

Hadron-hadron correlation functions and its relation to exotic hadron search

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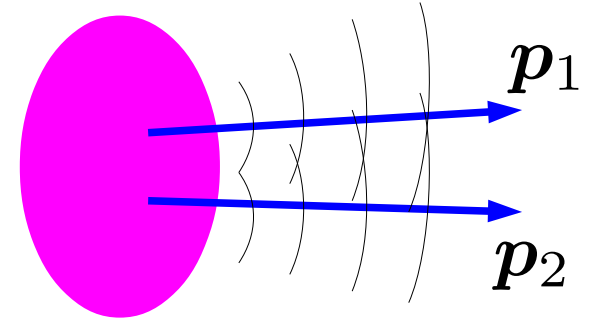
ALICE Week, 9 - 13 November, 2020 (Online)

- Introduction
- Survey of measured correlation function data
 - $\Lambda\Lambda$, $p\Omega^-$, pK^- , $p\Xi^-$.
- Implications of current correlation function data to the existence of hadronic molecule states
 - Source size dependence of the correlation function
- Correlation functions in the near future
 - $\Lambda\Xi^-$, Λpp , D^+p , D^-p , ...
- Summary

Correlation Function (CF): Standard and Non-Std usage

■ Correlation function

- Correlation from the quantum statistics and the final state int. under indep. particle production assumption lead KP formula,



Koonin('77), Pratt+('86), Lednicky+('82)

$$C(\mathbf{p}_1, \mathbf{p}_2) = \frac{N_{12}(\mathbf{p}_1, \mathbf{p}_2)}{N_1(\mathbf{p}_1)N_2(\mathbf{p}_2)} \simeq \int d\mathbf{r} \underbrace{S_{12}(\mathbf{r})}_{\text{source fn.}} \underbrace{|\varphi_q(\mathbf{r})|^2}_{\text{relative w.f.}}$$

■ Standard: Source size from CF (HBT-GGLP effects)

Hanbury Brown & Twiss ('56); Goldhaber, Goldhaber, Lee, Pais ('60)

- CF of free identical scalar bosons from spherical Gaussian source

$$\phi(\mathbf{r}) = \sqrt{2} \cos \mathbf{q} \cdot \mathbf{r} \rightarrow C(q) = 1 + \exp(-4R^2 q^2)$$

■ Non-standard: hadron-hadron interaction from CF

- CF of non-identical pair from Gaussian source

R. Lednicky, V. L. Lyuboshits ('82); K. Morita, T. Furumoto, AO ('15)

$$C(\mathbf{q}) = 1 + \int d\mathbf{r} S(r) \{ |\varphi_0(r)|^2 - |j_0(qr)|^2 \} \quad (\varphi_0 = \text{s-wave w.f.})$$

CF shows how much $|\varphi|^2$ is enhanced $\rightarrow V_{hh}$ effects !

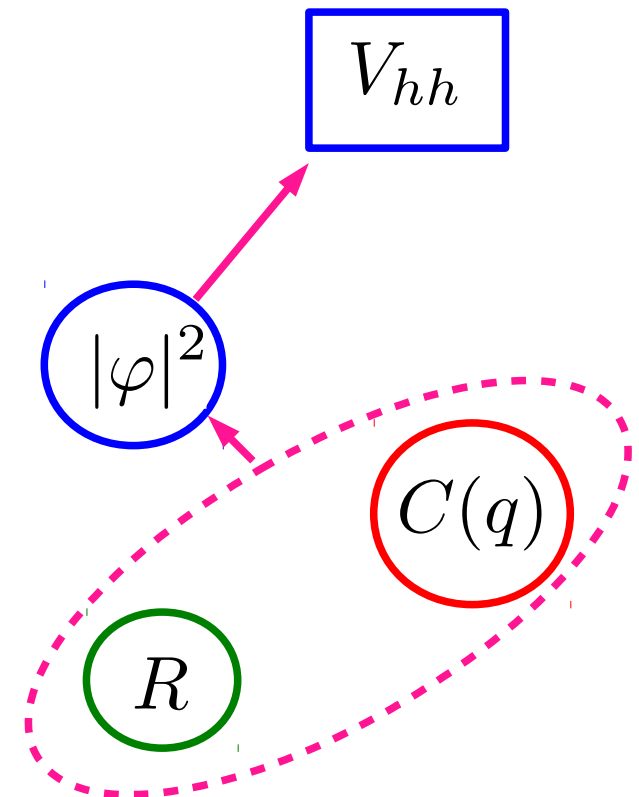
Fermtoscopic Study of Hadron-Hadron Interaction

- If CF is measured and source is well known, we may fit low E. scattering parameters to data by using Lednicky-Lybovits (LL) formula.

$$C_{LL}(q) = 1 + \frac{|f(q)|^2}{2R^2} F_3\left(\frac{r_{\text{eff}}}{R}\right) + \frac{2\text{Re}f(q)}{\sqrt{\pi}R} F_1(x) + \frac{2\text{Im}f(q)}{R} F_2(x)$$

non-identical pair, $f(q) = (e^{2i\delta_0} - 1)/2iq$ (scatt. ampl.), $x = 2qR$, (F_i = known functions)

- Asymptotic w.f. is assumed.
- If reliable $V(r)$ or $f(q)$ exists, we may fit the source size and other parameters to data, then we can examine $V(r)$ or $f(q)$.
 - J. Haidenbauer ('19): CF from $f(q)$
 - Our approach: CF from $V(r)$



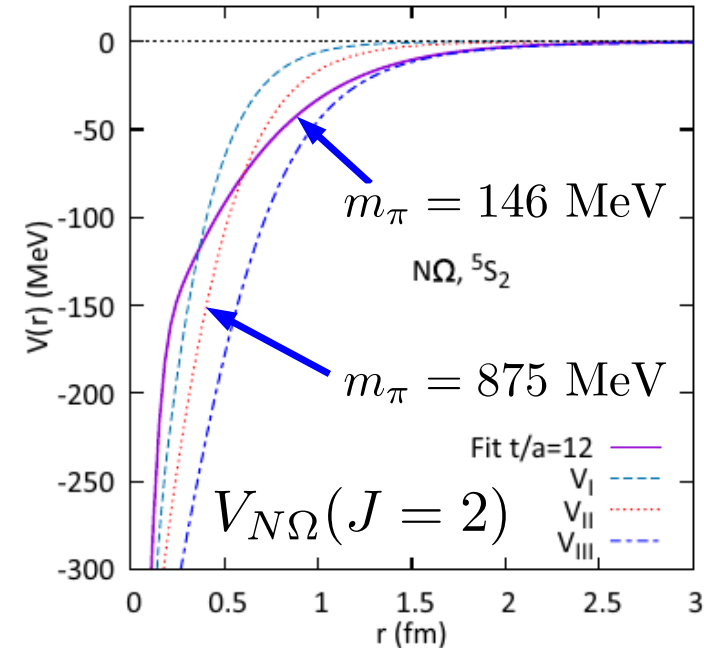
Modern Hadron-Hadron Interactions

■ Lattice QCD hh potential

- V_{hh} is obtained from the Schrödinger eq. for the Nambu-Bethe-Salpeter (NBS) amplitude.

N. Ishii, S. Aoki, T. Hatsuda, PRL99('07)022001.

→ $\Omega\Omega$, $N\Omega$, $\Lambda\Lambda$ - $N\Xi$ potentials
at phys. quark mass are published



■ Chiral EFT / Chiral SU(3) dynamics

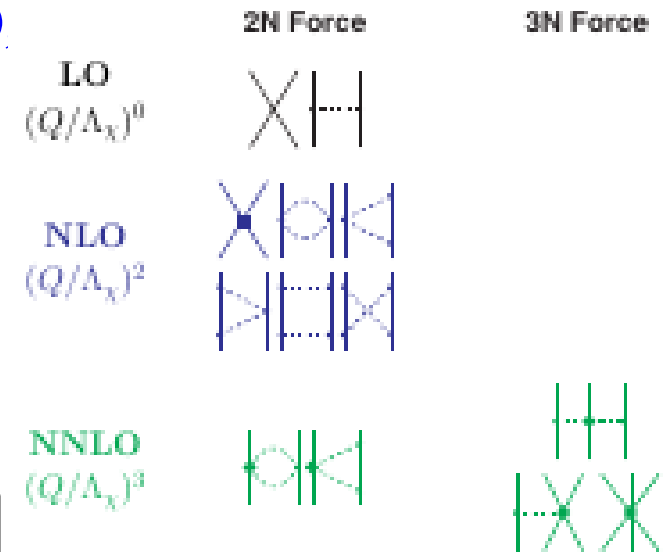
- V_{hh} at low E. can be expanded systematically in powers of Q/Λ .

S. Weinberg ('79); R. Machleidt, F. Sammarruca ('16)

Y. Ikeda, T. Hyodo, W. Weise ('12).

→ NN , NY , YY , \overline{KN} - $\pi\Sigma$ - $\pi\Lambda$, ...

- Quark cluster models,
- Meson exchange models,
- More phenomenological models, ...

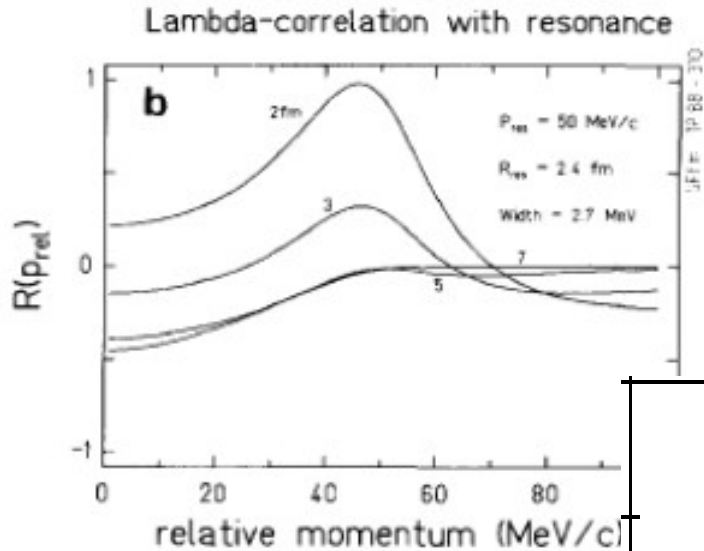


Let us examine modern hh interactions !

Survey of measured correlation function data

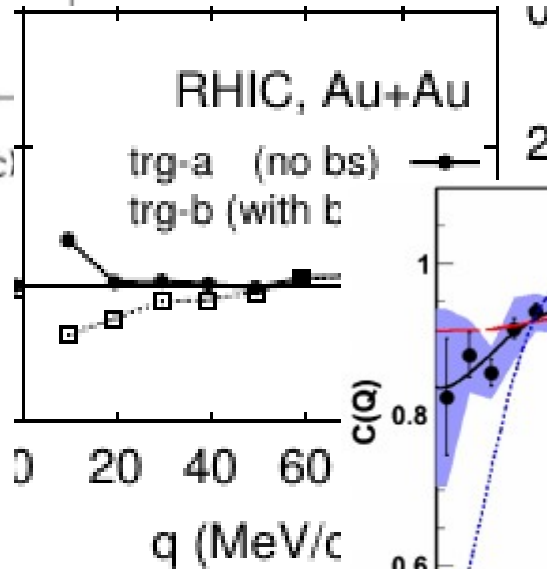
**I will show some of the correlation functions
which I investigated.
Details will be discussed by B. Hohlweger.**

$\Lambda\Lambda$ correlation and $\Lambda\Lambda$ interaction

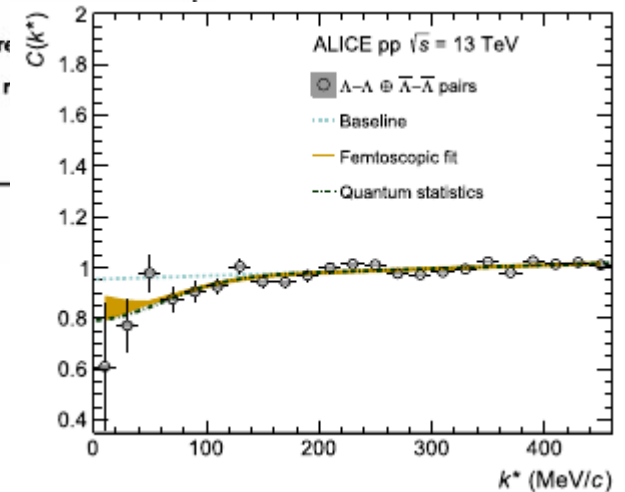
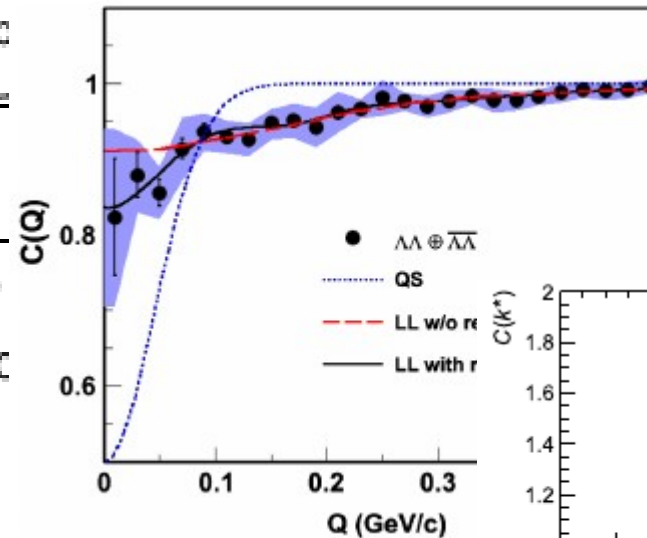


AO, Hirata, Nara,
Shinmura, Akaishi,
NPA670('00)297c

C. Greiner, B. Muller,
PLB219('89)199.



L. Adamczyk+[STAR],
PRL114('15)022301

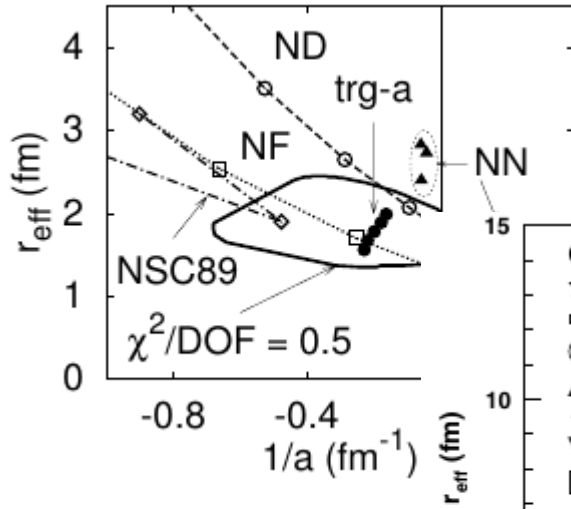


Weak enh. over quantum
statistical (HBT) CF.

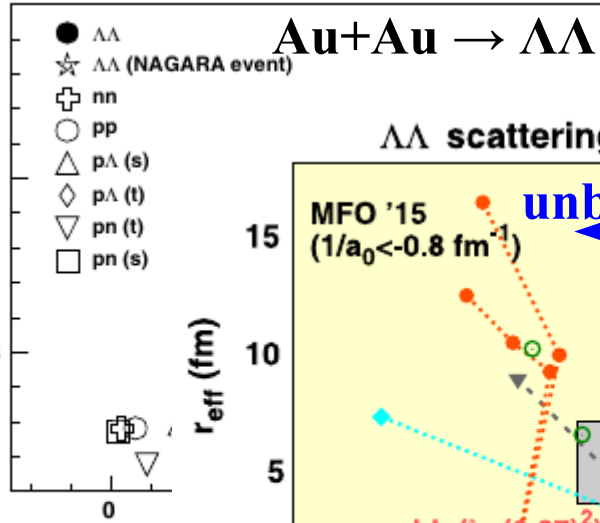
S. Acharya+[ALICE],
PLB797('19)134822

$\Lambda\Lambda$ correlation and $\Lambda\Lambda$ interaction

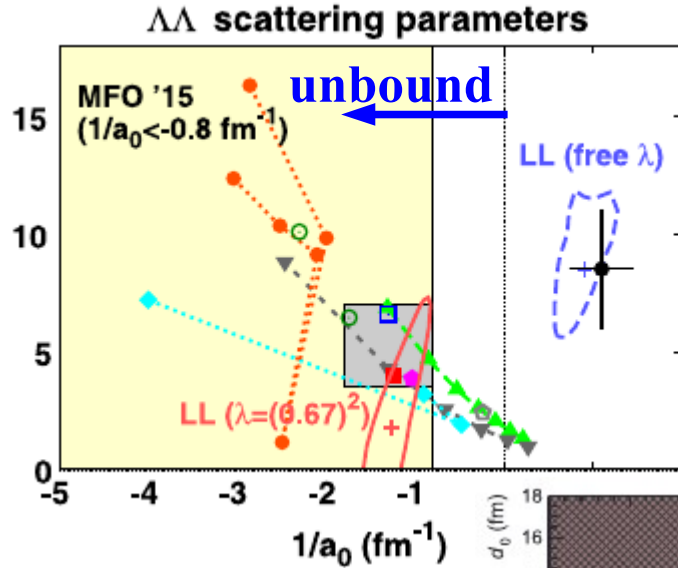
It is unlikely that $\Lambda\Lambda$ bound state exists.



L. Adamczyk+[STAR], PRL114('15)022301

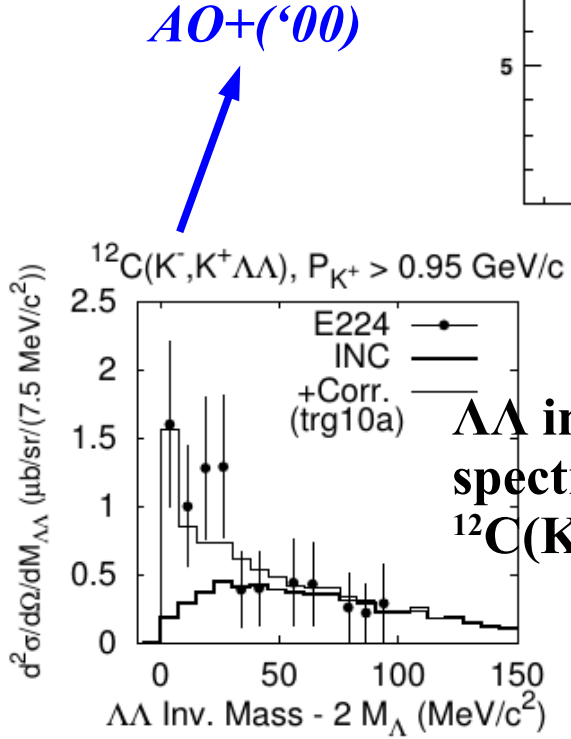


AO, K. Morita, K. Miyahara, T. Hyodo, NPA954('16)294



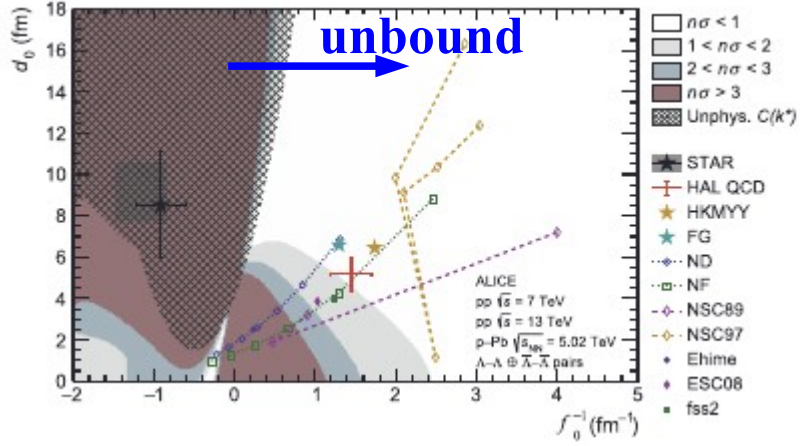
- ND —▲—
- NF -▲-
- NSC89 -▲-
- NSC97 -▲-
- ESC08c -▲-
- Ehime -▲-
- fss2 -▲-
- FG -▲-
- HKMY -▲-
- STAR -▲-

$$\delta \sim -a_0 q$$

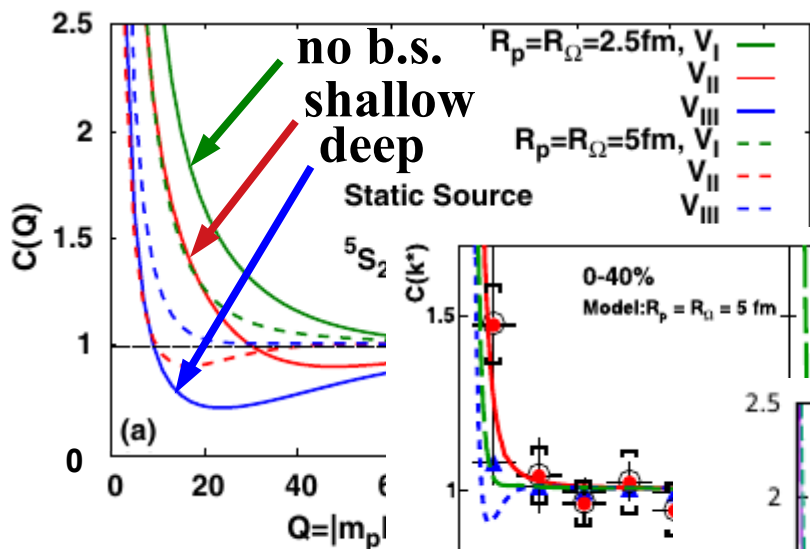


S. Acharya+[ALICE], PLB797('19)134822

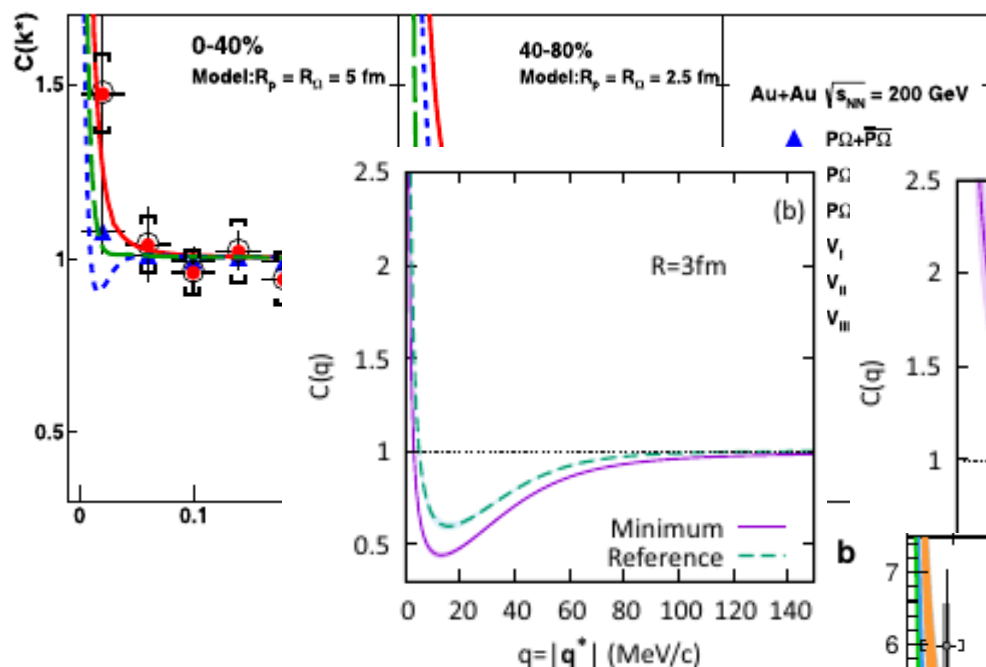
$$\delta \sim +a_0 q$$



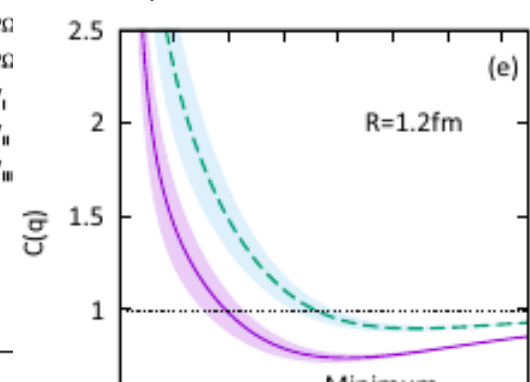
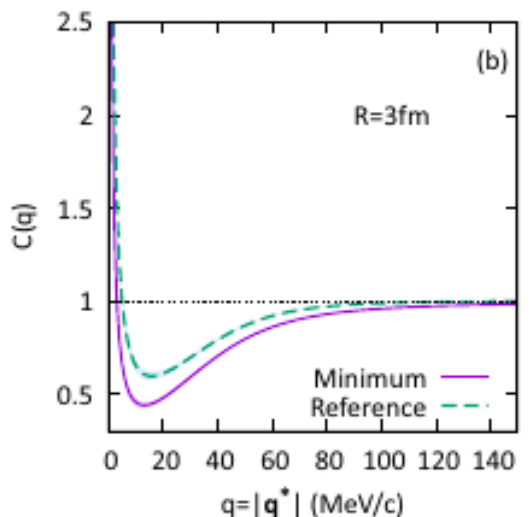
$p\Omega^-$ correlation



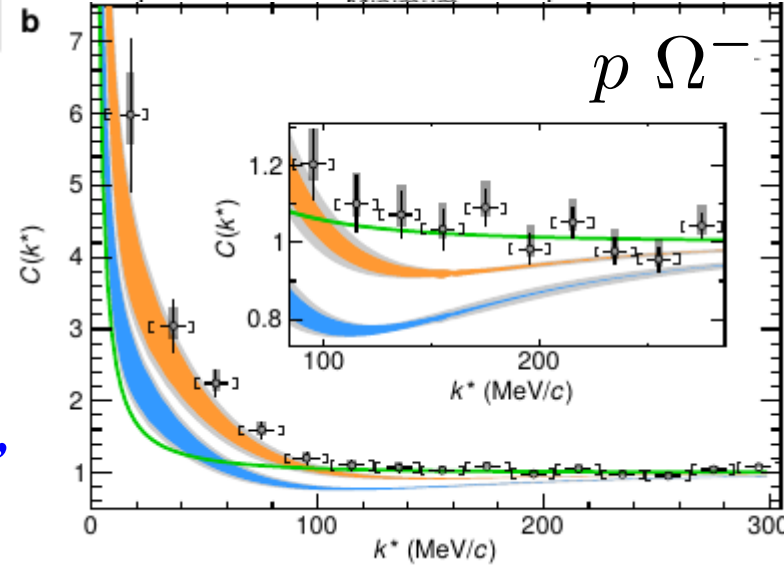
*K. Morita, AO, F. Etminan,
T. Hatsuda, PRC94('16)031901(R)
(w/ Lattice potential with heavier quark mass)*



*J. Adam+[STAR],
PLB790('19)490.*



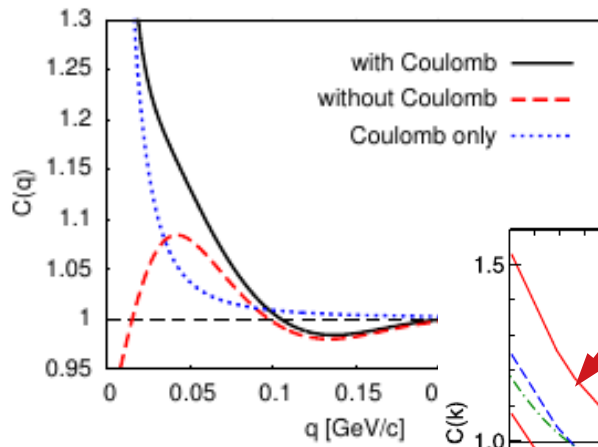
*K. Morita, S. Gongyo, T. Hatsuda,
T. Hyodo, Y. Kamiya, AO,
PRC 101('20)015201. (w/ Lattice
potential at physical quark mass)*



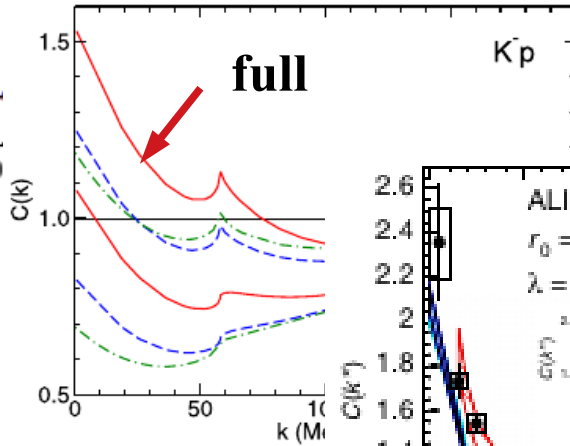
*S. Acharya+[ALICE],
2005.11495 [nucl-ex]
(pp 13 TeV)*

Bound state ?

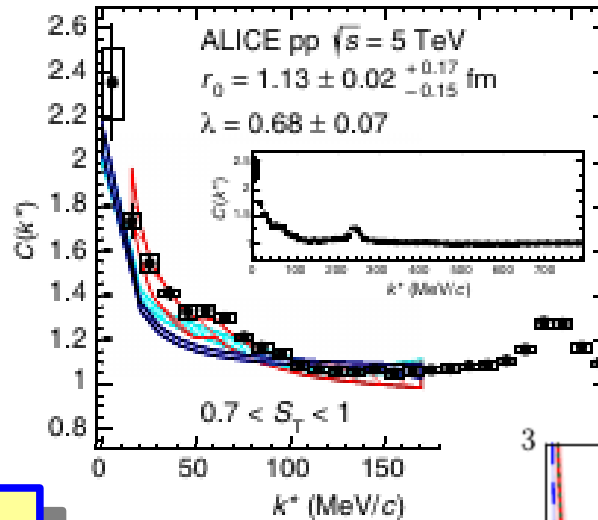
pK^- correlation



*S. Cho+ [ExHIC], PPNP95('17)279.
(Insufficient coupled-channel effects)*

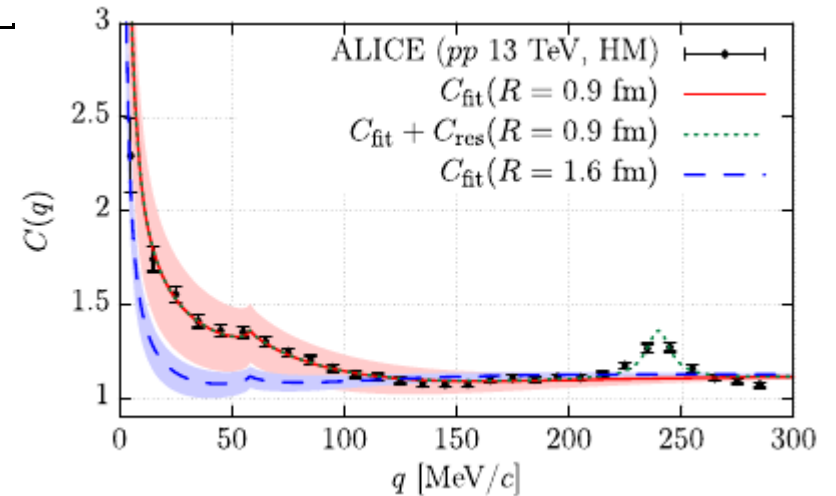


*J. Haidenbauer, NPA981('19)1.
(w/ CC effects, w/o Coulomb)*



*S. Acharya+[ALICE],
PRL124('20)092301*

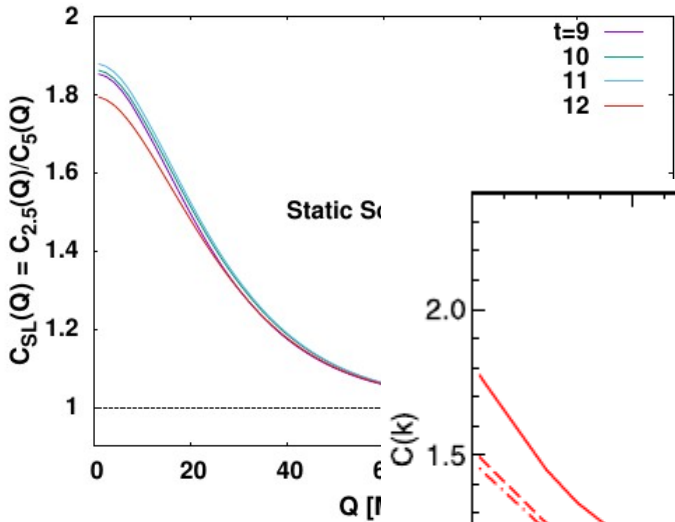
*Source size dep. shows
interesting feature.*



*Y. Kamiya, T. Hyodo, K. Morita, AO,
W. Weise, PRL124('20)132501.
(Chiral SU(3) dynamics)*

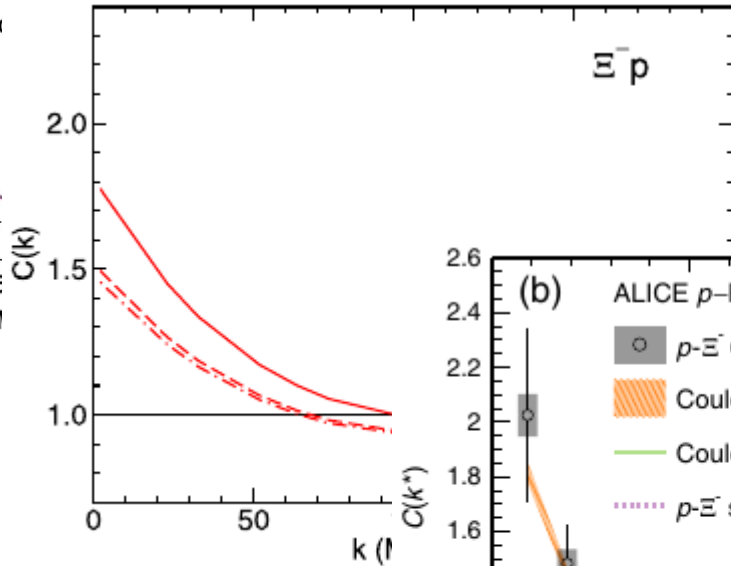
$p\bar{E}^-$ correlation

Where is H ?

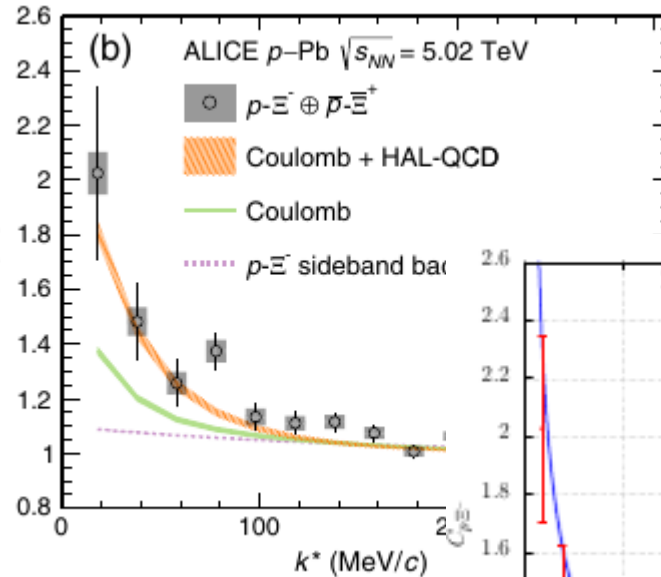


*T. Hatsuda, K. Morita, AO,
K. Sasaki, NPA967('17)856.*

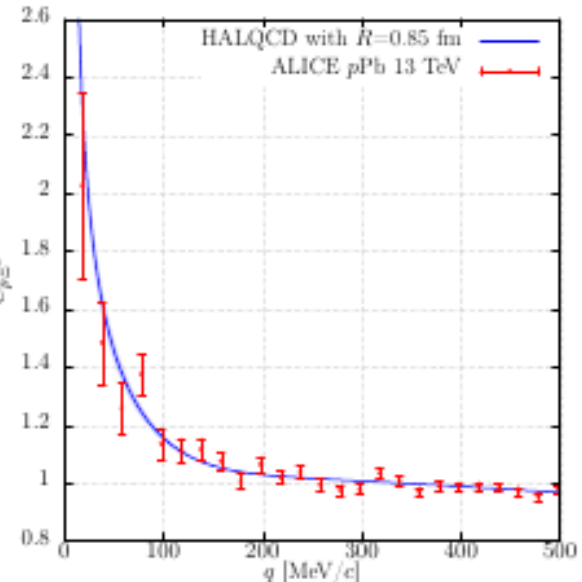
*J. Haidenbauer, NPA981('19)1.
(NLO(600), w/ CC effects,
w/o Coulomb)*



*S. Acharya+[ALICE],
PRL123('19)112002.*



*Y. Kamiya, K. Sasaki, T. Fukui, T. Hatsuda,
T. Hyodo, K. Morita, K. Ogata, AO, in prep.
(w/ Lattice BB pot. at phys. quark mass
and CC effects with $\Lambda\Lambda$)*



Fermtoscopic Study of Hadron-Hadron Interaction

- What did we learn from CF data of $\Lambda\Lambda$, $p\Omega^-$, pK^- and $p\Xi^-$?
 - CF($\Lambda\Lambda$) constrain (a_0, r_{eff}) region implying **small $|a_0|$** .
It is likely that $a_0 < 0$ (in nucl. phys. convention, $\delta \sim -a_0 q$).
(Consistent with double Λ hypernuclear data.)
 - CF($p\Omega^-$) show **strong enhancement** at small q , **imply large $|a_0|$** ,
and **examine the recent lattice $N\Omega$ potential.** (**First info. on $N\Omega$**)
 - CF(pK^-) show enhancement at small q , and **examine potentials from the Jülich model and chiral $SU(3)$ dynamics.**
Coupled-channel effects with $\bar{K}^0 n$ and $\pi\Sigma$ are large and visible, respectively (as discussed later).
 - CF($p\Xi^-$) show **strong enhancement** at small q , **imply large $|a_0|$** ,
and **examine the chiral EFT and recent lattice $N\Xi$ potentials.**
(Nijmegen $N\Xi$ potential seems to be inconsistent with data.)
Coupled-channel effects with $\Lambda\Lambda$ are not large (as discussed later).
(Consistent with Ξ -hypernuclear data ?)

Fermtoscopic Study of Hadron-Hadron Interaction

- These recent (2015-2020) observations of correlation functions are **great achievements of RHIC and LHC**. We (theoretical physicists) appreciate your efforts. (I hope that ALICE publishes the new data more slowly...)
- Many hadron physicists loves **peaks**, E and Γ of discrete poles, rather than smooth spectra. (Pc from LHCb is a good example !) How can we (CF lovers) satisfy and attract them ?
 - Relation of CF with the existence of **the bound state pole**.
 - Do coupled-channel effects modify CF and/or generate additional poles ?
- They also love hadronic states including heavy quarks. Does silicon vertex detector specifies decay point of **chamed hadrons** ?
- Many neutron star matter EOS physicists love **the three-body force including hyperons**. Can we access the three-body force ?

*Implications from
current correlation function data
to the existence of
hadronic molecule states*

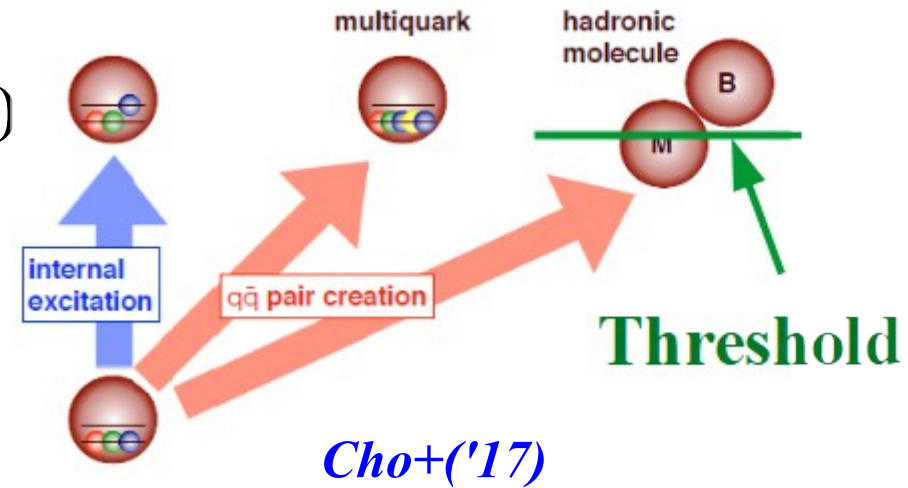
Trend in Hadron Physics

■ Hadron-Hadron interaction is closely related with ...

- Quark-gluon structure of hadrons (Multi-quark or Hadronic molecule)

*To be bound or not to be,
That is the problem.*

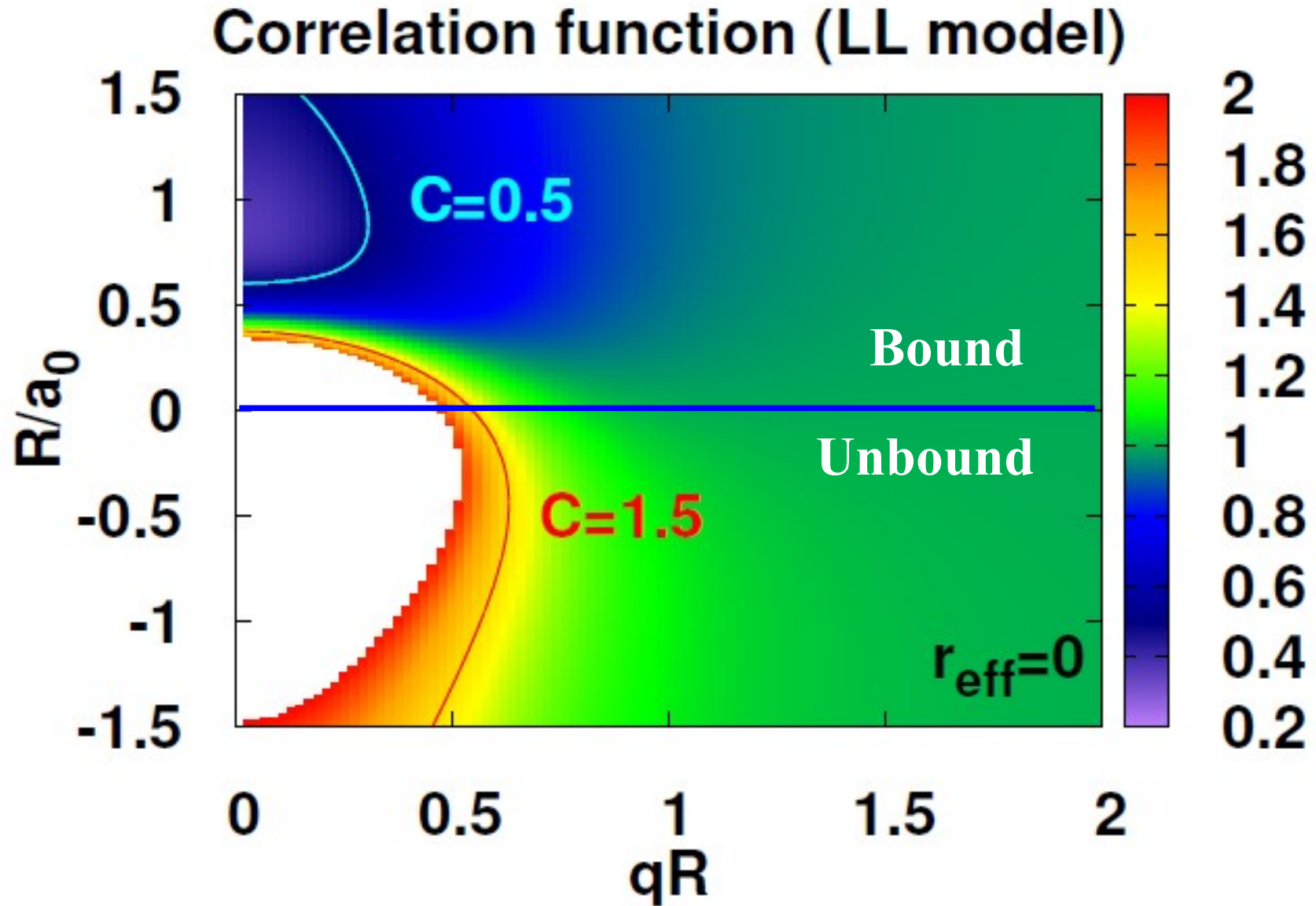
- Hadrons with heavy-quarks
- Hadrons in nuclear matter and EOS of nuclear matter



■ High-Energy Nuclear Collisions ($\sqrt{s_{NN}}=40$ GeV – 14 TeV) are favorable as a Hadron Factory !

- $dN/dy \sim 1000$ (RHIC, Au+Au) $\rightarrow 10^3$ - 10^5 hadrons in one event
- Various hadrons, nuclei ($A \leq 4$) and anti-nuclei are formed.
- Yield \sim Stat. Model calc.
(Formation processes are too complicated to be out of statistical.)

Source Size Dependence of Correlation Function



LL model: R. Lednicky, V. L. Lyuboshits ('82)

Wave function around threshold (S-wave, attraction)

Low energy w.f. and phase shift

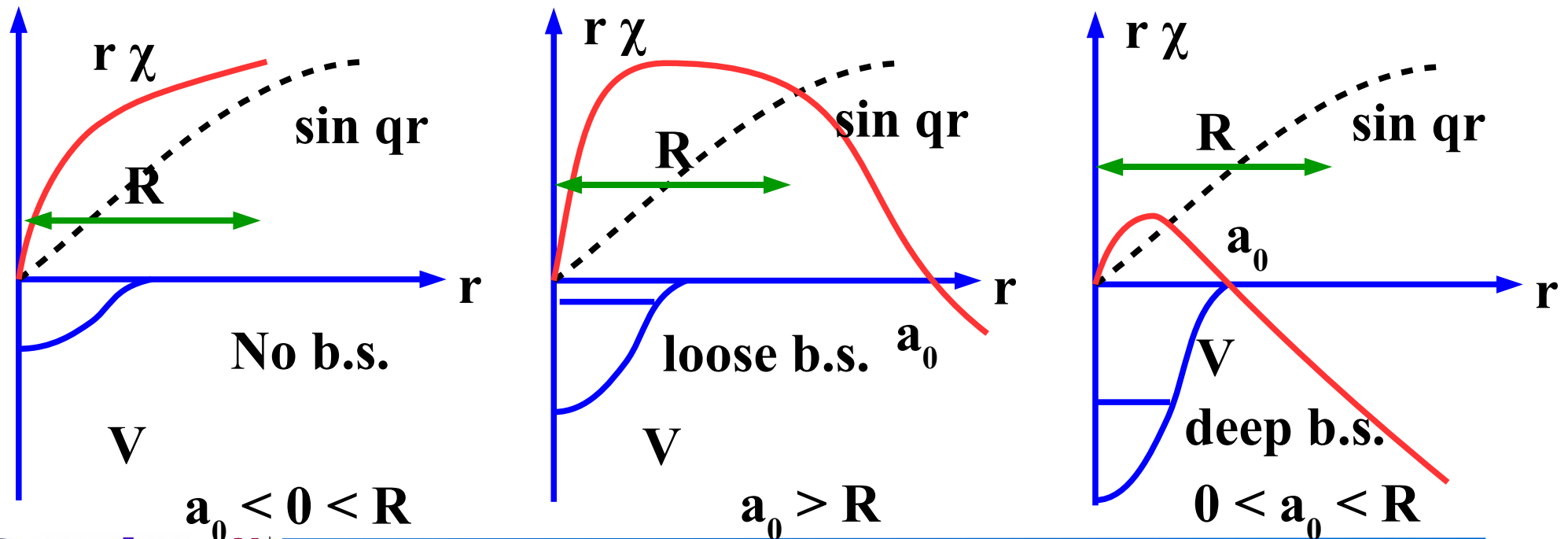
$$u(r) = qr\chi_q(r) \rightarrow \sin(qr + \delta(q)) \sim \sin(q(r - a_0))$$

$$q \cot \delta = -\frac{1}{a_0} + \frac{1}{2}r_{\text{eff}}q^2 + \mathcal{O}(q^4) \quad (\delta \sim -a_0q)$$

a_0 =scatt. length

r_{eff} =eff. range

- Wave function grows rapidly at small r with attraction.
- With a bound state ($a_0 > 0$), a node appears around $r = a_0$



From correlation function to hadron-hadron interaction

- Large $|a_0|$ ($|a_0| > R$)

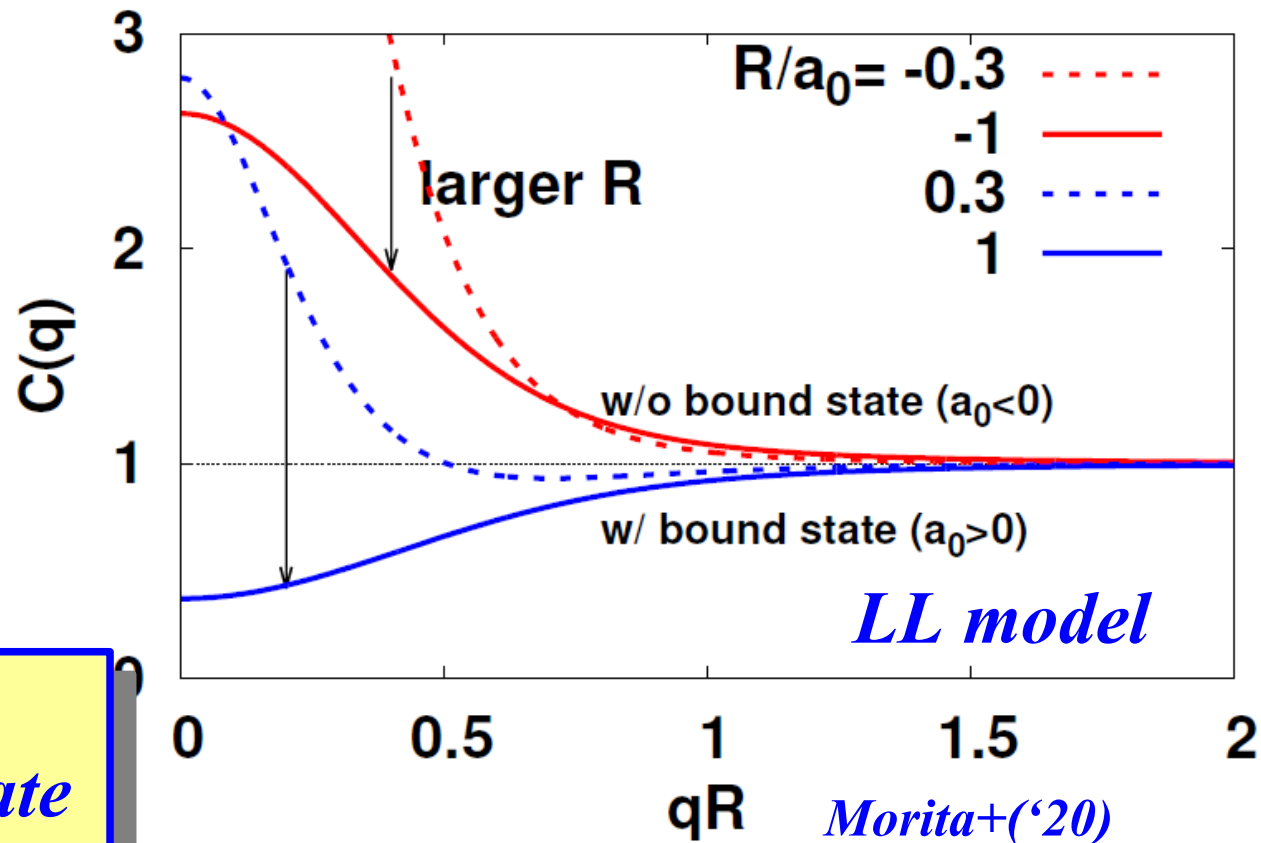
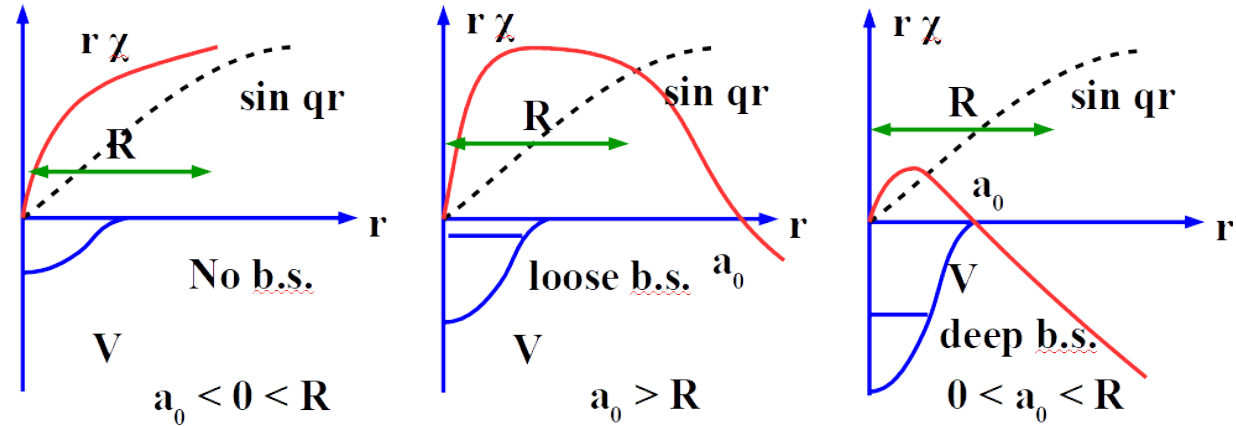
→ Large $C(q)$
(unitary regime)

- w/o bound state
($a_0 < 0$, $|a_0| \sim R$)

→ $C(q) > 1$

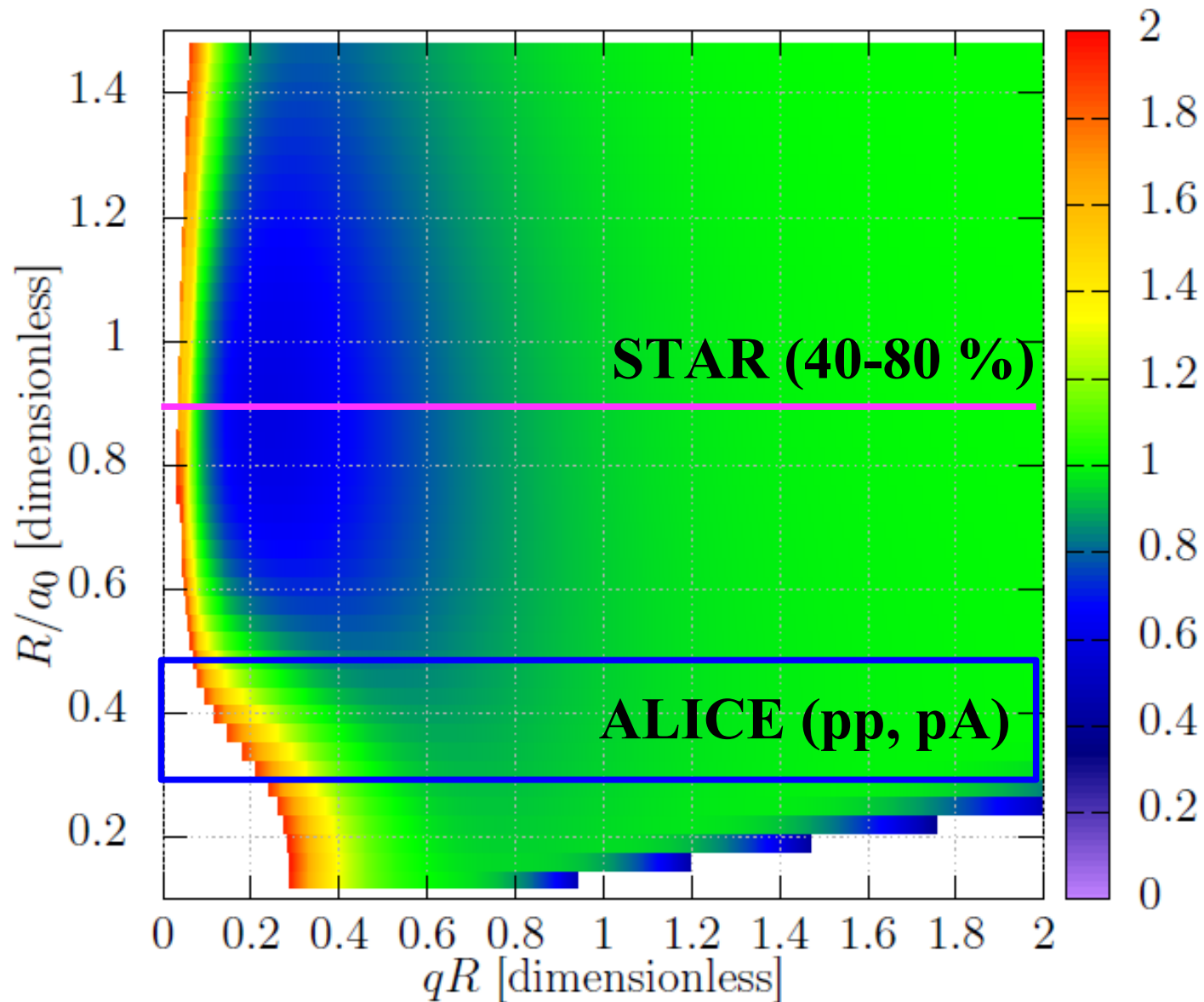
- With bound state
($a_0 > 0$, $|a_0| \sim R$)

→ Region with
 $C(q) < 1$ appears



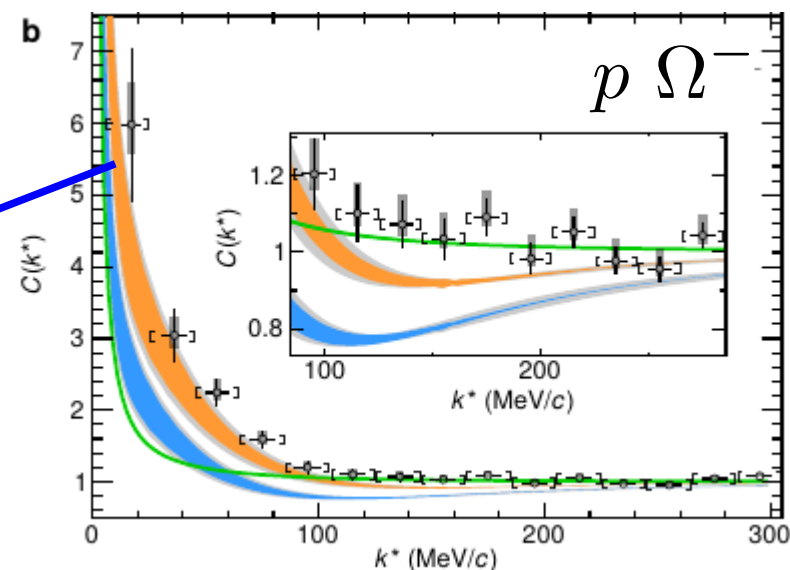
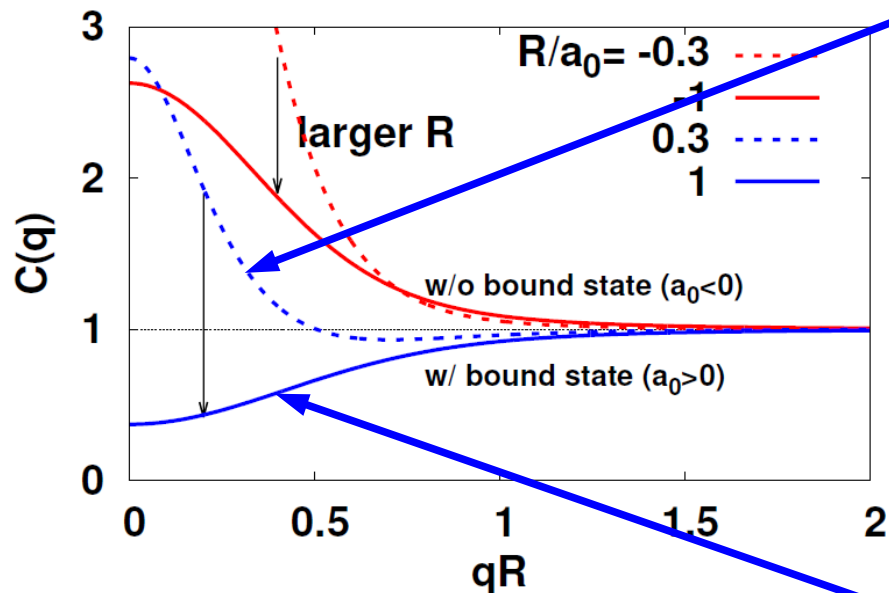
Source size dep. of CF
→ *Existence of bound state*

Correlation Function with Gaussian source



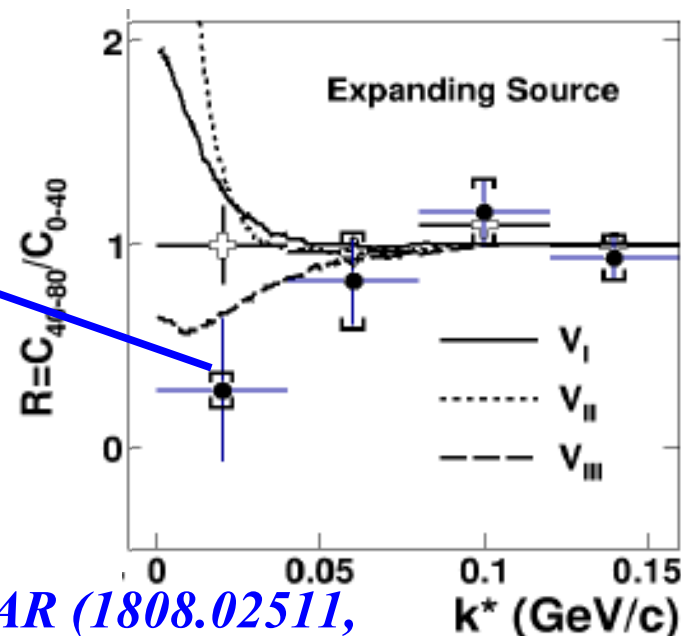
$N\Omega$ potential ($J=2$, HAL QCD, $a_0=3.4$ fm) + Coulomb

STAR + ALICE = $N\Omega$ Dibaryon



ALICE, 2005.11495

Do you know any mechanism to suppress $C(q)$ other than the existence of bound state, when the interaction is attractive ?
(Strong flow, ...)

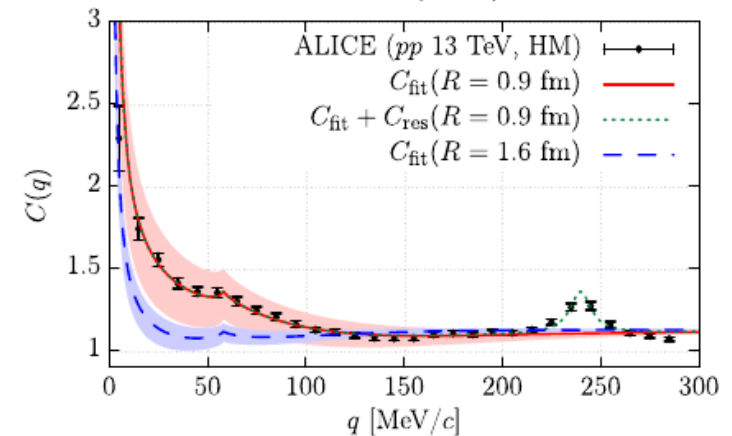
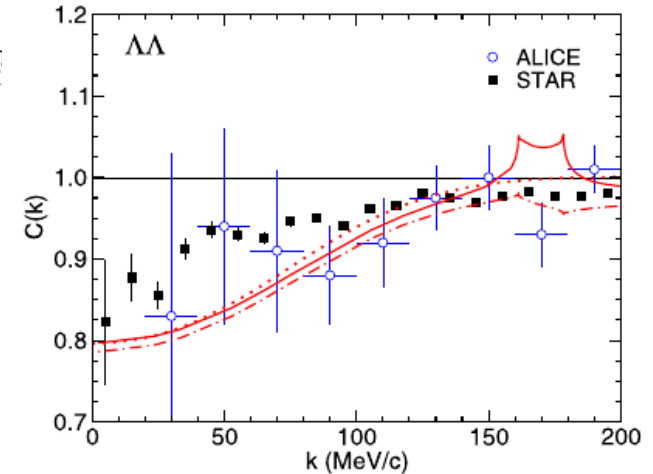
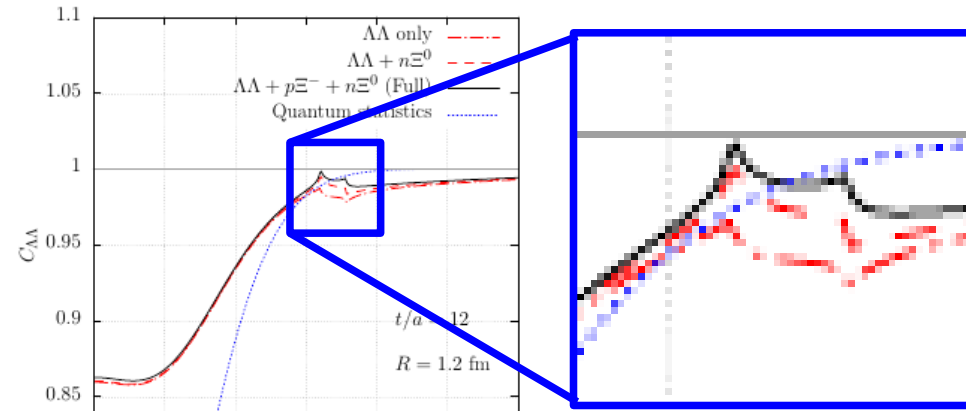


STAR (1808.02511, PLB790 ('19) 490)

Other bound states ?

■ $\Lambda\Lambda$ - $N\Xi$

- $C_{\Lambda\Lambda}(q)$ in AA(RHIC) and pp(LHC) are similar (No b.s. below $\Lambda\Lambda$).
 - LQCD predicts a virtual pole near $N\Xi$ threshold, which can be detected as the cusp in $C_{\Lambda\Lambda}(q)$.
- NLO(600) potential predicts the same.
(The fate of H particle)
K. Sasaki+[HAL QCD], NPA998('20)121737;
Y. Kamiya+, in prep.; Haidenbauer('19).

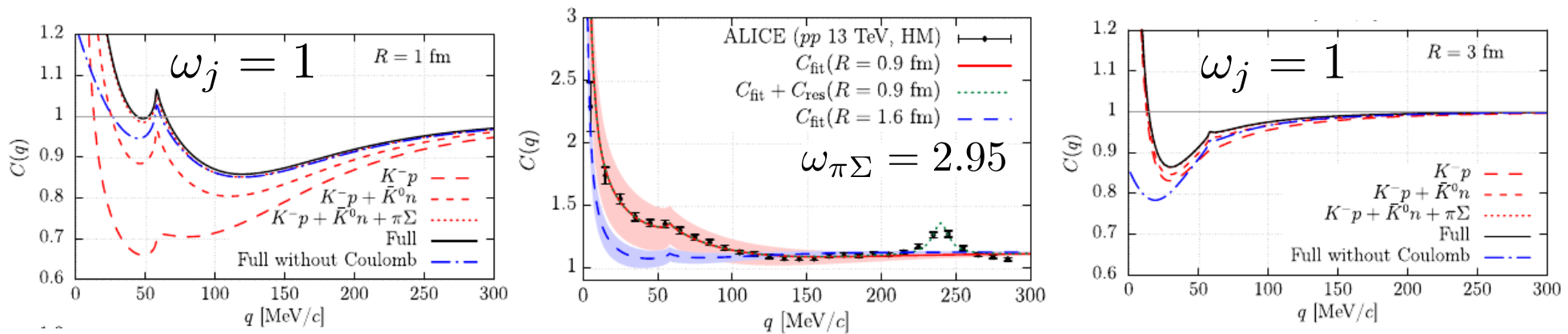


■ $\overline{K}N$

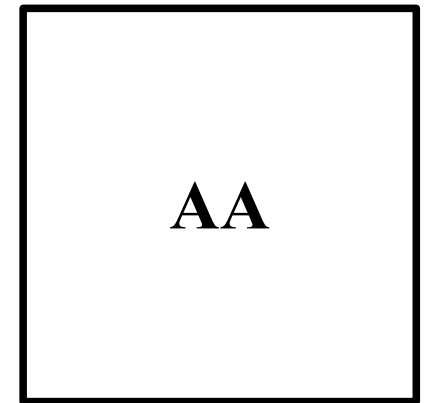
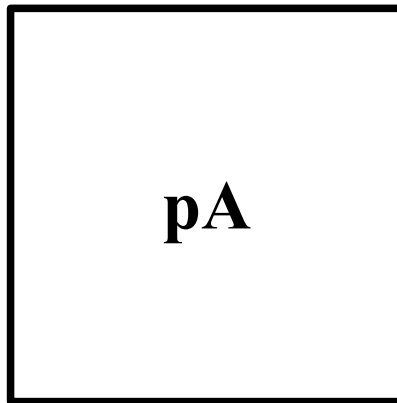
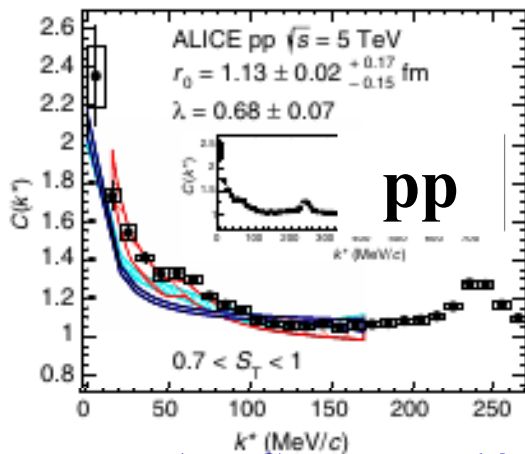
- $\Lambda(1405)$ is believed to be the bound state of $\overline{K}N$, and “dip” is expected at $R \sim a_0$.
 - However, Coulomb and coupled-channel effects modify the dip-like behavior.
- Kamiya+ ('20).*

Source Size Dependence of $C(pK^-)$

- Coupled-channel effects are suppressed when R is large, and “pure” pK^- wave function may be observed in HIC.
- Can we deduce $(\text{Re } a_0, \text{Im } a_0)$ at precision comparable to that in SIDDHARTA kaonic hydrogen data ?



Y. Kamiya, T. Hyodo, K. Morita, AO, W. Weise, PRL124('20)132501.



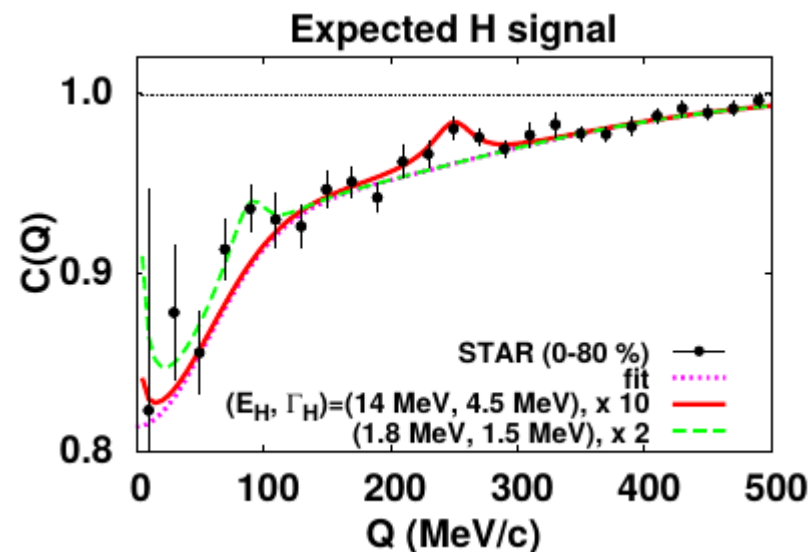
S. Acharya+[ALICE], PRL124('20)092301

*Correlation functions
in the near future*

CF from ALICE in the near future

■ S=-3 baryon-baryon correlation (e.g. $\Lambda\Xi^-$)

- Important to confirm $N\Omega$ bound state as a peak in $C_{\Lambda\Xi}(q)$.
- Statistically challenging. In $C_{\Lambda\Lambda}$ data from STAR, statistical fluc. is 10 times larger than expected signal from statistical model estimate.



■ Three-body correlation (e.g. Λpp)

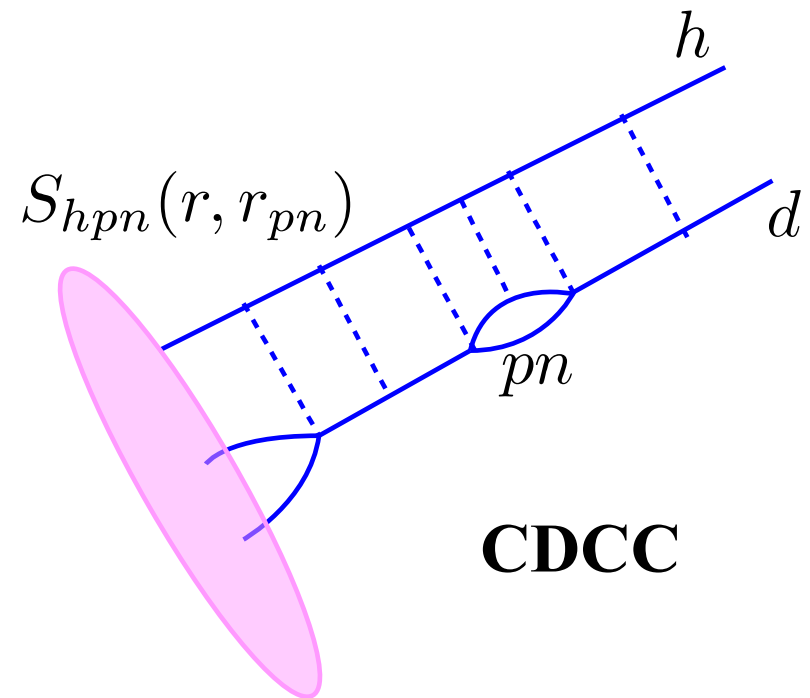
- Extremely important to neutron star matter EOS, if we can extract three-body force.
- We may need to develop a framework beyond the “Riverside approximation” to include hh and hhh interaction.

E. O. Alt, T. Csorgo, B. Lorstad, J. Schmidt-Sorensen, PLB458 ('99)407 for 3π .

(I got the info. mainly from Laura, Valentina and Oton, but I never told it to people other than CF collaborators of mine.)

CF from ALICE in the near future (cont.)

- Hadron-deuteron correlation (Λ -d, K^- -d, Ξ^- -d, Ω^- -d, ...)
 - Scattering length data of these are important to evaluate binding energy and lifetime of hyper triton (Λ -d), $I=1$ $\bar{K}N$ interaction (K^- -d), and the existence of a bound state. [Etminan+ \(2006.12771\)](#); [J. Haidenbauer, PRC102\('20\)034001](#).
 - For serious estimate, deuteron breakup effects ($d \leftrightarrow pn$) need to be accounted for. I asked two low-energy few-body nuclear physicists (K. Ogata, T. Fukui) to apply the few-body reaction framework (Continuum-discretized coupled-channels (CDCC)) to hadron-deuteron correlation.

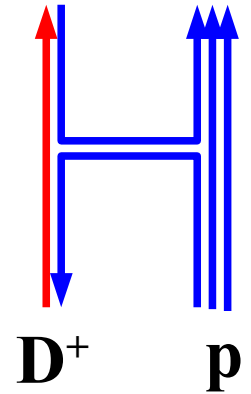


(I got the info. mainly from Laura, Valentina and Oton, but I never told it to people other than CF collaborators of mine.)

CF from ALICE in the near future (cont.)

■ CF including charmed hadron

- Extremely important in recent hadron physics.
- $D^+(\bar{c}d)$ - $p(uud)$ correlation
qq can annihilate, and D^+p couples with many other channels. (LQCD calc. is difficult.)



- $D^-(\bar{c}d)$ - $p(uud)$ correlation
Probes $\Theta_c(\bar{c}\text{-}ud\text{-}ud)$ state (replace \bar{s} in $\Theta(\bar{s}\text{-}ud\text{-}ud)$ with \bar{c})

Two pion exchange can induce attraction.

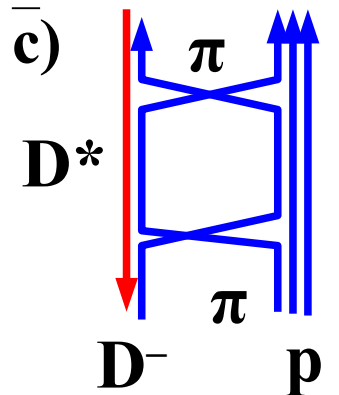
Provides production-mechanism-free (thermal) result.

Easy to calculate the potential in LQCD.

D. O. Riska, N. N. Scoccola, PLB299('93)338 (pred.);

A. Aktaset+ [H1], PLB588('04)17 (positive);

J. M. Linket+ [FOCUS], PLB622('05)229 (negative).



(I got the info. mainly from Laura, Valentina and Oton, but I never told it to people other than CF collaborators of mine.)

Summary

- **Correlation functions (CFs) can be utilized to constrain / examine the hadron-hadron interactions (V_{hh}), provided that**
 - the pair purity (λ) is not very small,
 - and V_{hh} dominates the correlation.
- **Recent data from ALICE and STAR enable us to access V_{hh} unexplored in previous works.**
- **Source size (R) dependence of CF is important to get knowledge of the existence of a bound state around the threshold.**
 - With a bound state, CF shows suppression at $qR \sim 1$ for $a_0/R \sim 1$ in single-channel problems.
 - Coupled-channel effects are suppressed for larger R.
- **CFs involving charmed hadrons are charming and would attract many hadron physicists.**

Thank you for attention !

Coauthors of *arXiv:1908.05414* ($p\Omega$, $\Omega\Omega$) and *arXiv:1911.01041* (pK^-)

K. Morita



S. Gongyo



T. Hatsuda



T. Hyodo



Y. Kamiya



K.Sasaki



ALICE

W. Weise



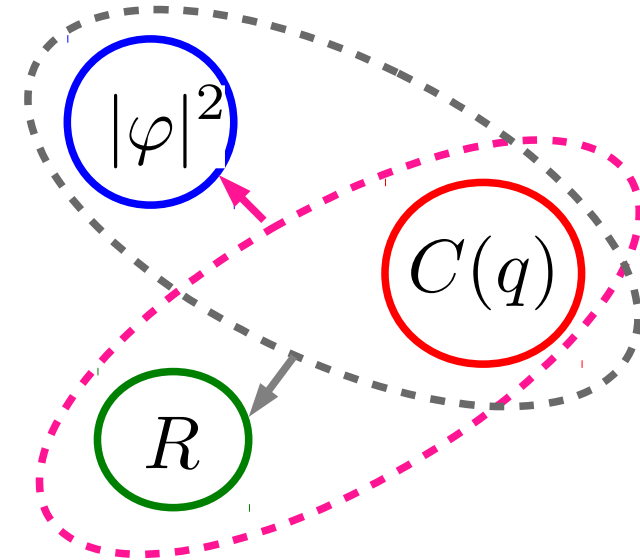
(My) Homeworks

- $p\Lambda$, $p\Sigma^0$ and $\Lambda\Xi^-$ correlation functions using the updated LQCD potentials.
- CF involving deuteron (K. Ogata, T. Fukui).
- Developing a framework to handle three-body correlations.
- Interactions between hadrons with charm.
- Simple Gaussian source function should be improved to include resonance decay effects as done in CATS.
(c.f. B. Hohlweger's talk)
- ...

Correlation Function (CF): Non-standard usage

- HBT, GGLP: CF + w.f. \rightarrow Source Size
Another way: CF + Source Size \rightarrow w.f. \rightarrow hh interaction
- Effect of hadron-hadron interaction on the wave function
 - Assumption: Only s-wave (L=0) is modified.
 - Non-identical particle pair, Gauss source.

$$\begin{aligned}\varphi_{\mathbf{q}}(\mathbf{r}) &= e^{i\mathbf{q}\cdot\mathbf{r}} - j_0(qr) + \chi_q(r) \\ \rightarrow C(\mathbf{q}) &= \int d\mathbf{r} S(r) |\varphi_{\mathbf{q}}(\mathbf{r})|^2 \\ &= 1 + \int d\mathbf{r} S(r) \{ |\chi_q(r)|^2 - |j_0(qr)|^2 \}\end{aligned}$$



K. Morita, T. Furumoto, AO, PRC91('15)024916

*Corr. Fn. shows how much squared w. f. is enhanced
 \rightarrow Large CF is expected with attraction*

Lednický-Lyuboshits (LL) model

■ Lednický-Lyuboshits analytic model

- **Asymp. w.f. + Eff. range corr. + $\psi^{(-)} = [\psi^{(+)}]^*$**

$$\psi_0(r) \rightarrow \psi_{\text{asy}}(r) = \frac{e^{-i\delta}}{qr} \sin(qr + \delta) = \mathcal{S}^{-1} \left[\frac{\sin qr}{qr} + f(q) \frac{e^{iqr}}{r} \right]$$

$$\begin{aligned} \Delta C_{\text{LL}}(q) &= \int dr S_{12}(r) (|\psi_{\text{asy}}(r)|^2 - |j_0(qr)|^2) \\ &= \frac{|f(q)|^2}{2R^2} F_3\left(\frac{r_{\text{eff}}}{R}\right) + \frac{2\text{Re}f(q)}{\sqrt{\pi}R} F_1(x) - \frac{\text{Im}f(q)}{R} F_2(x) \end{aligned}$$

($x = 2qR$, $R = \text{Gaussian size}$, F_1, F_2, F_3 : Known functions)

■ Phase shifts

$$q \cot \delta = -\frac{1}{a_0} + \frac{1}{2} r_{\text{eff}} q^2 + \mathcal{O}(q^4) \rightarrow \delta \simeq -a_0 q + \mathcal{O}(q^3)$$

$$\sin(qr + \delta) \simeq \sin(q(r - a_0) + \dots) \quad \text{Node at } \mathbf{r} \sim \mathbf{a}_0$$

for small q

$C(q)$ in the low momentum limit

- Correlation function at small q (and $r_{\text{eff}}=0$) $\rightarrow F_1=1, F_2=0, F_3=1$

$$\Delta C_{\text{LL}}(q) \rightarrow \frac{|f(0)|^2}{2R^2} + \frac{2\text{Re}f(0)}{\sqrt{\pi}R} \quad (q \rightarrow 0)$$

$$f(q) = (q \cot \delta - iq)^{-1} \simeq \left(-\frac{1}{a_0} + \frac{1}{2}r_{\text{eff}}q^2 - iq \right)^{-1} \rightarrow -a_0$$

$$C_{\text{LL}}(q \rightarrow 0) = 1 + \frac{a_0^2}{2R^2} - \frac{2a_0}{\sqrt{\pi}R} = 1 - \frac{2}{\pi} + \frac{1}{2} \left(\frac{a_0}{R} - \frac{2}{\sqrt{\pi}} \right)^2$$

$$1 - 2/\pi \simeq 0.36, \quad \sqrt{\pi}/2 \simeq 0.89$$

$C(q \rightarrow 0)$ takes a minimum of 0.36 at $R/a_0 = 0.89$ in the LL model.

$K^- p$ correlation

$K^- p$ interaction

- $\Lambda(1405)$ $\bar{K}N$ quasi-bound state

Dalitz, Tuan ('60); Koch ('94); Kaiser, Siegel, Weise ('95); AO, Nara, Koch ('97); Jido+('03)

- Positive scattering length in K^- atoms

*M.Iwasaki et al. PRL78('97)3067;
M.Bazzi et al. [SIDDHARTA Collab.],
PLB704('11)113.*

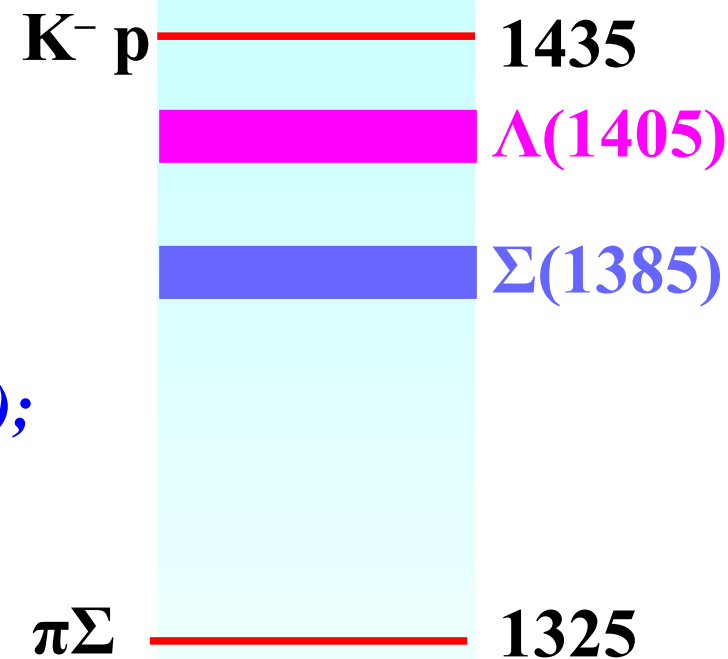
- Kaonic nuclei ?

*Nogami ('63); Akaishi, Yamazaki ('02);
Shevchenko, Gal, Mares ('07); Ikeda, Sato ('07);
Dote, Hyodo, Weise ('09)*

→ Needs precise info. on $\bar{K}N$ int.

- Scattering amplitude and Potential fitting scattering and SIDDARTA data in chiral approach

*Ikeda, Hyodo, Weise ('11,'12),
Miyahara, Hyodo ('16)*



How about $K^- p$ correlation ?

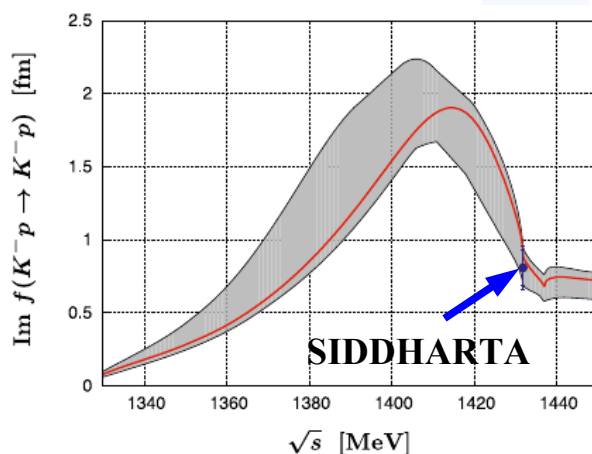
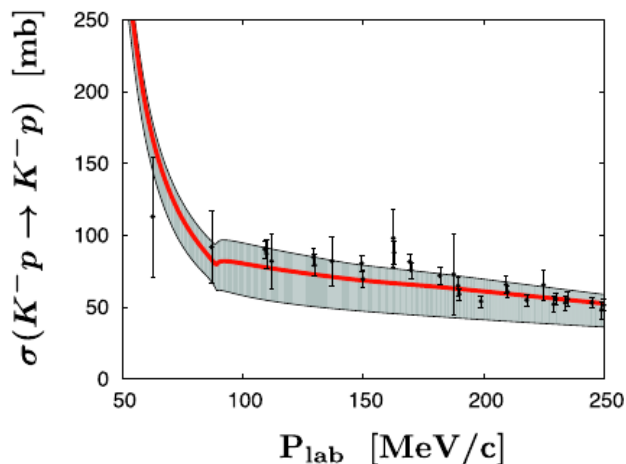
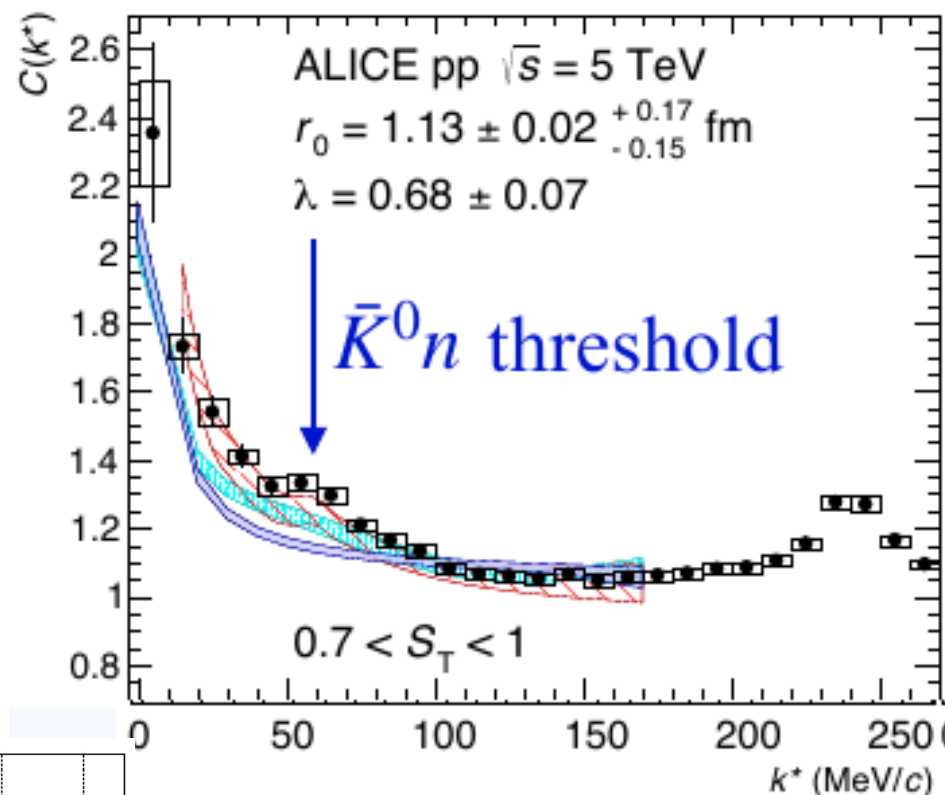
$K^- p$ correlation function data

■ $K^- p$ correlation function from high-multiplicity events of pp collisions

S. Acharya et al. (ALICE),

PRL124('20)092301 [1905.13470]

- High precision data from low to high momentum ! c.f. Previous scatt. data & Kaonic atom data.
- Enhanced at low k , cusp, $\Lambda(1520)$, ...



Red: Kyoto model
 Blue: Julich model
 grey: Coulomb

Y. Ikeda, T. Hyodo, W. Weise, NPA881 ('12) 98

$\bar{K}N-\pi\Sigma-\pi\Lambda$ Scattering Amplitude and Potential

■ Amplitude in chiral SU(3) coupled-channels dynamics

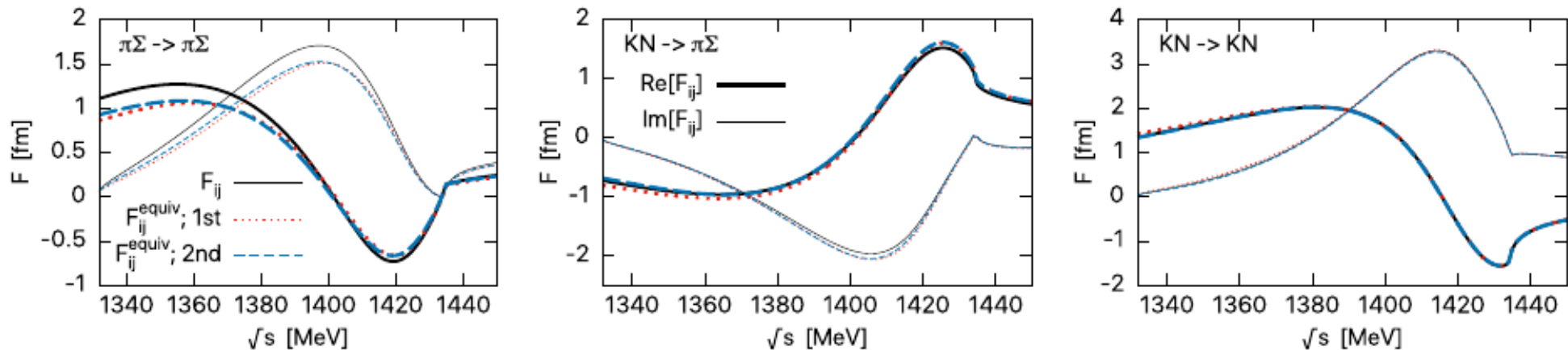
Y. Ikeda, T. Hyodo, W. Weise, NPA881 ('12) 98

- NLO meson-baryon effective Lagrangian ($\bar{K}N-\pi\Sigma-\pi\Lambda$)
+ fit of Kaonic Hydrogen, Cross Section, Threshold branching ratio

■ Coupled-channels potential

K. Miyahara, T. Hyodo, W. Weise, PRC98('18)025201

- Potential fitted to IHW amplitude



Y. Ikeda, T. Hyodo, W. Weise, NPA881 ('12) 98

K. Miyahara, T. Hyodo, W. Weise, PRC98('18)025201

Correlation Function with Coupled-Channels Effects

J. Haidenbauer, NPA 981('19)1; R. Lednicky, V. V. Lyuboshits, V. L. Lyuboshits, Phys. At. Nucl. 61('98)2950.

- **Single channel, w/o Coulomb (non-identical pair)**

$$C(\mathbf{q}) = \underline{1} + \int d\mathbf{r} S(\mathbf{r}) \left[\underline{|\chi^{(-)}(r, q)|^2} - \underline{|j_0(qr)|^2} \right]$$

- **Single channel, w/ Coulomb**

$$C(\mathbf{q}) = \int d\mathbf{r} S(\mathbf{r}) \left[\underline{|\varphi^{C,\text{full}}(\mathbf{q}, \mathbf{r})|^2} + \underline{|\chi^{C,(-)}(r, q)|^2} - \underline{|j_0^C(qr)|^2} \right]$$

Full free

Coulomb w.f.

s-wave w.f.

with Coul.

s-wave

Coul. w.f.

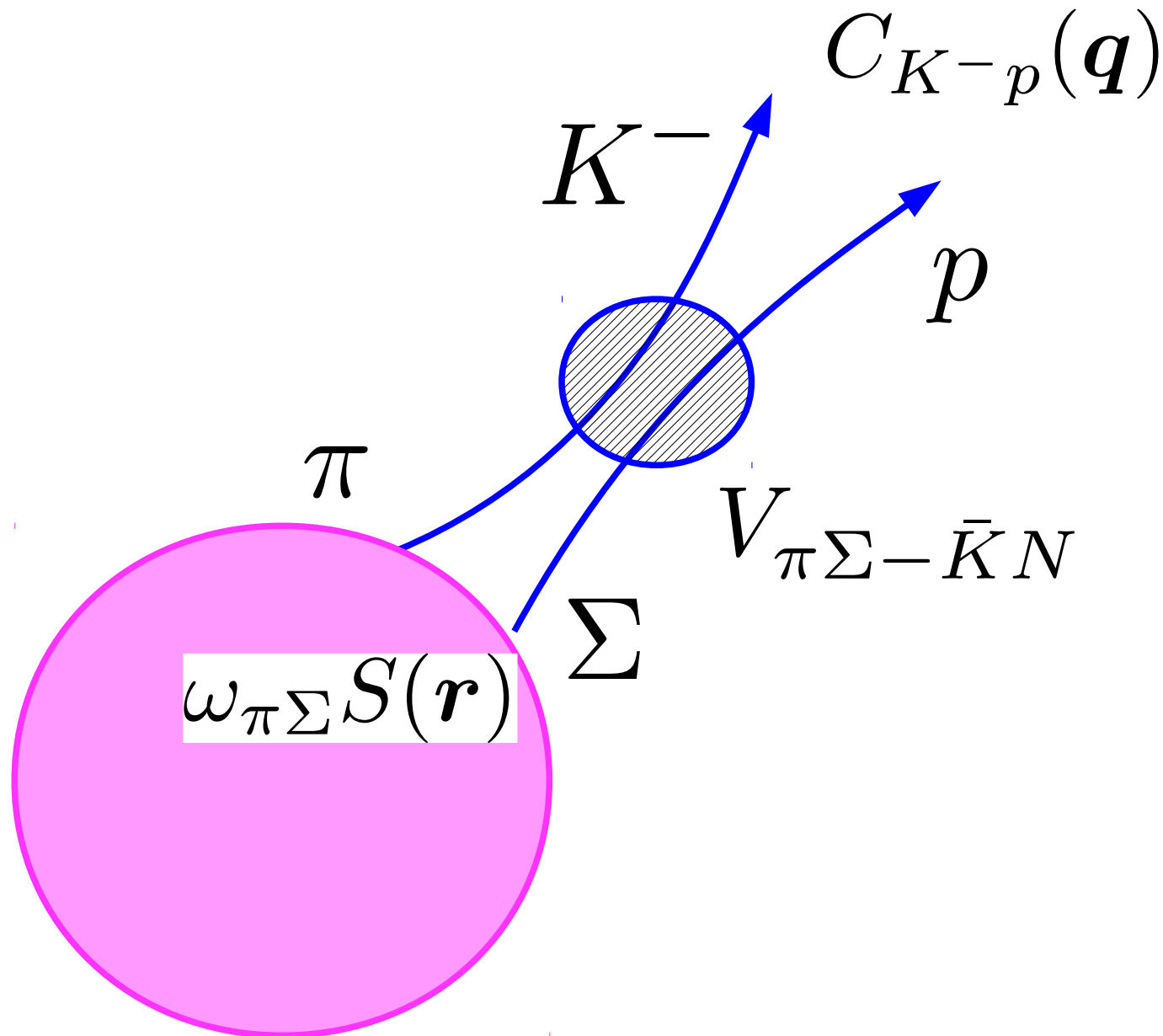
- **Coupled channel, w/ Coulomb**

$$C_i(\mathbf{q}) = \int d\mathbf{r} S_i(\mathbf{r}) \left[\underline{|\varphi^{C,\text{full}}(\mathbf{q}, \mathbf{r})|^2} + \underline{|\chi_i^{C,(-)}(r, q)|^2} - \underline{|j_0^C(qr)|^2} \right]$$

$$+ \sum_{j \neq i} \omega_j \int d\mathbf{r} S_j(\mathbf{r}) \underline{|\chi_j^{C,(-)}(r, q)|^2} \quad \text{s-wave w.f. in j-th channel}$$

Outgoing B.C. in the i-th channel, $\omega_j = \text{Source weight } (\omega_j=1)$

Correlation Function with Coupled-Channels Effects



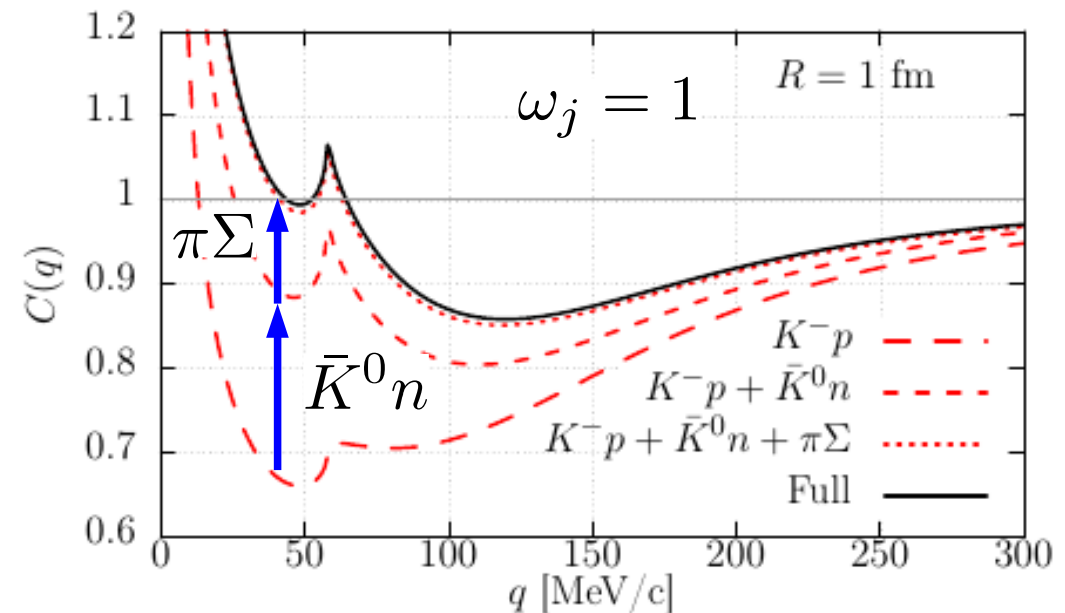
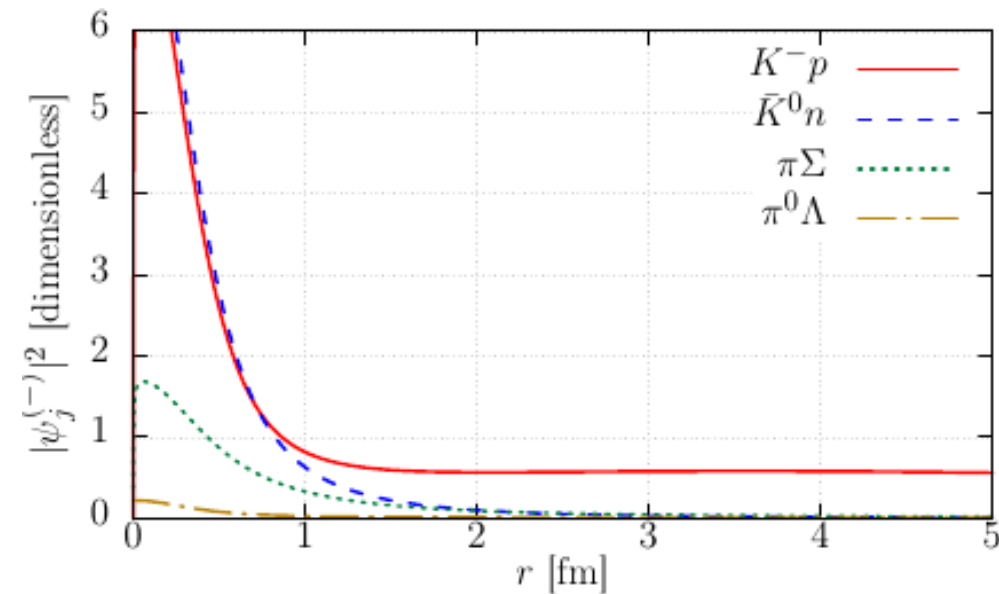
Correlation Function from Chiral SU(3) Potential (1)

- **Corr. Fn. from Chiral SU(3) coupled-channels potential + Coulomb + threshold difference (for the first time !)**

Y. Kamiya, T. Hyodo, K. Morita, AO, W. Weise, PRL124('20)132501 [1911.01041].

- **Coupled-channels effect**

- **W.f. of other channels than $K^- p$ decay in $r < 1$ fm.**
- **But they contribute to corr. fn. meaningfully.**



Correlation Function from Chiral SU(3) Potential (2)

- “Free” parameters

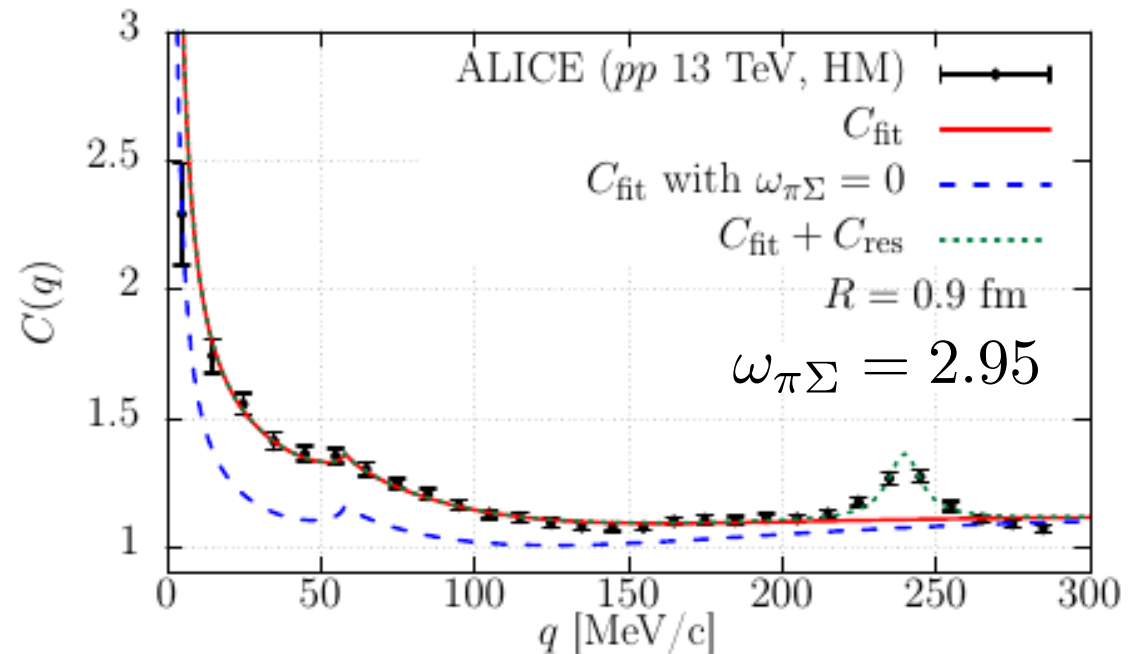
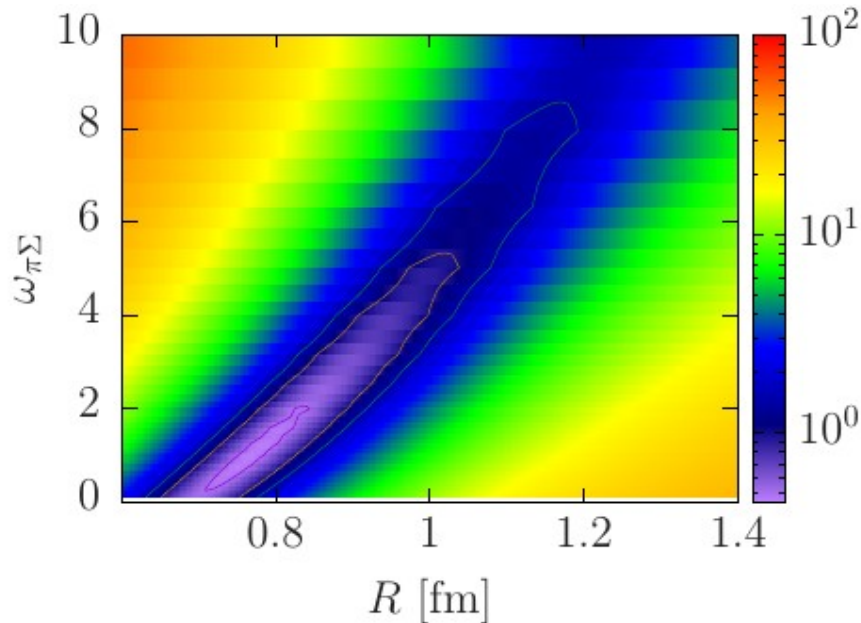
= Source Size R , Source Weight ω_j ← Th+Exp.

+ Normalization + Pair purity (λ) ← Exp.

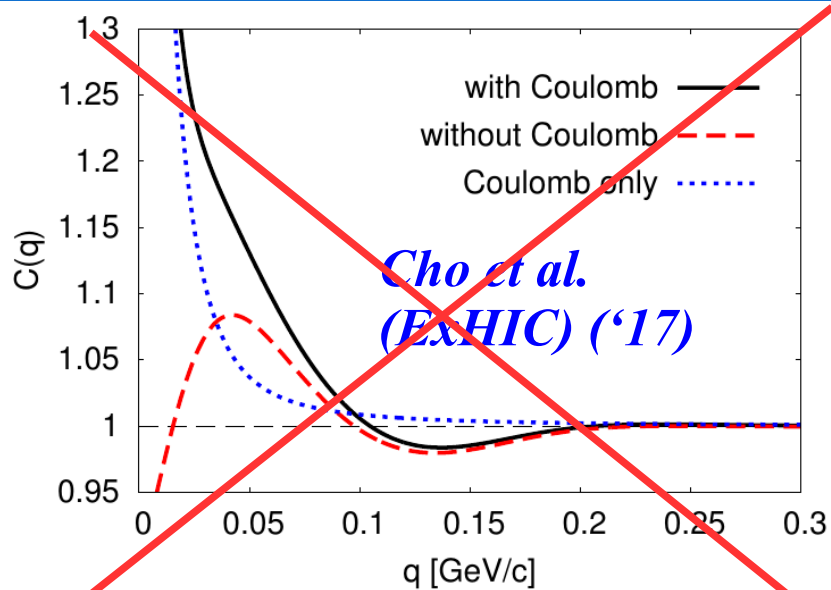
- Larger R → Smaller couple-channels effect from $\pi\Sigma$ (Favorable values of R and ω_j are correlated)

- Simple statistical model estimate

$$\omega_{\pi\Sigma} \sim \exp[(m_K + m_N - m_\pi - m_\Sigma)/T] \sim 2.$$

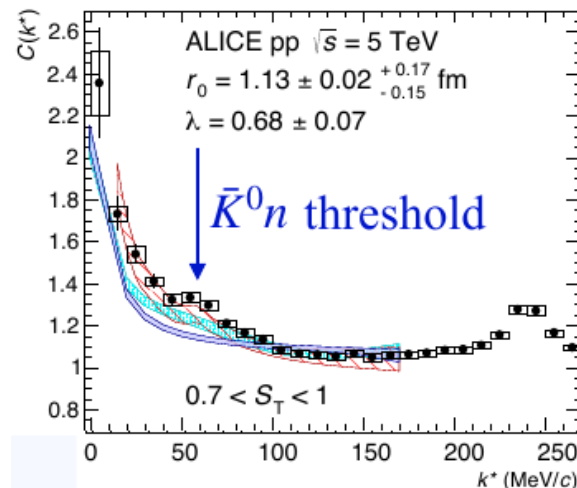


Comparison with other estimates



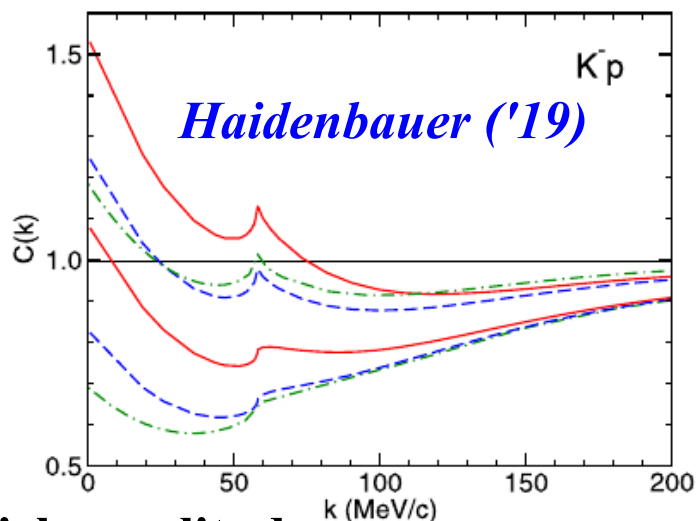
*Cho et al.
(ExHIC) ('17)*

No $\bar{K}^0n, \pi\Sigma$ source
No threshold diff.

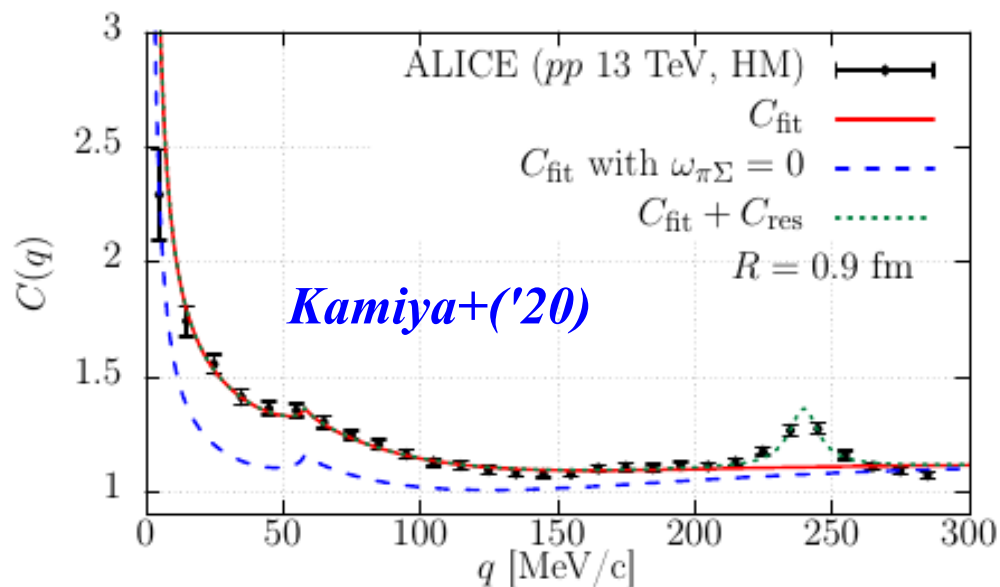


ALICE

Kyoto model (\sim Cho+('17)) \rightarrow accident
Julich model (Blue) Gamow corr. for Coulomb

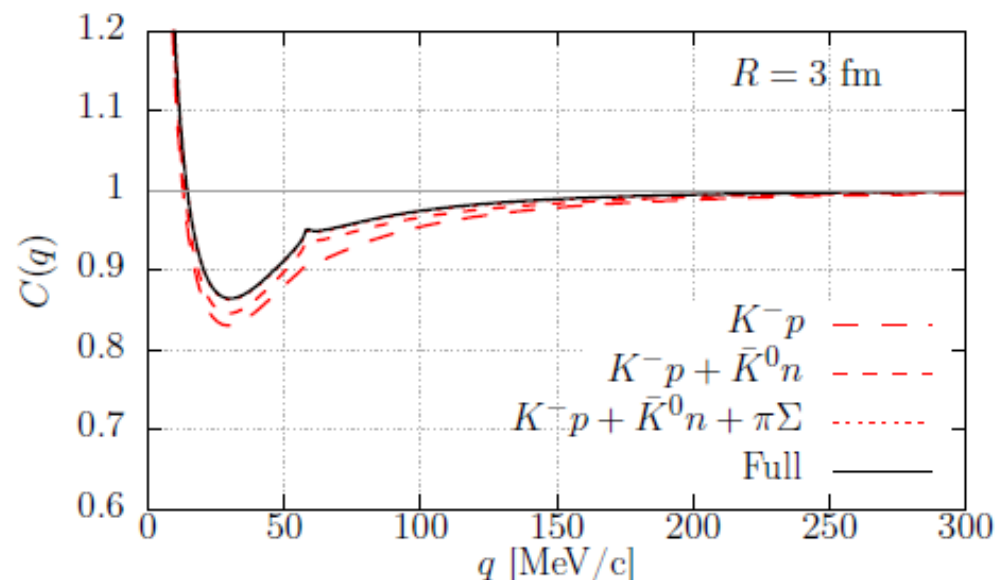
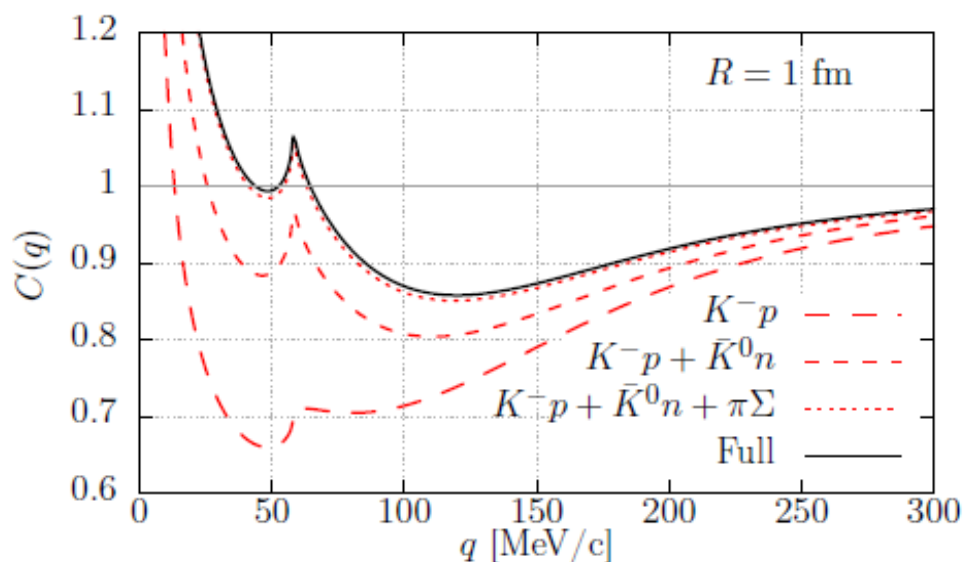


Julich amplitude
Consistent with Kamiya+('19) w/o Coulomb



Source Size Dependence (2)

- Experimental confirmation of coupled-channels contribution
→ Source size dependence
 - Channel w.f. other than $K^- p$ are localized at around $r=0$.
(Outgoing boundary condition for $K^- p$)
 - Contribution of $\pi\Sigma$ source is suppressed for larger R .



Source Size Dependence (2)

- Corr. Fn. from pA & AA collisions will elucidate the role of $\pi\Sigma$
 - $R \sim 1.6$ fm \rightarrow $\pi\Sigma$ effects are suppressed.

