Correlation functions of strange hadrons and their relevance to bound state search

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公募研究 19H05151: 2 粒子運動量相関から探るハドロン間相互作用としきい値近辺の散乱振幅

3rd Symposium on Clustering as a window on the hierarchical structure of quantum systems
May 18, 2020 (Online symposium)

Clusters & Hierarchies

http://www2.yukawa.kyoto-u.ac.jp/~akira.ohnishi/Slide/Cluster2020-AO.pdf

K. Morita, S. Gongyo, T. Hatsuda, T. Hyodo, Y. Kamiya, AO, PRC101 ('20) 015201 [1908.05414] (NΩ, ΩΩ)(Editors' Suggestion)

Y. Kamiya, T. Hyodo, K. Morita, AO, W. Weise, PRL 124 ('20) 132501 [1911.01041] [nucl-th] (K^-p)

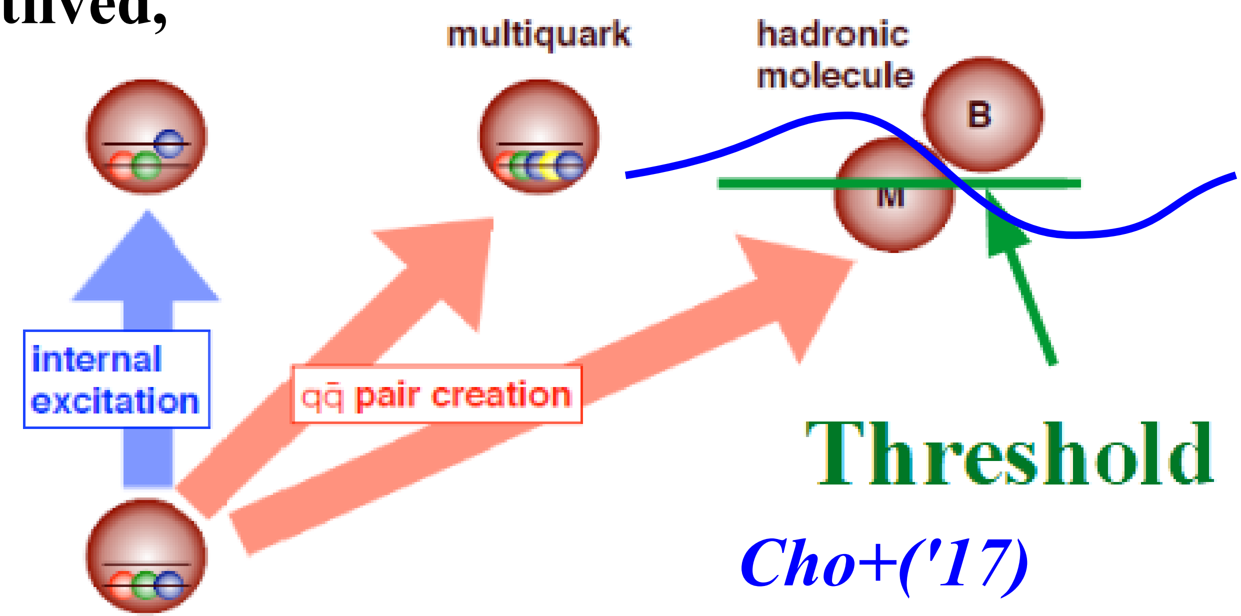
Y. Kamiya, K, Sasaki, T. Fukui, T. Hatsuda, T. Hyodo, K. Morita, K. Ogata, A. Ohnishi, work in progress (ΞN-ΛΛ)

Where do we find clusters ?

- Many cluster states appear around the threshold.
  - Long wave length → Easy for developed clustering states to appear
- What controls the scattering amplitude around the threshold ?
  → scattering length a_0

$$f(k) = [k \cot \delta(k) - ik]^{-1} \simeq \left(-\frac{1}{a_0} + \frac{1}{2} r_{\rm eff} k^2 - ik \right)^{-1}$$

- How can we obtain the scattering length in hadron-hadron int. ?
  - Many hadrons are shortlived, and scattering exp. is impossible.
    → scattering by final state int. (Correlation Function)



Cho+('17)

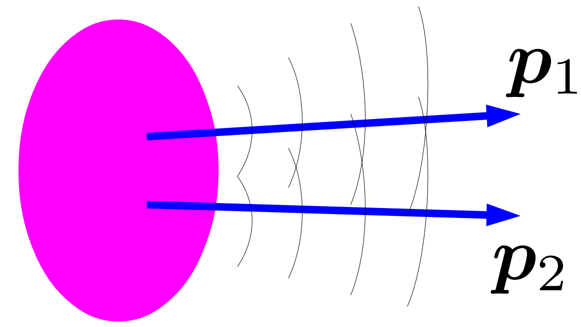
Outline

- **Introduction**
- **Correlation Function and Its Relevance to Interaction and Bound State**
  - **pΩ correlation from lattice QCD potential, STAR and ALICE**
- **Correlation Function Data and Hadron-Hadron Interaction – Coupled-channel effects –**
  - **$\bar{K}$N potential from chiral SU(3) dynamics and K^-p correlation from ALICE**
  - **ΞN-$\Lambda\Lambda$ potential from HAL QCD, and pΞ^- and $\Lambda\Lambda$ correlation**
- **Summary**

Correlation Function and Its Relevance to Interaction and Bound State

Correlation Function

- **Koonin-Pratt formula**



  - **Assumptions: independent particle production, corr. from final state int.**
    → Corr. Fn.= Scattering via FSI
    from chaotic initial cond. **(c.f. Ishikawa)**
    Koonin('77), Pratt+('86), Lednicky+('82)

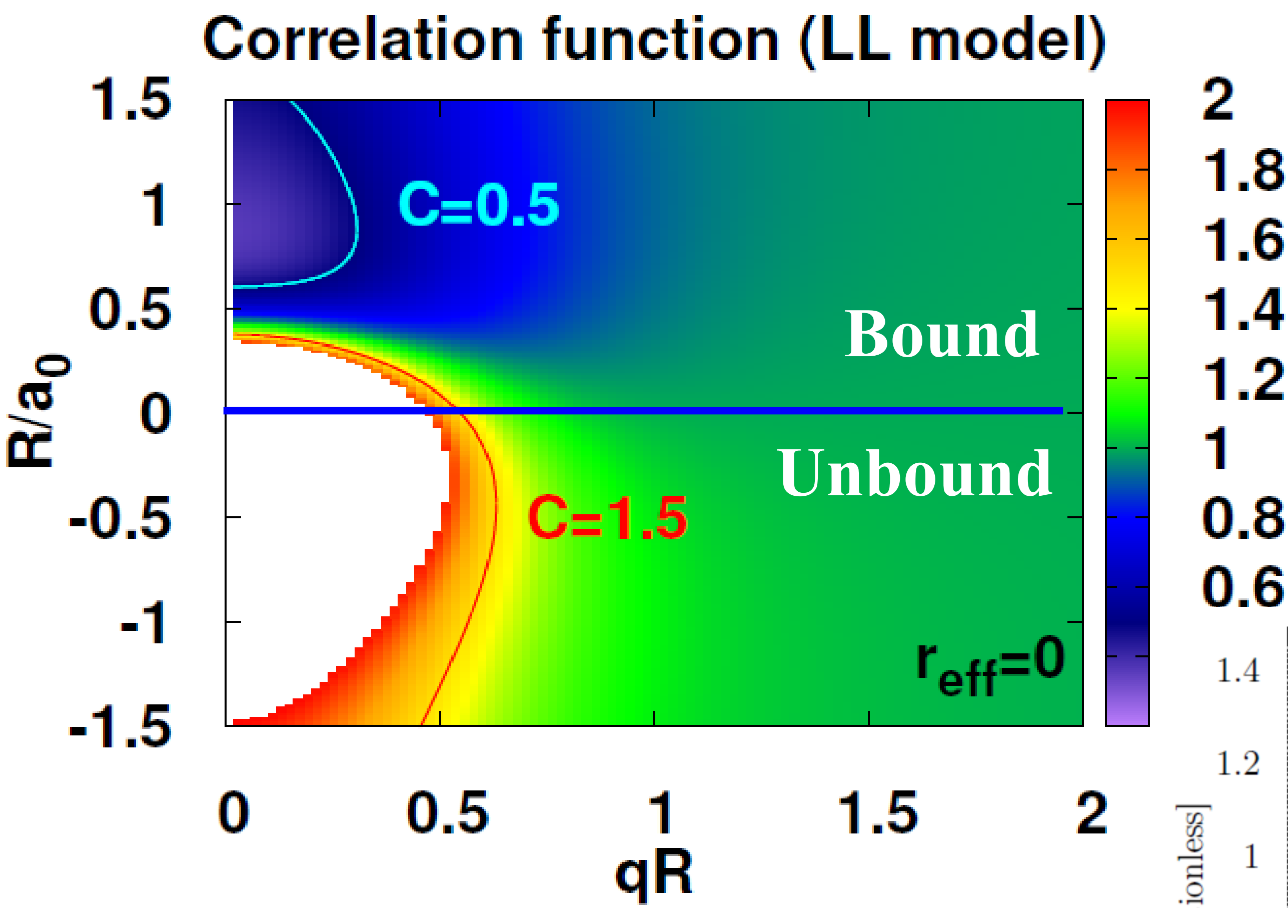
$$C(\boldsymbol{p}_1, \boldsymbol{p}_2) = \frac{N_{12}(\boldsymbol{p}_1, \boldsymbol{p}_2)}{N_1(\boldsymbol{p}_1) N_2(\boldsymbol{p}_2)} \simeq \int d\boldsymbol{r} S_{12}(\boldsymbol{r}) |\varphi_{\boldsymbol{q}}(\boldsymbol{r})|^2$$

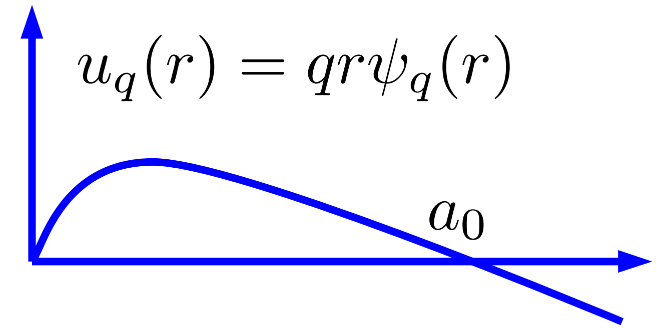
  - **Further assumptions: Only s-wave (L=0) is modified, Non-identical particle pair, Spherical source, w/o Coulomb**
    K. Morita, T. Furumoto, AO, PRC91('15)024916

$$C(\boldsymbol{q}) = 1 + \int d\boldsymbol{r} S(r) \left\{ |\chi_q(r)|^2 - |j_0(qr)|^2 \right\}$$

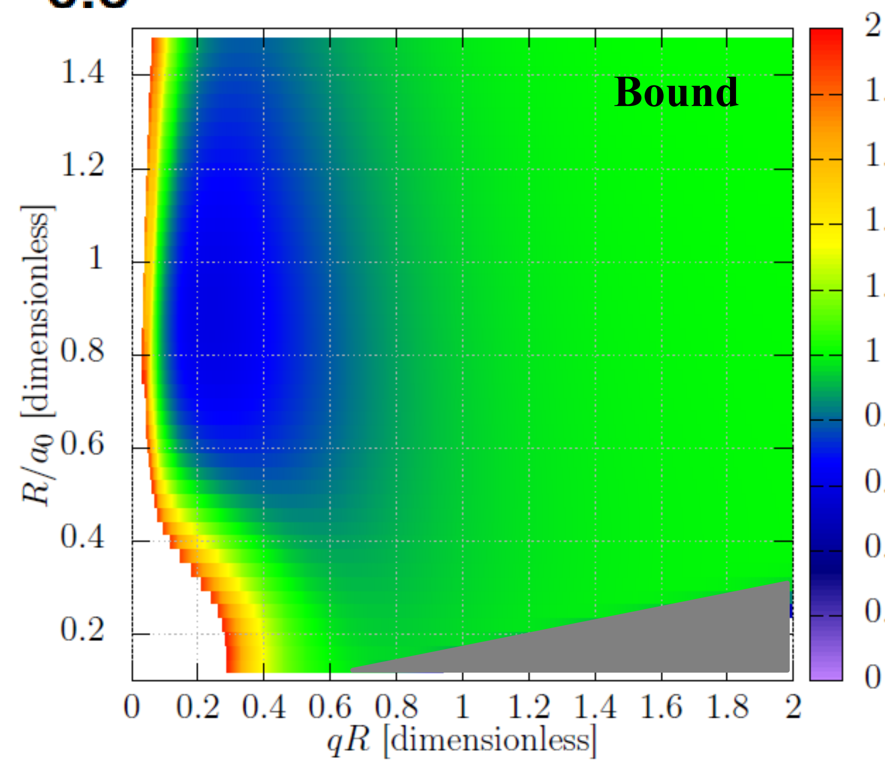
Corr. Fn. shows how much squared w. f. is enhanced
→ Large CF is expected with attraction

Source Size dependence of Correlation Function





HAL QCD (NΩ, J=2, a_0=3.4 fm) + Coulomb

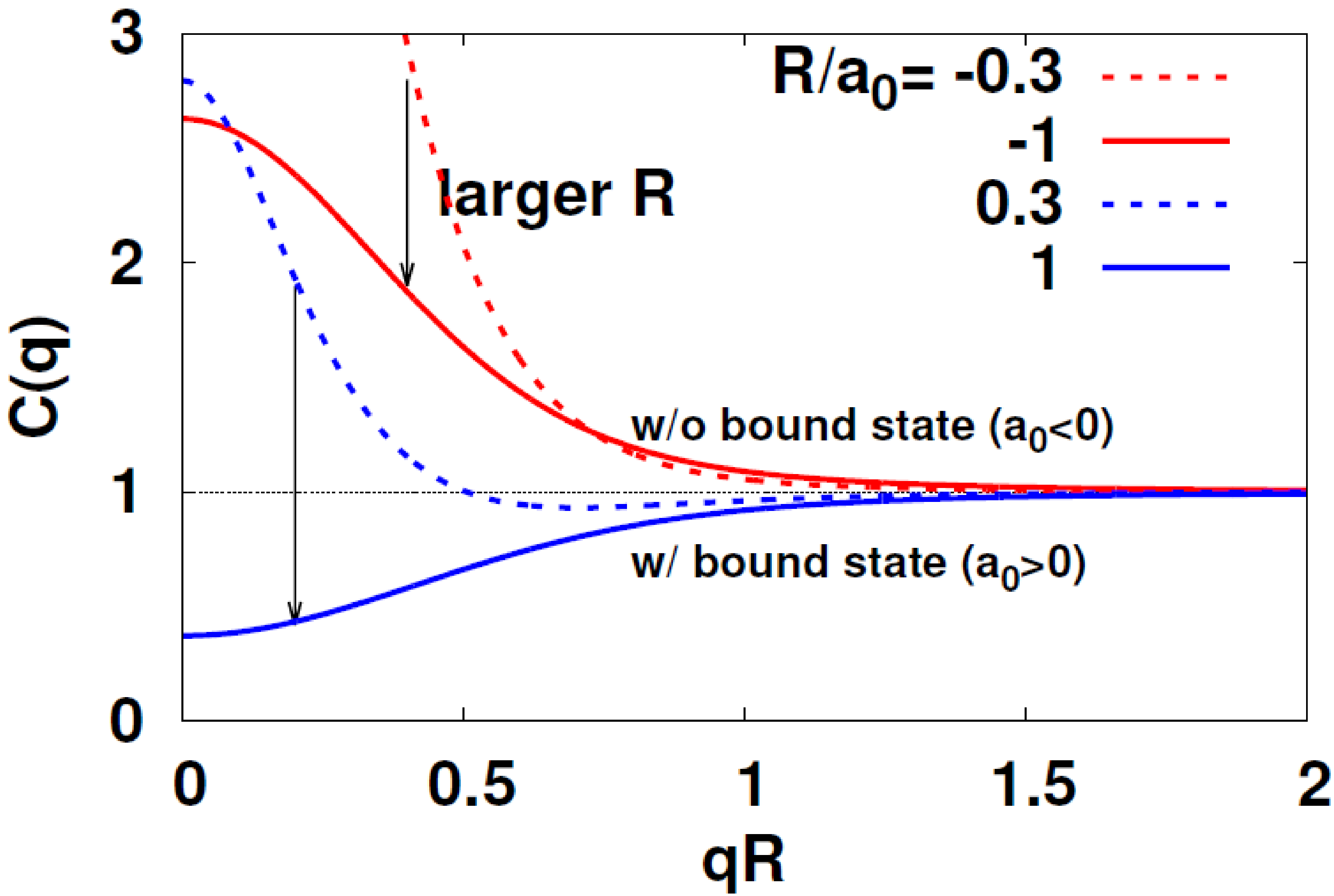


LL model: R. Lednicky, V. L. Lyuboshits ('82)
Similar Fig. is shown in AO, K. Morita,
K. Miyahara, T. Hyodo, NPA954('16), 294.

$$q \cot \delta = -\frac{1}{a_0} + \frac{1}{2} r_{\mathrm{eff}} q^2 + \mathcal{O}(q^4)$$

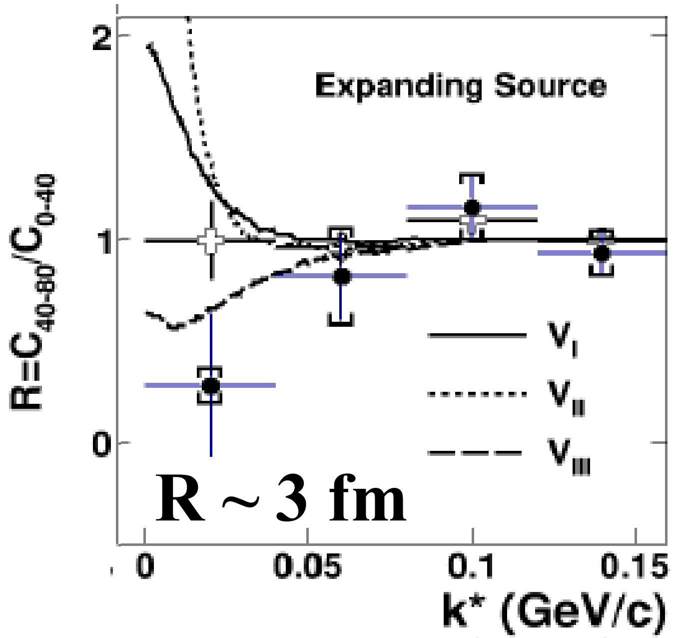
$$\to \delta \simeq -a_0 q$$

Source Size dependence of Correlation Function



K. Morita, S. Gongyo, T. Hatsuda, T. Hyodo, Y. Kamiya, AO, PRC101 ('20) 015201 [1908.05414] (NΩ, ΩΩ)(Editors' Suggestion)

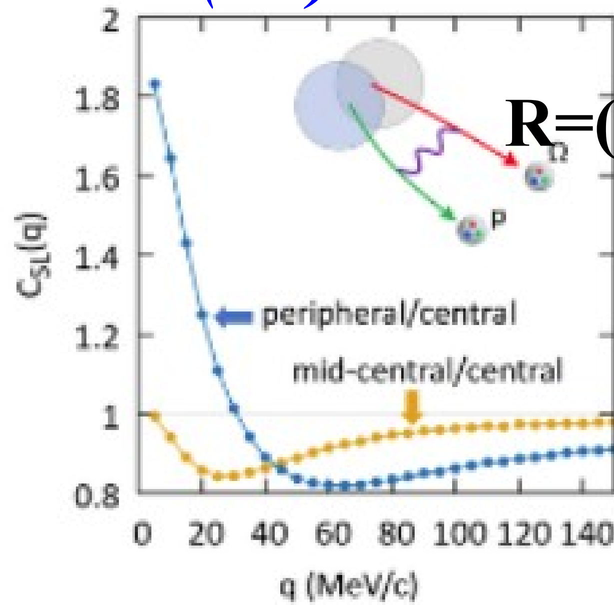
STAR + ALICE = NΩ Dibaryon

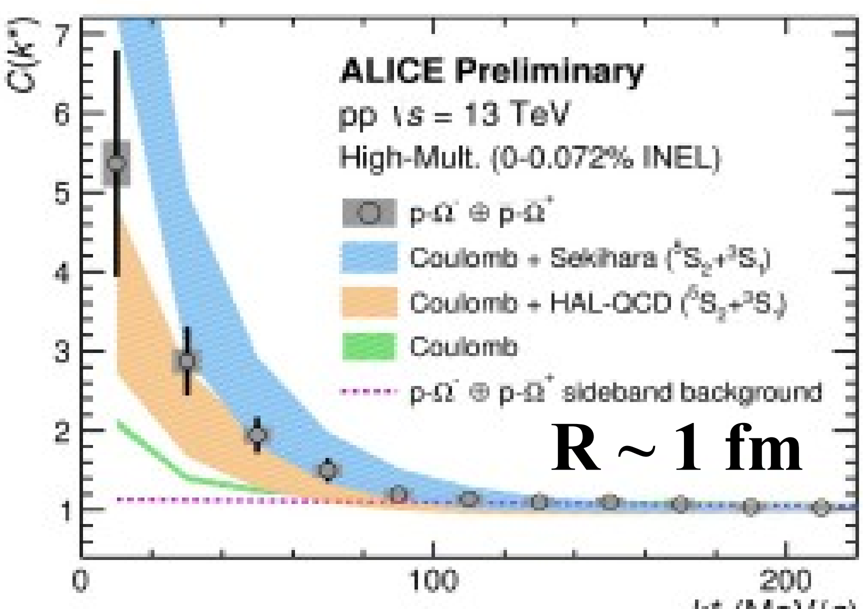


Au+Au (STAR, PLB790 ('19) 490)

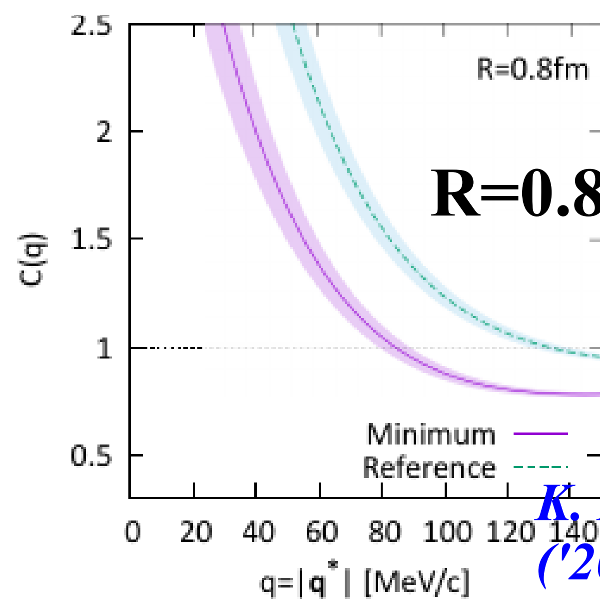
Corr. Fn. from HAL QCD pot.

Iritani+('19)





pp (ALICE, PRL123('19)112002)



K. Morita+, PRC101 ('20) 015201

Corr. Fn. Signal of a Bound State

- **Source size dep. of CF signals the existence of a bound state.**
  - **With a loosely bound state,**
    CF is enhanced at small R (a_0 << R)
    CF is suppressed at R ~ a_0
  - **pΩ Corr. Fn. data show the typical behavior with a bound state.**

> ***There should be (a) bound state(s) in pΩ,***
> ***whose scatt. length is around the source size of HIC.***
> ***Any other possibility ?***

- **Other hadronic molecule ?**
  - **Λ(1405) ~ $\bar{K}$N bound state**
  - **H dibaryon ~ pole near ΞN threshold ?**

→ We need a coupled-channel framework !

Correlation Function Data and Hadron-Hadron Interaction – Coupled-channel effects –

Correlation Function with Coupled-Channels Effects

J. Haidenbauer, NPA 981('19)1; R. Lednicky, V. V. Lyuboshits, V. L. Lyuboshits, Phys. At. Nucl. 61('98)2950.

- **Single channel, w/o Coulomb (non-identical pair)**

$$C(\boldsymbol{q}) = \underline{1} + \int d\boldsymbol{r} S(\boldsymbol{r}) \left[\underline{|\chi^{(-)}(r,q)|^2} - \underline{|j_0(qr)|^2} \right]$$

- **Single channel, w/ Coulomb**

$$C(\boldsymbol{q}) = \int d\boldsymbol{r} S(\boldsymbol{r}) \left[\underline{|\varphi^{C,\mathrm{full}}(\boldsymbol{q},\boldsymbol{r})|^2} + \underline{|\chi^{C,(-)}(r,q)|^2} - \underline{|j_0^C(qr)|^2} \right]$$

Full free Coulomb w.f. **s-wave w.f. with Coul.** **s-wave Coul. w.f.**

- **Coupled channel, w/ Coulomb**

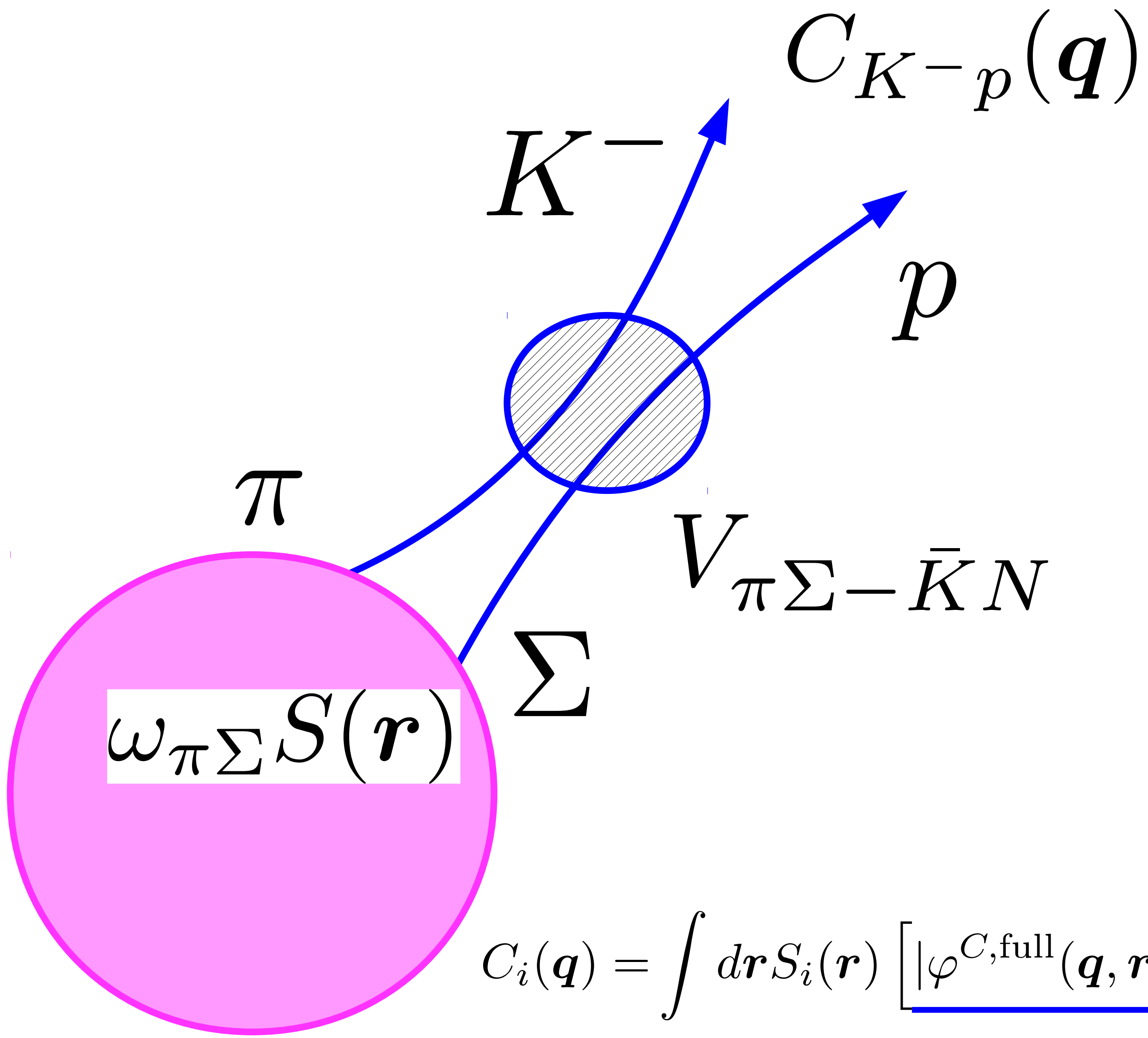
$$C_i(\boldsymbol{q}) = \int d\boldsymbol{r} S_i(\boldsymbol{r}) \left[\underline{|\varphi^{C,\mathrm{full}}(\boldsymbol{q},\boldsymbol{r})|^2} + \underline{|\chi_i^{C,(-)}(r,q)|^2} - \underline{|j_0^C(qr)|^2} \right]$$

$$+ \sum_{j \neq i} \omega_j \int d\boldsymbol{r} S_j(\boldsymbol{r}) \underline{|\chi_j^{C,(-)}(r,q)|^2}$$

s-wave w.f. in j-th channel

Outgoing B.C. in the i-th channel, ω_j = Source weight (ω_j=1)

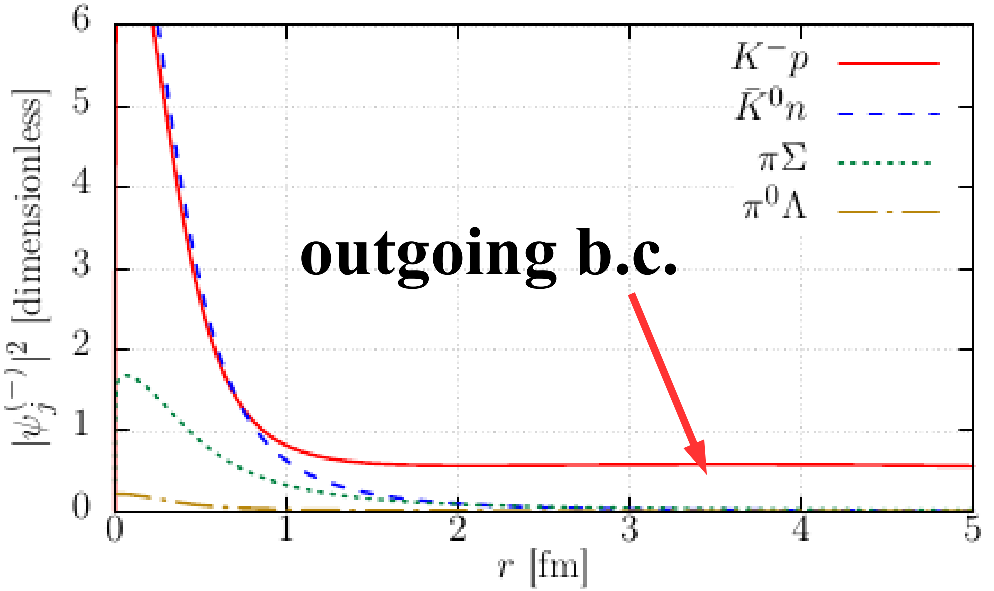
Correlation Function with Coupled-Channels Effects

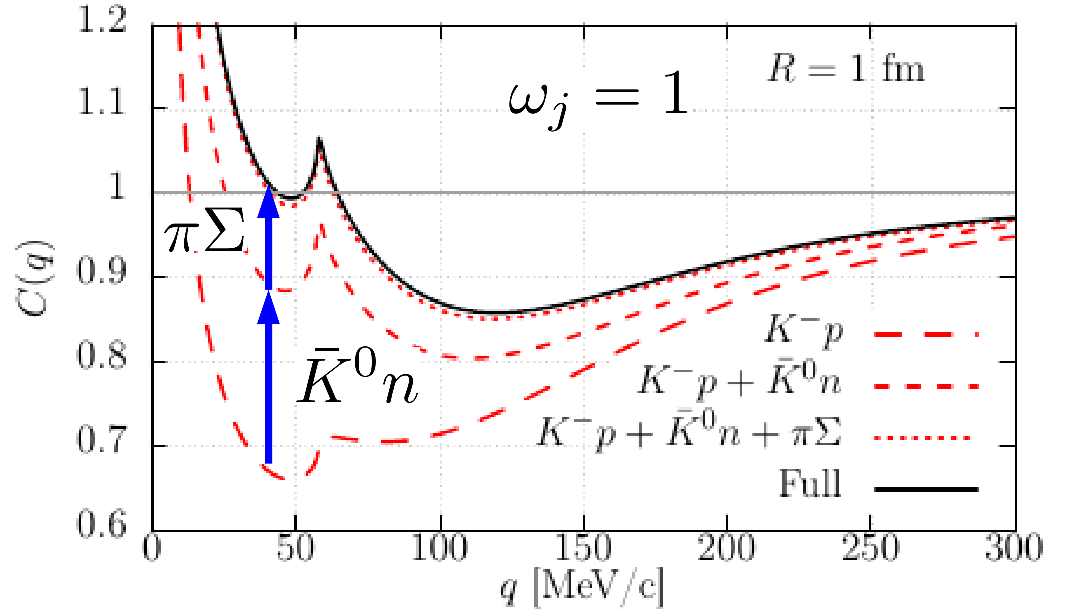


$$C_i(\boldsymbol{q}) = \int d\boldsymbol{r} S_i(\boldsymbol{r}) \left[|\varphi^{C,\mathrm{full}}(\boldsymbol{q},\boldsymbol{r})|^2 + |\chi_i^{C,(-)}(r,q)|^2 - |j_0^C(qr)|^2 \right] + \sum_{j\neq i} \omega_j \int d\boldsymbol{r} S_j(\boldsymbol{r}) |\chi_j^{C,(-)}(r,q)|^2$$

K^-p Correlation Function from Chiral SU(3) Potential (1)

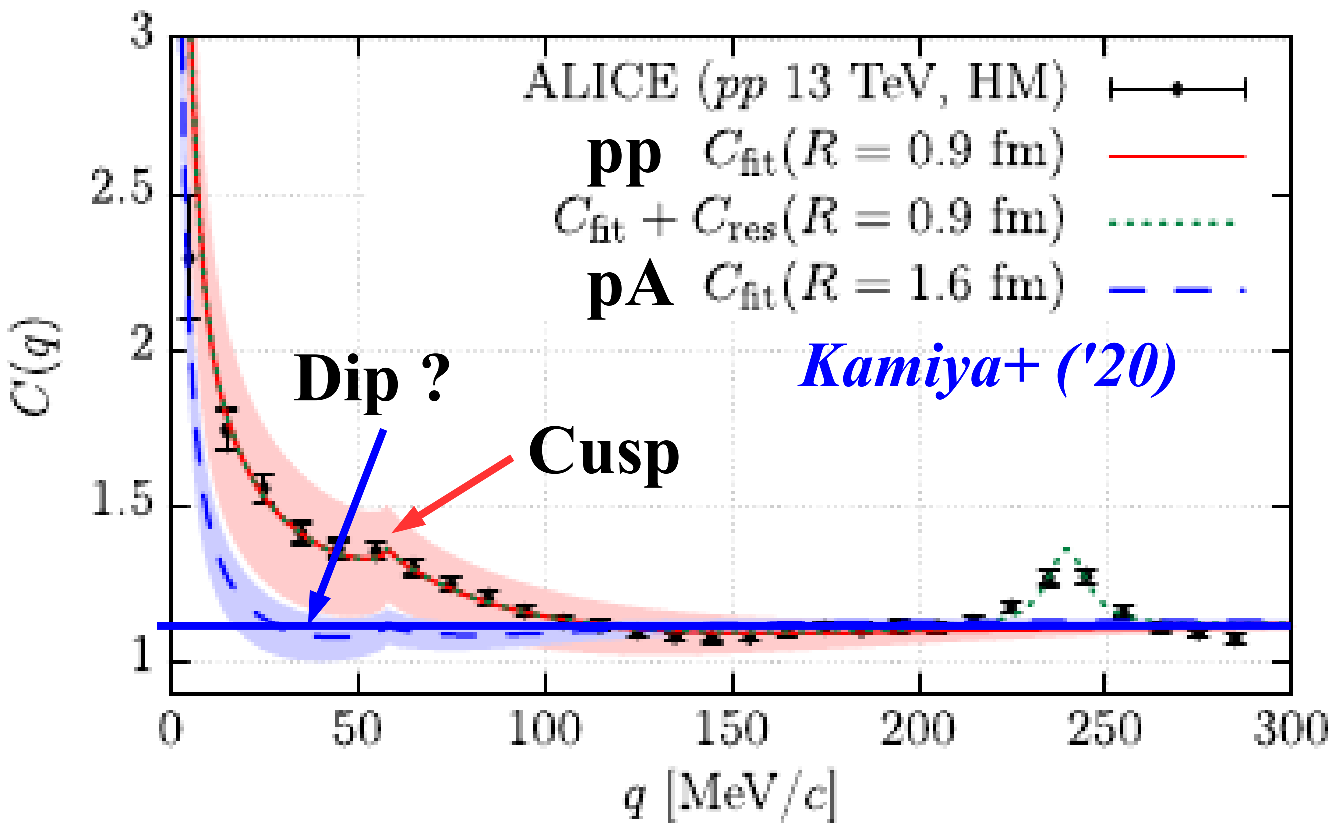
- **Chiral SU(3) potential**
  Ikeda, Hyodo, Weise ('12); Miyahara, Hyodo, Weise ('18)
- **Coupled-channels effect**
  - **W.f. of other channels than K^- p decay in r < 1 fm.**
  - **But they contribute to corr. fn. meaningfully.**
- **Corr. Fn. from Chiral SU(3) coupled-channels potential + Coulomb + threshold difference (for the first time !)**
  Y.Kamiya, T.Hyodo, K.Morita, AO, W.Weise, PRL124('20)132501





K^-p Correlation Function from Chiral SU(3) Potential (2)

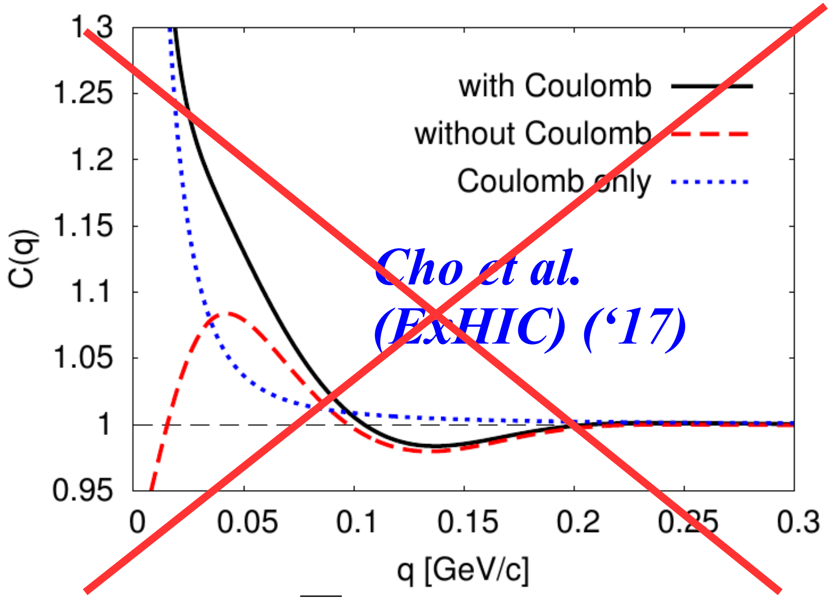
- Unknown (relevant) parameters = R (size), $\omega_{\pi\Sigma}$ ($\pi\Sigma$ weight)
  → R=0.9 fm, $\omega_{\pi\Sigma}$=2.95
  for pp 13 TeV high-multiplicitiy events
- CC effects are suppressed for larger size reactions
  → Corr. Fn. from pA reactions will examine CC effects !



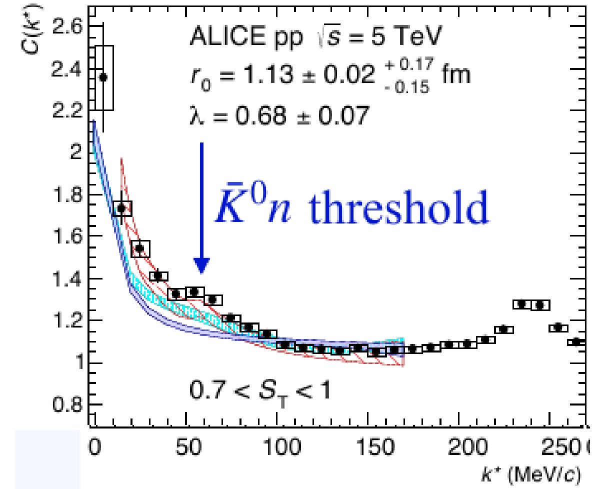
Data: S. Acharya et al. (ALICE), PRL124('20)092301

$\omega_{\pi\Sigma}$=2.95, N=1.13, λ=0.58

Comparison with other estimates

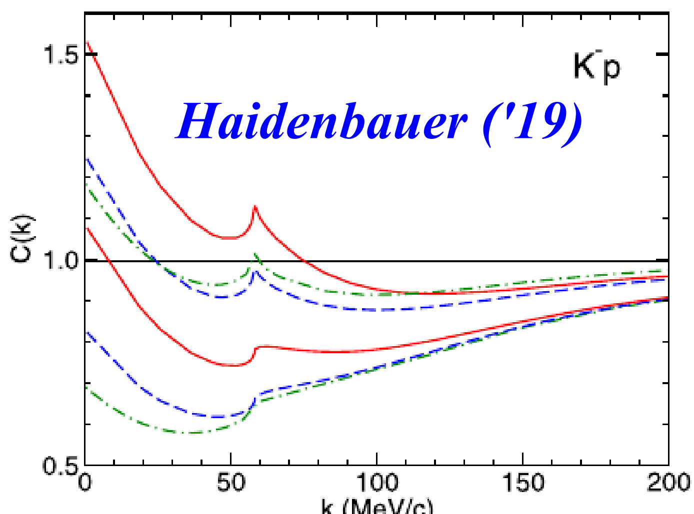


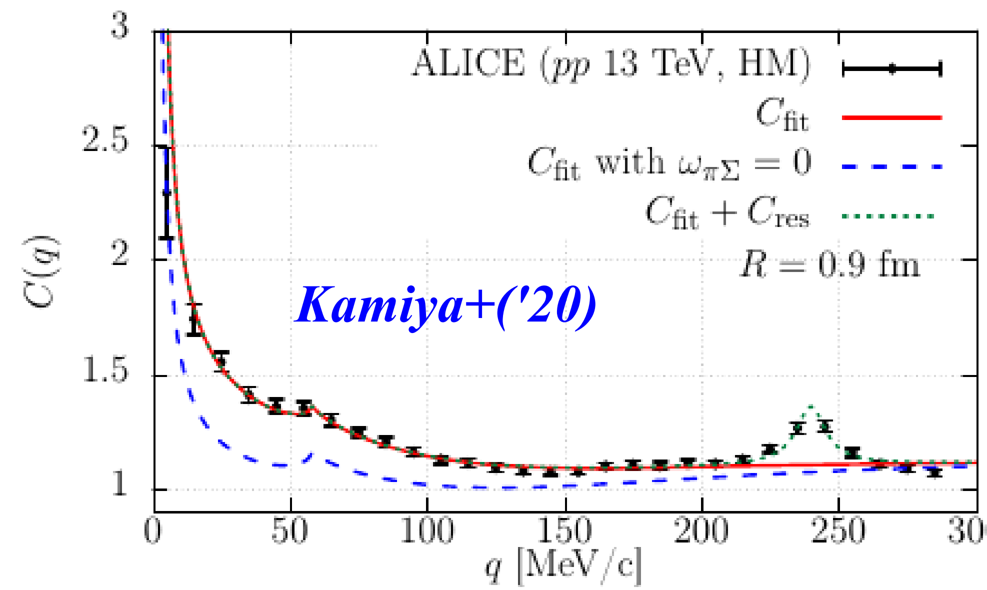
No $\overline{K}^0 n$, $\pi\Sigma$ source
No threshold diff.



ALICE

Kyoto model (~Cho+('17))→ accident
Julich model (Blue) Gamow corr. for Coulomb





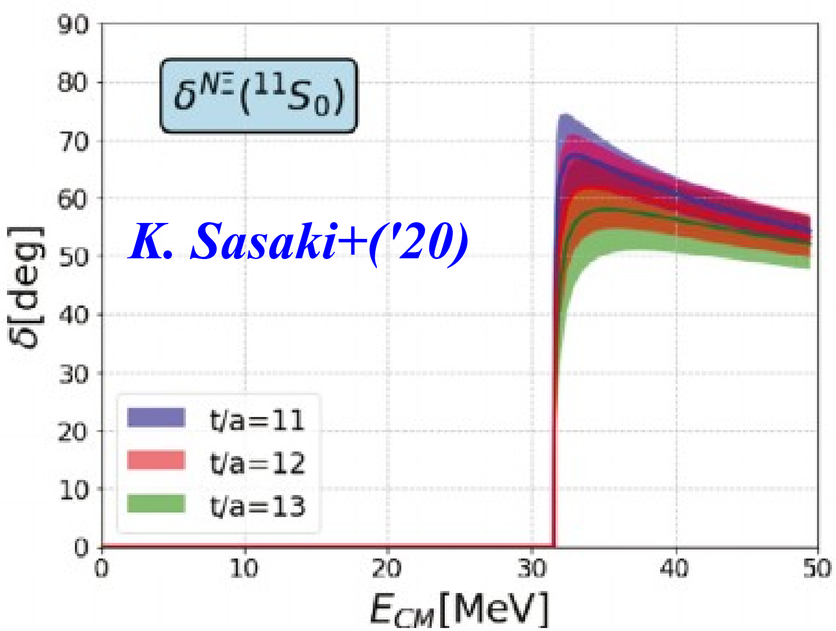
Julich amplitude
Consistent with Kamiya+('20) w/o Coulomb

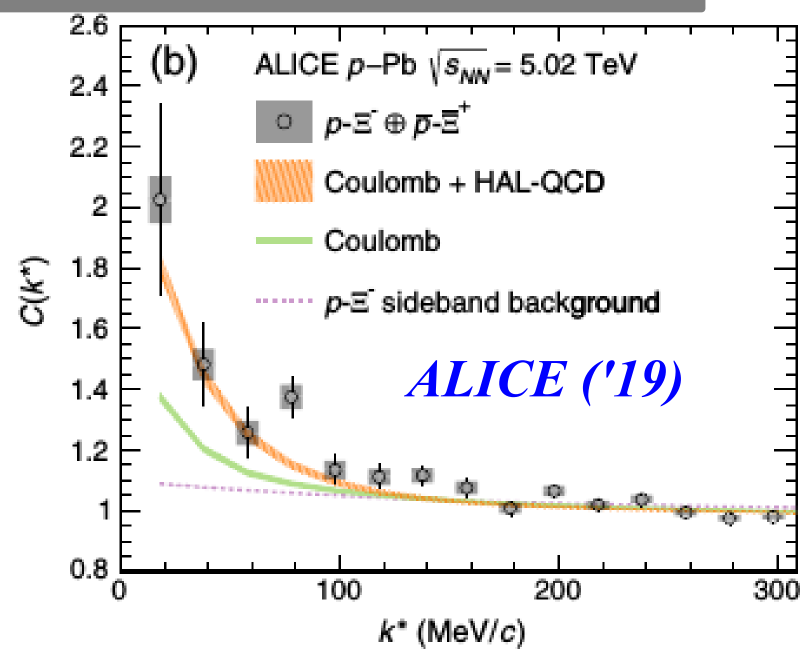
$p\Xi^-$ correlation from Lattice BB potential

- **S=–2 HAL QCD potential**
  H particle virtual pole in NΞ ($^{11}S_0$) channel
  K. Sasaki et al. (HAL QCD), PoS LATTICE2016 ('17) 116 (heavy quarks);
  K. Sasaki et al. (HAL QCD), NPA 998 ('20)121737 (~phys. mass)

- **$p\Xi^-$ correlation function**
  S. Acharya et al. (ALICE), PRL123('19)112002;
  T. Hatsuda, K. Morita, AO, K. Sasaki, NPA 967 ('17) 856 (← HAL('17));

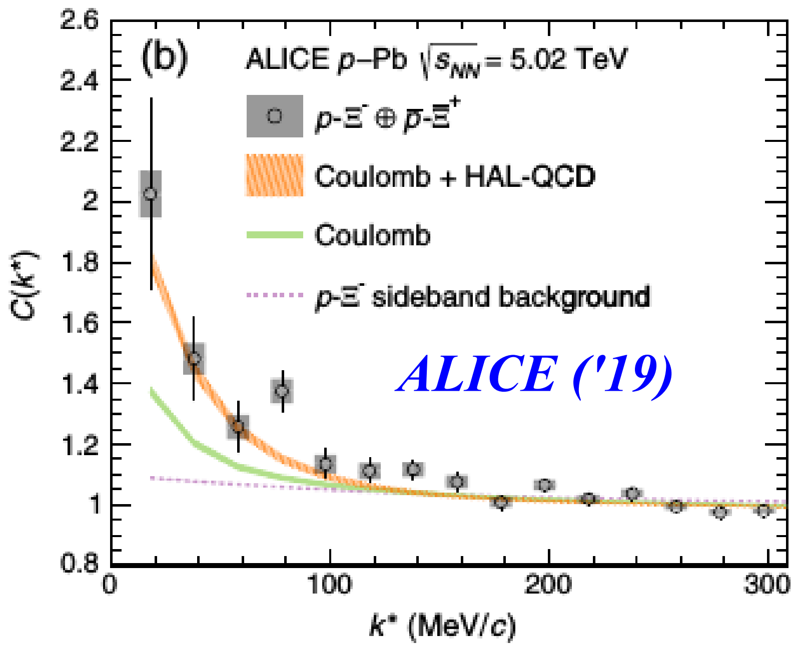
Let us calc. CF w/ updated HAL QCD pot. !

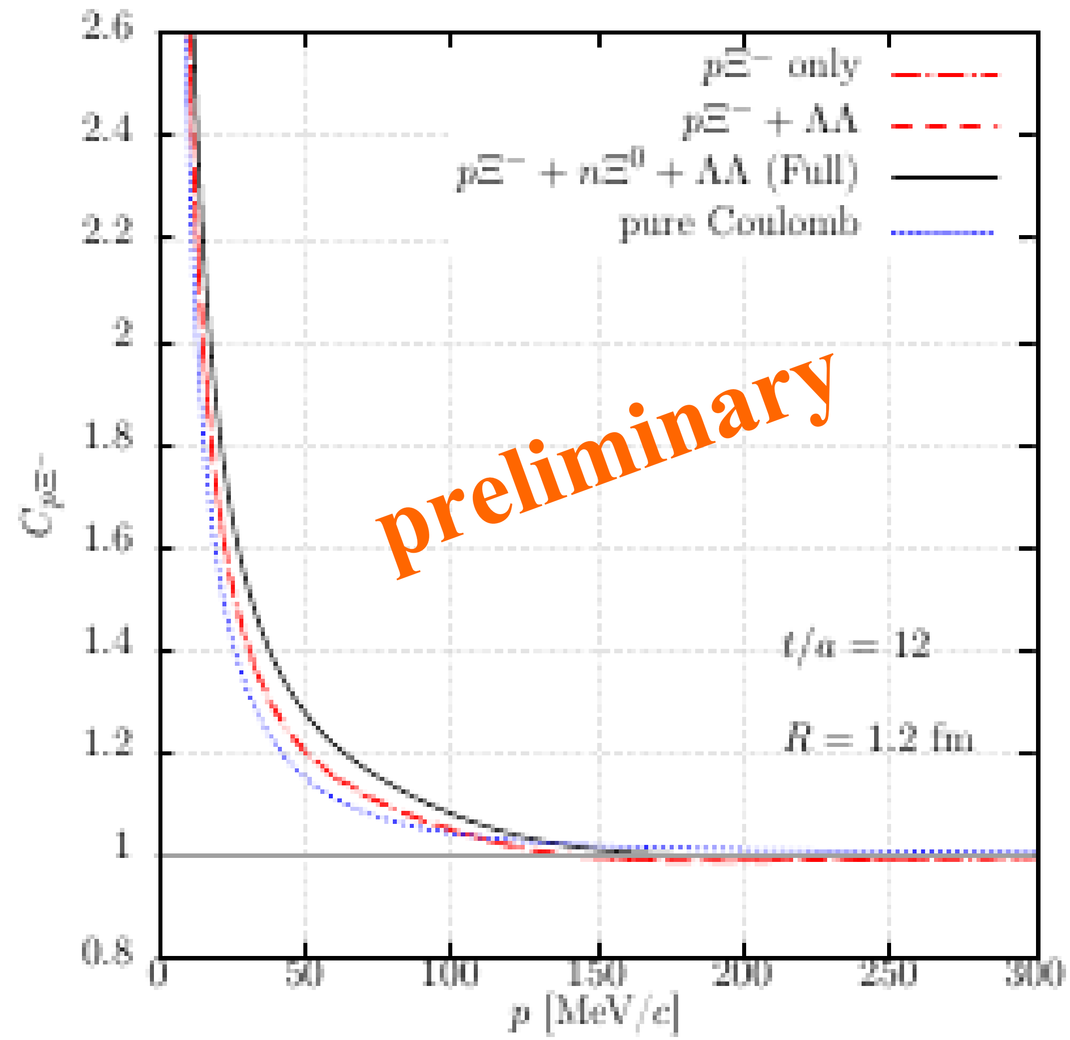




$p\Xi^-$ correlation from Lattice BB potential

- **$p\Xi^-$ correlation function with updated HAL QCD S=–2 potential**
  Y. Kamiya, K. Sasaki, T. Fukui, T. Hatsuda, T. Hyodo, K. Morita, K. Ogata, AO, work in prog.
  - **Coupling with ΛΛ channel is not very important in $p\Xi^-$ correlation.**





Summary

- **Hadron-hadron momentum correlation functions are useful to get knowledge on hadron-hadron interactions and the existence of a bound state.**
  - **Large corr. fn. at small q implies large $|a_0|/R$.**
    The source size dep. may show the sign of a_0,
    to be or not to be bound.
  - **ALICE and STAR data strongly suggest**
    the existence of S= –3 dibaryon as a bound state of NΩ.
- **Coupled-channel effects are discussed mainly for K^- p corr.**
  - **ALICE data are consistent with chiral SU(3) $\bar{K}N$-πΣ-πΛ amplitudes, while $\omega_{\pi\Sigma}$ needs to be assumed.**
    Corr. fn. from larger source size will elucidate the CC effects.
  - **ALICE data of Ξ^- p implies large $|a_0|/R$, as suggested by the (updated) HAL QCD potential.**

To do

- **Examine and complete CC results in the NΞ-ΛΛ system (Kamiya, Fukui, Ogata).**
- **ALICE seems to have h-deuteron correlation data ($h \sim K^-, \Xi^-, \Lambda, \ldots$)**
  → Continuum Discretized Coupled-Channels (CDCC) (Fukui, Ogata)
- **Is the dip structure associated with a loosely bound state generic ?**
  - **E.g. pn correlation ($a_0 \sim$ (5-6) fm) in AA at LHC ?**
  - **Detecting charge neutral hadrons is important, e.g. ηN.**
- **Three-body momentum correlation and three-body force.**
- **It is desired to re-develop quark cluster model hh force with the light of lattice hh potential.**
- **Can we determine the scattering length only from Corr. Fn. ?**

Thank you for attention !

Coauthors (except for ExHIC members)

K. Morita

S. Gongyo

T. Hatsuda

T. Hyodo

T. Fukui

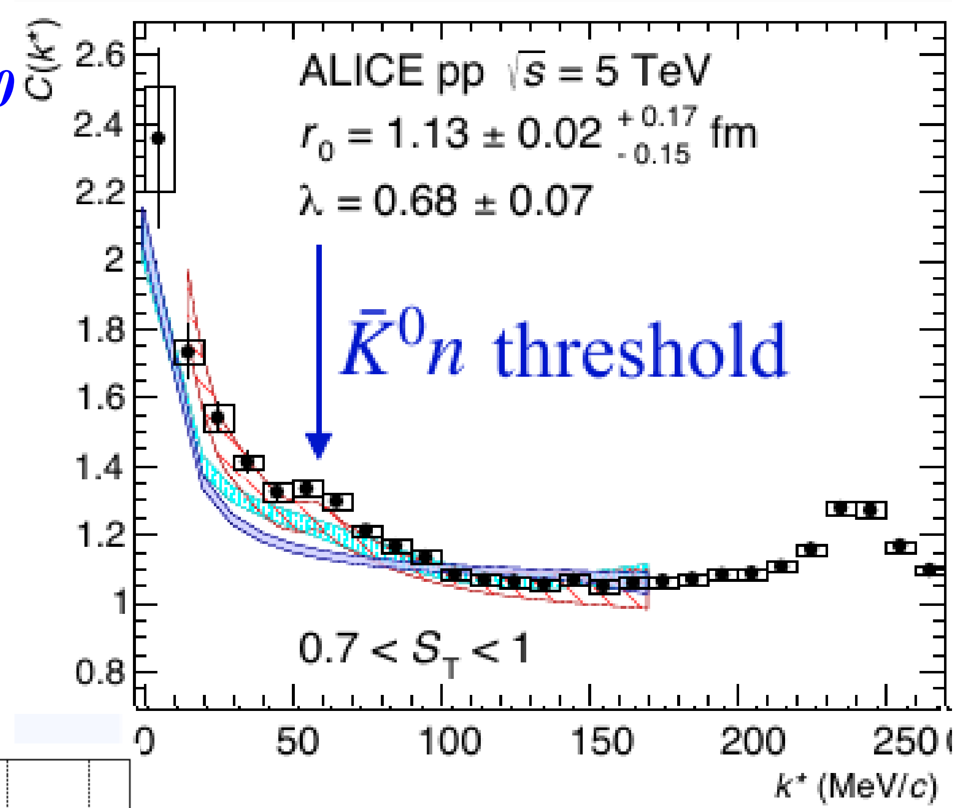
K. Ogata

K. Sasaki **AO** **Y. Kamiya**

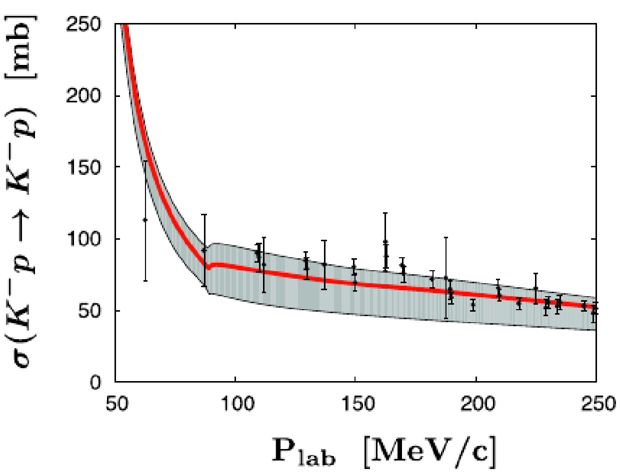
W. Weise

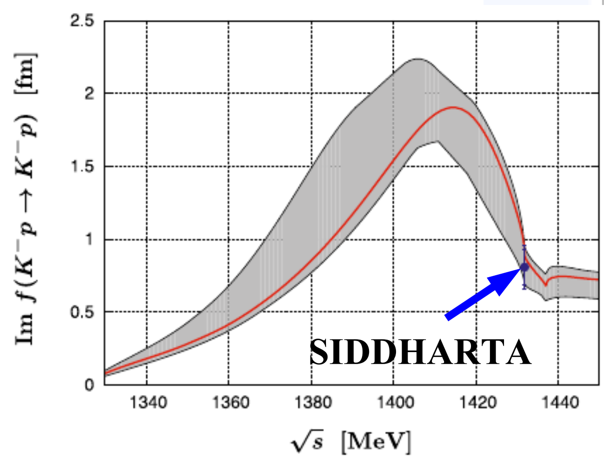
K^- p correlation function data

- **K – p correlation function from high-multiplicity events of pp collisions**
  S. Acharya et al. (ALICE), PRL124('20)09230 [1905.13470]
  - **High precision data from low to high momentum ! c.f. Previous scatt. data & Kaonic atom data.**
  - **Enhanced at low k, cusp, Λ(1520), ...**



Red:Kyoto model
Blue: Julich model
grey: Coulomb

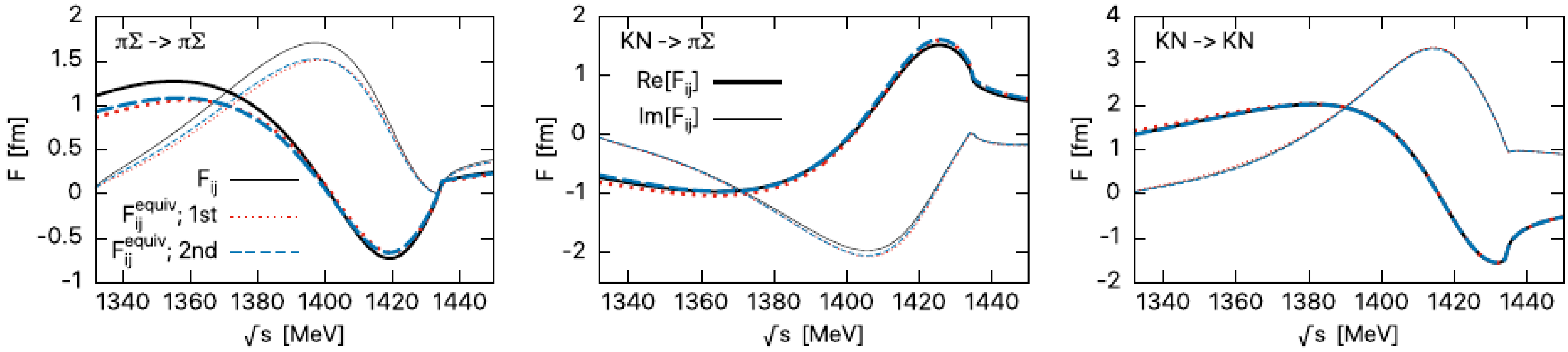




Y. Ikeda, T. Hyodo, W. Weise,NPA881 ('12) 98

$\bar{K}N$-$\pi\Sigma$-$\pi\Lambda$ Scattering Amplitude and Potential

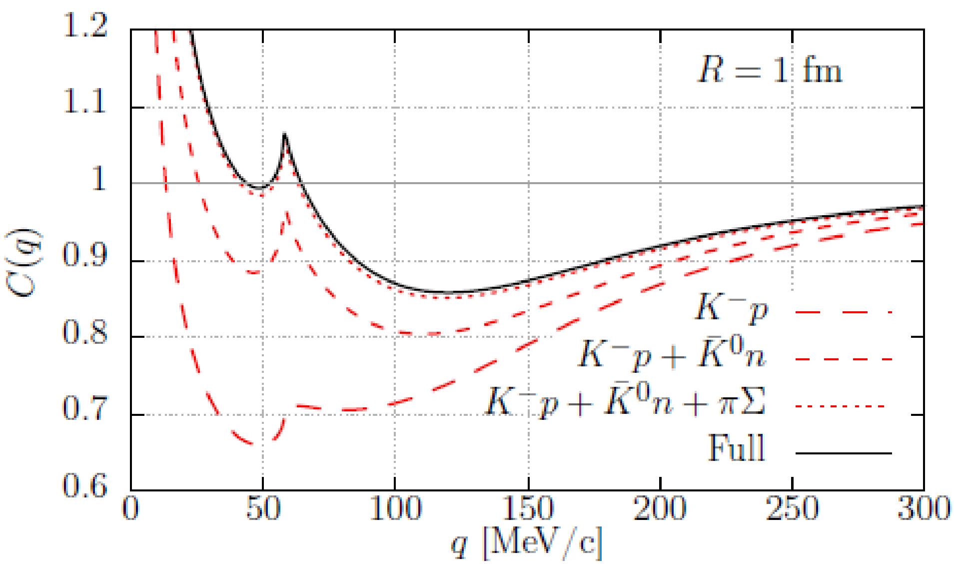
- **Amplitude in chiral SU(3) coupled-channels dynamics**
  Y. Ikeda, T. Hyodo, W. Weise,NPA881 ('12) 98
  - **NLO meson-baryon effective Lagrangian ($\bar{K}N$-$\pi\Sigma$-$\pi\Lambda$) + fit of Kaonic Hydrogen, Cross Section, Threshold branching ratio**
- **Coupled-channels potential**
  K. Miyahara, T. Hyodo, W. Weise, PRC98('18)025201
  - **Potential fitted to IHW amplitude**

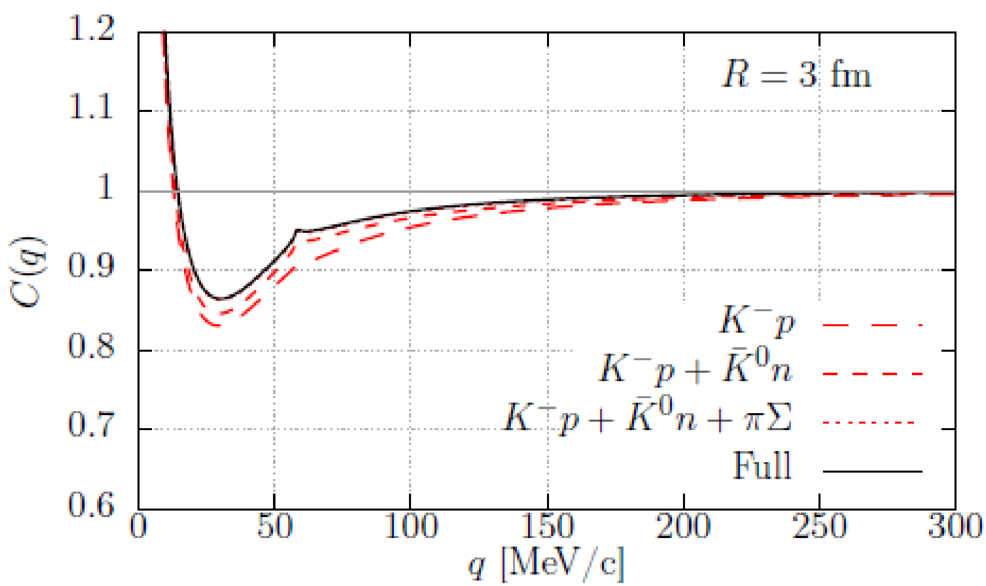


Y. Ikeda, T. Hyodo, W. Weise,NPA881 ('12) 98
K. Miyahara, T. Hyodo, W. Weise, PRC98('18)025201

Source Size Dependence

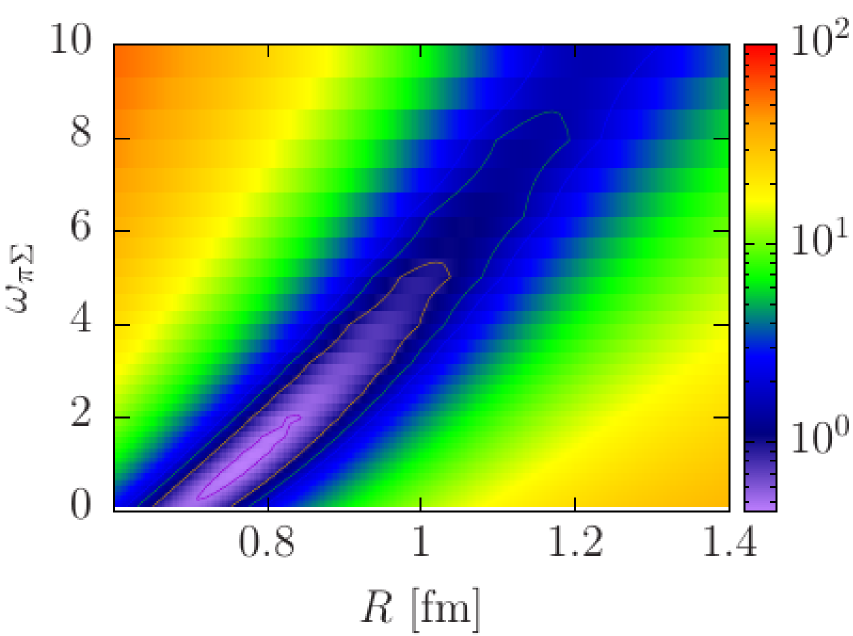
- **Experimental confirmation of coupled-channels contribution → Source size dependence**
  - **Channel w.f. other than K⁻ p are localized at around r=0. (Outgoing boundary condition for K⁻ p)**
  - **Contribution of πΣ source is suppressed for larger R.**

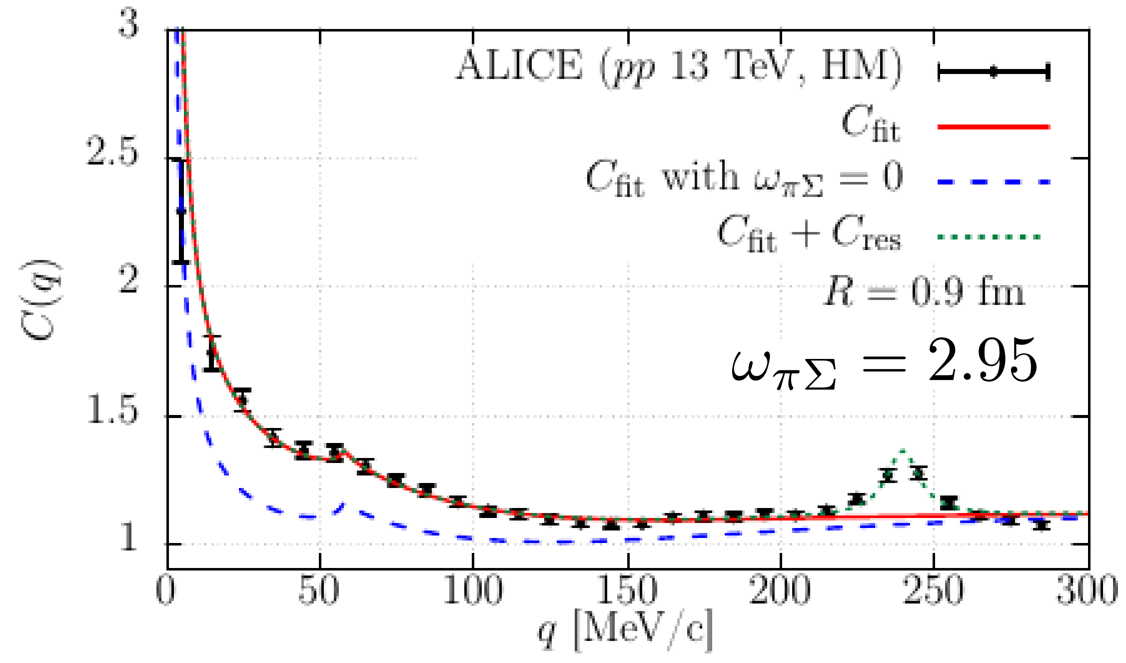




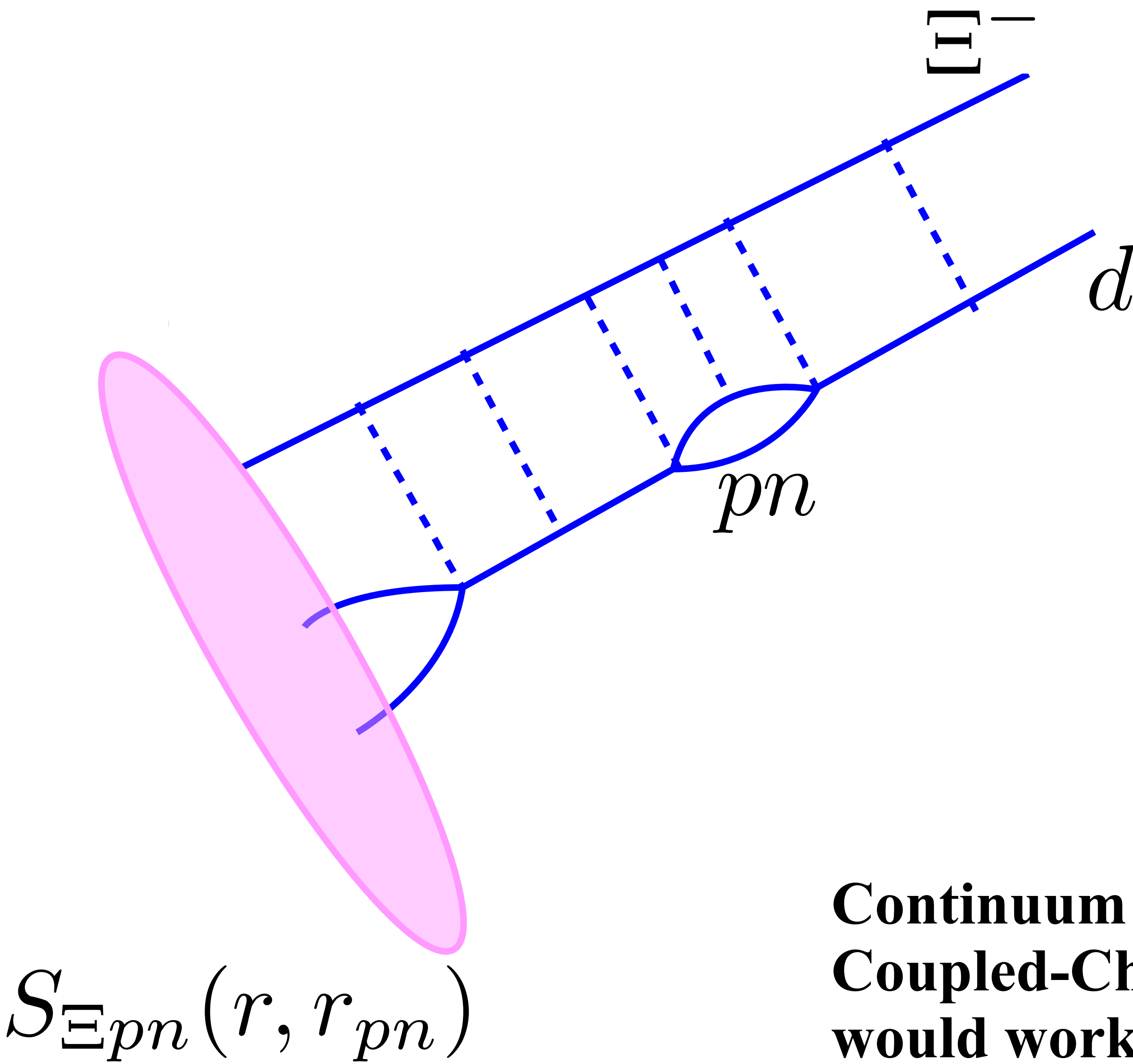
Correlation Function from Chiral SU(3) Potential (2)

- **"Free" parameters**
  = Source Size R, Source Weight ω_j ← Th+Exp.
  + Normalization + Pair purity (λ) ← Exp.
  - **Larger R → Smaller couple-channels effect from $\pi\Sigma$**
    (Favorable values of R and ω_j are correlated)
  - **Simple statistical model esitmate**
    $\omega_{\pi\Sigma} \sim \exp[(m_K+m_N-m_\pi-m_\Sigma)/T] \sim 2$.





Break up effect of deuteron



Continuum Discretized Coupled-Channel (CDCC) would work

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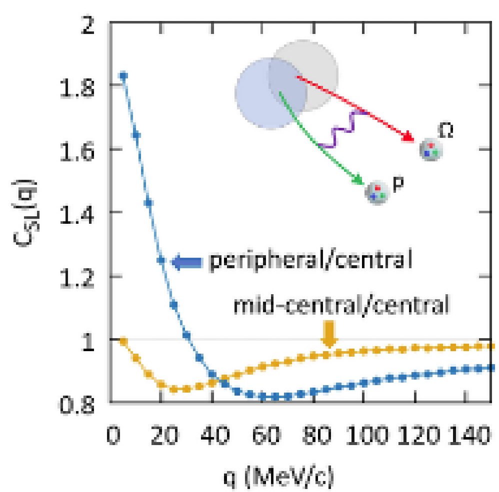


EDITORS' SUGGESTION

Probing $\Omega\Omega$ and $p\Omega$ dibaryons with femtoscopic correlations in relativistic heavy-ion collisions

The authors investigate correlations between protons and Ω baryons, and between two Ω baryons, in heavy-ion collisions at RHIC and LHC. Given sufficient statistics in upcoming experiments, such measurements could provide valuable information on the existence of strange dibaryons and on the equation of state relevant to neutron stars.

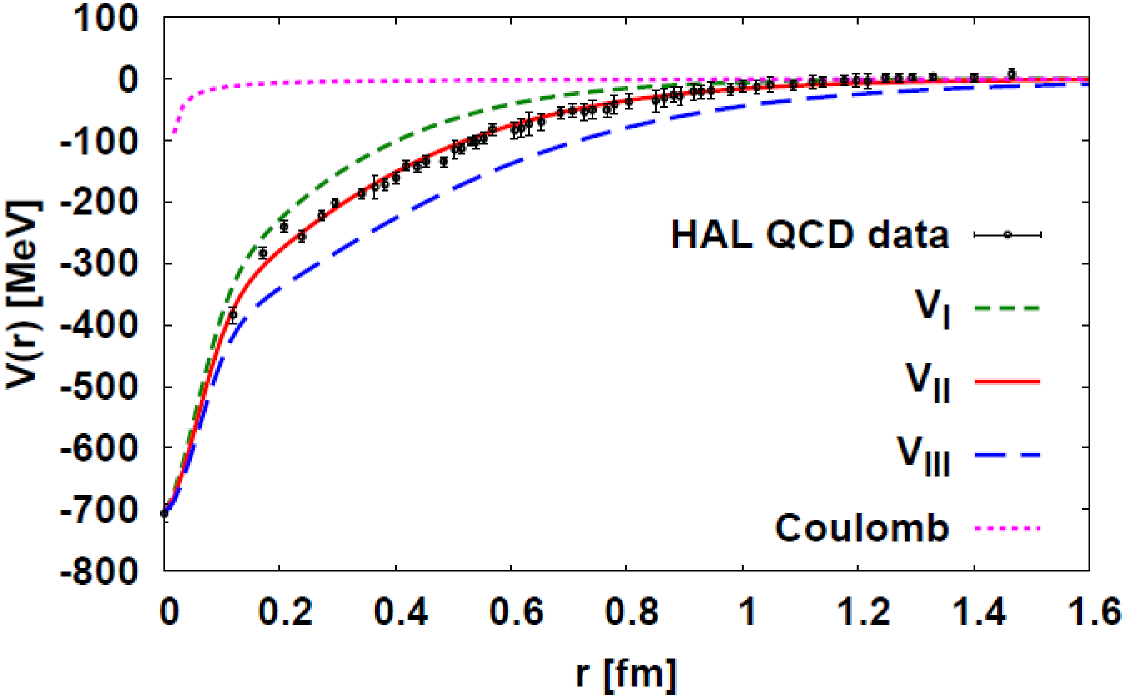
Kenji Morita *et al.*

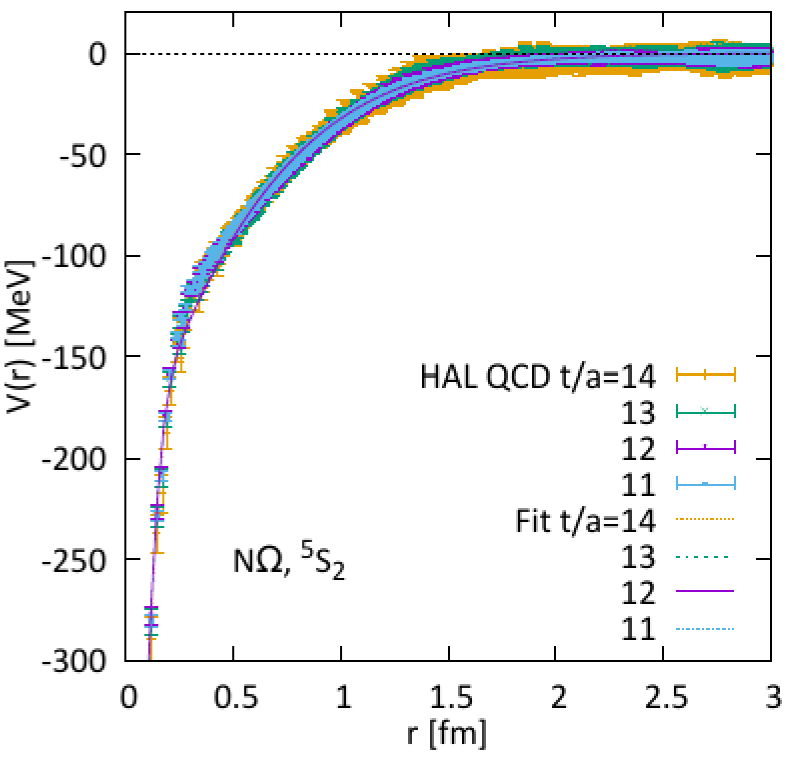
Phys. Rev. C **101**, 015201 (2020)

K. Morita, S. Gongyo, T. Hatsuda, T. Hyodo, Y. Kamiya, AO, PRC101('20)015201

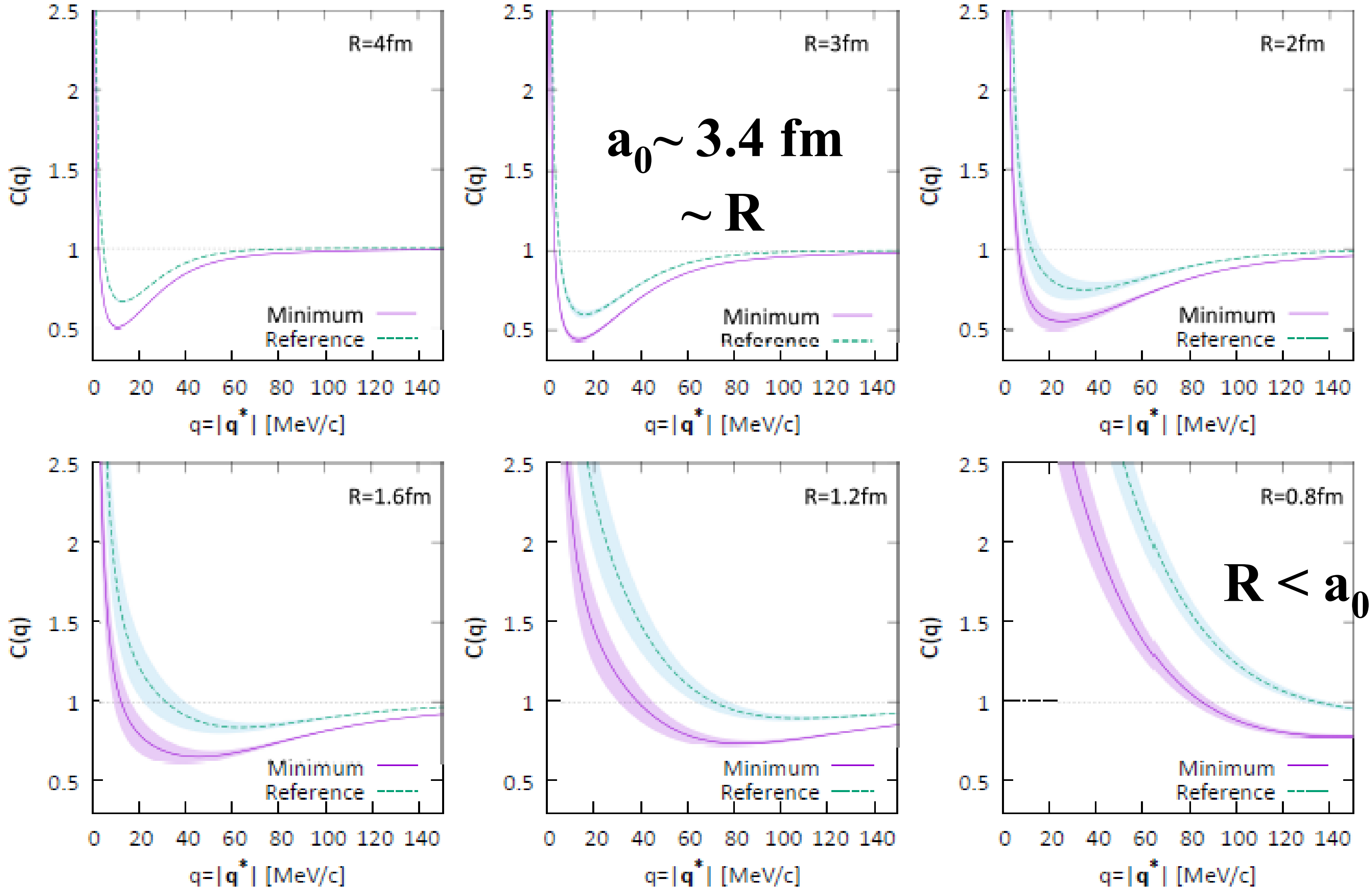
ΩN potential from lattice QCD

- **ΩN potential by HAL QCD Collab. (J=2)**
  - **m_π=875 MeV, B.E.~ 0.63 MeV**
    F.Etminan et al. (HAL QCD Collab.), NPA928('14)89.
  - **m_π=146 MeV, B.E.~ 2.2 MeV**
    T. Iritani et al. (HAL QCD Collab.), PLB 792('19)284.





Source Size Dependence of Correlation Function



Gaussian Source

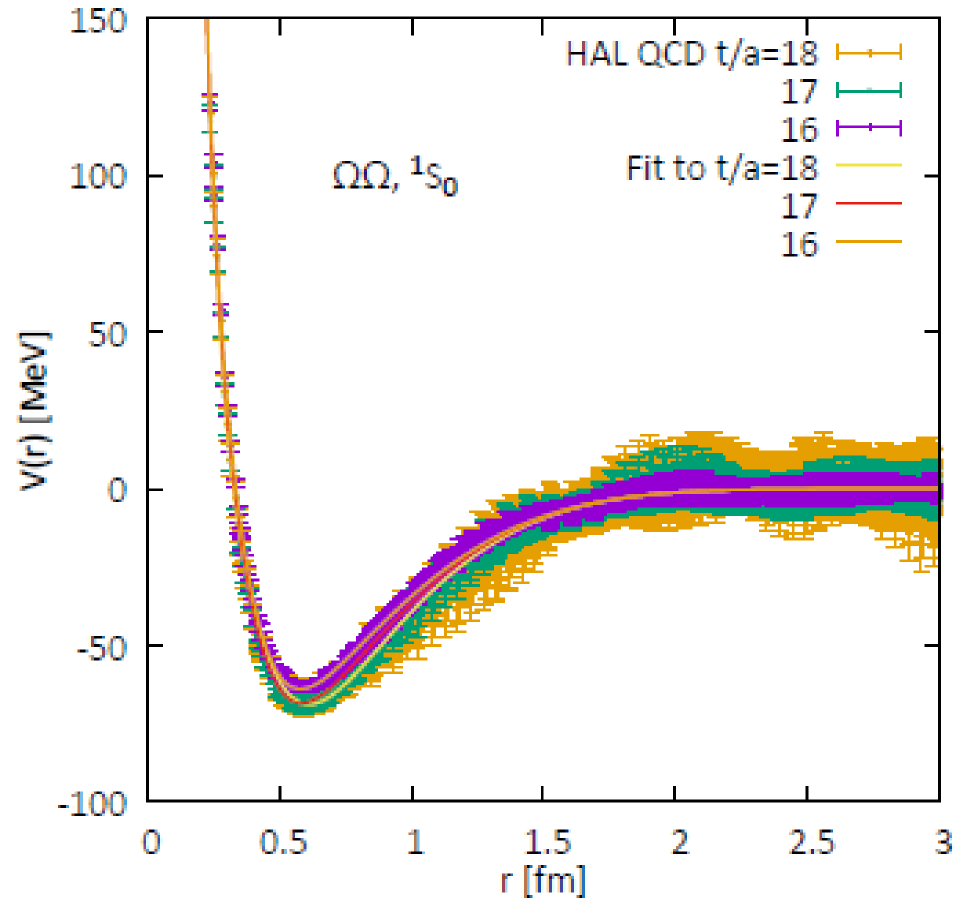
K. Morita, S. Gongyo, T. Hatsuda, T. Hyodo, Y. Kamiya, AO ('20)

ΩΩ dibaryon

- **ΩΩ potential from lattice QCD (J=0)**
  S. Gongyo et al. (HAL QCD Collab), Phys. Rev. Lett. 120, 212001 (2017).
  - **ΩΩ bounds for J=0 ! (Most strange dibaryon state)**
  - **B.E. is very small. B.E.=(0.1-1.0) MeV → a_0 > 10 fm**

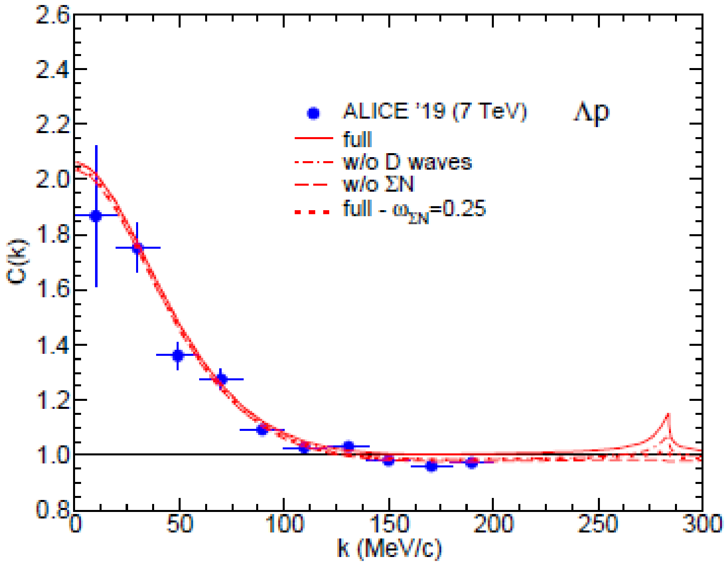
very big !

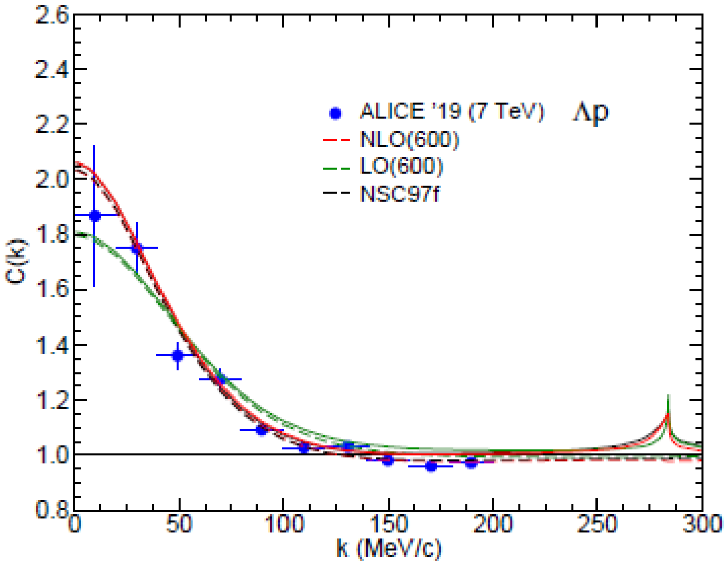
t/a	a_0 [fm]	$r_{\rm eff}$ [fm]	E_B [MeV]
16	65.28	1.29	0.1
17	17.59	1.24	0.54
18	11.69	1.26	1.0



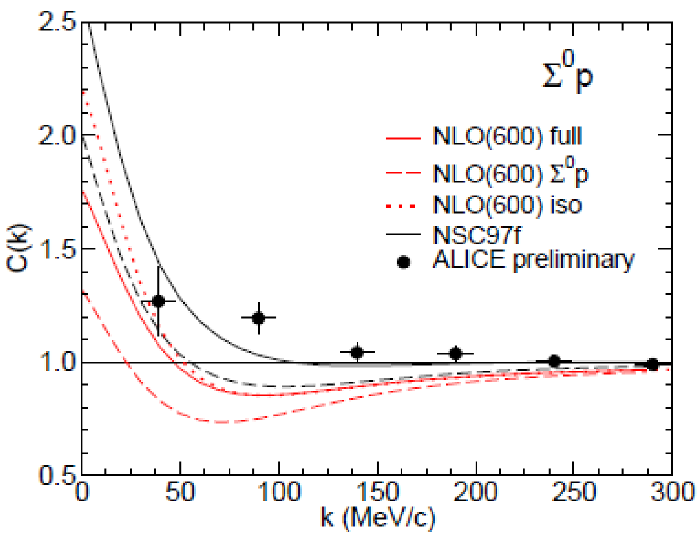
Correlation Function Studies by Jülich Group

- **Λp correlation (chiral EFT, NLO)** *J. Haidenbauer, FemTUM2019*





- **Σ^0p correlation**



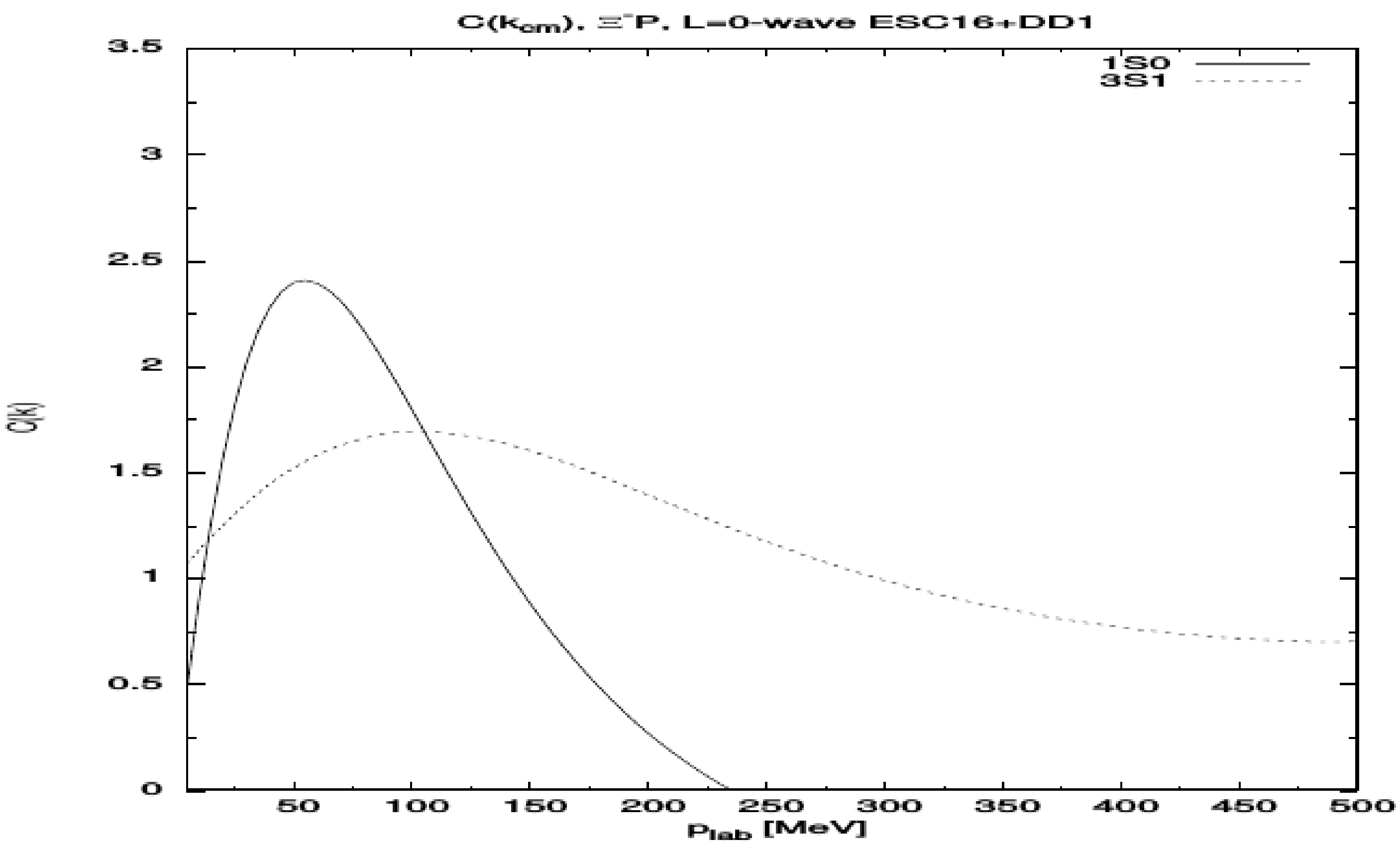
Correlation Function Studies using Nijmegen potential

21x Problem ESC with ALICE correlations II

Problem ESC and ALICE $\Xi^- p$-correlation II

- Inclusion short-range medium effects in Source
  via "Effective"two-body BB-interaction !?
- "Effective"ΞN-interaction needed for well-depth U_Ξ

T. Rijken @ FemTUM19



Recent Measurement of Hadron-Hadron Corr. Fn.

- **ΛΛ**
  Adamczyk et al. (STAR Collaboration), PRL 114 ('15) 022301.
  S. Achara et al. (ALICE), PRC99('19), 024001; arXiv:1905.07209
  Th: K.Morita, T.Furumoto, AO, PRC91('15)024916;
  AO, K.Morita, K.Miyahara, T.Hyodo, NPA954 ('16), 294 (ΛΛ, K⁻p)
  S. Cho et al. (ExHIC Collab.), Prog.Part.Nucl.Phys.95('17)279 (ΛΛ, K⁻p)
  J. Haidenbauer, NPA981 ('19) 1 (ΛΛ, Ξ⁻p, K⁻p); Greiner,Muller('89); AO+('98)

- **Ω⁻p**
  J. Adam et al. (STAR), PLB790 ('19) 490 [1808.02511]
  O. Vázquez Doce et al. (ALICE preliminary), Hadrons 2019
  Th: K. Morita, AO, F. Etminan, T. Hatsuda, PRC94('16)031901(R)
  K. Morita, S. Gongyo, T. Hatsuda, T. Hyodo, Y. Kamiya, AO, PRC101('20)015201

- **Ξ⁻p**
  S. Acharya et al. (ALICE), PRL123 ('19)112002
  Th: T. Hatsuda, K. Morita, AO, K. Sasaki, NPA967('17)856; J. Haidenbauer ('19)

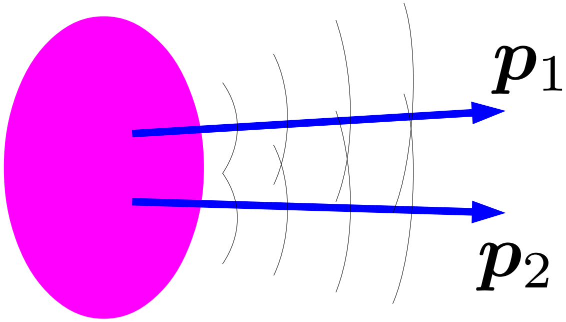
- **K⁻p**
  S. Acharya et al. (ALICE), PRL124('20)092301 [arXiv:1905.13470]
  Th: AO+('16), S. Cho+(ExHIC)('17), J. Haidenbauer ('19)
  Y. Kamiya, T. Hyodo, K. Morita, AO, W. Weise, PRL 124 ('20) 132501 [arXiv:1911.01041]

Correlation Function and Interaction

Correlation Function

- **Emitting source function**

$$N_i(\boldsymbol{p}) = \int d^4x S_i(x, \boldsymbol{p})$$



- **Two-particle momentum dist.**
  - **Assumption: Two particles are produced independently, and the correlation is generated by the final state int.**
    Koonin('77), Pratt+('86), Lednicky+('82)

$$N_{12}(\boldsymbol{p}_1, \boldsymbol{p}_2) \simeq \int d^4x d^4y S_1(x, \boldsymbol{p}_1) S_2(y, \boldsymbol{p}_2) \underline{|\Psi_{\boldsymbol{p}_1, \boldsymbol{p}_2}(x, y)|^2}$$

two-body w.f.

$$\simeq \int d^4x d^4y S_1(x, \boldsymbol{p}_1) S_2(y, \boldsymbol{p}_2) \underline{|\varphi_{\boldsymbol{q}}(\boldsymbol{r})|^2}$$

relative w.f.

- **Correlation function**

$$C(\boldsymbol{p}_1, \boldsymbol{p}_2) = \frac{N_{12}(\boldsymbol{p}_1, \boldsymbol{p}_2)}{N_1(\boldsymbol{p}_1) N_2(\boldsymbol{p}_2)} \simeq \int d\boldsymbol{r} S_{12}(\boldsymbol{r}) |\varphi_{\boldsymbol{q}}(\boldsymbol{r})|^2$$

Correlation Function

- **Example: Free identical bosons (spin 0, non-relativistic), Gaussian source (static, simultaneous, spherical)**

$$S(\boldsymbol{x},\boldsymbol{p}) \propto \exp\left[-\frac{\boldsymbol{x}^2}{2R^2}-\frac{\boldsymbol{p}^2}{2MT}\right]$$

$$S(\boldsymbol{x},\boldsymbol{p}_1)S(\boldsymbol{y},\boldsymbol{p}_2) \propto \exp\left[-\frac{\boldsymbol{R}_{\rm cm}^2}{R^2}-\frac{\boldsymbol{r}^2}{4R^2}-\frac{\boldsymbol{P}^2}{4MT}-\frac{\boldsymbol{q}^2}{2\mu T}\right]$$

$$\Psi_{\boldsymbol{p}_1,\boldsymbol{p}_2}(\boldsymbol{x},\boldsymbol{y}) \propto \frac{1}{\sqrt{2}}\left[e^{i\boldsymbol{p}_1\cdot\boldsymbol{x}+i\boldsymbol{p}_2\cdot\boldsymbol{y}}+e^{i\boldsymbol{p}_1\cdot\boldsymbol{y}+i\boldsymbol{p}_2\cdot\boldsymbol{x}}\right]$$

$$= e^{i\boldsymbol{P}\cdot\boldsymbol{R}_{\rm cm}} \times \sqrt{2}\cos\boldsymbol{q}\cdot\boldsymbol{r}$$

- **Correlation function**

$$C(\boldsymbol{q}) =(4\pi R^2)^{-3/2}\int d\boldsymbol{r}\exp\left[-\frac{\boldsymbol{r}^2}{4R^2}\right]2\cos^2\boldsymbol{q}\cdot\boldsymbol{r}$$

$$=1+\exp(-4q^2R^2)$$

Correlation Function → Source Size

How can we measure the radius of a star ?

- **Two photon intensity correlation**
  Hanbury Brown & Twiss, Nature 10 (1956), 1047.
  - **Simultaneous two photon observation probability is enhanced from independent emission cases**
    → angular diameter of Sirius=6.3 msec

Recent data (Wikipedia) 5.936±0.016 msec

A TEST OF A NEW TYPE OF STELLAR INTERFEROMETER ON SIRIUS

By R. HANBURY BROWN
Jodrell Bank Experimental Station, University of Manchester
AND
DR. R. Q. TWISS
Services Electronics Research Laboratory, Baldock

NATURE November 10, 1956 VOL. 178

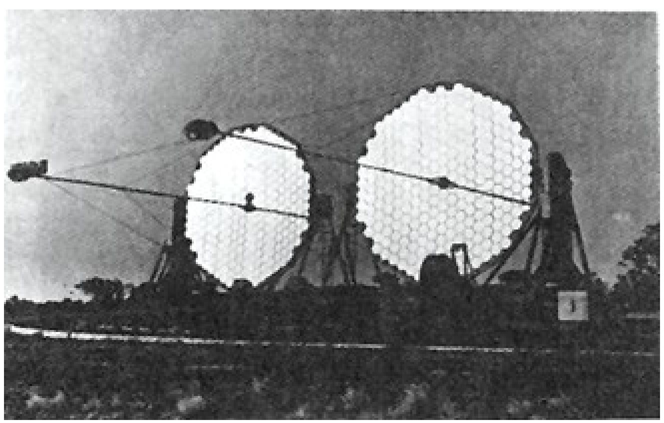
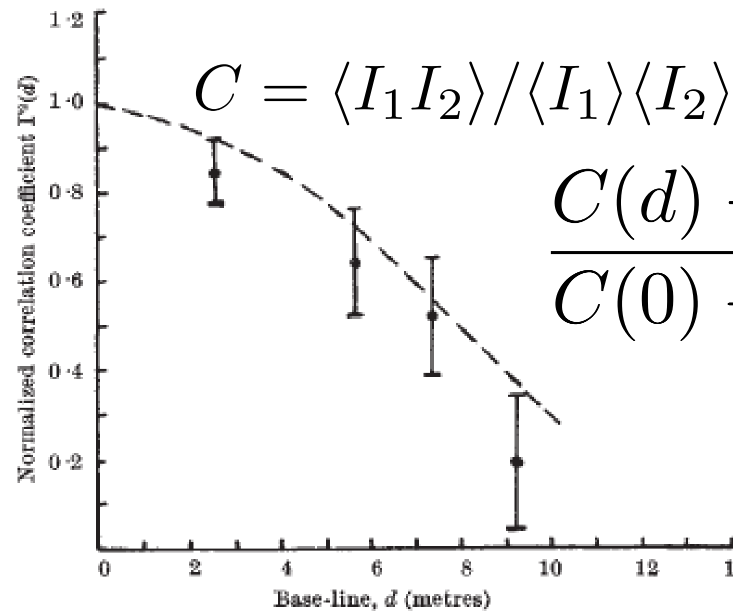


Figure 2. Picture of the two telescopes used in the HBT experiments. The figure was extracted from Ref.[1].

HBT telescope (from Goldhaber, ('91))



$$C = \langle I_1 I_2 \rangle / \langle I_1 \rangle \langle I_2 \rangle$$

$$\frac{C(d)-1}{C(0)-1}$$

Fig. 2. Comparison between the values of the normalized correlation coefficient $\Gamma^2(d)$ observed from Sirius and the theoretical values for a star of angular diameter 0·0063". The errors shown are the probable errors of the observations

HBT ('56)

Two particle intensity correlation

- **Wave function symmetrization from quantum statistics**

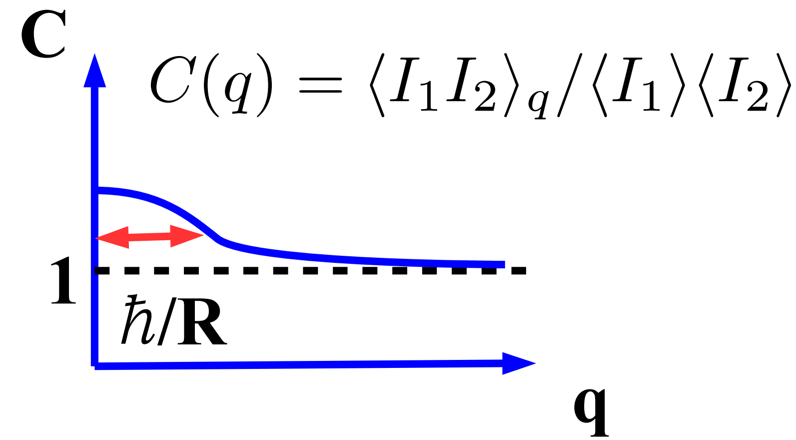
$$C(\mathbf{q}) = \int d^3r\, S(\mathbf{q},\mathbf{r}) \left| \frac{1}{\sqrt{2}}\left(e^{i\mathbf{q}\cdot\mathbf{r}} + e^{-i\mathbf{q}\cdot\mathbf{r}}\right) \right|^2 \simeq 1+\exp(-4q^2R^2)$$

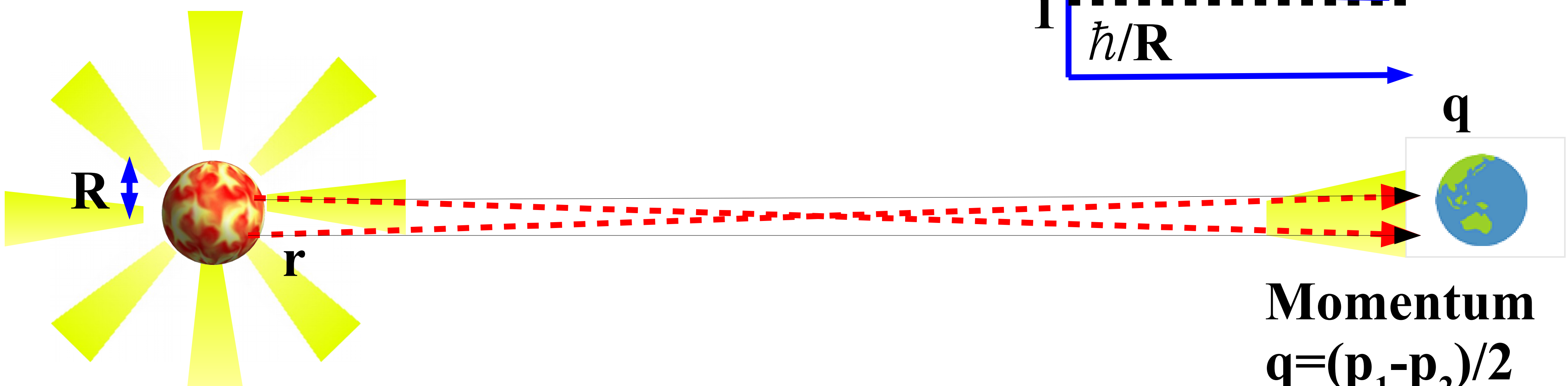
Source fn.
(r=relative coordinate)

(symmetrized w.f.)^2

Static spherical source case

→ **Small relative momenta are favored due to symmetrization of the relative wave function.**

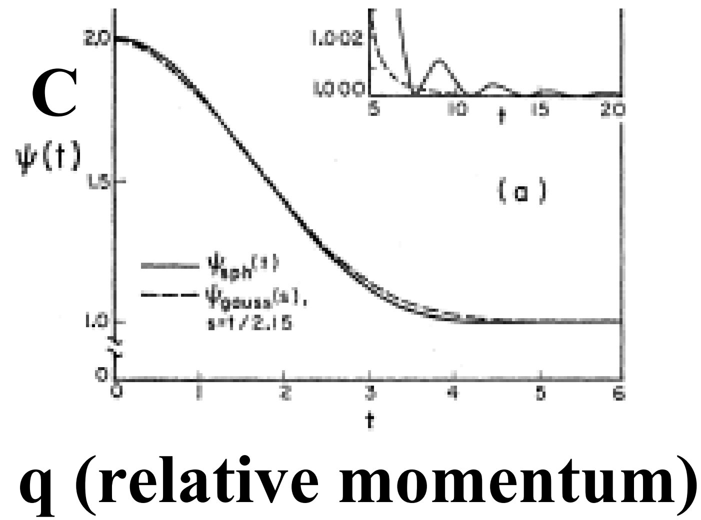




Momentum
$q=(p_1-p_2)/2$

How can we measure source size in nuclear reactions ?

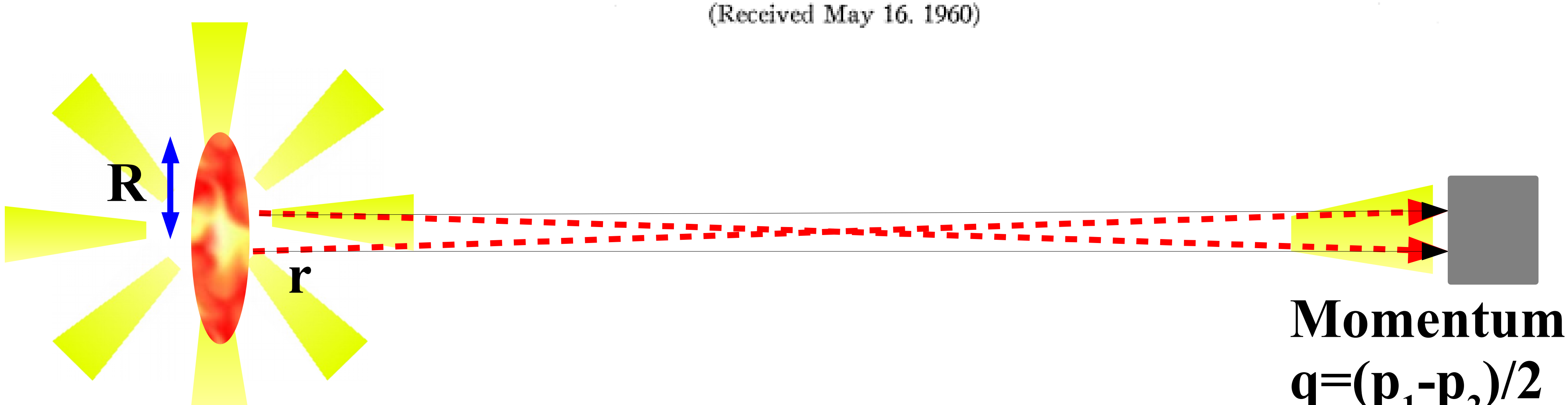
- **Two pion interferometry**
  G. Goldhaber, S. Goldhaber, W. Lee,
  A. Pais, Phys. Rev. 120 (1960), 300
  - **Two pion emission probability is enhanced at small relative momenta**
    → Pion source size ~ 0.75 ℏ / μc



PHYSICAL REVIEW VOLUME 120, NUMBER 1 OCTOBER 1, 1960

Influence of Bose-Einstein Statistics on the Antiproton-Proton Annihilation Process*

Gerson Goldhaber, Sulamith Goldhaber, Wonyong Lee, and Abraham Pais†
Lawrence Radiation Laboratory and Department of Physics, University of California, Berkeley, California
(Received May 16. 1960)



Femtoscopic Study of Hadron-Hadron Interaction

- **HBT, GGLP: Corr. Fn. + w.f. → Source Size**
  Another way: Corr. Fn. + Source Size
  → wave function → hadron-hadron interaction
- **Effect of hadron-hadron interaction on the wave function**
  - **Assumption: Only s-wave (L=0) is modified.**
  - **Non-identical particle pair, Gauss source.**

$$
\begin{aligned}
\varphi_{\boldsymbol{q}}(\boldsymbol{r}) =& e^{i\boldsymbol{q}\cdot\boldsymbol{r}} - j_0(qr) + \chi_q(r) \\
\to C(\boldsymbol{q}) =& \int d\boldsymbol{r} S(r) |\varphi_{\boldsymbol{q}}(\boldsymbol{r})|^2 \\
=& 1 + \int d\boldsymbol{r} S(r) \left\{ |\chi_q(r)|^2 - |j_0(qr)|^2 \right\}
\end{aligned}
$$

K. Morita, T. Furumoto, AO, PRC91('15)024916

Corr. Fn. shows how much squared w. f. is enhanced
→ Large CF is expected with attraction

Wave function around threshold (S-wave, attraction)

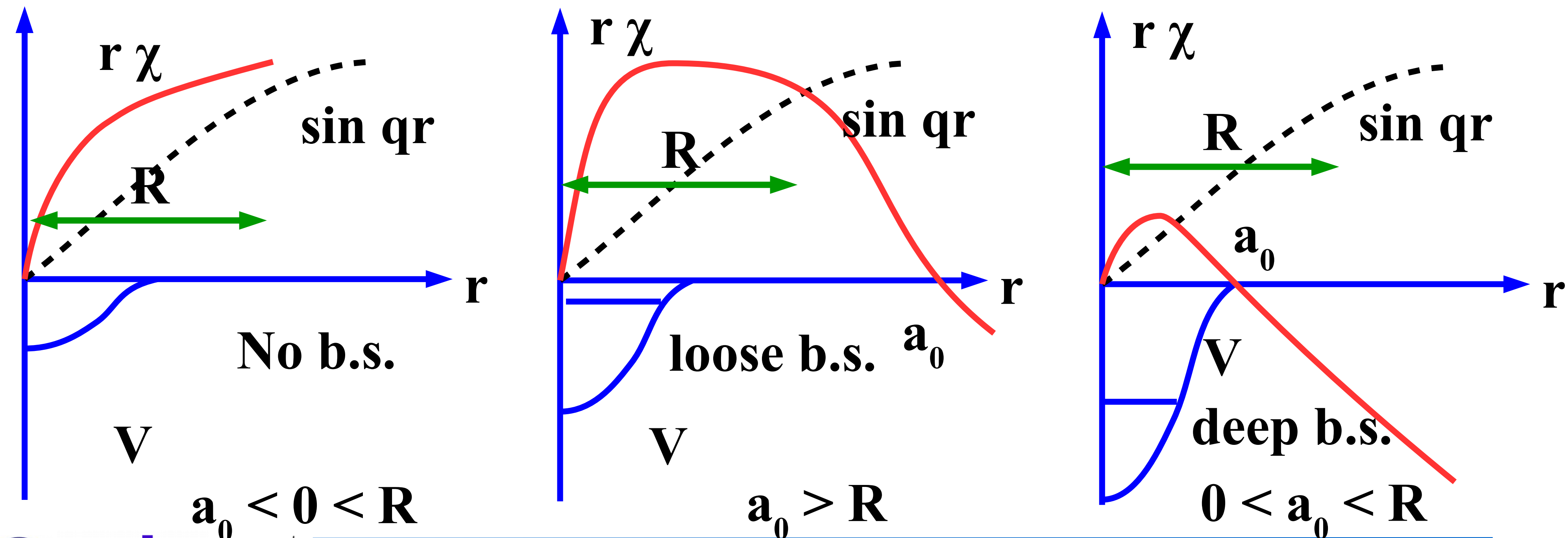
- **Low energy w.f. and phase shift**

$$u(r) = qr\chi_q(r) \to \sin(qr + \delta(q)) \sim \sin(q(r - a_0))$$

$$q \cot\delta = -\frac{1}{a_0} + \frac{1}{2} r_{\text{eff}} q^2 + \mathcal{O}(q^4) \ (\delta \sim -a_0 q)$$

a_0 =scatt. length
r_{eff}=eff. range

  - **Wave function grows rapidly at small r with attraction.**
  - **With a bound state ($a_0>0$), a node appears around $r=a_0$**



Lednicky-Lyuboshits (LL) model

- **Lednicky-Lyuboshits analytic model**
  - **Asymp. w.f. + Eff. range corr. + $\psi^{(-)} = [\psi^{(+)}]^*$**

$$\psi_0(r) \to \psi_{\rm asy}(r) = \frac{e^{-i\delta}}{qr}\sin(qr+\delta) = \mathcal{S}^{-1}\left[\frac{\sin qr}{qr} + f(q)\frac{e^{iqr}}{r}\right]$$

$$\Delta C_{\rm LL}(q) = \int d\boldsymbol{r} S_{12}(r)\left(|\psi_{\rm asy}(r)|^2 - |j_0(qr)|^2\right)$$
$$= \frac{|f(q)|^2}{2R^2} F_3\left(\frac{r_{\rm eff}}{R}\right) + \frac{2{\rm Re}f(q)}{\sqrt{\pi}R}F_1(x) - \frac{{\rm Im}f(q)}{R}F_2(x)$$

$$(x = 2qR, R = \text{Gaussian size}, \ F_1, F_2, F_3 : \text{Known functions})$$

- **Phase shifts**

$$q\cot\delta = -\frac{1}{a_0} + \frac{1}{2}r_{\rm eff}q^2 + \mathcal{O}(q^4) \to \delta \simeq -a_0 q + {\rm O}(q^3)$$

$$\sin(qr+\delta) \simeq \sin(q(r-a_0)+\cdots)$$

Node at r ~ a_0 for small q

C(q) in the low momentum limit

- **Correlation function at small q (and r_{eff}=0) → F_1=1, F_2=0, F_3=1**

$$\Delta C_{\rm LL}(q) \to \frac{|f(0)|^2}{2R^2} + \frac{2{\rm Re}f(0)}{\sqrt{\pi}R} \quad (q \to 0)$$

$$f(q) = (q\cot\delta - iq)^{-1} \simeq \left(-\frac{1}{a_0} + \frac{1}{2}r_{\rm eff}q^2 - iq\right)^{-1} \to -a_0$$

$$C_{\rm LL}(q \to 0) = 1 + \frac{a_0^2}{2R^2} - \frac{2a_0}{\sqrt{\pi}R} = 1 - \frac{2}{\pi} + \frac{1}{2}\left(\frac{a_0}{R} - \frac{2}{\sqrt{\pi}}\right)^2$$

$$1 - 2/\pi \simeq 0.36, \ \sqrt{\pi}/2 \simeq 0.89$$

C(q → 0) takes a minimum of 0.36 at R/a_0 = 0.89 in the LL model.