Bound states in femtoscopy – To be (bound) or not to be (*), that is the question – (Resonance, virtual pole, ..)*

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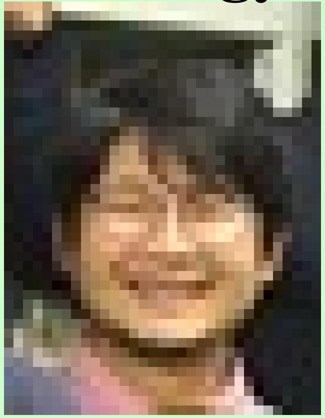
TUM, Sep. 18 (Wed), 2019, Munich, Germany

K. Morita, T. Furumoto, AO, PRC91('15)024916 (ΛΛ)
AO, K. Morita, K. Miyahara, T. Hyodo, NPA954 (‘16)294 (ΛΛ, K–p)
K. Morita, AO, F. Etminan, T. Hatsuda, PRC94(‘16), 031901(R) (ΩN)
S. Cho et al.(ExHIC Collab.), Prog.Part.Nucl.Phys.95(‘17)279 (ΛΛ, K–p)
T. Hatsuda, K. Morita, AO, K. Sasaki, NPA967(‘17), 856 (Ξ–p)
K. Morita, S. Gongyo, T. Hatsuda, T. Hyodo, Y. Kamiya, AO, arXiv:1908.05414 (NΩ, ΩΩ)

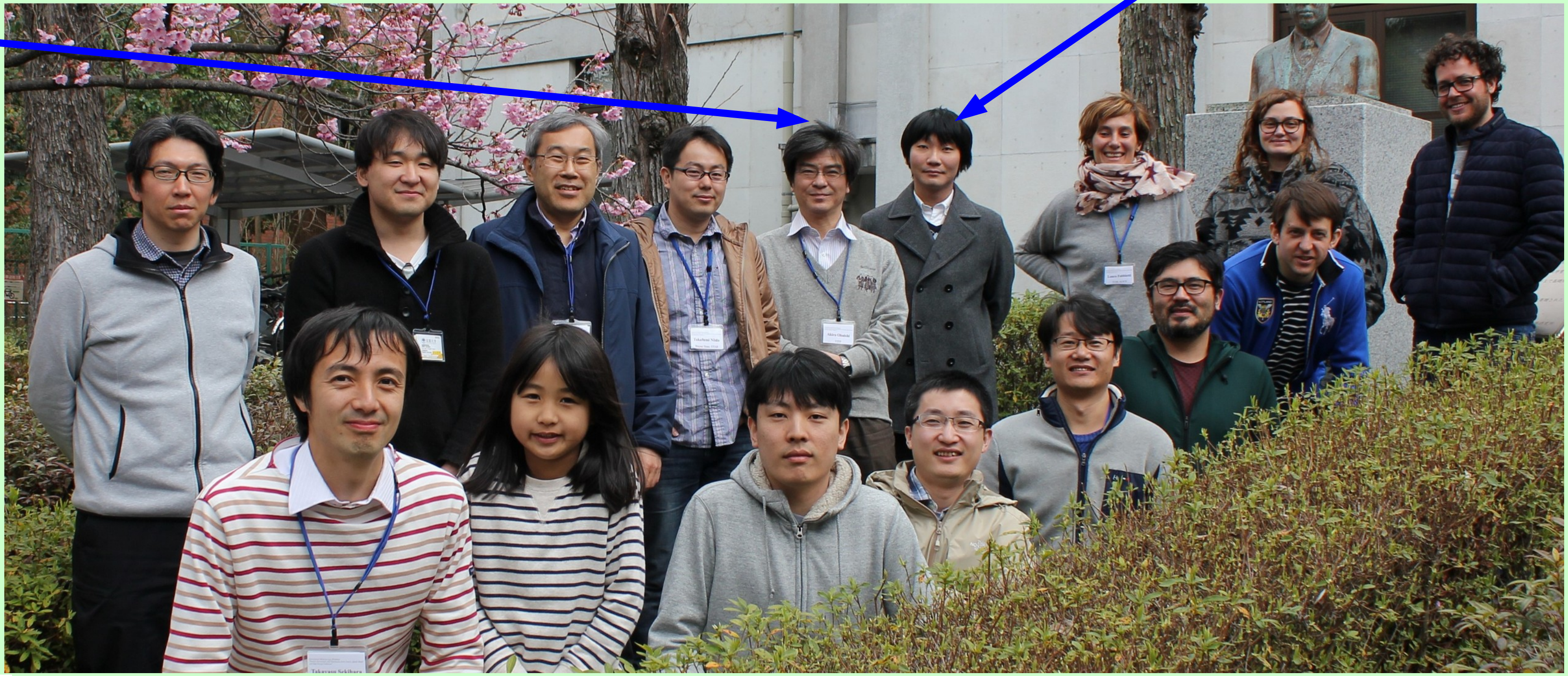
Coauthors of Morita+('19) [arXiv:1908.05414]


K. Morita


S. Gongyo


T. Hatsuda


T. Hyodo

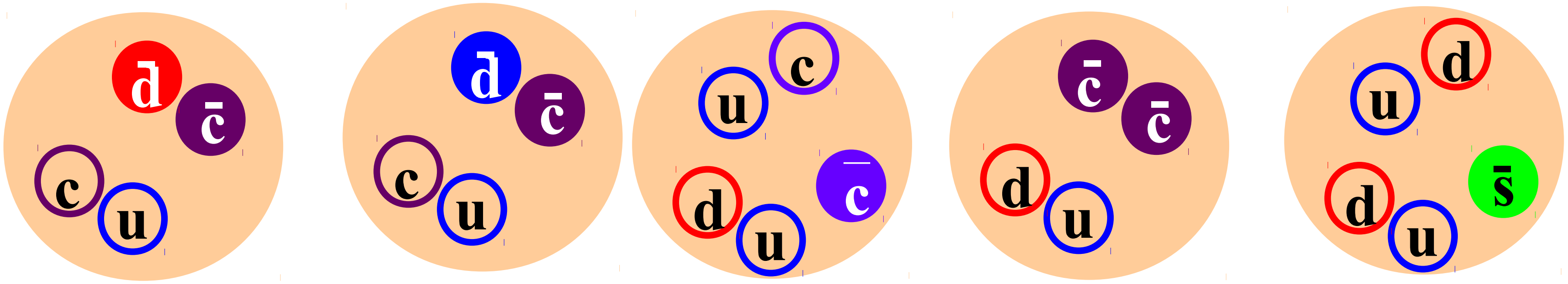


Outline

- Introduction
  - Candidates of hadronic bound state
- Correlation function and interaction
  - Scattering length and size dependence of the correlation function
- Bound States in Femotoscopy
  - Λ(1405) (→ Kamiya)
  - ΛΛ & ΞN
  - (NΩ ←- FemTUM talk)
  - ΩΩ
- Summary

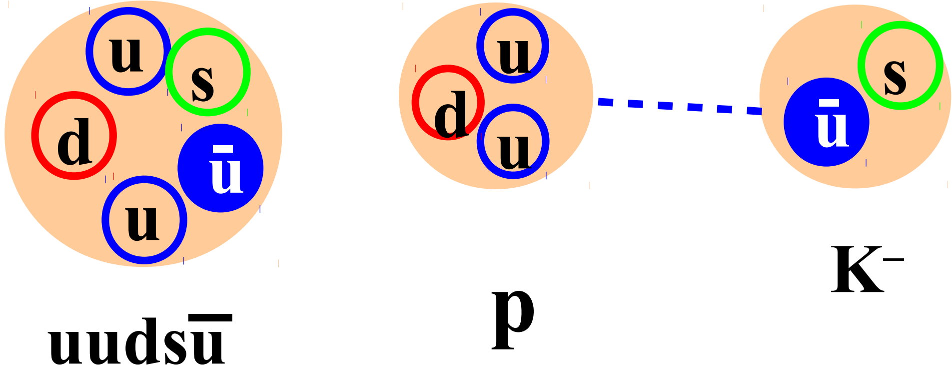
Exotic Hadrons

- Exotic hadrons: Z, X, Y, Pc, (Θ^+), …
  at LEPS, Belle, BaBar, LHCb, BES, ...



Z(4430) X(3872) Pc T_{cc} Θ^+

- Various pictures
  - Di-quark component
  - Hadronic molecule
  - $Q\overline{Q}$ couples with $Q\overline{Q}\,q\overline{q}$



uudsū p K^-

- Hadronic bound state candidates
  - Λ(1405), H, NΩ, ΩΩ,

Λ (1405)

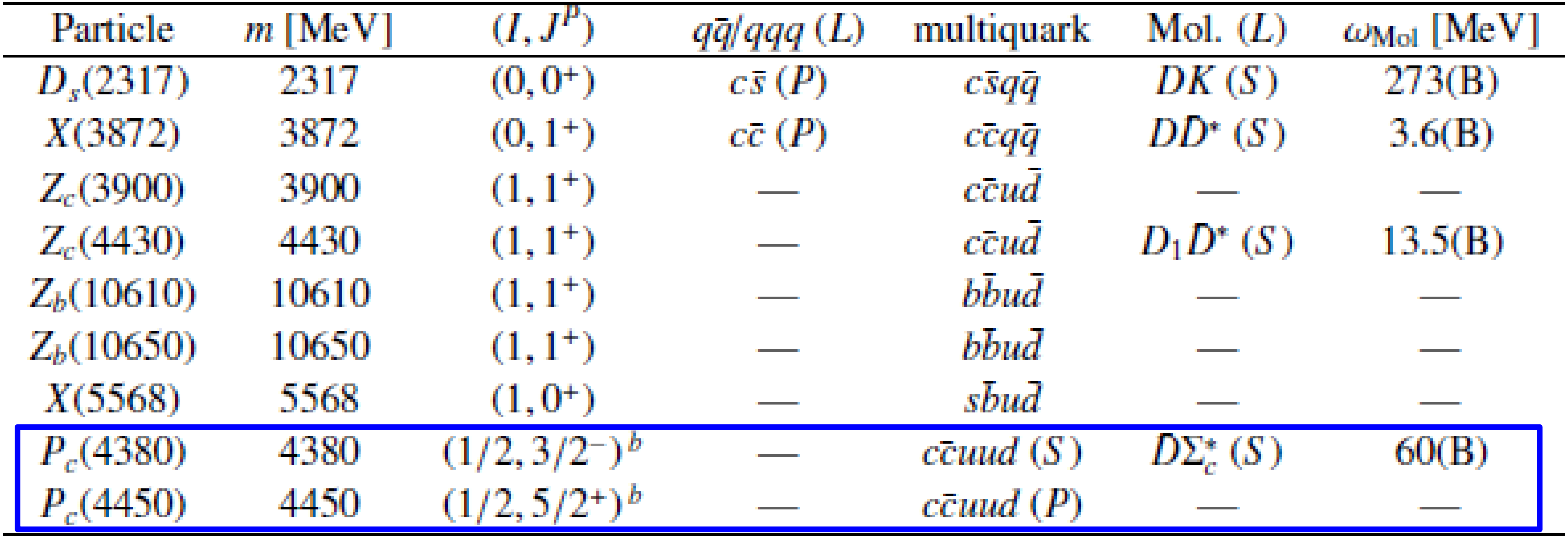
Hadronic Molecule Candidates: Light flavors (u,d,s)

Particle	m [MeV]	(I, J^P)	$q\bar{q}/qqq$ (L)	multiquark	Mol. (L)	$\omega_{\rm Mol}$ [MeV]
$f_0(980)$	980	$(0, 0^+)$	$q\bar{q}$ (P) $(s\bar{s}$ $(P))$	$qs\bar{q}\bar{s}$	$\bar{K}K$ (S)	67.8(B)
$a_0(980)$	980	$(1, 0^+)$	$q\bar{q}$ (P)	$qs\bar{q}\bar{s}$	$\bar{K}K$ (S)	67.8(B)
$K(1460)$	1460	$(1/2, 0^-)$	—	$qq\bar{q}\bar{s}$ (P)	$\bar{K}KK$ (P)	69.0(R)
$\Lambda(1405)$	1405	$(0, 1/2^-)$	uds (P)	$udsq\bar{q}$	$\bar{K}N$ (S)	20.5(R)
$\Delta\Delta$	2380	$(0, 3^+)$	—	q^6	—	—
$\Lambda\Lambda$-$N\Xi$ (H)	2245	$(0, 0^+)$	—	$uuddss$	$N\Xi$ (S)	73.2(B)
$N\Omega$	2592	$(1/2, 2^+)$	—	$uudsss$	—	—

$$\omega_{\rm mol} = 6\ \text{B.E. or } 3/2\mu\langle r^2\rangle$$

S. Cho, T. Hyodo, D. Jido, C. M. Ko, S. H. Lee, S. Maeda, K. Miyahara, K. Morita, M. Nielsen, A. Ohnishi, T. Sekihara, T. Song, S. Yasui, K. Yazaki (ExHIC Collaboration), Prog. Part. Nucl. Phys. 95 (2017), 279-322

Hadronic Molecule Candidates: Heavy flavors (c,b)



Particle	m [MeV]	(I, J^P)	$q\bar{q}/qqq\ (L)$	multiquark	Mol. (L)	ω_{Mol} [MeV]
$D_s(2317)$	2317	$(0, 0^+)$	$c\bar{s}\ (P)$	$c\bar{s}q\bar{q}$	$DK\ (S)$	273(B)
$X(3872)$	3872	$(0, 1^+)$	$c\bar{c}\ (P)$	$c\bar{c}q\bar{q}$	$D\bar{D}^*\ (S)$	3.6(B)
$Z_c(3900)$	3900	$(1, 1^+)$	—	$c\bar{c}u\bar{d}$	—	—
$Z_c(4430)$	4430	$(1, 1^+)$	—	$c\bar{c}u\bar{d}$	$D_1\bar{D}^*\ (S)$	13.5(B)
$Z_b(10610)$	10610	$(1, 1^+)$	—	$b\bar{b}u\bar{d}$	—	—
$Z_b(10650)$	10650	$(1, 1^+)$	—	$b\bar{b}u\bar{d}$	—	—
$X(5568)$	5568	$(1, 0^+)$	—	$s\bar{b}u\bar{d}$	—	—
$P_c(4380)$	4380	$(1/2, 3/2^-)^b$	—	$c\bar{c}uud\ (S)$	$\bar{D}\Sigma_c^*\ (S)$	60(B)
$P_c(4450)$	4450	$(1/2, 5/2^+)^b$	—	$c\bar{c}uud\ (P)$	—	—

ExHIC('17)

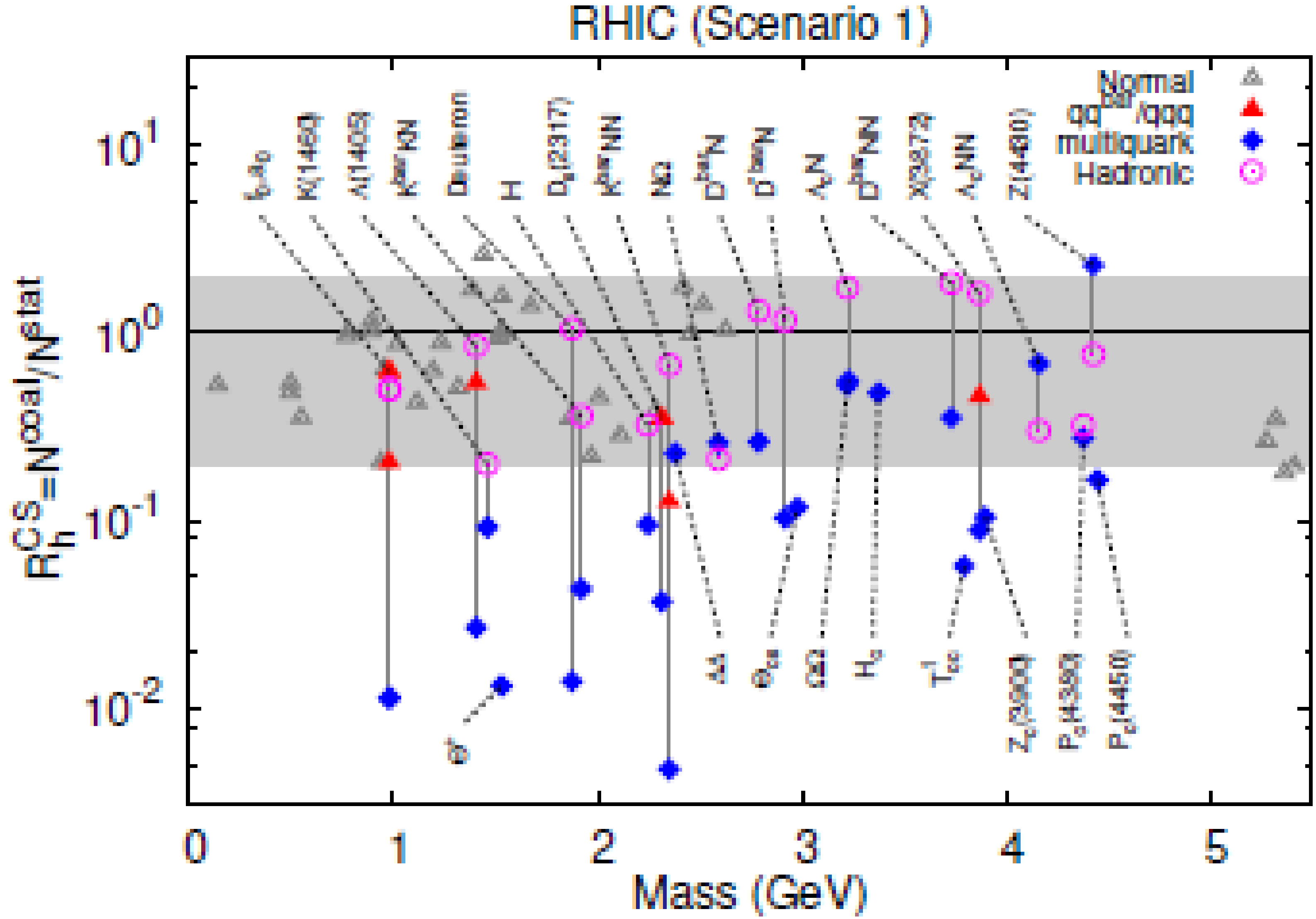
Hadronic Molecule Candidates: Others

Particle	m [MeV]	(I, J^P)	$q\bar{q}/qqq$	multiquark (L)	Mol. (L)	$\omega_{\rm Mol}$ [MeV]
$\Theta(1530)$	1530	$(0, 1/2^+)$	—	$qqqq\bar{s}$ (P)	—	—
$\bar{K}KN$	1920	$(1/2, 1/2^+)$	—	$qqqs\bar{s}$ (P)	$\bar{K}KN$	42(R)
$\bar{K}NN$	2352	$(1/2, 0^-)$	q^5s (P)	$q^6s\bar{q}$ (S)	$\bar{K}NN$	20.5(T)
$\Omega\Omega$	3228	$(0, 0^+)$	—	s^6	—	—
T^1_{cc}	3797	$(0, 1^+)$	—	$ud\bar{c}\bar{c}$	—	—
DN	2790	$(0, 1/2^-)$	—	$qqqq\bar{c}$	DN	6.48(R)
D^*N	2919	$(0, 3/2^-)$	—	$qqqq\bar{c}$(D)	D^*N	6.48(R)
Θ_{cs}	2980	$(1/2, 1/2^+)$	—	$qqqs\bar{c}$ (P)	—	—
H_c^{++}	3377	$(1, 0^+)$	—	$qqqqsc$	—	—
DNN	3734	$(1/2, 0^-)$	—	$q^7\bar{c}$	DNN	6.48(T)
Λ_cN	3225	$(1/2, 1^+)$	—	$cuduud$	Λ_cN	4.24(R)
Λ_cNN	4164	$(0, 3/2^+)$	—	$cuduududd$	Λ_cNN	33.16(R)
T^0_{cb}	7123	$(0, 0^+)$	—	$ud\bar{c}\bar{b}$	—	—

Many loosely bound hadronic molecules are expected with charm quarks.

ExHIC('17)

Production rate of hadronic molecules



Hadronic molecules would be produced as normal hadrons (R^{cs} ~ 1)

ExHIC('17)

Many of exotic hadrons
may have hadronic molecule structure
(hadronic bound states).

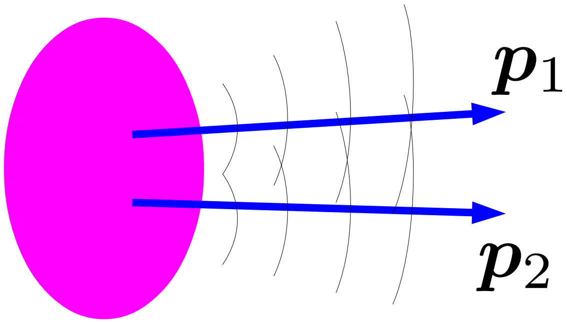
Femtoscopic correlation may reveal
their bound state nature !

Correlation Function and Interaction

Correlation Function

- **Emitting source function**

$$N_i(\boldsymbol{p}) = \int d^4x S_i(x,\boldsymbol{p})$$



- **Two-particle momentum dist.**
  - **Assumption: Two particles are produced independently, and the correlation is generated by the final state int.**
    Koonin('77), Pratt+('86), Lednicky+('82)

$$N_{12}(\boldsymbol{p}_1,\boldsymbol{p}_2) \simeq \int d^4xd^4yS_1(x,\boldsymbol{p}_1)S_2(y,\boldsymbol{p}_2)\underline{|\Psi_{\boldsymbol{p}_1,\boldsymbol{p}_2}(x,y)|}^2$$

two-body w.f.

$$\simeq \int d^4xd^4yS_1(x,\boldsymbol{p}_1)S_2(y,\boldsymbol{p}_2)\underline{|\varphi_{\boldsymbol{q}}(\boldsymbol{r})|^2}$$

relative w.f.

- **Correlation function**

$$C(\boldsymbol{p}_1,\boldsymbol{p}_2) = \frac{N_{12}(\boldsymbol{p}_1,\boldsymbol{p}_2)}{N_1(\boldsymbol{p}_1)N_2(\boldsymbol{p}_2)} \simeq \int d\boldsymbol{r}S_{12}(\boldsymbol{r})|\varphi_{\boldsymbol{q}}(\boldsymbol{r})|^2$$

Wave function around threshold (S-wave, attraction)

- **Low energy w.f. and phase shift**

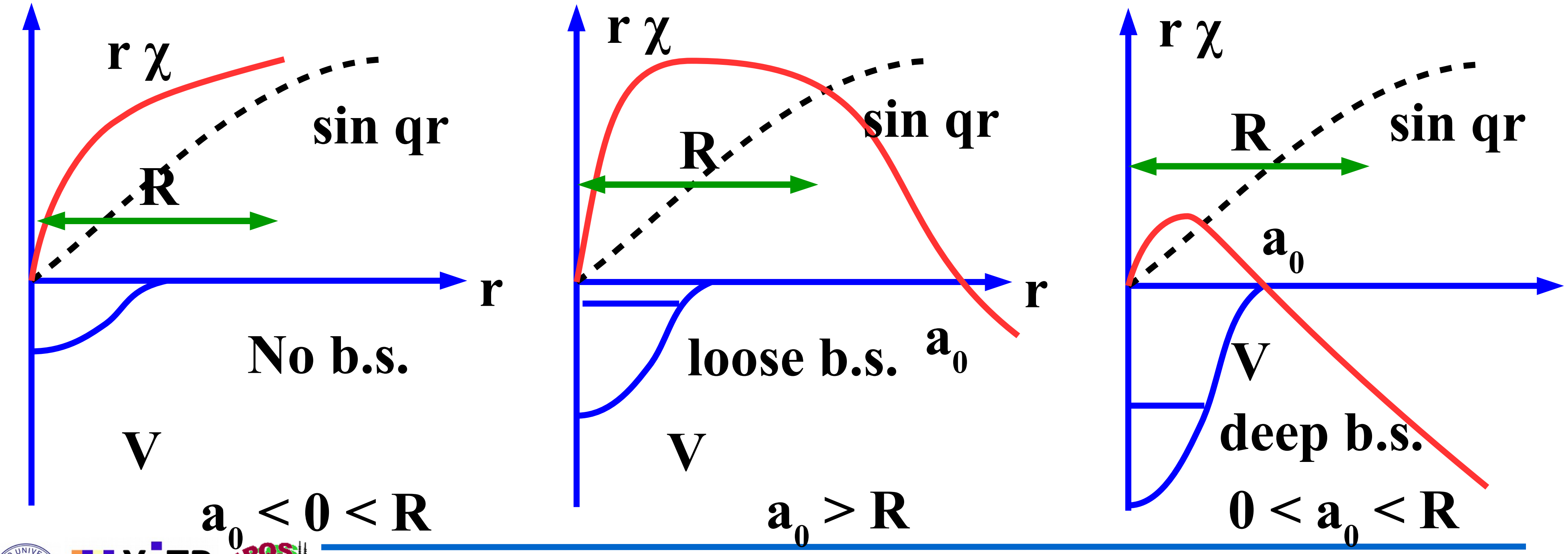
$$u(r) = qr\chi_q(r) \to \sin(qr + \delta(q)) \sim \sin(q(r - a_0))$$

$$q\cot\delta = -\frac{1}{a_0} + \frac{1}{2}r_{\rm eff}q^2 + \mathcal{O}(q^4)\ \ (\delta \sim -a_0 q)$$

a_0 =scatt. length

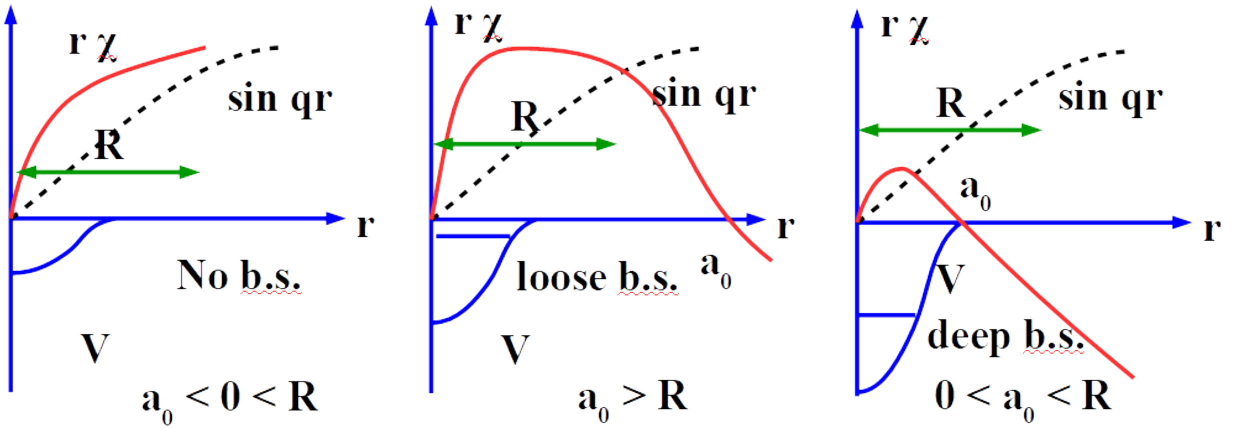
r_{eff}=eff. range

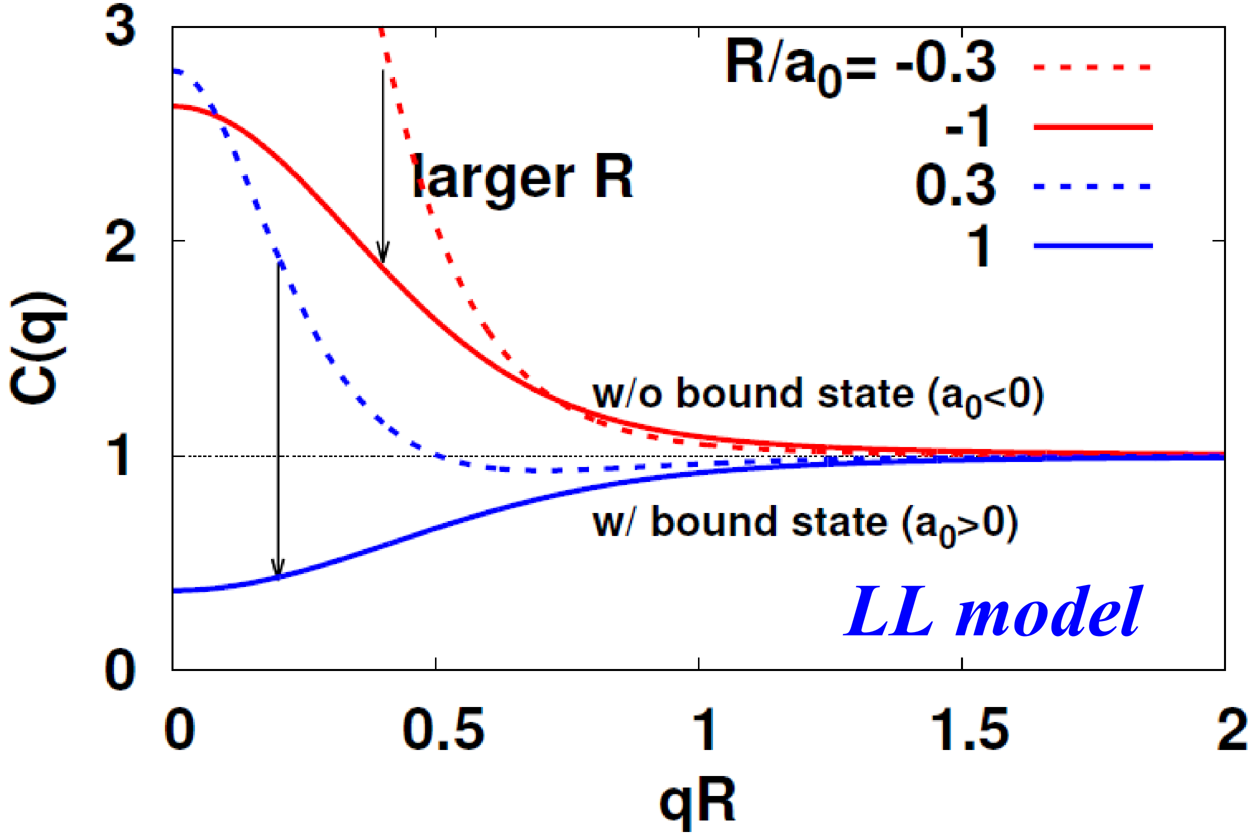
  - **Wave function grows rapidly at small r with attraction.**
  - **With a bound state (a_0>0), a node appears around r=a_0**



From correlation function to hadron-hadron interaction

- **Small q behavior**
  - **$R/a_0 < 0 \quad \rightarrow C(q) > 1$**
  - **$-1 < R/a_0 < 0.3$**
    $\rightarrow C(q) >> 1$
  - **$0.5 < R/a_0 \rightarrow C(q) < 1$**
- **Loosely bound state (large a_0)**
  $\rightarrow$ C(q)>>1 at small R
  C(q)< 1 at large R
  in the case
  w/o Coulomb,
  w/o coupled channel,
  w/o quantum stat.





Lednicky-Lyuboshits (LL) model

- **Lednicky-Lyuboshits analytic model**
  - **Asymp. w.f. + Eff. range corr. + $\psi^{(-)} = [\psi^{(+)}]^*$**

$$\psi_0(r) \to \psi_{\mathrm{asy}}(r) = \frac{e^{-i\delta}}{qr}\sin(qr+\delta) = \mathcal{S}^{-1}\left[\frac{\sin qr}{qr} + f(q)\frac{e^{iqr}}{r}\right]$$

$$\boxed{\begin{aligned}\Delta C_{\mathrm{LL}}(q) &= \int d\boldsymbol{r}\, S_{12}(r)\left(|\psi_{\mathrm{asy}}(r)|^2 - |j_0(qr)|^2\right)\\ &= \frac{|f(q)|^2}{2R^2}F_3\left(\frac{r_{\mathrm{eff}}}{R}\right) + \frac{2\mathrm{Re}f(q)}{\sqrt{\pi}R}F_1(x) - \frac{\mathrm{Im}f(q)}{R}F_2(x)\end{aligned}}$$

$$(x = 2qR, R = \text{Gaussian size, } F_1, F_2, F_3 : \text{Known functions})$$

- **Phase shifts**

$$q\cot\delta = -\frac{1}{a_0} + \frac{1}{2}r_{\mathrm{eff}}q^2 + \mathcal{O}(q^4) \to \delta \simeq -a_0 q + \mathrm{O}(q^3)$$

$$\sin(qr+\delta) \simeq \sin(q(r-a_0)+\cdots)$$

Node at r ~ a_0
for small q

C(q) in the low momentum limit

- **Correlation function at small q (and $r_{\rm eff}$=0) → F_1=1, F_2=0, F_3=1**

$$\Delta C_{\rm LL}(q) \to \frac{|f(0)|^2}{2R^2} + \frac{2{\rm Re}f(0)}{\sqrt{\pi}R} \quad (q \to 0)$$

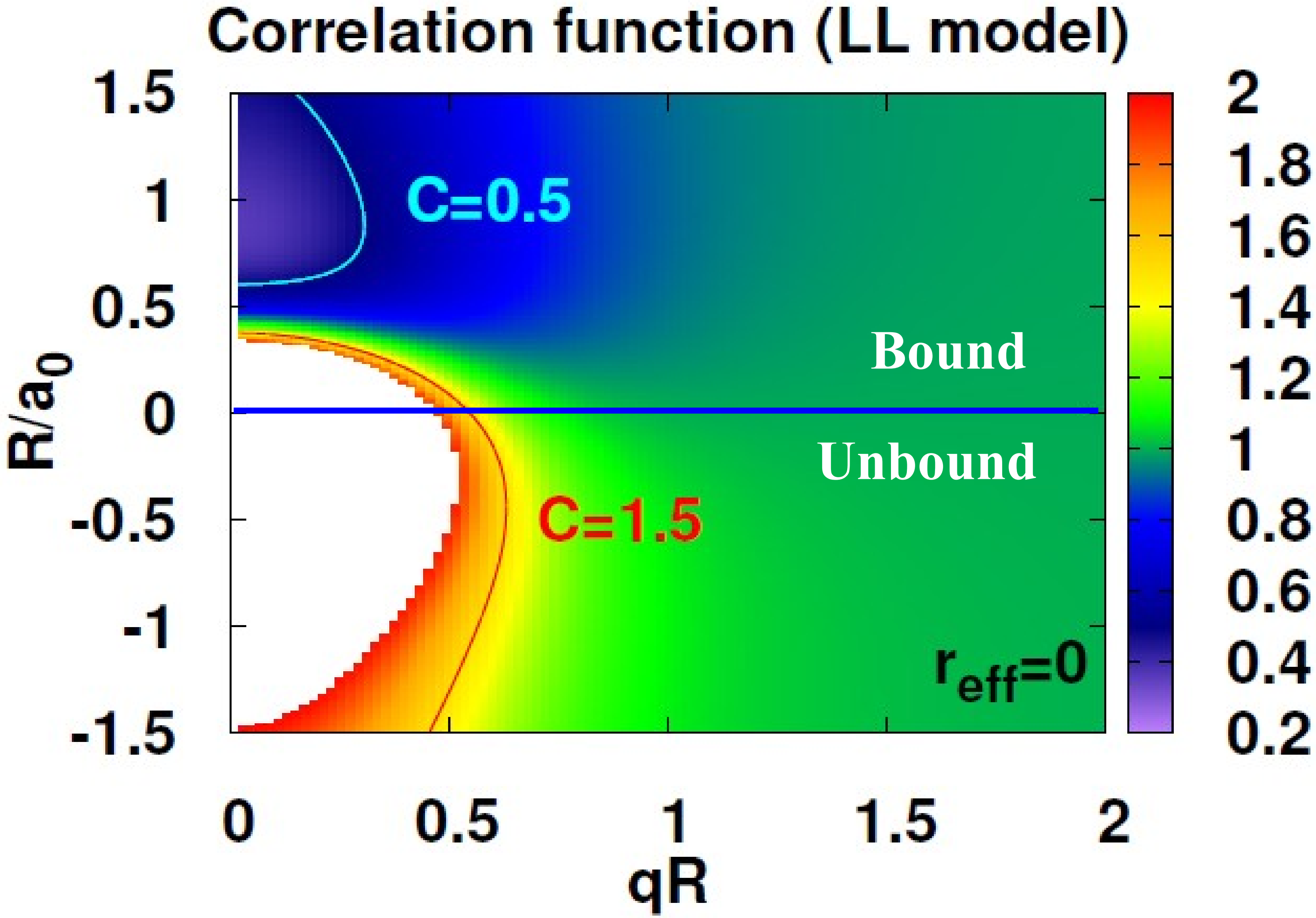
$$f(q) = (q\cot\delta - iq)^{-1} \simeq \left(-\frac{1}{a_0} + \frac{1}{2}r_{\rm eff}q^2 - iq\right)^{-1} \to -a_0$$

$$C_{\rm LL}(q\to 0) = 1 + \frac{a_0^2}{2R^2} - \frac{2a_0}{\sqrt{\pi}R} = 1 - \frac{2}{\pi} + \frac{1}{2}\left(\frac{a_0}{R} - \frac{2}{\sqrt{\pi}}\right)^2$$

$$1 - 2/\pi \simeq 0.36,\ \sqrt{\pi}/2 \simeq 0.89$$

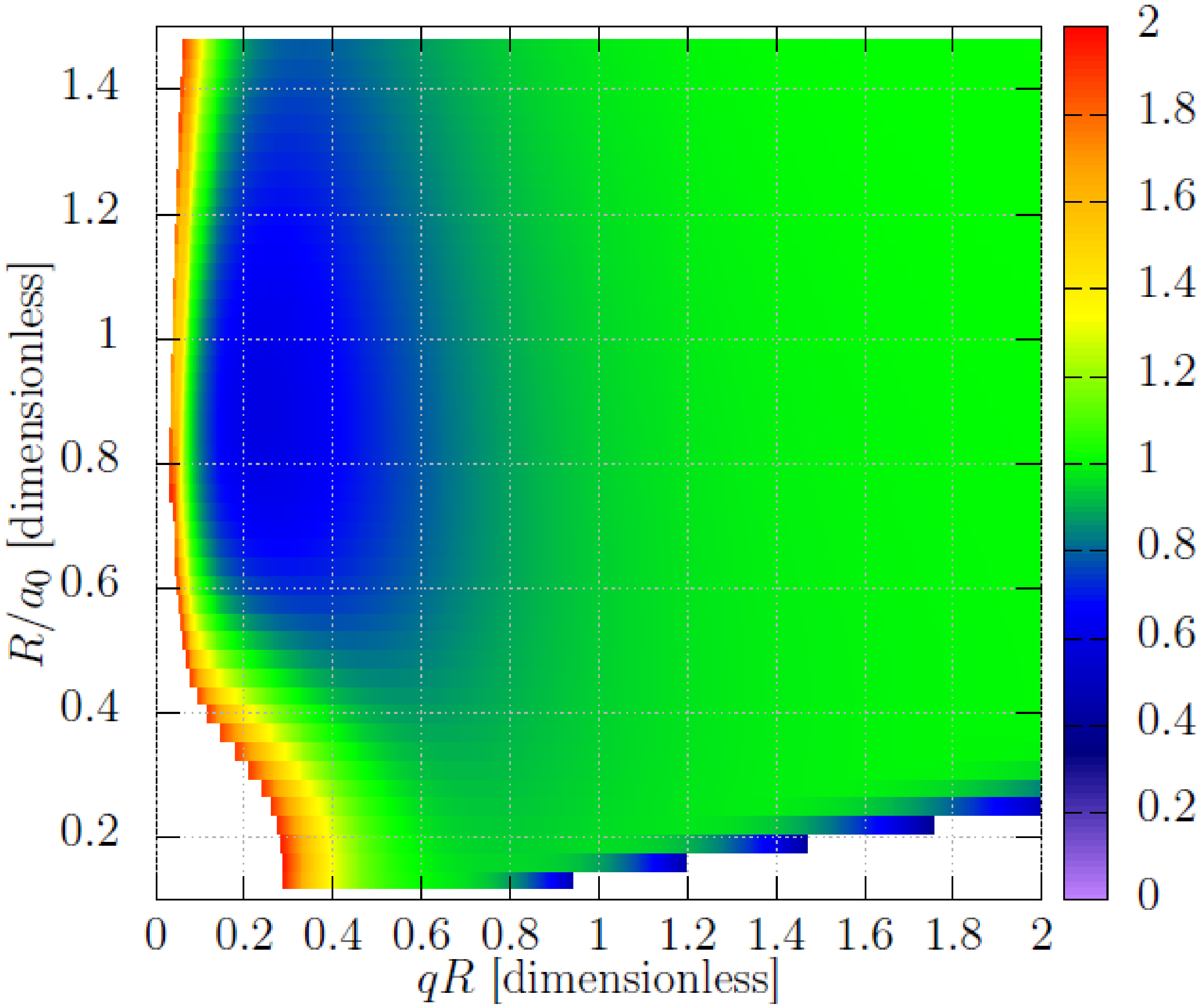
C(q → 0) takes a minimum of 0.36 at R/a_0 = 0.89 in the LL model.

Correlation Function in LL model



LL model: R. Lednicky, V. L. Lyuboshits ('82)

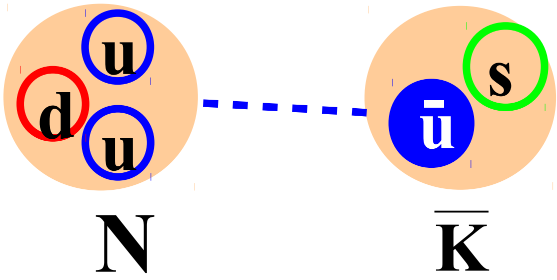
Correlation Function with Gaussian source



NΩ potential (J=2, HAL QCD) + Coulomb

Bound states in Fermtoscopy

Example 1: Λ(1405)



- **Λ(1405) as a bound state of $\bar{K}$N**

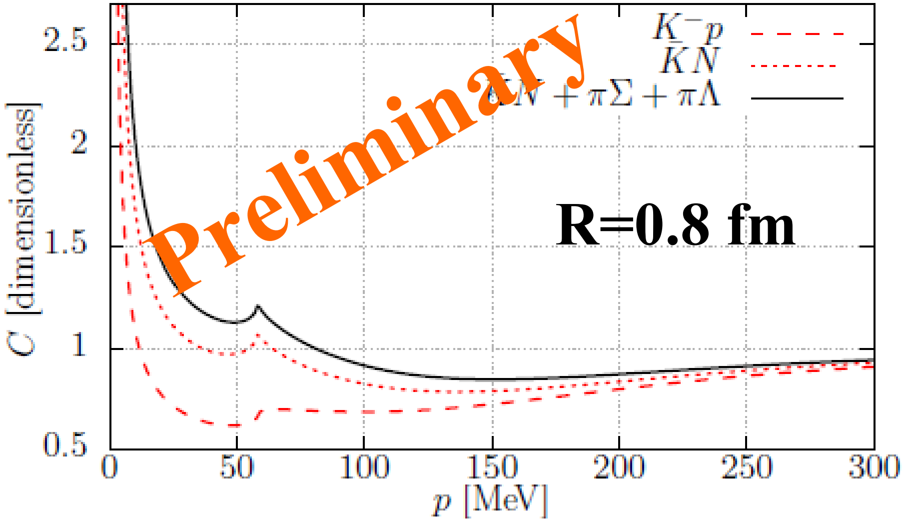
  Dalitz, Wong ('67); Siegel, Weise ('88)

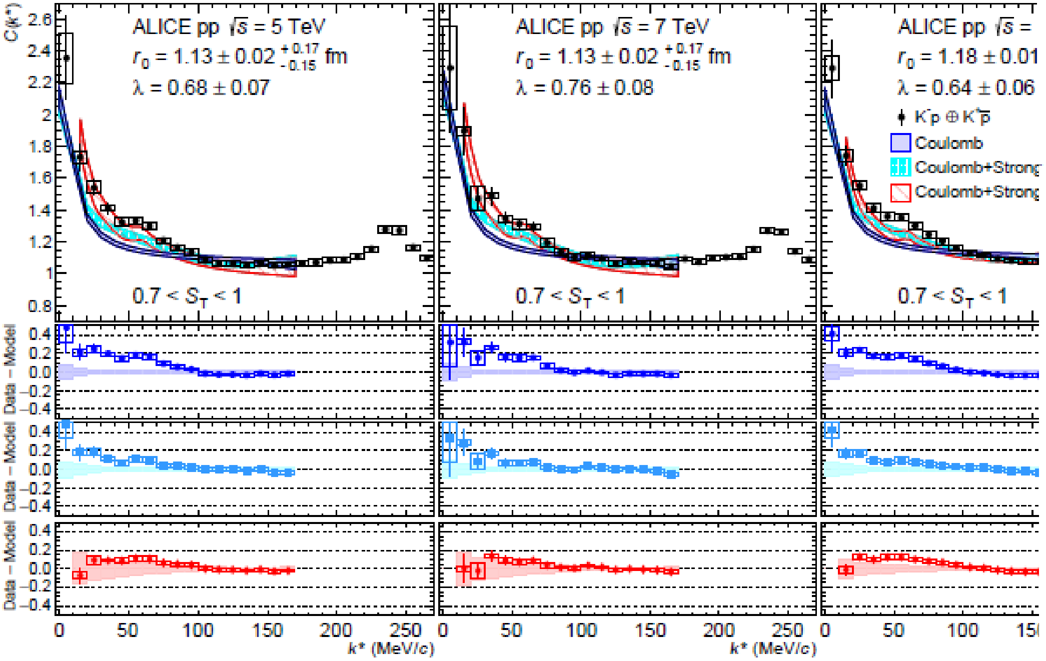
- **How can we confirm it ?**
  - **Binding Energy**
    → Deep (~ 30 MeV) or Shallow (~ 10 MeV) binding
    Akaishi, Yamazaki ('02); Jido, Oller, Oset, Ramos, Meissner ('03); Hyodo, Weise ('08); Shevchenko, Gal, Mares ('07); Ikeda, Sato ('07), …
  - **Medium effects → Upward mass shift or No change**
    Koch ('94); Waas, Kaiser, Weise ('96); Lutz ('98)
  - **In-medium Branching Ratio Change → Indirect (?)**
    Vander Velde-Wilquet et al. ('77); AO, Nara, Koch ('98)
  - **Form factor → Not measured yet**
    Sekihara, Hyodo, Jido ('08, '11)
  - **Correlation Function**
    AO+('16), S. Cho et al.(ExHIC)('17), S. Acharya et al. (ALICE), arXiv:1905.13470, Y. Kamiya, T. Hyodo, K. Morita, AO, in prep.

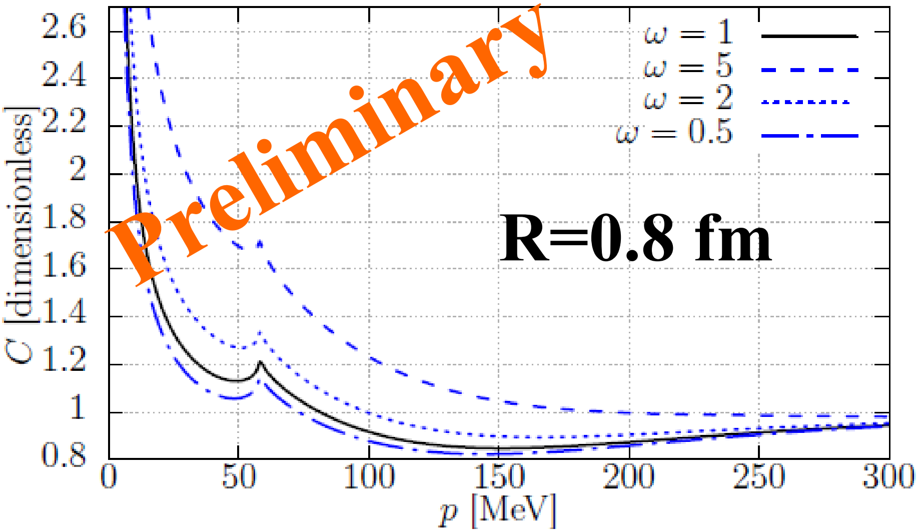
$K - p$ correlation function

- **Strong source size dependence seems to be observed in pA collisions, which may signal the bound state nature of Λ(1405)**

Ramona Lea (FemTUM),Kamiya+, in prep.





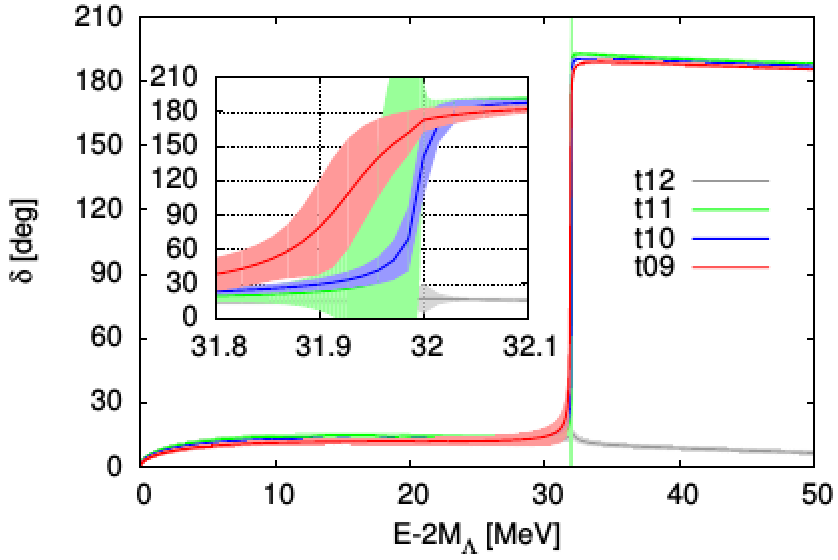


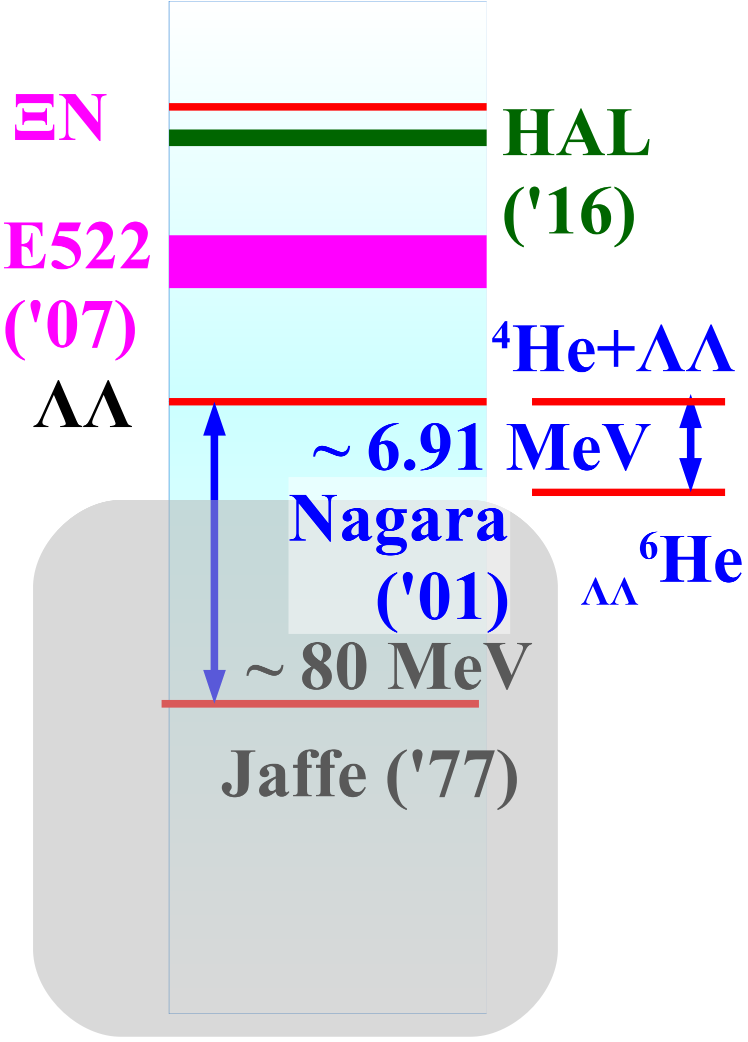
S. Acharya et al. (ALICE), arXiv:1905.13470

Kamiya, Morita, Hyodo, AO (in prep.)

Example 2: H particle

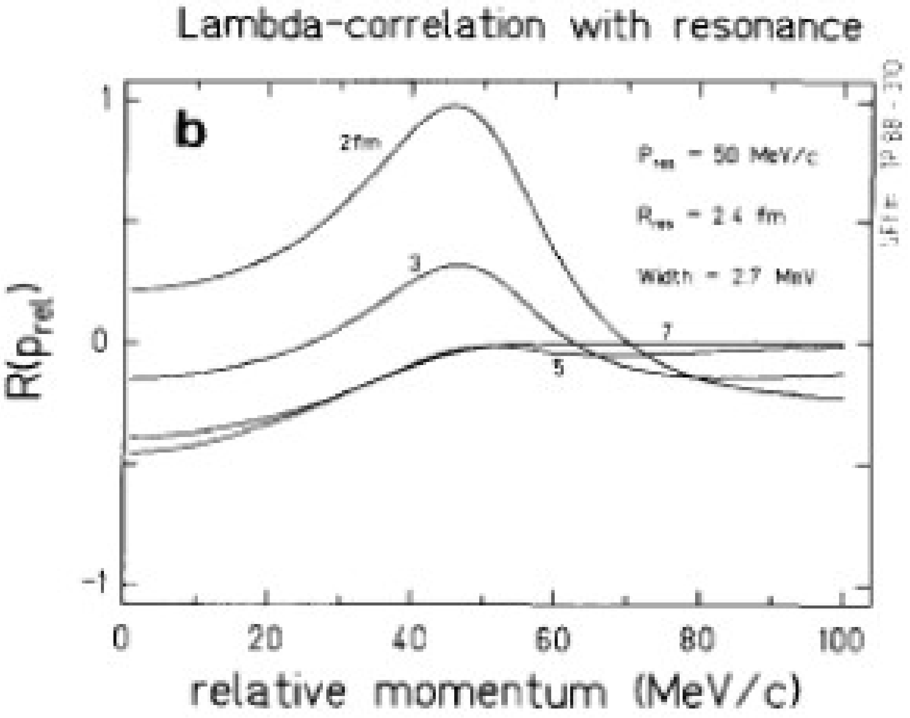
- **H-particle: 6-quark state (uuddss)**
  - **Prediction:** *R.L.Jaffe, PRL38(1977)195*
  - **Ruled-out by double Λ hypernucleus**
    Takahashi et al.,PRL87('01) 212502
  - **Resonance or Bound "H" ?**
    Yoon et al.(KEK-E522)+AO ('07)
  - **Lattice QCD**
    HAL QCD & NPLQCD ('11)
    K. Sasaki et al. (HAL QCD, '16-'18)



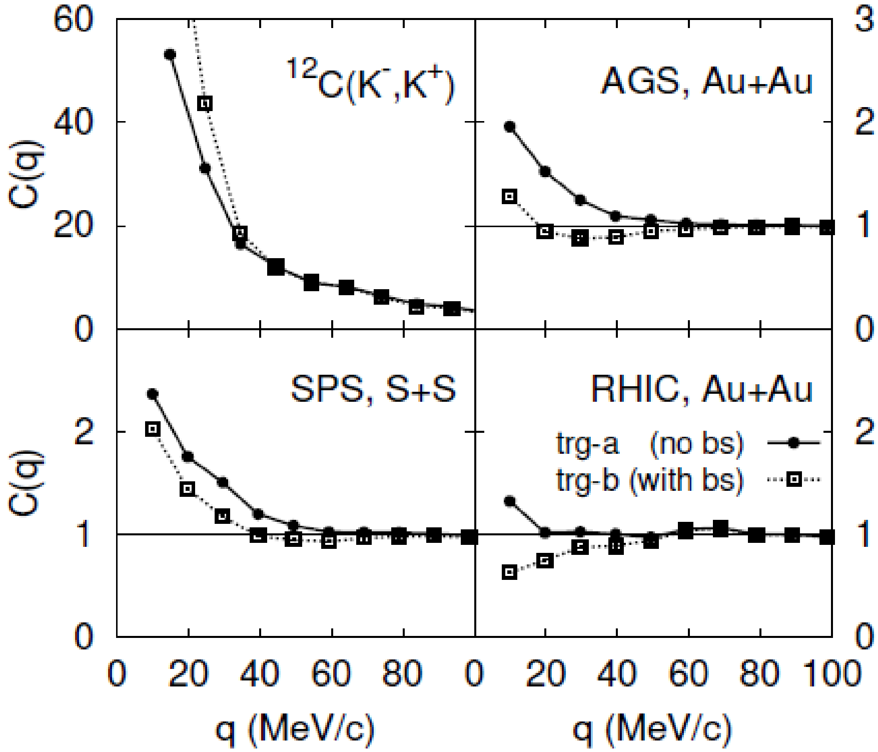


$\Lambda\Lambda$ correlation in HIC

- **Resonance H particle search via femtoscopy**
  C. Greiner, B. Muller, PLB219('89),199
- **Interaction study via $\Lambda\Lambda$ correlation in HIC**
  AO, Y. Hirata, Y. Nara, S. Shinmura, Y. Akaishi, NPA670 ('00), 297c



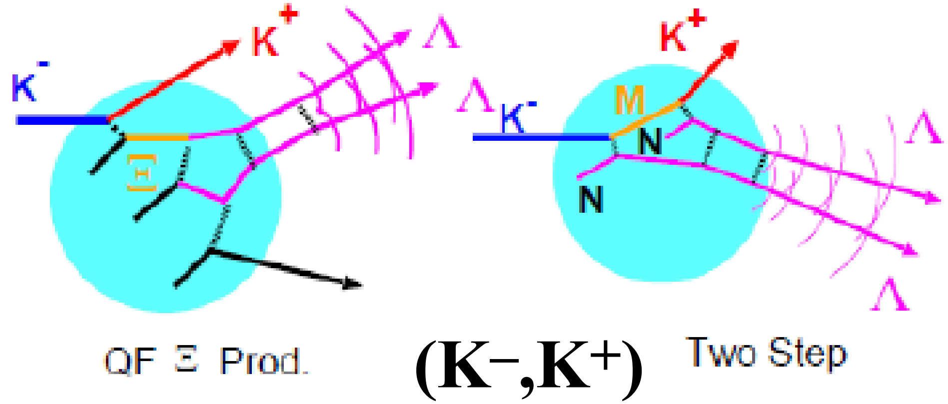
C. Greiner, B. Muller, PLB219('89)199.



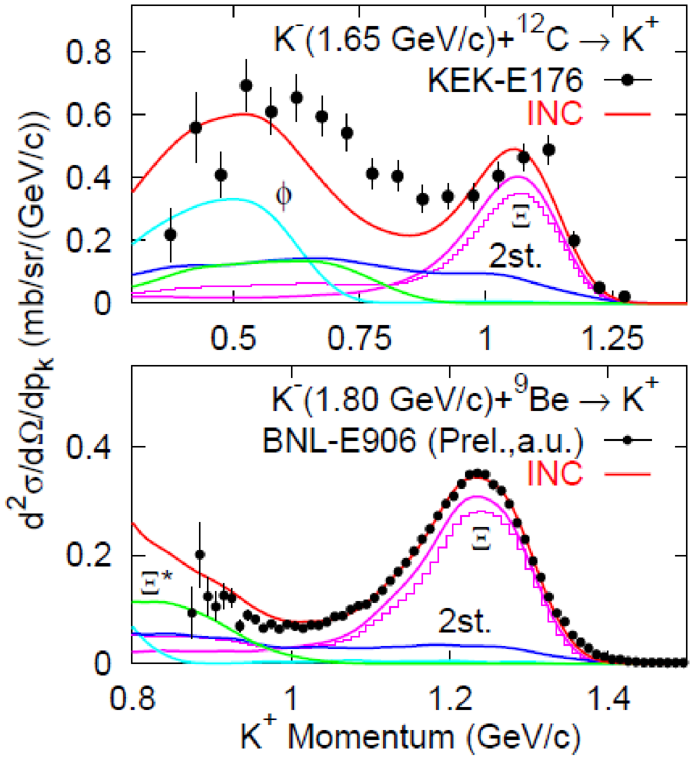
AO, Hirata, Nara, Shinmura, Akaishi, NPA670('00)297c

$\Lambda\Lambda$ correlation from $(K^-,K^+\Lambda\Lambda)$ reaction

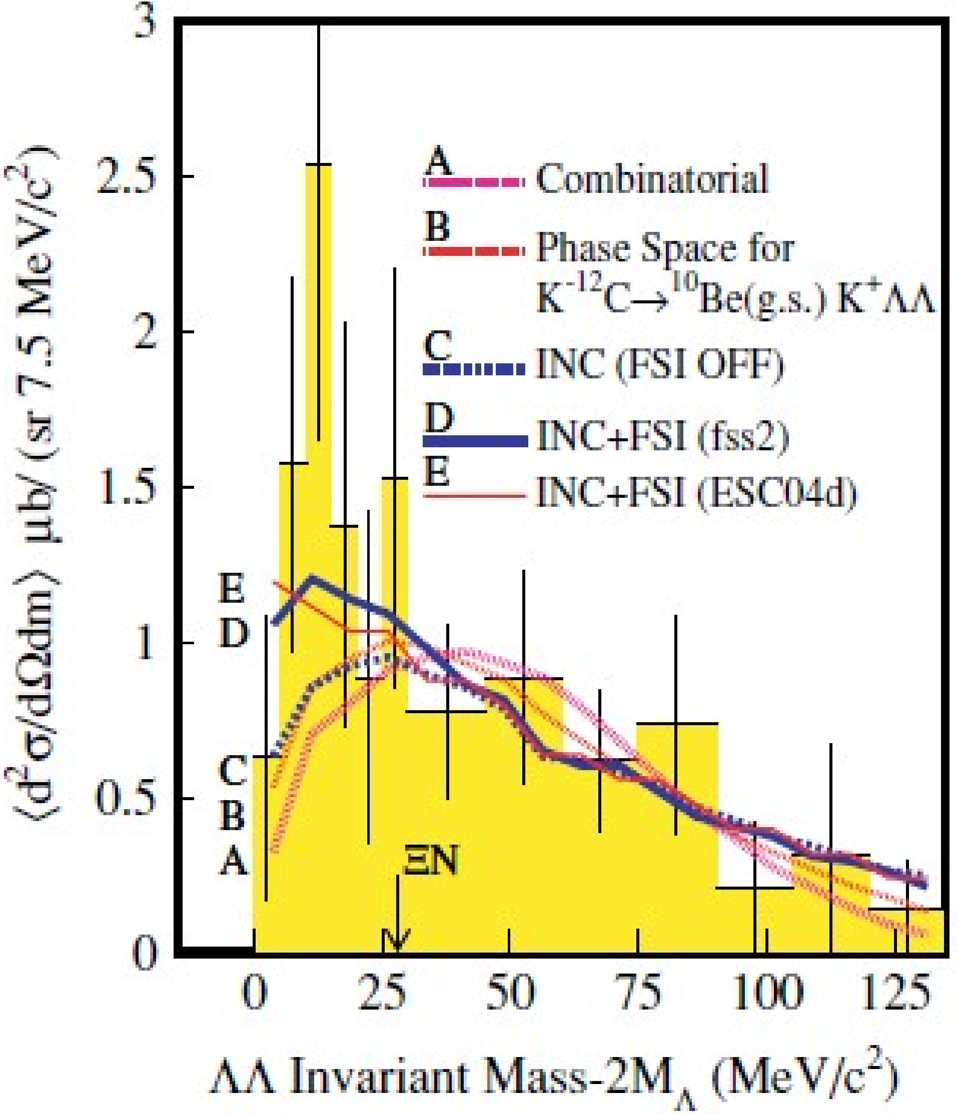
- **Enhancement at ~ 2 M(Λ)+ 10 MeV,**



(K^-,K^+)



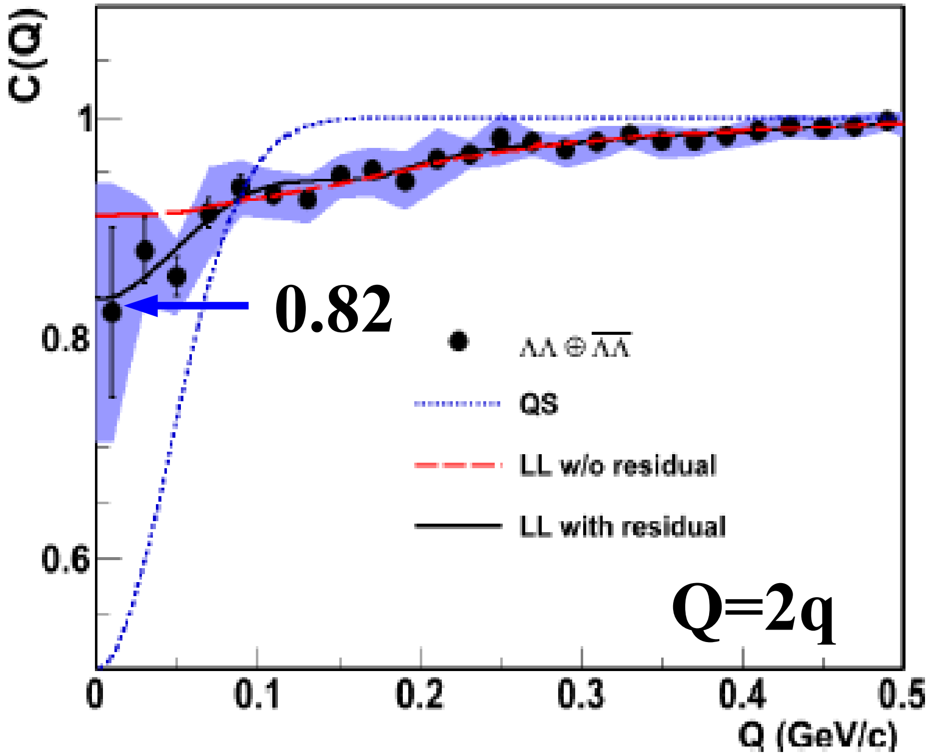
$(K^-,K^+\Lambda\ \Lambda)$



C.J.Yoon, …, (KEK-E522),AO, PRC75 (2007) 022201(R)
J. K. Ahn et al. (KEK-E224).

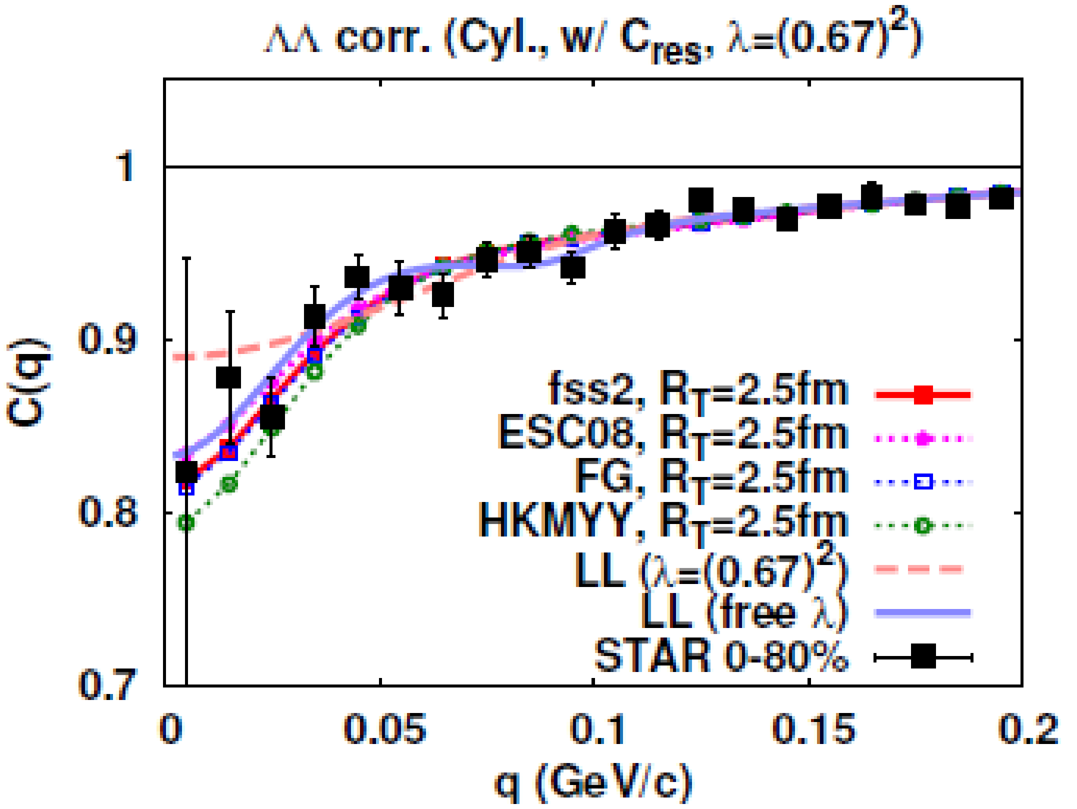
ΛΛ correlation at RHIC

- **STAR collaboration at RHIC measured ΛΛ correlation !**
  Adamczyk et al. (STAR Collaboration), PRL 114 ('15) 022301.
  - **RHIC, Au+Au ($\sqrt{s_{NN}}$=200 GeV), Weak decay vertex analysis.**
- **Theoretical Analysis well explains the data**
  K.Morita et al., T.Furumoto, AO, PRC91('15)024916;
  AO, K.Morita, K.Miyahara, T.Hyodo, NPA954 ('16), 294.



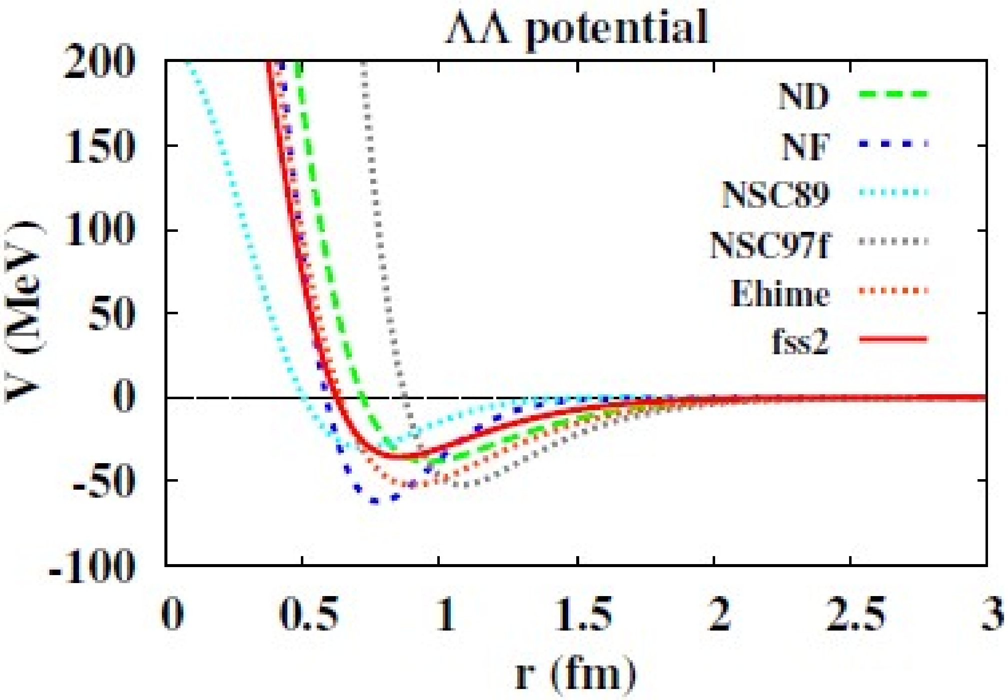
0.82

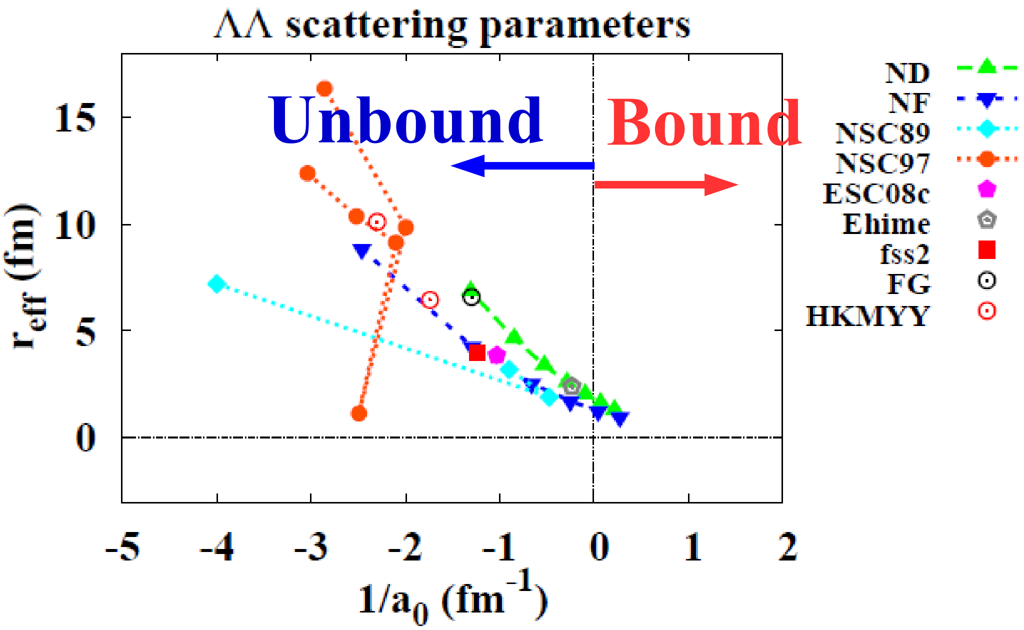
Q=2q



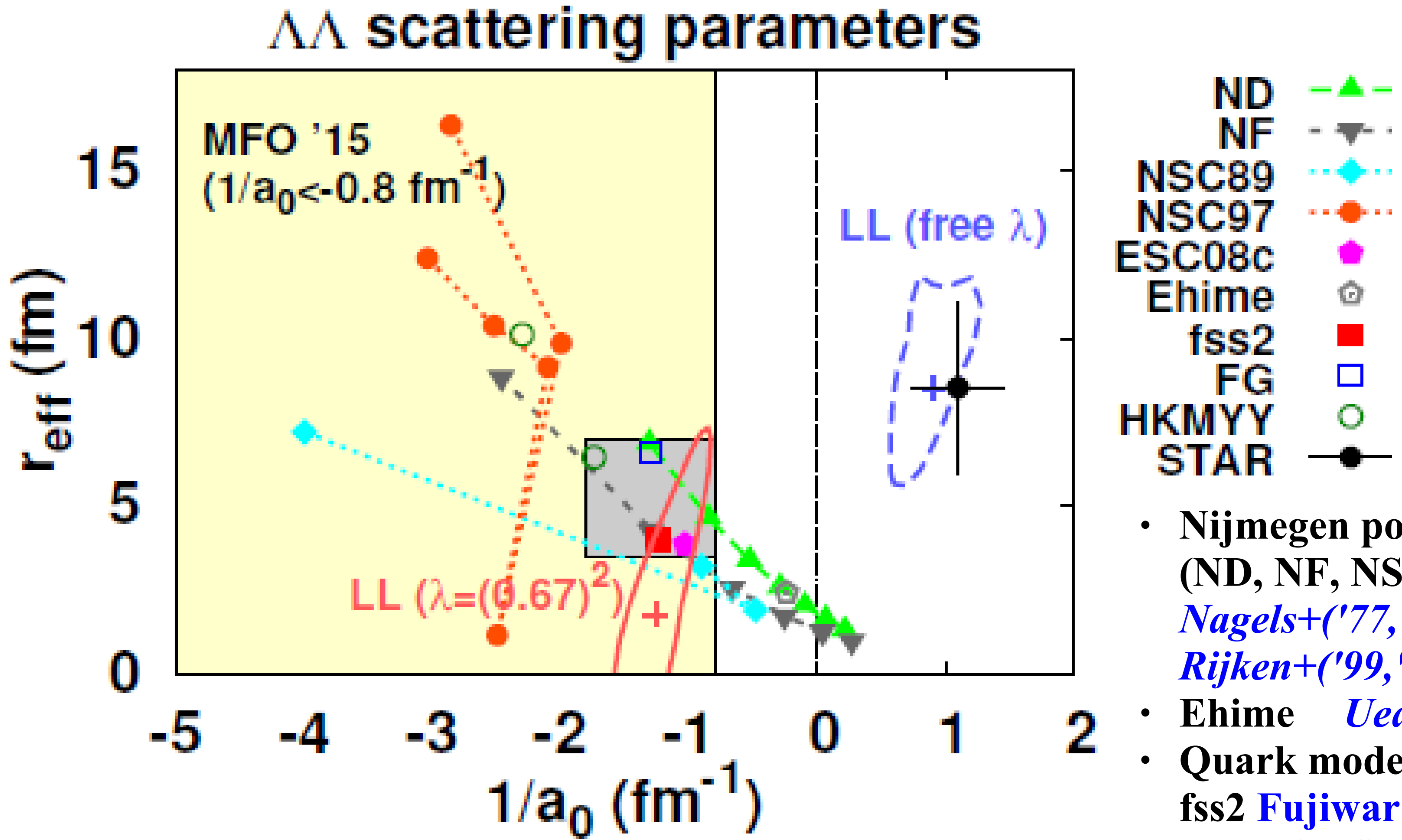
ΛΛ interaction

- **Propsed ΛΛ interactions**
  - **Meson Ex. models: Nijmegen model D, F, Soft Core (89, 97), ESC08**
    Nagels, Rijken, de Swart ('77, '79), Maessen, Rijken, de Swart ('89), Rijken, Stoks, Yamamoto ('99); Rijken, Nagels, Yamamoto ('10).
  - **Quark cluster model interaction: fss2**
    Fujiwara, Fujita, Kohno, Nakamoto, Suzuki ('00)
  - **Phenomenological model: Ehime** **T. Ueda et al. ('99).**
- **Two (or three) range gaussian fit results are used in the analysis.**





ΛΛ interaction from ΛΛ correlation



- **Nijmegen potentials** **(ND, NF, NSC89, NSC97, ESC08)** *Nagels+('77, '79), Maessen+('89), Rijken+('99,'10)*
- **Ehime** *Ueda et al. ('98)*
- **Quark model interaction:** fss2 Fujiwara et al.('07)
- **Potential fitted to Nagara** *Filikhin, Gal ('02) (FG), Hiyama et al. ('02, '10)(HKMYY)*

$$q \cot \delta = -\, 1/a_0 + r_{\mathrm{eff}}\, q^2/2 + O(q^4)$$

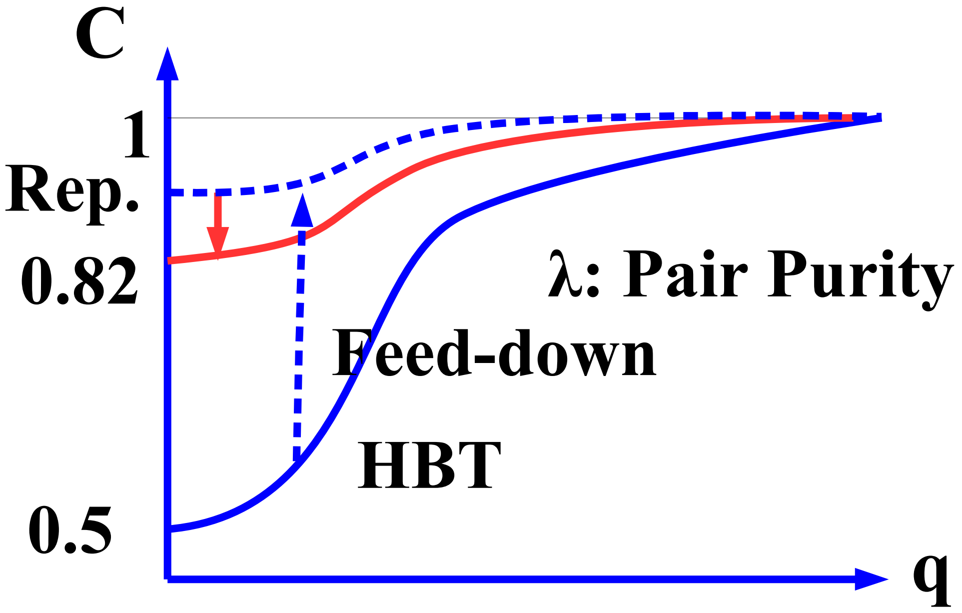
Positive a_0 (STAR) ⟵⟶ Negative a_0 (MFO'15)

Difference comes from the pair purity

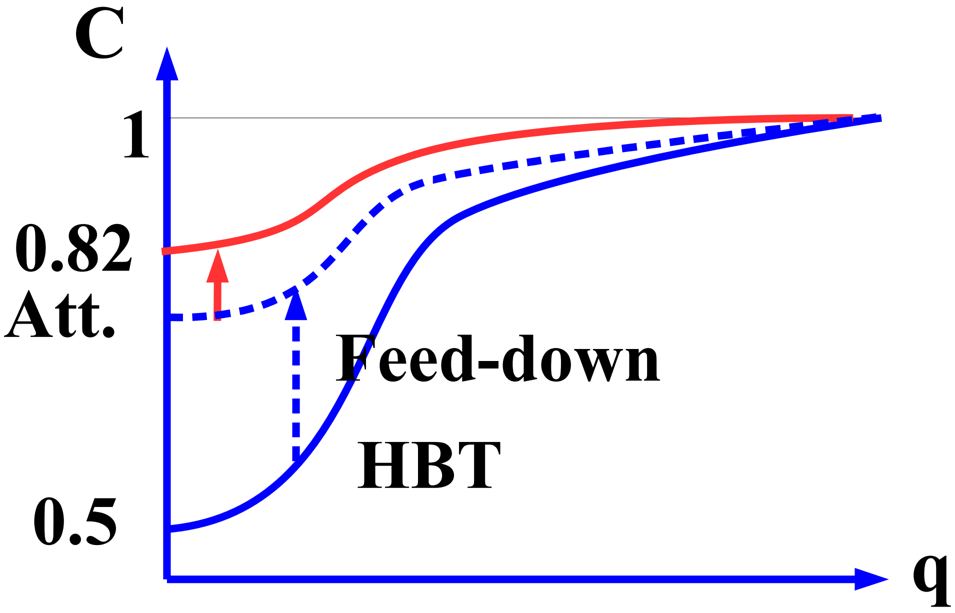
Feed-Down Effects & Residual Source

- **Correlation Fn. w/ Feed-down & Residual source effects.**

$$C_{\mathrm{corr}}(q) = 1 + \lambda(C_{\mathrm{bare}}(q) - 1) + a_{\mathrm{res}} \exp(-4r_{\mathrm{res}}^2 q^2)$$



STAR:
λ ~ 0.18 (free para.)



Morita et al. (MFO15):
λ ~ 0.45

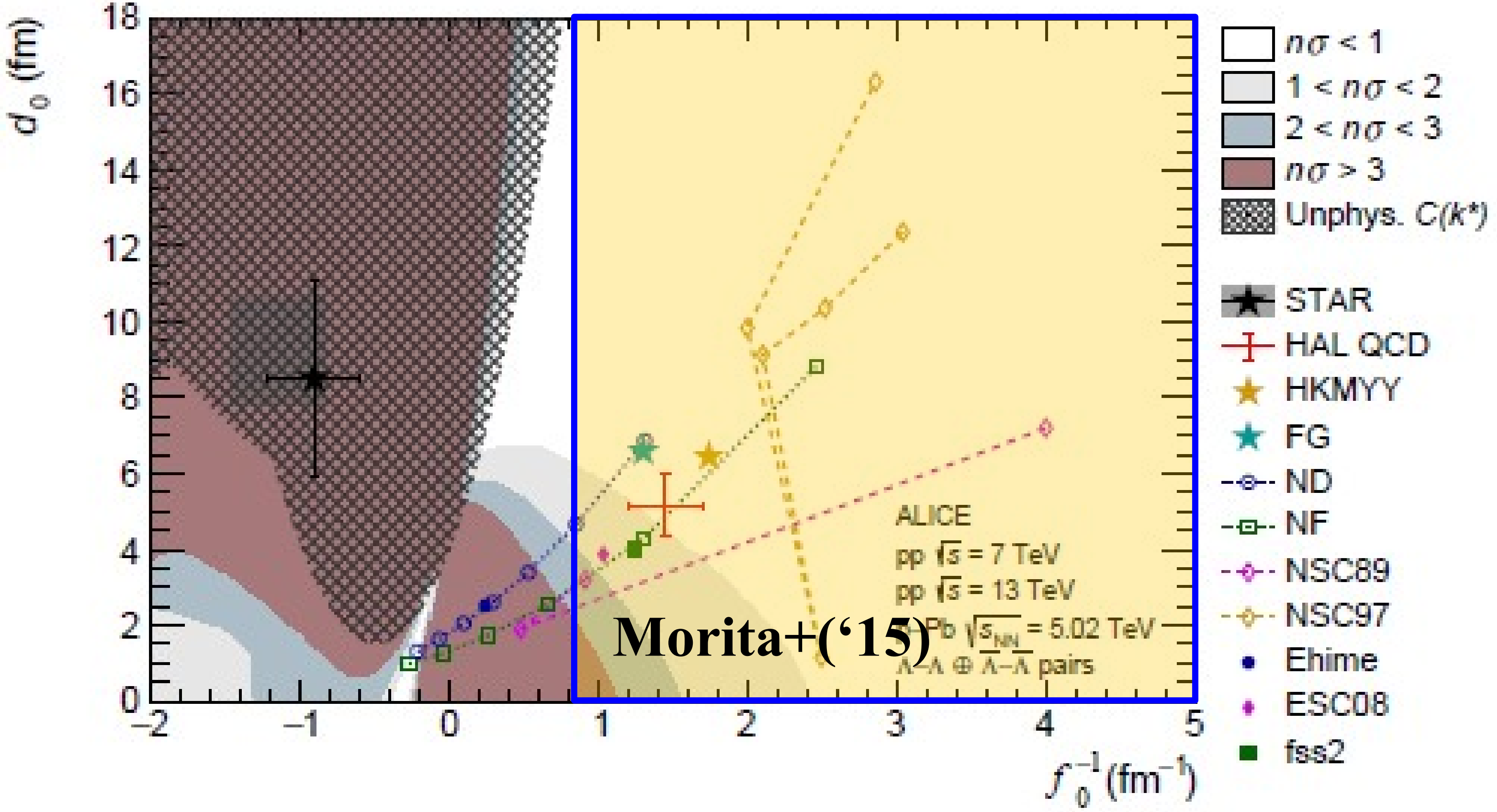
Pair purity (λ) should be determined experimentally !
Puzzle: Residual source

Σ^0/Λ=0.278 (p+Be, 28.5 GeV/c)
Sullivan et al. ('87)
Ξ/Λ = 15 % (RHIC)

AO, Morita, Mihayara, Hyodo ('16)

New Data from LHC-ALICE

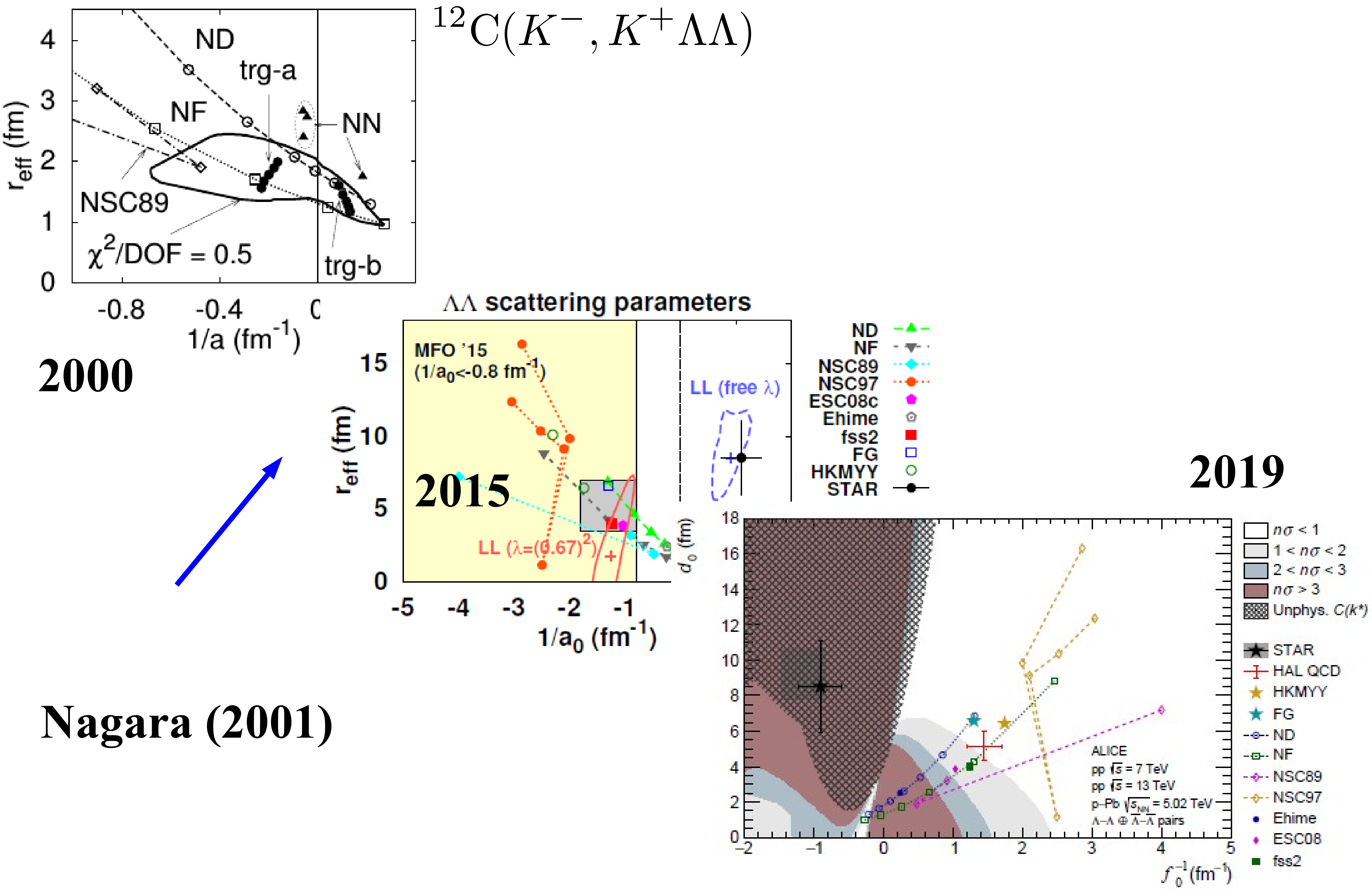
S. Acharya+ (ALICE), PLB 797('19), 134822 [1905.07209]



Weakly attractive $V_{\Lambda\Lambda}$.

Large reff → Becomes repulisve at relatively low density.

Time dependence of ΛΛ interaction



$^{12}{\rm C}(K^-, K^+\Lambda\Lambda)$

2000

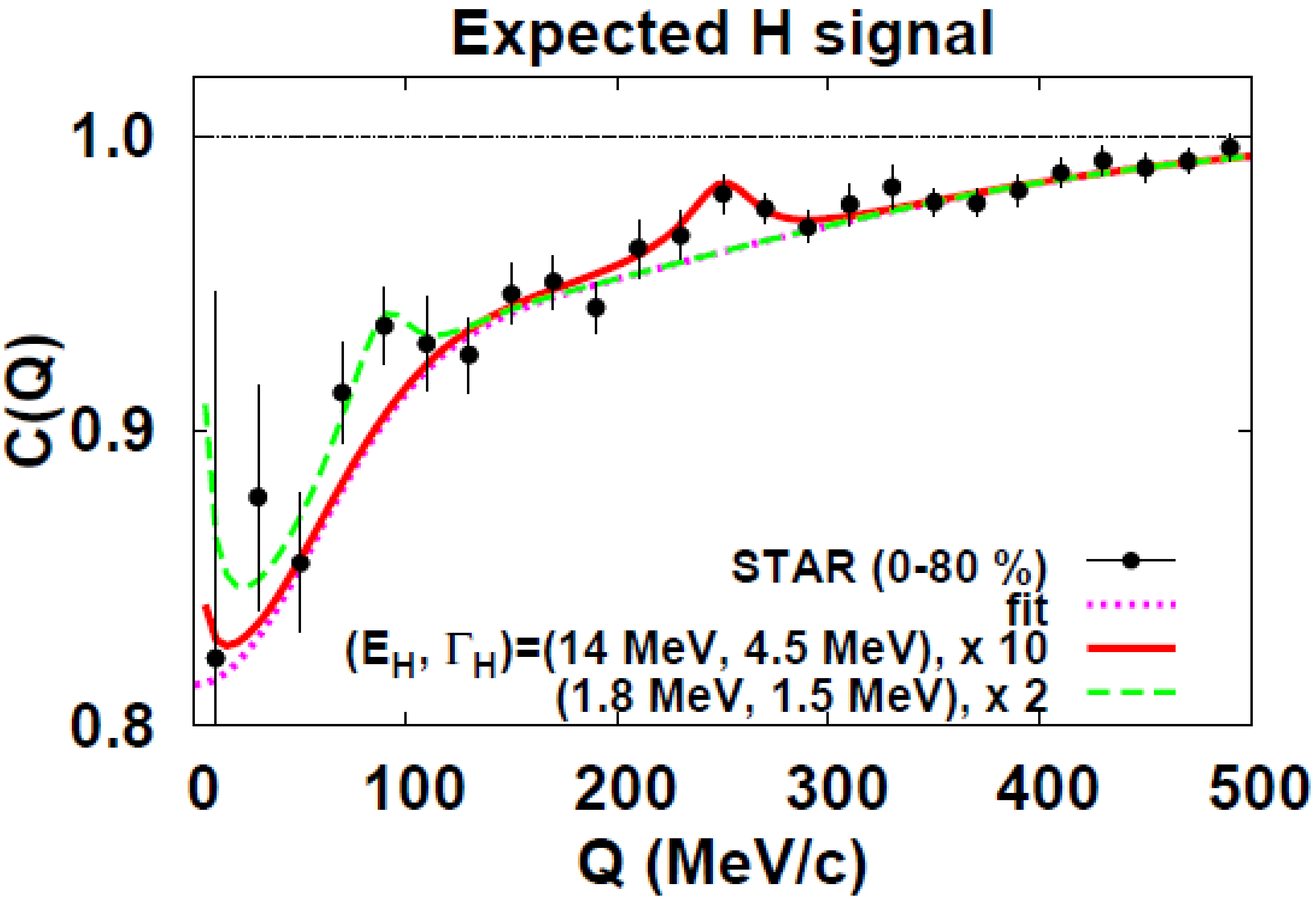
2015

2019

Nagara (2001)

Detecting Dibaryon State from Invariant Mass Spectrum ?

- **Strong signal of dibaryon(s) in correlation function**
  → How about invariant mass spectrum
  → Needs much more statistics



Morita+ ('15)

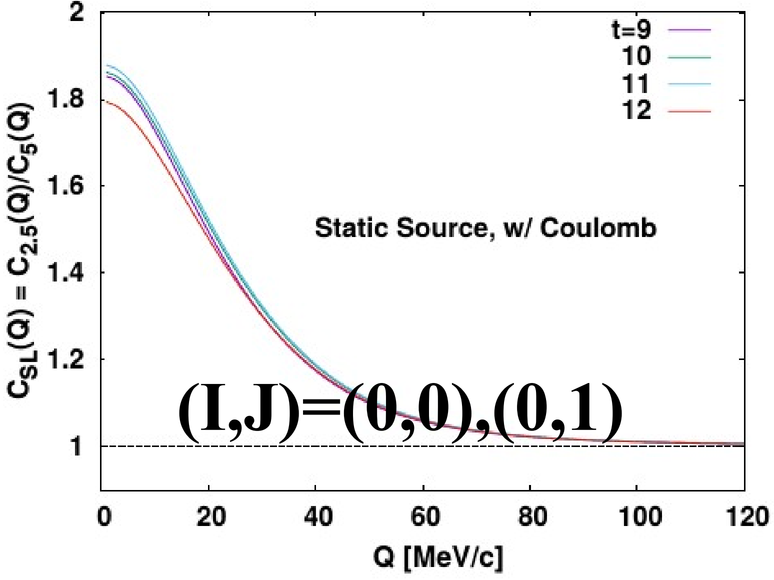
J. Haidenbauer will make an objection ...

$\Xi^- p$ correlation

- **Prediction of the correlation function by using ΞN potential (HAL QCD Collab.) + Coulomb potential**

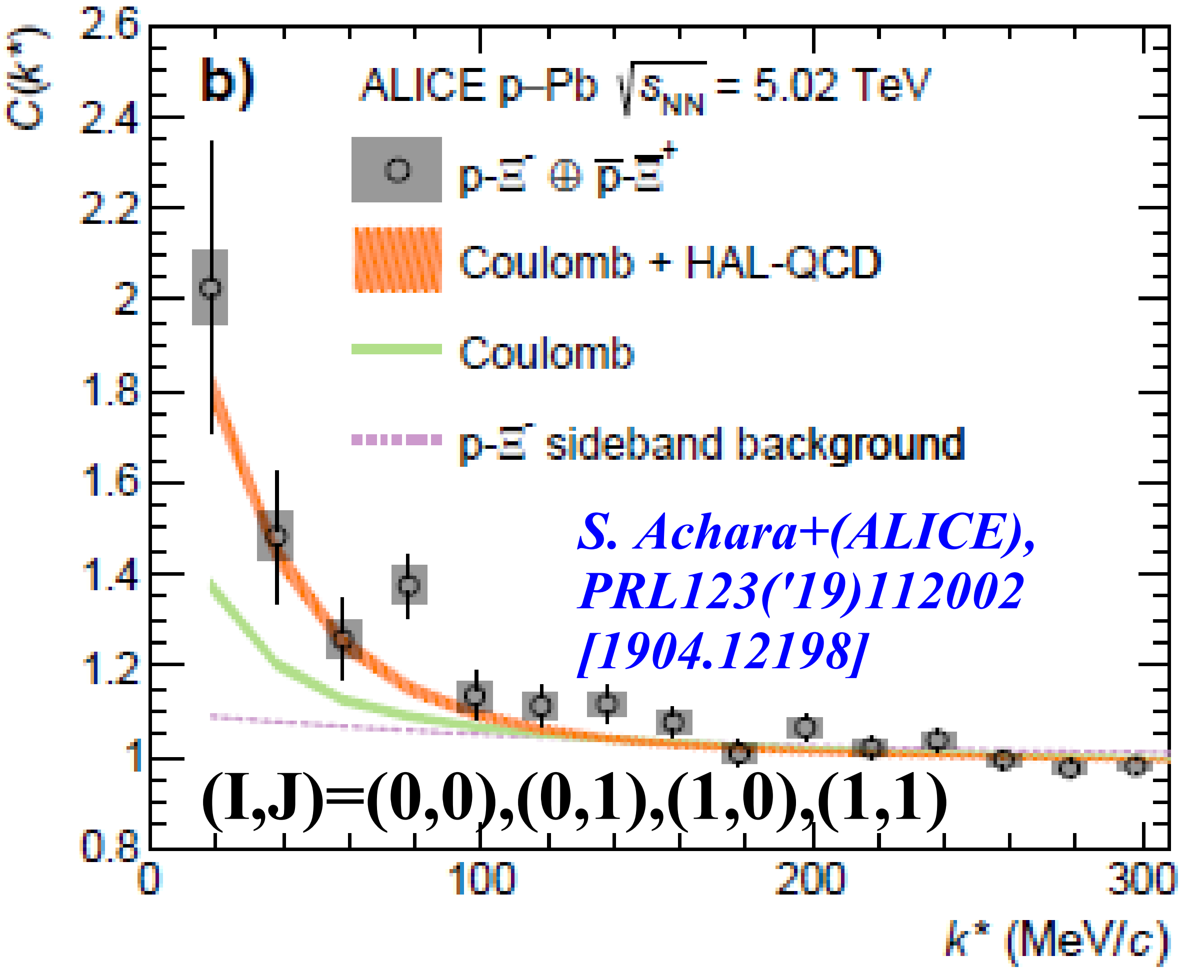
$$|\psi|^2_{\text{spin av.}} = \frac{1}{2}\sum_{I=0,1}\left[\frac{1}{4}|\psi_I^{J=0}|^2 + \frac{3}{4}|\psi_I^{J=1}|^2\right]$$

HAL prediction is examined !



(I,J)=(0,0),(0,1)

T. Hatsuda, K. Morita, AO, K. Sasaki, NPA967('17), 856.

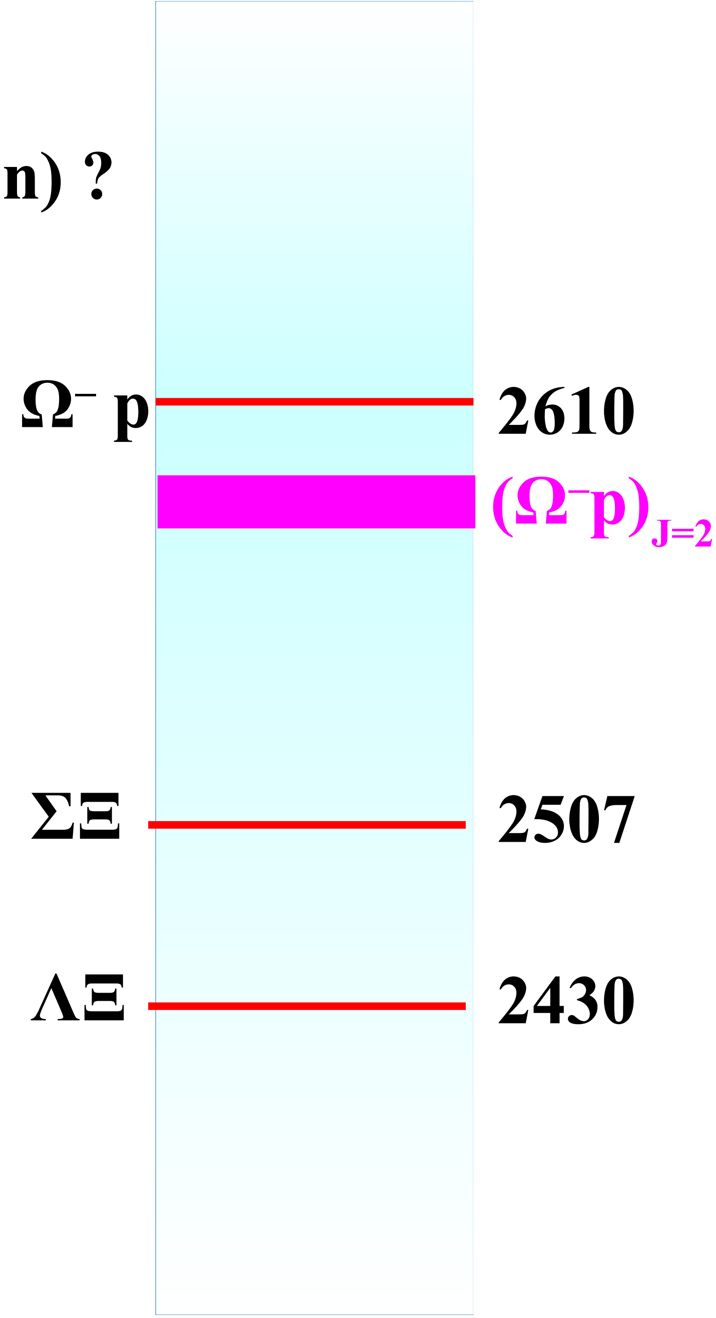


S. Achara+(ALICE), PRL123('19)112002 [1904.12198]

(I,J)=(0,0),(0,1),(1,0),(1,1)

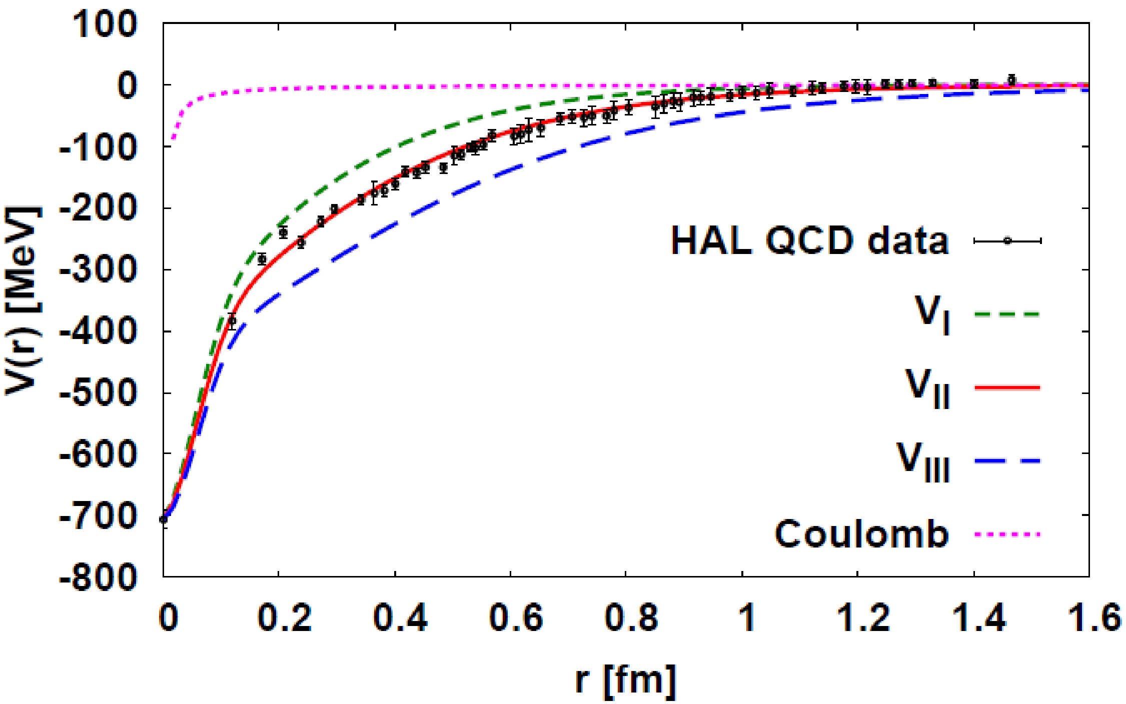
Example 3: Ω N dibaryon

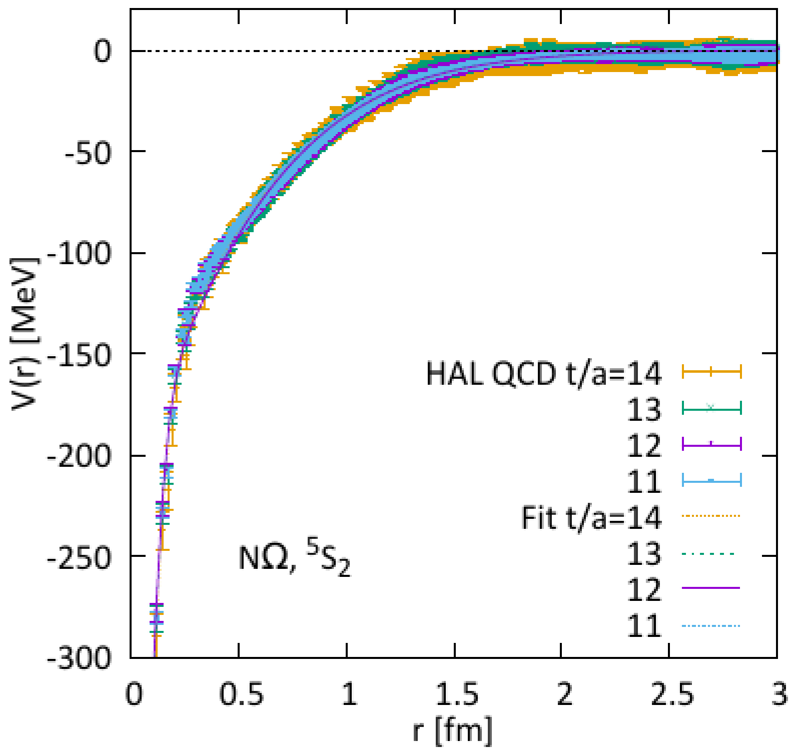
- Ω : sss, $J\pi=3/2+$, M=1672 MeV
- Is there an ΩN bound state (S= –3 dibaryon) ?
  - Predicted as a dibaryon candidate
    Goldman+ (‘87), Oka ('88), Gal ('16)
  - Lattice QCD predicts a bound state with narrow width for J=2 ($^5S_2$)
    Etminan+ (HAL QCD)('14),
    Iritani+ (HAL QCD) (‘19)
  - Correlation function is measurable !
    Adam+ (STAR)(‘19), Oton (FemTUM),
    ALICE, in prep.



ΩN potential from lattice QCD

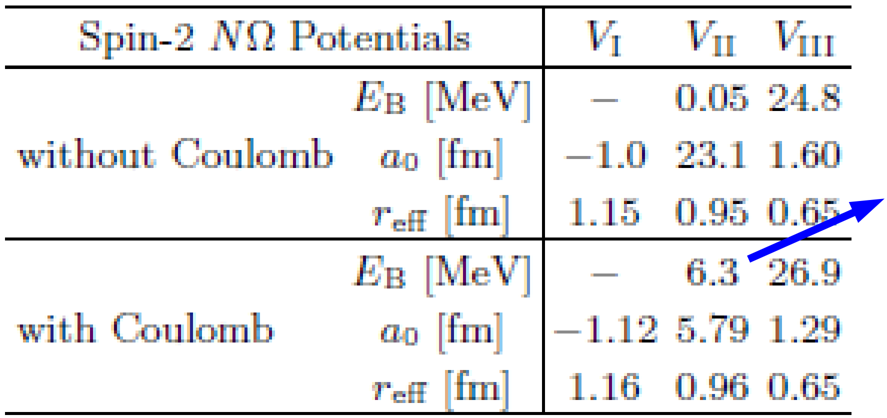
- ΩN potential by HAL QCD Collab. (J=2)
  - m_π=875 MeV, B.E.~ 19 MeV
    F.Etminan et al. (HAL QCD Collab.), NPA928('14)89.
  - m_π=146 MeV, B.E.~ 2.2 MeV
    T. Iritani et al. (HAL QCD Collab.), PLB 792('19)284.





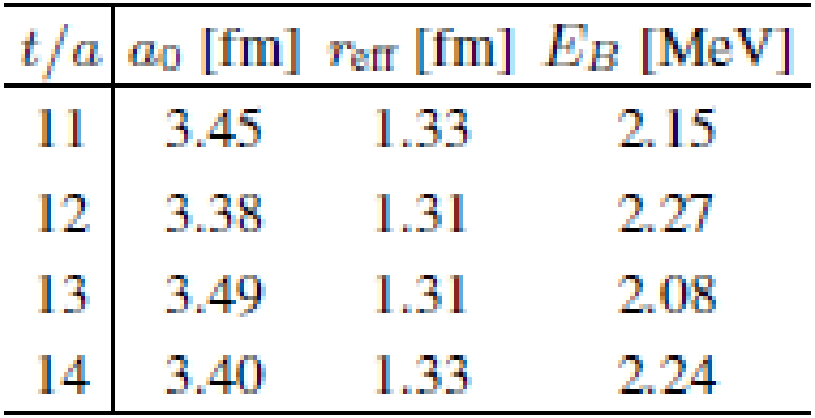
Binding Energy, Scattering Length, Effecitve Range (pΩ)

- **m_π=875 MeV, B.E.~ 19 MeV**
  F.Etminan et al. (HAL QCD Collab.), NPA928('14)89.
- **m_π=146 MeV, B.E.~ 2.2 MeV**
  T. Iritani et al. (HAL QCD Collab.), PLB 792('19)284.

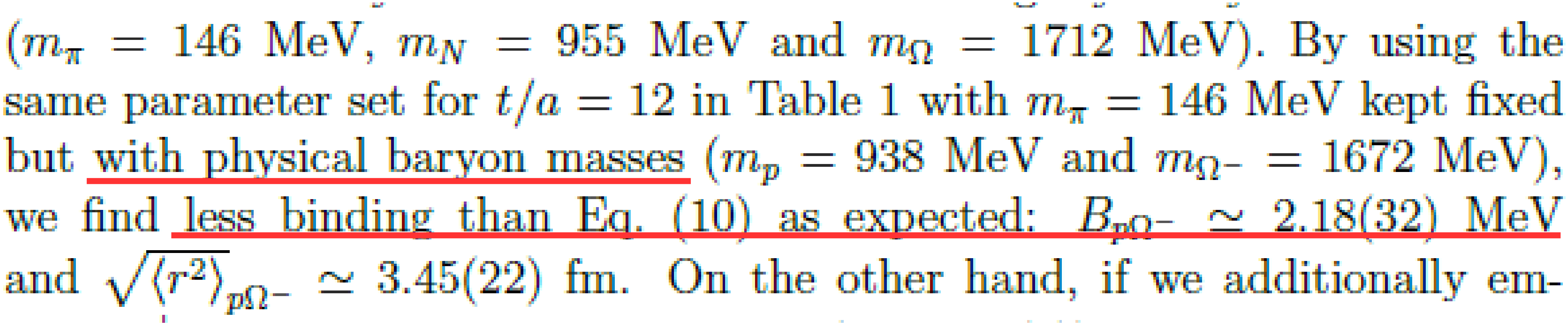


Spin-2 $N\Omega$ Potentials		$V_{\rm I}$	$V_{\rm II}$	$V_{\rm III}$
without Coulomb	$E_{\rm B}$ [MeV]	–	0.05	24.8
	a_0 [fm]	−1.0	23.1	1.60
	$r_{\rm eff}$ [fm]	1.15	0.95	0.65
with Coulomb	$E_{\rm B}$ [MeV]	–	6.3	26.9
	a_0 [fm]	−1.12	5.79	1.29
	$r_{\rm eff}$ [fm]	1.16	0.96	0.65

0.63



t/a	a_0 [fm]	$r_{\rm eff}$ [fm]	E_B [MeV]
11	3.45	1.33	2.15
12	3.38	1.31	2.27
13	3.49	1.31	2.08
14	3.40	1.33	2.24



(m_π = 146 MeV, m_N = 955 MeV and m_Ω = 1712 MeV). By using the same parameter set for $t/a = 12$ in Table 1 with m_π = 146 MeV kept fixed but with physical baryon masses (m_p = 938 MeV and m_{Ω^-} = 1672 MeV), we find less binding than Eq. (10) as expected: $B_{p\Omega^-} \simeq 2.18(32)$ MeV and $\sqrt{\langle r^2\rangle_{p\Omega^-}} \simeq 3.45(22)$ fm. On the other hand, if we additionally em-

Calculation Details

- **NΩ potential from HAL QCD Collab.**
  Etminan+(HAL QCD) ('14), Iritani+ (HAL QCD)('19)
  - **J=1 potential is uncertain → Three models**
    Strong abs. at $r < r_0$ ($r_0 \sim 2$ fm) (*Morita+('16)*) (Standard)
    Complete absorption $\chi(J=1) = 0$ (Minimum)
    Same w.f. as that with J=2, $\chi(J=1) = \chi(J=2)$ (Reference)
  - **Statistical Error can be evaluated by using Jackknife potentials.**
- **Coulomb potential enhances CF even without strong int.**
  → Small-Large ratio of CF (*Morita+('16)*)
  - **Large source → Coulomb force dominate**
    Small source → Visible strong interaction effects
- **Source function: Blast wave, Gaussian source**

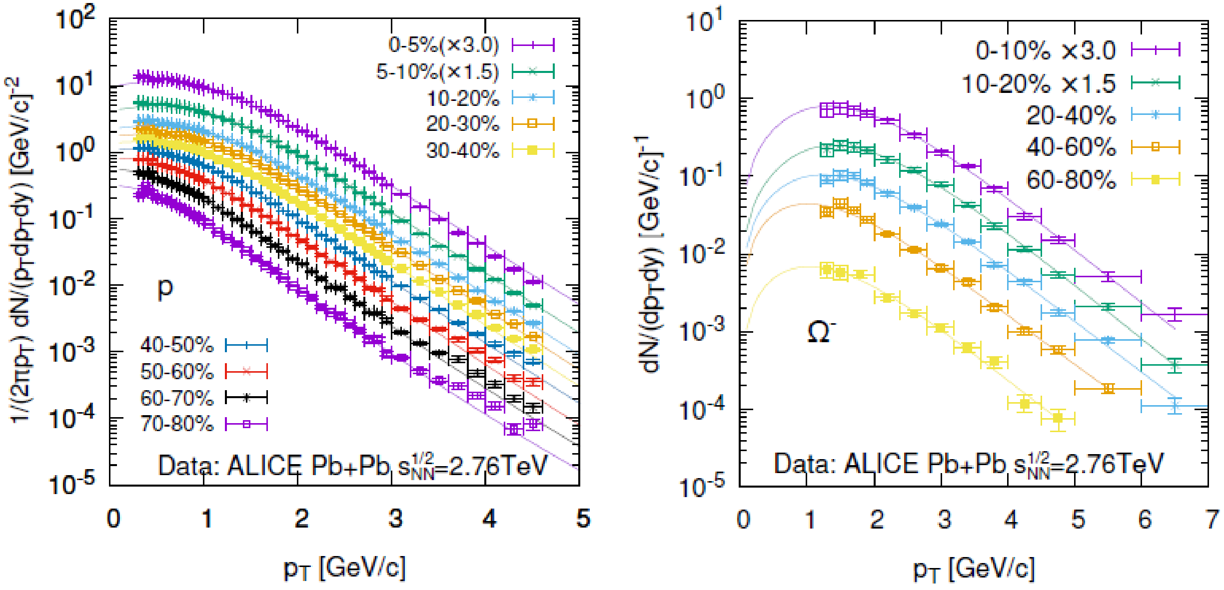
Emission Source Function

- **Gaussian Source ∝ exp(-$r^2/4R^2$), R=(0.8-4) fm [Simple and convenient]**
- **Expanding source model [Reasonably realistic]**

Flow velocity $u_\mu(x)$

$$d^4x S_i(x,p) = \tau_0 d\eta_s d^2 r_T \frac{d}{(2\pi)^3} \underline{n_f(u\cdot p, T)} \exp\left(-\frac{r_T^2}{2R_T^2}\right)$$

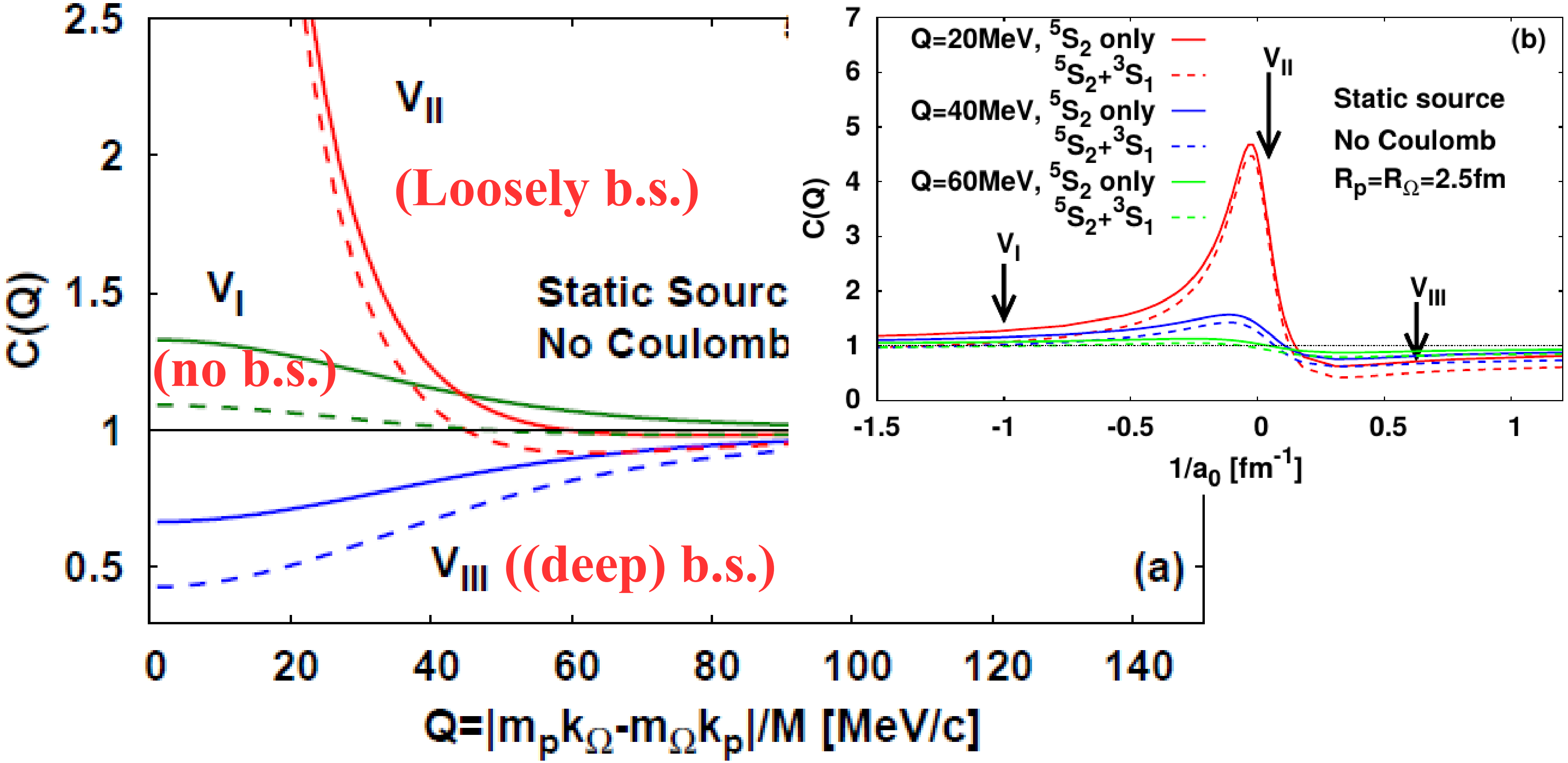
Fermi dist.



Morita+('19)

- **Transport model result [should be realistic] → Future work**

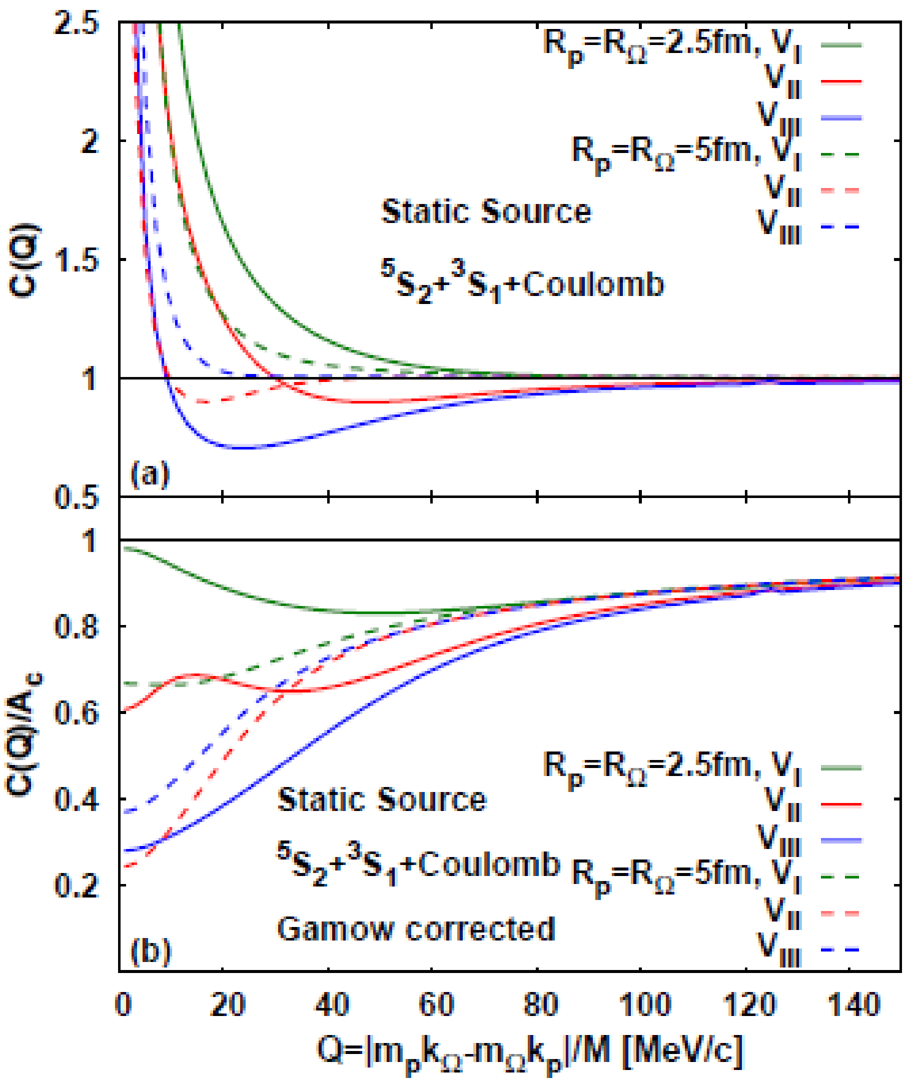
Ω^- p correlation



(w/o Coulomb, Strong absorption at r< 2 fm in $^3S_1$ (decay to 8-8 in S-wave))

K. Morita, AO, F. Etminan, T. Hatsuda, PRC94('16)031901(R) [arXiv:1605.06765 [hep-ph]]

Ω^- p correlation w/ Coulomb

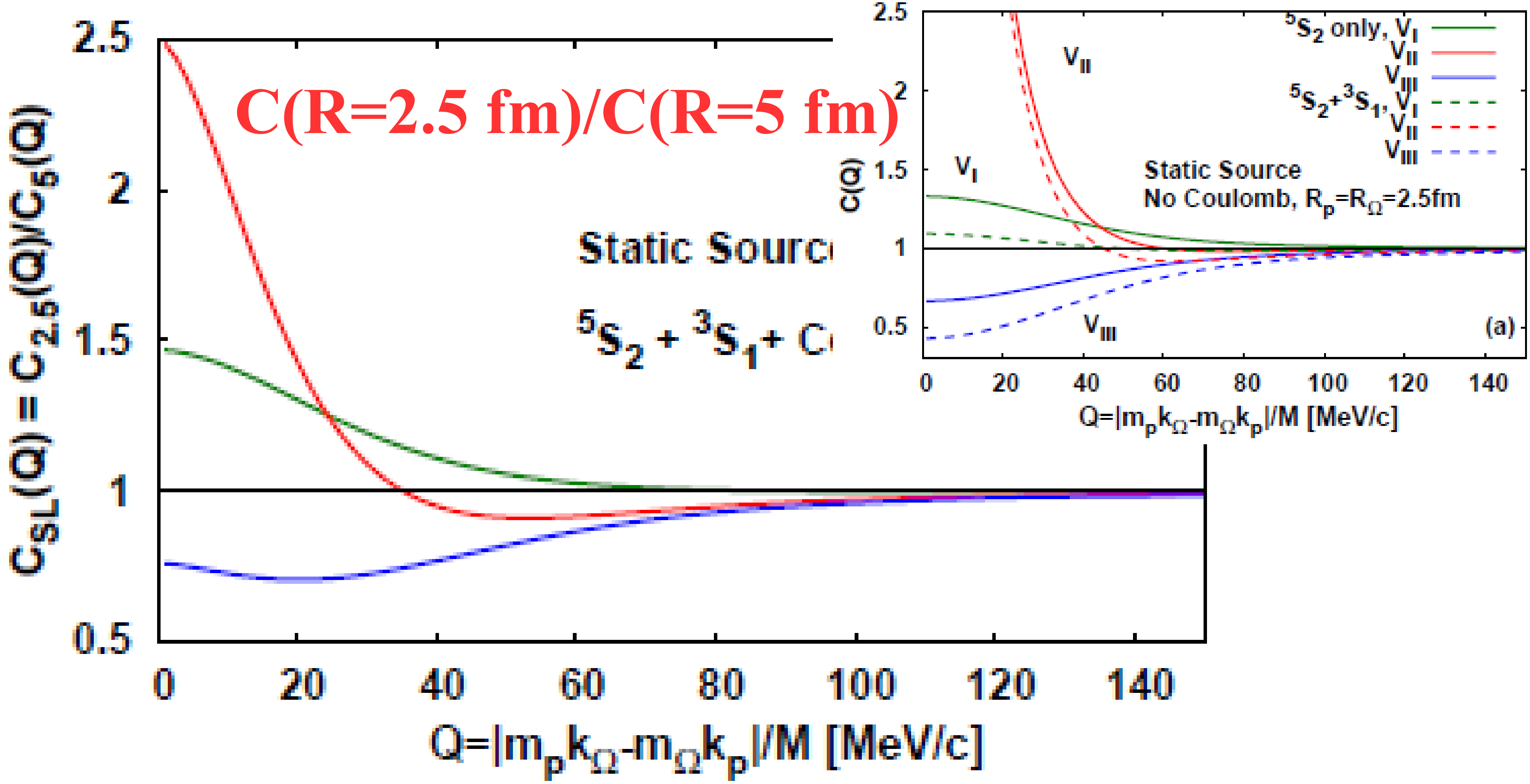


With Coulomb

Coulomb + Gamow corr.

Coulomb potential washes out the features of V_I , V_{II} , V_{III} , and Gamow correction is too strong.

Ω^- p correlation: Small / Large Ratio



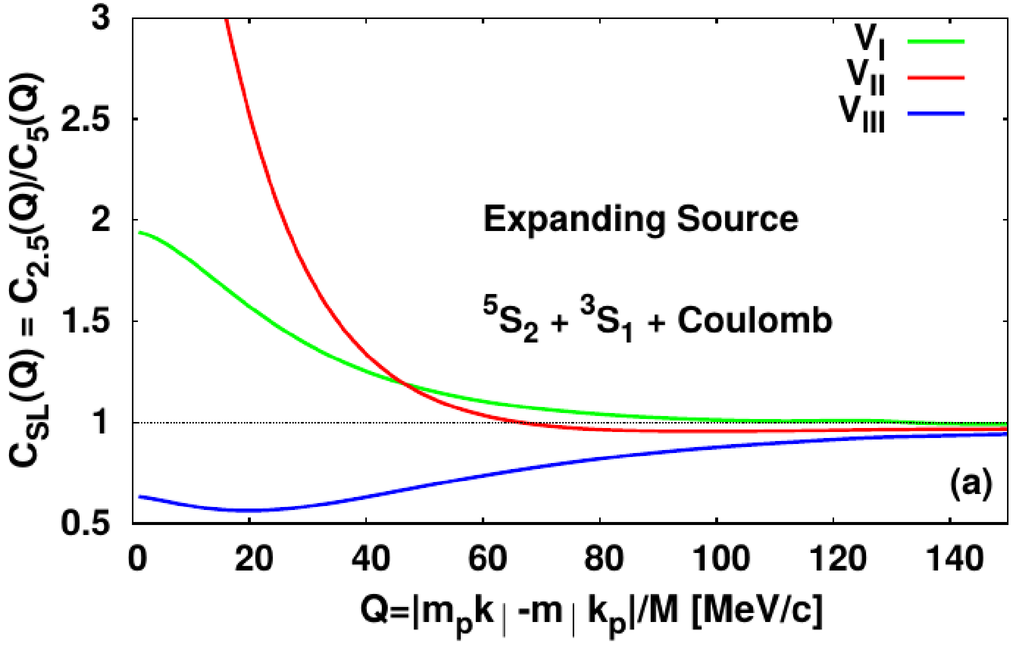
C(R=2.5 fm)/C(R=5 fm)

By taking small (R=2.5 fm) / large (R=5 fm) ratio, we approximately see the corr. fn. w/o Coulomb !

Comparison with STAR data

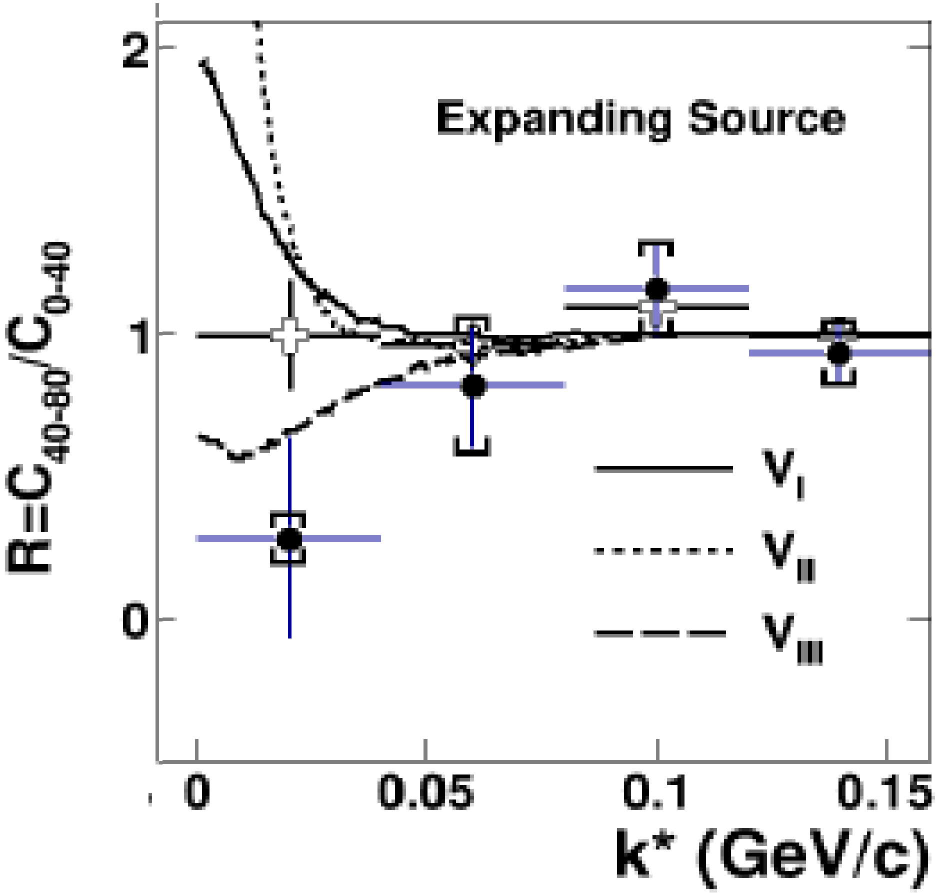
- **Dip structure in Small-Large ratio of CF**
  → We may have a dibaryon state in ΩN channel

C(R=2.5 fm)/C(R=5 fm)

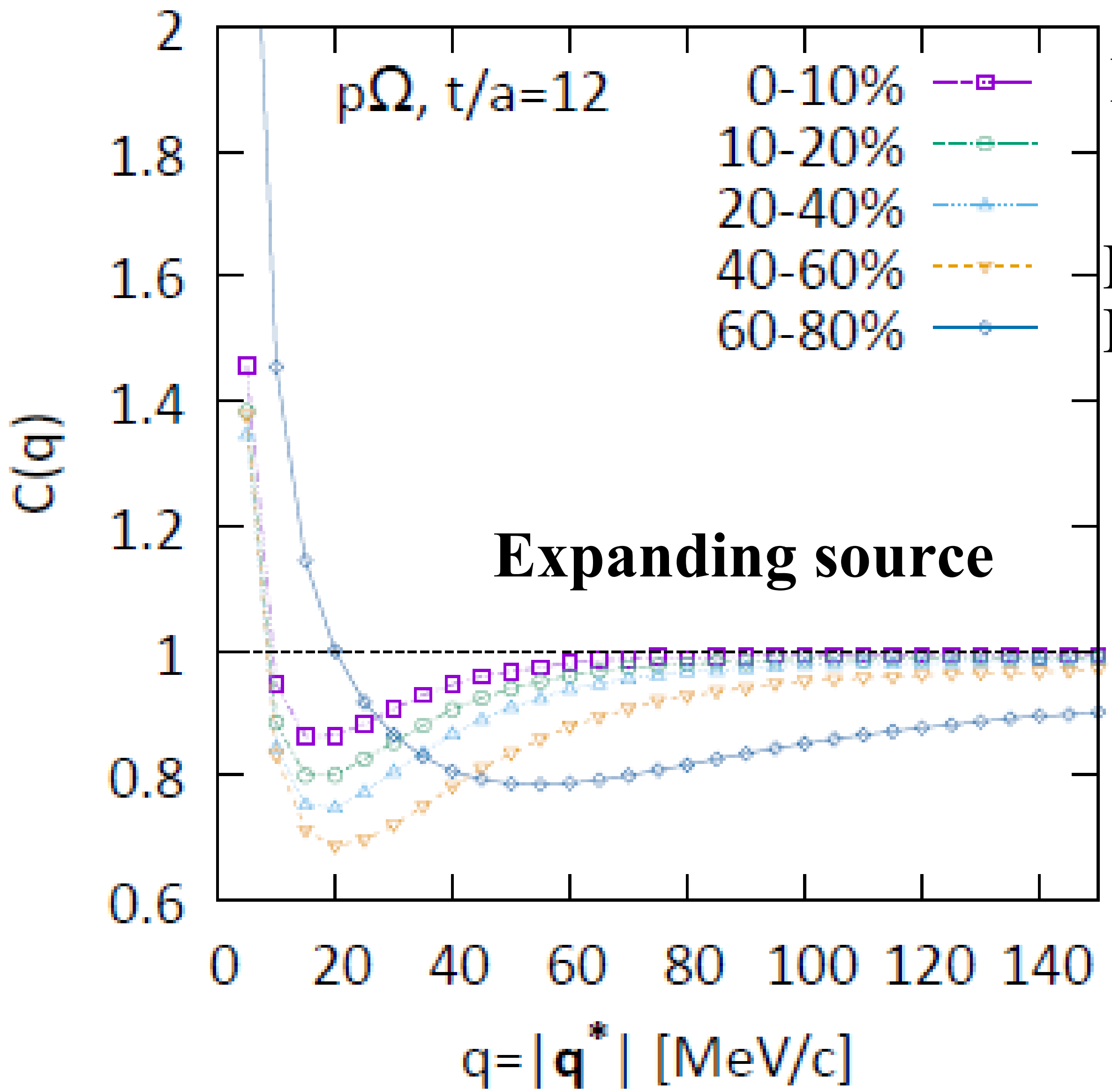


Morita, AO, Etminan, Hatsuda ('16)

STAR (1808.02511, PLB790 ('19) 490)



Source Size Dependence of Correlation Function



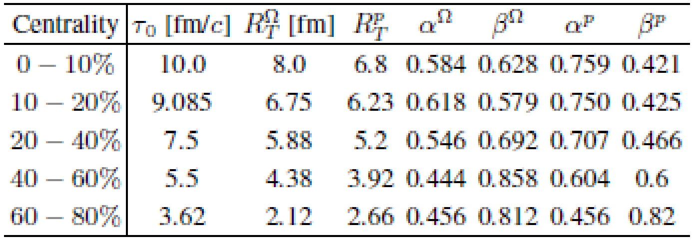
Expanding source

R ~ 8 fm

R ~ 4 fm

R ~ 2.5 fm

a_0 (pΩ)~ 3.4 fm



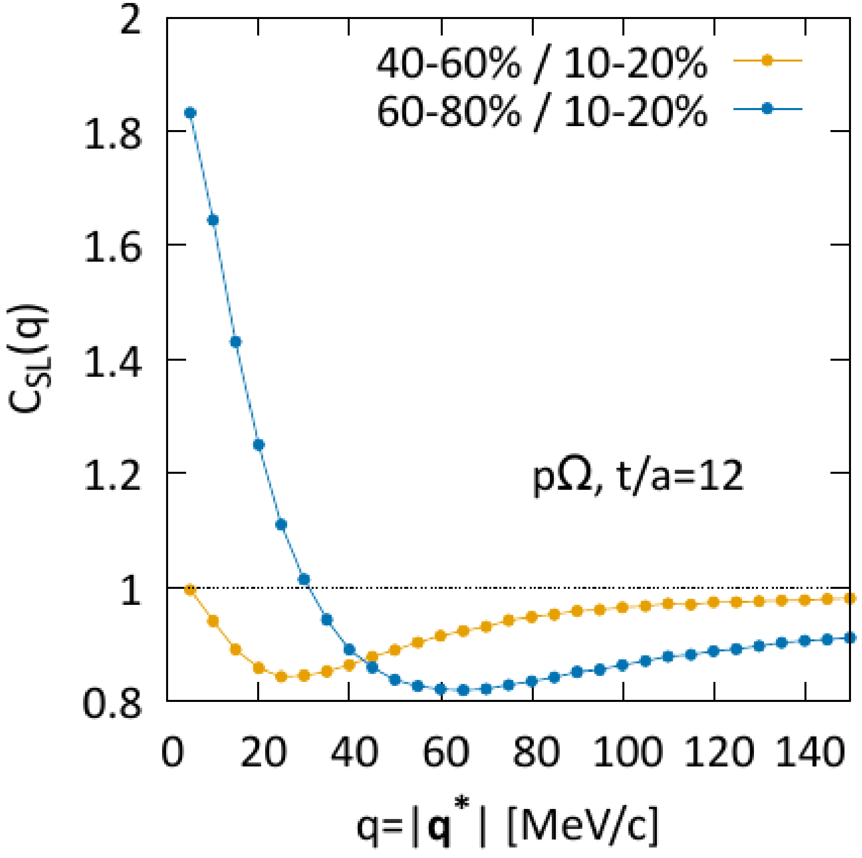
Centrality	τ_0 [fm/c]	R_T^Ω [fm]	R_T^p	α^Ω	β^Ω	α^p	β^p
0 – 10%	10.0	8.0	6.8	0.584	0.628	0.759	0.421
10 – 20%	9.085	6.75	6.23	0.618	0.579	0.750	0.425
20 – 40%	7.5	5.88	5.2	0.546	0.692	0.707	0.466
40 – 60%	5.5	4.38	3.92	0.444	0.858	0.604	0.6
60 – 80%	3.62	2.12	2.66	0.456	0.812	0.456	0.82

K. Morita, S. Gongyo, T. Hatsuda,
T. Hyodo, Y. Kamiya, AO ('19)

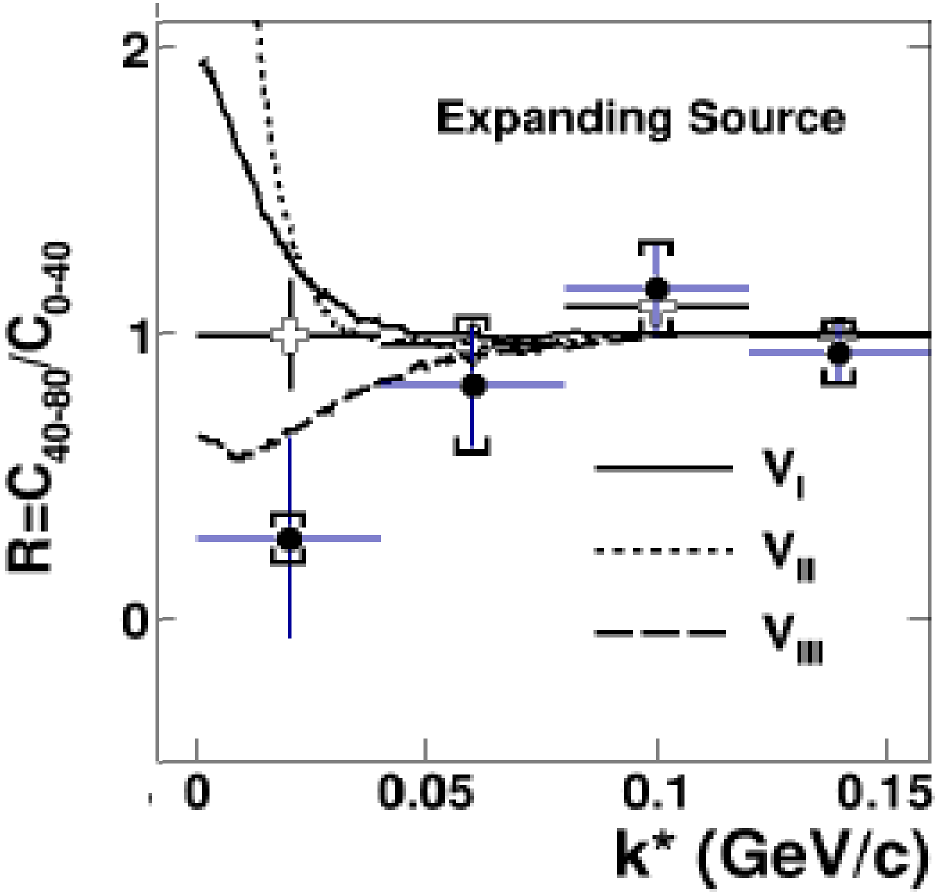
Comparison with STAR data

- **Results with potential at nearly physical quark mass (= between V_{II} and V_{III})**
  → Dip is seen but is not deep enough to explain STAR data.

Morita, Gongyo, Hatsuda, Hyodo, Kamiya, AO, arXiv:1908.05414



STAR (1808.02511, PLB790 ('19) 490)

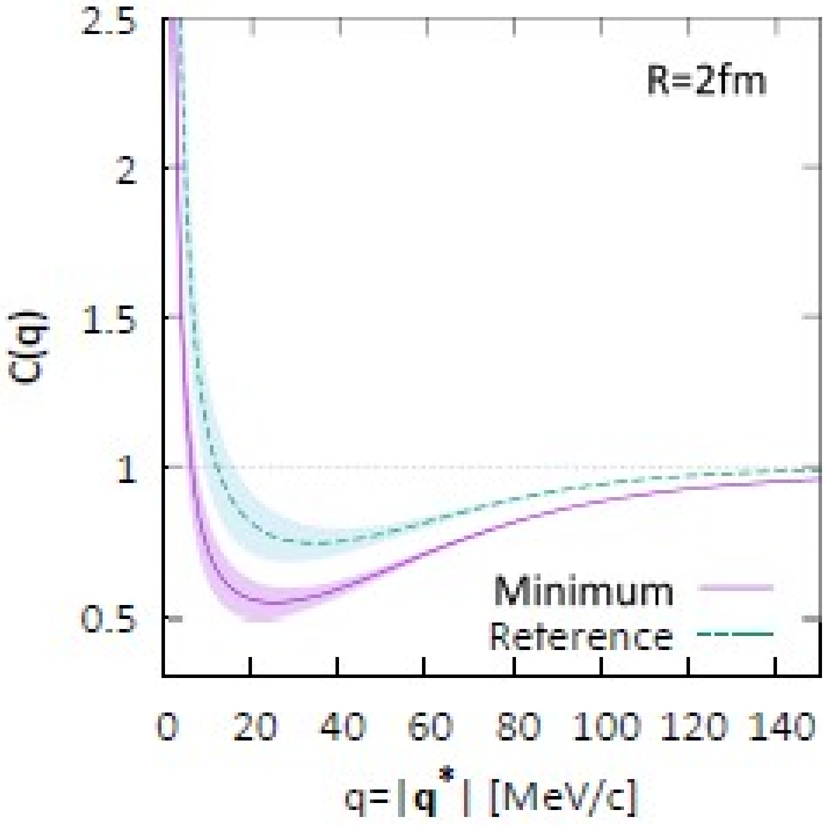


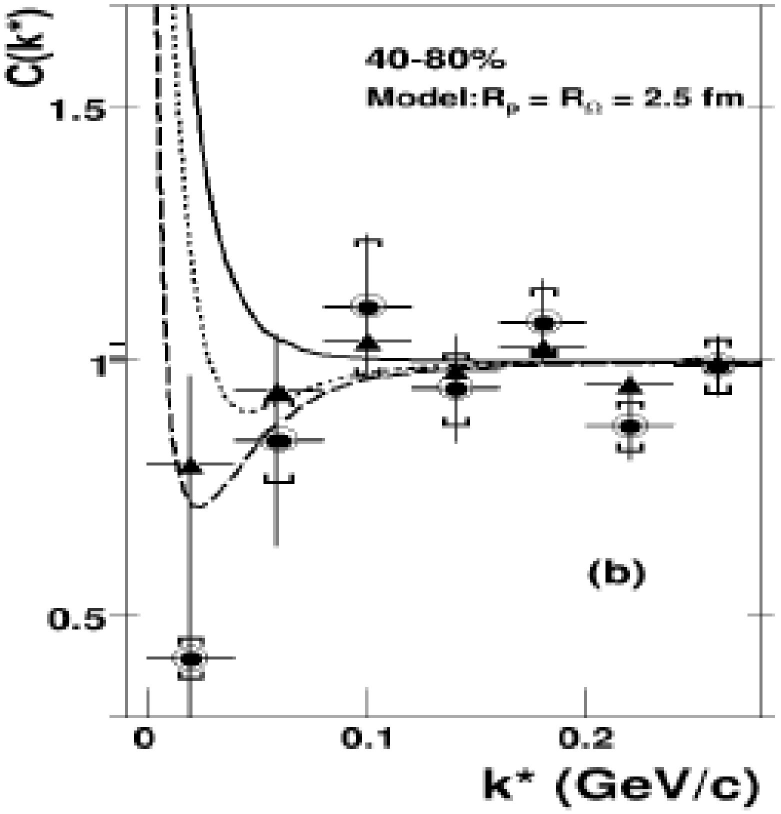
Comparison with STAR data

- **Results with Gaussian source**
  → Agrees with STAR data within 1σ
- **Detail discussion of source is also necessary….**

Morita, Gongyo, Hatsuda, Hyodo, Kamiya, AO, arXiv:1908.05414

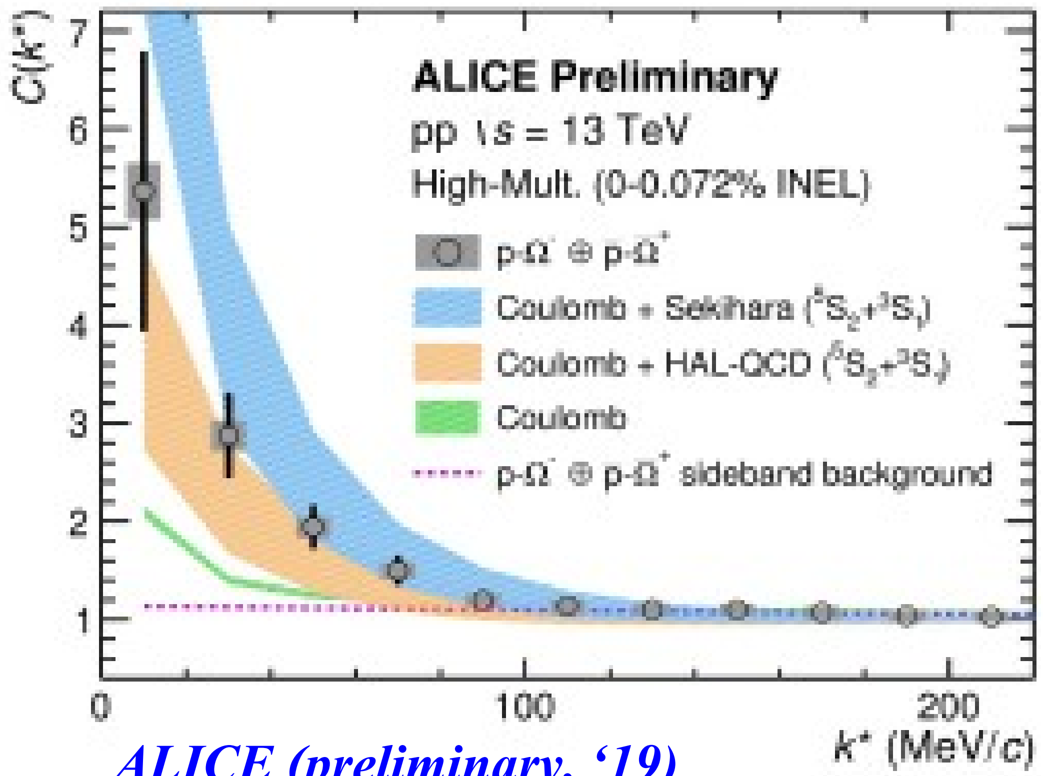
STAR (1808.02511, PLB790 ('19) 490)



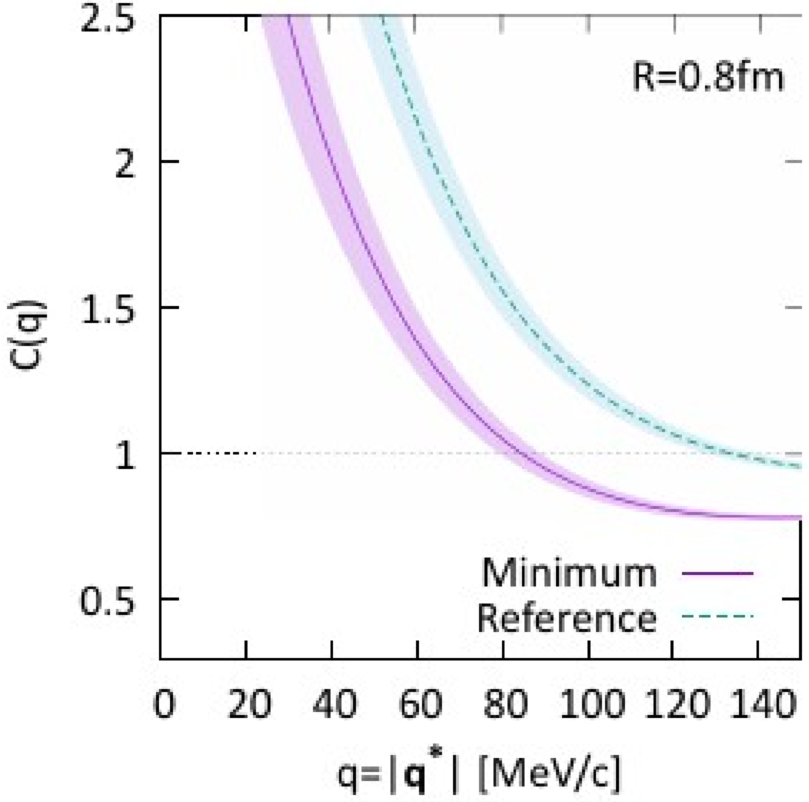


ALICE Preliminary

- **pp 13 TeV high-multiplicity events in ALICE**
  → Strong enhancement of CF at small q
  O. Vázquez Doce et al. (ALICE), Hadrons 2019

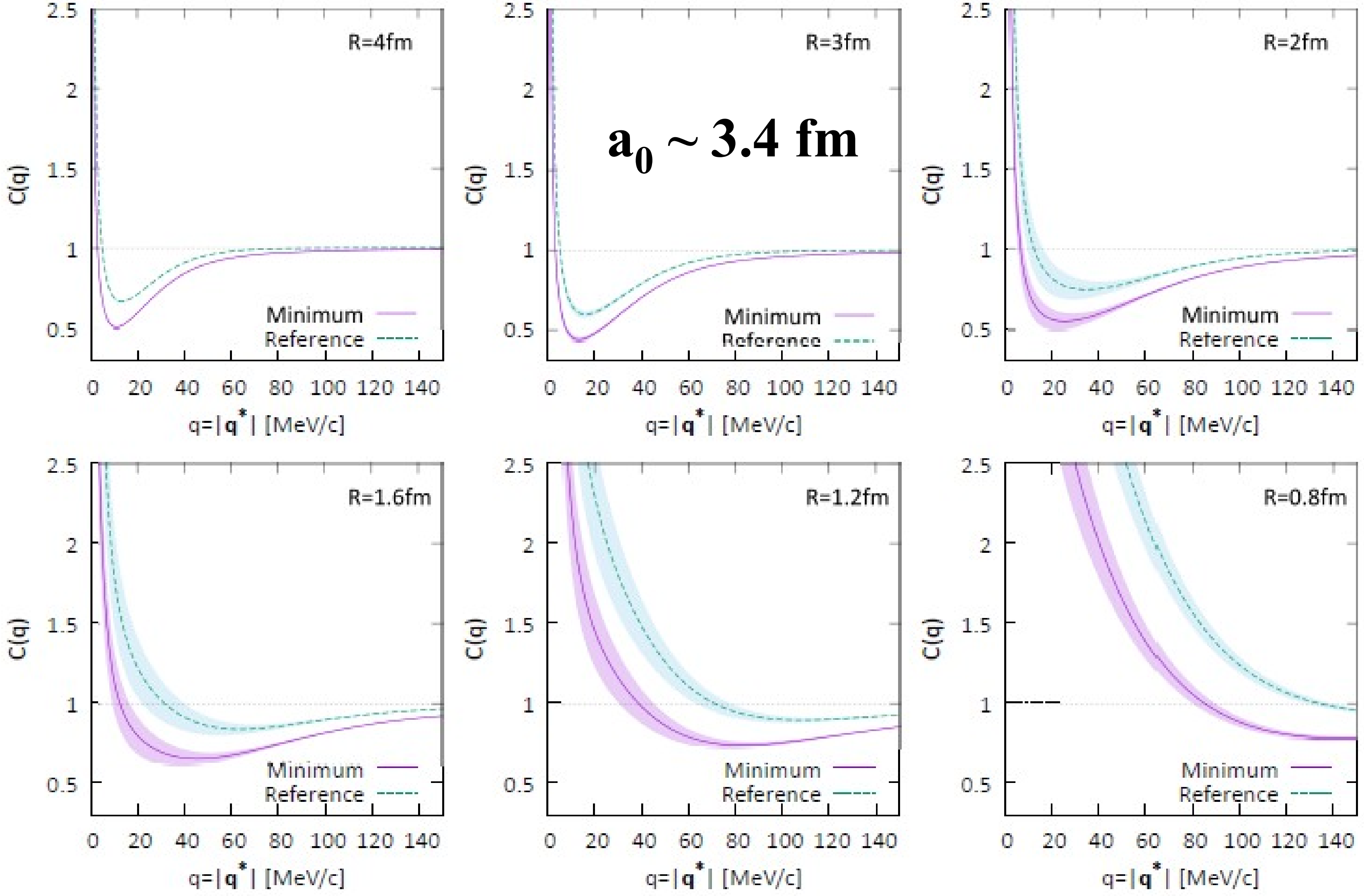


ALICE (preliminary, '19)



Morita+, arXiv:1908.05414

Source Size Dependence of Correlation Function

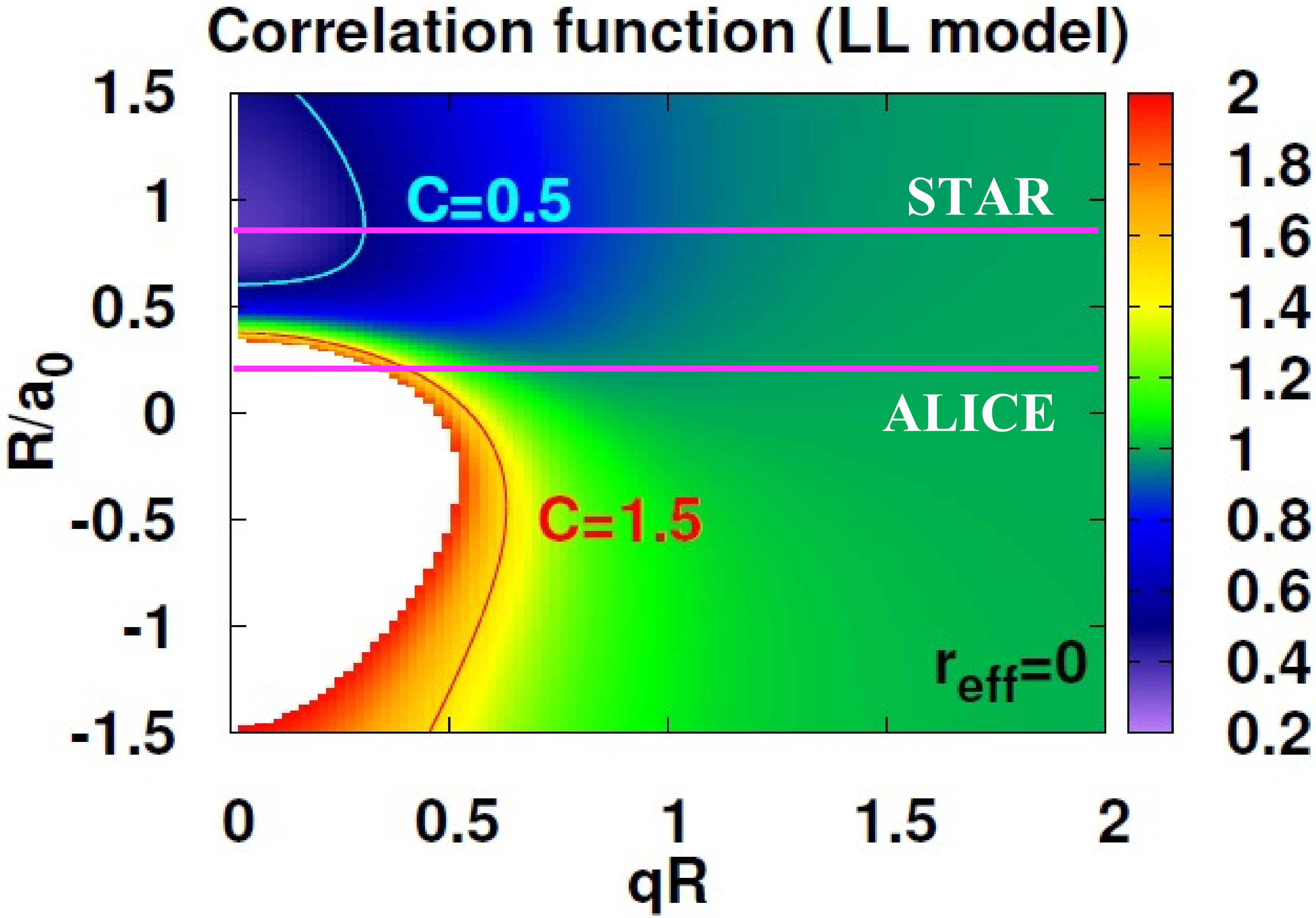


$a_0 \sim 3.4$ fm

Gaussian Source

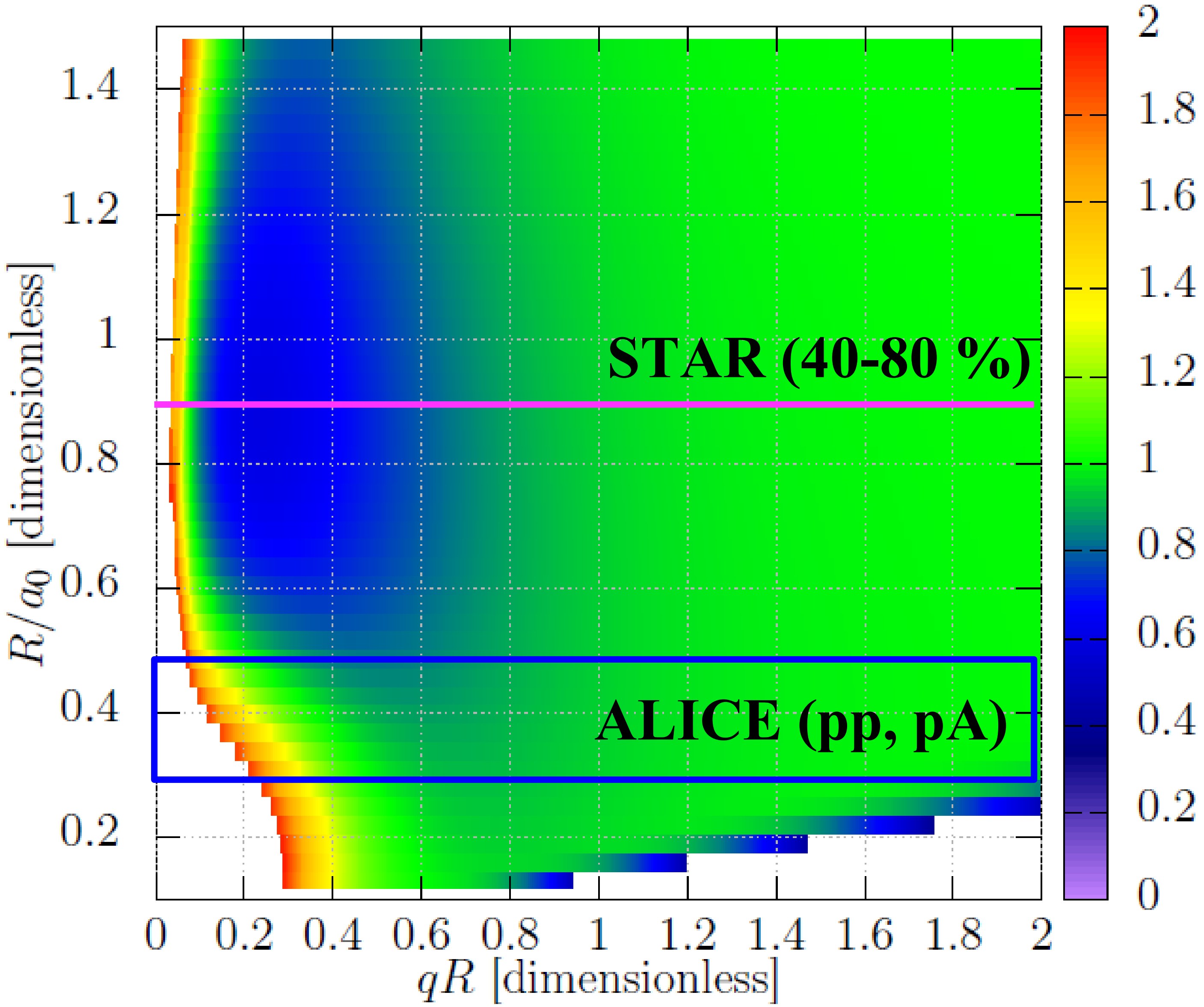
K. Morita, S. Gongyo, T. Hatsuda,
T. Hyodo, Y. Kamiya, AO ('19)

Correlation Function in LL model



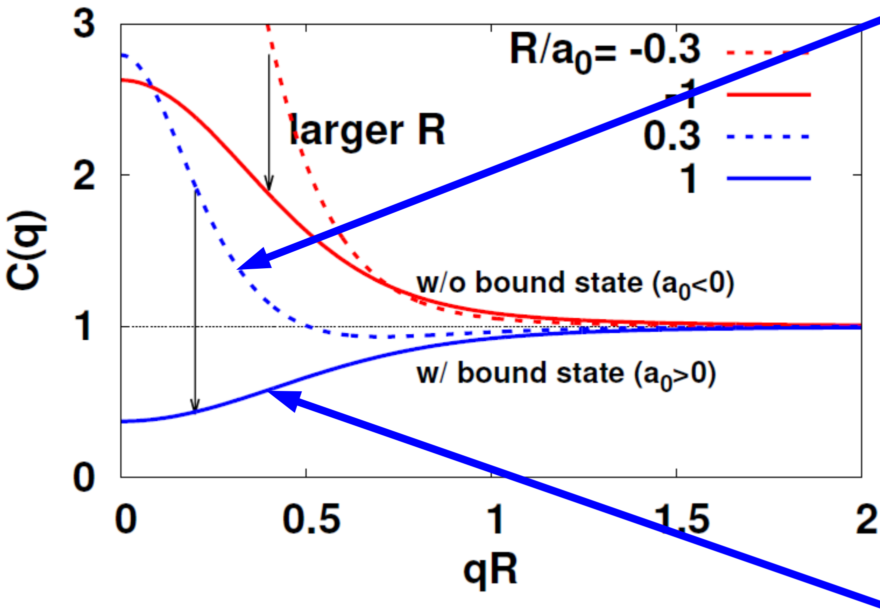
a_0 (pΩ)~ 3.4 fm, R(ALICE)~0.7 fm, R(STAR)~3 fm

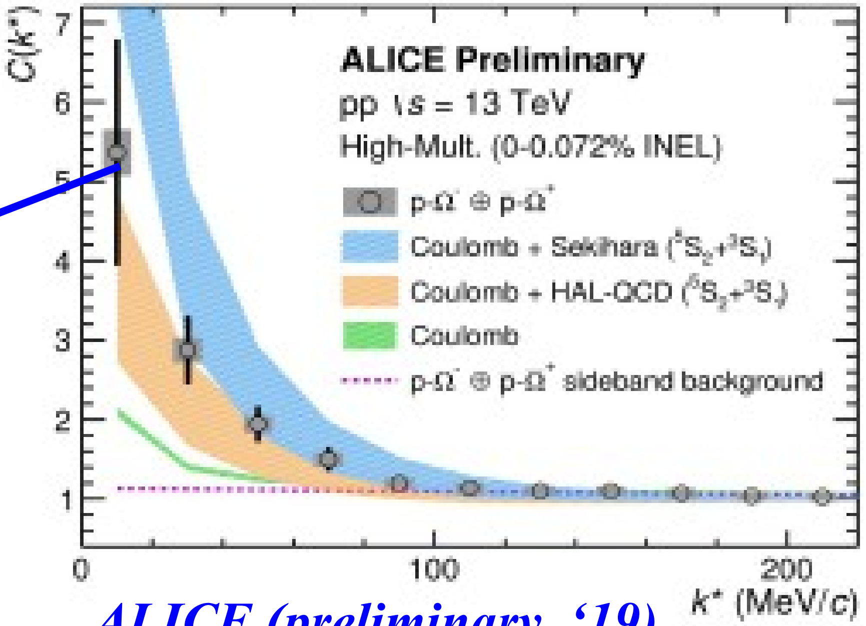
Correlation Function with Gaussian source



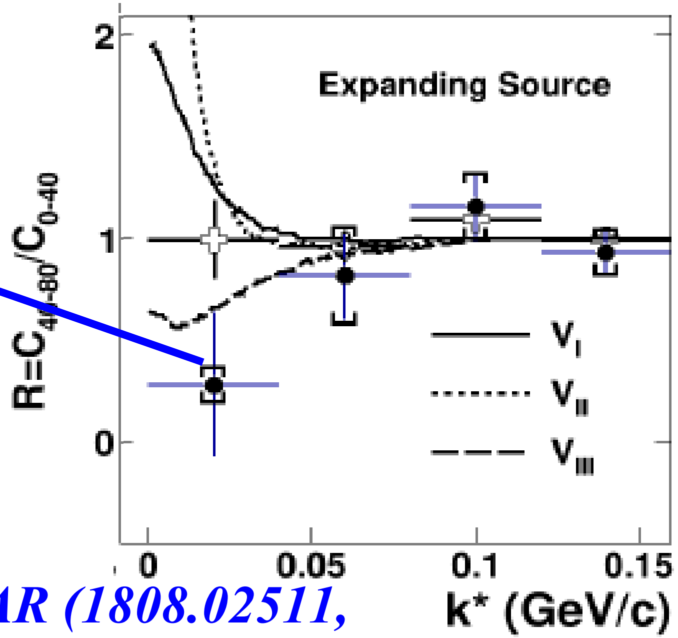
NΩ potential (J=2, HAL QCD, a_0=3.4 fm) + Coulomb

STAR + ALICE = NΩ Dibaryon





ALICE (preliminary, '19)



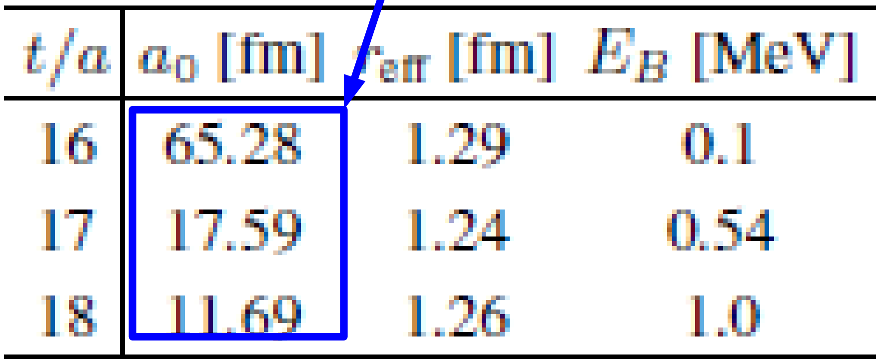
STAR (1808.02511, PLB790 ('19) 490)

Do you know any mechanism to suppress C(q) other than the existence of bound state ? (Strong flow, ...)

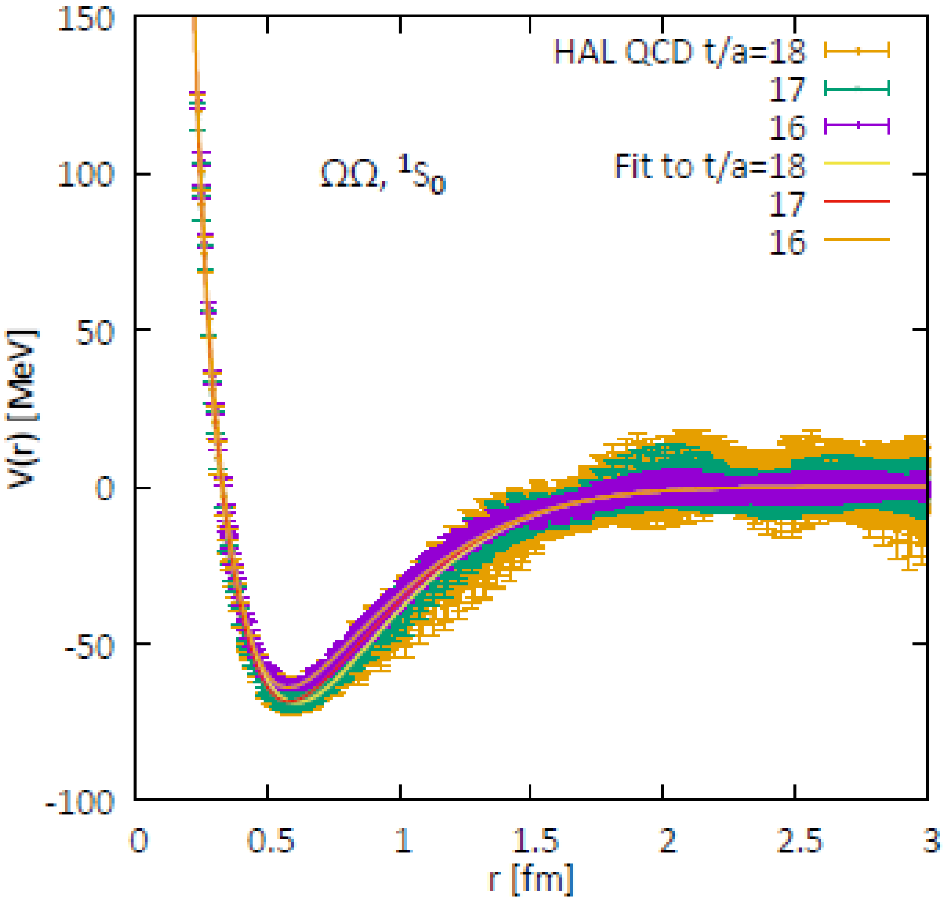
Example 4: ΩΩ dibaryon

- **ΩΩ potential from lattice QCD (J=0)**
  S. Gongyo et al. (HAL QCD Collab), Phys. Rev. Lett. 120, 212001 (2017).
  - **ΩΩ bounds for J=0 ! (Most strange dibaryon state)**
  - **B.E. is very small. B.E.=(0.1-1.0) MeV → $a_0 > 10$ fm**

very big !



t/a	a_0 [fm]	$r_{\rm eff}$ [fm]	E_B [MeV]
16	65.28	1.29	0.1
17	17.59	1.24	0.54
18	11.69	1.26	1.0

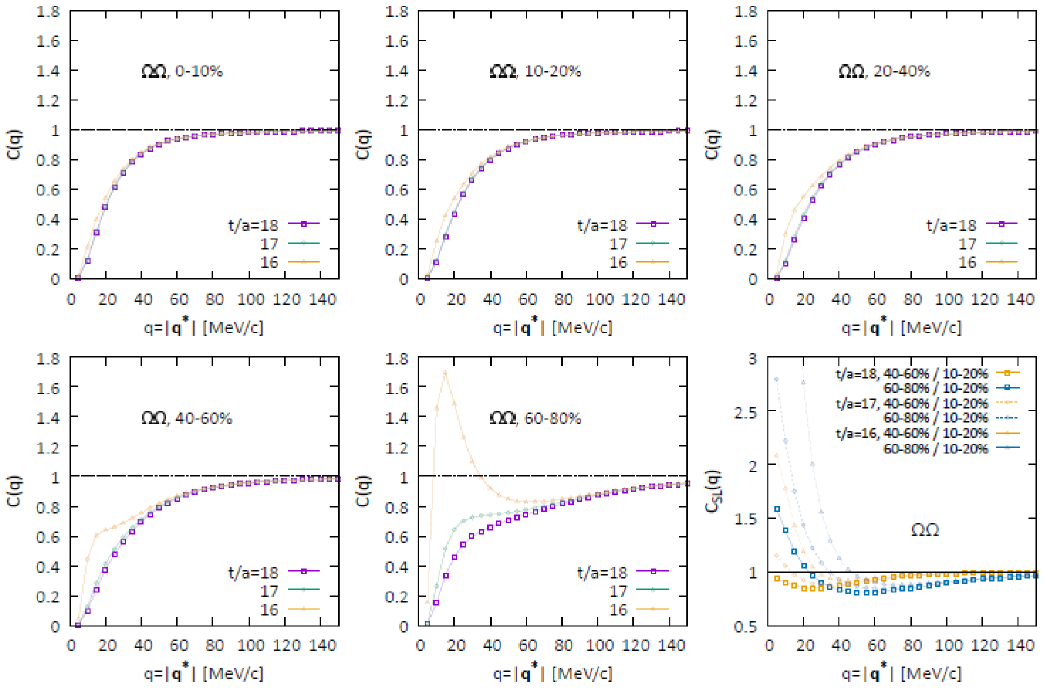


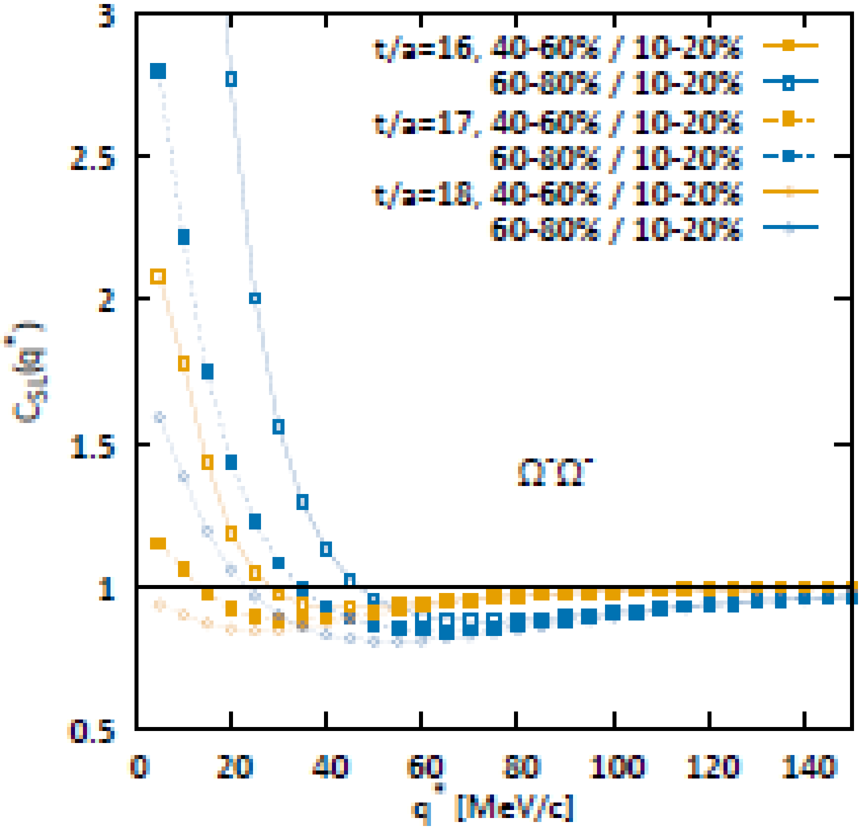
ΩΩ correlation

- **ΩΩ correlation**

  K. Morita, S. Gongyo, T. Hatsuda, T. Hyodo, Y. Kamiya, AO, arXiv:1908.05414

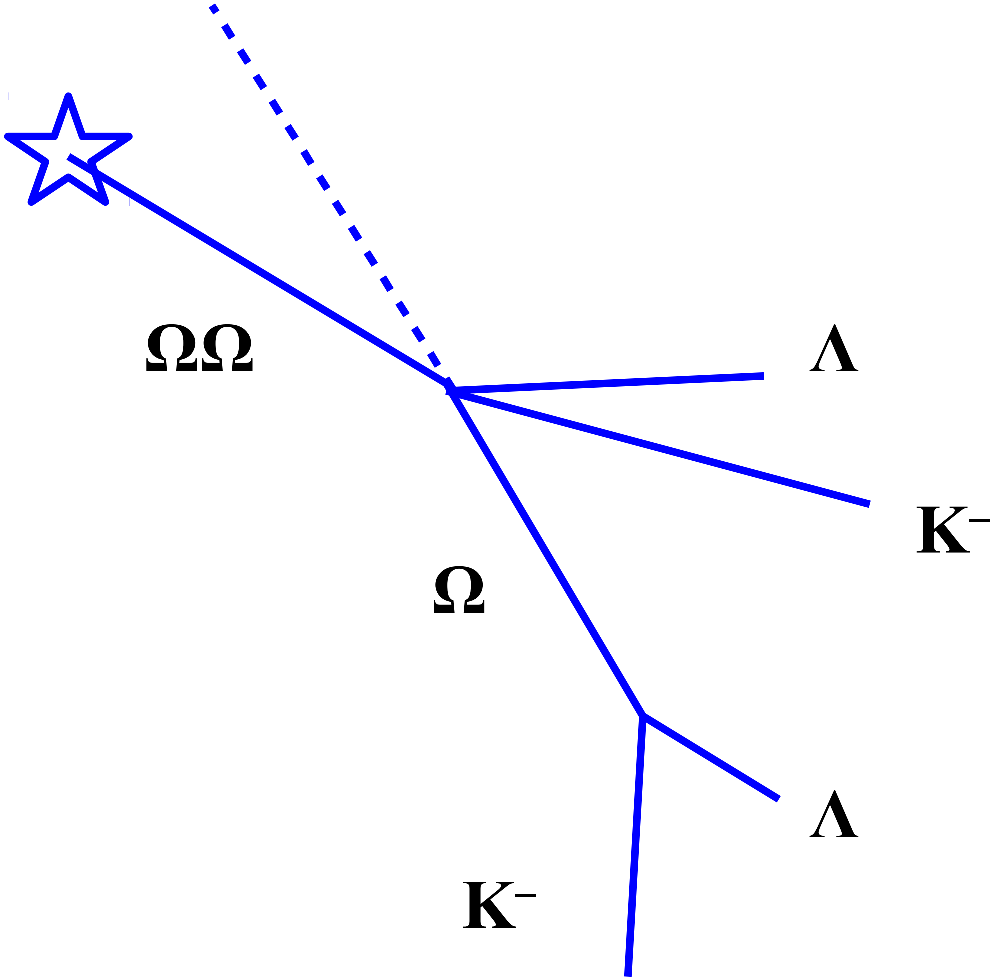
  - J ≠ 0 potentials are assumed to be zero.
  - Large scattering length
    → $R/a_0 < 0.5$ (unitary regime), but Coulomb suppressed





Detecting ΩΩ dibaryon ?

- **Large DCA Ω ? (J. Schaffner-Bielich)**
  - **Implies a weakly decaying ΩX bound state.**
  - **Fourier transf. of w.f. in ΩΩ → Ω momentum dist.**
  - **Production rate of ΩΩ ~ 1.8 x 10^{-5} at LHC ExHIC ('17)**



Summary

- It is fun to find hadronic bound states.
- Bound states show characteristic features in C(q) at small q,
  - Enhanced C(q) for $R/a_0 < 0.4$,
  - Suppressed C(q) for $0.5 < R/a_0 < 2$
- ALICE and STAR data strongly suggest the existence of S= –3 dibaryon as a bound state of NΩ.
  - Strong enhancement of pΩ corr. fn. found in ALICE data implies large $|a_0|$.
  - Coulomb potential enhances C(q).
  - Then I do not know the mechanism to show suppressed C(q) other than the existence of a bound state.
- Source size dep. of $p\Xi^-$ and pK^- corr. fn. would be interesting.
- Detecting diomega ? Fantastic.

To do (or Can do)

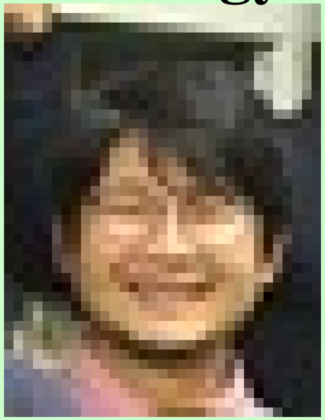
- **Coupled channel effects → Talk by Haidenbauer, Kamiya**
- **Ξ^- p correlation with updated HAL QCD potential (K. Sasaki et al.) → Sasaki's talk**
- **Λp correlation with various potentials (χEFT, Nijmegen, fss2, lattice) → Talk by Heidenbauer, Rijken**
- **K^- p correlation with amplitude from chiral SU(3) dynamics (e.g. Ikeda, Hyodo, Weise) → Kamiya's talk**
- **J=1 Ω^- p potential → Hyodo's talk**
- ***Let us use deuteron !***
  K^- d corr. (I=0 ampl.), Λd corr. ($^3_\Lambda$H B.E., 3BF), Ξ^- d, …
- ***Can we go to heavy-quarks ?***
  $c\tau(D) = 0.3$ mm → $\gamma c\tau(D) = c\tau(D) \cosh y \sim 15$ cm ($y=7$)
  We may have enough D mesons at y=7 in fixed target LHC.
  (N. Yamanaka)

Thank you for your attention !

Coauthors of Morita+('19) [arXiv:1908.05414]

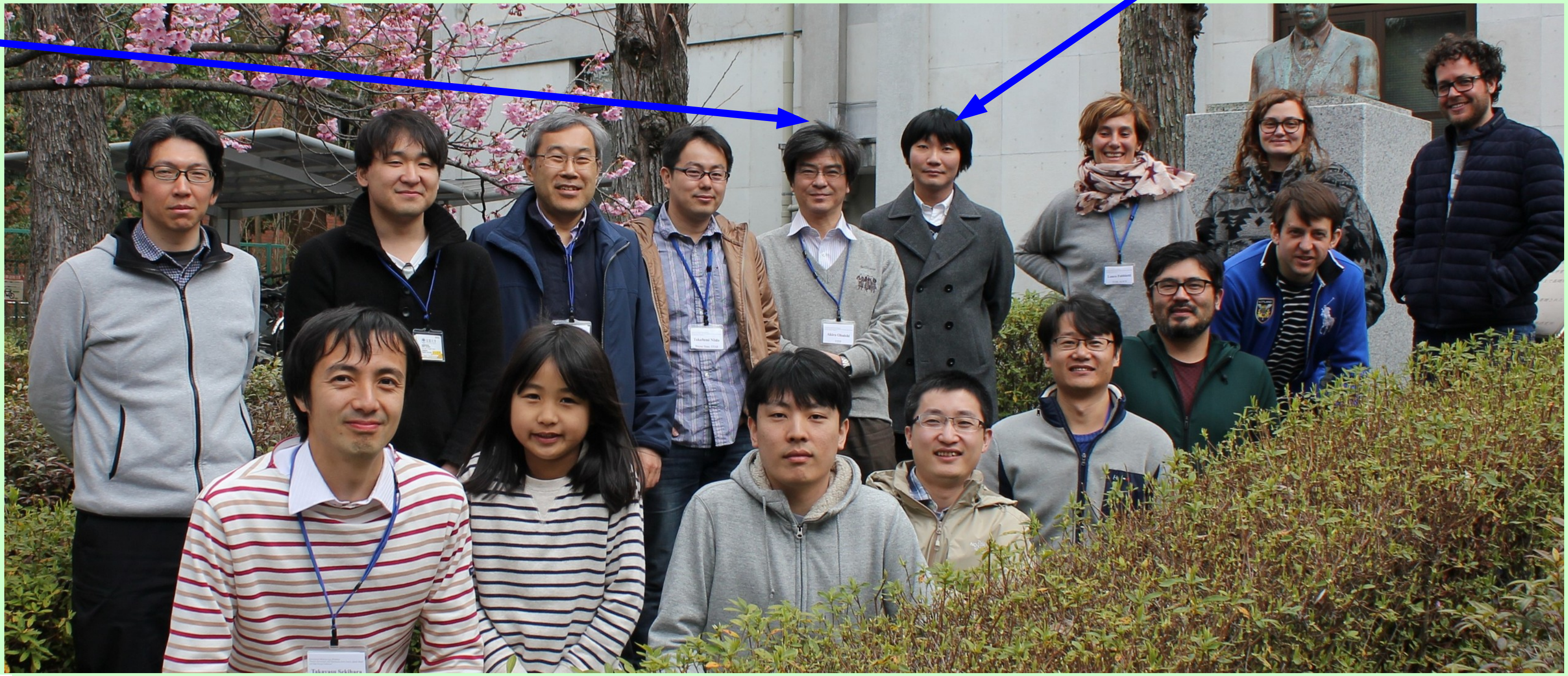
K. Morita

S. Gongyo



T. Hatsuda

T. Hyodo

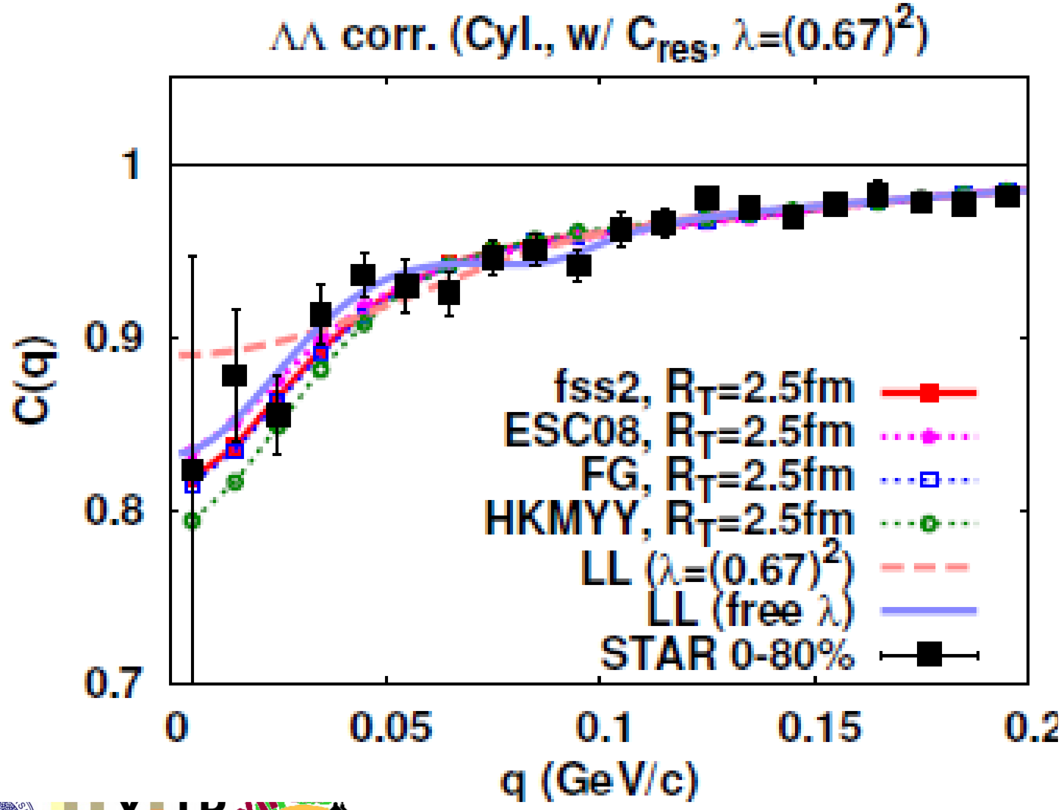


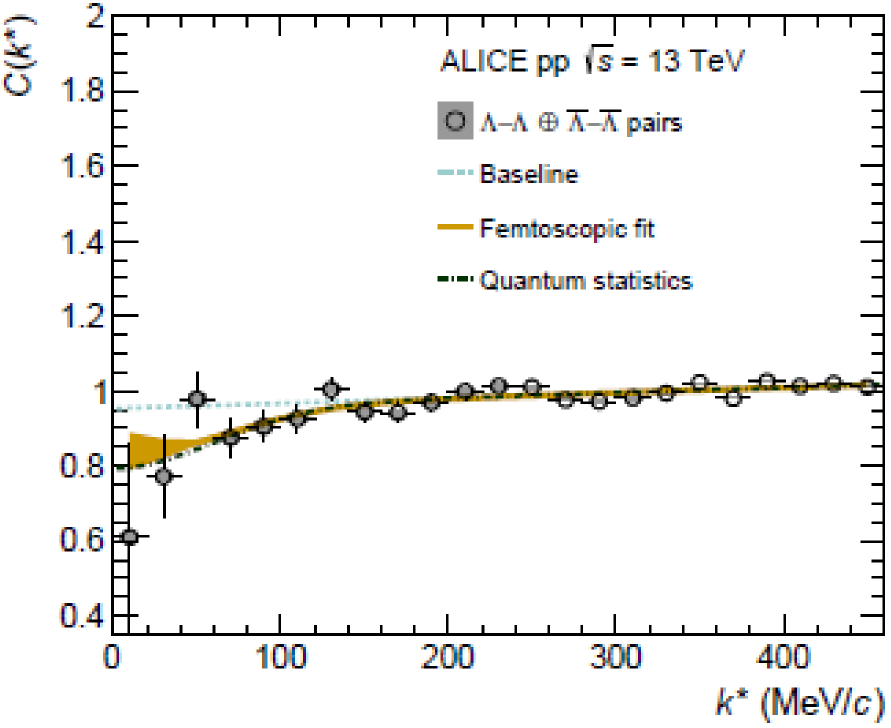
Y. Kamiya

AO

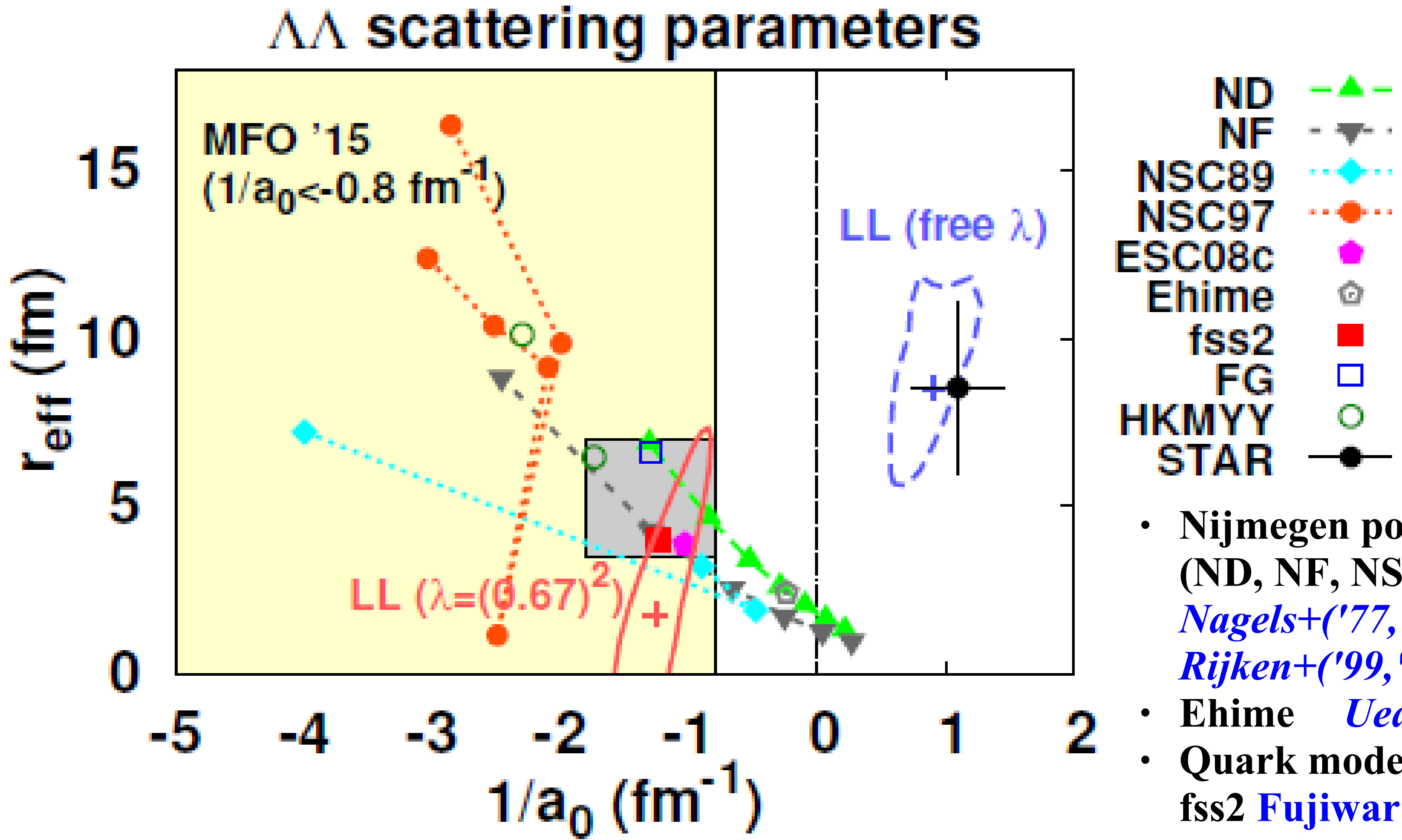
ΛΛ correlation at RHIC

- **STAR collaboration at RHIC measured ΛΛ correlation !**
  Adamczyk et al. (STAR Collaboration), PRL 114 ('15) 022301.
- **Theoretical Analysis well explains the data**
  K.Morita et al., T.Furumoto, AO, PRC91('15)024916;
  AO, K.Morita, K.Miyahara, T.Hyodo, NPA954 ('16), 294.
- **New Data from ALICE**
  S. Acharya+ (ALICE), PLB 797('19), 134822 [1905.07209]





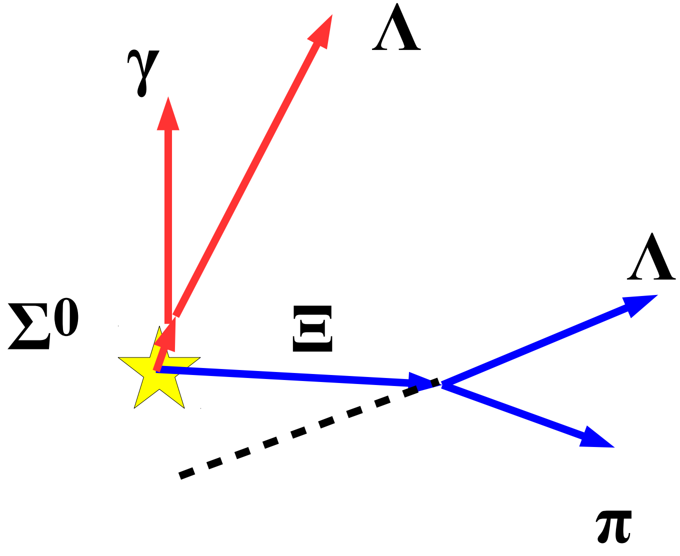
ΛΛ interaction from ΛΛ correlation



- **Nijmegen potentials (ND, NF, NSC89, NSC97, ESC08)** *Nagels+('77, '79), Maessen+('89), Rijken+('99,'10)*
- **Ehime** *Ueda et al. ('98)*
- **Quark model interaction:** fss2 Fujiwara et al.('07)
- **Potential fitted to Nagara** *Filikhin, Gal ('02) (FG), Hiyama et al. ('02, '10)(HKMYY)*

$$q \cot \delta = -1/a_0 + r_{\text{eff}}\, q^2/2 + O(q^4)$$

Additional Source



- **Feed down effects**

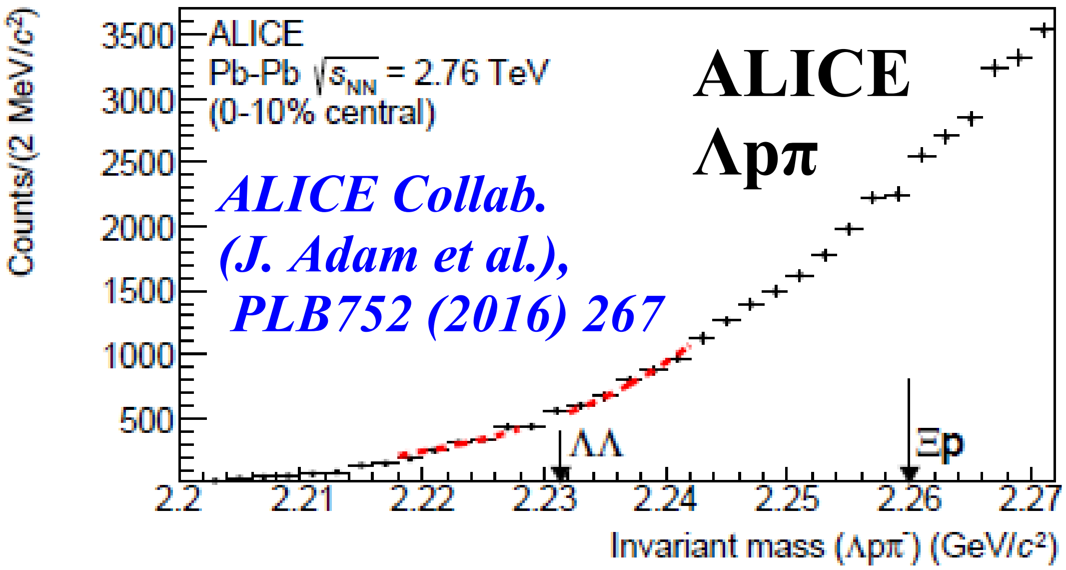
$$C_{\mathrm{corr}}(Q) = 1 + \lambda(C_{\mathrm{bare}}(Q) - 1)$$

λ = Purity of ΛΛ pair

  - **Short-lived Y* → mod. of source fn.**
  - **Ξ → Λπ can be excluded (cτ=8.71 cm)**
  - **Σ^0 → Λγ is difficult to reject**
  - **Data based purity $\lambda=(0.67)^2$**
    Σ^0/Λ=0.278 (p+Be, 28.5 GeV/c) ***Sullivan et al. ('87)***
    Ξ/Λ = 15 % (RHIC)

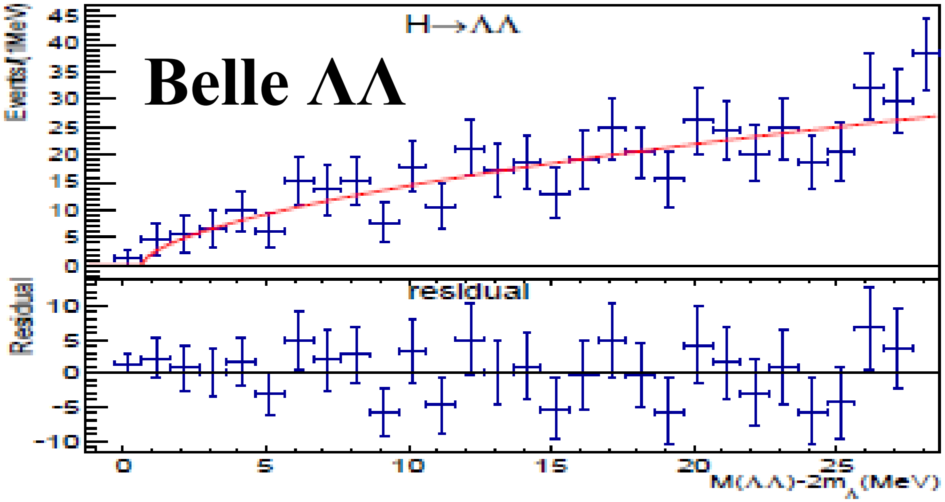
- **"Residual" source**
  - **High-momentum tail → R_{res} ~ 0.5 fm (STAR collab.)**

ΛΛ invariant mass / BB correlation function (as of 2016)



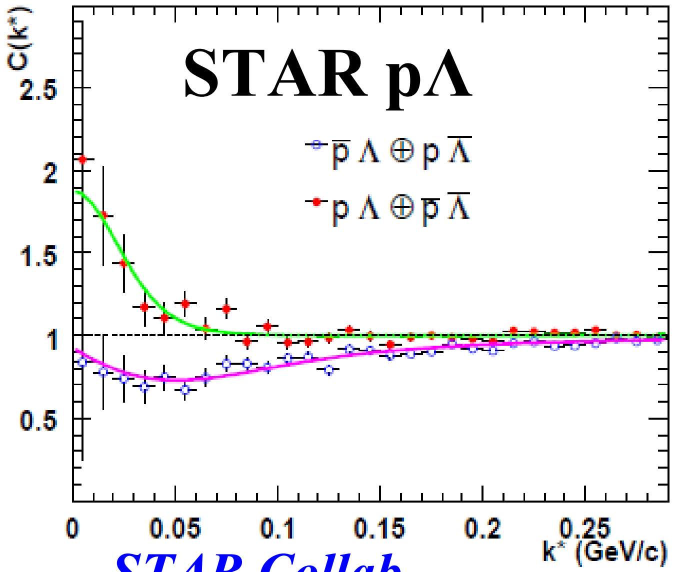
ALICE Λpπ

ALICE Collab. (J. Adam et al.), PLB752 (2016) 267



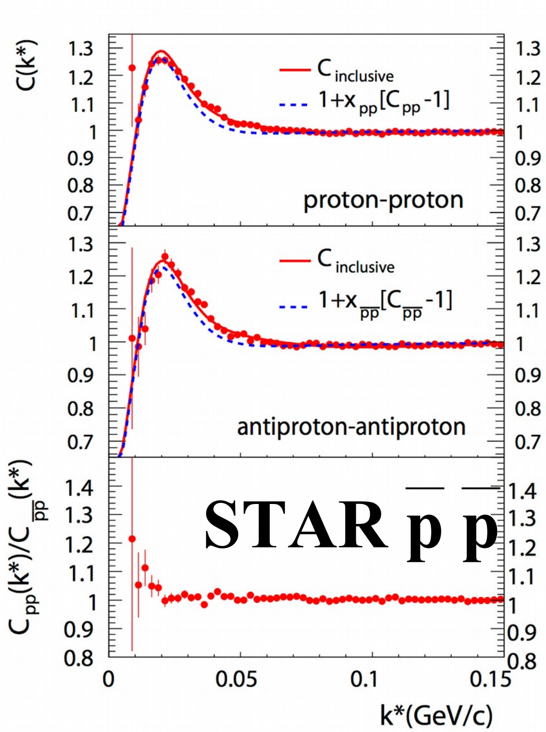
Belle ΛΛ

Belle Collaboration (Kim, B.H. et al.), PRL110('13)222002.



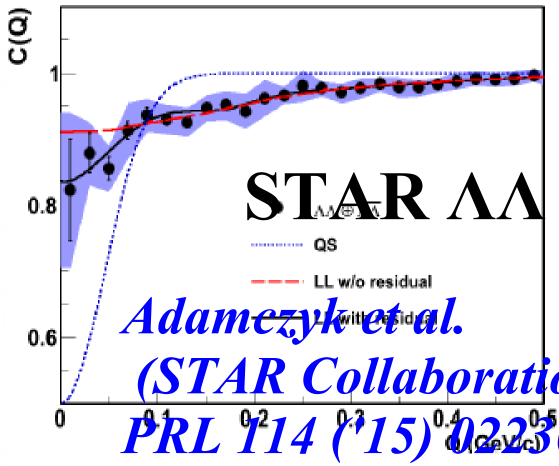
STAR pΛ

STAR Collab. (J. Adams et al.), PRC74('06)064906.



STAR p̄ p̄

STAR Collab. (J. Adams et al.) Nature 527('15)345



STAR ΛΛ

Adamczyk et al. (STAR Collaboration), PRL 114 ('15) 022301.

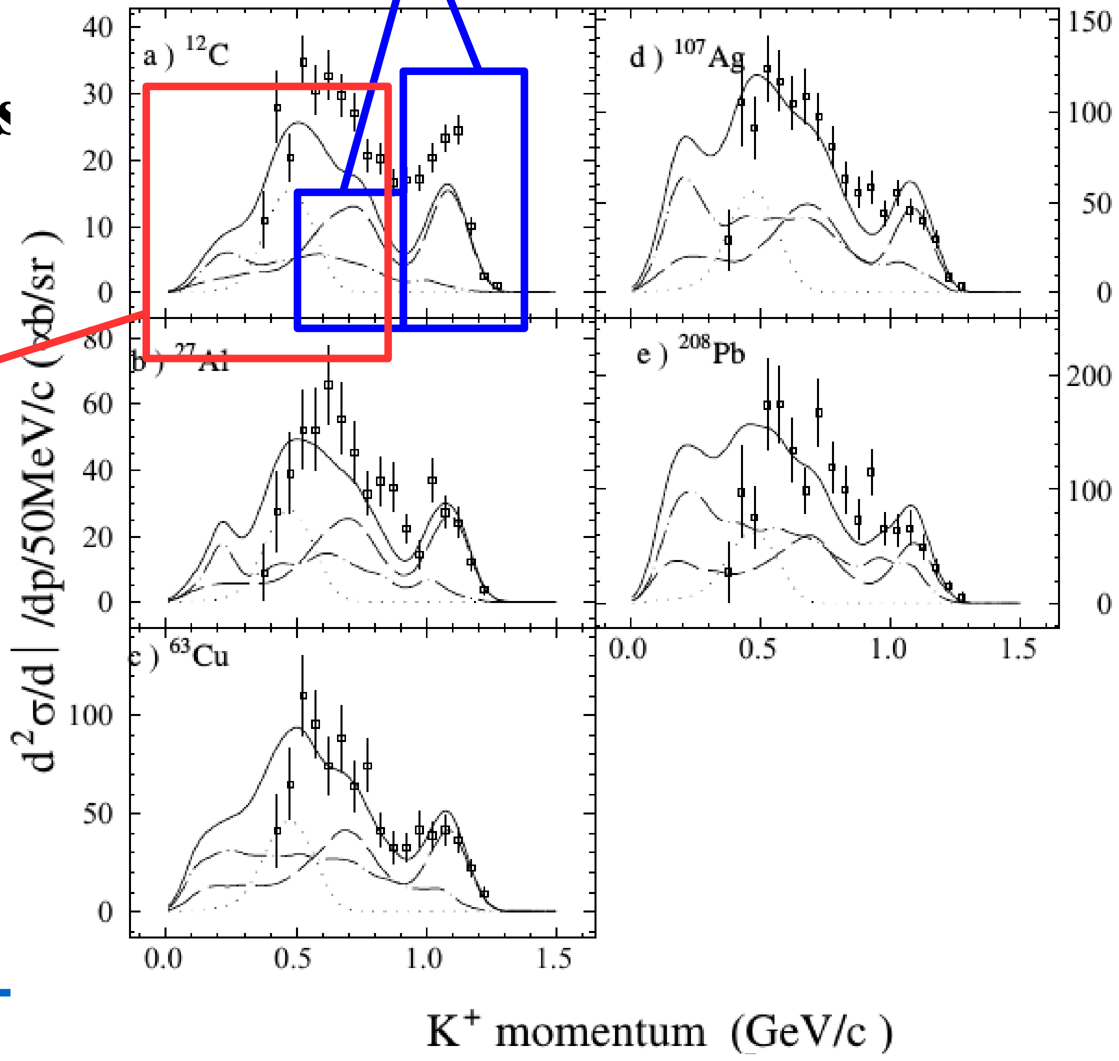
(K^-, K^+) reaction

- **(K^-, K^+) reaction = doorway to produce S=-2 systems**
  - **$K^- p \rightarrow K^+ \Xi^- \rightarrow \Xi$ nuclei, stopped Ξ to ΛΛ nuclei, …**
- **What is the origin of large yield at smaller K+ momentum ?**

  T. Iijima et al., NPA546('92) 588.

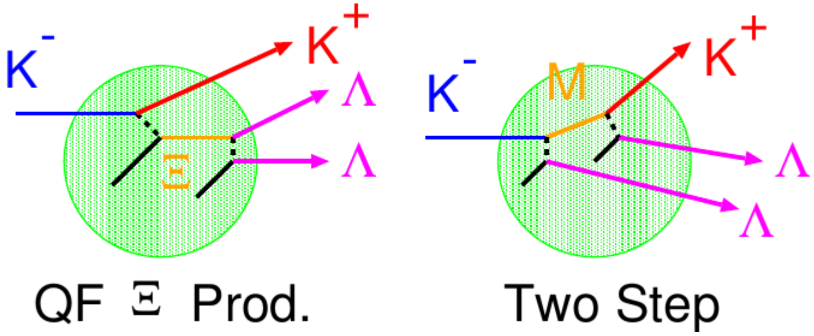
  → Various two step processes

  Y. Nara, AO, T. Harada, A. Engel, NPA614('97)433



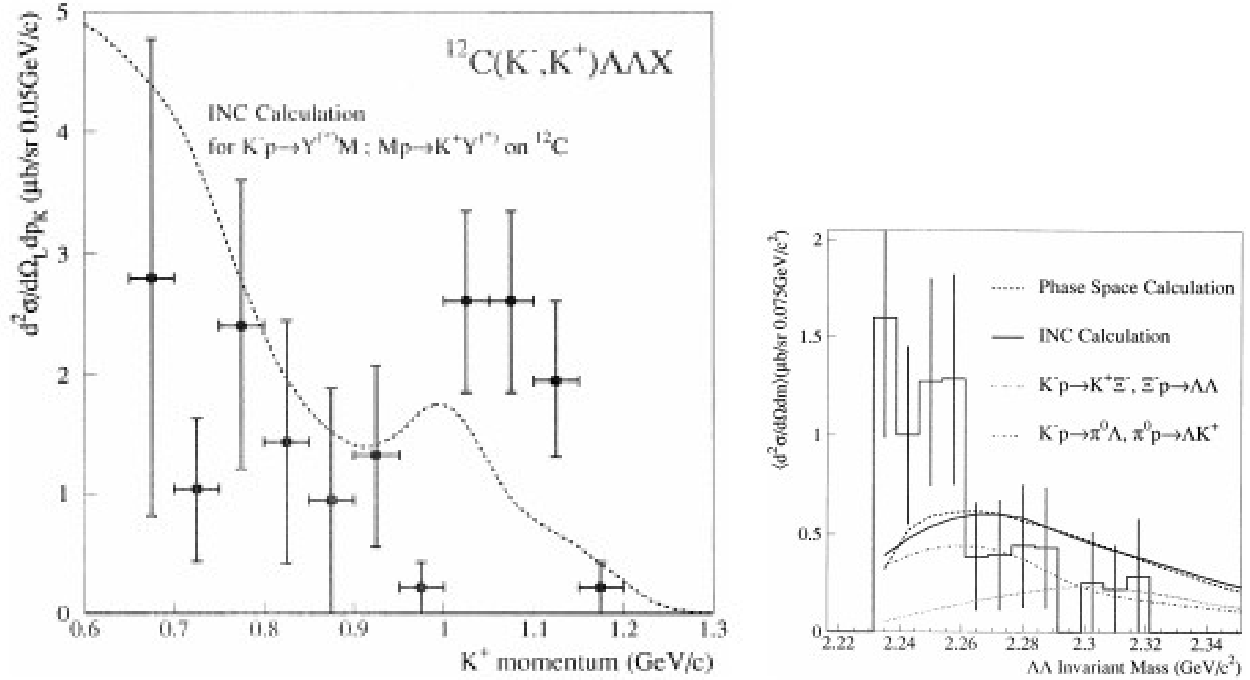
Quasi Free Ξ(Ξ*)

Multi-step

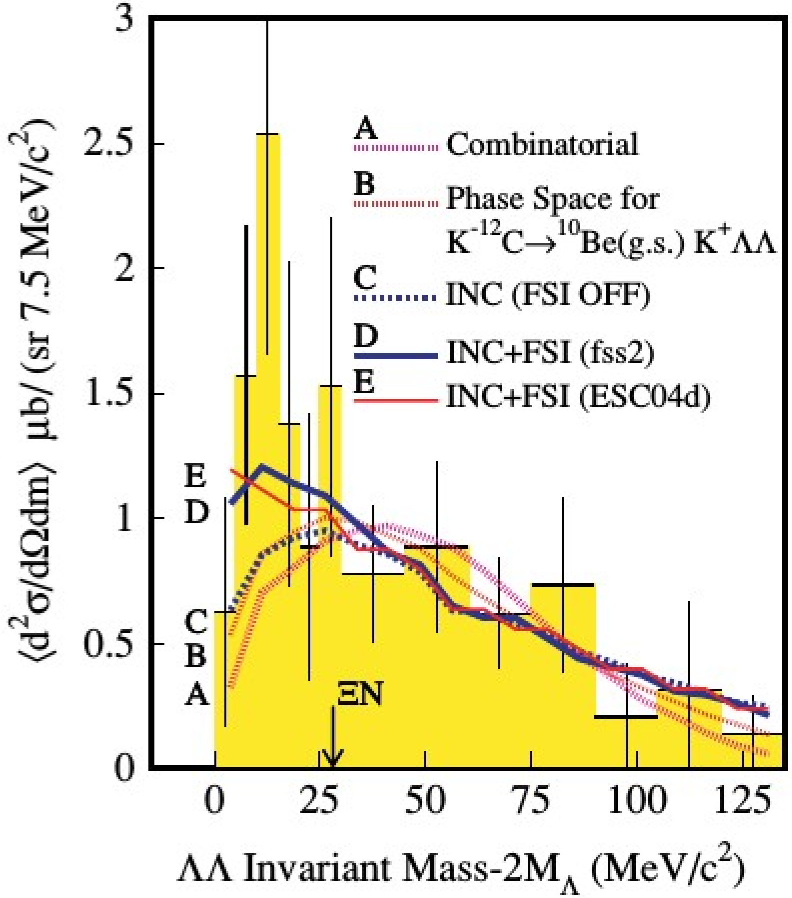


Two Λ production in (K^-, K^+) reactions

- **Experimentalists really measured two Λ emission !**
  J.K.Ahn et al. (KEK-PS E224 Collab.), PLB444('98)267.
  C. J. Yoon et al. (KEK-PS E522 Collab. +AO), PRC75('07)022201
- **Invariant mass spectrum of ΛΛ is enhanced from our cascade calculation.**
  → FSI enhancement ? or H particle ?



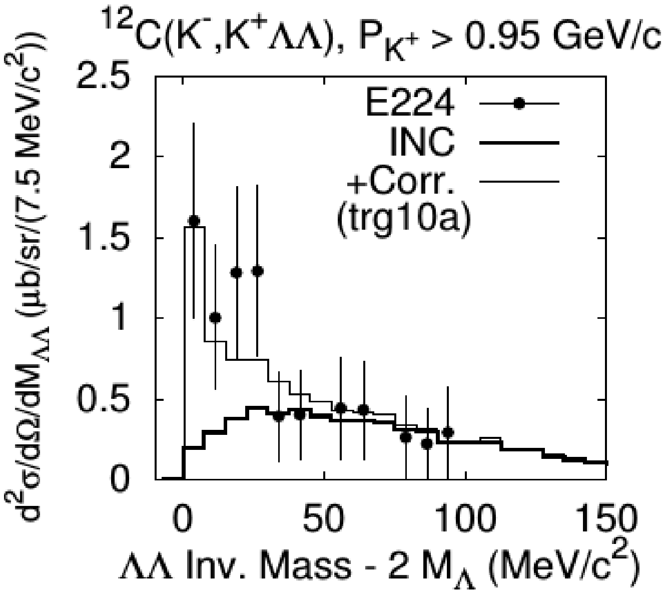
Ahn et al. (E224)('98)

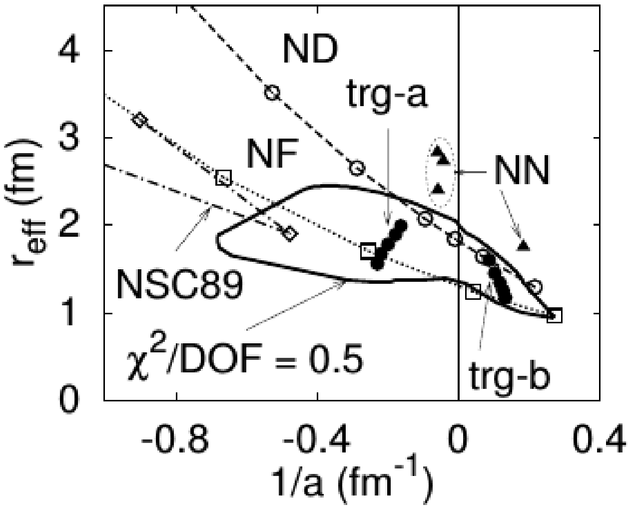


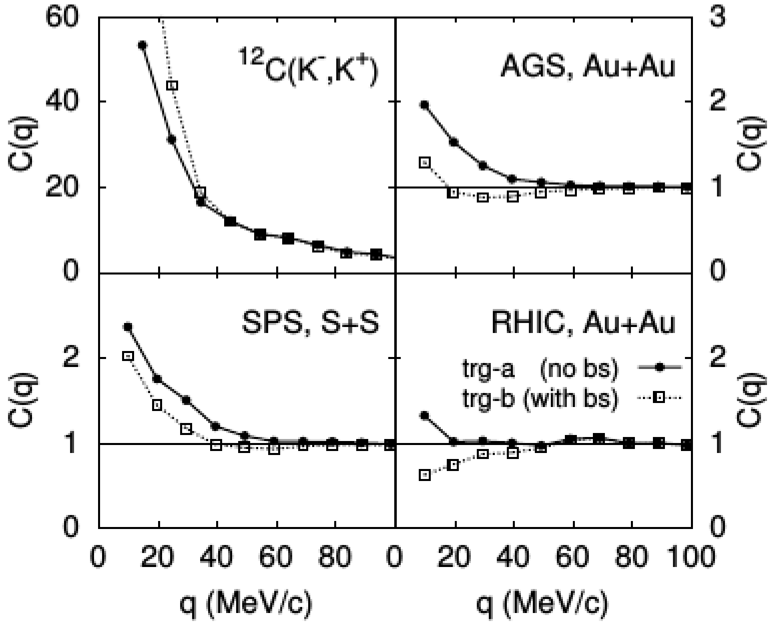
Yoon et al. (E522)('07)

From correlation to interaction

- **Enhancement of ΛΛ may show ΛΛ interaction effects !**
  AO, Y. Hirata, Y. Nara, S. Shinmura, Y. Akaishi,
  NPA670('00)297c; NPA684('01)595; NPA691('01)242c
  - **Enh. is roughly explained by ΛΛ final state int.**
  - **It should be clearer to measure in heavy-ion collisions.**
    Enh. w/o bound state, Suppression w/ bound state.
    → I asked Prof. Huan Z. Hunag (STAR) in ExHIC 2010 meeting and they measured it !







AO, Hirata, Nara, Shinmura, Akaishi ('00)