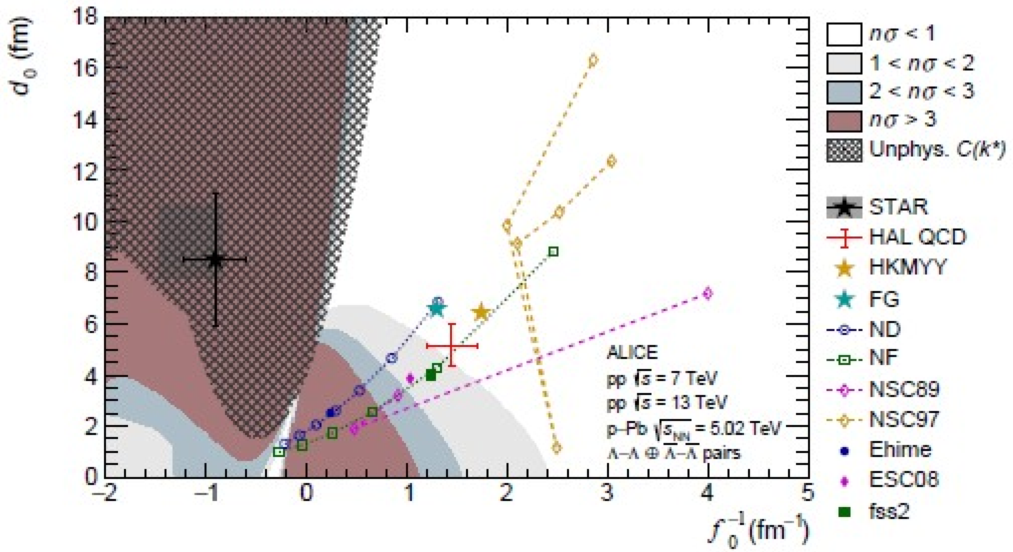


2000



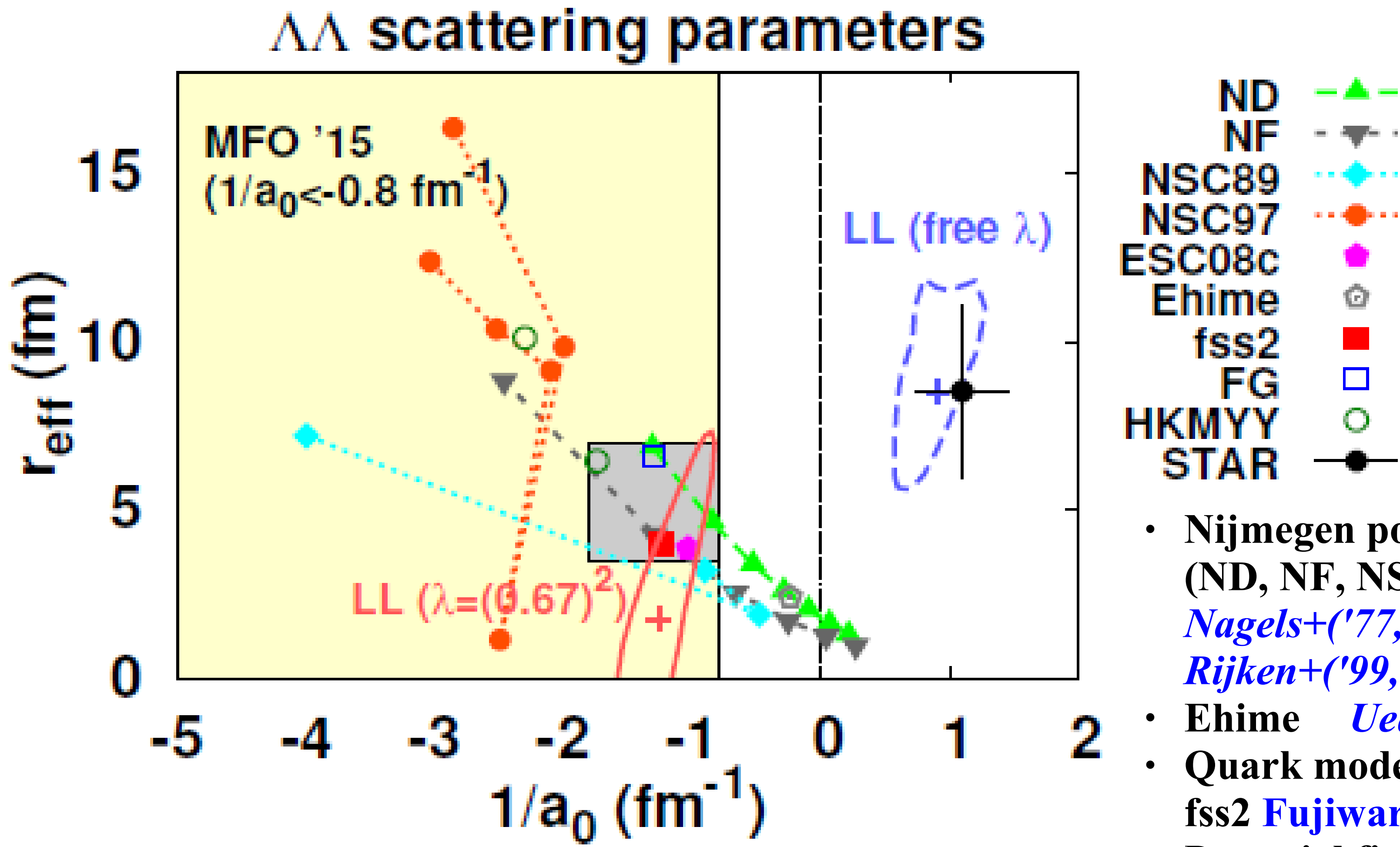
2015

2019



Nagara (2001)

ΛΛ interaction from ΛΛ correlation



- Nijmegen potentials (ND, NF, NSC89, NSC97, ESC08)
  Nagels+('77, '79), Maessen+('89), Rijken+('99,'10)
- Ehime *Ueda et al. ('98)*
- Quark model interaction:
  fss2 Fujiwara et al.('07)
- Potential fitted to Nagara
  Filikhin, Gal ('02) (FG), Hiyama et al. ('02, '10)(HKMYY)

$$q \cot \delta = -1/a_0 + r_{eff}\, q^2/2 + O(q^4)$$

Positive a_0 (STAR) ⟷ Negative a_0 (MFO'15)

Difference comes from the pair purity

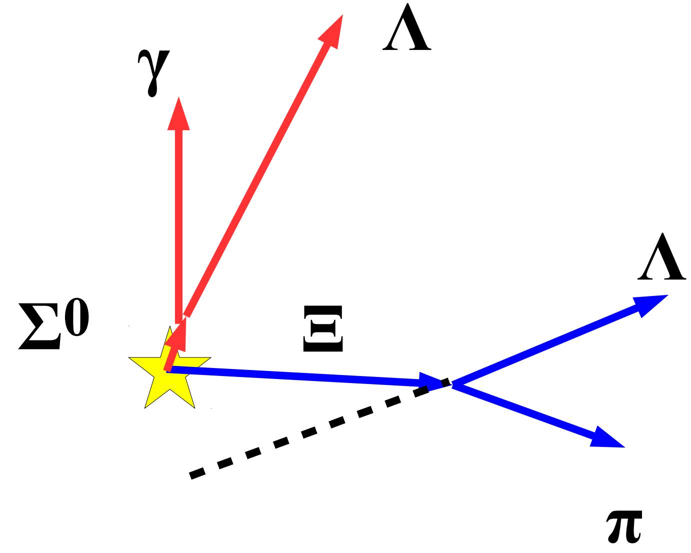
Additional Source

- **Feed down effects**

$$C_{\rm corr}(Q) = 1 + \lambda(C_{\rm bare}(Q) - 1)$$

λ = Purity of ΛΛ pair

- **Short-lived Y* → mod. of source fn.**
- **Ξ → Λπ can be excluded (cτ=8.71 cm)**
- **$\Sigma^0 \to \Lambda\gamma$ is difficult to reject**
- **Data based purity $\lambda=(0.67)^2$**
  Σ^0/Λ=0.278 (p+Be, 28.5 GeV/c) ***Sullivan et al. ('87)***
  Ξ/Λ = 15 % (RHIC)

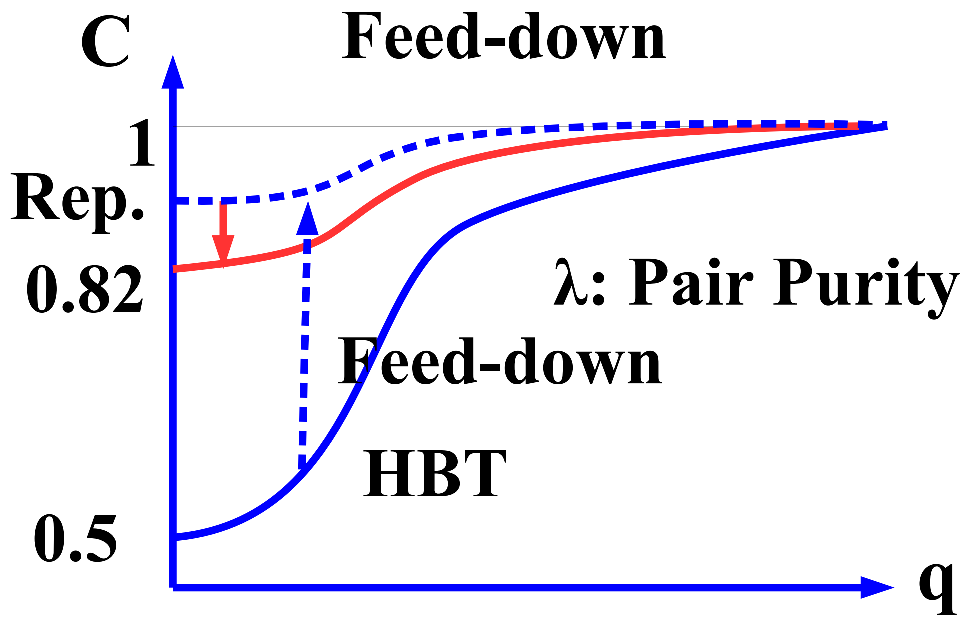


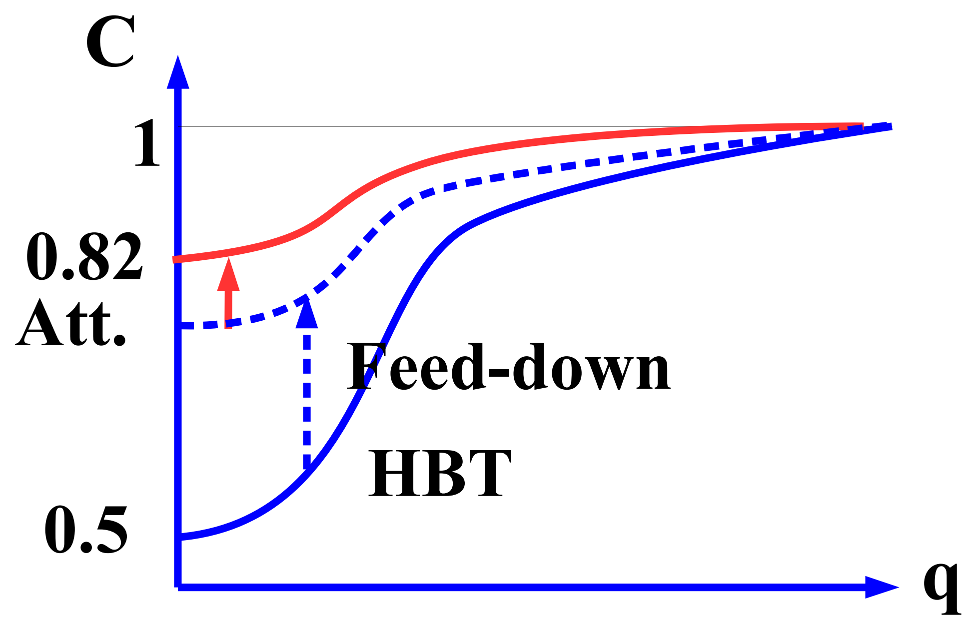
- **"Residual" source**
  - **High-momentum tail → $R_{\rm res}$ ~ 0.5 fm (STAR collab.)**

Feed-Down Effects & Residual Source

- **Correlation Fn. w/ Feed-down & Residual source effects.**

$$C_{\rm corr}(q) = 1 + \lambda(C_{\rm bare}(q) - 1) + a_{\rm res}\exp(-4r_{\rm res}^2 q^2)$$





STAR:
λ ~ 0.18 (free para.)

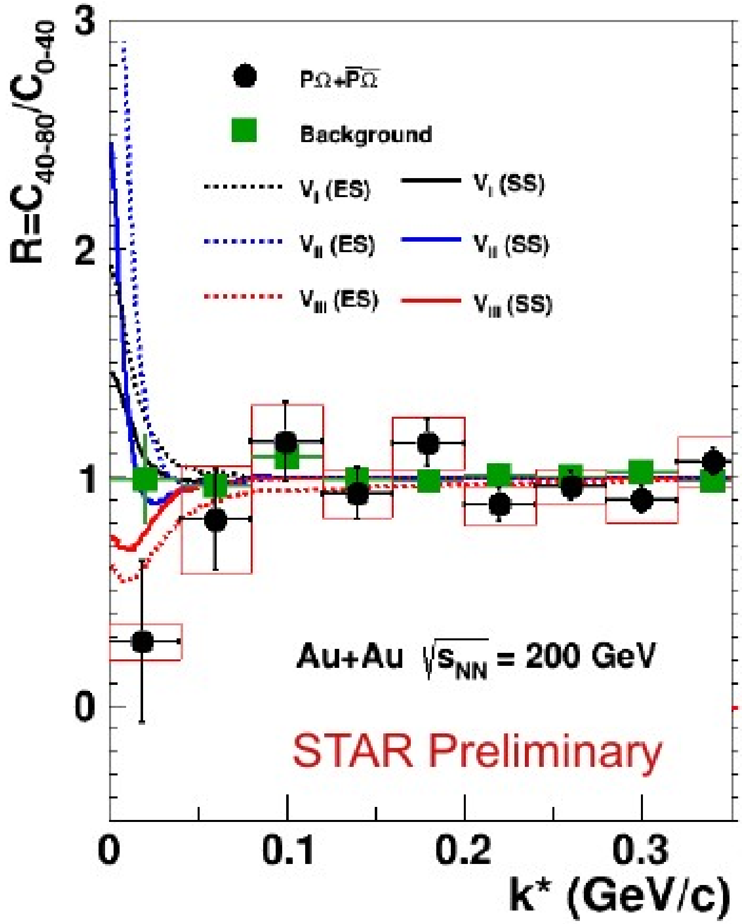
Morita et al. (MFO15):
λ ~ 0.45

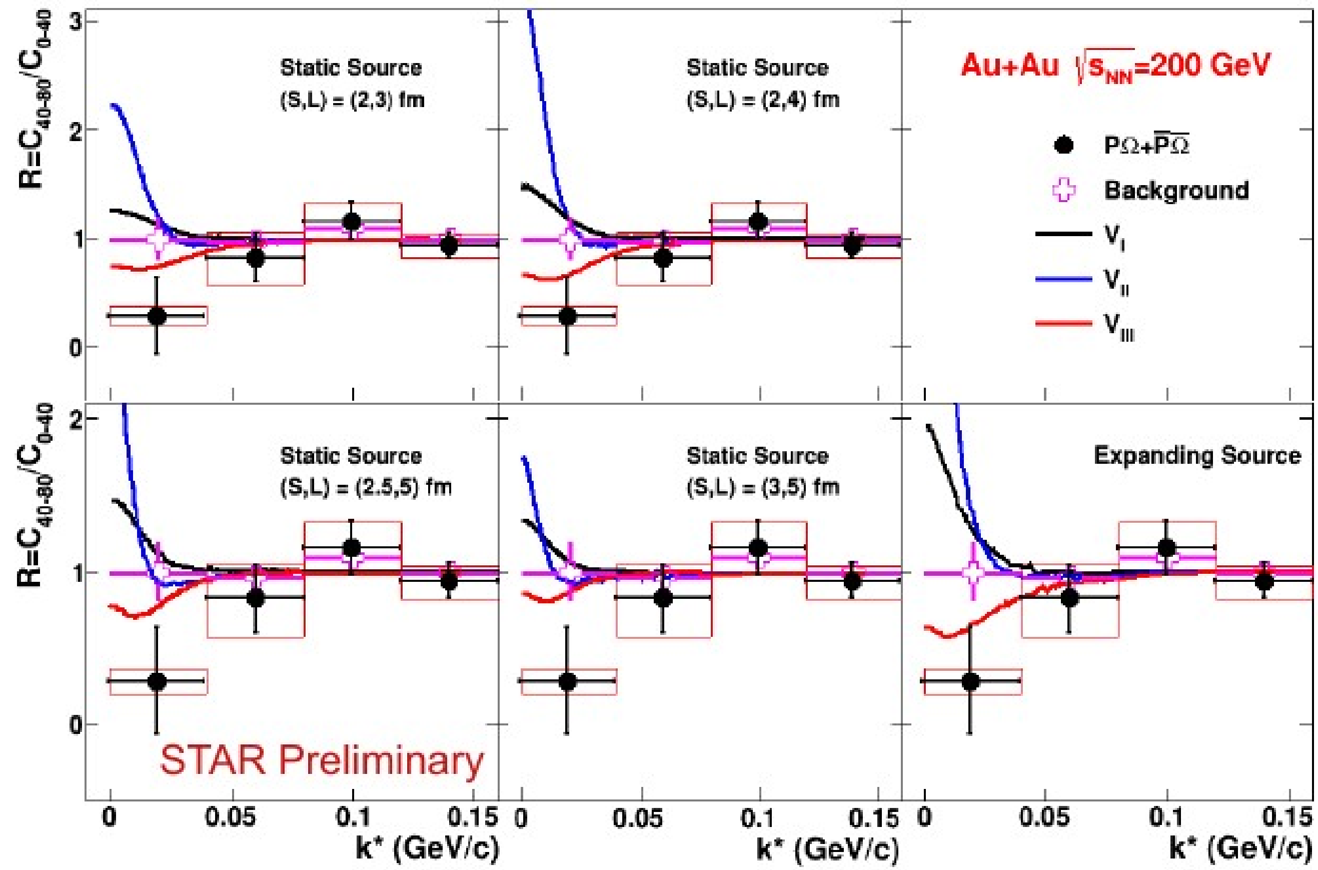
Pair purity (λ) should be determined experimentally !
Puzzle: Residual source

Σ^0/Λ=0.278 (p+Be, 28.5 GeV/c)
Sullivan et al. ('87)
Ξ/Λ = 15 % (RHIC)

AO, Morita, Mihayara, Hyodo ('16)

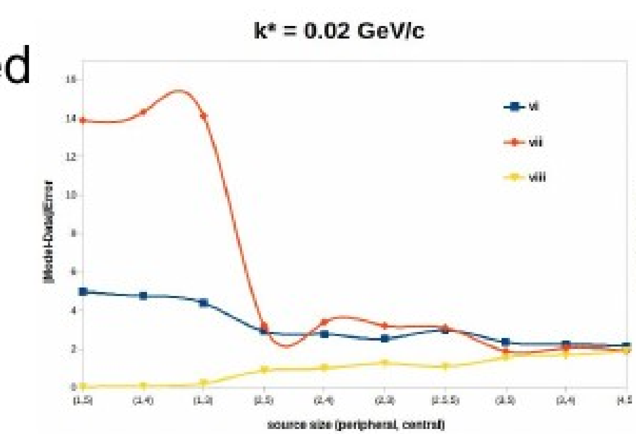
STAR Source Size Analysis on pΩ Correlation Function

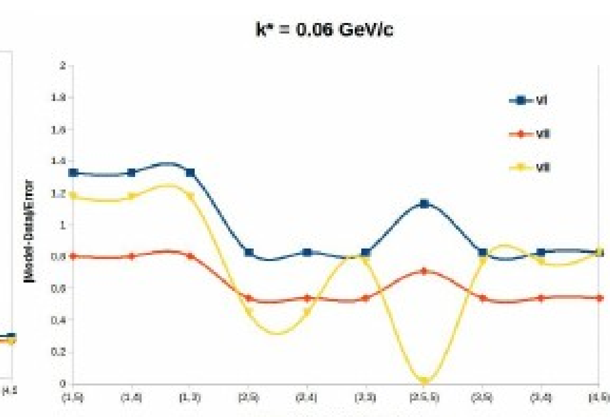




- ☑ The source parameters are derived from other PID and centrality dependence measurements at RHIC

https://drupal.star.bnl.gov/STAR/files/Zbroszczyk_Hanna.pdf

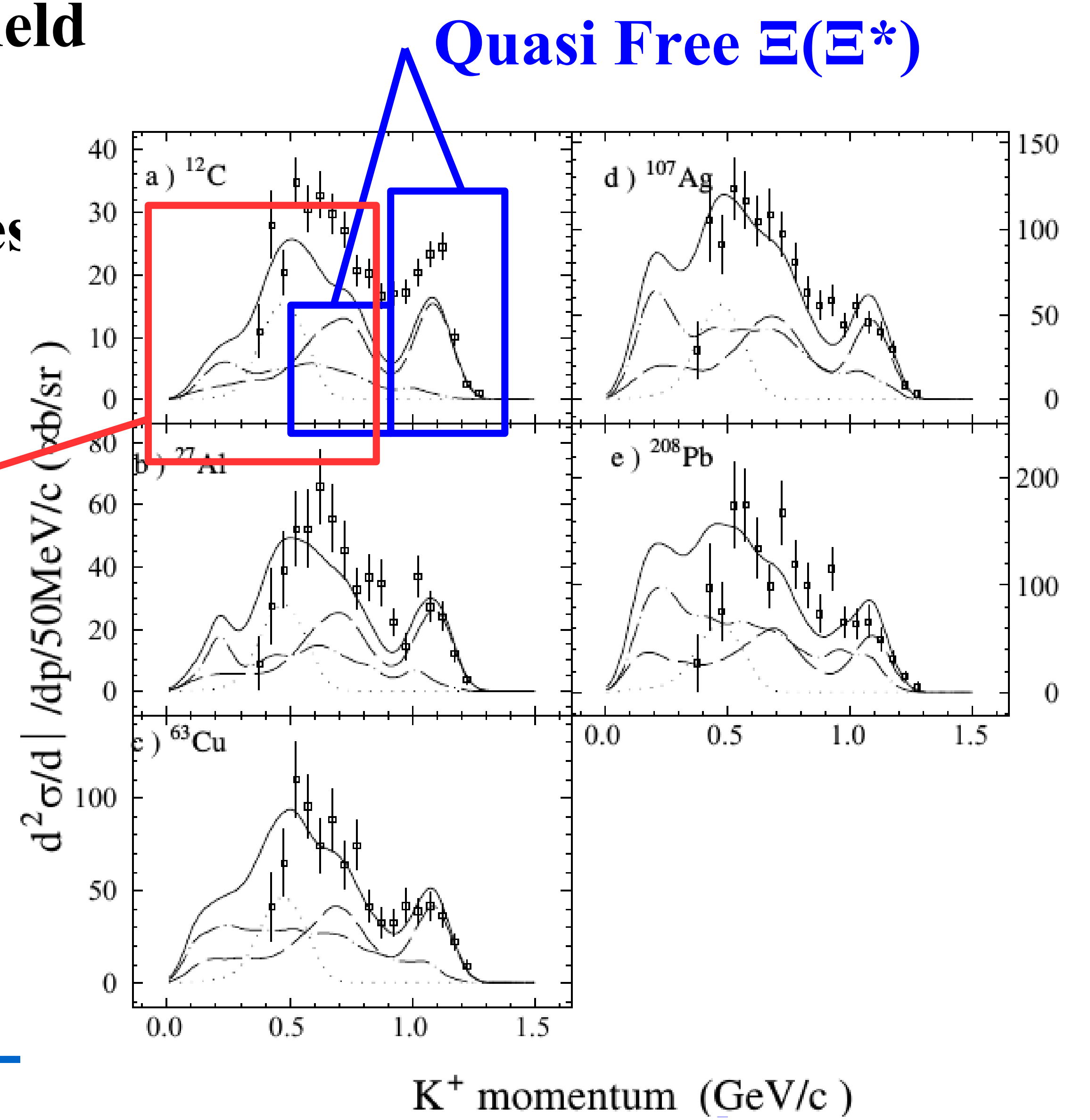


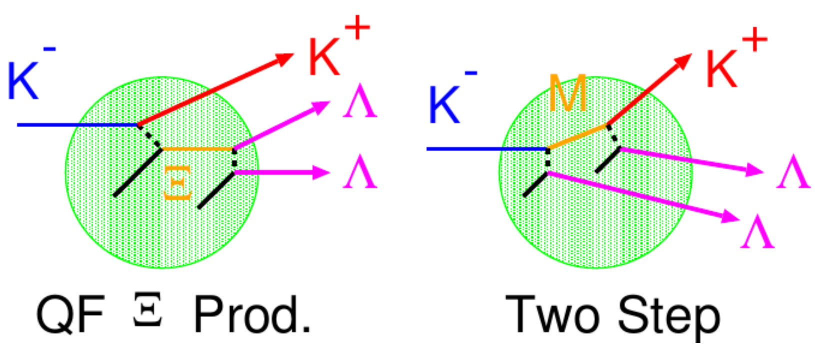


(K^-, K^+) reaction

- (K^-, K^+) reaction = doorway to produce S=-2 systems
  - $K^- p \rightarrow K^+ \Xi^- \rightarrow \Xi$ nuclei, stopped Ξ to $\Lambda\Lambda$ nuclei, …
- What is the origin of large yield at smaller K+ momentum ?
  T. Iijima et al., NPA546('92) 588.
  → Various two step processes
  Y. Nara, AO, T. Harada, A. Engel, NPA614('97)433

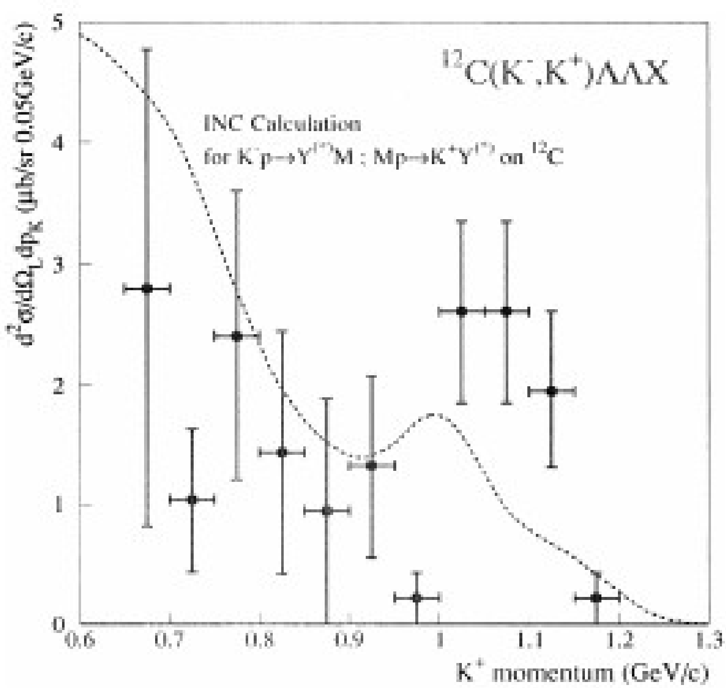
Multi-step

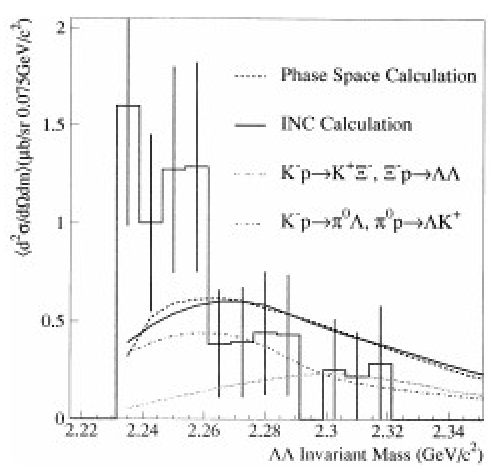


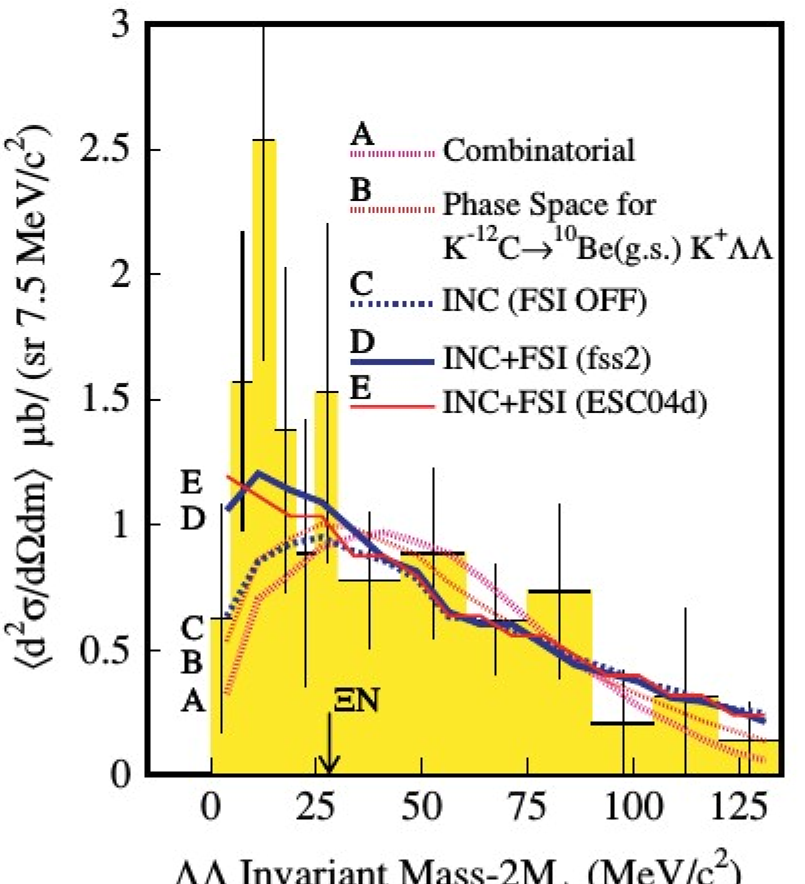


Two Λ production in (K⁻, K⁺) reactions

- **Experimentalists really measured two Λ emission !**
  J.K.Ahn et al. (KEK-PS E224 Collab.), PLB444('98)267.
  C. J. Yoon et al. (KEK-PS E522 Collab. +AO), PRC75('07)022201
- **Invariant mass spectrum of ΛΛ is enhanced from our cascade calculation.**
  → FSI enhancement ? or H particle ?



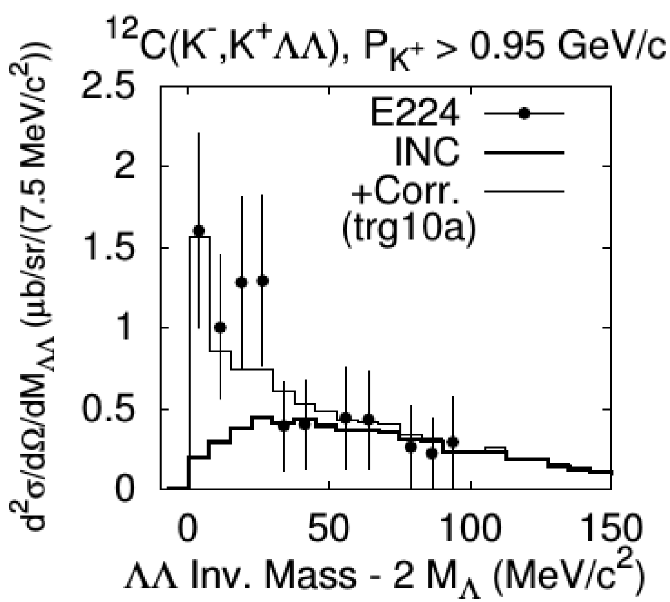


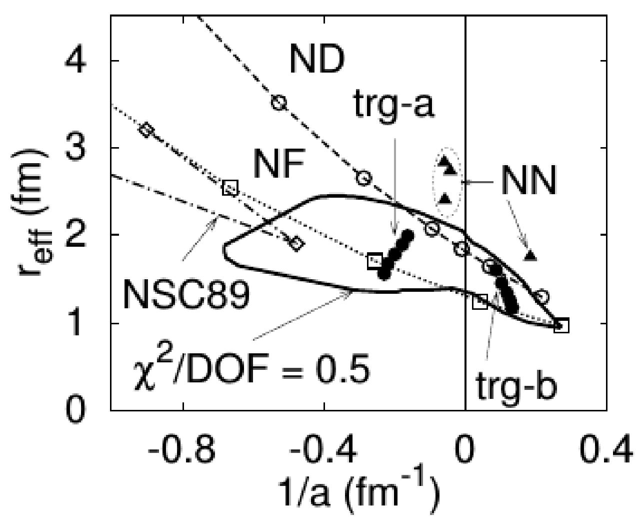


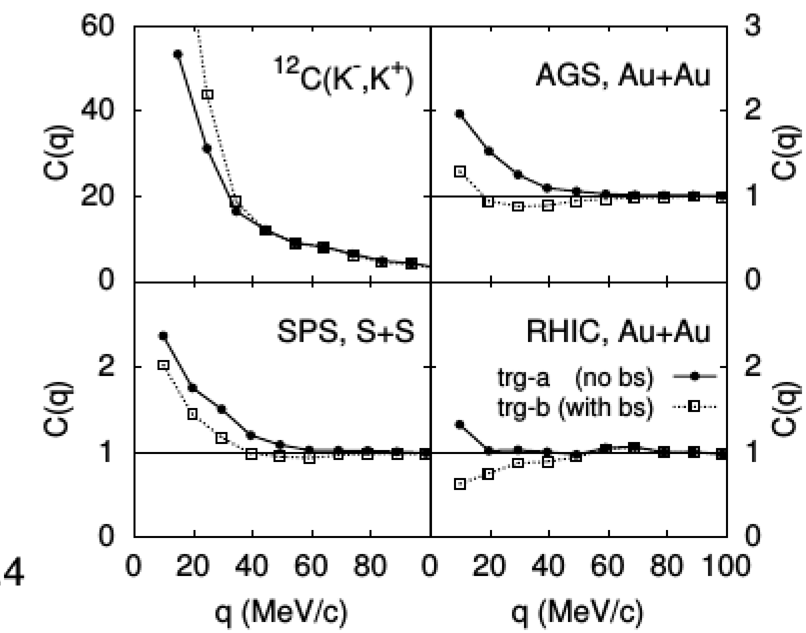
Ahn et al. (E224)('98) *Yoon et al. (E522)('07)*

From correlation to interaction

- **Enhancement of ΛΛ may show ΛΛ interaction effects !**
  AO, Y. Hirata, Y. Nara, S. Shinmura, Y. Akaishi,
  NPA670('00)297c; NPA684('01)595; NPA691('01)242c
  - **Enh. is roughly explained by ΛΛ final state int.**
  - **It should be clearer to measure in heavy-ion collisions.**
    Enh. w/o bound state, Suppression w/ bound state.
    → I asked Prof. Huan Z. Hunag (STAR) in ExHIC 2010 meeting
    and they measured it !







AO, Hirata, Nara, Shinmura, Akaishi ('00)