

# *Hadron-Hadron Correlation and Interaction from High-Energy Collisions*

**Akira Ohnishi (YITP, Kyoto U.)**

*Hadron Interactions and Polarization  
from Lattice QCD, Quark Model, and Heavy-Ion Collisions,  
Mar. 25- Apr. 5, 2019, Kyoto, Japan*

*K. Morita, T. Furumoto, AO, PRC91('15)024916 ( $\Lambda\Lambda$ )*

*AO, K. Morita, K. Miyahara, T. Hyodo, NPA954 ('16)294 ( $\Lambda\Lambda$ ,  $K-p$ ).*

*K. Morita, AO, F. Etminan, T. Hatsuda, PRC94('16), 031901(R) ( $\Omega N$ ).*

*S. Cho et al.(ExHIC Collab.), Prog.Part.Nucl.Phys.95('17)279 ( $\Lambda\Lambda$ ,  $K-p$ ).*

*T. Hatsuda, K. Morita, AO, K. Sasaki, NPA967('17), 856 ( $\Xi-p$ ).*

*K. Morita, S. Gongyo, T. Hatsuda, T. Hyodo, T. Iritani, AO, K. Sasaki, in  
prep. ( $N\Omega$ ,  $\Omega\Omega$ )*



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# *Pre-History*

# $(K^-, K^+)$ reaction

■  $(K^-, K^+)$  reaction = doorway to produce  $S=-2$  systems

●  $K^- p \rightarrow K^+ \Xi^- \rightarrow \Xi$  nuclei, stopped  $\Xi$  to  $\Lambda\Lambda$  nuclei, ...

■ What is the origin of large yield at smaller  $K^+$  momentum ?

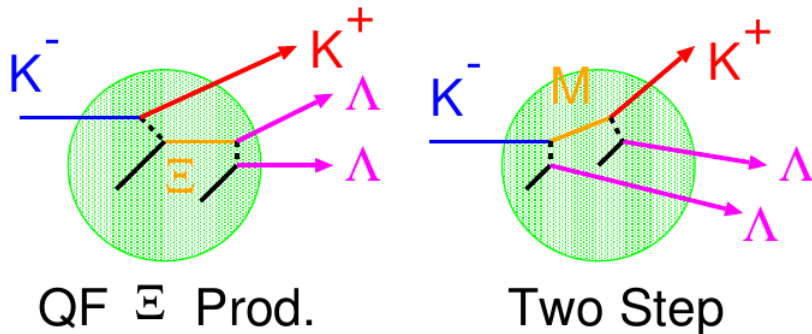
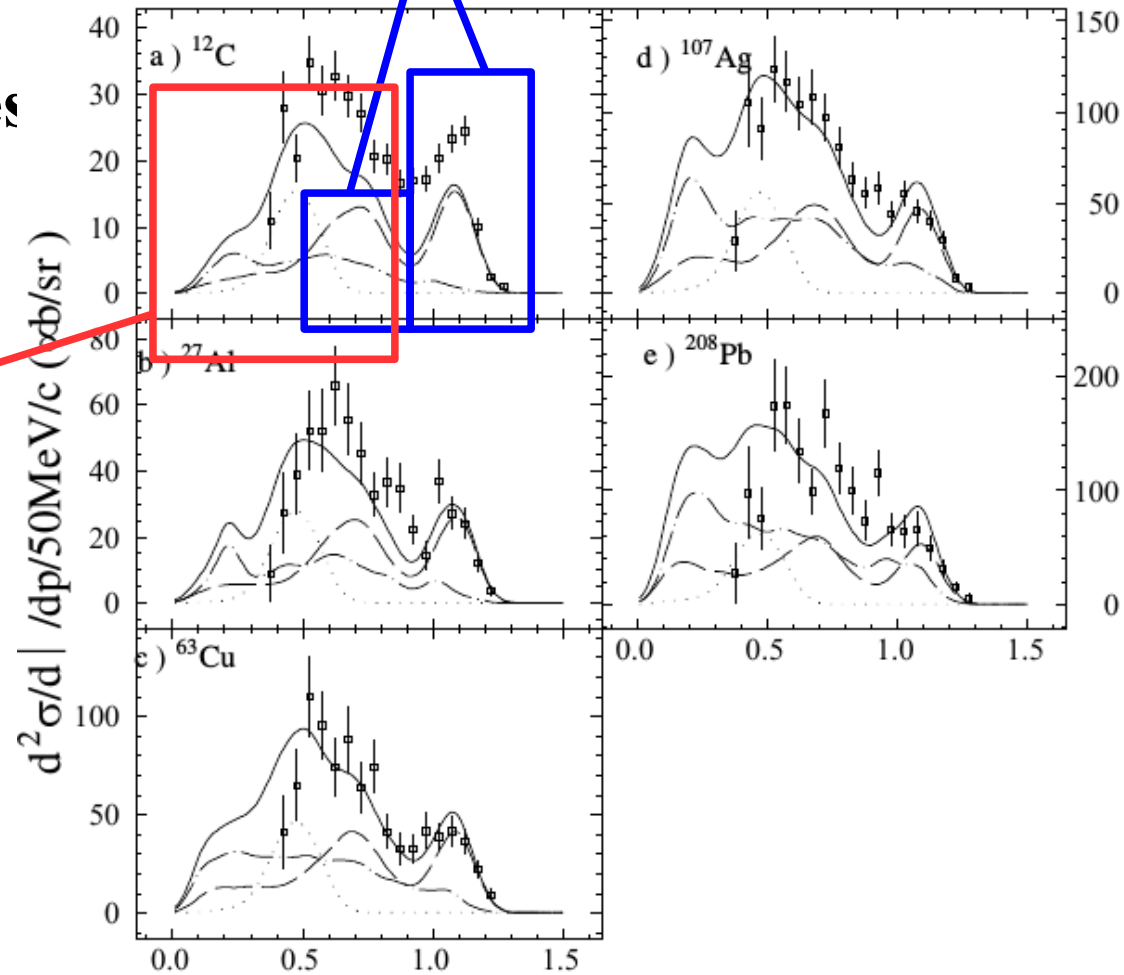
*T. Iijima et al., NPA546('92) 588.*

→ Various two step processes

*Y. Nara, AO, T. Harada, A. Engel, NPA614('97)433*

Quasi Free  $\Xi(\Xi^*)$

Multi-step



$K^+$  momentum (GeV/c)

# Two $\Lambda$ production in ( $K^-$ , $K^+$ ) reactions

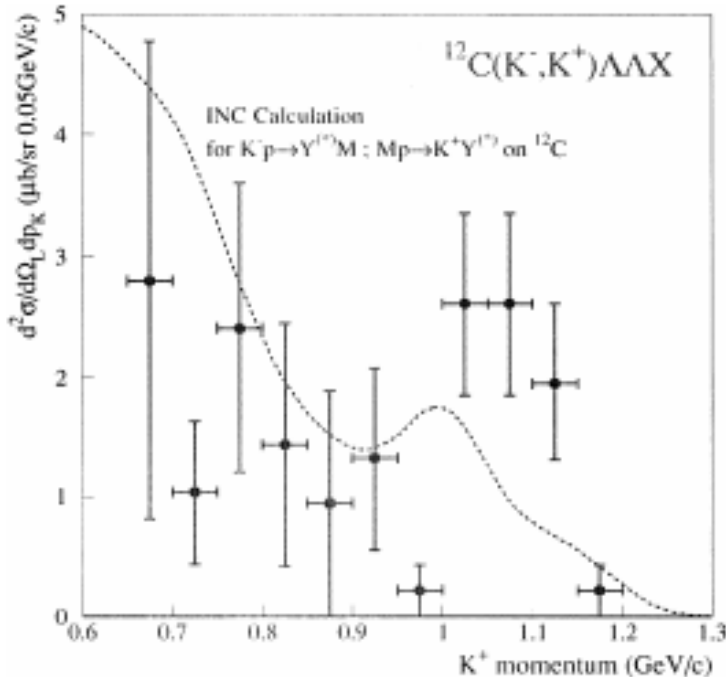
- Experimentalists really measured two  $\Lambda$  emission !

*J.K.Ahn et al. (KEK-PS E224 Collab.), PLB444('98)267.*

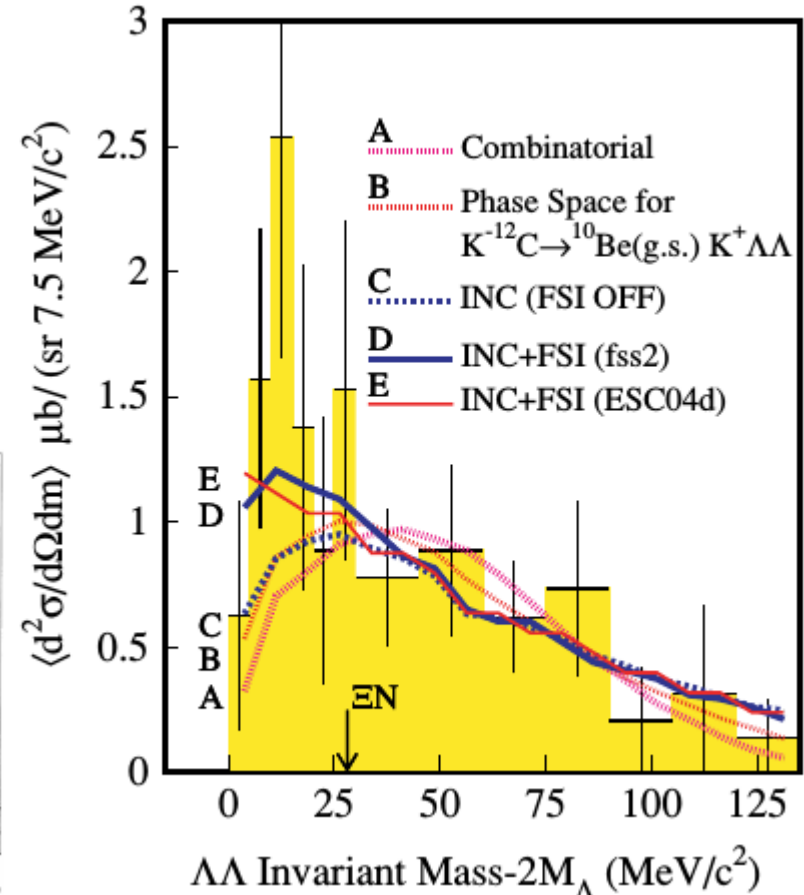
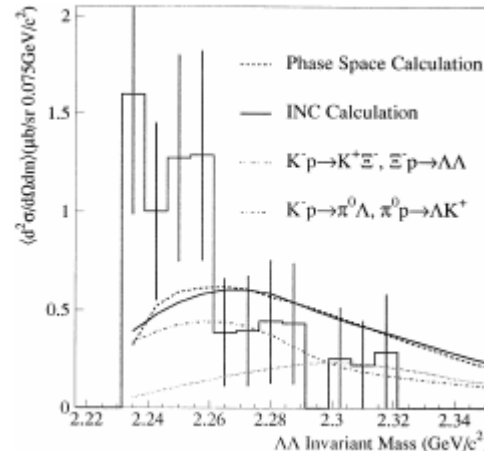
*C. J. Yoon et al. (KEK-PS E522 Collab. +AO), PRC75('07)022201*

- Invariant mass spectrum of  $\Lambda\Lambda$  is enhanced from our cascade calculation.

→ FSI enhancement ? or H particle ?



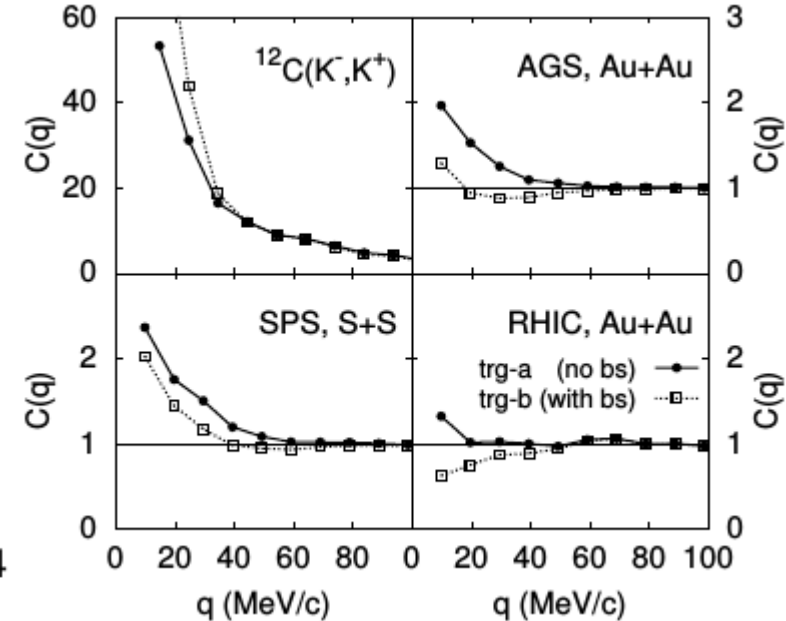
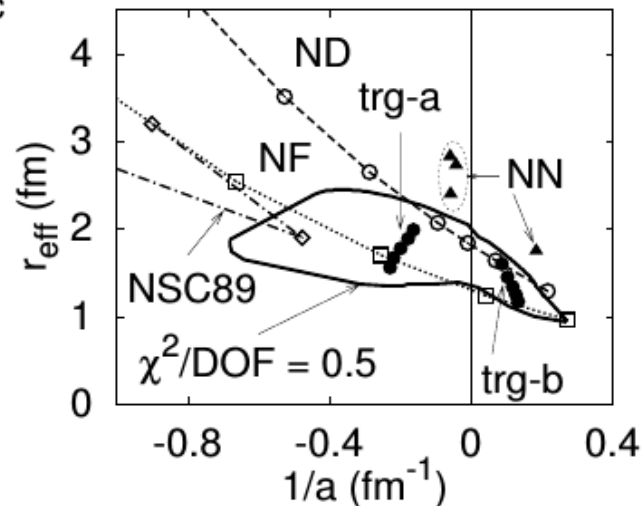
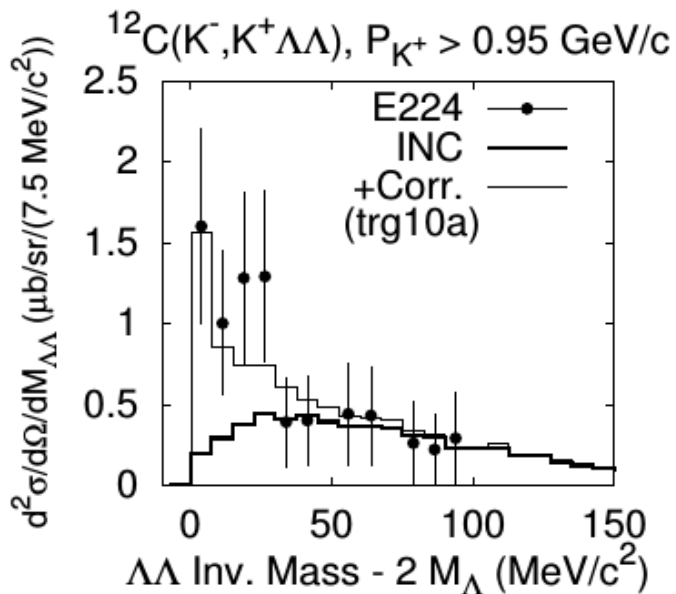
*Ahn et al. (E224)('98)*



*Yoon et al. (E522)('07)*

# From correlation to interaction

- Enhancement of  $\Lambda\Lambda$  may show  $\Lambda\Lambda$  interaction effects !  
 AO, Y. Hirata, Y. Nara, S. Shinmura, Y. Akaishi,  
*NPA670('00)297c; NPA684('01)595; NPA691('01)242c*
- Enh. is roughly explained by  $\Lambda\Lambda$  final state int.
- It should be clearer to measure in heavy-ion collisions.  
 Enh. w/o bound state, Suppression w/ bound state.  
 → I asked Prof. Huan Z. Hunag (STAR) in **ExHIC 2010 meeting**  
 and they measured it !



AO, Hirata, Nara, Shinmura, Akaishi ('00)

# Contents

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  - **$\Lambda\Lambda$  correlation and  $\Lambda\Lambda$  interaction**
  - **$\Omega^-$  p correlation and lattice  $\Omega N$  interaction**
  - **$K^-$  p correlation from chiral SU(3) dynamics**
  - **$\Xi^-$  p correlation and lattice  $\Xi N$  interaction**
- **Summary**

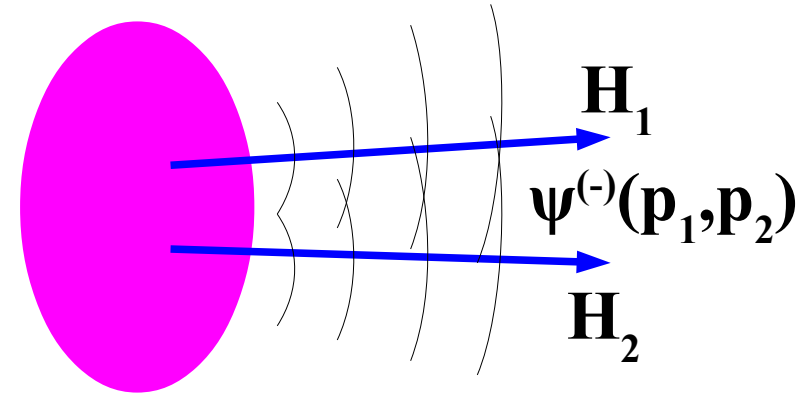
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# *Two Particle correlation and Correlation Function*

# Hadron-Hadron Correlation in HIC

## Hadron-Hadron Correlation Func. (Koonin-Pratt (KP) formula)

*S. E. Koonin, PLB 70 ('77) 43; S. Pratt, T. Csorgo and J. Zimanyi, PRC42 ('90) 2646; W. Bauer, C.-K. Gelbke, S. Pratt, Annu. Rev. Nucl. Part. Sci. 42 ('92)77; R. Lednicky, V. L. Lyuboshits, Sov. J. Nucl. Phys. 35 ('82) 770.*



$$C(\mathbf{q}) = \frac{E_1 E_2 dN_{12}/d\mathbf{p}_1 d\mathbf{p}_2}{(E_1 dN_1/d\mathbf{p}_1)(E_2 dN_2/d\mathbf{p}_2)} \simeq \int d\mathbf{r} \underbrace{S_{12}(\mathbf{r})}_{\text{Source fn.}} \underbrace{\left| \psi_{12}^{(-)}(\mathbf{r}, \mathbf{q}) \right|^2}_{\text{int.} \rightarrow \text{relative w.f.}}$$

$\mathbf{q}$ : rel. mom. (referred to also as  $\mathbf{k}^*$ )

## Static sph. Gaussian source, Int. for s-wave, Identical fermions

$$C(q) \simeq 1 - \frac{1}{2} \exp(-4q^2 R^2) + \frac{1}{2} \int d^3r \underbrace{S_{12}(r)}_{\text{Source}} \left[ \underbrace{|\psi_0(r)|^2}_{\text{s-wave w.f.}} - \underbrace{|j_0(qr)|^2}_{\text{free}} \right]$$

↖ **Fermion (Quant. Stat.)**



# Lednický-Lyuboshits (LL) model

## ■ Lednický-Lyuboshits analytic model

- **Asymp. w.f. + Eff. range corr. +  $\psi^{(-)} = [\psi^{(+)}]^*$**

$$\psi_0(r) \rightarrow \psi_{\text{asy}}(r) = \frac{e^{-i\delta}}{qr} \sin(qr + \delta) = \mathcal{S}^{-1} \left[ \frac{\sin qr}{qr} + f(q) \frac{e^{iqr}}{r} \right]$$

$$\begin{aligned} \Delta C_{\text{LL}}(q) &= \int dr S_{12}(r) (|\psi_{\text{asy}}(r)|^2 - |j_0(qr)|^2) \\ &= \frac{|f(q)|^2}{2R^2} F_3\left(\frac{r_{\text{eff}}}{R}\right) + \frac{2\text{Re}f(q)}{\sqrt{\pi}R} F_1(x) - \frac{\text{Im}f(q)}{R} F_2(x) \end{aligned}$$

( $x = 2qR$ ,  $R = \text{Gaussian size}$ ,  $F_1, F_2, F_3$  : Known functions)

## ■ Phase shifts

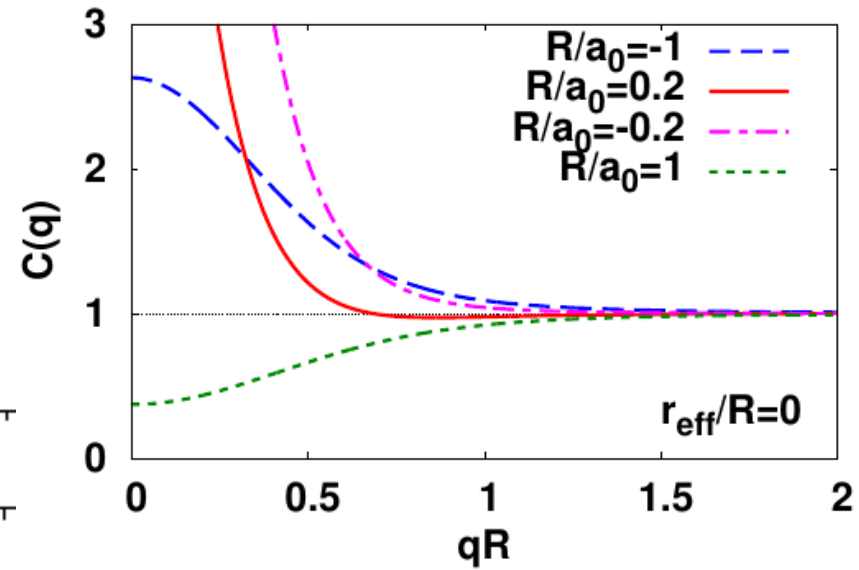
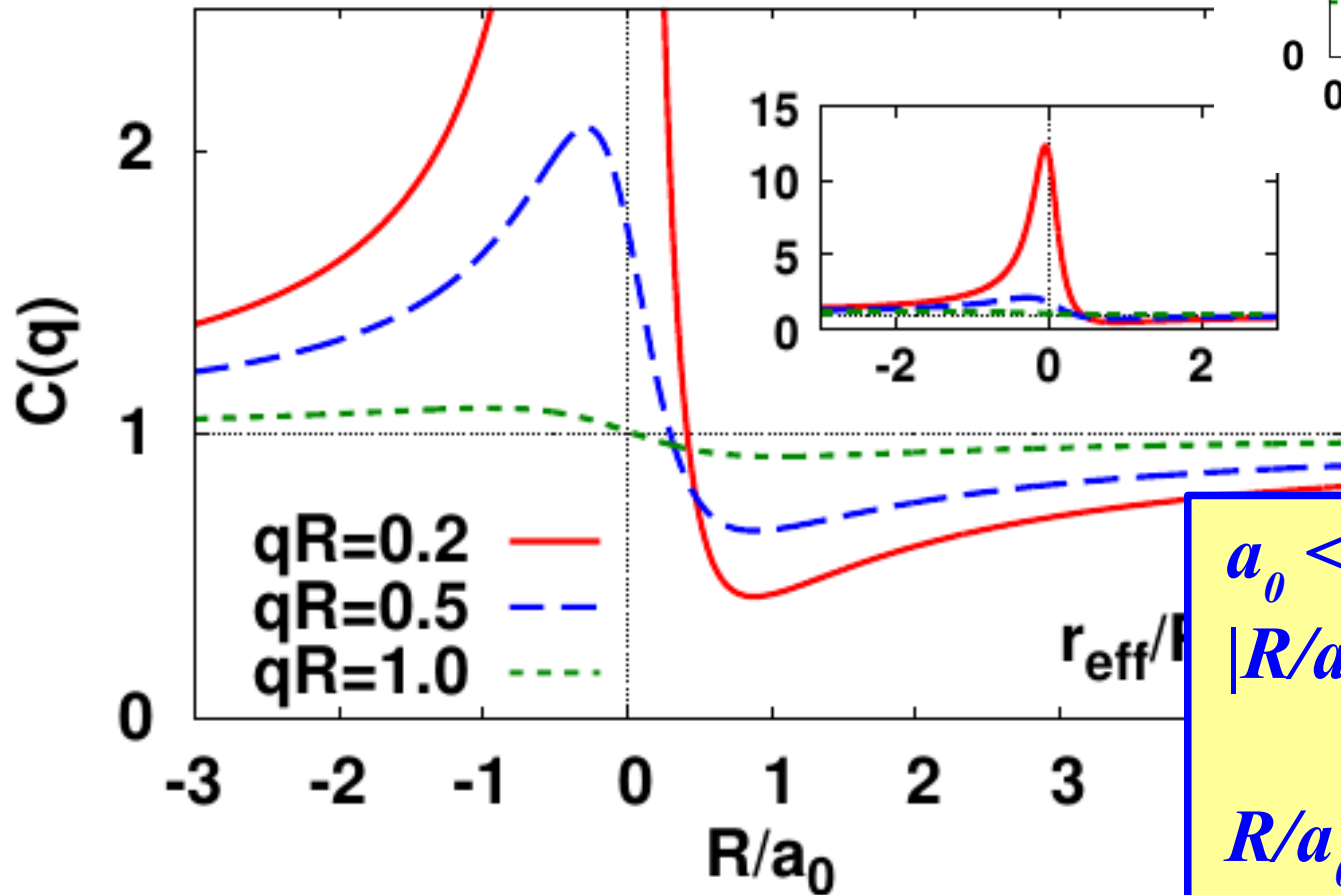
$$q \cot \delta = -\frac{1}{a_0} + \frac{1}{2} r_{\text{eff}} q^2 + \mathcal{O}(q^4) \rightarrow \delta \simeq -a_0 q + \mathcal{O}(q^3)$$

$$\sin(qr + \delta) \simeq \sin(q(r - a_0) + \dots) \quad \text{Node at } \mathbf{r} \sim \mathbf{a}_0$$

**for small  $q$**

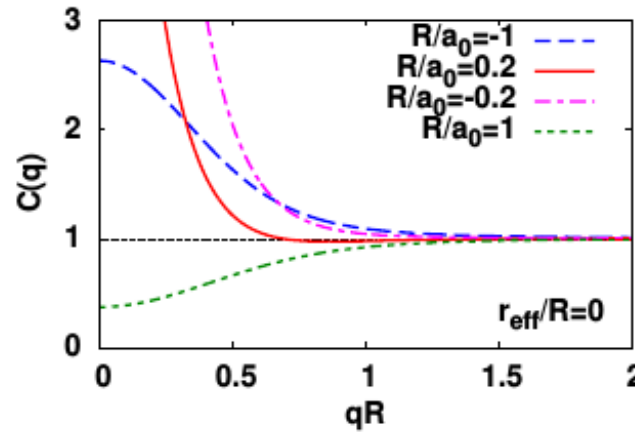
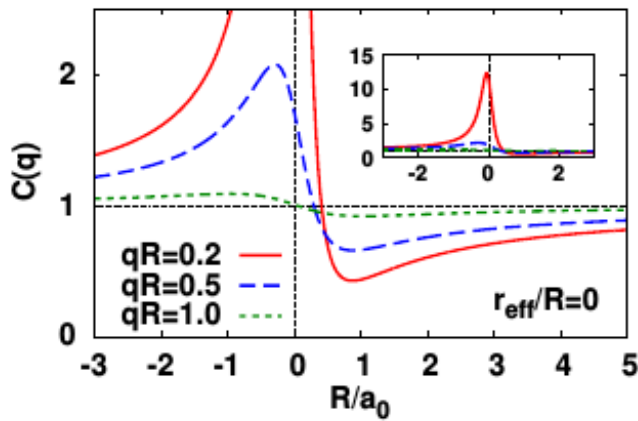
# Interaction Dependence of Correlation Function

$$q \cot \delta = -1/a_0 + r_{\text{eff}} q^2/2 + O(q^4)$$



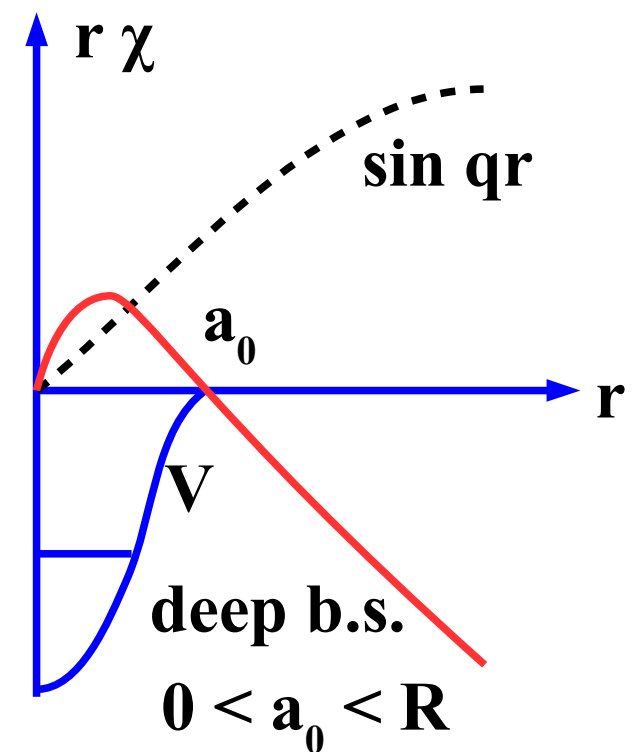
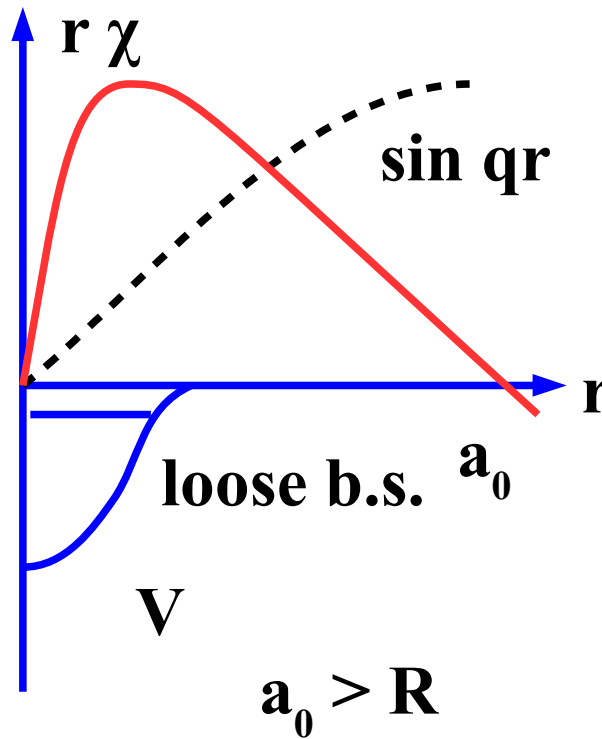
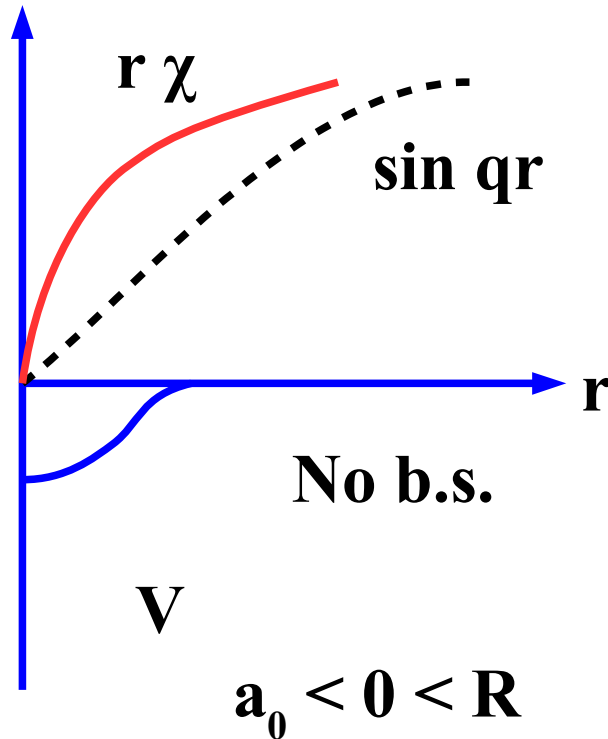
$a_0 < 0 \rightarrow$  *Enhancement*  
 $|R/a_0| \ll 1 \rightarrow$  *Strong enh.*  
*at small q*  
 $R/a_0 > 1 \rightarrow$  *Suppression*

# Interaction Dependence of Correlation Function



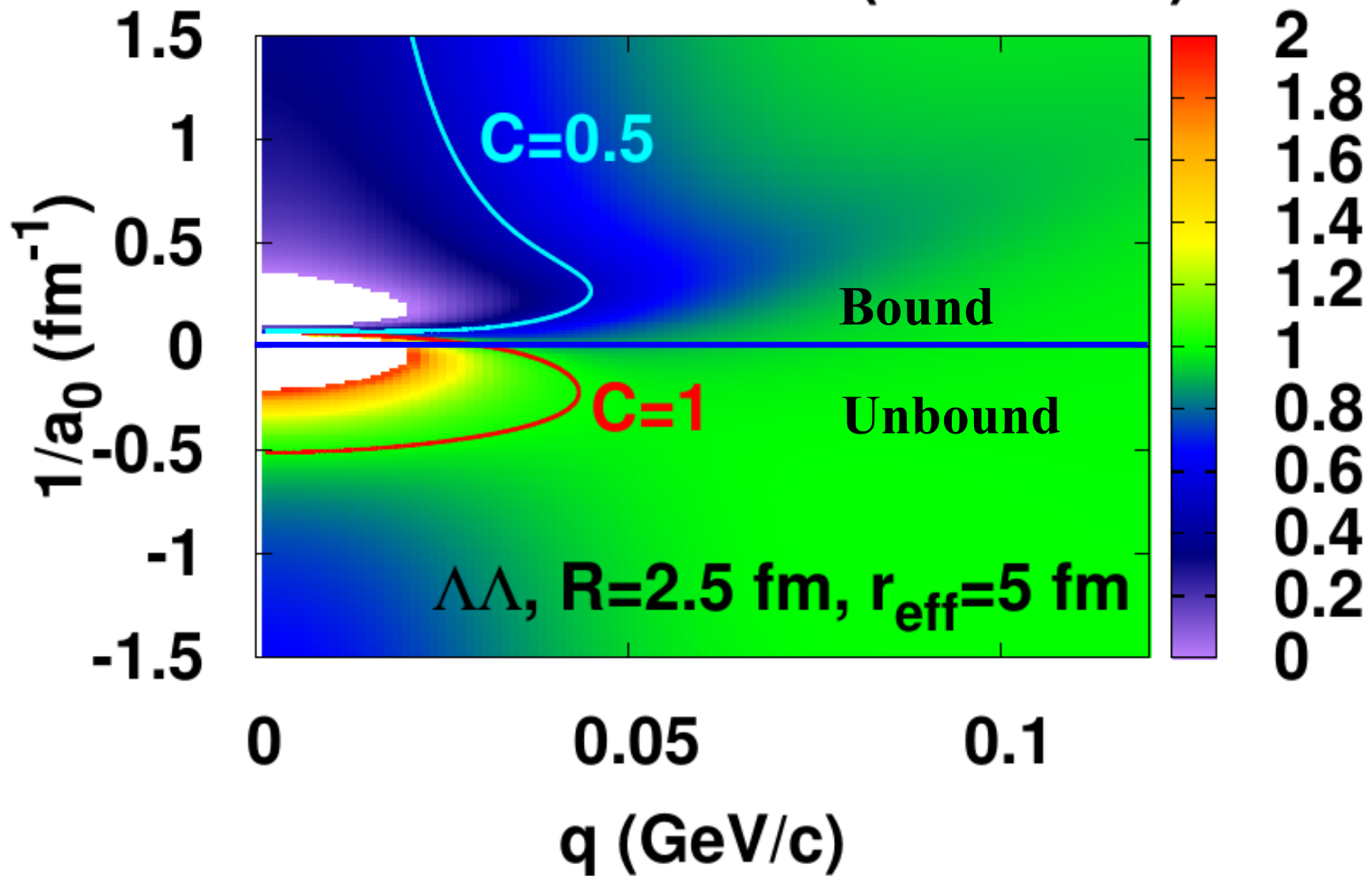
$$q \cot \delta = -1/a_0 + r_{\text{eff}} q^2/2 + O(q^4)$$

*w.f. node suppresses CF*



# Interaction Dependence of Correlation Function

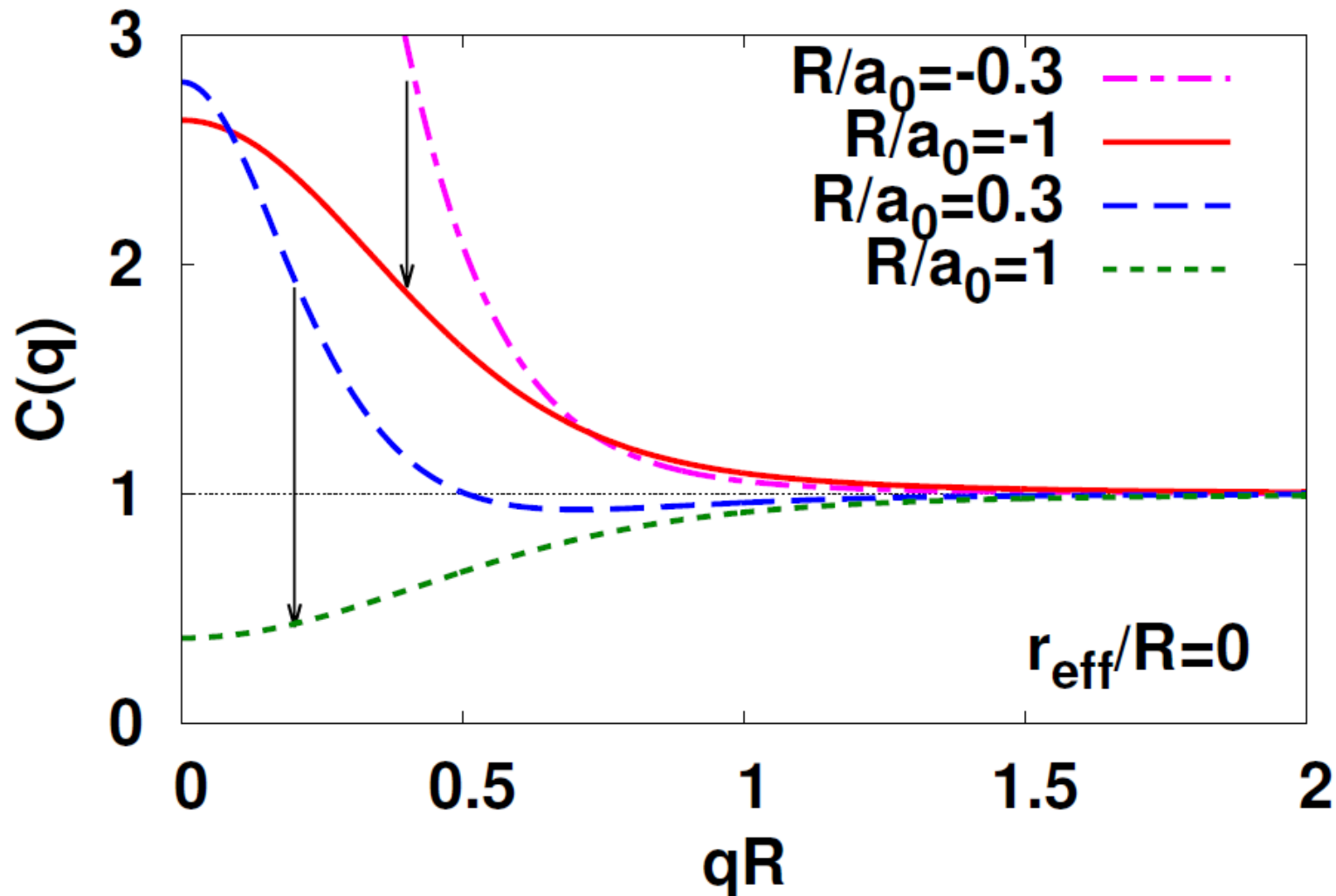
## Correlation function (LL model)



$$q \cot \delta = -1/a_0 + r_{\text{eff}} q^2/2 + O(q^4)$$

AO, K. Morita, K. Miyahara, T. Hyodo, NPA954 ('16), 294.

# How can we distinguish positive and negative $a_0$ ?



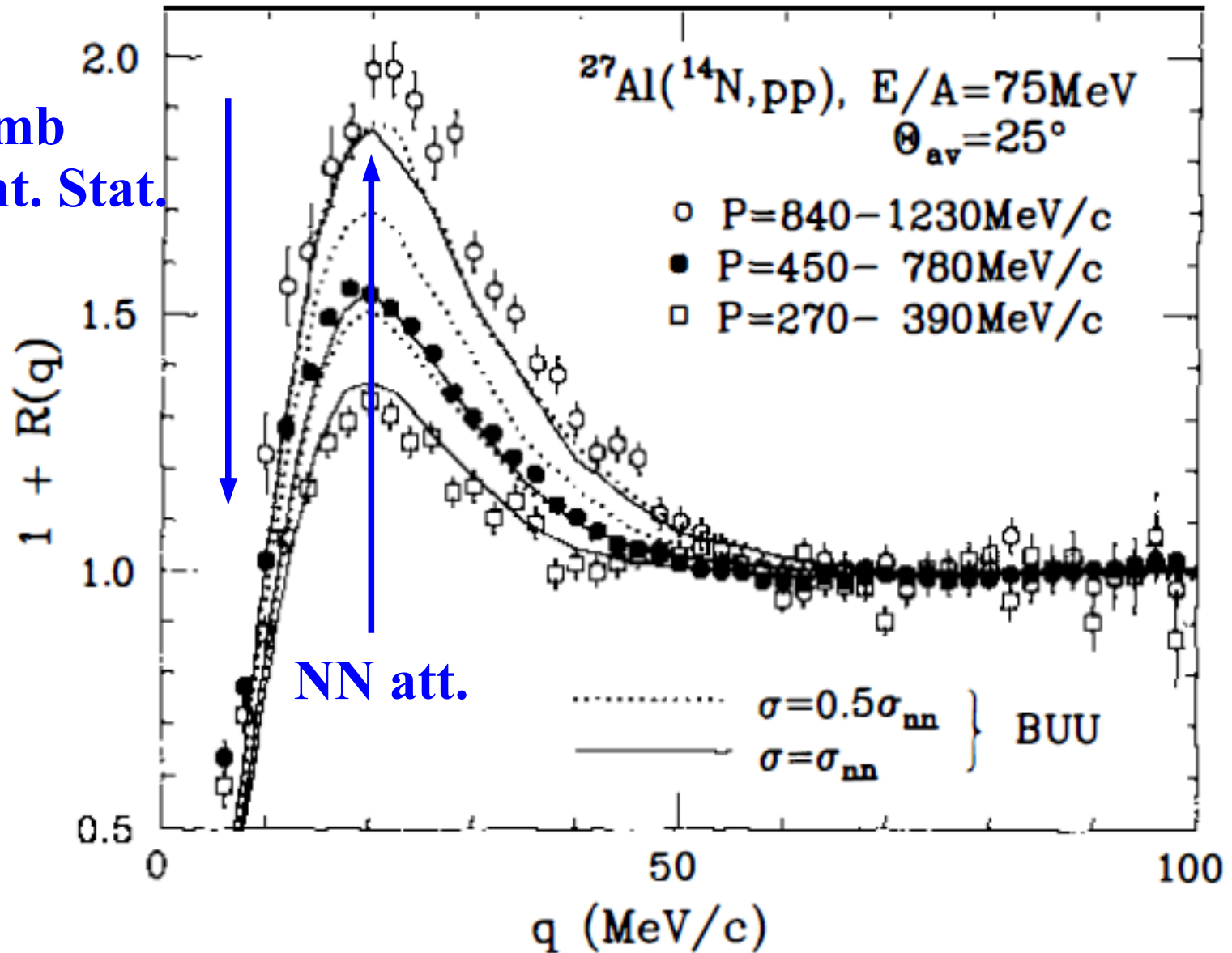
*Size dependence will help to tell the sign of  $a_0$ .*

*Positive  $a_0 \rightarrow$  CF decreases and becomes negative for large  $R$*

*Negative  $a_0 \rightarrow$  CF decreases but stay above unity.*

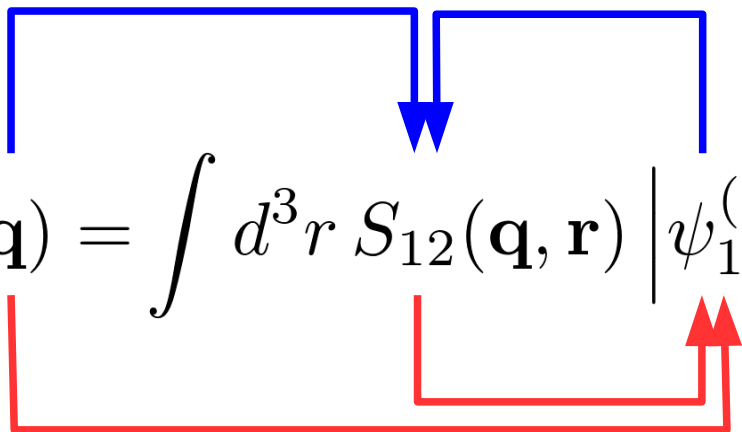
# How does interaction modifies correlation ?

Coulomb  
+Quant. Stat.



Bauer et al. ('92)

*Standard Usage*  
*Known Int. + Corr. Fn. → Source Size*


$$C(\mathbf{q}) = \int d^3r S_{12}(\mathbf{q}, \mathbf{r}) \left| \psi_{12}^{(-)}(\mathbf{r}; \mathbf{q}) \right|^2$$

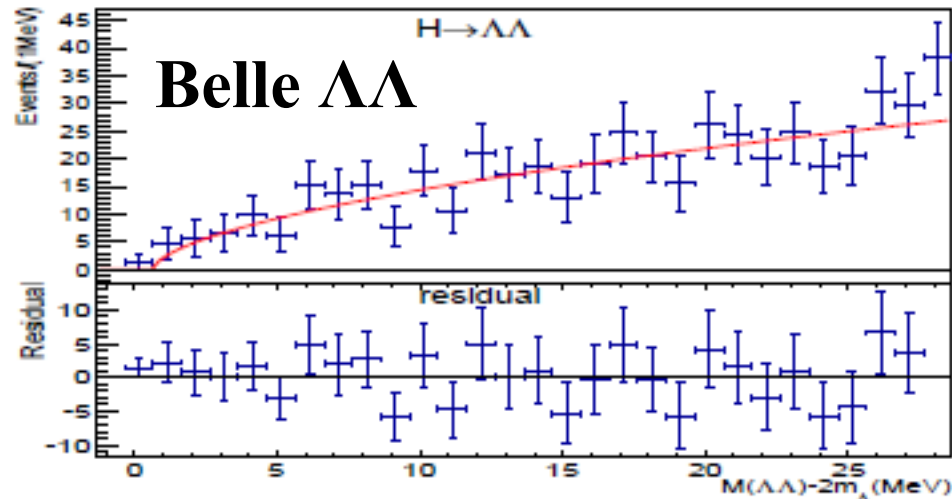
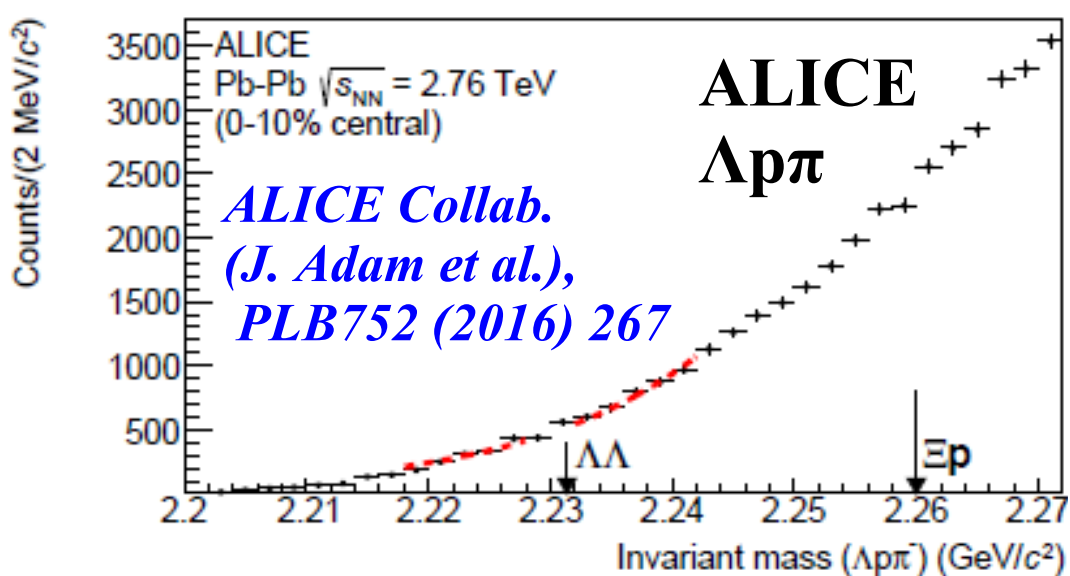
*Idea of Reversal:*  
*Can we determine hh interaction*  
*from hh correlation ?*

*Let us try ! Examples:  $\Lambda\Lambda$ ,  $\Omega^- p$ , and  $K^- p$*

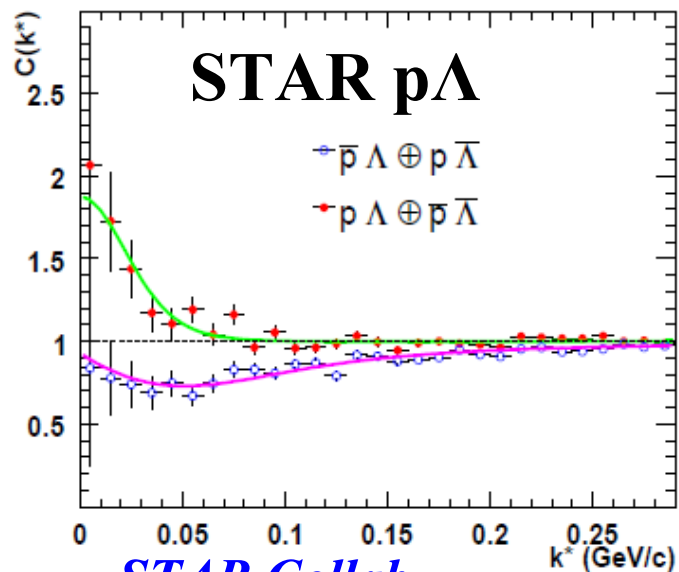
*Previous works ( $\Lambda\Lambda$ ): C. Greiner, B. Muller, PLB219('89)199.;*

*AO, Y. Hirata, Y. Nara, S. Shinmura, Y. Akaishi, NPA 670 ('00), 297c*

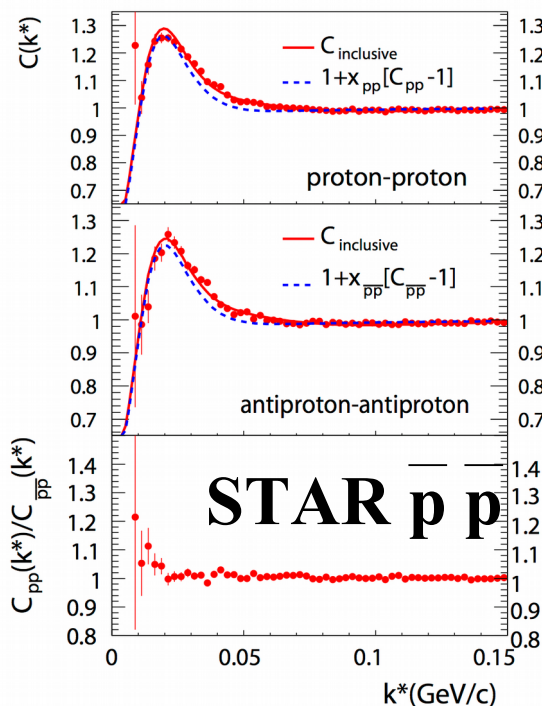
# $\Lambda\Lambda$ invariant mass / BB correlation function (as of 2016)



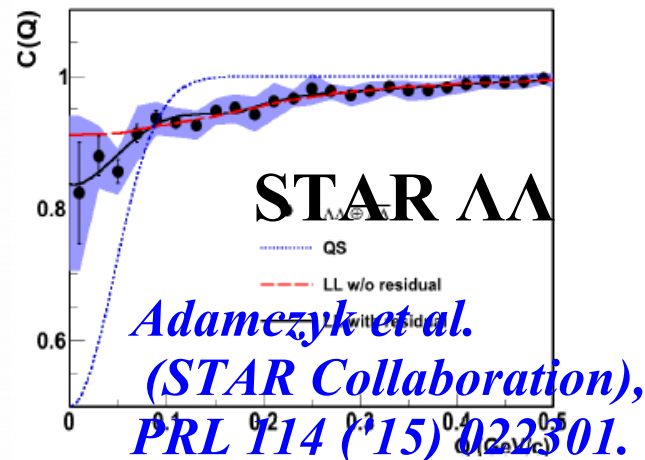
*Belle Collaboration (Kim, B.H. et al.),  
PRL110('13)222002.*



*STAR Collab.*  
(J. Adams et al.),  
*PRC74('06)064906.*



*STAR Collab.*  
(J. Adams et al.)  
*Nature 527('15)345*



*Adamezyk et al.*  
(STAR Collaboration),  
*PRL 114 ('15) 022301.*



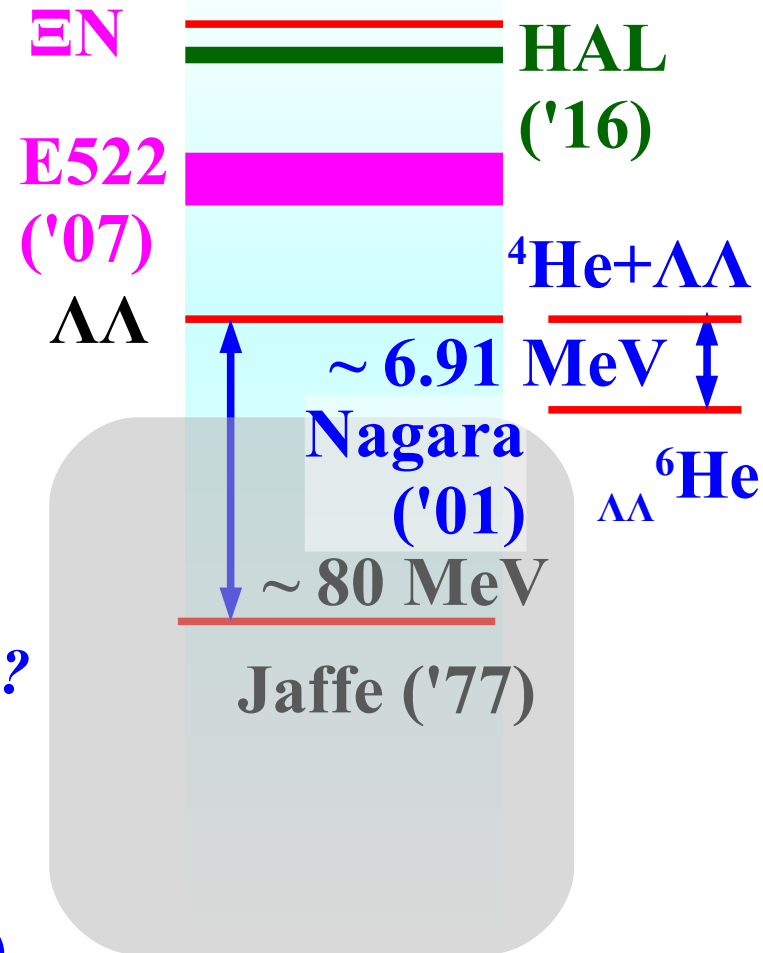
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# *$\Lambda\Lambda$ correlation and $\Lambda\Lambda$ interaction*

# Relevance of $\Lambda\Lambda$ interaction to physics

## ■ H-particle: 6-quark state (uuddss)

- Prediction: *R.L.Jaffe, PRL38(1977)195*
- Ruled-out by double  $\Lambda$  hypernucleus  
*Takahashi et al., PRL87('01) 212502*
- Resonance or Bound “H” ?  
*Yoon et al.(KEK-E522)+AO ('07)*
- Lattice QCD  
*HAL QCD & NPLQCD ('11)*  
*HAL QCD ('16): H as a loosely bound EN ?*



## ■ Neutron Star Matter EOS

- Hyperon Puzzle  
*Demorest et al. ('10), Antoniadis et al. ('13)*
- Cooling Puzzle ( $\Lambda\Lambda$  superfluidity)  
*T. Takatsuka, R. Tamagaki, PTP 112('04)37*

# $\Lambda\Lambda$ correlation in HIC

## ■ Merit of HIC to measure $\Lambda\Lambda$ correlation

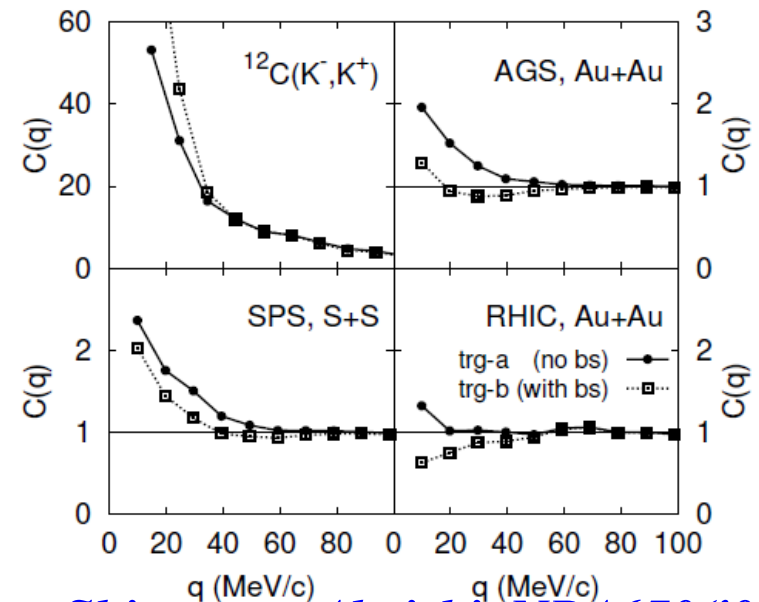
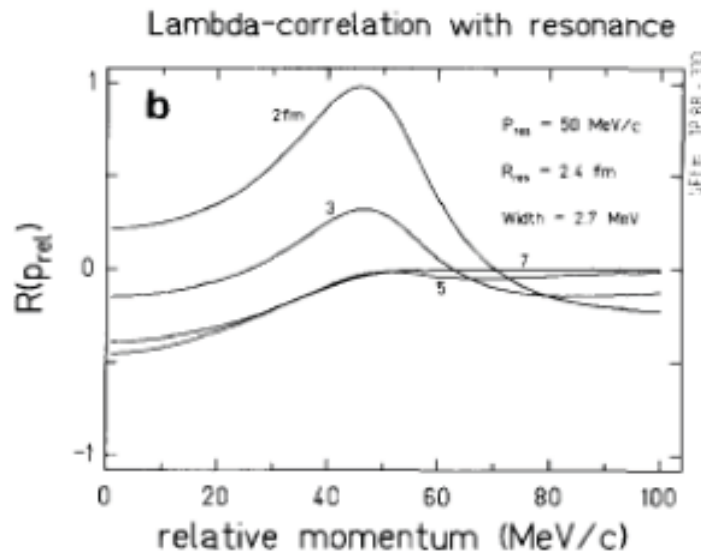
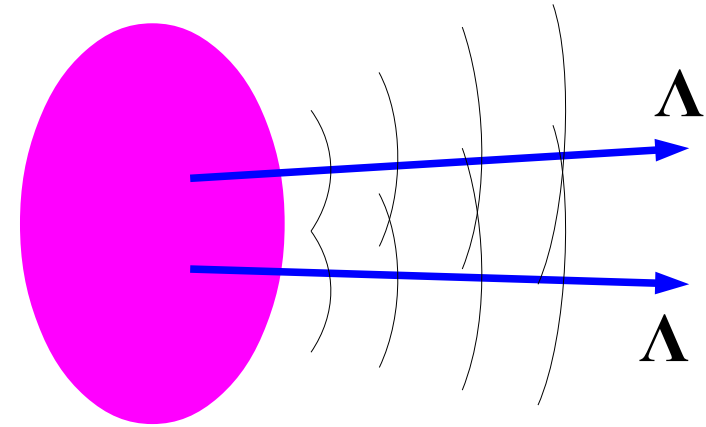
- Source is “Simple and Clean” !  
T,  $\mu$ , flow, size, ... are well-analyzed.

- Nearly Stat. prod.  
→ Many exotics will be produced.

*Schaffner-Bielich, Mattiello, Sorge ('00), Cho et al.(ExHIC Collab.) ('11)*

- Discovery of “H” and/or Constraint on  $\Lambda\Lambda$  int.

*Bound state exhaust the low q strength → suppressed  $C(q)$ .*



*C. Greiner, B. Muller, PLB219('89)199.*

*AO, Hirata, Nara, Shinmura, Akaishi, NPA670('00)297c*

# $\Lambda\Lambda$ correlation at RHIC

- STAR collaboration at RHIC measured  $\Lambda\Lambda$  correlation !

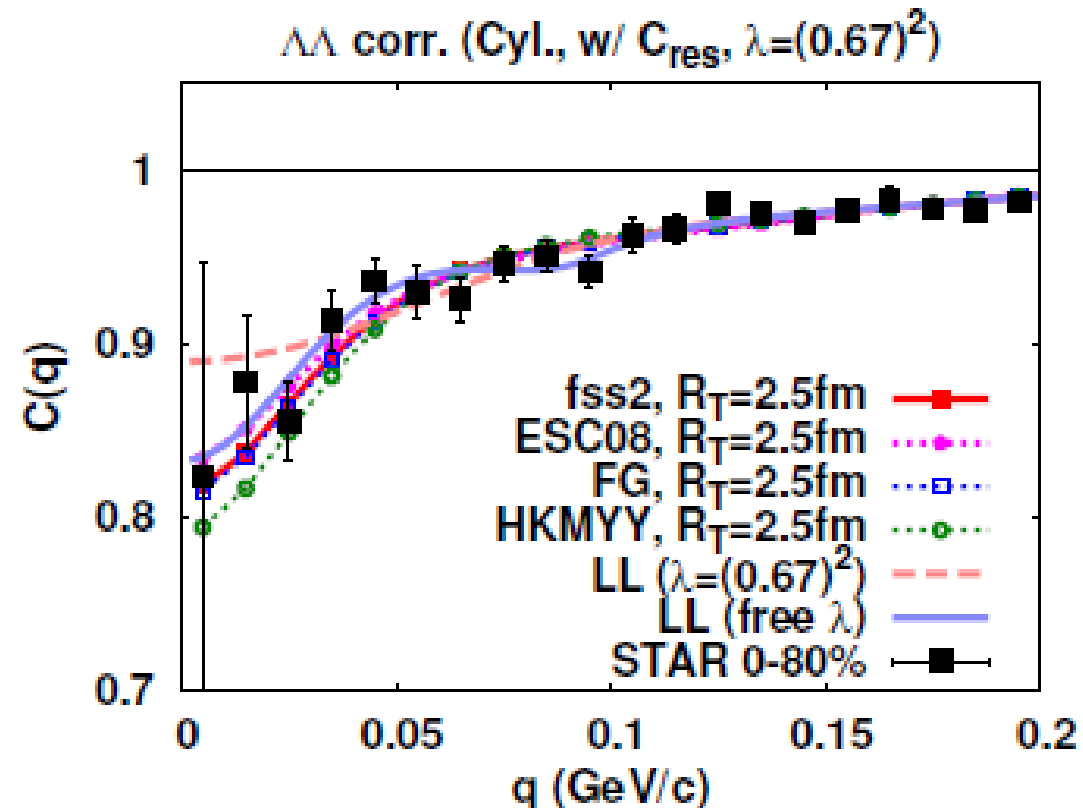
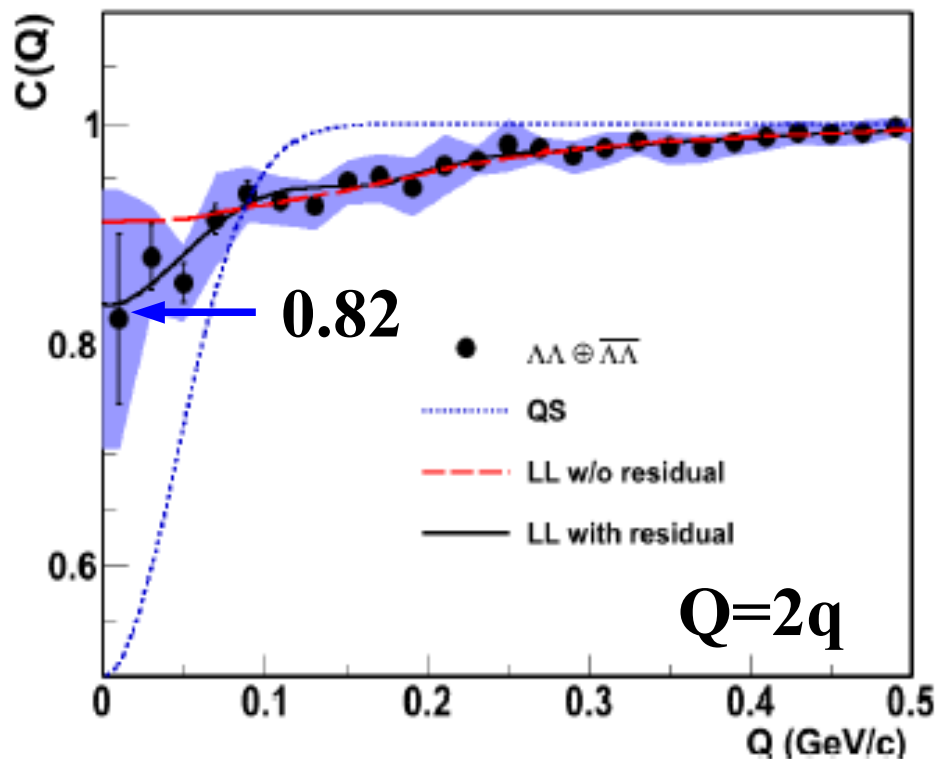
*Adamczyk et al. (STAR Collaboration), PRL 114 ('15) 022301.*

- RHIC, Au+Au ( $\sqrt{s_{NN}}=200$  GeV), Weak decay vertex analysis.

- Theoretical Analysis well explains the data

*K.Morita et al., T.Furumoto, AO, PRC91('15)024916;*

*AO, K.Morita, K.Miyahara, T.Hyodo, NPA954 ('16), 294.*

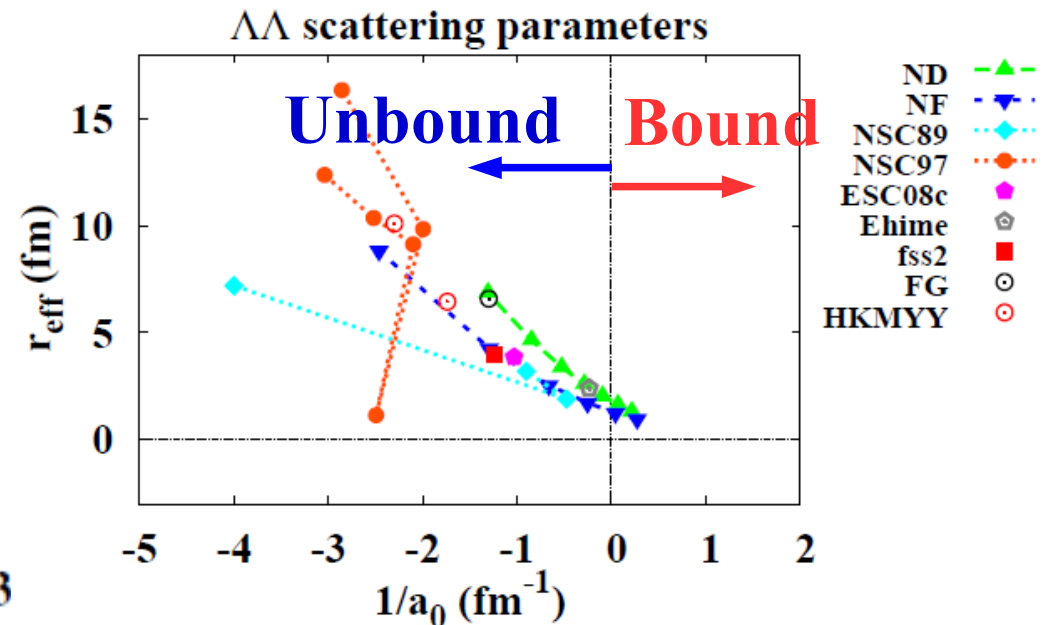
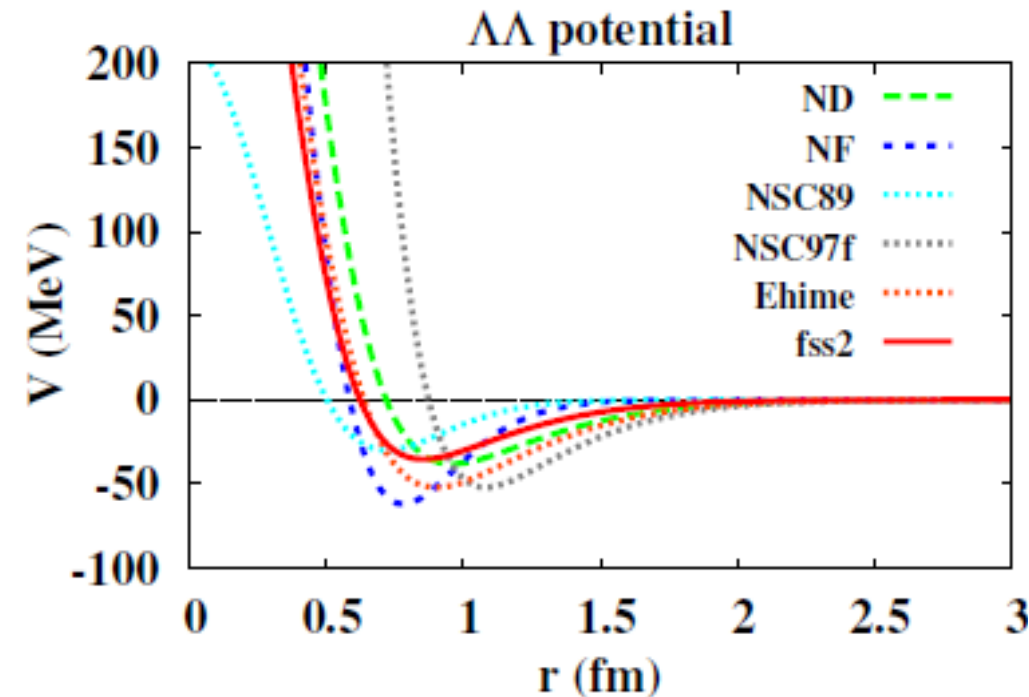


# $\Lambda\Lambda$ interaction

## Proposed $\Lambda\Lambda$ interactions

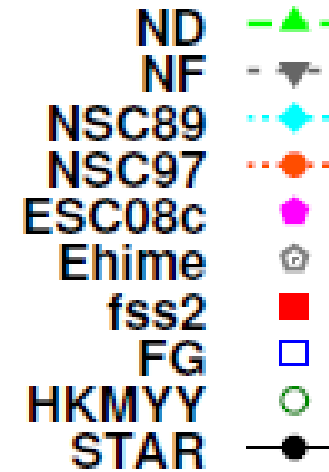
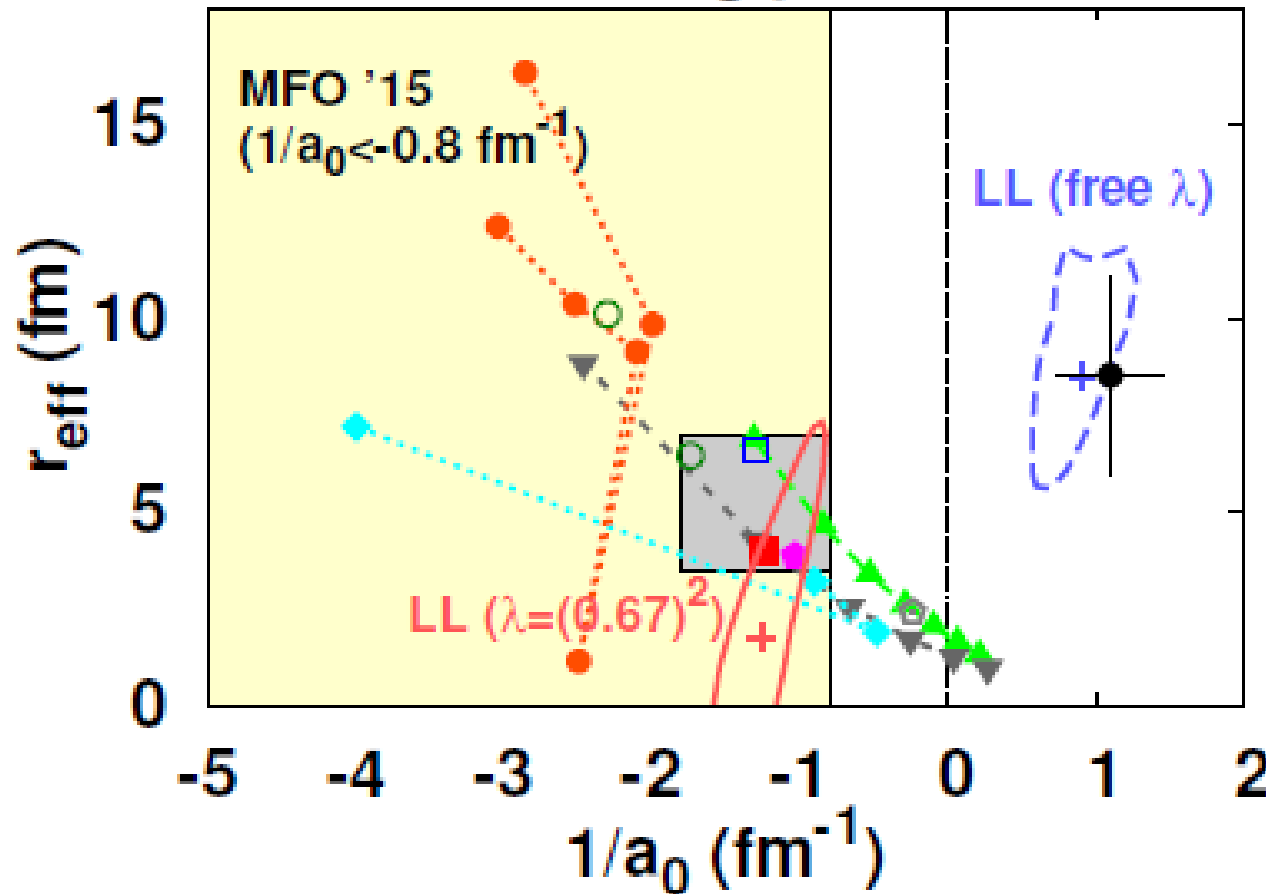
- **Meson Ex. models:** Nijmegen model D, F, Soft Core (89, 97), ESC08 Nagels, Rijken, de Swart ('77, '79), Maessen, Rijken, de Swart ('89), Rijken, Stoks, Yamamoto ('99); Rijken, Nagels, Yamamoto ('10).
- **Quark cluster model interaction:** fss2 Fujiwara, Fujita, Kohno, Nakamoto, Suzuki ('00)
- **Phenomenological model:** Ehime T. Ueda et al. ('99).

## Two (or three) range gaussian fit results are used in the analysis.



# $\Lambda\Lambda$ interaction from $\Lambda\Lambda$ correlation

## $\Lambda\Lambda$ scattering parameters



- Nijmegen potentials (ND, NF, NSC89, NSC97, ESC08) *Nagels+('77, '79), Maessen+('89), Rijken+('99, '10)*
- Ehime *Ueda et al. ('98)*
- Quark model interaction: fss2 *Fujiwara et al.('07)*
- Potential fitted to Nagara *Filikhin, Gal ('02) (FG), Hiyama et al. ('02, '10)(HKMY Y)*

$$q \cot \delta = -1/a_0 + r_{\text{eff}} q^2/2 + \mathcal{O}(q^4)$$

*Positive  $a_0$  (STAR)  $\longleftrightarrow$  Negative  $a_0$  (MFO'15)*  
*Difference comes from the pair purity*

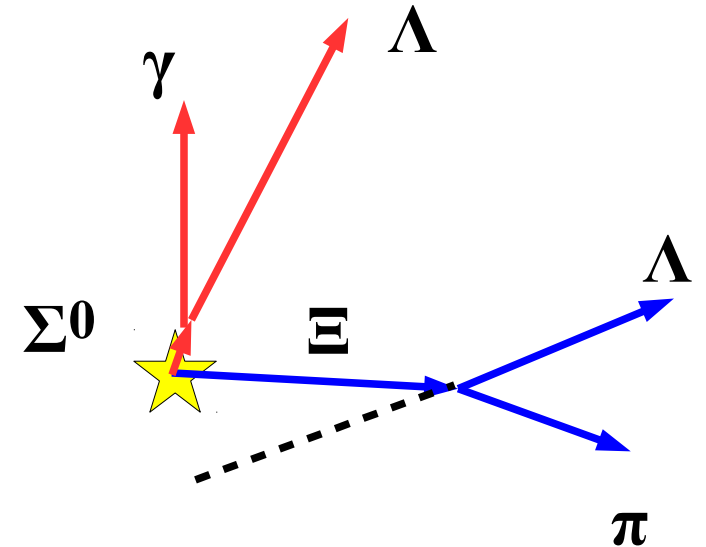
# Additional Source

## ■ Feed down effects

$$C_{\text{corr}}(Q) = 1 + \lambda(C_{\text{bare}}(Q) - 1)$$

$\lambda =$  Purity of  $\Lambda\Lambda$  pair

- Short-lived  $Y^* \rightarrow$  mod. of source fn.
- $\Xi \rightarrow \Lambda\pi$  can be excluded ( $c\tau=8.71$  cm)
- $\Sigma^0 \rightarrow \Lambda\gamma$  is difficult to reject
- Data based purity  $\lambda=(0.67)^2$   
 $\Sigma^0/\Lambda=0.278$  (p+Be, 28.5 GeV/c) *Sullivan et al. ('87)*  
 $\Xi/\Lambda = 15\%$  (RHIC)



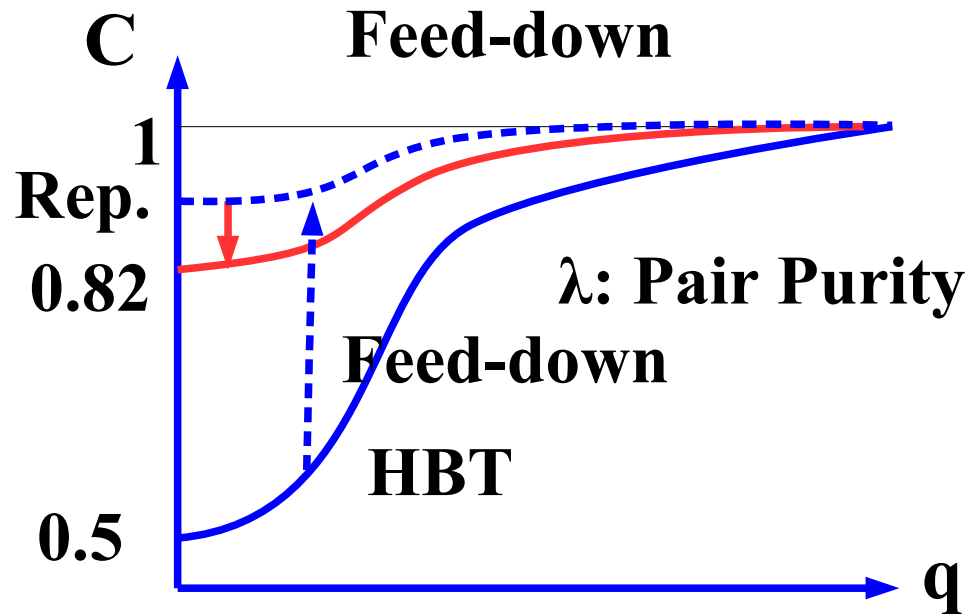
## ■ “Residual” source

- High-momentum tail  $\rightarrow R_{\text{res}} \sim 0.5$  fm (STAR collab.)

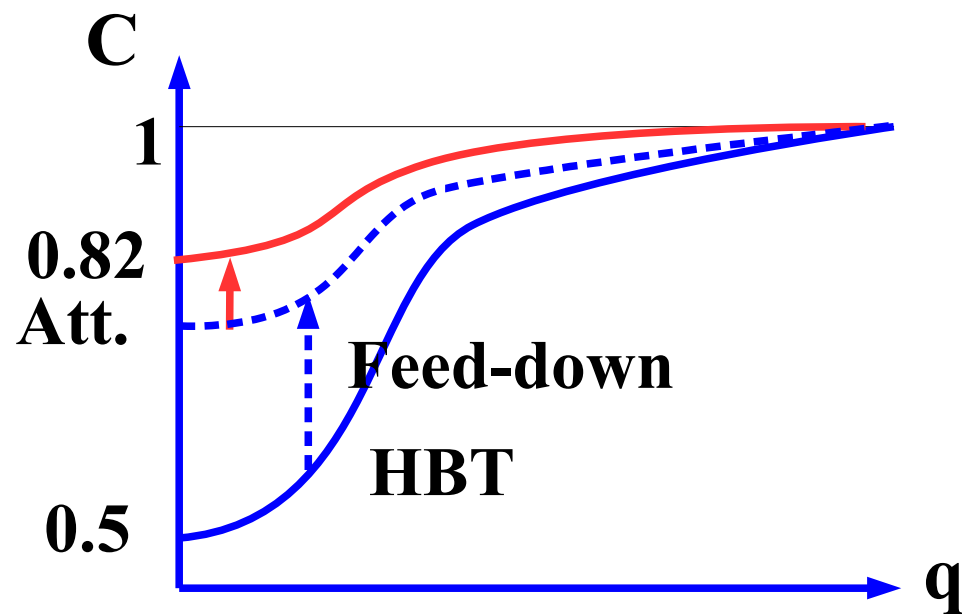
# Feed-Down Effects & Residual Source

- Correlation Fn. w/ Feed-down & Residual source effects.

$$C_{\text{corr}}(q) = 1 + \lambda(C_{\text{bare}}(q) - 1) + a_{\text{res}} \exp(-4r_{\text{res}}^2 q^2)$$



STAR:  
 $\lambda \sim 0.18$  (free para.)



Morita et al. (MFO15):  
 $\lambda \sim 0.45$

$\Sigma^0/\Lambda=0.278$  (p+Be, 28.5 GeV/c)  
*Sullivan et al. ('87)*  
 $\Xi/\Lambda = 15 \%$  (RHIC)

*AO, Morita, Mihayara, Hyodo ('16)*

*Pair purity ( $\lambda$ ) should be determined experimentally !  
 Puzzle: Residual source*



# New Data from LHC-ALICE

V. Mantovani-Sarti (ALICE Collab.), MESONS 2018

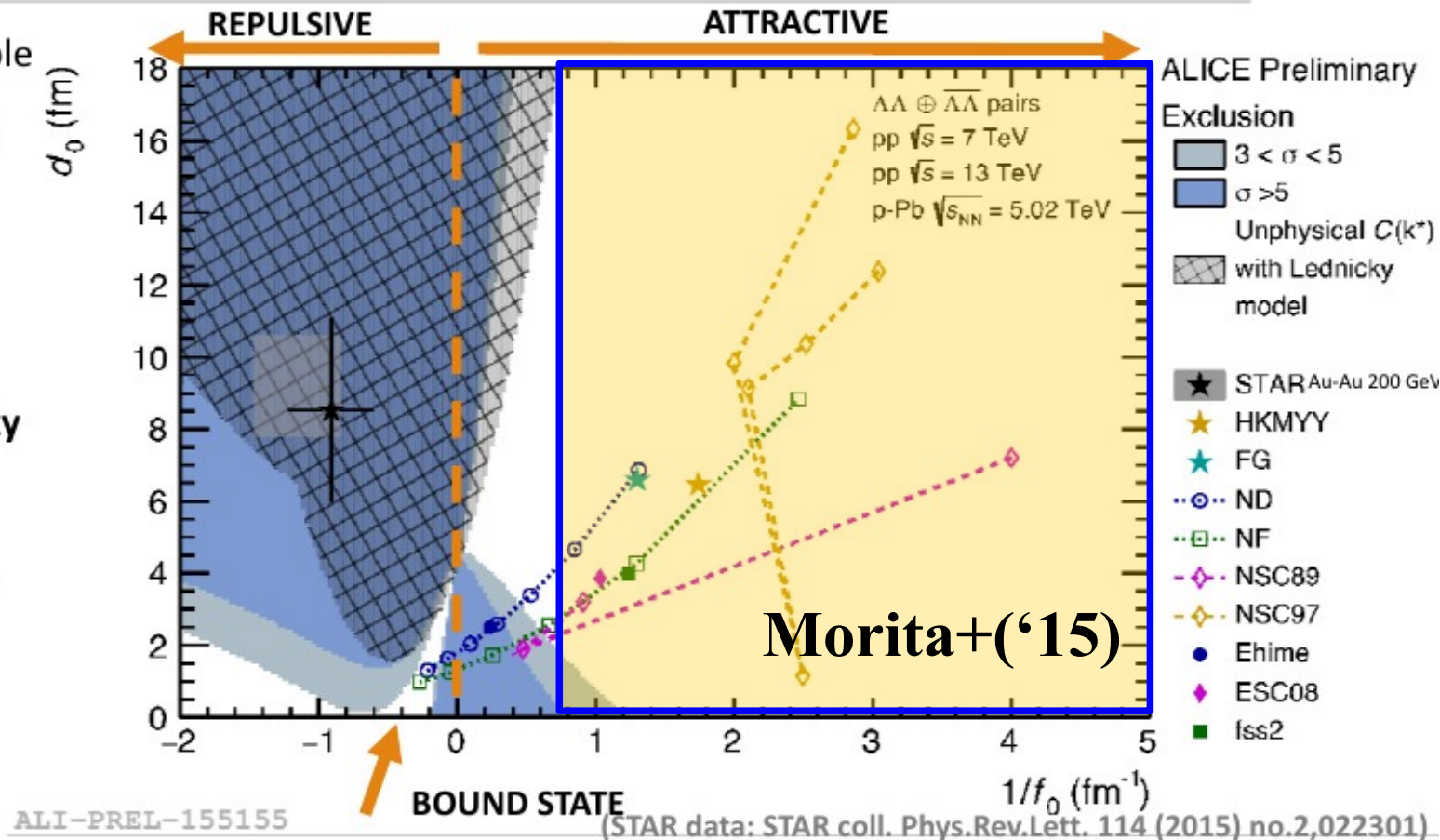


ALICE

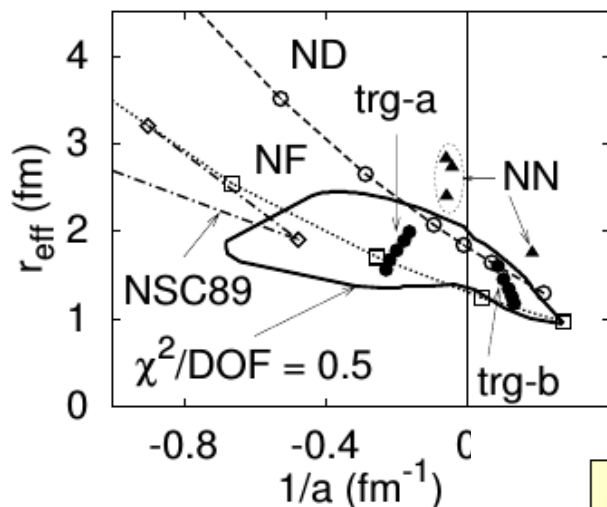
## $\Lambda$ - $\Lambda$ Correlations: Combined Exclusion Plot



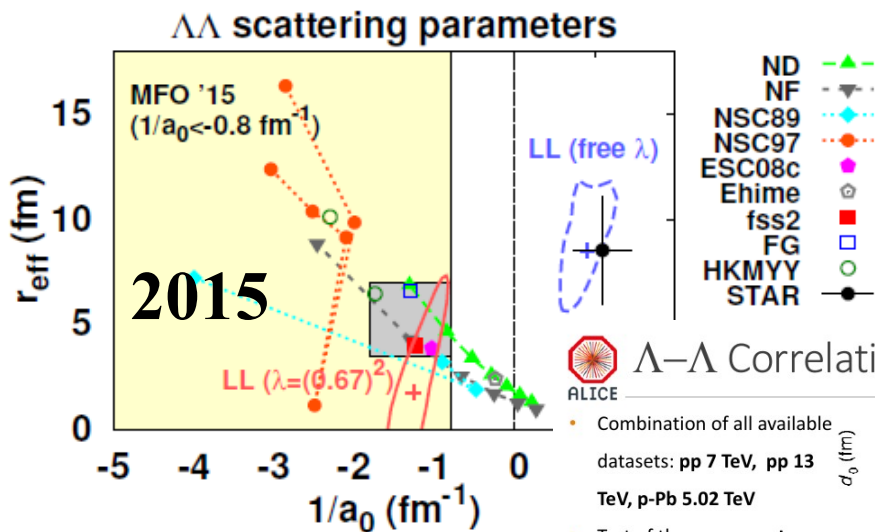
- Combination of all available datasets: **pp 7 TeV, pp 13 TeV, p-Pb 5.02 TeV**
- Test of the **agreement between data and the prediction by the Lednicky model by  $n\sigma$**
- Small source size, large  $d_0$  and negative  $f_0$  limit the prediction power of Lednicky



# Time dependence of $\Lambda\Lambda$ interaction



2000



2015

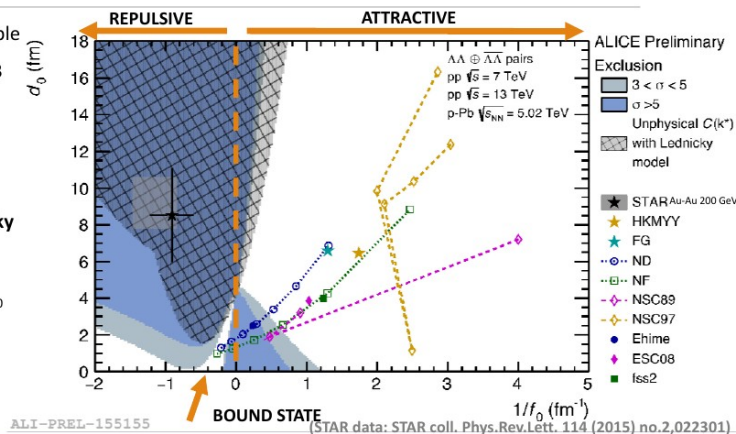
2018

Nagara (2001)

$\Lambda$ - $\Lambda$  Correlations: Combined Exclusion Plot



- Combination of all available datasets: pp 7 TeV, pp 13 TeV, p-Pb 5.02 TeV
- Test of the agreement between data and the prediction by the Lednický model by  $n\sigma$
- Small source size, large  $d_0$  and negative  $f_0$  limit the prediction power of Lednický



ALI-PREL-155155

(STAR data: STAR coll. Phys.Rev.Lett. 114 (2015) no.2,022301)



Valentina Mantovani Sarti (TUM Physics Department - E62)

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# $\Omega^- p$ correlation

# $\Omega^- p$ interaction

■  $\Omega^-$  : quark content=sss,  $J^\pi=3/2^+$ ,  $M=1672$  MeV

■  $\Omega^- p$  bound state as a  $S = -3$  dibaryon ?

*F.Etminan et al. (HAL QCD Collab.),  
NPA928('14)89.*

- No quark Pauli blocking in  $\Omega N$ ,  $H=uuddss$ , and  $d^*=\Delta\Delta$  channels.

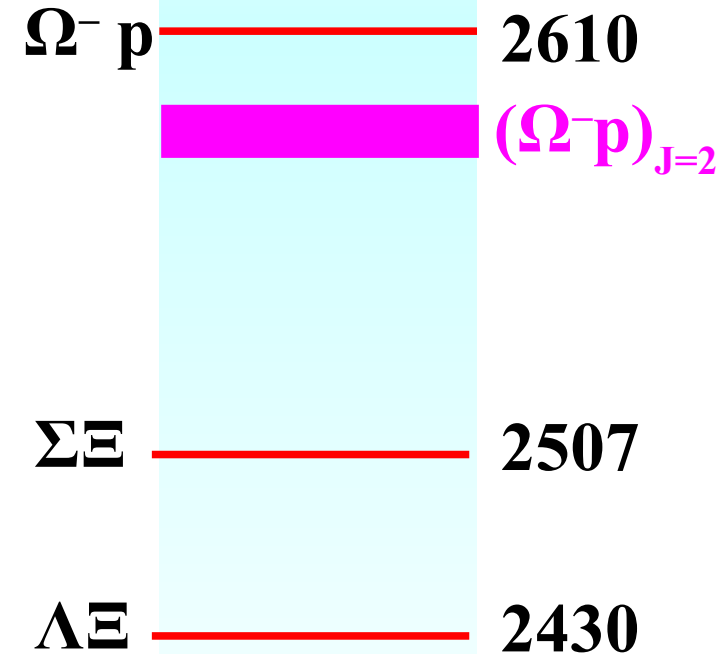
*Oka ('88), Gal ('16)*

- $J=2$  state ( ${}^5S_2$ ) couples to Octet-Octet baryon pair only with  $L \geq 2$   
→ Small width is expected.

*Etminan et al. (HAL QCD)('14)*

- Correlation is measurable at RHIC !

*Neha Shah (STAR), private commun.*



*Let us try to discover the first(?) dibaryon (after deuteron) !  
(First dibaryon with  $S < 0$  !)*

# $\Omega^- p$ potential from lattice QCD

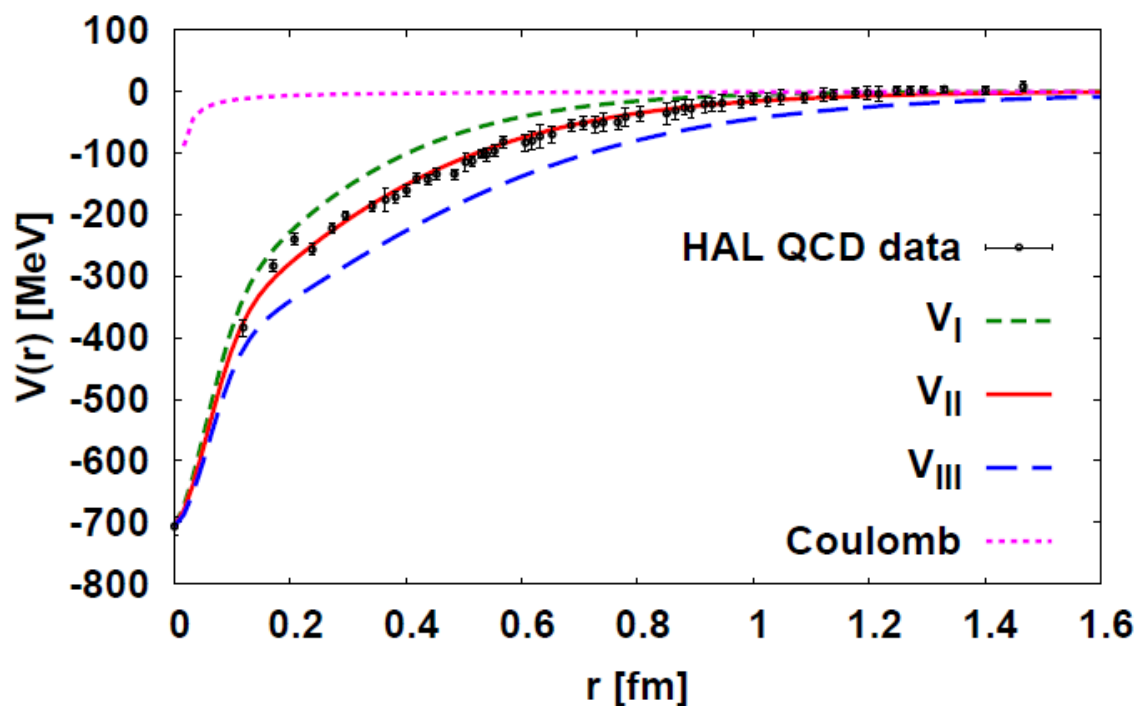
- Lattice QCD predicts  $\Omega^- p$  bound state at large quark mass,  $m_\pi=875$  MeV (B.E.~ 19 MeV) in  ${}^5S_2$  channel.

*F.Etminan et al. (HAL QCD Collab.), NPA928('14)89.*

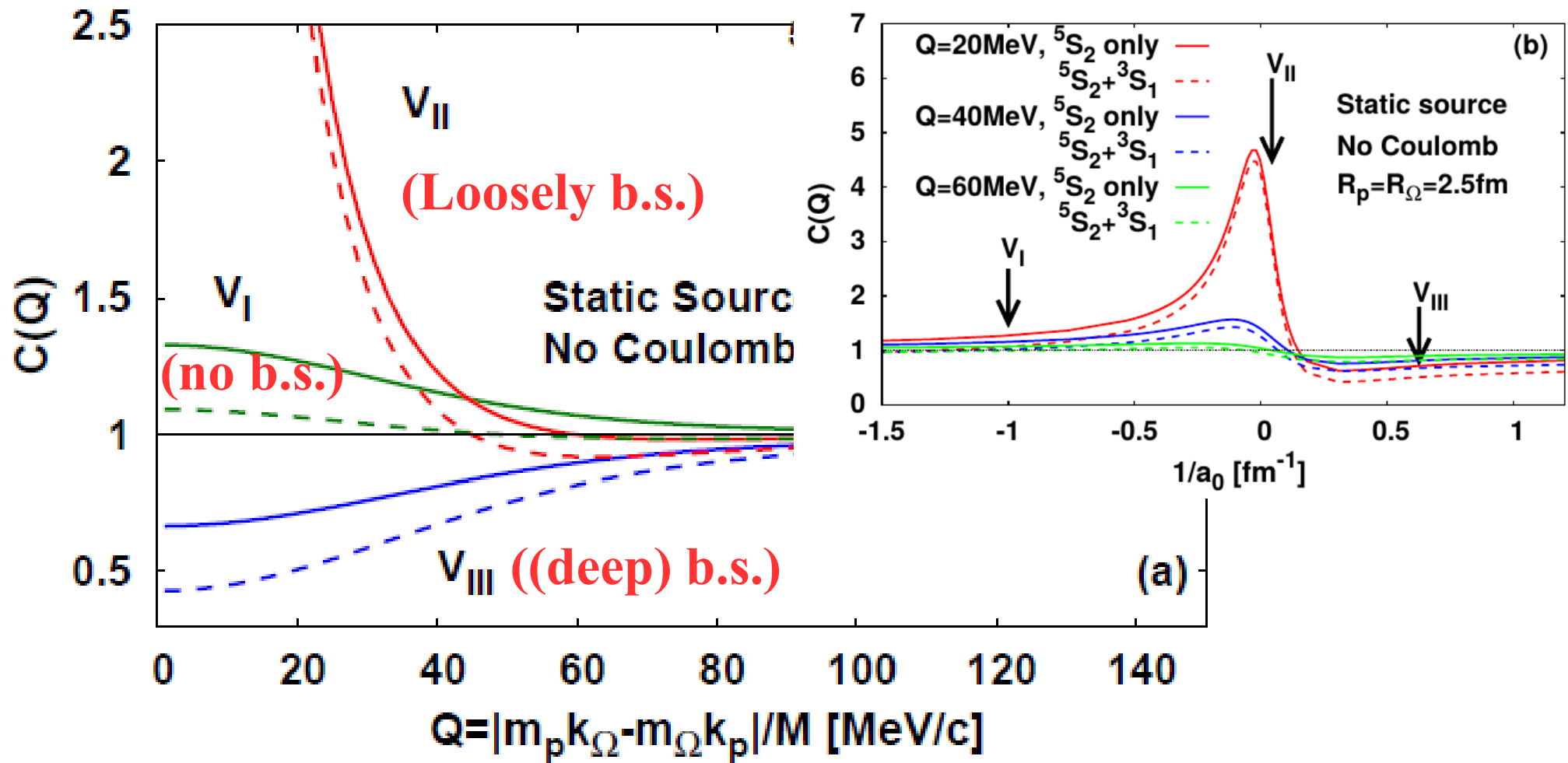
- Extrapolation to physical quark mass

- VI → Weaker potential (no b.s.)
- VII → Same potential (shallow b.s.)
- VIII → Stronger potential (deep b.s.)

Spin-2 $N\Omega$ Potentials		$V_I$	$V_{II}$	$V_{III}$
without Coulomb	$E_B$ [MeV]	–	0.05	24.8
	$a_0$ [fm]	–1.0	23.1	1.60
	$r_{\text{eff}}$ [fm]	1.15	0.95	0.65
with Coulomb	$E_B$ [MeV]	–	6.3	26.9
	$a_0$ [fm]	–1.12	5.79	1.29
	$r_{\text{eff}}$ [fm]	1.16	0.96	0.65



# $\Omega$ - $p$ correlation

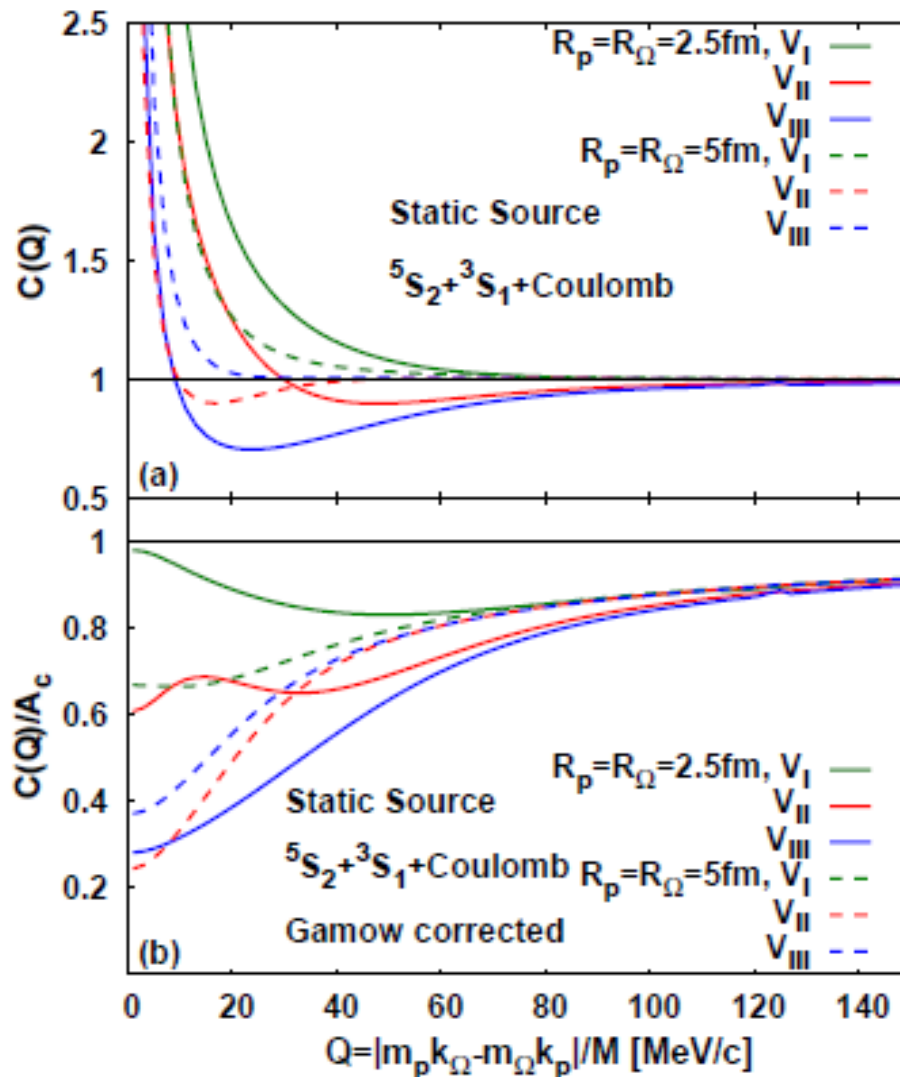


(w/o Coulomb, Strong absorption at  $r < 2$  fm in  $^3S_1$  (decay to 8-8 in S-wave))

*K. Morita, AO, F. Etminan, T. Hatsuda, PRC94('16)031901(R) [arXiv:1605.06765 [hep-ph]]*



# $\Omega$ - $p$ correlation w/ Coulomb

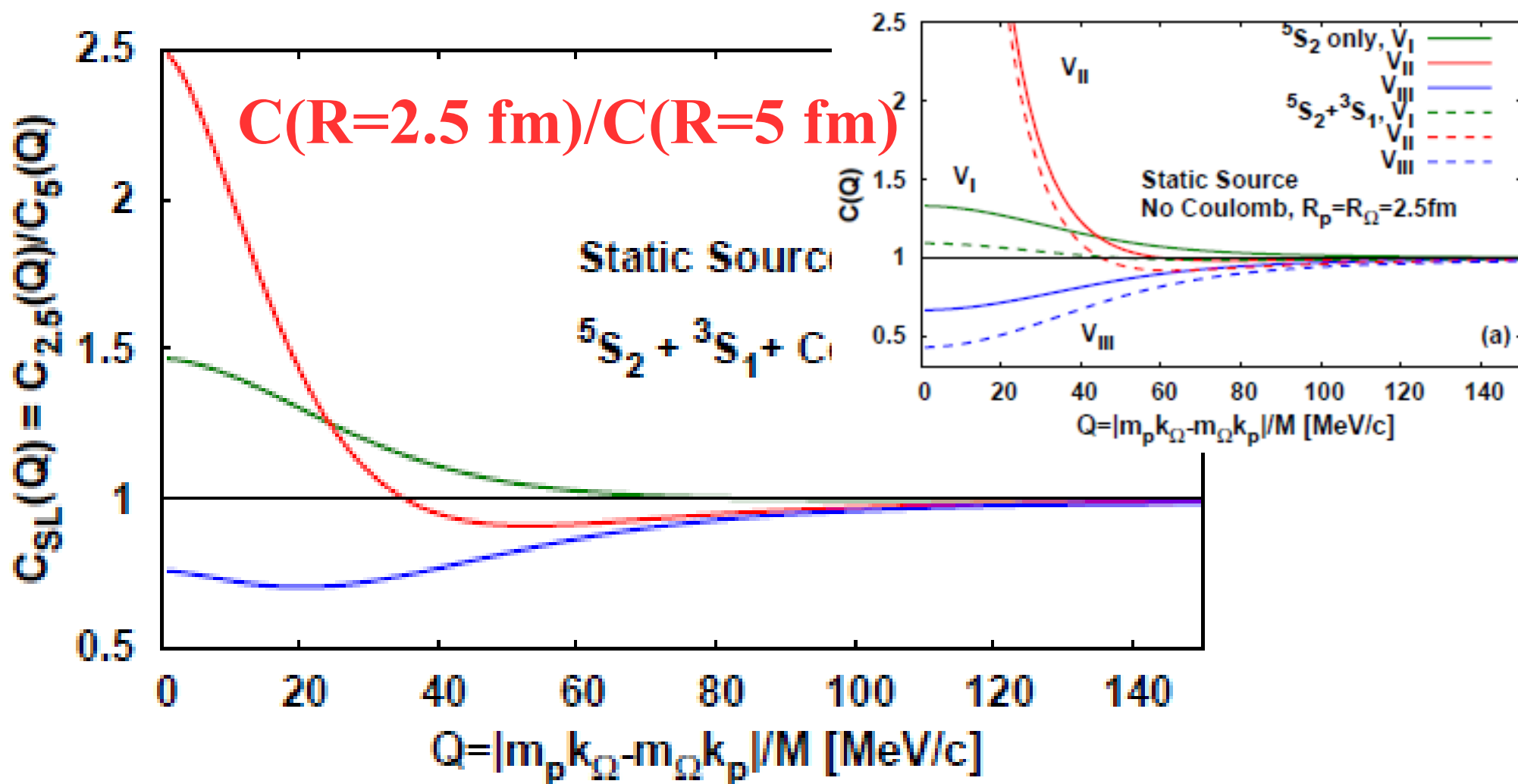


**With Coulomb**

**Coulomb + Gamow corr.**

*Coulomb potential washes out the features of  $V_I$ ,  $V_{II}$ ,  $V_{III}$ , and Gamow correction is not enough.*

# $\Omega$ - $p$ correlation: Small / Large Ratio

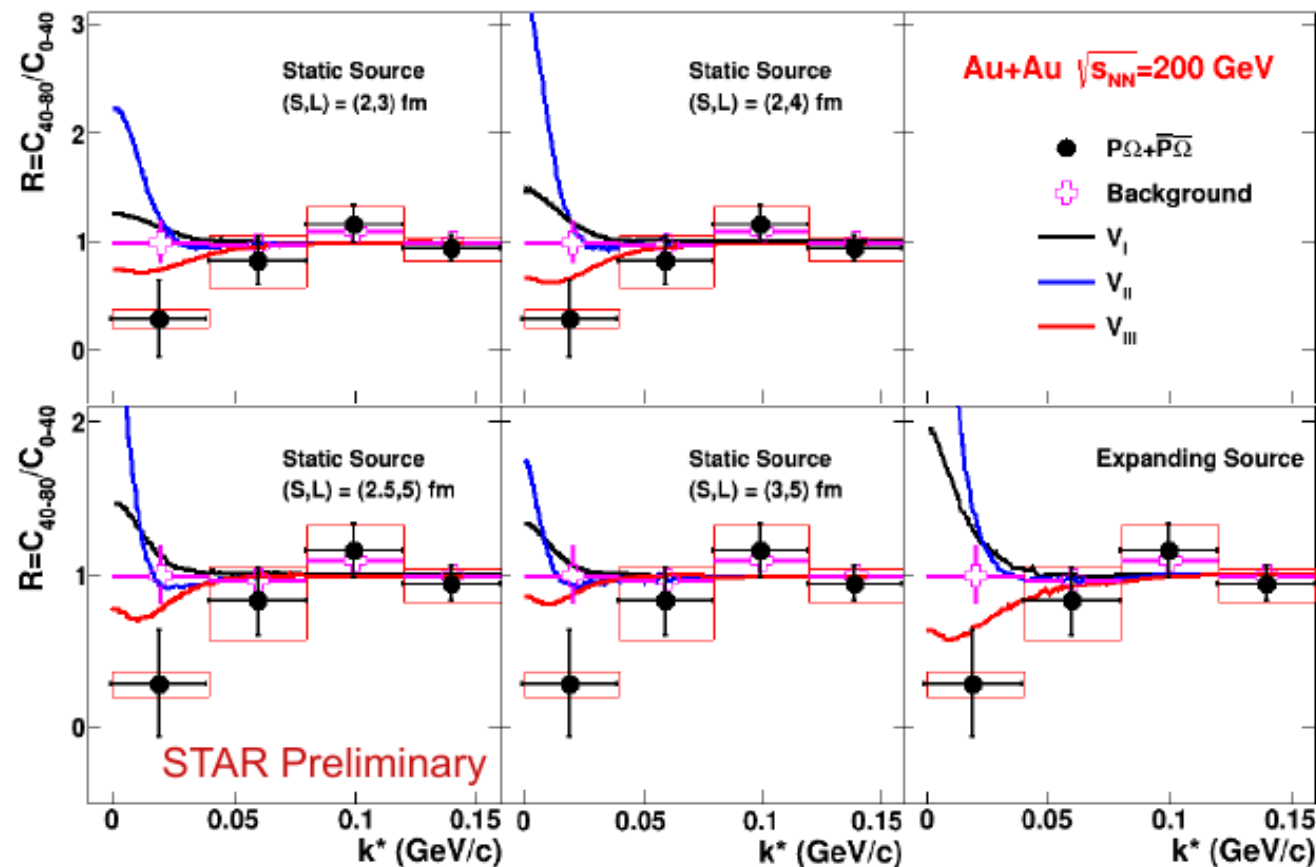
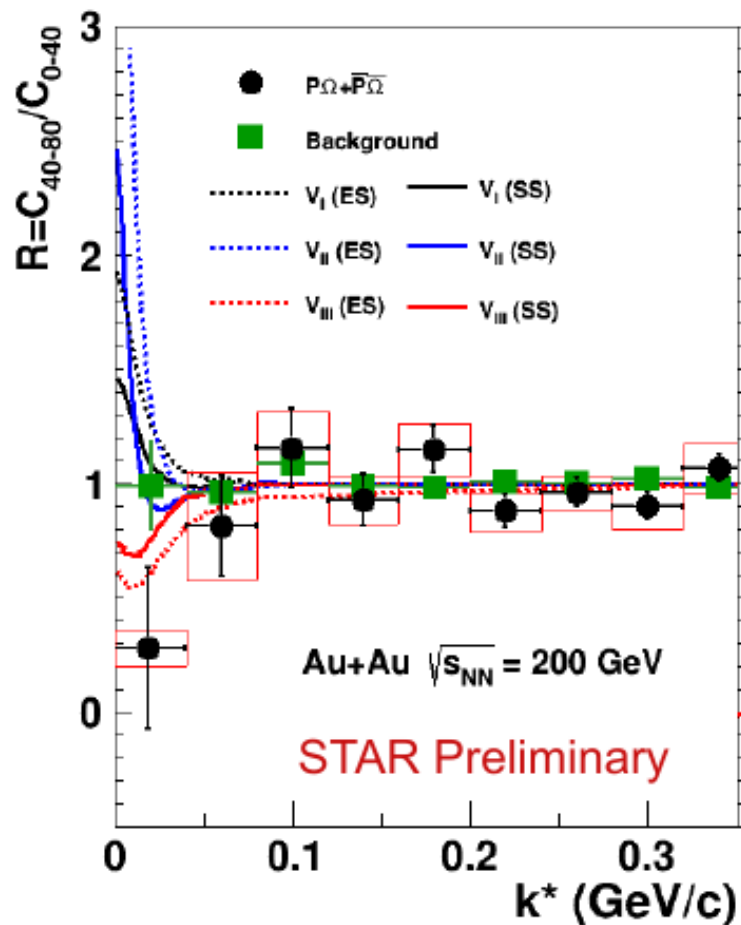


*By taking small ( $R=2.5 \text{ fm}$ ) / large ( $R=5 \text{ fm}$ ) ratio, we approximately see the corr. fn. w/o Coulomb !*



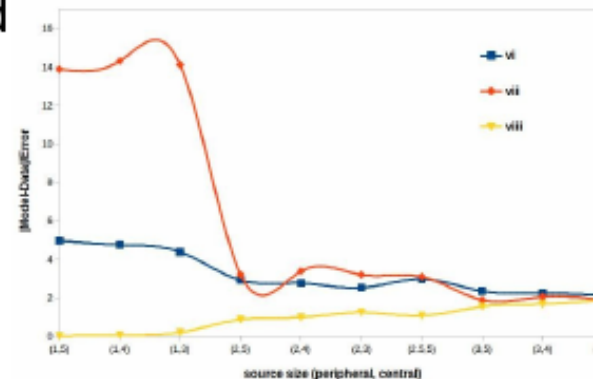


## Source Size Analysis on $p\Omega$ Correlation Function

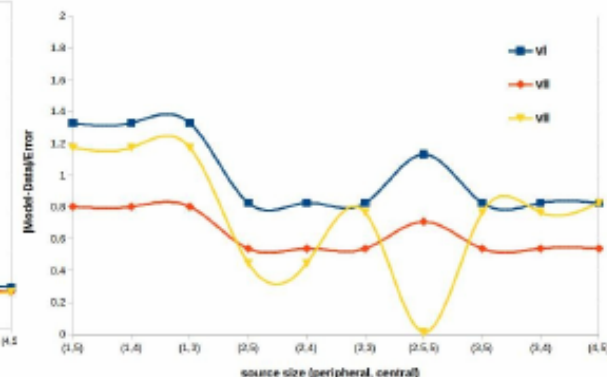


The source parameters are derived from other PID and centrality dependence measurements at RHIC

$k^* = 0.02 \text{ GeV}/c$



$k^* = 0.06 \text{ GeV}/c$



# Results with ppdated HAL QCD potential

*K. Morita, S. Gongyo, T. Hatsuda, T. Hyodo, T. Iritani, AO, K. Sasaki, in prep.*

## ■ Updated HAL QCD $N\Omega$ potential

*T. Iritani et al. (HAL QCD Collab.), 1810.03416*

Almost physical point  $m_\pi = 146$  MeV

→ K. Sasaki's talk

$t/a$	$a_0$ [fm]	$r_{\text{eff}}$ [fm]	$E_B$ [MeV]
11	3.45	1.33	2.15
12	3.38	1.31	2.27
13	3.49	1.31	2.08
14	3.40	1.33	2.24

## ■ Cylindrical source with radial transverse flow

→ pT spectra of protons and  $\Omega$ s

## ■ Small-Large ratio to suppress the Coulomb effects & Absorptive potential in J=1 channel

*K. Morita, AO, F. Etminan, T. Hatsuda, PRC94('16)031901(R) [arXiv:1605.06765 [hep-ph]]*

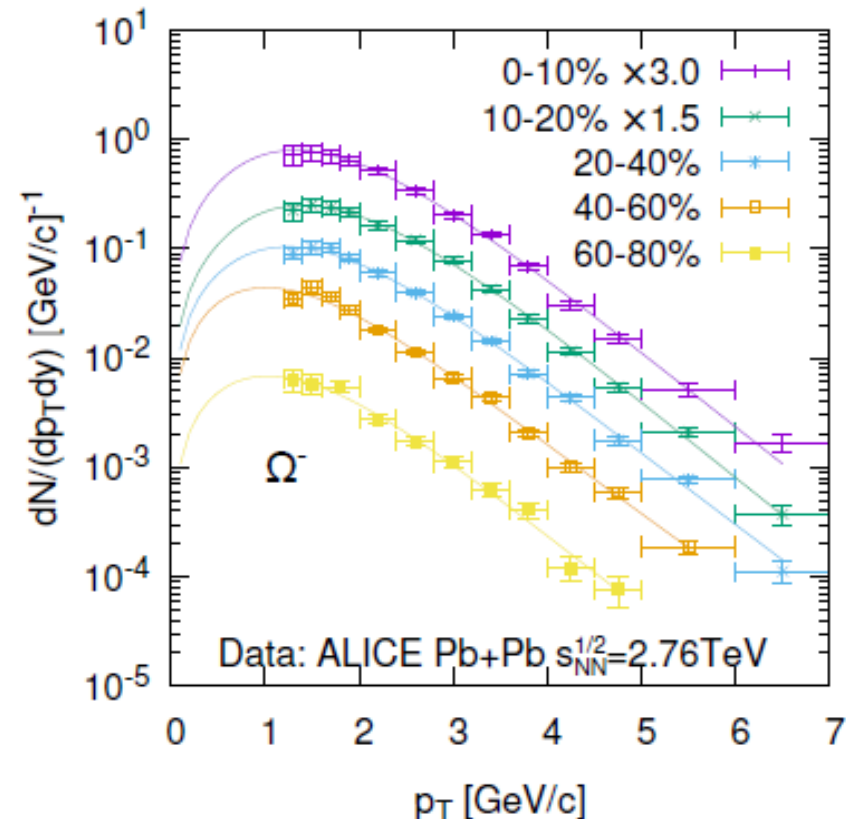
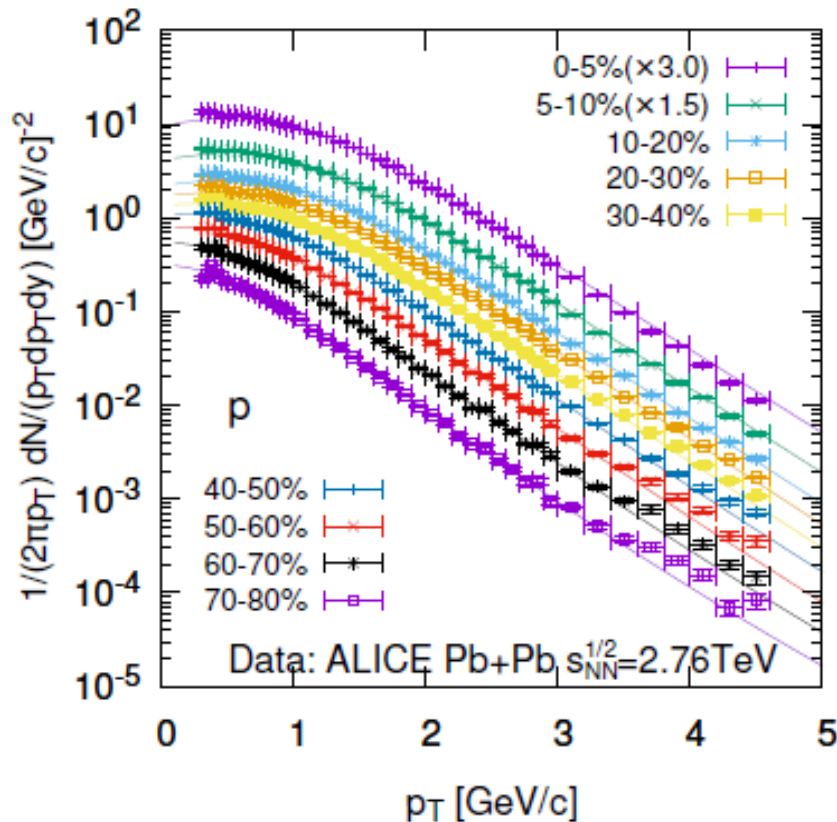
# Source function

## ■ Blast wave model fit

Flow velocity

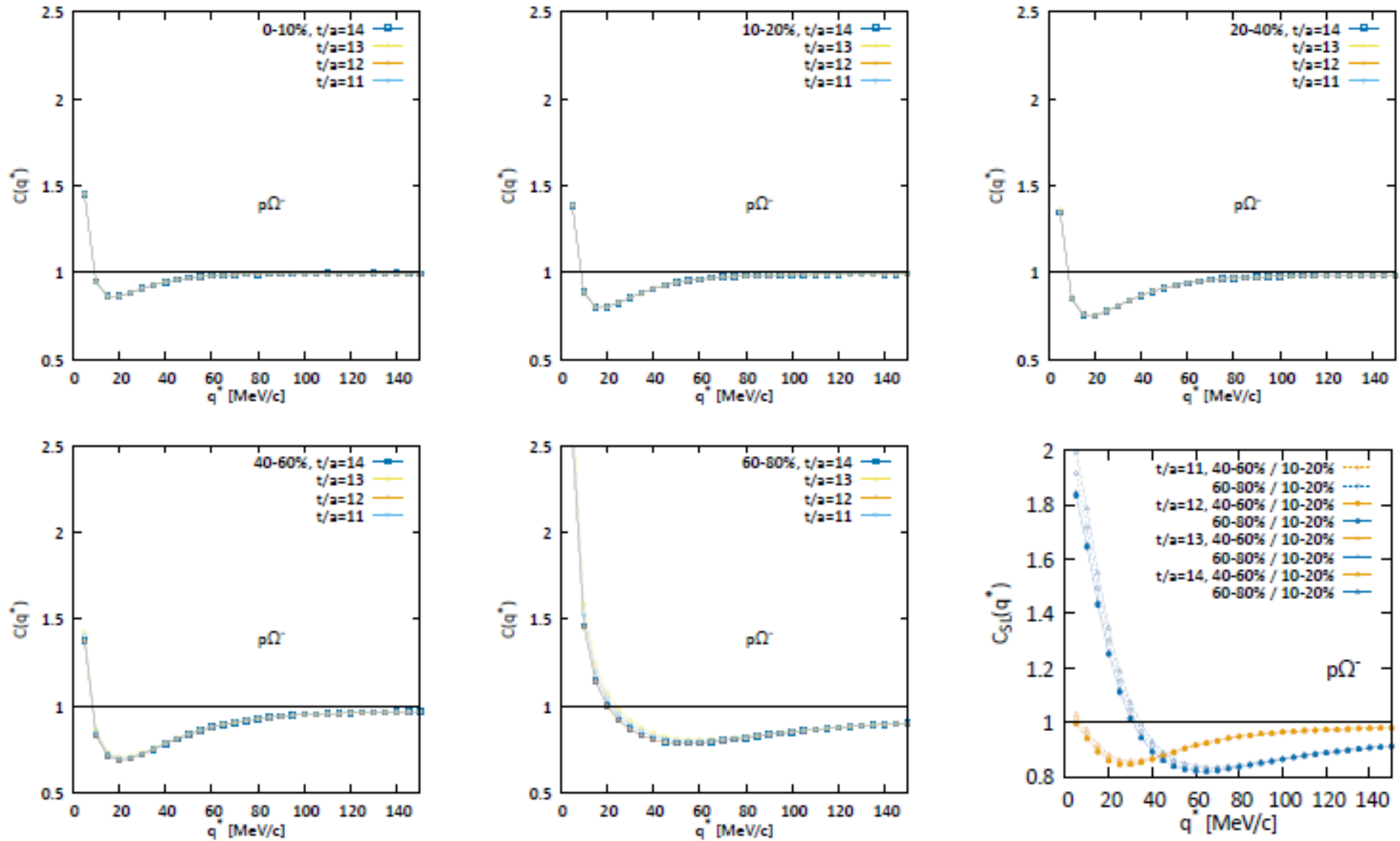
$$d^4 x S_i(x, \mathbf{p}) = \tau_0 d\eta_s d^2 r_T \frac{d}{(2\pi)^3} \underline{n_f(u \cdot p, T)} \exp\left(-\frac{r_T^2}{2R_T^2}\right)$$

Fermi dist.



*K. Morita, S. Gongyo, T. Hatsuda, T. Hyodo, T. Iritani, AO, K. Sasaki, in prep.*

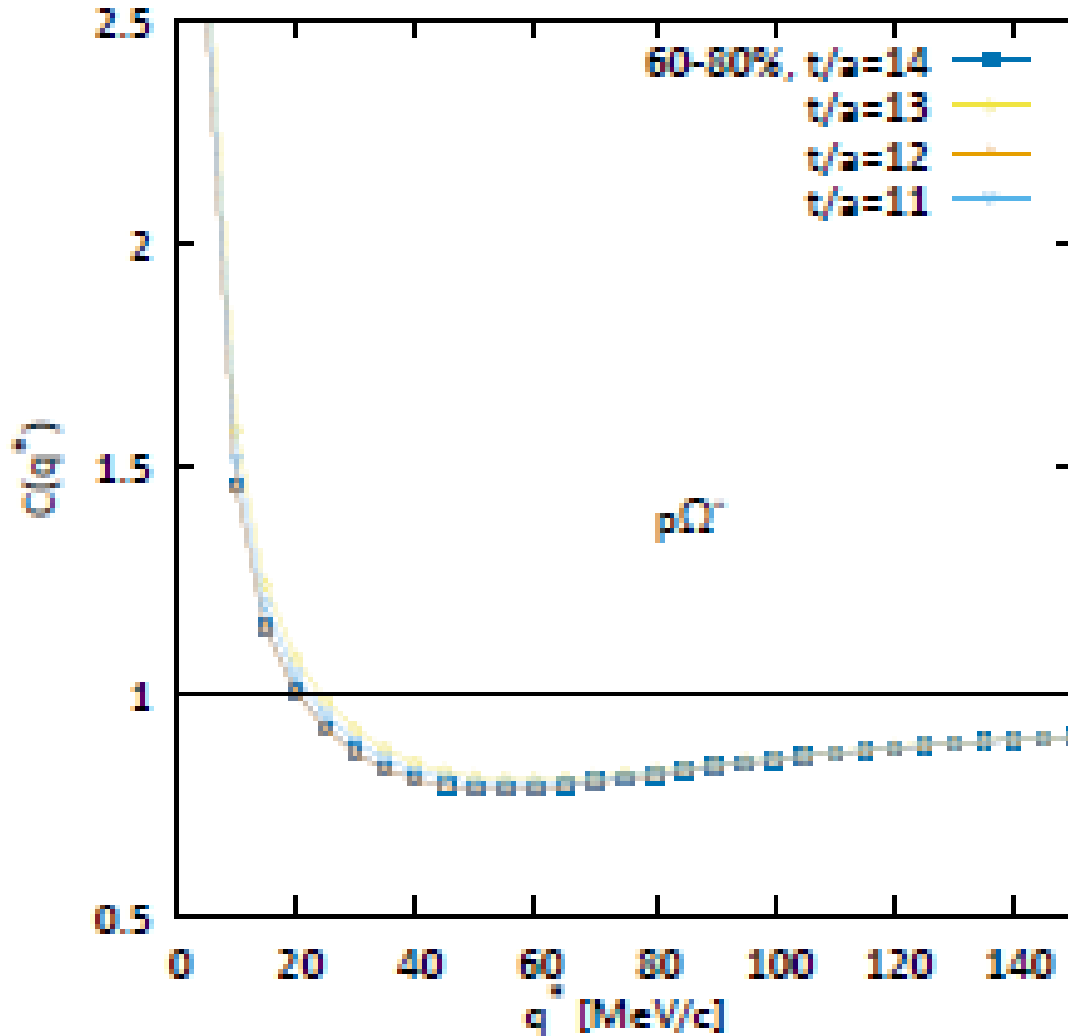
# Correlation function from heavy-ion collisions



*K. Morita, S. Gongyo, T. Hatsuda, T. Hyodo, T. Iritani, AO, K. Sasaki, in prep.*

*A. Ohnishi, HIPLQH, Apr. 3, 2019 36*

# Correlation function from heavy-ion collisions



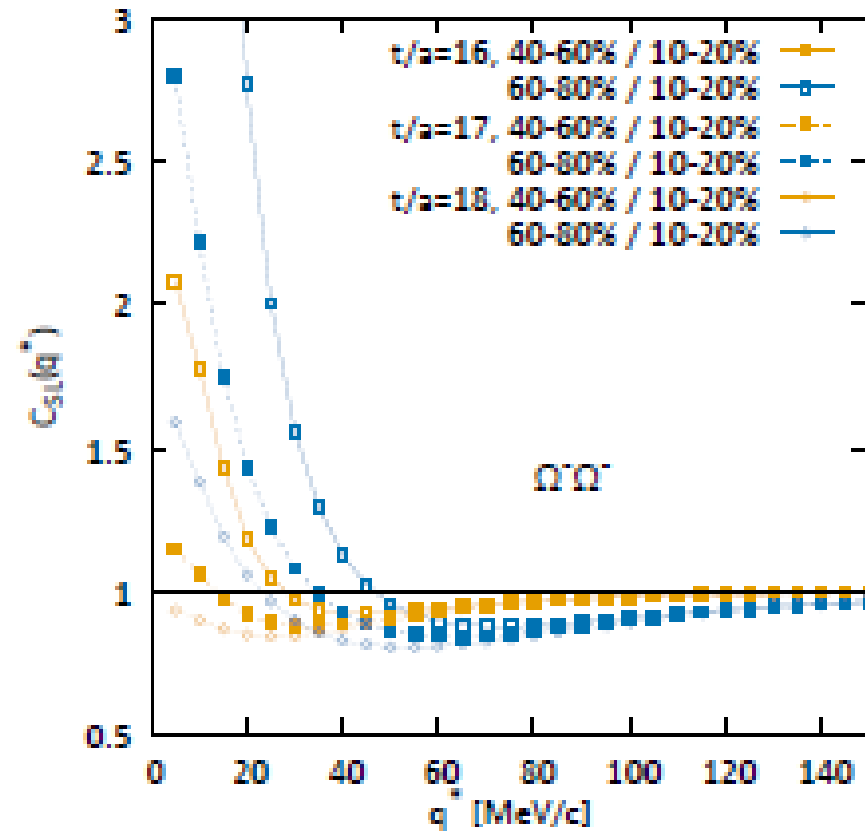
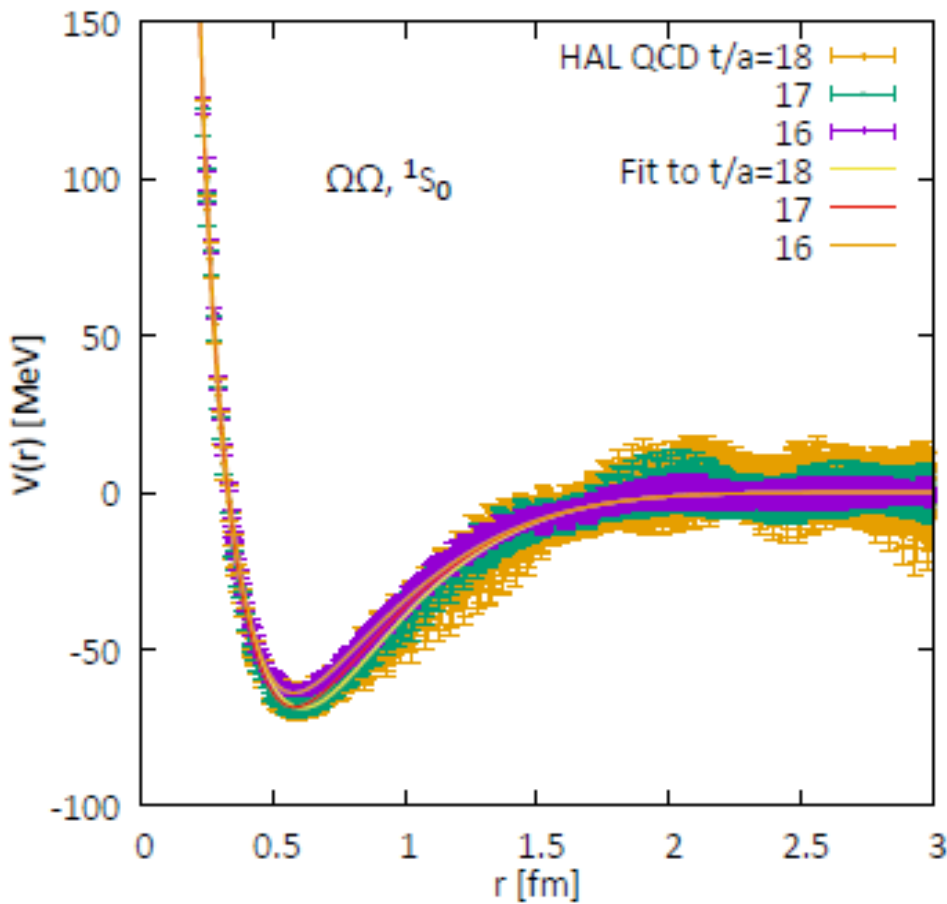
**Peripheral collisions**  
**( $R \sim (2-3)$  fm)**  
**→ Strong enhancement**  
**at small  $q$ ,**  
**+ Suppressed CF**  
**at finite  $q$**

Centrality	$\tau_0$ [fm/c]	$R_T^\Omega$ [fm]	$R_T^p$	$\alpha^\Omega$	$\beta^\Omega$	$\alpha^p$	$\beta^p$
0 – 10%	10.0	8.0	6.8	0.584	0.628	0.759	0.421
10 – 20%	9.085	6.75	6.23	0.618	0.579	0.750	0.425
20 – 40%	7.5	5.88	5.2	0.546	0.692	0.707	0.466
40 – 60%	5.5	4.38	3.92	0.444	0.858	0.604	0.6
60 – 80%	3.62	2.12	2.66	0.456	0.812	0.456	0.82

*K. Morita, S. Gongyo, T. Hatsuda, T. Hyodo, T. Iritani, AO, K. Sasaki, in prep.*

# $\Omega\Omega$ correlation

$\Omega\Omega$  potential: S. Gongyo et al. (HAL QCD Collab),  
Phys. Rev. Lett. 120, 212001 (2017), 1709.00654.



K. Morita, S. Gongyo, T. Hatsuda, T. Hyodo, T. Iritani, AO, K. Sasaki, in prep.

---

# *$K^-p$ correlation*

# $K^- p$ interaction

- $\Lambda(1405)$   $\bar{K}N$  quasi-bound state

*Dalitz, Tuan ('60); Koch ('94); Kaiser, Siegel, Weise ('95); AO, Nara, Koch ('97)*

- Positive scattering length in  $K^-$  atoms

*M.Iwasaki et al. PRL78('97)3067;  
M.Bazzi et al. [SIDDHARTA Collab.],  
PLB704('11)113.*

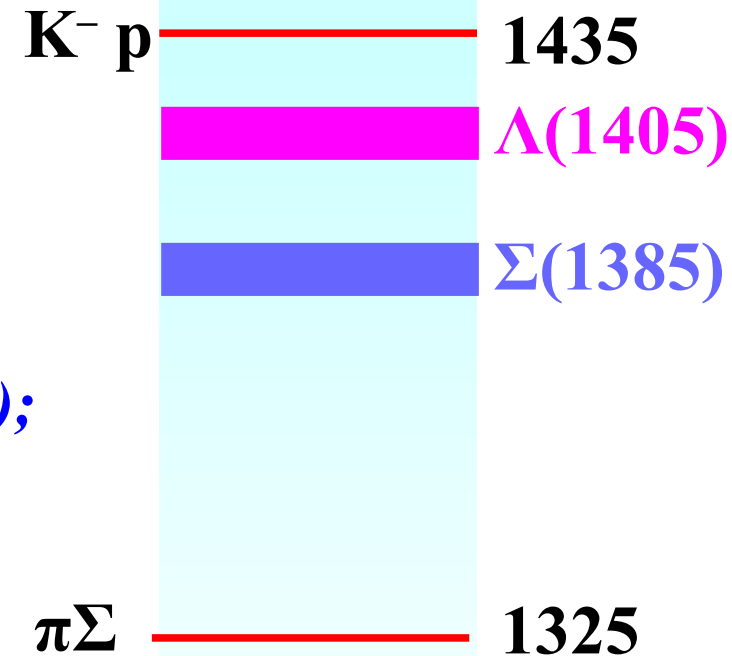
- Kaonic nuclei ?

*Nogami ('63); Akaishi, Yamazaki ('02);  
Shevchenko, Gal, Mares ('07); Ikeda, Sato ('07);  
Dote, Hyodo, Weise ('09)*

→ Needs precise info. on  $\bar{K}N$  int.

- Scattering amplitude and Potential fitting scattering and SIDDARTA data in chiral approach

*Ikeda, Hyodo, Weise ('11,'12),  
Miyahara, Hyodo ('16)*



*How about  $K^- p$  correlation ?*



# $K^- p$ scattering and $K^- p$ correlation

- $K^- p$  scattering: Plane wave + Outgoing spherical wave

$$\Psi_{K^- p}^{(+)}(\mathbf{r}) \rightarrow \exp(i\mathbf{q} \cdot \mathbf{r})\chi(K^- p) + \frac{e^{iqr}}{r} [f_{K^- p \rightarrow K^- p}\chi(K^- p) + f_{K^- p \rightarrow \bar{K}^0 n}\chi(\bar{K}^0 n)]$$

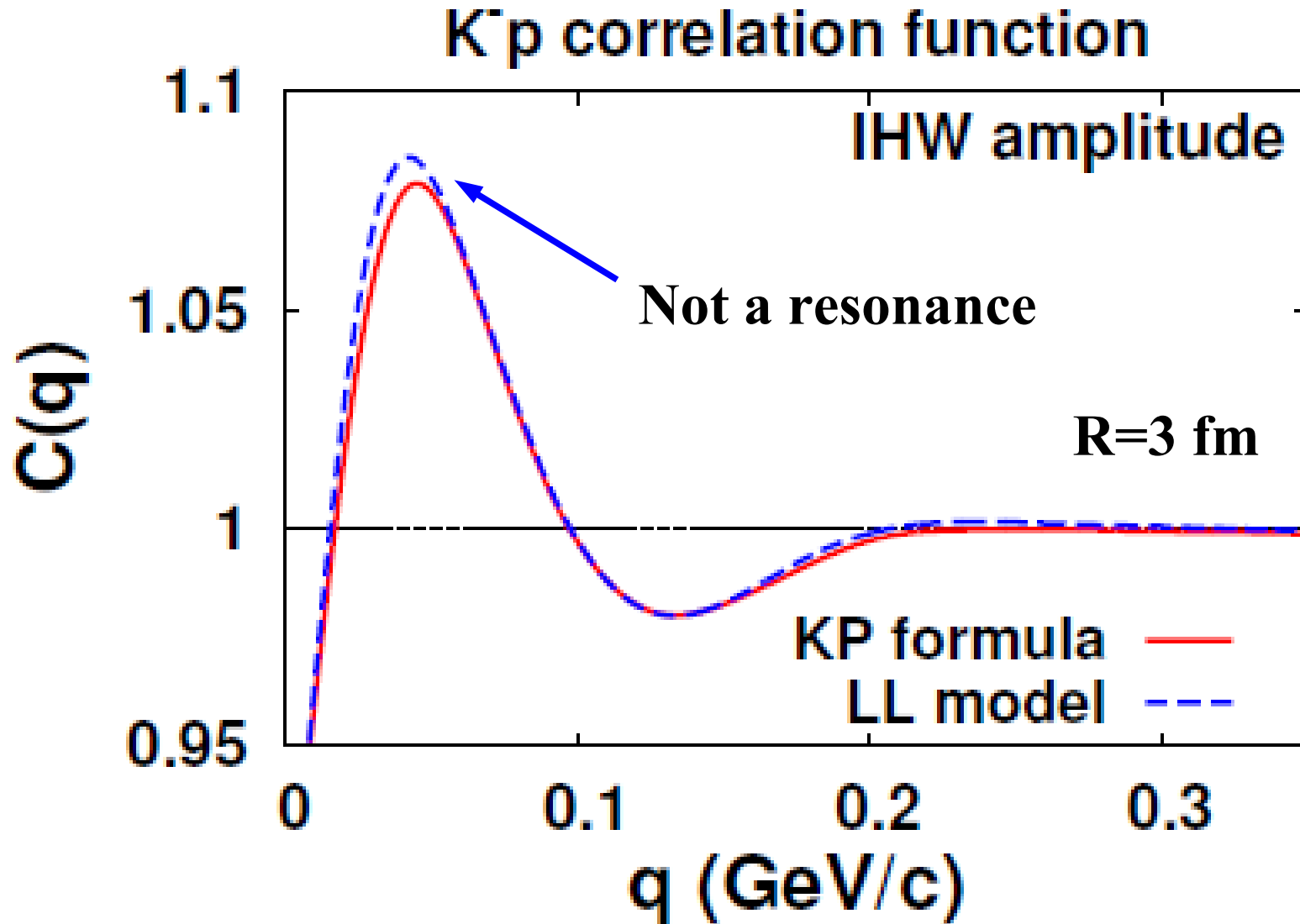
- $K^- p$  correlation: **Plane wave + Incoming spherical wave**

$$\Psi_{K^- p}^{(-)}(\mathbf{r}) \rightarrow \exp(i\mathbf{q} \cdot \mathbf{r})\chi(K^- p) + \frac{e^{-iqr}}{r} [\tilde{f}_{K^- p \rightarrow K^- p}\chi(K^- p) + \tilde{f}_{K^- p \rightarrow \bar{K}^0 n}\chi(\bar{K}^0 n)]$$

$$\tilde{f}_{K^- p \rightarrow K^- p} = \frac{\tilde{\mathcal{S}} - 1}{2iq} \neq f_{K^- p \rightarrow K^- p}, \quad \tilde{\mathcal{S}} = \left[ \frac{\mathcal{S}_{I=0}^{-1} + \mathcal{S}_{I=1}^{-1}}{2} \right]^{-1}$$

$\Psi^{(-)} \neq (\Psi^{(+)})^* \rightarrow K^- p$  correlation probes  
different combination of  $I=0, 1$  from  $K^- p$  scattering

# $K^-p$ correlation



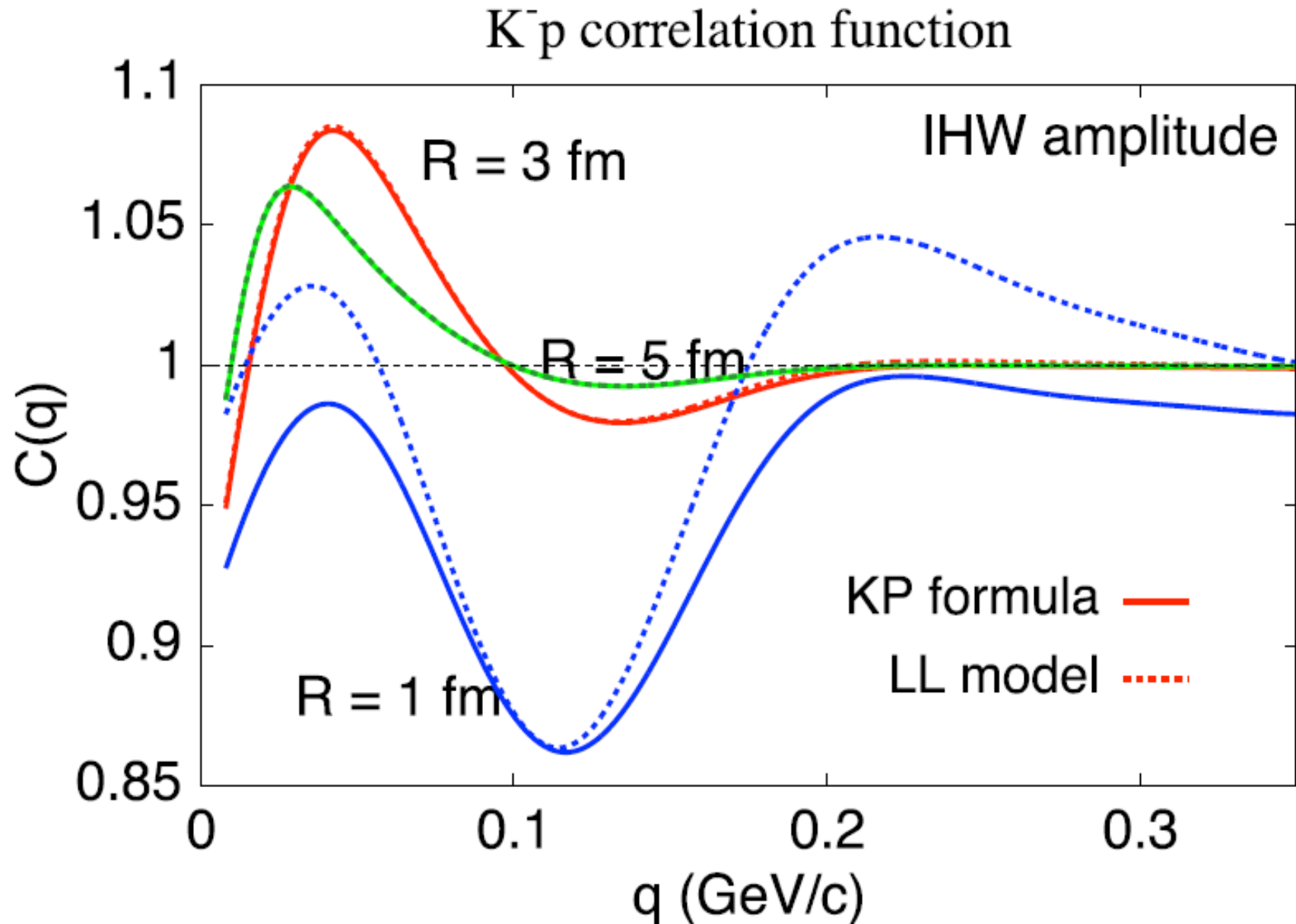
*Calc: AO, K. Morita, K. Miyahara, T. Hyodo, NPA954 ('16)294 ( $\Lambda\Lambda$ ,  $K^-p$ ).*

*Potential: K. Miyahara, T. Hyodo, PRC93 ('16) 015201.*

*Amplitude: Y. Ikeda, T. Hyodo, W. Weise, PLB 706 ('11) 63; NPA 881 ('12) 98.*

*Fitting SIDDHARTA data: M. Bazzi et al. [SIDDHARTA Collab.], PLB 704 ('11) 113.*

# $K^-p$ correlation



$\psi^{(-)}$ : Another source of fake peak ?

K. Miyahara

# New Data from LHC-ALICE

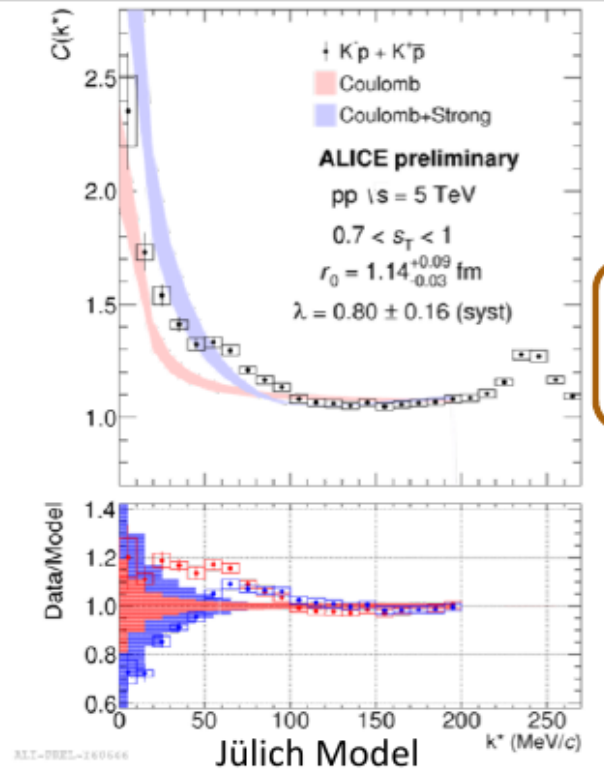
V. Mantovani-Sarti (ALICE Collab.), MESONS 2018



## K<sup>-</sup> - p Correlation - pp 5 TeV, 7 TeV, 13 TeV

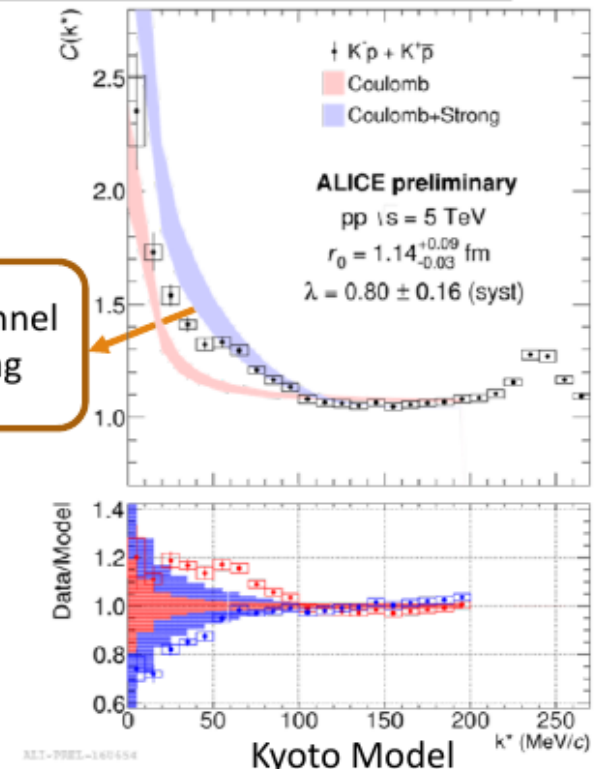


- Analysis on datasets:
  - pp 5 TeV, 7 TeV, 13 TeV
- Short range  $\bar{K}N$  interaction:
  - $\Lambda(1405)$ , kaonic atoms and kaonic clusters
- Kaonic atoms and scattering data



(Haidenbauer et al., Phys.Rev. C66 (2002) 055214)

$\bar{K}^0n$  channel opening

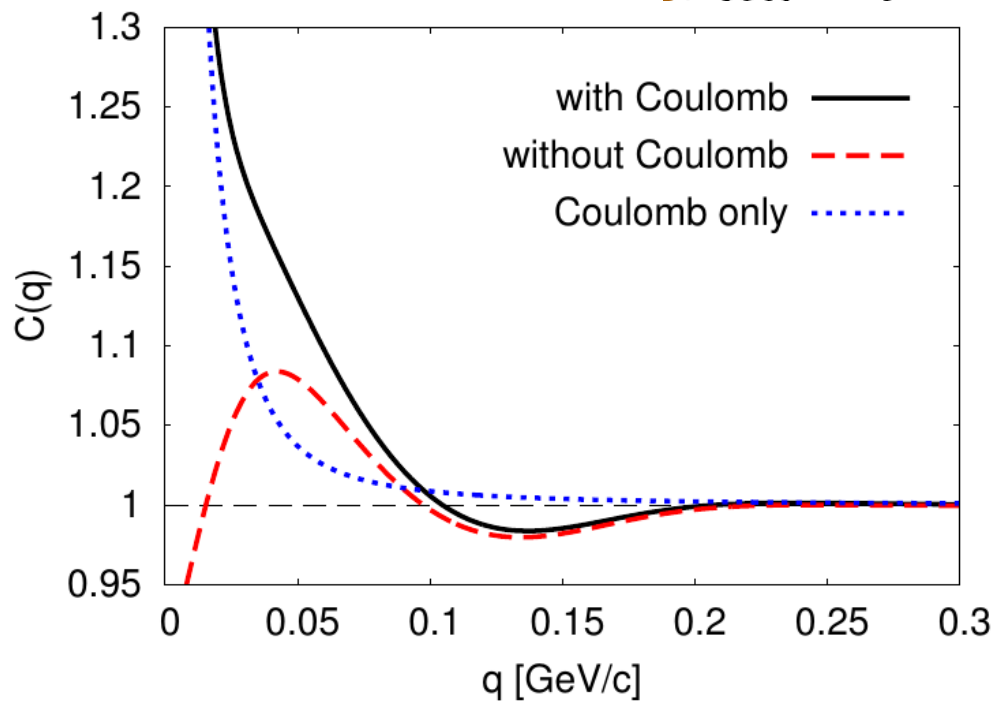


(Hyodo et al., Phys.Rev. C95 (2017) no.6,065202)

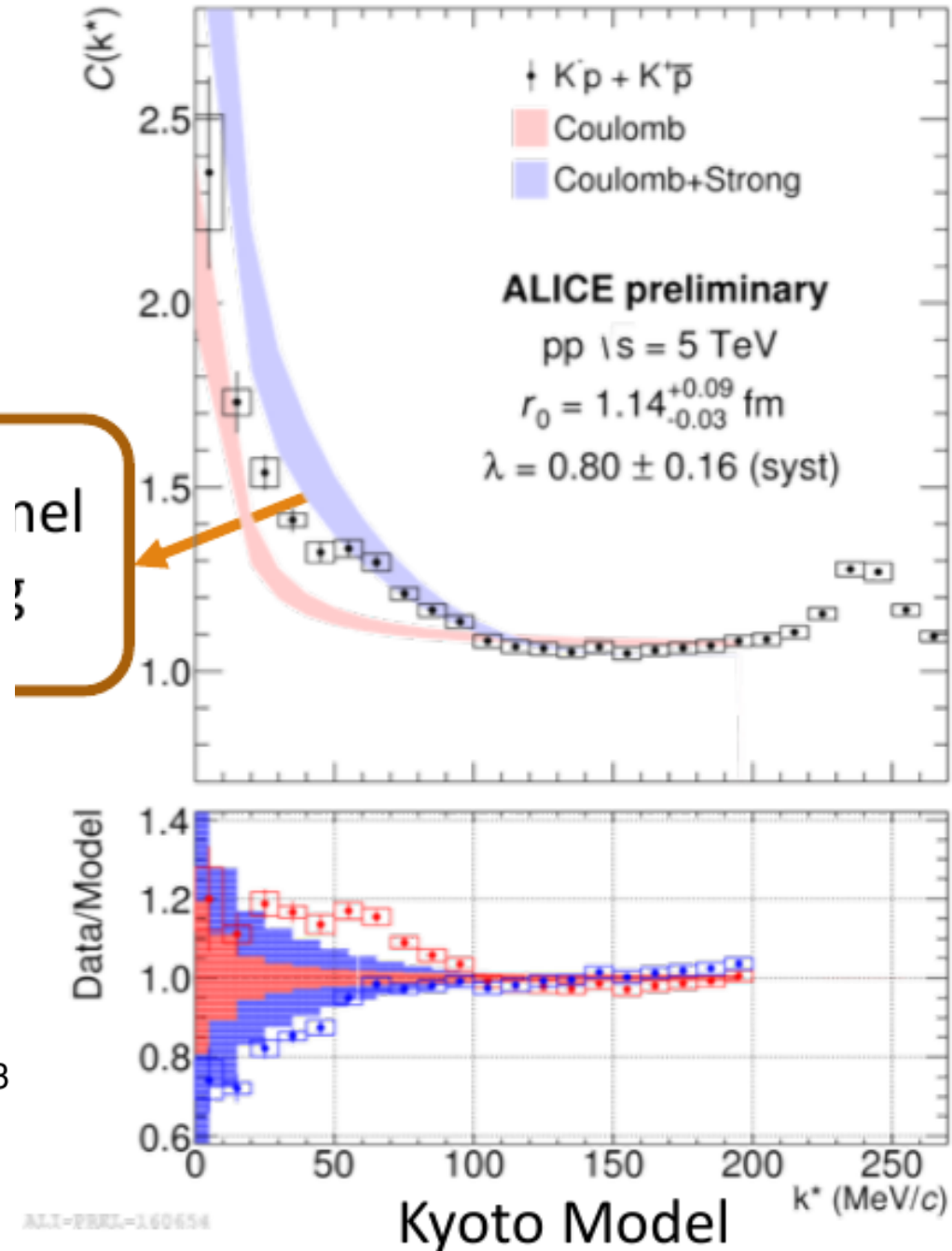
Analysis performed by R.Lea (INFN-TS)

# New Data from LHC-ALICE

- Data shows shoulder at  $q \sim 50$  MeV
  - Threshold effect ?
  - Interference of  $I=0$  and  $I=1$  amplitudes ? (Cho et al. ('17))



Cho et al. (ExHIC Collab.) ('17)



# Coupled Channels Effects Revisited

## ■ J. Haidenbauer (arXiv:1808.05049)

- Coupled channels effects strongly modifies the correlation function of  $K^- p$  and  $\Xi^- p$ .
- Threshold difference needs to be taken care of.
- Source of other channels need to be added.
- No Coulomb, No Flow, Gaussian source with a fixed size, ...

## ■ Two channel closure approximation is used in our previous works, and source function of other channels were missed (forgotten to add).

*Cho et al. ('17), AO et al. ('16)*

$$\tilde{S} = (S_{2 \times 2})^{-1} \neq (S^\dagger)_{2 \times 2}$$

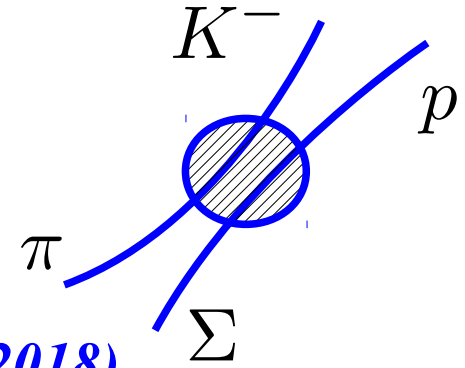
→ We need to revisit  $K^- p$  correlation ...

# Correlation Function with Coupled Channels

- Contribution of the source in other channels

$$C_\alpha(\mathbf{q}) = \sum_\beta \int d\mathbf{r} S_\beta(\mathbf{r}) \left| \psi_{\beta\alpha}^{(-)}(\mathbf{r}, \mathbf{q}) \right|^2$$

Source fn.



- Asymptotic wave function

*K. Miyahara, T. Hyodo, W. Weise, Phys. Rev. C 98, 025201 (2018), arXiv:1804.08269.*

$$|\Psi_\alpha^{(\pm)}\rangle = \sum_\beta \psi_{\beta\alpha}^{(\pm)} |\beta\rangle$$

No outgoing w.f. for  $\beta \neq \alpha$

$$\psi_{\beta\alpha}^{(+)}(r) \rightarrow \frac{-1}{2ik_\alpha} \left[ \delta_{\beta\alpha} \frac{e^{-ik_\beta r}}{r} - \sqrt{\frac{v_\alpha}{v_\beta}} S_{\beta\alpha} \frac{e^{ik_\beta r}}{r} \right] \quad (v_\alpha = k_\alpha/\mu_\alpha)$$

- Asymptotically outgoing wave function

$$\psi_{\beta\alpha}^{(-)}(r) = \frac{1}{k_\alpha} \sum_\gamma \psi_{\beta\gamma}^{(+)}(r) S_{\gamma\alpha}^\dagger k_\gamma \sqrt{\frac{v_\alpha}{v_\gamma}} \rightarrow \frac{1}{2ik_\alpha} \left[ \delta_{\beta\alpha} \frac{e^{ik_\beta r}}{r} - \sqrt{\frac{v_\alpha}{v_\beta}} S_{\beta\alpha}^\dagger \frac{e^{-ik_\beta r}}{r} \right]$$

- Difference of  $\tilde{S}$  and  $S^\dagger$  may lead to different results....

---

# $E^-p$ correlation



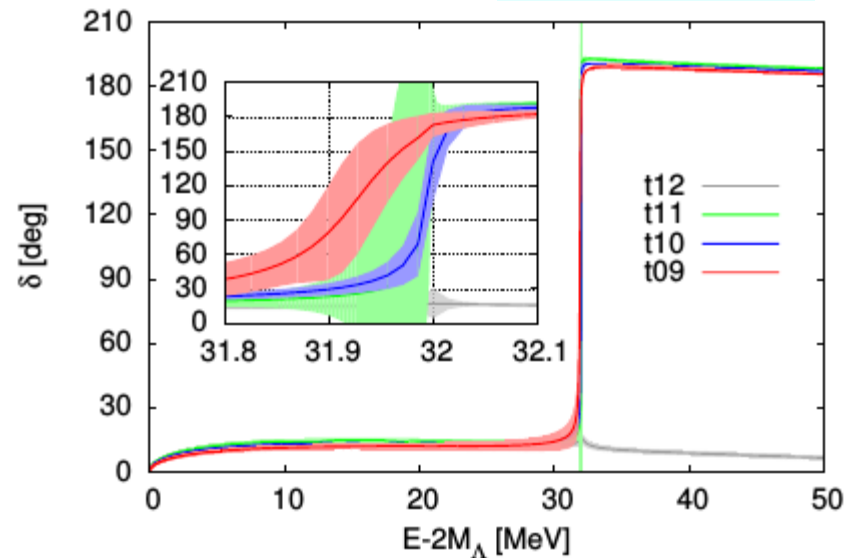
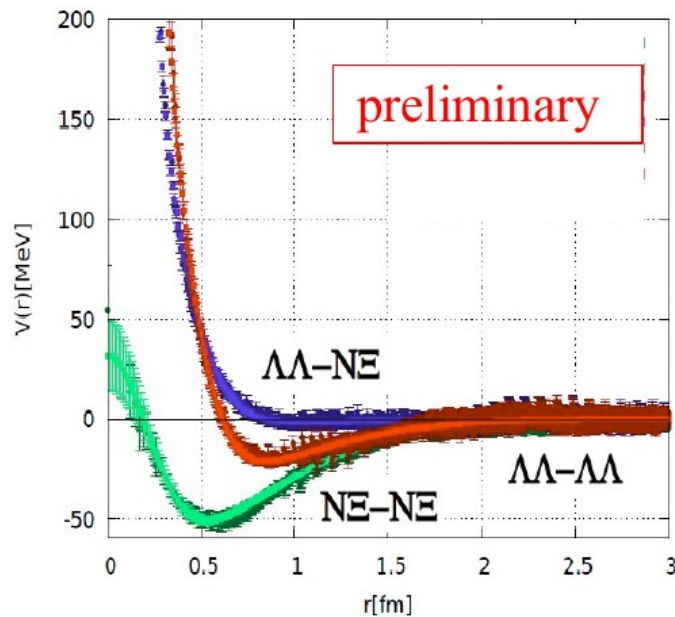
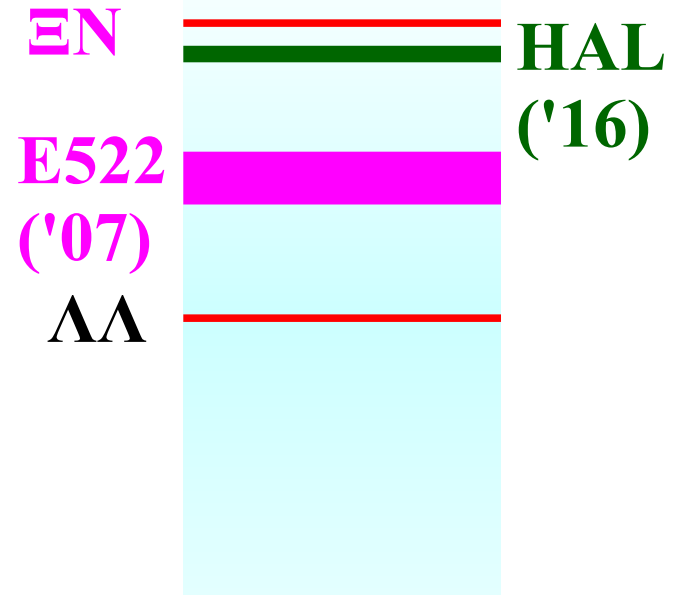
# Relevance of $\Xi N$ interaction to physics

- H-particle: 6-quark state (uuddss) may be realized as a loosely bound state of  $\Xi N$  ( $I=0$ )

*K. Sasaki et al. (HAL QCD, '16, '17)*

- Repulsive  $\Xi N$  interaction ( $I=1$ ) may help to support  $2 M_{\odot}$  Neutron Star

*Weissborn et al., NPA881 ('12) 62.*



*K. Sasaki et al. (HAL QCD Collab.), EPJ Web Conf. 175 ('18) 05010.*

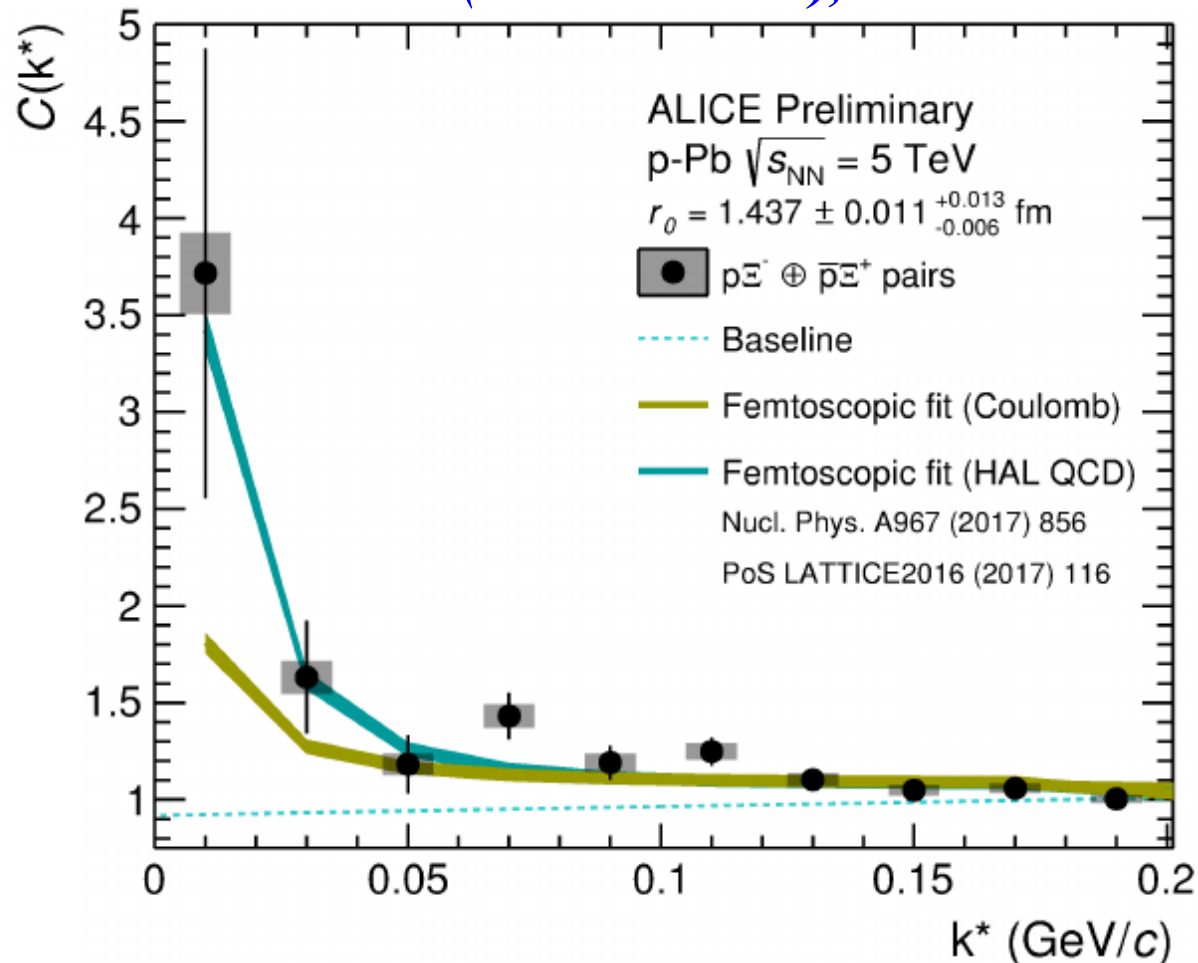
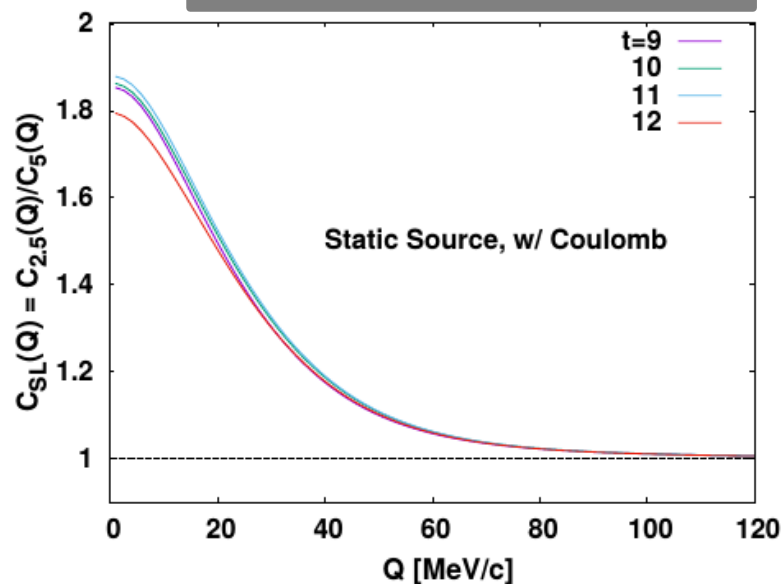
# $E^- p$ correlation

- Prediction of the correlation function by using  $\Xi N$  potential (HAL QCD Collab.) + Coulomb potential

$$|\psi|_{\text{spin av.}}^2 = \frac{1}{2} \sum_{I=0,1} \left[ \frac{1}{4} |\psi_I^{J=0}|^2 + \frac{3}{4} |\psi_I^{J=1}|^2 \right]$$

*HAL prediction is examined!*

*V. Mantovani-Sarti (ALICE Collab.), MESONS 2018*



*T. Hatsuda, K. Morita, AO,  
K. Sasaki, NPA967('17), 856.*

# Summary

- High energy collisions (incl. heavy-ion collisions) are hadron factories, and can be utilized to perform hadron physics as well.
- Hadron-Hadron correlation contains information on interactions.
  - Correlations in various pairs have been measured:  $\pi\pi$ ,  $KK$ ,  $pp$ ,  $nn$ ,  $\bar{p} \bar{p}$ ,  $\Lambda\Lambda$ ,  $\Lambda p$ ,  $K^- p$ ,  $\Omega^- p$ ,  $\Xi^- p$ , ...
  - When the pair purity is large enough, corr. fn. has sensitivity to hh interaction.
- Some of hh correlations have been discussed.
  - $\Lambda\Lambda$  correlation data constrain  $(a_0, r_{\text{eff}})$  region of  $\Lambda\Lambda$  interaction.
  - $\Omega^- p$  correlation suggests the existence of a  $S=-3$  dibaryon.
  - $K^- p$  correlation shows “shoulder” structure.
  - $\Xi^- p$  correlation has examined the prediction by HAL QCD Collab.
- Many other type of pairs are waiting for us.

# Challenge, Lessons, and Reflections

---

## ■ Challenge

- “Static Gaussian source + single channel problem” can be now done by experimentalists.  
*D. L. Mihaylov et al. EPJC78('18), 394.*
- Coupled channels problem needs to be investigated further, including the effects of threshold difference.
- Source fn. needs to be studied more.  
Combination with transport models  
Gaussian source for pp and  $e^+e^-$  collisions ?

## ■ Lessons

- Let's draw some graph on observables which Experimentalists have not shown ! It can be a prediction !
- It can take 20 years before your prediction is examined.

- Reflections: We need to write “original” papers rather than proceedings.

---

*Thank you  
for participating HIPLQH !*

# How can we measure the radius of a star ?

## Two photon intensity correlation

*Hanbury Brown & Twiss, Nature 10 (1956), 1047.*

- Simultaneous two photon observation probability is enhanced from independent emission cases  
 → angular diameter of Sirius=0.0063”

### A TEST OF A NEW TYPE OF STELLAR INTERFEROMETER ON SIRIUS

By R. HANBURY BROWN

Jodrell Bank Experimental Station, University of Manchester

AND

DR. R. Q. TWISS

Services Electronics Research Laboratory, Baldock

NATURE November 10, 1956 Vol. 178

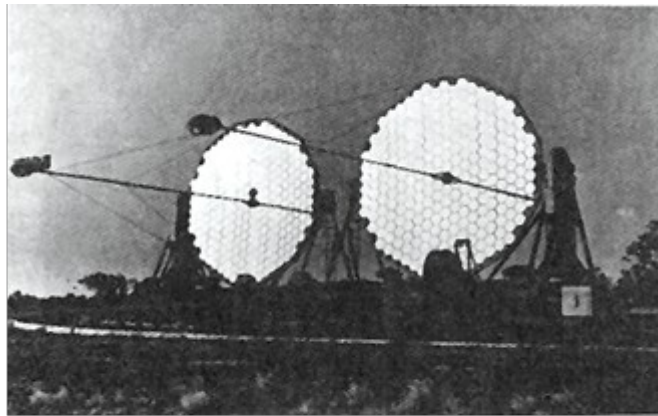


Figure 2. Picture of the two telescopes used in the HBT experiments. The figure was extracted from Ref.[1].

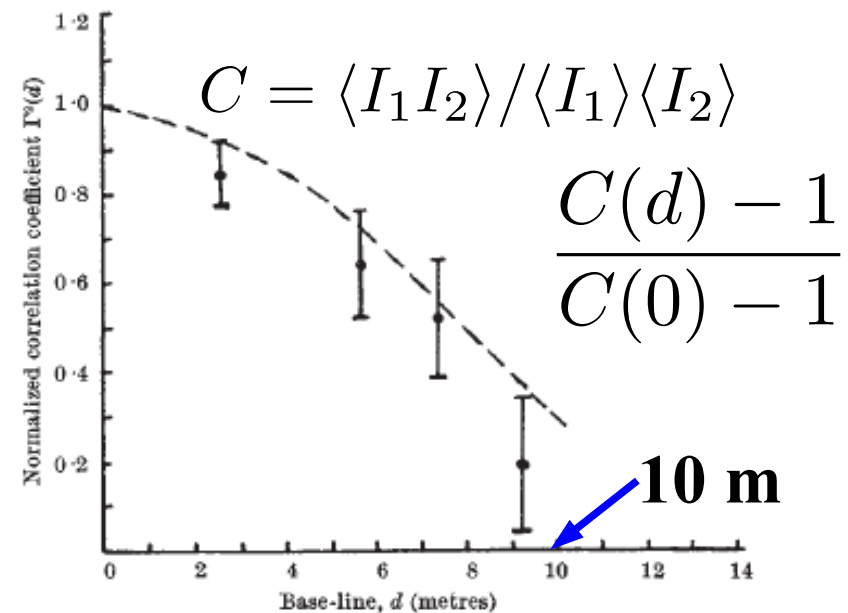


Fig. 2. Comparison between the values of the normalized correlation coefficient  $\Gamma^2(d)$  observed from Sirius and the theoretical values for a star of angular diameter 0.0063”. The errors shown are the probable errors of the observations

*HBP telescope (from Goldhaber, ('91))*

*HBT ('56)*

# Two particle intensity correlation

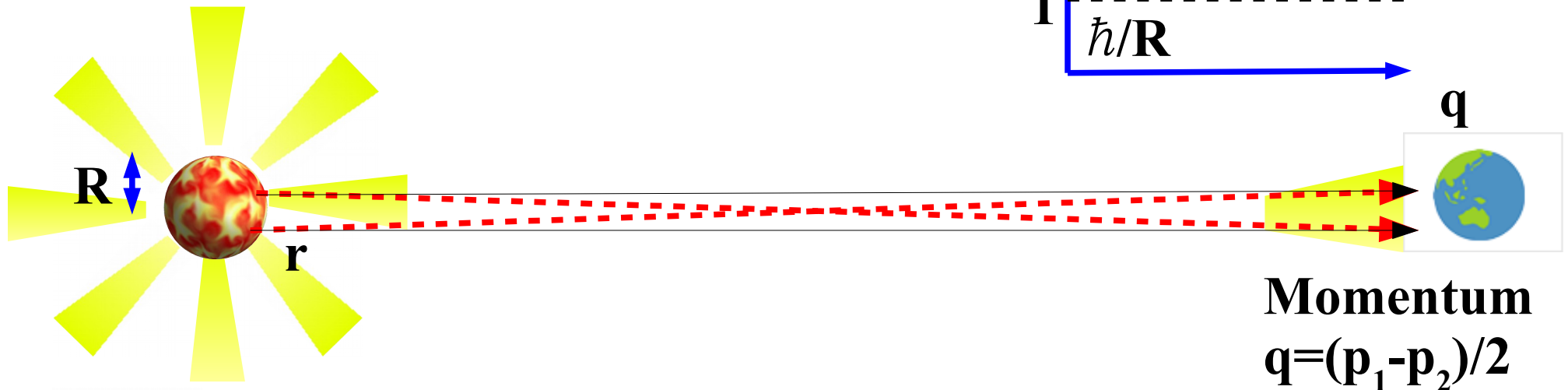
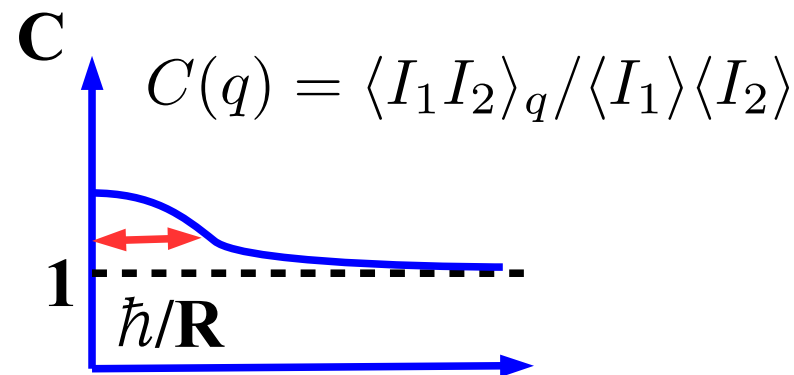
## Wave function symmetrization from quantum statistics

$$C(\mathbf{q}) = \int d^3r S(\mathbf{q}, \mathbf{r}) \left| \frac{1}{\sqrt{2}} (e^{i\mathbf{q}\cdot\mathbf{r}} + e^{-i\mathbf{q}\cdot\mathbf{r}}) \right|^2 \simeq \underline{1 + \exp(-4q^2 R^2)}$$

Source fn.  
( $\mathbf{r}$ =relative coordinate)  
(symmetrized w.f.)<sup>2</sup>

Static spherical source case

→ Small relative momenta are favored due to symmetrization of the relative wave function.



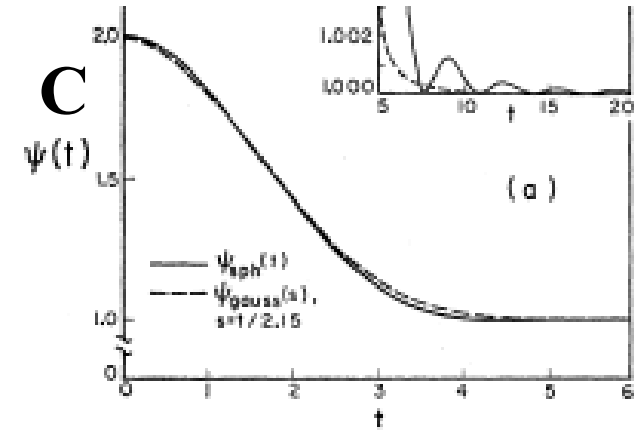
# How can we measure source size in nuclear reactions ?

## ■ Two pion interferometry

*G. Goldhaber, S. Goldhaber, W. Lee, A. Pais, Phys. Rev. 120 (1960), 300*

- Two pion emission probability is enhanced at small relative momenta

→ Pion source size  $\sim 0.75 \hbar / \mu c$



$q$  (relative momentum)

PHYSICAL REVIEW

VOLUME 120, NUMBER 1

OCTOBER 1, 1960

## Influence of Bose-Einstein Statistics on the Antiproton-Proton Annihilation Process\*

GERSON GOLDHABER, SULAMITH GOLDHABER, WONYONG LEE, AND ABRAHAM PAIS†

*Lawrence Radiation Laboratory and Department of Physics, University of California, Berkeley, California*

(Received May 16, 1960)

